Numerical Bootstrap of Holographic CFTs

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Intro

- Conformal bootstrap is currently the most powerful/general non-perturbative method of studying conformal field theories (CFT) in $d \geq 2$ dimensions.
- Its used to solve long-standing questions in condensed matter systems (mostly non-susy CFTs in 2+1 dimensions).
- Also used to study quantum gravity via AdS/CFT.
- There is much more to do, many of the biggest questions remain open, and both technical and conceptual progress is needed!

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Outline

- Basics of CFT in $d > 2$
  - Correlation functions
  - Unitarity constraints
- Bootstrap algorithm
  - Bounding scaling dimensions and OPE coefficients.
  - Practical implementation of algorithm
- Application to holographic CFTs
  - Bootstrapping superconformal CFTs
  - Review of bootstrap results for max susy CFTs

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Conformal symmetry

- Conformal transformations change metric up to overall factor $g_{\mu\nu}(x_\mu) \to \Omega^2(x_\mu) g_{\mu\nu}(x_\mu)$. In $d > 2$ it consists of:
  1. Translations P_μ: $x_\mu \to x_\mu + \epsilon_\mu$
  2. Rotations $M_{\mu\nu}$: $x_\mu \to \epsilon^{\mu\nu} x_\nu$
  3. Dilations D: $x_\mu \to \epsilon x_\mu$
  4. SCT (inversion-translation-inversion) K_μ: $\frac{x_\mu}{x^2} \to \frac{x_\mu}{x^2} - \epsilon_\mu$
- Conformal group is isomorphic to $SO(d, 2)$ in $d > 2$.
  - In $d = 2$ have infinitely more generators, group is Virasoro $\times$ Virasoro, with global subgroup $SO(2, 2)$.

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Representations of conformal group

- Operators $\mathcal{O}(x)$ labeled by weights of Euclidean conformal group $SO(d+1,1)$, given by the two sets of labels:
  1. Irrep of rotation group $SO(d)$. "spin ℓ" is traceless symmetric rank ℓ.
  2. Scaling dimension Δ of the dilation generator: $[D,\mathcal{O}(0)]=\Delta\mathcal{O}(0)$. Raised by translation generator P_μ, lowered by SCT generator K_μ.
- Irreps of conformal group are called primaries, defined by $[K_\mu,\mathcal{O}(0)]=0$.
  - Descendents with $\Delta+n$ can be obtained by acting n times with P_μ.
  - Conformal multiplet consists of primary plus infinite descendents.

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Correlation functions

- $\langle \mathcal{O}(x) \rangle = 0$, bc there is no scale in CFT.
- Can fix 2 and 3-point functions up to physical constants λ_{ijk}:

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2) \rangle = \frac{\delta_{ij}}{x_{12}^{2\Delta_i}}$$

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- Can reduce n-point functions to lower point functions using OPE:

$$\mathcal{O}_i(x_1)\mathcal{O}_j(x_2) = \sum_k \lambda_{ijk} C_{ijk}(x_{12}, \partial_2) \mathcal{O}_k(x_2)$$

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4-point function

- Apply OPE twice to 4-point to get in terms of $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$, $v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2}$:

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{x_{12}^{2\Delta_\phi} x_{34}^{2\Delta_\phi}} \sum_{\Delta,\ell} \lambda_{\Delta,\ell}^2 g_{\Delta,\ell}(u,v)$$

  - Conformal blocks $g_{\Delta,\ell}(u,v)$ fixed by conformal symmetry for each Δ,ℓ, compute as expansion in $r(u,v)$ [Kos, Poland, DSD '13] .

- Can apply OPE in $1,3$ and $2,4$ channel instead of $1,2$ and $3,4$, demanding equality gives the crossing equations:

$$v^{\Delta_\phi} \sum_{\Delta,\ell} \lambda_{\Delta,\ell}^2 g_{\Delta,\ell}(u,v) - u^{\Delta_\phi} \sum_{\Delta,\ell} \lambda_{\Delta,\ell}^2 g_{\Delta,\ell}(v,u) = 0$$

  - If ϕ transforms under global symmetry with index a, then 4-point has T_{abcd} tensor structure, which leads to multiple crossing equations.

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$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{x_{12}^{2\Delta_\phi} x_{34}^{2\Delta_\phi}} \sum_{\Delta,\ell} \lambda_{\Delta,\ell}^2 g_{\Delta,\ell}(u,v)$$

  - Conformal blocks $g_{\Delta,\ell}(u,v)$ fixed by conformal symmetry for each Δ, ℓ, compute as expansion in $r(u,v)$ [Kos, Poland, DSD '13] .

- Can apply OPE in $1,3$ and $2,4$ channel instead of $1,2$ and $3,4$, demanding equality gives the crossing equations:

$$v^{\Delta_\phi} \sum_{\Delta,\ell} \lambda_{\Delta,\ell}^2 g_{\Delta,\ell}(u,v) - u^{\Delta_\phi} \sum_{\Delta,\ell} \lambda_{\Delta,\ell}^2 g_{\Delta,\ell}(v,u) = 0$$

  - If ϕ transforms under global symmetry with index a, then 4-point has T_{abcd} tensor structure, which leads to multiple crossing equations.

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- Unitarity imposes lower bounds on scaling dimensions Δ of spin ℓ operators in d dimensions (exception is unit operator with $\Delta = 0$):

$$\ell = 0: \qquad \Delta \geq \frac{d-2}{2} \qquad\qquad \ell > 0: \qquad \Delta \geq d - 2 + \ell$$

- When Δ saturates bound for $\ell = 0$, CFT is free. When saturates for $\ell > 0$, operator is conserved current.
  - All local CFTs have $\ell = 2$ conserved stress tensor. All local CFTs with continuous symmetry have $\ell = 1$ conserved current.
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Bootstrap algorithms: ingredients

- First rewrite $1 \leftrightarrow 3$ crossing equation as

$$\sum_{\Delta,\ell} \lambda^2_{\phi\phi\mathcal{O}_{\Delta,\ell}} F^{\Delta_\phi}_{\Delta,\ell}(u,v) = 0\,, \qquad F^{\Delta_\phi}_{\Delta,\ell}(u,v) \equiv v^{\Delta_\phi} g_{\Delta,\ell}(u,v) - u^{\Delta_\phi} g_{\Delta,\ell}(v,u)\,.$$

  - Sum runs over all even spin operators that appear in $\phi \times \phi$ OPE.
  - If ϕ transforms in global symmetry, then multiple such equations.
- $\lambda^2_{\phi\phi\mathcal{O}_{\Delta,\ell}} \geq 0$ bc of unitarity.
- All Δ (except identity with $\Delta = 0$) have lower bound from unitarity.
- Think of $F^{\Delta_\phi}_{\Delta,\ell}(u,v)$ as infinite-dimensional vectors, labeled by Δ, ℓ, in the vector space u, v, so crossing imposes that infinite sum of infinite dimensional vectors with positive coefficients must be zero.
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1. Normalize α such that $\alpha[F_{0,0}^{\Delta_\phi}(u,v)] = 1$.
2. Assume that the scaling dimensions Δ_ℓ of all spin ℓ operators in $\phi \times \phi$ except $\mathcal{O}_{0,0}$ obey lower bounds $\Delta_\ell \geq \Delta_\ell^B$. The unitarity bound provides a minimal choice.
3. Look for α that satisfies $\alpha[F_{\Delta,\ell}^{\Delta_\phi}(u,v)] \geq 0$ for all $\mathcal{O}_{\Delta,\ell}$ except $\mathcal{O}_{0,0}$.
4. If such α exists, then by positivity of $\lambda^2_{\phi\phi\mathcal{O}_{\Delta,\ell}}$ we have

$$\alpha\left[\sum_{\Delta,\ell} \lambda^2_{\phi\phi\mathcal{O}_{\Delta,\ell}} F_{\Delta,\ell}^{\Delta_\phi}(u,v)\right] > 0\,,$$

which contradicts α acting on the crossing equations, so our assumptions $\Delta \geq \Delta_\ell^B$ must be false, i.e. $\Delta < \Delta_\ell^B$. If we cannot find such an α, then we conclude nothing.

- e.g. set Δ_ℓ^B to their unitarity values for all ℓ except some ℓ^G, then by varying $\Delta_{\ell^G}^B$ and Δ_ϕ we get upper bound on $\Delta_{\ell^G}^B$, i.e. lowest dimension operator with spin ℓ^G, as function of Δ_ϕ.

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- To get both upper/lower bound on some $\Delta^I_{\ell G}$, add gap assumption:

$$\alpha[F^{\Delta_\phi}_{\Delta^I,\ell G}(u,v)] \geq 0 \qquad \text{for} \quad \Delta^I_{\ell G} < \Delta^B_{\ell G}\,,$$

  - $\Delta^B_{\ell G}$ is the gap, above which we allow for continuum of operators (as in previous slide).
  - $\Delta^I_{\ell G}$ is the operator we insert below the gap. Note it has fixed Δ.
- Only specific values of $\Delta^I_{\ell G}$ correspond to a physical CFT, so we will find interval of allowed $\Delta^I_{\ell G}$ (based on precision of search) around physical $\Delta^I_{\ell G}$ for given Δ_ϕ.
- E.g.: assume exists only one relevant scalar in $\phi \times \phi$ by setting $\ell^G = 0$, $\Delta^B_0 = d$, and then vary Δ^I_0 in the range $\frac{d-2}{2} \leq \Delta^I_0 \leq d$ that is allowed by the unitarity bound and the gap above it.

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- Only specific values of $\Delta^I_{\ell G}$ correspond to a physical CFT, so we will find interval of allowed $\Delta^I_{\ell G}$ (based on precision of search) around physical $\Delta^I_{\ell G}$ for given Δ_ϕ.

- E.g.: assume exists only one relevant scalar in $\phi \times \phi$ by setting $\ell^G = 0$, $\Delta^B_0 = d$, and then vary Δ^I_0 in the range $\frac{d-2}{2} \leq \Delta^I_0 \leq d$ that is allowed by the unitarity bound and the gap above it.

Bootstrap algorithms: λ upper/lower bounds

1. Normalize α such that $\alpha[F^{\Delta_\phi}_{\Delta_O,\ell_O}(u,v)] = s$, where $s = \pm 1$ for upper/lower bounds.
2. Assume that the scaling dimensions Δ_ℓ of all operators in $\phi \times \phi$ except $\mathcal{O}_{0,0}$ and $\mathcal{O}_{\Delta_O,\ell_O}$ obey lower bounds $\Delta_\ell \geq \Delta^B_\ell$.
3. Require that $\alpha[F^{\Delta_\phi}_{\Delta,\ell}(u,v)] \geq 0$ for all $\mathcal{O}_{\Delta,\ell}$ except $\mathcal{O}_{0,0}$ and $\mathcal{O}_{\Delta_O,\ell_O}$.
4. Maximize $\alpha[F^{\Delta_\phi}_{0,0}(u,v)]$ to get the upper/lower bounds (from α on crossing equations, positivity of $\lambda^2_{\phi\phi\mathcal{O}_{\Delta,\ell}}$, and steps 1 and 3):

$$\text{Upper}: \qquad \lambda^2_{\phi\phi\mathcal{O}_{\Delta_O,\ell_O}} \leq -\alpha[F^{\Delta_\phi}_{0,0}(u,v)]\,,$$

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- **Lower bound requires gap** above $\mathcal{O}_{\Delta_O,\ell_O}$, otherwise $\alpha[F^{\Delta_\phi}_{\Delta_O,\ell_O}(u,v)] = -1$ in step 1 is then inconsistent with $\alpha[F^{\Delta_\phi}_{\Delta,\ell}(u,v)] \geq 0$ from step 3 bc continuum of operators $\mathcal{O}_{\Delta,\ell_O}$ with Δ arbitrarily close to Δ^O.

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Truncated crossing equations

- Crossing equations involve 4 different kinds of infinities:
  1. Spins ℓ, which can be any even positive number (for the specific single correlator of identical operators that we discuss so far).
  2. Scaling dimensions Δ, which can be any real number greater than the unitarity bounds.
  3. Blocks written as infinite series in $r(u, v)$ (at finite $\eta(u, v)$).
  4. The infinite vector space of real u, v (or r, η) in $F^{\Delta_\phi}_{\Delta,\ell}(u, v)$ that we act on with the functional α.
- Must truncate each of these to get finite problem that can be solved numerically.

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Truncation of (u, v) space

- Define variables $u \equiv z\bar{z}, v \equiv (1-z)(1-\bar{z})$ s.t. $z = \bar{z} = \frac{1}{2}$ is crossing symetric under $u \leftrightarrow v$ (i.e. $1 \leftrightarrow 3$).
- Truncate (u, v) space as Taylor series around $z = \bar{z} = \frac{1}{2}$:

$$\sum_{\Delta,\ell} \lambda^2_{\phi\phi\mathcal{O}_{\Delta,\ell}} \partial_z^m \partial_{\bar{z}}^n F^{\Delta_\phi}_{\Delta,\ell}(z,\bar{z})|_{z=\bar{z}=\frac{1}{2}} = 0 \qquad \text{for} \qquad m+n \leq \Lambda\,.$$

- $\alpha_{m,n}$ acts on elements of finite vector space V_Λ of derivatives.
  - Bounds improve monotonically with $|V_\Lambda|$, so bootstrap bounds rigorous! $|V_\Lambda|$ is most important parameter in accuracy of bootstrap.
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Truncation of block expansion in r

- Expand blocks up to some $r_{\max}$ (expansion is convergent).
- At crossing symmetric point $z = \bar{z} = \frac{1}{2}$, we have $\eta_c = 1$, $r_c = 3 - 2\sqrt{2} \approx .17$, so block expansion converges quickly.
  - Another motivation for truncating (u, v) by expanding around $z = \bar{z} = \frac{1}{2}$.
- Truncation non-rigorous (unlike truncation in (u, v)), so bootstrap bounds could change in either direction as $r_{\max}$ increase (bc for small $r_{\max}$ blocks are essentially random functions).
- In practice, convergence for $r_{\max}$ is so fast that bounds for given Λ do not change at all for sufficiently high $r_{\max}$.
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Truncation of spins ℓ

- Include spins up to some ℓ_{max}.
- Tail of block expansion (i.e. including all operators with $\Delta > \Delta^*$) is bounded as [Pappadopulo, Rychkov, Espin, Rattazzi] :

$$\sum_{\Delta>\Delta^*} \lambda^2_{\phi\phi\mathcal{O}_{\Delta,\ell}} g_{\Delta,\ell}(r,\eta) < \frac{(2\Delta^*)^{4\Delta_\phi}}{\Gamma(4\Delta_\phi+1)} r^{\Delta^*} \, .$$

- Since $r_c \approx .17$, large Δ operators contribute very little to bootstrap.
- Since unitarity bounds relate $\Delta \geq \ell + d - 2$, thus large ℓ operators also contribute little.
- Like r truncation, ℓ truncation is non-rigorous, but converges so quickly that bounds for given Λ dont change for large enough ℓ_{max}.
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Truncation of spins ℓ

- Include spins up to some ℓ_{max}.
- Tail of block expansion (i.e. including all operators with $\Delta > \Delta^*$) is bounded as [Pappadopulo, Rychkov, Espin, Rattazzi] :

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Truncation of Δ with linear programming

- Original [Rattazzi, Rychkov, Tonni, Vichi '08] accounted for infinite Δ by imposing Δ_{max}, and looking at discrete list of $\Delta \leq \Delta_{\text{max}}$.
- Truncation to Δ_{max} converges well as shown in previous slide, while discretization works in practice.
- After truncations, bootstrap algorithms become finite systems of linear inequalities, e.g. for each $m, n \in V_\Lambda$ we look for

$$\alpha_{m,n}\partial_z^m\partial_{\bar{z}}^n F_{\Delta,\ell}^{\Delta_\phi}(z,\bar{z})|_{z=\bar{z}=\frac{1}{2}} \geq 0 \qquad \text{for all } \Delta \leq \Delta_{\text{max}} \text{ and } \ell \leq \ell_{\text{max}}\,.$$

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Semidefinite approach to Δ

- Avoid non-rigorous truncation in Δ by writing constraints as polynomial in Δ [Poland, DSD, Vichi '11] .
- Expansion of blocks in r takes form

$$g_{\Delta,\ell}(r,\eta) = (4r)^{\Delta} \sum_{m=0}^{\infty} r^m \sum_n \frac{f_{m,n}(\eta)}{\Delta - \Delta_{m,n}} .$$

- After truncating to $r_{\max}$, define Δ polynomial as

$$p^{r_{\max}}_{\Delta,\ell}(r,\eta) \equiv (4r)^{-\Delta} g^{r_{\max}}_{\Delta,\ell}(r,\eta) \prod_{m,n} (\Delta - \Delta_{m,n}) .$$

- Write $\partial_z^m \partial_{\bar{z}}^n F^{\Delta_\phi}_{\Delta,\ell}(r_c, 1)$ as polynomial in Δ by redefining $\lambda^2_{\Delta,\ell}$ to absorb (positive) product of poles and $(4r_c)^{\Delta}$.
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Quantum gravity from CFT

- Only non-perturbatively defined quantum gravity given by AdS/CFT: QG (usually string/M-theory) on $\text{AdS}_{d+1} \Leftrightarrow \text{CFT}_d$.
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Superconformal field theories

- Have global symmetry called R symmetry.
- Primaries related by susy form multiplet with same λ.
  - E.g. stress tensor in same multiplet as scalar for max susy.
  - Protected multiplets have fixed Δ.
- Expand correlator in multiplets with superblocks G^{IJKL}:

$$\langle\phi^I(x_1)\phi^J(x_2)\phi^K(x_3)\phi^L(x_4)\rangle = \frac{1}{x_{12}^{2\Delta_\phi}x_{34}^{2\Delta_\phi}}\sum_{\Delta,\ell}\lambda^2_{\Delta,\ell}G^{IJKL}_{\Delta,\ell}(u,v)$$

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QG: 4d $\mathcal{N} = 4$ CFTs

- All max susy $\mathcal{N} = 4$ CFTs have $SU(4)$ R-symmetry and 1-complex dimensional conformal manifold.
- All known theories are super-Yang-Mill (SYM) with compact gauge group (e.g. $SU(N)$) and complexified coupling $\tau \equiv \frac{\theta}{2\pi} + i\frac{4\pi^2}{g_{\text{YM}}^2}$.
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- Perturbative methods at small g_{YM} (weak coupling) and leading large N finite $\lambda \equiv g_{\text{YM}}^2 N$ (integrability dual to classical string).
- Question: How do we study subleading and finite N (dual to quantum string)?

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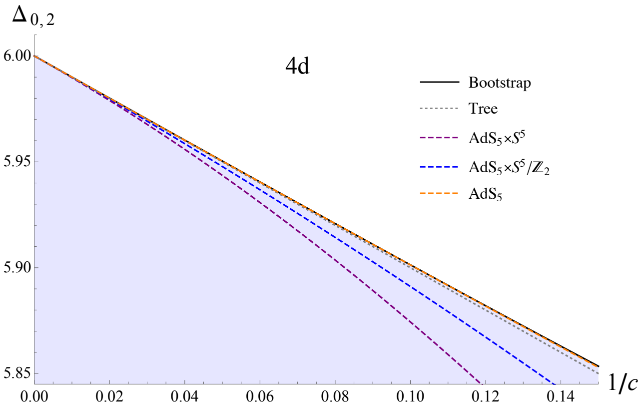

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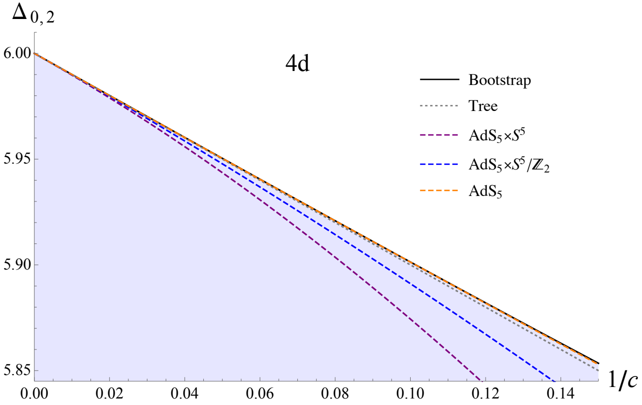

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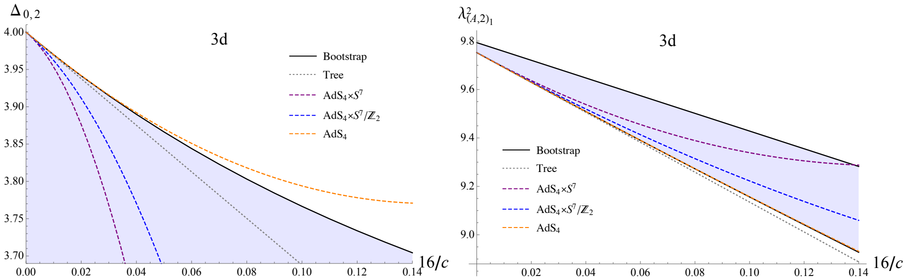

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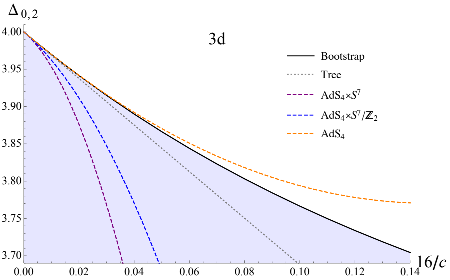

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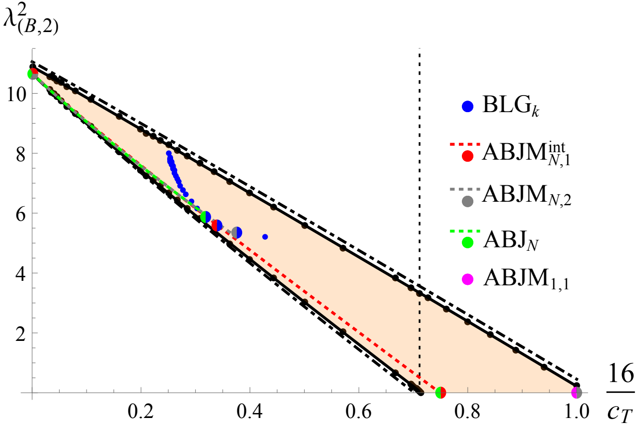

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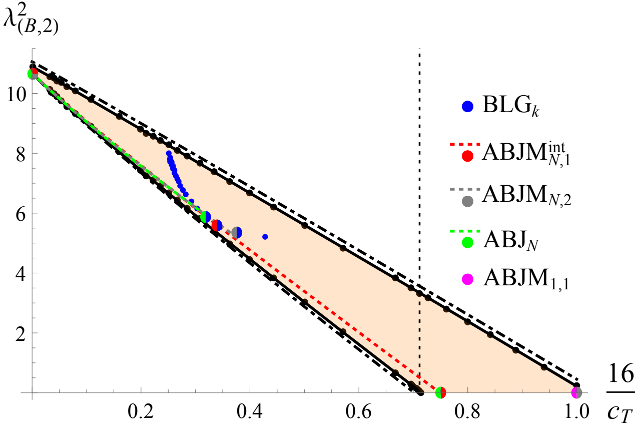

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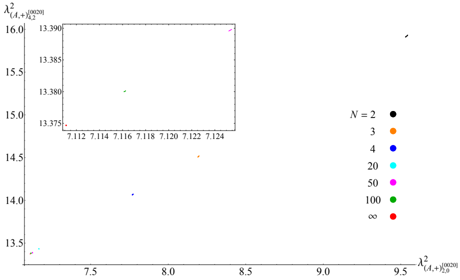

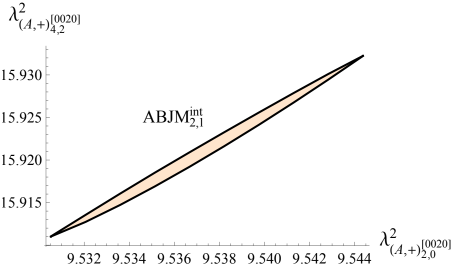

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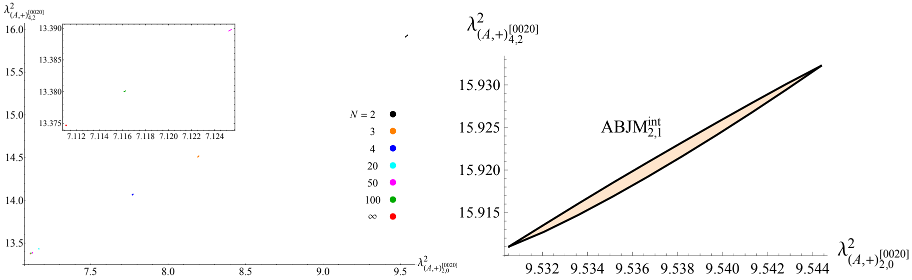

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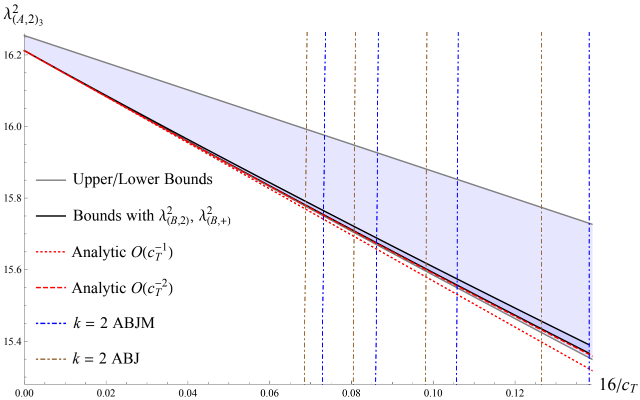

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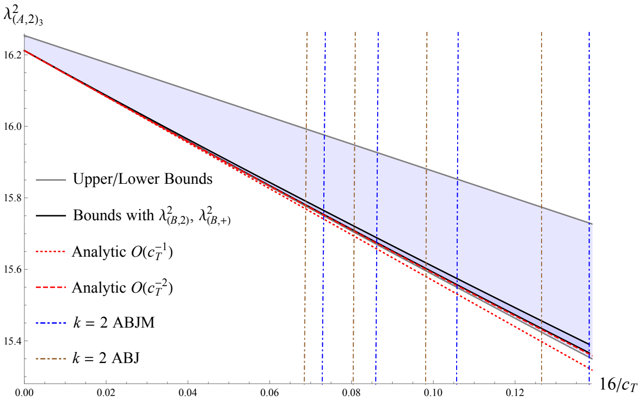

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6d $(2,0)$ CFTs

- No Lagrangian, all known theories constructed from string theory [Witten '95; Seiberg '98] , classified by groups $A_N \equiv SU(N+1)$, $D_N \equiv SO(2N)$, and E_N.
  - A_N is dual to M-theory on $AdS_7 \times S^4$.
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- Relate CFT to bulk parameters by $L_{\text{AdS}}/\ell_{11} \sim \mathfrak{o}16c$ where $\mathfrak{o}=2$ for orbifold.
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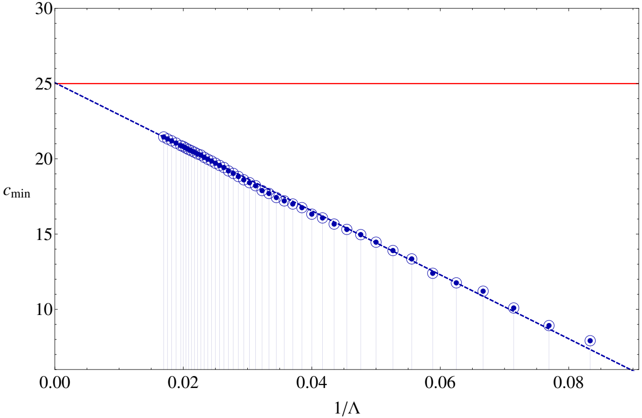

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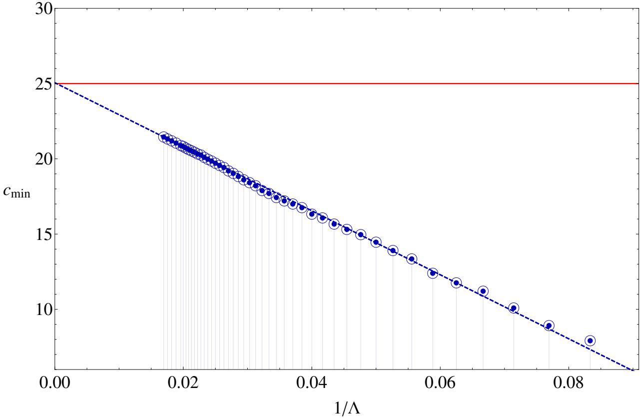

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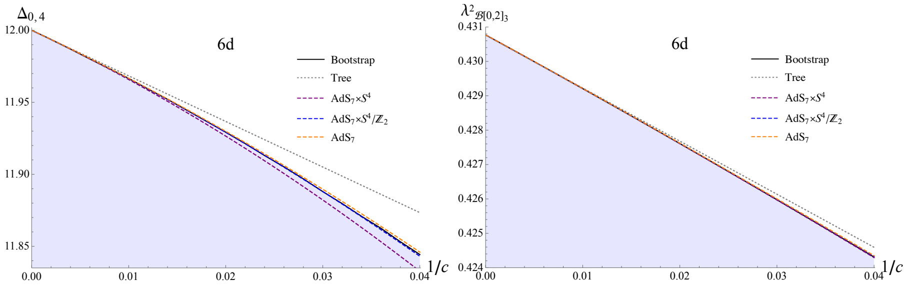

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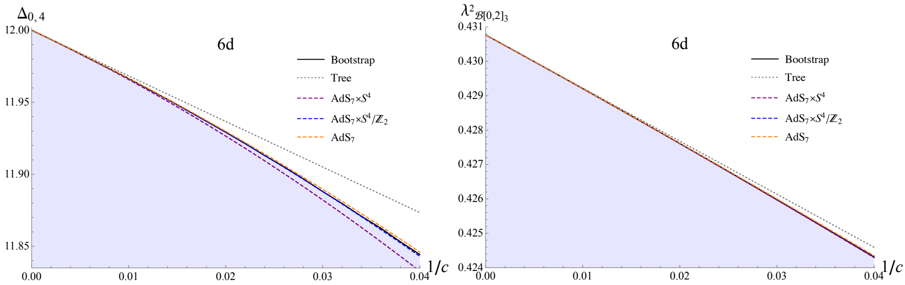

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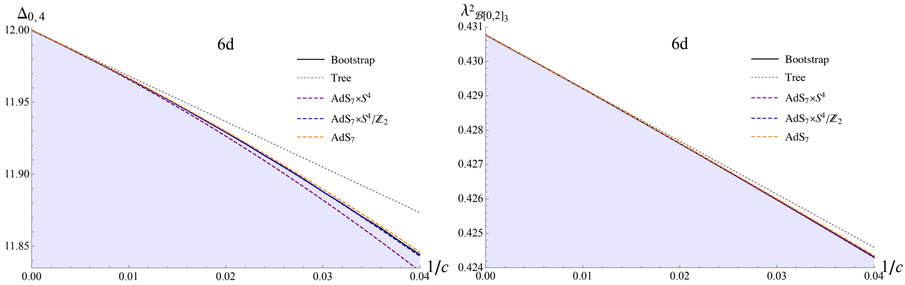

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- Recall that boundary of allowed region can be used to read off all CFT data in OPE in principle.
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  - In holographic CFTs, high twist operator dual to black hole states.
- I think this is the MOST IMPORTANT open general problem. We can only claim to have solved a CFT if we can extract ALL its CFT data, not just low twist.

General future directions: Higher point functions

- Most bootstrap studies so far for 4-point functions.
- Higher point functions in principle would impose new non-perturbative constraints.
- Many point functions alternative way of probing high twist operators.
- Many ingredients needed for higher point numerical bootstrap:
  - Product of multiple OPE coefficients not gauranteed to be real, need some kind of positivity of tensors.
  - Some first steps in [Rosenhaus '18, Fortin, Ma, Skiba '20; Poland, Prilepina, Tadic '23; Antunes, Harris, Kaviraj, Schomerus '23] .

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Whats the point?

- We have discussed many examples of CFTs that have almost been solved numerically, as well as prospects of solving other CFTs.
- What ultimately is the general lesson of computing all local CFT data say for $\mathcal{N} = 4$ SYM?
- We discussed partial answers before (e.g. black hole states for holographic CFTs), but this needs more thought.
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Bootstrap Manifesto

Conformal bootstrap program disdains to conceal its views and aims.

It openly declares that its ends can be attained only by numerical solutions to all existing CFTs.

Let perturbative methods tremble at a bootstrap revolution.

Physicists have nothing to lose but their Lagrangians.

They have a non-perturbative world to win.

Working theorists of all disciplines, bootstrap!

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