Global Symmetry and Localization Constraints on Superconformal Impurities

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iTHEMS-YITP workshop on Bootstrap, Localization, and Holography
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Based on 2405.10914 w/ S. Pufu, R. Dempsey and B. Offertaler
2305.08297 w/ S. Pufu and V. Rodriguez
(See also 2405.10862 and 2308.16575 from Billò, Frau, Galvagno, Lerda)

Plan of the Talk

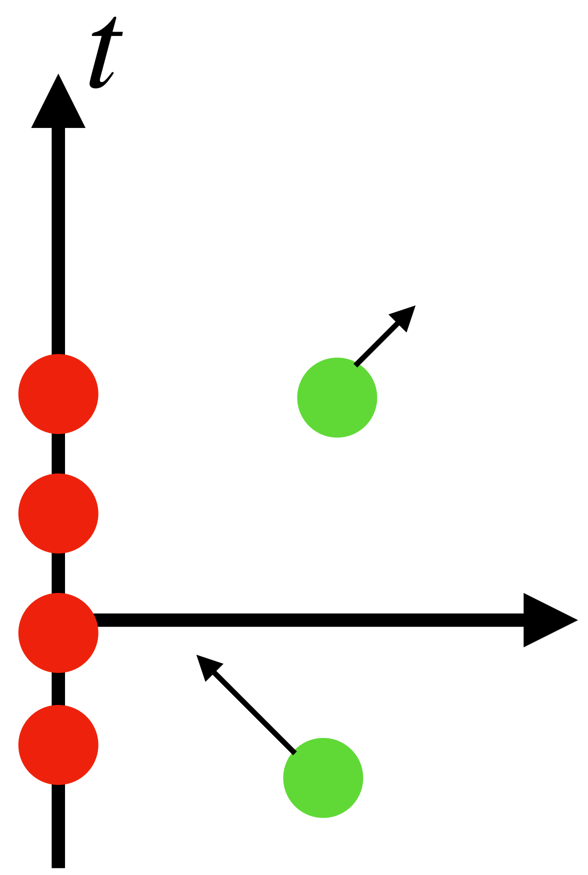
- Motivation and Background on Impurities/Defects
- Review: Superconformal Impurities
- Global Symmetry Constraint on Superconformal Lines
- Superconformal Ward Identity on Current multiplet 2-point function
- Supersymmetric Mass Deformation and Integral Constraint
- Applications and Discussions

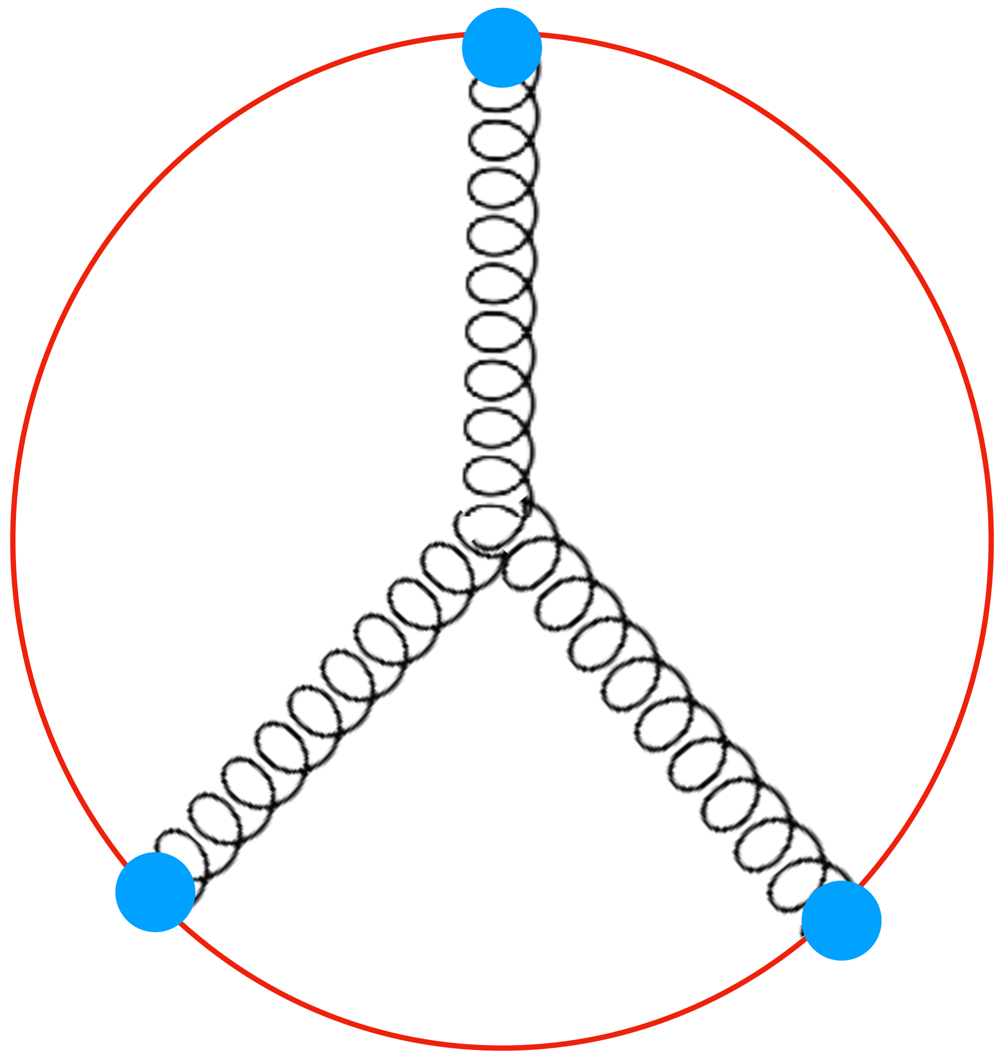
Impurities are Ubiquitous

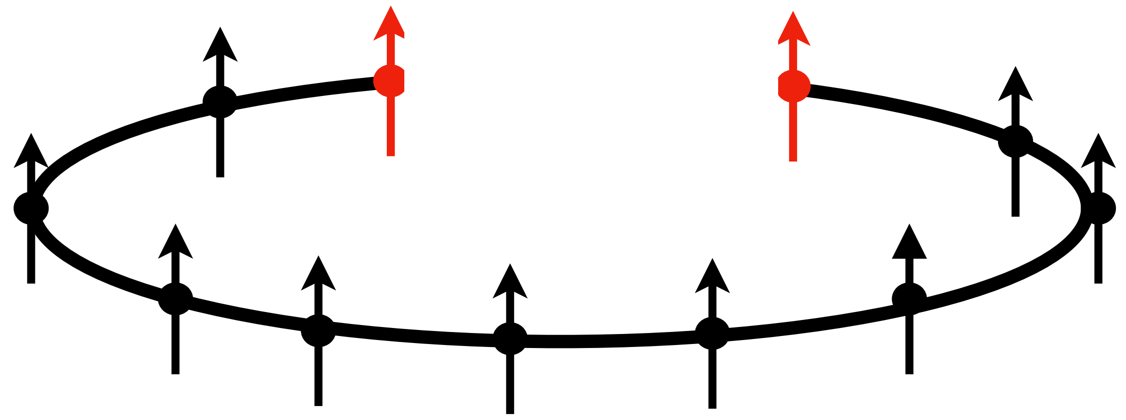
- Kondo effect: magnetic impurities in metal
- Worldlines of heavy charged particles
- Lattice systems (spin chain) with boundaries
- D-Branes in worldsheet string theory

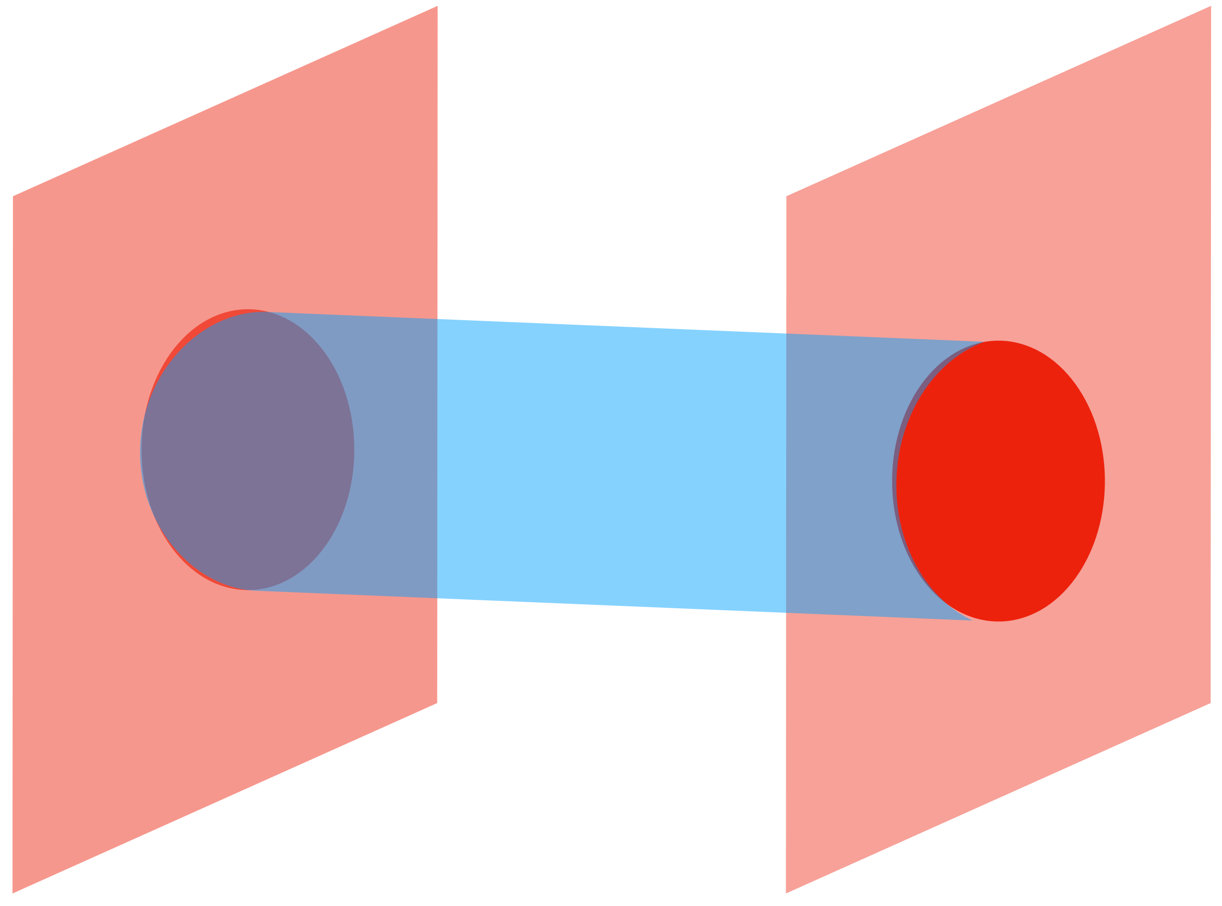
Descriptions in QFT →

- Defect lines in SU(2) WZW [Affleck-Ludwig,…]
- Wilson/’t Hooft loops [Wilson, t’ Hooft, Kapustin,…]
- Conformal boundaries [Cardy, Diehl,…]
- 2d Cardy states [Cardy,…]





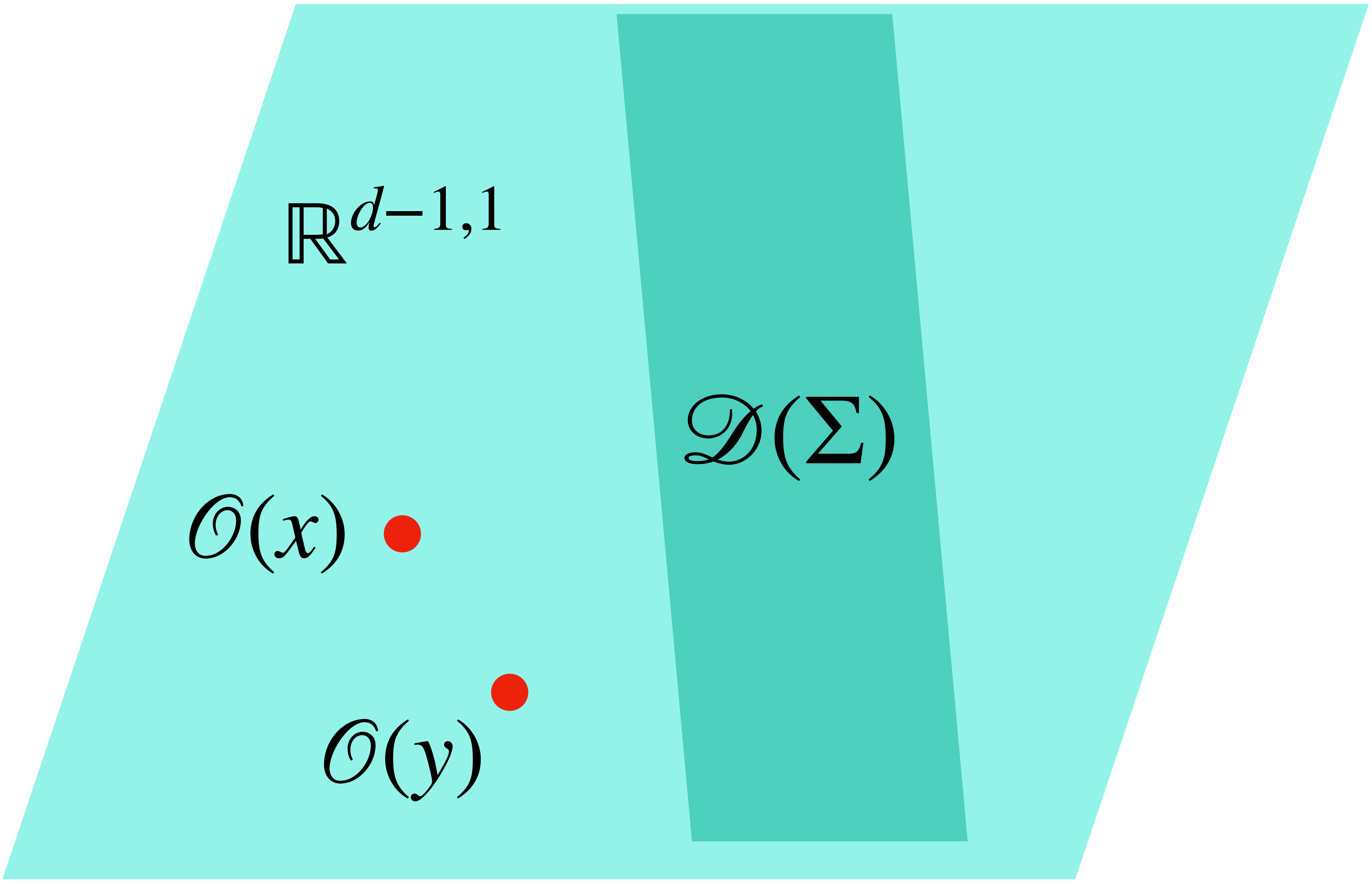




Beyond Local Point Operators

- Basic observables in QFT: correlation functions of local operators $\mathscr{O}(x)$
- Defects: extended operators $\mathscr{D}$ (e.g. Wilson loops, boundary conditions)

Vast enrichment of QFT observables!
(describing impurities)



Extended Quantum Field Theory

- Refined classification of QFT (distinguish otherwise identical theories)
- Elucidate bulk phase structures (e.g. confinement-deconfinement)
- New phase diagram on the defects (defect field theory or DFT, nontrivial even when bulk is free)
- Constructing lower dimensional QFTs (by gapping and decoupling)
- Full structure in correspondence w/ the holographic dual string/M theory (e.g. branes)
- Special case: topological defects -> Generalized symmetries
- Non-topological defect -> (higher) representations of generalized (higher) symmetries

Many general results

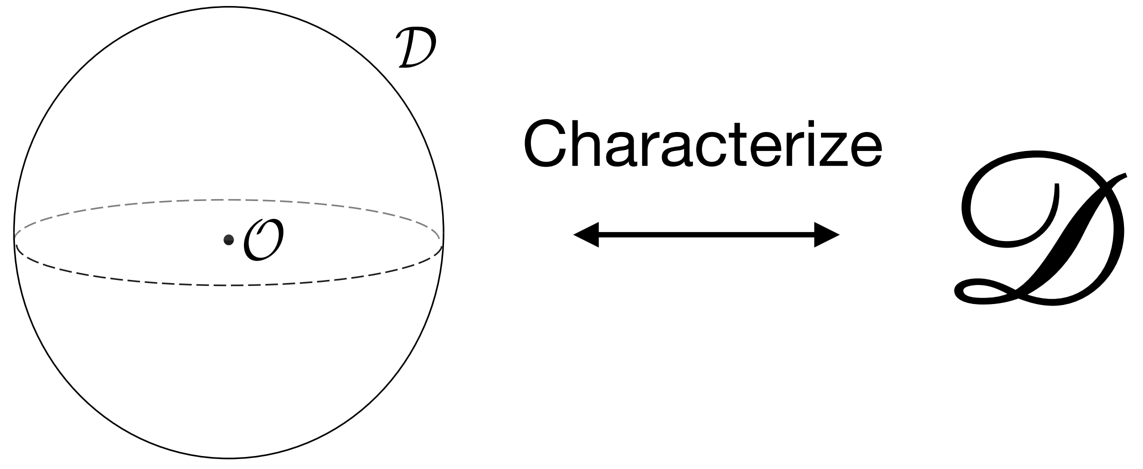
Precise dynamical questions?

Conformal Defects (DCFT)

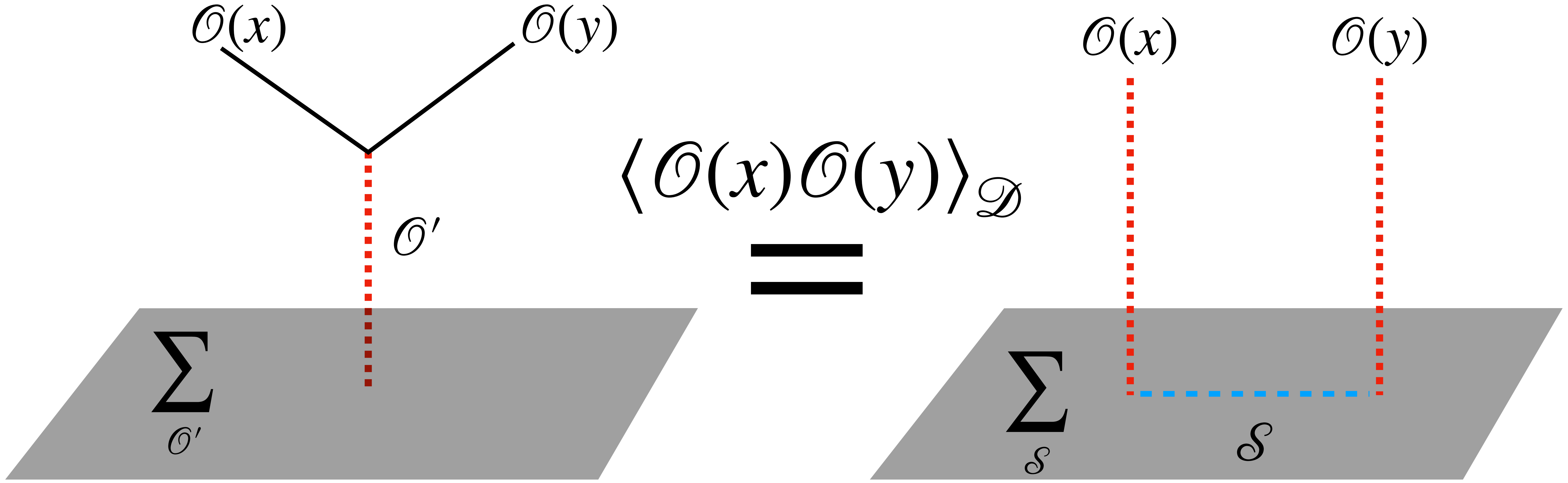
- **Critical phase** in the presence of boundaries/defects
- **Universality classes** of defect RG flows
- **No local p-dimensional** stress tensor or currents (generically)
- **New critical exponents and OPE data**
  (e.g. defect local ops $\mathscr{S}$ and bulk local op. 1pf $\langle \mathscr{O} \rangle_{\mathscr{D}}$)
- Constrained by **defect bootstrap** equations
  (e.g. residual conf symmetry, crossing and unitarity)

Symmetry of p dimensional conformal defect

$$SO(p,2) \times SO(d-p) \subset SO(d,2)$$



All bulk op $\mathscr{O}$



No positivity 🙁

[Cardy-Lewellen,
Liendo-Rastelli-van Rees,
Gaiotto-Mazac-Paulos,
Liendo-Meneghelli,
Billo-Goncalves-Lauria-Meineri,…]

Superconformal Defects

- Most examples of **nontrivial conformal defects** from SUSY setups (especially in $d>4$ where the only known unitary interacting CFTs are SCFTs)
- Exact techniques for studying these defects: in particular **SUSY localization** (also integrability) **Nikolay's talk**
- Map to **branes** in string/M-theory under top down AdS/CFT constructions

Goal: unpackage DCFT data contained in the localization results for defects

Case Study: Superconformal Defects in the SYM

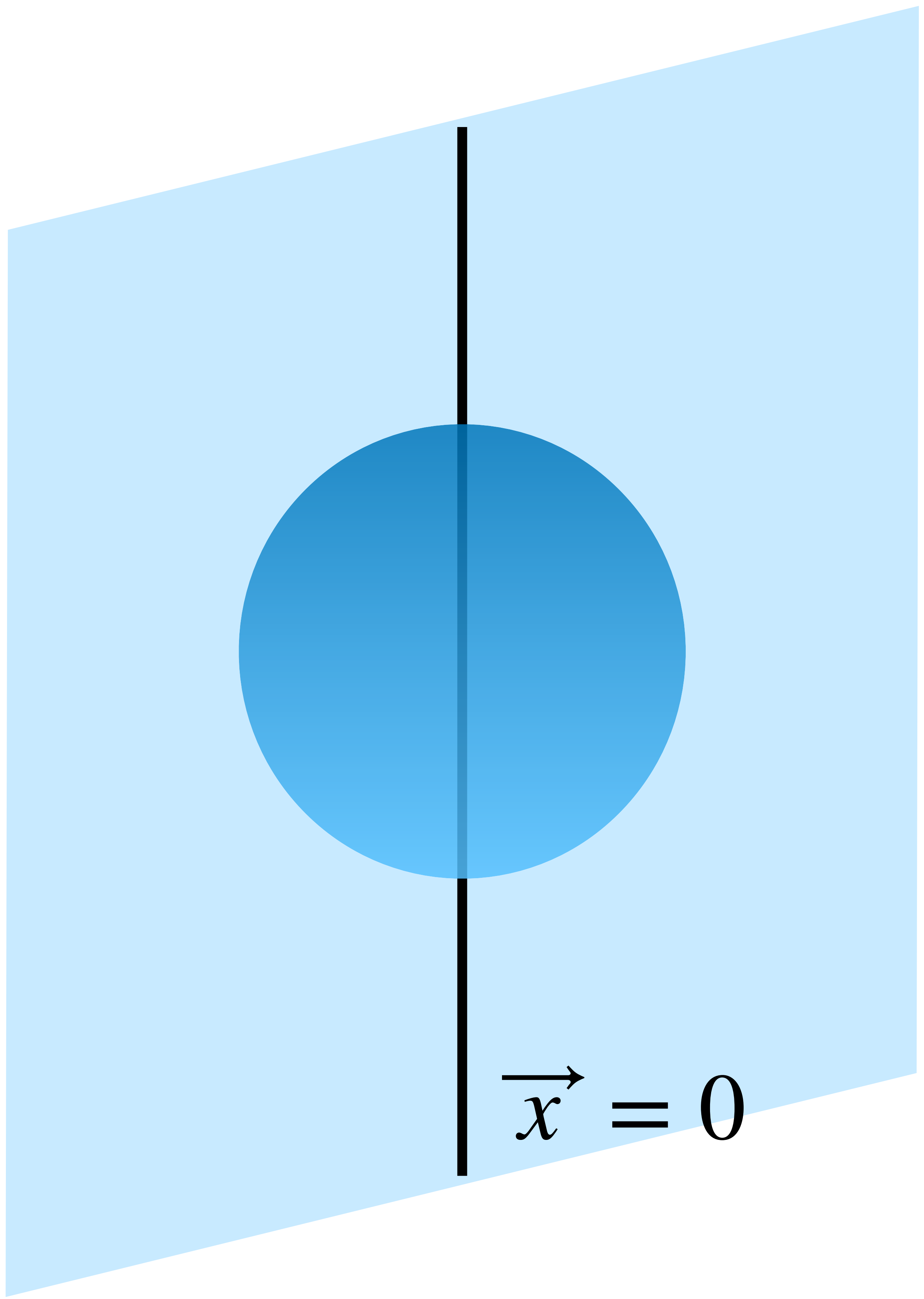
$$S_{\text{SYM}} = -\frac{1}{2g_4^2}\int_{\mathbb{R}^4} d^4x\,\text{tr}\left(\frac{1}{2}(F_{\mu\nu})^2 + \frac{1}{2}[\Phi_I,\Phi_J]^2 + (D_\mu\Phi_I)^2 - \Psi\gamma^\mu D_\mu\Psi - \Psi\gamma^I[\Phi_I,\Psi]\right)$$

- Rich zoo of superconformal defects:

  Lines: supersymmetric Wilson-'t Hooft loops,

$$\mathbb{W}_R = \text{tr}_\text{R}\text{P}\exp i\oint ds\left(A_\mu(x(s))\dot{x}^\mu(s) + \Phi_6(x(s)|\dot{x}(s)|\right)$$

$$\mathbb{T}_{\vec{m}} \quad F_{ij}(x) = \frac{1}{2}\epsilon_{ijk}\frac{x_k}{|x|^3}T_{\vec{m}} \quad \Phi_6(x) = -\frac{1}{2|x|}T_{\vec{m}}$$

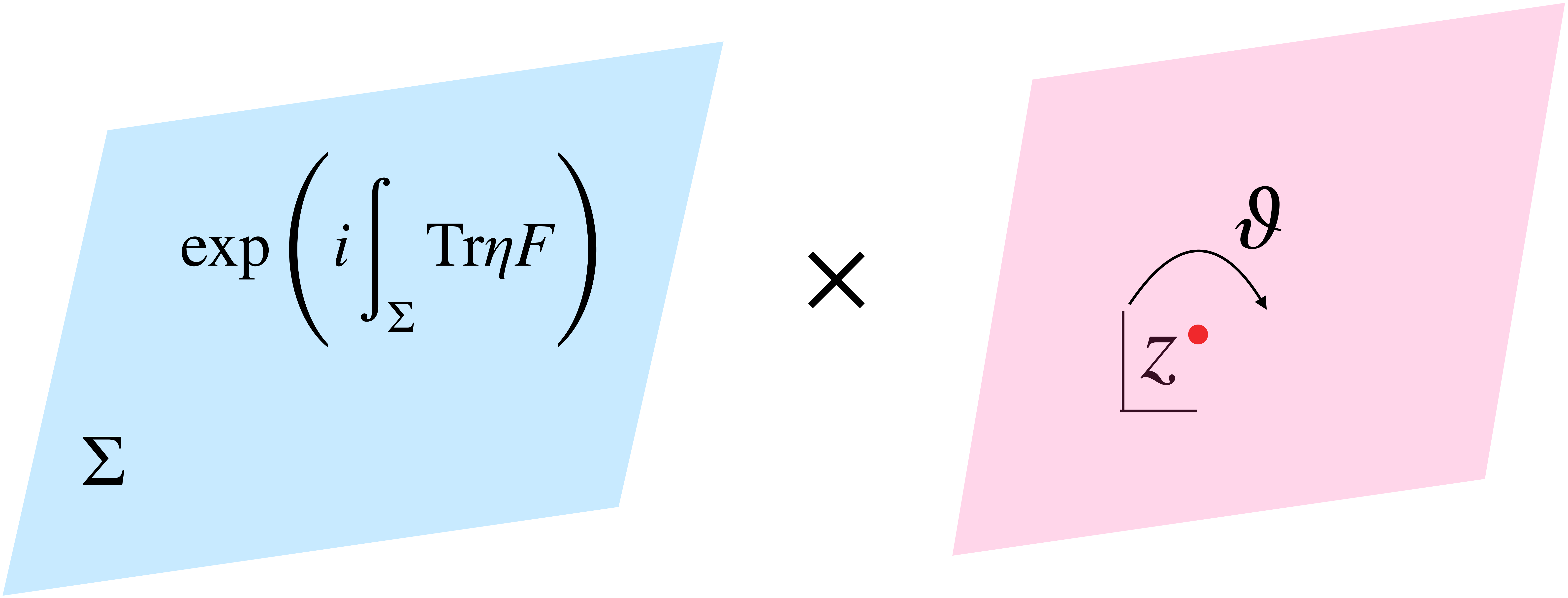


Case Study: Superconformal Defects in the SYM

$$S_{\mathrm{SYM}} = -\frac{1}{2g_4^2}\int_{\mathbb{R}^4} d^4x\, \mathrm{tr}\left(\frac{1}{2}(F_{\mu\nu})^2 + \frac{1}{2}[\Phi_I, \Phi_J]^2 + (D_\mu \Phi_I)^2 - \Psi\gamma^\mu D_\mu \Psi - \Psi\gamma^I[\Phi_I, \Psi]\right)$$

- Rich zoo of superconformal defects:

  Surfaces: half-BPS Gukov-Witten surface operators,



$$\Phi^5 + i\Phi^6 = \frac{\beta + i\gamma}{z}$$

$$A = \alpha d\vartheta$$

Marginal parameters $\alpha, \beta, \gamma, \eta$ valued in the Cartan subalgebra

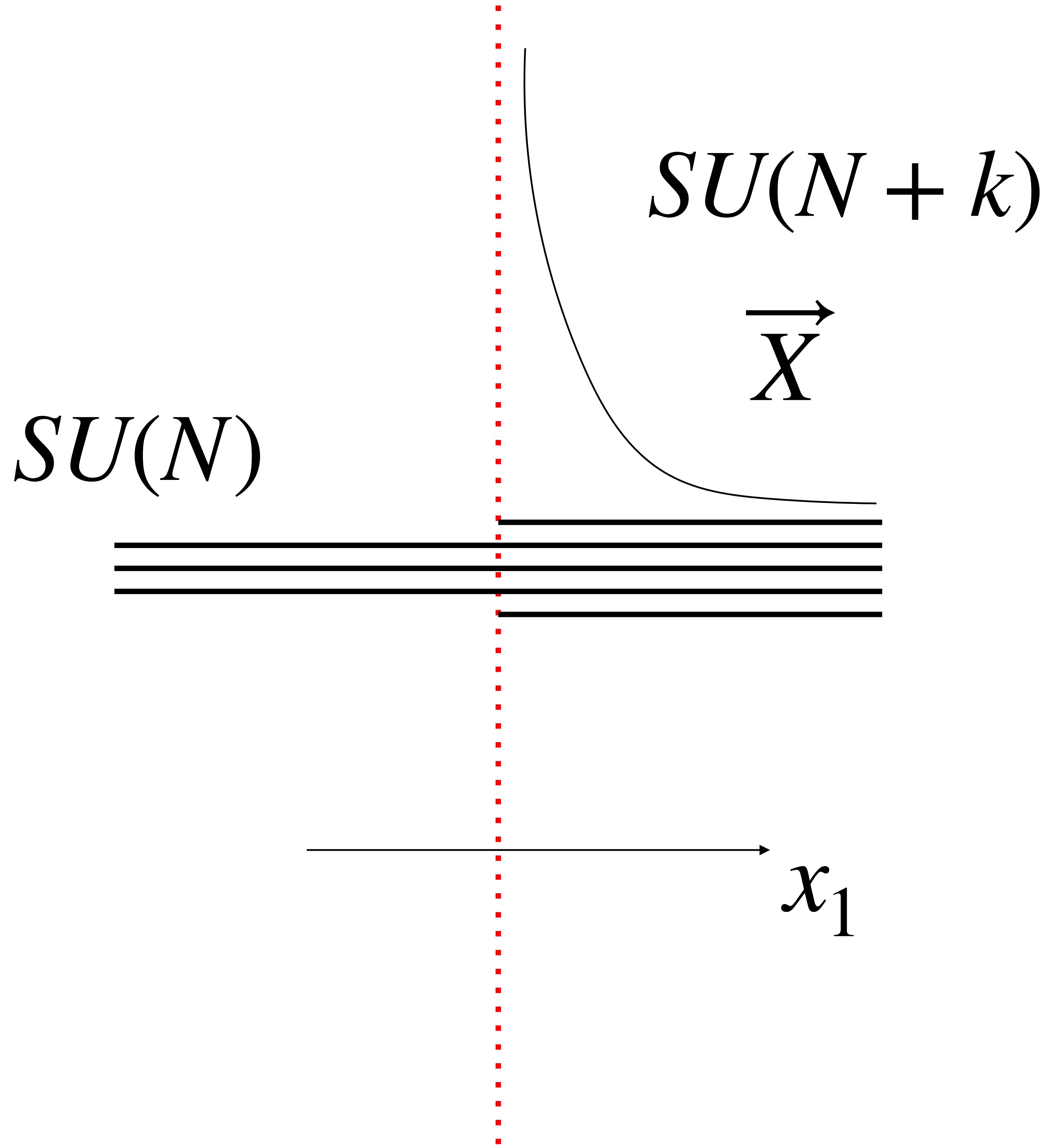
Case Study: Superconformal Defects in the SYM

$$S_{\text{SYM}} = -\frac{1}{2g_4^2}\int_{\mathbb{R}^4} d^4x \,\text{tr}\left(\frac{1}{2}(F_{\mu\nu})^2 + \frac{1}{2}[\Phi_I, \Phi_J]^2 + (D_\mu \Phi_I)^2 - \Psi\gamma^\mu D_\mu \Psi - \Psi\gamma^I[\Phi_I, \Psi]\right)$$

- Rich zoo of superconformal defects:

interfaces (boundaries): Gukov-Witten boundary/interface.

$$\vec{X} = (\Phi_1, \Phi_2, \Phi_3), \quad \vec{Y} = (\Phi_4, \Phi_5, \Phi_6)$$



Nahm pole configuration

$k \times k$ block

$$A_\mu^+ = \begin{pmatrix} A_\mu^- & \cdot \\ \cdot & \cdot \end{pmatrix}, \quad \vec{Y}^+ = \begin{pmatrix} \vec{Y}^- & \cdot \\ \cdot & \cdot \end{pmatrix}, \quad \vec{X}^+ = \begin{pmatrix} \vec{X}^- & \cdot \\ \cdot & \frac{-\vec{t}}{x_1} \end{pmatrix}$$

$$[t_i, t_j] = \epsilon_{ijk} t_k$$

Case Study: Superconformal Defects in the SYM

$$S_{\text{SYM}} = -\frac{1}{2g_4^2}\int_{\mathbb{R}^4} d^4x \,\text{tr}\left(\frac{1}{2}(F_{\mu\nu})^2 + \frac{1}{2}[\Phi_I, \Phi_J]^2 + (D_\mu\Phi_I)^2 - \Psi\gamma^\mu D_\mu\Psi - \Psi\gamma^I[\Phi_I, \Psi]\right)$$

- Rich zoo of superconformal defects:

  lines: supersymmetric Wilson-'t Hooft loops,

  surfaces: Gukov-Witten surface operators,

  interfaces (boundaries): Gukov-Witten boundary/interface.

Defect Networks

- A variety of methods to study: e.g. supersymmetric localization, integrability, bootstrap
- Close connection to branes in string theory via AdS/CFT

Case Study: Superconformal Defects in the SYM

$$S_{\text{SYM}} = -\frac{1}{2g_4^2}\int_{\mathbb{R}^4} d^4x \,\text{tr}\left(\frac{1}{2}(F_{\mu\nu})^2 + \frac{1}{2}[\Phi_I, \Phi_J]^2 + (D_\mu \Phi_I)^2 - \Psi\gamma^\mu D_\mu \Psi - \Psi\gamma^I[\Phi_I, \Psi]\right)$$

- Rich zoo of superconformal defects:

AdS/CFT

lines: supersymmetric Wilson-'t Hooft loops, → (p, q) strings: AdS_2

surfaces: Gukov-Witten surface operators, → D3 branes: $\text{AdS}_3 \times \text{S}^1$

interfaces (boundaries): Gukov-Witten boundary/interface. → D5/NS5 branes: $\text{AdS}_4 \times \text{S}^2$

- A variety of methods to study: e.g. supersymmetric localization, integrability, bootstrap
- Close connection to branes in string theory via AdS/CFT

Supersymmetric DCFT observables?

Quasi-Topological DCFT Sector from Supersymmetric Localization

$$S_{\rm SYM} = -\frac{1}{2g_4^2}\int_{\mathbb{R}^4} d^4x\, {\rm tr}\left(\frac{1}{2}(F_{\mu\nu})^2 + \frac{1}{2}[\Phi_I,\Phi_J]^2 + (D_\mu\Phi_I)^2 - \Psi\gamma^\mu D_\mu\Psi - \Psi\gamma^I[\Phi_I,\Psi]\right)$$

SUSic Defects in 4d SYM on $\mathbb{R}^4$

$$\langle \mathscr{D}_1 \mathscr{O}_1(x_1)\mathscr{O}_2(x_2)\ldots\mathscr{D}_2\ldots\rangle_{\rm SYM}$$

$$g_{\rm YM}^2 = -\frac{8\pi}{g_4^2 R^2}$$

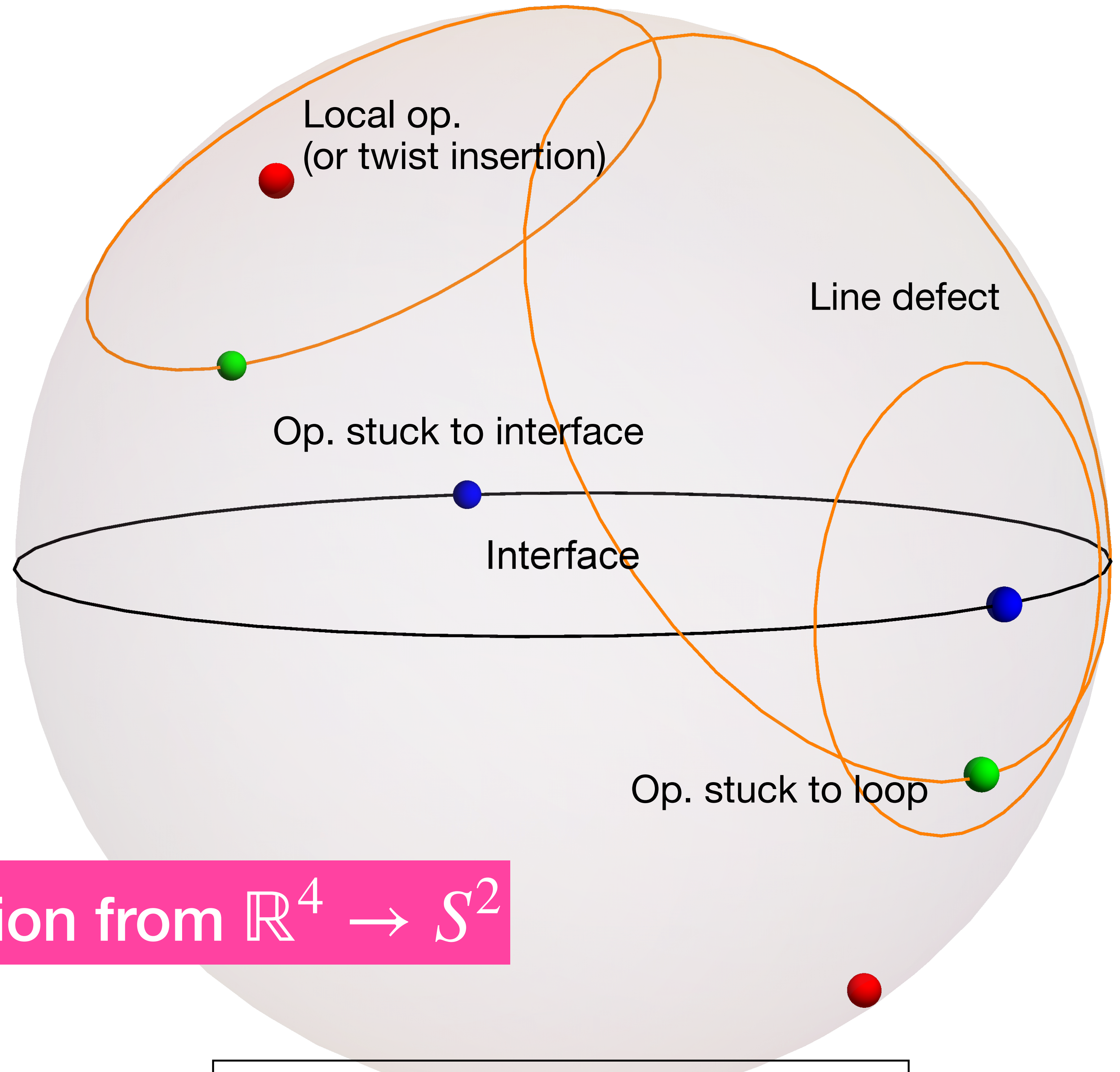
$$\mathscr{A} \equiv A + i\epsilon_{ijk}\phi_i x^k dx^j$$

(coupled to 3 of the 6 scalars)

$$S_{\rm YM}(\mathscr{A}) \equiv -\frac{1}{g_{\rm YM}^2}\int_{S^2} d^2z\sqrt{g}\,{\rm tr}(\star\mathscr{F})^2$$

Defects in 2d YM on S^2

$$\langle \mathscr{D}_1 \mathscr{O}_1(z_1)\mathscr{O}_2(z_2)\ldots\mathscr{D}_2\ldots\rangle_{\rm 2dYM}$$



Projection from $\mathbb{R}^4 \to S^2$

1/16 BPS defect networks

- Dynamics of BPS observables captured by an emergent lower dimensional theory
- Reduce complicated 4d observables to solvable 2d observables
- Matrix model techniques for 2d YM + defects
- Path integral -> Finite integral
- Rich DCFT data (but no 4d instanton contributions)

[Pestun 09, YW 20] also previous works [Drukker-Giombi-Ricci-Trancanelli, Giombi-Pestun, Giombi-Komatsu…]

More Localization Results for N=2 DCFT: Integrated Correlators

- So far: **localized** insertions of local and defect operators in the $\mathcal{Q}$ cohomology (for localizing supercharge $\mathcal{Q}$

- More generally: **integrated** insertions preserving supercharge $\mathcal{Q}$

$$S \to S + \int d^d x \sqrt{g} (\lambda \mathcal{O}(x) + \ldots)$$

- Equivalently from $\mathcal{Q}$ preserving deformations of the CFT action

- For $\mathcal{Q}_{\text{Pestun}}$ that localizes 4d N=2 theories on S^4 to a matrix model: [Pestun 07]

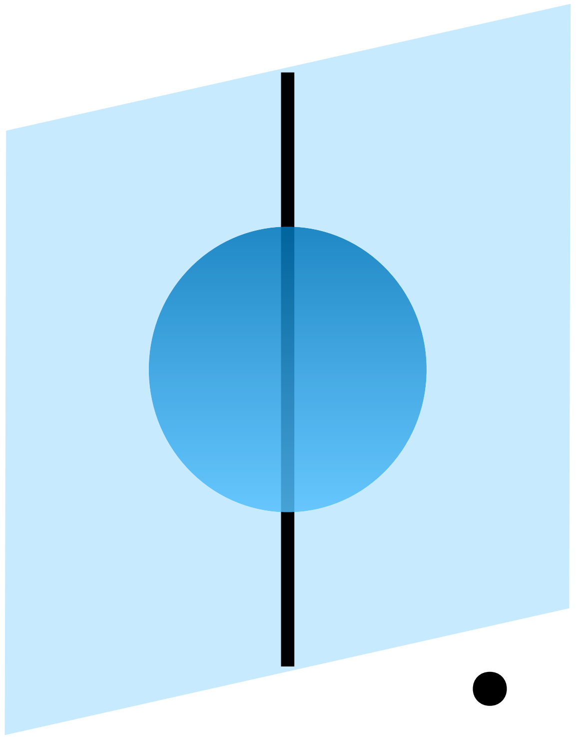
Supersymmetric mass m ⟶ Flavor current multiplet

Supersymmetric squashing b ⟶ Stress tensor multiplet

- Integrated Correlators from derivatives of supersymmetric partition function

Integral Constraints on Superconformal Lines in N=2 SCFTs

- Consider N=2 SCFTs with continuous global symmetry G_F
- Superconformal line $\mathbb{L}$ preserve $\mathfrak{osp}(4^*|2) \subset \mathfrak{su}(2,2|2)$ half-BPS



$$\mathfrak{so}(1,2) \times \mathfrak{so}(3) \times \mathfrak{su}(2)_R$$

- $\mathbb{L}$ must preserve the global symmetry G_F by unitarity and locality! [Agmon-YW 21]
- Preserved SUSY + Global symmetry -> **compatible** SUSlc mass deformation with defect $\mathbb{L}$ insertion [Pestun 07]

$$\langle \mathbb{L} \rangle (m) \equiv \frac{Z_{\mathbb{L}}(m)}{Z(m)} \qquad \partial_m^2 \log \langle \mathbb{L} \rangle |_{m=0}$$

Integrated Defect Correlator

[Pufu-Roderiguez-YW 23]

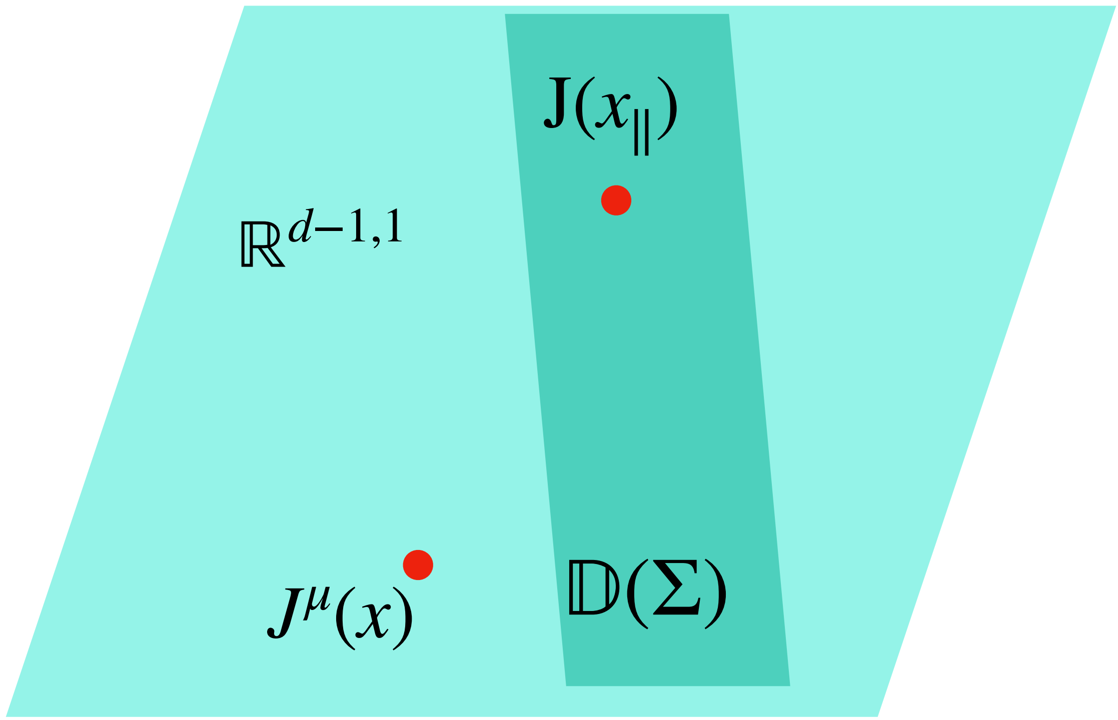
No Global Symmetry Breaking on 4d Superconformal Line [Agmon-YW 21]

$$x = (x_{\|}, x_{\perp})$$

Distinguished local operators $\mathrm{J}(x_{\|})$ on defect $\mathbb{D}$ associated to **broken** bulk symmetry currents J^{μ}

Local modification of the Ward identities by operator valued contact terms on the defect compatible with the residual $SO(p,2)$ conformal symmetry

$$\partial_{\mu} J^{\mu}(x) = \delta_{\Sigma}(x_{\perp}) \mathrm{J}(x_{\|}) \xrightarrow[\text{radially-ordered}]{\text{integrated}} [U, \mathbb{D}] = \int_{\Sigma} dx_{\|}^{p}\, \mathrm{J}(x_{\|}) \mathbb{D}$$



Quantum numbers **protected by bulk Ward identities**

Most basic example is the **displacement operator** (broken transverse translation)

$$\partial_{\mu} T^{\mu i}(x) = -\, \delta_{\Sigma}(x_{\perp}) D^{i}(x_{\|}) \qquad [P_i, \mathscr{D}] = \int_{\Sigma} d^{p} x_{\|}\, D_i(x_{\|}) \mathscr{D}$$

$$\partial_{\mu} T^{\mu a}(x) = T^{\mu}_{\mu} = 0$$

Complete into **superconformal multiplet** when defect preserves superconformal algebra

When not possible by unitarity: symmetry breaking is forbidden!

N=2 Flavor Symmetry

- Half-BPS conserved current multiplet of $\mathfrak{su}(2,2|2)$

Flavor adjoint

$$J_{ij}^{I} \xrightarrow{Q_{i\alpha}, Q_{\alpha}^{i}} \xi_{\alpha}^{iI}, \xi_{i\alpha}^{I} \xrightarrow{Q_{i\alpha}, Q_{\alpha}^{i}} K^{I}, \bar{K}^{I}, j_{\mu}^{I}$$

$\mathfrak{su}(2)_R$ triplet and $\Delta = 2$

Moment map operators

Conserved Currents

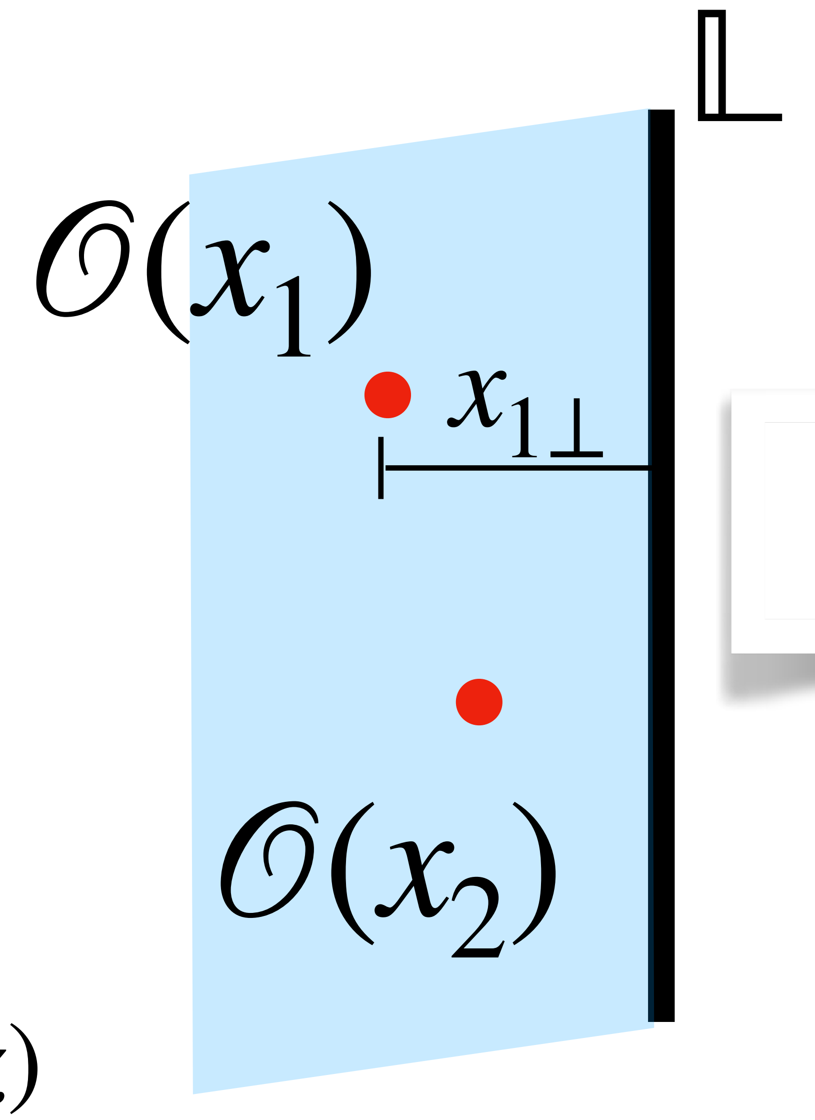
- Supersymmetric mass deformation in a Cartan direction of G_F (preserve $\mathcal{Q}_{\text{Pestun}}$ on S^4)

$$S \to S + m \int d^4x \sqrt{g} \left((K(x) + \bar{K}(x) + \frac{i}{r} \left(J_{11}(x) + J_{22}(x) \right) \right)$$

Two-point function with Line Defect

- Similar kinematics as for the 4pf w/o defect

$$\frac{\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathbb{L}\rangle}{\langle \mathbb{L}\rangle} = \frac{1}{x_{1\perp}^{\Delta} x_{2\perp}^{\Delta}} F_{\mathcal{O}\mathcal{O}\mathbb{L}}(z,\bar{z})$$



Bulk OPE: $z \to 1$
Defect OPE: $z \to 0$

Conformal transformation $\longrightarrow$ $x_1 = (0,0,\mathrm{Re}z,\mathrm{Im}z)$, $x_2 = (0,0,1,0)$

- Integrated defect correlator for half-BPS line defect

$$\partial_m^2 \log\langle \mathbb{L}\rangle\,|_{m=0} = \int dz d\bar{z}\, \rho(z,\bar{z}) F_{JJ\mathbb{L}}(z,\bar{z})$$

See also [Billò-Frau-Galvagno-Lerda]

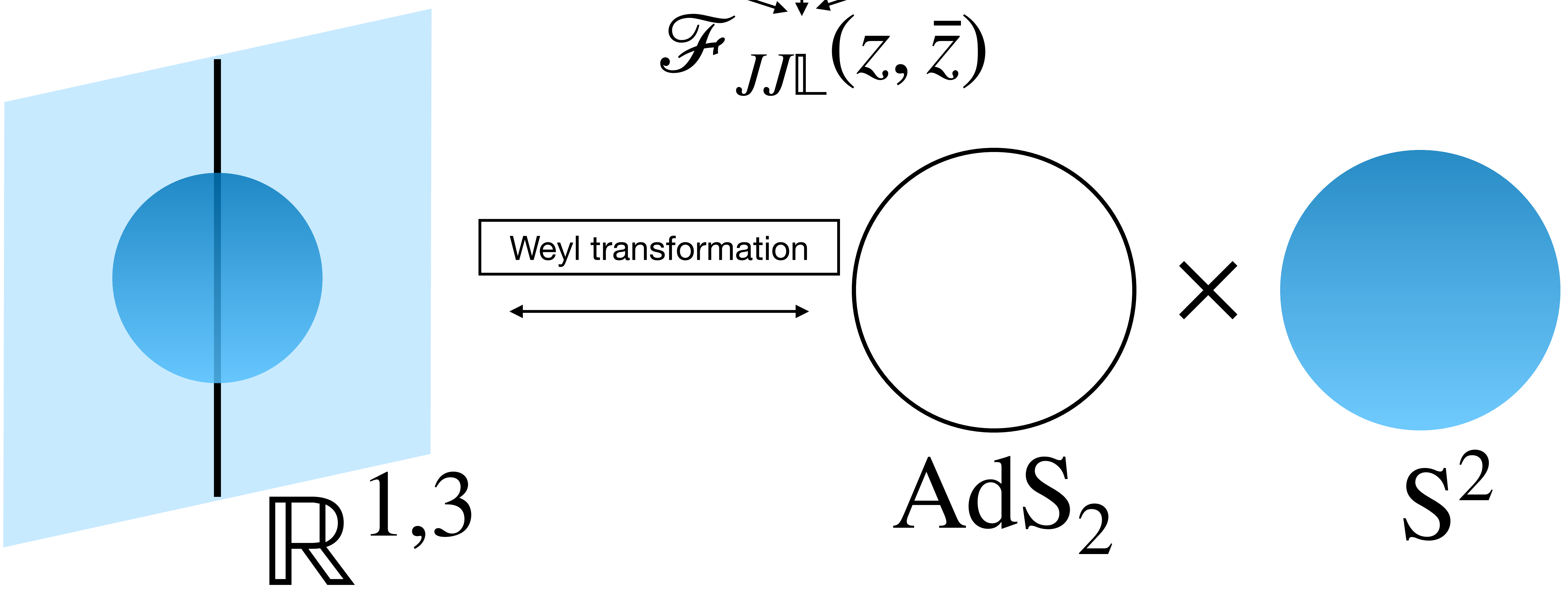
Goal: identify the measure $\rho(z,\bar{z})$

Superconformal Ward Identities and Integral Constraint

Made Easy by Hyperbolic Space

$$\langle JJ \rangle_{\mathbb{L}} \quad \langle JK \rangle_{\mathbb{L}} \quad \langle KK \rangle_{\mathbb{L}}$$

$$\mathcal{F}_{JJ\mathbb{L}}(z, \bar{z})$$



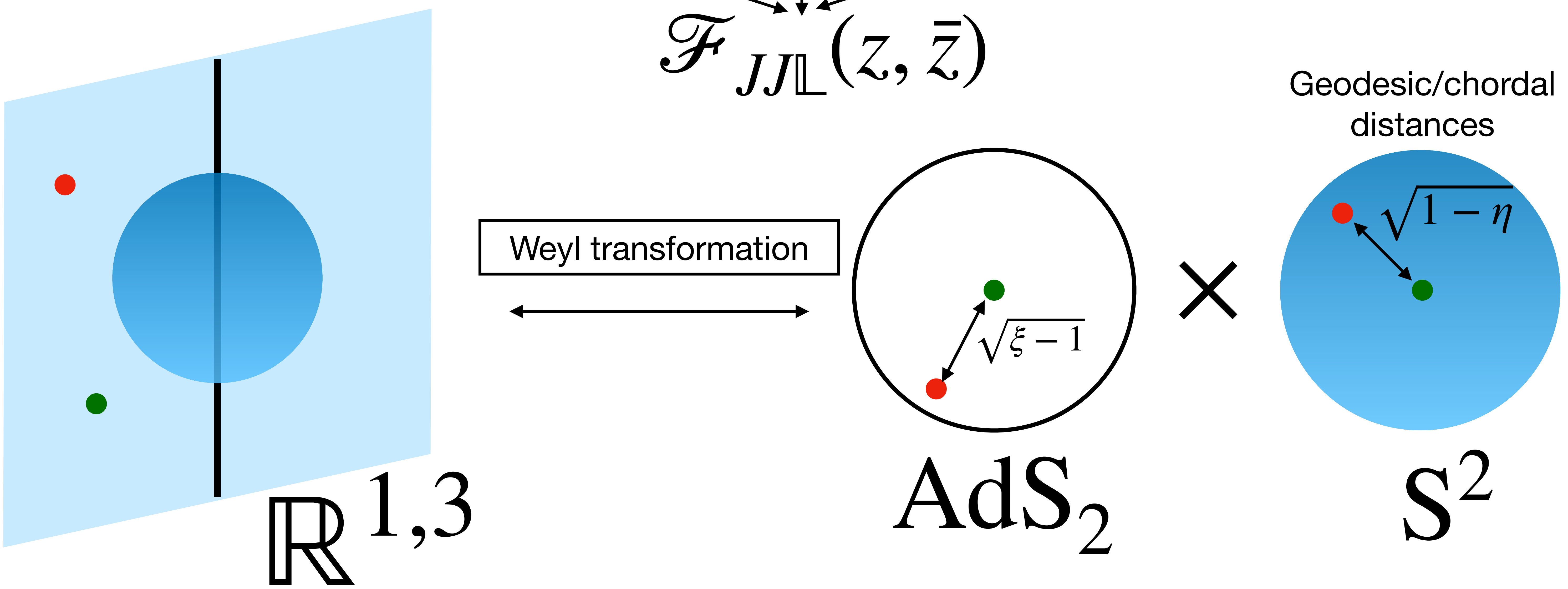
$$\partial_m^2 \log\langle \mathbb{L} \rangle \,|_{m=0} = -\,4\pi^2 \int_{\mathbb{H}^2\times \mathrm{S}^2} d^4x \sqrt{g}\, \langle J_{11}(x) J_{22}(y) \rangle_{\mathbb{L},c}$$

Superconformal Ward Identities and Integral Constraint

Made Easy by Hyperbolic Space

$$\langle JJ \rangle_{\mathbb{L}} \quad \langle JK \rangle_{\mathbb{L}} \quad \langle KK \rangle_{\mathbb{L}}$$

$$\mathcal{F}_{JJ\mathbb{L}}(z, \bar{z})$$



$$\partial_m^2 \log\langle \mathbb{L} \rangle \,|_{m=0} = -\, 16\pi^4 \int_1^\infty d\xi \int_{-1}^{1} d\eta \, F_{JJ\mathbb{L}}(\xi, \eta)$$

Superconformal Ward Identities

Made Easy by Hyperbolic Space

Where is the Magic?

- Residual (super)conformal symmetry as (super)isometry
- Defect at asymptotic boundary
- Ward identity as differential identity with Laplace type operators

$\mathfrak{osp}(4^*|2)$
transformations δ, δ'

$$\langle K\delta\delta' J\rangle_{\mathbb{L}}$$

Supersymmetric $\mathbb{L}$

SUSY Ward identities

Nontrivial but simple Killing spinors

$$\langle KK\rangle_{\mathbb{L}} = \langle \delta\delta' KJ\rangle_{\mathbb{L}} \propto \Box(\cdots)\langle JJ\rangle_{\mathbb{L}}$$

SUSY algebra on $\mathrm{AdS}_2 \times \mathrm{S}^2$

A bit more details…

Superconformal symmetry on curved space

$$\delta \equiv \bar{\epsilon}^i Q_i + \bar{\epsilon}_i Q^i + \bar{\eta}^i S_i + \bar{\eta}_i S^i \qquad D_\mu \epsilon^i = \gamma_\mu \eta^i \quad D_\mu \epsilon_i = \gamma_\mu \eta_i$$

On $\mathrm{AdS}_2 \times \mathrm{S}^2$

$$ds^2 = \frac{-dt^2 + d\vec{x}^2}{|\vec{x}|^2}$$

$$\mathfrak{su}(2,2|2)$$

$$\mathfrak{osp}(4^*|2)$$

$$\epsilon^i = \frac{\alpha^i + x^a \gamma_a \beta^i}{\sqrt{|\vec{x}|}}, \quad \epsilon_i = \frac{\alpha_i + x^a \gamma_a \beta_i}{\sqrt{|\vec{x}|}}$$

$$\alpha^i = \varepsilon^{ij} \gamma^0 \alpha_j, \quad \beta^i = \varepsilon^{ij} \gamma^0 \beta_j$$

Strategy

Choose δ, δ' such that $\langle K \delta \delta' J \rangle_{\mathbb{L}} \propto \langle KK \rangle_{\mathbb{L}}$ (quadratic equations for $\alpha_i, \beta_i, \alpha^i, \beta^i$)

Superconformal Ward Identities w/ Line Defect

$\mathrm{AdS}_2 \times \mathrm{S}^2$ Laplacian $\Box f = (\xi^2 - 1)\partial_\xi^2 f + 2\xi\partial_\xi f + (1-\eta^2)\partial_\eta^2 f - 2\eta\partial_\eta f$

$$F_{KK\mathbb{L}}(\xi,\eta) = \frac{1}{2}\Box\left[(\xi-\eta)F_{JJ\mathbb{L}}(\xi,\eta)\right]$$

$$F_{K\bar{K}\mathbb{L}}(\xi,\eta) = \frac{1}{2}\Box\left[(\xi+\eta)F_{JJ\mathbb{L}}(\xi,\eta)\right]$$

$$F_{\bar{K}\bar{K}\mathbb{L}}(\xi,\eta) = \frac{1}{2}\Box\left[(\xi-\eta)F_{JJ\mathbb{L}}(\xi,\eta)\right]$$

By-product: defect 2-pt superconformal blocks

- Same form as SUSY Ward identity for two-point function without defect!
- **Sanity Check 1**: trivial defect

$$F_{K\bar{K}\mathbb{L}} \propto \frac{1}{(\xi-\eta)^3}\,,\quad F_{KK\mathbb{L}} = F_{\bar{K}\bar{K}\mathbb{L}} = 0\,,\quad F_{JJ\mathbb{L}} \propto \frac{1}{(\xi-\eta)^2}$$

- **Sanity Check 2**: Stress tensor block

1. Stress tensor multiplet operators $M, T_{\mu\nu}$ exchanged in bulk OPE channel
2. Decompose into bosonic blocks for $M, T_{\mu\nu}$

$$\text{eg } F_{JJ\mathbb{L}}|_{\mathrm{ST}} = a_{JJ}f_{2,0}(\xi,\eta) + b_{JJ}f_{4,2}(\xi,\eta)$$

3. There exist solutions to a, b

Integral Constraint

$$S \to S + m \int d^4x \sqrt{g} \left((K(x) + \bar{K}(x) + \frac{i}{r} \left(J_{11}(x) + J_{22}(x) \right) \right)$$

- Wick rotate Ward identities to Euclidean space
- Apply to integrated insertions of current multiplet operators
- Natural/trivial integration measure on $\mathbb{H}^2 \times \mathrm{S}^2$ (Weyl equivalent to S^4)

Combine and integrate over one location

$$\partial_m^2 \log\langle \mathbb{L} \rangle \,|_{m=0} = -4\pi^2 \int_{\mathbb{H}^2 \times \mathrm{S}^2} d^4x \sqrt{g} \, \langle J_{11}(x) J_{22}(y) \rangle_{\mathbb{L},c}$$

$$\partial_m^2 \log\langle \mathbb{L} \rangle \,|_{m=0} = -16\pi^4 \int_1^\infty d\xi \int_{-1}^1 d\eta \, F_{JJ\mathbb{L}}(\xi, \eta)$$

Taking Care of Contact Terms

- Important subtlety in integrated correlators (w/ or w/o defect)

$$K(x)\bar{K}(y) \sim \alpha_{K\bar{K}M} M(x)\delta^4(x-y) \quad \langle K(x)\bar{K}(y)\rangle_{\mathbb{L}} \sim \alpha_{K\bar{K}M}\langle M(x)\rangle_{\mathbb{L}}\delta^4(x-y)$$

Stress tensor multiplet primary

- Scheme dependence of regularization and renormalization

- Equivalently, in terms of generating functional $\log Z[\text{sources}]$, by redefinitions of the sources and additions of local counter-terms (e.g. seagull term from source redefinition)

- Here scheme choice (contact term) fixed by SUSY. i.e. SUSY Ward identities hold at coincide point

$$F_{K\bar{K}\mathbb{L}}(\xi,\eta) = \frac{1}{2}\Box\left[(\xi+\eta)F_{JJ\mathbb{L}}(\xi,\eta)\right] \longrightarrow \Box\frac{1}{\xi-\eta} = -8\pi^2\delta(x,y)$$

Bulk OPE limit $\xi,\eta\to 1$ dominated by exchanging $M(x)$

$$\alpha_{K\bar{K}M} \sim C_{JJM}$$

Application to Wilson line in N=4 SYM

$$\mathbb{W} = \mathrm{tr}_{\mathrm{fund}} \mathrm{P} \exp i \oint ds \left(A_\mu(x(s)) \dot{x}^\mu(s) + \Phi_6(x(s) \,|\, \dot{x}(s) \,| \right)$$

- N=4 SYM as N=2 SCFT with $G_F = SU(2)$
- N=2 current multiplet completed by N=4 SUSY to the stress tensor multiplet
- Exact results for $\mathbb{W}$ in SU(N) SYM at large N and finite coupling $\tau = \dfrac{8\pi^2}{g^2} + \dfrac{i\theta}{2\pi}$
- Non-perturbative integral constraint on stress-tensor two-point function with WL

Victor's talk

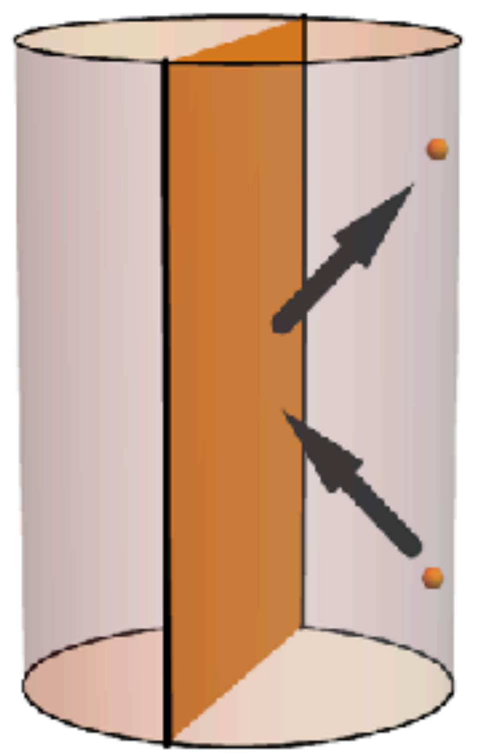
Application to Wilson line in N=4 SYM

Simple demonstration

Witten diagram at
Leading large N ’t Hooft limit

[Barrat-Gimenez-Grau-Liendo 21,
Gimenez-Grau 23]

$$F_{JJ\mathbb{W}}(\xi,\eta) = \frac{\sqrt{\lambda}}{32\pi^4}\frac{\log(\xi+\sqrt{\xi^2-1}) - \xi\sqrt{\xi^2-1}}{(\xi-\eta)(\xi^2-1)^{3/2}} + O(1,\lambda^{3/2}/N^2)$$



$$-16\pi^4\int_1^\infty d\xi\int_{-1}^{1} d\eta$$

Also assessible from the
quasi-topological sector

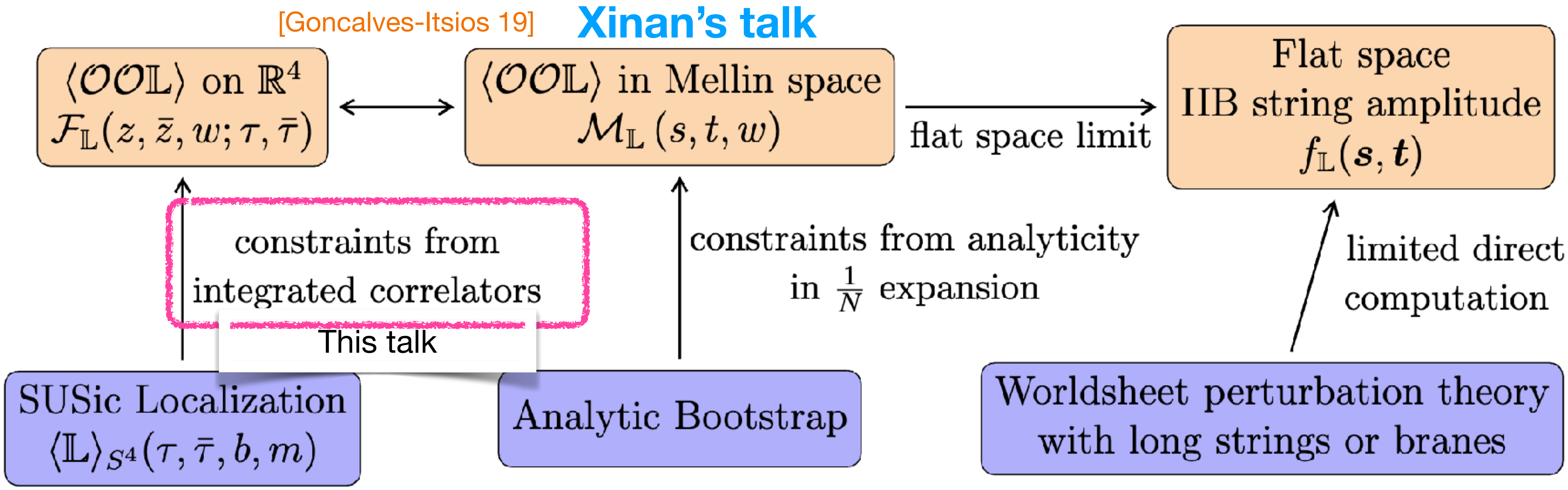
Integrated correlator at
Leading large N ’t Hooft limit

[Pufu-Roderiguez-YW 23]

$$\partial_m^2 \log\langle\mathbb{L}\rangle\,|_{m=0} = \sqrt{\lambda} + O(1,\lambda^{3/2}/N^2)$$

Victor’s talk

Roadmap for Solving N=4 Wilson Loops/Long Strings in AdS



Nikolay's talk

Explore complementary approaches: integrability (planar and nonplanar) and numerical bootstrap (finite N and less SUSY)

Shai and Ross's talks

Marco's talk

Future Directions

- Integral constraints for N=2 superconformal lines from **squashing**
- Integral constraints on correlators with **other superconformal defects** Take Advantage of Hyperbolic Space!
  Tomoki's talk
- General program to learn about worldvolume effective actions of **branes in string/M theory** coupled to bulk supergraviton (complementary to e.g. local interactions on the brane **Maria, Pietro, Tobias's talks**
- **Modularity properties** of the integrated correlators and relations between them in N=4 SYM **Daniele, Victor's talks**
- **New localization procedure** and more integrated correlators **Joe's talk**
- **Multiple impurities** (e.g. pair of BPS anti-BPS defects, open-string tachyon condensation in string theory) as well as junctions between or bent impurities (dual to junctions between strings/branes in string theory) **Nikolay's talk**
- General conformal impurities **without SUSY**: New Ideas?

Thank you!