(High Energy) scattering of strings in AdS

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Based on work with L. F. Alday and T. Hansen

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Outline

- Introduction
- Mathematical structure of scattering amplitudes
- The (AdS) Virasoro-Shapiro amplitude
- The High Energy limit of string scattering in AdS
- Summary & Conclusions

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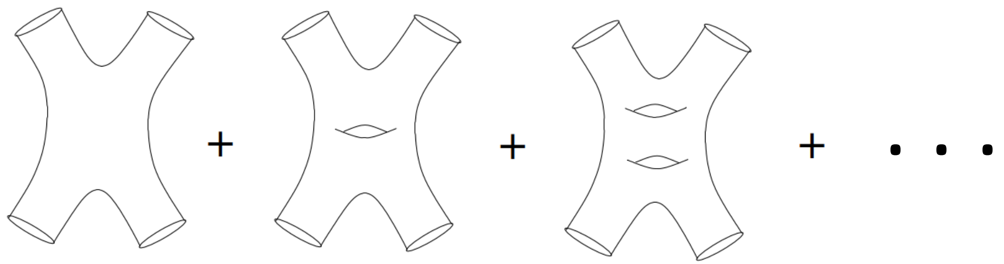
Introduction

Introduction



- Scattering amplitudes encode the differential probability for a certain process to happen. This predictive power makes them an essential object in Particle Physics, Mathematics, and String Theory.

- String scattering in flat space
  → perturbative String Theory
  → worldsheet **methods**

$$A^{(n)}(\Lambda_i, p_i) = \sum_{\text{topologies}} g_s^{-\chi} \frac{1}{\text{Vol}} \int DX Dg \; e^{-S_{\text{Poly}}} \prod_{i=1}^{n} V_{\Lambda_i}(p_i)$$

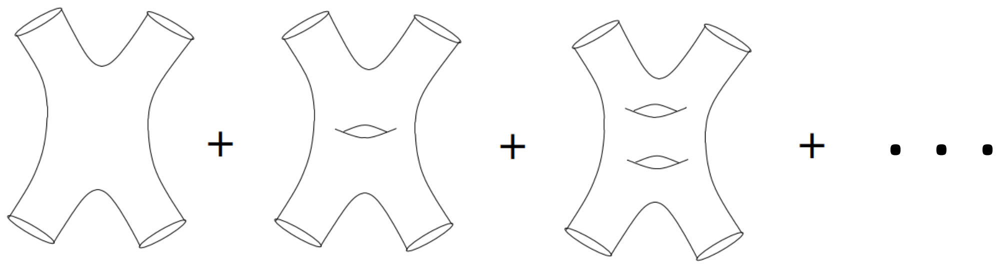
- When considering scattering processes, we sum over all possible configurations of WSs.

- Sum over Riemann surfaces of i**ncreasing genus** with insertions of vertex operators for the initial/final states (different topologies).

- In some regimes, the sum is dominated by a **saddle point**.

Parameters:
g_s (string coupling constant)
α' (size of the string)

Introduction



- What about curved spacetimes?
  → difficulties with standard formulations
  → **perturbative genus expansion** but no direct worldsheet approach, even at tree-level

- AdS/CFT: tool to compute string scattering amplitudes on AdS from CFT correlators of the dual boundary theory.

- Strategy: combine worldsheet intuition with other powerful tools.

This program: how to compute string amplitudes on AdS

→ Analyze and include curvature corrections systematically and efficiently.
→ Exploit and emphasize the interplay between String Theory and **Number Theory.**

Main object of study of this program

- Scattering of four graviton states at tree level.

- **Flat space** : Virasoro Shapiro amplitude

Prefactor: polarisation vectors

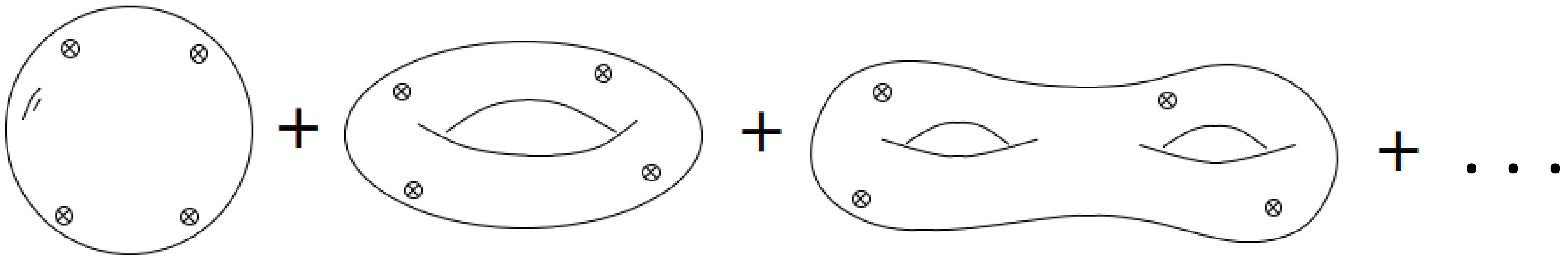
$$A_4(\varepsilon_i, p_i) = K(\varepsilon_i, p_i) \int d^2 z |z|^{-2S-2} |1-z|^{-2T-2}$$

The integrand is a **single-valued** function of z.

- $AdS_5 \times S^5$: correlator of 4 stress-tensor multiplets, to leading non-trivial order in a *1/c* expansion.

- Right language: Mellin space.

- Then, Borel transform: AdS analog of the Virasoro Shapiro amplitude.



Key Takeaway

In flat space, we can use the **worldsheet theory** to compute string amplitudes. For curved backgrounds, we need additional tools.

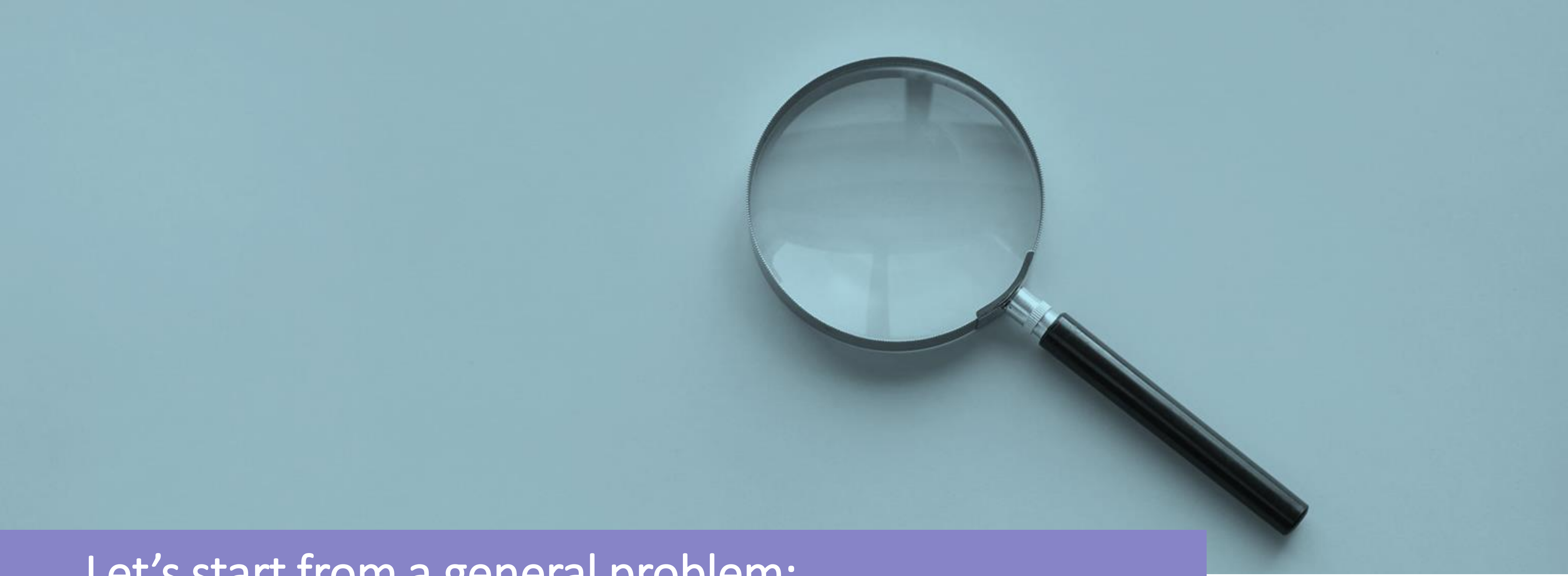
Relate the AdS Virasoro Shapiro amplitude to a **worldsheet action**.

High Energy limit: further step in this program.

Goal of this project

[2312.02261]

2 Mathematical structure of scattering amplitudes



Let’s start from a general problem:

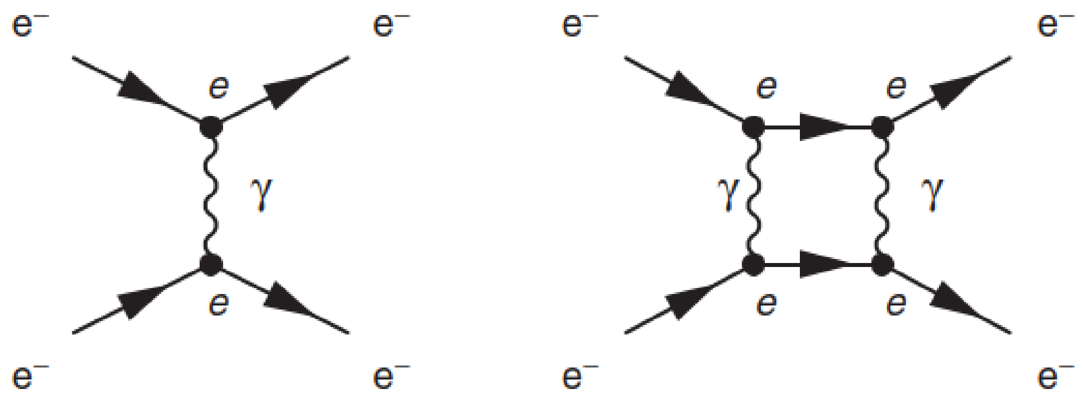
Reveal and understand the hidden **mathematical structure of scattering amplitudes** in Field Theory/String Theory.

Scattering amplitudes in QFT

Perturbation theory:

- sum over Feynman diagrams
- **loop** integrals
- **complicated functions with branch cuts**

(intermediate virtual particles going on-shell)



Strategy: study loop integrals from a purely mathematical and algebraic point of view.

Special numbers and functions in loop computations

$$B(p^2) = e^{\gamma_E \epsilon} \int \frac{d^D k}{i\pi^{D/2}} \frac{1}{k^2\,(k+p)^2}\,,$$

$$T(p_1^2, p_2^2, p_3^2) = e^{\gamma_E \epsilon} \int \frac{d^D k}{i\pi^{D/2}} \frac{1}{k^2\,(k+p_1)^2\,(k+p_1+p_2)^2}$$

$$B(p^2) = \frac{1}{\epsilon} + 2 - \log(-p^2) + \epsilon\left[\frac{1}{2}\log^2(-p^2) - 2\log(-p^2) - \frac{1}{2}\zeta_2 + 4\right] + \mathcal{O}(\epsilon^2)$$

$$T(p_1^2, p_2^2, p_3^2) = \frac{2}{\sqrt{\lambda}}\left[\mathrm{Li}_2(z) - \mathrm{Li}_2(\bar{z}) - \log(z\bar{z})\,\log\frac{1-z}{1-\bar{z}}\right] + \mathcal{O}(\epsilon)\,,$$

Rational functions are insufficient to write down the answer!

Notice the appearance of **zeta values** (Riemann ζ function at integer values):

$$\zeta_n = \sum_{k=1}^{\infty} \frac{1}{k^n}\,, \quad n > 1$$

and the (powers) of logarithms as well as their generalisations (polylogs):

$$\log z = \int_1^z \frac{dt}{t}$$

Special numbers and functions in loop computations

Can arbitrarily complicated functions appear in the Feynman integrals computation?

Feynman integrals evaluate to a restricted set of numbers and functions called periods, such as zeta values and polylogs!

Let's focus on a specific class of special functions, the **(single-valued) polylogs.**

- SVMPLS are of great interest in pure mathematics.
- They appear in loop computations: Feynman integrals with massless propagators and 3 off-shell external legs, conformal 4 pt functions in 4d.
- They describe multi-Regge limit of scattering amplitudes in planar N=4 SYM, and much more.

Polylogarithms

Classical polylogs: $Li_m(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^m}$. Converges on the unit disk $|z| < 1$.

Can be continued to the cut plane $\mathbb{C} \setminus [1, \infty)$ by an iterated integral representation:

$$Li_m(z) = \int_0^z dz' \, \frac{Li_{m-1}(z')}{z'} \qquad Li_m(1) = \zeta_m \, , m > 1$$

Can define more general classes of polylogs by changing the kernel ⇒ harmonic polylogs

- $\partial_z H_{x_0 w}(z) = \frac{H_w(z)}{z}$
- $\partial_z H_{x_1 w}(z) = \frac{H_w(z)}{1-z}$
- $H_e(z) = 1$
- $H_{x_0^n}(z) = \frac{log^n z}{n!}$
- $\lim_{z \to 0} H_{w \neq x_0^n}(z) = 0$

w = word
$\{x_0, x_1\}$ = alphabet
WEIGHT $|w|$ = lenght of the word

HPLs: analytic functions of a single complex variable, with branch points (**multi-valued** functions on the complex plane).

Single-valued polylogs

$\log|z|^2 = \log z + \log \bar{z}$

We can build weight-preserving linear combinations of $H_{w_1}(z)$ and $H_{w_2}(\bar{z})$ such that all the discontinuities cancel and they are single-valued in the $(z, \bar{z})$ plane.

Polylogs are examples of periods. SVPLs are their images under the *sv* projection [F.Brown].

- $\frac{\partial}{\partial z}\mathcal{L}_{0w}(z) = \frac{1}{z}\mathcal{L}_w(z)$
- $\frac{\partial}{\partial z}\mathcal{L}_{1w}(z) = \frac{1}{z-1}\ \mathcal{L}_w(z)$
- $\mathcal{L}_{0^n}(z) = \frac{log^n(z\bar{z})}{p!}$
- $\lim_{z\to 0}\ \mathcal{L}_{w\neq x_0^n}(z) = 0$

[Brown]: there exists a unique family of solutions that's single-valued in the complex plane.

At any given weight, there is a finite-dimensional vector space of available functions.

Single-valued multiple zetas

- The traditional polylog of a single variable can be generalized to the multiple version:

$$\mathrm{Li}_{n_1,\ldots,n_r}(z_1,\ldots,z_r) := \sum_{0<k_1<\cdots<k_r} \frac{z_1^{k_1}\ldots z_r^{k_r}}{k_1^{n_1}\ldots k_r^{n_r}}$$

- Multiple zeta values: real numbers defined by the absolutely convergent nested series

$$\zeta(n_1,\ldots,n_r) = \sum_{0<k_1<\cdots<k_r} \frac{1}{k_1^{n_1}\ldots k_r^{n_r}}$$

- Single-valued multiple zeta values = single-valued projection of MZVs = SVMPLs at unity.

Scattering amplitudes in String Theory

- Scattering amplitudes in open/closed string theory: correlation functions of vertex operators inserted at/in the boundary/bulk of a Riemann surface (worldsheet).

$$A_4(\varepsilon_i, p_i) = K(\varepsilon_i, p_i) \int d^2 z |z|^{-2S-2} |1-z|^{-2T-2}$$

- The nature of the underlying worldsheet describing the string interaction is fundamental!

- We want to motivate why the dependence on α' (inverse string tension) encodes a rich analytic structure of the amplitude.

Single-valuedness & String Theory

- Building blocks of **closed string theory** amplitudes at genus 0 (tree-level):

$$M_{N+3}(\mathbf{s},\mathbf{n},\tilde{\mathbf{n}}) = \left(\frac{i}{2\pi}\right)^N \int_{\mathbb{C}^N} \prod_{0<i<j<N+1} |z_i - z_j|^{2s_{ij}} (z_i - z_j)^{n_{ij}} (\bar{z}_i - \bar{z}_j)^{\tilde{n}_{ij}} \prod_{i=1}^{N} dz_i d\bar{z}_i$$

- Functions of the complex variables s_{ij}.

  s : collection of Mandelstam kinematic invariants: $s_{ij} = \alpha' p_i \cdot p_j$

  $z_0 = 0\,, \quad z_{N+1} = 1, \quad N \in \mathbb{N}, \quad n_{ij}, \tilde{n}_{ij} \in \mathbb{Z}$

$N = 1,\ n_{12} = \tilde{n}_{12} = -1$: Virasoro-Shapiro amplitude

- [Vanhove, Zerbini]
  The global (any *s*) and local properties are related to the **theory of SV periods**.

Single-valuedness & String Theory

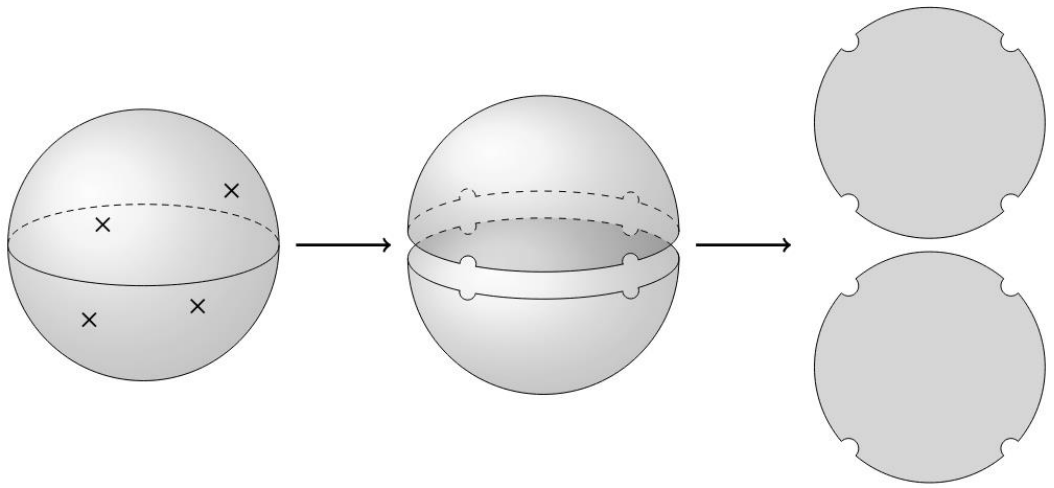
- The **Low Energy expansion** $(\alpha' \to 0)$ of closed string amplitudes contains only SVMZVs. → Let's see this for Virasoro-Shapiro.

- **Theory of integration of SVMPLs** to compute algorithmically the coefficients of the asymptotic expansion.

- Moreover, the Low Energy expansion can be obtained from the open string amplitude by replacing MZVs by their SV image.

- **Tree-level** open and closed strings are related by the **KLT** relations.

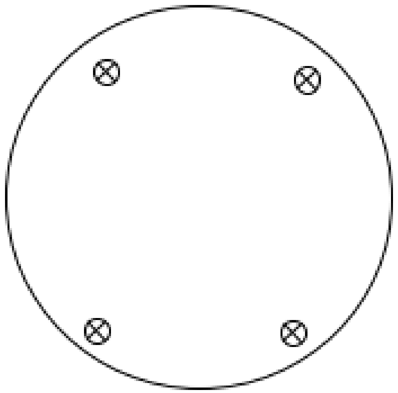
- Relation between gauge and gravity amplitudes.



Picture from Braune, Broedel.

Here: Virasoro-Shapiro amplitude

$$A^{(0)}(S,T) = -\frac{\Gamma(-S)\Gamma(-T)\Gamma(-U)}{\Gamma(S+1)\Gamma(T+1)\Gamma(U+1)}$$



- Crossing symmetry in the 3 Mandelstam variables $S+T+U=0$
- Fix T and vary S.
  Poles at mass of the tachyon + higher states of the closed string

⇒ STRING AMPLITUDE AS AN INFINITE NUMBER OF (s-channel) TREE-LEVEL QFT DIAGs

- Regge behaviour (large $|S|$)
- Low/High Energy $S = -\frac{\alpha'}{4}(p_1+p_2)^2\,, \qquad T = -\frac{\alpha'}{4}(p_1+p_3)^2\,, \qquad U = -\frac{\alpha'}{4}(p_1+p_4)^2$

Virasoro-Shapiro amplitude and single-valued periods

- Low Energy expansion of VS

$$A^{(0)}(S,T) = \frac{1}{STU} + 2\sum_{a,b=0}^{\infty} \sigma_2^a \sigma_3^b \alpha^{(0)}_{a,b} \qquad \sigma_2 = \frac{1}{2}(S^2+T^2+U^2)\ ,\sigma_3 = STU$$

SUGRA + TOWER OF STRINGY CORRECTIONS

- Only **odd ζ values** appear!
  The Wilson coefficients live in the ring of **SVMZVs.**

$$A^{(0)}(S,T) = \frac{\exp\left(\sum_{n=1}^{\infty} \frac{\zeta^{\text{sv}}(2n+1)(S^{2n+1}+T^{2n+1}+U^{2n+1})}{2n+1}\right)}{STU}$$

- This reflects the single-valued nature of the integral representation.

$$A^{(0)}(S,T) = \frac{1}{U^2}\int d^2z|z|^{-2S-2}|1-z|^{-2T-2}$$

Key Takeaways

- The infinite number of vibration modes in string spectra introduces transcendental numbers already at tree-level!
- The Low Energy expansion of closed string amplitudes contains only SVMZVs.

Let's use what we learnt to compute **strings amplitudes on curved backgrounds**, where we lack a worldsheet technology.

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The AdS Virasoro-Shapiro amplitude

The AdS Virasoro-Shapiro amplitude

Scattering of massless strings: 4 **gravitons** on $\boldsymbol{AdS_5 \times S_5}$ in Type IIB superstring theory

AdS/CFT

Correlator of four **stress-tensor** multiplets in Mellin space, to leading order in inverse powers of the central charge

Borel

$$A(S,T) = A^{(0)}(S,T) + \frac{\alpha'}{R^2} A^{(1)}(S,T) + \dots$$

VS in flat space

Curvature corrections

AdS amplitude as a curvature expansion around flat space

The AdS Virasoro-Shapiro amplitude

We reviewed the Low Energy expansion of the flat space VS.

What about the higher corrections $A^{(k)}(S,T)$?

- Each of them admits a **Low Energy expansion**: assume the unknown coefficients to be single-valued zetas as in flat space!
- **Intuition from the worldsheet**: $A^{(k)}(S,T)$ from WS integrals similar to the one in flat space.

$$\int d^2z |z|^{-2S-2} |1-z|^{-2T-2} G(z,\bar{z})$$

- **Structure of poles** (from the expansion of the AdS propagator around flat-space and dispersive sum-rules). [Alday, Hansen, Silva]

The AdS Virasoro-Shapiro amplitude

What is the relevant space of functions? Linear combination of single-valued functions such that the Low Energy expansion contains only SVMZVs.

The k-th order answer takes the form of a genus 0 WS integral involving **weight 3k** SVMPLs. [Alday, Hansen]

$$A(S,T) = \int d^2z |z|^{-2S}|1-z|^{-2T} W_0(z,\bar{z}) \left(1 + \frac{S^2}{R^2} W_3(z,\bar{z}) + \frac{S^4}{R^4} W_6(z,\bar{z}) + \ldots\right)$$

$$W_0(z,\bar{z}) = \frac{1}{2\pi U^2 |z|^2 |1-z|^2}$$

$$\alpha' = 1$$

Tools:
-crossing symmetry
-SUGRA limit
-structure of poles (dispersive sum rules)
-CFT data (from integrability)

NOTE: This is NOT the result of a direct worldsheet computation!

Key Takeaways

- Single valuedness plays a fundamental role in the construction of AdS scattering amplitudes, as in flat space!
- Can extract the CFT-data and compare with integrability results for planar $N = 4$ SYM at strong coupling!

Next step towards the worldsheet theory: investigate the High Energy regime!

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The High Energy limit of string scattering in AdS

Motivations

- Difficult to investigate String Theory in general. We can explore its mathematical structure ⇒ formulate questions and carry out computations that probe String Theory in different regimes.
- QFT: the short-distance behavior of the theory plays a crucial role (OPE, RG flow…)
  What about String Theory?
- Here: after the Low Energy analysis ($\alpha' \to 0$, field theory limit) of the AdS VS amplitude, the next step towards the WS theory is the High Energy limit ($\alpha' \to \infty$).

How to make connections to more direct WS computations?

High Energy = regime in which such a connection can be made, at least classically!

Flat space result [Gross & Mende]

- HE limit: $|S|, |T| \gg 1$ and S/T fixed

$$A^{(0)}(S,T) = -\frac{\Gamma(-S)\Gamma(-T)\Gamma(-U)}{\Gamma(S+1)\Gamma(T+1)\Gamma(U+1)}$$

- Use Stirling's formula to access this regime:

$$A^{(0)}(S,T)_{HE} \sim e^{-2S\log|S| - 2T\log|T| - 2U\log|U|}$$

→ soft exponential behavior!

Analogous to the universality of singularities of the OPE in field theory.

- The exponential behavior is universal: independent of the particular String Theory and the quantum numbers of the scattered particles.

- We can understand this also from the WS integral representation:

$$A(S,T) \sim \int d^2z |z|^{-2S} |1-z|^{-2T} W_0(z,\bar{z})$$

HE limit: saddle point approximation $z = \bar{z} = \frac{S}{S+T} = z_0$

What do we expect for AdS?

Given the "WS representation" for AdS, given that the transcendental functions $W_n(z,\bar{z})$ are polynomials in S, T, the location of the saddle is not modified in a $1/R$ expansion!

The AdS VS in the HE limit can be computed by evaluating the WS integral representation on the saddle point:

$$A_4^{AdS}(S,T)_{\mathrm{HE}} \sim e^{-2S\log|S|-2T\log|T|-2U\log|U|} W_0(z_0)\left(1+\frac{S^2}{R^2}W_3(z_0)+\frac{S^4}{R^4}W_6(z_0)+\cdots\right)$$

where we keep **the leading large energy** contribution at each order.

We are looking at a regime with large R, S and S^2/R^2 finite.

Flat space result [Gross & Mende]

Alternatively, we can understand HE from the point of view of spacetime.

$$A^{(0)}(S,T) \sim \int Dg\ DX \exp\left[-\frac{1}{4\pi}\int d\zeta_1 d\zeta_2 \sqrt{g} g^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu\right]\prod_i V_i(p_i)$$

WS coordinates: $(\zeta, \bar{\zeta})$
Punctures: $z_k \in \mathbb{R}$

$$V_i(p_i) \sim \int d^2 z_i \sqrt{g} e^{ip_i \cdot X(z_i)}$$

$$p_i^2 = 0\ ,\ p_1 \cdot p_2 = -2S\ ,\ p_1 \cdot p_3 = -2T\ ,\ p_1 \cdot p_4 = -2U$$

At HE, the path integral is dominated by a classical solution:

$$X^\mu(\zeta) = -i\sum_k p_k^\mu \log\left|1 - \frac{\zeta}{z_k}\right|$$

Plug the classical solution into the PI: correct HE result!

Key Takeaways

High Energy limit:

- WS representation → saddle point
- spacetime (path integral) → classical solutions

Let's carry this classical analysis for AdS!

Classic scattering problem in AdS_d

Embedding coordinates labeled by $M = (0;\mu) = (0, 1, \ldots, d)$

Constraint: $X^M X_M = -R^2$

Vertex operators: $V_i(P_i) \sim \int d^2 z_i \sqrt{g} e^{iP_i^M X_M(z_i)}$

$$\mathcal{L} = \frac{1}{2\pi}\partial X^M \bar{\partial} X_M + \Lambda(X^M X_M + R^2) - i\sum_k P_k^M X_M \delta^{(2)}(\zeta - z_k)$$

Expectation: the HE behavior of the amplitude is captured by classical solutions, now in AdS!

- Virasoro constraints: $\partial X^N \partial X_N = \bar{\partial} X^N \bar{\partial} X_N = 0$
- Equations of motion away from the punctures: $\partial\bar{\partial} X^M = \frac{\partial X^N \bar{\partial} X_N}{R^2} X^M$
- Boundary conditions: $X^M = -iP_k^M \log\left|1 - \frac{\zeta}{z_k}\right| + Q_k^M + \ldots$

Classic scattering problem in AdS_d

The scattering problem in flat space arises as a limit of the AdS problem.

We solve for X^0 using the constraint and take $R \to \infty$. The X^μ coordinates are constant in this limit and identified with the flat space coordinates.

$$X^0 = R + \frac{1}{R} X_1^0 + \cdots \,, \qquad P_k^0 = \frac{1}{R} p_{k,1}^0 + \cdots \,,$$

$$X^\mu = X_0^\mu + \frac{1}{R^2} X_1^\mu + \cdots \,, \qquad P_k^\mu = p_{k,0}^\mu + \frac{1}{R^2} p_{k,1}^\mu + \cdots$$

$$X_0^\mu = -i \sum_k p_{k,0}^\mu \log \left| 1 - \frac{\zeta}{z_k} \right| + q_0^\mu$$

→ Plug into VIRASORO CONSTRAINTS →

$$\frac{(z_1 - z_3)(z_2 - z_4)}{(z_1 - z_2)(z_3 - z_4)} = -\frac{T}{S}$$

Classic scattering problem in AdS_d

Flat space solution: single-valued as we move around each puncture on the worldsheet

$$X_0^\mu = -\frac{i}{2}\sum_k p_{k,0}^\mu \mathcal{L}_{z_k}(\zeta)$$

Higher orders?

-solve EOMs and Virasoro constraints in a ***1/R*** **expansion**

-write the solution in terms of **SVMPLs whose letters are the locations of the punctures**

$$\partial\bar{\partial}X^M = \frac{\partial X^N \bar{\partial} X_N}{R^2} X^M$$

Integrate $\partial\bar{\partial}X$ at each order with the rules:

$$\int d\zeta \frac{\mathcal{L}_w(\zeta)}{\zeta - z_i} \to \mathcal{L}_{z_i w}(\zeta)$$

$$\int d\bar{\zeta} \frac{\mathcal{L}_w(\zeta)}{\bar{\zeta} - z_i} \to \mathcal{L}_{w z_i}(\zeta) + \dots$$

Sum of terms of uniform weight $|\boldsymbol{w}| + \mathbf{1}$

Solution for the first correction

$$\partial\bar{\partial}X_1^\mu = \partial X_0 \cdot \bar{\partial} X_0 \, X_0^\mu = \frac{i}{8}\sum_{i,j,k}\frac{p_{i,0}\cdot p_{j,0}}{(\zeta - z_i)(\bar{\zeta} - z_j)} p_{k,0}^\mu \mathcal{L}_{z_k}(\zeta)$$

$$\int d\bar{\zeta}\,\frac{\mathcal{L}_{z_k}(\zeta)}{(\bar{\zeta} - z_j)} \to \mathcal{L}_{z_k z_j}(\zeta) + \mathcal{L}_{z_k}(z_j)\mathcal{L}_{z_j}(\zeta) - \mathcal{L}_{z_j}(z_k)\mathcal{L}_{z_k}(\zeta)$$

$$X_1^\mu = \frac{i}{8}\sum_{i,j,k=1}^{4} p_{i,0}\cdot p_{j,0}\; p_{k,0}^\mu \left(\mathcal{L}_{z_i z_k z_j}(\zeta) + \mathcal{L}_{z_k}(z_j)\mathcal{L}_{z_i z_j}(\zeta) - \mathcal{L}_{z_j}(z_k)\mathcal{L}_{z_i z_k}(\zeta)\right)$$

Classic scattering problem in AdS_d

$$X^\mu = \mathcal{L}_1(\zeta) + \frac{1}{R^2}\mathcal{L}_3(\zeta) + \frac{1}{R^4}\mathcal{L}_5(\zeta) + \dots$$

FINAL SOLUTION

$$X^0 = \sqrt{R^2 + X_\mu X^\mu} \Rightarrow X^0 = R\mathcal{L}_0(\zeta) + \frac{1}{R}\mathcal{L}_2(\zeta) + \frac{1}{R^3}\mathcal{L}_4(\zeta) + \dots$$

$\mathcal{L}_n(\zeta)$ are linear combinations of pure SVMPLs of weight n, with either ζ or z_i as their arguments and letters in the alphabet $\{z_1, z_2, z_3, z_4\}$.

Once the seed solution X_0^μ is given, the whole tower in $1/R$ is fixed by the EOMs and integration.

$$X_0^\mu \to X^\mu = X_0^\mu + \frac{1}{R^2}X_1^\mu + \dots$$

X_n^μ has weight $2n+1$.

Evaluate the action

Plugging our classical solution into the action:

$$A_{4,\text{bos}}^{AdS}(S,T)_{\text{HE}} \sim A_4^{\text{flat}}(S,T)_{\text{HE}} \times e^{\frac{S^2}{R^2}V_3(z_0)+\frac{S^3}{R^4}V_5(z_0)+\cdots}$$

$V_i(z_0)$ = combinations of transcendental functions of weight i

Our result correctly reproduces the HE limit (after an appropriate redefinition of the Mandelstam variables):

$$A_4^{AdS}(S,T)_{\text{HE}} \sim e^{-2S\log|S|-2T\log|T|-2U\log|U|}W_0(z_0)\left(1+\frac{S^2}{R^2}W_3(z_0)+\frac{S^4}{R^4}W_6(z_0)+\cdots\right)$$

$$W_3(z_0) = V_3(z_0)$$

$$\boxed{W_6(z_0) = \frac{1}{2}V_3(z_0)^2}$$

HIGHLY NON-TRIVIAL!!! Test of exponentiation at quadratic order!

Main result

$$A_4^{AdS}(S,T)_{HE} \sim A_4^{flat}(S,T)_{HE} \times e^{\frac{S^2}{R^2} W_3(z_0)}$$

- Curvature corrections in the HE limit exponentiate!
- The **full** High Energy limit of AdS VS to **all orders** in S^2/R^2 is determined by the subleading exponent.
- High Energy limit: regime where the amplitude can be computed to all orders in the curvature expansion.
- This result can be explicitly checked to order $1/R^4$ by comparison with AdS VS.

Non-universality

- Momenta in a $1/R$ expansion from the flat space momenta: $P_k^\mu = p_{k,0}^\mu + \frac{1}{R^2} p_{k,1}^\mu + \cdots$

- Can consider deformations of our solution consistent with EOM and Virasoro such that the flat space momenta are invariant (next slide).

- Can rescale flat space momenta by a constant:

$$p_0^\mu + \frac{1}{R^2} p_1^\mu + \frac{1}{R^4} p_2^\mu + \cdots \to \lambda p_0^\mu + \frac{\lambda^3}{R^2} p_1^\mu + \frac{\lambda^5}{R^4} p_2^\mu + \cdots$$

- Mandelstam variables: $S \to \lambda^2 S$, with $\lambda^2 = \left(1 + \frac{\alpha}{R^2} + \cdots\right)^2$

- Ratios of Mandelstam variables are invariant.

Non-universality

- Now, let's go back to the evaluation of the action on our classical solution.

$$A_{4,\text{bos}}^{AdS}(S,T)_{\text{HE}} \sim e^{-\mathcal{S}} = e^{SV_1(z_0)+\frac{S^2}{R^2}V_3(z_0)+\frac{S^3}{R^4}V_5(z_0)+\cdots}$$

- Rescale momenta (keep ratios invariant): $S \to S(1+\frac{SF_2(z_0)}{R^2}+\cdots)$

This comes from quantum corrections (such as contributions from fermionic fields...).

- The first correction enters the leading High Energy behavior.

$$SV_1(z_0) = -\mathcal{S}^{(0)}\,, \qquad S^2V_3(z_0) = -\mathcal{S}^{(1)} - 2SF_2(z_0)\mathcal{S}^{(0)}$$

- $F_2(z_0)$ is a subleading non-universal quantity: cannot be determined with our classical bosonic model, but can be fixed by comparing with AdS VS!

$$F_2(z_0) = \frac{1}{4}\left(-\mathcal{L}_{00}\left(z_0\right) + \frac{2}{z_0}\mathcal{L}_{01}\left(z_0\right) + \frac{z_0-1}{z_0}\mathcal{L}_{11}\left(z_0\right)\right)$$

Our HE result does not depend on the AdS dimension, but $F_2(z_0)$ can change for different theories.

Summary and conclusions

- Explore the mathematical structure of String Theory by probing it in different regimes.

- **Compute string amplitudes on AdS** from
  - *AdS/CFT*
  - *Number Theory*
  - *Integrability*
  - *Worldsheet intuition*

- **Single valuedness** to understand/construct scattering amplitudes in AdS (as in flat space).

- AdS Virasoro-Shapiro amplitude as a «worldsheet» integral.

Summary and conclusions

- Further step towards the worldsheet theory: High Energy limit.
- Curvature corrections exponentiate!
- The leading behavior at High Energy is captured by a bosonic model describing scattering of classical strings on AdS.
- Universality of our result: in our regime, only the first-order curvature corrections around flat space are important!

⇨ Strong constraints on curvature corrections at higher orders.

Thanks for your attention!