Hadrons and hadron interactions in QCD 2024 Kyoto, 06.11.2024

Surprises on the way to the QCD phase diagram from the lattice

Owe Philipsen

- I. Chiral phase transition, massless limit: 40 of common wisdom + resolution
- II. Chiral spin symmetry: emergent intermediate temperature regime
- III. Cold+dense, qualitatively: nuclear liquid gas transition + quarkyonic matter

Tool: QCD away from physical point, study limits + parameter dependence: constraints!

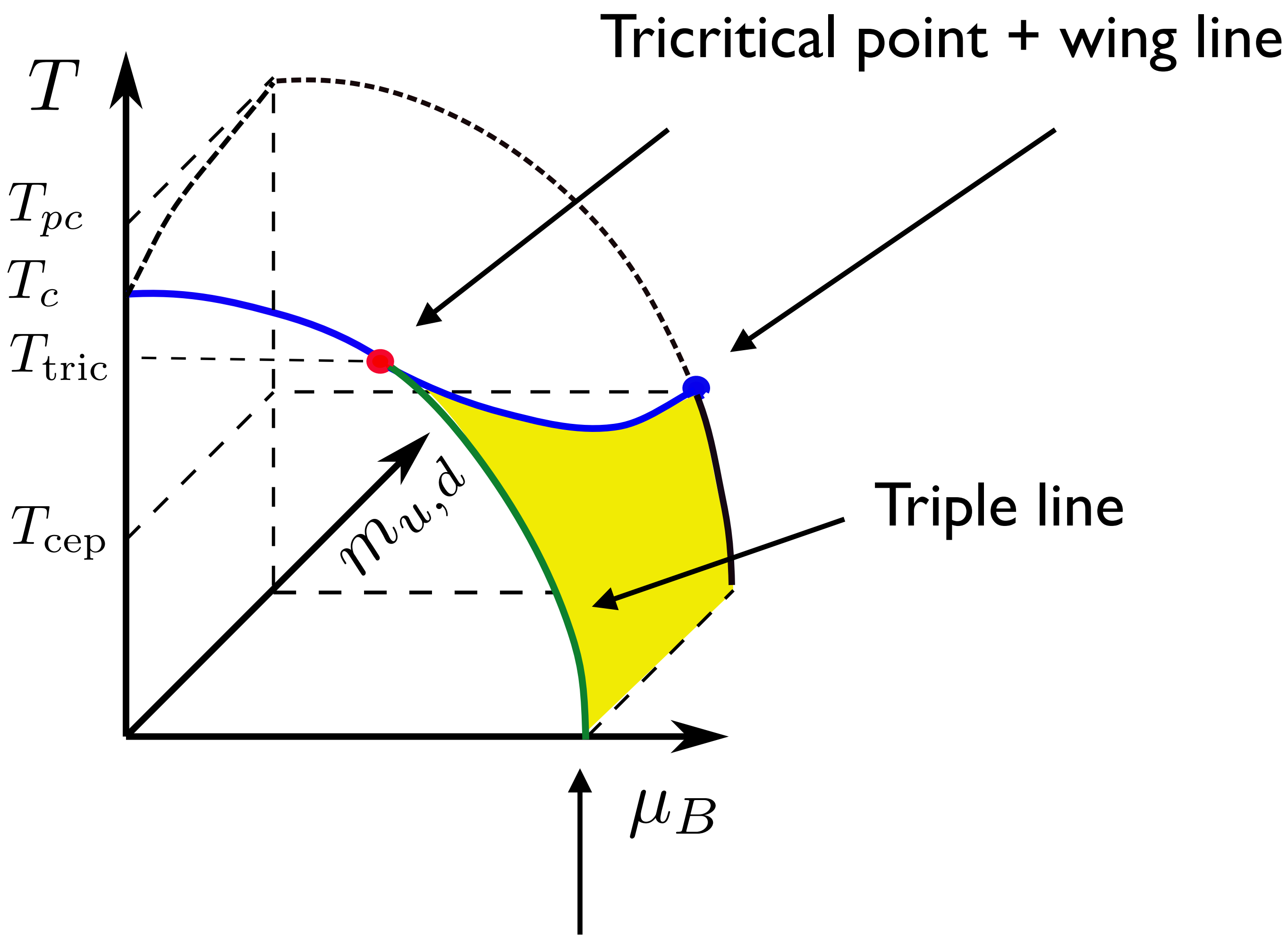
History: motivation for the critical endpoint

[Rajagopal 95, Halasz et al., PRD 98, Stephanov, Rajagopal, Shuryak PRL 98, Rajagopal, Wilczek 00, Hatta, Ikeda, PRD 03,…]

Breaking/restoration of exact chiral symmetry requires a (non-analytic) phase transition

$N_f = 2:$

Model predictions,
early lattice results

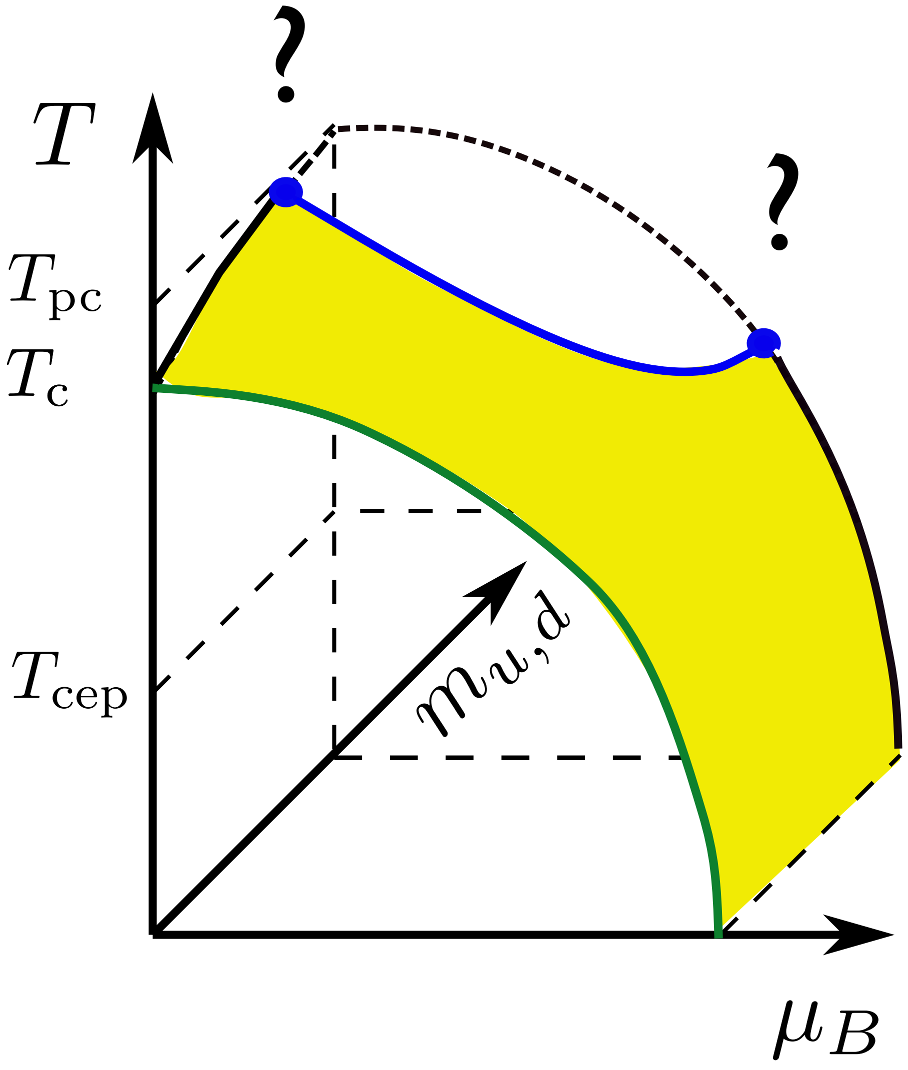


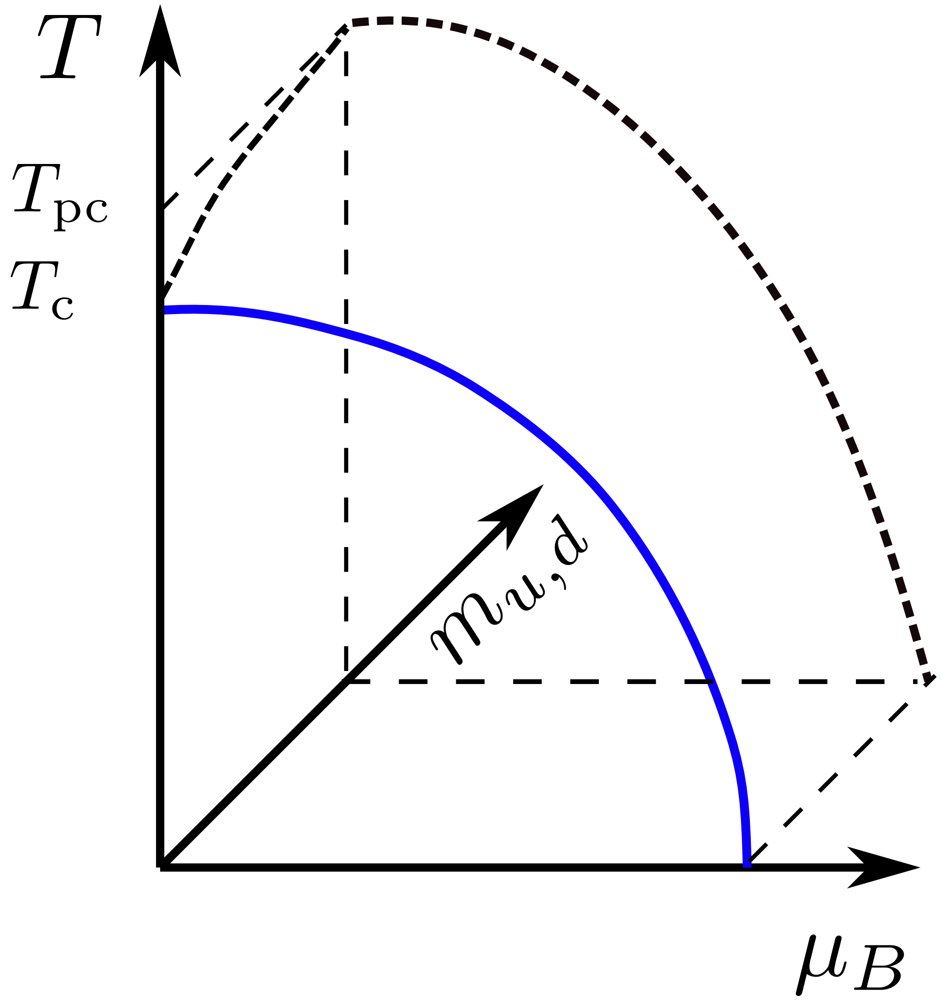
Tricritical point + wing line

Triple line

Model predictions, no full QCD information

Other (mostly ignored) possibilities



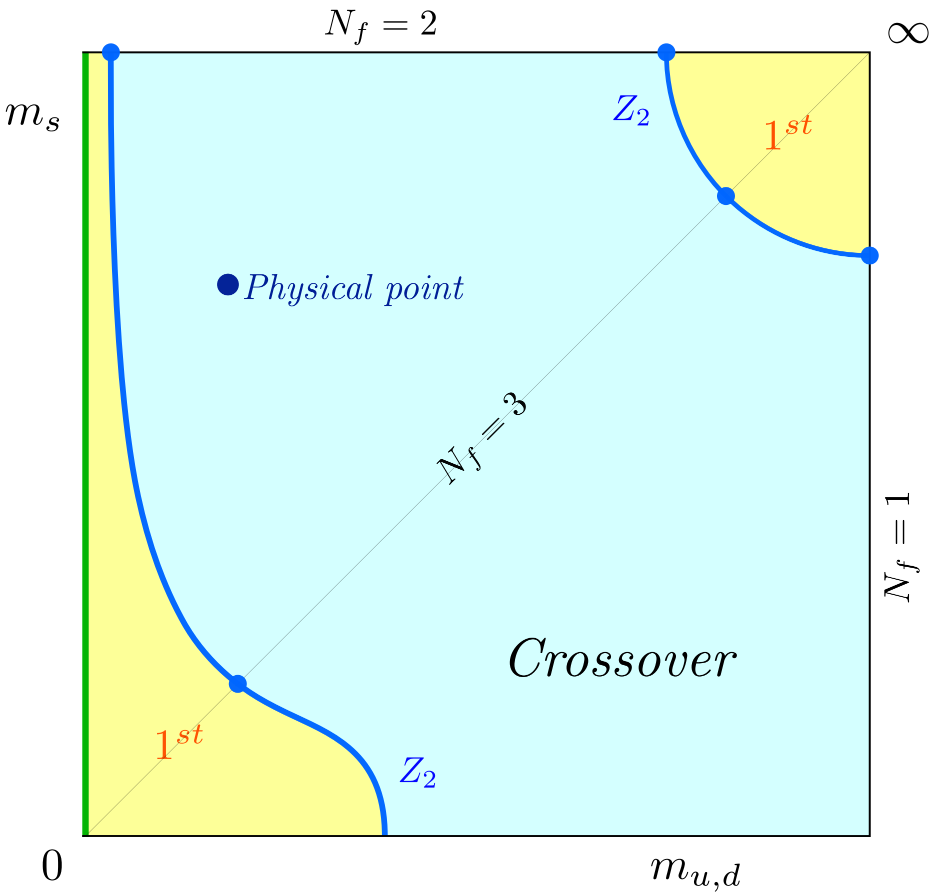


Knowledge of the chiral phase transition at $\mu_B = 0$ narrows down possibilities

I. Nature of the QCD thermal transition at a) zero density

$N_f = 2+1$

deconfinement p.t.:
breaking of global $Z(3)$ symmetry

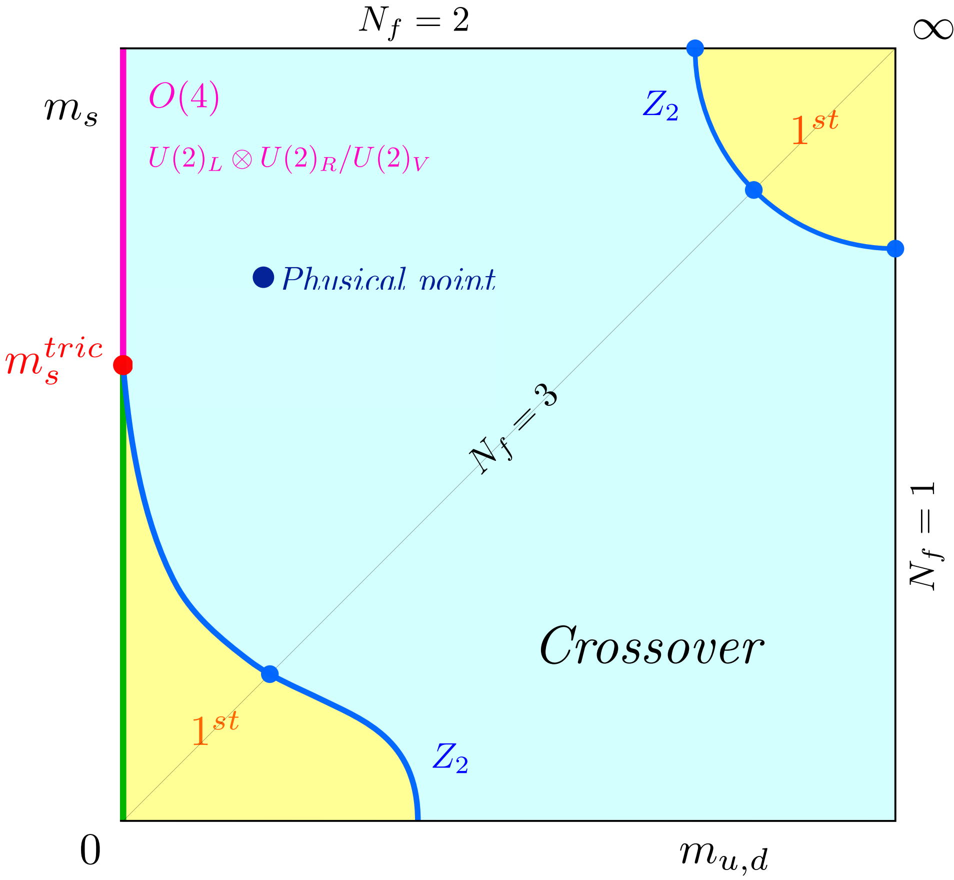


[Pisarski, Wilczek, PRD 84]:
(Linear sigma model in 3d)

$N_f = 2$ depends on $U(1)_A$

restored broken

$N_f \geq 3$ 1st order

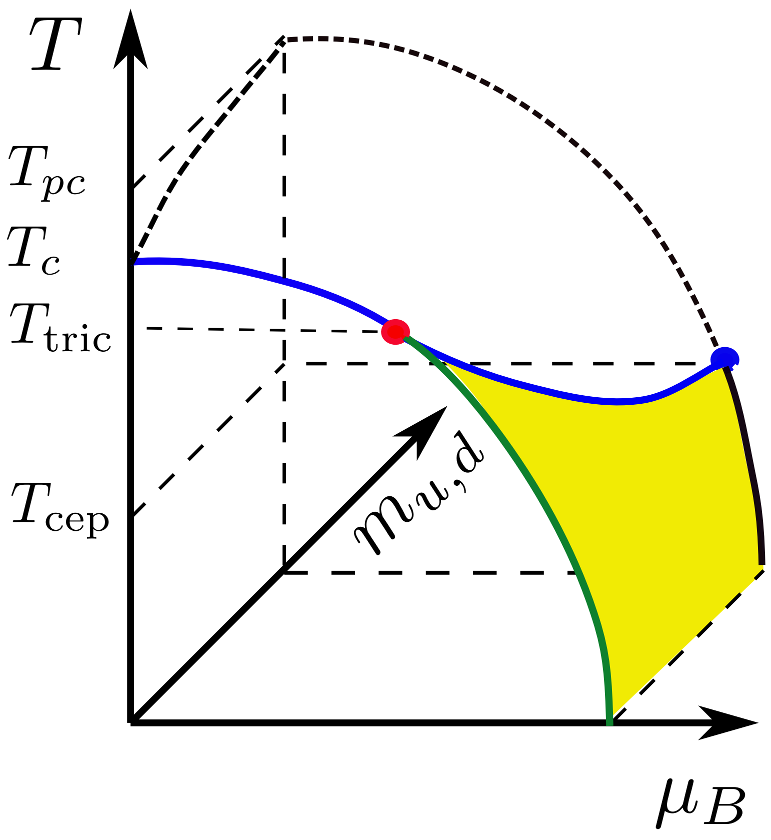


chiral p.t.
restoration of global symmetry in flavour space

$$SU(2)_L \times SU(2)_R \times \underset{\substack{\uparrow \\ \text{anomalous}}}{U(1)_A}$$

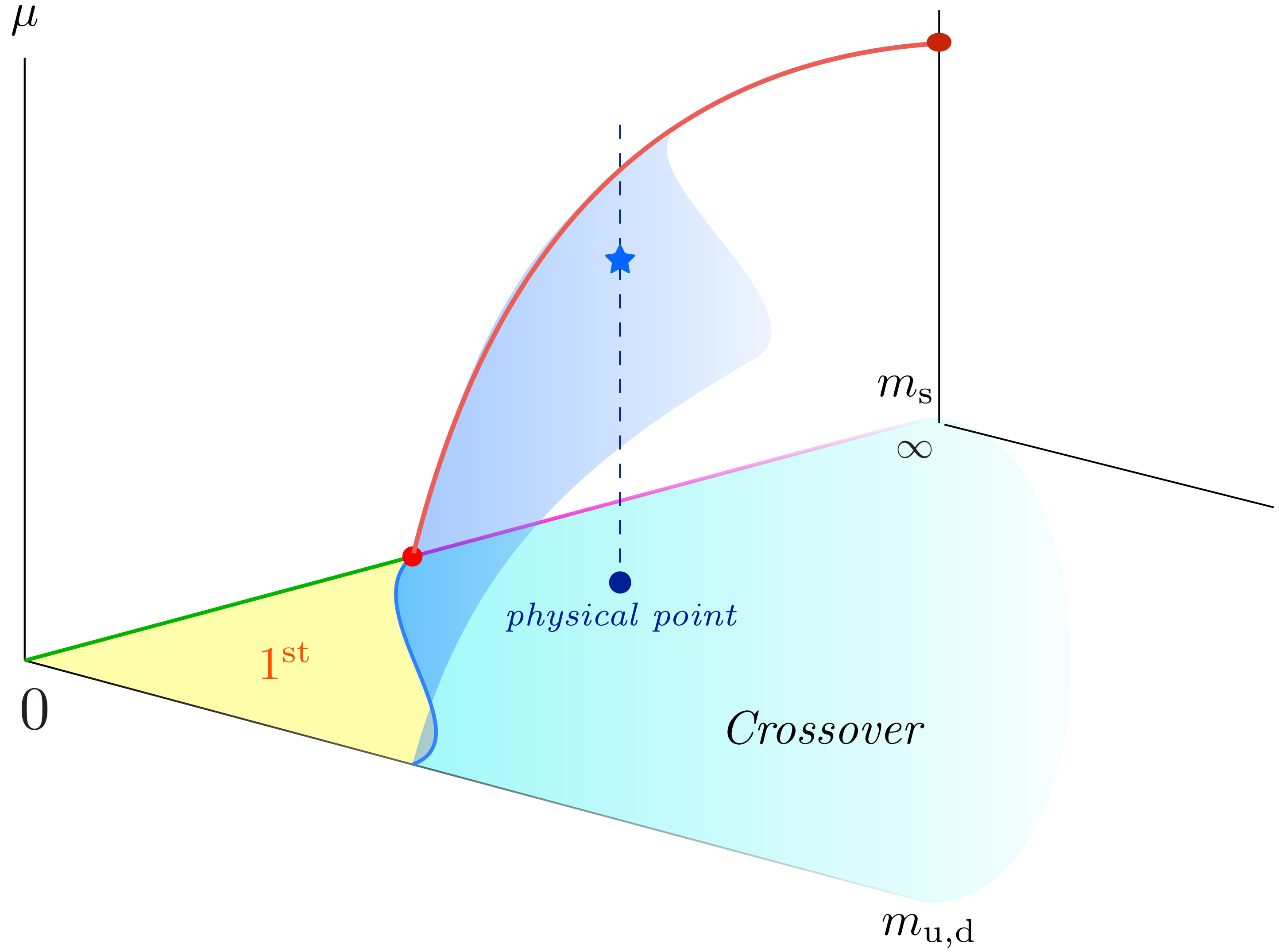
The Columbia plot with chemical potential

$N_f = 2$



$N_f = 2 + 1$

[edited from Sciarra, PhD thesis 2016]

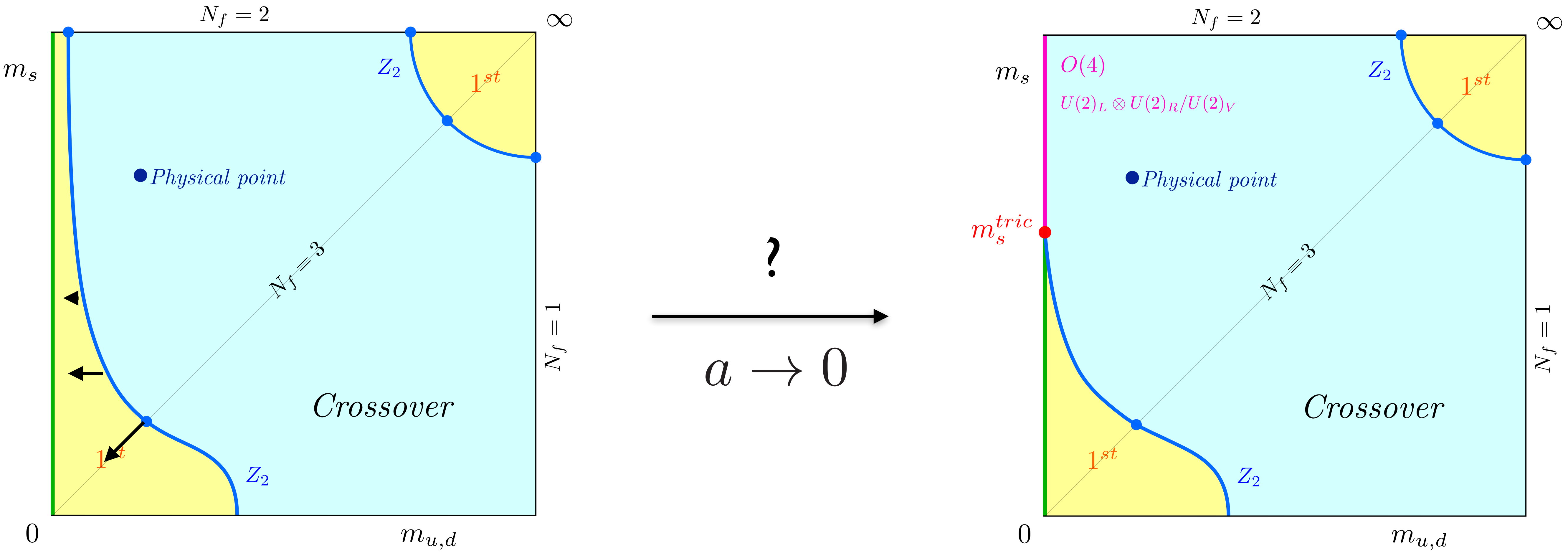


[Stephanov, Rajagopal, Shuryak PRL 98]: (models + early lattice results)

"As m_s is reduced from infinity, the tricritical point … moves to lower μ until it reaches the T-axis and can be identified with the tricritical point in the (T, m_s)-plane"

The nature of the QCD chiral transition at zero density

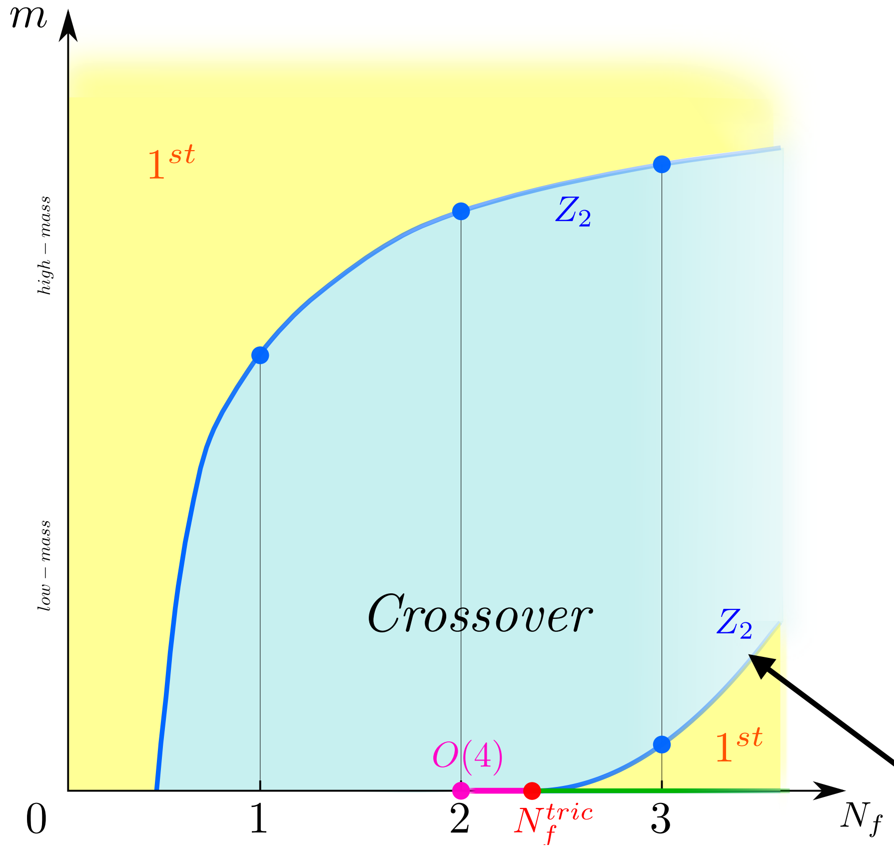
…is elusive, massless limit not simulable!

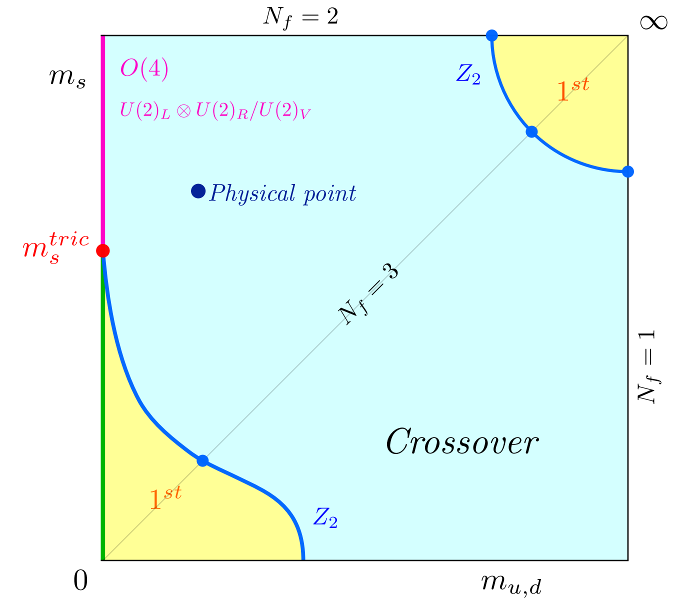


- Coarse lattices with unimproved actions: 1st order for $N_f = 2, 3$
- 1st order region shrinks rapidly as $a \to 0$, no 1st order for improved staggered actions
- Apparent contradictions between different lattice actions?

Details and reference list: [O.P., Symmetry 13, 2021]

Different view point: mass degenerate quarks





$$Z(N_\mathrm{f}, g, m) = \int \mathcal{D}A_\mu \, (\det M[A_\mu, m])^{N_\mathrm{f}} \, e^{-\mathcal{S}_\mathrm{YM}[A_\mu]}$$

$$N_\mathrm{f}^c(am) = N_\mathrm{f}^\mathrm{tric} + \mathcal{B}_1 \cdot (am)^{2/5} + \mathcal{O}\left((am)^{4/5}\right)$$

- Consider analytic continuation to continuous N_f
- Tricritical point guaranteed to exist if there is 1st order at any N_f
- Known exponents for critical line entering tric. point!
- Continuation to $a \neq 0:$ Z(2) surface ends in tricritical line

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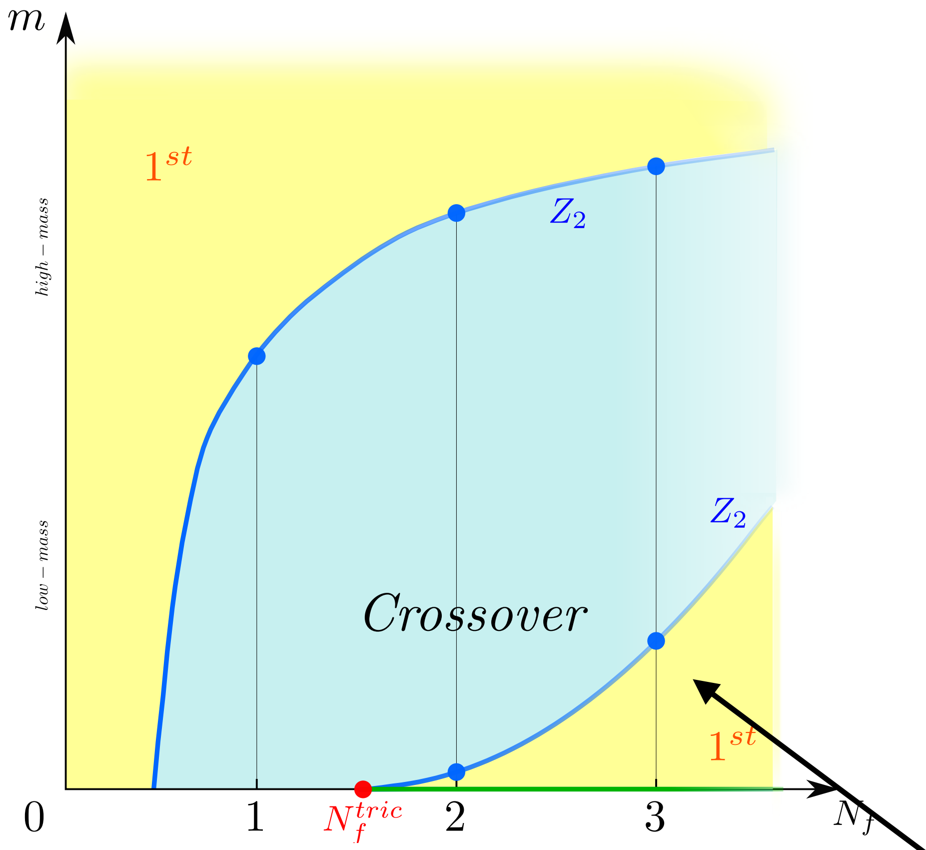
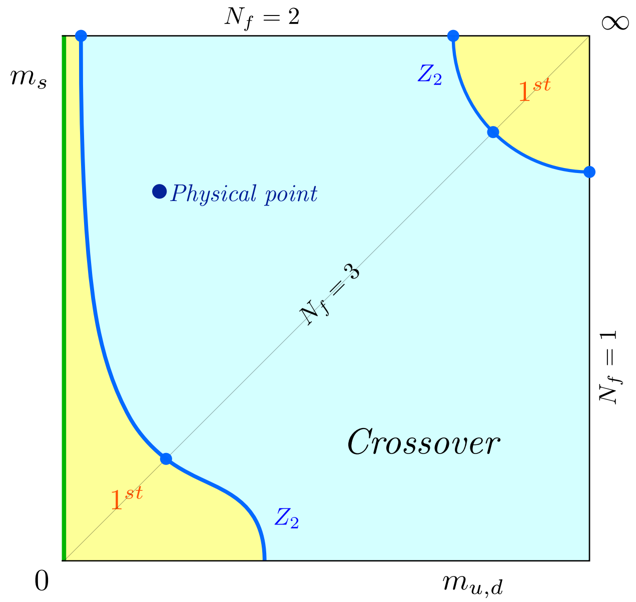
[Cuteri, O.P., Sciarra PRD 18]

Different view point: mass degenerate quarks





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[Cuteri, O.P., Sciarra PRD 18]

Methodology to determine order of transition

Finite size scaling of generalised cumulants

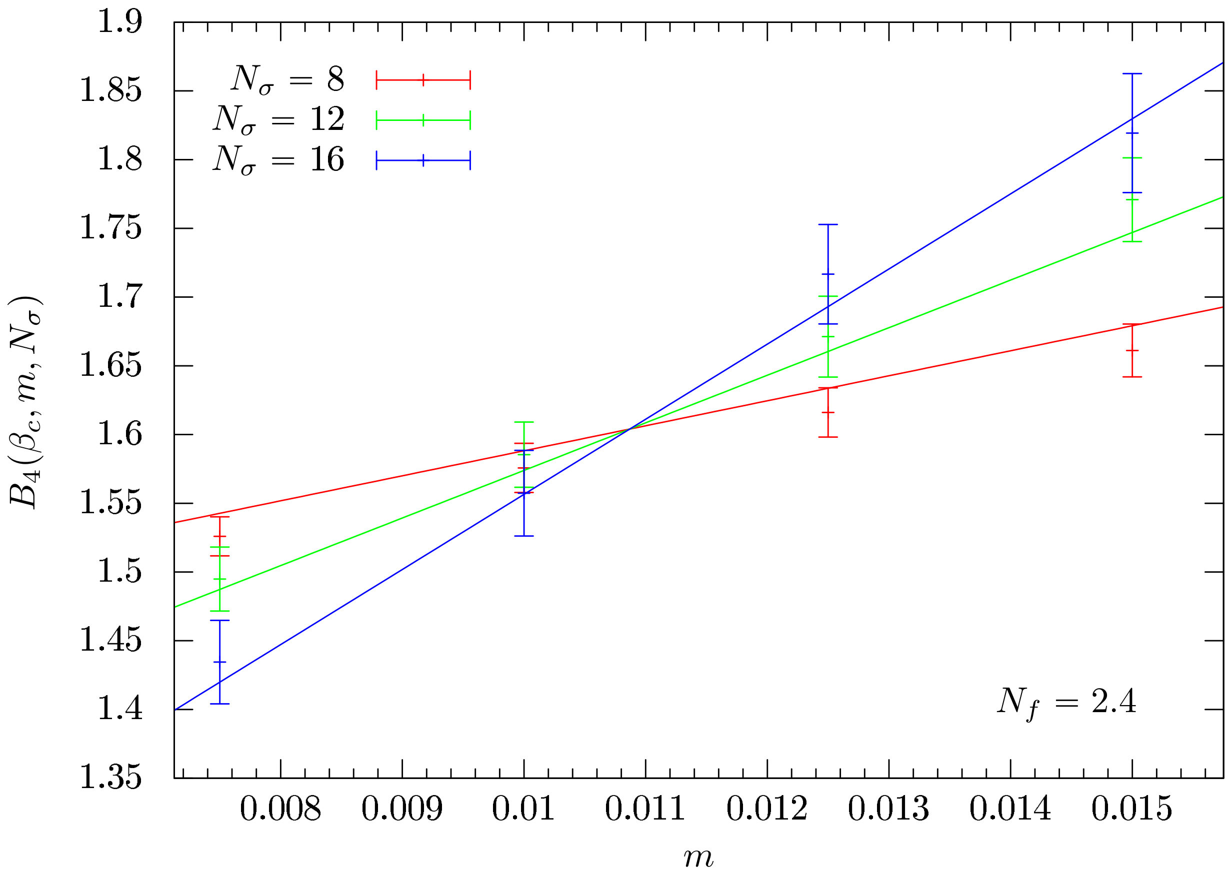
$$B_n = \frac{\langle(\bar\psi\psi - \langle\bar\psi\psi\rangle)^n\rangle}{\langle(\bar\psi\psi - \langle\bar\psi\psi\rangle)^2\rangle^{n/2}}$$

Standard staggered fermions, bare parameters: β, am, N_f, N_τ

(Pseudo-critical) phase boundary: $B_3 = 0$ 3d manifold

Second-order 3d Ising:

2d chiral critical surface separates 1st order from crossover



$$B_4(\beta_c, am, N_\sigma) \approx 1.604 + c\,(am - am_c)\,N_\sigma^{1/0.6301}$$

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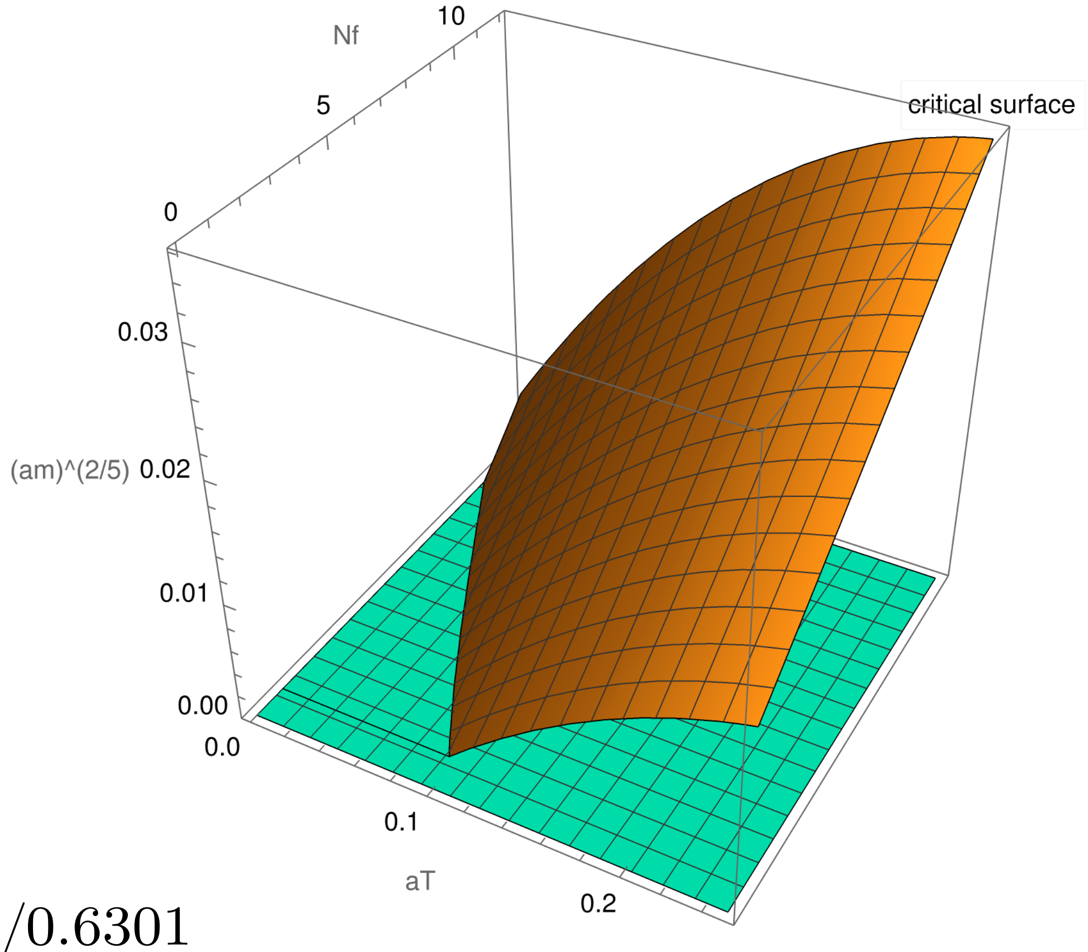

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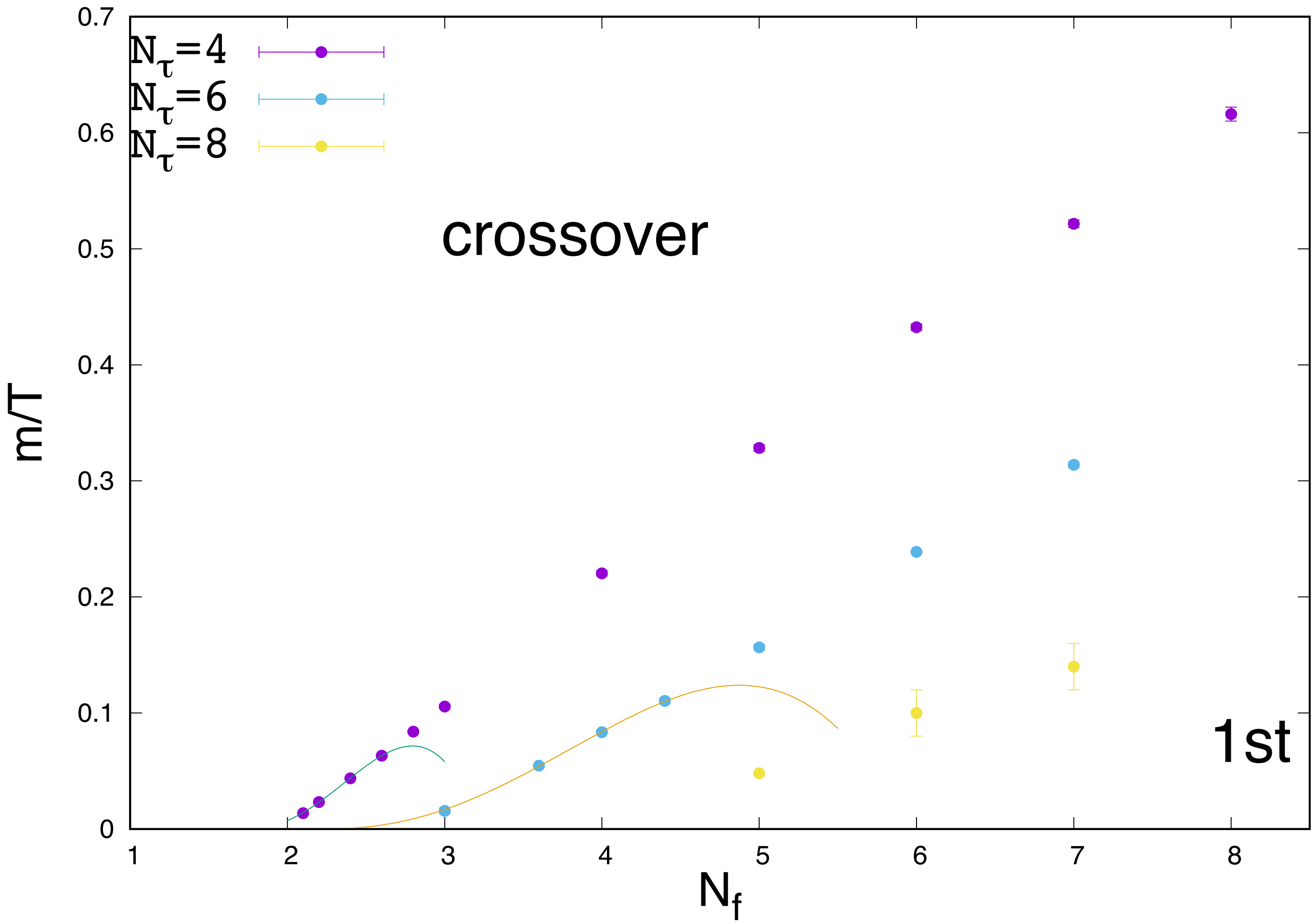
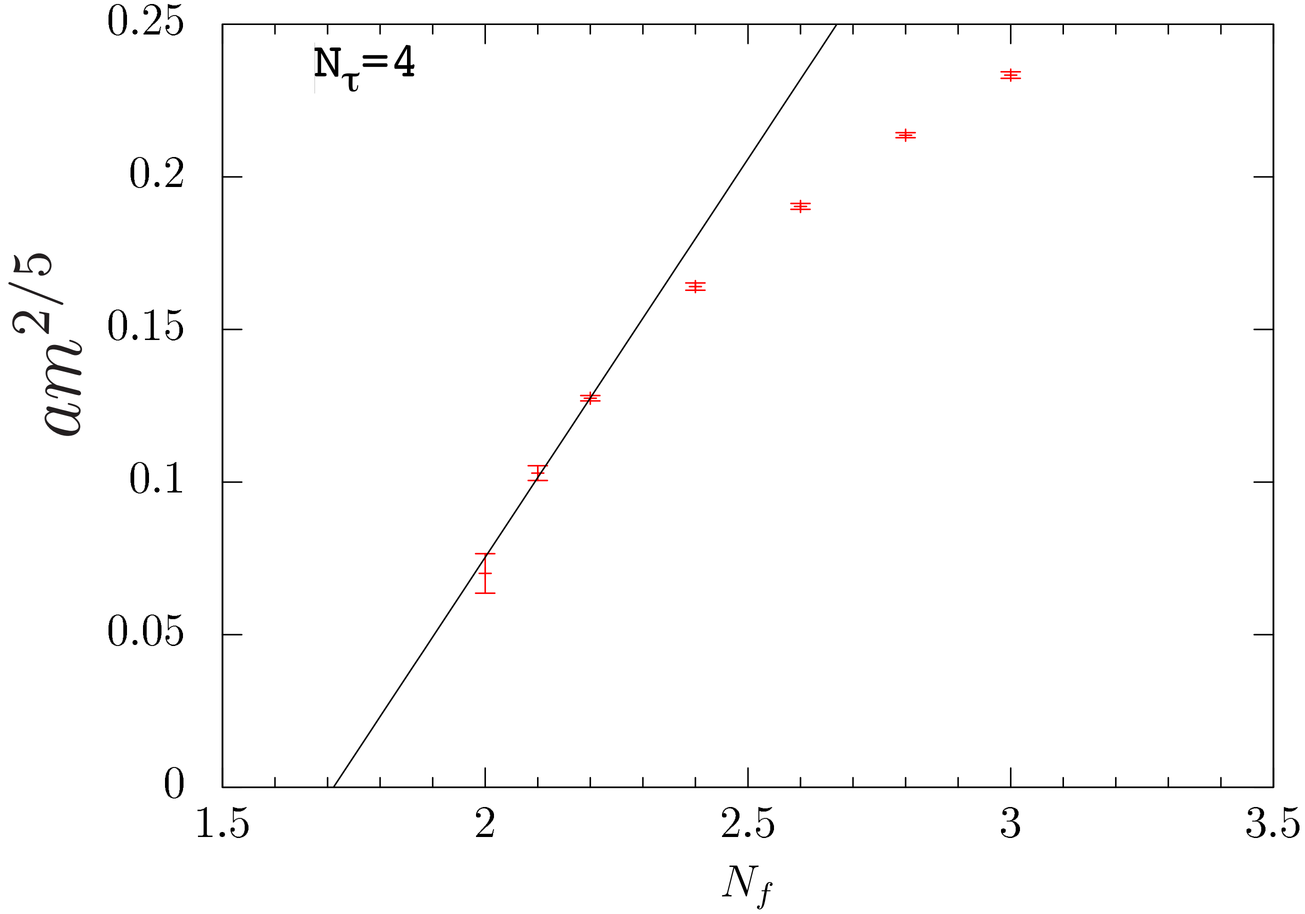
Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra 21] ~120 M Monte Carlo trajectories with light fermions,
aspect ratios 3,4,5

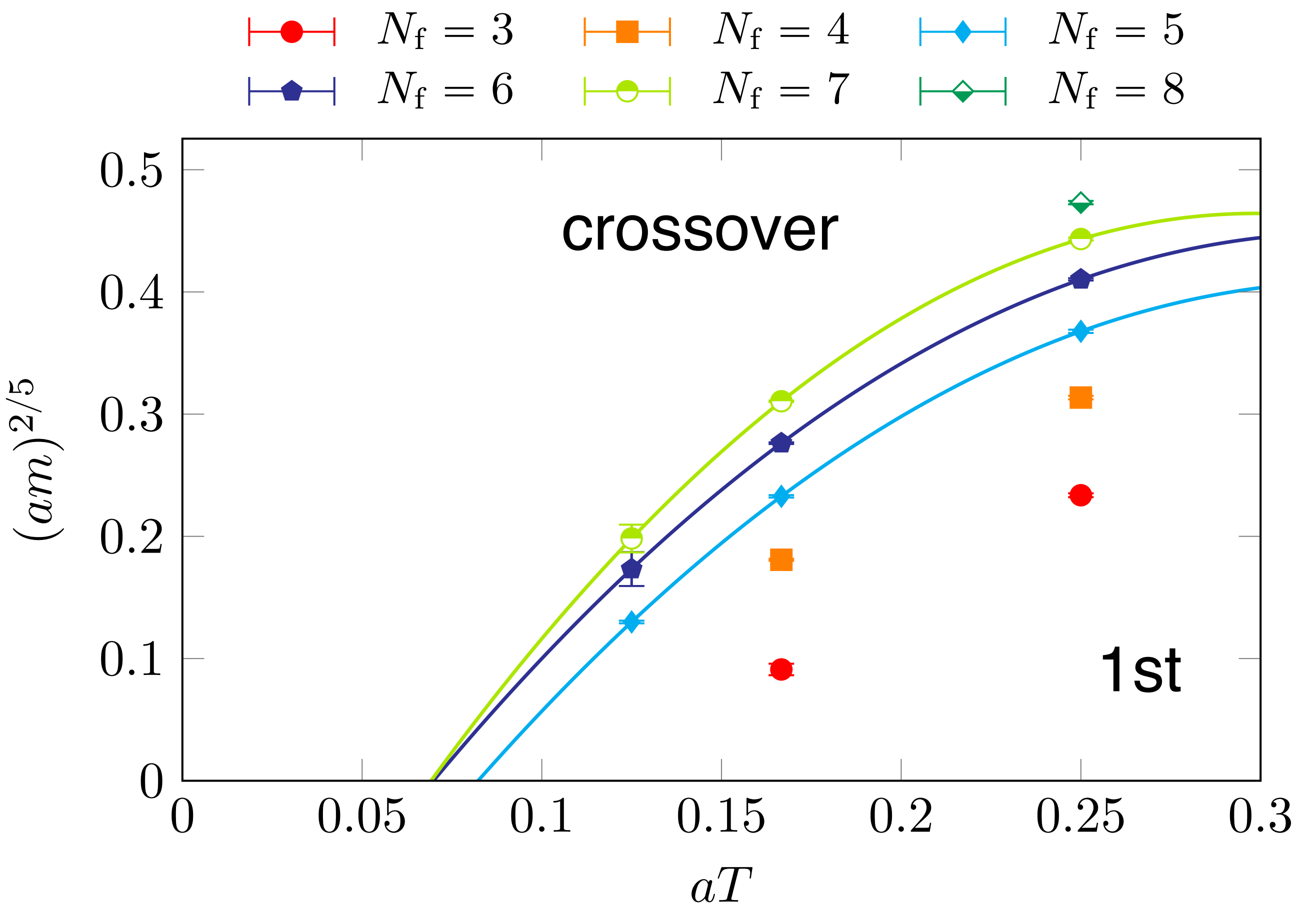




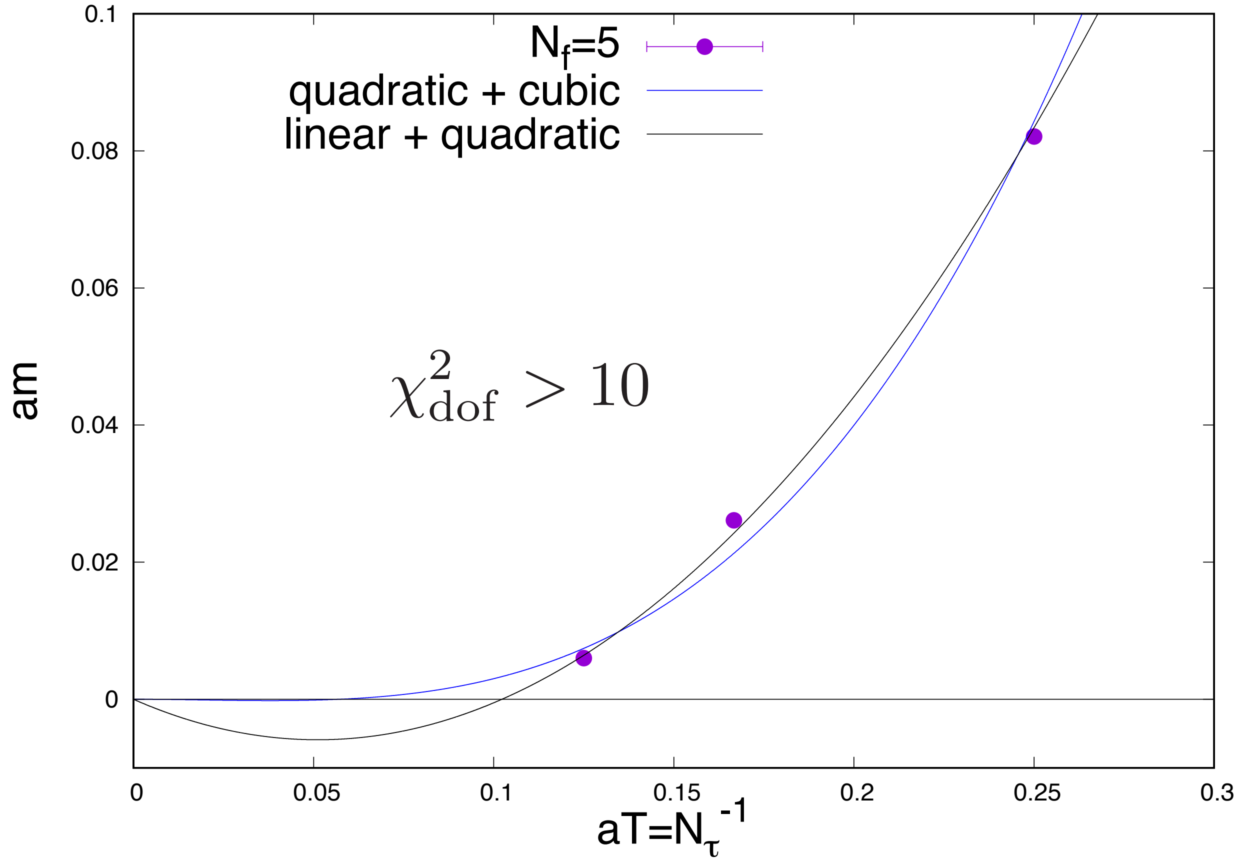
- Tricritical scaling observed in different variable pairings
- Old question: $m_c/T = 0 \text{ or } \neq 0$? Answered for $N_f = 2$
- Surprising new question: will N_f^{tric} slide beyond $N_f = 3$?

Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra 21]



1st order scenario does not fit!

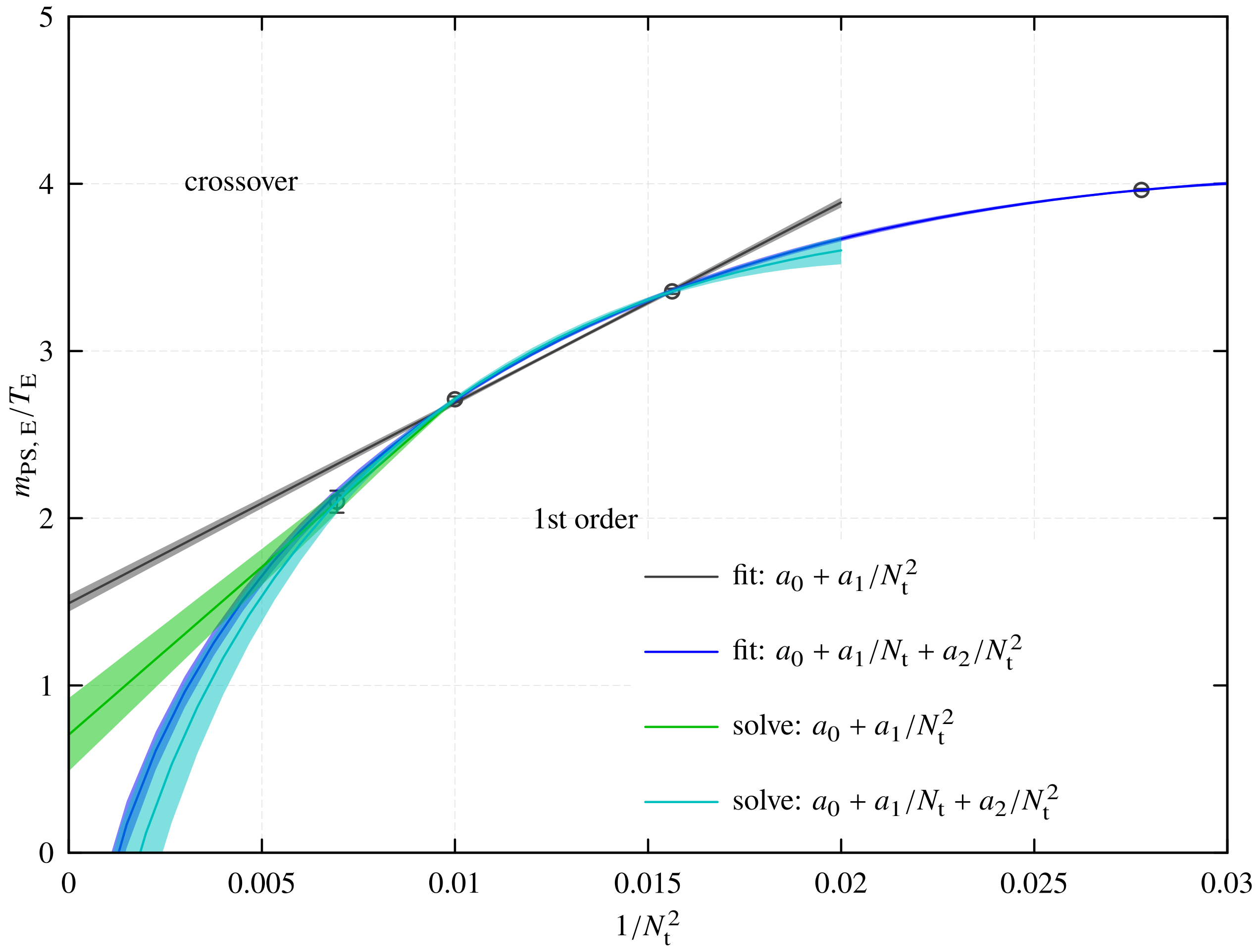


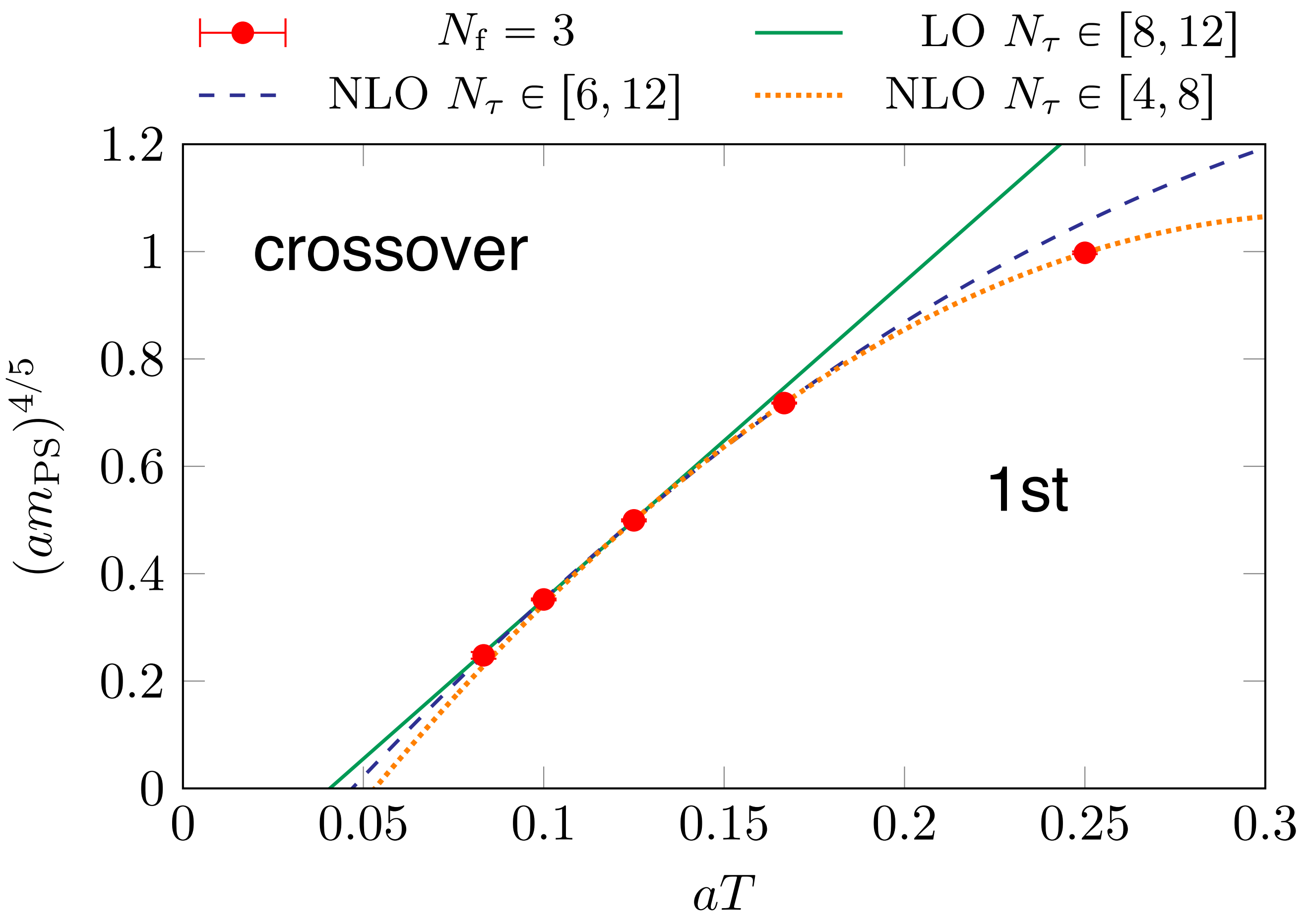
- Tricritical scaling observed also in plane of mass vs. lattice spacing, 2nd order in continuum
- Allows extrapolation to lattice chiral limit, tricritical points $N_\tau^\mathrm{tric}(N_f)$
- 1st order scenario: $m_c(a) = m_c(0) + c_1(aT) + c_2(aT)^2 + \dots$ incompatible!
- The chiral transition is second order for $N_f = 2-6$

Nf=3 O(a)-improved Wilson fermions

[Kuramashi et al. PRD 20] $m_\pi^c \leq 110\ \mathrm{MeV} \quad N_\tau = 4, 6, 8, 10, 12$

Re-analysis using: $am_{PS}^2 \propto am_q$





[Cuteri, O.P., Sciarra, JHEP 21]

Tricritical scaling!

Nf=3 consistent with staggered, 2nd order in chiral continuum limit!

The Columbia plot in the continuum

[Cuteri, O.P., Sciarra JHEP 21]

[HotQCD PRL 19]

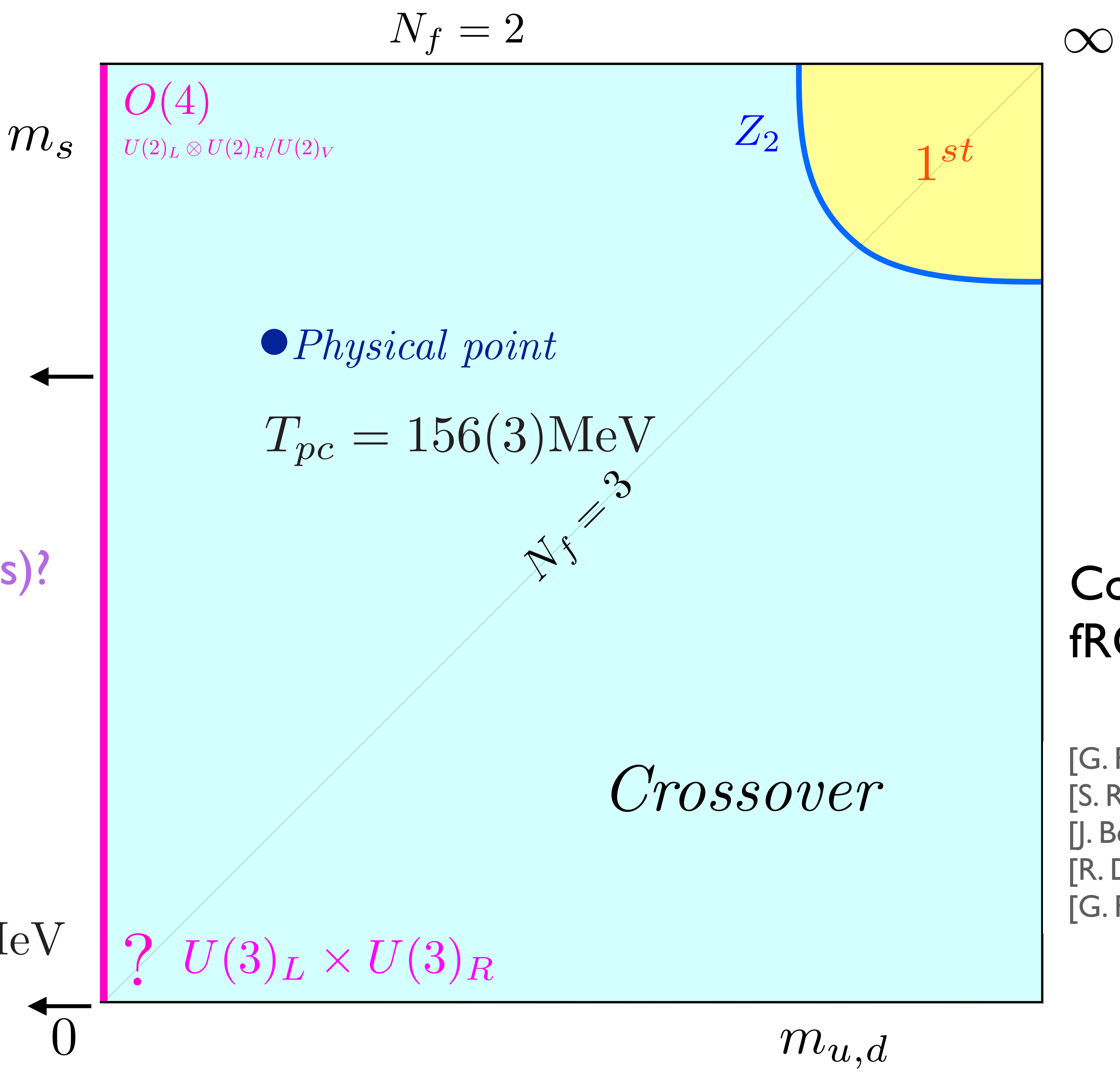
$T_c^0 = 135(8)\mathrm{MeV}$

Universality class(es)?

[HotQCD PRD 22]

HISQ, crossover
down to $m_\pi \approx 80$ MeV

$T_c^0 = 98(6)\mathrm{MeV}$



Consistent with:
fRG, DSE, bootstrap…

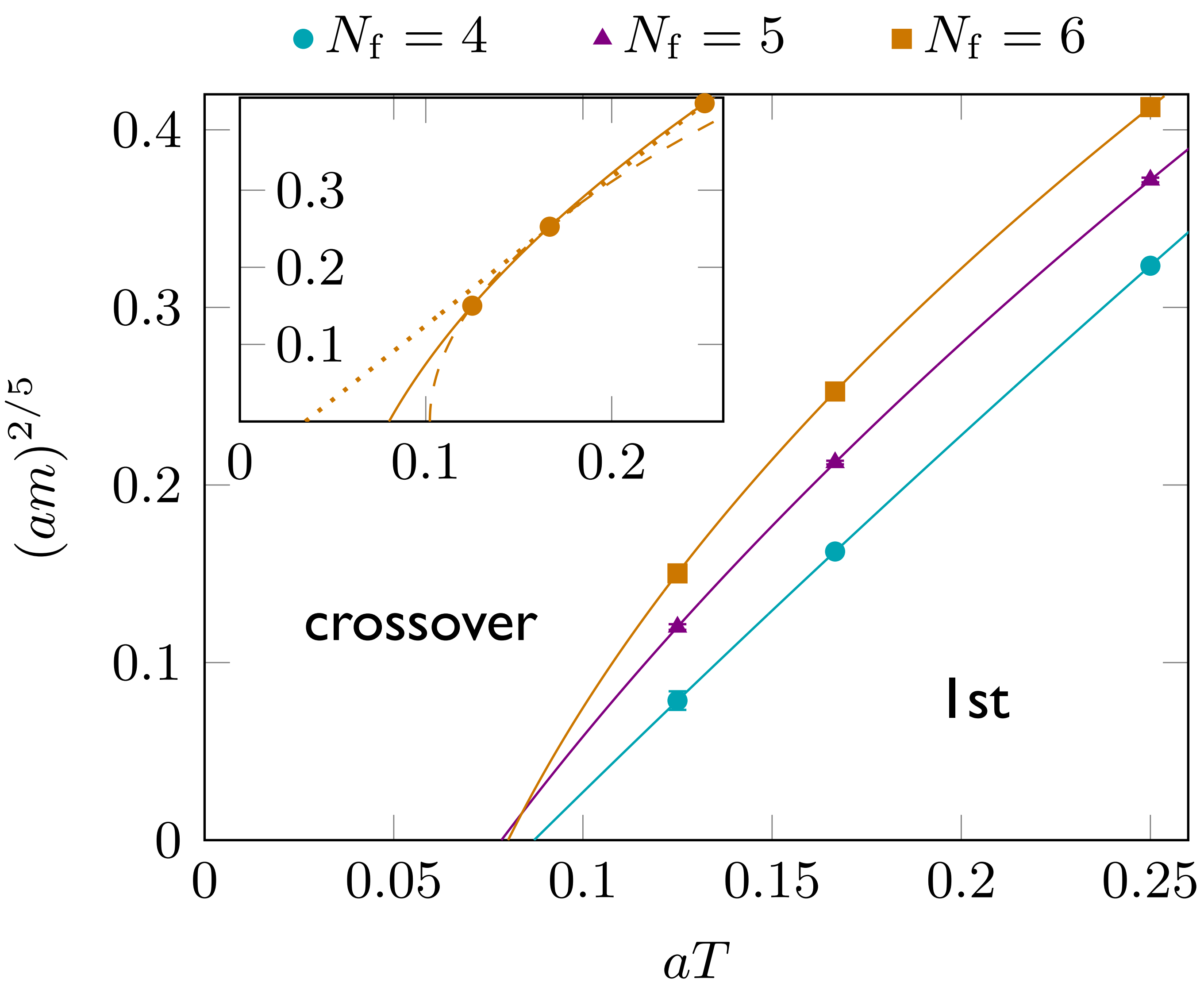
[G. Fejos, PRD 22]
[S. R. Kousvos, A. Stergiou SciPost 23]
[J. Bernhardt, C. S. Fischer PRD 23]
[R. D. Pisarski, F. Rennecke PRL 24]
[G. Fejos, T. Hatsuda PRD 24]

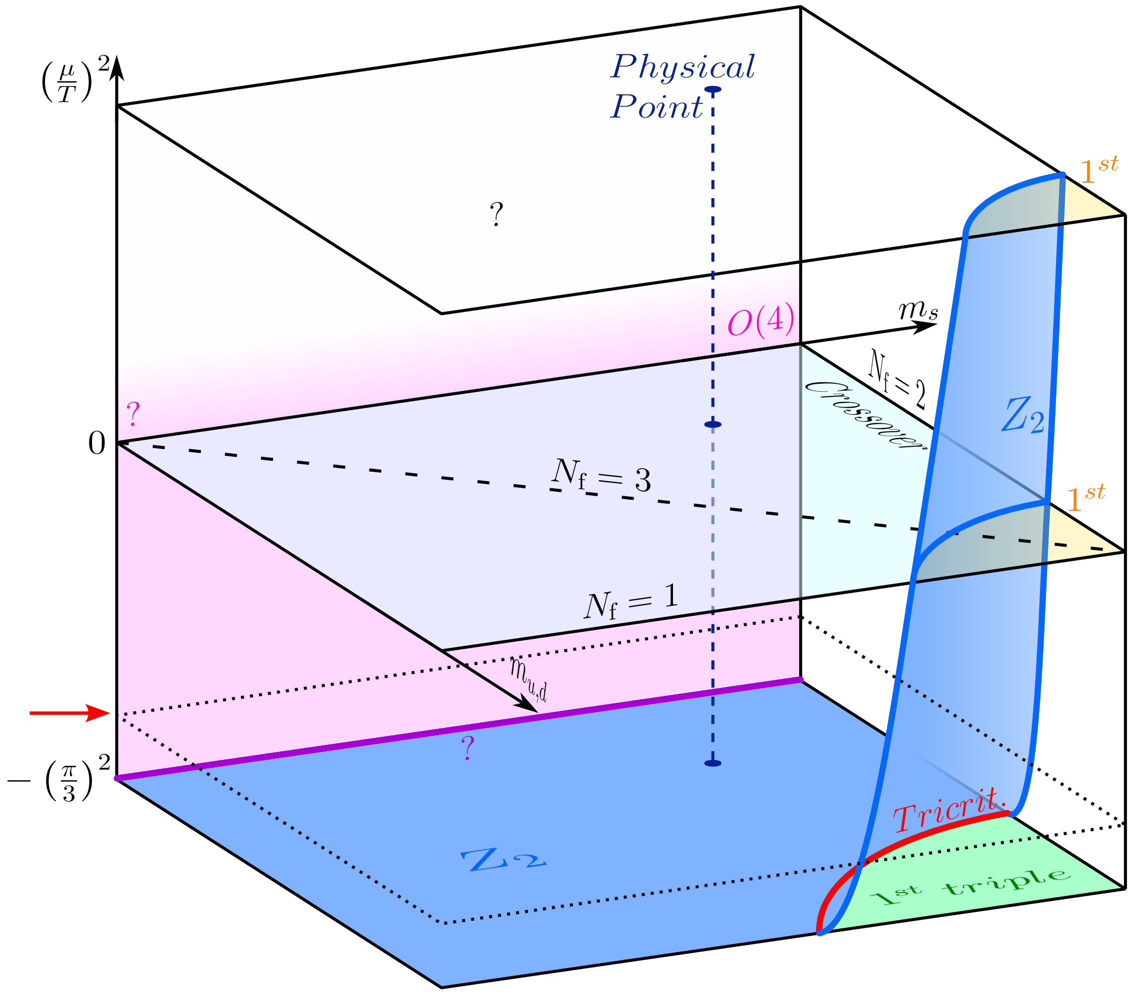
Crossover for DW fermions, Nf=3, $m_q \sim m_{phys}$ [Zhang et al., PoS LAT22, 23]

b) Imaginary chemical potential

[D'Ambrosio, Kaiser, O.P., PoS LAT 22 + in preparation]

Repeat study of Columbia plot with $\mu = i\, 0.81\pi T/3$ Same situation as $\mu = 0$





1st-order region not connected to continuum limit!

Imaginary chemical potential, improved actions

$\mu = i\pi T/3$ Roberge-Weiss boundary

- [Bonati et al., PRD 19] $N_\tau = 4$
  stout-smeared staggered

  quark mass scan down to $m_\pi \approx 50\ \mathrm{MeV}$
  fixed m_{ud}/m_s

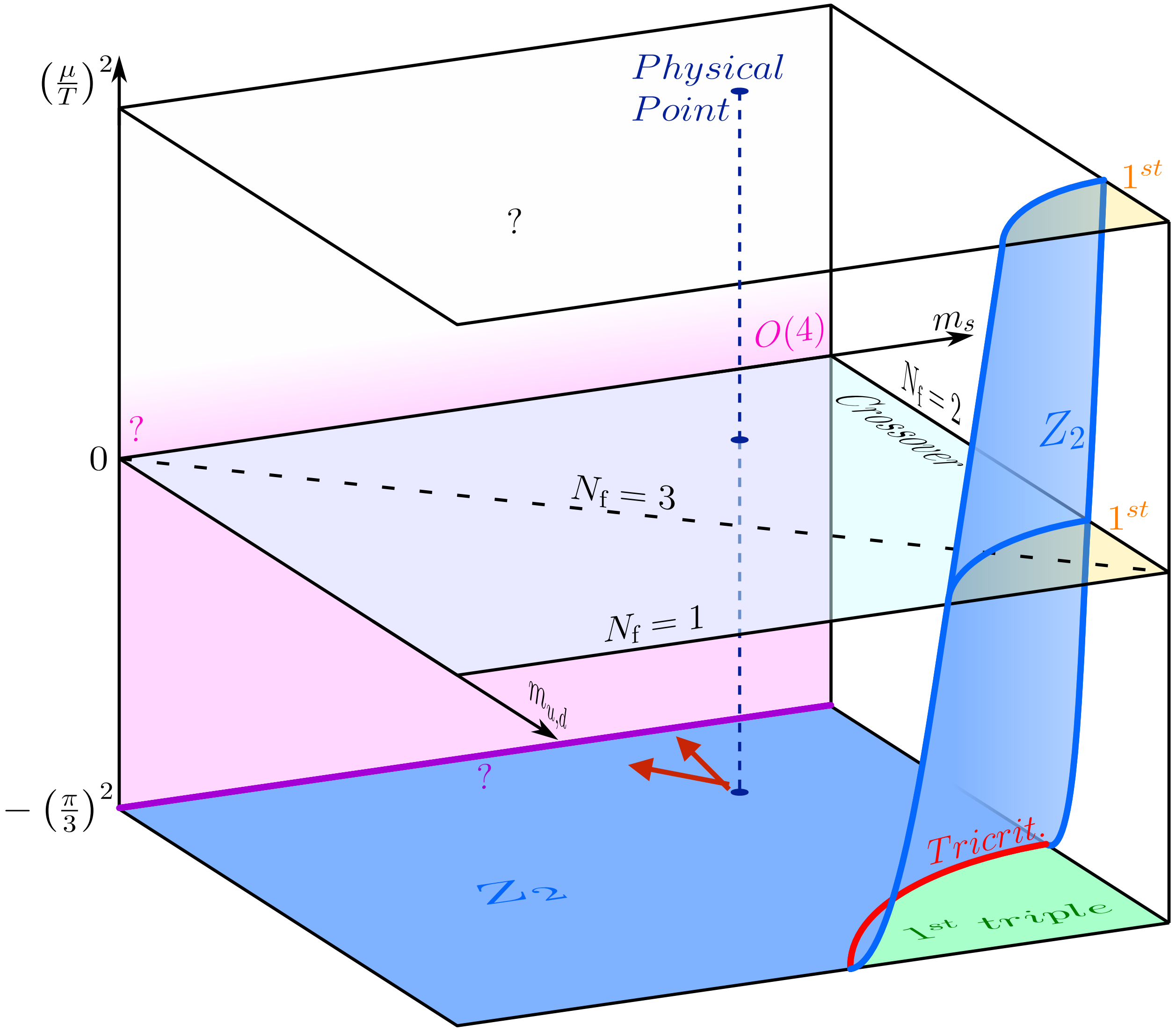
- [Bielefeld+Frankfurt, PRD 22]
  HISQ $N_\tau = 4$

  quark mass scan down to $m_\pi \approx 55\ \mathrm{MeV}$
  fixed m_s

- No sign of 1st-order phase transition!

- Consistent with DSE approach [Bernhardt, Fischer, PRD 23]

- Entire chiral critical surface moves to massless limit

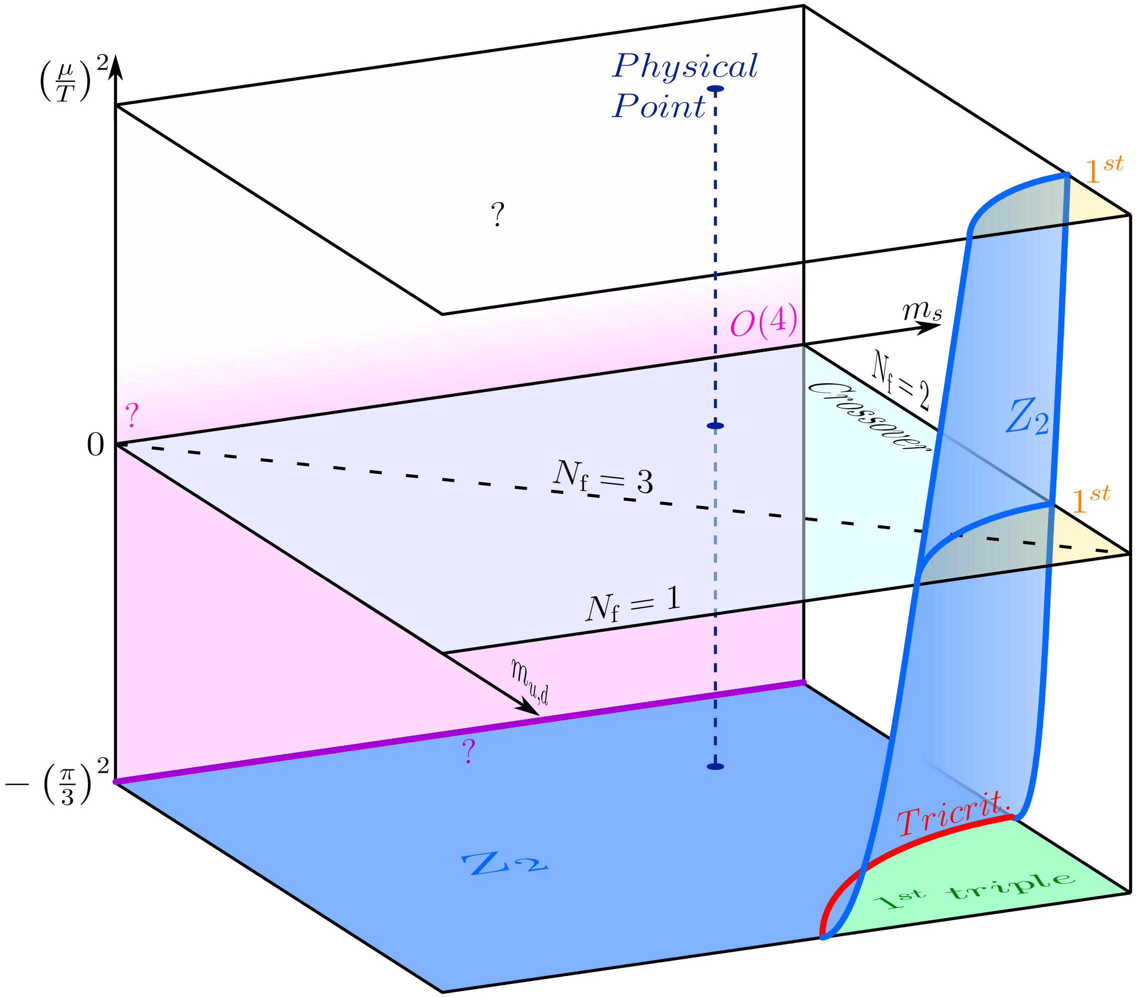


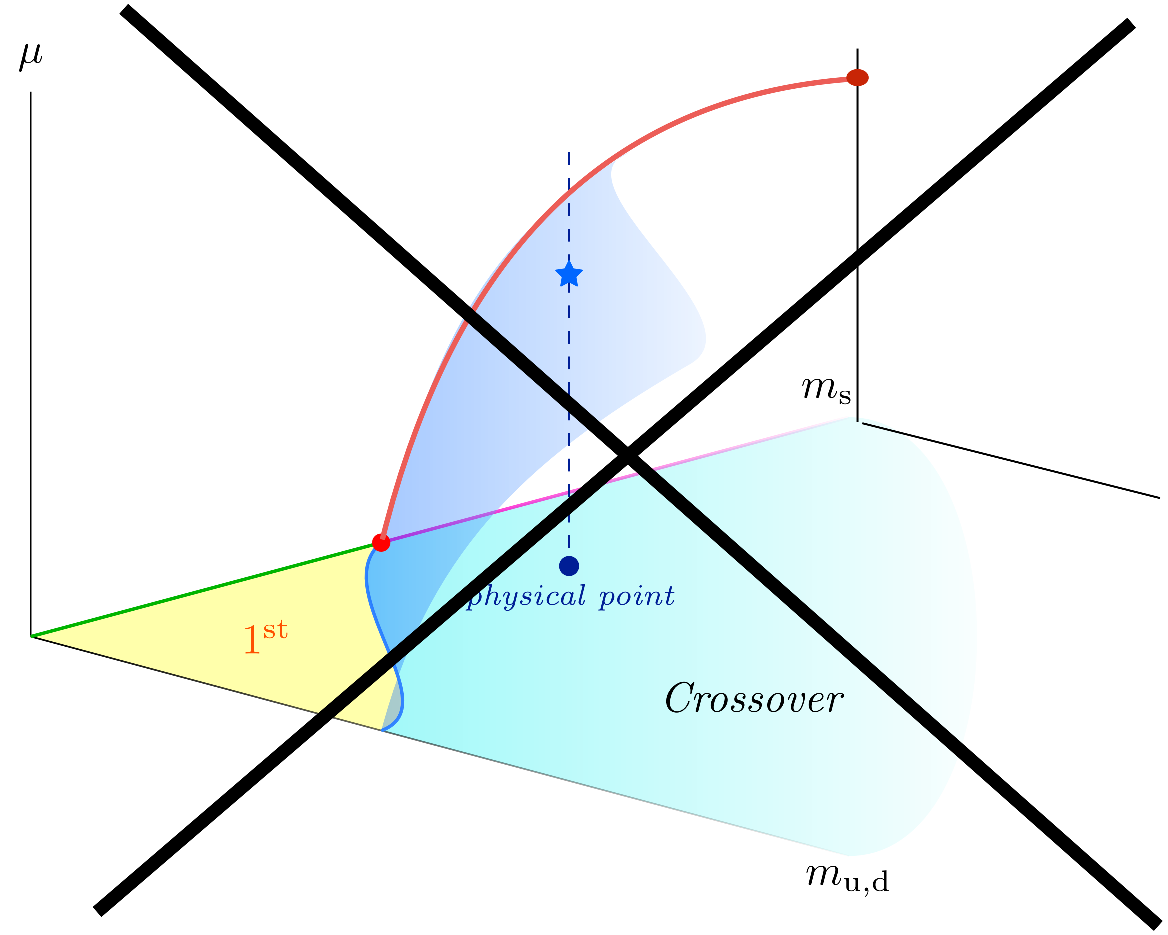
Columbia plot with chemical potential

If we take these results seriously:

Critical point not ruled out,
requires additional critical surface

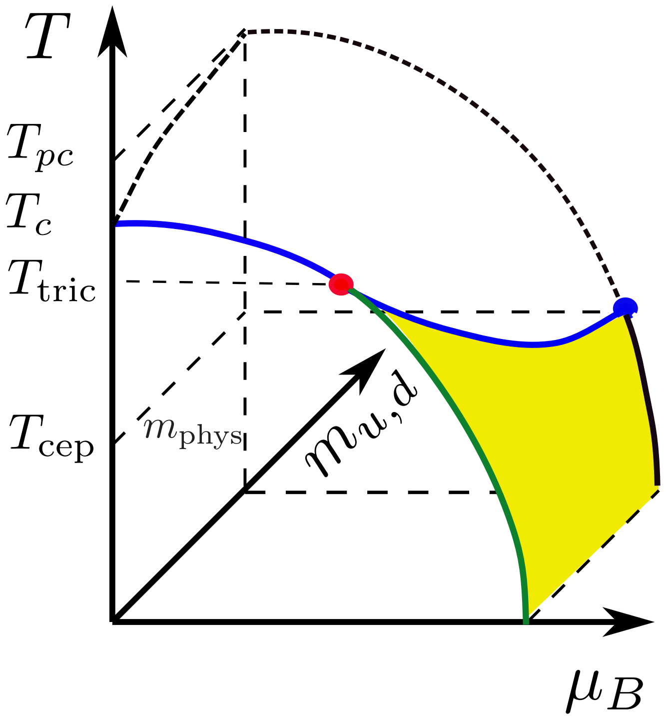
Class of low energy models now ruled out!





$\nexists$ Tuning of parameters for $N_f = 2+1$ theory with critical point at $\mu = 0$!

Summary: constraints on the critical point



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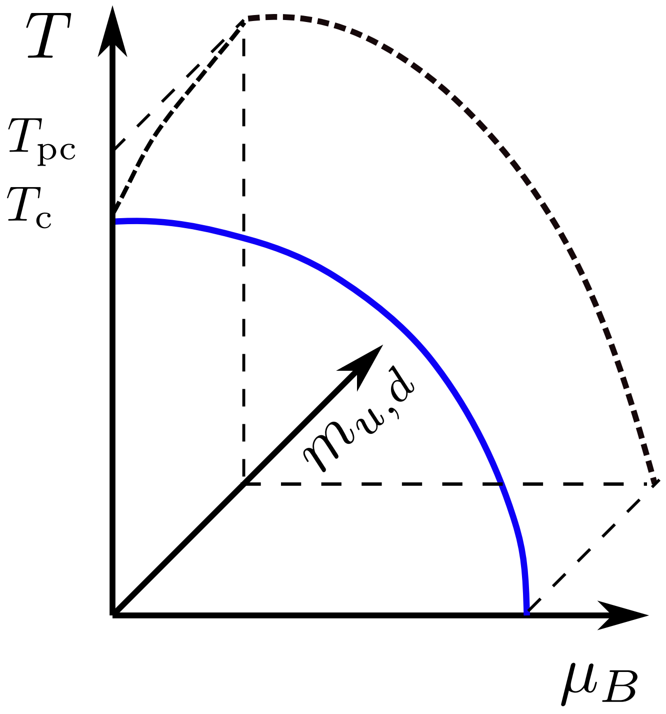


- Ordering of critical temperatures $\mu_B^{\rm cep} > 3.1\ T_{pc}(0) \approx 485\ \text{MeV}$ [O.P. Symmetry 21]
- Cluster expansion model of lattice fluctuations $\mu_B^{\rm cep} > \pi T$ [Vovchenko et al. PRD 18]
- Singularities, Pade-approx. fluctuations $\mu_B^{\rm cep} > 2.5T, T < 125\ \text{MeV}$ [Bollweg et al. PRD 21]
- Direct simulations with refined reweighting $\mu_B^{\rm cep} > 2.5T$ [Wuppertal-Budpest collaboration, PRD 21]

- Consistent with DSE, fRG [Fischer PPNP 19; Fu, Pawlowski, Rennecke PRD 20; Gao, Pawlowski PRD 21]

CEP seen at larger density, but "not yet controlled" $(T_{\rm CEP}, \mu_{B_{\rm CEP}}) = (98, 643)\,\text{MeV}$

II. Emergent chiral spin symmetry

Chiral spin transformation, $SU(2)_{CS}$: $\quad \psi \to \psi' = \exp\left(i\frac{\varepsilon^n \Sigma^n}{2}\right)\psi \qquad \Sigma^n = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}$

$$SU(2)_{CS} \otimes SU(2)_V \simeq \ SU(4) \ \supset \ SU(2)_L \times SU(2)_R \times U(1)_A$$

QCD quark action, chiral limit:

$$\bar\psi\gamma^\mu D_\mu\psi = \underset{\substack{\big\uparrow \\ \text{CS invariant}}}{\bar\psi\gamma^0 D_0\psi} + \underset{\substack{\big\uparrow \\ \text{breaks CS}}}{\bar\psi\gamma^i D_i\psi}$$

Necessary condition for approximate CS symmetry:

Quantum effective action dynamically dominated by colour-electric interactions!

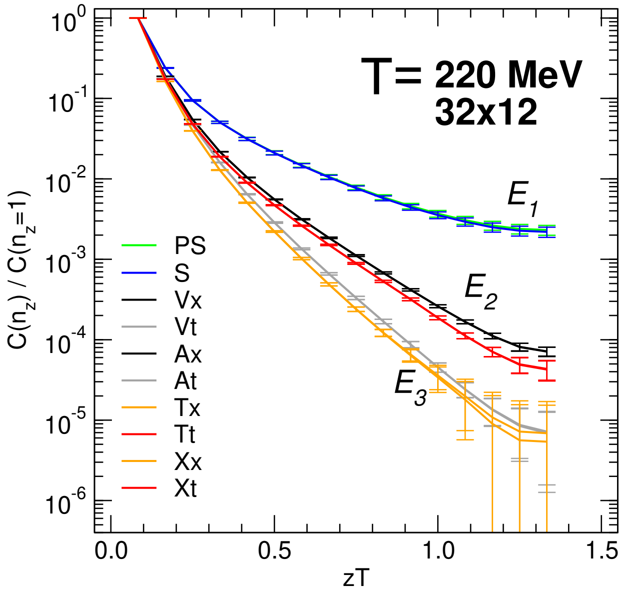
CS-symmetry observed in meson correlators

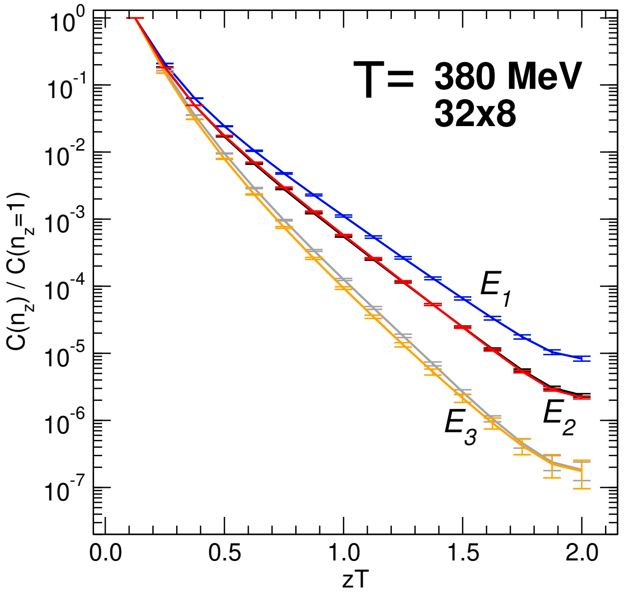
JLQCD domain wall fermions at phys. point

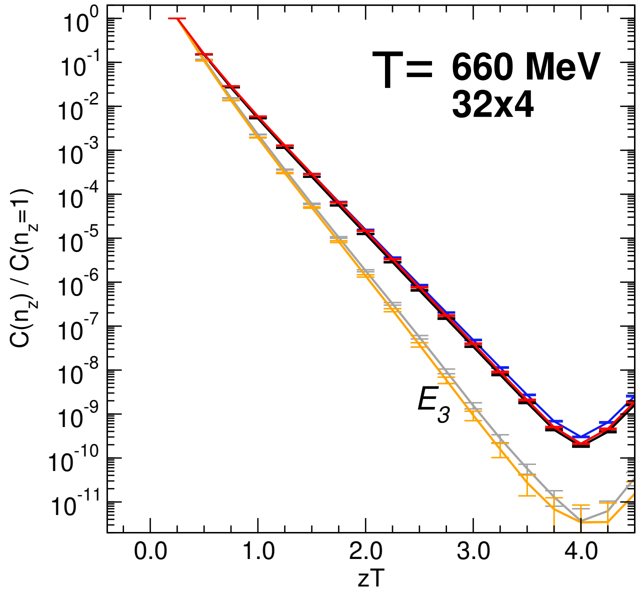
Spatial correlators: [Rohrhofer et al., PRD 19]

Nf=2+1+1 DW [Chiu, PRD 23]

$$\begin{array}{lcl} E_1: & PS \leftrightarrow S\,, & U(1)_A \\ E_2: & V_x \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x\,, & SU(4) \\ E_3: & V_t \leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t\,. & SU(2)_L \times SU(2)_R \times U(1)_A \end{array}$$





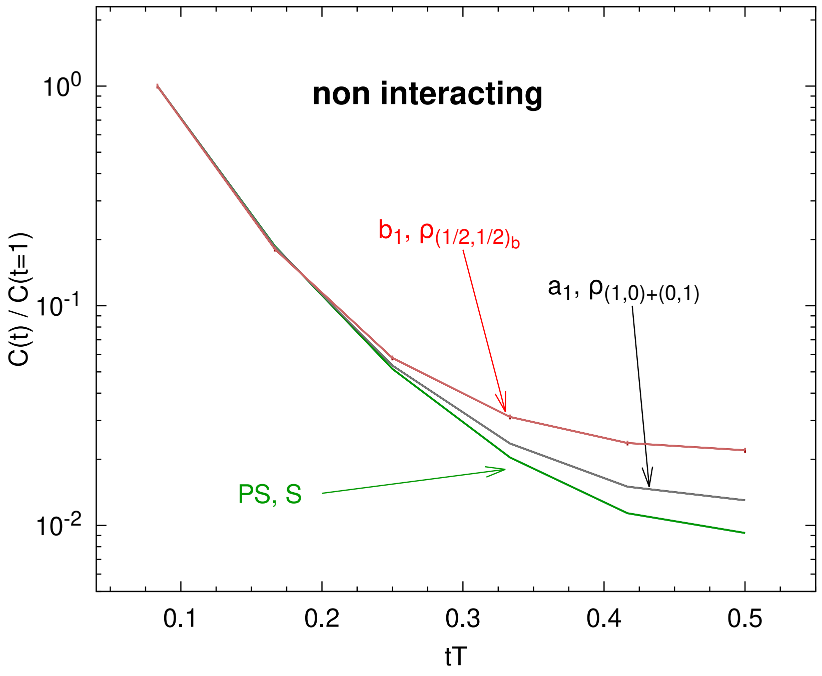


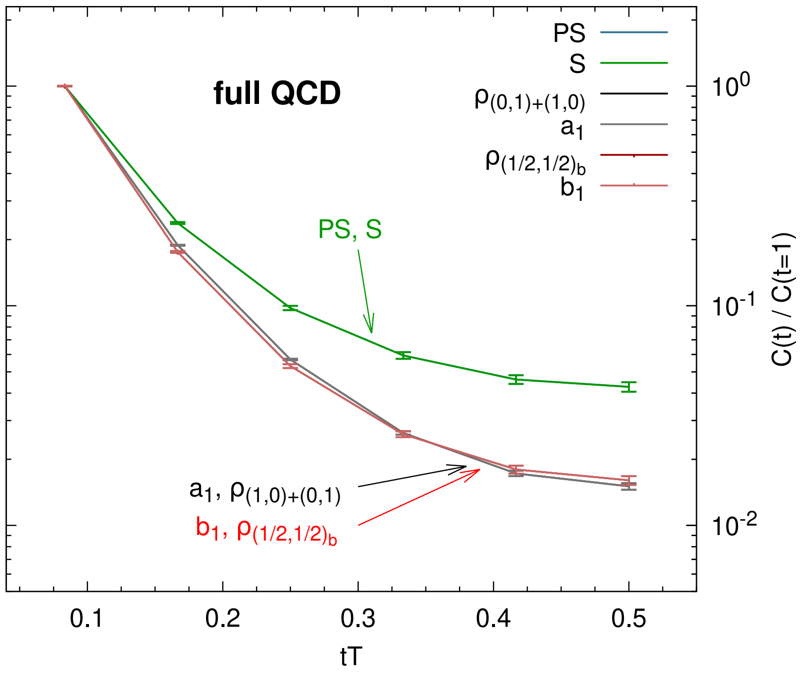
Temporal correlators:

[Rohrhofer et al., PLB 20]

$$T = 220\mathrm{MeV}\ (1.2T_c)$$

$$48^3 \times 12 \qquad (a = 0.075\ \mathrm{fm})$$

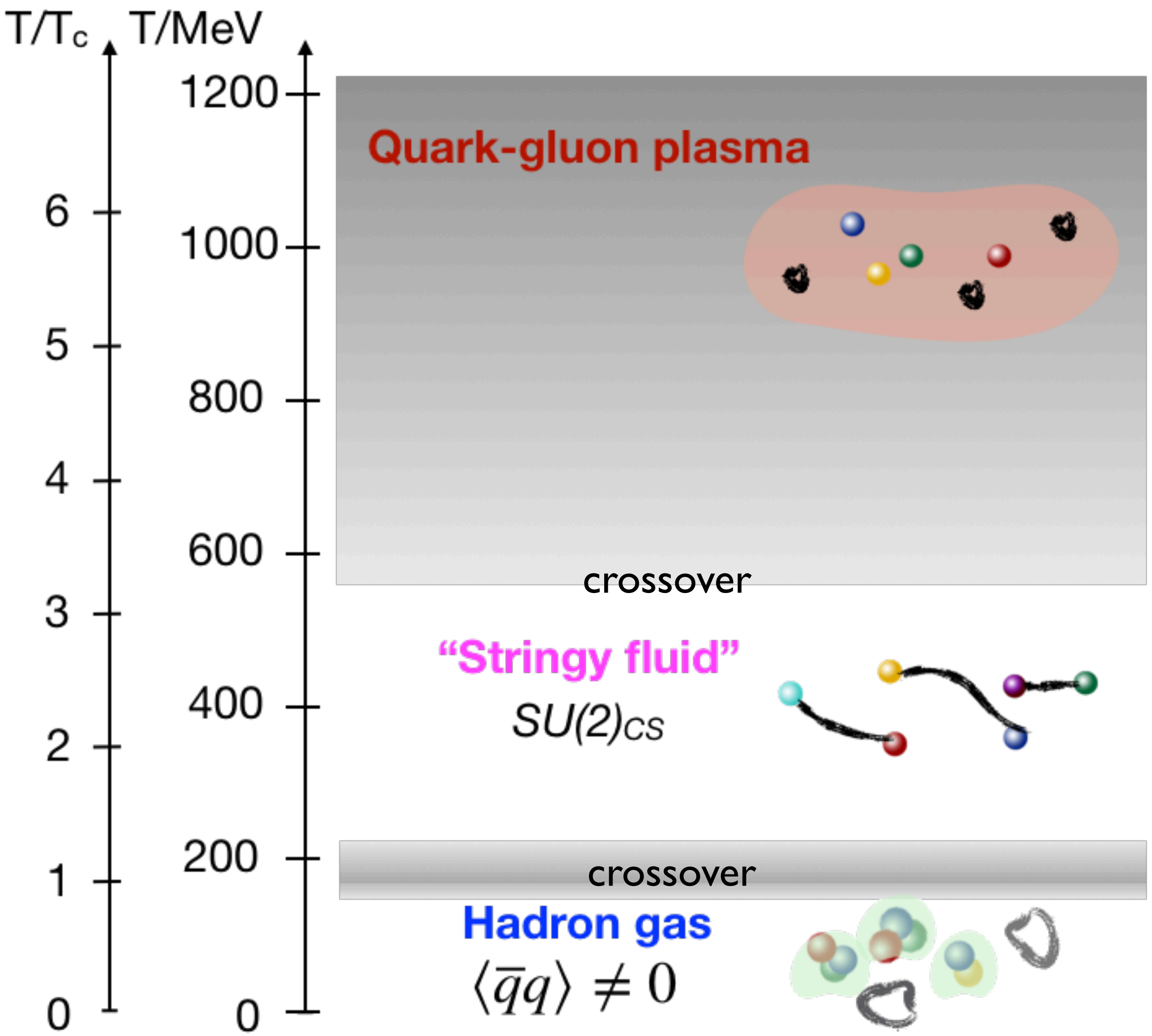




Three temperature regimes of QCD

Symmetries (verified):

Degrees of freedom (to be verified):



Chiral symmetry (approximate)

Chiral spin symmetry (approximate)

Chiral symmetry broken

Rohrhofer et al., Phys. Rev. D100 (2019)

Check well-studied observables: screening masses

$$C^s_\Gamma(z) = \sum_{x,y,\tau} C_\Gamma(\tau, \boldsymbol{x}) \quad \xrightarrow{z\to\infty} \text{const. } e^{-m_{scr} z}$$

Directly related to the partition function and equation of state

by transfer matrices: $T = e^{-aH}, T_z = e^{-aH_z}$

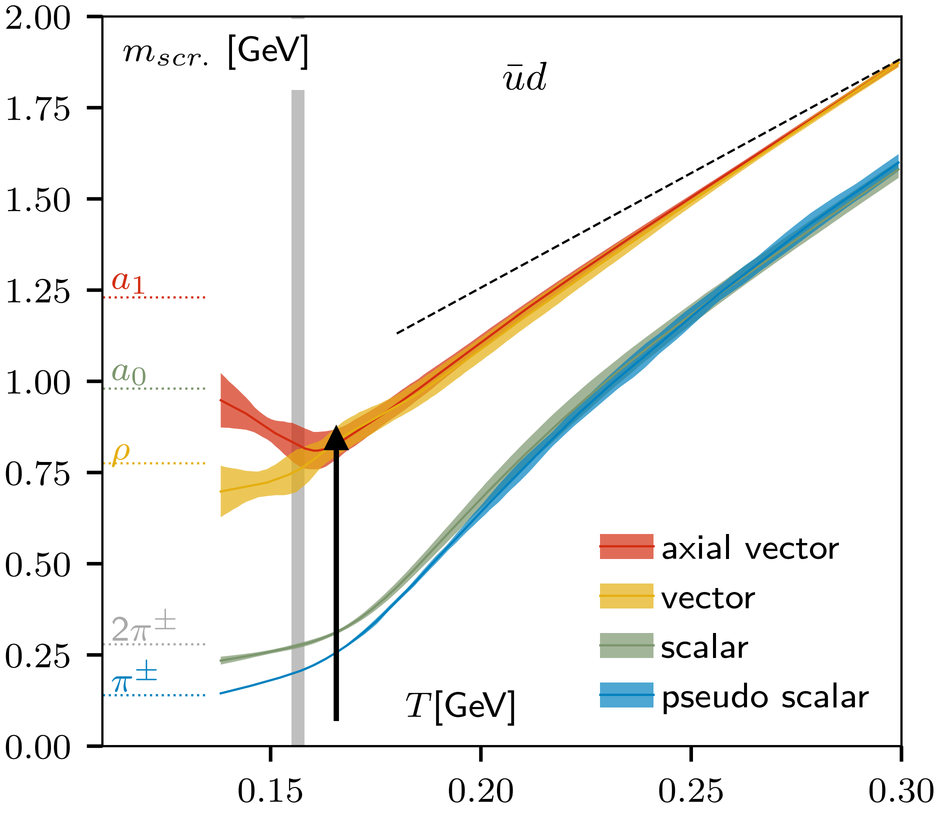
$$e^{pV/T} = Z = \mathrm{Tr}(e^{-aHN_\tau})$$
$$= \mathrm{Tr}(e^{-aH_z N_z}) = \sum_{n_z} e^{-E_{n_z} N_z}$$

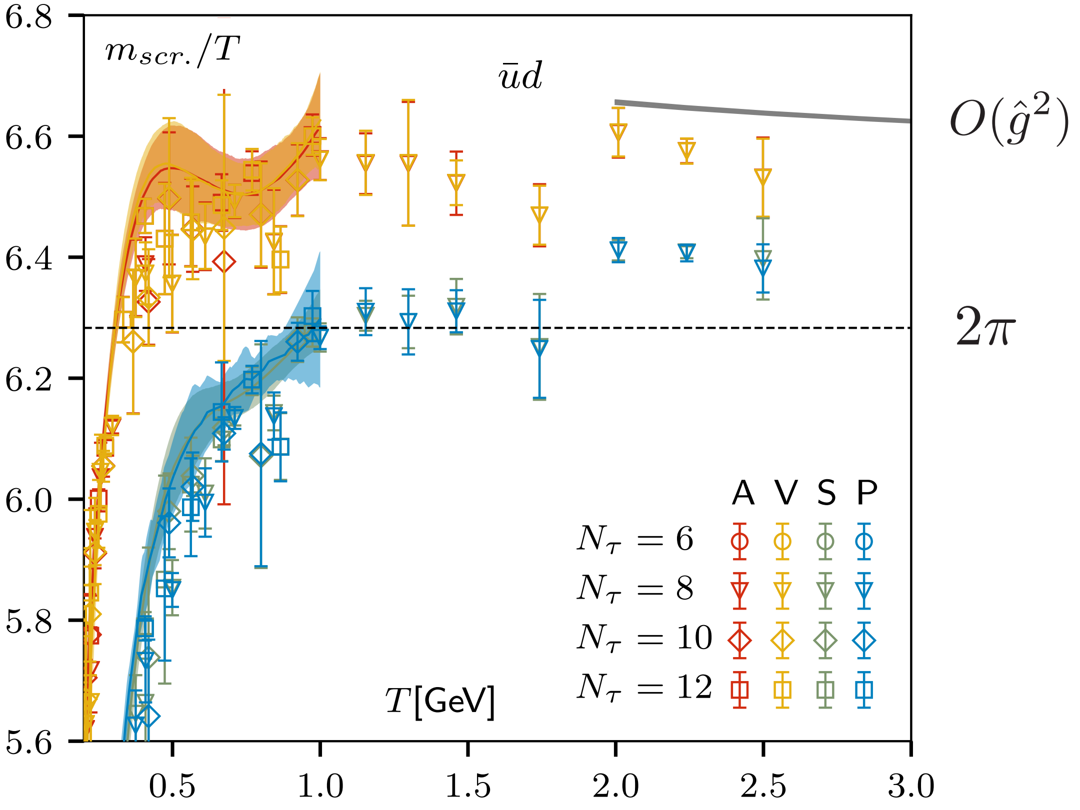
Screening masses: eigenvalues of H_z

For T=0 equivalent to eigenvalues of H, for $T \neq 0$ temperature effects

Meson screening masses at intermediate temperatures

[HotQCD, PRD 19]





Chiral symmetry restoration

Heavy chiral partners "come down" in all flavour combinations

➡ pressure increases

Resummed pert. theory:

$$\frac{m_{PS}}{2\pi T} = 1 + p_2\,\hat{g}^2(T) + p_3\,\hat{g}^3(T) + p_4\,\hat{g}^4(T)\ ,$$

$$\frac{m_V}{2\pi T} = \frac{m_{PS}}{2\pi T} + s_4\,\hat{g}^4(T)\ ,$$

Cannot describe the "bend"

No quark hadron duality for T<0.5 GeV in 12 lightest meson channels! CS symmetry!

[Glozman, O.P., Pisarski, EPJA 22]

Spectral functions at finite T

General euclidean correlator: $$C(\tau, \mathbf{p}) \ = \int_0^\infty \frac{d\omega}{2\pi} \frac{\cosh(\omega(|\tau| - \beta/2))}{\sinh(\beta\omega/2)} \rho(\omega, \mathbf{p})$$

- Inversion problem ill-defined on a discrete lattice
- Statistical approaches to find "most likely" spectral function:

Maximum entropy, Bayesian, Backus-Gilbert methods,….

[Asakawa, Hatsuda, Nakahara, PPNP 01
Meyer, PoS INPC 16
Spriggs et al., EPJ Web Conf. 22
…]

- Alternative: microcausality + KMS [Bros, Buchholz, Ann. Inst. Poincare Phys, Theo 96]

$$\rho(\omega, \mathbf{p}) = \int_0^\infty ds \int \frac{d^3\mathbf{u}}{(2\pi)^2} \, \epsilon(\omega) \, \delta\big(\omega^2 - (\mathbf{p} - \mathbf{u})^2 - s\big) \, \widetilde{D}_\beta(\mathbf{u}, s)$$ ← Thermal spectral density

$$C(x_3) \ = \frac{1}{2} \int_0^\infty ds \int_{|x_3|}^\infty dR \; e^{-R\sqrt{s}} D_\beta(R, s)$$

[Lowdon, O.P. JHEP 22]

Thermal spectral density + thermoparticles

- The thermal spectral density $\widetilde{D}_{\beta}(\boldsymbol{u},s)$ holds the key to understanding in-medium phenomena, but what structure does it have?
- A natural decomposition [Bros, Buchholz, *NPB* 627 (2002)] is:

$$\boxed{\widetilde{D}_{\beta}(\vec{u},s) = \widetilde{D}_{m,\beta}(\vec{u})\,\delta(s-m^2) + \widetilde{D}_{c,\beta}(\vec{u},s)}$$

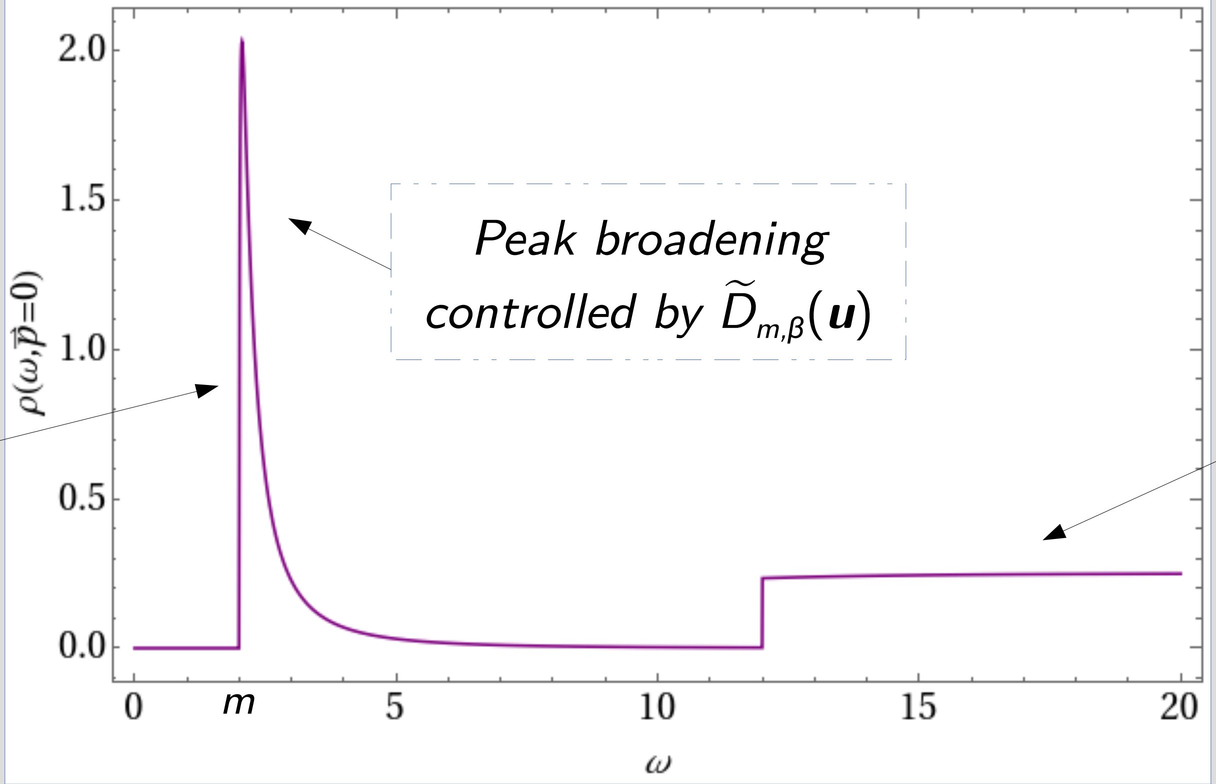
(Negligible at low T)

Thermoparticle component

"Damping factor"

Continuous component

Causes $T=0$ mass pole m to be screened by thermal effects



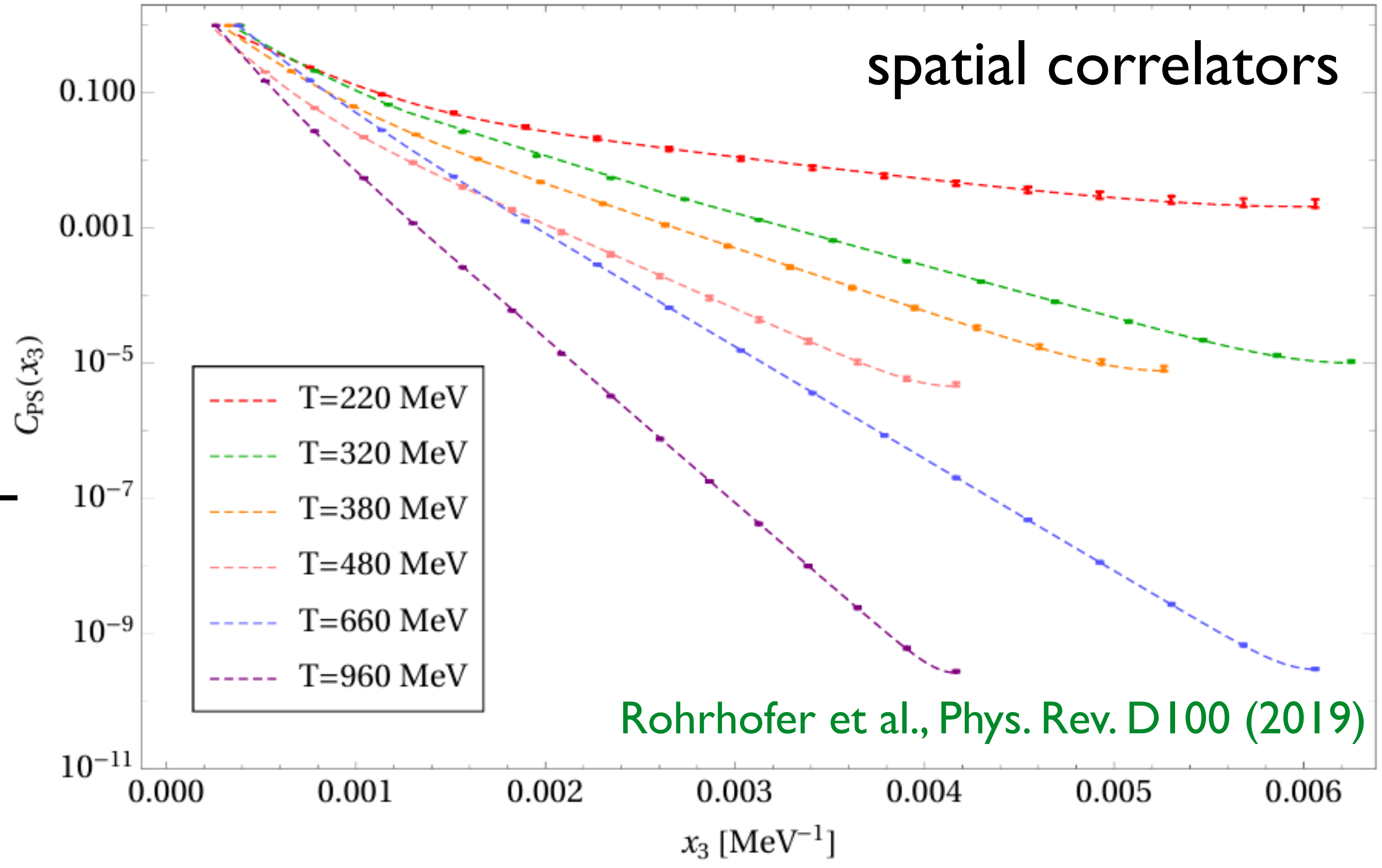
Peak broadening controlled by $\widetilde{D}_{m,\beta}(\boldsymbol{u})$

Fixes T-dependence of continuous spectral contributions

[slide by P. Lowdon]

The pion spectral function

[Lowdon, O.P., JHEP 22]



spatial correlators

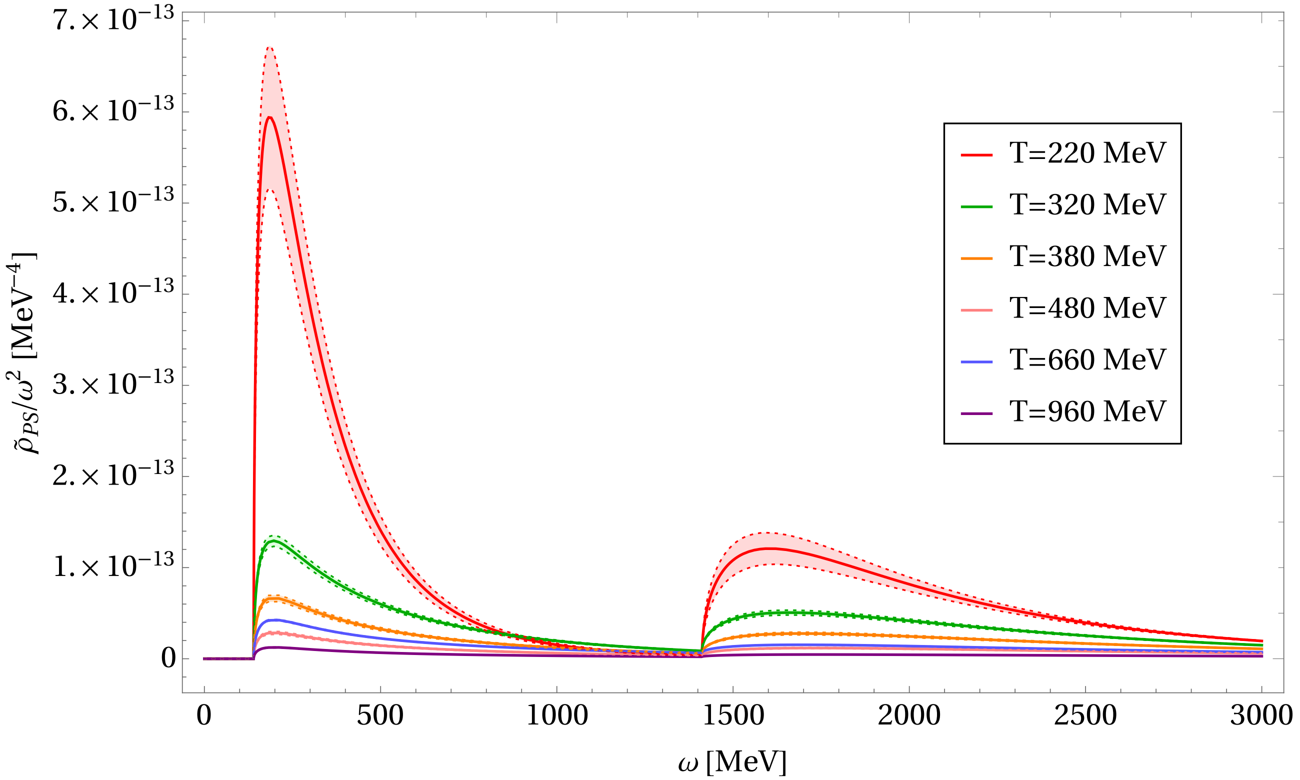
Rohrhofer et al., Phys. Rev. D100 (2019)

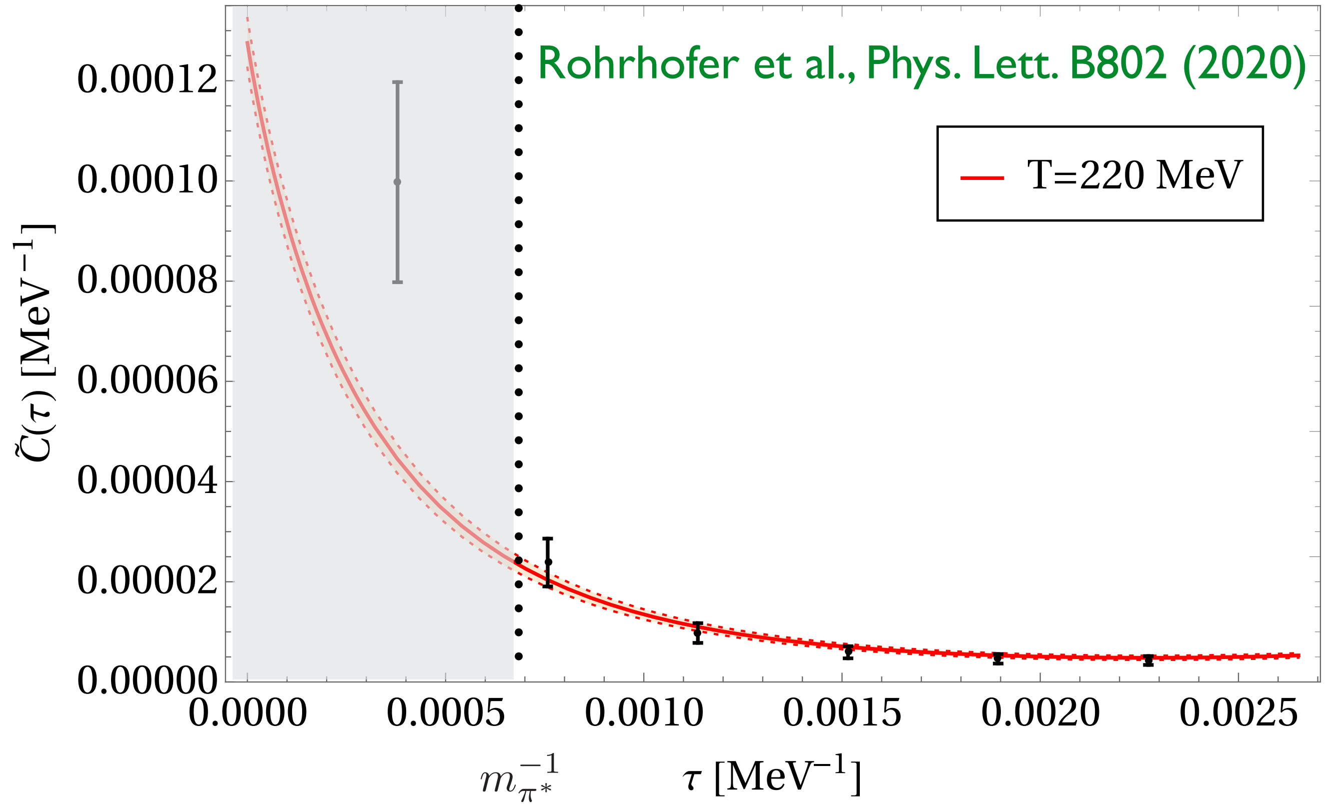
2-state fits π, π^*

$$D_{m_{\pi^{(*)}},\beta} = \alpha_{\pi^{(*)}} e^{-\gamma_{\pi^{(*)}} x_3}$$

spectral functions

predict temporal correlators, compare with data





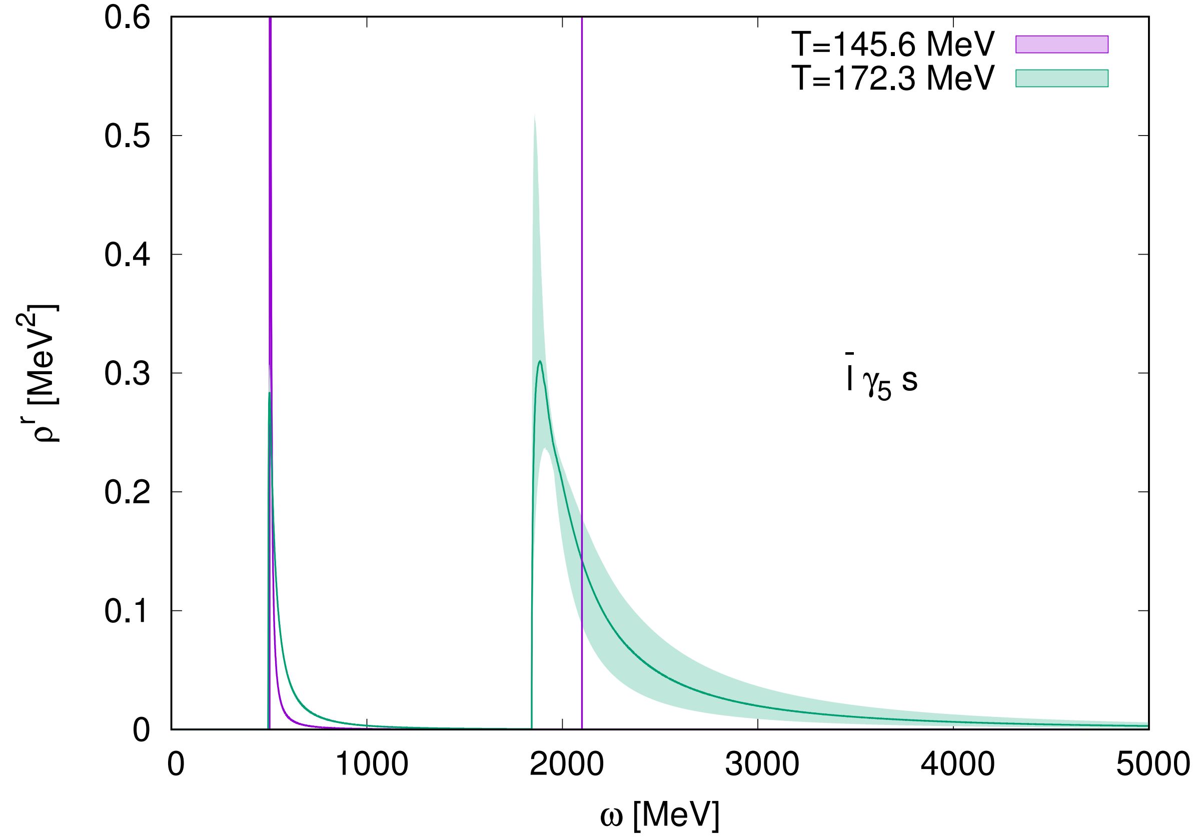
Rohrhofer et al., Phys. Lett. B802 (2020)

Does QCD deconfine across the chiral crossover ?

Kaon + Kaon* in full QCD

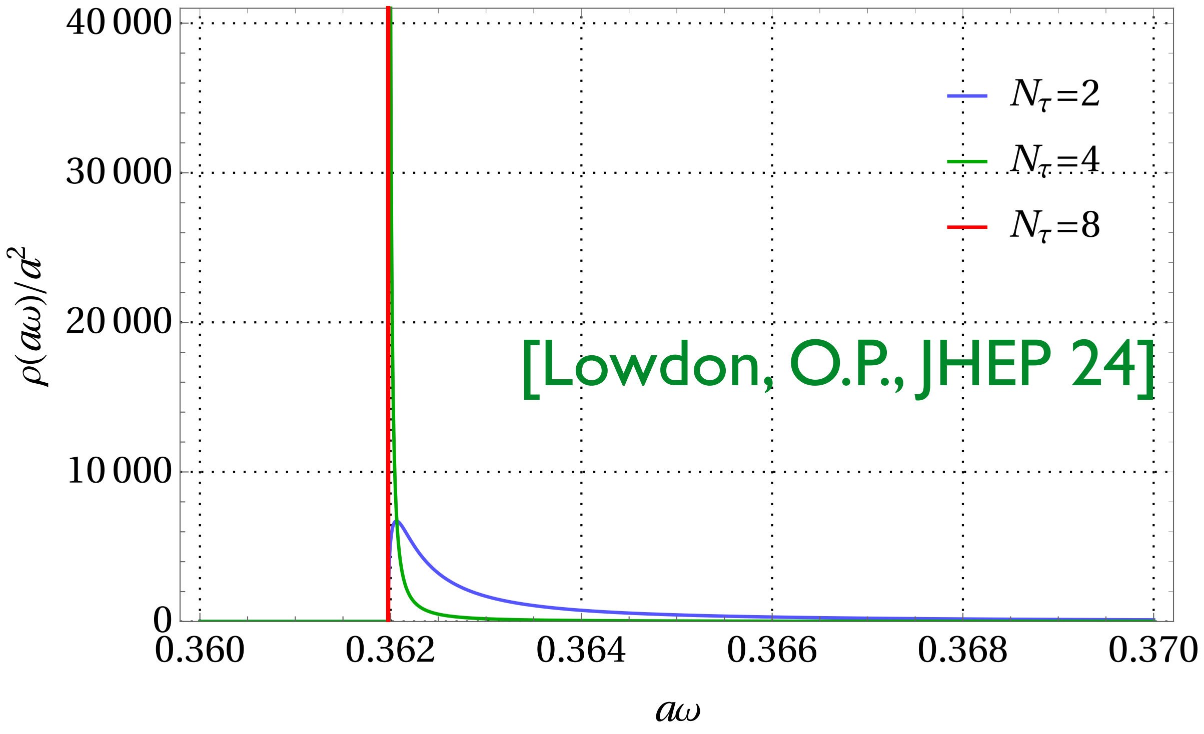
slightly below and above chiral crossover

[Bala et al., JHEP 24]



Scalar point particle in ϕ^4

no phase transition, no "melting",
only "collisional broadening"



[Lowdon, O.P., JHEP 24]

III. Effective heavy (dense) lattice theory from Wilson action

Pure gauge part: character expansion

$$u(\beta) = \frac{\beta}{18} + \frac{\beta^2}{216} + \ldots < 1$$

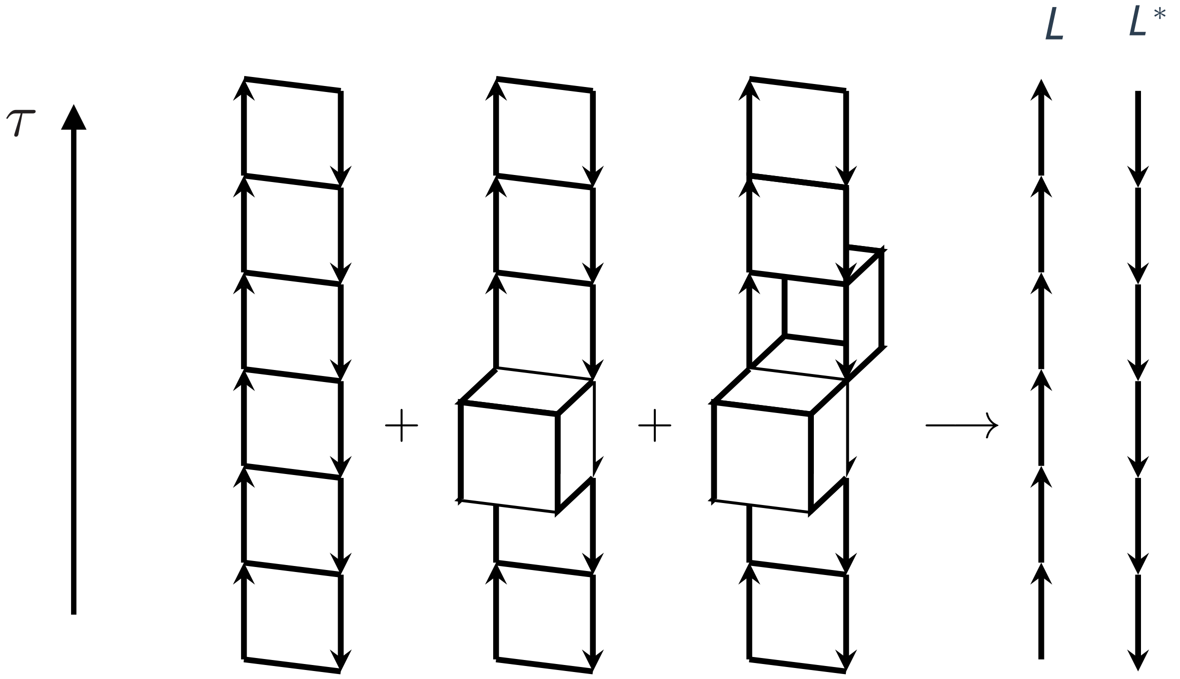
$$\beta = \frac{2N_c}{g^2} \qquad T = \frac{1}{aN_\tau}$$

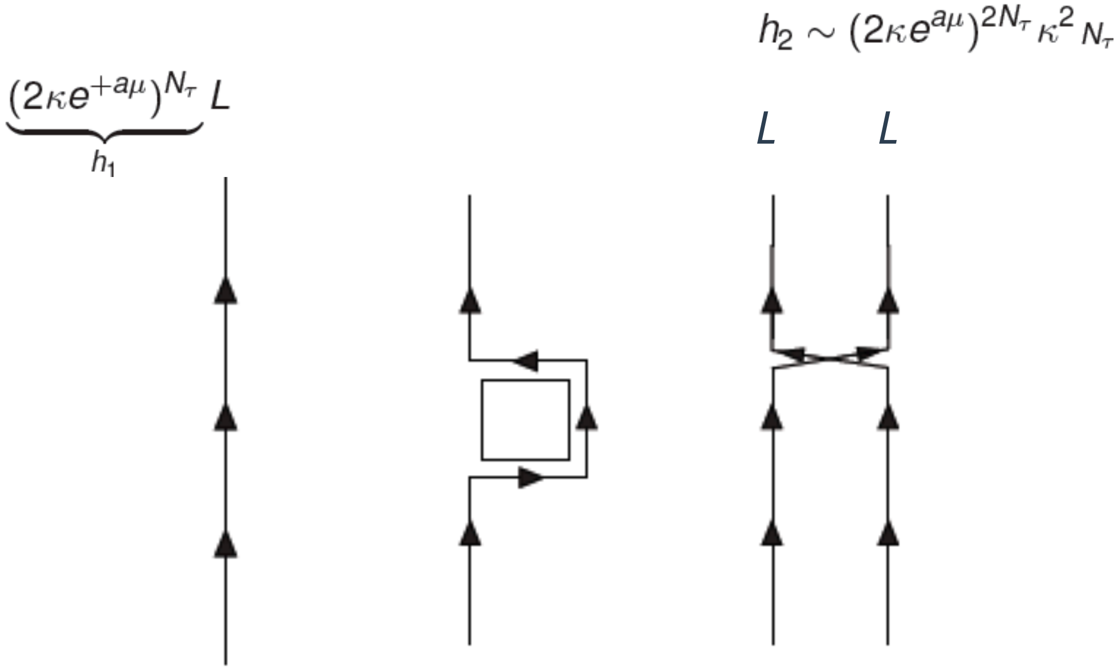
Fermion determinant:

expansion in spatial hops,
temporal hops fully included

$$\kappa = \frac{1}{2am+8}$$

Generates couplings over all distances, n-pt. couplings, higher reps….:





Example LL* : $\lambda(u, N_\tau \geq 5) \; = \; u^{N_\tau} \exp\left[N_\tau\left(4u^4 + 12u^5 - 14u^6 - 36u^7 + \frac{295}{2}u^8 + \frac{1851}{10}u^9 + \frac{1055797}{5120}u^{10}\right)\right]$

The 3d effective lattice theory, leading interactions

$$Z = \int DW \prod_{<\mathbf{x},\mathbf{y}>} \left[1 + \lambda_1 (L_{\mathbf{x}} L_{\mathbf{y}}^* + L_{\mathbf{x}}^* L_{\mathbf{y}})\right] \qquad \text{pure gauge}$$

$$\times \prod_{\mathbf{x}} [1 + h_1 L_{\mathbf{x}} + h_1^2 L_{\mathbf{x}}^* + h_1^3]^{2N_f} [1 + \bar{h}_1 L_{\mathbf{x}}^* + \bar{h}_1^2 L_{\mathbf{x}} + \bar{h}_1^3]^{2N_f} \qquad \text{stat. det.} \sim \kappa_s^0$$

$$\times \prod_{<\mathbf{x},\mathbf{y}>} \left[1 - 2N_f h_2 \left(\mathrm{Tr}\frac{h_1 W_{\mathbf{x}}}{1 + h_1 W_{\mathbf{x}}} - \mathrm{Tr}\frac{\bar{h}_1 W_{\mathbf{x}}^\dagger}{1 + \bar{h}_1 W_{\mathbf{x}}^\dagger}\right) \left(\mathrm{Tr}\frac{h_1 W_{\mathbf{y}}}{1 + h_1 W_{\mathbf{y}}} - \mathrm{Tr}\frac{\bar{h}_1 W_{\mathbf{y}}^\dagger}{1 + \bar{h}_1 W_{\mathbf{y}}^\dagger}\right)\right] \qquad \text{kinetic det.} \sim \kappa_s^2$$

$$\times \ldots \qquad O(\kappa_s^4)$$

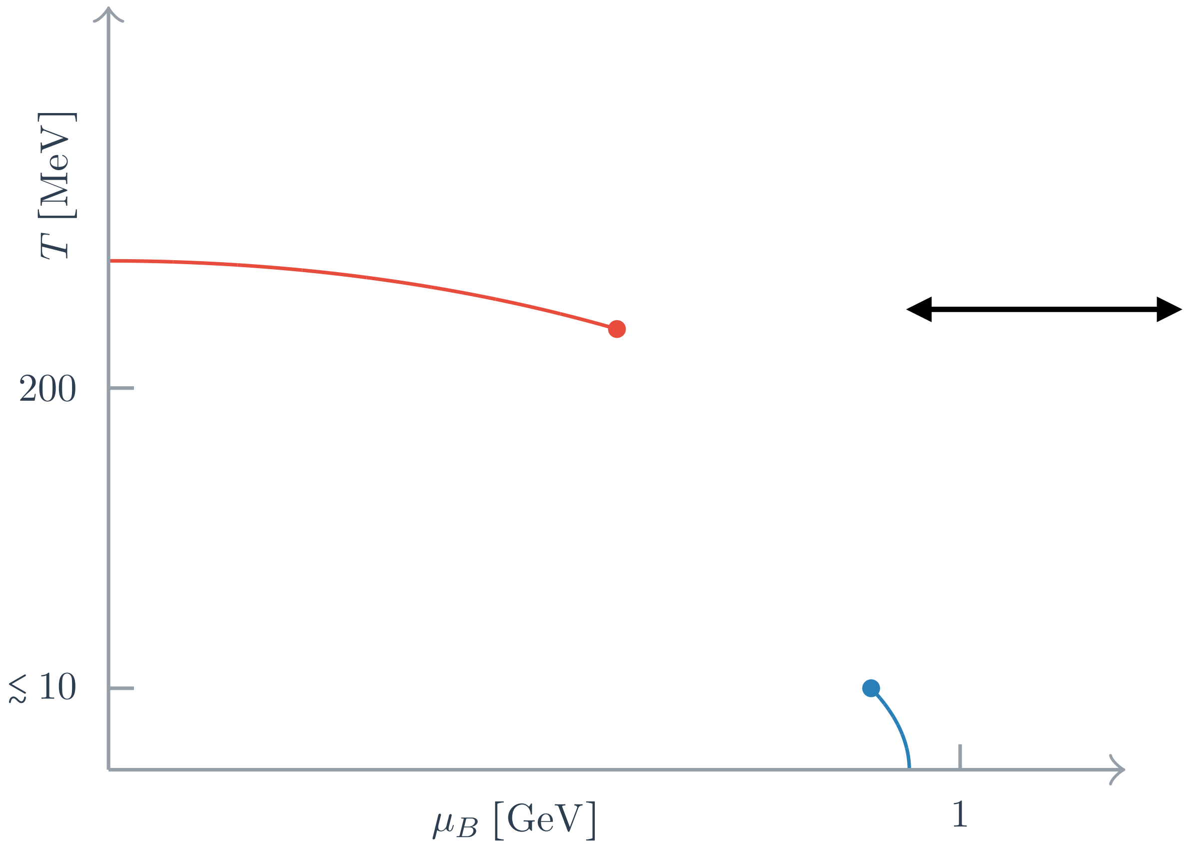
$$W_{\mathbf{x}} = \prod_{\tau=1}^{N_\tau} U_0(\tau, \mathbf{x}), \quad L(\mathbf{x}) = \mathrm{Tr} W(\mathbf{x}), \quad DW = \prod_{\mathbf{x}} dW(\mathbf{x})$$

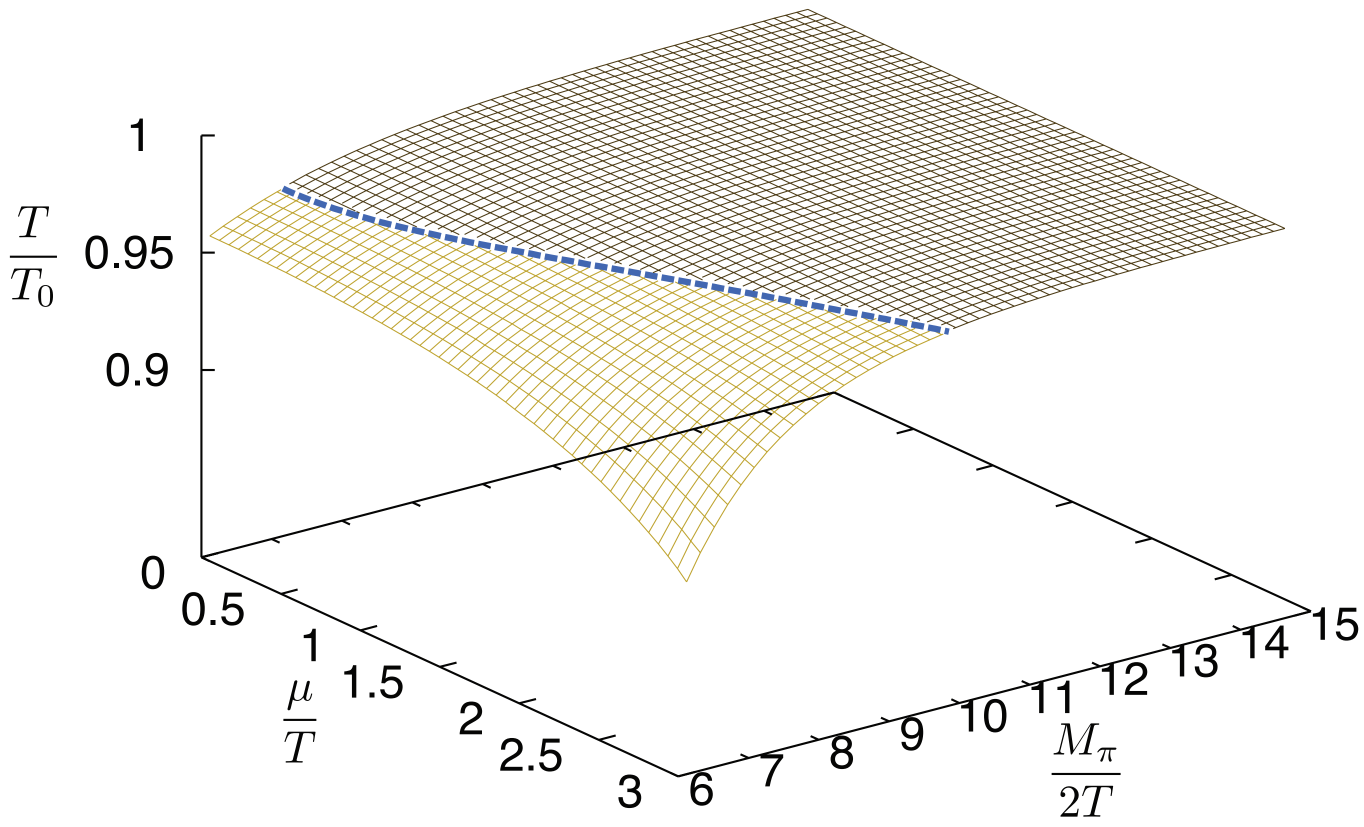
This is a class of 3d SU(3) spin models!

The fully calculable deconfinement transition

(no continuum limit yet)

"Heavy QCD" phase diagram



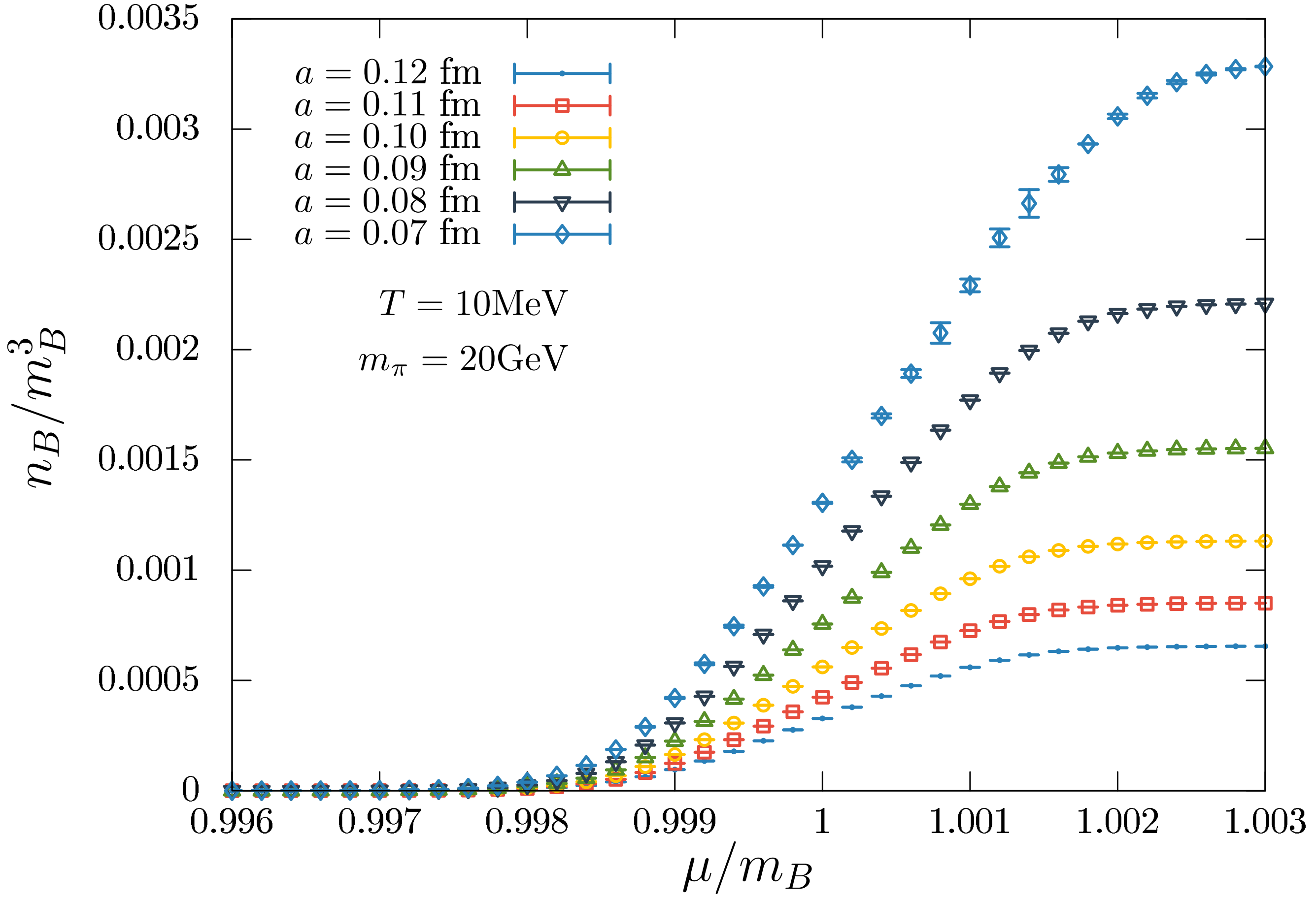


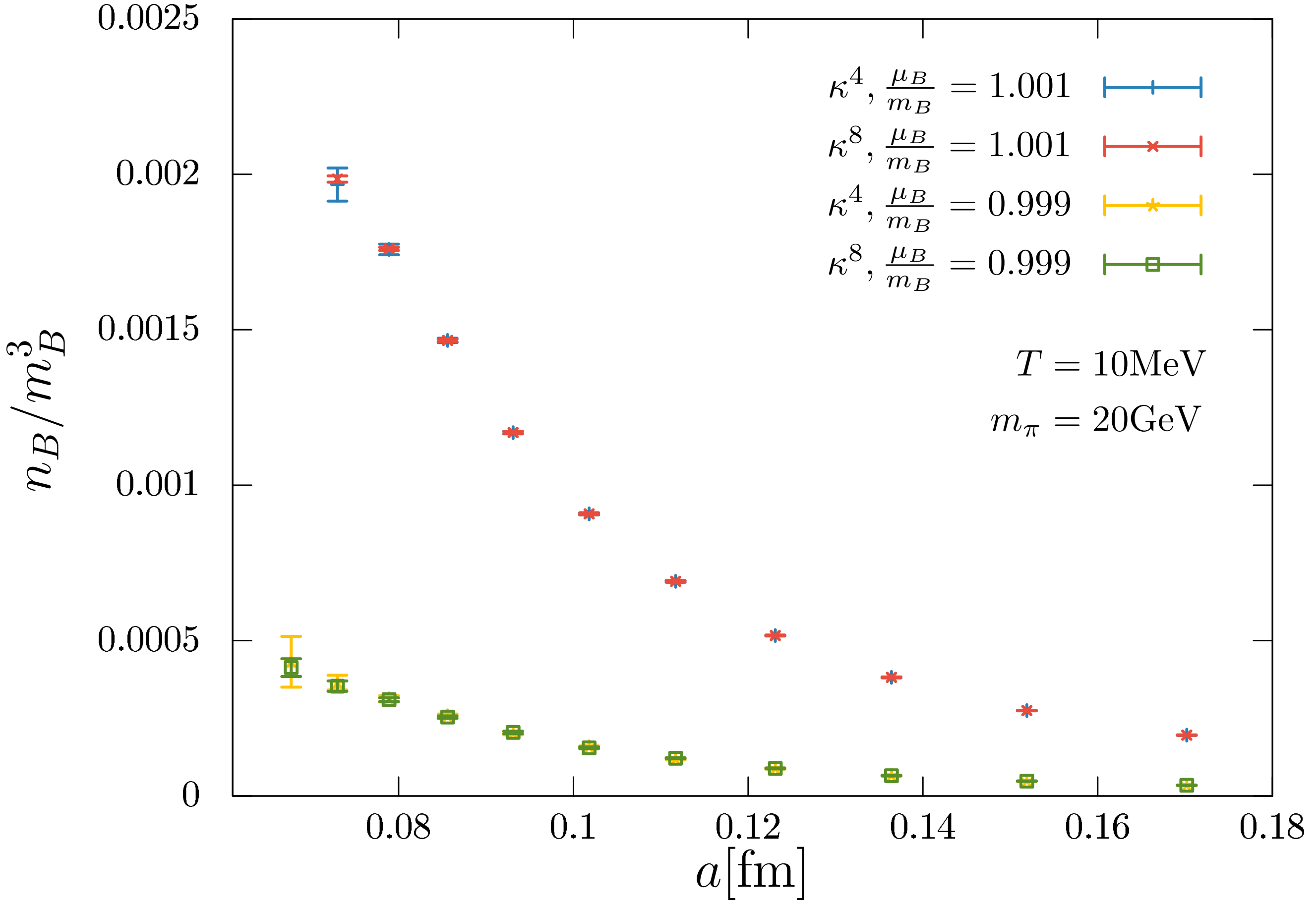
Zero density agrees within 10% with full lattice simulations on Nt=6!

[Fromm, Langelage, Lottini, O.P. JHEP (2012)]

Cold and dense regime

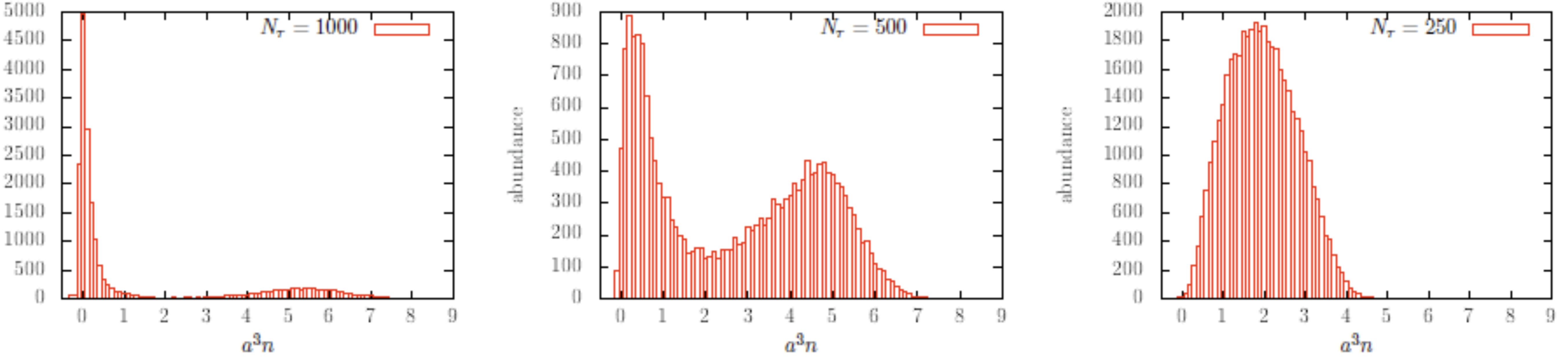
[Fromm, Langelage, Lottini, Neuman, O.P. , PRL 13, Glesaaen, Neuman, O.P. , JHEP 15]

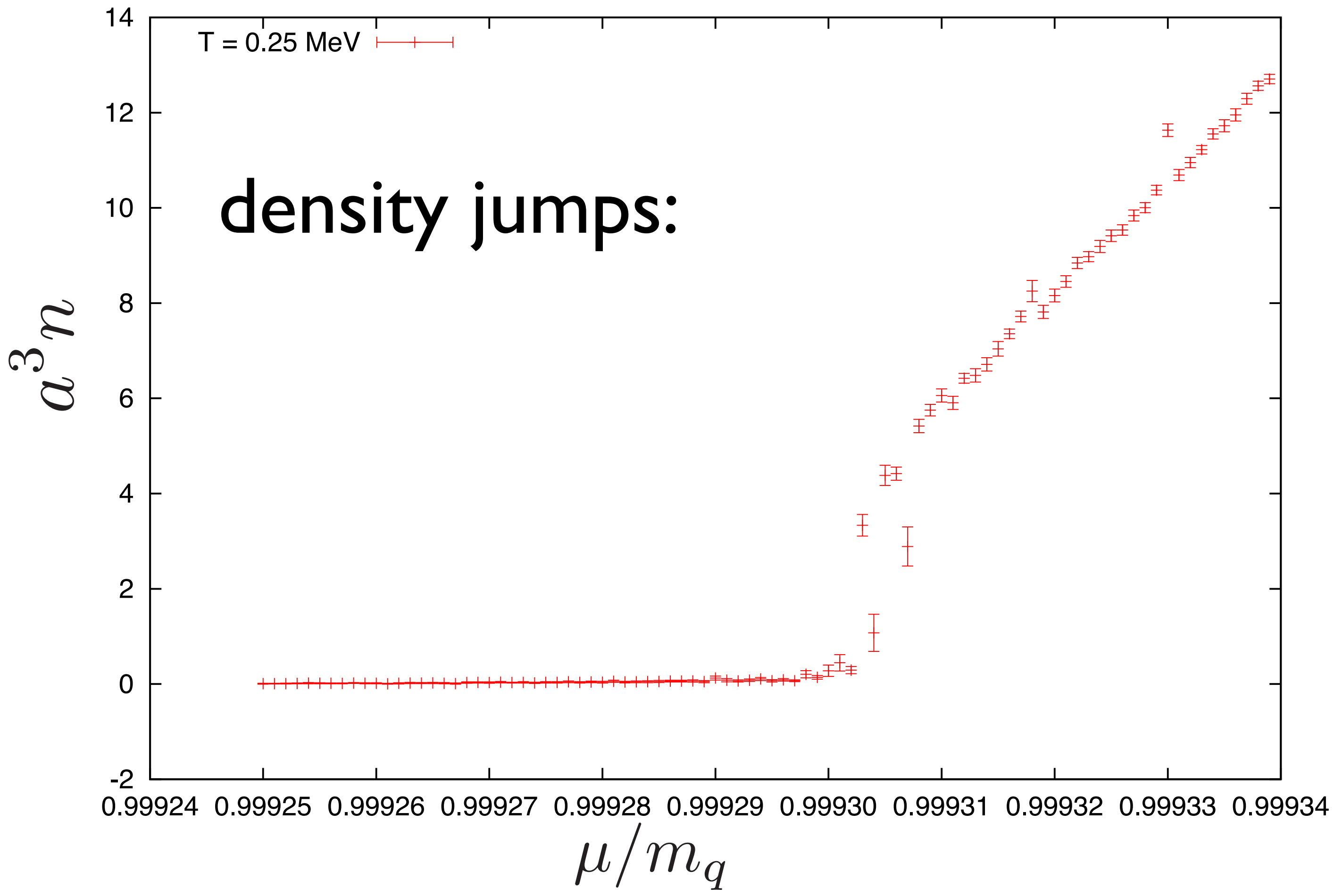




- Continuum approach ~a as expected for Wilson fermions
- Cut-off effects grow rapidly beyond onset transition: lattice saturation!
- Finer lattice necessary for larger densities!

Light quarks: first order transition + endpoint

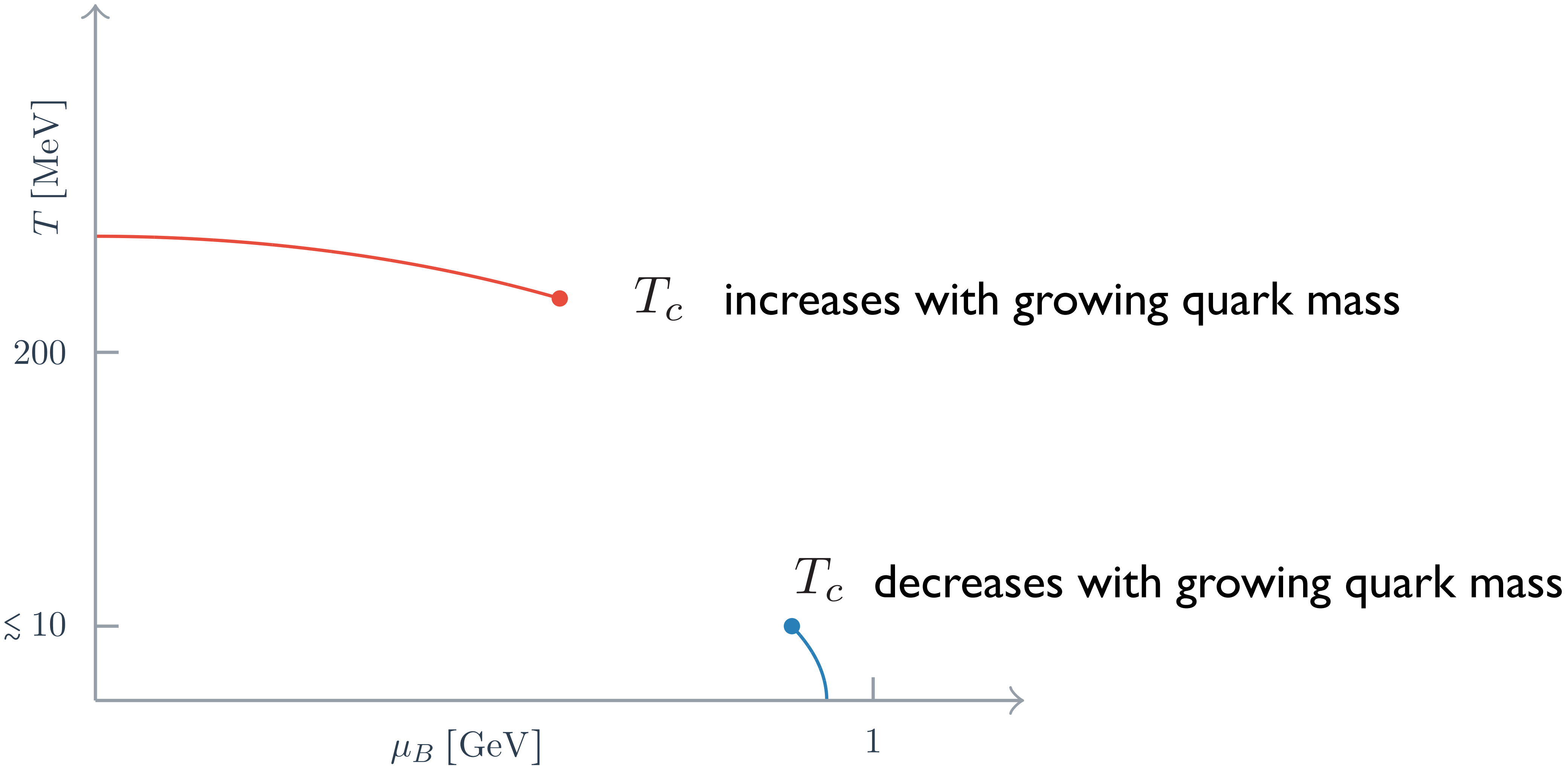




density jumps:

- phase coexistence: first order
- for higher $T = \frac{1}{aN_\tau}$ crossover
- nuclear liquid gas transition!

Phase diagram of heavy quark QCD



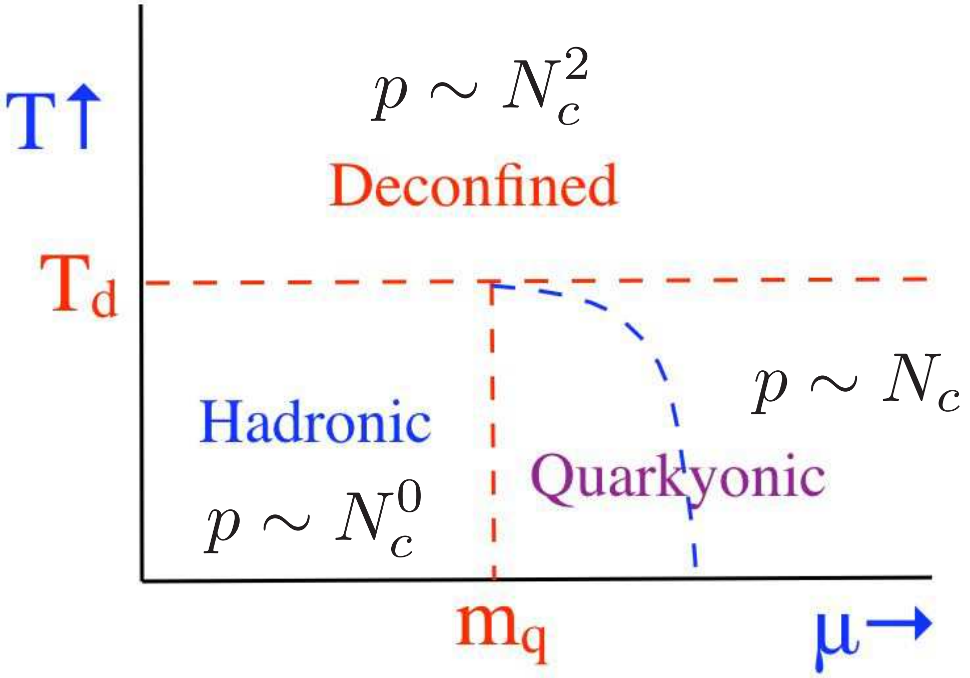
T_c increases with growing quark mass

T_c decreases with growing quark mass

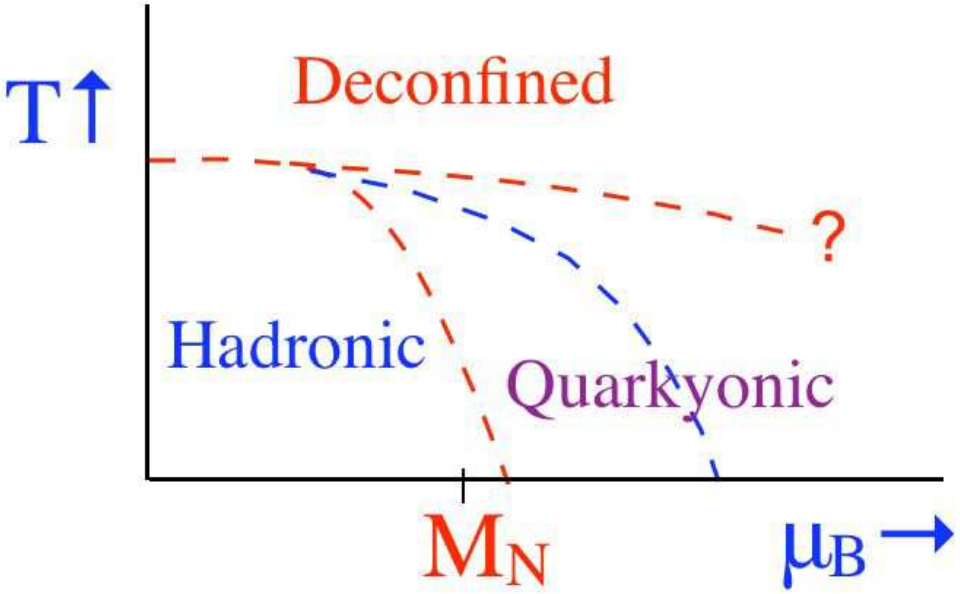
The large N_c QCD phase diagram

[McLerran, Pisarski NPA (2007), …]

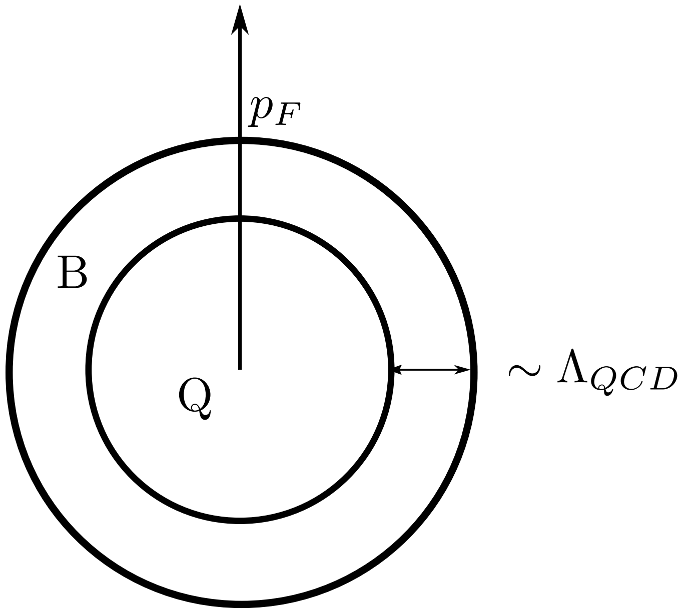
large N_c



$N_c = 3$



Quarkyonic matter in momentum space:



Fermi sea of quarks, surrounded by Fermi shell of baryons;

$p_F \sim \mu$ can interpolate from purely baryonic to quark matter

From conjecture to calculation: eff. theory for general N_c

Strong coupling limit

[O.P., Scheunert JHEP (2019)]

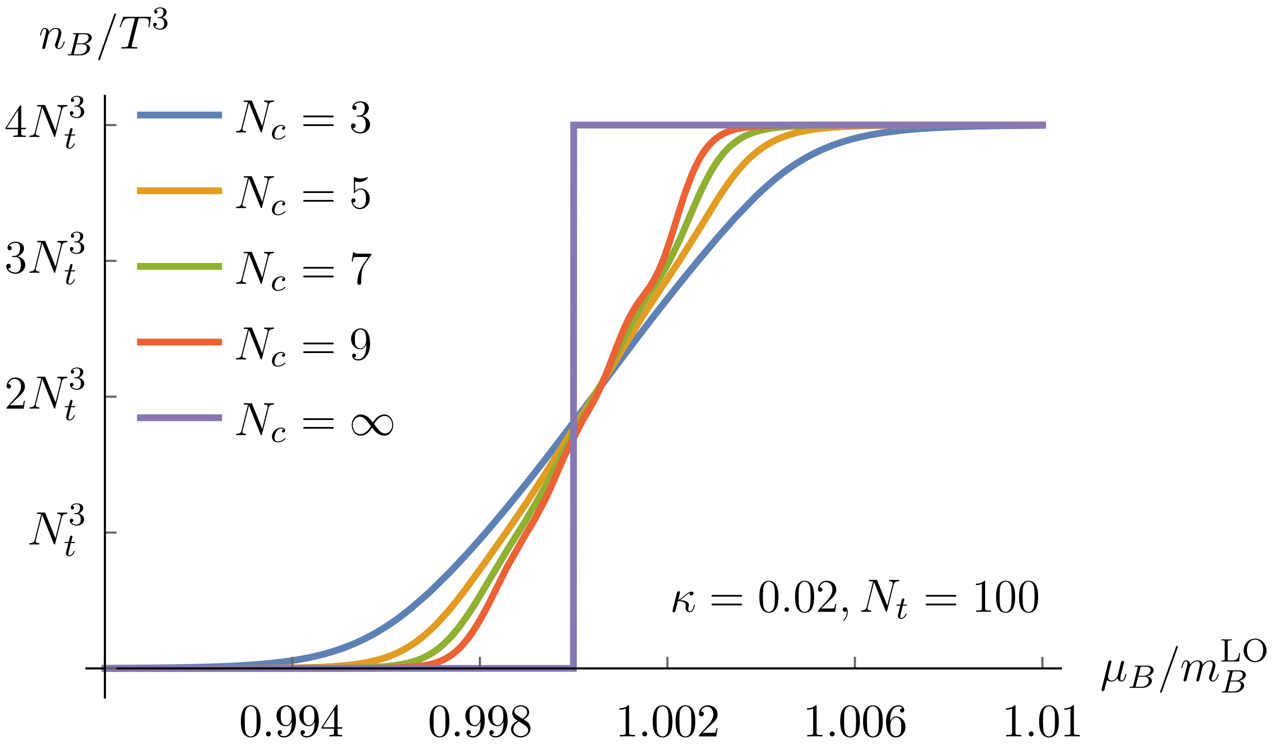
Order hopping expansion		κ^0	κ^2	κ^4
$h_1 < 1$ $(\mu_B < m_B)$	$a^4 p$	$\sim \frac{1}{6N_\tau} N_c^3 h_1^{N_c}$	$\sim -\frac{1}{48} N_c^7 h_1^{2N_c}$	$\sim \frac{3N_\tau \kappa^4}{800} N_c^8 h_1^{2N_c}$
	ϵ	0	$\sim -\frac{1}{4} N_c^3 h_1^{N_c}$	
$h_1 > 1$ $(\mu_B > m_B)$	$a^4 p$	$\sim \frac{4\ln(h_1)}{N_\tau} N_c$	$\sim -12 N_c$	$\sim 198 N_c$
	ϵ	0	~ -6	

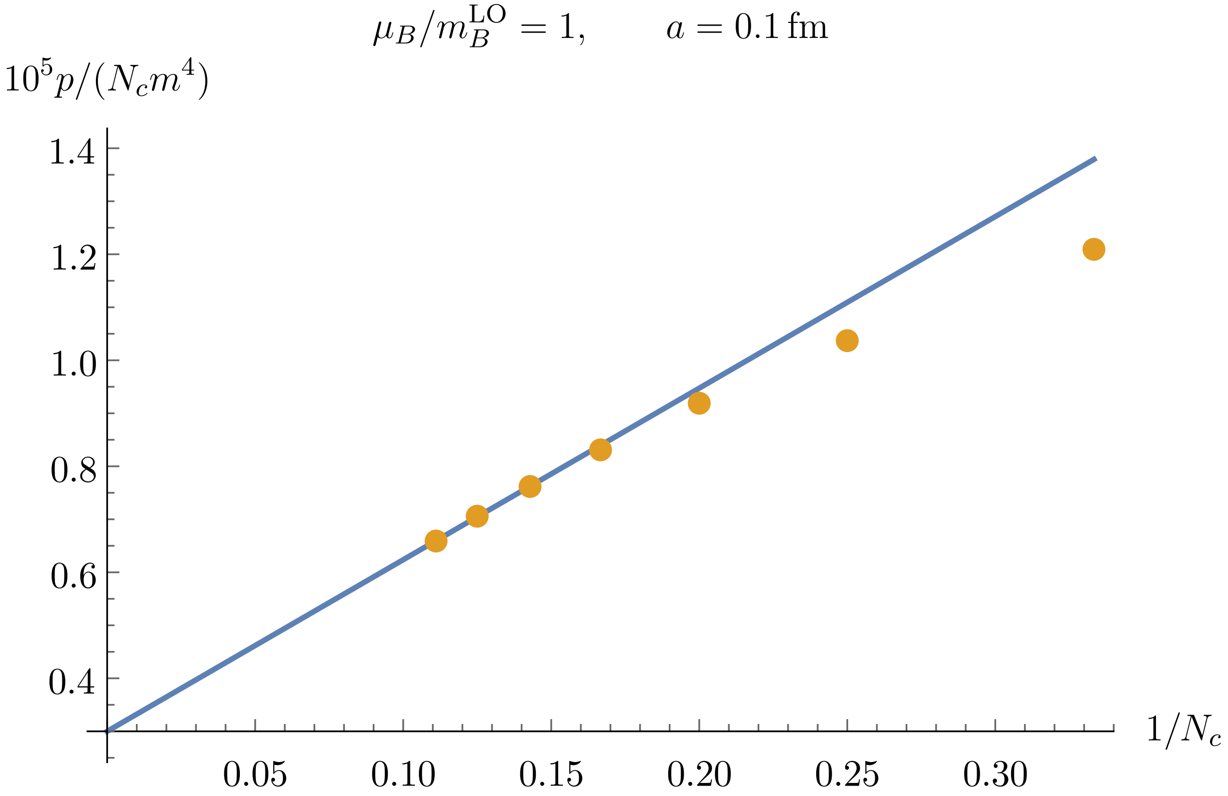
Beyond the onset transition: $p \sim N_c$ definition of quarkyonic matter!

The baryon onset transition for growing N_c

Transition becomes more strongly 1st-order for every T!

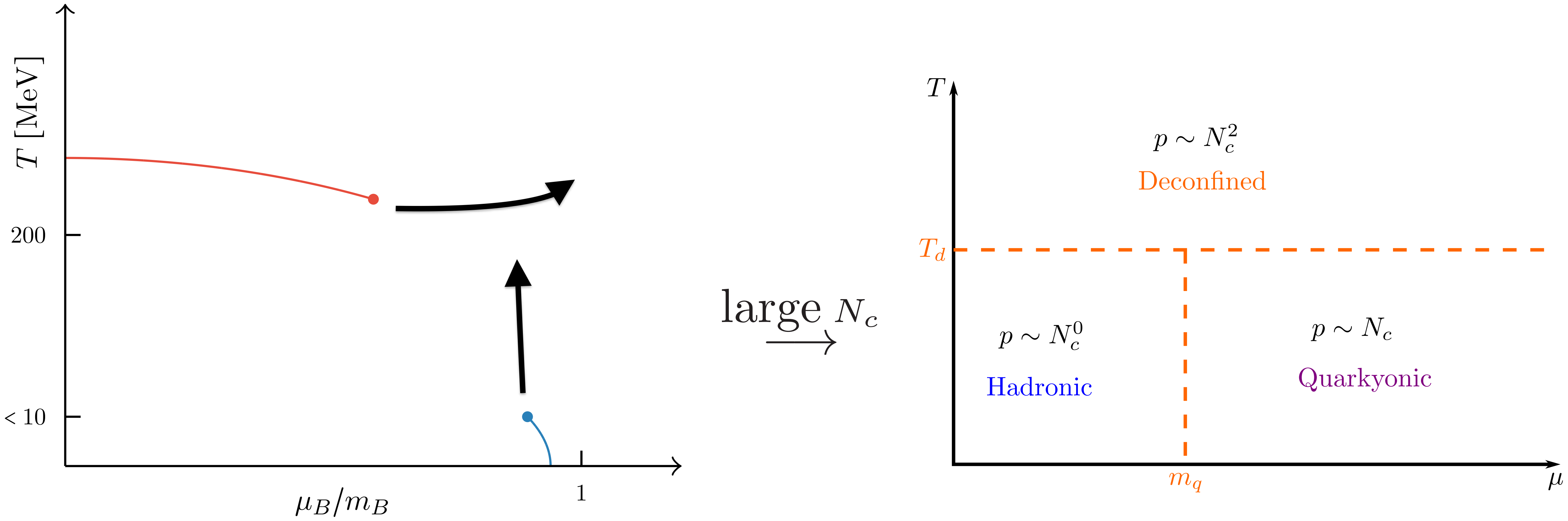
Pressure scaling right after onset





$$p \sim N_c(1 + \mathrm{const.} N_c^{-1})$$

Altogether:



- Large N_c phase diagram emerges smoothly
- Varying N_c : dense QCD is consistent with quarkyonic matter
- Should also hold for light quarks, N_c-scaling is property of expansion coefficients!

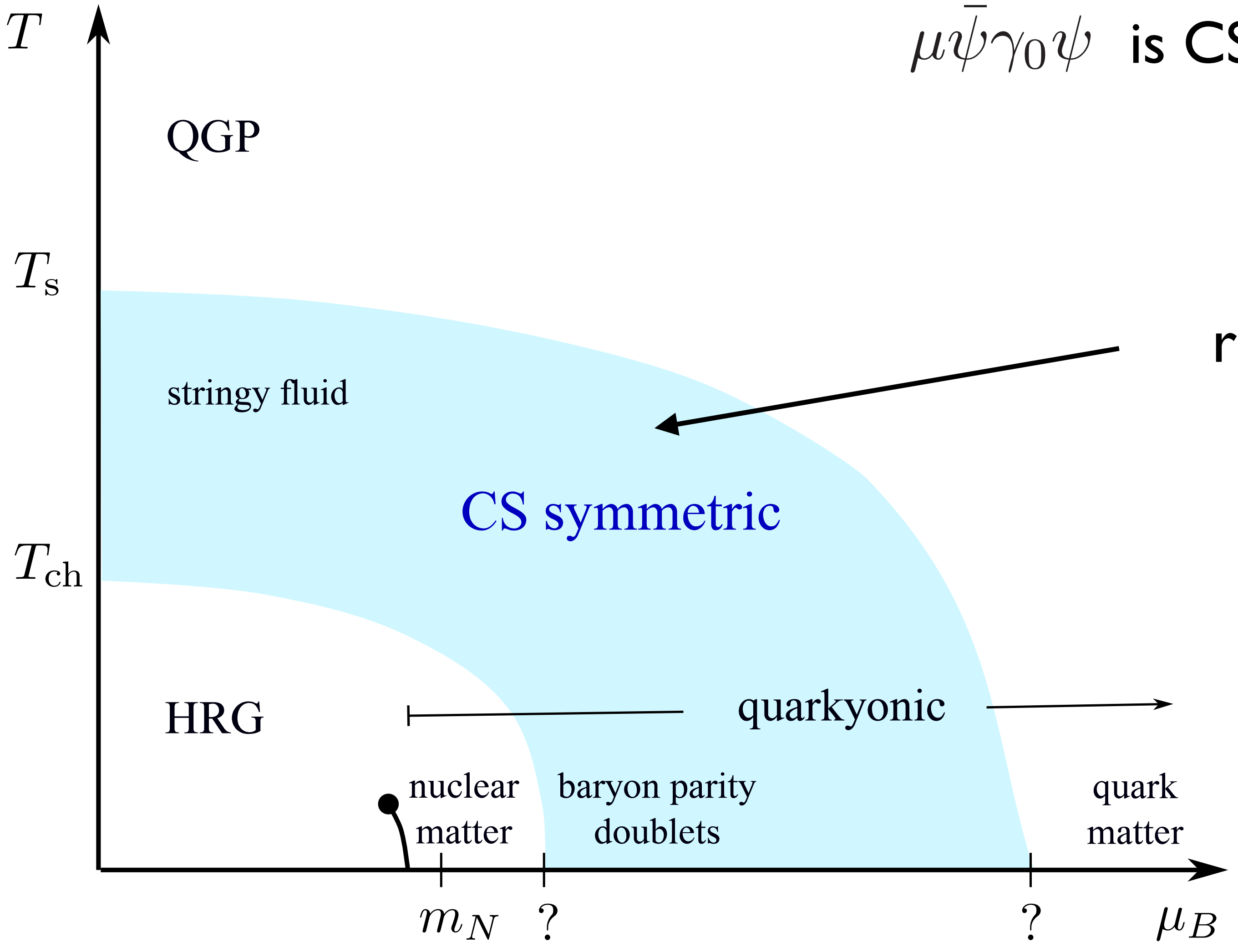
If so: physical baryon matter is special case of quarkyonic matter!

See also phenomenological evidence: [Koch, McLerran, Miller, Vovchenko, PRC 24]

Implications for physical QCD?

One viable scenario (more possibilities): [L. Glozman, R. Pisarski, O.P., EJPA 22]

$\mu\bar{\psi}\gamma_0\psi$ is CS symmetric



resonance-like states

Consistent with neutron star data analysis in Wolfram Weise's talk!

- 1st order transition in neutron star "unlikely", consistent with quark-hadron continuity
- Ordinary nuclear matter (a few times nucl. density) consistent with mass-radius relations

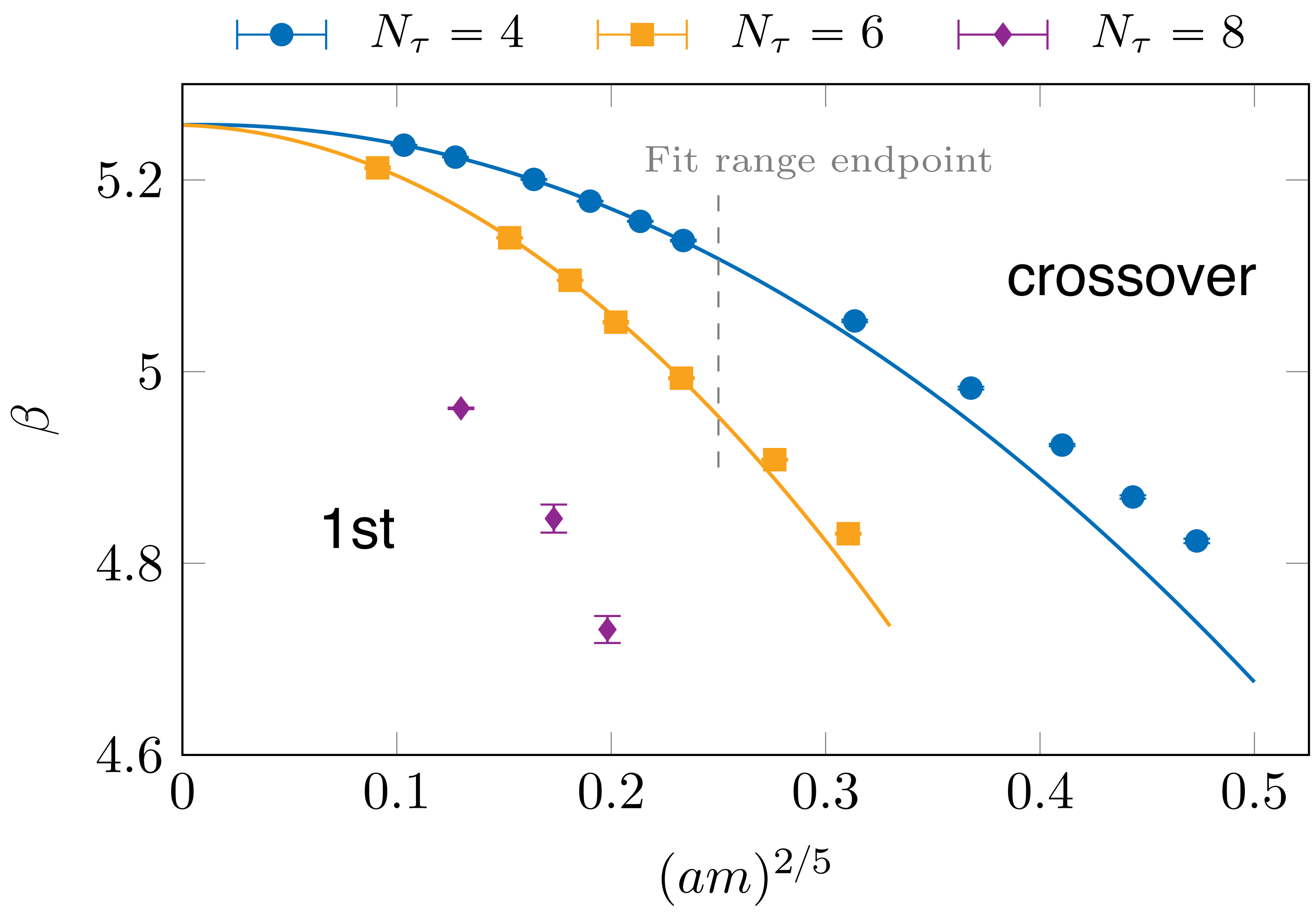
Conclusions

- Chiral transition at zero density is likely 2nd order for Nf=2-7 massless quark flavours
- Imaginary chemical potential has no effect on the order of the chiral transition
- There is an intermediate T-regime with chiral-spin symmetry and predominantly resonance-like degrees of freedom: correlator multiplets, screening masses, spectral functions
- Heavy mass LQCD consistent with quarkyonic matter and quark-hadron continuity

Backup slides

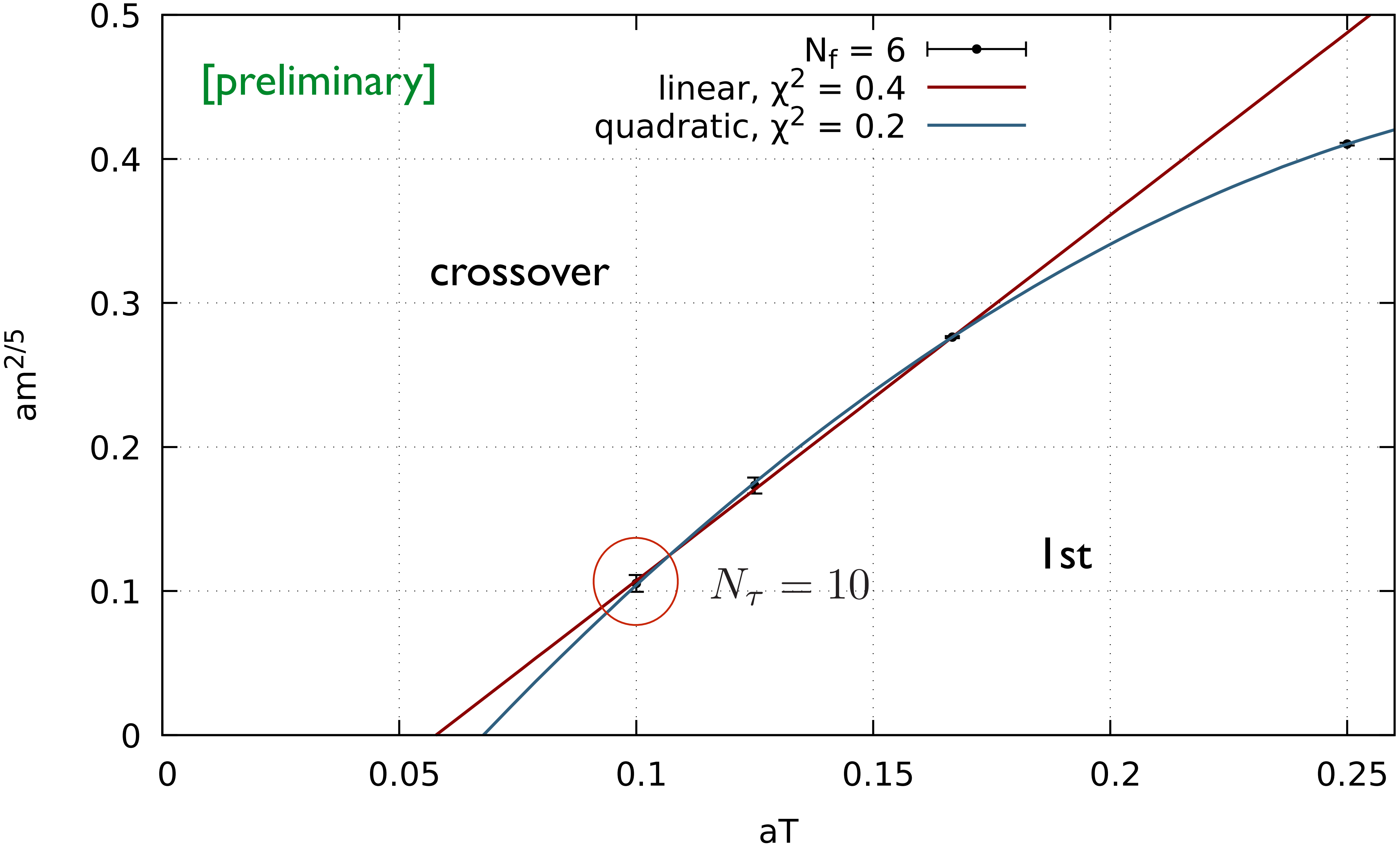
Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra JHEP 21] ~120 M Monte Carlo trajectories with light fermions,
aspect ratios 3,4,5



- Data points implicitly labeled by Nf
- Tricritical scaling observed in lattice bare parameter space
- Tricritical extrapolation always possible!

progressing to finer lattices



New $N_\tau = 10$ result on predicted scaling curve!

Meson screening masses at high temperatures

[Dalla Brida et al., JHEP 22]

Nf=3, T=1 GeV -160 GeV

Highly non-trivial technically:
shifted b.c. + step-scaling techniques

$$\frac{m_{PS}}{2\pi T} = 1 + p_2\,\hat{g}^2(T) + p_3\,\hat{g}^3(T) + p_4\,\hat{g}^4(T)$$

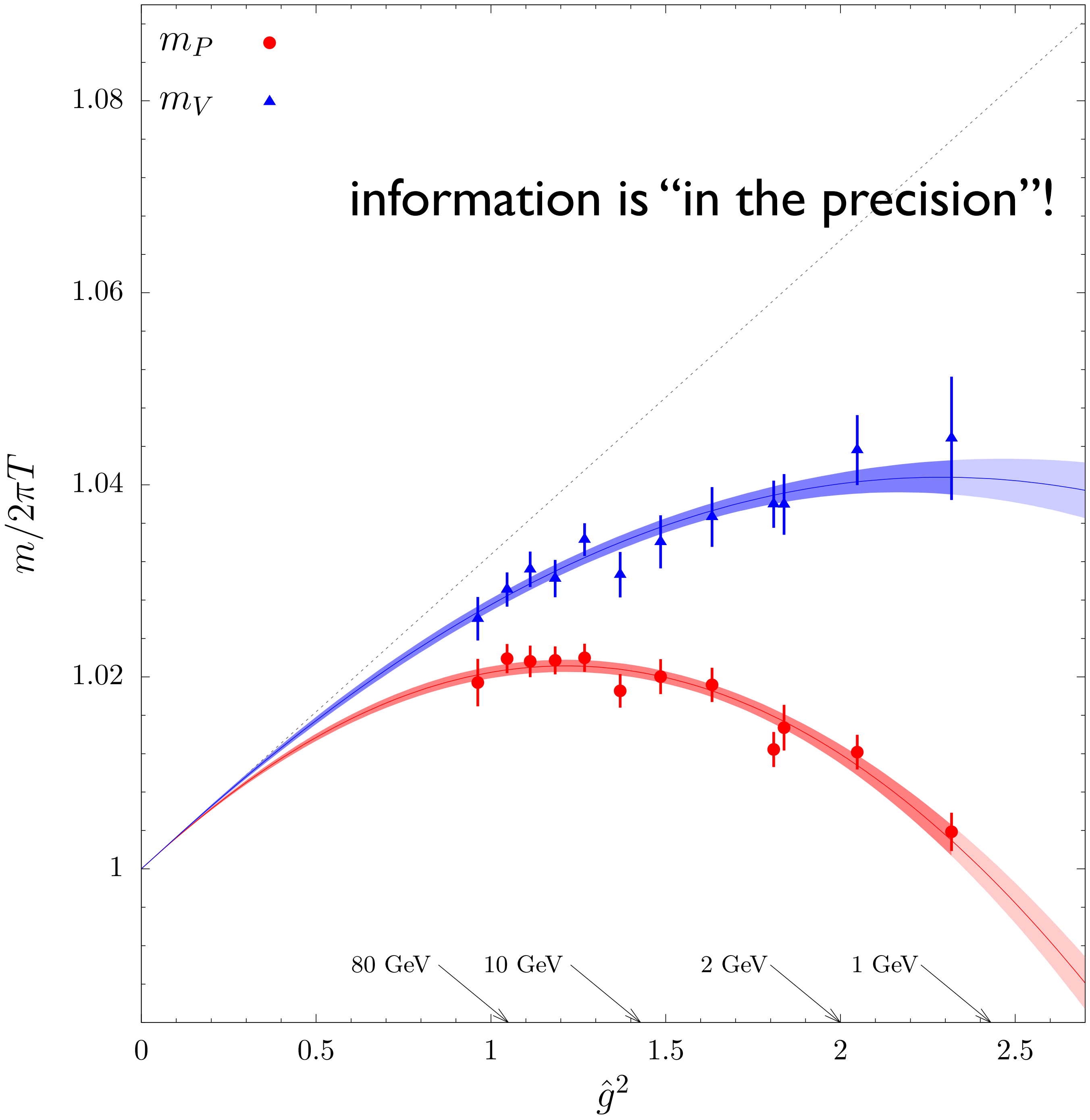
$$\frac{m_V}{2\pi T} = \frac{m_{PS}}{2\pi T} + s_4\,\hat{g}^4(T)$$

$p_2 = 0.032739961$

[Laine, Vepsäläinen., JHEP 04]

p_3, p_4, s_4 fitted, excellent $\chi^2_{\rm dof}$

Quark hadron duality holds



$$\frac{1}{\hat{g}^2(T)} \equiv \frac{9}{8\pi^2}\ln\frac{2\pi T}{\Lambda_{\overline{\rm MS}}} + \frac{4}{9\pi^2}\ln\left(2\ln\frac{2\pi T}{\Lambda_{\overline{\rm MS}}}\right)$$

Comparison with plasmon ansatz

Bros+Buchholz Ansatz

Perturbative plasmon: Breit-Wigner shape

Both fit spatial correlator

$$\rho_{\rm PS}(\omega,\mathbf{p}=0)=\epsilon(\omega)\left[\theta(\omega^2-m_\pi^2)\,\frac{4\,\alpha_\pi\,\gamma_\pi\sqrt{\omega^2-m_\pi^2}}{(\omega^2-m_\pi^2+\gamma_\pi^2)^2}+\theta(\omega^2-m_{\pi^*}^2)\,\frac{4\,\alpha_{\pi^*}\,\gamma_{\pi^*}\sqrt{\omega^2-m_{\pi^*}^2}}{(\omega^2-m_{\pi^*}^2+\gamma_{\pi^*}^2)^2}\right]$$

$$\rho_{PS}^{BW}(\omega,\mathbf{p}=0)=\frac{4\alpha_\pi\omega\Gamma_\pi}{(\omega^2-m_\pi^2-\Gamma_\pi^2)^2+4\omega^2\Gamma_\pi^2}+\frac{4\alpha_\pi^*\omega\Gamma_{\pi^*}}{(\omega^2-m_{\pi^*}^2-\Gamma_{\pi^*}^2)^2+4\omega^2\Gamma_{\pi^*}^2}$$

Predicted temporal correlators: