User driven quantum simulations with Rydberg atoms

Yannick Meurice

The University of Iowa
with James Corona, Avi Kaufman, Zane Ozzello and M. Asaduzzaman
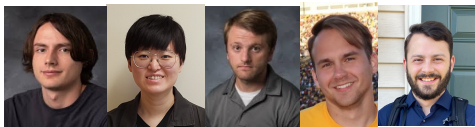QuLAT collaboration
yannick-meurice@uiowa.edu

HHIQCD2024 YITP 11/14

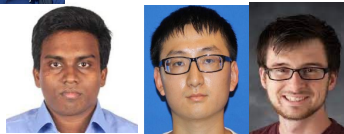# Students and postdocs

U. Iowa Group

- **Undergrads:** Avi Kaufman, Aditya Venkatesh, Jaeho Kim
- **Grad. students:** James Corona, Zheyue Hang, Michael Hite, Zane Ozzello, Blake Senseman, ...
- **PostDocs:** M. Asaduzzaman, Jin Zhang (now Chongqing U.) and Kenny Heitritter (now qBraid)

High-Energy Physics:
- B-meson decays with lattice gauge theory
- Composite models for the Higgs boson
- Applications of the Renormalization Group
- Tensor formulations of lattice field theory
- Machine learning in MC simulations

Quantum computing:
- Real-time evolution of field theory models
- Quantum computing (IBMQ, trapped ions)
- Quantum simulations (Rydberg, QuEra)

Former graduate students
Yuzhi Liu (Ph. D. 2013): postdocs at U. Colorado Boulder and Indiana U.; Software engineer at Google

Haiyuan Zou (Ph. D. 2014): postdocs at Pittsburgh U. and T. D. Lee Center; Assistant Prof. at East China Normal Un.

J. Unmuth-Yockey (Ph. D. 2017): postdocs at Syracuse U. and Fermilab.

Z. Gelzer (Ph. D. 2017): postdoc at U. Illinois UC.

D. Floor (Ph. D. 2018), Software engineer in Brazil.

S. Foreman (Ph. D. 2019): postdoc at Argonne Nat. Lab.

E. Gustafson (Ph. D. 2021): postdoc at Fermilab

D. Simons (Ph. D. 2023); R. Maxton (Ph. D. 2023)

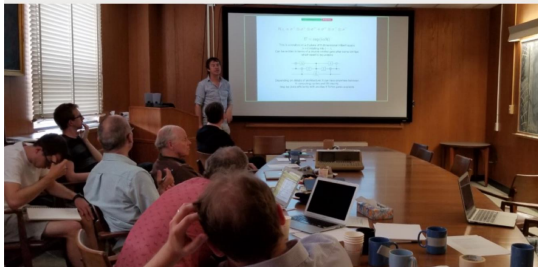# QuLAT Collaboration

https://qulat.sites.uiowa.edu/

IOWA

**Foundations of Quantum
Computing for Gauge Theories and
Quantum Gravity - The QuLAT
Collaboration**

About   Abstrac

Supported by the Department of Energy (QuantISED HEP)

Goals of the collaboration

Quantum computers are expected to exceed the capacity of classical computers and to revolutionize several aspects of computation especially for the simulation of quantum systems. We develop new methods for using quantum computers to study aspects of the evolution of strongly interacting particles in collisions, the quantum behavior of gravitational systems and the emergence of space-time which are beyond the reach of classical computing. Our goal is to design the building blocks of universal quantum computers relevant for these problems and develop algorithms which scale reasonably with the size of the system.

Principal Investigators

Alexei Bazavov, Michigan State University
David Berenstein and Xi Dong, UCSB
Richard Brower, Boston University
Simon Catterall, Jay Hubisz, Alex Maloney
Jason Pollack, Syracuse University
Patrick Dreher, NCSU
Yannick Meurice, Univ. of Iowa (Spokesperson)

Contents

- Short review of the compact Abelian Higgs Model (Scalar QED) and its Rydberg atom simulator (PRD 104, 094513 (2021))
- Effective theory for the simulator (with J. Zhang and S.W. Tsai PRD 110, 034513 (2024))
- Phase diagram of the simulator
- First experimental observation of a floating phase (with QuEra,J. Zhang and S.W. Tsai, arXiv 2401.08087)
- Estimation of entanglement using mutual information of experimental bitstrings (arXiv 2404.09935)
- Improved entanglement estimators using "filtered" probabilities (arXiv 2411.07092)
- Asymptotic behavior of low probabilities (divergent in a non-normalizable way in the infinite volume limit?)
- Classical competition for real-time evolution: HOTRG (arXiv 2411.05301)
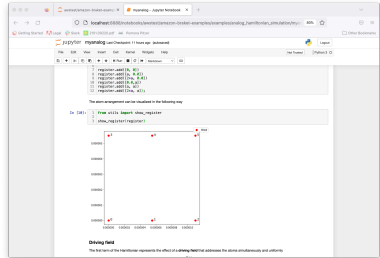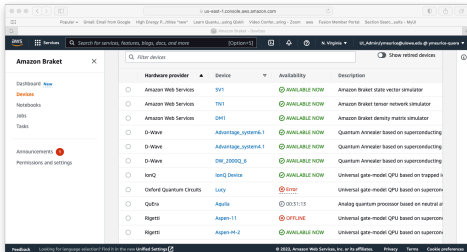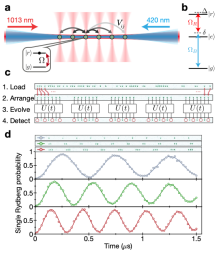
QuEra
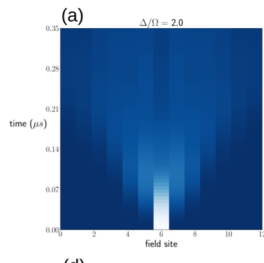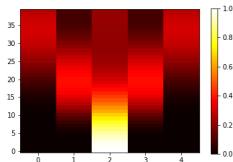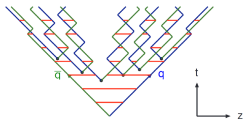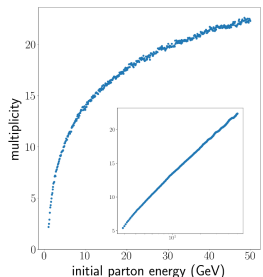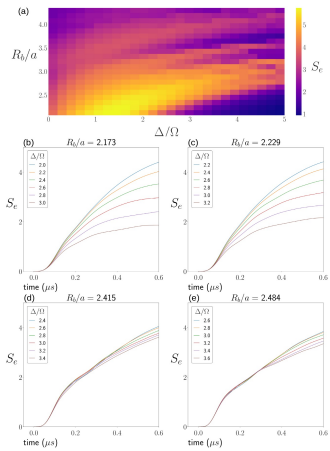
Evolution of a particle-antiparticle pair with the Lund model/Pythia (left), the Abelian Higgs model in 1+1 dimensions (middle), and a Rydberg atom simulator for this model (right).
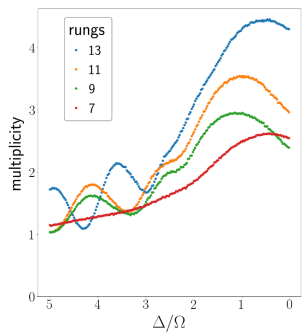
Pythia/Lund

Entanglement Entropy (simulator)

First attempt on a small lattice
with detuning playing the role
of energy
(see arxiv 2212.02476)

Compact Abelian Higgs Model (CAHM)

The lattice compact Abelian Higgs model is a non-perturbative regularized formulation of scalar quantum electrodynamics (scalar electrons-positrons + photons with compact fields).

$$Z_{CAHM} = \prod_x \int_{-\pi}^{\pi} \frac{d\varphi_x}{2\pi} \prod_{x,\mu} \int_{-\pi}^{\pi} \frac{dA_{x,\mu}}{2\pi} e^{-S_{gauge} - S_{matter}},$$

$$S_{gauge} = \beta_{plaquette} \sum_{x,\mu < \nu} (1 - \cos(A_{x,\mu} + A_{x+\hat{\mu},\nu} - A_{x+\hat{\nu},\mu} - A_{x,\nu})),$$

$$S_{matter} = \beta_{link} \sum_{x,\mu} (1 - \cos(\varphi_{x+\hat{\mu}} - \varphi_x + A_{x,\mu})).$$

- local invariance: $\varphi'_x = \varphi_x + \alpha_x$ and $A'_{x,\mu} = A_{x,\mu} - (\alpha_{x+\hat{\mu}} - \alpha_x)$.
- φ is the Nambu-Goldstone mode of the original model. The Brout-Englert-Higgs mode is decoupled (heavy).

The continuous-time limit yields the Hamiltonian

$$H = \frac{U}{2} \sum_{i=1}^{N_s} (L_i^z)^2 + \frac{Y}{2} \sum_i (L_{i+1}^z - L_i^z)^2 - X \sum_{i=1}^{N_s} U_i^x$$

with $U^x \equiv \frac{1}{2}(U^+ + U^-)$ and $L^z|m\rangle = m|m\rangle$ and $U^{\pm}|m\rangle = |m \pm 1\rangle$.

- m is a discrete electric field quantum number ($-\infty < m < +\infty$)
- In practice, we need to apply truncations: $U^{\pm}| \pm m_{max}\rangle = 0$.
- We focus on the spin-1 truncation ($m = \pm 1, 0$ and $U^x = L^x/\sqrt{2}$.)
- U-term: electric field energy.
- Y-term: matter charges (determined by Gauss's law)
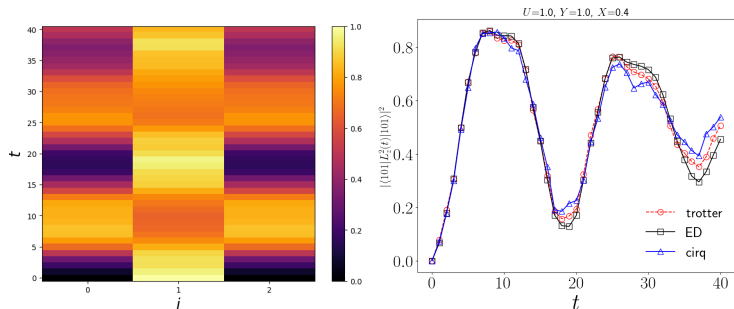- X-term: currents inducing temporal changes in the electric field.

$$H = \frac{U}{2} \sum_{i=1}^{N_s} \left(L_i^z\right)^2 + \frac{Y}{2} \sum_i (L_{i+1}^z - L_i^z)^2 - X \sum_{i=1}^{N_s} U_i^x$$

Initial state: particle-antiparticle (connected by an electric field +1)
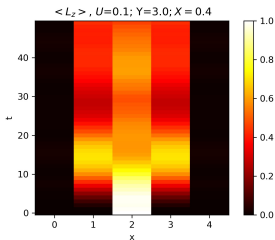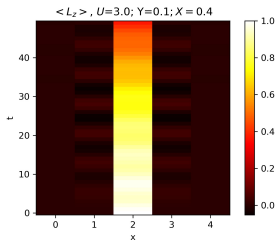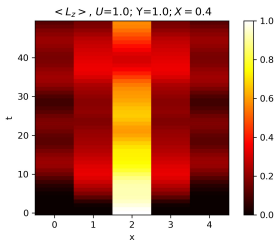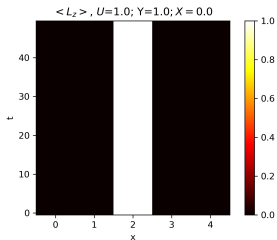
Figure: Left: Trotter evolution of the electric field in 3-spin scalar QED. Right: Comparison with ED and Cirq for the left site (M. Asaduzzaman). Experimental work in progress with M. Blok group (Rochester) plus N. Cross and R. Naik (Advanced Quantum Testbed, LBNL).

Configurable Arrays of Rydberg Atoms (CARA)

- One can adapt (Y.M., PRD 104) the optical lattice construction (J. Zhang et al. PRL 121, A. Bazavov et al. PRD 92) using ^{87}Rb atoms separated by controllable (but not too small) distances, coupled to the excited Rydberg state $|r\rangle$ with a detuning Δ to quantum simulate the Abelian Higgs model.
- The ground state is denoted $|g\rangle$ and the two possible states $|g\rangle$ and $|r\rangle$ can be seen as a qubit. $n|g\rangle = 0$, $n|r\rangle = |r\rangle$.
- The Hamiltonian reads

$$H = \frac{\Omega}{2} \sum_i (|g_i\rangle\langle r_i| + |r_i\rangle\langle g_i|) - \Delta \sum_i n_i + \sum_{i<j} V_{ij} n_i n_j,$$

with

$$V_{ij} = \Omega R_b^6 / r_{ij}^6,$$

for a distance r_{ij} between the atoms labelled as i and j. Note: when $r = R_b$, $V = \Omega$.
- This repulsive interaction prevents two atoms close enough to each other to be both in the $|r\rangle$ state (blockade mechanism)
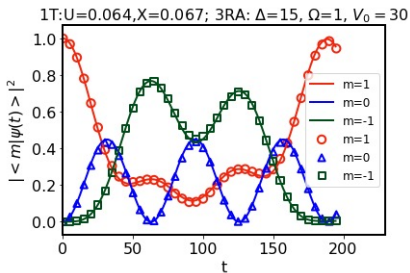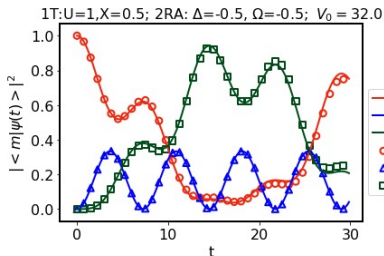
One site spin-1 with 2 and 3 atoms (Y.M. PRD 104)

Solid line: target, Symbols: simulator

QuEra (as seen by users)

Figure: Left: The phase diagram of a single chain quantum simulator with continuously tunable parameters, composed of Rydberg atoms in a study of the Kimble-Zurek mechanism (Keesling et al.). Middle: A heatmap of the specific heat from the Extended-O(2) model with a symmetry-breaking $h_q \cos(q\phi)$ (Hostetler, Bazavov et al., PRD 109, 054514 (2024)). Right: Phase diagram for the Rydberg ladder (Zhang et al., arXiv 2401.08087).
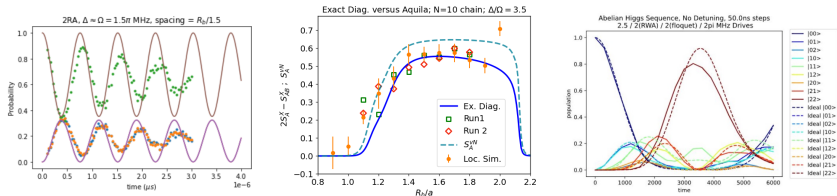
Figure: Left: Beta test of QuEra for two atoms starting in the ground state. Middle: Entanglement proxy $2S_A^X - S_{AB}^X$ vs. R_b/a_x with three methods described in arXiv 2404.09935, empty symbols are Aquila results with 1000 shots (tomorrow's seminar); Right: Example of Floquet engineering for the two qutrit systems developed by M. Blok with undergraduate student Max Niederbach and Ray Parker. The dash lines correspond to a simulation with QuTIP including noise that they expect to implement on their facilities, while the continuous lines are obtained with exact diagonalization.

QuLAT had 2 awards: simulations on hyperbolic spaces (Goksu Con Toga), multipartite entanglement in 2D arays (YM).

Teaching quantum mechanics with public quantum devices?

It is possible to implement "gedanken experiments" in Feynman, Sakurai and Townsend textbooks using universal quantum computers

6 A MODERN APPROACH TO QUANTUM MECHANICS
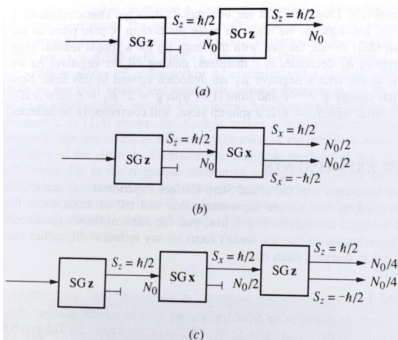
FIGURE 1.3
A block diagram of (a) Experiment 1, (b) Experiment 2, and (c) Experiment 3. N_0 is the number of particles in the beam exiting the first SG device with $S_z = \hbar/2$.


```
In [17]:  1  #Ex[periment 3]
          2  mci = Circuit().h(0).cnot(0,1).ry(0,-np.pi/2).cnot(0,2).ry(0,np.pi/2).cnot(0,3)
          3  print(mci)
          4  # run circuit
          5  nshots=1000
          6  result = device.run(mci, shots=nshots).result()
          7  # get measurement shots
          8  counts = result.measurement_counts
          9  # print counts
         10  print(counts)
         11  # plot using Counter
         12  plt.bar(counts.keys(), counts.values())
         13  plt.xlabel('bitstrings')
         14  plt.ylabel('counts')

T  : |0|1|    2    |3|  4    |5|

q0 : -H-C-Ry(-1.57)-C-Ry(1.57)-C-
        |           |           |
q1 : ---X-----------|-----------|-
                    |           |
q2 : ---------------X-----------|-
                                |
q3 : ---------------------------X-

T  : |0|1|    2    |3|  4    |5|
Counter({'1101': 132, '0000': 129, '1001': 129, '0110': 128, '1011': 124, '0010': 124, '1111': 118, '0100': 116

Out[17]:  Text(0, 0.5, 'counts')
```
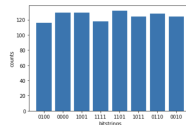
Effective Hamiltonian (with J. Zhang and S. W. Tsai)

represent the four projected states in the z direction for spin 3/2; then, the on-rung interaction can be expressed by $\sum_{u=0}^{3} A_u (\hat{L}_i^z)^u$, where the coefficients A_u can be found by matching the energy spectrum. However, in real quantum systems, the most common terms are the linear term, which is coupled to the external field, and the quadratic term, which is the single-ion anisotropy. In addition, the Rydberg interaction is strong when the rung size is smaller than the Rydberg blockade radius, such that the $|r_{i,1} r_{i,2}\rangle$ state is not likely to appear. Here we only consider a spin-1 realization by mapping the first three states to the spin-1 projected states $|0\rangle$, $|+1\rangle$, and $|-1\rangle$, respectively. The onsite interaction term in spin language is thus $-\Delta (\hat{L}_i^z)^2$. The relation of the z-component spin operator to the Rydberg number operator is defined as [15]

$$\hat{L}_i^z = \hat{n}_{i,+1} - \hat{n}_{i,-1}. \tag{6}$$

If we take the square of this equation, use the property $\hat{n}^2 = \hat{n}$, and drop the term $\hat{n}_{i,+1}\hat{n}_{i,-1}$, which is zero in the low-energy sector, we obtain effectively

$$(\hat{L}_i^z)^2 = \hat{n}_{i,+1} + \hat{n}_{i,-1}. \tag{7}$$

Solving for $\hat{n}_{i,m}$, we get

$$\hat{n}_{i,+1(-1)} = \left[(\hat{L}_i^z)^2 \pm \hat{L}_i^z \right]/2. \tag{8}$$

between atoms in different legs $V_2 = V_0 \rho^6/(1+\rho^2)^3$. The interactions between the $(i, i+1)$ rungs are

$$\hat{H}_{2LR,NN} = \frac{V_1 - V_2}{2} \hat{L}_i^z \hat{L}_{i+1}^z + \frac{V_1 + V_2}{2} (\hat{L}_i^z)^2 (\hat{L}_{i+1}^z)^2. \tag{9}$$

For generic values of ρ, it is necessary to include the long-range interactions. The interactions between the spin at site i and that at site $i+k$ take the same form as Eq. (9) by replacing V_1, V_2 by $V_1^{(k)} = V_0 \rho^6/k^6$, $V_2^{(k)} = V_0 \rho^6/(k^2 + \rho^2)^3$.

Finally, we note that the Rydberg Rabi term can flip the spin projections between $|0\rangle$ and $|\pm 1\rangle$, but there is no direct flipping channel between $|+1\rangle$ and $|-1\rangle$, where the Rabi term is equivalent to the spin-1 ladder operator. In summary, if the rung size of the two-leg ladder is smaller than the Rydberg blockade radius, or $V_0 \gg \Delta, \Omega$, the two-leg Rydberg ladder is an effective spin-1 chain:

$$\begin{aligned}
\hat{H}_{2LR}^{eff} = &-\Delta \sum_{i=1}^{N_s} (\hat{L}_i^z)^2 + \sum_k \left(\frac{V_1^{(k)} - V_2^{(k)}}{2} \sum_{i=1}^{N_s-k} \hat{L}_i^z \hat{L}_{i+k}^z \right. \\
&+ \left. \frac{V_1^{(k)} + V_2^{(k)}}{2} \sum_{i=1}^{N_s-k} (\hat{L}_i^z)^2 (\hat{L}_{i+k}^z)^2 \right) \\
&+ \frac{\Omega}{2} \sum_{i=1}^{N_s} (\hat{U}_i^+ + \hat{U}_i^-). \tag{10}
\end{aligned}$$

We now consider the case where the long-range interactions have a negligible effect and keep only the

Comparison between ladder $H_{eff.}$ and H_{CAHM}
Phys.Rev.D 110 (2024) 3, 034513 arXiv 2312.04436

$H_{CAHM} = \frac{U}{2} \sum_{i=1}^{N_s} \left(L_i^z\right)^2 - Y\sum_{i=1}^{N_s-1} L_{i+1}^z L_i^z - X \sum_{i=1}^{N_s} U_i^x$

$H_{eff.} = -\Delta \sum_{i=1}^{N_s} (L_i^z)^2 + \frac{V_1 - V_2}{2} \sum_{i=1}^{N_s-1} L_i^z L_{i+1}^z + \Omega \sum_{i=1}^{N_s} U_i^x + H_{quartic}$

$H_{quartic} = \frac{V_1 + V_2}{2} \sum_{i=1}^{N_s-1} \left(L_i^z\right)^2 \left(L_{i+1}^z\right)^2.$

Matching

- $\Delta = -U/2$ (sign matters!)
- The coefficient for $L_i^z L_{i+1}^z$ is positive ($V_1 > V_2$) for the simulator (repulsive/antiferromagnetic) but the CAHM has ferromagnetic interactions. This can be remedied by redefining the observable $L_{2i+1}^z \rightarrow -L_{2i+1}^z$ (staggered)
- After redefinition $V_1 = -V_2 = Y > 0$ but $V_2 > 0$
- $\Omega = -X$ (sign does not matter)

Everything agrees with two-rung results (YM, PRD104)
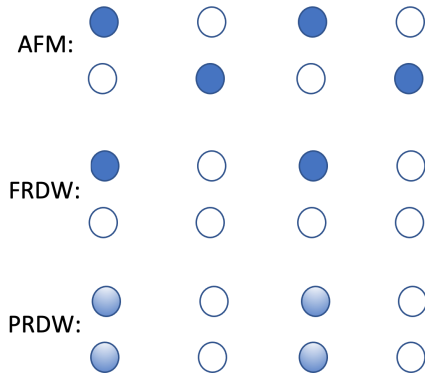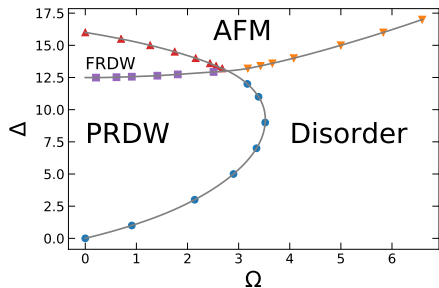
Figure: Ground-state phase diagram for the effective Hamiltonian of the two-leg Rydberg ladder. Here $L = 512$, $V_0 = 1000$, $\rho = dy/dx = 0.5$. The PRDW phase is disordered in even or odd sites, and the FRDW phase is FM in even or odd sites.

Probing quantum floating phases in Rydberg atom arrays

Jin Zhang[1,2,*] Sergio H. Cantú[3,†] Fangli Liu[3,‡] Alexei Bylinskii[3], Boris Braverman[3], Florian Huber[3], Jesse Amato-Grill[3], Alexander Lukin[3], Nathan Gemelke[3], Alexander Keesling[3], Sheng-Tao Wang[3], Y. Meurice[1], and S.-W. Tsai[4]

[1] *Department of Physics and Astronomy, University of Iowa, Iowa City, IA 52242, USA*
[2] *Department of Physics and Chongqing Key Laboratory for Strongly Coupled Physics, Chongqing University, Chongqing 401331, China*
[3] *QuEra Computing Inc., 1284 Soldiers Field Road, Boston, MA, 02135, USA and*
[4] *Department of Physics and Astronomy, University of California, Riverside, CA 92521, USA*
(Dated: February 13, 2024)

The floating phase, a critical incommensurate phase, has been theoretically predicted as a potential intermediate phase between crystalline ordered and disordered phases. In this study, we investigate the different quantum phases that arise in ladder arrays comprising up to 92 neutral-atom qubits and experimentally observe the emergence of the quantum floating phase. We analyze the site-resolved Rydberg state densities and the distribution of state occurrences. The site-resolved measurement reveals the formation of domain walls within the commensurate ordered phase, which subsequently proliferate and give rise to the floating phase with incommensurate quasi-long-range order. By analyzing the Fourier spectra of the Rydberg density-density correlations, we observe clear signatures of the incommensurate wave order of the floating phase. Furthermore, as the experimental system sizes increase, we show that the wave vectors approach a continuum of values incommensurate with the lattice. Our work motivates future studies to further explore the nature of commensurate-incommensurate phase transitions and their non-equilibrium physics.
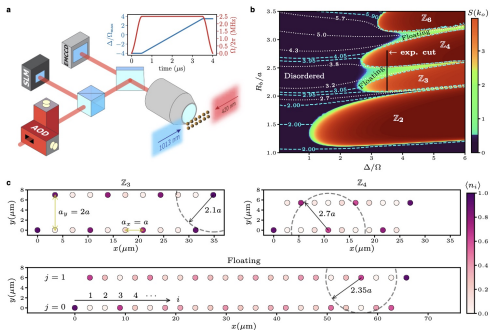
FIG. 1. **Quantum phases of Rydberg atoms arranged in a two-leg ladder.** **a**, Atoms are loaded into a two-leg ladder of optical tweezer traps generated using a SLM and rearranged into defect-free patterns by a second set of moving tweezers using a pair of crossed AODs. Coherent transitions are driven between the ground state $|g\rangle = |5S_{1/2}\rangle$ and the Rydberg state $|r\rangle = |70S_{1/2}\rangle$ in each atom with a two-photon transition induced by lasers at 420 nm and 1013 nm. The inset shows a linear detuning sweep $\Delta(t)$ at a constant Rabi frequency $\Omega_{max} = 2\pi \times 2.5$ MHz for preparing the ground states of the phase diagram via adiabatic evolution. Projection of the many-body quantum state into bitstrings of $|g\rangle$ and $|r\rangle$ for each atom can be detected on an EMCCD camera with the Rydberg state $|r\rangle$ detected as loss of atom. **b**, The ground-state phase diagram for the Rydberg Hamiltonian [Eq. (1)] in a two-leg ladder is shown with lattice spacings $a_x = a$ and $a_y = 2a$. Structure factors $S(k)$ are numerically computed for $1 \le R_b/a \le 3.5$ using DMRG (Supplementary Information). The color map depicts the peak height $S(k_o)$ at $k_o = 2\pi/p$ with p being the wavelength in units of the lattice constant a, while contour lines show the constant-p lines. The $\mathbb{Z}_p$ orders have constant values of integer p, while the floating phase exhibits a continuously varying p. The black line cut corresponds to experimentally measured parameters shown in subsequent figures. **c**, Experimentally measured Rydberg densities illustrate the $\mathbb{Z}_p$ orders and the floating phase. The radius of the dashed circle illustrates the Rydberg blockade radius R_b. The $\mathbb{Z}_3$ and $\mathbb{Z}_4$ orders exhibit Rydberg density oscillations with periods of $p = 3$ and $p = 4$ lattice spacings, respectively. The incommensurate floating phase displays no discernible periodicity in its density oscillations.
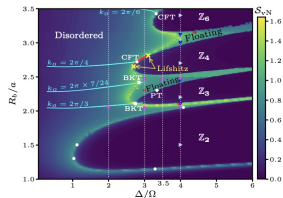

FIG. 7. **Ground-state phase diagram using von Neumann entanglement entropy.** The results for $1 \le R_b/a \le 2.5$, $2.5 < R_b/a \le 3.15$, and $3.15 < R_b/a \le 3.5$ are computed for systems with $L = 288$, 285, and 290, respectively (Fig. 1b in the main text also employs this technique). Slight variations in L in different regimes ensure compatibility with the periods of the crystalline orders. The dark lobes represent crystalline orders, and the green areas between crystalline orders, bounded by the bright yellow lines, show the floating phase. The bright yellow lines are BKT transition lines, separating the floating phases and the disordered phase. The boundaries between the floating phases and the crystalline orders are PT transition lines. The red line labels the direct phase transition between the $\mathbb{Z}_4$ order and the disordered phase, which is a chiral transition line with continuously varying critical exponents plus a single CFT point (white circle). The chiral transition line terminates at two Lifshitz points (yellow dots) where the floating phases emerge. There also exist direct phase transitions between the $\mathbb{Z}_6$ order and the disordered phase, which also include a single CFT point. On the equal-k_o lines (cyan lines) in the disordered phase, the peak position of the structure factor $S(k)$ remains constant at k_o. Commensurate lines, where $2\pi/k_o$ is an integer, intersect with the $\mathbb{Z}_{4(6)}$ boundary at the CFT point. The floating phase fully encompasses the $\mathbb{Z}_3$ order, and the $k_o = 2\pi/3$ line goes from the disordered phase into a critical phase and then into the $\mathbb{Z}_3$ order (see Fig. 21). White circles on the $\mathbb{Z}_2$ boundary denote four Ising critical points studied in Fig. 18. Triangles, squares, and the diamond represent the points dis-

Experimental and Numerical Fourier transforms

The floating phase is characterized by a continuous dependence in the lattice spacing (at infinite volume!).
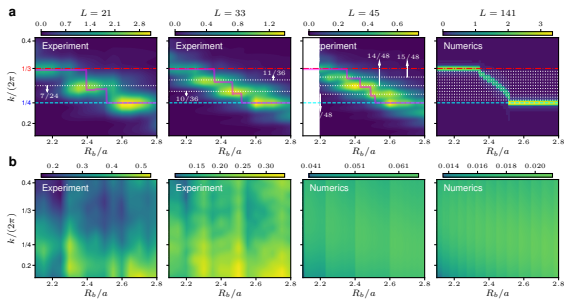
Figure: **The structure factor for different system sizes. a**, The structure factor as a function of R_b/a is measured along the cut $\Delta/\Omega = 3.5$ for $L = 21, 33, 45$. The magenta curves are peak positions from numerical results. The results for $L = 141$ is from numerics. **b**, The structure factor is measured along the cut $\Delta/\Omega = 0$ in the disordered phase.

Entanglement entropy

Starting with

$$\rho_{AB} = |vac.\rangle\langle vac.|$$

We assume that the N_q qubits are split into N_{qA} and N_{qB} qubits for a bi-partition A-B. Writing the 2^{N_q} dimensional vector $c_{\{n\}}$ corresponding to the vacuum as a $2^{N_{qA}} \times 2^{N_{qB}}$ matrix

$$C_{\{n\}_A, \{n\}_B} = c_{\{n\}},$$

we find that the reduced density matrix $\rho_A = Tr_B \rho_{AB}$ can be written as

$$\rho_{A\{n\}_A, \{n'\}_A} = (CC^\dagger)_{\{n\}_A, \{n'\}_A},$$

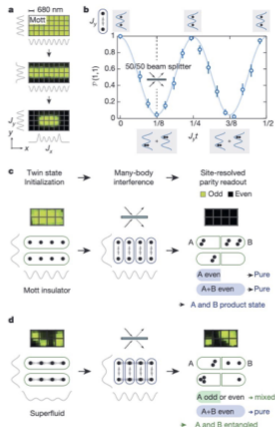in the computational basis. The von Neuman entropy is

$$S_A^{vN} = -Tr_A \rho_A \ln(\rho_A) = -\sum_m \lambda_m \ln(\lambda_m),$$

with λ_m the eigenvalues of ρ_A which are independent of the basis used in A.

Experimental entanglement without twin copy (YM, arxiv 2404.09935)

- Preparing and interfering twin copies is not easy experimentally!
- We propose to use the entropy associated with the experimental measurements of the vacuum of a single system of entangled qubits S_{AB}^X and the reduced entropy S_A^X obtained by tracing the experimental probabilities over one half of the system.
- Using Rydberg arrays, we found from exact diagonalization and local QuEra simulations that the von Newman entanglement entropy $S_{vN} \simeq 1.25(2S_A^X - S_{AB}^X)$.
- The errors subtract to some extend making $2S_A^X - S_{AB}^X$ a robust quantity.
- We conjectured that $S_{vN} \geq 2S_A^X - S_{AB}^X$ should hold in general.

Experimental entropy S_{AB}^X

Given an arbitrary prepared state $|\psi\rangle$, we can expand it in the computational basis

$$|\psi\rangle = \sum_{\{n\}} c_{\{n\}} |\{n\}\rangle.$$

This implies that the state $|\{n\}\rangle$ will be observed with a probability

$$p_{\{n\}} = |c_{\{n\}}|^2.$$

These probabilities define an "experimental" (Shannon) entropy

$$S_{AB}^X \equiv - \sum_{\{n\}} p_{\{n\}} \ln(p_{\{n\}})$$

It is clear that this quantity depends on the computational basis and that it contains no information about entanglement.

Experimental reduced entropy S_A^X

We can define a reduced probability in the subsystem A by tracing over B:

$$p_{\{n\}_A} = \sum_{\{n\}_B} p_{\{n\}_A \{n\}_B},$$

and define the corresponding reduced experimental entropy

$$S_A^X \equiv - \sum_{\{n\}_A} p_{\{n\}_A} \ln(p_{\{n\}_A}).$$

Again, this quantity depend on the computational basis used and cannot be identified with the von Neuman entropy S_A^{vN}. However, it is possible to show that $S_A^X \geq S_A^{vN}$.
Similarly, we can define S_B^X by interchanging A ad B.

First model calculation

N_s=10 chain; $\Omega = 5\pi$ MHz; $\Delta = 17.5\pi$ MHz; $R_b = 8.375\mu$; A and B are the left and right halves so $S_A^X = S_B^X$.

Geometry: 10 atom chain
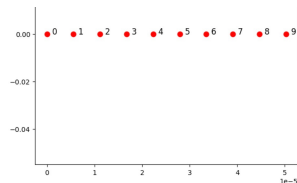
Bitstrings for 1000 shots

```
{'grgrgrgrgr': 16,
 'rggrggrggr': 2,
 'rggrgrgrgr': 209,
 'rgrggrgrgr': 278,
 'rgrgrggrgr': 288,
 'rgrgrgrggr': 188,
 'rgrgrgrgrg': 19}
```

Adiabatic vacuum preparation
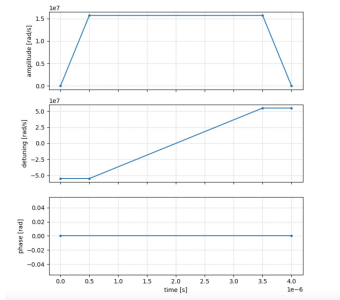
Average Rydberg occupations

$S_A^{vN} \propto 2S_A^X - S_{AB}^X$ with Exact Diagonalization

Figure: Entropies for a chain of 10 atoms with $\Delta/\Omega = 3.5$ as a function of R_b/a_x. Top: S_{AB}^X, S_A^X and S_A^{vN}; the vertical line is at $R_b/a_x = 1.11$; Bottom: $1.25(2S_A^X - S_{AB}^X)$ and S_A^{vN}.

Observations/Conjectures

- For large Shannon entropies: $2S_A^X \simeq S_{AB}^X$
- In the limiting case where all the probabilities are equal to 2^{-N_q}, the reduced probabilities are all equal to $2^{-N_q/2}$ and $2S_A^X - S_{AB}^X \to 0$.
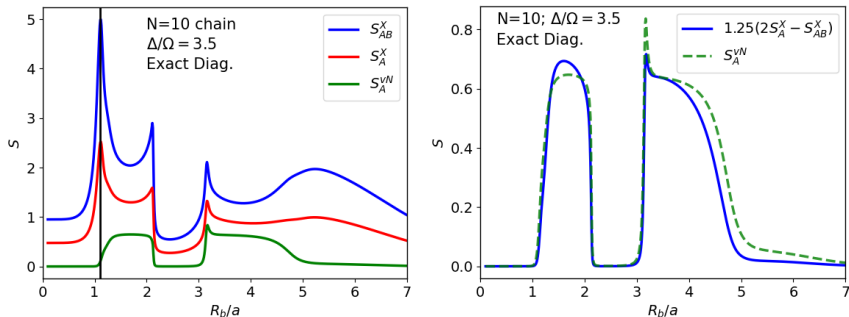- This limiting case can be realized for a state which is the sum of all the computational states with equal coefficients $2^{-N_q/2}$. This state is a product state

$$|PS\rangle = \bigotimes_{j=0}^{N_q-1} (|0\rangle_j + |1\rangle_j)/\sqrt{2},$$

and the corresponding density matrix has zero entanglement.
- We observed that in good approximation

$$S_A^{vN} \propto 2S_A^X - S_{AB}^X.$$

- The important point is that it can be extracted from the bitstring measurements of a single copy of AB.
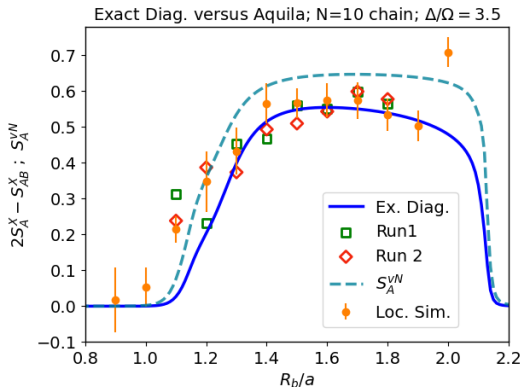
Figure: $2S_A^X - S_{AB}^X$ vs. R_b/a_x with three methods: 1) exact diagonalization (continuous curve); 2) local simulator with no ramping down of Ω at the end (filled circles), the errors bars are calculated using 10 independent runs of 1000 shots; 3) 2 runs with 1000 shots with Aquila (empty symbols); S_A^{vN} (dashed line) is also given for reference.

Rigorous inequalities

The classical mutual information is defined as

$$I_{AB}^X \equiv S_A^X + S_B^X - S_{AB}^X.$$

It was found by Shannon that this quantity is always positive or zero.

Upper bounds can also be found by reducing ρ_A to the diagonal part ρ_A^X

$$S_B^{vN} = S_A^{vN} \leq S_A^X (\text{or} S_B^X).$$

Holevo bound:

$$0 \leq I_{AB}^X \leq S_B^{vN} = S_A^{vN} \leq S_A^X (\text{or} S_B^X).$$

Probabilility filtering and mutual information

YM arXiv 2404.09935 (supplementary material)

Method \ State	Exact	LSNRD1	LSNRD2	LSST1	LSST2	Aquila1	Aquila2
grgrgrgrgr	12	14	19	24	18	16	18
rggggrgrgr	16	21	18	< 10 (0)	< 10 (1)	15	14
rggrgrgrgr	138	142	152	201	222	136	149
rgrgggrggr	23	18	28	< 10 (0)	< 10 (0)	19	24
rgrggrgrgg	< 10 (6)	11	< 10 (7)	< 10 (0)	< 10 (0)	14	< 10 (5)
rgrggrgrgr	276	232	239	274	264	174	175
rgrgrggggr	16	14	18	< 10 (0)	< 10 (0)	19	12
rgrgrgrgr	276	248	244	278	266	190	180
rgrgrgrggr	138	179	168	207	200	151	179
rgrgrgrgrg	12	13	18	12	25	23	16
Total	907	892	904	996	995	804(*)	812(**)

TABLE I. States with at least 10 observations for 1000 shots. For Aquila, (*) five addtional states are not in the table: ggrgrggrgr (12), rgggrgrggr (12), rggrgrgggr (12), rgrggrgggr (14), rgrgrgrgrr (11) for run 1, and (**) and three additional states are not in the table: rgggrggrgr (12), rggrgrgrgr (11), rgrgrgrggg (12) for run 2.
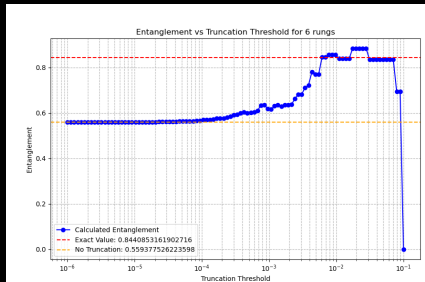
Method \ Entropy	Exact	LSNRD1	LSNRD2	LSST1	LSST2	Aquila1	Aquila2
S_{AB}^X	2.126	2.241	2.198	1.527	1.558	2.944	2.874
S_{AB}^X(Trunc.)	1.6476	1.734	1.740	1.504	1.527	2.035	1.901
S_A^X	1.337	1.374	1.354	1.117	1.134	1.685	1.681
S_A^X(Trunc.)	1.131	1.149	1.167	1.115	1.108	1.288	1.233
$1.25(2S_A^X - S_{AB}^X)$	0.686	0.634	0.639	0.883	0.888	0.531	0.611
$1.25(2S_A^X - S_{AB}^X)$(Trunc.)	0.769	0.706	0.742	0.907	0.861	0.676	0.707

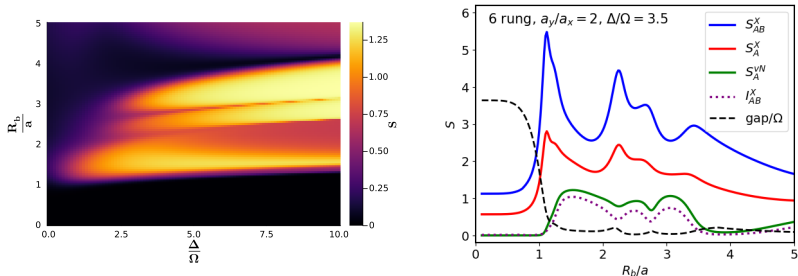TABLE II. Entropies calculated with the full data set and truncated data sets (Trunc.) where observations with less than 10 events are discarded.

Entanglement entropy and mutual information (ladder)

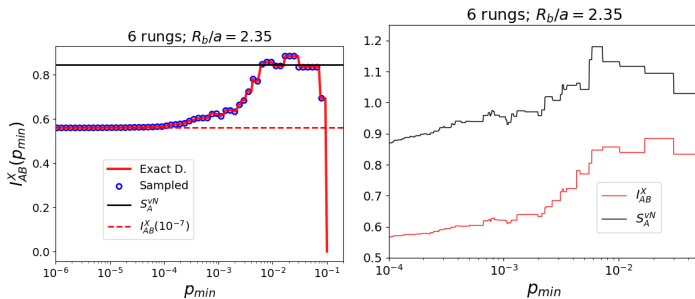Figure: Left: von Neumann entanglement entropy for a six-rung ladder with $a_y = 2a_x$ as a function of R_b/a_x and Δ/Ω. Right: Comparison of S_{AB}^X, S_A^X, S_A^{vN} and I_{AB}^X at a fixed $\Delta/\Omega = 3.5$.

Effect of low probability truncation (filtering)

arXiv 2411.07092 with A. Kaufman, J. Corona, Z. Ozzello, and M. Asaduzzaman (Asad). Truncation: remove states with $p \leq p_{min}$ and renormalize.

Figure: Left: $I^X_{AB}(p_{min})$ for a six-rung ladder with exact diagonalization (continuous line) and DMRG sampling (open circles). Right: $I^X_{AB}(p_{min})$ and $S^{vN}_A(p_{min})$ for a six-rung ladder with exact diagonalization. The ground state is also modified if we remove the corresponding states and renormalize so $I^X_{AB}(p_{min}) \leq S^{vN}_A(p_{min})$ and no contradiction.
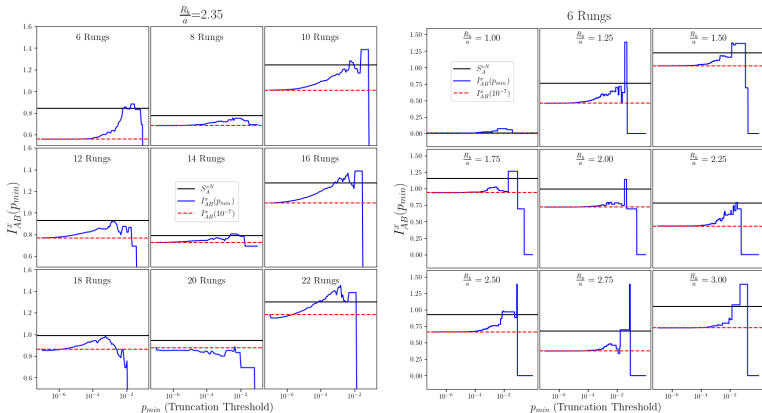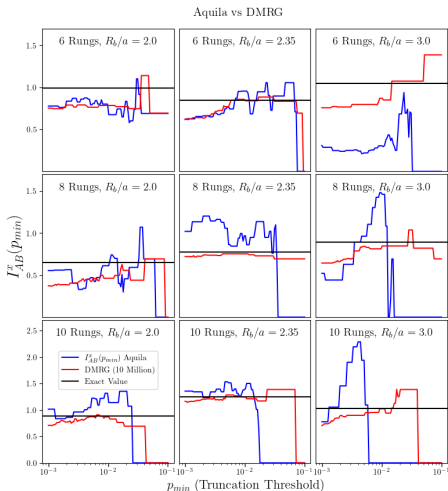
Figure: Effects of the system size on $I_{AB}^{X}(p_{min})$ for 6, 8, ..., 22 rungs, $R_b/a = 2.35$ and 10 million counts obtained via DMRG. Effects of the lattice spacing on $I_{AB}^{X}(p_{min})$ for $R_b/a = 1.0, 1.25, \ldots, 3.0$, 6 rungs and 10 million counts obtained via DMRG.

Figure: Filtered mutual information obtained from DMRG (10 million counts) vs Aquilla (1000 shots) for varying *Rb*/*a* and system sizes.

Distribution of probability values

- When the computational basis has a large dimension, many states have a low probability.
- We define the number of states $NS(p, dp)$ having a probabilty between $p - dp/2$ and $p + dp/2$.
- For systems large enough and bins small but not too small, it is tempting to try to use a continuous approximation

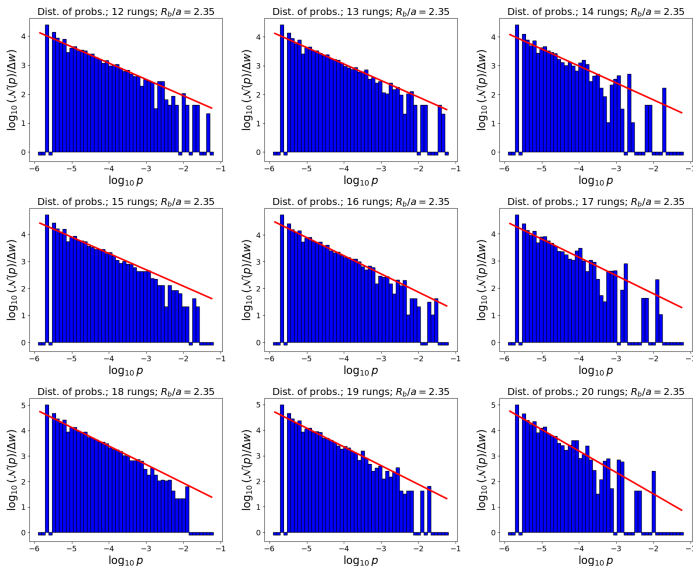$$NS(p, dp) \simeq \mathcal{N}_{lin.}(p)dp.$$

- In the following, we will see that in good approximation for small p,
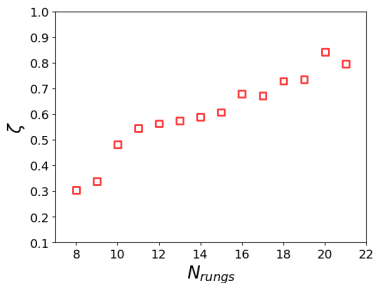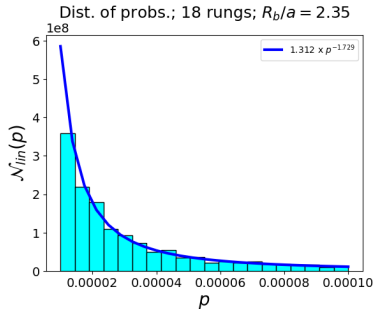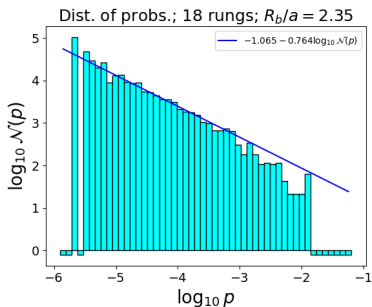
$$\mathcal{N}_{lin.}(p) \simeq Cp^{-1-\zeta},$$

- If we assume that in the infinite volume limit, the power behavior extends to arbitrarily small values of p, then the normalization of the probability requires that $\zeta < 1$.
- What should we do if $\zeta \geq 1$?

log-log fits (using J. Corona DMRG sampling)

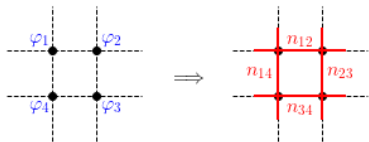# $\mathcal{N}_{lin.}(p) \simeq Cp^{-1-\zeta}$ (preliminary)

TLFT: From compact to discrete (O(2) example)

$$Z_{O(2)} = \prod_x \int_{-\pi}^{\pi} \frac{d\varphi_x}{2\pi} e^{\beta \sum_{x,\mu} \cos(\varphi_{x+\hat{\mu}} - \varphi_x)} = \text{Tr} \prod_x T^{(x)}_{n_{x-\hat{1},1}, n_{x,1}, \ldots, n_{x,D}}.$$

$$e^{\beta \cos(\varphi_{x+\hat{\mu}} - \varphi_x)} = \sum_{n_{x,\mu}=-\infty}^{\infty} e^{i n_{x,\mu} \varphi_{x+\hat{\mu}}} I_{n_{x,\mu}}(\beta) e^{-i n_{x,\mu} \varphi_x}.$$

Tensor : $T^{(x)}_{n_{x-\hat{1},1}, n_{x,1}, \ldots, n_{x-\hat{D},D}, n_{x,D}} = \sqrt{I_{n_{x-\hat{1},1}} I_{n_{x,1}}, \ldots, I_{n_{x-\hat{D},D}} I_{n_{x,D}}} \times \delta_{n_{x,\text{out}}, n_{x,\text{in}}},$

$$\prod_x \int_{-\pi}^{\pi} d\varphi_x \Longrightarrow \sum_{\{n\}}$$

The gauged version is the Abelian Higgs model.

Tensor lattice field theory for renormalization and quantum computing

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Judah Unmuth-Yockey
Department of Physics, Syracuse University, Syracuse, New York 13244, USA
and Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA

(published 26 May 2022)

The successes and limitations of statistical sampling for a sequence of models studied in the context of lattice QCD are discussed and the need for new methods to deal with finite-density and real-time evolution is emphasized. It is shown that these lattice models can be reformulated using tensorial methods where the field integrations in the path-integral formalism are replaced by discrete sums. These formulations involve various types of duality and provide exact coarse-graining formulas that can be combined with truncations to obtain practical implementations of the Wilson renormalization group program. Tensor reformulations are naturally discrete and provide manageable transfer matrices. Truncations with the time continuum limit are combined, and Hamiltonians suitable for performing quantum simulation experiments, for instance, using cold atoms, or to be programmed on existing quantum computers, are derived. Recent progress concerning the tensor field theory treatment of noncompact scalar models, supersymmetric models, economical four-dimensional algorithms, noise-robust enforcement of Gauss's law, symmetry preserving truncations, and topological considerations are reviewed. Connections with other tensor network approaches are also discussed.

DOI: 10.1103/RevModPhys.94.025005

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Quantum Field Theory
A quantum computation approach

Yannick Meurice

IOP ebooks

Interdisciplinary effort

SVD and TN methods developed in condensed matter (T. Nishino, X-G. Wen, M. Levin, Tao Xiang, ...). Character expansions used in strong coupling expansions (C. Itzykson, ...). Cutting edge effort in Japan: S. Akiyama, D. Kadoh, Y. Kuramashi, R. Sakai, S. Takeda, Y. Yoshimura, ...

INT WORKSHOP INT-21R-1C

Tensor Networks in Many Body and Quantum Field Theory

April 3, 2023 - April 7, 2023

ORGANIZERS

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Georgia Institute of Technology
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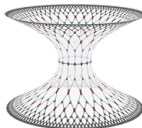**Yannick Meurice**
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DIVERSITY COORDINATOR

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APPLICATION FORM - FOR FULL CONSIDERATION, APPLY BY NOVEMBER 27, 2022

OVERVIEW

Note to applicants: This is an in-person workshop. There is no virtual/online option for this event at this time. Please be aware that all participants must show proof of vaccination against COVID-19 upon arrival to the INT.

Disclaimer: Please also be aware that due to ongoing concerns regarding the COVID-19 pandemic, the workshop may be cancelled.
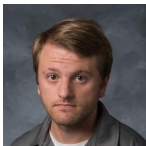
Tensor network methods are rapidly developing and evolving in many areas of
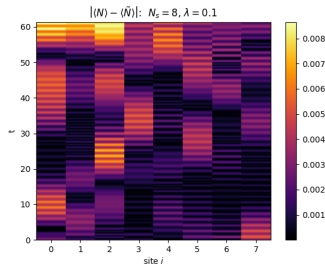
Gaussian Wave Packets (Michael Hite)

arXiv 2411.05301

Michael Hite: Two-particle Scattering

$$\hat{H} = -\sum_{j=0}^{N_s-1} \left(\lambda \hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \hat{\sigma}_j^z\right)$$

- Initialize a right moving wave packet centered at site 0 and left moving wave packet centered at site 4.
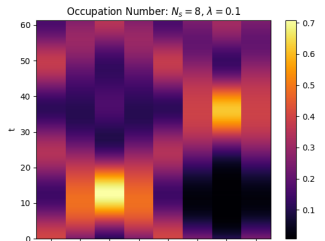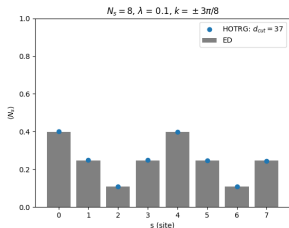
Occupation Number: $N_s = 8, \lambda = 0.1$

$N_s = 8, \lambda = 0.1, k = \pm 3\pi/8$

$|\langle N\rangle - \langle\hat{N}\rangle|$: $N_s = 8, \lambda = 0.1$

$SU(2)$ principal chiral model with tensor renormalization group on a cubic lattice

Shinichiro Akiyama [1,2,*] Raghav G. Jha [3,†] and Judah Unmuth-Yockey [4,‡]

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[2] Graduate School of Science, The University of Tokyo, Bunkyo-ku, Tokyo, 113-0033, Japan
[3] Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA
[4] Fermi National Accelerator Laboratory, Batavia, Illinois, USA

We study the continuous phase transition and thermodynamic observables in the three-dimensional Euclidean $SU(2)$ principal chiral field model with the triad tensor renormalization group (tTRG) and the anisotropic tensor renormalization group (ATRG) methods. Using these methods, we find results that are consistent with previous Monte Carlo estimates and the predicted renormalization group scaling of the magnetization close to criticality. These results bring us one step closer to studying finite-density QCD in four dimensions using tensor network methods.

Tensor renormalization group for fermions

Shinichiro Akiyama (Tsukuba U., CCS and Tokyo U., ICEPP), Yannick Meurice (Iowa U.), Ryo Sakai (Tokyo U.)

Jan 16, 2024

Conclusions

- QC/QIS in HEP and NP: big goals with many intermediate steps
- Tensor Lattice Field Theory (TLFT): generic tool to discretize path integral formulations of lattice model with compact variables (truncations preserve symmetries).
- Ladder-shaped Rydberg arrays with two (or three) atoms per site proposed as simulators for the compact Abelian Higgs model.
- Matching between simulator and target model should be understood in the continuum limit (universal behavior).
- Effective Hamiltonians for the simulator: same three types of terms as the target model plus an extra quartic term.
- The two-leg ladder has a very rich phase diagram that can be studied with the mutual information.
- Implementations with AWS/QuEra
- Progress with hybrid hadronization, evolution in AdS.
- Thanks for listening!
- For questions, email: yannick-meurice@uiowa.edu .

Thanks for listening!

Figure: Isingized version of Yukawa