Z_3 lattice gauge theory as a toy model of QCD

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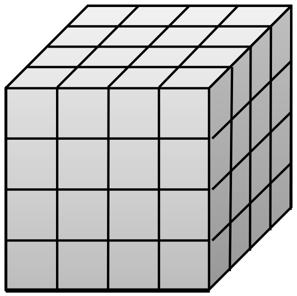
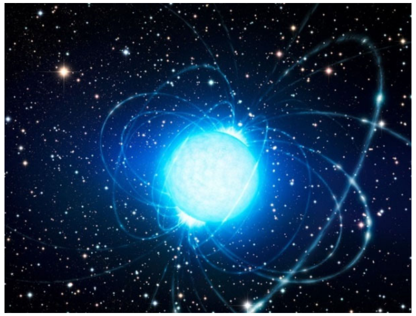
Introduction



https://en.wikipedia.org/wiki/Magnetar



dense QCD

→

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quantum computer

Introduction

continuous gauge group is difficult for qubit devices continuous gauge group is difficult for qubit devices

Z_2 gauge $|g\rangle = c_0|+1\rangle + c_1|-1\rangle$ $\leftrightarrow$ 1 qubit

Z_4 gauge $|g\rangle = c_0|e^{i0}\rangle + c_1|e^{i\pi/2}\rangle + c_2|e^{i\pi}\rangle + c_3|e^{i3\pi/2}\rangle$ $\leftrightarrow$ 2 qubits

$\vdots$

Z_n gauge $|g\rangle = c_0|e^{i0}\rangle + c_1|e^{i2\pi/n}\rangle + \cdots + c_{n-1}|e^{i2\pi(n-1)/n}\rangle$ $\leftrightarrow$ $\log_2 n$ qubits

$\xrightarrow[n\to\infty]{}$ $U(1)$ gauge

Introduction

let's use a toy model with discrete gauge group

Z_3 lattice gauge theory (instead of Z_2 lattice gauge theory)

for near-term quantum simulation

Contents

2. Z_3 lattice gauge theory

Z_3 lattice gauge theory

Z_3 gauge field

state vector $|g\rangle = c_0 \left| e^{i0} \right\rangle + c_1 \left| e^{i2\pi/3} \right\rangle + c_2 \left| e^{i4\pi/3} \right\rangle$

link operator $U = e^{iA}$

conjugate operator $\Pi = e^{iE}$

Z_3 lattice gauge theory

Z_3 gauge field

state vector $$|g\rangle = c_0\left|e^{i0}\right\rangle + c_1\left|e^{i2\pi/3}\right\rangle + c_2\left|e^{i4\pi/3}\right\rangle = \begin{pmatrix} c_0 \\ c_1 \\ c_2 \end{pmatrix}$$

link operator $$U = e^{iA} = \begin{pmatrix} e^{i0} & 0 & 0 \\ 0 & e^{\frac{i2\pi}{3}} & 0 \\ 0 & 0 & e^{\frac{i4\pi}{3}} \end{pmatrix}$$

conjugate operator $$\Pi = e^{iE} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Gauss law operator

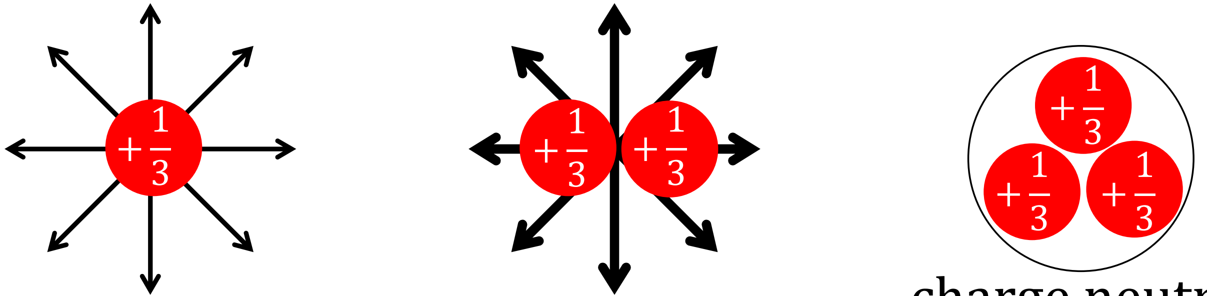
$$\sum_k \Pi_k(\boldsymbol{x})\Pi_k^+(\boldsymbol{x}-\boldsymbol{e}_k) = e^{i2\pi\rho(\boldsymbol{x})/3} \quad \leftrightarrow \quad \sum_k \partial_k E_k(\boldsymbol{x}) = 2\pi\rho(\boldsymbol{x})/3$$

Z_3 lattice gauge theory

Gauss law operator

$$\sum_k \Pi_k(\boldsymbol{x})\Pi_k^+(\boldsymbol{x}-\boldsymbol{e}_k) = e^{i2\pi\rho(\boldsymbol{x})/3} \qquad \leftrightarrow \qquad \sum_k \partial_k E_k(\boldsymbol{x}) = 2\pi\rho(\boldsymbol{x})/3$$

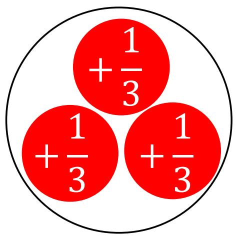
$$\text{electric charge} = 0, \frac{1}{3}, \frac{2}{3} \pmod 1$$



charge neutral

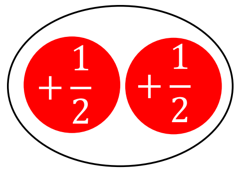
Z_3 lattice gauge theory

Z_3 gauge theory



fermionic baryon

Z_2 gauge theory



bosonic baryon

3. Phase diagram in 3 dimensions

Phase diagram in 3 dimensions

Hamiltonian

$$H = H_{\text{electric}} + H_{\text{magnetic}} + H_{\text{quark}}$$

parameters

gauge coupling g

quark mass m (assumed to be nonzero)

Phase diagram in 3 dimensions

strong coupling limit $g \gg 1$

$$H = H_{\text{electric}} + H_{\text{magnetic}} + H_{\text{quark}}$$

$$O(g^2) \qquad O(1/g^2) \qquad O(1)$$

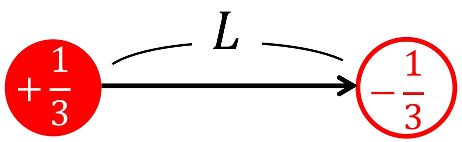
Phase diagram in 3 dimensions

strong coupling limit $g \gg 1$

$$H = H_{\text{electric}} + H_{\text{magnetic}} + H_{\text{quark}}$$

$$O(g^2) \qquad O(1/g^2) \qquad O(1)$$

confinement (+ string breaking)



$$\langle H_{\text{electric}} \rangle \propto g^2 L$$

Phase diagram in 3 dimensions

strong coupling limit $g \gg 1$

$$H = H_{\text{electric}} + H_{\text{magnetic}} + H_{\text{quark}}$$

$$O(g^2) \qquad O(1/g^2) \qquad O(1)$$

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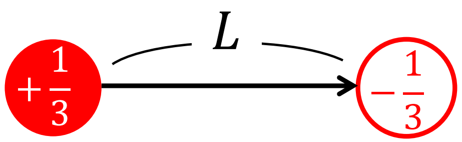
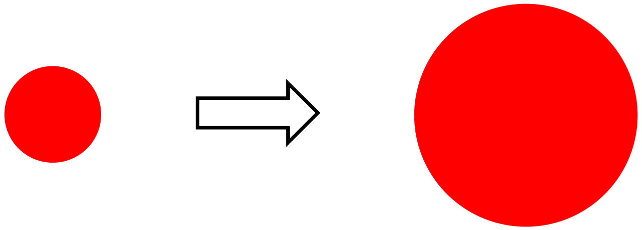


$\langle H_{\text{electric}} \rangle \propto g^2 L$

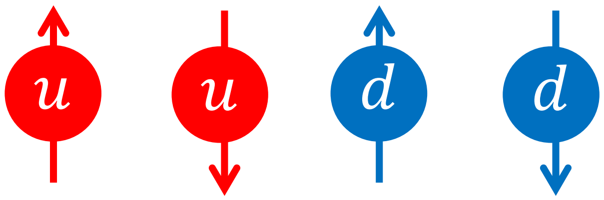
chiral symmetry breaking



m $\qquad$ $m + O(1/g^2)$

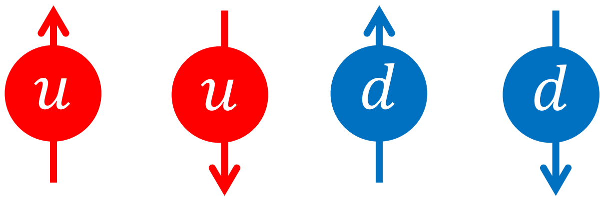
Phase diagram in 3 dimensions

two-flavor quarks

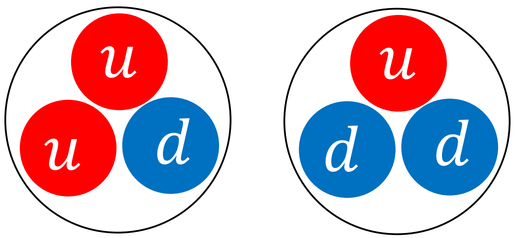


Phase diagram in 3 dimensions

two-flavor quarks

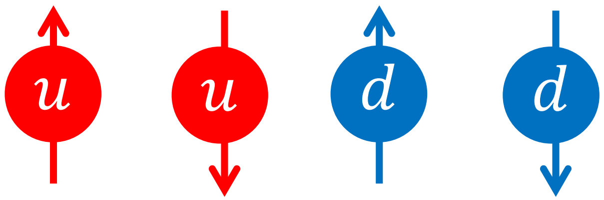


proton & neutron

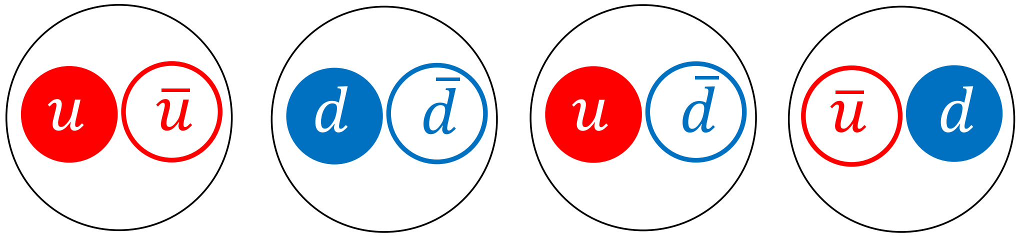


Phase diagram in 3 dimensions

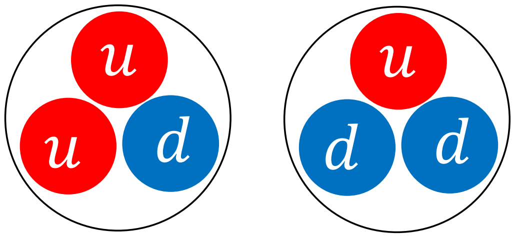
two-flavor quarks



mesons

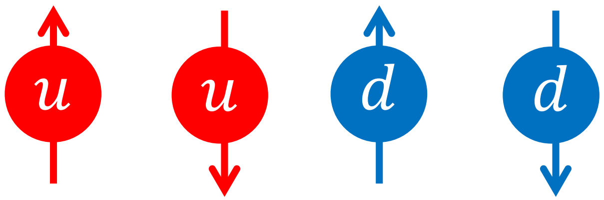


proton & neutron

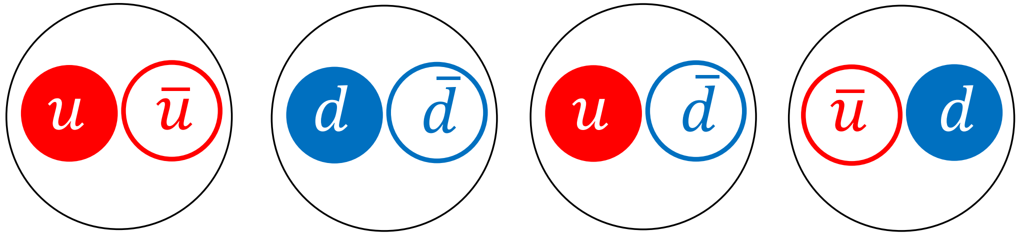


Phase diagram in 3 dimensions

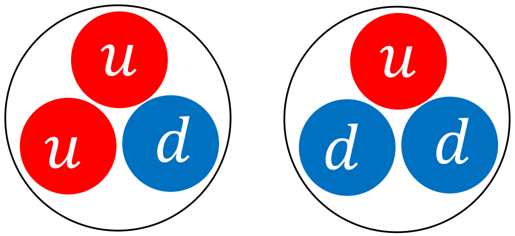
two-flavor quarks



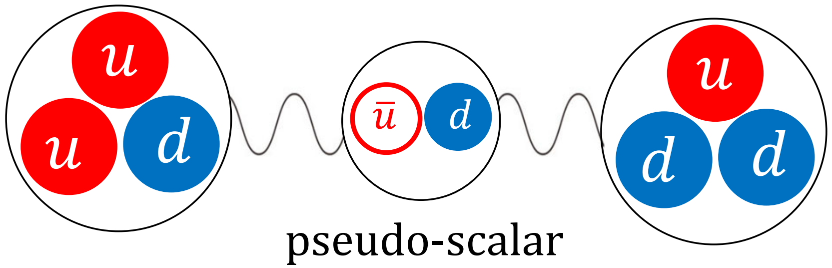
mesons



proton & neutron



baryon-baryon interaction

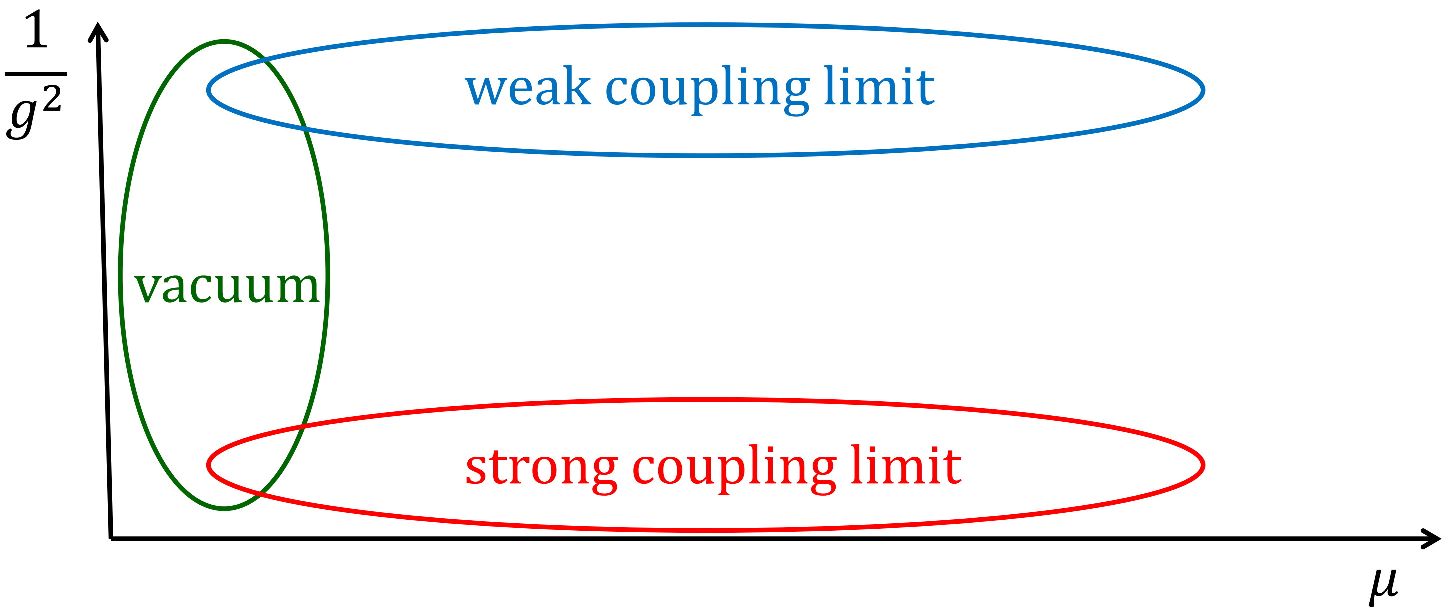


Phase diagram in 3 dimensions

μ vs g phase diagram (zero temperature)

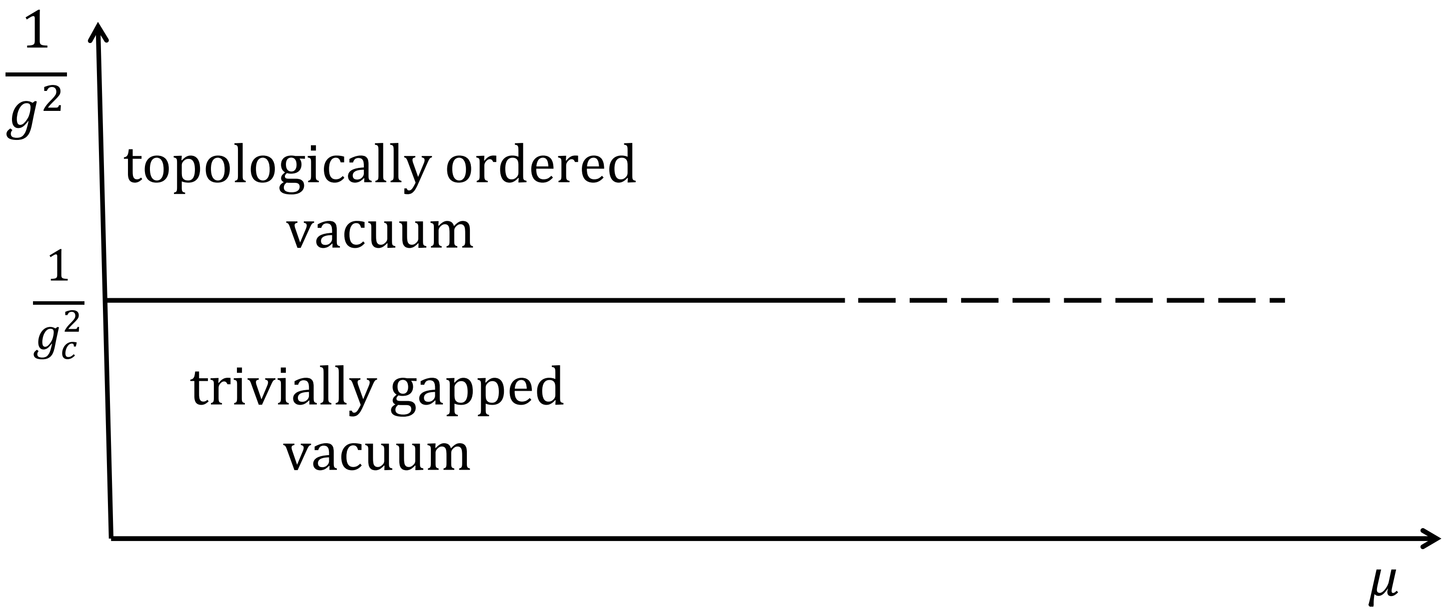
Phase diagram in 3 dimensions

μ vs g phase diagram (zero temperature)

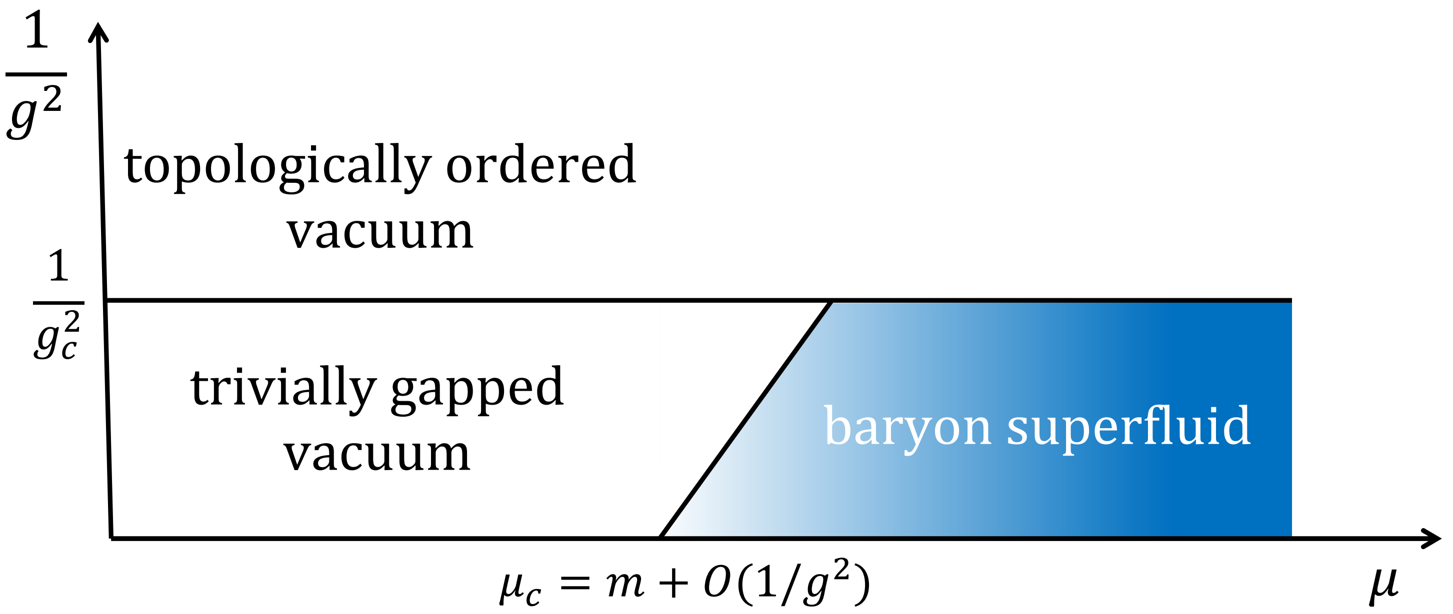


Phase diagram in 3 dimensions

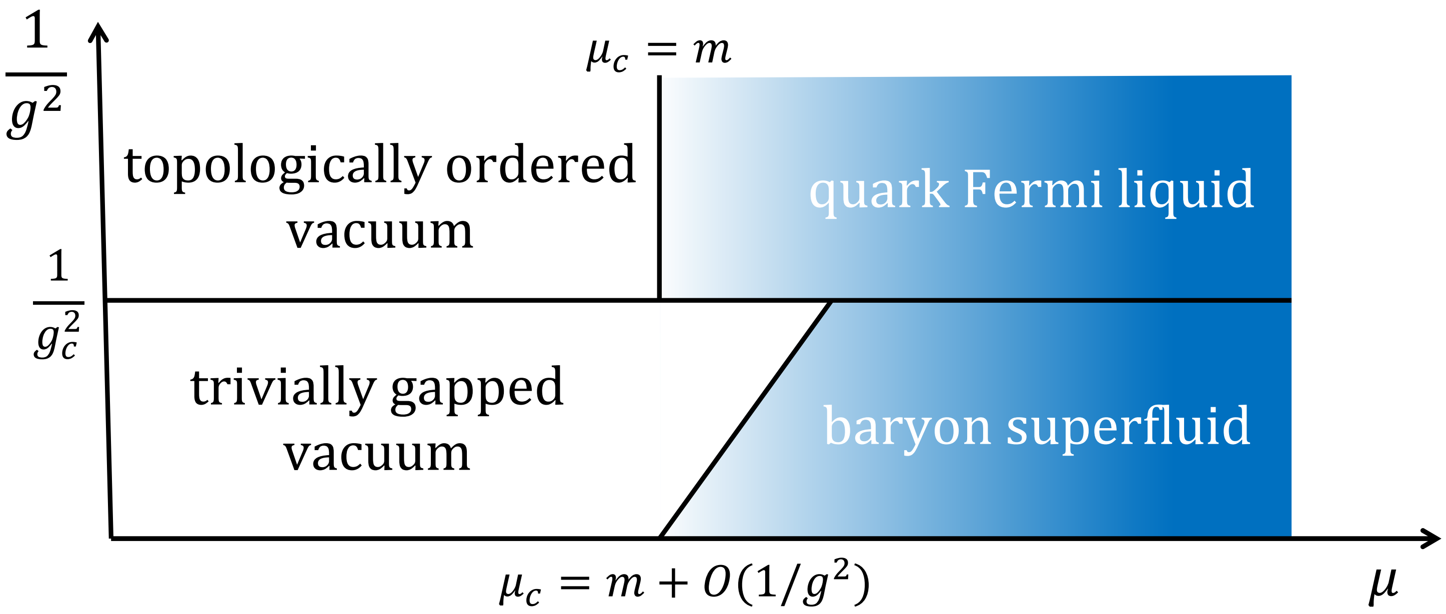
μ vs g phase diagram (zero temperature)



μ vs g phase diagram (zero temperature)



μ vs g phase diagram (zero temperature)



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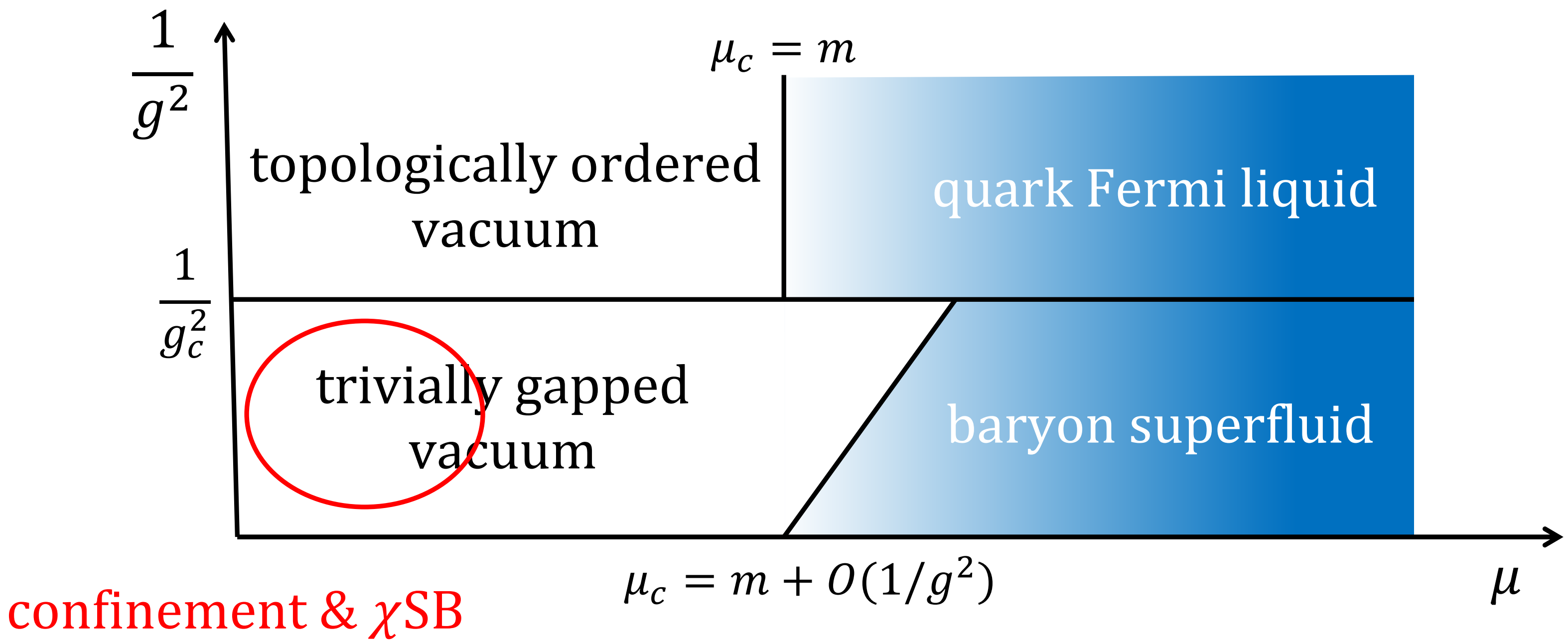
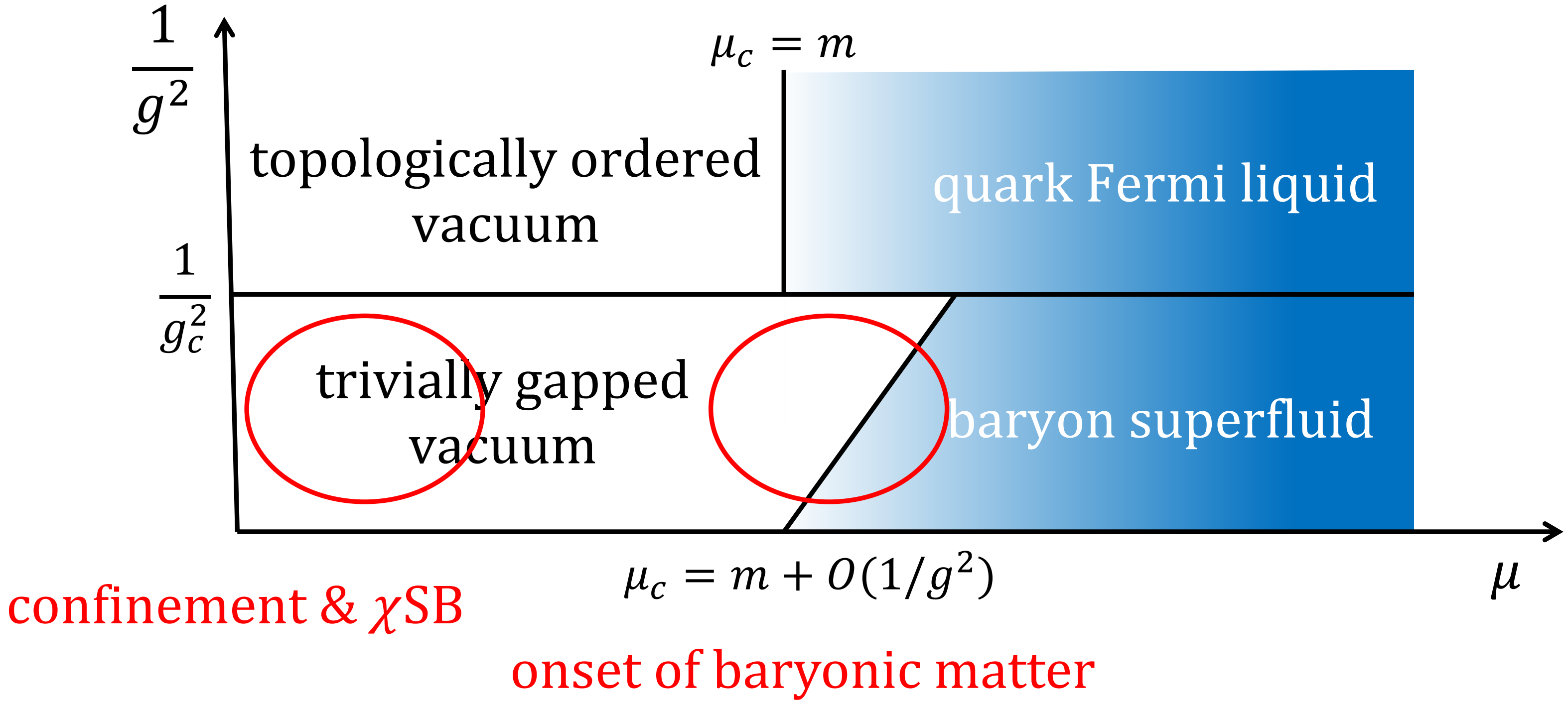
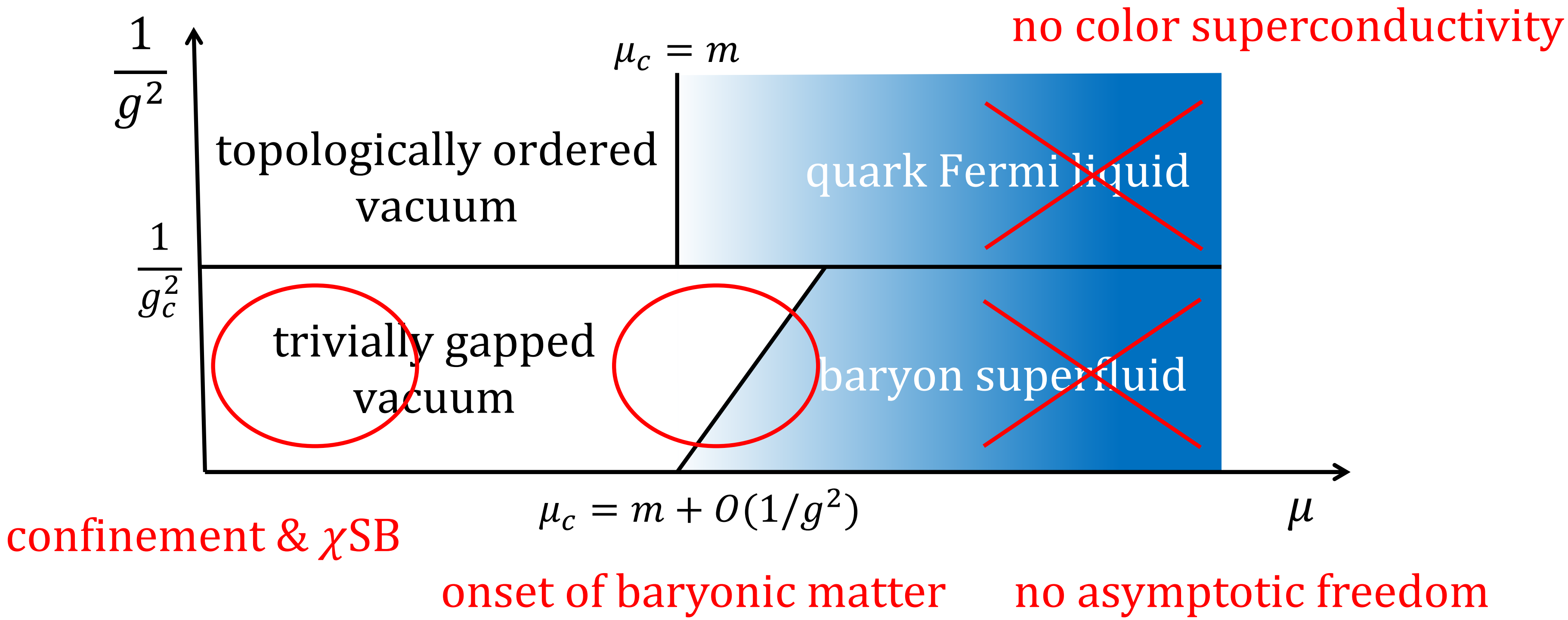


μ vs g phase diagram (zero temperature)



Phase diagram in 3 dimensions

μ vs g phase diagram (zero temperature)



4. Emulator test in 1 dimension

Emulator test in 1 dimension

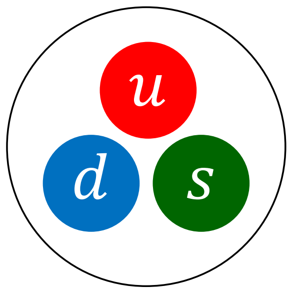
Florio, Weichselbaum, Valgushev, Pisarski (2024)

Hamiltonian

$$H = H_{\text{electric}} + H_{\text{quark}}$$

three-flavor quarks

SU(3) singlet baryon



Emulator test in 1 dimension

quark numbers ($q = u, d, s$)

$$\left[H, Q_q\right] = 0$$

$$\rightarrow \quad Q_q|\Psi\rangle = N_q|\Psi\rangle$$

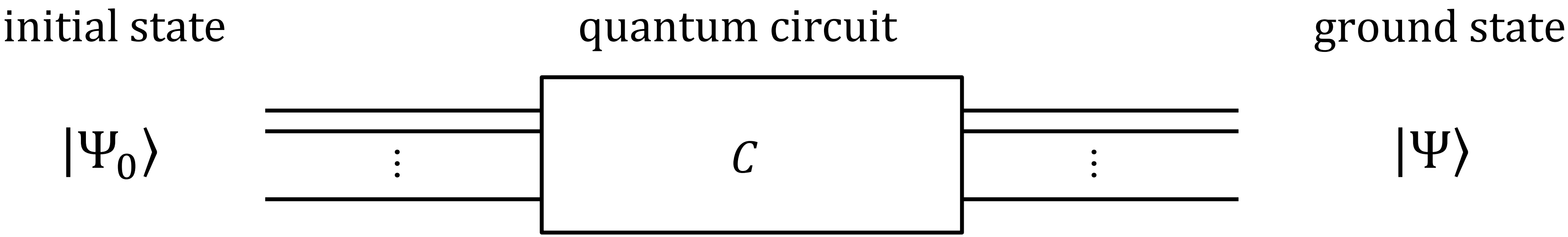
Z_3 gauge symmetry

$$[H, G(x)] = \left[H, \Pi(x)\Pi^{+}(x-1) - e^{i2\pi\rho(x)/3}\right] = 0$$

$$\rightarrow \quad G(x)|\Psi\rangle = 0$$

Emulator test in 1 dimension

ground state is obtained by adiabatic (or variational) algorithm

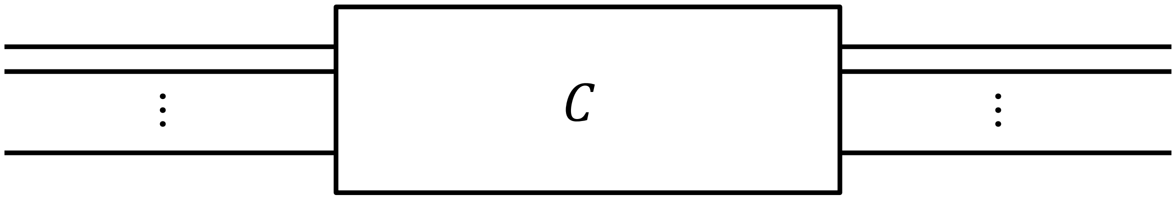


Emulator test in 1 dimension

ground state is obtained by adiabatic (or variational) algorithm

initial state | quantum circuit | ground state

$|\Psi_0\rangle$



$|\Psi\rangle$

$Q_q|\Psi_0\rangle = N_q|\Psi_0\rangle$

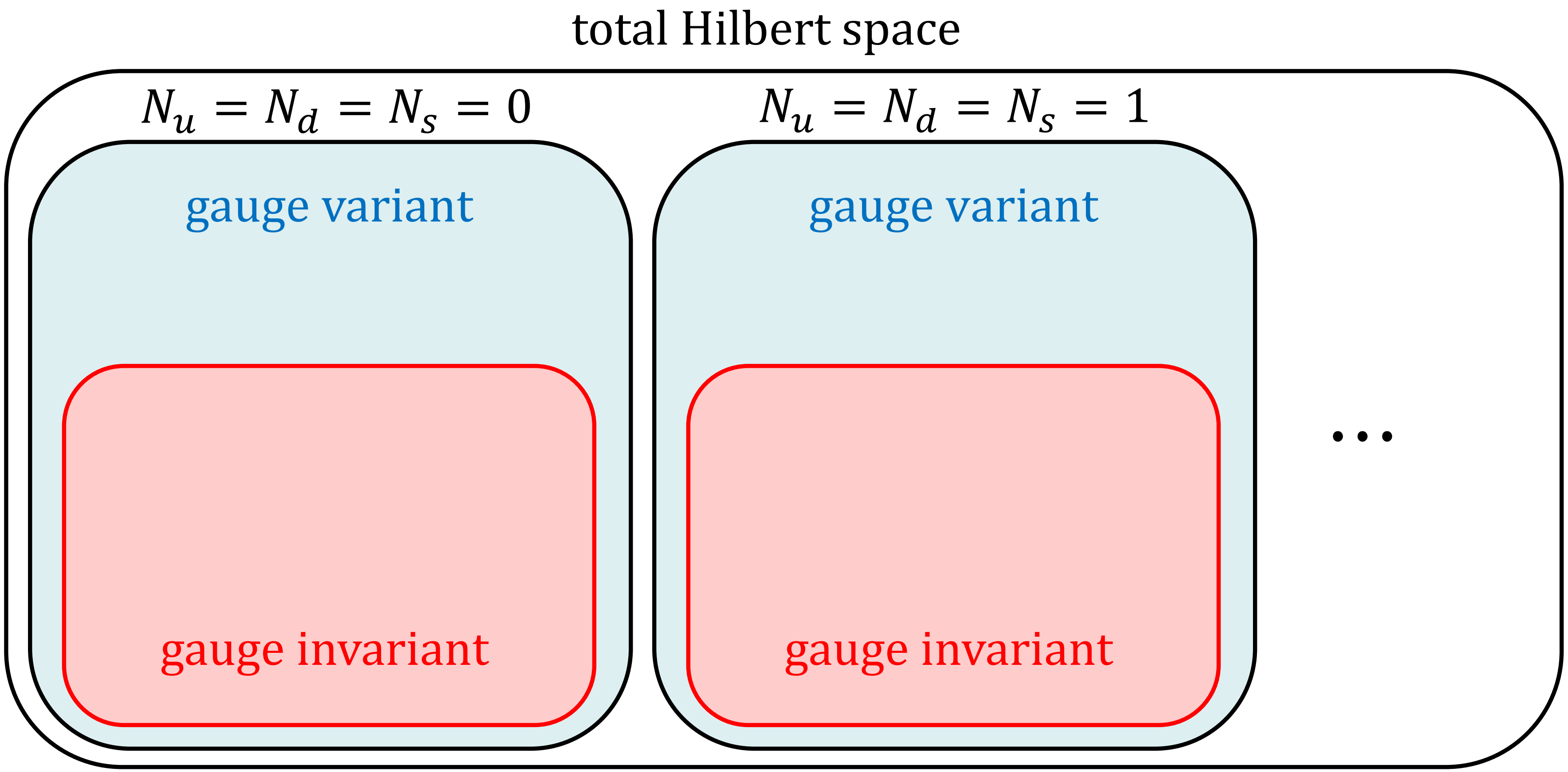
$G(x)|\Psi_0\rangle = 0$

$[C, Q_q] = [C, G(x)] = 0$

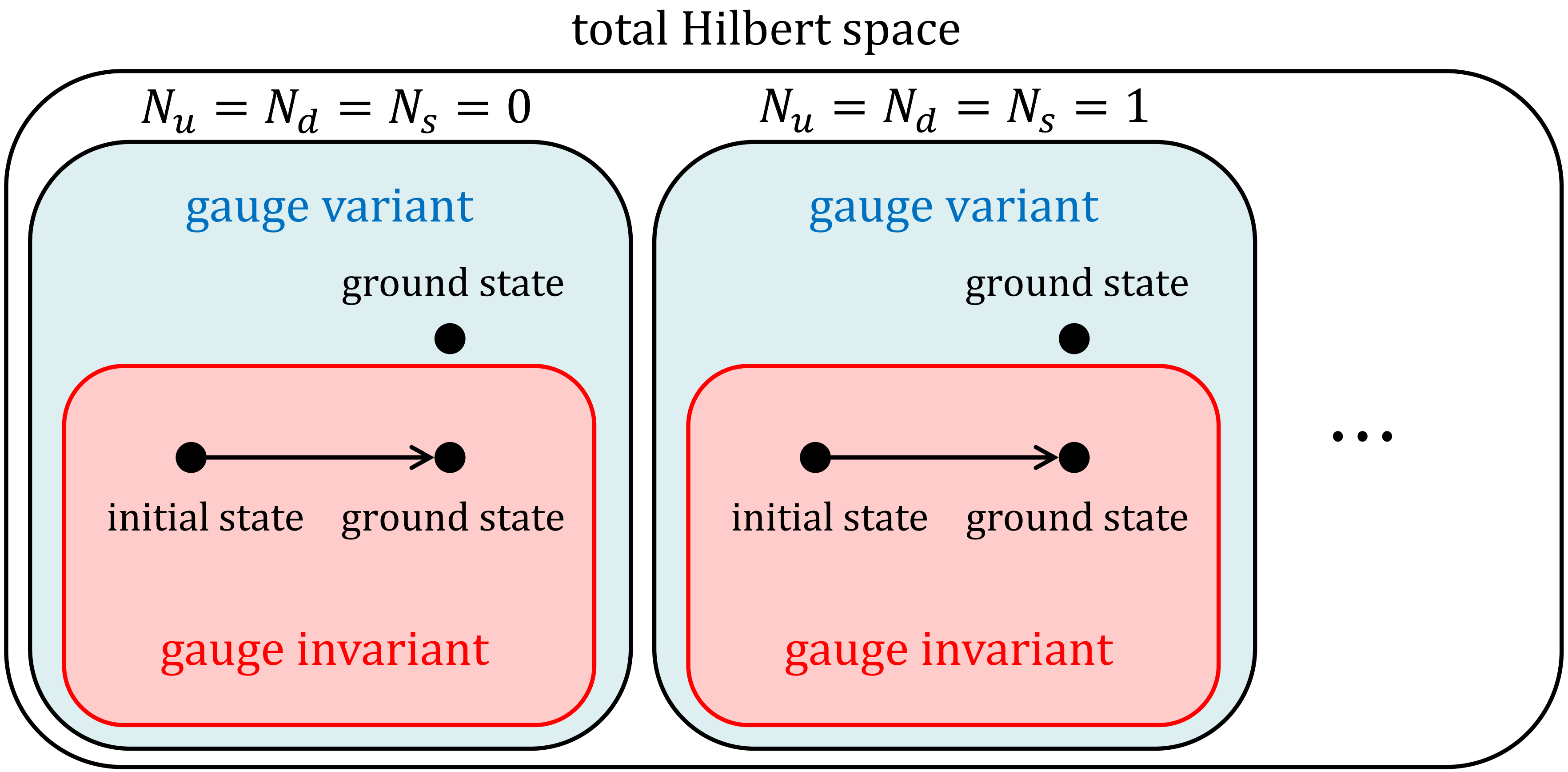
$Q_q|\Psi\rangle = N_q|\Psi\rangle$

$G(x)|\Psi\rangle = 0$

Emulator test in 1 dimension

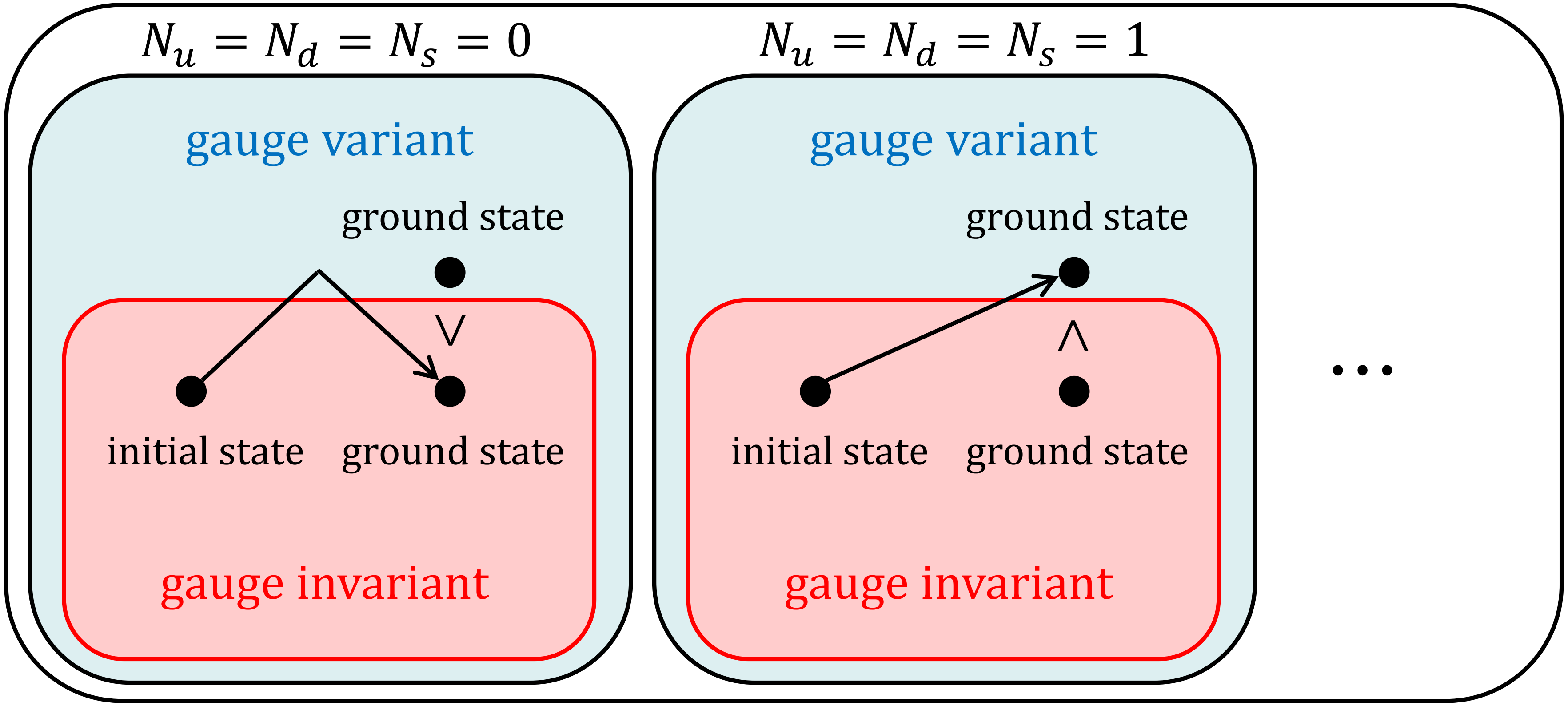


Emulator test in 1 dimension

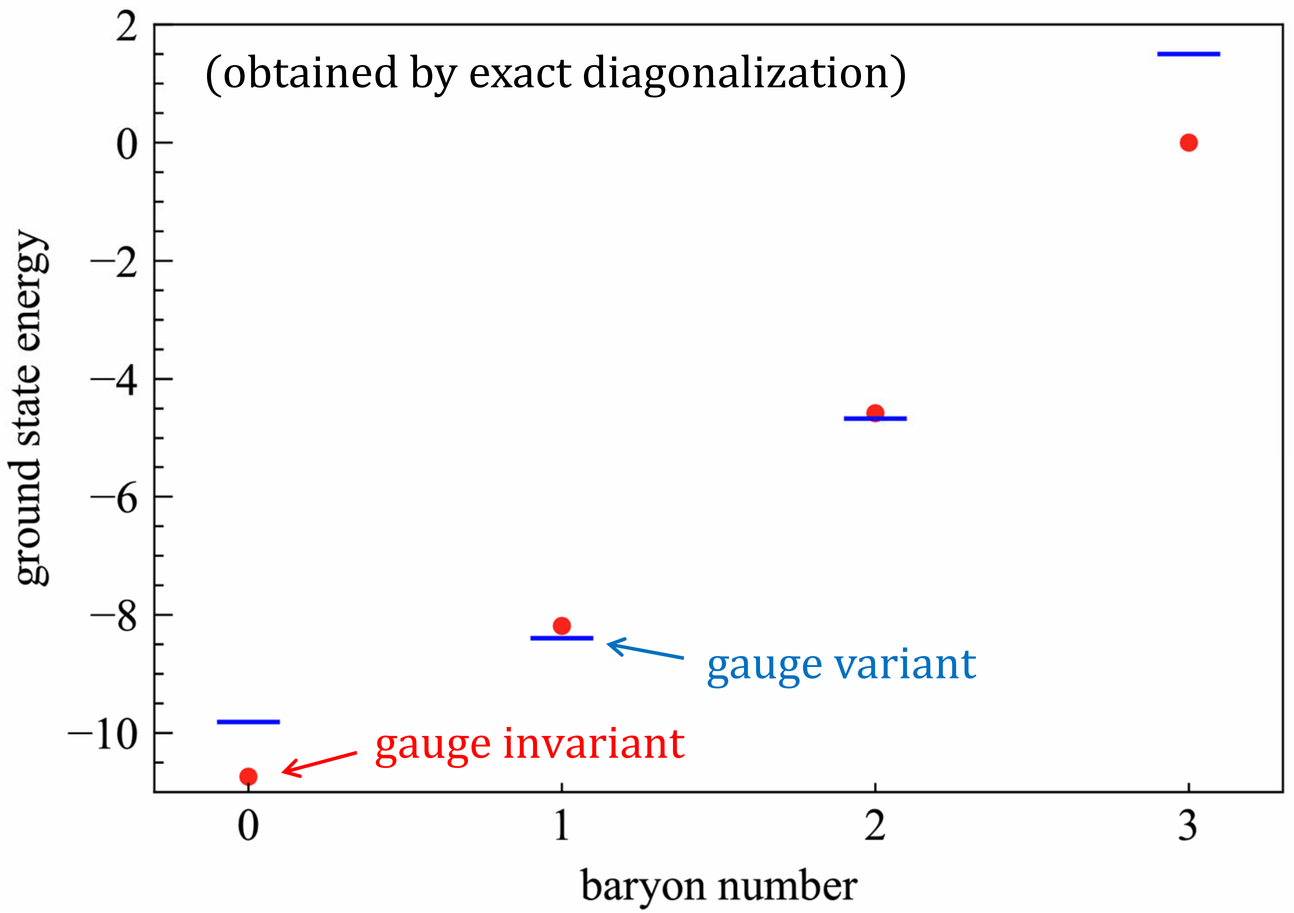


Emulator test in 1 dimension

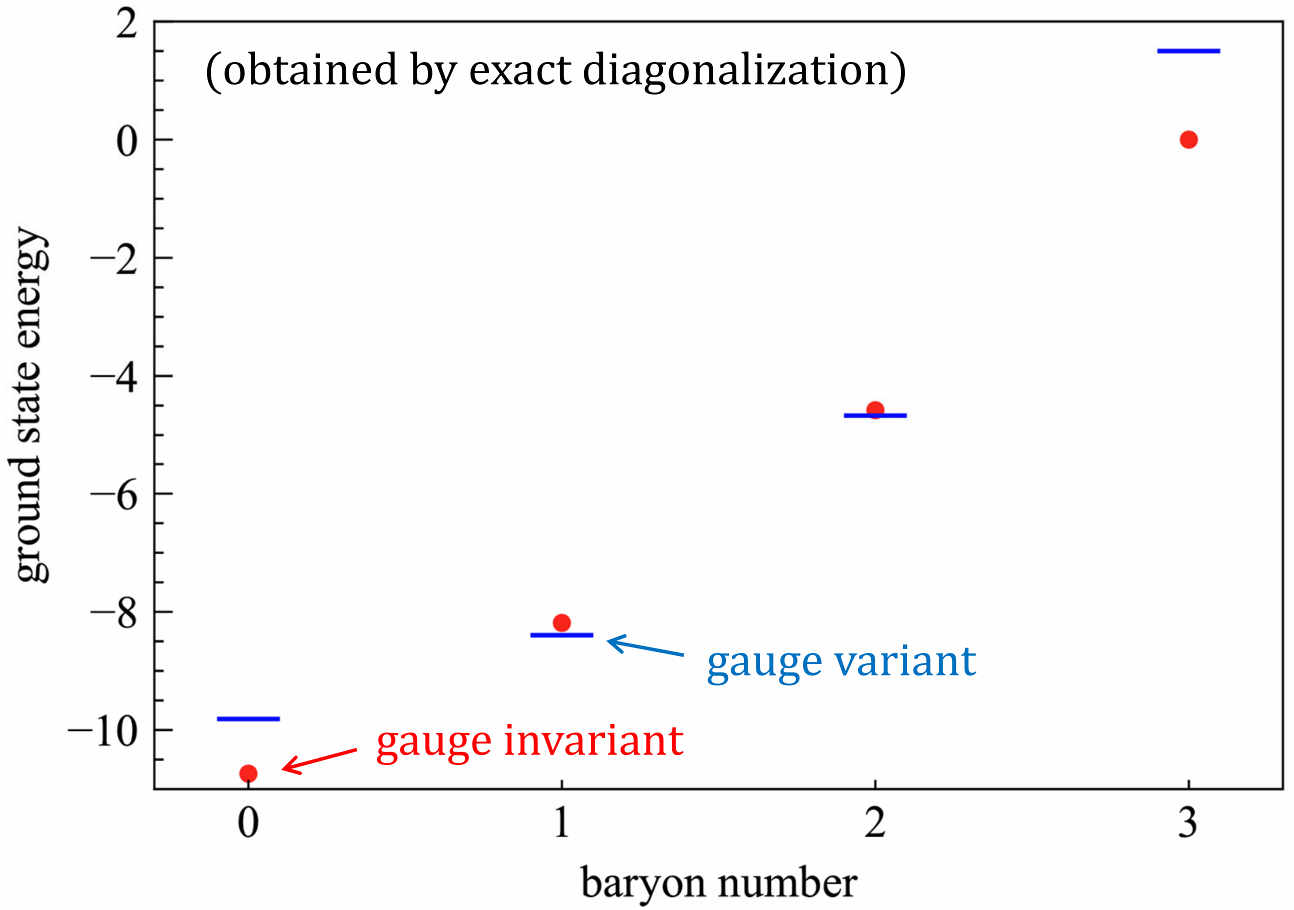
if circuit is not gauge invariant...



Emulator test in 1 dimension

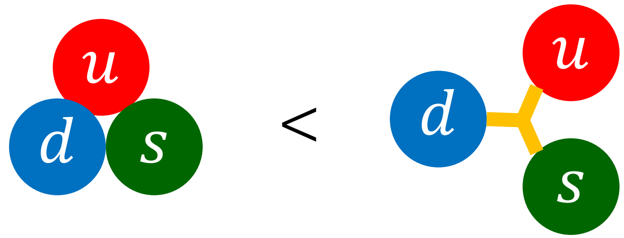


Emulator test in 1 dimension

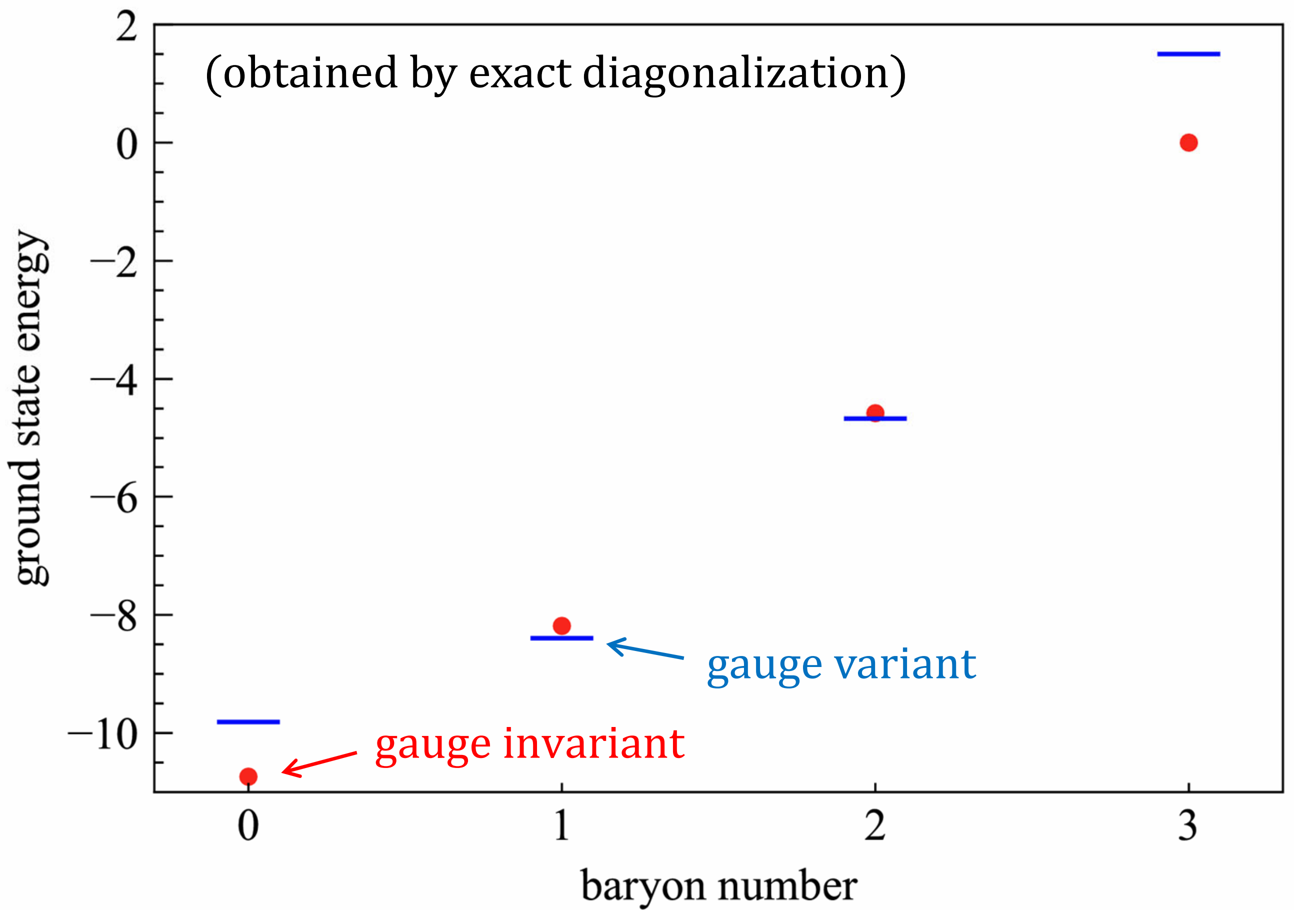


$g \gg 1$

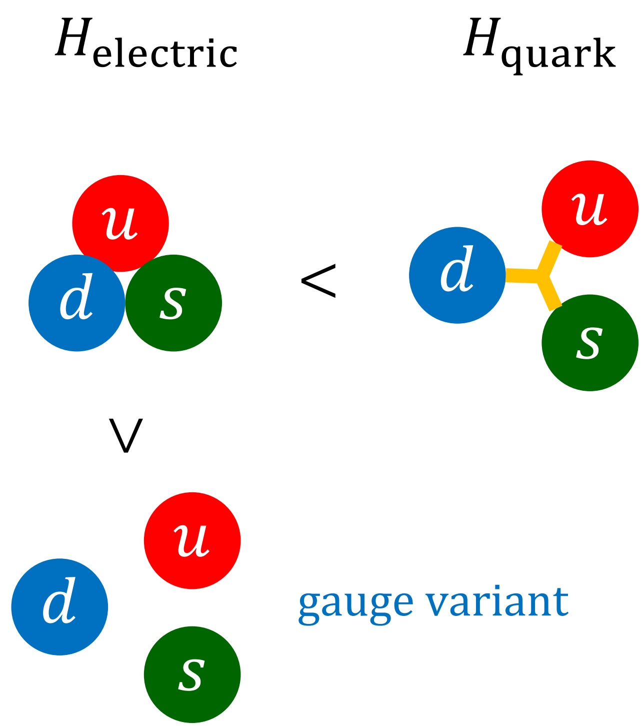
H_{electric} H_{quark}



Emulator test in 1 dimension

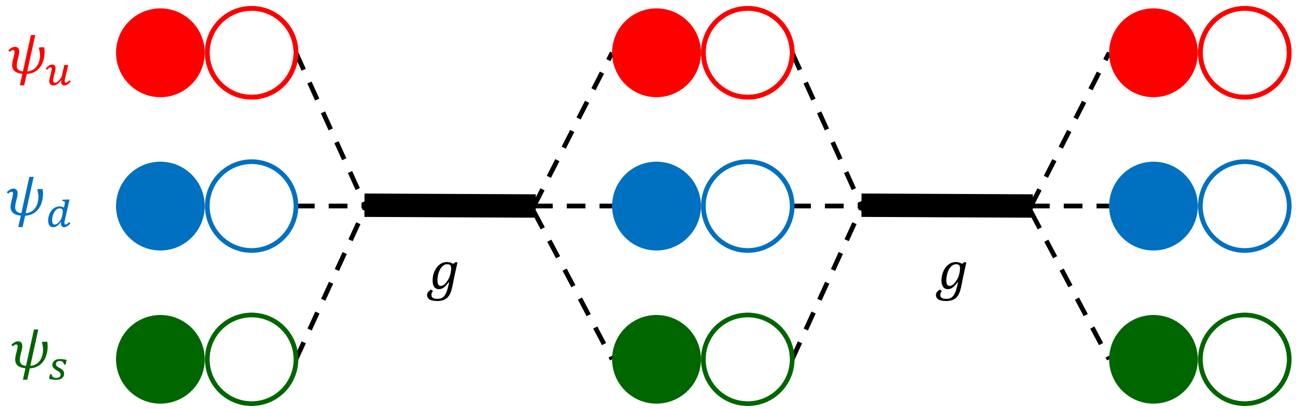


$g \gg 1$



Emulator test in 1 dimension

$$|\Psi\rangle = \prod_{\text{site}} |\psi_{u1}\rangle|\psi_{u2}\rangle|\psi_{d1}\rangle|\psi_{d2}\rangle|\psi_{s1}\rangle|\psi_{s2}\rangle \prod_{\text{link}} |g\rangle$$



Emulator test in 1 dimension

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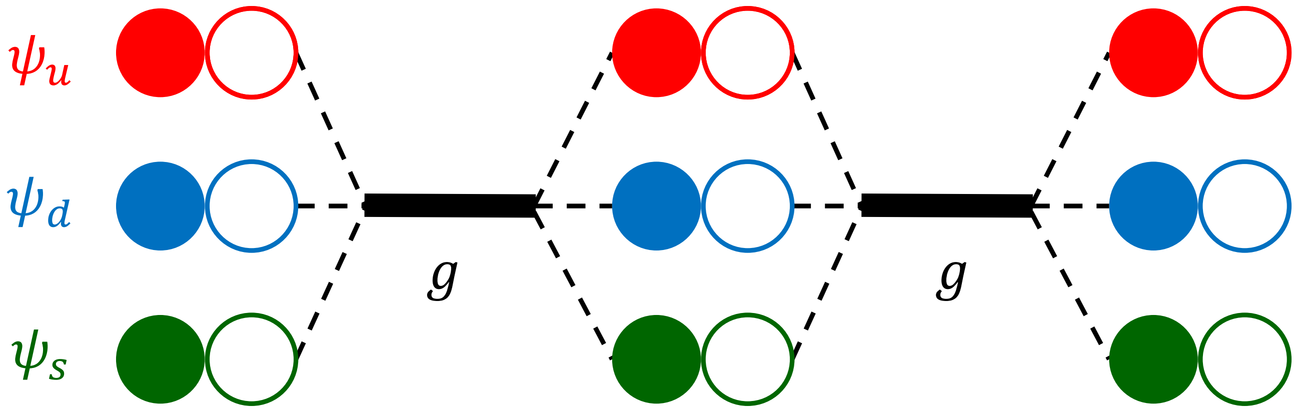
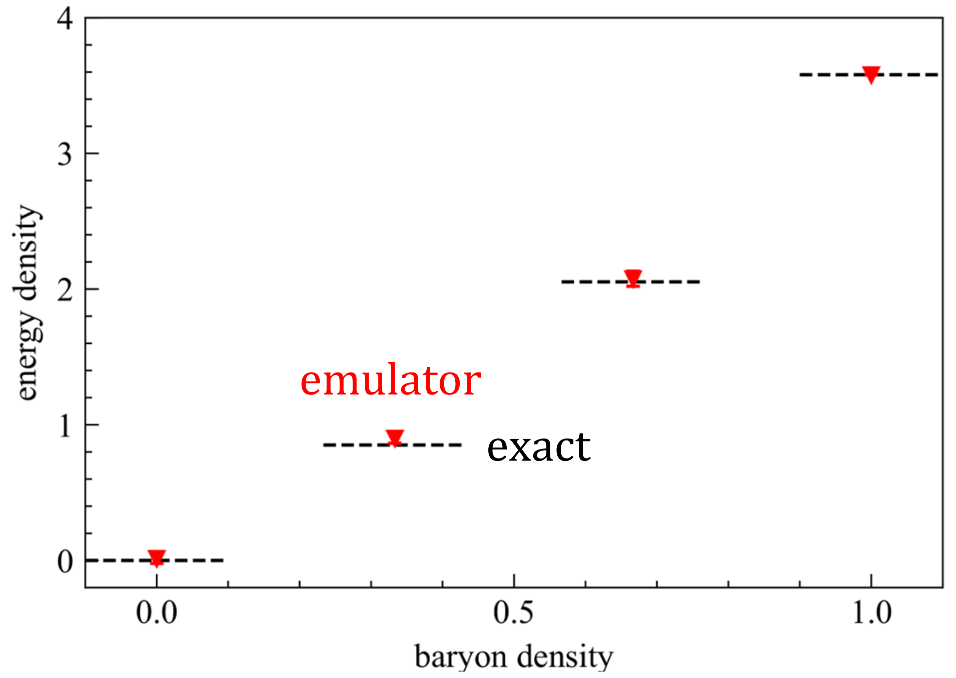


$$6 \quad + \quad 2 \quad + \quad 6 \quad + \quad 2 \quad + \quad 6 \quad = \quad 22 \text{ qubits}$$

Emulator test in 1 dimension



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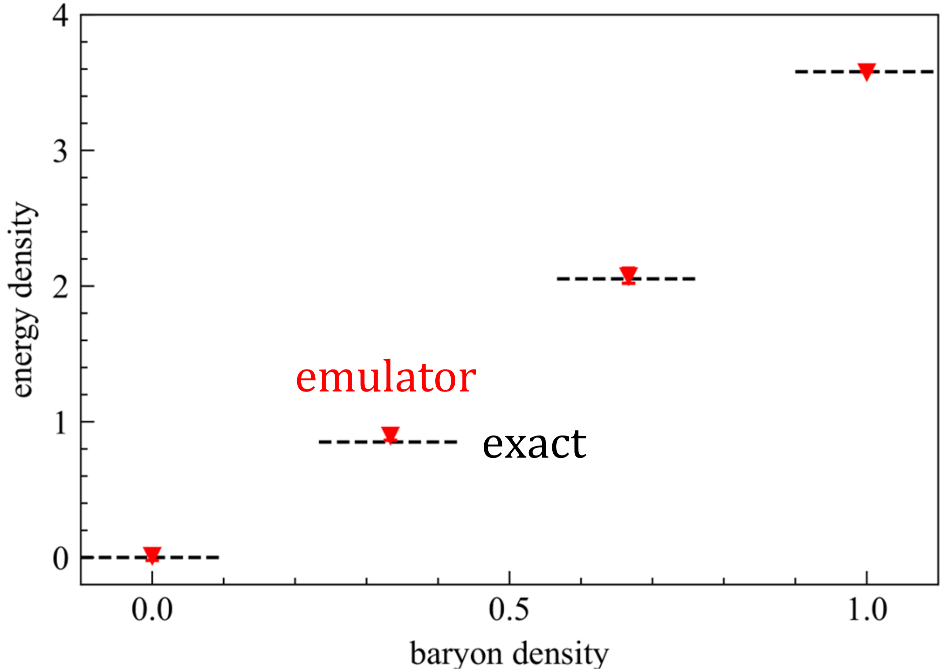

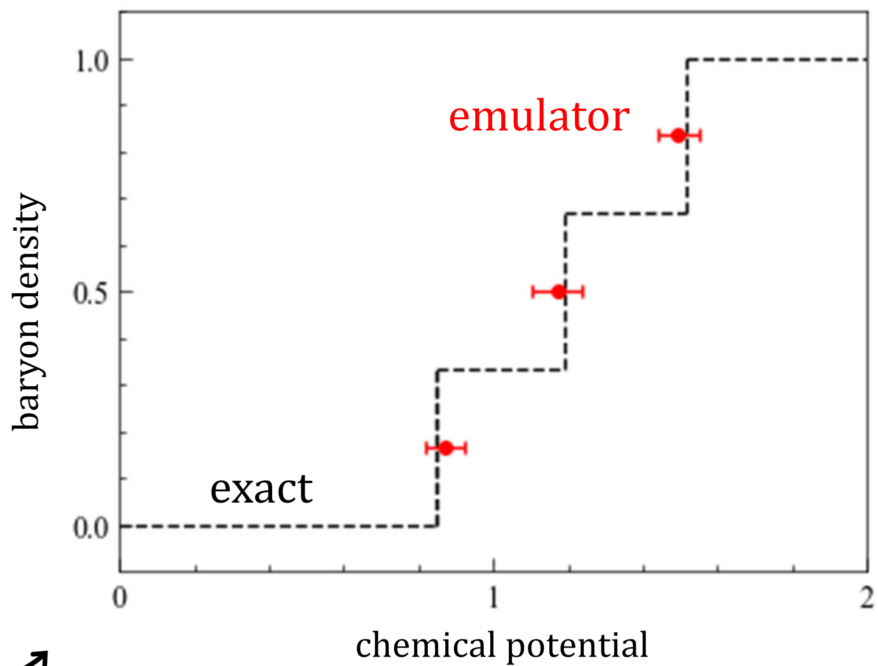

$$\min[E - \mu(N_u + N_d + N_s)]$$

Summary

Z_3 lattice gauge theory is a nice model for benchmarking quantum simulation

- ✓ 1D Z_3 will be possible now
- ✓ 3D Z_3 will be possible in several years
- ✓ dense QCD is the ultimate goal