# Quantum Simulations of the Schwinger Model

*From vacuum to dense matter*

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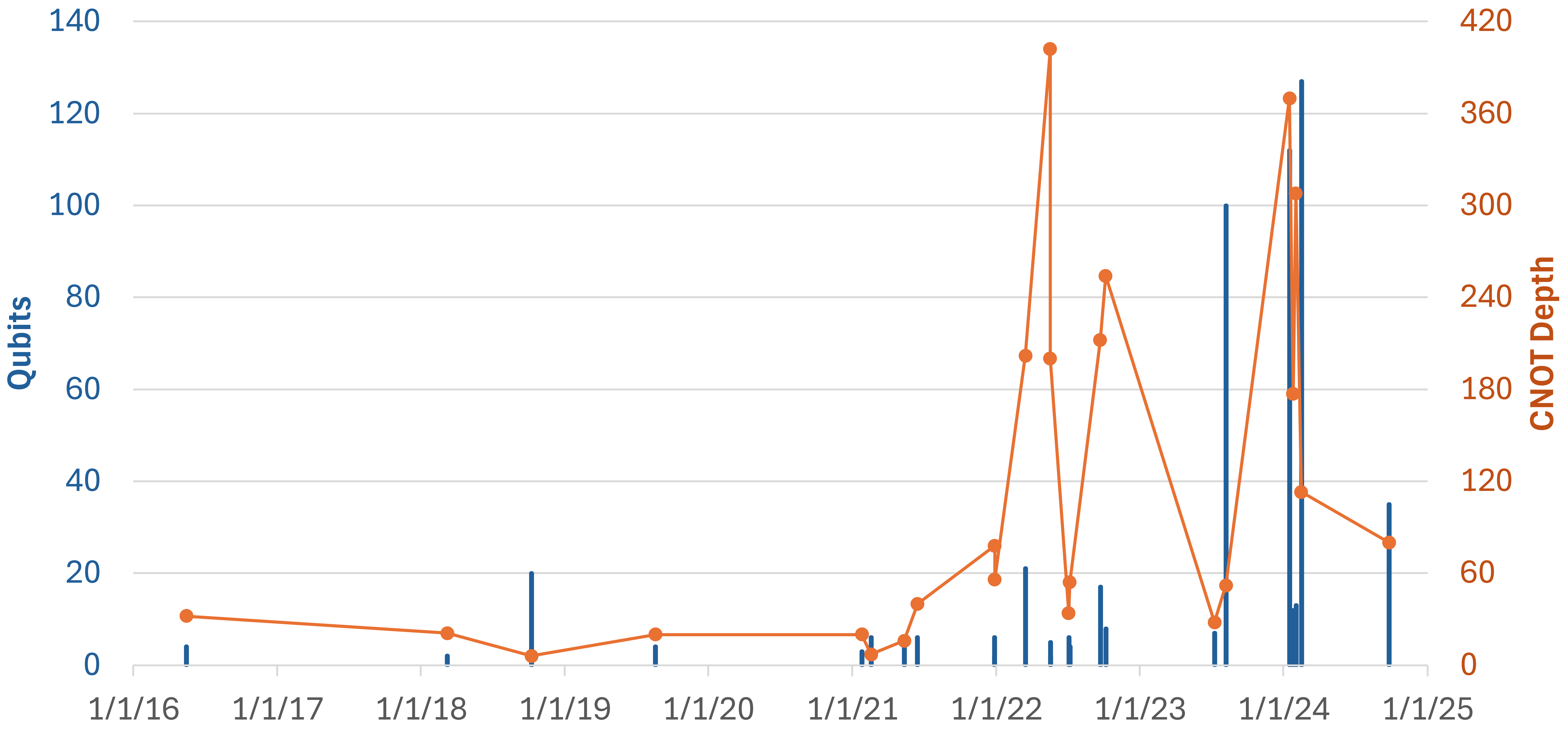
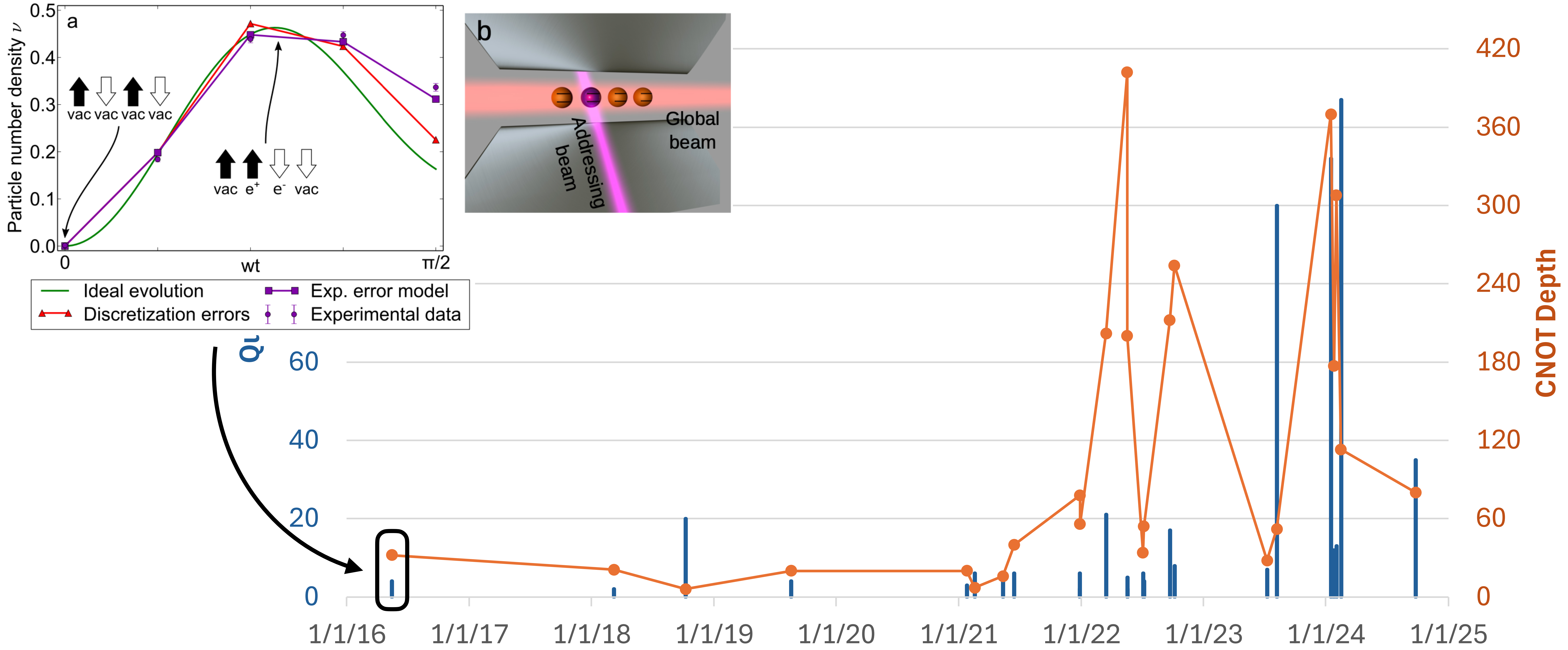
**YITP, Kyoto**

# Progess in digital quantum simulations of LGT



# Progess in digital quantum simulations of LGT

E. A. Martinez et al. Nature (2016)



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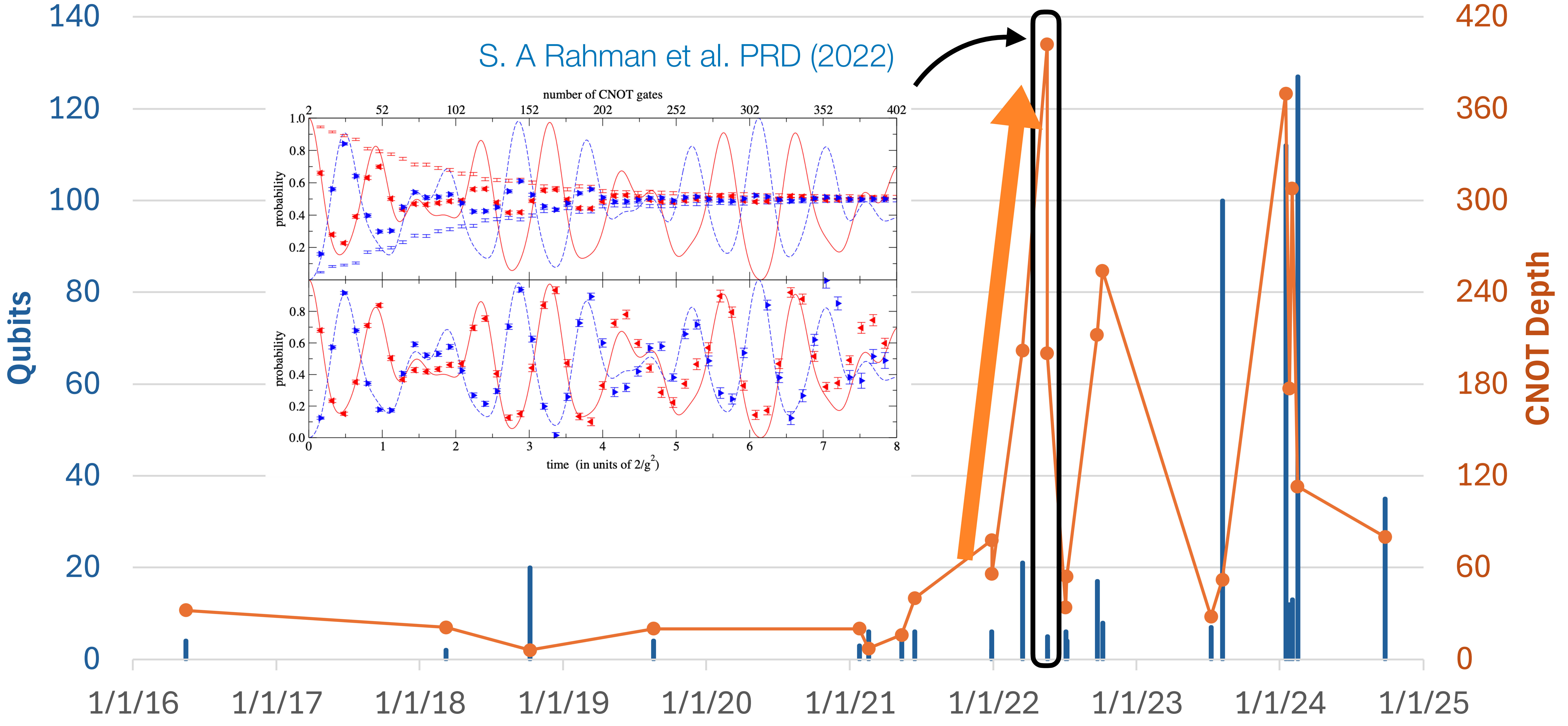

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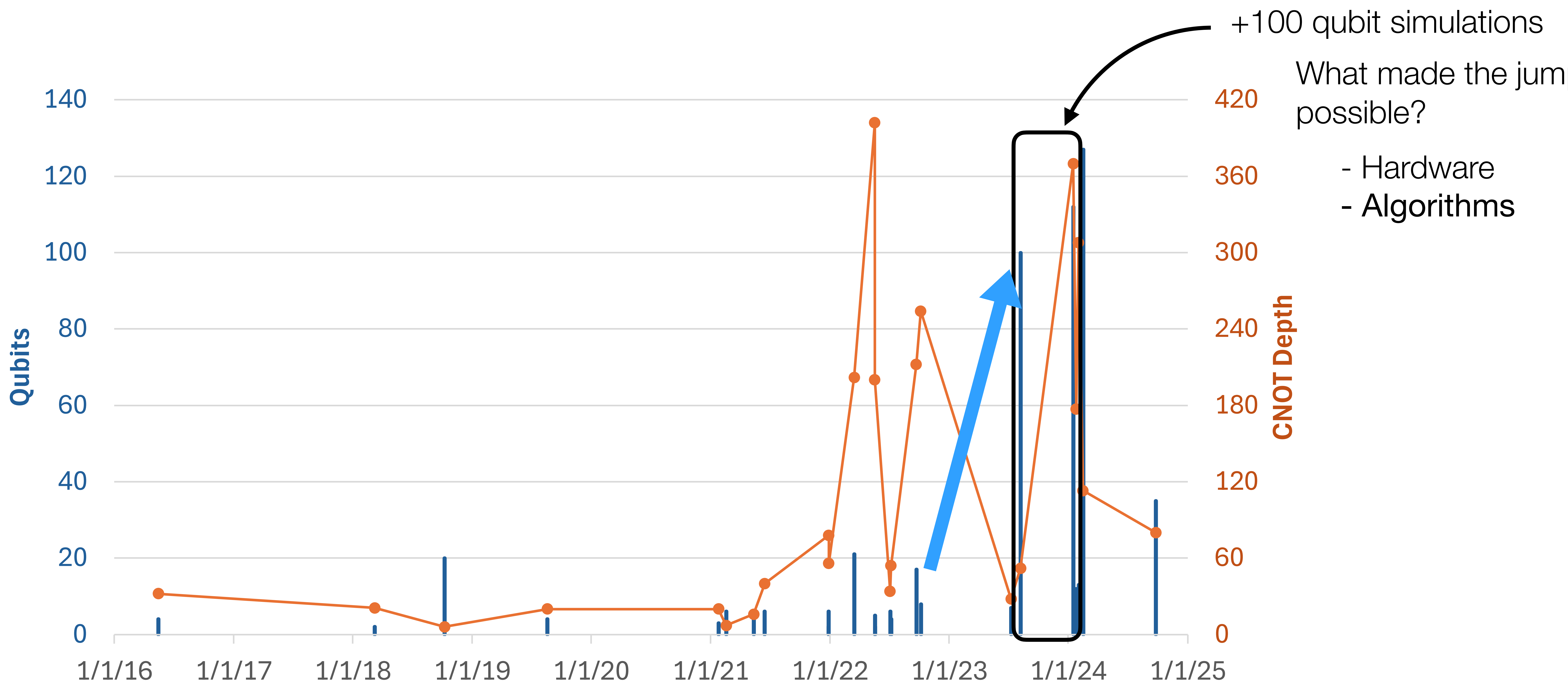



What made the jump possible?

- Hardware
- Algorithms

# The Main Elements

- 1+1D Quantum Electrodynamics (QED) - The Schwinger Model
    - Gapped and translational invariant
    - Confinement (hadrons) - similarities with QCD
- ADAPT-VQE with scalable operator pool: SC-ADAPT-VQE
    - Classical resources to exponentially converge on small/modest systems
- Efficient circuits for state preparation and time evolution
- Error mitigation techniques for running on 100+ qubits
    - Operator decoherence renormalization
- Fragmentation and hadronization with classical sources: discretization effects

# Connection between the Schwinger Model and QCD

- Confinement $V(x) \propto x$ $\longrightarrow$ "Hadrons" (bound states) can form

- Non-zero gap $\Lambda$ $\longrightarrow$ Characteristic length scale $\xi = 1/\Lambda$

  $\downarrow$

  State-preparation circuits only need structure for qubits separated by $r \lesssim \xi$

- Highly non-trivial vacuum, but it has translational invariance

- Perfect sandbox to study hadronization, fragmentation and finite-density phases

- One of the most studied systems using QC

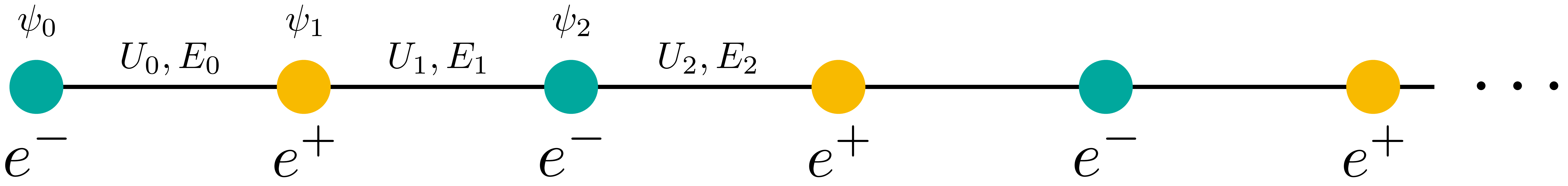
E. A. Martinez et al., Nature 2016
N. H. Nguyen et al., PRXQ 2022
…
N. Klco et al., PRA 2018
F. M. Surace et al., PRX 2020
A. Mil et al., Science 2020

# The Schwinger Model (1+1D QED)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$$

Banks, Kogut & Susskind PRD 1975-76 Lattice version, with staggered fermions in Weyl gauge ($A_0 = 0$)

$$H = \frac{1}{2}\sum_{n=0}^{2L-2}(\psi_n^{\dagger}U_n\psi_{n+1} + \text{h.c.}) + m\sum_{n=0}^{2L-1}(-1)^n\psi_n^{\dagger}\psi_n + \frac{g^2}{2}\sum_{n=0}^{2L-2}|E_n|^2$$




We have:
- ✦ Local interactions
- ✦ Fermionic degrees of freedom (electrons and positrons) : 2 level-systems → qubits
- ✦ Bosonic degrees of freedom (gauge links): n level-systems → multiple qubits or qudits

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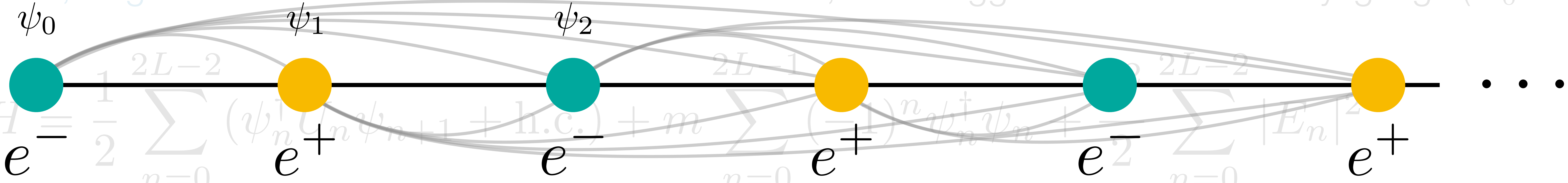



Change to Axial gauge ($A_1 = 0$)

P. Sala et al., PRD 2018
R. C. Farrel et al., PRD 2023

In 1+1D, the gauge field is not a degree of freedom, completely constrained by Gauss's law

$$H = \frac{1}{2}\sum_{n=0}^{2L-2}(\psi_n^{\dagger}\psi_{n+1} + \text{h.c.}) + m\sum_{n=0}^{2L-1}(-1)^n\psi_n^{\dagger}\psi_n + \frac{g^2}{2}\sum_{n=0}^{2L-2}(\sum_{m\leq n}Q_m)^2$$

We have:
- ✦ Non-local interactions
- ✦ Fermionic degrees of freedom (electrons and positrons) : 2 level-systems → qubits

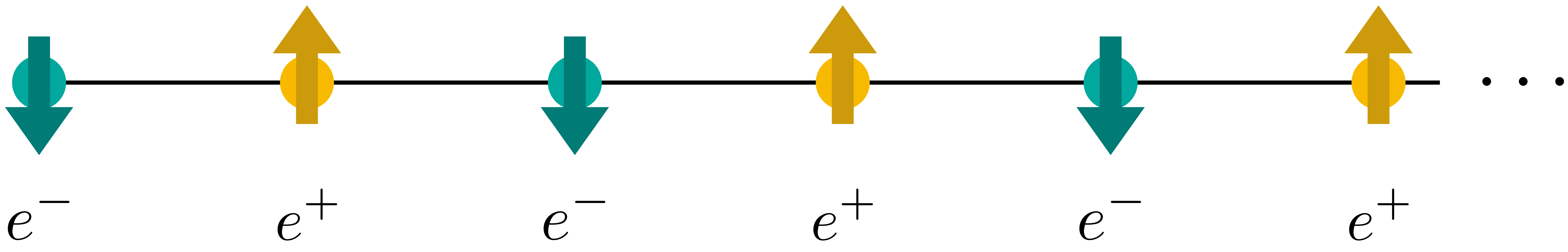
# The Schwinger Model (1+1D QED)

Apply the Jordan-Wigner transformation to map fermion field operators to spins,

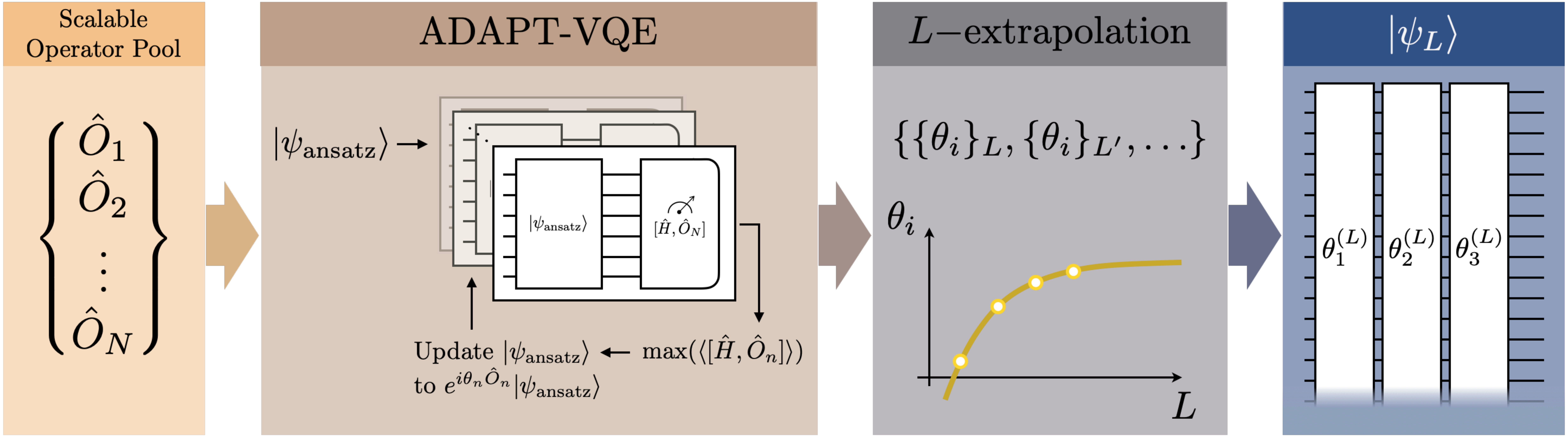
$$H = \frac{1}{2}\sum_{j=0}^{2L-2}(\sigma_j^+\sigma_{j+1}^- + \text{h.c.}) + \frac{m}{2}\sum_{j=0}^{2L-1}[(-1)^j Z_j + I] + \frac{g^2}{2}\sum_{j=0}^{2L-2}(\sum_{k\leq j} Q_k)^2$$

$$Q_k = -\frac{1}{2}[Z_k + (-1)^k I]$$

Example: strong-coupling vacuum (all empty)



# Preparing the Vacuum: SC-ADAPT-VQE



# Preparing the Vacuum: Scalable Operators

The operators from the pool are constrained to be charge neutral, symmetric under CP, and invariant under time reversal

$$\left.\begin{aligned}\hat{\Theta}_m^V &= \frac{1}{2}\sum_{n=0}^{2L-1}(-1)^n \hat{Z}_n \\ \hat{\Theta}_h^V(d) &= \frac{1}{4}\sum_{n=0}^{2L-1-d}\left(\hat{X}_n \hat{Z}^{d-1}\hat{X}_{n+d} + \hat{Y}_n \hat{Z}^{d-1}\hat{Y}_{n+d}\right)\end{aligned}\right\}\text{Volume operators}$$

$$\left.\begin{aligned}\hat{\Theta}_m^S(d) &= (-1)^d \frac{1}{2}\left(\hat{Z}_d - \hat{Z}_{2L-1-d}\right) \\ \hat{\Theta}_h^S(d) &= \frac{1}{4}\left(\hat{X}_1 \hat{Z}^{d-1}\hat{X}_{d+1} + \hat{Y}_1 \hat{Z}^{d-1}\hat{Y}_{d+1} \;+\; \hat{X}_{2L-2-d}\hat{Z}^{d-1}\hat{X}_{2L-2} + \hat{Y}_{2L-2-d}\hat{Z}^{d-1}\hat{Y}_{2L-2}\right)\end{aligned}\right\}\text{Surface operators}$$

# Preparing the Vacuum: Scalable Operators

Time reversal symmetry ⟷ Real wavefunction ⟷ $e^{i\theta_i \hat{O}_i}$ real ⟷ $\hat{O}_i$ imaginary and anti-symmetric

These are not imaginary and anti-symmetric ✗

↓

$$\hat{\Theta}_m^V = \frac{1}{2}\sum_{n=0}^{2L-1}(-1)^n \hat{Z}_n$$

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# Preparing the Vacuum: Scalable Operators

Time reversal symmetry $\longleftrightarrow$ Real wavefunction $\longleftrightarrow$ $e^{i\theta_i \hat{O}_i}$ real $\longleftrightarrow$ $\hat{O}_i$ imaginary and anti-symmetric

These are imaginary and anti-symmetric

$$\{\hat{O}\} = \left\{\hat{O}^V_{mh}(d)\ ,\ \hat{O}^S_{mh}(0,d)\ ,\ \hat{O}^S_{mh}(1,d)\right\}$$

$$\hat{O}^V_{mh}(d) \equiv i\left[\hat{\Theta}^V_m, \hat{\Theta}^V_h(d)\right] = \frac{1}{2}\sum_{n=0}^{2L-1-d}(-1)^n\left(\hat{X}_n\hat{Z}^{d-1}\hat{Y}_{n+d} - \hat{Y}_n\hat{Z}^{d-1}\hat{X}_{n+d}\right)$$

$$\hat{O}^S_{mh}(0,d) \equiv i\left[\hat{\Theta}^S_m(0), \hat{\Theta}^V_h(d)\right] = \frac{1}{4}\left(\hat{X}_0\hat{Z}^{d-1}\hat{Y}_d - \hat{Y}_0\hat{Z}^{d-1}\hat{X}_d - \hat{Y}_{2L-1-d}\hat{Z}^{d-1}\hat{X}_{2L-1} + \hat{X}_{2L-1-d}\hat{Z}^{d-1}\hat{Y}_{2L-1}\right)$$

$$\hat{O}^S_{mh}(1,d) \equiv i\left[\hat{\Theta}^S_m(1), \hat{\Theta}^S_h(d)\right] = \frac{1}{4}\left(\hat{Y}_1\hat{Z}^{d-1}\hat{X}_{d+1} - \hat{X}_1\hat{Z}^{d-1}\hat{Y}_{d+1} + \hat{Y}_{2L-2-d}\hat{Z}^{d-1}\hat{X}_{2L-2} - \hat{X}_{2L-2-d}\hat{Z}^{d-1}\hat{Y}_{2L-2}\right)$$

These operators create and annihilate $e^+e^-$ pairs separated by a distance $d$

# Preparing the Vacuum: Scalable Operators

$$\{\hat{O}\} = \left\{\hat{O}^V_{mh}(d)\ ,\ \hat{O}^S_{mh}(0,d)\ ,\ \hat{O}^S_{mh}(1,d)\right\}$$

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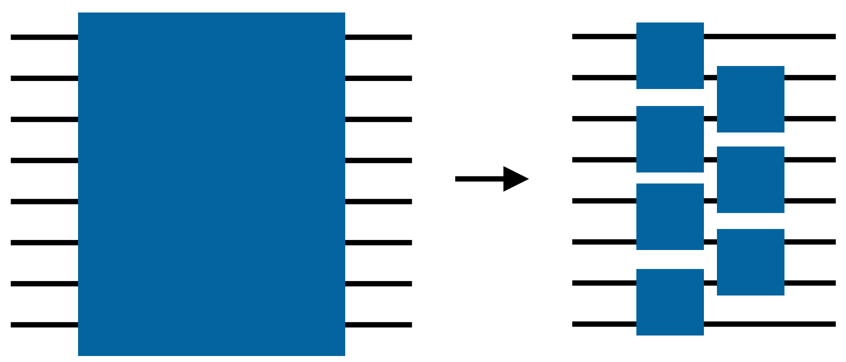
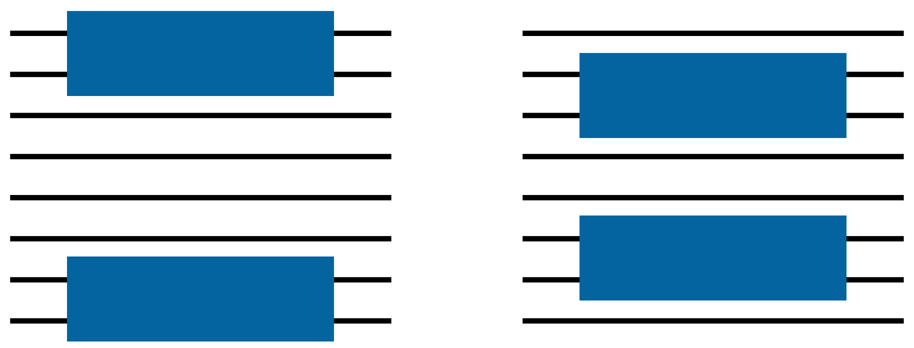


$$\hat{O}^S_{mh}(0,d) = \frac{1}{4}\left(\hat{X}_0\hat{Z}^{d-1}\hat{Y}_d - \hat{Y}_0\hat{Z}^{d-1}\hat{X}_d - \hat{Y}_{2L-1-d}\hat{Z}^{d-1}\hat{X}_{2L-1} + \hat{X}_{2L-1-d}\hat{Z}^{d-1}\hat{Y}_{2L-1}\right)$$

$$\hat{O}^S_{mh}(1,d) = \frac{1}{4}\left(\hat{Y}_1\hat{Z}^{d-1}\hat{X}_{d+1} - \hat{X}_1\hat{Z}^{d-1}\hat{Y}_{d+1} + \hat{Y}_{2L-2-d}\hat{Z}^{d-1}\hat{X}_{2L-2} - \hat{X}_{2L-2-d}\hat{Z}^{d-1}\hat{Y}_{2L-2}\right)$$



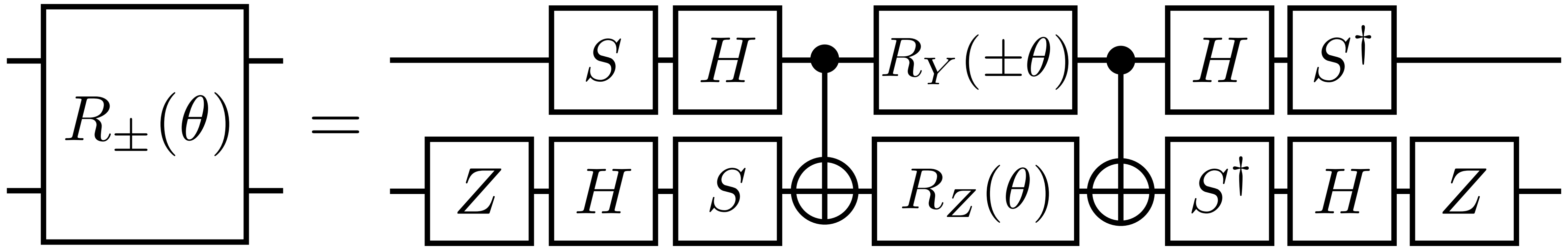
Need efficient circuits for $e^{i\theta(XZ^{d-1}Y - YZ^{d-1}X)}$

These operators create and annihilate $e^+e^-$ pairs separated by a distance $d$

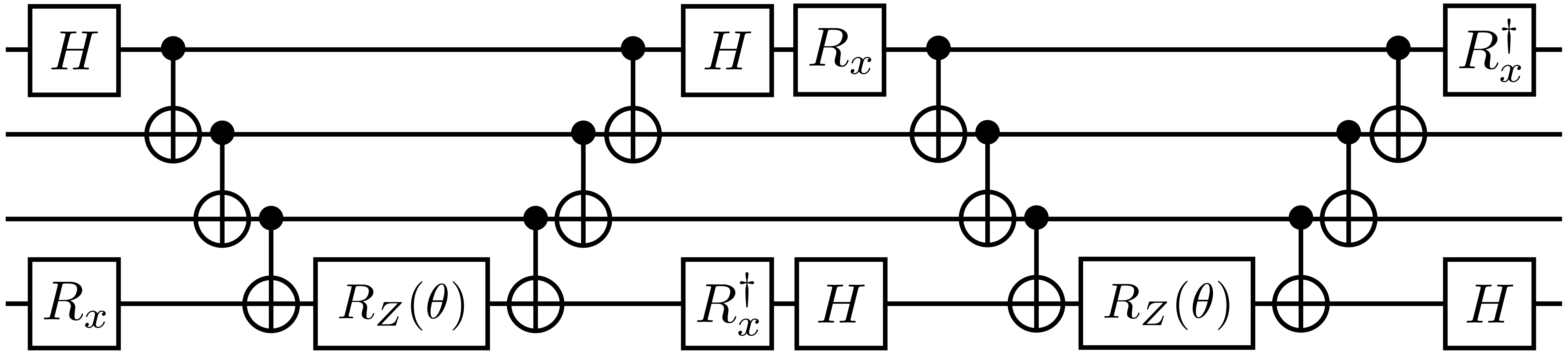
# Preparing the Vacuum: Efficient Circuts

Using the tools developed in Algaba et al., Quantum 2024

$$e^{i\frac{\theta}{2}(XY \pm YX)} =$$




$$e^{-i\frac{\theta}{2}(XZ^2Y - YZ^2X)} =$$

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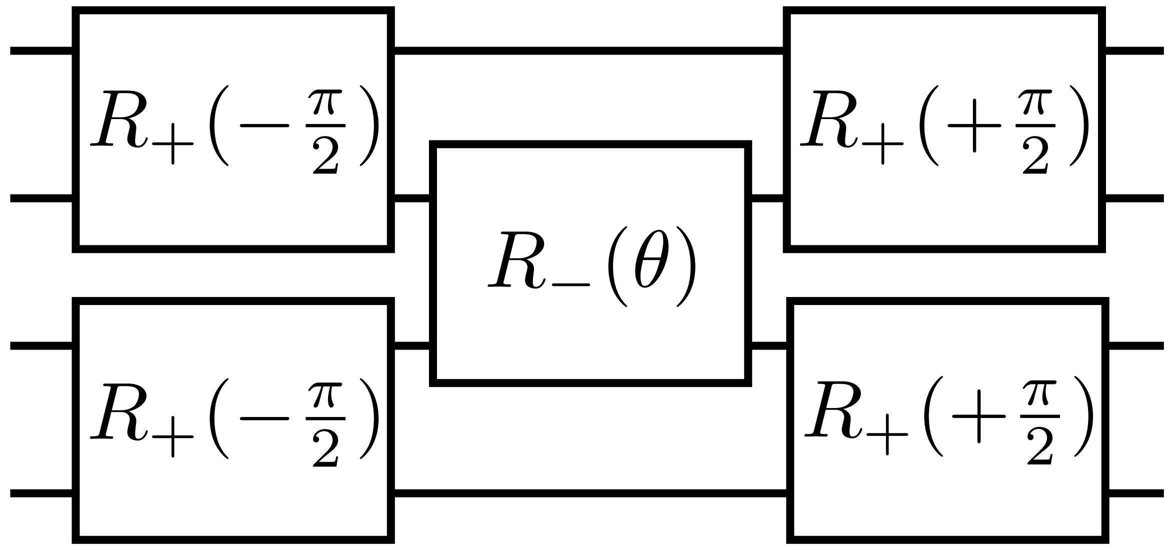

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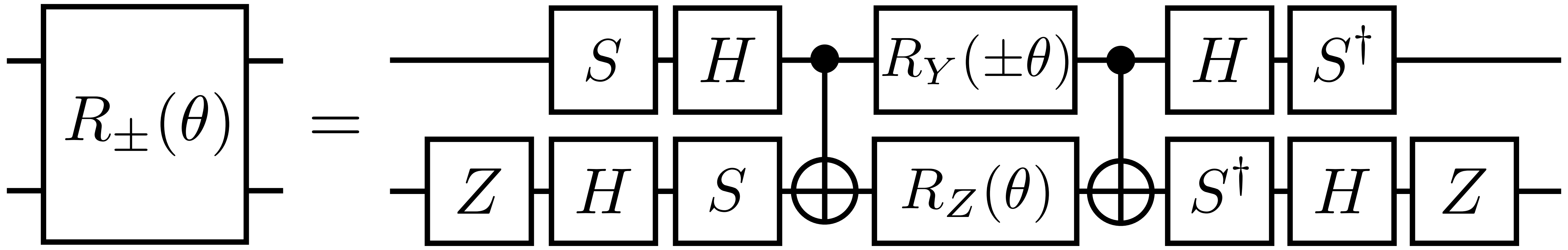


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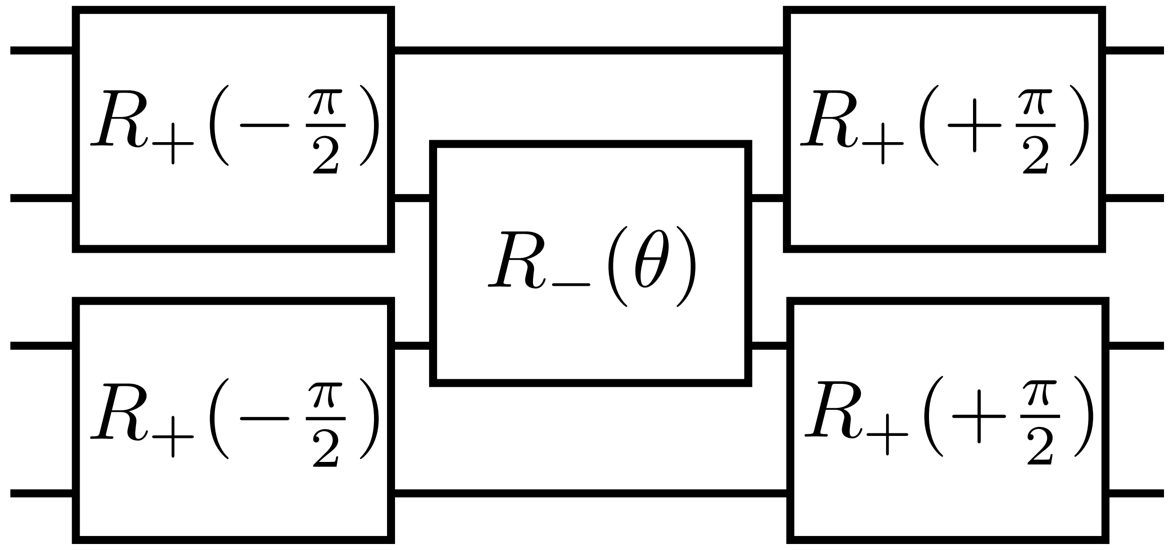
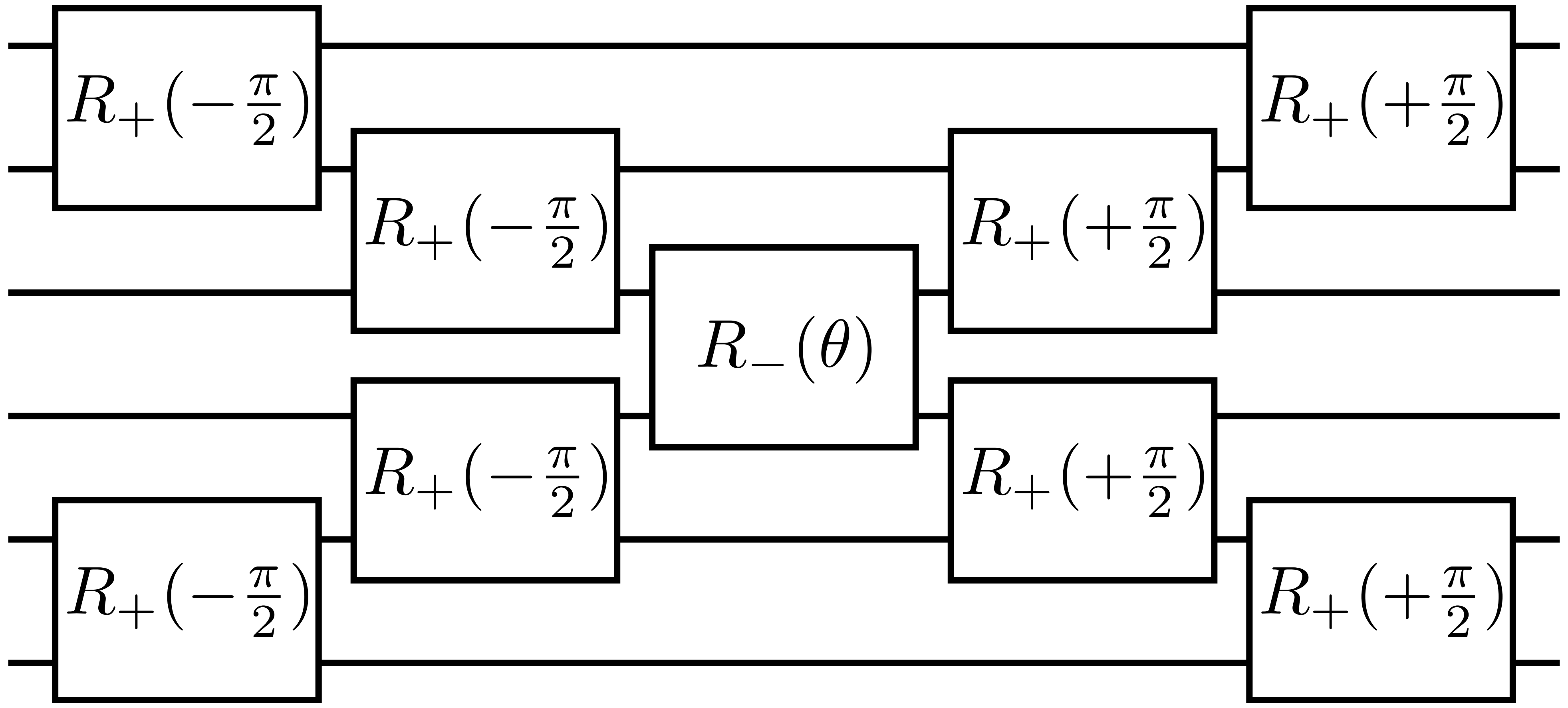



$$e^{i\frac{\theta}{2}(XZ^4Y - YZ^4X)} =$$



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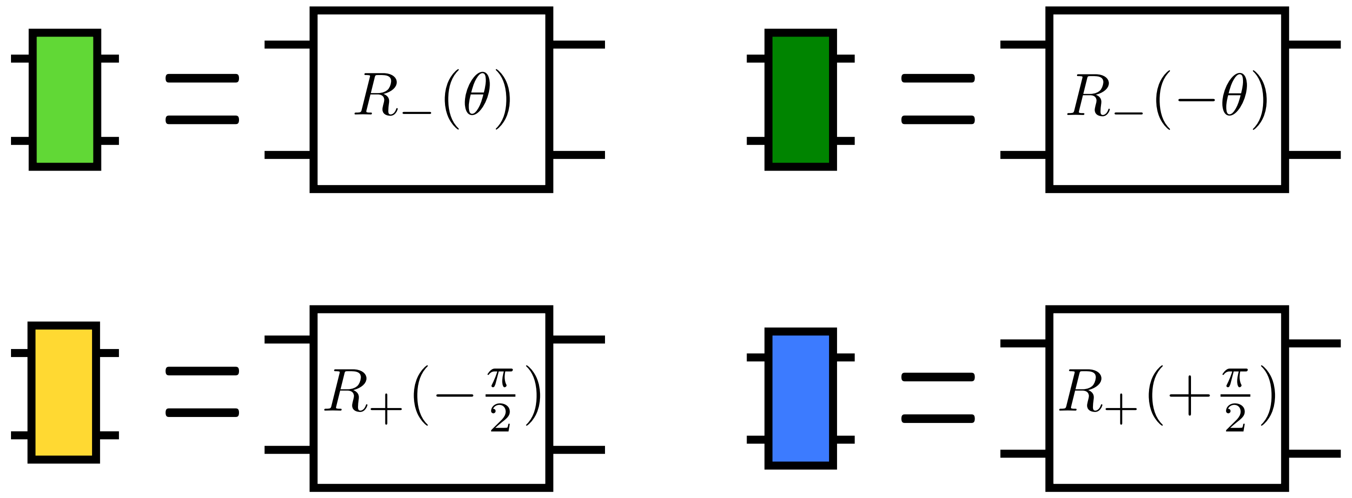
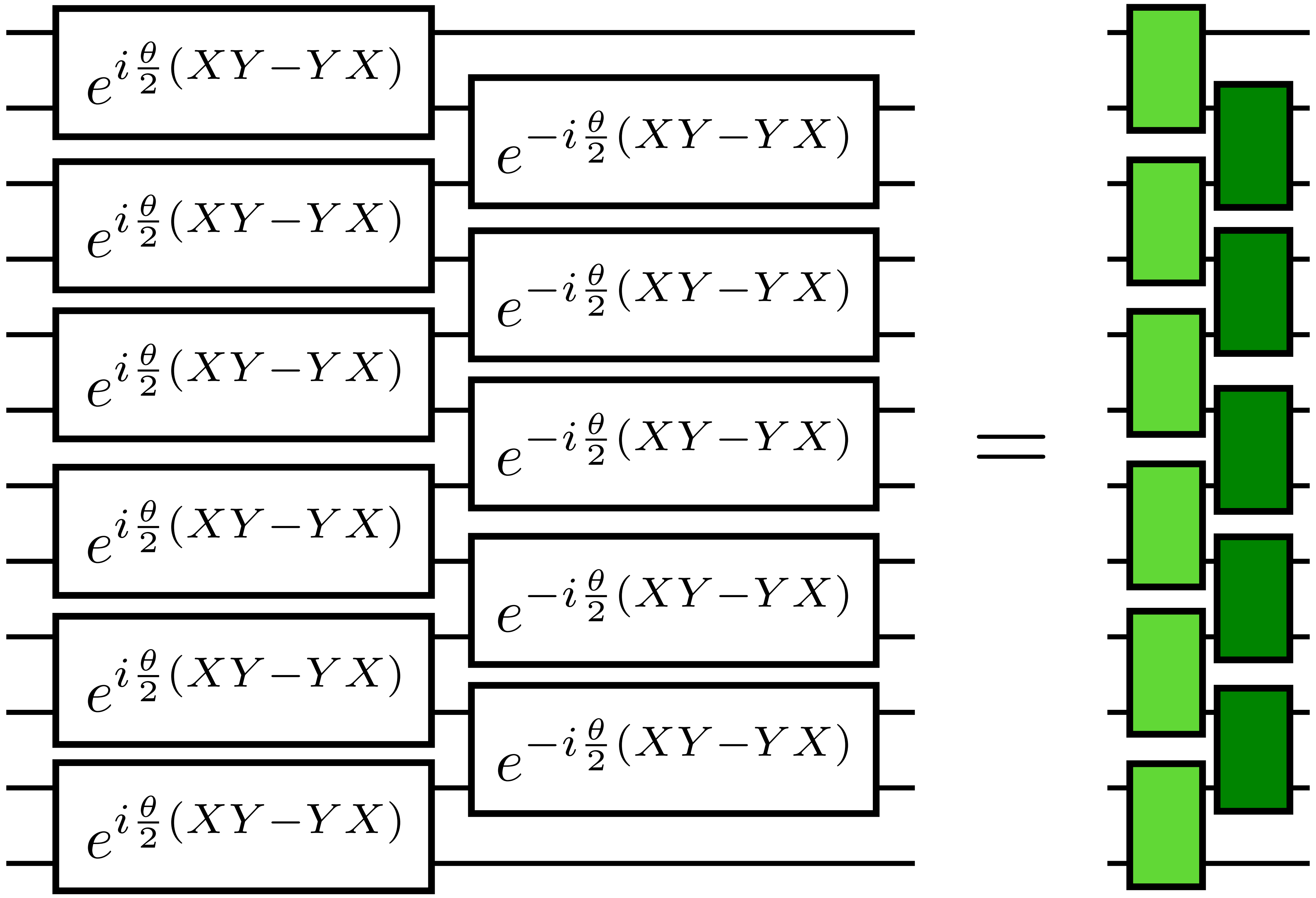



$$\hat{O}^V_{mh}(1) = \frac{1}{2}\sum_{n=0}^{2L-2}(-1)^n\left(\hat{X}_n\hat{Y}_{n+1} - \hat{Y}_n\hat{X}_{n+1}\right) \longrightarrow e^{i\theta\hat{O}^V_{mh}(1)}$$

Example for $L = 6$



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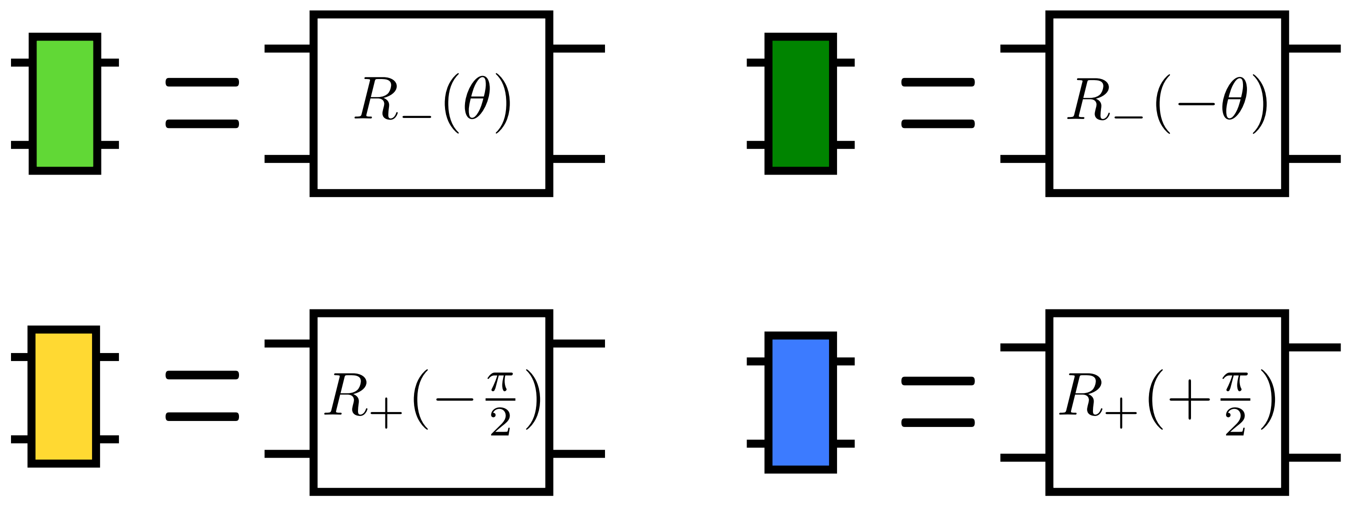


$$\hat{O}^V_{mh}(3) = \frac{1}{2}\sum_{n=0}^{2L-4}(-1)^n\left(\hat{X}_n\hat{Z}^2\hat{Y}_{n+3} - \hat{Y}_n\hat{Z}^2\hat{X}_{n+3}\right) \longrightarrow e^{i\theta\hat{O}^V_{mh}(3)}$$

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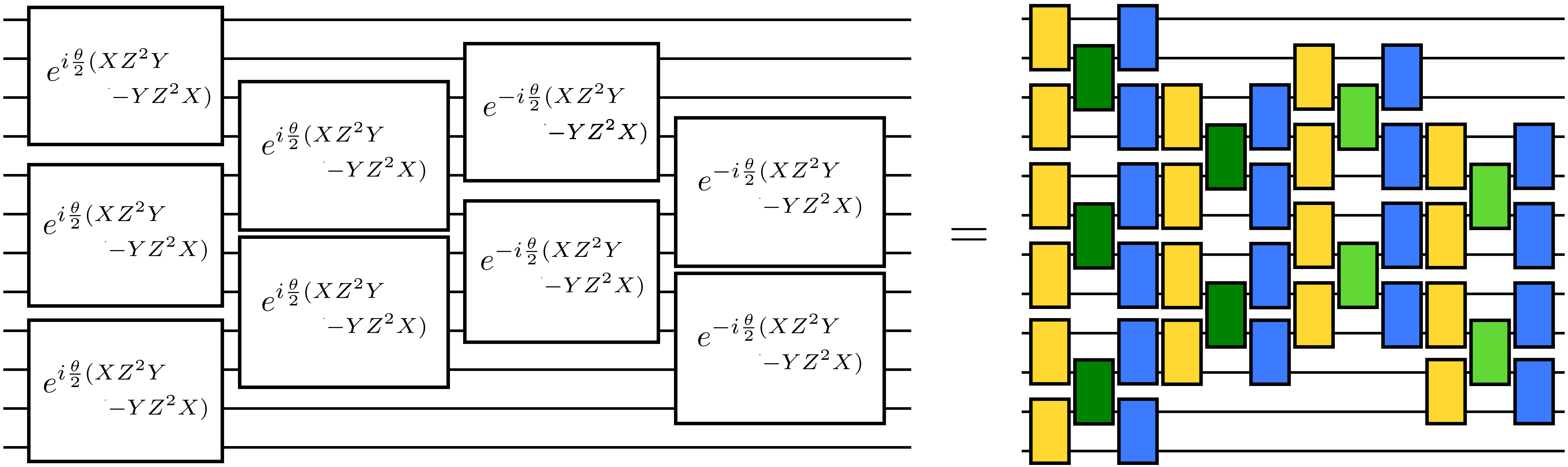

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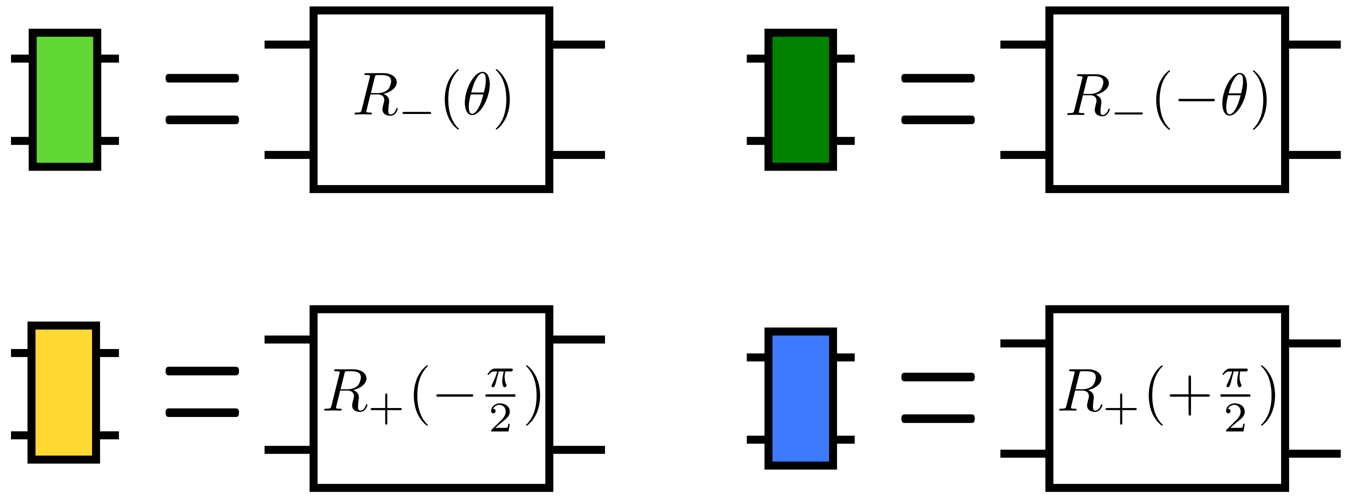


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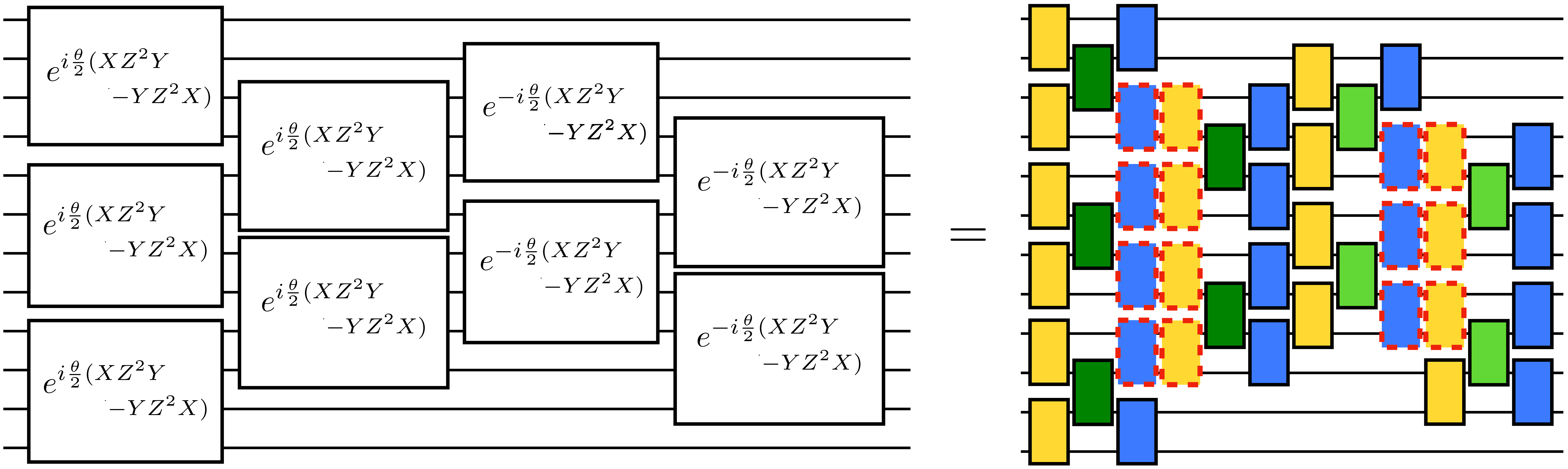

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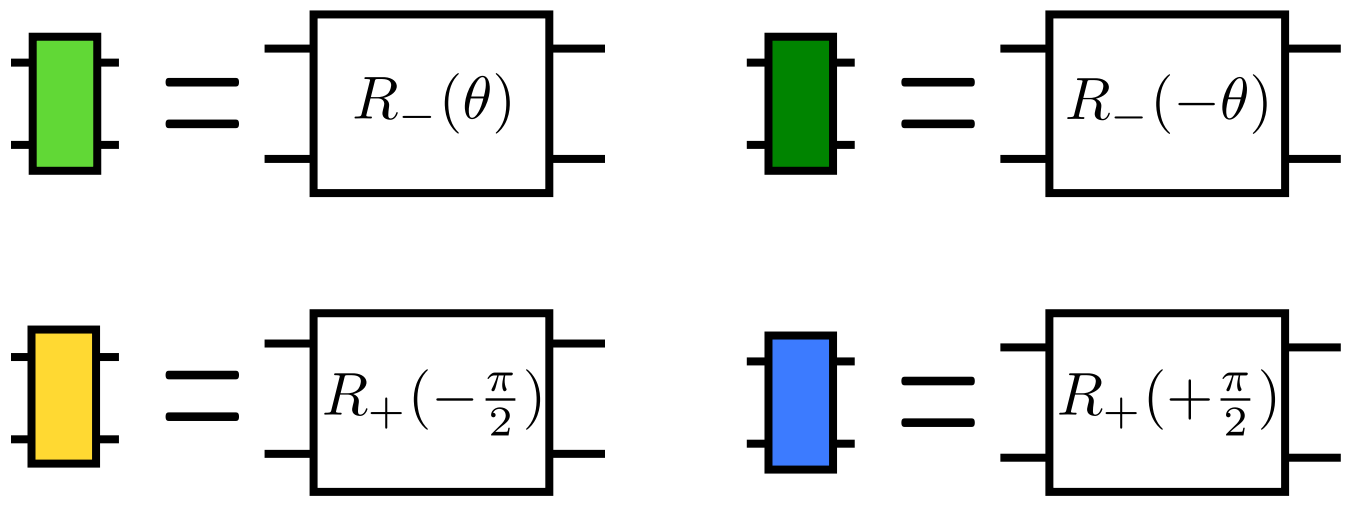


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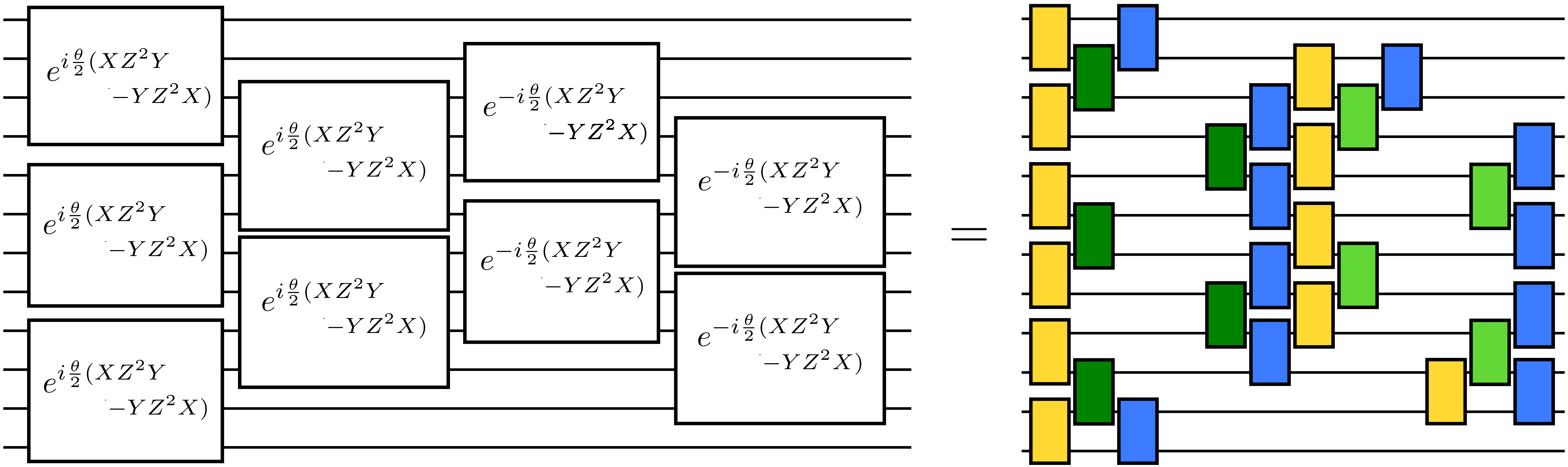

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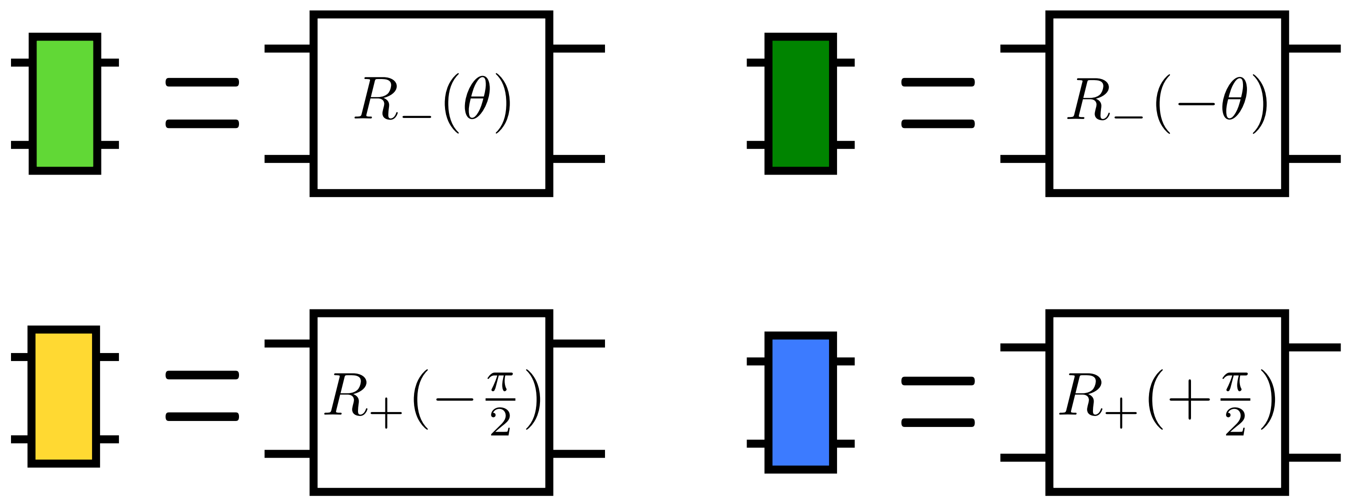


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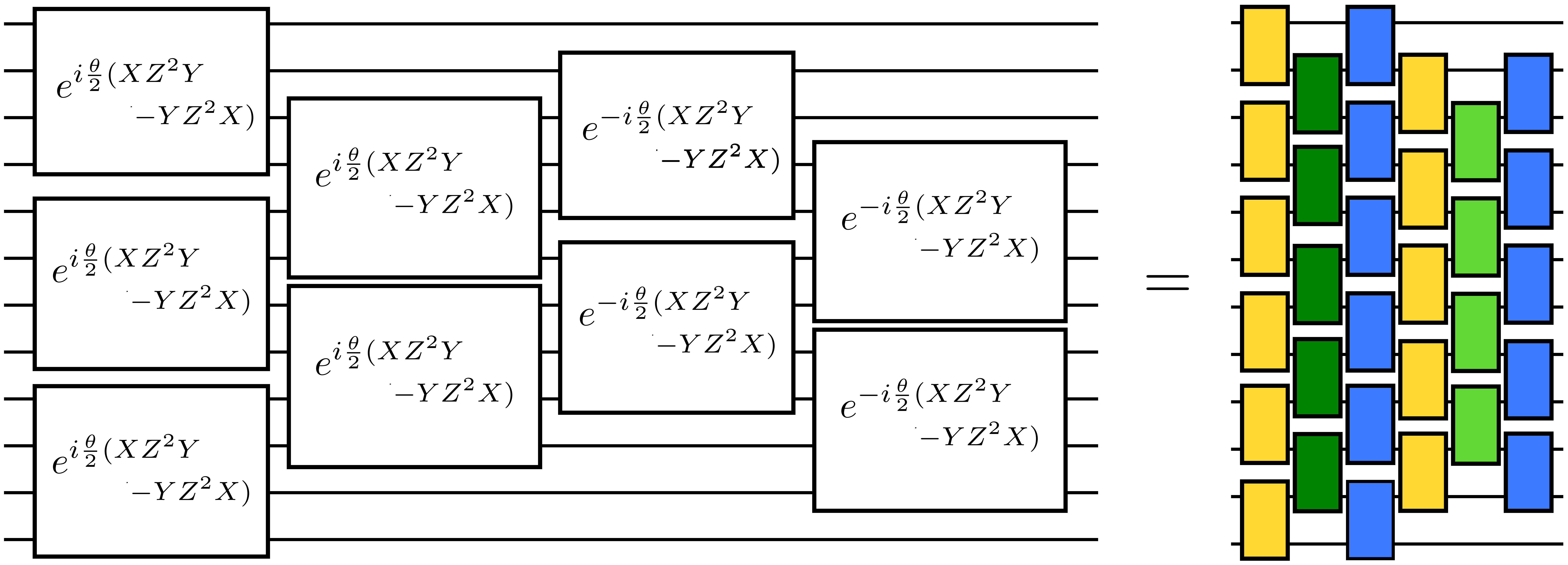
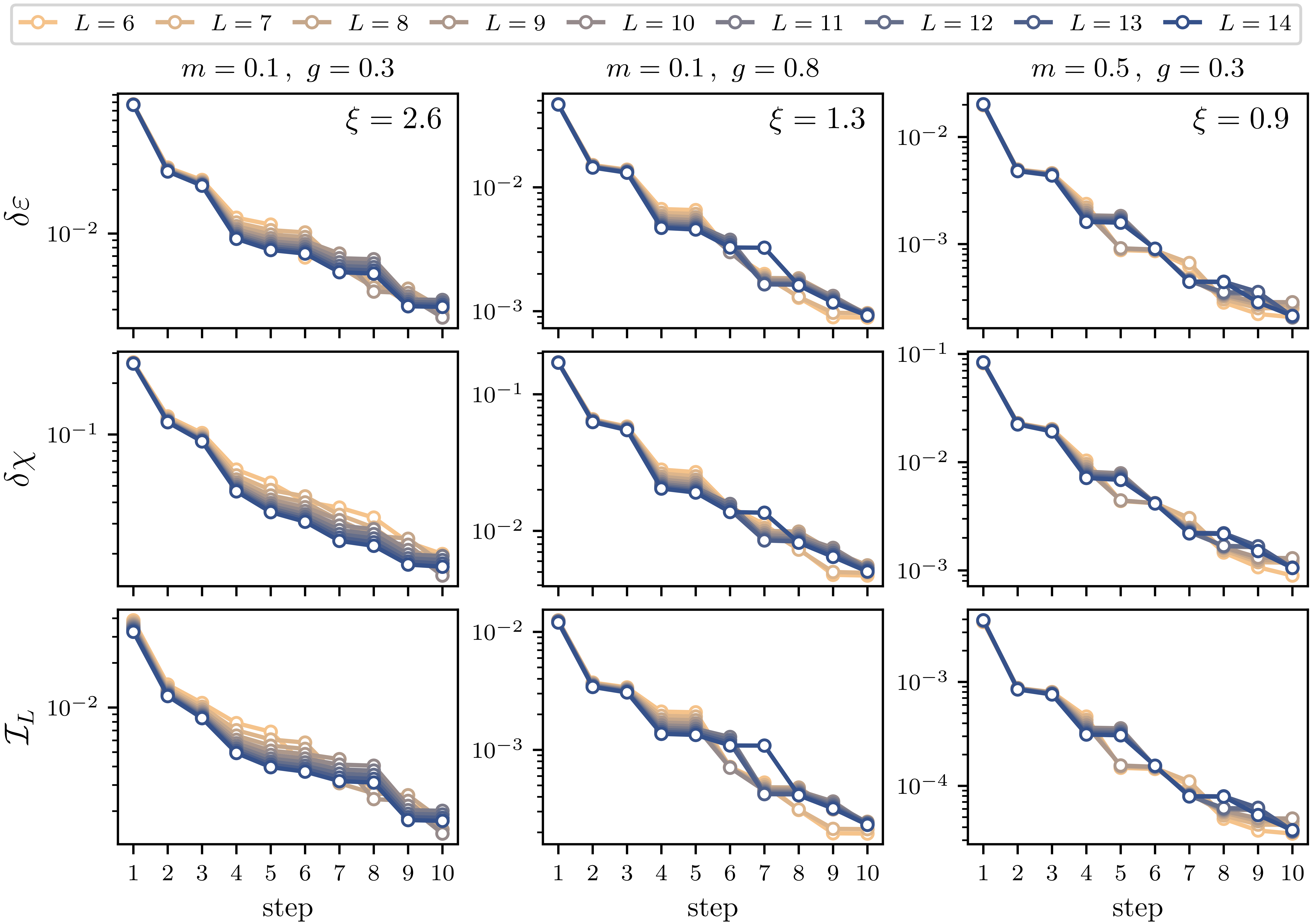


# Convergence as a funtion of Layers (using classical resources)




$$\delta x = \left| \frac{x^{(\mathrm{aVQE})} - x^{(\mathrm{exact})}}{x^{(\mathrm{exact})}} \right|$$

$$\varepsilon = \langle \hat{H} \rangle / L$$

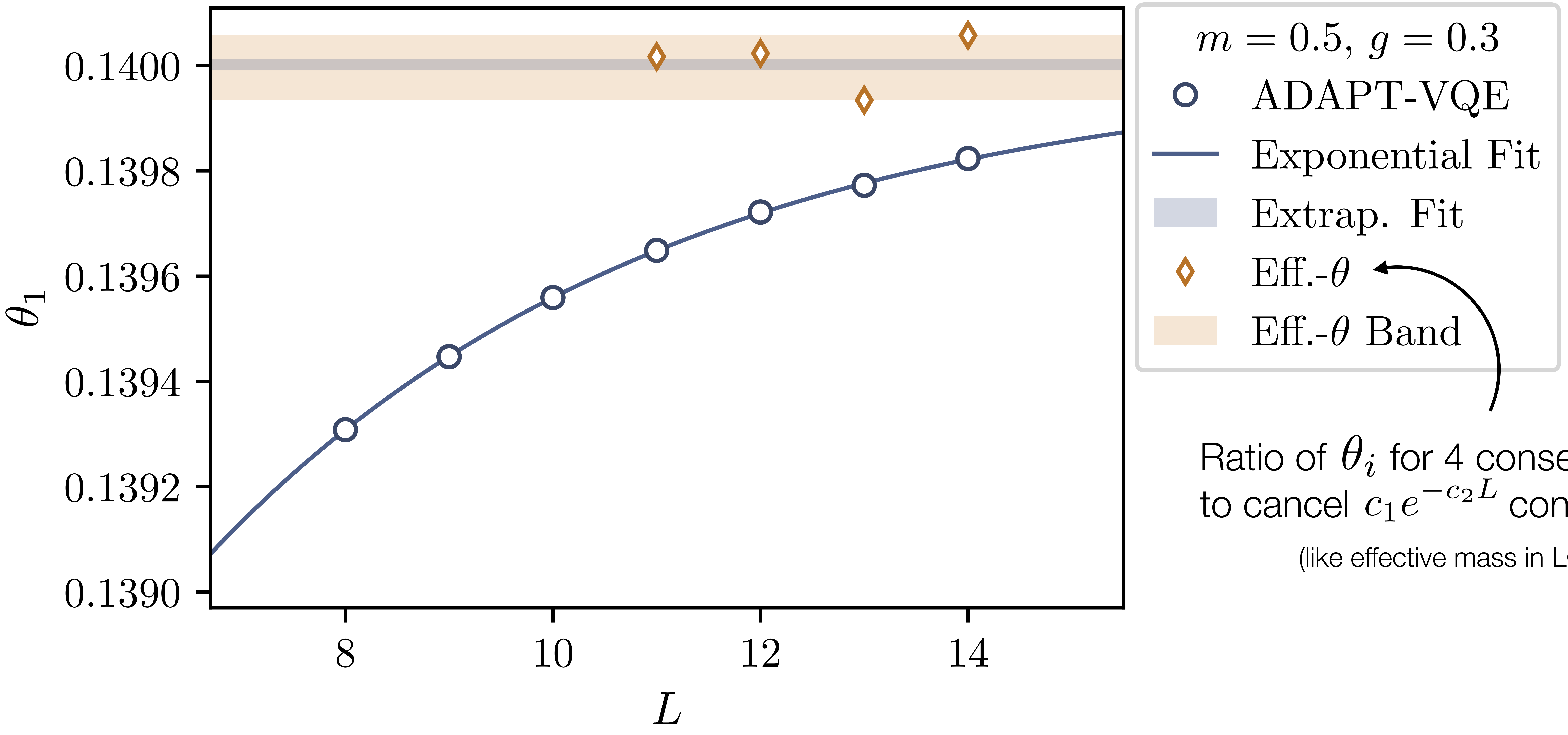
$$\chi = \langle \overline{\psi}\psi \rangle \to \frac{1}{L} \sum_i \langle \overline{\psi}_i \psi_i \rangle$$

$$\mathcal{I}_L = \frac{1}{L}\left(1 - |\langle \psi_{\mathrm{ansatz}} | \psi_{\mathrm{exact}} \rangle|^2\right)$$

# Convergence as a funtion of Layers (using classical resources)

Having exponentially-decaying correlations, and in volumes large enough to contain the longest correlation length, the variational parameters are expected to be exponentially close to their infinite-volume values.

$$\theta_i(L) = \theta_i^\infty + c_1 e^{-c_2 L}$$



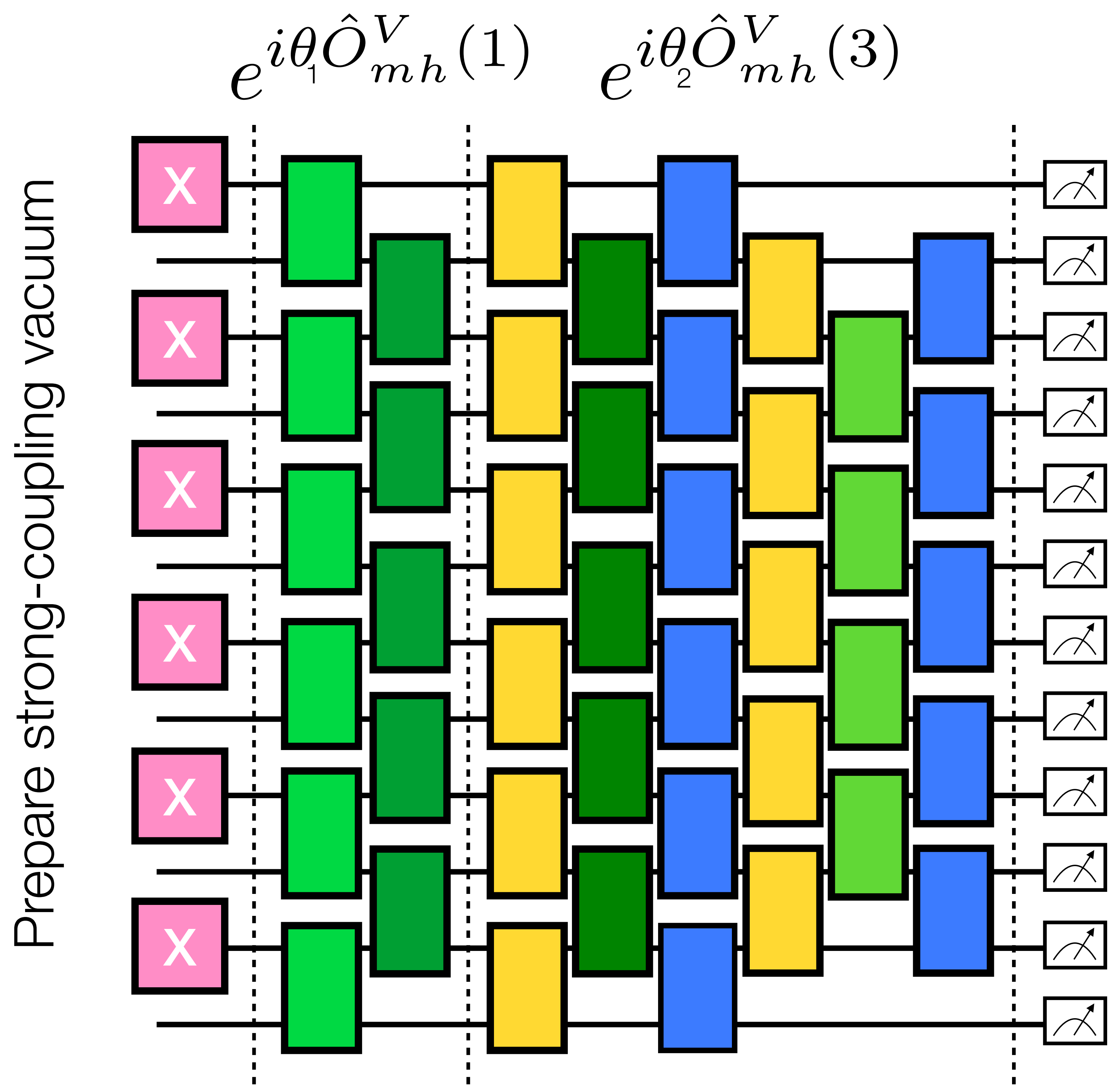

Ratio of $\theta_i$ for 4 consecutive Ls to cancel $c_1 e^{-c_2 L}$ contribution

(like effective mass in LQCD)

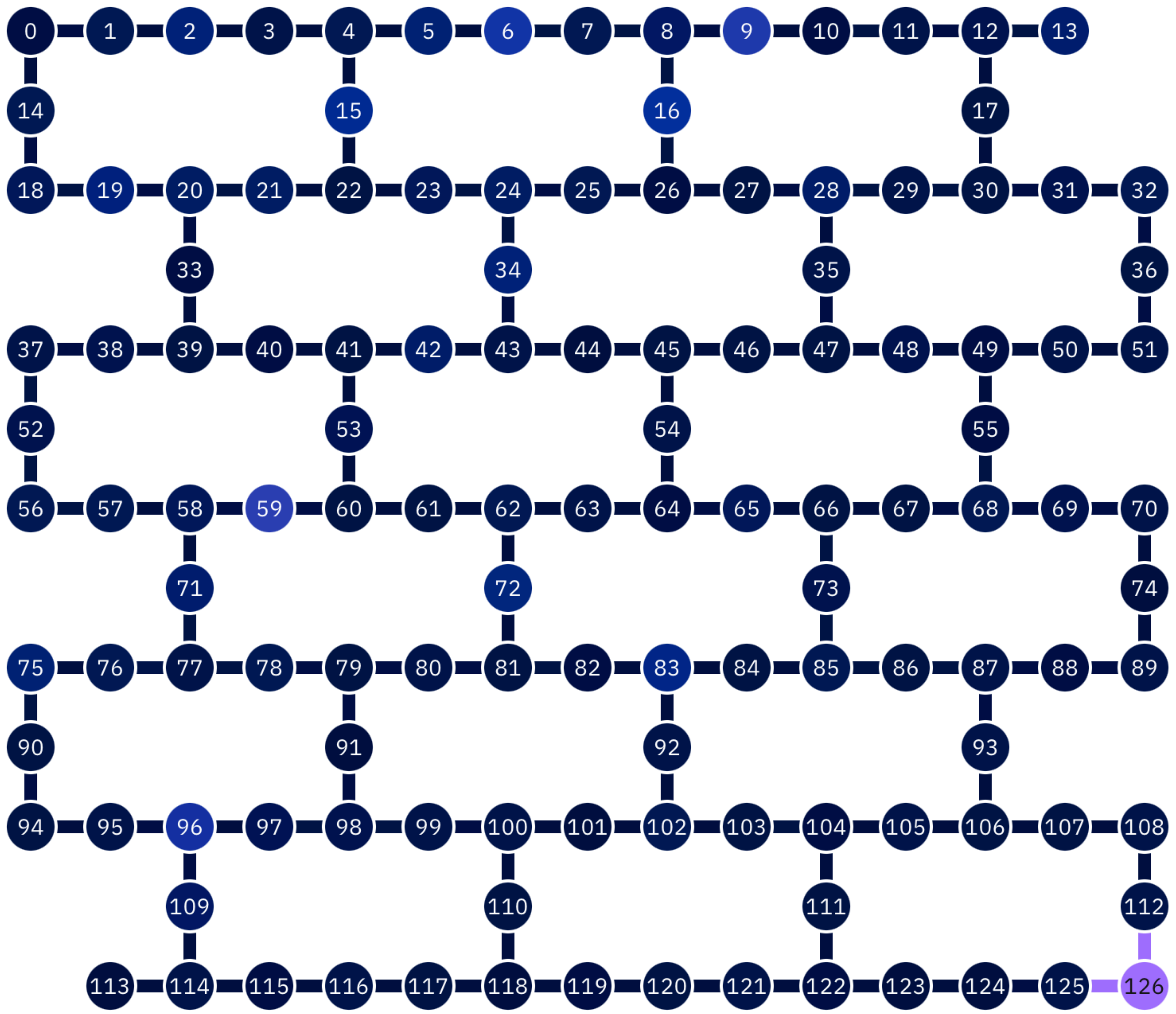
# Running on a Quantum Computer

Once we have the angles of the operators up to L=14 computed classically, we can extrapolate them to arbitrarily large L

We run circuits with 2 layers for L=14, 20, 30, 40 and 50 (which means 28, 40, 60, 80 and 100 qubits)




We need to map the circuit on the chip



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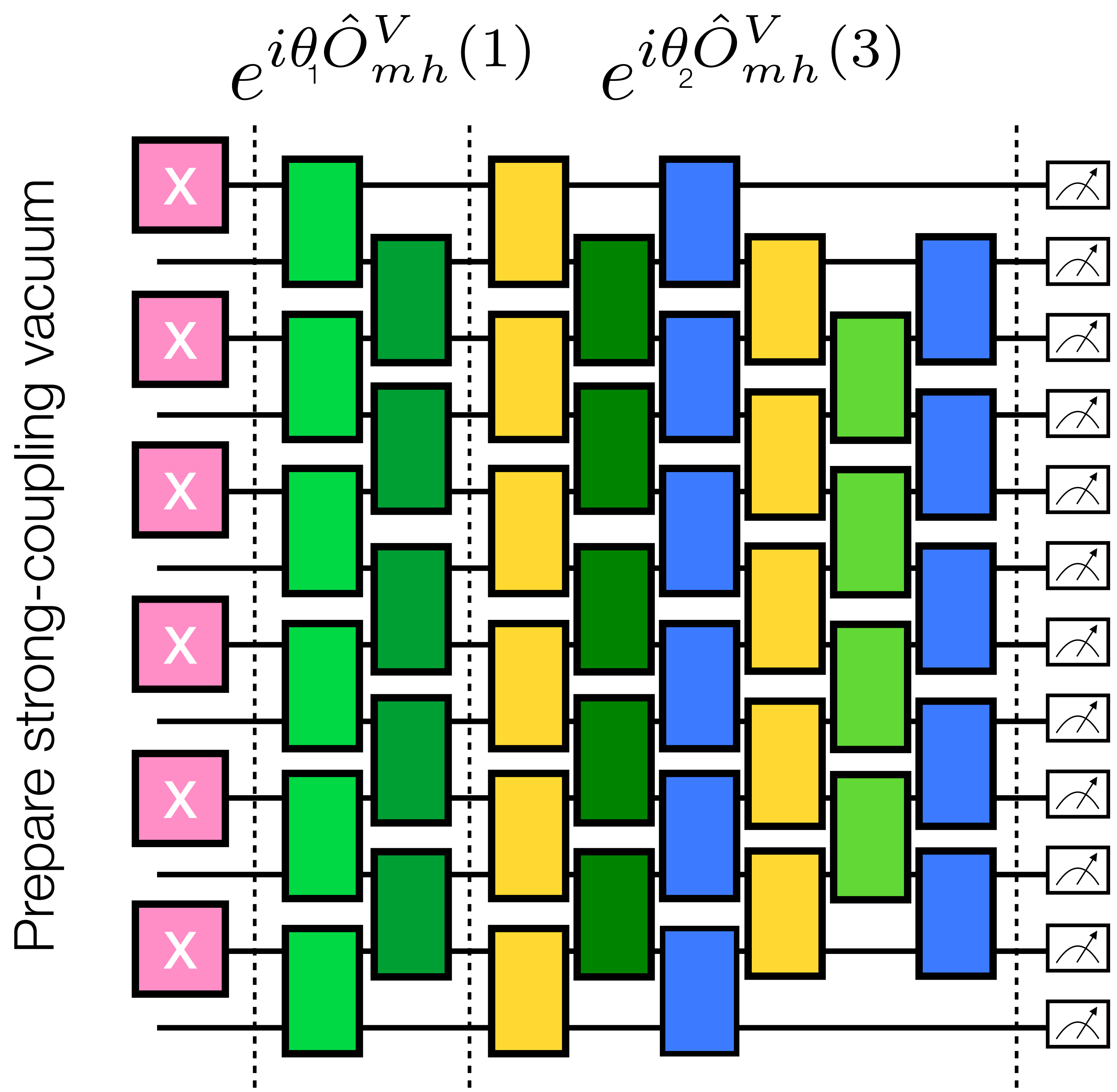


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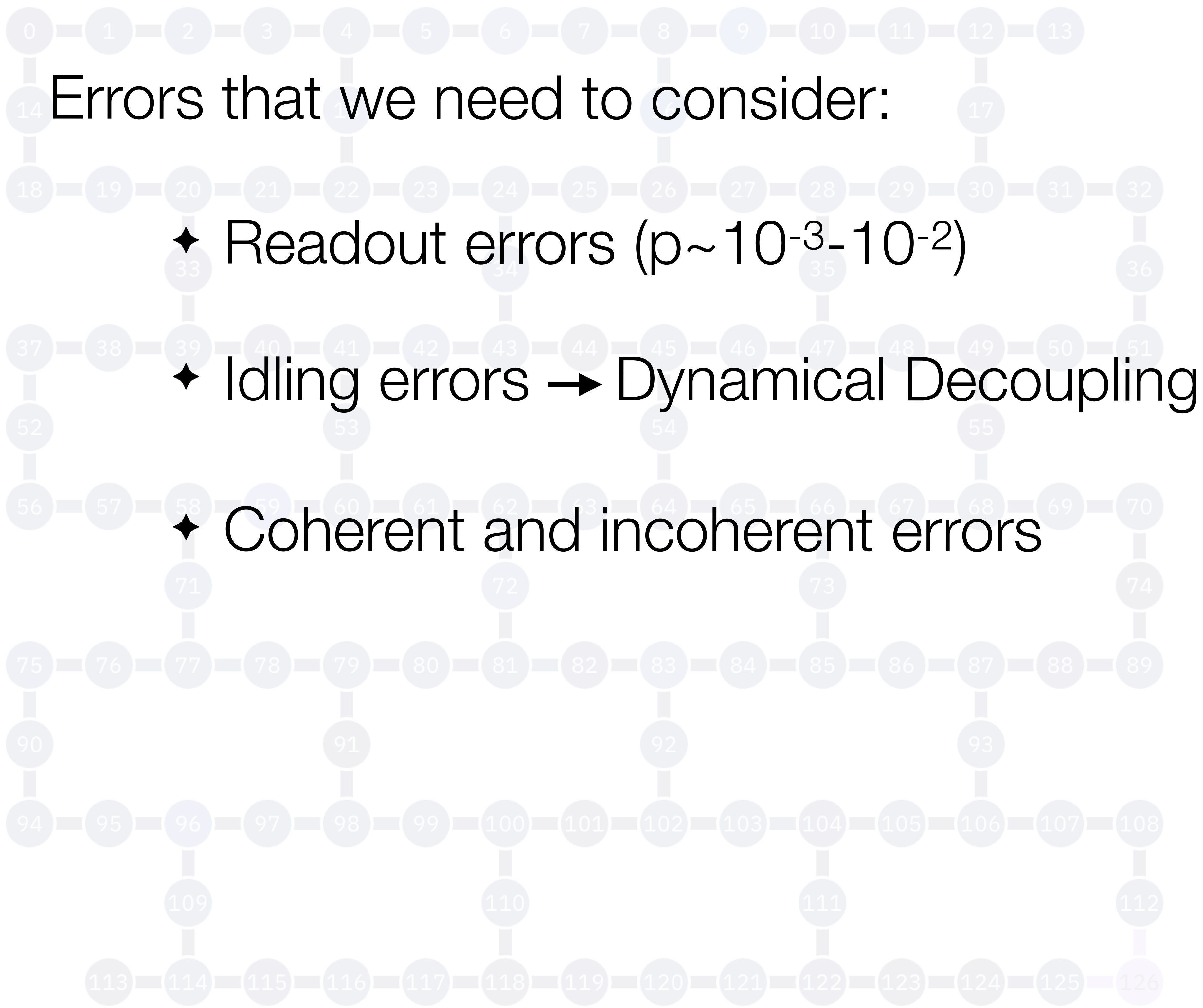


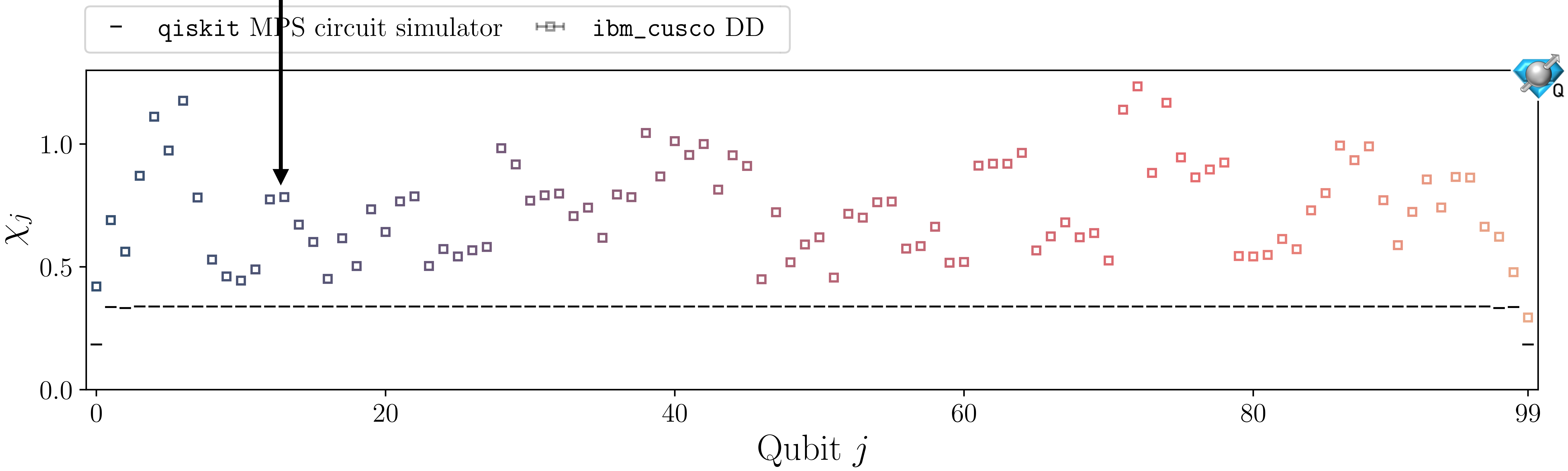
Errors that we need to consider:

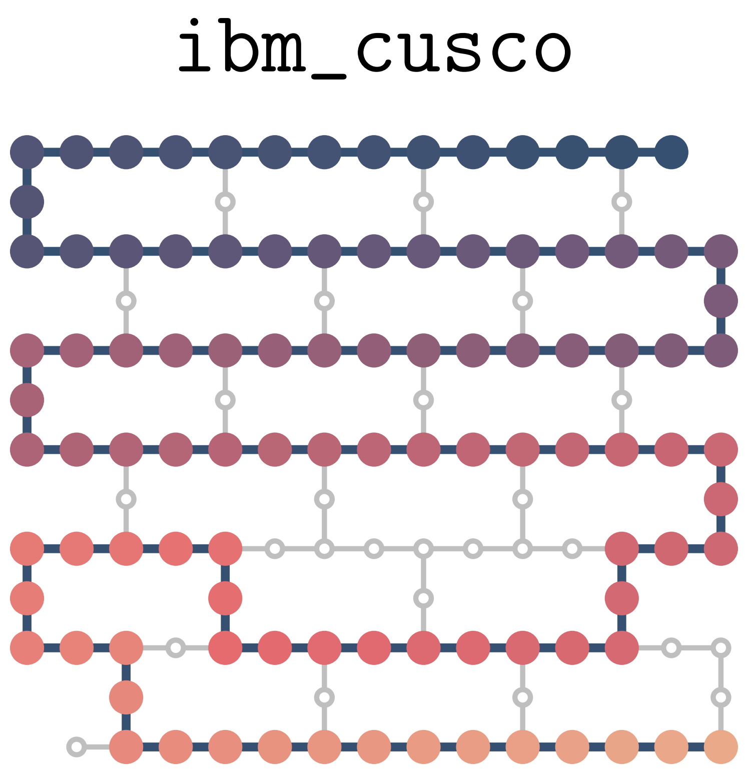
- ✦ Readout errors (p~$10^{-3}$-$10^{-2}$)
- ✦ Idling errors → Dynamical Decoupling
- ✦ Coherent and incoherent errors

# Running on a Quantum Computer: Chiral Condensate

$$\chi = \frac{1}{L}\sum_i \langle \overline{\psi}_i \psi_i \rangle = \frac{1}{2L}\sum_i [(-1)^i Z_i + I] \equiv \frac{1}{2L}\sum_i \chi_i$$

There are still additional errors ⟶ Main contribution coming from 2-qubit gates, i.e., CNOTs



# Mitigating the noise

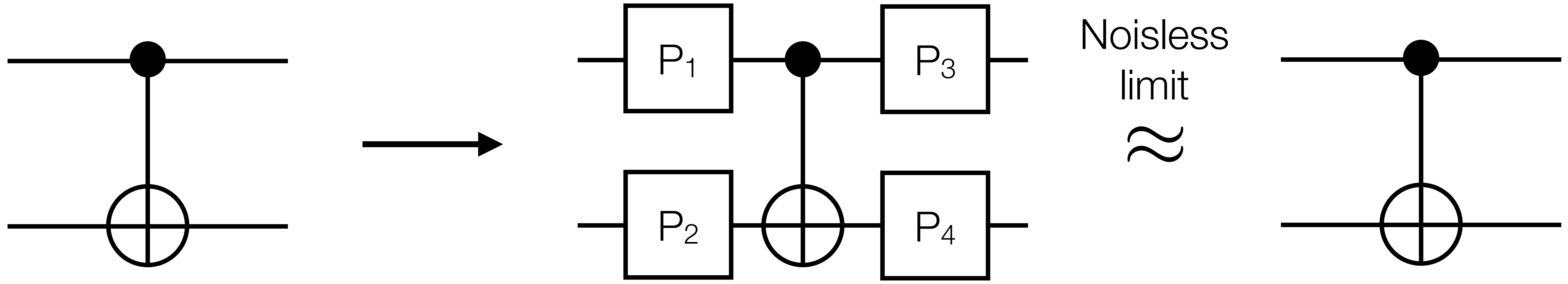
On a quantum computer, we have to deal with:

- ✦ Incoherent (stochastic) noise: relaxation and dephasing (T1 and T2 times)
- ✦ Coherent noise: unitary rotations caused by miscalibrations or cross-talk

We know how to deal with incoherent noise (later), but not with coherent noise. However, there is a way to transform coherent noise into incoherent noise

↓

**Pauli Twirling (or randomized compiling)**




$P_i = \{I, X, Y, Z\}$

# Mitigating the noise

$$\chi = \frac{1}{L}\sum_i \langle \overline{\psi}_i \psi_i \rangle = \frac{1}{2L}\sum_i [(-1)^i Z_i + I] \equiv \frac{1}{2L}\sum_i \chi_i$$

The remaining errors are incoherent (stochastic) → We can correct them using **Operator Decoherence Renormalization (ODR)**

# Mitigating the noise

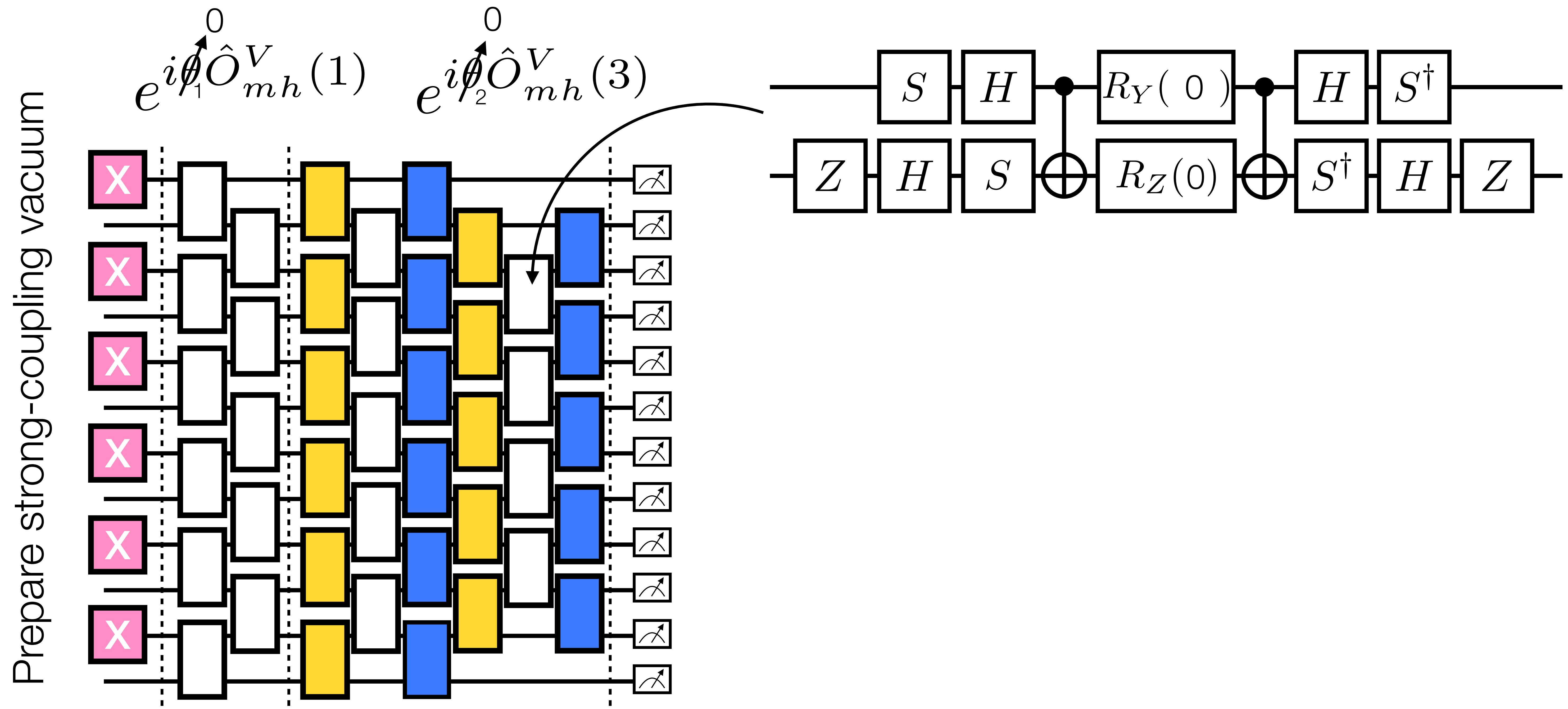
M. Urbanek et al., PRL 2021
S. A Rahman et al., PRD 2022
R. C. Farrel et al., PRD 2023
A. N. Ciavarella, PRD 2023
⋮

## Operator Decoherence Renormalization (ODR)

If we only have incoherence noise, we can assume that our measured observable is related to the noiseless one via the following relation:

$$\langle \hat{O} \rangle_{\text{meas}} = (1 - \eta_O) \langle \hat{O} \rangle_{\text{pred}}$$

We need a way estimate the noise parameter $\eta_O$ ⟶ We have to know $\langle \hat{O} \rangle_{\text{meas}}$ and $\langle \hat{O} \rangle_{\text{pred}}$



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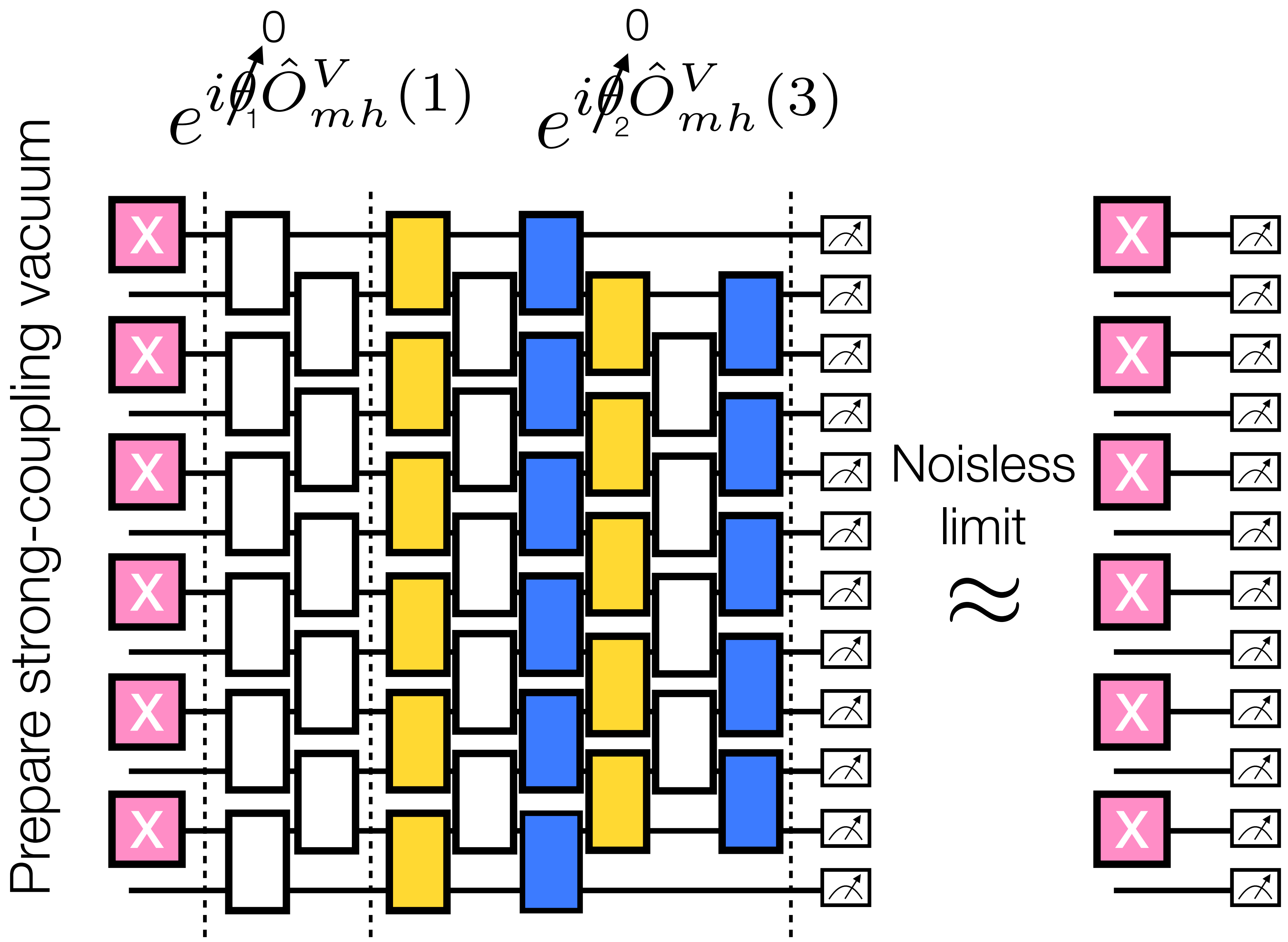



We call this the **mitigation circuit**, which has the same structure as the physics circuit, but we know the noiseless output

# Mitigating the noise

$$\chi = \frac{1}{L}\sum_i \langle \overline{\psi}_i \psi_i \rangle = \frac{1}{2L}\sum_i [(-1)^i Z_i + I] \equiv \frac{1}{2L}\sum_i \chi_i$$

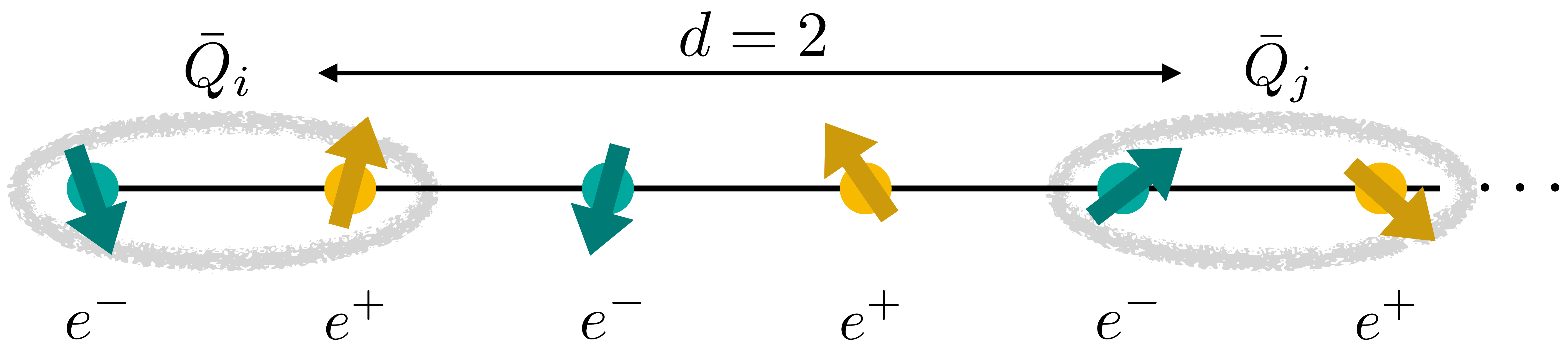
After applying Pauli Twirling + ODR,
results agree within statistical uncertainty ✓

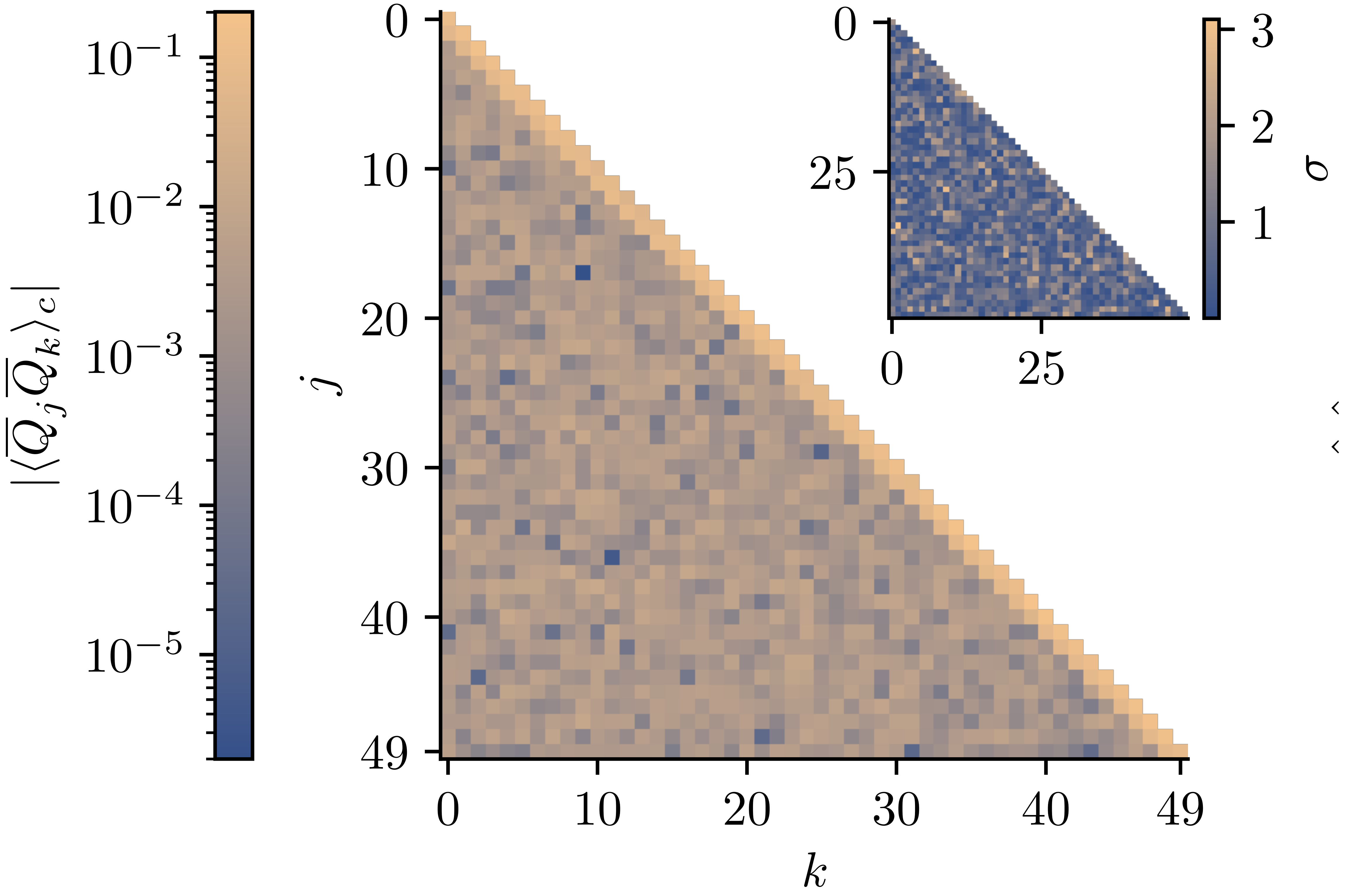
40 mitigation circuits + 40 physics circuits
= 80 circuts

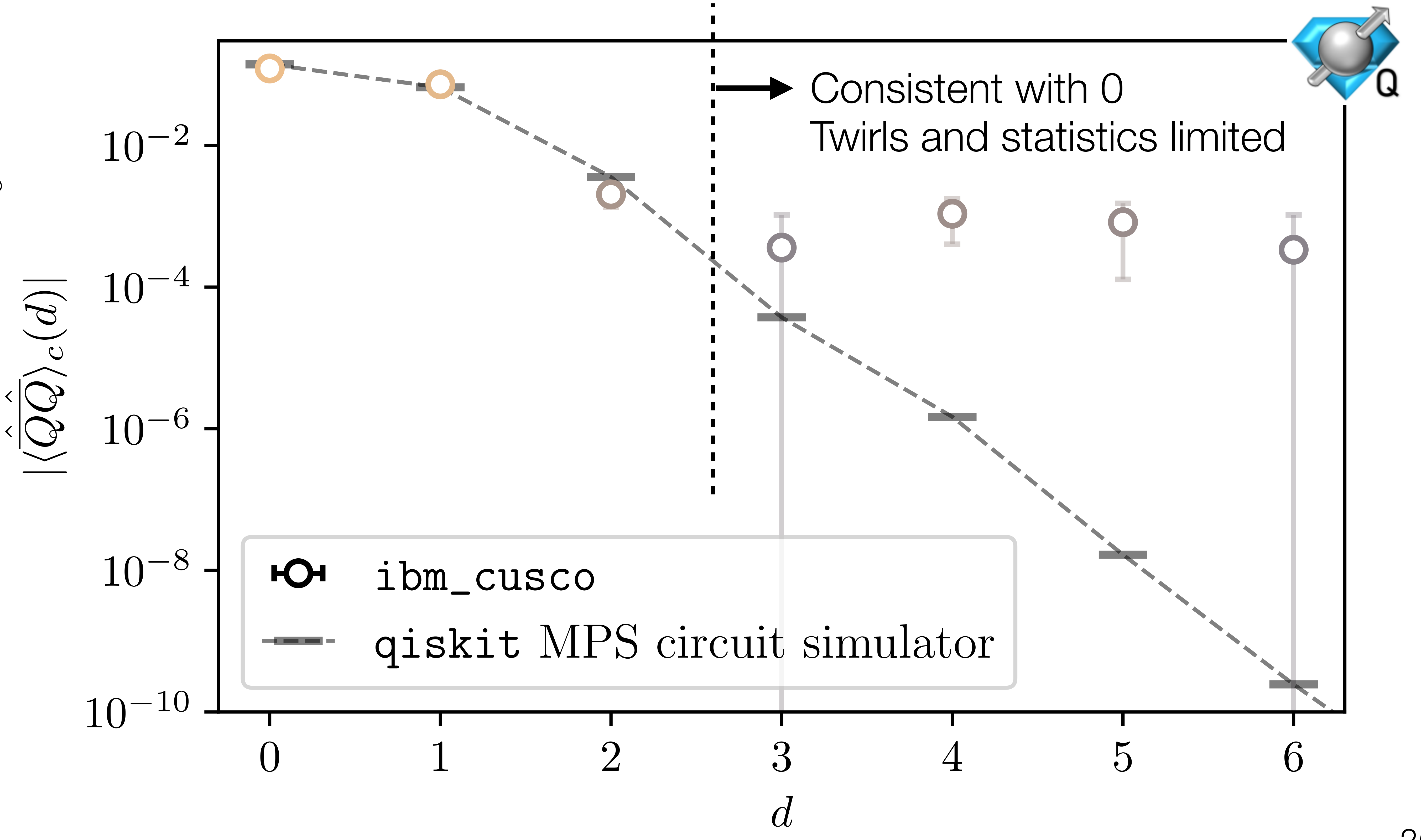
w/ 8000 shots each

# Running on a Quantum Computer: QQ Correlations

$$\langle \hat{\bar{Q}}_j \hat{\bar{Q}}_k \rangle_c = \langle \hat{\bar{Q}}_j \hat{\bar{Q}}_k \rangle - \langle \hat{\bar{Q}}_j \rangle \langle \hat{\bar{Q}}_k \rangle$$



## Once we have prepared the vacuum, we can study some dynamics

# Preparing a Hadron: Adiabatic method à la JLP

S. P. Jordan, K. S. M. Lee, J. Preskill, QIC (2014)

For lattice scalar field theory:

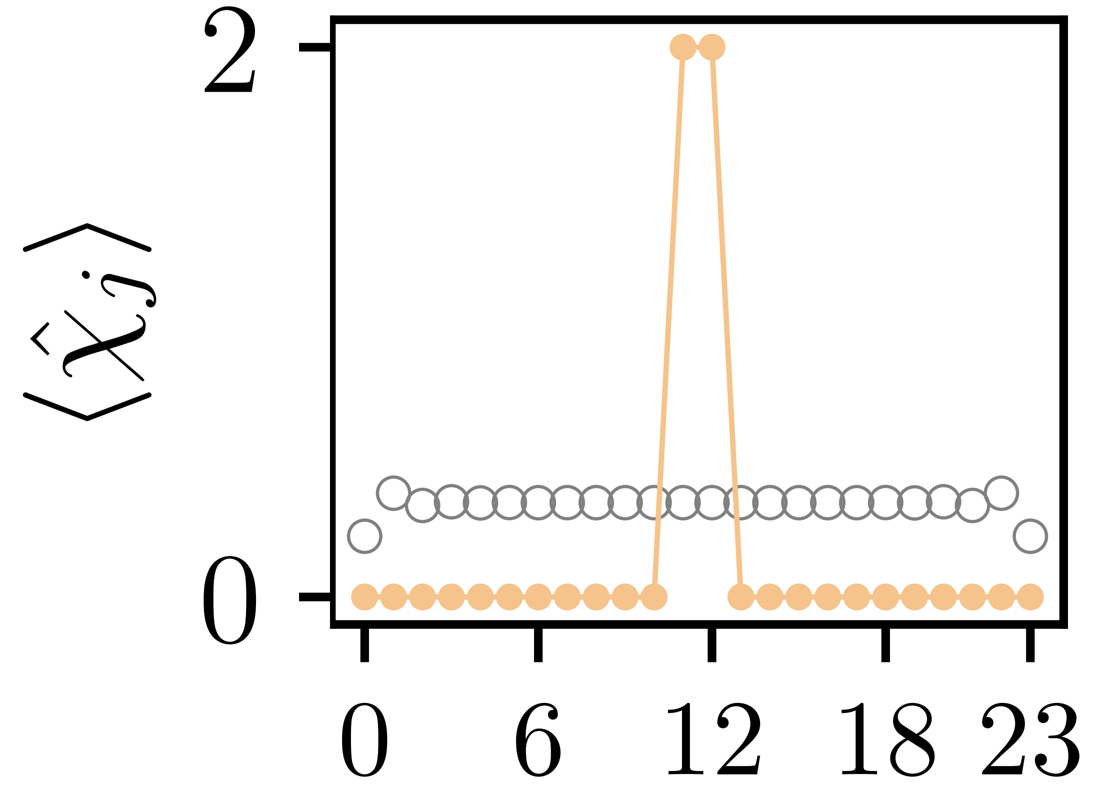
Start with the free theory → Adiabatic time evolution → End with the interacting theory

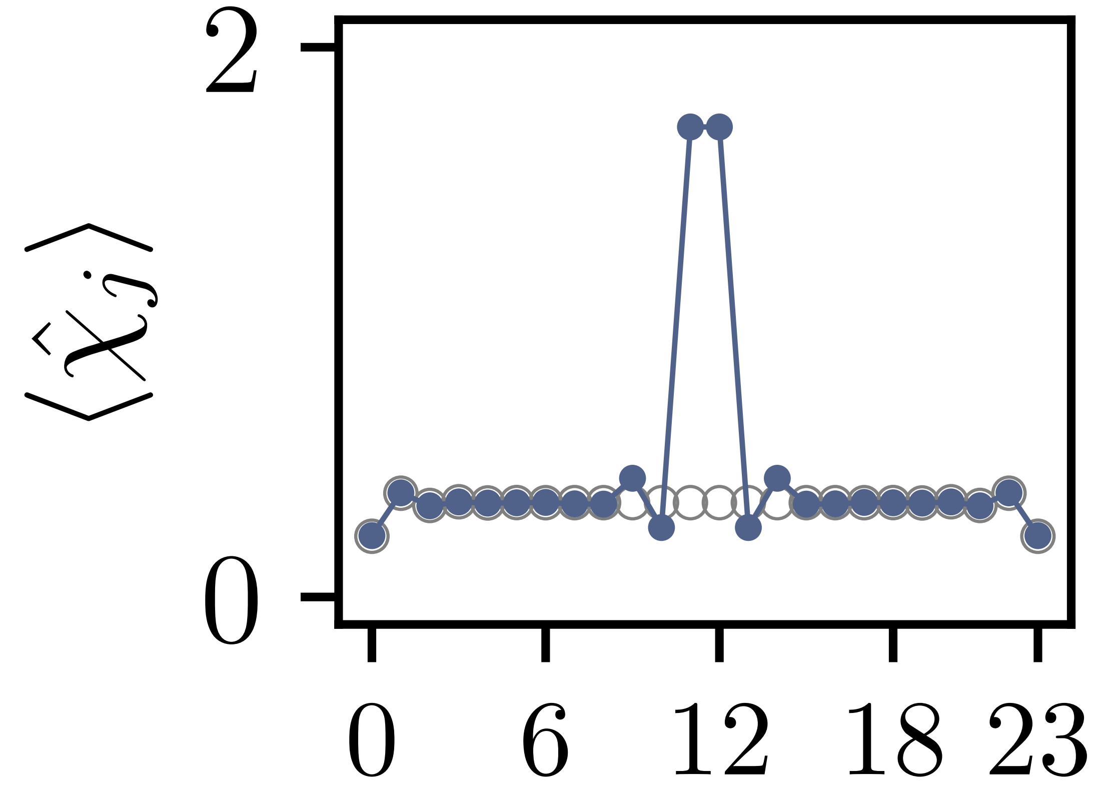
(slowly turn on $\lambda\phi^4$ )

For lattice Schwinger model:

Start with the strong-coupling theory → Adiabatic time evolution → End with the full theory

(slowly turn on kinetic term)



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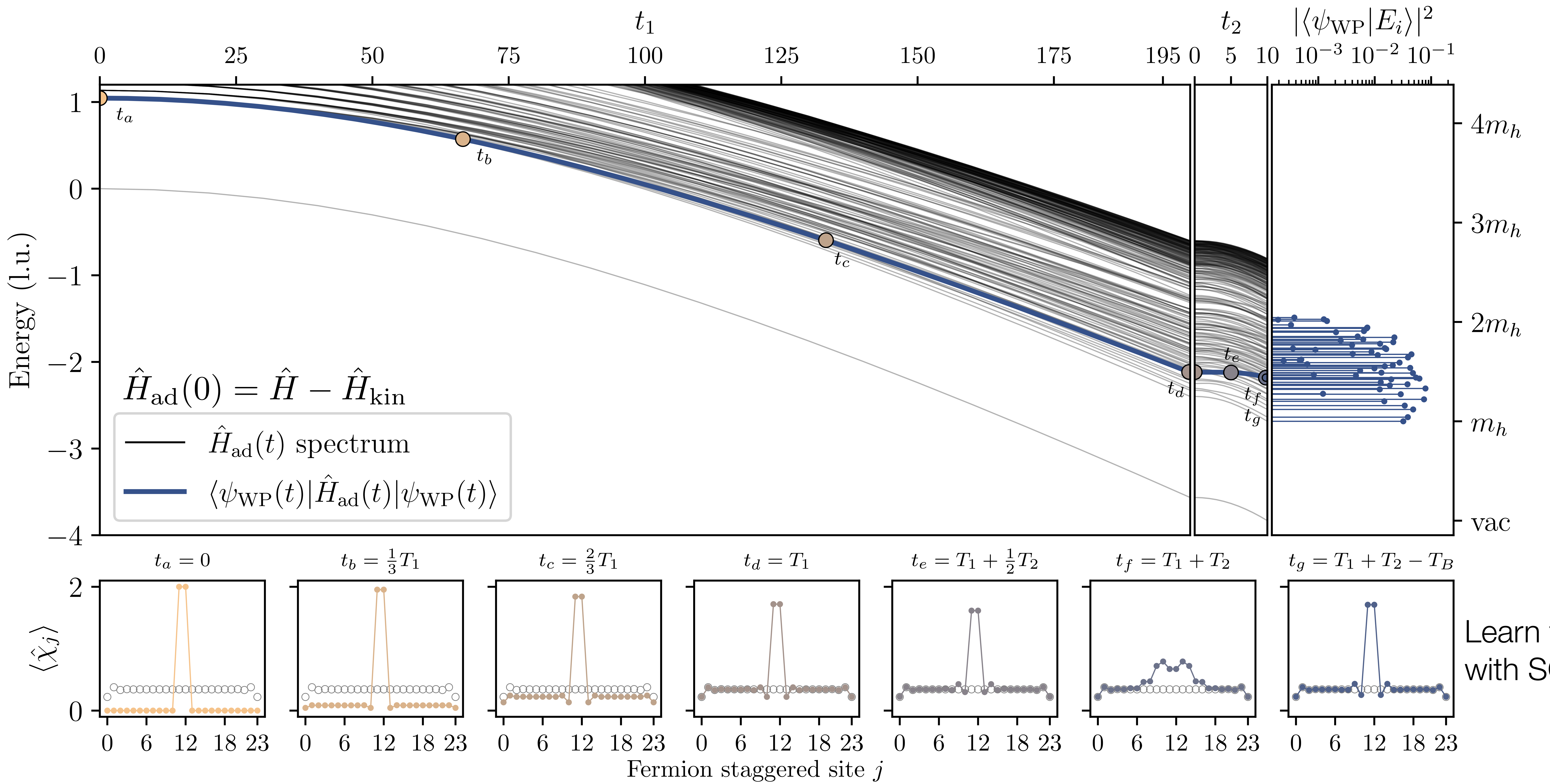
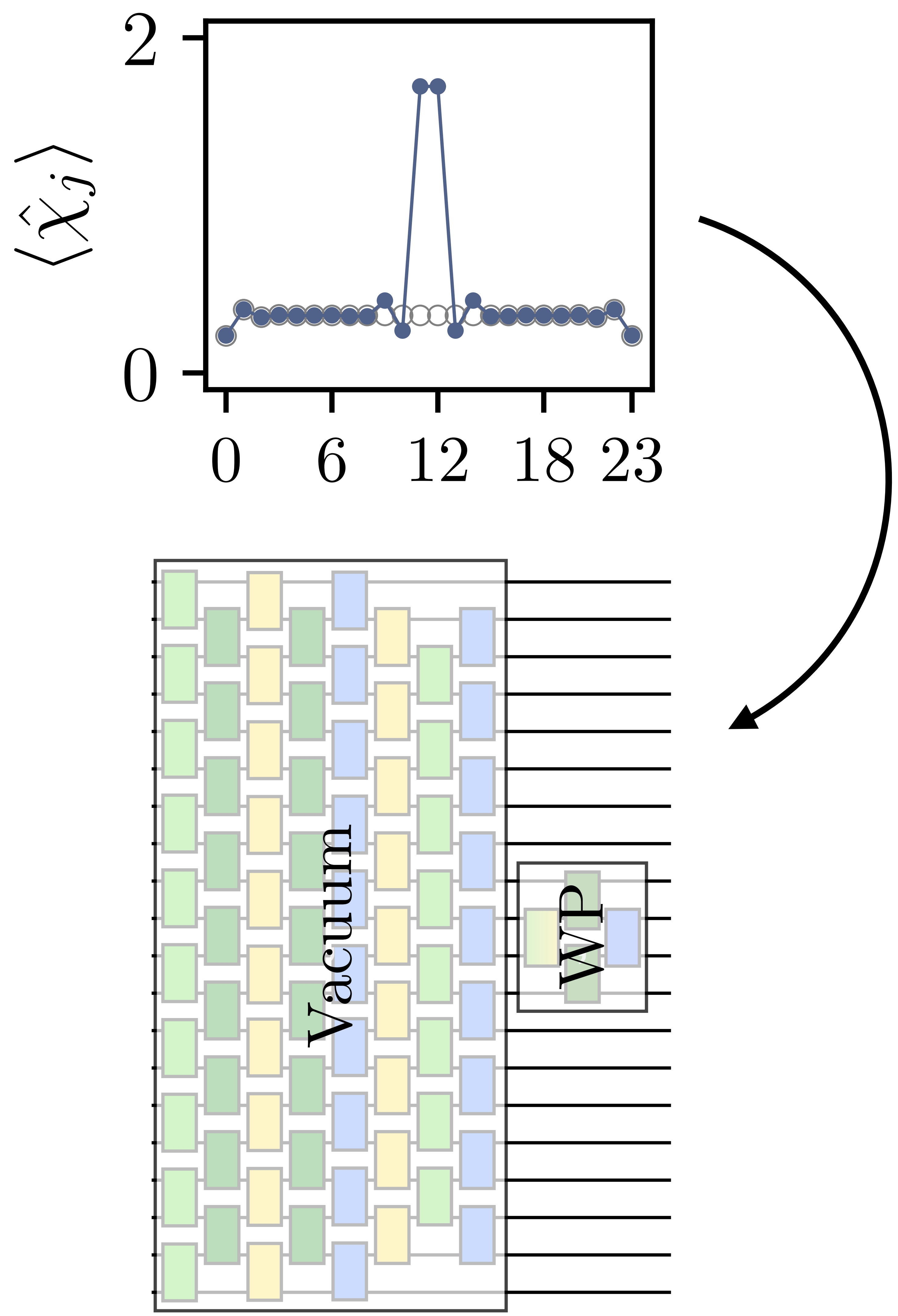


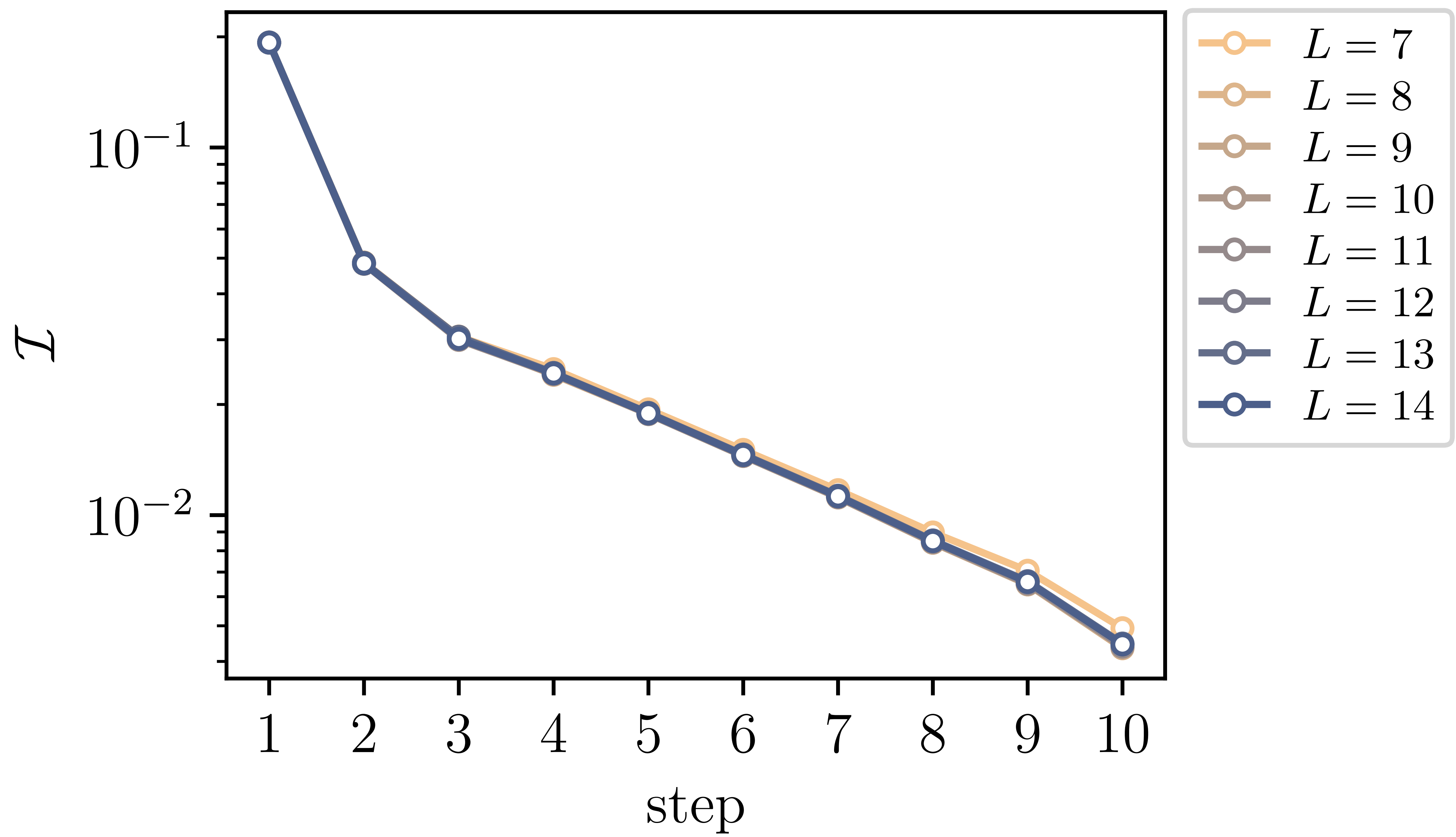

Learn the final state with SC-ADAPT-VQE

# Preparing a Hadron: using SC-ADAPT-VQE




(localized state)

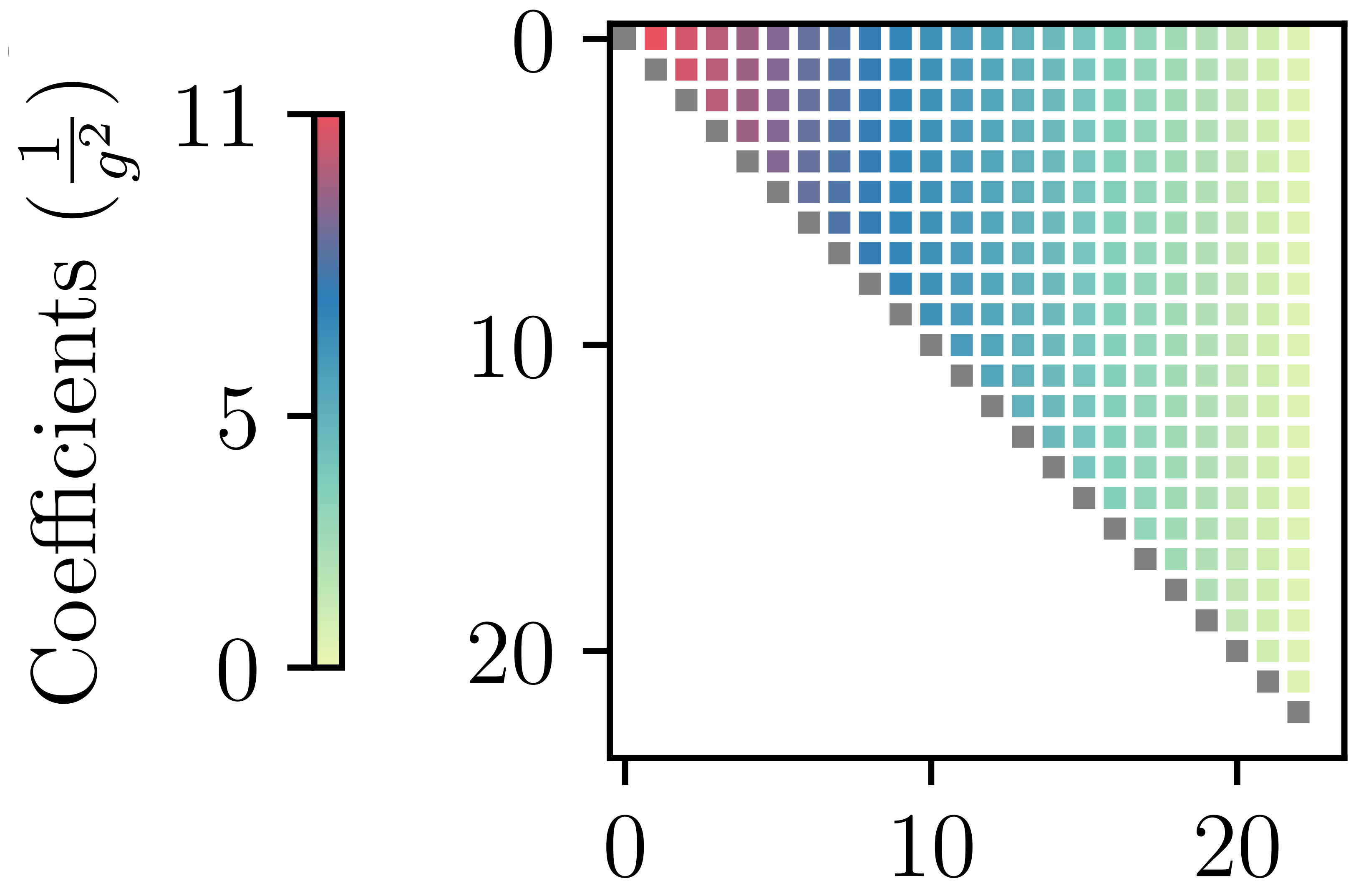
$$\mathcal{I} = 1 - |\langle \psi_{\text{ansatz}} | \psi_{\text{WP}} \rangle|^2$$

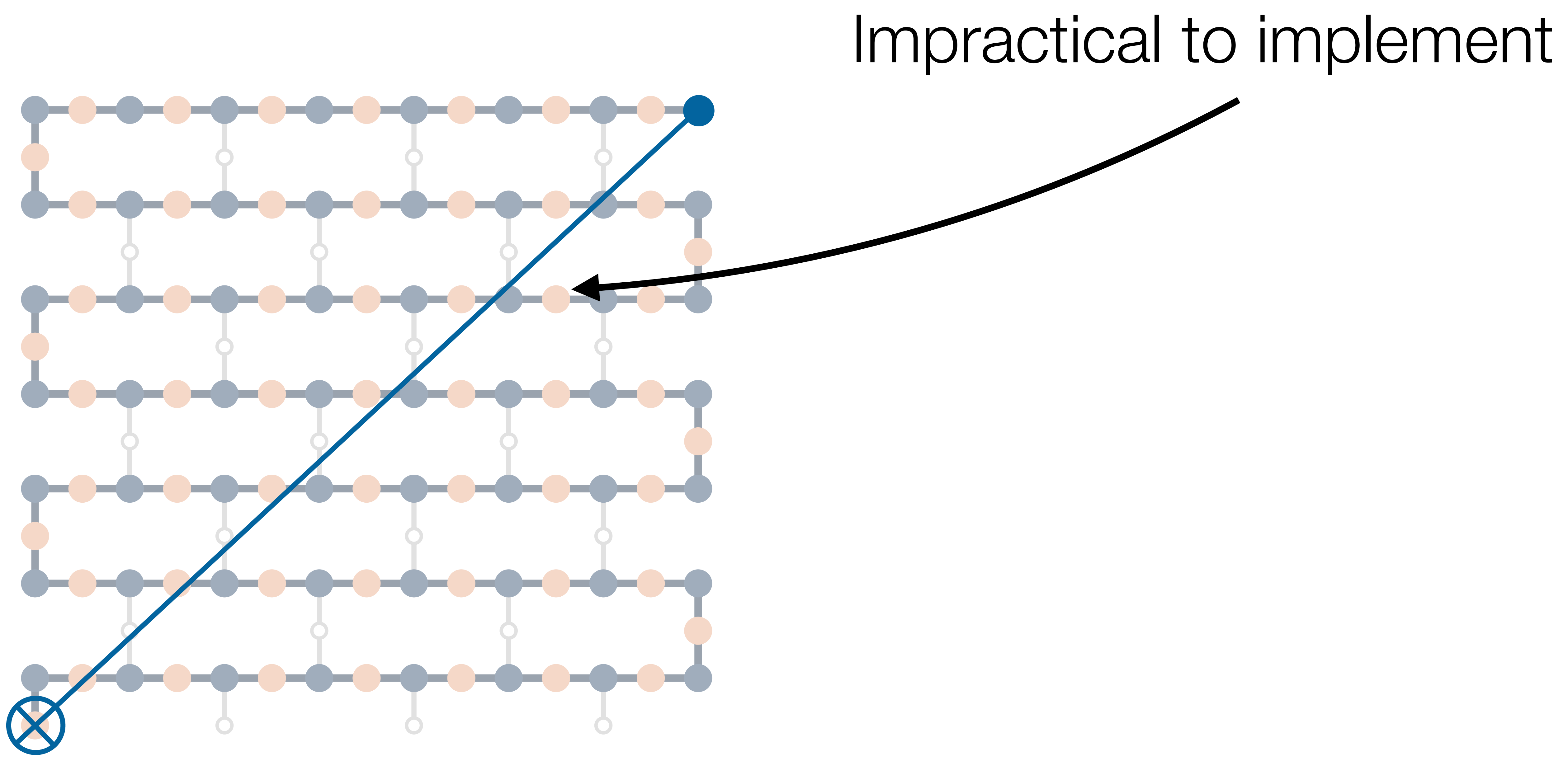


# Performing time evolution: truncating the interaction

$$H = \underbrace{\frac{1}{2}\sum_{j=0}^{2L-2}(\sigma_j^+\sigma_{j+1}^- + \text{h.c.})}_{\text{Easy } \mathcal{O}(L)} + \underbrace{\frac{m}{2}\sum_{j=0}^{2L-1}[(-1)^j Z_j + I]}_{\text{Easy } \mathcal{O}(L)} + \underbrace{\frac{g^2}{2}\sum_{j=0}^{2L-2}(\sum_{k\leq j} Q_k)^2}_{\text{Hard } \mathcal{O}(L^2)}$$

Number of gates: Easy $\mathcal{O}(L)$ Easy $\mathcal{O}(L)$ Hard $\mathcal{O}(L^2)$



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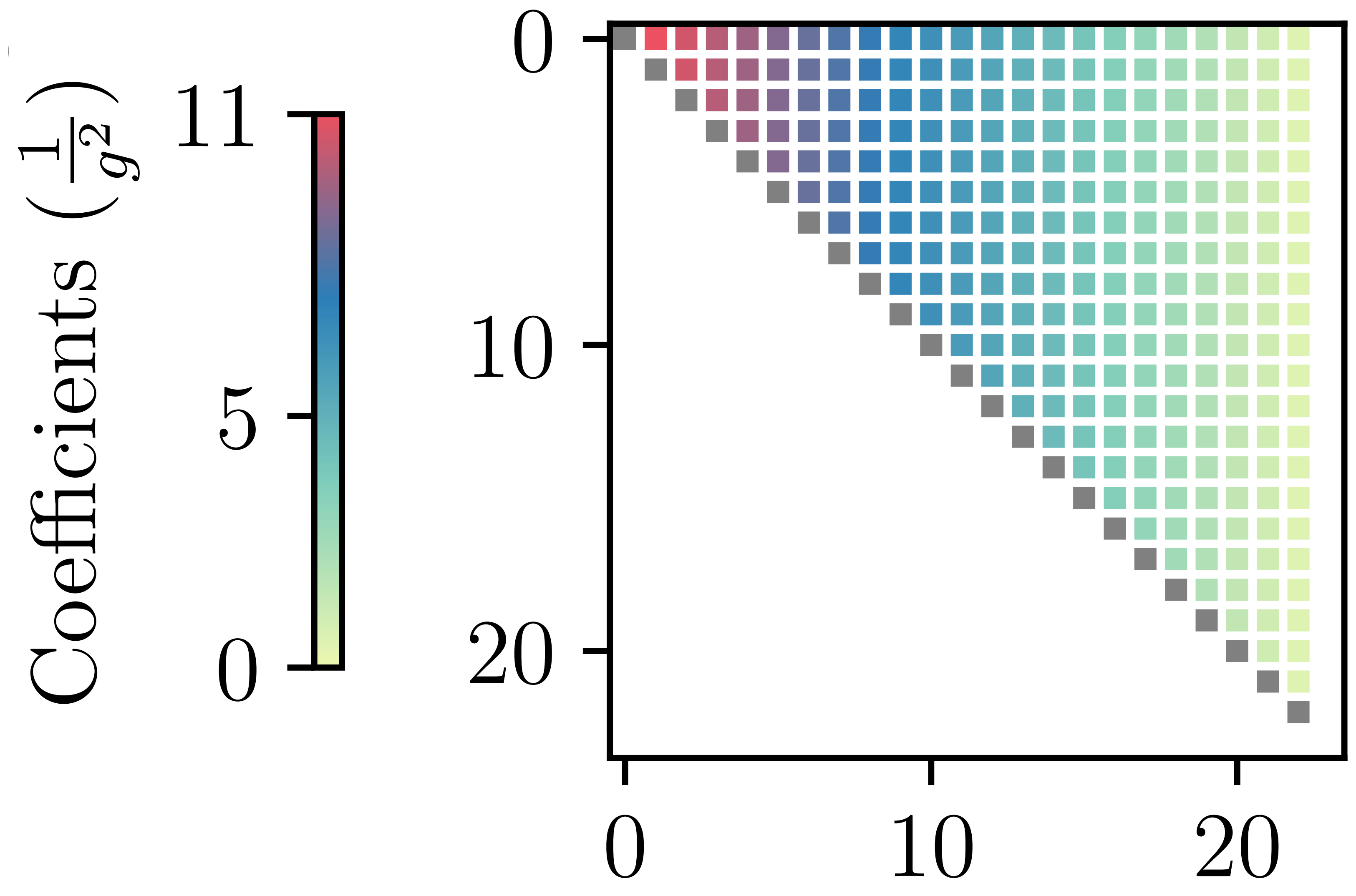
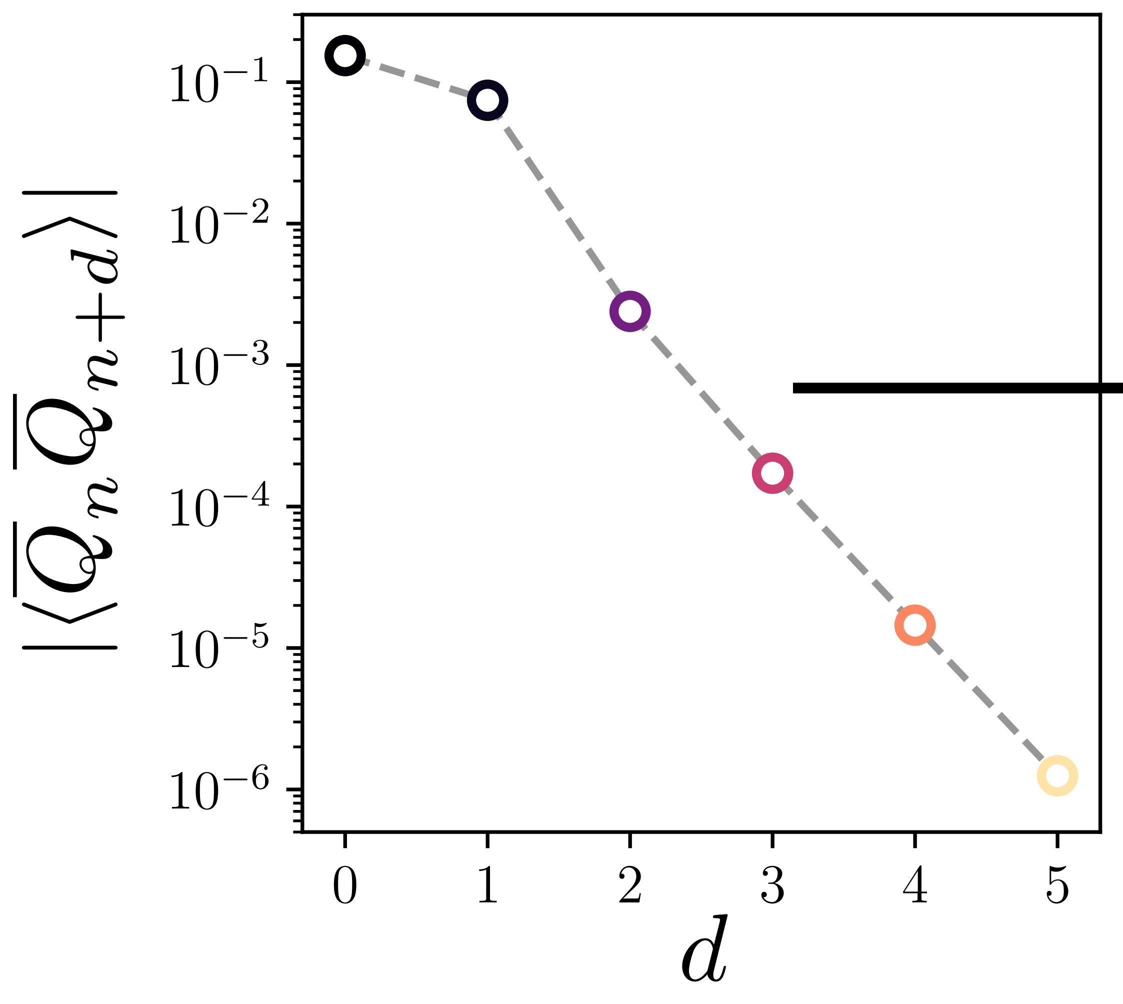



Charges are screened

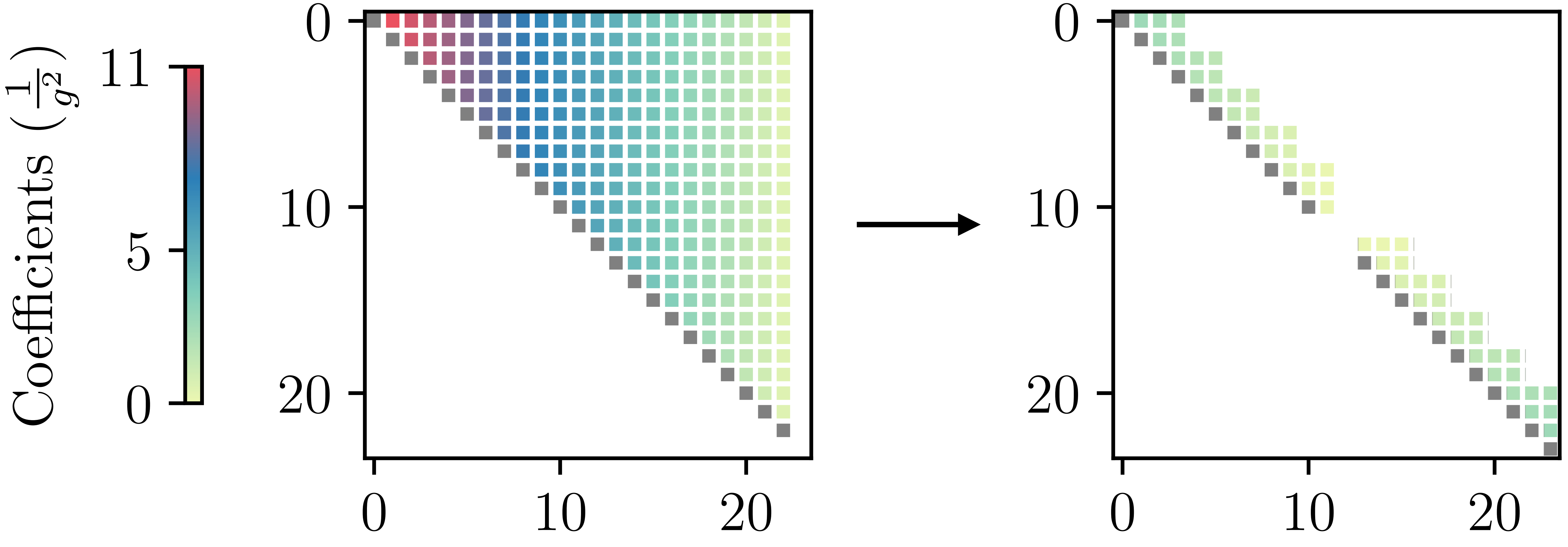



→ We can truncate the interaction beyond confinement length

# Performing time evolution: truncating the interaction

$$H = \underbrace{\frac{1}{2}\sum_{j=0}^{2L-2}(\sigma_j^+\sigma_{j+1}^- + \text{h.c.})}_{\text{Easy } \mathcal{O}(L)} + \underbrace{\frac{m}{2}\sum_{j=0}^{2L-1}[(-1)^j Z_j + I]}_{\text{Easy } \mathcal{O}(L)} + \underbrace{\frac{g^2}{2}\sum_{j=0}^{2L-2}(\sum_{k\leq j} Q_k)^2}_{\text{Hard } \mathcal{O}(L^2)}$$

Number of gates: Easy $\mathcal{O}(L)$ Easy $\mathcal{O}(L)$ Hard ~~$\mathcal{O}(L^2)$~~ → $\mathcal{O}(\bar{\lambda}L)$

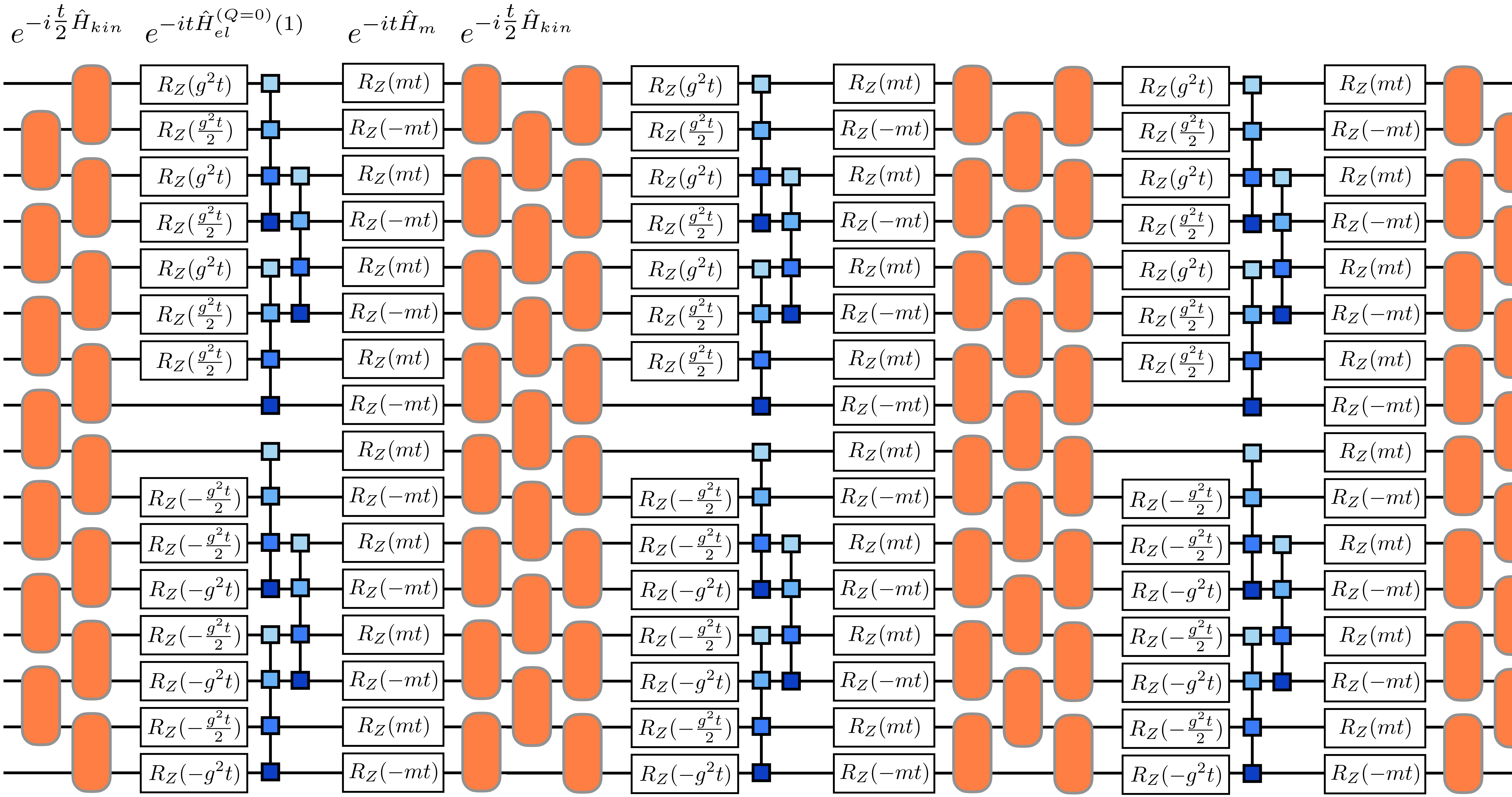



For demonstration, we use $\bar{\lambda} = 1$

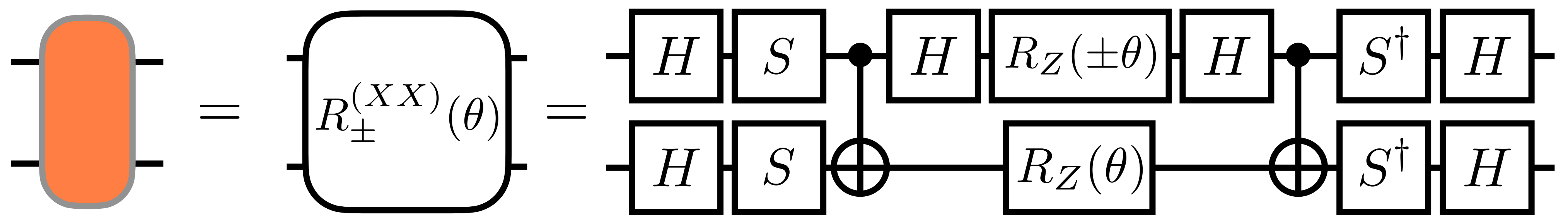
# Performing time evolution: Trotterized operator

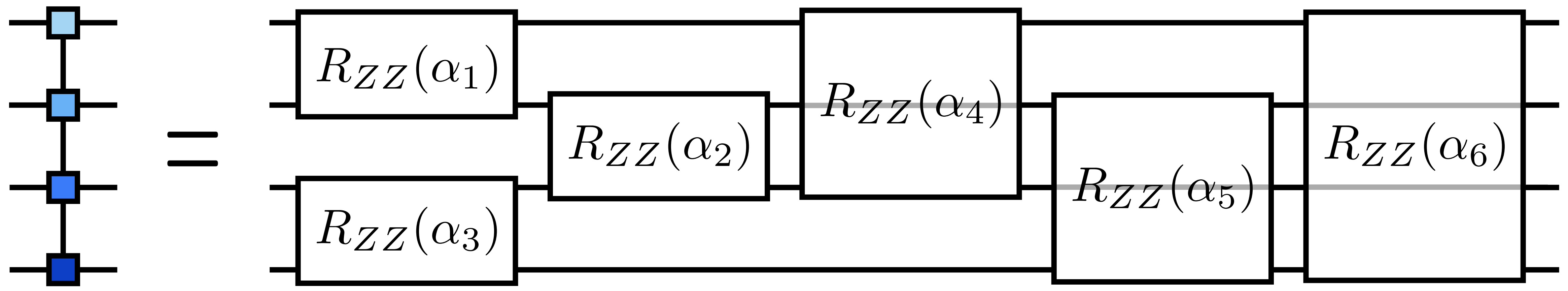
$$e^{-it\hat{H}} \approx$$

2nd order Trotter



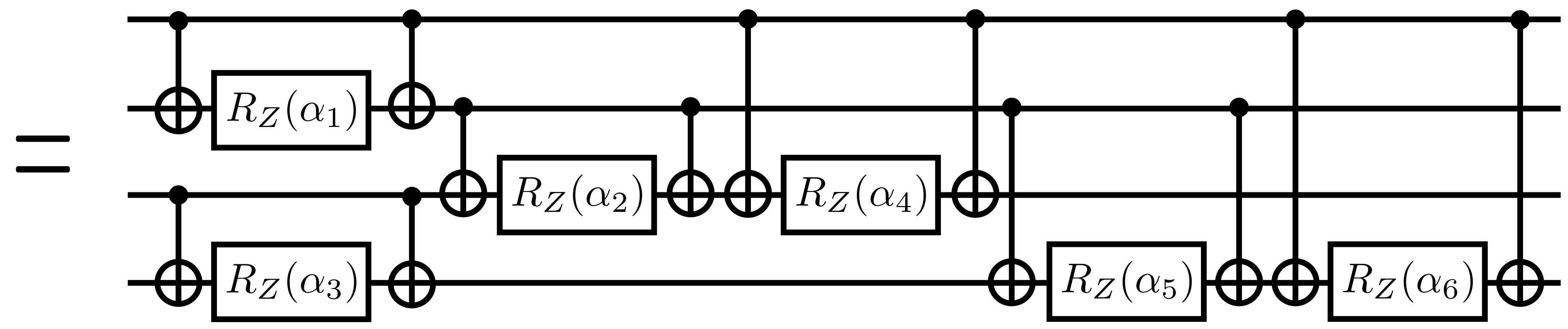
# Performing time evolution: Trotterized operator

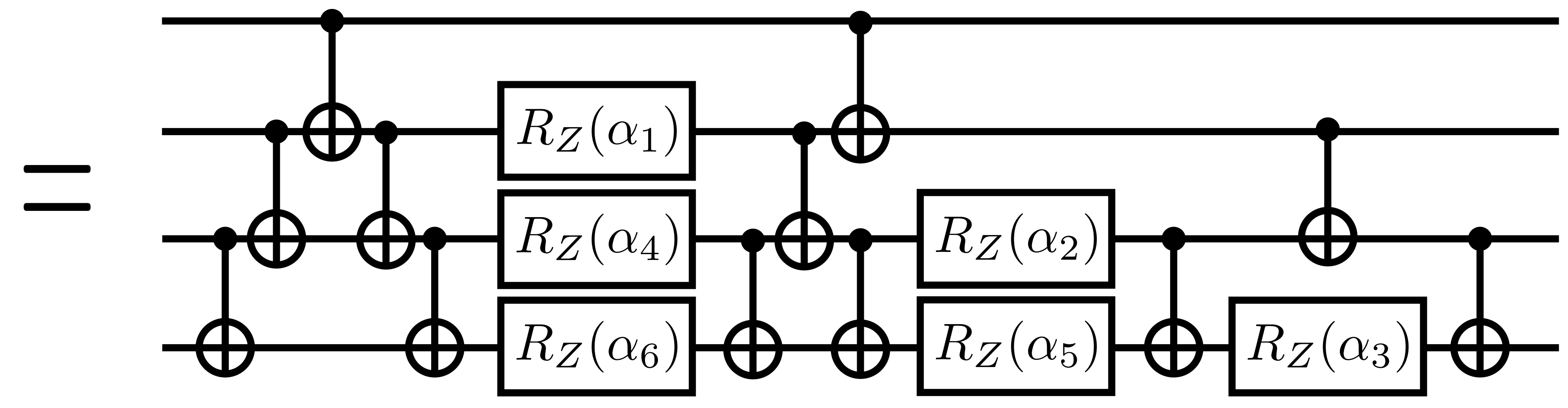



With all-to-all connectivity,

$$\text{depth} = N_q$$




With linear connectivity, we have the same number of CNOTS, but

$$\text{depth} \propto N_q^2$$

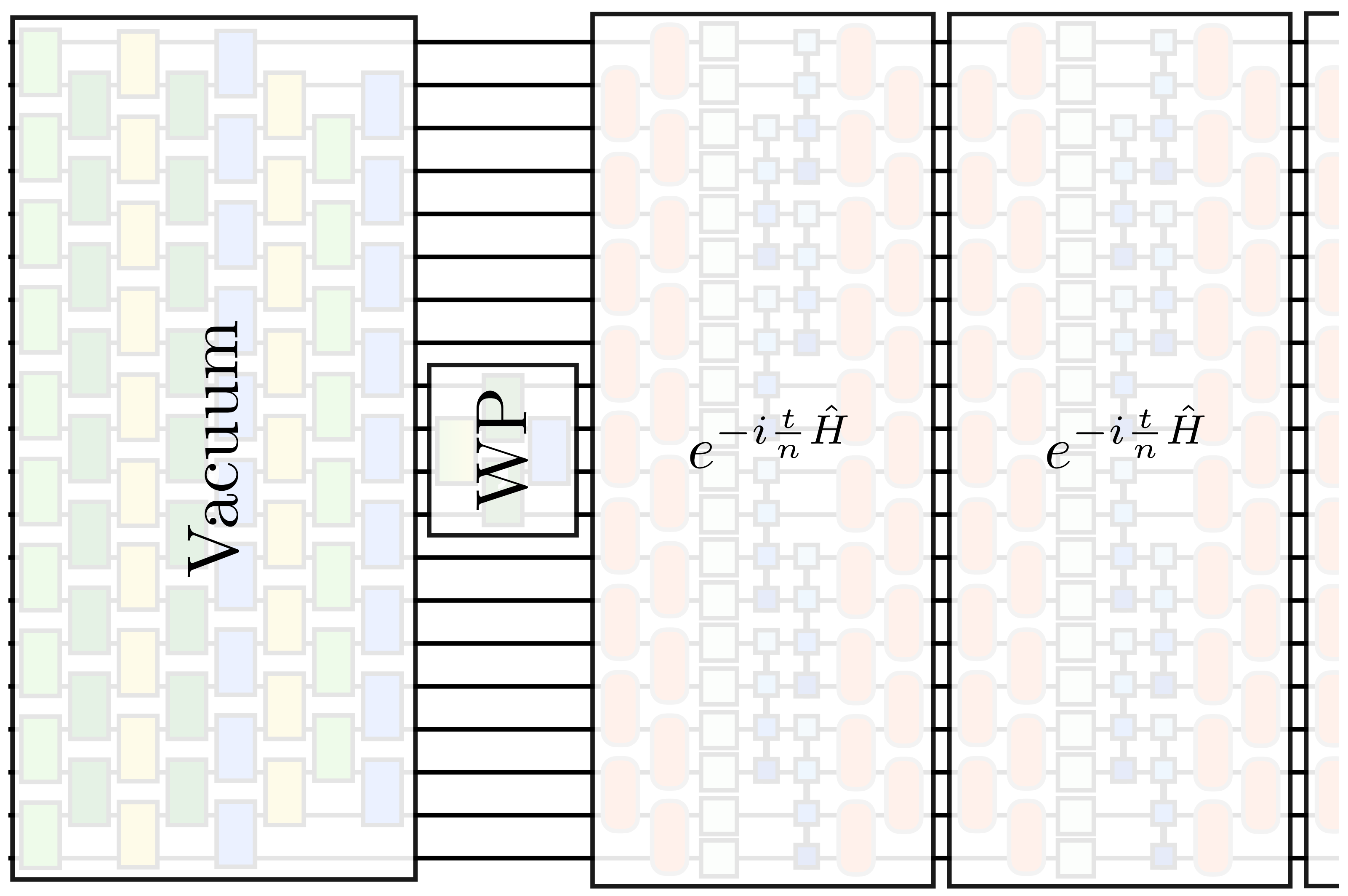
# Performing time evolution: Results

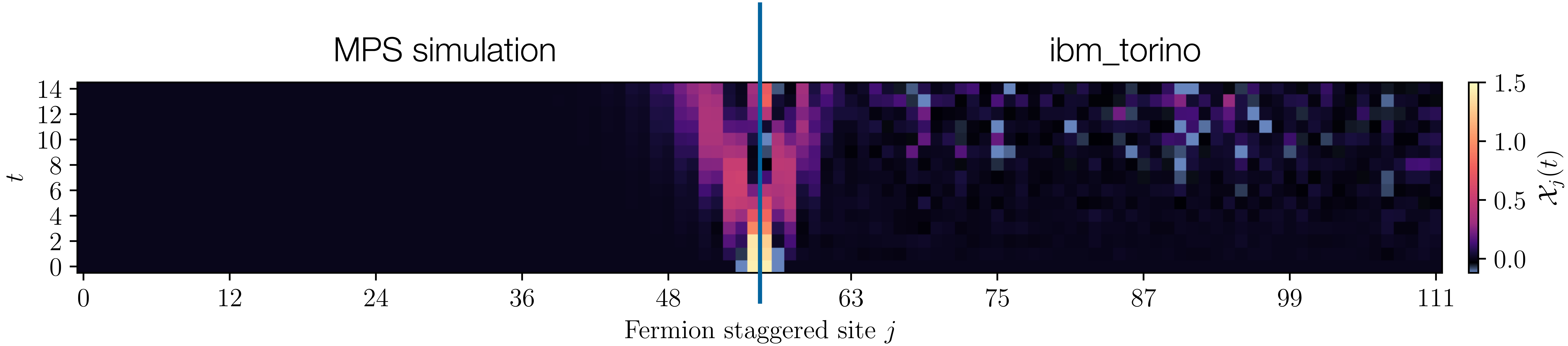
1) Prepare vacuum and hadron wavepacket

2) Apply Trotterized time evolution operator

3) Measure chiral condensate (removing vacuum contribution)

$$\mathcal{X}_j(t) = \langle \psi_{\mathrm{WP}}(t) | \chi_j | \psi_{\mathrm{WP}}(t) \rangle - \langle \psi_{\mathrm{vac}}(t) | \chi_j | \psi_{\mathrm{vac}}(t) \rangle$$



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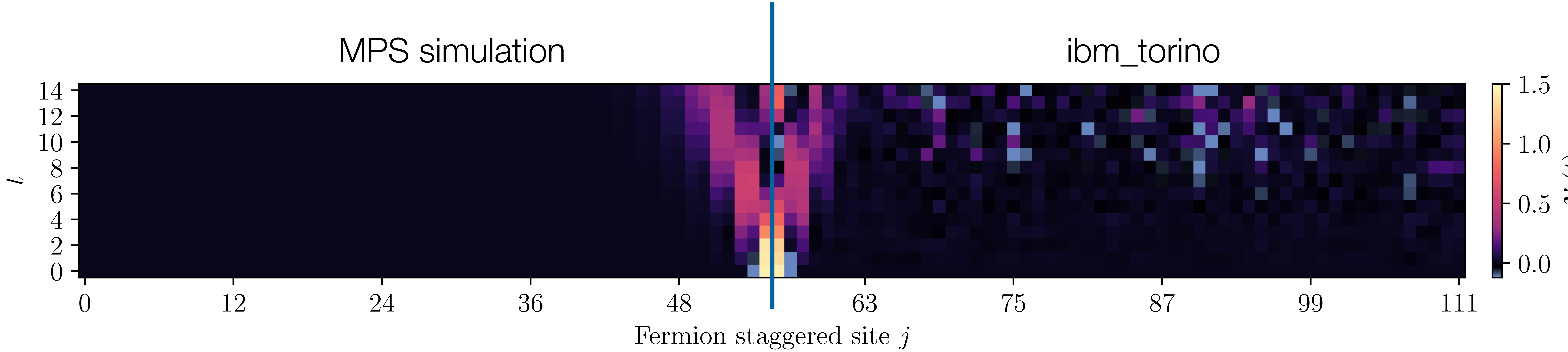
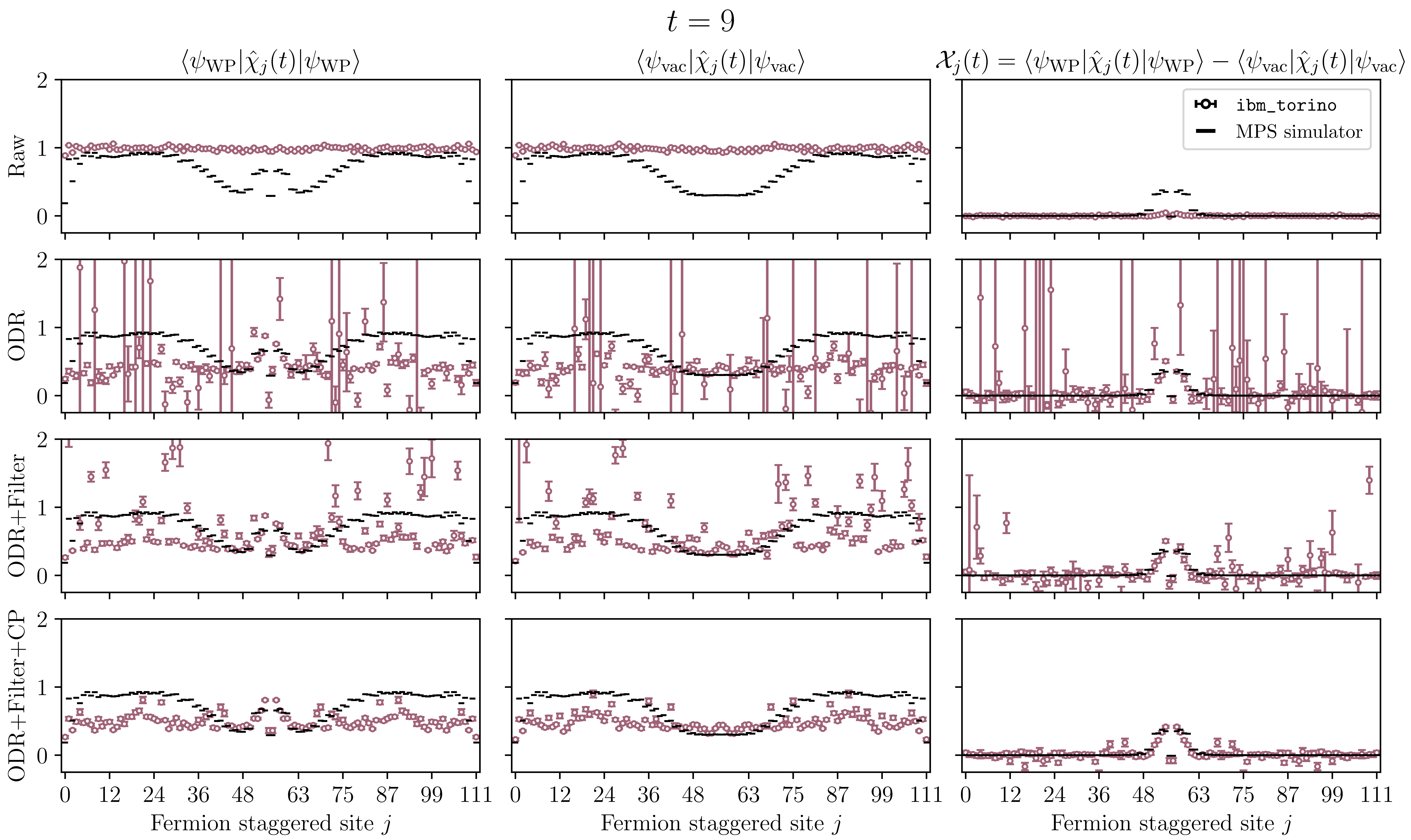
$$\mathcal{X}_j(t) = \langle \psi_{\mathrm{WP}}(t)|\chi_j|\psi_{\mathrm{WP}}(t)\rangle - \langle \psi_{\mathrm{vac}}(t)|\chi_j|\psi_{\mathrm{vac}}(t)\rangle$$

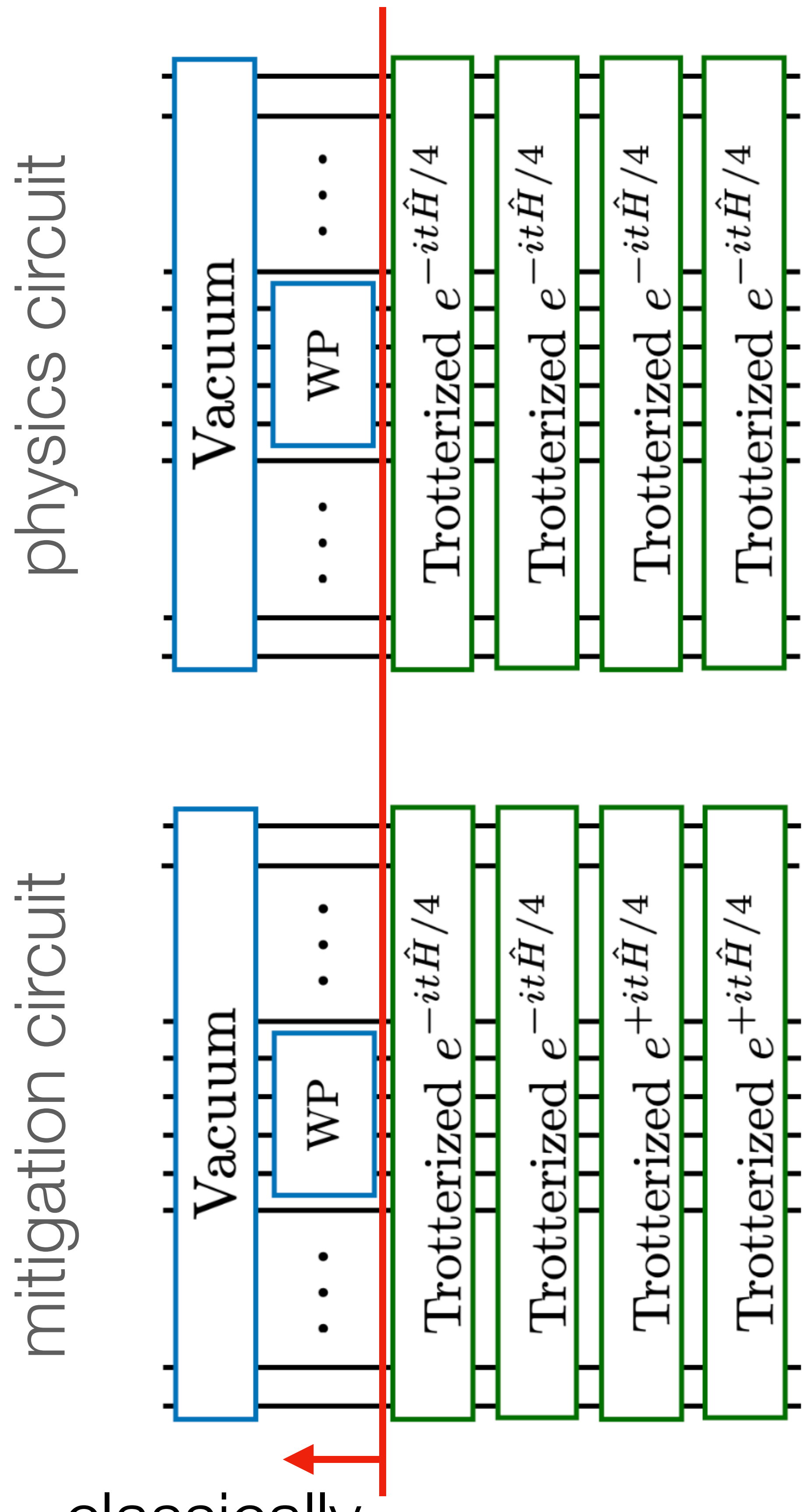
| $t$ | $N_T$ | # of CNOTs (per $t$) | CNOT depth (per $t$) | Total # of shots ($\times 10^6$) |
|---|---|---|---|---|
| 1 & 2 | 2 | 2,746 | 70 | $4 \times 2 \times 3.8$ |
| 3 & 4 | 4 | 4,598 | 120 | $4 \times 2 \times 3.8$ |
| 5 & 6 | 6 | 6,450 | 170 | $4 \times 2 \times 3.8$ |
| 7 & 8 | 8 | 8,302 | 220 | $4 \times 2 \times 3.8$ |
| 9 & 10 | 10 | 10,154 | 270 | $4 \times 2 \times 1.3$ |
| 11 & 12 | 12 | 12,006 | 320 | $4 \times 2 \times 1.3$ |
| 13 & 14 | 14 | 13,858 | 370 | $4 \times 2 \times 1.3$ |
| **Totals** | | | | $1.54 \times 10^8$ |



# Mitigating the noise (again)

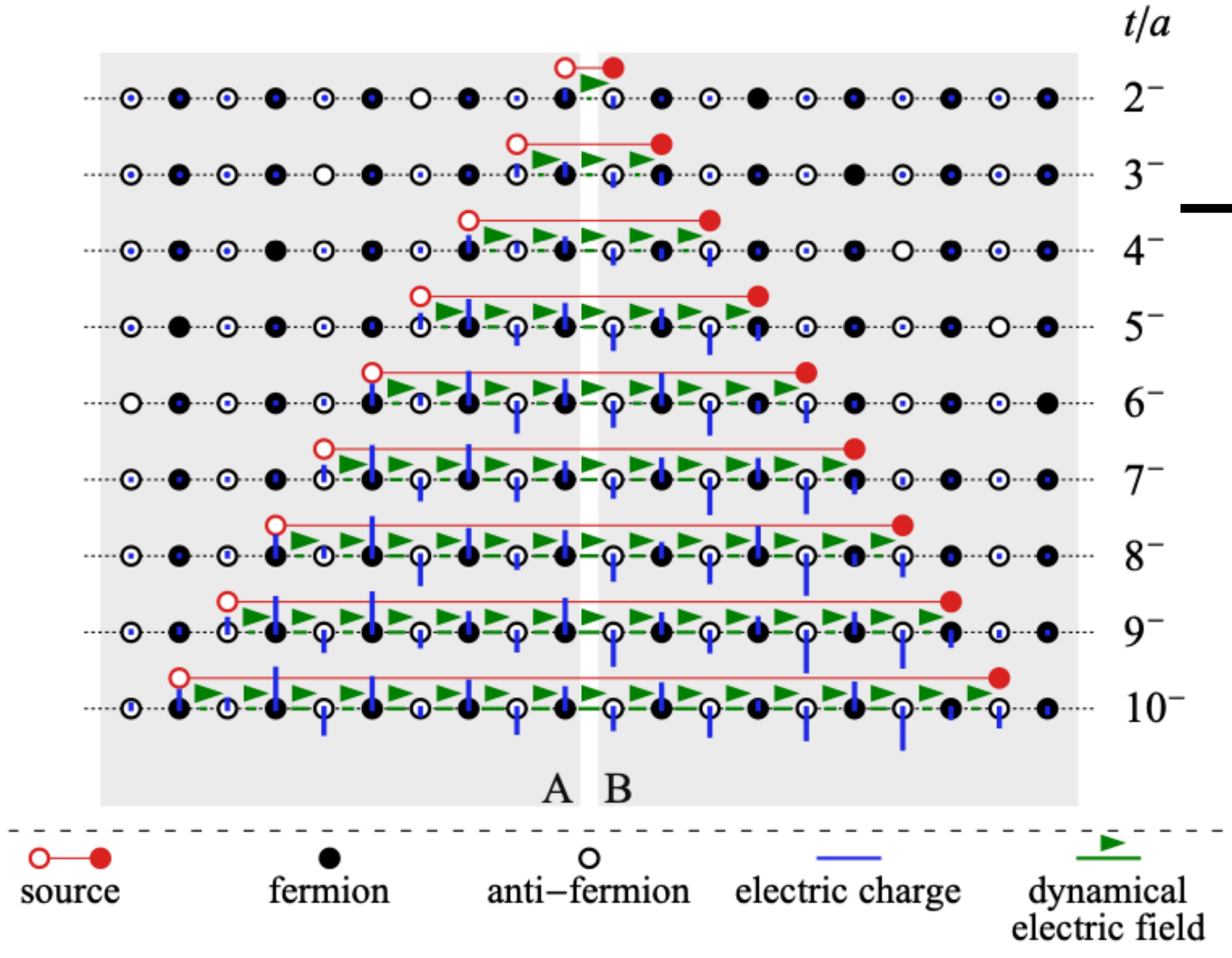



Different mitigation circuit:



# Schwinger model with external charges

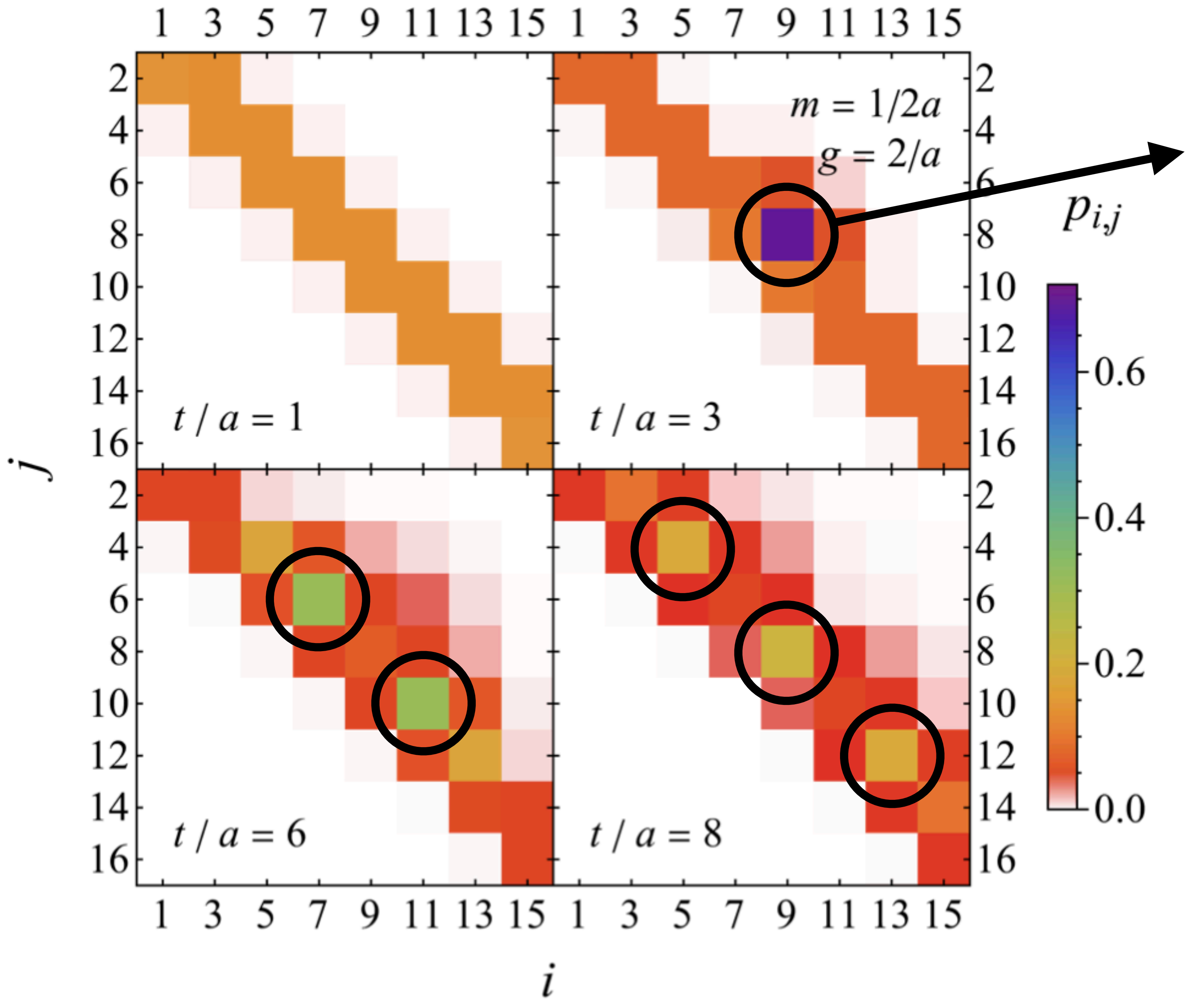
A. Florio et al. PRL (2023), arXiv:2404.00087




$$q_m^{(\mathrm{ext})}(t) = -\Theta\left(\frac{t-t_0}{a} - |m-L|\right)$$

2 back-to-back charges moving **at the speed of light**

$$p_{i,j} = |\langle \Psi_t | \chi_i \chi_j^\dagger | \mathrm{Neel} \rangle|$$



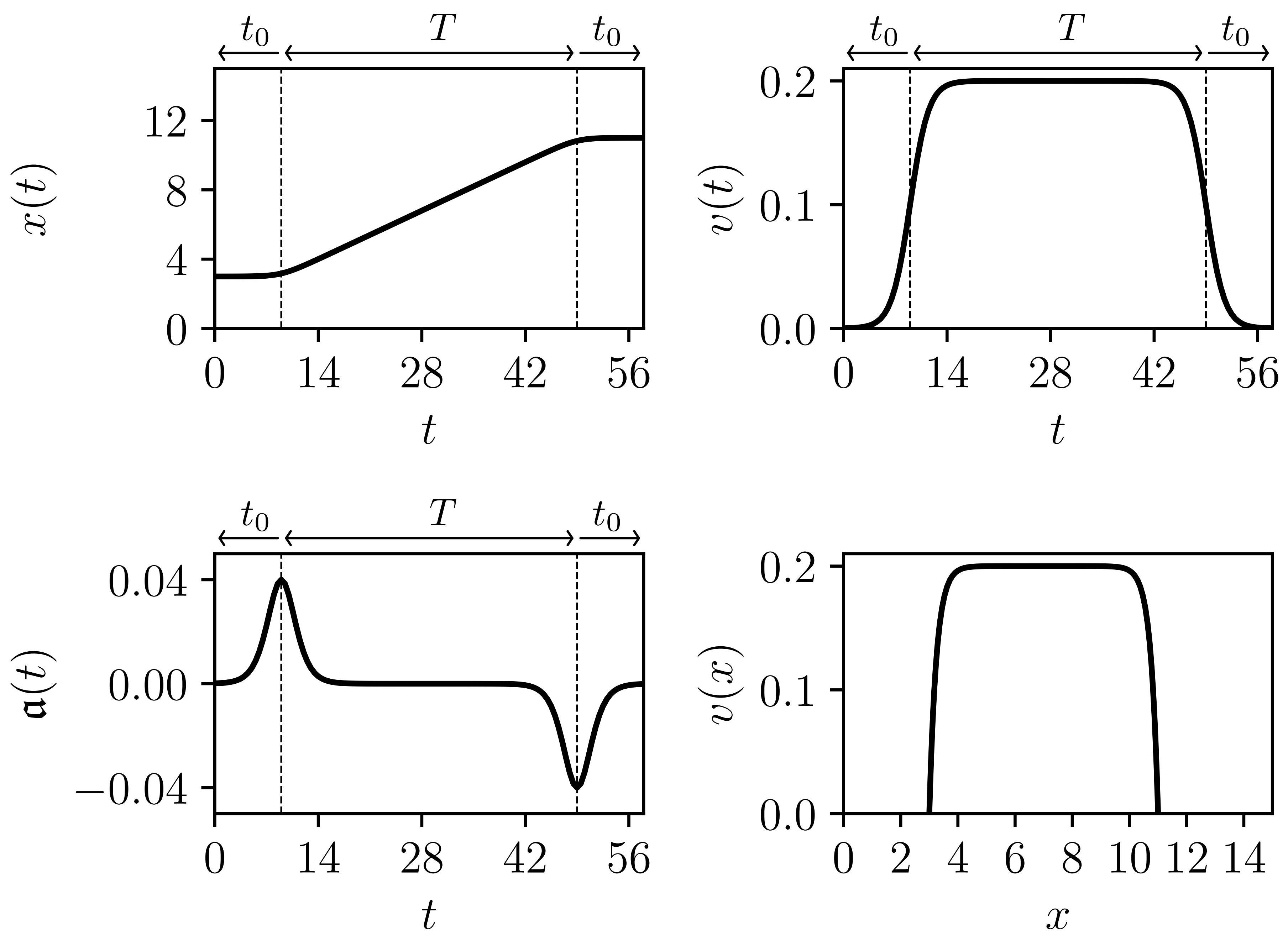

creation of meson states

$$H_{el} = \frac{g^2}{2}\sum_n \left(\sum_{m\le n} Q_m\right)^2 \rightarrow \frac{g^2}{2}\sum_n \left(\sum_{m\le n} Q_m + q_m^{(\mathrm{ext})}(t)\right)^2$$

Add external charge in the system

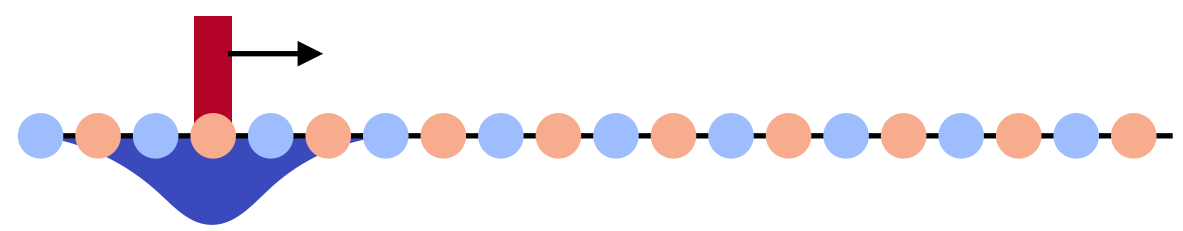
# Schwinger model with external charges

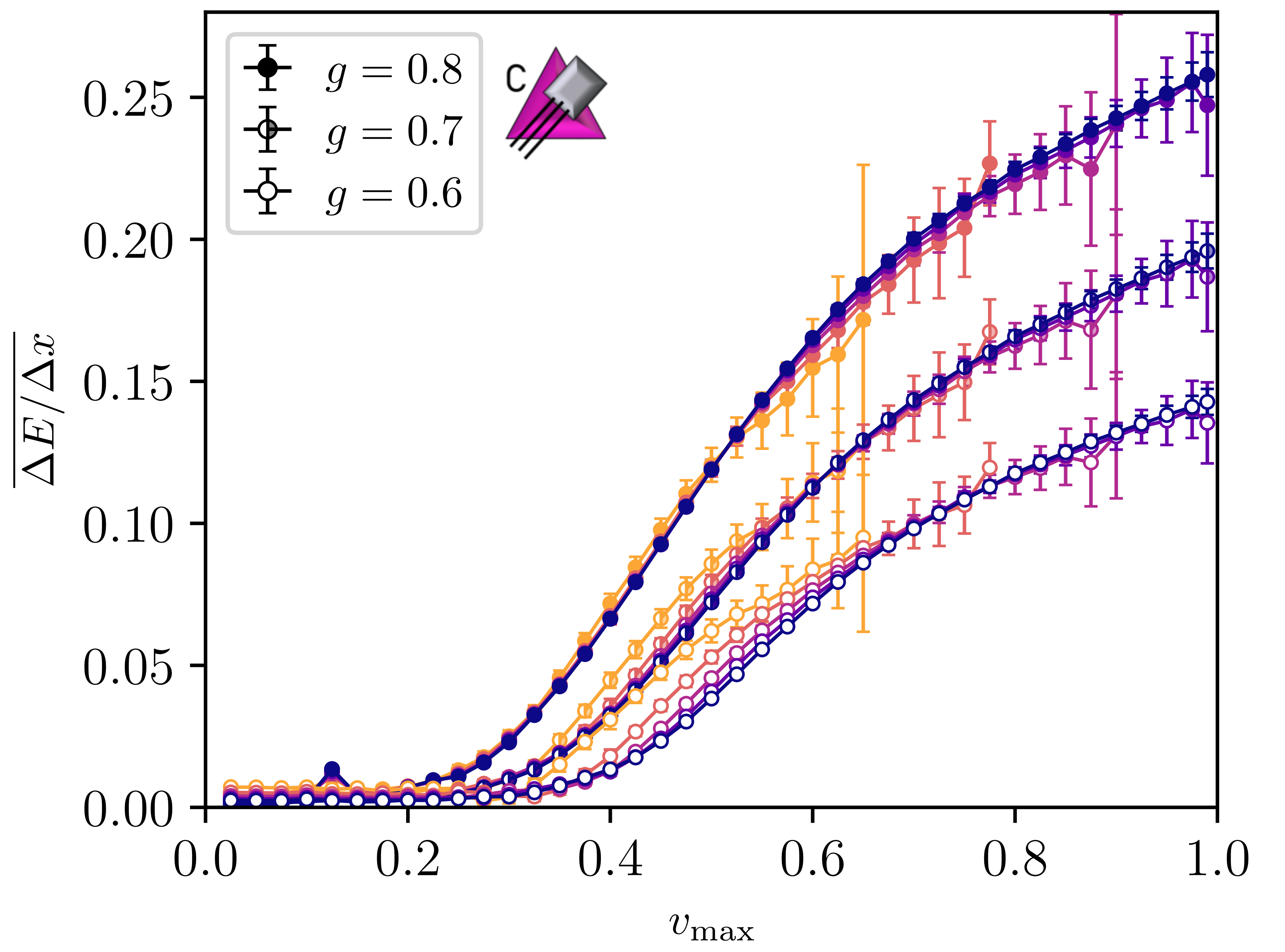
Going at the speed of light seems to be too fast on the lattice.
Are we seeing discretization effects?




Charge moving at a constant speed
(with minimal acceleration)

Due to Lorentz invariance, energy should be conserved at constant speed





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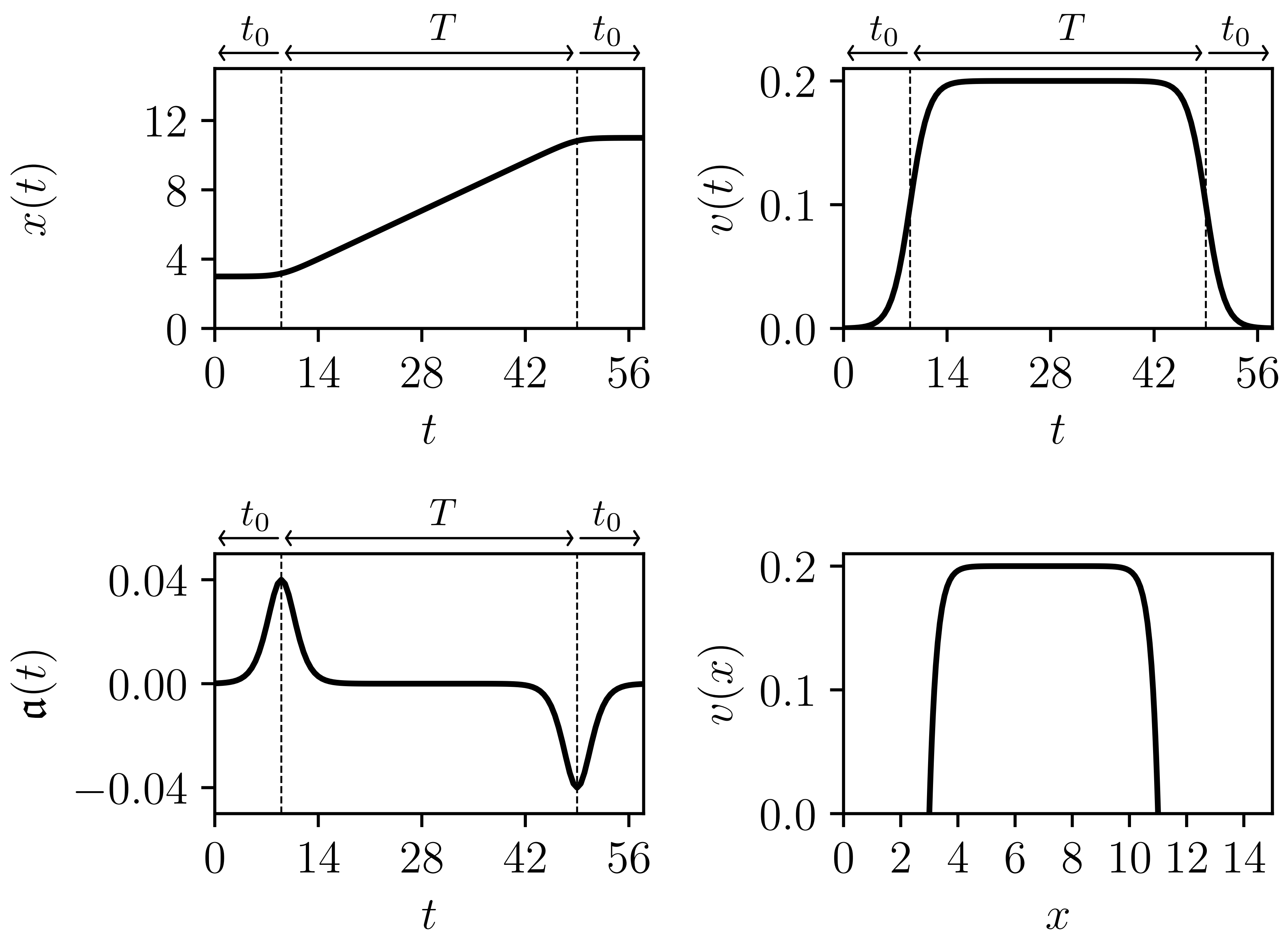


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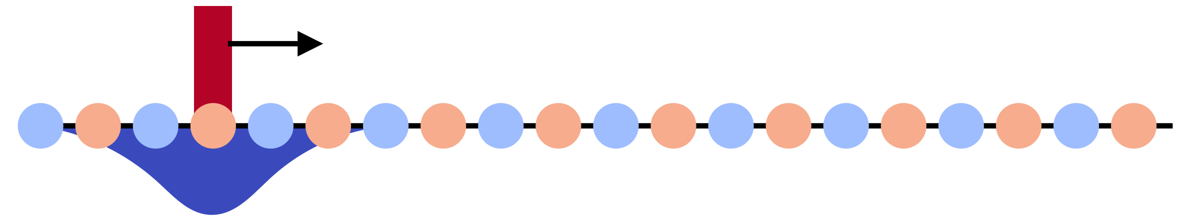

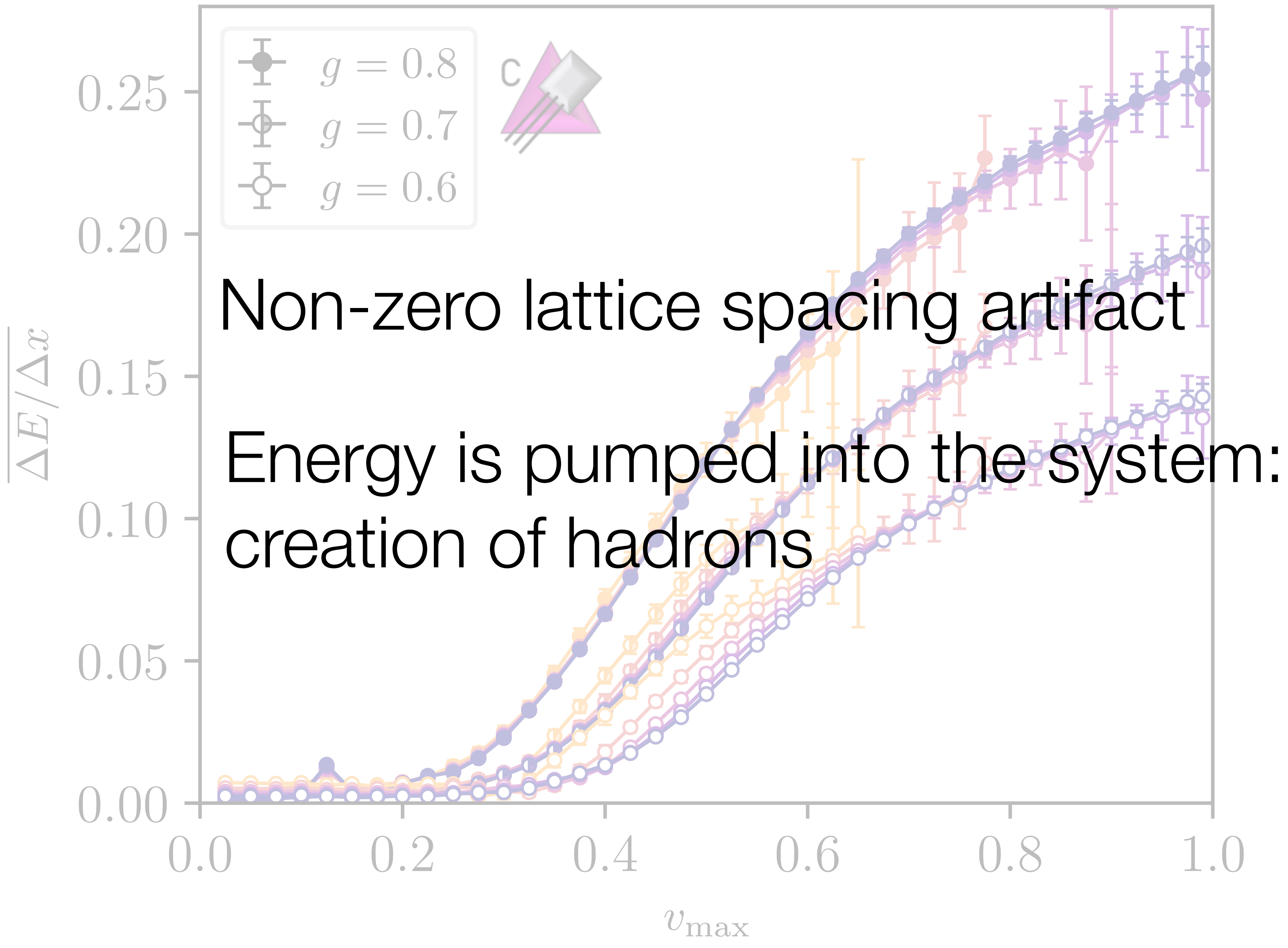

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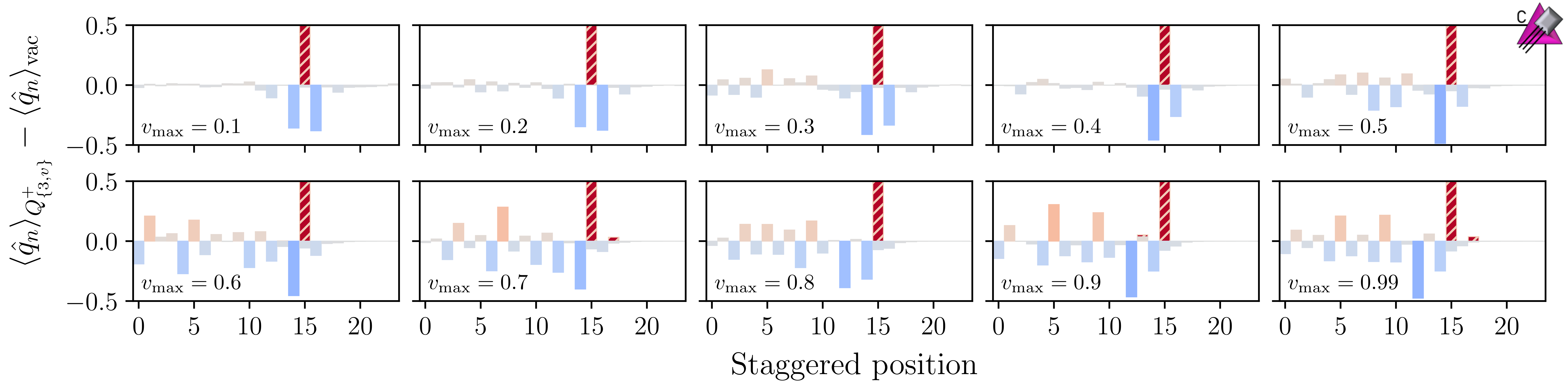

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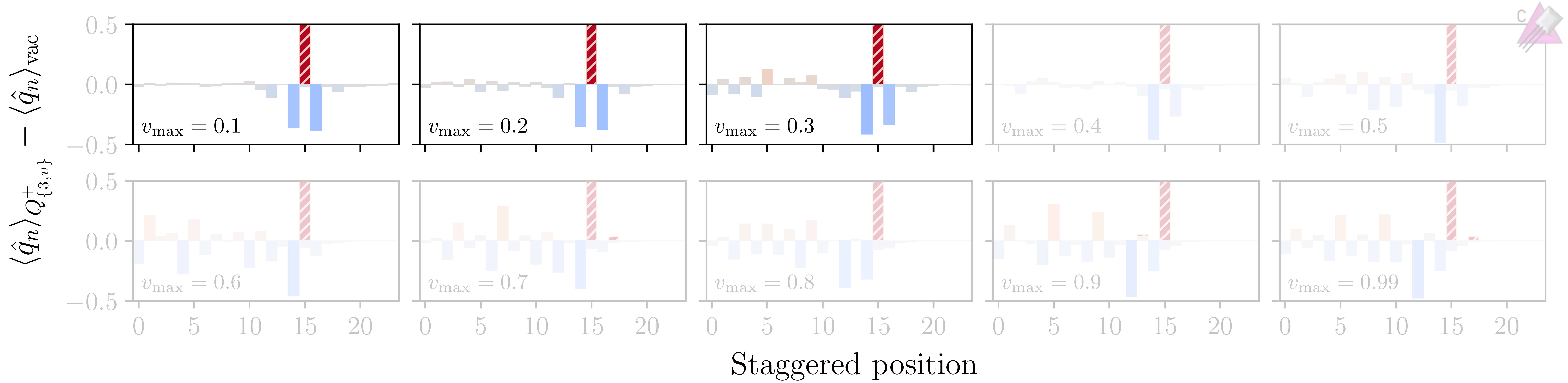


For small velocities, we see the expected behavior (nothing is happening)

# Schwinger model with external charges

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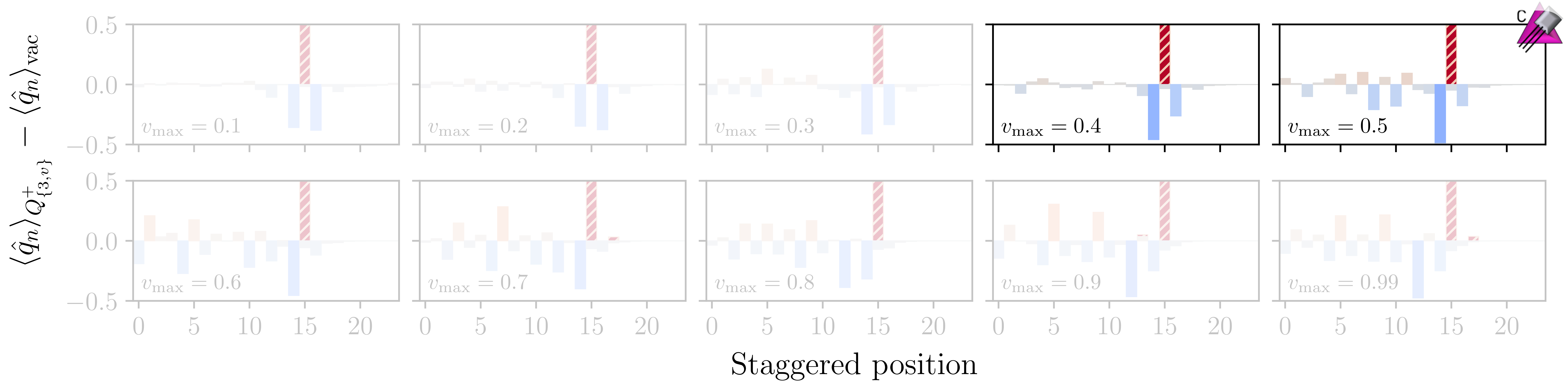


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As we increase the speed, we see that the light charges screening the heavy charge start to be left behind

# Schwinger model with external charges

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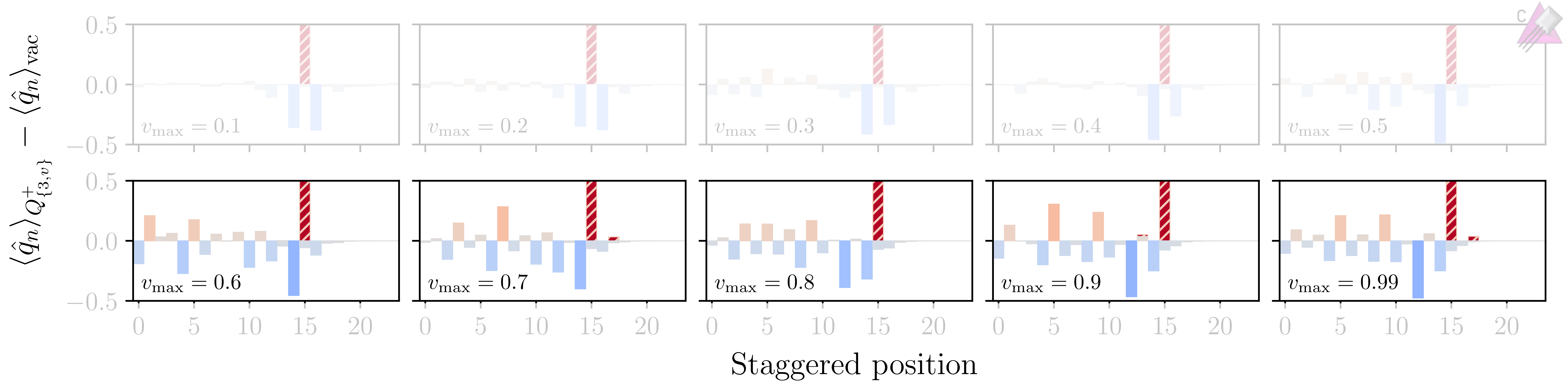



For small velocities, we see the expected behavior (nothing is happening)

As we increase the speed, we see that the light charges screening the heavy charge start to be left behind

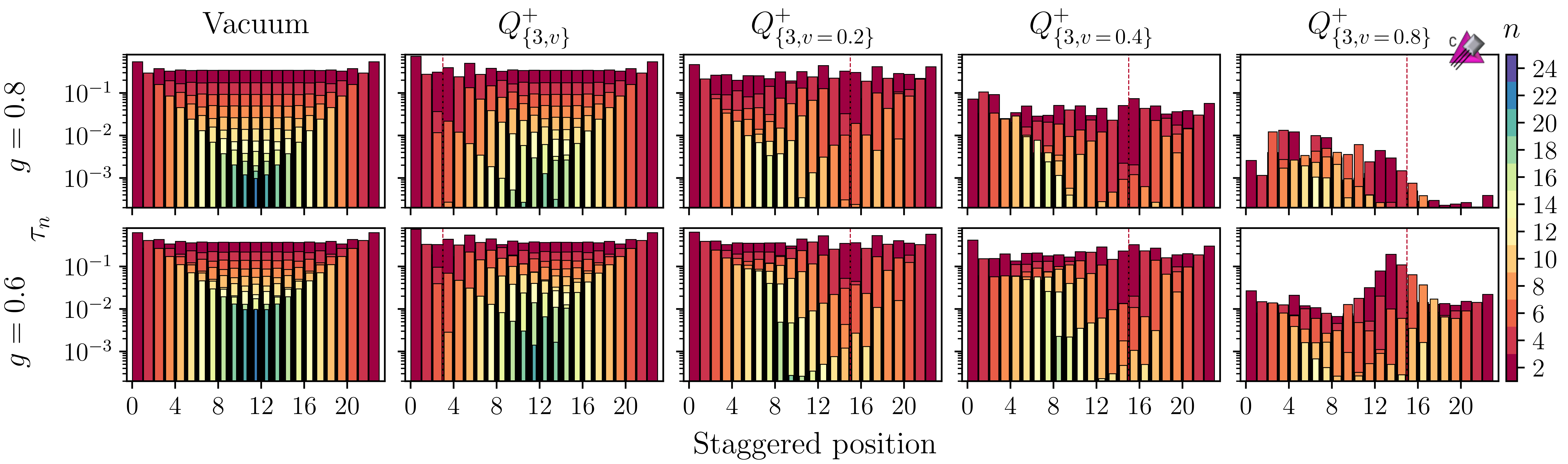
At high velocities, the light charges cannot keep up with the heavy charge

# Schwinger model with external charges

Going at the speed of light seems to be too fast on the lattice.
Are we seeing discretization effects?

n-tangles

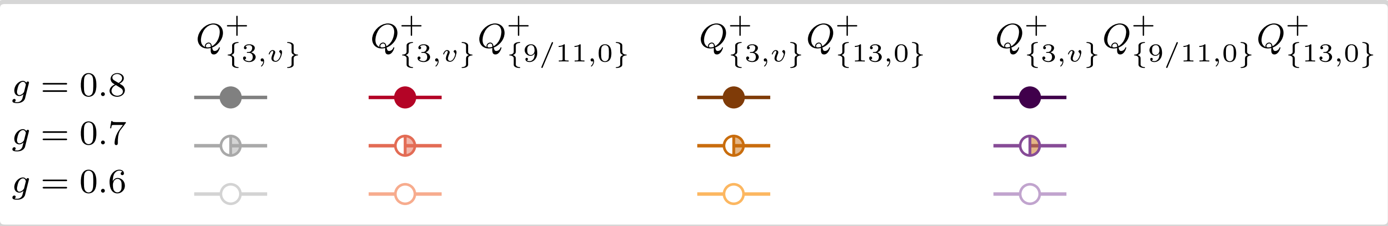
$$\tau_n = |\langle\psi|\sigma^y_{i_1}\sigma^y_{i_2}\cdots\sigma^y_{i_n}|\psi\rangle^*|^2$$

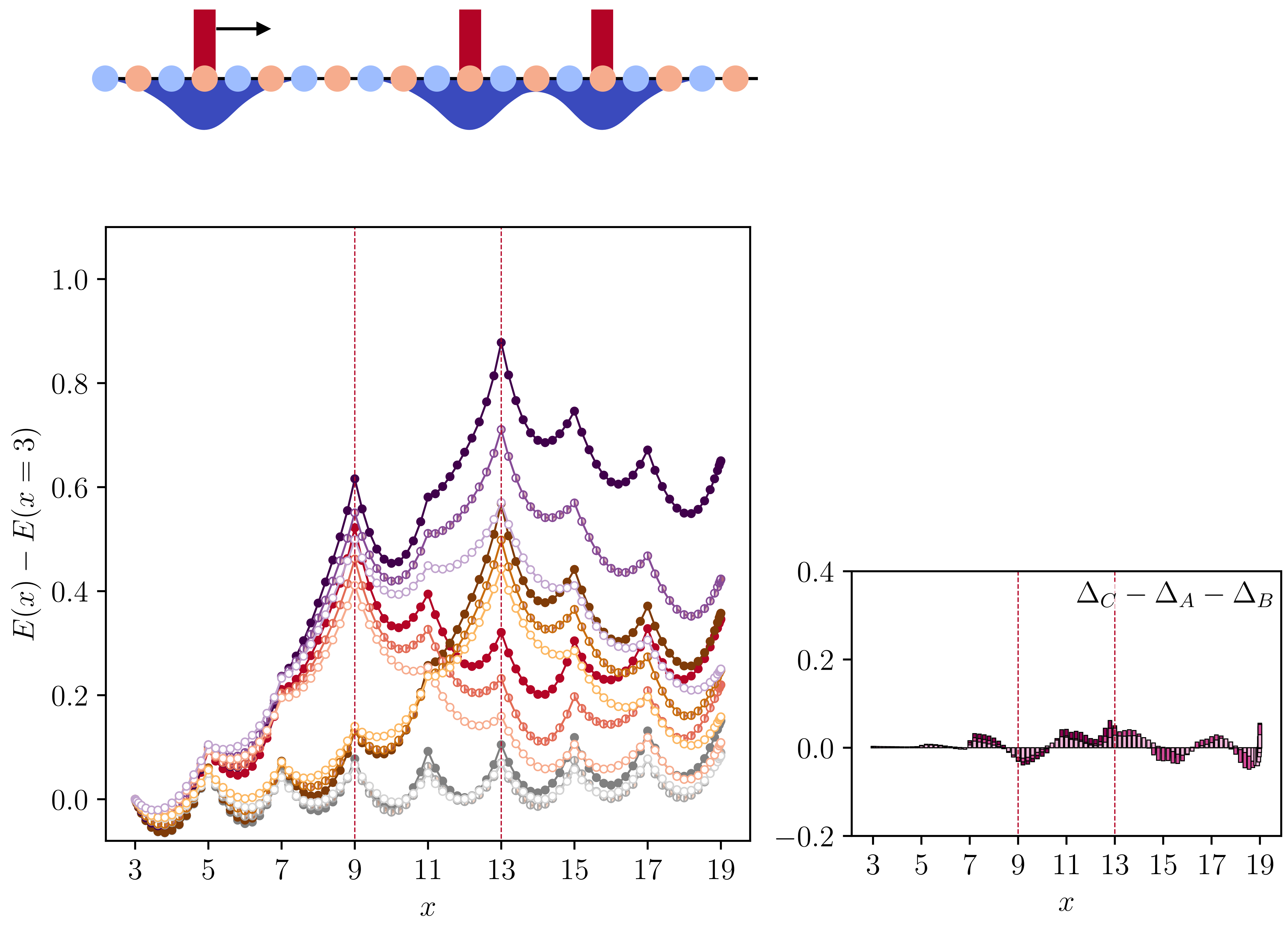


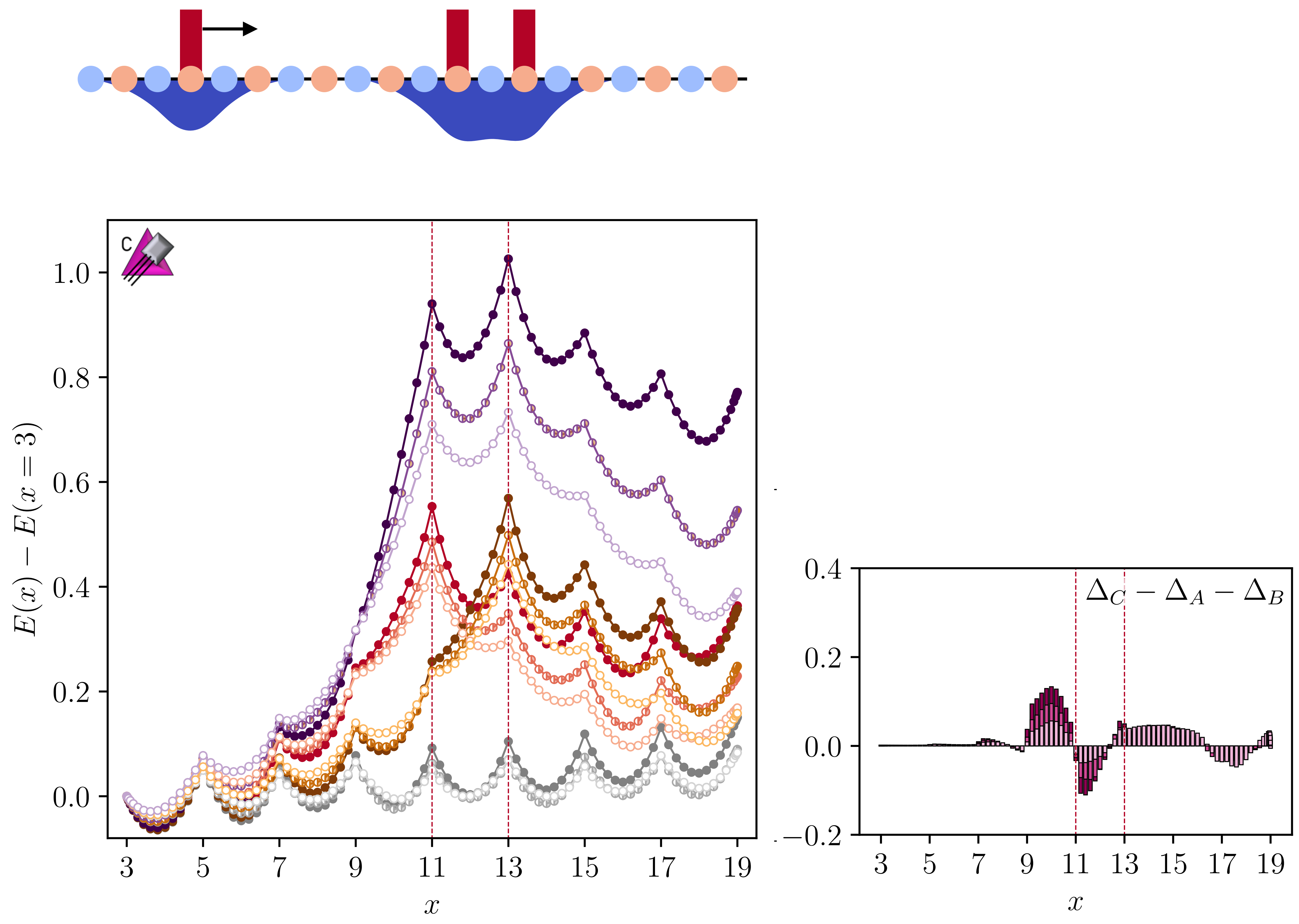

The entanglement also seems to be affected by discretization effects ⟶ Complete destruction

# Schwinger model with external charges

What happens if we collide two (or more) of these heavy charges?



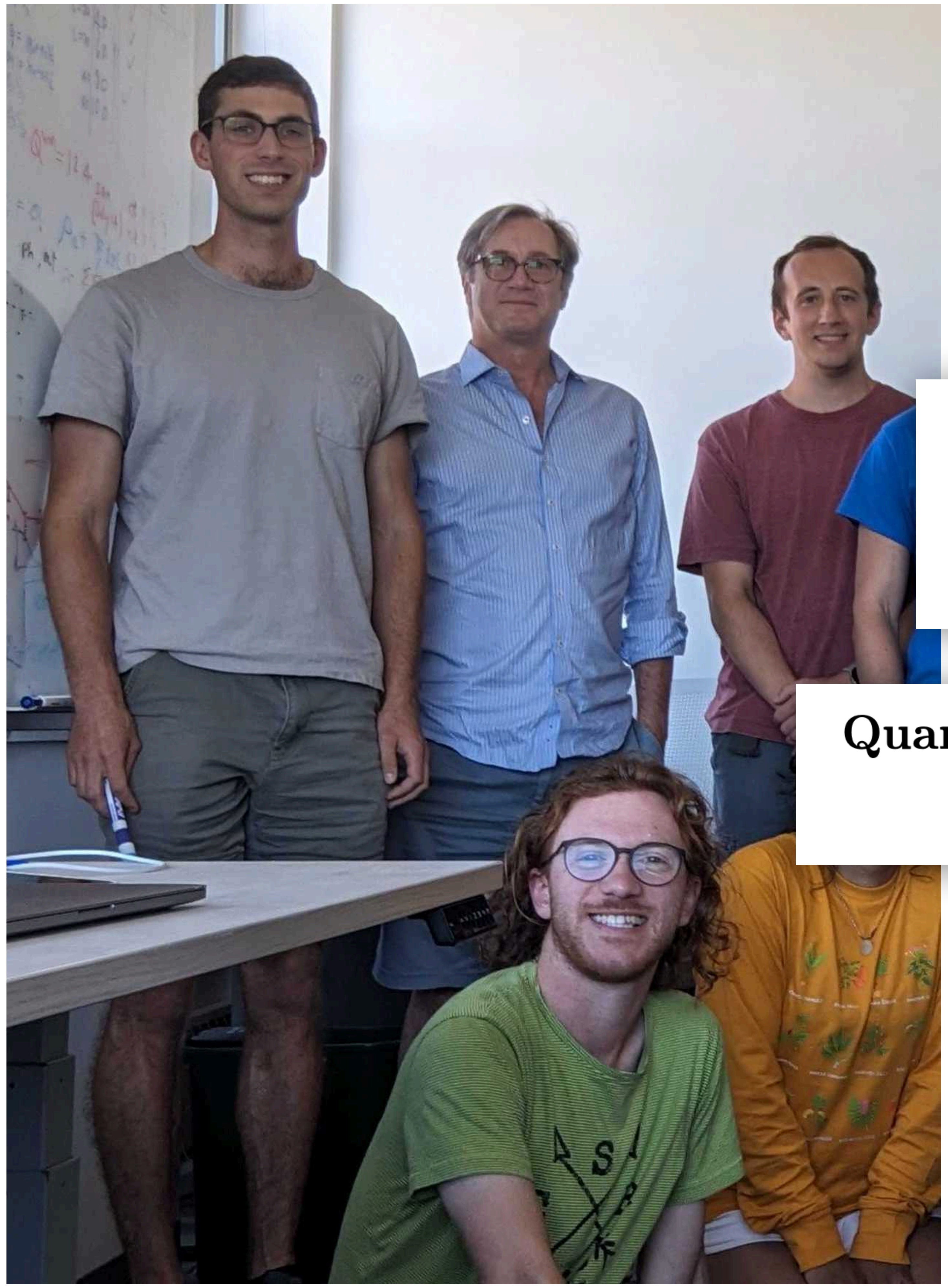

In-medium quantum coherence effects start to become visible (small v=0.2)

# Summary

- 1+1D QED is a great testbed for quantum simulations of subatomic physics
- We have introduced SC-ADAPT-VQE, which is “physics guided”, making use of the properties of the theory (expectation is it will work with QCD)
- Good results obtained from 100+ qubits, error mitigation is essential
- We showed how to prepare the vacuum, excite a hadron wavepacket on top, and evolve it in time.
- One needs to be carefull of the discretization effects (as in LQCD)
- Next step is to go to higher dimensions

# Thank you

ありがとうございます



PRX Quantum 5 (2024), arXiv:2308.04481 [quant-ph]

**Scalable Circuits for Preparing Ground States on Digital Quantum Computers: The Schwinger Model Vacuum on 100 Qubits**

Roland C. Farrell,* Marc Illa,† Anthony N. Ciavarella,‡ and Martin J. Savage§

**Quantum Simulations of Hadron Dynamics in the Schwinger Model using 112 Qubits**

Roland C. Farrell[1,*] Marc Illa[1,†] Anthony N. Ciavarella[1,2,‡] and Martin J. Savage[1,§]

Phys. Rev. D (2024), arXiv:2401.08044 [quant-ph]

**Steps Toward Quantum Simulations of Hadronization and Energy-Loss in Dense Matter**

Roland C. Farrell[1,2,*] Marc Illa[1,†] and Martin J. Savage[1,‡]

arXiv:2405.06620 [quant-ph]