Baryon-baryon scattering in SU(3)-flavour-symmetric QCD

Jeremy R. Green

Zeuthen Particle Physics Theory, DESY

Hadrons and Hadron Interactions in QCD 2024
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto, Japan

Questions in nuclear physics

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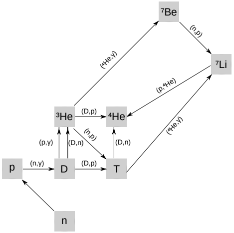

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$\rightarrow$ controls abundances of light elements.

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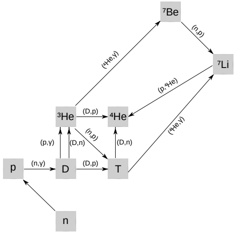

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$\longrightarrow$ controls abundances of light elements.

How strongly does deuteron binding depend on quark masses?
Could pp or nn bind?

Nuclei as tools in experiments

In practice, nuclei instead of free nucleons are often used.

- Argon in neutrino experiments (MicroBooNE, DUNE).
- Xenon for dark matter direct detection (XENONnT, LUX-ZEPLIN).

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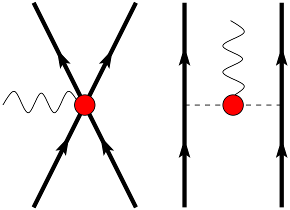

e.g. EMC effect:
distribution of quarks is different inside nucleus compared with proton and neutron

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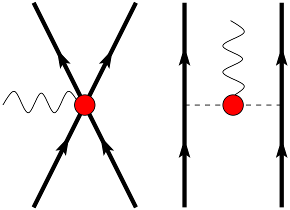

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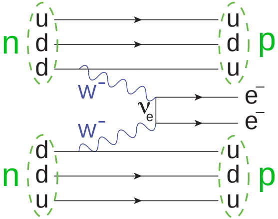

Long-term challenge: neutrinoless double beta decay.

Are neutrinos Majorana?

Hyperon interactions

NN interaction thoroughly studied in experiments. What about strange baryons (*hyperons*)?
Hyperon interactions with $S = -1$ or -2 less well known.

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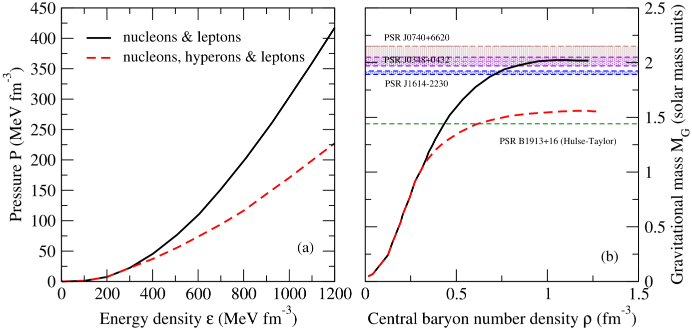

I. Vidaña, EPJ Web Conf. **271**, 09001 (2022)

Λ baryons can reduce Fermi pressure in neutron stars.

Contradicted by detection of neutron stars with $M \approx 2M_\odot$.

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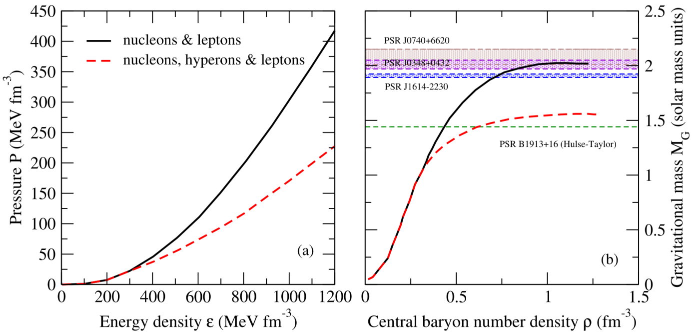

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Do hyperon-hyperon (YY) or NNY interactions play a role?

Outline

1. Methodology and challenges
2. NN: old versus new calculations
3. H dibaryon at SU(3) symmetric point
4. NN at SU(3) symmetric point
5. Outlook

Methods for baryon-baryon scattering

Standard approach:

1. Compute the finite-volume spectra for various quantum numbers: flavour, total momentum $\boldsymbol{P}$, little-group irrep Λ.
2. Use finite-volume quantization to constrain model for scattering amplitude.
3. Find bound-state poles, resonances, etc. in model.

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In all cases:

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Find an interpolating operator $\mathcal{O}$ with the desired quantum numbers.

Compute the two-point function

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Inserting a complete set of states, we get

$$\begin{aligned} C(t) &= \sum_n e^{-E_n t} \left| \langle n | \mathcal{O}^\dagger | \Omega \rangle \right|^2 \\ &\xrightarrow{t\to\infty} e^{-E_0 t} \left| \langle 0 | \mathcal{O}^\dagger | \Omega \rangle \right|^2 . \end{aligned}$$

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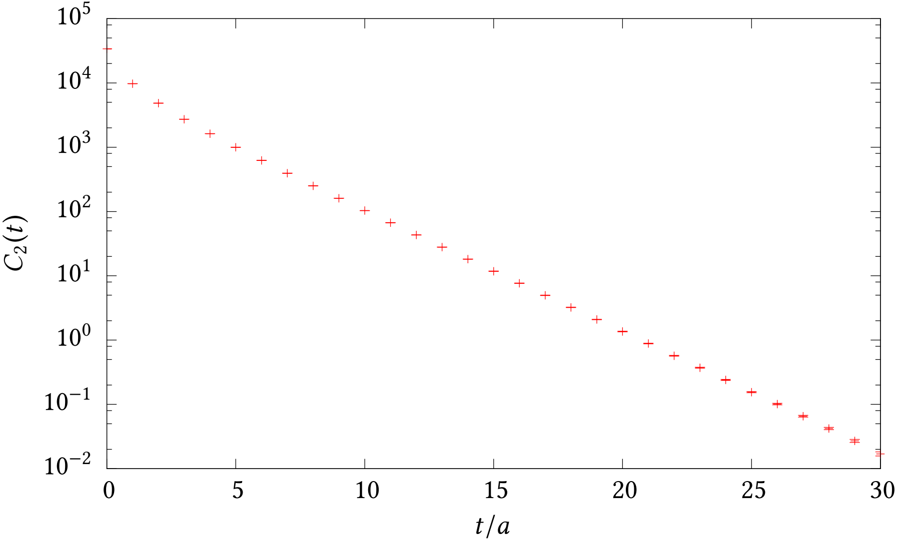
Then take the effective mass,

$$m_{\text{eff}}(t) = \frac{1}{\Delta}\log\frac{C(t)}{C(t+\Delta)} \longrightarrow E_0 + O(e^{-(E_1-E_0)t}).$$

Spectroscopy (simple approach)



proton
two-point function

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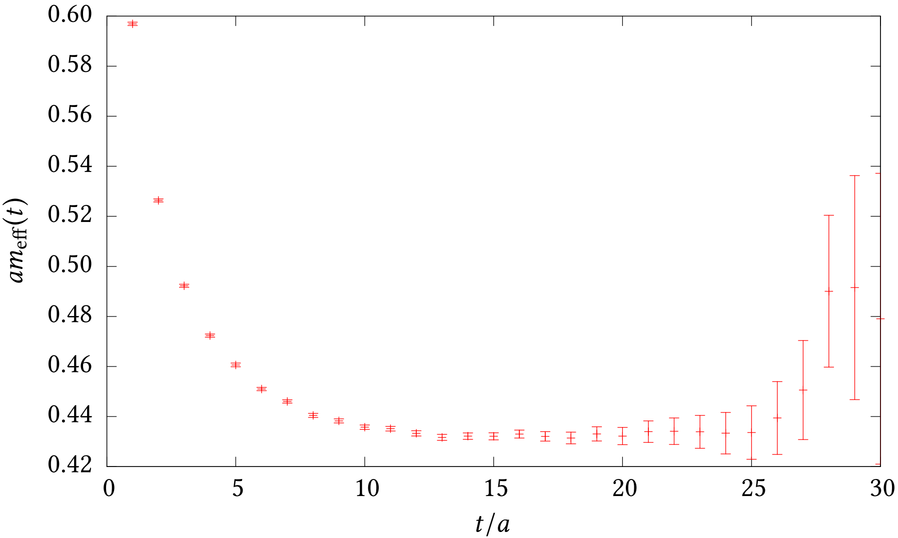
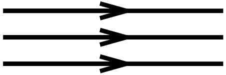


proton
effective mass

Signal-to-noise problem

Nucleon correlator:



$$
\begin{aligned}
C_{2\mathrm{pt}}(t) &= \left\langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \right\rangle \sim \langle\!\langle \mathfrak{R}[S(t,0)^3] \rangle\!\rangle \\
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\end{aligned}
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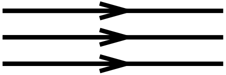

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In general $\sigma^2(X) = \langle\!\langle X^2 \rangle\!\rangle - \langle\!\langle X \rangle\!\rangle^2$.

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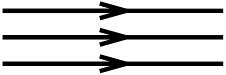

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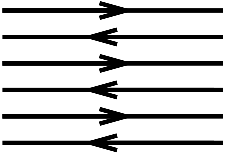
$$\sigma^2(C_{2\text{pt}}(t)) \sim \langle\!\langle S(t,0)^3 S^*(t,0)^3 \rangle\!\rangle + \cdots$$
$$\to e^{-3m_\pi t}$$



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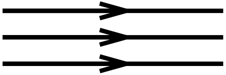

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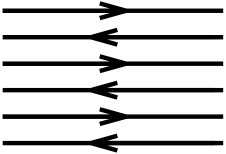

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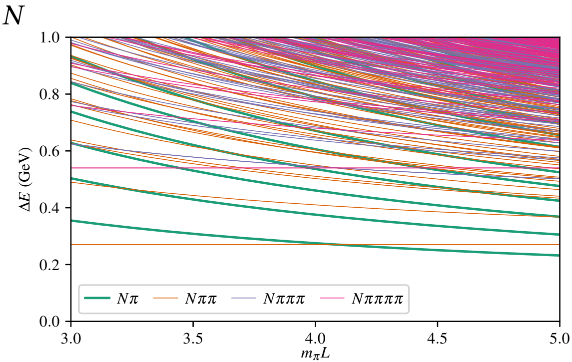
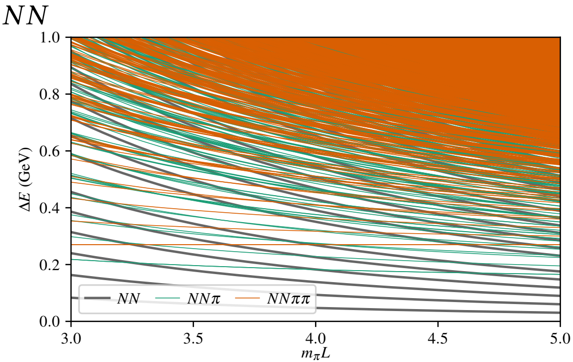
$$\to e^{-3m_\pi t}$$

Signal-to-noise ratio:

$$S/N \equiv \frac{C_{2\text{pt}}(t)}{\sigma(C_{2\text{pt}}(t))} \to e^{-(m_N - \frac{3}{2}m_\pi)t} \quad \text{single nucleon}$$
$$\to e^{-2(m_N - \frac{3}{2}m_\pi)t} \quad \text{two nucleons}$$

Excited-state spectrum (noninteracting)





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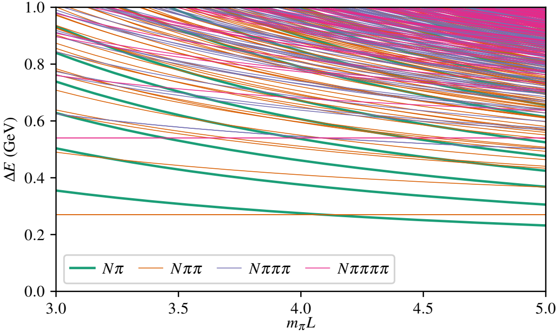
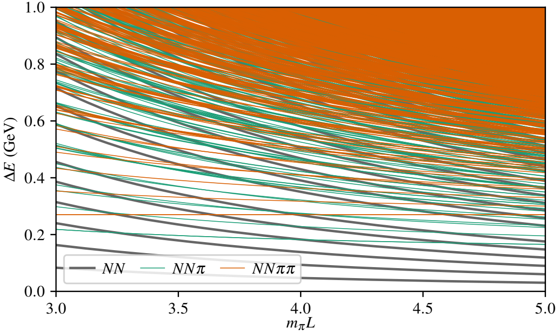


$$S/N \sim e^{-(m_N - \frac{3}{2} m_\pi) t}$$

NN



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Given a set of N interpolating operators $\{\mathcal{O}_i\}$,
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$$\mathrm{C}_{ij}(t) \equiv \langle \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \rangle,$$

$$\mathrm{C}(t+\Delta) v_n = \lambda_n \mathrm{C}(t) v_n.$$

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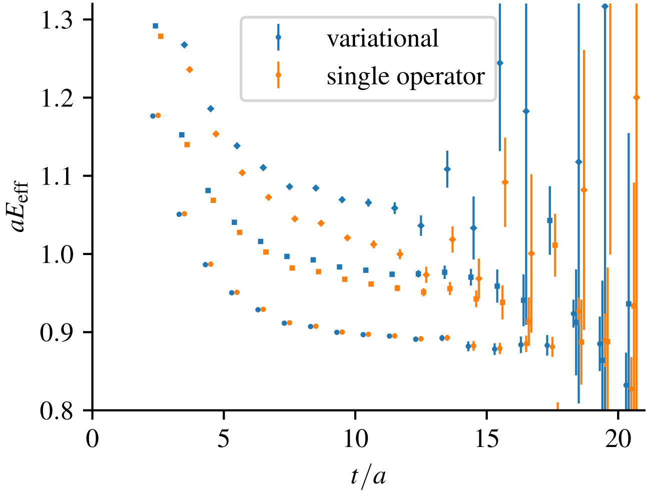
For each of the lowest N states, this gives an effective mass and an optimized interpolating operator:

$$m_{\text{eff},n} = \frac{-1}{\Delta} \log \lambda_n, \qquad \tilde{\mathcal{O}}_n = v_{ni}^\dagger \mathcal{O}_i,$$

with faster approach to plateau $\sim e^{-(E_N - E_n)t}$.

Importance of variational method



Variational approach essential for excited states.

Single operators can also fail to obtain ground state.

Interpolating operators for dibaryon

Typically use “smeared” quark fields with Gaussian-like profile. Simplest choices:

Hexaquark

$$\mathcal{O}_H(t, \boldsymbol{P}) = \sum_{\boldsymbol{x}} e^{-i\boldsymbol{P}\cdot\boldsymbol{x}} (qqqqqq)(t, \boldsymbol{x})$$

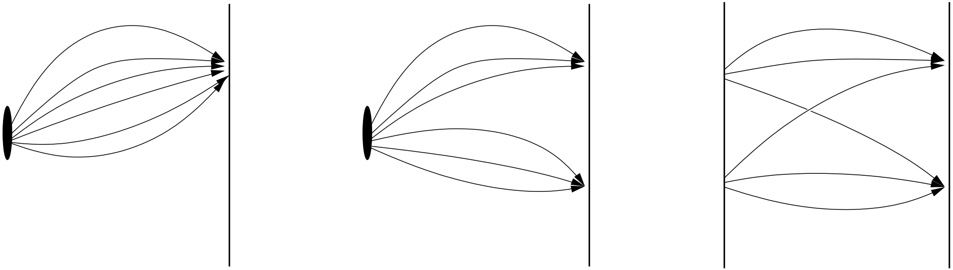
- Looks like quark-model state.

Two-baryon

$$\mathcal{O}_{BB}(t, \boldsymbol{P}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{-i\boldsymbol{p}_1\cdot\boldsymbol{x}} e^{-i(\boldsymbol{P}-\boldsymbol{p}_1)\cdot\boldsymbol{y}} (qqq)(t, \boldsymbol{x})(qqq)(t, \boldsymbol{y})$$

- Looks like noninteracting baryon-baryon state.
- Varying $\boldsymbol{p}_1$ yields many different operators with same total $\boldsymbol{P}$.

Correlation functions



$\langle \mathcal{O}_H(t)\mathcal{O}_H^\dagger(0)\rangle$ $\langle \mathcal{O}_{BB}(t)\mathcal{O}_H^\dagger(0)\rangle$ $\langle \mathcal{O}_{BB}(t)\mathcal{O}_{BB}^\dagger(0)\rangle$

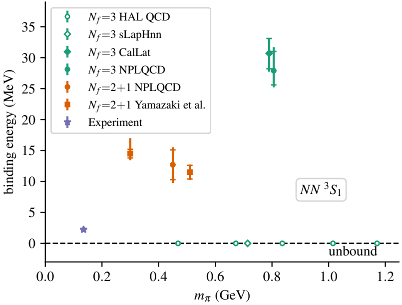
How to compute?

- Point-source propagator $\longrightarrow \langle \mathcal{O}_H(t)\mathcal{O}_H^\dagger(0)\rangle$ or $\langle \mathcal{O}_{BB}(t)\mathcal{O}_H^\dagger(0)\rangle$.
- Nonlocal methods like *distillation* $\longrightarrow \langle \mathcal{O}_{BB}(t)\mathcal{O}_{BB}^\dagger(0)\rangle$.

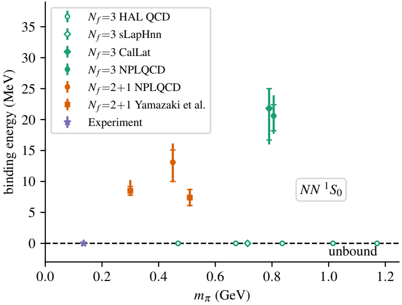
Many early calculations used only $\langle \mathcal{O}_{BB}(t)\mathcal{O}_H^\dagger(0)\rangle$ asymmetric correlators.

Nucleon-nucleon scattering from LQCD: past calculations

Decade-long controversy over presence of bound states at heavy quark masses.



deuteron



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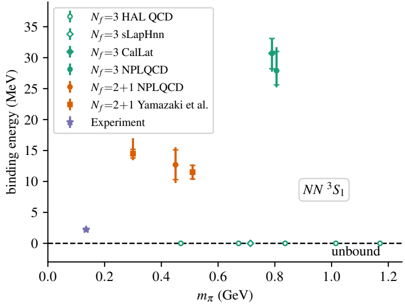

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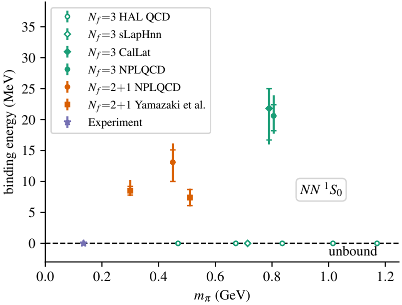

dineutron

Disagreement about simplest warm-up problem for nuclear physics on the lattice.

Experiment: $B_d \approx 2.2$ MeV known for 90 years. J. Chadwick and M. Goldhaber, Nature **134**, 237–238 (1934)

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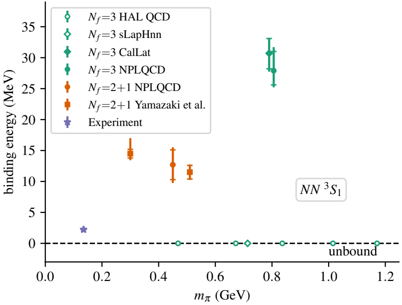

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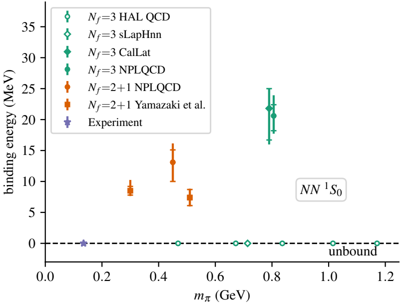

dineutron

No calculation performed using more than one lattice spacing.

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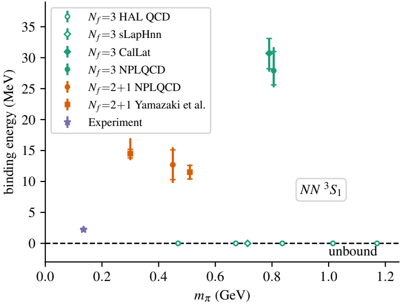

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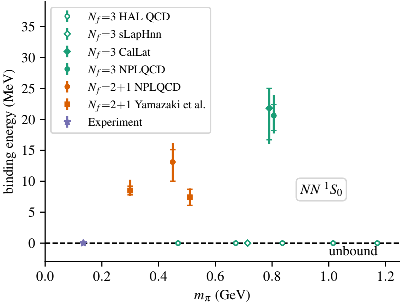

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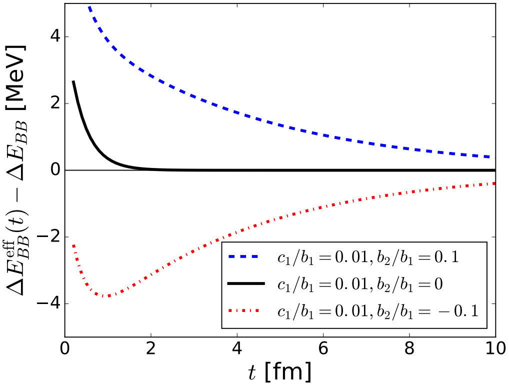
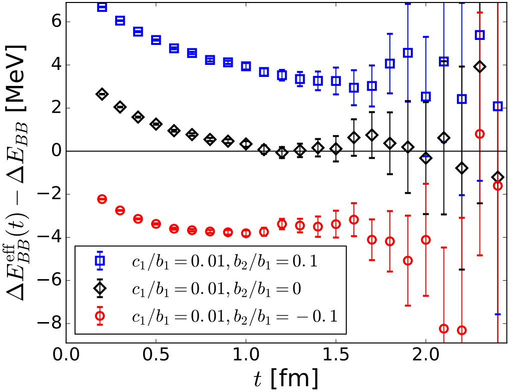
No calculation performed using more than one lattice spacing.

All calculations that obtain bound states use $\langle O_{BB}(t) O_H^\dagger(0) \rangle$ asymmetric correlation functions.

What can go wrong?

T. Iritani *et al.* (HAL QCD), Mirage in temporal correlation functions for baryon-baryon interactions in lattice QCD, JHEP **2016**, 101 (2016) [1607.06371] (CC BY 4.0)





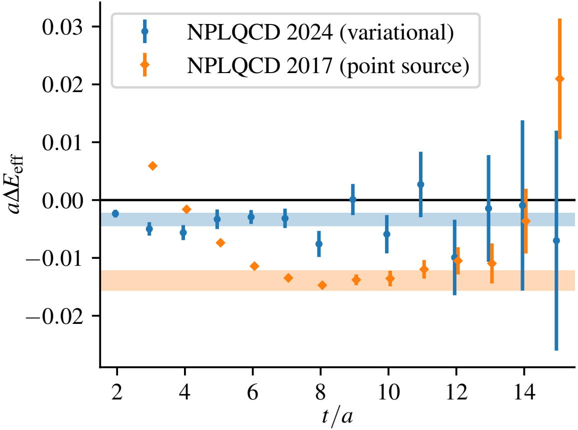
Mock data: effective mass for correlator $C(t) = b_1 + b_2 e^{-\delta E_{\mathrm{el}} t} + c_1 e^{-\delta E_{\mathrm{inel}} t}$.

"elastic" excitation $\delta E_{\mathrm{el}} = 50$ MeV

"inelastic" excitation $\delta E_{\mathrm{inel}} = 500$ MeV

Point sources versus variational method with bilocal interpolators

Old and new methods used on same ensemble.



$m_\pi \approx 800$ MeV.

Old calculation:

$^1S_0$ bound state with $B_{nn} \approx 21$ MeV.

New calculation consistent with unbound NN.

Data extracted from

W. Detmold *et al.* (NPLQCD), 2404.12039

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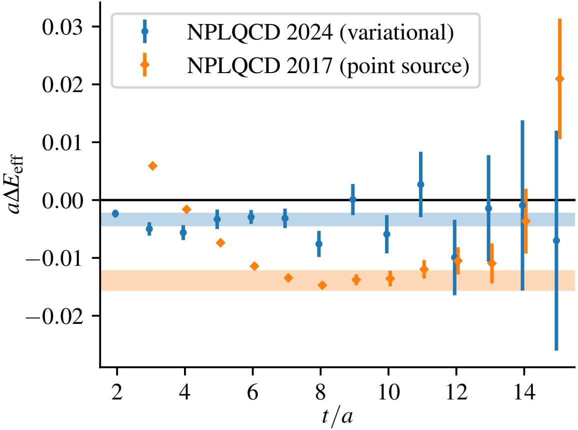

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Several variational baryon-baryon calculations done:

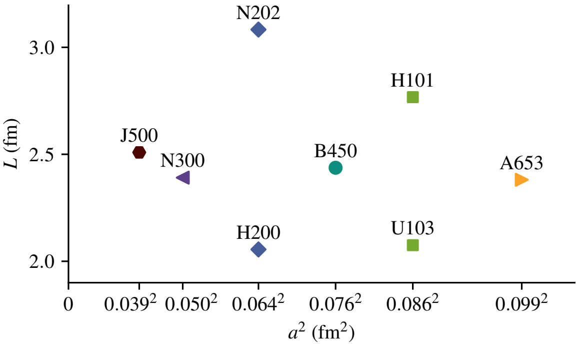
A. Francis, JRG *et al.*, PRD 99, 074505 (2019) [1805.03966]
B. Hörz *et al.* (sLapHnn), PRC 103, 014003 (2021) [2009.11825]
JRG *et al.*, PRL 127, 242003 (2021) [2103.01054]
S. Amarasinghe *et al.* (NPLQCD), PRD 107, 094508 (2023) [2108.10835]
W. Detmold *et al.* (NPLQCD), 2404.12039
Z.-Y. Wang @ Lattice 2024 Y. Geng (CLQCD) @ Lattice 2024

Largely consistent picture:

no NN bound state at heavy m_π.

Calculations at light SU(3)-symmetric point

JRG, A. D. Hanlon, P. M. Junnarkar, H. Wittig: Phys. Rev. Lett. **127**, 242003 (2021); PoS LATTICE **2021**, 294; PoS LATTICE **2022**, 200;
M. Padmanath, J. Bulava, JRG, A. D. Hanlon, B. Hörz, P. Junnarkar, C. Morningstar, S. Paul, H. Wittig, PoS LATTICE **2021**, 459
+ ongoing work (BaSc collaboration)



Ensembles with $O(a)$ improved Wilson-clover fermions from CLS.

SU(3)-symmetric point with physical $m_u + m_d + m_s$.

$m_\pi = m_K = m_\eta \approx 420$ MeV.

Two octet baryons: $(\mathbf{8} \otimes \mathbf{8})_S = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27}$, $\quad (\mathbf{8} \otimes \mathbf{8})_A = \mathbf{8} \oplus \mathbf{10} \oplus \overline{\mathbf{10}}$.
H dibaryon: $\mathbf{1}$; $\quad NN$: $\mathbf{27}$, $\overline{\mathbf{10}}$.

H dibaryon

VOLUME 38, NUMBER 5 PHYSICAL REVIEW LETTERS 31 JANUARY 1977

Perhaps a Stable Dihyperon*

R. L. Jaffe†
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305, and Department of Physics and Laboratory of Nuclear Science,‡ Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 1 November 1976)

In the quark bag model, the same gluon-exchange forces which make the proton lighter than the Δ(1236) bind six quarks to form a stable, flavor-singlet (with strangeness of −2) $J^P = 0^+$ dihyperon (H) at 2150 MeV. Another isosinglet dihyperon (H^*) with $J^P = 1^+$ at 2335 MeV should appear as a bump in ΛΛ invariant-mass plots. Production and decay systematics of the H are discussed.

TABLE I. Quantum numbers and masses of S-wave dibaryons.

$SU(6)_{cs}$ representation	C_6	J	$SU(3)_f$ representation	Mass in the limit $m_s = 0$ (MeV)
490	144	0	1	1760
896	120	1,2	8	1986
280	96	1	10	2165
175	96	1	10*	2165
189	80	0,2	27	2242
35	48	1	35	2507
1	0	0	28	2799

Proposed *uuddss* flavour-singlet dibaryon with $J^P = 0^+$.

Bound state of two Λ hyperons with $B_H \approx 80$ MeV.

H dibaryon: Experimental searches

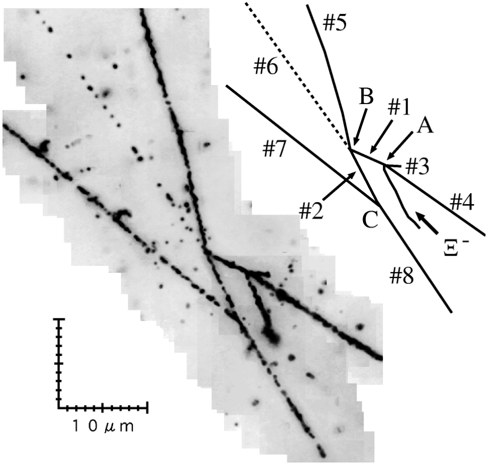


FIG. 2. Photograph and schematic drawing of NAGARA event. See text for detailed explanation.

H. Takahashi *et al.*, PRL **87**, 212502 (2001)

Strongest constraint comes from "Nagara" event from E373 at KEK, which found a ${}^{6}_{\Lambda\Lambda}\mathrm{He}$ double-hypernucleus with $\Lambda\Lambda$ separation energy

$$B_{\Lambda\Lambda}^{\text{Nagara}} = 6.91 \pm 0.16 \text{ MeV}.$$

Absence of strong decay ${}^{6}_{\Lambda\Lambda}\mathrm{He} \to {}^{4}\mathrm{He} + H$ implies

$$B_H < B_{\Lambda\Lambda}^{\text{Nagara}}.$$

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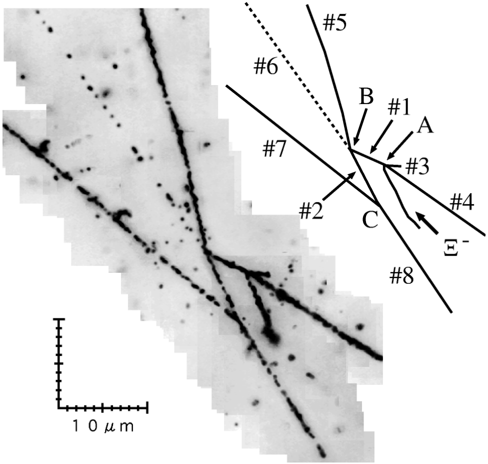

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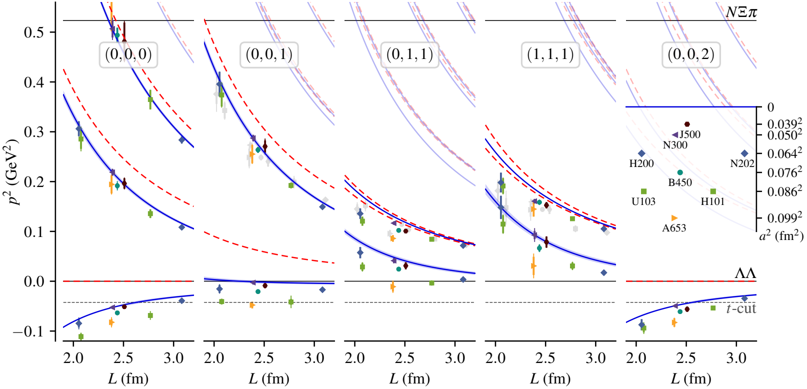
Also studied using "femtoscopy" method at LHC.

ALICE, PLB **797**, 134822 (2019)

H dibaryon: spectrum summary

Weakly bound H dibaryon from SU(3)-flavor-symmetric QCD
JRG, A. D. Hanlon, P. M. Junnarkar, H. Wittig, Phys. Rev. Lett. **127**, 242003 (2021)



SU(3) singlet.

Trivial (A1g or A1) irreps.

p^2 is back-to-back scattering momentum: $E_{\text{cm}} = 2\sqrt{p^2 + m^2}$
Points: lattice energy levels.
Red dashed curves: noninteracting levels.
Blue curves: interacting levels in continuum.

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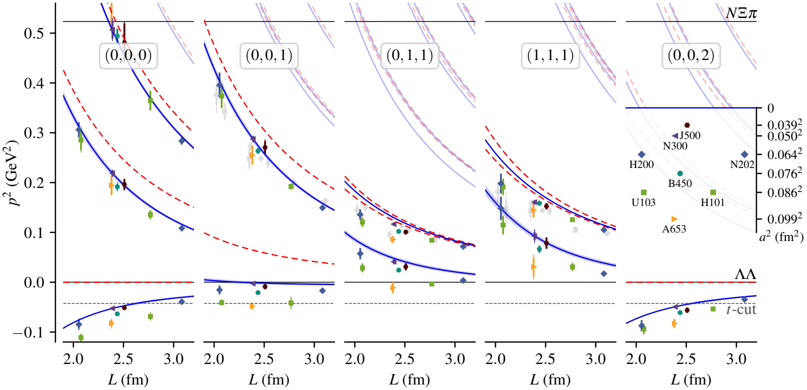

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Strong dependence on a^2!
Levels lie on left-hand cut!

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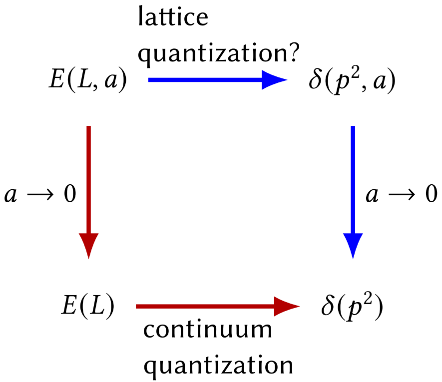
Quantization condition and continuum limit



Continuum extrapolation:
follow blue path, applying continuum quantization condition at nonzero lattice spacing.

Combined fits to multiple lattice spacings: let

$$p \cot \delta(p^2, a) = \sum_{i=0}^{N-1} c_i(a) p^{2i}, \quad c_i(a) = c_{i0} + c_{i1} a^2.$$

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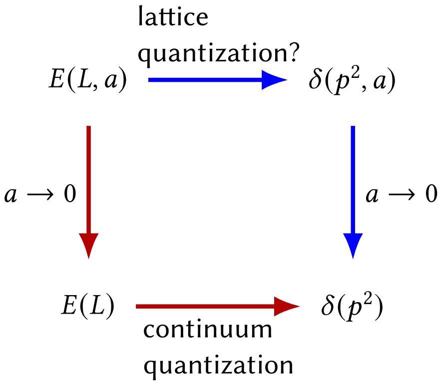

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Recent work on including discretization effects in quantization condition:

M. T. Hansen and T. Peterken, 2408.07062

Combined phase shift fits

S-wave quantization condition:

$$p \cot \delta(p) = \frac{2}{\sqrt{\pi L \gamma}} Z_{00}^{\boldsymbol{P}L/(2\pi)}\left(1, \left(\frac{pL}{2\pi}\right)^2\right)$$

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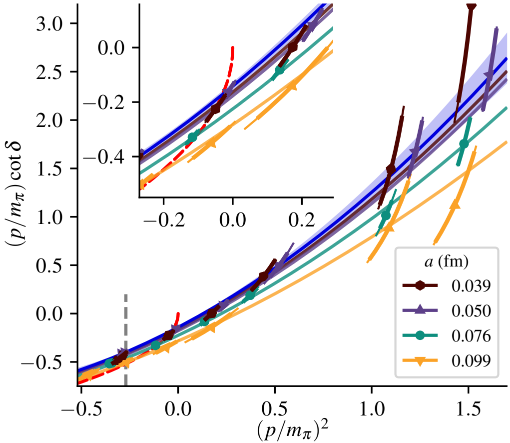

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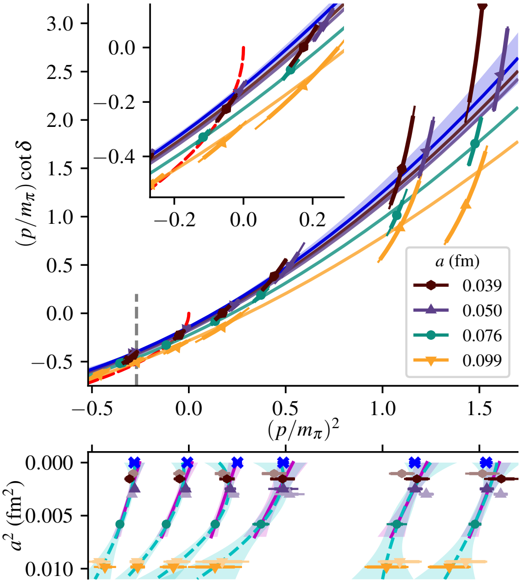
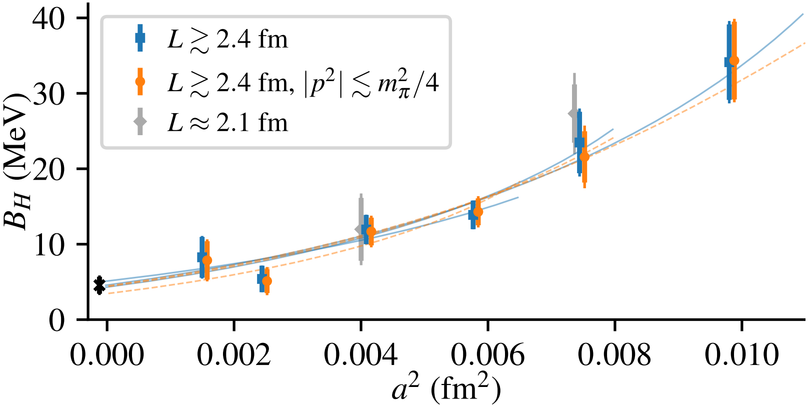
Cross check: extrapolate energies at fixed volume.



H dibaryon binding energy versus lattice spacing



Fits to spectrum with different cuts on a and p^2.
Strong dependence on lattice spacing.

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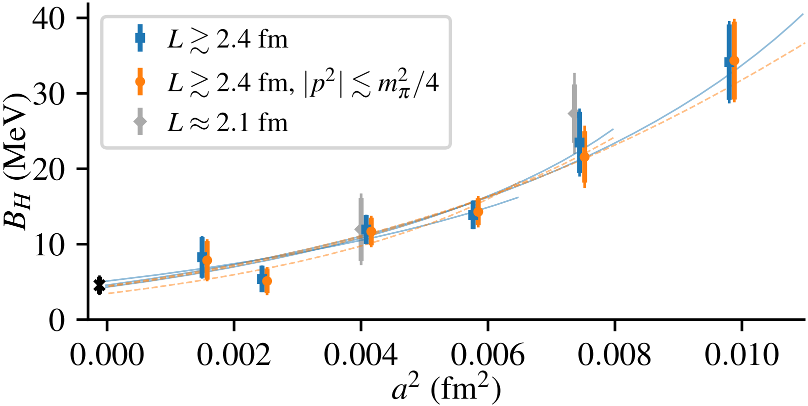

Strong dependence on a^2 also found by HAL QCD and NPLQCD at heavier pion mass.

T. Inoue, Few Body Syst. **65**, 34 (2024)
R. Perry @ Lattice 2024

Fits to spectrum with different cuts on a and p^2.
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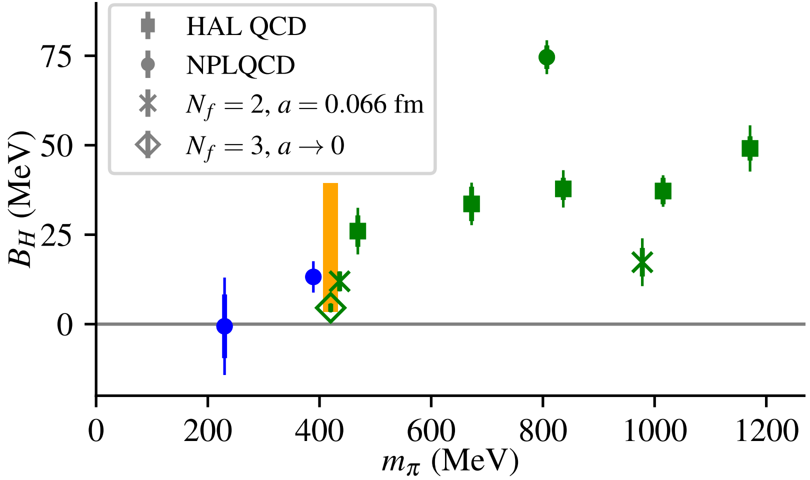
H dibaryon binding energy: comparison with literature



green: $m_s = m_{ud}$

blue: $m_s = m_s^{\text{phys}} > m_{ud}$

H dibaryon: breaking of SU(3) flavour

With broken SU(3), H dibaryon can couple to three baryon-baryon channels: $\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$.

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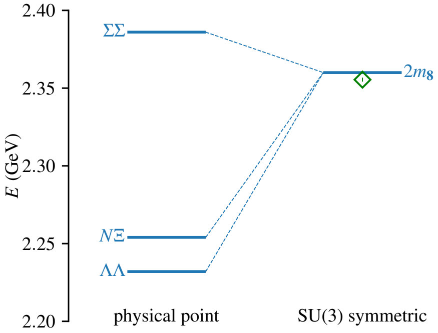

Splitting is large compared with binding at SU(3) point.

SU(3)-broken lattice data currently being analyzed.

M. Padmanath, Lattice 2021

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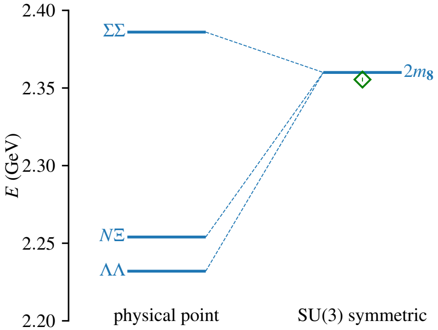

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M. Padmanath, Lattice 2021

Previous EFT-based extrapolations suggest physical bound state is unlikely.

Symanzik theory: EFT describing lattice QCD at $a > 0$

With $O(a)$ improved action, corrections start at a^2:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} + a^2 \sum_i \mathcal{O}_i + O(a^3).$$

Dimension-six operators $\mathcal{O}_i$ are gluonic, $\bar{q}q$, or $(\bar{q}q)^2$ satisfying symmetries of lattice action:

- Some break $O(4)$ rotational symmetry $\rightarrow$ modified dispersion relations.
- Some break chiral symmetry.

Logarithmic corrections also understood. N. Husung *et al.*, 2022

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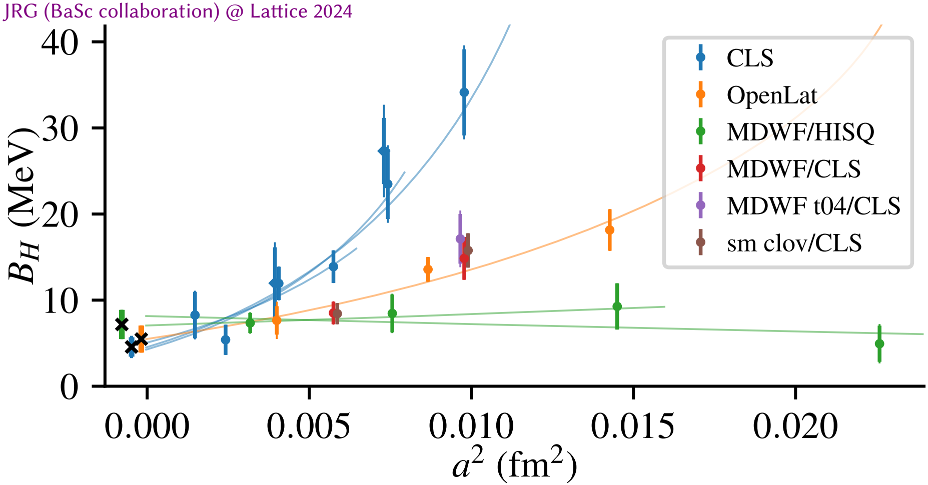
We see percent-level effects on baryon-baryon energies
but O(100%) effects on scattering observables such as the scattering length.

Can we understand what is causing these large effects? Study using different actions.

Binding energy of H dibaryon: different lattice actions



Three independent $a \to 0$ extrapolations agree. Size of lattice artifacts varies significantly.

Nucleon-nucleon: finite-volume quantization

Notation: partial wave ${}^{2s+1}\ell_J$.

Four categories of partial waves:

- Even ℓ, spin zero $\implies I = 1$. ${}^1S_0, {}^1D_2, {}^1G_4, \ldots$
- Even ℓ, spin one $\implies I = 0$. ${}^3S_1$–${}^3D_1, {}^3D_2, {}^3D_3$–${}^3G_3, \ldots$
- Odd ℓ, spin zero $\implies I = 0$. ${}^1P_1, {}^1F_3, {}^1H_5, \ldots$
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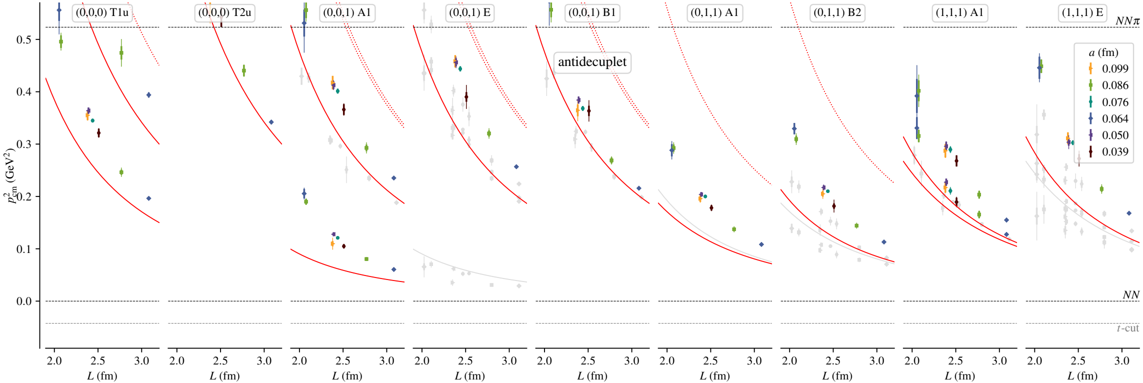
Still a work in progress; these results are PRELIMINARY.

Antidecuplet ($NN\ I = 0$): spin 0 spectrum

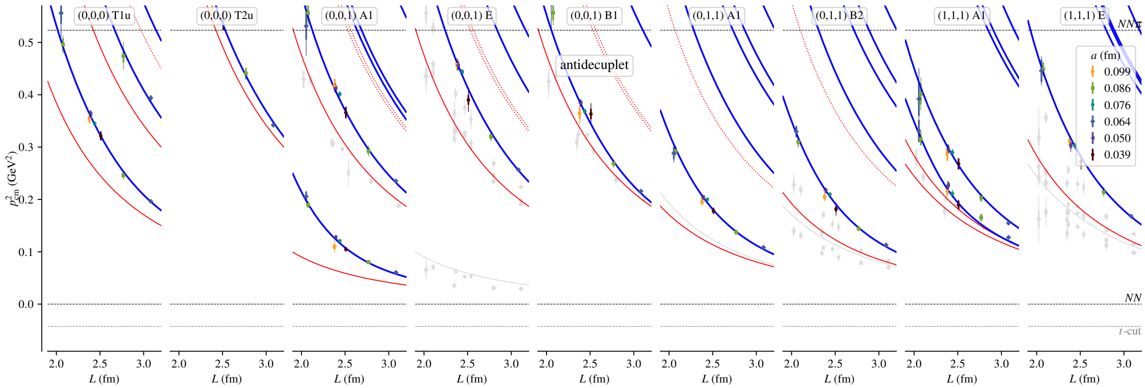


Operators constructed with definite spin. Spin-1 states (gray) identified via overlaps.

Quantization condition factorizes in spin. Here $^1P_1$ and $^1F_3$ are relevant.

Red curves: noninteracting levels.

Antidecuplet ($NN\ I = 0$): spin 0 spectrum, example fit 1

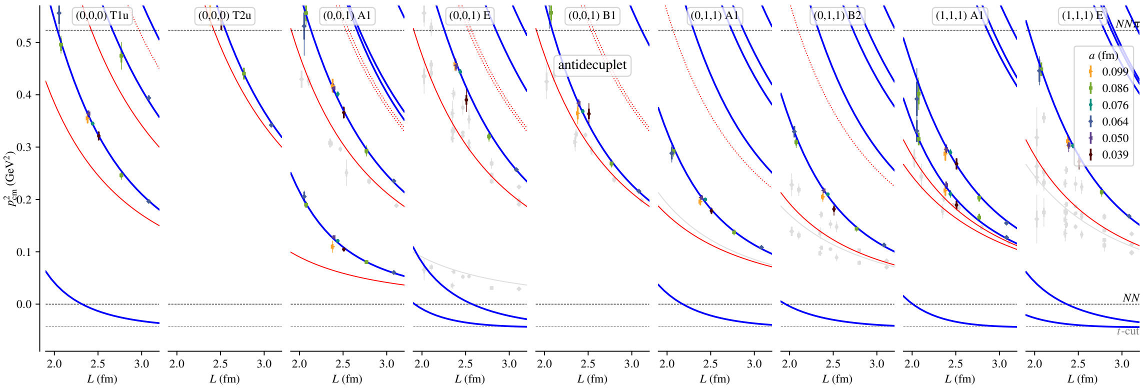


Fit ansatz:

$$p^3 \cot \delta_{{}^1P_1} = c_1 + c_2 p^2, \qquad p^7 \cot \delta_{{}^1F_3} = c_3 + c_4 p^8,$$

assuming no discretization effects.

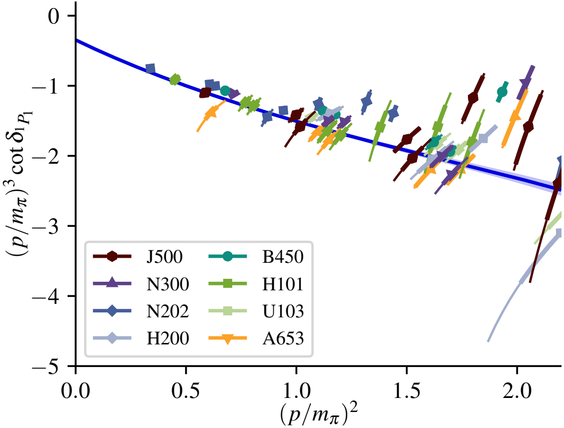
Antidecuplet ($NN\ I = 0$): spin 0 spectrum, example fit 2

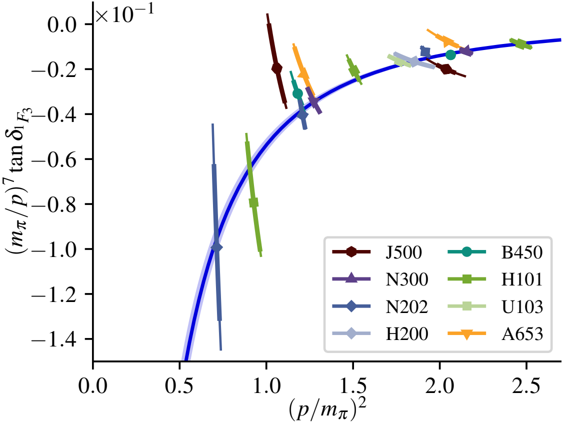


Fit ansatz: solutions to Lippmann-Schwinger equation for ${}^1P_1$ and ${}^1F_3$ with one-pion-exchange potential and contact terms, $\Lambda = 1.5 m_\pi$, assuming no discretization effects.

Note: spurious solutions to quantization condition near left-hand cut.

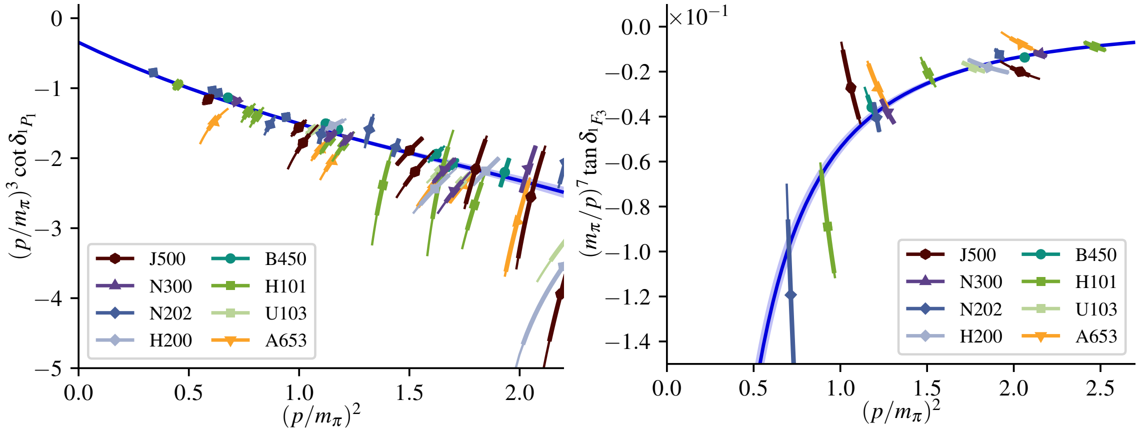
Spin 0 phase shifts: P and F waves (fit 2)





Points: energy levels under single-partial-wave approximation.

Spin 0 phase shifts: P and F waves (fit 2)



Points: energy levels taking other partial wave into account.

Data lie on single curve. Nontrivial consistency check of spectrum!

Spin 1: coupled partial waves

Use Blatt-Biedenharn decomposition of 2×2 scattering matrix:

$$S_J = \begin{pmatrix} \cos\epsilon_J & -\sin\epsilon_J \\ \sin\epsilon_J & \cos\epsilon_J \end{pmatrix} \begin{pmatrix} e^{2i\delta_{J\alpha}} & 0 \\ 0 & e^{2i\delta_{J\beta}} \end{pmatrix} \begin{pmatrix} \cos\epsilon_J & \sin\epsilon_J \\ -\sin\epsilon_J & \cos\epsilon_J \end{pmatrix}.$$

Near threshold:

$$\delta_{J\alpha} \sim p^{2J-1}, \qquad \delta_{J\beta} \sim p^{2J+3}, \qquad \epsilon_J \sim p^2.$$

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Below threshold, a (bound or virtual) deuteron pole exists where $p \cot\delta_{1\alpha}(p) = ip$.
The deuteron's asymptotic D/S-wave ratio is given by $-\tan\epsilon_1(p)$.

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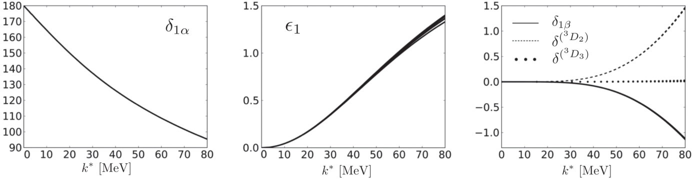
Phenomenology often uses Stapp parametrization:

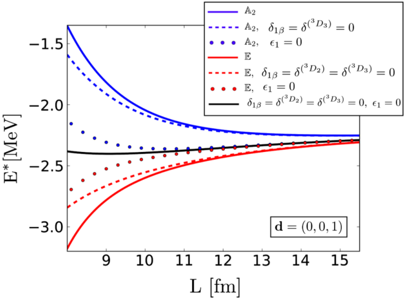
$$S_J = \begin{pmatrix} e^{i\delta_{3(J-1)_J}} & 0 \\ 0 & e^{i\delta_{3(J+1)_J}} \end{pmatrix} \begin{pmatrix} \cos 2\bar{\epsilon}_J & i\sin 2\bar{\epsilon}_J \\ i\sin 2\bar{\epsilon}_J & \cos 2\bar{\epsilon}_J \end{pmatrix} \begin{pmatrix} e^{i\delta_{3(J-1)_J}} & 0 \\ 0 & e^{i\delta_{3(J+1)_J}} \end{pmatrix}.$$

$J = 1$ mixing angle

Sign of mixing angle is physical: affects scattering amplitudes.
Effect on finite-volume spectrum? Worked out in a decade ago.
R. A. Briceño, Z. Davoudi, T. C. Luu, M. J. Savage, Phys. Rev. D **88**, 114507 (2013)

Phase shifts / mixing angles from experiment (input).



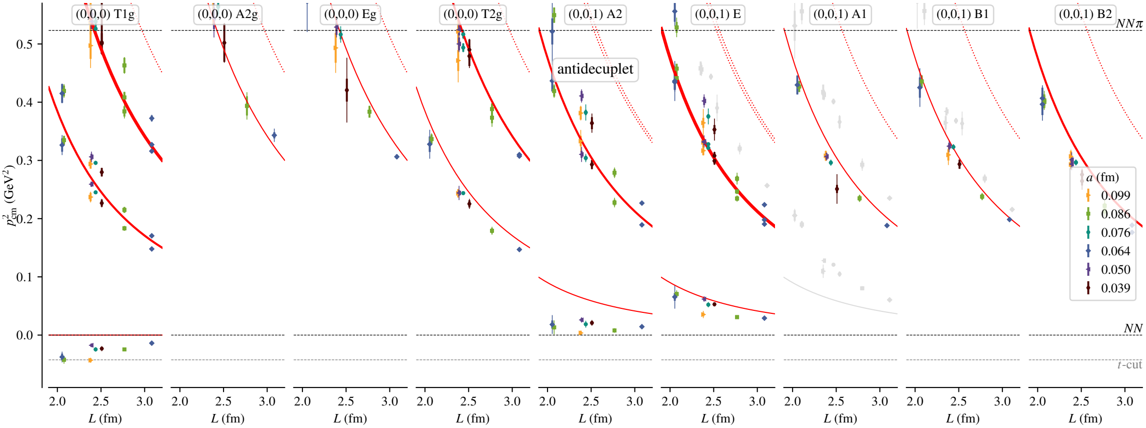


Ground states in frame $\boldsymbol{P} = \frac{2\pi}{L}(0, 0, 1)$.

For $J^P = 1^+$:
A_2 irrep has helicity 0
E irrep has helicity ± 1

If this is correct, at physical point expect $E_{A_2} > E_E$.

Antidecuplet (NN $I = 0$): spin 1 spectrum (1)



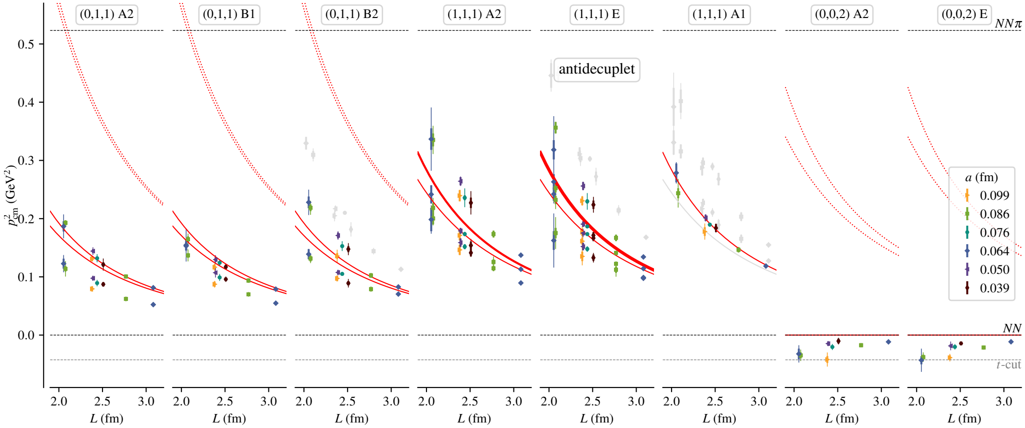
Spin-zero states shown in gray.

Thickness of red curves proportional to degeneracy of noninteracting level.

(39 levels) × (8 ensembles) = 312, although some lie above $N N \pi$ threshold.

In frame $\boldsymbol{P} = \frac{2\pi}{L}(0, 0, 1)$, get $E_{A_2} < E_E$.

Antidecuplet ($NN\ I = 0$): spin 1 spectrum (2)



Spin-zero states shown in gray.

Thickness of red curves proportional to degeneracy of noninteracting level.

(39 levels) × (8 ensembles) = 312, although some lie above $N N\pi$ threshold.

In frame $\boldsymbol{P} = \frac{2\pi}{L}(0,0,1)$, get $E_{A_2} < E_E$.

Scattering amplitude with partial wave mixing

Nuclear physics convention:
define in position space.

J. M. Blatt and L. C. Biedenharn, Rev. Mod. Phys. **24**, 258 (1952)

$$\mathcal{Y}_{Jls}{}^{M} = \sum_{m_l=-l}^{l} \sum_{m_s=-s}^{s} (lsm_l m_s | lsJM)\, Y_{l,m_l}(\theta,\phi)\, \chi_{s,m_s}$$

In terms of these definitions, the most general wave function in channel α, s with total angular momentum quantum numbers J, M consists of the superposition of an ingoing and outgoing spherical wave, each with spin-angle-dependence (3.2). At sufficiently large distances, we can write

$$\Psi_{\alpha s}(JM) = \frac{1}{r_\alpha (v_\alpha)^{\frac{1}{2}}} \mathcal{Y}_{Jls}{}^{M}\, \Phi_{\alpha s} \times \{ A_{\alpha s l}{}^{JM} \exp[-i(k_\alpha r_\alpha - \tfrac{1}{2} l\pi)] - B_{\alpha s l}{}^{JM} \exp[+i(k_\alpha r_\alpha - \tfrac{1}{2} l \pi)] \}. \quad (3.3)$$

The coefficients $A_{\alpha s l}{}^{JM}$ and $B_{\alpha s l}{}^{JM}$ are not independent of each other. Rather, if the amplitudes of the ingoing waves are known, the amplitudes of the outgoing waves are determined uniquely by the wave equation. The relation between them defines the *scattering matrix*:

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R. A. Briceño, Phys. Rev. D **89**, 074507 (2014)

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Relate using plane-wave expansion:

$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}} = 4\pi \sum_{\ell, m} i^\ell j_\ell(kr) Y_\ell^m(\hat{k}) Y_\ell^{m*}(\hat{r}),$$

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Mismatch! Differ by factor $i^{\ell-\ell'}$.

Flips sign of mixing.

Analyzing coupled $^3S_1$ and $^3D_1$

Quantization condition: $\det(\tilde{K}^{-1} - B) = 0$. Briceño, Davoudi, Luu 2013; Morningstar *et al.* 2017

Blatt-Biedenharn parametrization including $i^{\ell-\ell'}$ due to convention mismatch:

$$\tilde{K}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & p^2 \end{pmatrix} \begin{pmatrix} \cos\epsilon_1 & \sin\epsilon_1 \\ -\sin\epsilon_1 & \cos\epsilon_1 \end{pmatrix} \begin{pmatrix} p\cot\delta_{1\alpha} & 0 \\ 0 & p\cot\delta_{1\beta} \end{pmatrix} \begin{pmatrix} \cos\epsilon_1 & -\sin\epsilon_1 \\ \sin\epsilon_1 & \cos\epsilon_1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & p^2 \end{pmatrix}.$$

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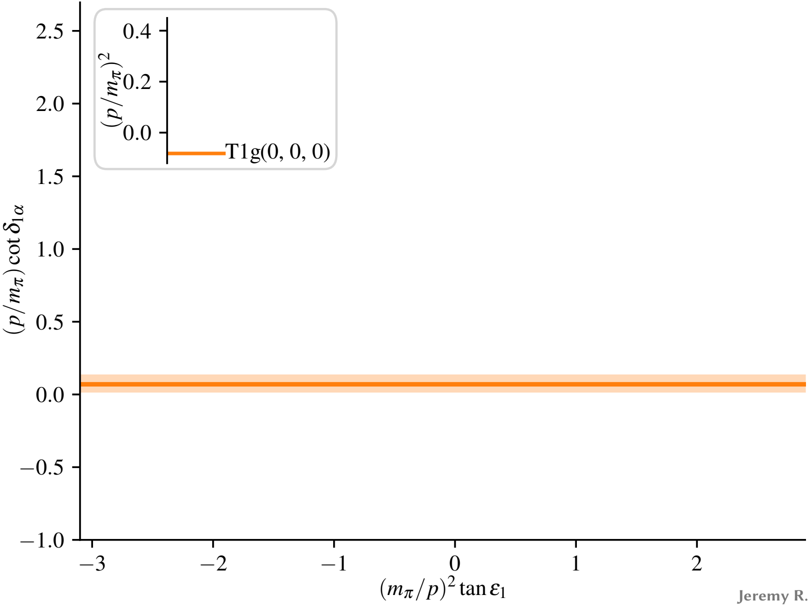
Each energy level imposes constraint on $(p^{-2}\tan\epsilon_1, p\cot\delta_{1\alpha})$ plane:

$$p\cot\delta_{1\alpha} = \frac{B_{00} - (B_{01} + B_{10})x + B_{11}x^2}{1 + p^4x^2}, \qquad x = p^{-2}\tan\epsilon_1.$$

$\delta_{1\alpha}$ and ϵ_1 on N202



Assume $\delta_{1\beta} = 0$.
Also neglect ${}^3D_2$, ${}^3D_3$.

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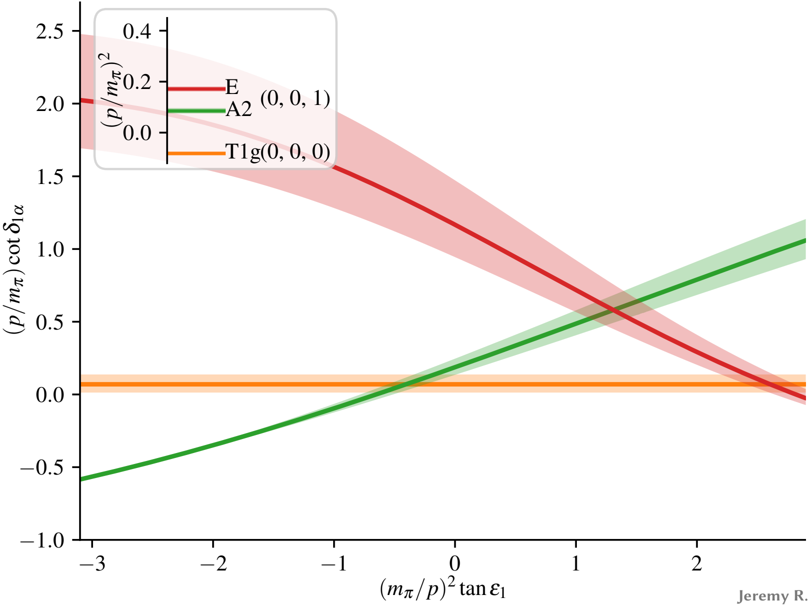

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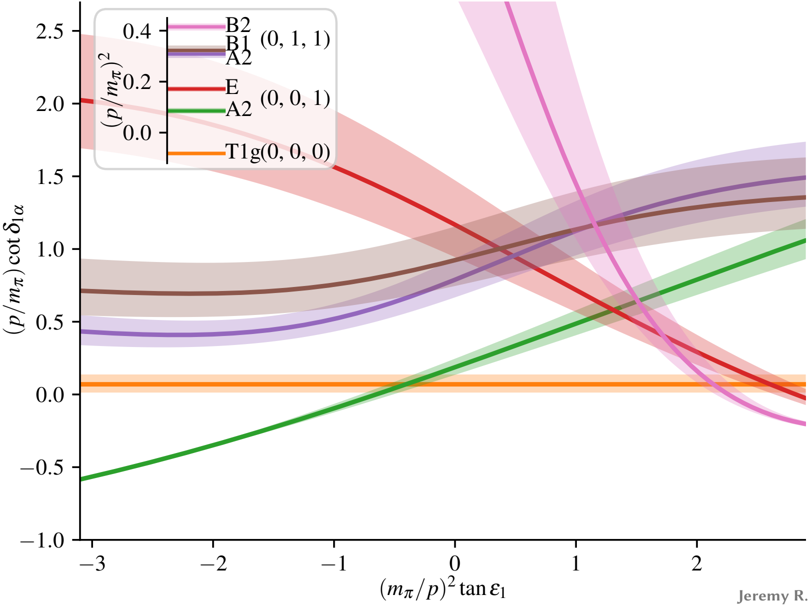

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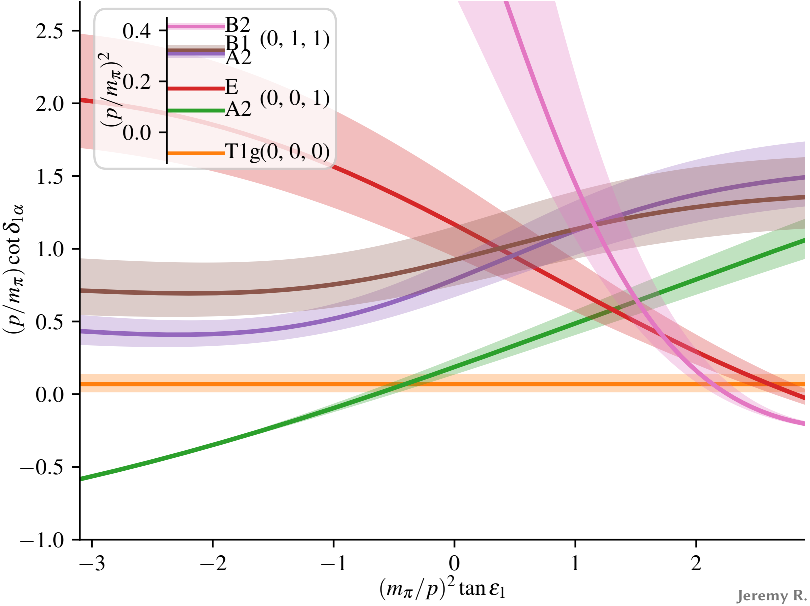

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Fit spectrum using

$$p\cot\delta_{1\alpha} = c_1 + c_2 p^2,$$
$$p^{-2}\tan\epsilon_1 = c_3.$$

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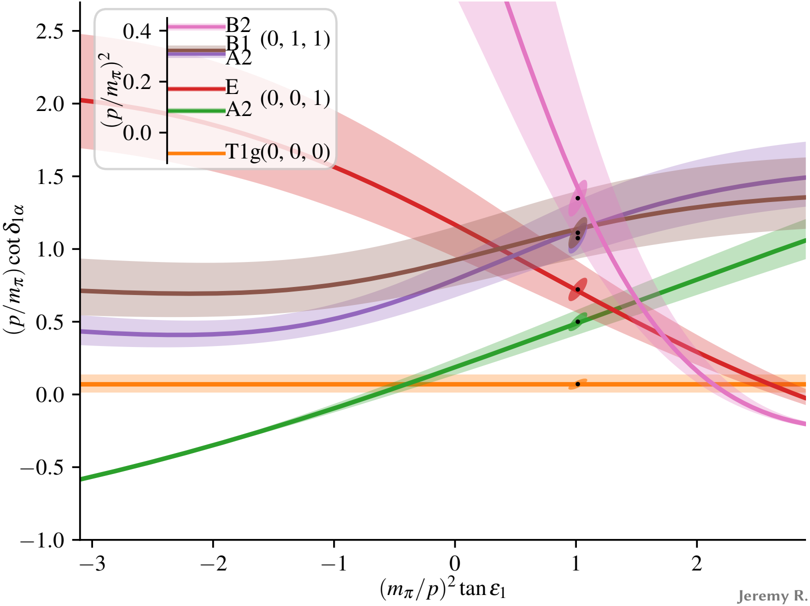
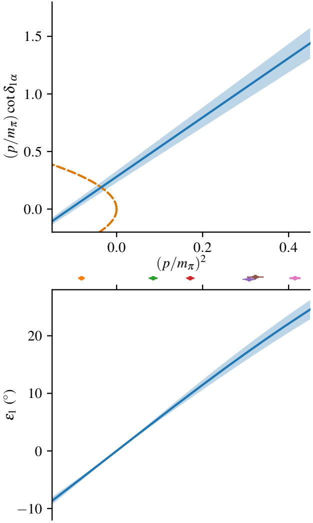
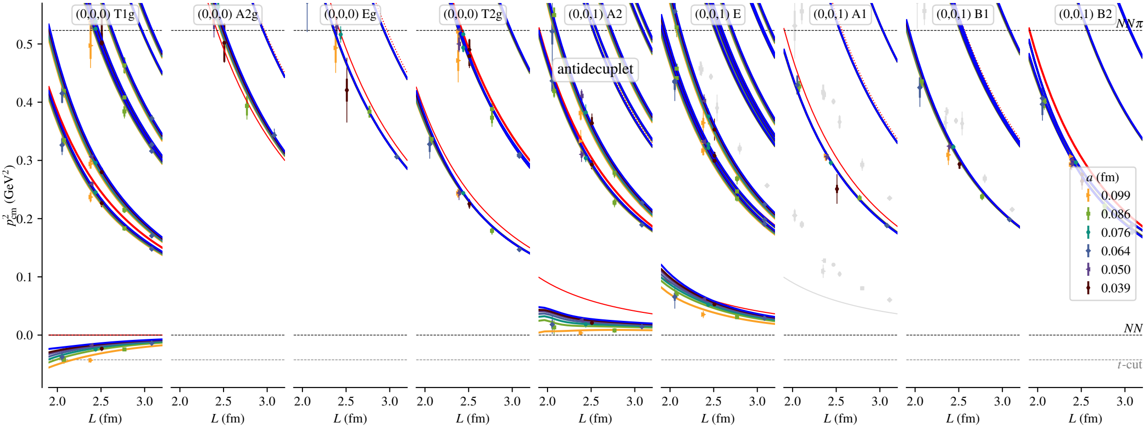




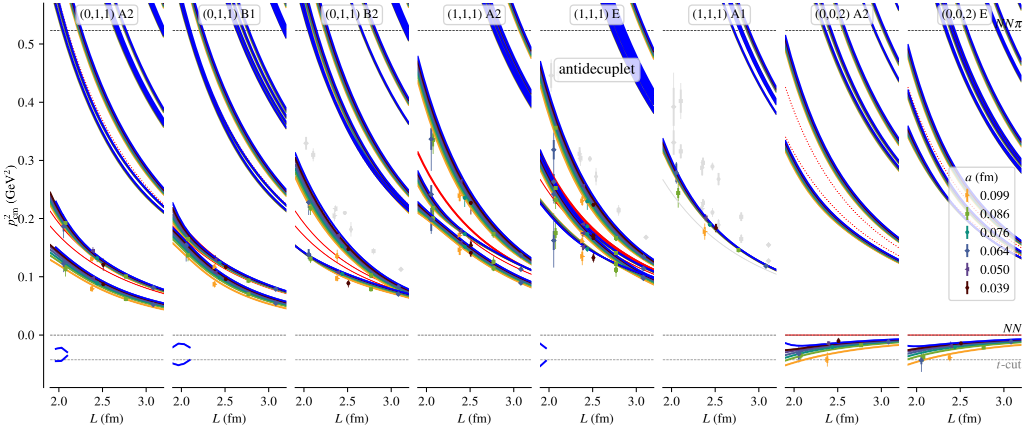
Deuteron is virtual state.

Antidecuplet ($NN\ I = 0$): spin 1 spectrum (1), example fit



Model $(\delta_{1\alpha}, \epsilon_1, \delta_{1\beta})$, $\delta_{^3D_2}$, $(\delta_{3\alpha}, \epsilon_3, \delta_{3\beta})$, $\delta_{^3G_4}$, $\delta_{^3G_5}$ using 35 parameters, including a^2-dependence up to $\delta_{3\alpha}$.

Antidecuplet (NN $I = 0$): spin 1 spectrum (2), example fit



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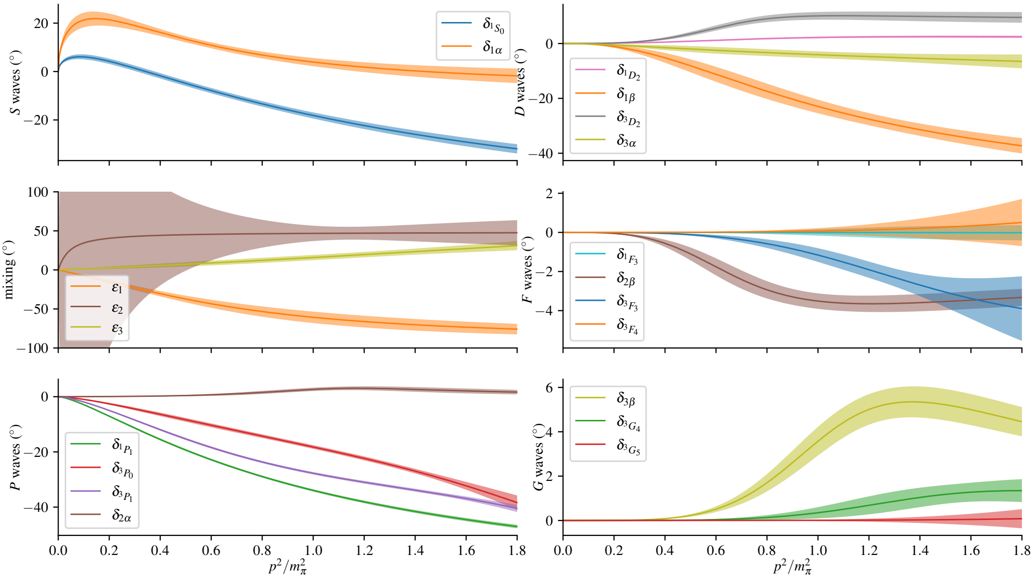
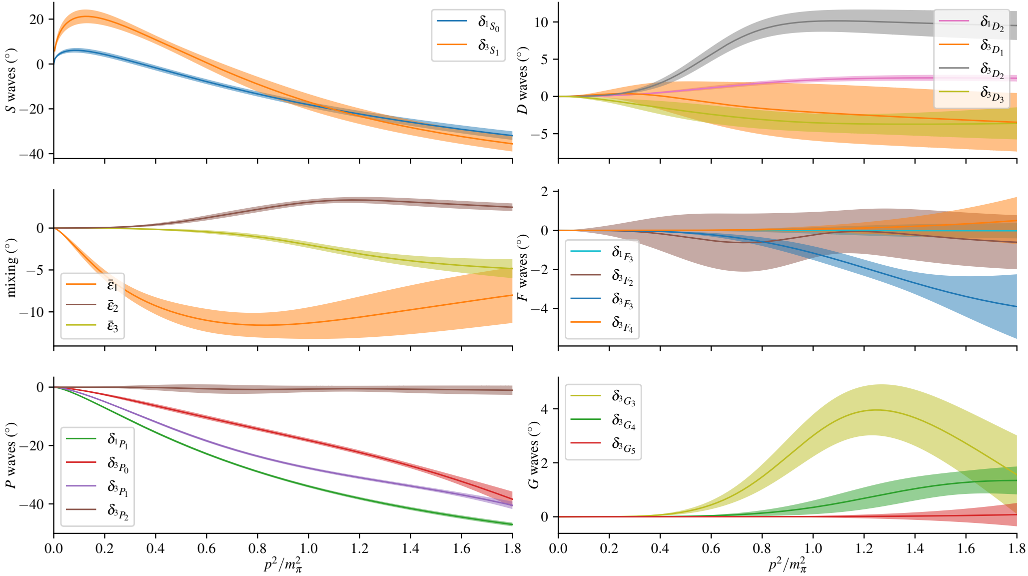
Example fitted phase shifts (Blatt-Biedenharn)



Example fitted phase shifts (Stapp)



Summary and outlook

Findings:

- Variational methods are essential for obtaining correct finite-volume spectrum.
- Contrary to earlier calculations, probably no NN bound state at heavy m_π.
- H dibaryon is bound by ~ 5 MeV at SU(3)-symmetric point.
- Discretization effects can be surprisingly important, particularly in S waves.

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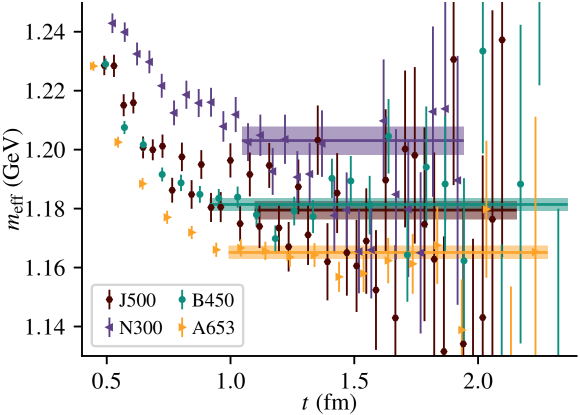
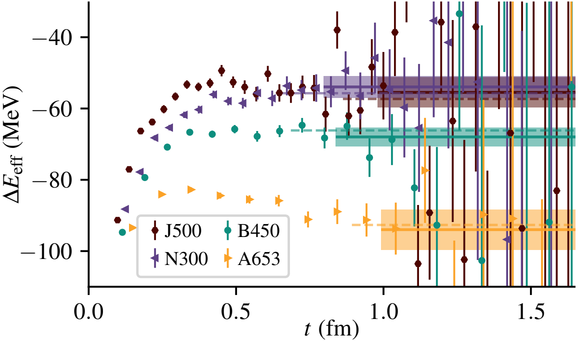
- Discretization effects can be surprisingly important, particularly in S waves.

Important next steps:

- Better understanding of lattice artifacts.
- Use EFT-inspired models for full set of NN data.
- Inclusion of left-hand cut in finite-volume quantization.
- More detailed cross-checks between collaborations and with HAL QCD.
- Lighter quark masses.

Plateau fits





Distillation

M. Peardon *et al.* (HadSpec), Phys. Rev. D **80**, 054506 (2009)

Define smeared quark fields using projector to lower-dimensional subspace:

$$\psi_{\text{sm}}(\boldsymbol{x}, t) = \sum_{\boldsymbol{x}'} P(\boldsymbol{x}, \boldsymbol{x}'; t)\psi(\boldsymbol{x}', t).$$

Standard choice: Laplacian-Heaviside (LapH) smearing. Use N lowest eigenmodes $v_n^{(t)}$ of smeared 3d gauge-covariant Laplacian $\Delta(t)$. Typically scale $N \propto L^3$ to keep smearing radius fixed.

$$P(\boldsymbol{x}, \boldsymbol{x}'; t) = I_{\text{spin}} \otimes \sum_{n=1}^{N} v_n^{(t)}(\boldsymbol{x}) v_n^{(t)\dagger}(\boldsymbol{x}').$$

Since $N \ll 3(L/a)^3$ it is feasible to compute and save full timeslice-to-all or all-to-all propagator within this subspace: this is the *perambulator*

$$\tau_{n'n}(t', t) \equiv \sum_{\boldsymbol{x}', \boldsymbol{x}} v_{n'}^{(t')\dagger}(\boldsymbol{x}') D^{-1}(\boldsymbol{x}', t'; \boldsymbol{x}, t) v_n^{(t)}(\boldsymbol{x}).$$

Timeslice-to-all requires $4N$ propagator solves.

Hadrons with distillation

Computing hadron correlation functions also requires mode doublets and triplets,

$$\Phi_{n'n}(t,\boldsymbol{p}) \equiv \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} v_{n'}^{(t)\dagger}(\boldsymbol{x}) v_n^{(t)}(\boldsymbol{x}), \quad T_{n_1n_2n_3}(t,\boldsymbol{p}) \equiv \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \epsilon_{abc} v_{n_1a}^{(t)}(\boldsymbol{x}) v_{n_2b}^{(t)}(\boldsymbol{x}) v_{n_3c}^{(t)}(\boldsymbol{x})$$

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$\to$ contraction cost for any number of mesons is $O(N^3)$.

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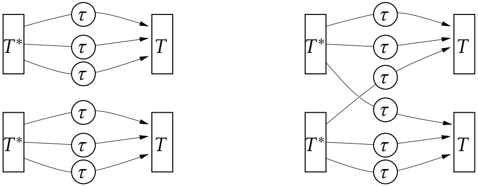
Baryons require rank-3 tensor T. For up to 2 baryons get $O(N^4)$ cost. [3 baryons is $O(N^6)$.]



Baryon-baryon: two classes of Wick contractions.

$N^4 \propto L^{12}$ scaling is a problem! Try to keep N small but eventually need alternative strategies.