Evgeny Epelbaum, Ruhr University Bochum

RUB

Chiral EFT for nuclear interactions using Gradient Flow

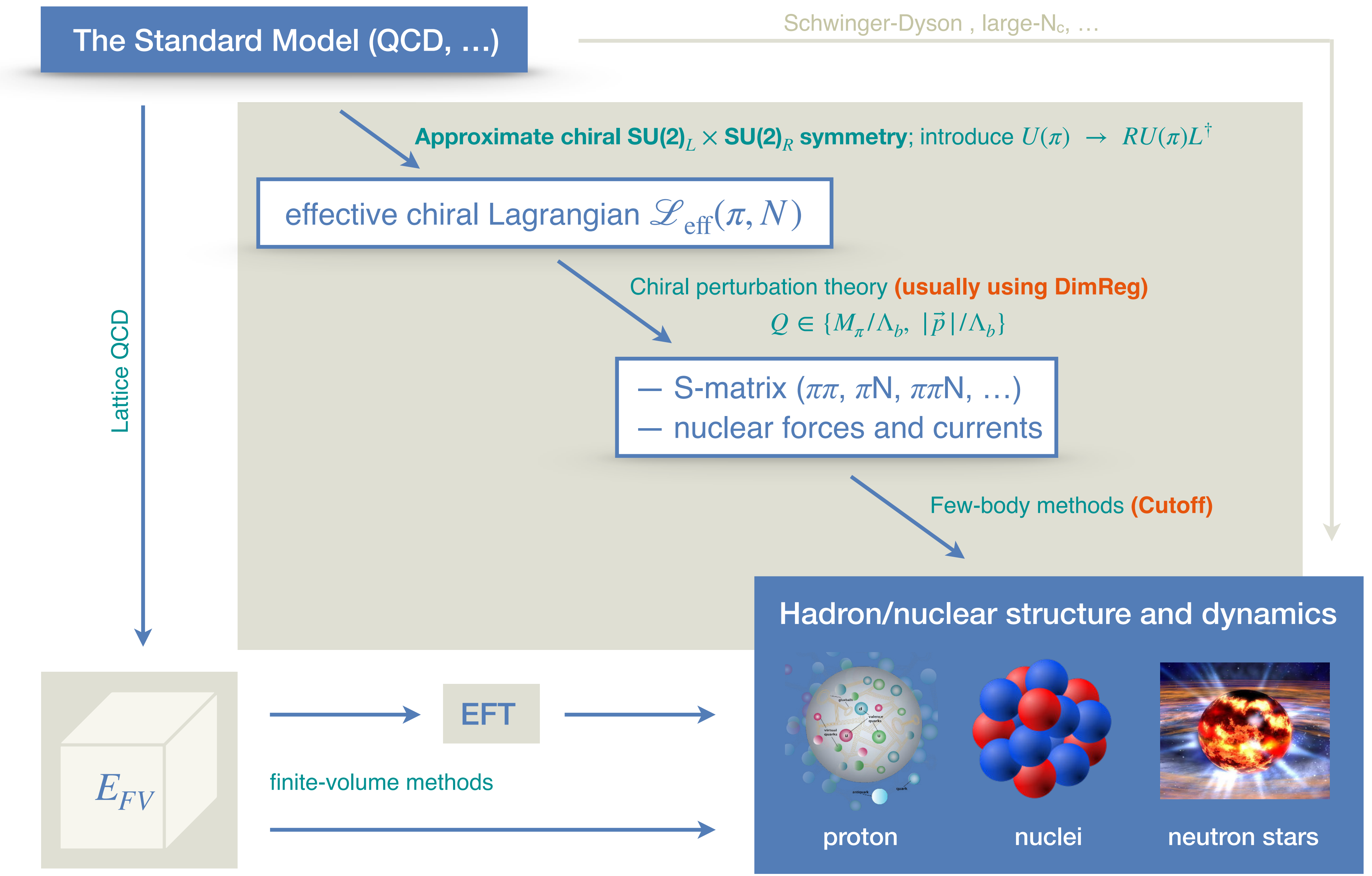
based on work done with Hermann Krebs, PRC 110 (2024) 044003; 044004

…opens an avenue for accurate χEFT calculations beyond the 2N system

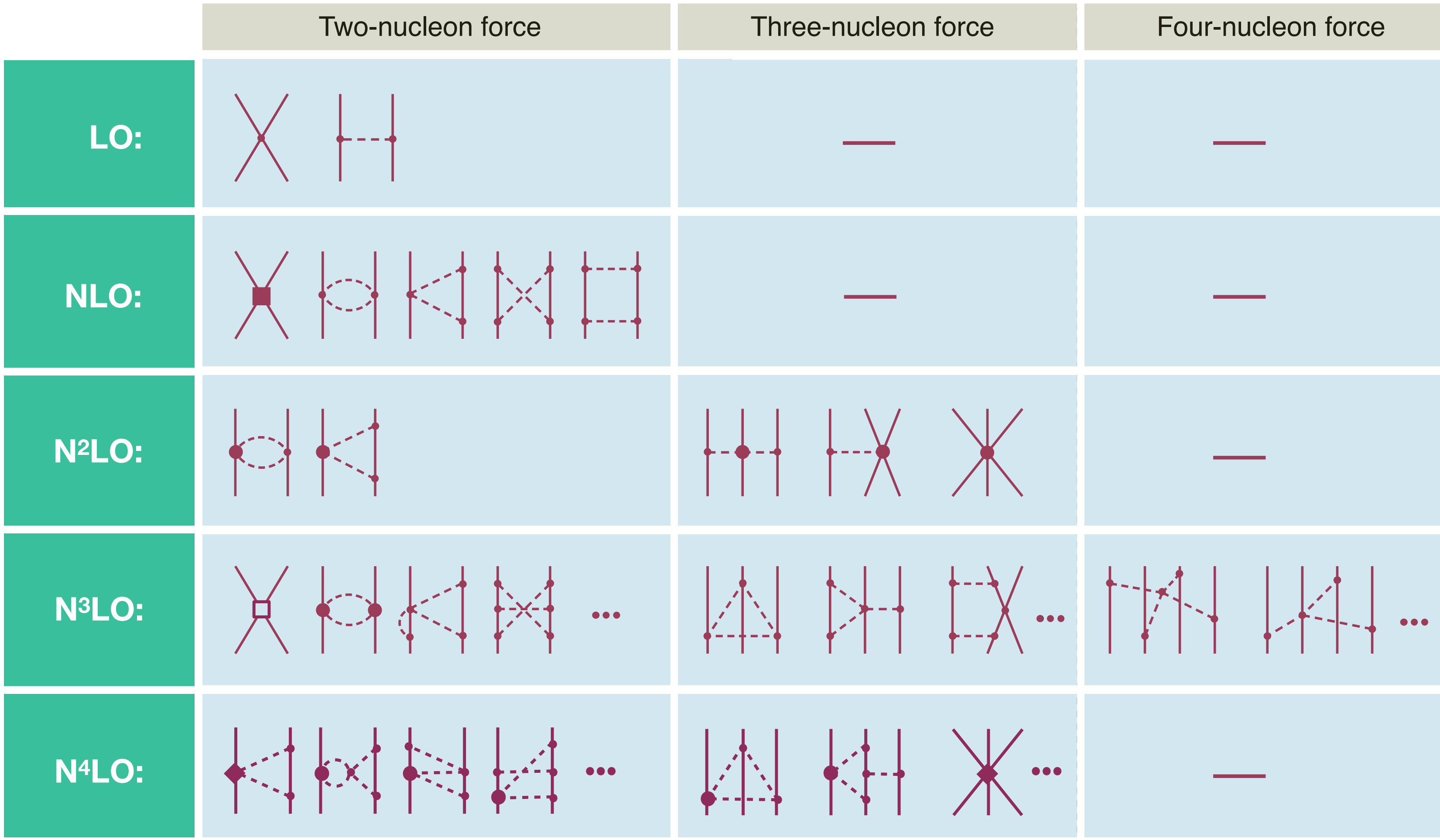
- Introduction & state-of-the-art
- The need for a symmetry-preserving regulator
- Chiral EFT using gradient flow
- Summary & outlook

erc European Research Council
Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
Ministerium für Kultur und Wissenschaft des Landes Nordrhein-Westfalen
ERATO Exploratory Research for Advanced Technology

From QCD to nuclear physics



Chiral expansion of nuclear forces



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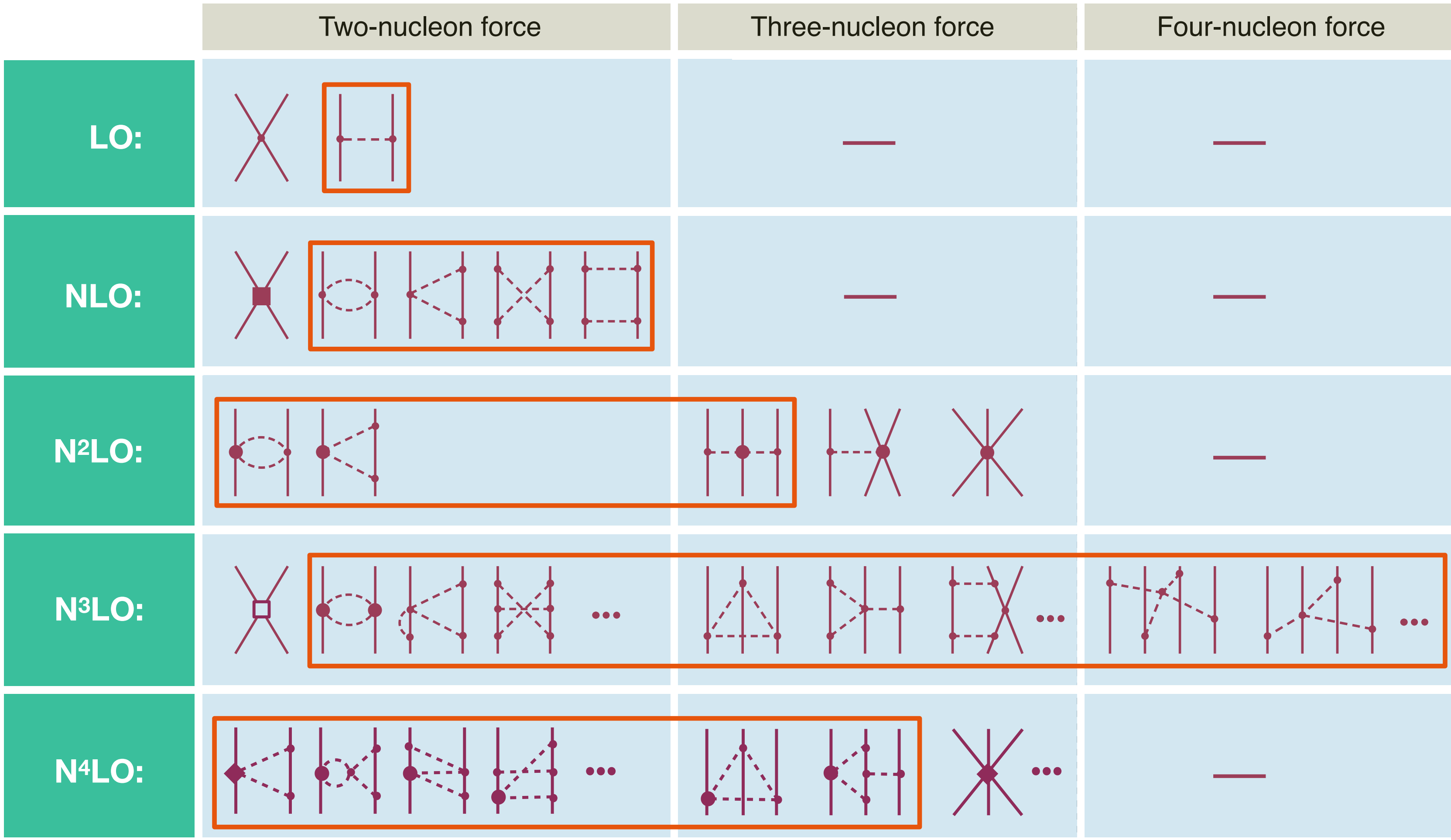
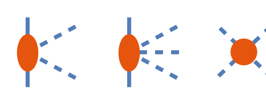


Chiral dynamics: Long-range interactions are predicted in terms of on-shell amplitudes 

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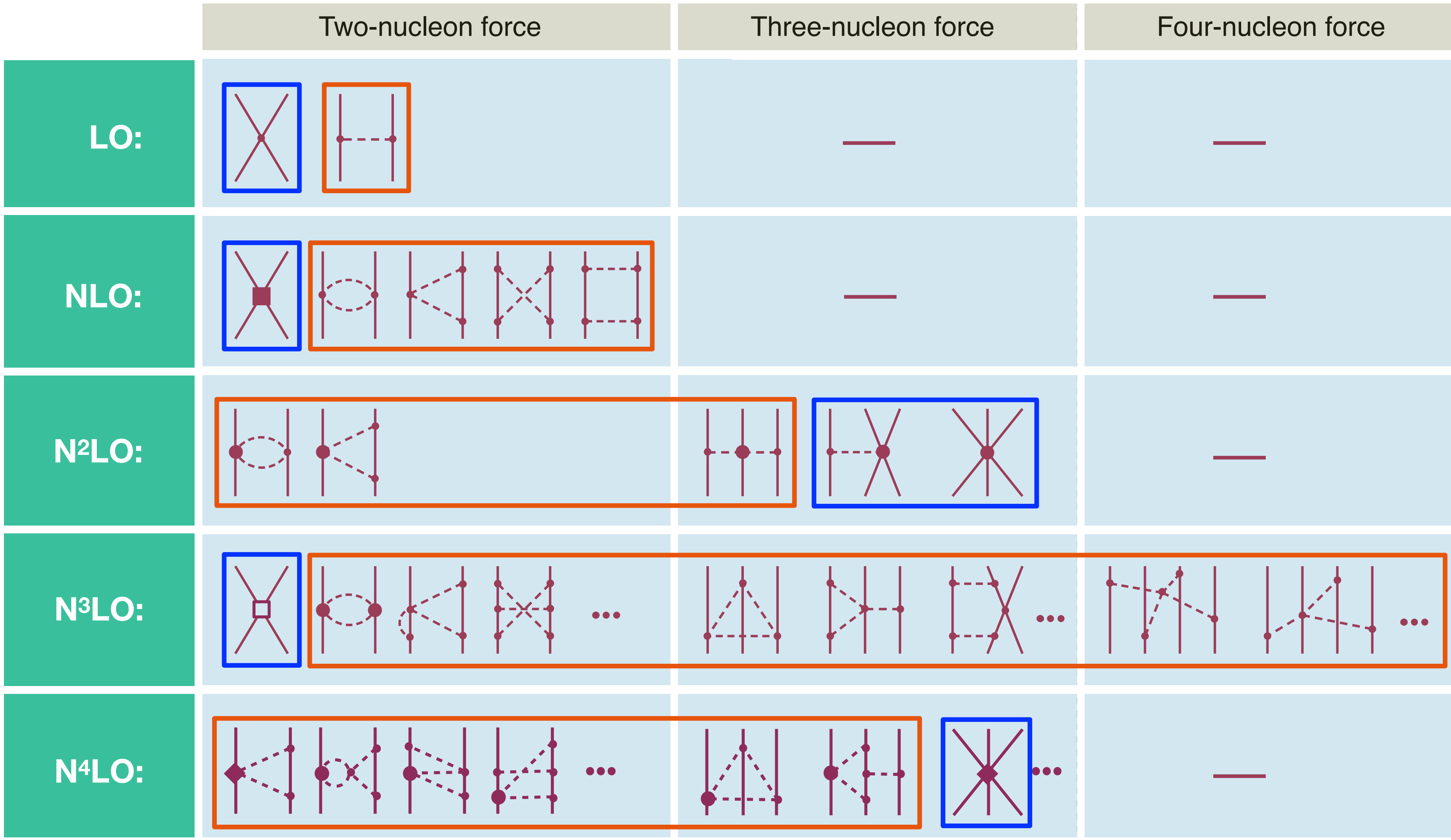
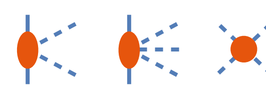
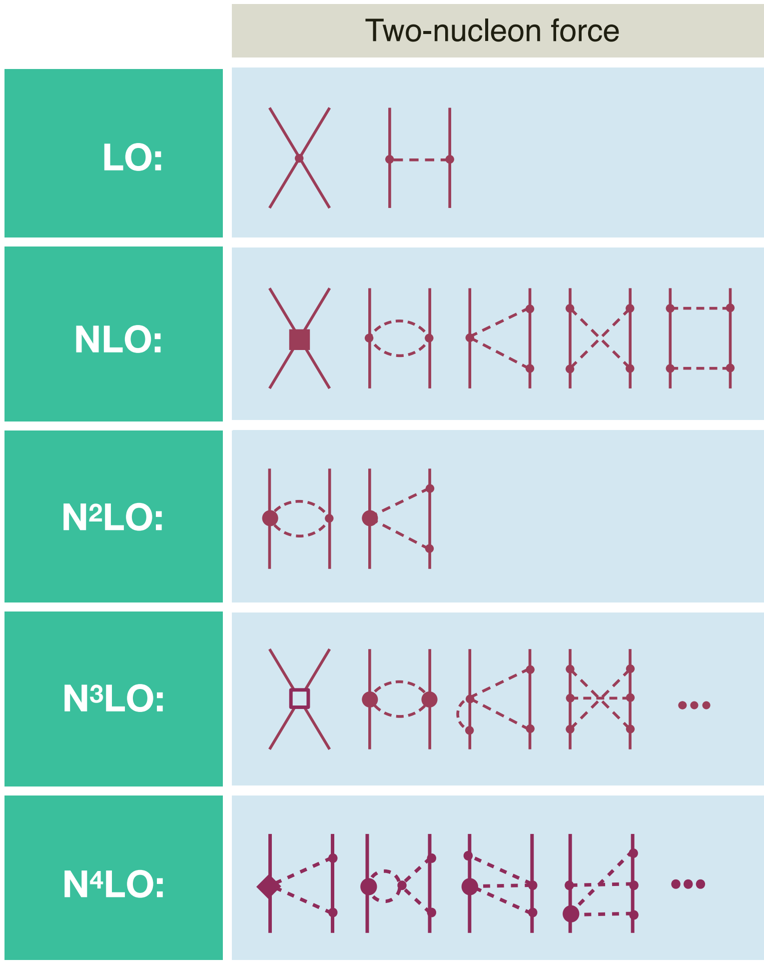


Chiral dynamics: Long-range interactions are predicted in terms of on-shell amplitudes 

Short-range few-N interactions are tuned to experimental data

Chiral expansion of nuclear forces



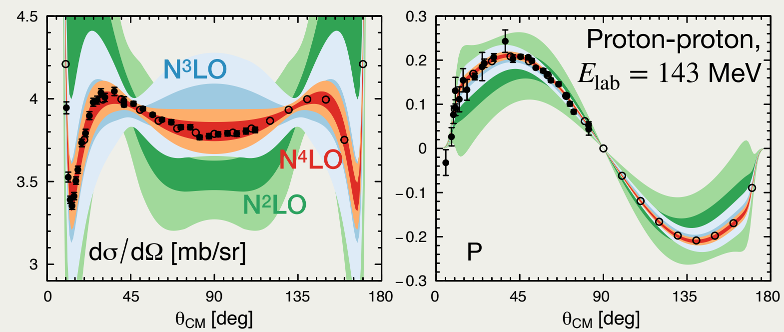
χEFT as a precision tool in the 2N sector

— N4LO+: currently most accurate and precise NN interactions on the market

— clear evidence of the TPEP from NN data

— almost no residual cutoff dependence

— Bayesian truncation-error estimation



— Precision calculations for 2 nucleons:

$g_{\pi NN} = 13.24 \pm 0.04$ Reinert, Krebs, EE '20

$r_{\text{str}}^{^2\text{H}} = 1.9729^{+0.0015}_{-0.0012}$ fm Filin et al., '21

Semi-local regularization in momentum space Reinert, Krebs, EE, EPJA 54 (2018) 86; PRL 126 (2021) 092501

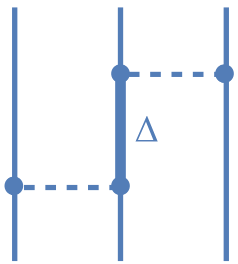
$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^2 + M_\pi^2}\, e^{-\frac{\vec{q}^2 + M_\pi^2}{\Lambda^2}} + \text{subtraction,} \qquad V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\,\mu\, \frac{\rho(\mu)}{\vec{q}^2 + \mu^2}\, e^{-\frac{\vec{q}^2 + \mu^2}{2\Lambda^2}} + \text{subtractions}$$

+ nonlocal (Gaussian) cutoff for contacts

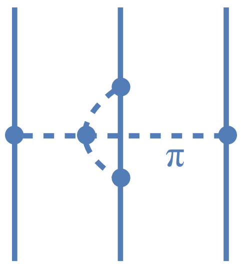
3-body force: A frontier in nuclear & atomic physics

Endo, EE, Naidon, Nishida, Sekiguchi, Takahashi, e-Print: 2405.09807 [nucl-th]

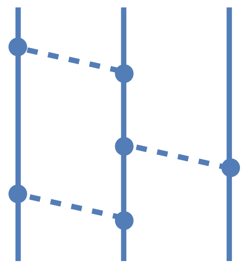
- Three-nucleon forces (3NF) are small **<u>but important</u>** corrections to the dominant NN forces
- 3NF mechanisms:



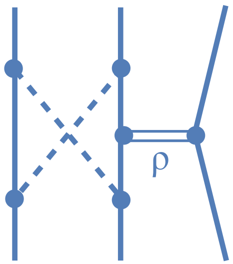
intermediate Δ-excitation
Fujita, Miyazawa '57



multi-pion interactions



off-shell behavior of the V_{NN}
$V_{\text{ring}} = \mathscr{A}_{3\pi} - V_\pi G_0 V_\pi G_0 V_\pi$



short-range

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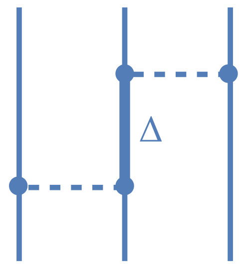

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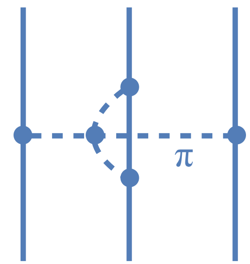

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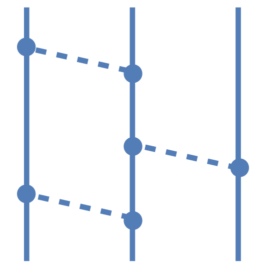

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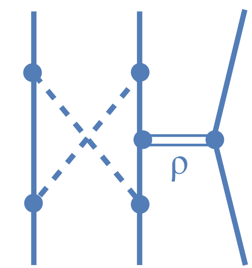

short-range

- **Difficult to model: None of the existing 3NFs allow to describe of 3N data…**
  - — scarcer database compared to the NN sector
  - — high computational cost of solving the Faddeev equation
  - — complicated structure:

$$V_{3N}^{\text{non-local}} = \sum_{i=1}^{320} O_i \times f_i = \sum_{i=1}^{68} O_i \times \tilde{f}_i + \text{perm.} \xrightarrow{\text{antisymm.}} \sum_{i=1}^{14} O_i \times \tilde{\tilde{f}}_i + \text{perm.}$$

Topolnicki ’17

spin-momentum-isospin (O_i); functions of 5 momenta ($\tilde{\tilde{f}}_i$)

Krebs, EE, in preparation

⇒ Guidance from theory indispensable — an opportunity for χEFT!

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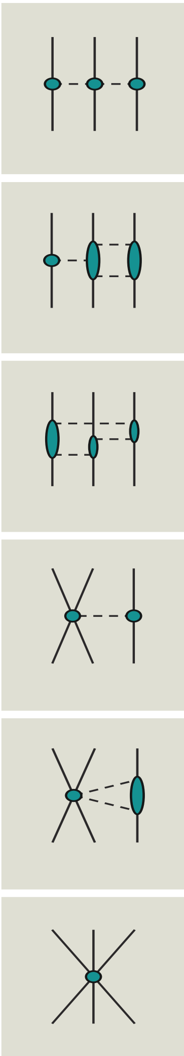

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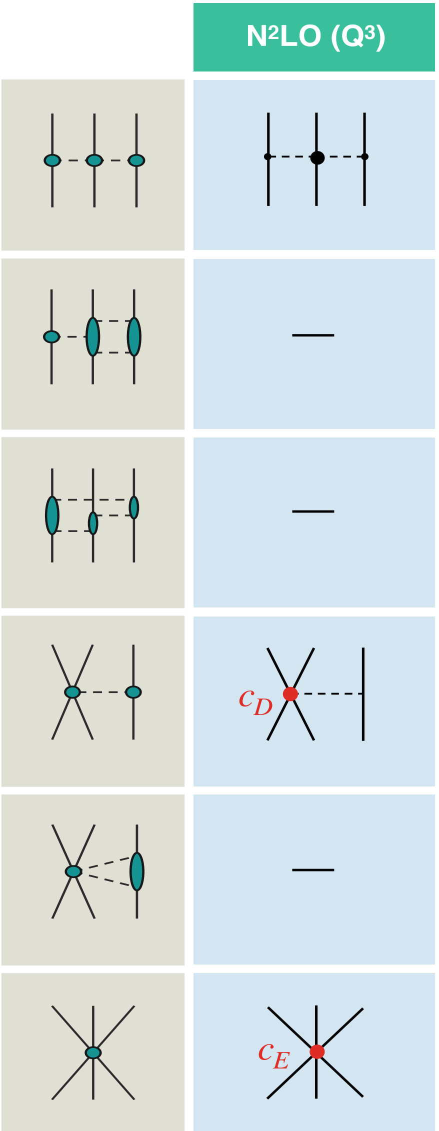
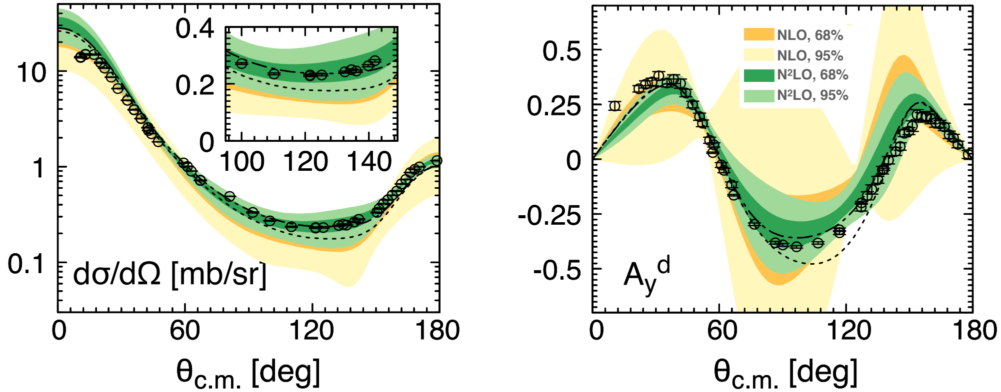
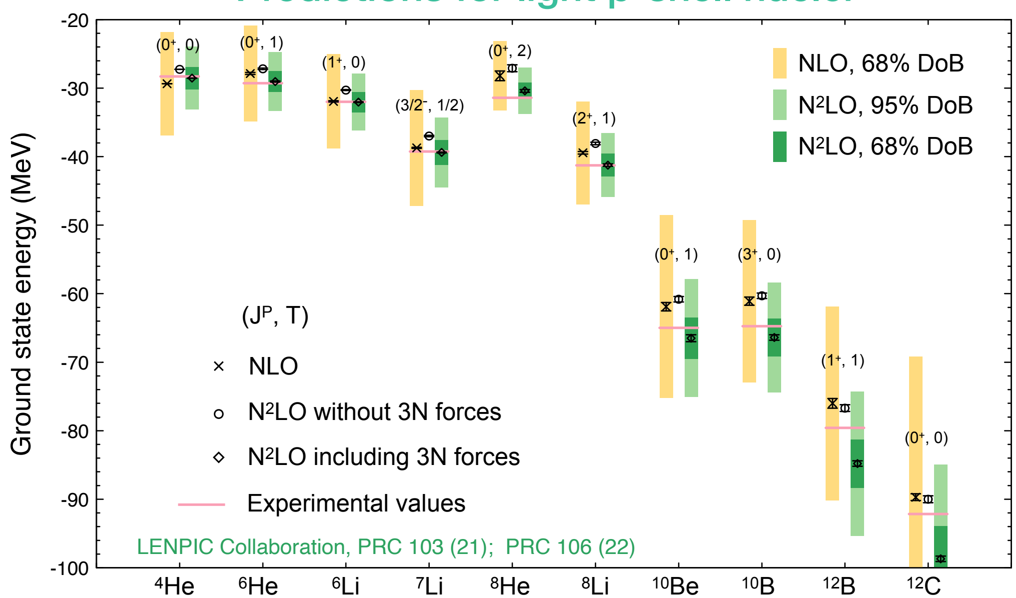


Elastic Nd scattering at 135 MeV



Predictions for light p-shell nuclei



LENPIC

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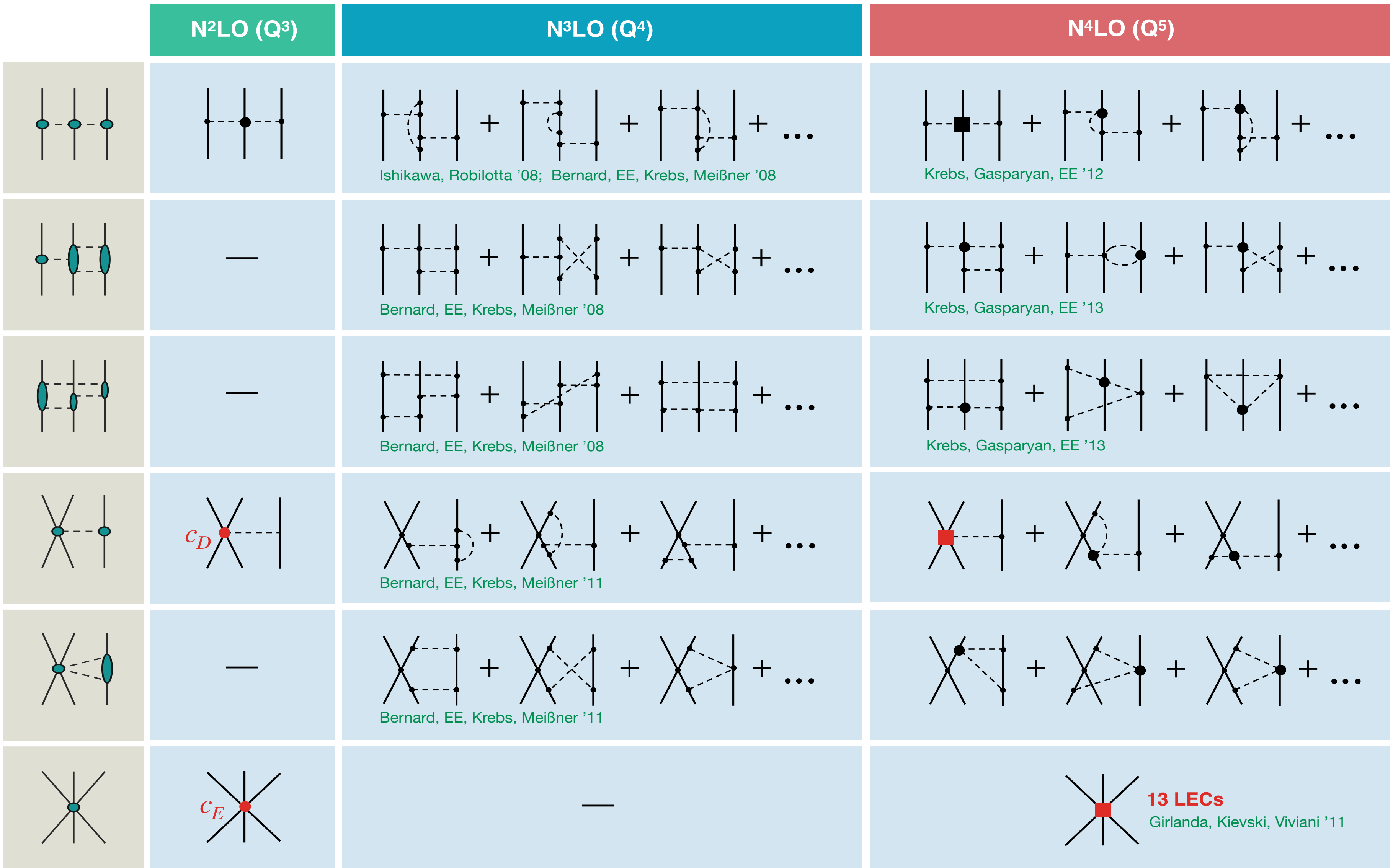

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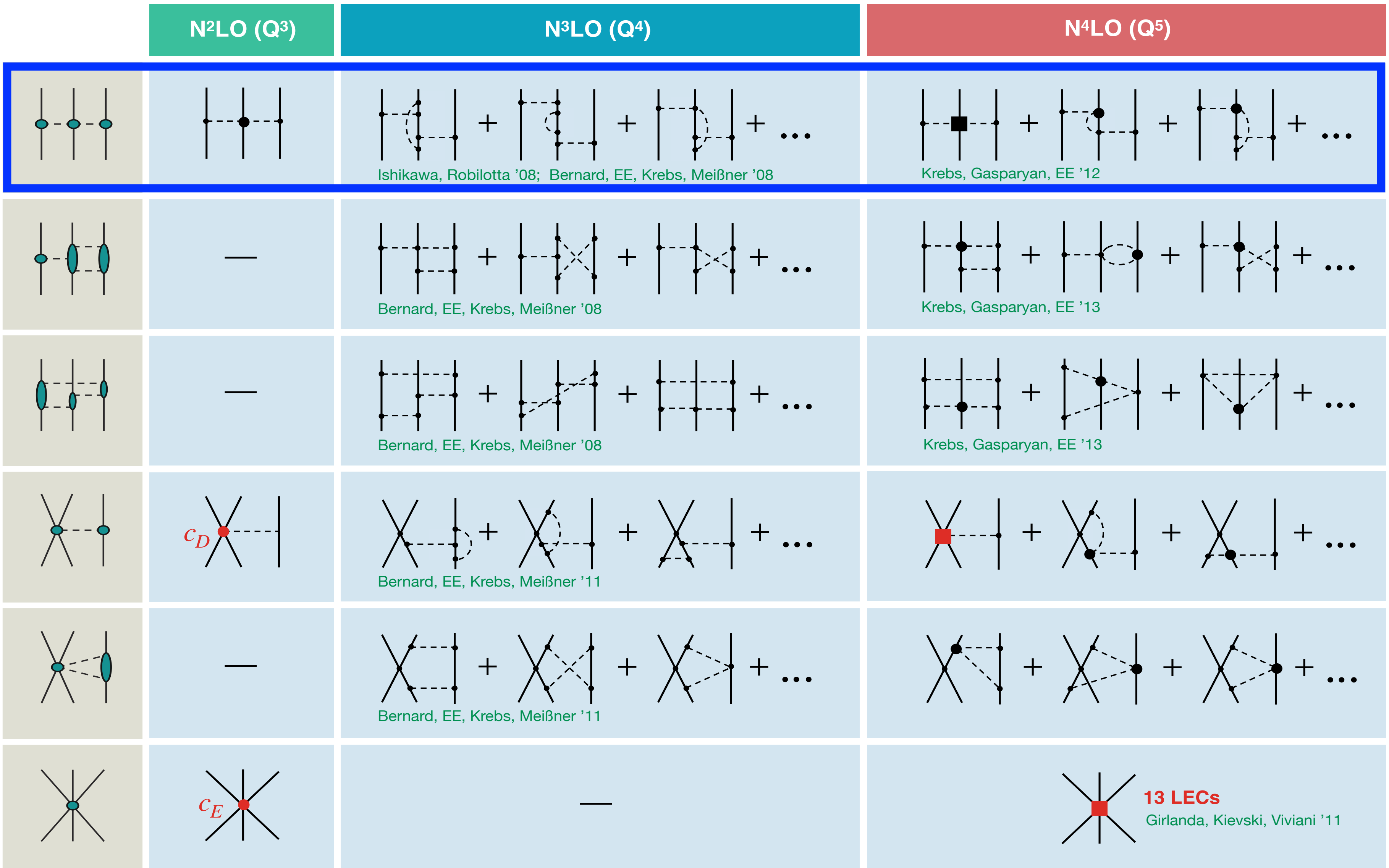

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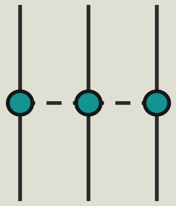
Example: 2π-exchange 3NF



$$V_{3\mathrm{N}} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{(q_1^2 + M_\pi^2)\,(q_3^2 + M_\pi^2)} \Big[\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big]$$

$$+ \quad \text{short-range terms} \; + \; \text{permutations}$$

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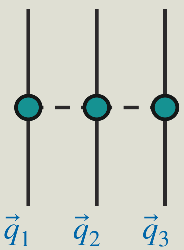

$$V_{3N} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{(q_1^2 + M_\pi^2)\,(q_3^2 + M_\pi^2)} \Big[\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big]$$

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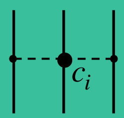
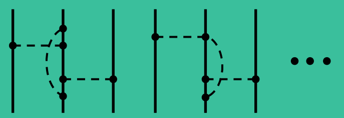
N²LO (Q³)



$$\mathcal{A}^{(3)} = \frac{g_A^2}{8F_\pi^4}\Big[(2c_3 - 4c_1)M_\pi^2 + c_3 q_2^2\Big], \qquad \mathcal{B}^{(3)} = \frac{g_A^2 c_4}{8F_\pi^4}$$

N³LO (Q⁴)

Bernard, EE, Krebs, Meißner ’08



$$\mathcal{A}^{(4)} = \frac{g_A^4}{256\pi F_\pi^6}\Big[\left(4g_A^2+1\right)M_\pi^3 + 2\left(g_A^2+1\right)M_\pi q_2^2 + A(q_2)\left(2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4\right)\Big]$$

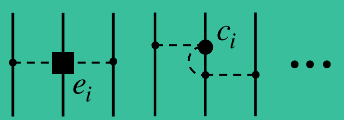
$$A(q_2) = \frac{1}{2q_2}\arctan\frac{q_2}{2M_\pi}$$

$$\mathcal{B}^{(4)} = -\frac{g_A^4}{256\pi F_\pi^6}\Big[A(q_2)\left(4M_\pi^2 + q_2^2\right) + (2g_A^2+1)M_\pi\Big]$$

calculated using DimReg

N⁴LO (Q⁵)

Krebs, Gasparyan, EE ’12



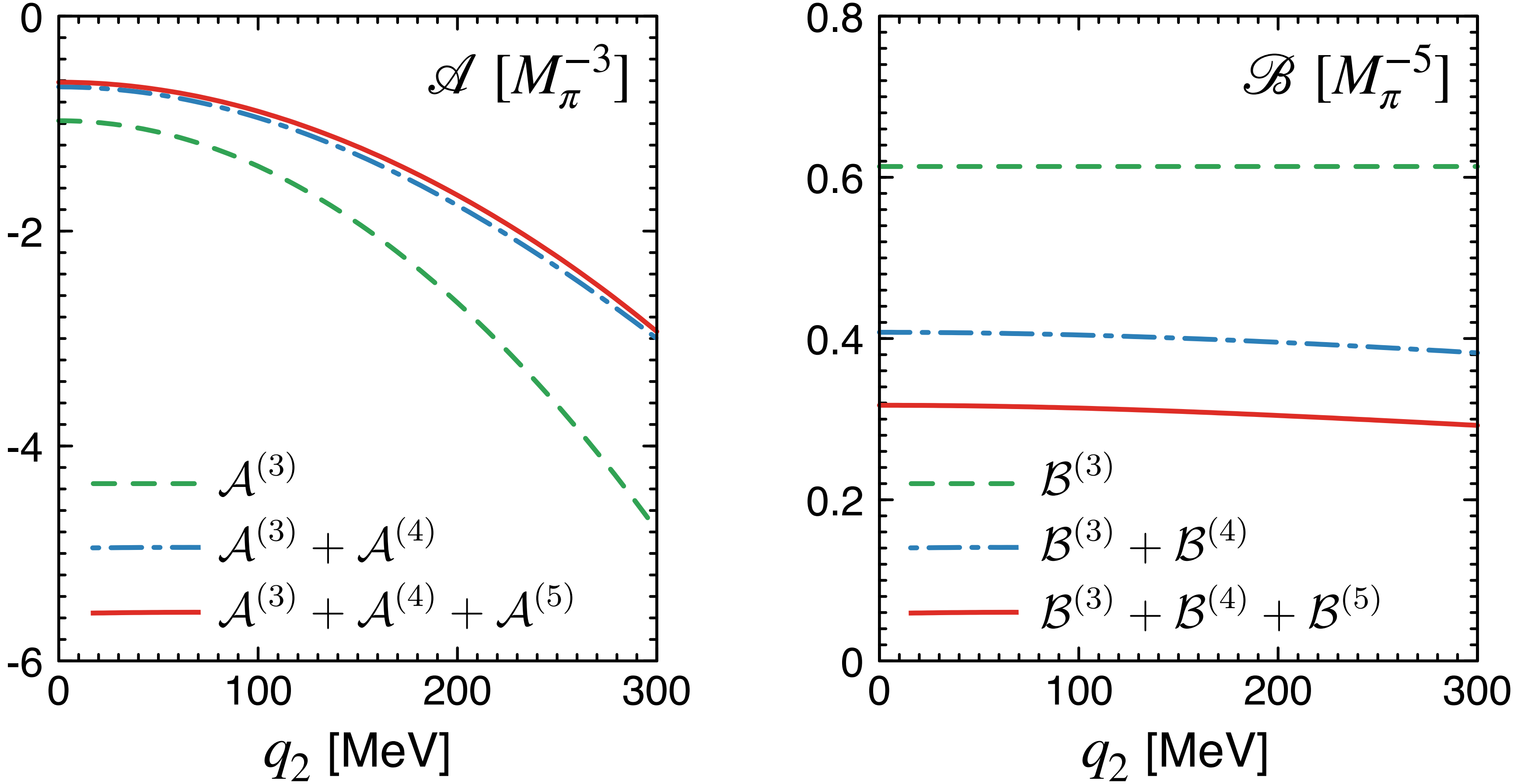
$$\mathcal{A}^{(5)} = \frac{g_A^2\left(M_\pi^2 + 2q_2^2\right)}{4608\pi^2 F_\pi^6}\Big\{\big[6c_1 - 2c_2 - 3c_3 - 2(6c_1 - c_2 - 3c_3)L(q_2)\big]12M_\pi^2 - q_2^2\big[5c_2 + 18c_3 - 6L(q_2)(c_2 + 6c_3)\big]\Big\} + \frac{g_A^2 \bar{e}_{14}}{2F_\pi^4}\left(2M_\pi^2 + q_2^2\right)^2$$

$$L(q_2) = \frac{\sqrt{q_2^2 + 4M_\pi^2}}{q_2}\log\frac{\sqrt{q_2^2+4M_\pi^2}+q_2}{2M_\pi}$$

$$\mathcal{B}^{(5)} = \frac{g_A^2 \bar{e}_{17}}{2F_\pi^4}\left(2M_\pi^2 + q_2^2\right) - \frac{g_A^2 c_4}{2304\pi^2 F_\pi^6}\Big\{q_2^2\big[5 - 6L(q_2)\big] + 12M_\pi^2\big[2 + 9g_A^2 - 2L(q_2)\big]\Big\}$$

calculated using DimReg

Example: 2π-exchange 3NF



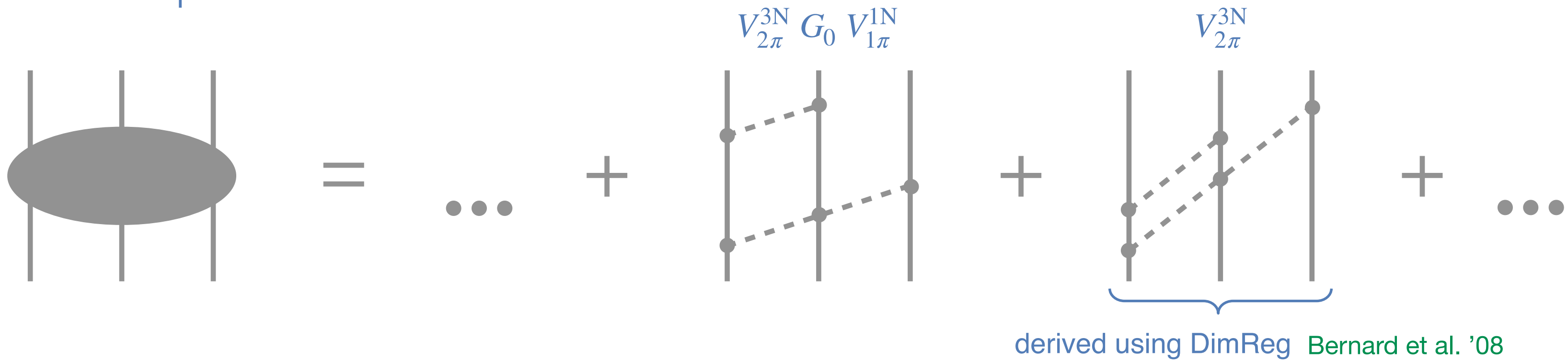
— all LECs c_i and $\bar{e}_i$ are known from the Roy-Steiner-equation analysis of the πN system

— the results are only meaningful (converged) at small momenta $\Rightarrow$ cutoff needed

Problem: Mixing DimReg with Cutoff violates chiral symmetry

Essence of the problem

Faddeev equation:



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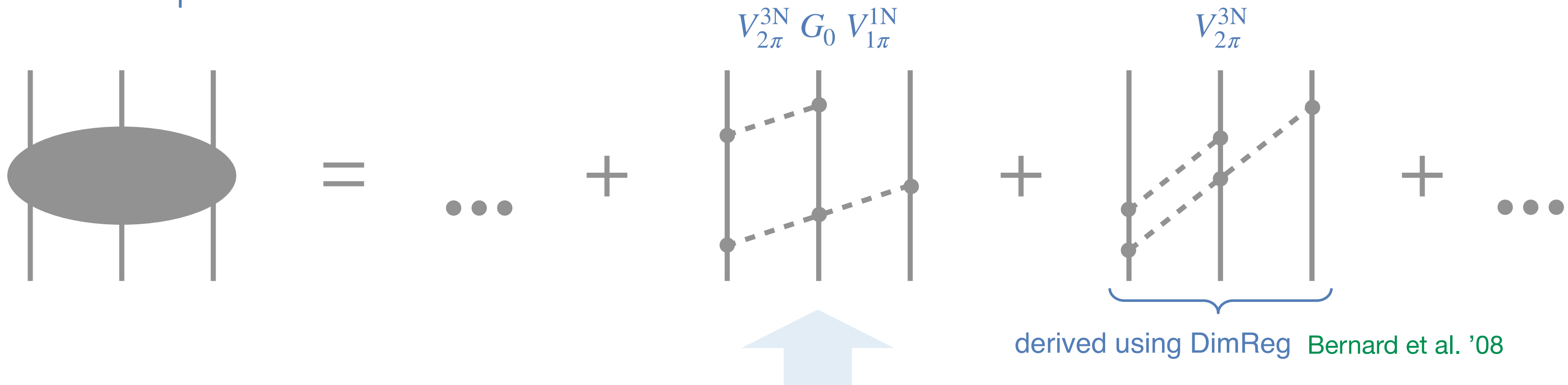

$$-\Lambda \frac{g_A^4}{96\sqrt{2\pi^3}F_\pi^6}\Big[\underbrace{\tau_1\cdot\tau_3\,(\vec{q}_3\cdot\vec{\sigma}_1)}_{\text{absorbable into } c_D} - \underbrace{\frac{4}{3}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)(\vec{q}_2\cdot\vec{\sigma}_3)}_{\text{violates chiral symmetry}}\Big]\frac{\vec{q}_3\cdot\vec{\sigma}_3}{q_3^3+M_\pi^2}+\ldots$$

The problematic divergence would cancel if $V_{2\pi}^{3N}$ were calculated using Cutoff EE, Krebs, Reinert '19

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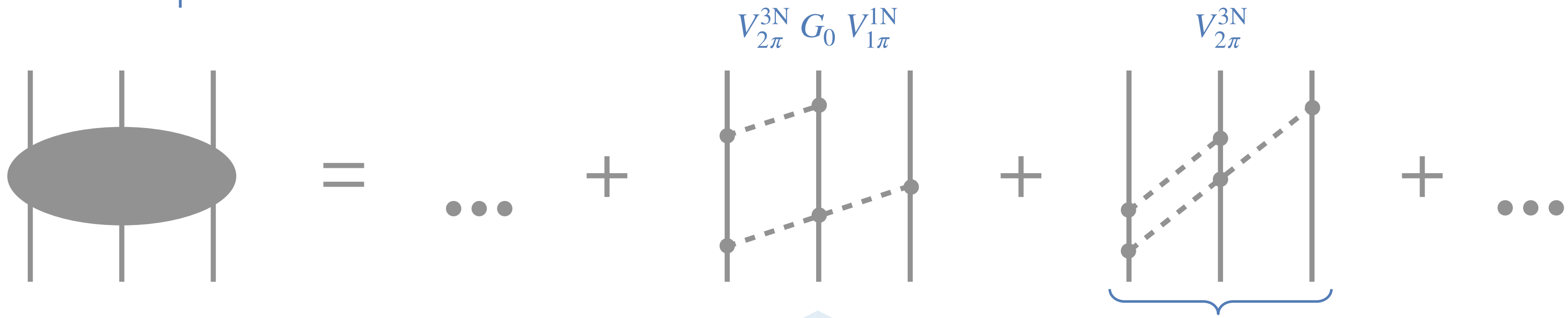

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absorbable into c_D: X--| violates chiral symmetry

The problematic divergence would cancel if $V_{2\pi}^{3N}$ were calculated using Cutoff EE, Krebs, Reinert '19

$\Rightarrow$ loop contributions to the 3NF, 4NF and MECs must be re-derived using symmetry preserving cutoff (2NF ok at fixed M_π)

Gradient flow for chiral interactions

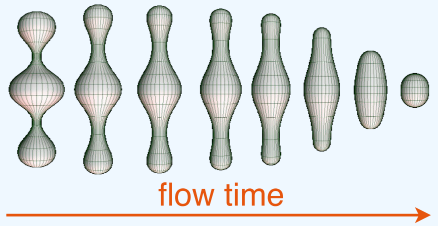
unpublished work by DBK

- Gradient flow as regulator
- Nucleons on the brane: regulating interactions in an extra dimension

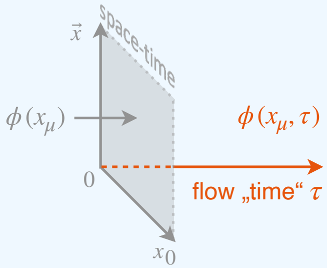
D. B. Kaplan ~ HHIQCD2015 ~ 13/3/15

Gradient flow

Gradient flows: methods for smoothing manifolds
(e.g., Ricci flow used in the proof of the Poincaré conjecture)



Gradient flow as a regulator in field theory



Flow equation: $$\frac{\partial}{\partial \tau}\phi(x,\tau) = -\left.\frac{\delta S[\phi]}{\delta \phi(x)}\right|_{\phi(x)\to\phi(x,\tau)}$$

subject to the boundary condition $\phi(x,0) = \phi(x)$

Gradient flow

Gradient flows: methods for smoothing manifolds
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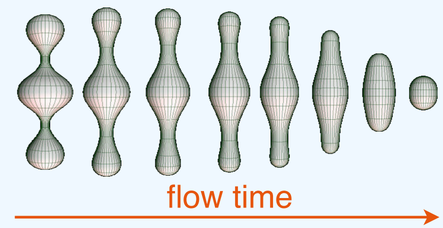

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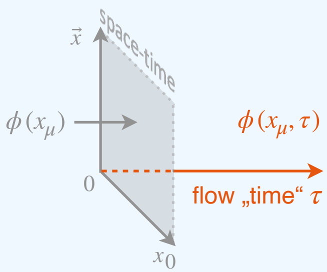

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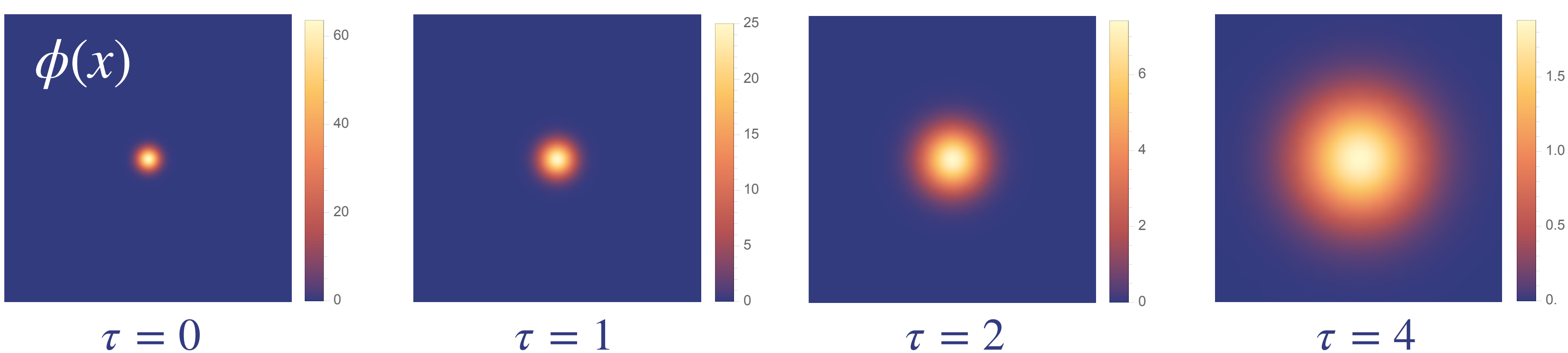
Free scalar field:

$$G(x,\tau) = \frac{\theta(\tau)}{16\pi^2\tau^2} e^{-\frac{x^2+4M^2\tau^2}{4\tau}}$$

$$\left[\partial_\tau - (\partial^x_\mu \partial^x_\mu - M^2)\right]\phi(x,\tau) = 0 \quad \Rightarrow \quad \phi(x,\tau) = \int d^4y\, \overbrace{\underbrace{G(x-y,\tau)}_{\text{heat kernel}}}\phi(y)$$



Gradient flow

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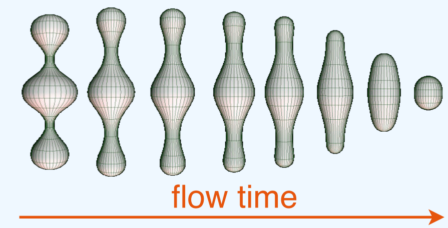
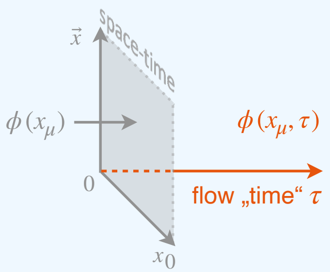


Gradient flow as a regulator in field theory



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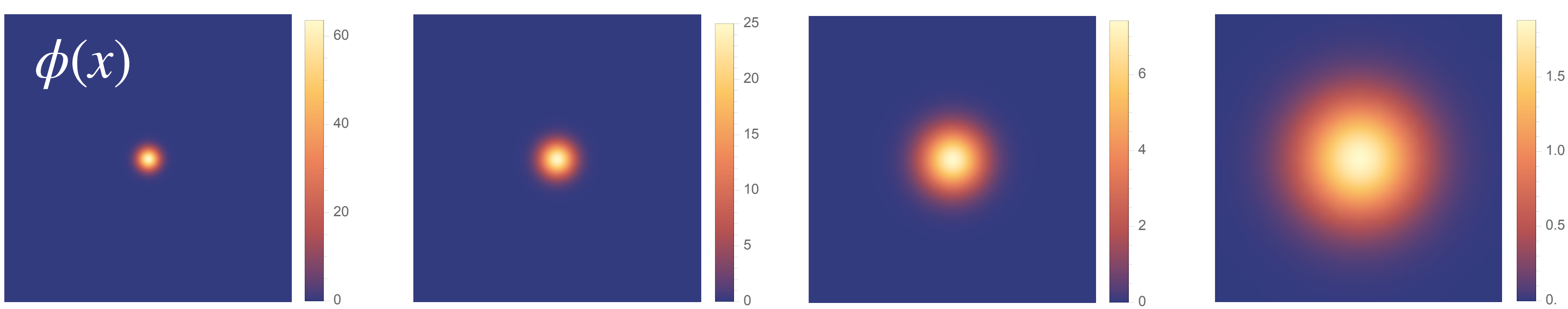

$\tau = 0$ $\tau = 1$ $\tau = 2$ $\tau = 4$

Gradient flow

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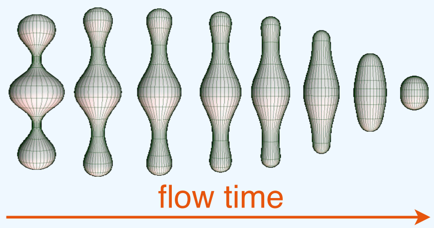

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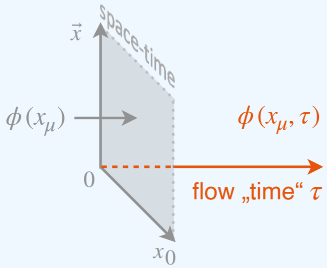


Flow equation: $$\frac{\partial}{\partial\tau}\phi(x,\tau) = -\frac{\delta S[\phi]}{\delta\phi(x)}\bigg|_{\phi(x)\to\phi(x,\tau)}$$

subject to the boundary condition $\phi(x,0) = \phi(x)$

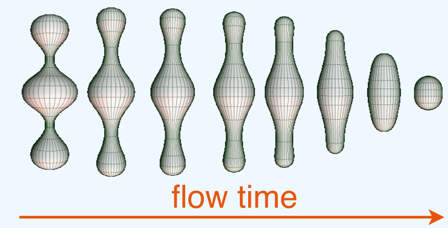
Free scalar field:

$$\left[\partial_\tau - (\partial_\mu^x\partial_\mu^x - M^2)\right]\phi(x,\tau) = 0 \quad\Rightarrow\quad \phi(x,\tau) = \int d^4y\,\overbrace{\underbrace{G(x-y,\tau)}_{\text{heat kernel}}}^{G(x,\tau)=\frac{\theta(\tau)}{16\pi^2\tau^2}e^{-\frac{x^2+4M^2\tau^2}{4\tau}}}\phi(y) \quad\Rightarrow\quad \tilde{\phi}(q,\tau) = e^{-\tau(q^2+M^2)}\tilde{\phi}(q)$$

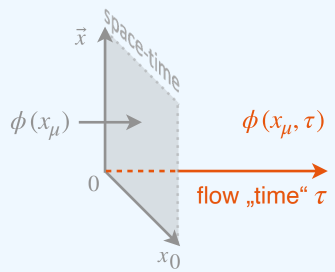
YM gradient flow Narayanan, Neuberger ’06, Lüscher, Weisz ’11: $\partial_\tau A_\mu(x,\tau) = D_\nu G_{\nu\mu}(x,\tau)$ ← extensively used in LQCD

Gradient flow

Gradient flows: methods for smoothing manifolds
(e.g., Ricci flow used in the proof of the Poincaré conjecture)



Gradient flow as a regulator in field theory



Flow equation: $\dfrac{\partial}{\partial \tau}\phi(x,\tau) = -\dfrac{\delta S[\phi]}{\delta \phi(x)}\Big|_{\phi(x)\to\phi(x,\tau)}$

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Free scalar field:

$$G(x,\tau) = \frac{\theta(\tau)}{16\pi^2\tau^2} e^{-\frac{x^2+4M^2\tau^2}{4\tau}}$$

$$\left[\partial_\tau - (\partial^x_\mu \partial^x_\mu - M^2)\right]\phi(x,\tau) = 0 \quad \Rightarrow \quad \phi(x,\tau) = \int d^4y \, \underbrace{\overbrace{G(x-y,\tau)}}_{\text{heat kernel}} \phi(y) \quad \Rightarrow \quad \tilde{\phi}(q,\tau) = e^{-\tau(q^2+M^2)}\tilde{\phi}(q)$$

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Chiral gradient flow Hermann krebs, EE, PRC 110 (2024) 044004

Generalize $U(x),\; U(x) \to R U(x) L^\dagger$ to $W(x,\tau)$: $\partial_\tau W = -i \underbrace{w}_{\sqrt{W}} \overbrace{\mathrm{EOM}(\tau)}^{[D_\mu, w_\mu] + \frac{i}{2}\chi_-(\tau) - \frac{i}{4}\mathrm{Tr}\,\chi_-(\tau)} w, \quad W(x,0) = U(x)$

We have proven $\forall \tau$: $W(x,\tau) \in \mathrm{SU}(2), \;\; W(x,\tau) \;\to\; R W(x,\tau) L^\dagger$

Chiral gradient flow

Solving the chiral gradient flow equation $\partial_\tau W = -iw\,\mathrm{EOM}(\tau)\,w$

— most general parametrization of U: $U = 1 + \frac{i}{F}\boldsymbol{\tau}\cdot\boldsymbol{\pi} - \frac{\boldsymbol{\pi}^2}{2F^2} - \alpha\frac{i}{F^3}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\boldsymbol{\pi}^2 + \ldots$

— similarly, write $W = 1 + i\boldsymbol{\tau}\cdot\boldsymbol{\phi} - \boldsymbol{\phi}^2 - i\,\alpha\,\boldsymbol{\tau}\cdot\boldsymbol{\phi}\boldsymbol{\phi}^2 + \ldots$ and make an ansatz $\boldsymbol{\phi} = \sum_{n=0}^{\infty}\frac{\boldsymbol{\phi}^{(n)}}{F^n}$

$\Rightarrow$ recursive (perturbative) solution of the GF equation in $1/F$

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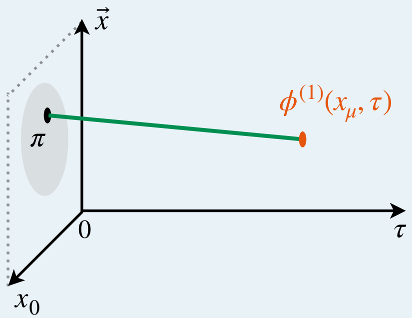
Leading order $\boldsymbol{\phi}^{(1)}$ (no external sources):

$$\left.\begin{array}{r}\left[\partial_\tau - (\partial_\mu^x\partial_\mu^x - M^2)\right]\boldsymbol{\phi}^{(1)}(x,\tau) = 0\\ \boldsymbol{\phi}^{(1)}(x,0) = \boldsymbol{\pi}(x)\end{array}\right\} \quad\Rightarrow\quad \boldsymbol{\phi}^{(1)}(x,\tau) = \int d^4y\,\overbrace{G(x-y,\tau)}^{G(x,\tau)=\frac{\theta(\tau)}{16\pi^2\tau^2}e^{-\frac{x^2+4M^2\tau^2}{4\tau}}}\boldsymbol{\pi}(y)$$

$\phi(x_\mu,\tau) \;=$  $+$

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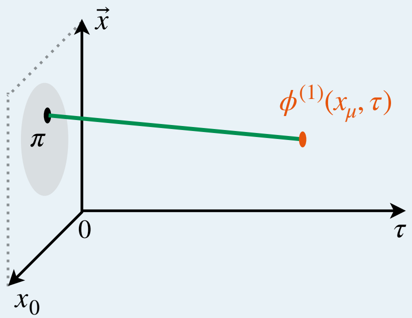
$\phi(x_\mu,\tau) =$



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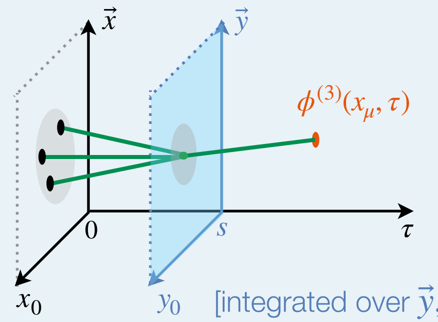

[integrated over $\vec{y}$, y_0, s]

$+ \ \ldots$

Nuclear forces using chiral gradient flow

Regularization is achieved by requiring N to „live“ at a fixed τ: $\mathscr{L}_{\pi N} \;\to\; \mathscr{L}_{\phi N}(\tau) \;=\; \mathscr{L}_{\pi N}\Big|_{U \to W(\tau)}$

Notice: chiral symmetry manifest since $W(\tau) \;\to\; RW(\tau)L^\dagger$ for all τ.

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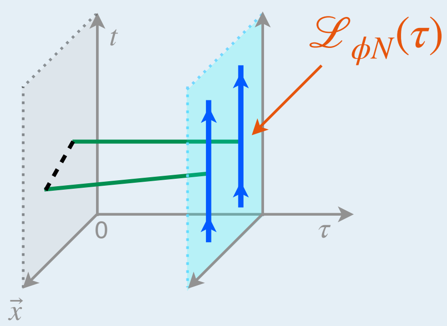
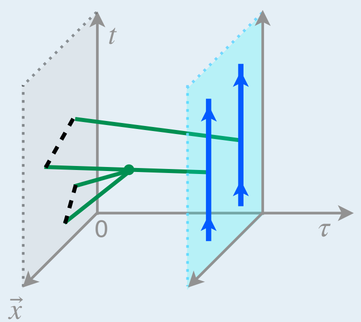
Local field theory in 5d





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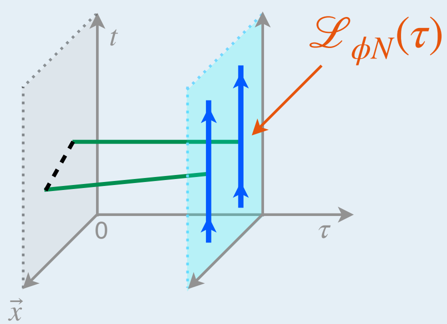
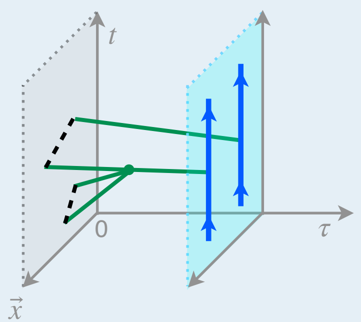
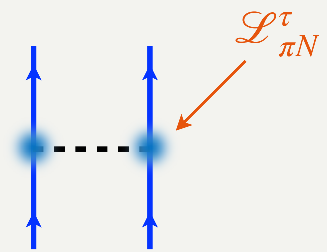
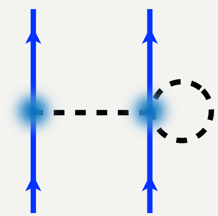




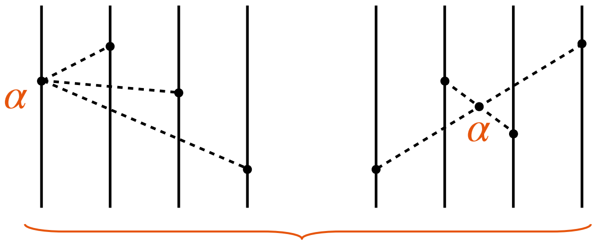
Smeared (non-local) theory in 4d





Chiral symmetry and the 4N force

unregularized



The sum of two diagrams ***must*** *be α-independent*

Unregularized expression for this 4NF EE, EPJA 34 (2007):

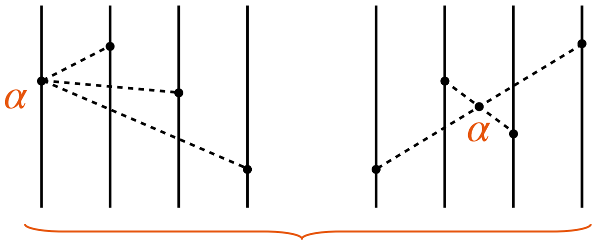
$$\hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_4 \, \vec{\sigma}_2 \cdot \vec{q}_2 \, \vec{\sigma}_3 \cdot \vec{q}_3 \, \vec{\sigma}_4 \cdot \vec{q}_4$$

$$V^{4N} = -\frac{g^4}{64F^6} \frac{\hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]}}{(\vec{q}_2^{\,2}+M^2)(\vec{q}_3^{\,2}+M^2)(\vec{q}_4^{\,2}+M^2)} \vec{\sigma}_1 \cdot \vec{q}_{12}$$

$$+ \frac{g^4}{128F^6} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]}}{(\vec{q}_1^{\,2}+M^2)(\vec{q}_2^{\,2}+M^2)(\vec{q}_3^{\,2}+M^2)(\vec{q}_4^{\,2}+M^2)} \left(M^2 + \vec{q}_{12}^{\,2}\right) + 23 \text{ perm.}$$

Chiral symmetry and the 4N force

unregularized



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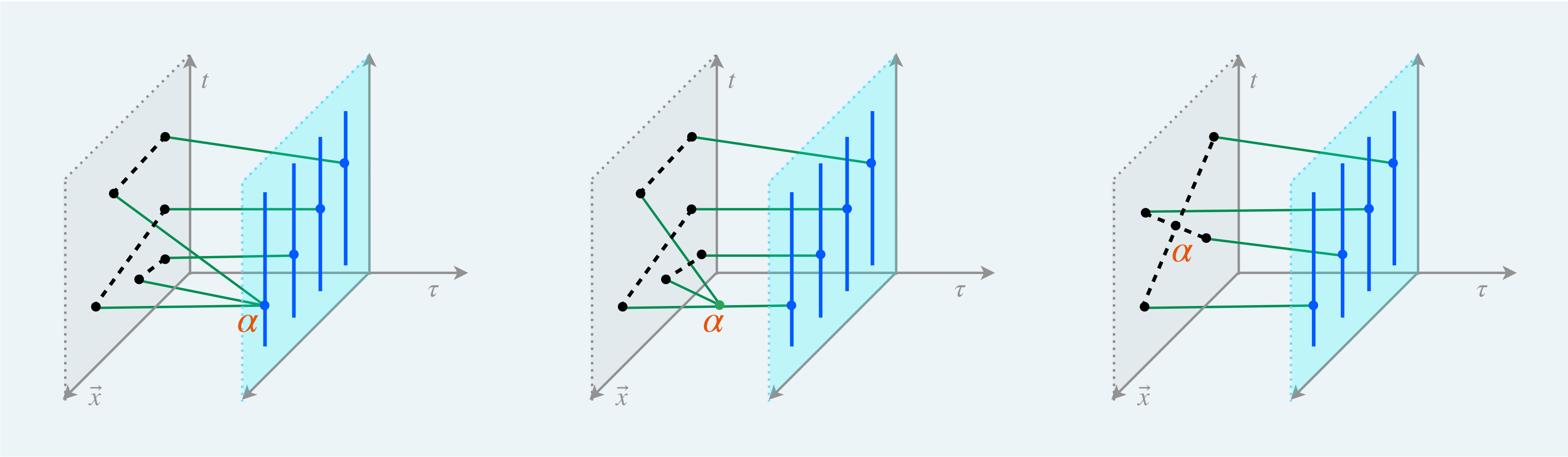

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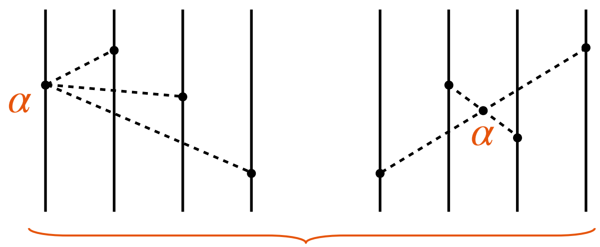

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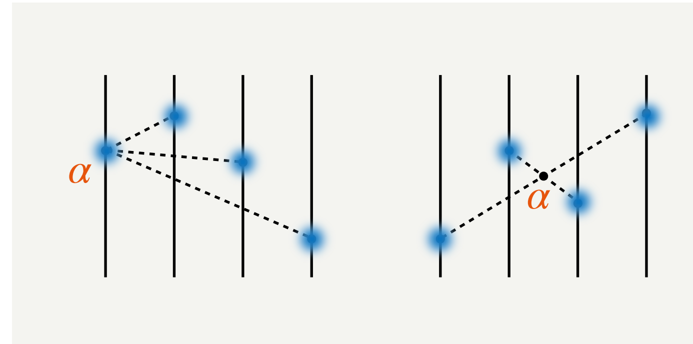

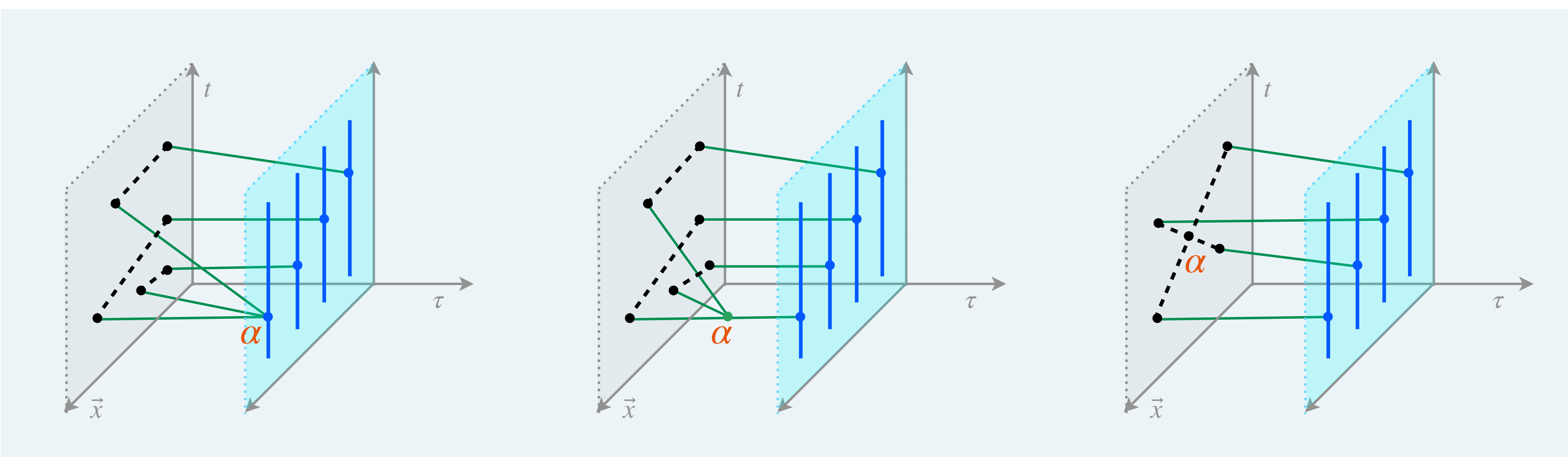

Chiral symmetry and the 4N force

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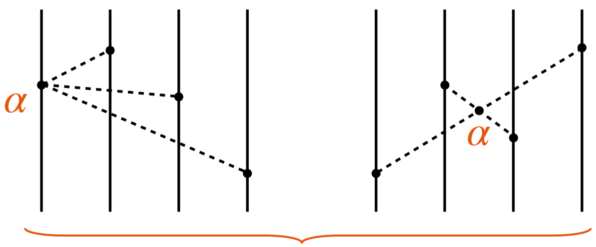
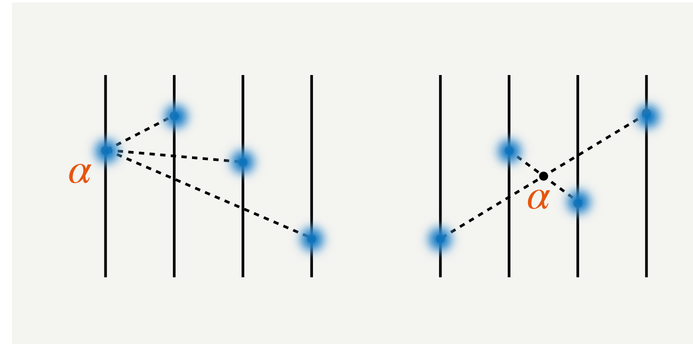


*The sum of two diagrams **must** be α-independent*



Regularized expression (ready to use in the A-body Schrödinger equation):

$$
\begin{aligned}
V_\Lambda^{4N} = \; & \frac{g^4}{64F^6}\frac{\hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]}}{(\vec{q}_2^{\,2}+M^2)(\vec{q}_3^{\,2}+M^2)(\vec{q}_4^{\,2}+M^2)}\Big[\vec{\sigma}_1\cdot\vec{q}_1\big(2g_\Lambda-4f_\Lambda^{123}+2f_\Lambda^{134}-f_\Lambda^{234}\big)-\vec{\sigma}_1\cdot\vec{q}_2 f_\Lambda^{234} \\
& +2\vec{\sigma}_1\cdot\vec{q}_1\big(5M^2+\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+\vec{q}_3^{\,2}+\vec{q}_4^{\,2}+\vec{q}_{34}^{\,2}\big)\frac{g_\Lambda-f_\Lambda^{134}}{2M^2+\vec{q}_1^{\,2}+\vec{q}_3^{\,2}+\vec{q}_4^{\,2}-\vec{q}_2^{\,2}} \\
& -4\vec{\sigma}_1\cdot\vec{q}_1\big(3M^2+\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+\vec{q}_3^{\,2}+\vec{q}_4^{\,2}-\vec{q}_{34}^{\,2}\big)\frac{g_\Lambda-f_\Lambda^{124}}{2M^2+\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+\vec{q}_4^{\,2}-\vec{q}_3^{\,2}}\Big] \\
+ \; & \frac{g^4}{128F^6}\frac{\vec{\sigma}_1\cdot\vec{q}_1\,\hat{O}_{[\sigma_i,\tau_i,\vec{q}_i]}}{(\vec{q}_1^{\,2}+M^2)(\vec{q}_2^{\,2}+M^2)(\vec{q}_3^{\,2}+M^2)(\vec{q}_4^{\,2}+M^2)}\big(M^2+\vec{q}_{12}^{\,2}\big)\big(4f_\Lambda^{123}-3g_\Lambda\big)+23\text{ perm.},
\end{aligned}
$$

$$f_\Lambda^{ijk} = e^{-\frac{\vec{q}_i^{\,2}+M^2}{\Lambda^2}}e^{-\frac{\vec{q}_j^{\,2}+M^2}{\Lambda^2}}e^{-\frac{\vec{q}_k^{\,2}+M^2}{\Lambda^2}}$$

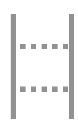
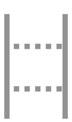
$$g_\Lambda = e^{-\frac{\vec{q}_1^{\,2}+M^2}{2\Lambda^2}}e^{-\frac{\vec{q}_2^{\,2}+M^2}{2\Lambda^2}}e^{-\frac{\vec{q}_3^{\,2}+M^2}{2\Lambda^2}}e^{-\frac{\vec{q}_4^{\,2}+M^2}{2\Lambda^2}}$$

(reduces to the unregularized result in the $\Lambda\to\infty$ limit)

Nuclear interactions from path integral

Hermann Krebs, EE, PRC 110 (2024) 044003

The considered 4NFs were calculated using Feynman diagrams. But more generally,

Potential  $\neq$ Feynman diagram 

Nuclear interactions from path integral

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Potential  $\neq$ Feynman diagram 

$\mathcal{L}_{\pi N}$ —*canonical quan.*→ $\mathcal{H}_{\pi N}$ —→ $V_{2\mathrm{N}},\; V_{3\mathrm{N}},\; \ldots$ / $J^{\mu}_{2\mathrm{N}},\; J^{\mu}_{3\mathrm{N}},\; \ldots$ / $A^{\mu}_{2\mathrm{N}},\; A^{\mu}_{3\mathrm{N}},\; \ldots$

TOPT Weinberg, van Kolck, Pastore, Schiavilla, …

Method of UT EE, Glöckle, Meißner, Krebs, Bernard, …

$\Rightarrow$ impractical for ***regularized*** Lagrangians, which involve $e^{-\tau(-\partial_x^2+M^2)}\boldsymbol{\pi}(x)$

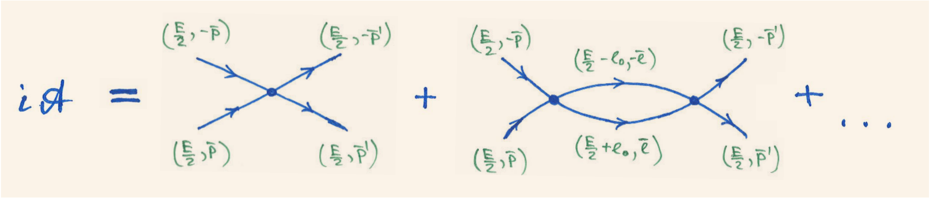
$\Rightarrow$ new method to derive nuclear interactions using the path integral approach Krebs, EE, PRC 110 (24) 044003

The idea

Pion-less EFT:

$$\mathcal{L} = N^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{2m_N} \right] N - \frac{C_S}{2} (N^\dagger N)^2 + \ldots$$

$$\Rightarrow \quad \mathcal{A}_{\rm tree} = [C_0 + C_2(\vec{p}^{\,2} + \vec{p}^{\,\prime 2}) + \ldots]$$



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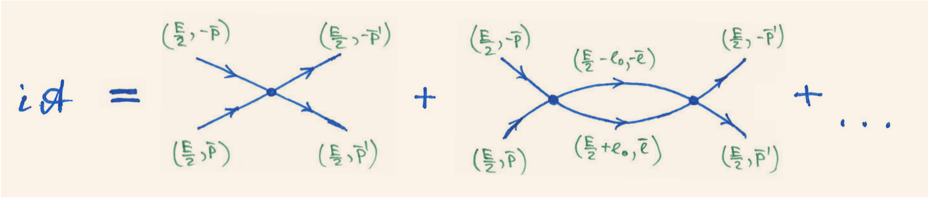

$$\mathcal{L} = N^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{2m_N} \right] N - \frac{C_S}{2}(N^\dagger N)^2 \; + \ldots$$

$$\Rightarrow \quad \mathcal{A}_{\rm tree} = [C_0 + C_2(\vec{p}^{\,2} + \vec{p}^{\,\prime 2}) + \ldots]$$

Scattering amplitude to 1 loop:

$$-i\mathcal{A}_{1-{\rm loop}} \;=\; \int \frac{d^4 l}{(2\pi)^4} \, [C_0 + C_2(\vec{p}^{\,2} + \vec{l}^{\,2}) + \ldots] \, \frac{1}{\left(\frac{E}{2} + l_0 - \frac{\vec{l}^{\,2}}{2m_N} + i\epsilon\right)\left(\frac{E}{2} - l_0 - \frac{\vec{l}^{\,2}}{2m_N} + i\epsilon\right)} \, [C_0 + \ldots]$$

$$= \; -i \int \frac{d^3 l}{(2\pi)^3} \, [C_0 + C_2(\vec{p}^{\,2} + \vec{l}^{\,2}) + \ldots] \, \frac{1}{E - \frac{\vec{l}^{\,2}}{m_N} + i\epsilon} \, [C_0 + (\vec{l}^{\,2} + \vec{p}^{\,\prime 2}) \ldots]$$

All l_0-integrals factorize $\Rightarrow$ Lippmann-Schwinger eq. $\mathcal{A} = \mathcal{V} + \mathcal{V}\, G_0\, \mathcal{A}$ with $\mathcal{V} = -\mathcal{L}_{\rm int}$

The idea

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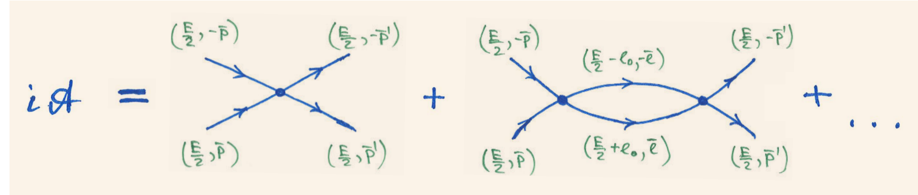

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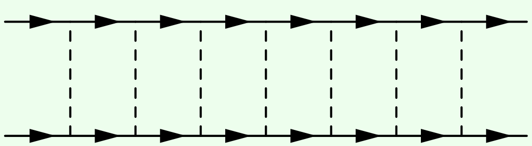

But l_0-integrals do not factorize for pions due to l_0-dependence of π-propagators…

The idea

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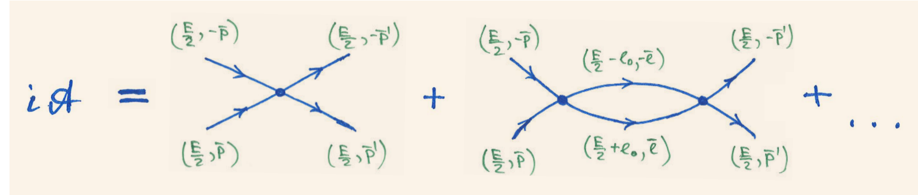

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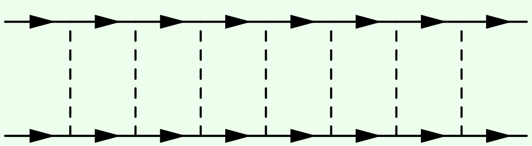
$$= -i \int \frac{d^3 l}{(2\pi)^3} [C_0 + C_2(\vec{p}^{\,2} + \vec{l}^{\,2}) + \ldots] \frac{1}{E - \frac{\vec{l}^{\,2}}{m_N} + i\epsilon} [C_0 + (\vec{l}^{\,2} + \vec{p}^{\,\prime 2}) \ldots]$$

All l_0-integrals factorize $\Rightarrow$ Lippmann-Schwinger eq. $\mathcal{A} = \mathcal{V} + \mathcal{V}\, G_0\, \mathcal{A}$ with $\mathcal{V} = -\mathcal{L}_{\text{int}}$



But l_0-integrals do not factorize for pions due to l_0-dependence of π-propagators…

Idea: $Z[\eta^\dagger, \eta] = A \int \mathscr{D}N^\dagger\, \mathscr{D}N\, \mathscr{D}\pi \exp\left(iS^\Lambda_{\text{eff}} + i \int d^4x [\eta^\dagger N + N^\dagger \eta] \right)$

Hermann Krebs, EE, 2311.10893

nonlocal redefinitions of $N, N^\dagger$ $\longrightarrow$ $A \int \mathscr{D}\tilde{N}^\dagger\, \mathscr{D}\tilde{N} \exp\left(iS^\Lambda_{\text{eff, N}} + i \int d^4x [\eta^\dagger \tilde{N} + \tilde{N}^\dagger \eta] \right)$

instantaneous $\rightarrow$ $S^\Lambda_{\text{eff, N}}$

Toy-model example

Regularized toy model: $$\mathcal{L}^{\rm E}_{\pi N} = N^\dagger \left[\partial_0 - \frac{\vec\nabla^2}{2m} - \frac{g}{2F} \vec\sigma \cdot \vec\nabla \boldsymbol{\pi} \cdot \boldsymbol{\tau} \right] N + \frac{1}{2} \boldsymbol{\pi} \cdot \left(-\partial^2 + M^2 \right) e^{\frac{-\partial^2 + M^2}{\Lambda^2}} \boldsymbol{\pi}$$

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Nonlocal action $S^{\rm E}_N$ after integrating out pion fields (Gaussian):

$$Z[\eta^\dagger,\eta] = \int DN^\dagger DN D\boldsymbol{\pi}\, e^{-S^{\rm E}_{\pi N}+\int d^4x\,\left[\eta^\dagger N + N^\dagger \eta\right]} = A\int DN^\dagger DN\, e^{-S^{\rm E}_{N}+\int d^4x\,\left[\eta^\dagger N + N^\dagger \eta\right]}$$

non-static regularized pion propagator

where $$S^{\rm E}_N = \underbrace{N^\dagger_x\left[\partial_0 - \frac{\vec{\nabla}^{\,2}_x}{2m}\right]N_x}_{\text{integration over } d^4x \text{ not shown}} + \underbrace{\frac{g^2}{8F^2}\left[N^\dagger\vec{\sigma}\boldsymbol{\tau}N\right]_{x_1}\cdot\left[\vec{\nabla}_{x_1}\otimes\vec{\nabla}_{x_1}\overbrace{\Delta^{\rm E}_\Lambda(x_1-x_2)}\right]\cdot\left[N^\dagger\vec{\sigma}\boldsymbol{\tau}N\right]_{x_2}}_{\text{integration over } d^4x_1\, d^4x_2 \text{ not shown}}$$

Toy-model example

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Nonlocal action $S_N^{\rm E}$ after integrating out pion fields (Gaussian):

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where $S_N^{\rm E} \; = \; \underbrace{N_x^\dagger\left[\partial_0 - \frac{\vec\nabla_x^2}{2m}\right]N_x}_{\text{integration over } d^4x \text{ not shown}} + \underbrace{\frac{g^2}{8F^2}\left[N^\dagger\vec\sigma\boldsymbol{\tau}N\right]_{x_1}\cdot\left[\vec\nabla_{x_1}\otimes\vec\nabla_{x_1}\overbrace{\Delta^{\rm E}_\Lambda(x_1 - x_2)}^{\text{non-static regularized pion propagator}}\right]\cdot\left[N^\dagger\vec\sigma\boldsymbol{\tau}N\right]_{x_2}}_{\text{integration over } d^4x_1\, d^4x_2 \text{ not shown}}$

Rewrite the pion propagator to the static one plus rest:

$$\Delta^{\rm E}_\Lambda(x) \; = \; \int\frac{d^4q}{(2\pi)^4} e^{iq\cdot x}\frac{e^{-\frac{q_0^2+\vec q^{\,2}+M^2}{\Lambda^2}}}{q_0^2+\vec q^{\,2}+M^2} \; = \; \underbrace{\Delta^{\rm S}_\Lambda(x)}_{\delta(x_0)\bar\Delta^{\rm S}_\Lambda(\vec x)} + \Delta^{\rm E}_\Lambda(x) - \Delta^{\rm S}_\Lambda(x) \; =: \; \Delta^{\rm S}_\Lambda(x) + \underbrace{\partial_0^2\Delta^{\rm ES}_\Lambda(x)}_{-\int\frac{d^4q}{(2\pi)^4}\frac{e^{iq\cdot x}}{q_0^2}[\tilde\Delta^{\rm E}_\Lambda(q) - \tilde\Delta^{\rm S}_\Lambda(\vec q)]}$$

Toy-model example

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Nucleon field redefinition: $N_x = \tilde{N}_x - \frac{g^2}{8F^2} \boldsymbol{\tau} \vec{\sigma} \tilde{N}_x \cdot \underbrace{\left[\vec{\nabla}_x \otimes \vec{\nabla}_x \, \partial_0 \Delta^{\rm ES}_\Lambda (x - x_2) \right] \cdot \left[\tilde{N}^\dagger \vec{\sigma} \boldsymbol{\tau} \tilde{N} \right]_{x_2}}_{\text{integration over } d^4x_2 \text{ not shown}}, \quad N_x^\dagger = \ldots$

$$\Rightarrow S^{\rm E}_{\tilde{N}} = \tilde{N}_x^\dagger \left[\partial_0 - \frac{\vec{\nabla}_x^2}{2m} \right] \tilde{N}_x + \frac{g^2}{8F^2} \left[\tilde{N}^\dagger \vec{\sigma} \boldsymbol{\tau} \tilde{N} \right]_{x_1} \cdot \left[\vec{\nabla}_{x_1} \otimes \vec{\nabla}_{x_1} \Delta^{\rm S}_\Lambda (x_1 - x_2) \right] \cdot \left[\tilde{N}^\dagger \vec{\sigma} \boldsymbol{\tau} \tilde{N} \right]_{x_2} + S^{1/m}_{\rm 2N} + S_{\rm 3N}$$

Toy-model example

To summarize:

$$Z[\eta^\dagger,\eta] \;=\; \int DN^\dagger DN D\boldsymbol{\pi}\, e^{-S^{\rm E}_{\pi N}+\int d^4x\,[\eta^\dagger N+N^\dagger\eta]} \;=\; \ldots \;=\; A\int D\tilde N^\dagger D\tilde N\, e^{-S^{\rm E}_{\tilde N}+\int d^4x\,[\eta^\dagger \tilde N+\tilde N^\dagger\eta]}$$

where the many-body action is now instantaneous (up to higher-order corrections):

$$S^{\rm E}_{\tilde N} \;=\; \tilde N^\dagger_x\left[\partial_0-\frac{\vec\nabla^2_x}{2m}\right]\tilde N_x \;+\; \frac{g^2}{8F^2}\left[\tilde N^\dagger\vec\sigma\,\boldsymbol{\tau}\tilde N\right]_{x_1}\cdot\left[\vec\nabla_{x_1}\otimes\vec\nabla_{x_1}\Delta^{\rm S}_\Lambda(x_1-x_2)\right]\cdot\left[\tilde N^\dagger\vec\sigma\,\boldsymbol{\tau}\tilde N\right]_{x_2} \;+\; S^{1/m}_{\rm 2N} \;+\; S_{\rm 3N}$$

$\Rightarrow$ read out $V_{\rm NN}$ directly from the action: $V^\Lambda_{\rm 2N}(\vec x_{12})=\dfrac{g^2}{4F^2}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,(\vec\sigma_1\cdot\vec\nabla)\,(\vec\sigma_2\cdot\vec\nabla)\,\Delta^{\rm S}_\Lambda(\vec x_{12})$

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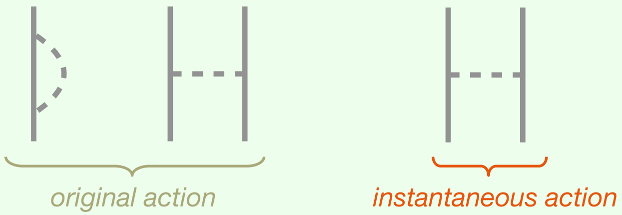
On the other hand, to order g^2:

⇒ something is missing…



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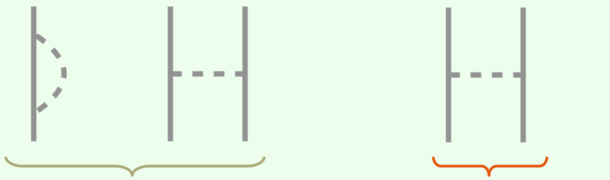

$$\int DN^\dagger DN\; e^{(\ldots)} \;=\; \int D\tilde N^\dagger D\tilde N\;\det\left[\boldsymbol{J}_{N,N^\dagger}(\tilde N,\tilde N^\dagger)\right]e^{(\ldots)} \;=\; \int D\tilde N^\dagger D\tilde N\; e^{(\ldots)+\int d^4x\tilde N^\dagger_x\Sigma_\Lambda\tilde N_x+\ldots}$$

with $$\Sigma_\Lambda \;=\; -\frac{3g^2}{8F^2}\int\frac{d^3p}{(2\pi)^3}\,\vec p^{\,2}\,\frac{e^{-\frac{\vec p^{\,2}+M^2}{\Lambda^2}}}{\vec p^{\,2}+M^2} \;=\; -\frac{3g^2}{64\pi^{3/2}F^2}\Lambda^3 \;+\; \frac{9g^2M^2}{64\pi^{3/2}F^2}\Lambda \;-\; \underbrace{\frac{3g^2M^3}{32\pi F^2}}_{\text{the leading non-analytic contribution to } m_N} \;+\; \mathcal{O}(\Lambda^{-1})$$

the leading non-analytic contribution to m_N

Summary and outlook

New formulation of nuclear chiral EFT:

— gradient flow regularized formulation of chiral EFT Krebs, EE, PRC 110 (2024) 044004

— path integral method to perform QM reduction of QFT Krebs, EE, PRC 110 (2024) 044003

⇒ regularized 3N, 4N forces and currents, which are consistent with the SMS NN potentials and respect chiral & gauge symmetries

Already done:

— NN at N^2LO, long-range 3NF (still needs to be implemented…) and 4NF at N^3LO

Work in progress:

— πN scattering inside the Mandelstam triangle (LECs), 3N scattering at N^3LO

The new method can also be useful for improving convergence of SU(3) BChPT

Thank you for your attention

Spares

The two-nucleon system

How far in the EFT expansion does one need to go to precisely describe low-energy NN data?

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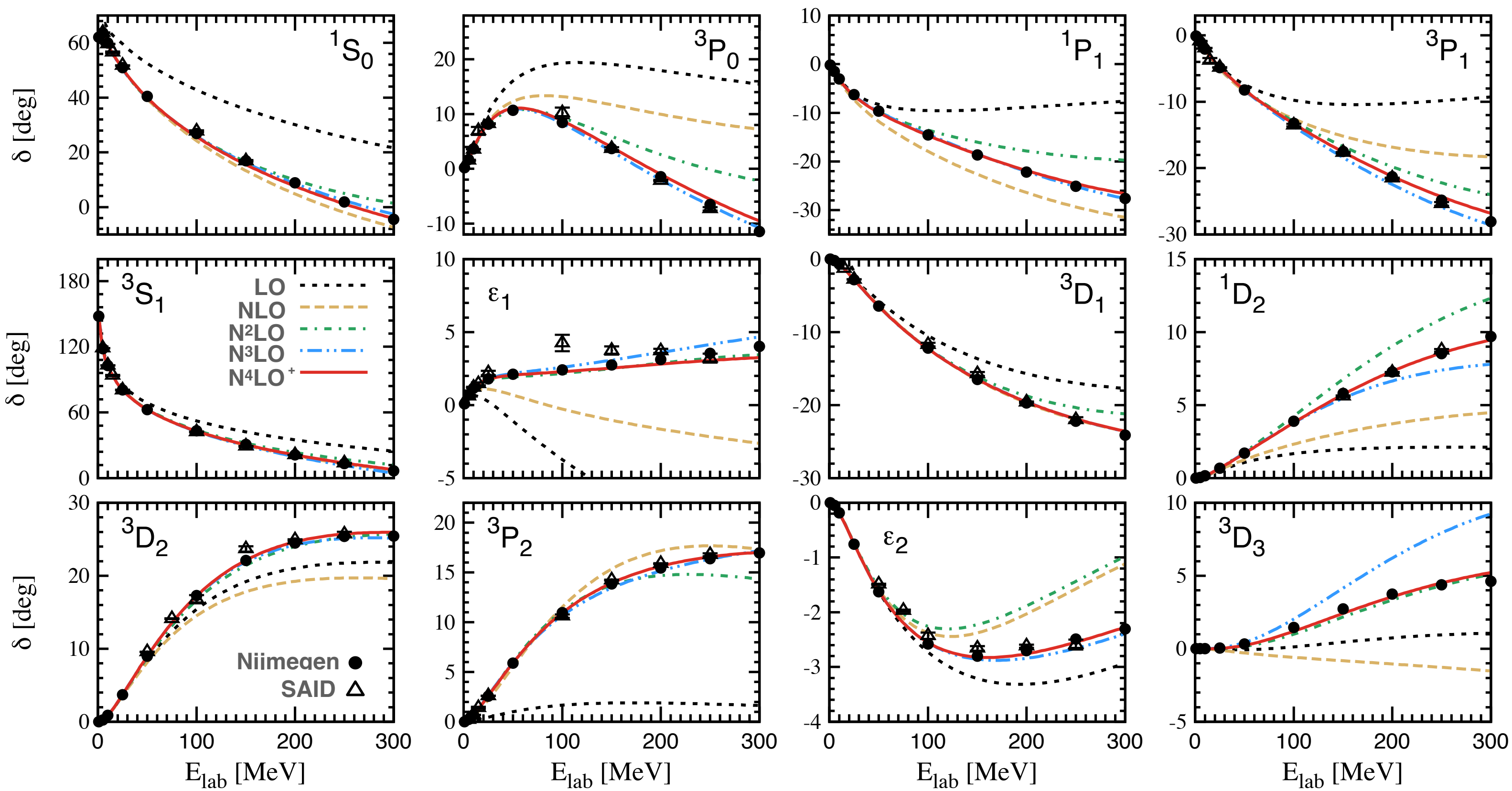
Results for $\Lambda = 450$ MeV

from: P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

	LO (Q^0)	NLO (Q^2)	N^2LO (Q^3)	N^3LO (Q^4)	N^4LO (Q^5)	N^4LO^+
χ^2/datum (np, 0 – 300 MeV)	75	14	4.1	2.01	1.16	1.06
χ^2/datum (pp, 0 – 300 MeV)	1380	91	41	3.43	1.67	1.00
	2 LECs	+ 7 + 1 IB LECs		+ 12 LECs	+ 1 LEC (np)	+ 4 LECs

Chiral expansion of the neutron-proton phase shifts [Λ = 450 MeV]



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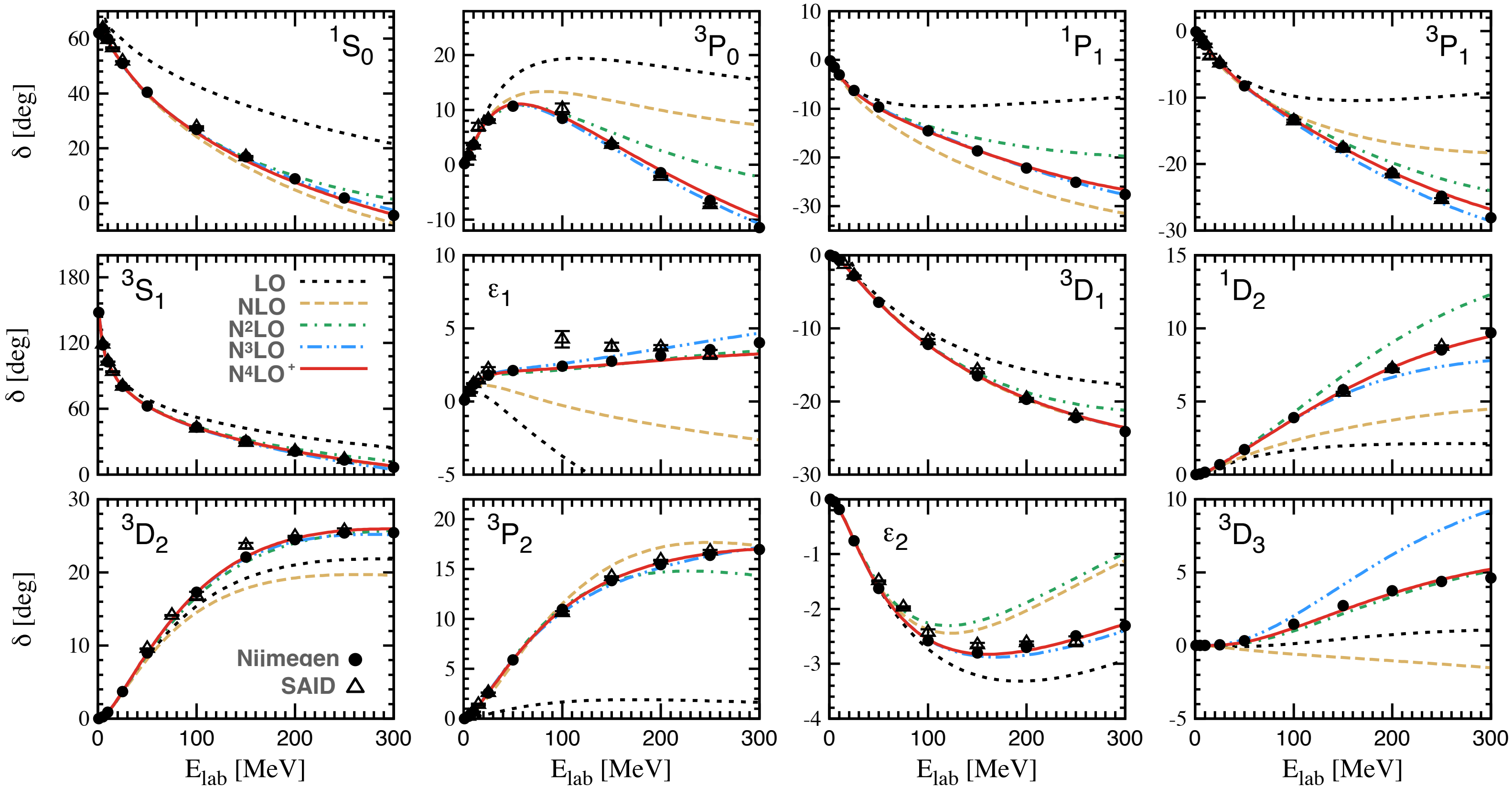
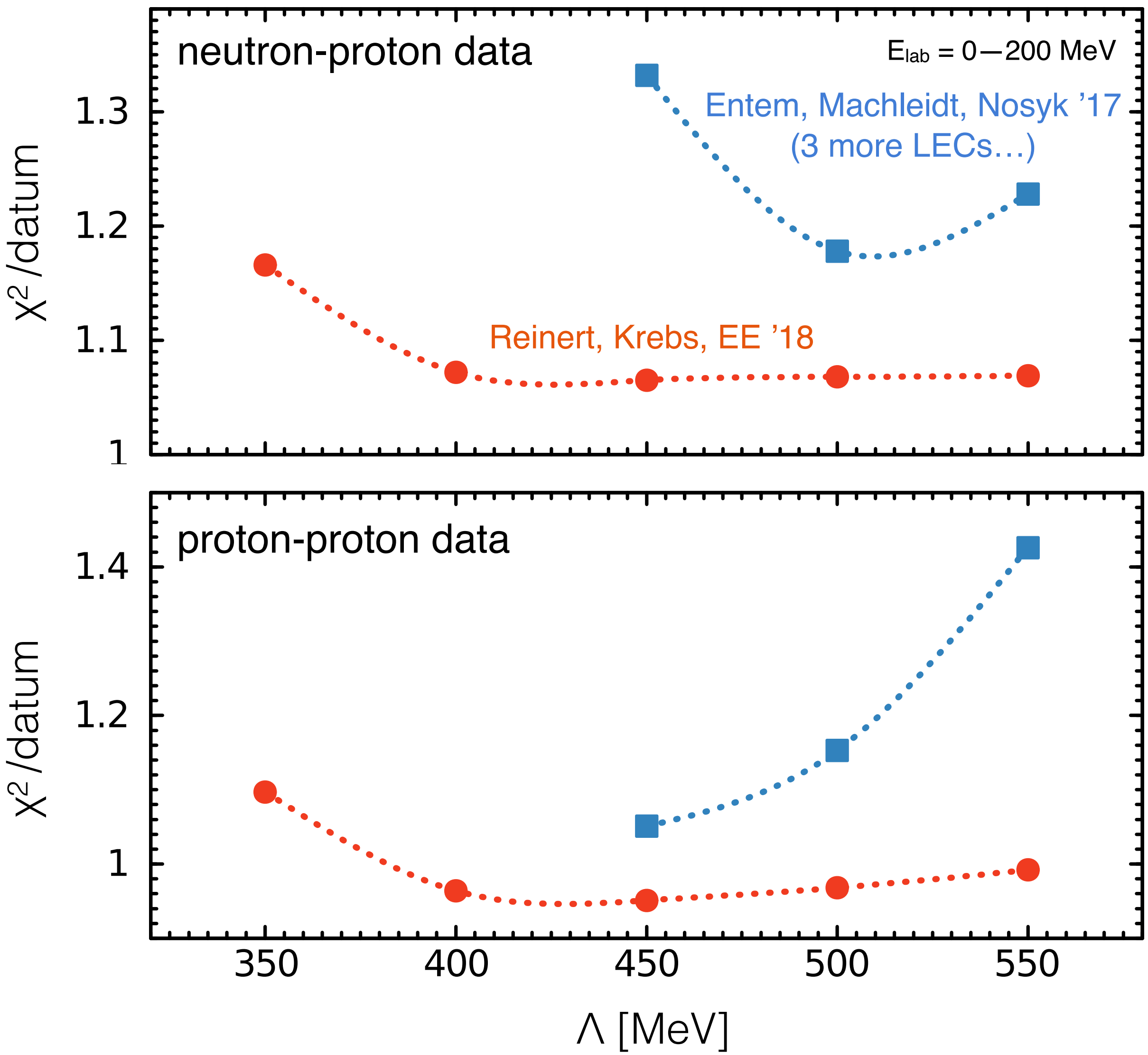


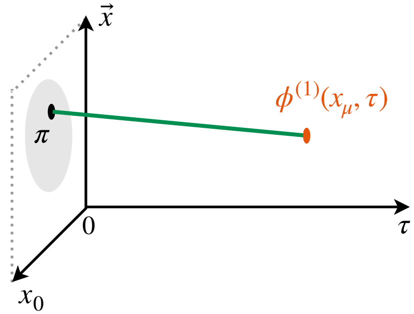
Regulator (in)dependence

χ^2/datum for the description of the NN data in the range of 0 – 200 MeV at N^{4}LO$^+$

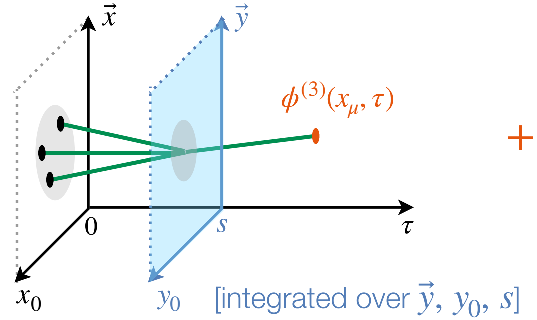


Solving the chiral gradient flow equation

$\phi(x_\mu, \tau)$ =



+



+ …

$$G(x,\tau) = \frac{\theta(\tau)}{16\pi^2\tau^2} e^{-\frac{x^2+4M^2\tau^2}{4\tau}}$$

$$\left.\begin{aligned} \left[\partial_\tau - (\partial^x_\mu \partial^x_\mu - M^2)\right] \boldsymbol{\phi}^{(1)}(x,\tau) &= 0 \\ \boldsymbol{\phi}^{(1)}(x,0) &= \boldsymbol{\pi}(x) \end{aligned}\right\} \Rightarrow \quad \boldsymbol{\phi}^{(1)}(x,\tau) = \int d^4y \, \overbrace{G(x-y,\tau)}\boldsymbol{\pi}(y) \quad \Rightarrow \quad \tilde{\boldsymbol{\phi}}^{(1)}(q,\tau) = e^{-\tau(q^2+M^2)} \tilde{\boldsymbol{\pi}}(q)$$

SMS regulator for $\tau = 1/(2\Lambda^2)$

$$\left[\partial_\tau - (\partial^x_\mu \partial^x_\mu - M^2)\right] \phi_b^{(3)}(x,\tau) = \overbrace{(1-2\alpha)\partial_\mu \boldsymbol{\phi}^{(1)} \cdot \partial_\mu \boldsymbol{\phi}^{(1)} \phi_b^{(1)} - 4\alpha \, \partial_\mu \boldsymbol{\phi}^{(1)} \cdot \boldsymbol{\phi}^{(1)} \partial_\mu \phi_b^{(1)} + \frac{M^2}{2}(1-4\alpha) \boldsymbol{\phi}^{(1)} \cdot \boldsymbol{\phi}^{(1)} \phi_b^{(1)}}^{\equiv \mathrm{RHS}_b(x,\tau)}$$

$$\phi_b^{(3)}(x,0) = 0$$

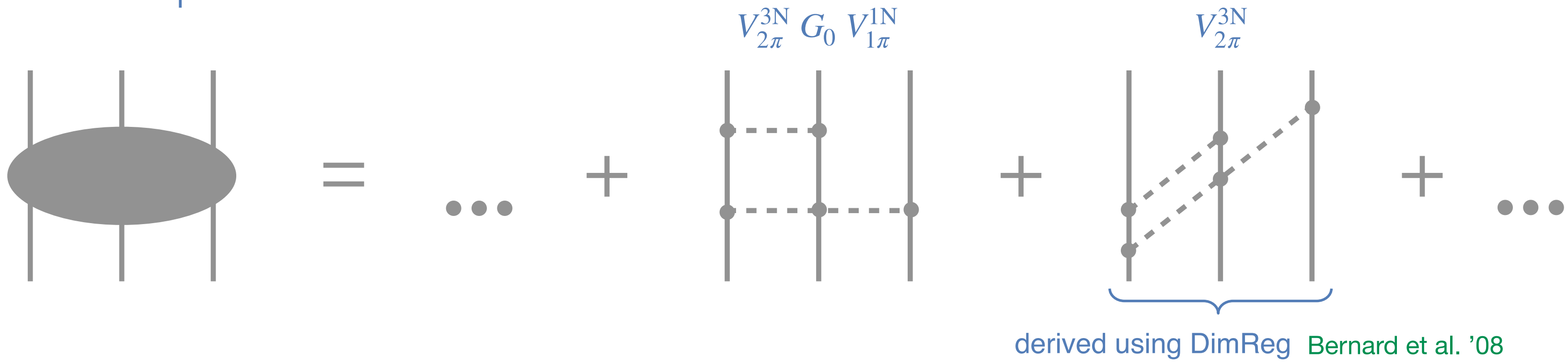
$$\Rightarrow \quad \phi_b^{(3)}(x,\tau) = \int_0^\tau ds \int d^4y \, G(x-y, \tau-s) \, \mathrm{RHS}_b(y,s)$$

In momentum space, this solution takes the form:

$$\tilde{\phi}_b^{(3)}(q,\tau) = \int \prod_{i=1}^{3} \frac{d^4q_i}{(2\pi)^4} (2\pi)^4 \delta^4(q - q_1 - q_2 - q_3) \underbrace{f_\Lambda(\{q_i\})}_{\frac{e^{-\tau(q^2+M^2)} - e^{-\tau\sum_{j=1}^3 (q_j^2+M^2)}}{q_1^2+q_2^2+q_3^2-q^2+2M^2}} \left[4\alpha \, q_1 \cdot q_3 - (1-2\alpha) q_1 \cdot q_2 + \frac{M^2}{2}(1-4\alpha)\right] \tilde{\boldsymbol{\pi}}(q_1) \cdot \tilde{\boldsymbol{\pi}}(q_2) \, \tilde{\pi}_b(q_3)$$

Essence of the problem

Faddeev equation:



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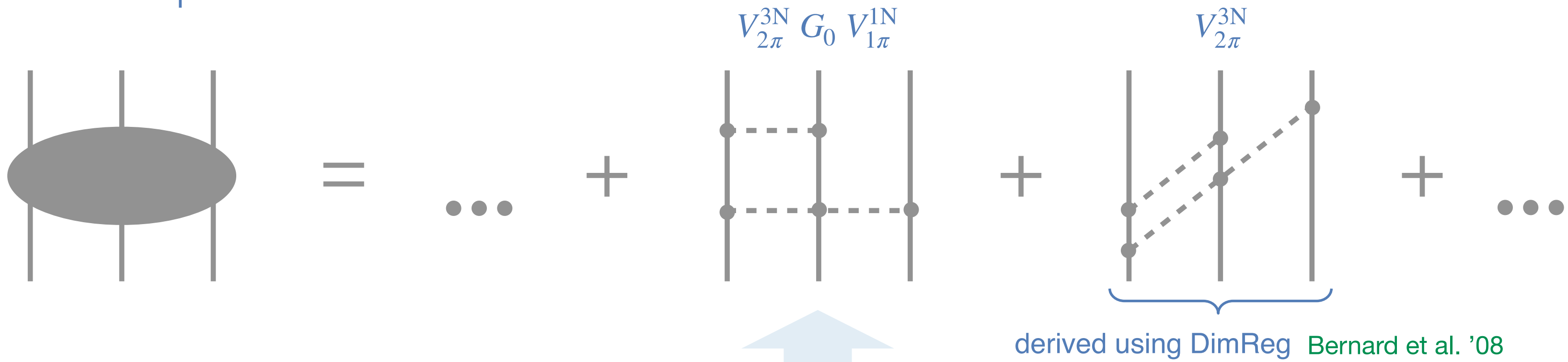

$$-\Lambda \frac{g_A^4}{96\sqrt{2\pi^3}F_\pi^6}\Big[\underbrace{\tau_1\cdot\tau_3\,(\vec{q}_3\cdot\vec{\sigma}_1)}_{\text{absorbable into } c_D} - \underbrace{\frac{4}{3}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)(\vec{q}_2\cdot\vec{\sigma}_3)}_{\text{violates chiral symmetry}}\Big]\frac{\vec{q}_3\cdot\vec{\sigma}_3}{q_3^3+M_\pi^2} + \ldots$$

The problematic divergence would cancel if $V_{2\pi}^{3N}$ were calculated using Cutoff EE, Krebs, Reinert '19

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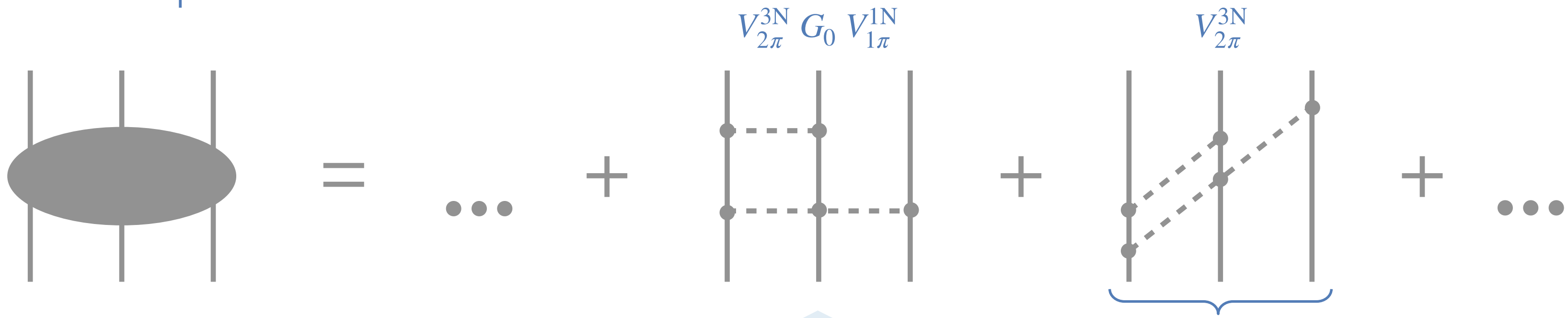

derived using DimReg Bernard et al. '08

$$-\Lambda \frac{g_A^4}{96\sqrt{2\pi^3}F_\pi^6}\left[\underbrace{\tau_1\cdot\tau_3\,(\vec{q}_3\cdot\vec{\sigma}_1)}_{\text{absorbable into } c_D} - \underbrace{\frac{4}{3}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)(\vec{q}_2\cdot\vec{\sigma}_3)}_{\text{violates chiral symmetry}}\right]\frac{\vec{q}_3\cdot\vec{\sigma}_3}{q_3^3+M_\pi^2} + \ldots$$

absorbable into cD: ╳-┤ violates chiral symmetry

The problematic divergence would cancel if $V_{2\pi}^{3N}$ were calculated using Cutoff EE, Krebs, Reinert '19

⇒ loop contributions to the 3NF, 4NF and MECs must be re-derived using symmetry preserving cutoff (2NF ok at fixed M_π)