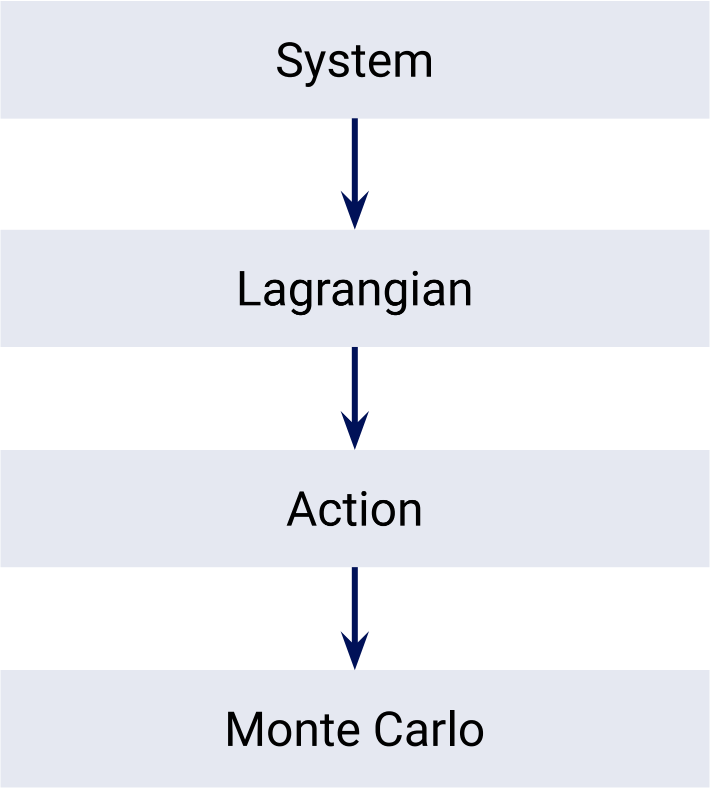
Tensor Networks for Lattice Gauge Theories

Patrick Emonts | HHIQCD 2024 | 01.11.2024 | Kyoto

Universiteit Leiden
The Netherlands

NWO

The usual pipeline



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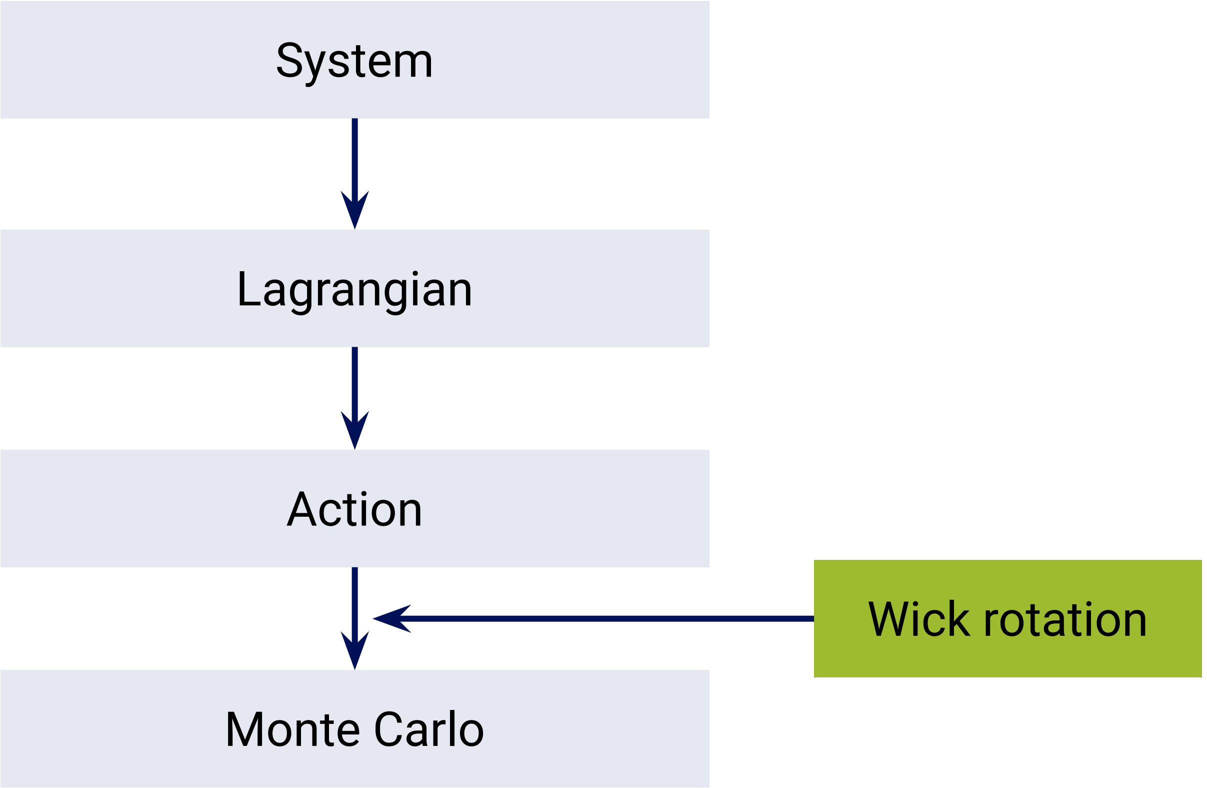


Wick rotation

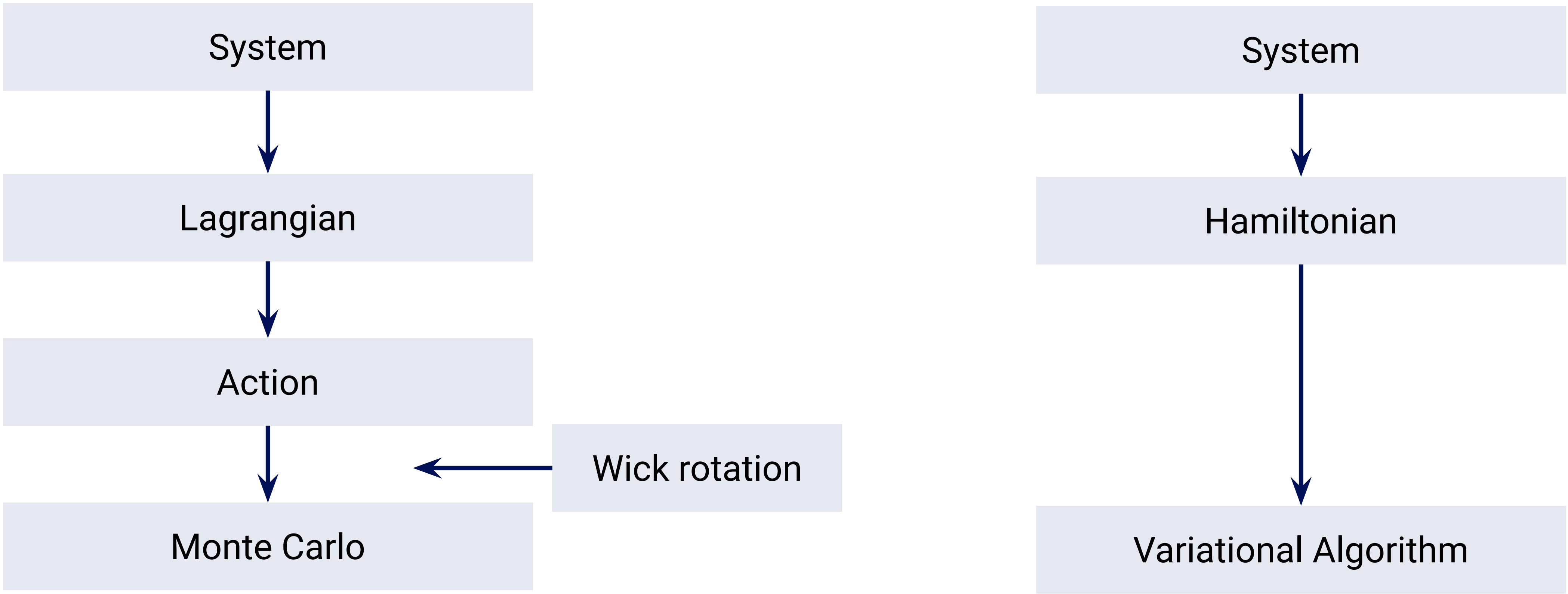
$$t \to -i\tau$$

Problems

- Possibility of a sign problem
- Time dynamics are not accessible

Mari Carmen Bañuls and Krzysztof Cichy (2020) Rep. Prog. Phys. 83 p. 024401;
John Kogut and Leonard Susskind (1975) Phys. Rev. D 11 pp. 395–408;
Kenneth G. Wilson (1974) Phys. Rev. D 10 pp. 2445–2459

Our Approach



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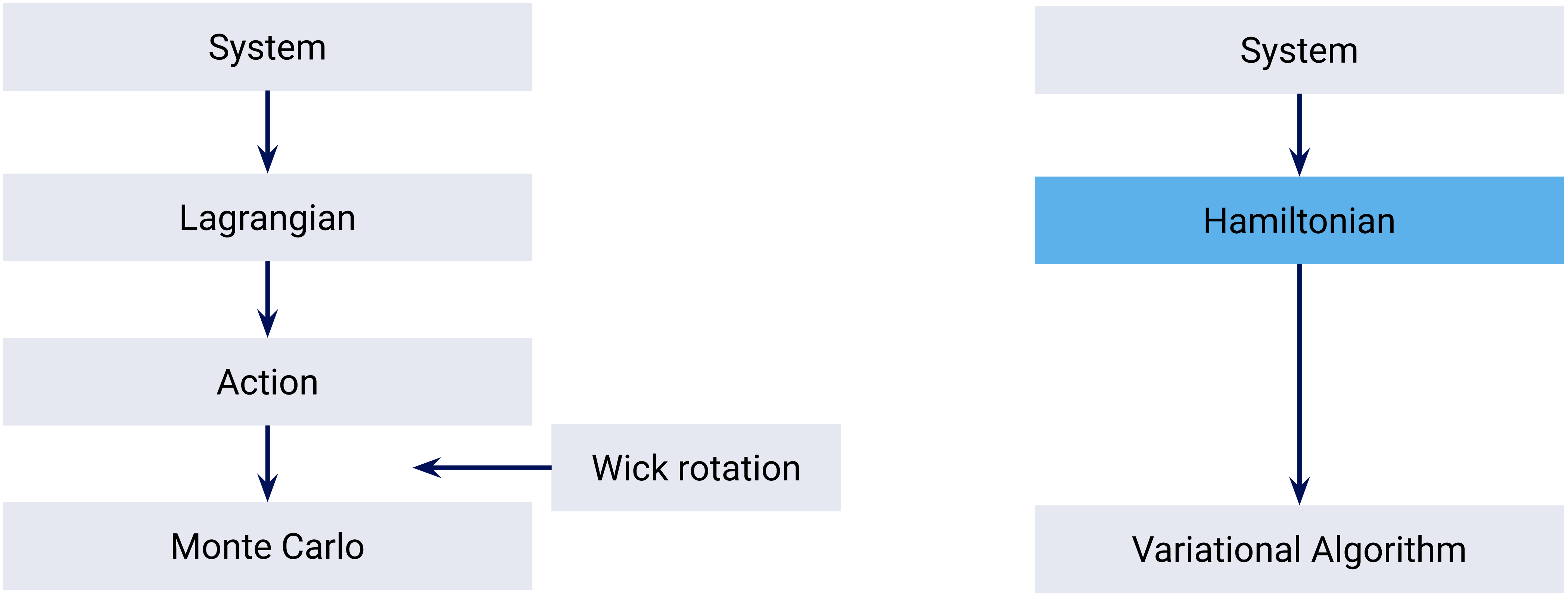

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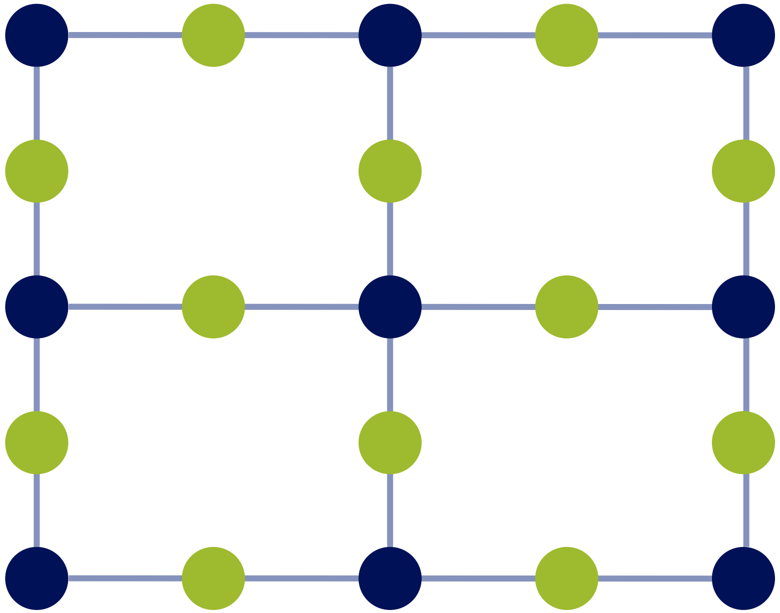
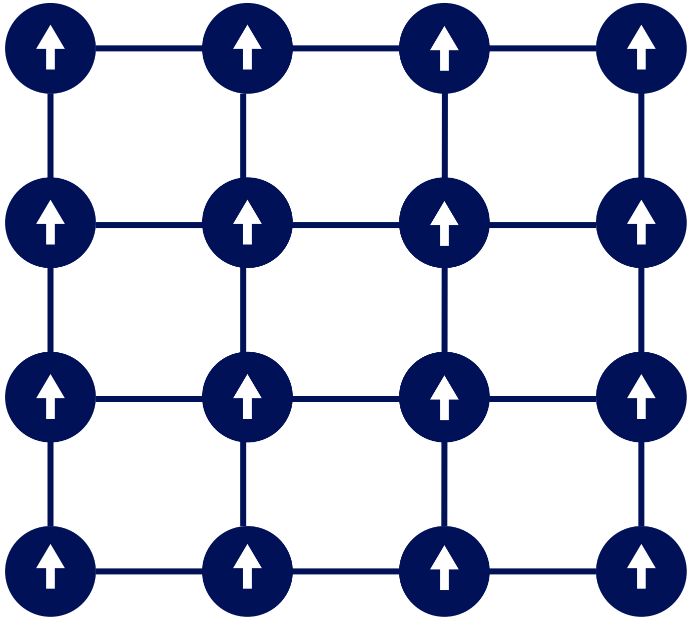
Hilbert spaces and Lattices



Hilbert space

$$\mathcal{H} \subset \mathcal{H}_{\text{gauge fields}} \bigotimes \mathcal{H}_{\text{fermions}}$$

Many-body physics – How hard can it be?



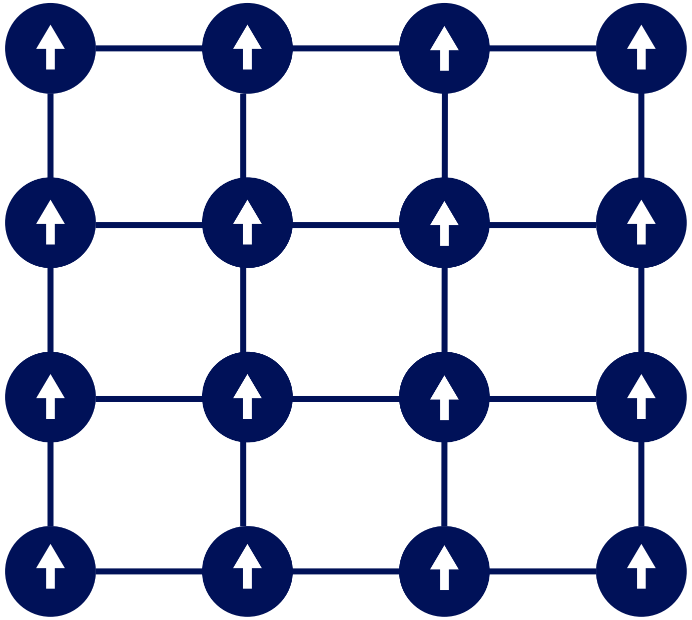
Number of Possibilities

$$Z = 2^N$$

Storage of minimal configuration (classical)

$$|\psi_0\rangle = 0101101101010011$$

Many-body physics – How hard can it be?



Number of Basis States

$$Z = 2^N$$

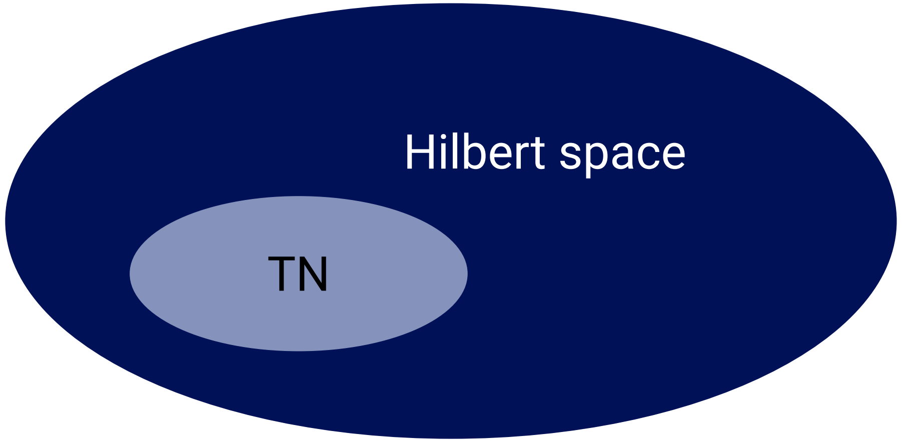
Storage of minimal configuration

$$|\psi_0\rangle = \sum_{\{i\}} c_{i_0,\ldots,i_{N-1}} |i_0, i_1, \ldots, i_{N-1}\rangle$$

Finding an Ansatz

Idea

Use an Ansatz with polynomially many parameters although the Hilbert space has exponentially many states



We explore only a small part of the Hilbert space

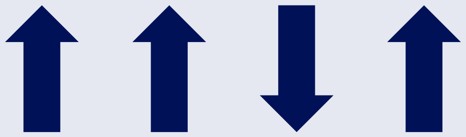
M. Fannes, B. Nachtergaele, and R. F. Werner (1992) Commun.Math. Phys. 144 pp. 443–490
J. I. Cirac, D. Pérez-García, N. Schuch, and F. Verstraete, Rev. Mod. Phys. 93, 045003 (2021).

Tensor Networks: A motivation

A general superposition state

$$|\psi_0\rangle = \sum_{\{i\}} c_{i_1,\ldots i_N} |i_1 \ldots i_N\rangle$$

An example system: Ising spins



How to get to polynomial scaling?

Can we just skip the small coefficients?

$$c^{0,0,1,0,1} = 0.3623$$
$$c^{0,1,1,0,1} = 0.0003$$
$$c^{1,0,0,0,0} = -0.0004$$
$$c^{0,1,0,0,1} = 0.5203$$

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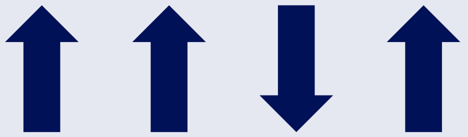

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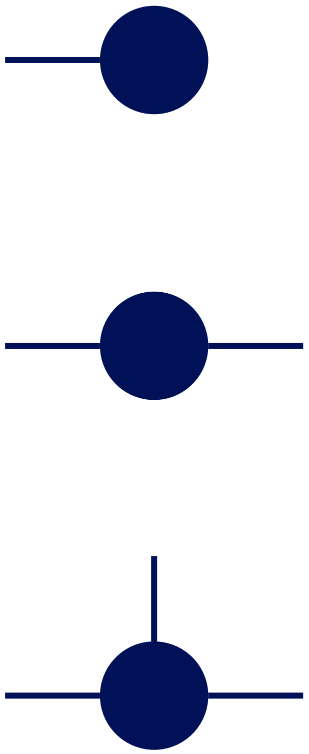
Tensor Network Notation



Vector

Matrix

Tensor

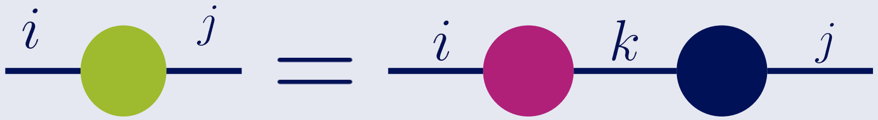
Dictionary

# of legs = rank of the object

Calculations with Pictures

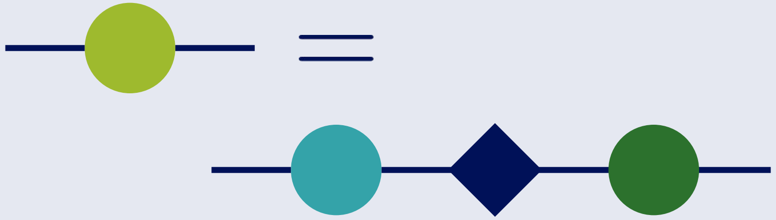
Matrix-Matrix Multiplication

$$C_{ij} = \sum_k A_{ik} B_{kj}$$



Singular Value Decomposition

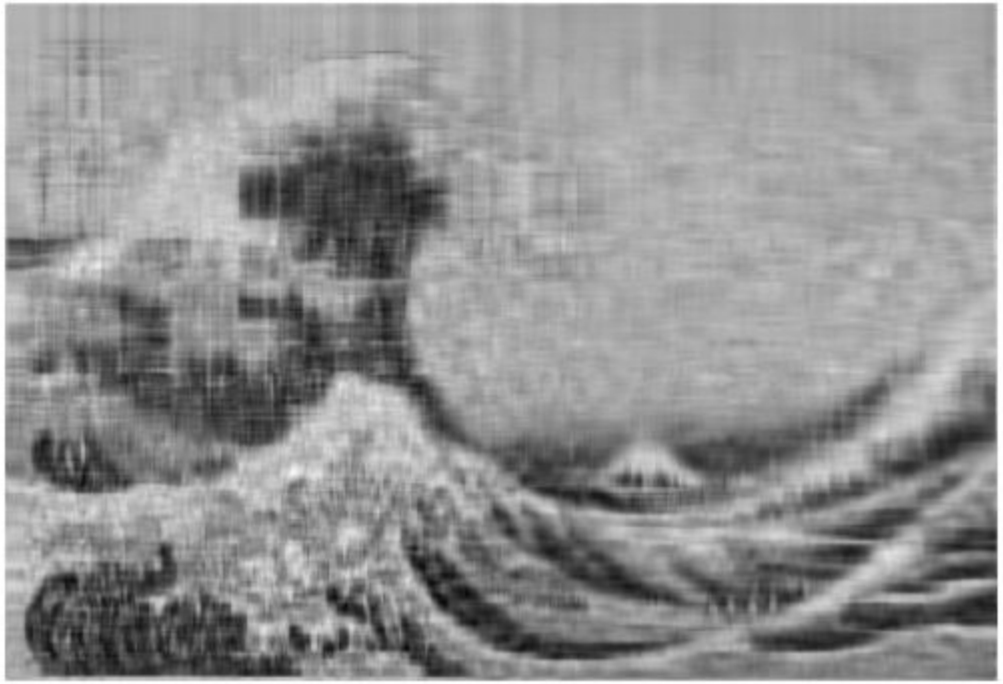
$$M = USV^\dagger$$



An idea of an SVD

Original Image

Truncated Image (20 SV)



An idea of an SVD

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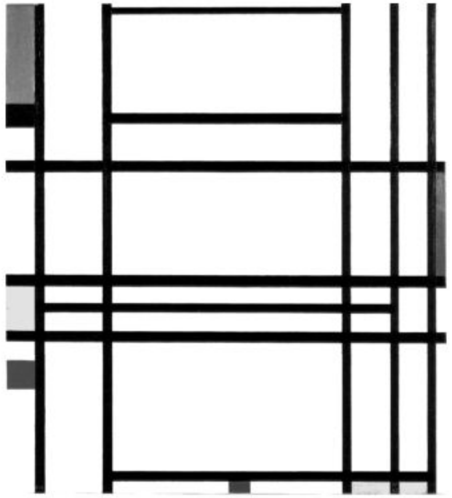

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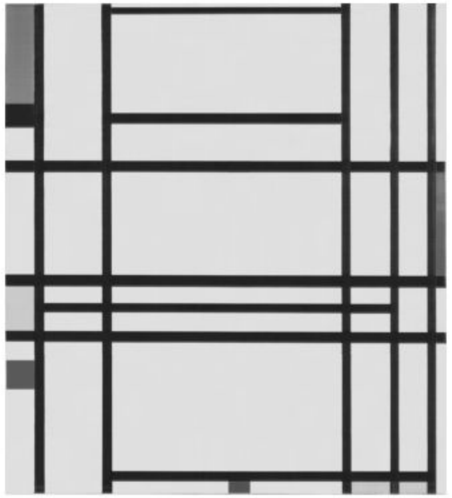
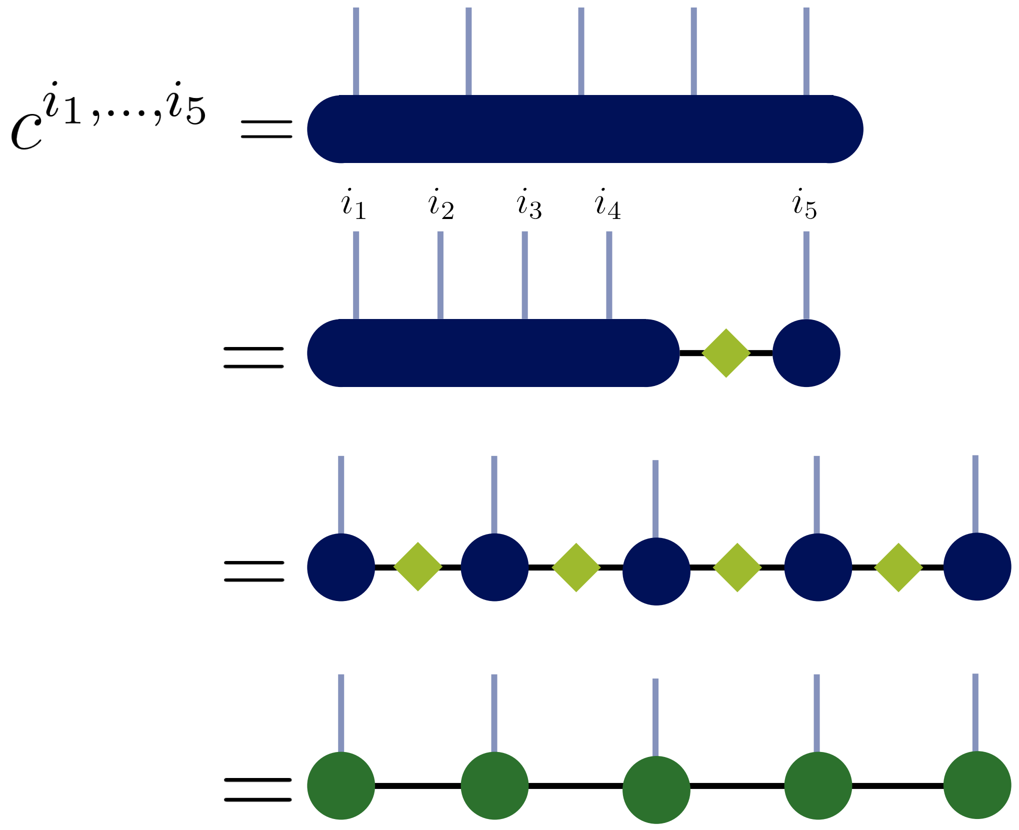


Construction of a state



A general superposition state

$$|\psi_0\rangle = \sum_{\{i\}} c_{i_1,\dots i_N} |i_1 \dots i_N\rangle$$

Matrix Product State

$$|\psi\rangle = \sum_{\{i\}} \sum_{\{\alpha\}} A^{i_1}_{1,\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} \cdots A^{i_N}_{\alpha_{N-1},1} |i_1, \cdots, i_N\rangle$$

Jacob C. Bridgeman and Christopher T. Chubb (2017) 50 p. 223001
Schollwöck, U. Annals of Physics 326, 96–192 (2011).

Putting it all together

Reducing to polynomially many parameters

Truncate to a virtual bond dimension D to reduce to polynomially many parameters.

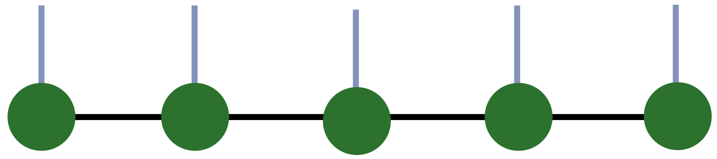
Notation

Physical index: i_j

Virtual Index: α_j

Tensors: D x D x d $A^{i}_{\alpha,\beta}$

$$|\psi\rangle = \sum_{\{i\}} \sum_{\{\alpha\}} A^{i_1}_{1,\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} \cdots A^{i_N}_{\alpha_{N-1},1} |i_1, \cdots, i_N\rangle$$

$=$ 

Take Home Message

The bond dimension controls the approximation.

What did we gain? – Counting Parameters

A general superposition state

$$|\psi_0\rangle = \sum_{\{i\}} c_{i_1,\ldots i_N} |i_1 \ldots i_N\rangle$$

Number of Parameters

$$d^N$$

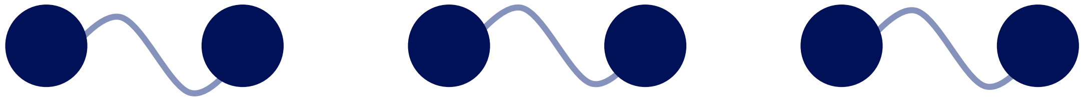
Matrix Product State

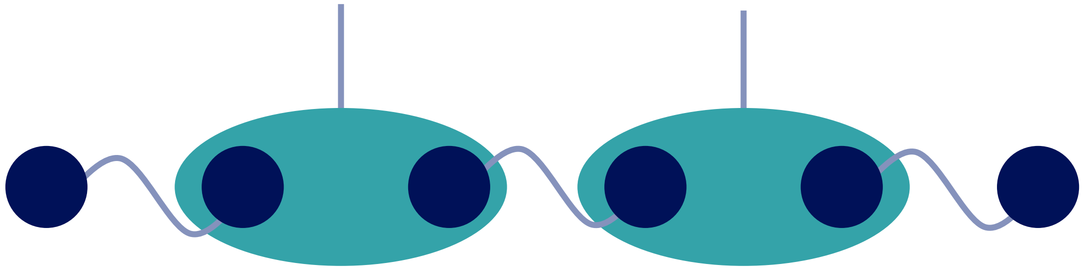
$$|\psi\rangle = \sum_{\{i\}} \sum_{\{\alpha\}} A^{i_1}_{1,\alpha_1} A^{i_2}_{\alpha_1,\alpha_2} \cdots A^{i_N}_{\alpha_{N-1},1} |i_1, \cdots, i_N\rangle$$

Number of Parameters

$$N(D \times D \times d)$$

MPS as 1D Projected Entangled Pair State (PEPS)





Entangled Pairs

$$|\Phi\rangle = \sum_{j=0}^{D-1} |jj\rangle$$

Example: Bell state

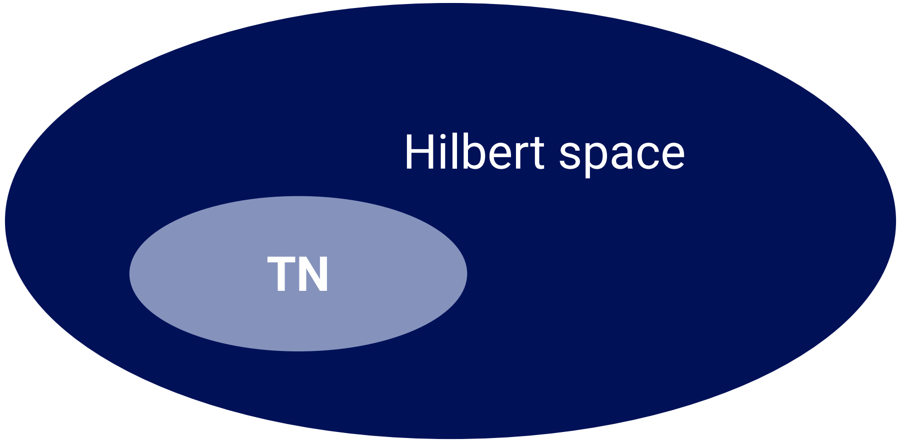
$$|\Phi\rangle = \frac{1}{\sqrt{2}} |00\rangle + |11\rangle$$

Projector

$$\omega = \sum_{i,\alpha,\beta} A^{i}_{\alpha,\beta} |i\rangle\langle\alpha\beta|$$

Affleck, I., Kennedy, T., Lieb, E. H. & Tasaki, H. Phys. Rev. Lett. 59, 799–802 (1987).

Why should we care?



Tensor networks efficiently approximate…

…ground states of local, gapped Hamiltonians.

M. B. Hastings, Phys. Rev. B 76, 035114 (2007).

II. Arad, A. Kitaev, Z. Landau, and U. Vazirani, arXiv:1301.1162.

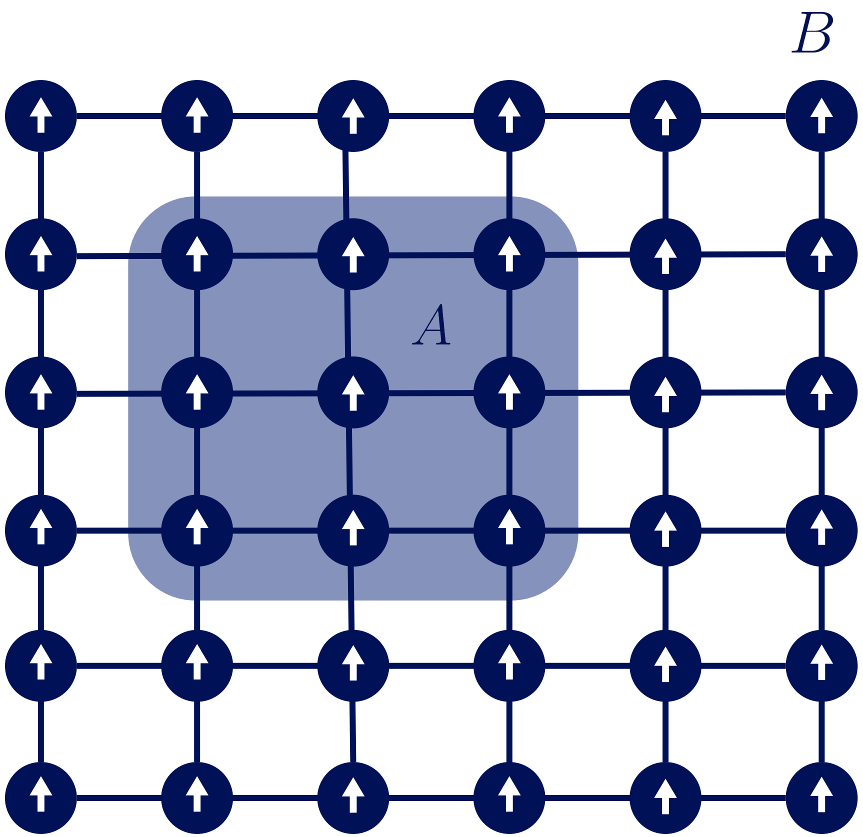
… states if their correlators decay quickly enough.

F. G. S. L. Brandao and M. Horodecki, Nature Phys **9**, 721 (2013).

… states with low entropy.

F. Verstraete and J. I. Cirac, Phys. Rev. B 73, (2006).

Entanglement Entropy

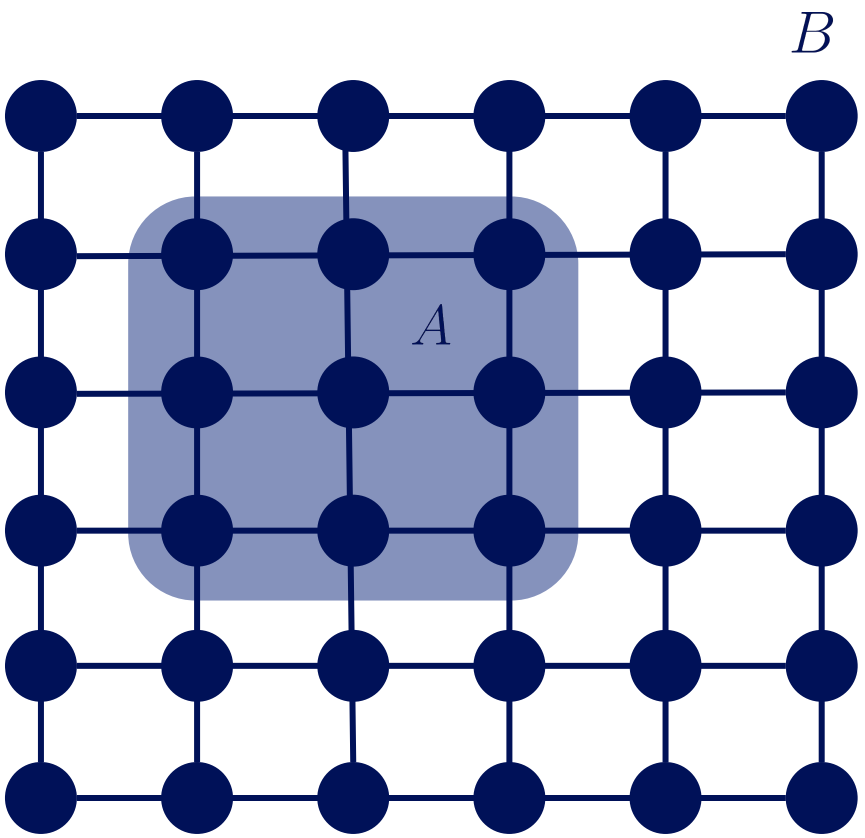


How does the entanglement scale?

Volume Law? $S \propto V$

Area Law? $S \propto \partial V$

Entanglement Entropy



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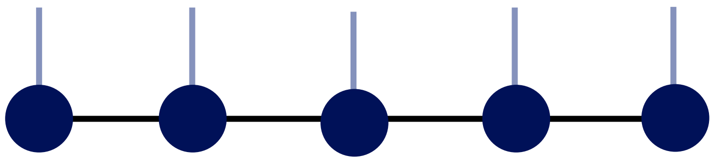
~~Volume Law?~~ $S \propto V$

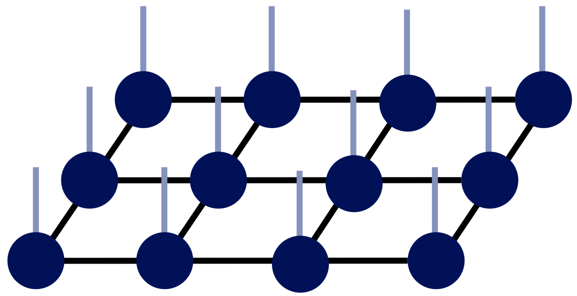
Area Law! $S \propto \partial V$

Different Families of Tensor Networks



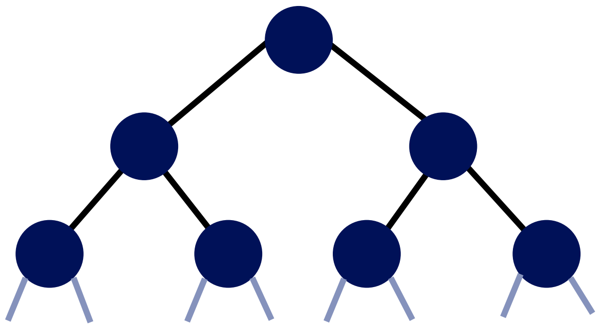
Matrix Product States (MPS)

M. Fannes, B. Nachtergaele, and R. F. Werner (1992)
Commun.Math. Phys. 144 pp. 443–490



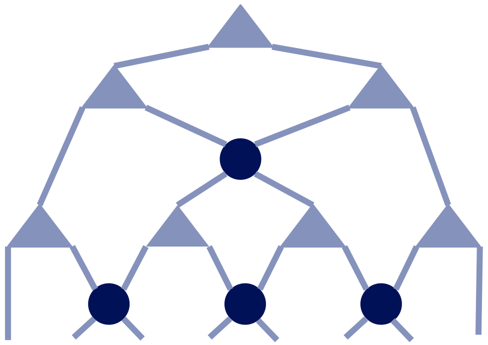
Projected Entangled Pair States (PEPS)

F. Verstraete and J. I. Cirac, arXiv:cond-mat/0407066.



Tree Tensor Network

Y.-Y. Shi, L.-M. Duan, and G. Vidal, Phys. Rev. A 74, 022320 (2006).



Multiscale Entanglement Renormalization Ansatz (MERA)

G. Vidal, Phys. Rev. Lett. 101, 110501 (2008).

Tensor Networks in Lattice Theories

Variational Methods

Compute an upper bound to the energy

$$E_{\text{var}} = \min_{\theta} \langle \psi(\theta) | H | \psi(\theta) \rangle$$

Renormalization Group Methods

Compute the partition sum

$$Z = \sum_{\{\sigma\}} e^{-\beta E(\sigma_1, \sigma_2, \ldots, \sigma_N)}$$

M. C. Bañuls and K. Cichy, Rep. Prog. Phys. **83**, 024401 (2020).

M. C. Bañuls, R. Blatt, J. Catani, et al., Eur. Phys. J. D **74**, 165 (2020).

Y. Meurice, R. Sakai, and J. Unmuth-Yockey, Rev. Mod. Phys. **94**, 025005 (2022).

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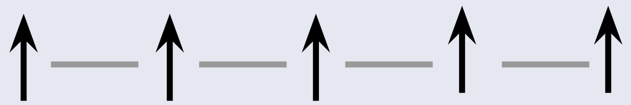
A short Intermezzo: Tensor renormalization group

The system



$$E = \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

M. C. Bañuls and K. Cichy, Rep. Prog. Phys. **83**, 024401 (2020).

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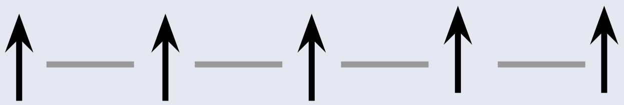


$$E = \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

The partition sum

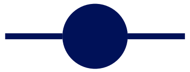
$$Z = \sum_{\{\sigma\}} \exp\left(-\beta \sum_i \sigma_i \sigma_{i+1}\right)$$

$$= \sum_{\{\sigma\}} \prod_i e^{-\beta \sigma_i \sigma_{i+1}}$$

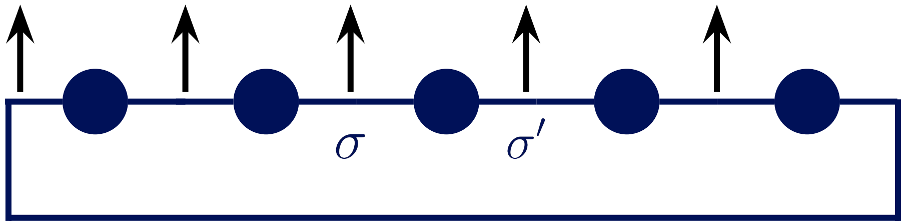
$$= \mathrm{Tr}(M^N)$$

$$M_{\sigma\sigma'} = e^{-\beta\sigma\sigma'}$$

$$=$$



$$Z =$$



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007).

Tensor renormalization group

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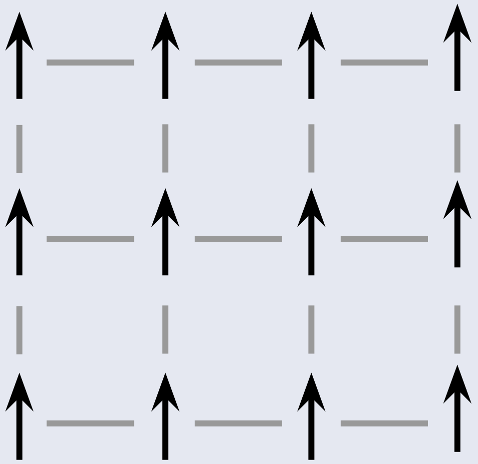
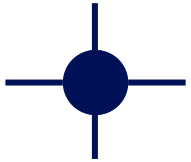
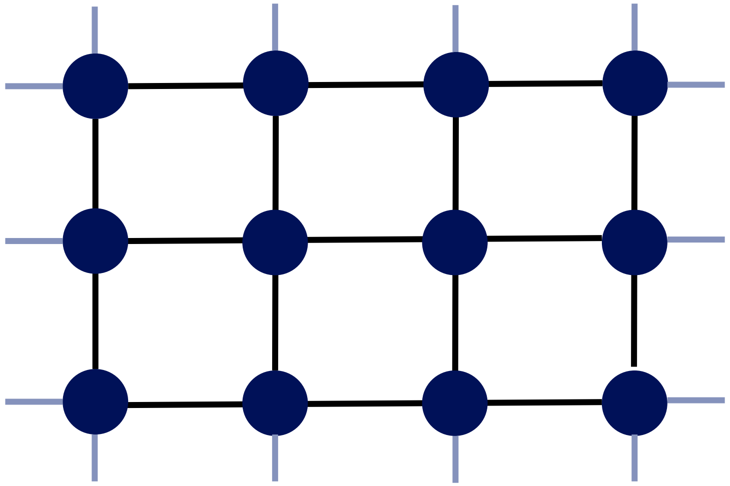


$$E = \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

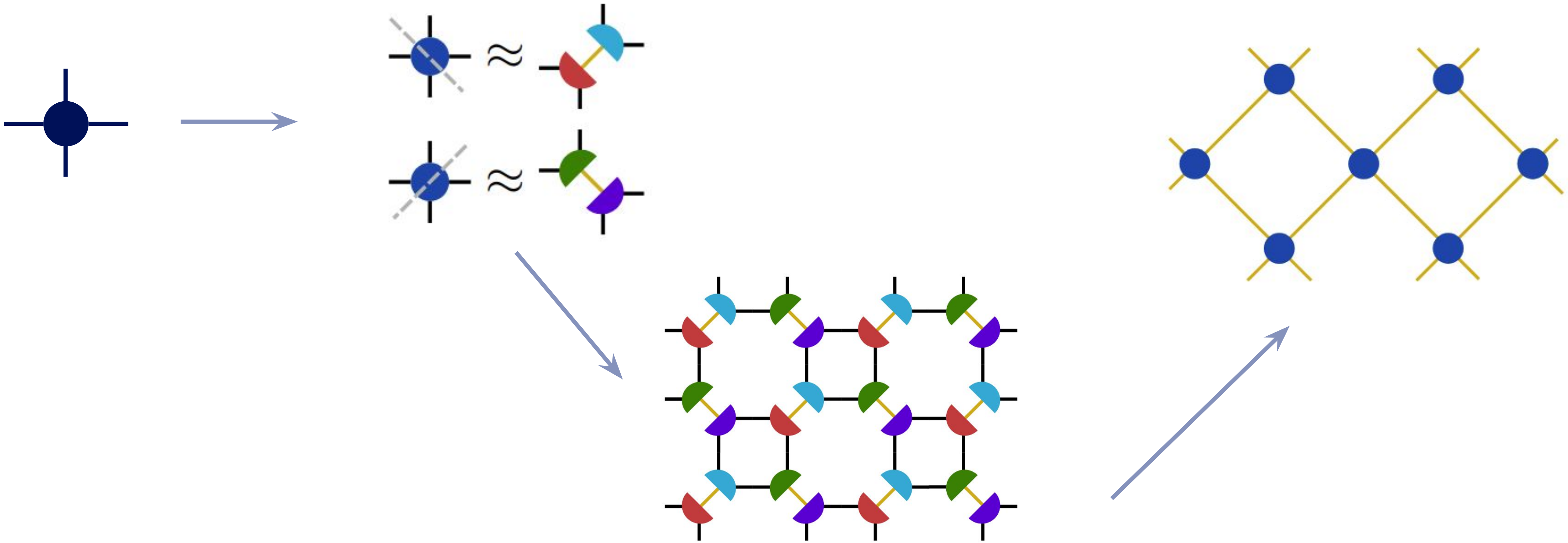
$$M_{\sigma_l \sigma_r \sigma_t \sigma_b} = e^{-\beta(\sigma_t\sigma_r + \sigma_r\sigma_b + \sigma_b\sigma_l + \sigma_l\sigma_t)}$$

$=$ 

$Z =$ 

M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007).

Tensor Renormalization Group



Z. Y. Xie, J. Chen, M. P. Qin, J. W. Zhu, L. P. Yang, and T. Xiang, Phys. Rev. B **86**, 045139 (2012).

D. Adachi, T. Okubo, and S. Todo, Phys. Rev. B 102, 054432 (2020).

S. Akiyama, D. Kadoh, Y. Kuramashi, T. Yamashita, and Y. Yoshimura, J. High Energ. Phys. 2020, 177 (2020).

Y. Meurice, R. Sakai, and J. Unmuth-Yockey, Rev. Mod. Phys. **94**, 025005 (2022).

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Y. Meurice, R. Sakai, and J. Unmuth-Yockey, Rev. Mod. Phys. **94**, 025005 (2022).

Back to Variational Approaches

early works with DMRG/TNS

Bymes PRD (2002)
Suihara NPB (2004)
Tagliacozzo PRB (2011)
Sugihara JHEP (2005)
Meurice PRB (2013)

3+1 d

Magnifico et al., Nat. Comm. 12 (2021)

2+1 d

Felser et al., PRX 10 (2020)
Robaina et al,. PRL 126 (2021)
Emonts et al., PRD 102 (2020)
Kelman et al., PRD 110 (2024)

Schwinger Model U(1) in 1+1 d

Banuls et al., JHEP 11 158 (2013)
Rico et al., PRL (2014)
Buyens et al., PRL (2014)
Kühn et al., PRA 90 (2014)
Banuls et al., PRD (2015)
Buyens et al., PRD (2016)
Picher alt al., PRX (2016)

Non-Abelian in 1D
string breaking dynamics

Kühn et al., JHEP 07 (2015)
Silvi et al., Quantum (2017)
Kühn et al., PRX (2017)

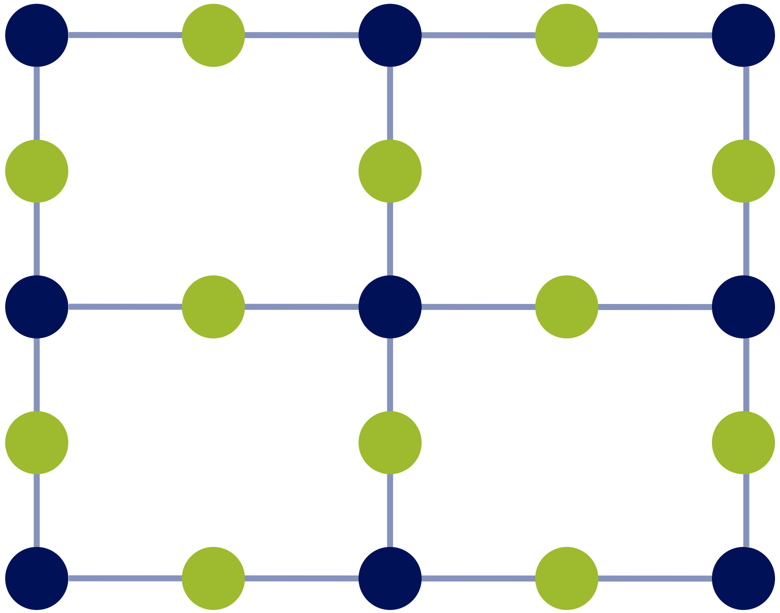
SU(3) Quantum Link Model

Sivli et al. PRD (2019)

and more …

Photo by Jack Anstey on Unsplash

Hilbert spaces and Lattices



Hilbert space

$$\mathcal{H} \subset \mathcal{H}_{\text{gauge fields}} \otimes \mathcal{H}_{\text{fermions}}$$

A general state

$$|\Psi\rangle = \int D\mathcal{G} |\mathcal{G}\rangle |\psi_F(\mathcal{G})\rangle$$

with $$D\mathcal{G} = \prod_{x,k} dg(x,k)$$

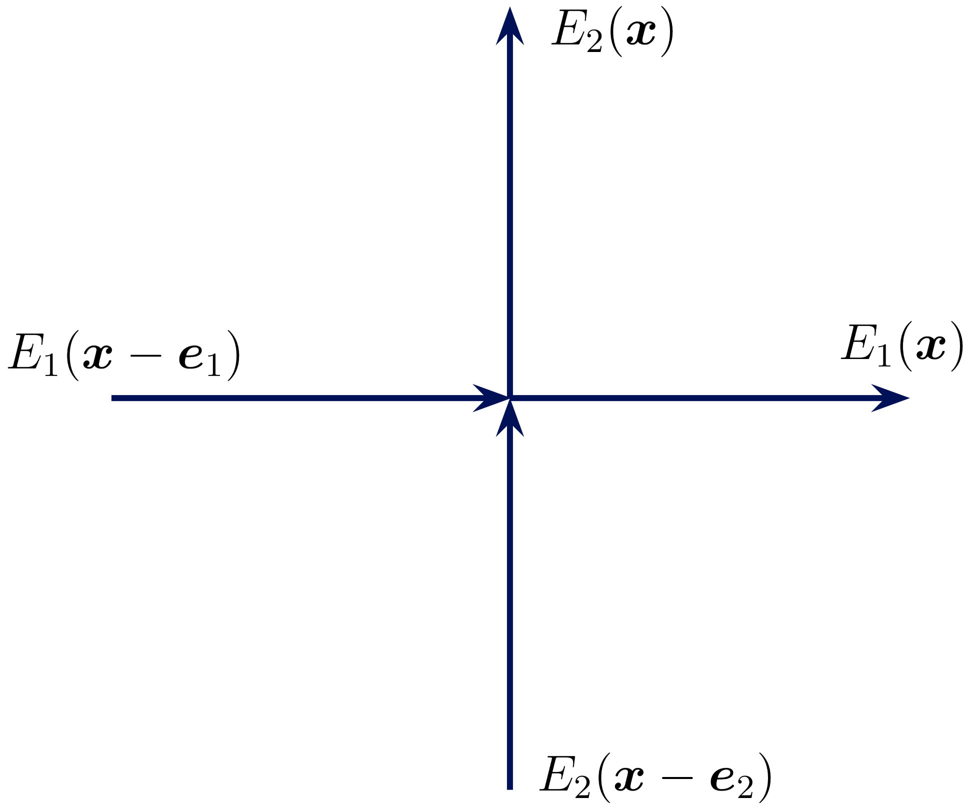
Gauss Law: A special system

Gauss Law

$$\sum_k \left(E_k(\boldsymbol{x}) - E_k(\boldsymbol{x} - \boldsymbol{e}_i)\right) |\text{phys}\rangle = 0 \quad \forall \boldsymbol{x}$$

Classical Analogue in Electrodynamics

$$\nabla \boldsymbol{E} = 0$$



Computing an Expectation Value

Assume that the observable acts only on the gauge field and is diagonal in the group element basis:

$$\langle\Psi|O|\Psi\rangle = \frac{\langle\Psi|O|\Psi\rangle}{\langle\Psi|\Psi\rangle}$$

$$= \frac{\int D\mathcal{G}\langle\mathcal{G}|O|\mathcal{G}\rangle\langle\psi_F(\mathcal{G})|\psi_F(\mathcal{G})\rangle}{\int \mathcal{G}'\langle\psi_F(\mathcal{G}')|\psi_F(\mathcal{G}')\rangle}$$

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$$\langle \Psi | O | \Psi \rangle = \frac{\langle \Psi | O | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

$$= \frac{\int D\mathcal{G} \langle \mathcal{G} | O | \mathcal{G} \rangle \langle \psi_F(\mathcal{G}) | \psi_F(\mathcal{G}) \rangle}{\int \mathcal{G}' \langle \psi_F(\mathcal{G}') | \psi_F(\mathcal{G}') \rangle}$$

$$= \int D\mathcal{G} \mathcal{F}_O(\mathcal{G}) p(\mathcal{G})$$

with $$p(\mathcal{G}) = \frac{\langle \psi_F(\mathcal{G}) | \psi_F(\mathcal{G}) \rangle}{\int D\mathcal{G}' \langle \psi_F(\mathcal{G}') | \psi_F(\mathcal{G}') \rangle}$$

A wishlist

Expectation value

$$\langle O \rangle = \int D\mathcal{G} \mathcal{F}_O(\mathcal{G}) p(\mathcal{G})$$

with $$p(\mathcal{G}) = \frac{\langle \psi_F(\mathcal{G}) | \psi_F(\mathcal{G}) \rangle}{Z}$$

ToDo List

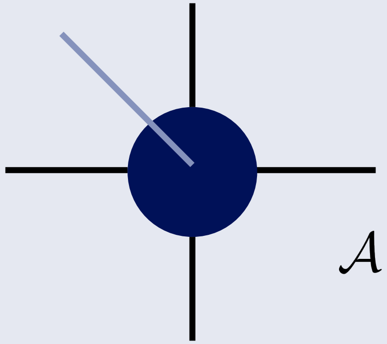
1. How do we construct $|\psi_F(\mathcal{G})\rangle$?
2. How do we efficiently calculate the probabilities?
3. Are those states useful?

Using a tensor network

Idea

Local tensors can lead to local gauge symmetry

Tensor



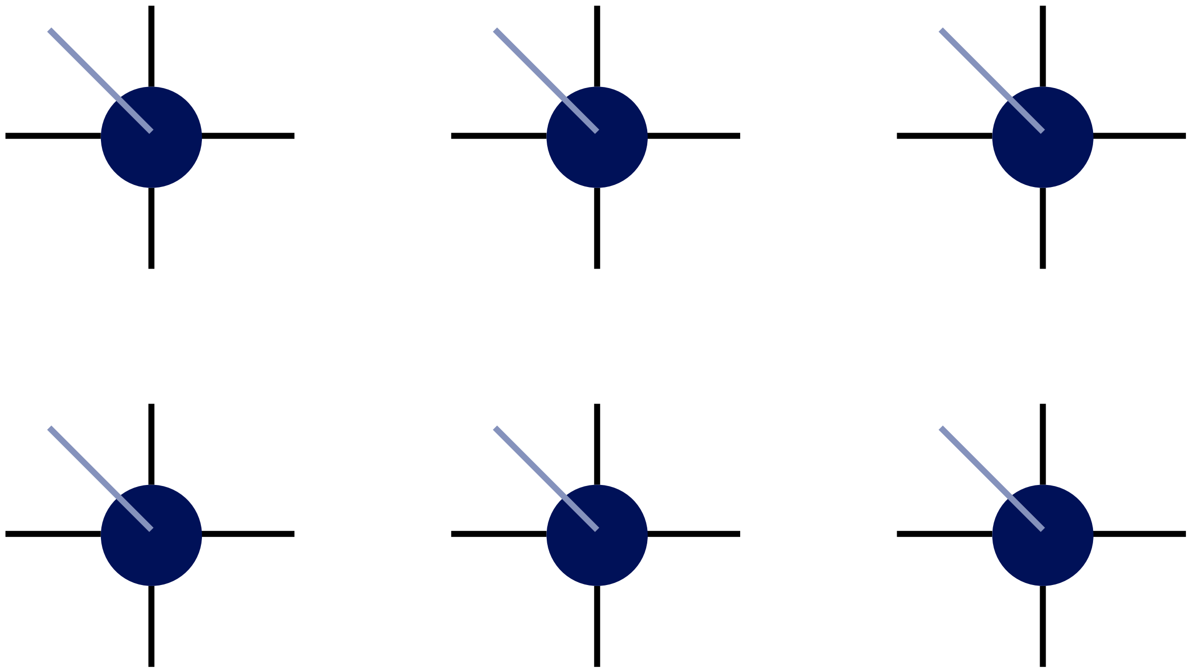
$$\mathcal{A}(\boldsymbol{x}) = \exp(T_{ij} a_i^\dagger(\boldsymbol{x}) b_j^\dagger(\boldsymbol{x}))$$

Projector

$$\omega(\boldsymbol{x}, k) = \omega_k(\boldsymbol{x}) \Omega_k(\boldsymbol{x}) \omega_k^\dagger(\boldsymbol{x})$$

$$\omega_0(\boldsymbol{x}) = \exp(l_+^\dagger(\boldsymbol{x}+\boldsymbol{e}_1) r_-^\dagger(\boldsymbol{x})) \exp(l_-^\dagger(\boldsymbol{x}+\boldsymbol{e}_1) r_+^\dagger(\boldsymbol{x}))$$

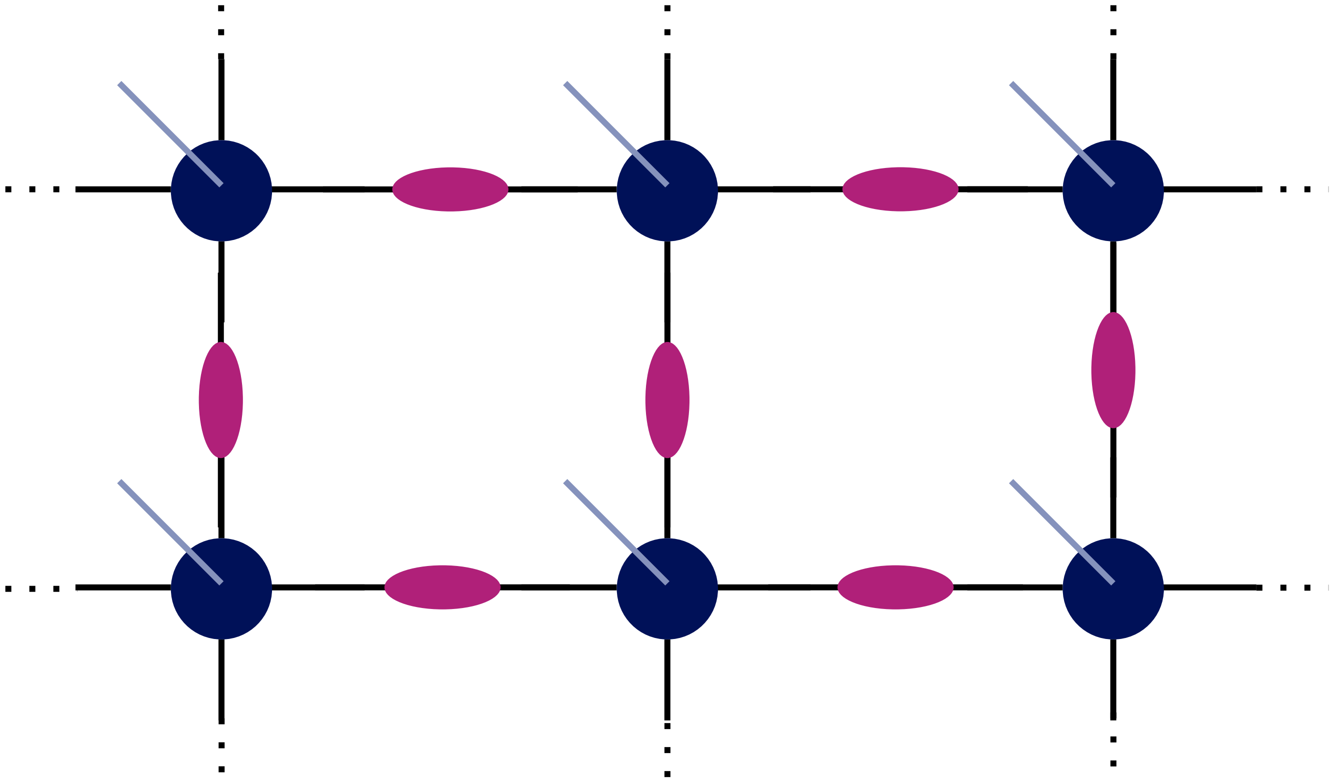
Building a state



Construction

$$\prod_{\boldsymbol{x}} \mathcal{A}(\boldsymbol{x}) |\Omega\rangle$$

Building a state

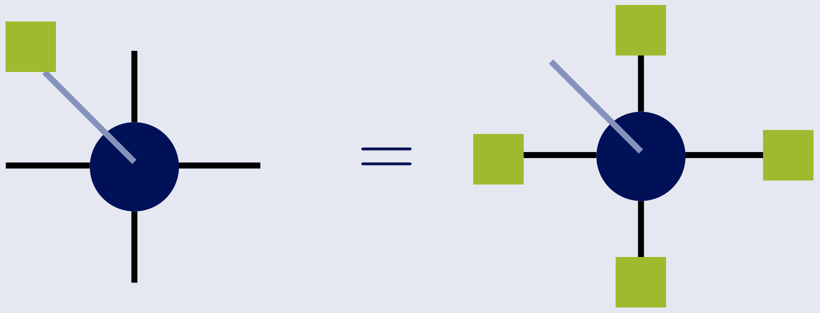


Construction

$$|\psi_0\rangle = \langle\Omega_v|\prod_{\ell}\omega_\ell\prod_{\boldsymbol{x}}\mathcal{A}(\boldsymbol{x})|\Omega\rangle$$

Global Gauge Invariance

Gauge invariance of the tensors



Acting on a physical degree of freedom with a gauge transformation is equivalent to acting on all auxiliary degrees of freedom.

Gauge invariance of the projectors

The projectors are invariant under auxiliary gauge transformations.

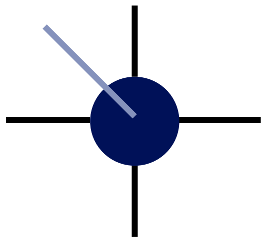
Local Gauge Invariance

Goal

Couple the gauge field to the state such that it is locally invariant under gauge transformations.

Gauging Procedure

$$|\psi_0\rangle = \langle\Omega_v| \prod_{\ell} \omega_\ell \prod_{x} \mathcal{A}(x)|\Omega\rangle$$



Local Gauge Invariance

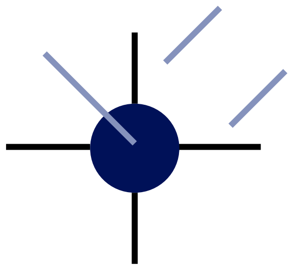
Goal

Couple the gauge field to the state such that it is locally invariant under gauge transformations.

Gauging Procedure

$$|\psi_0\rangle = \langle\Omega_v| \prod_\ell \omega_\ell \prod_x \mathcal{A}(x)|\Omega\rangle$$

$$\to \langle\Omega_v| \prod_\ell \omega_\ell \prod_x \mathcal{A}(x)|0\rangle_{x,1}|0\rangle_{x,2}|\Omega\rangle$$



Local Gauge Invariance

Goal

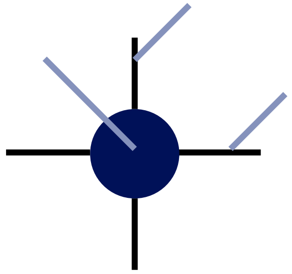
Couple the gauge field to the state such that it is locally invariant under gauge transformations.

Gauging Procedure

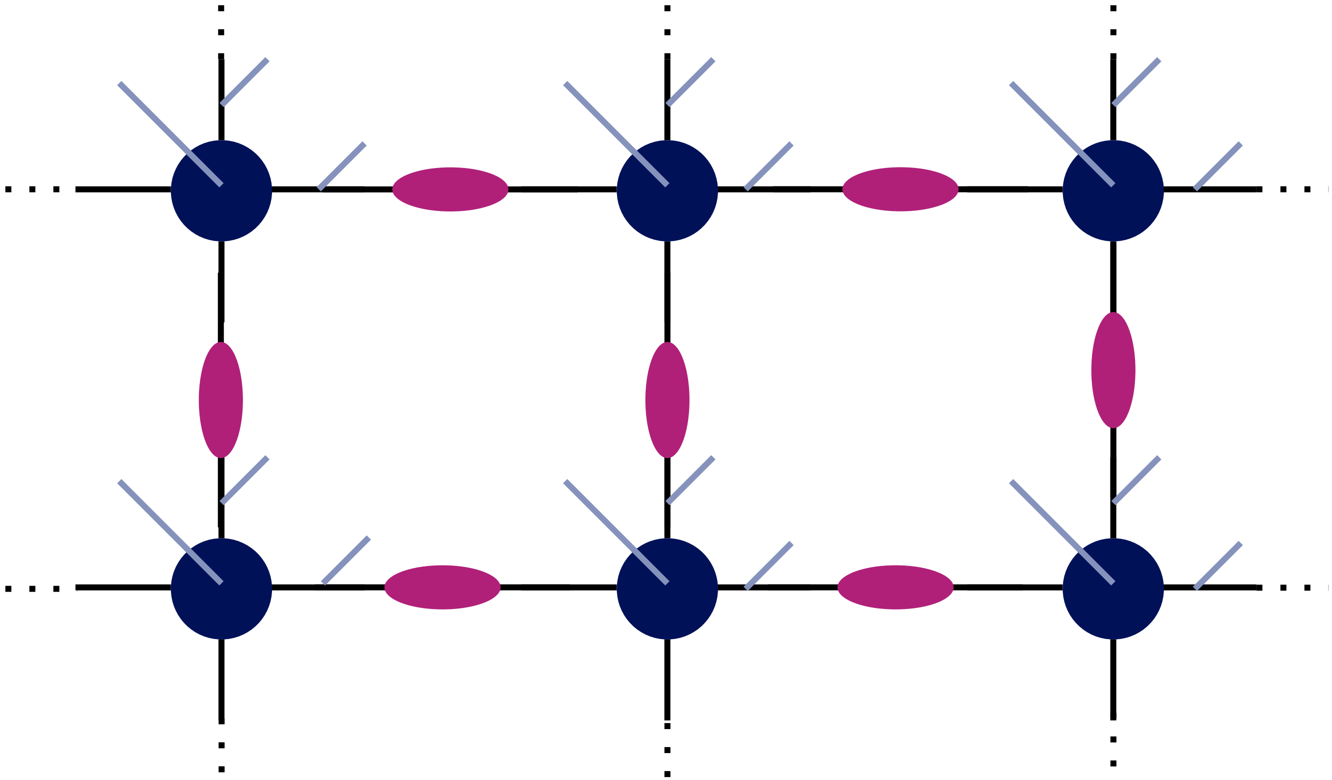
$$|\psi_0\rangle = \langle\Omega_v| \prod_{\ell} \omega_\ell \prod_{x} \mathcal{A}(x)|\Omega\rangle$$

$$\rightarrow \langle\Omega_v| \prod_{\ell} \omega_\ell \prod_{x} \mathcal{A}(x)|0\rangle_{x,1}|0\rangle_{x,2}|\Omega\rangle$$

$$|\psi\rangle = \langle\Omega_v| \prod_{\ell} \omega_\ell \prod_{x} \mathcal{U}_G(x,1)\mathcal{U}_G(x,2) \prod_{x} \mathcal{A}(x)|0\rangle_{x,1}|0\rangle_{x,2}|\Omega\rangle$$



Adding a local degree of freedom



Construction

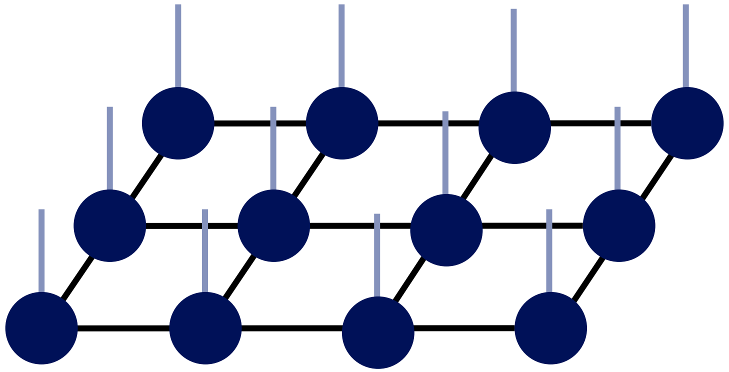
$$|\psi_F(\mathcal{G})\rangle = \langle \Omega_v| \prod_{\ell} \omega_\ell \prod_{\ell} U_\ell(\mathcal{G}) \prod_{\boldsymbol{x}} \mathcal{A}(\boldsymbol{x}) |\Omega\rangle$$

Who is afraid of norms?

ToDo List

1. How do we construct $|\psi_F(\mathcal{G})\rangle$? ✓
2. How do we efficiently calculate the probabilities?
3. Are those states useful?

$$p(\mathcal{G}) = \frac{\langle\psi_F(\mathcal{G})|\psi_F(\mathcal{G})\rangle}{\int D\mathcal{G}'\langle\psi_F(\mathcal{G}')|\psi_F(\mathcal{G}')\rangle}$$

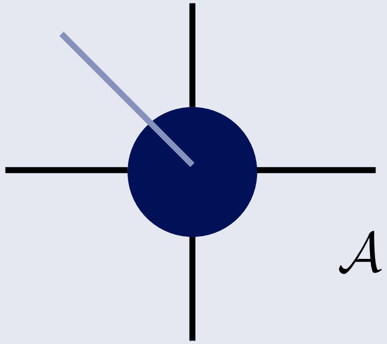


Exact PEPS contractions are #P hard

N. Schuch, M. M. Wolf, F. Verstraete, and J. I. Cirac, Phys. Rev. Lett. **98**, 140506 (2007).

A Gauged Gaussian PEPS

Tensor



$$\mathcal{A}(\boldsymbol{x}) = \exp(T_{ij} a_i^\dagger(\boldsymbol{x}) b_j^\dagger(\boldsymbol{x}))$$

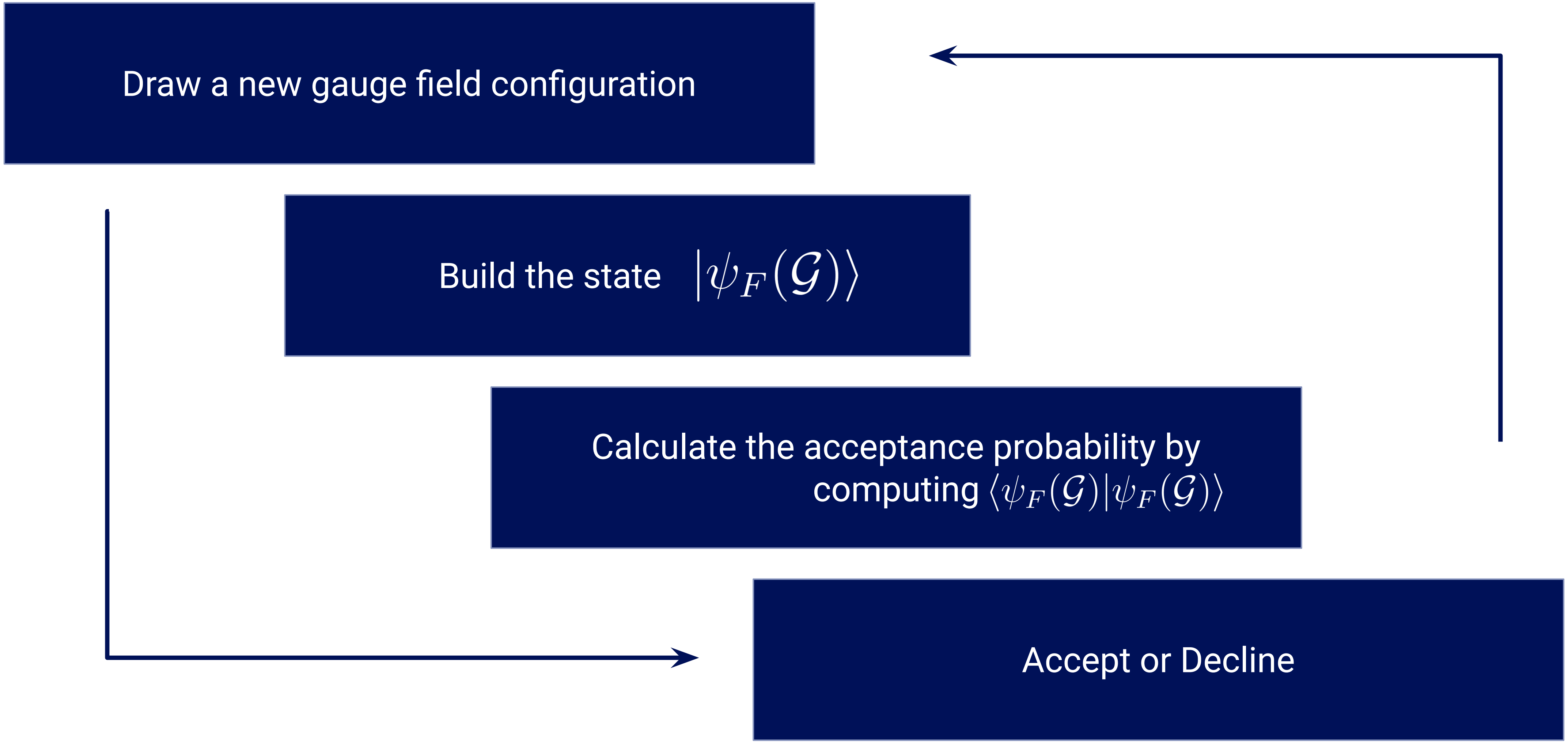
Projector

$$\omega(\boldsymbol{x}, k) = \omega_k(\boldsymbol{x}) \Omega_k(\boldsymbol{x}) \omega_k^\dagger(\boldsymbol{x})$$

$$\omega_0(\boldsymbol{x}) = \exp(l_+^\dagger(\boldsymbol{x} + \boldsymbol{e}_1) r_-^\dagger(\boldsymbol{x})) \exp(l_-^\dagger(\boldsymbol{x} + \boldsymbol{e}_1) r_+^\dagger(\boldsymbol{x}))$$

Use covariance matrices instead

The Algorithm



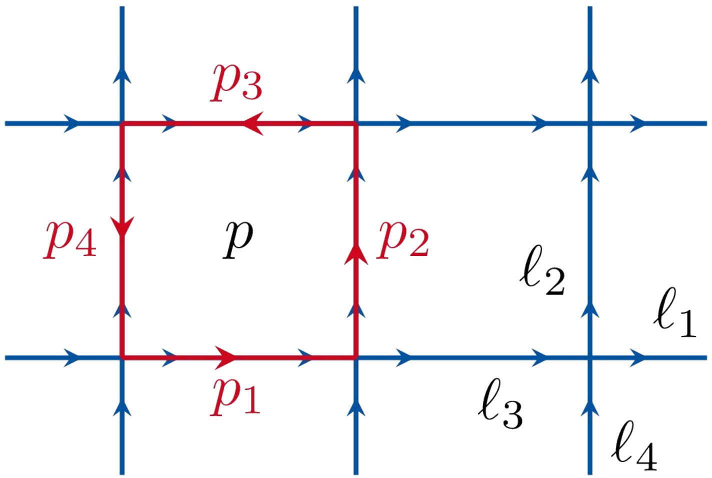
Numerical evidence

ToDo List

1. How do we construct $|\psi_F(\mathcal{G})\rangle$? ✓
2. How do we efficiently calculate the probabilities? ✓
3. Are those states useful?

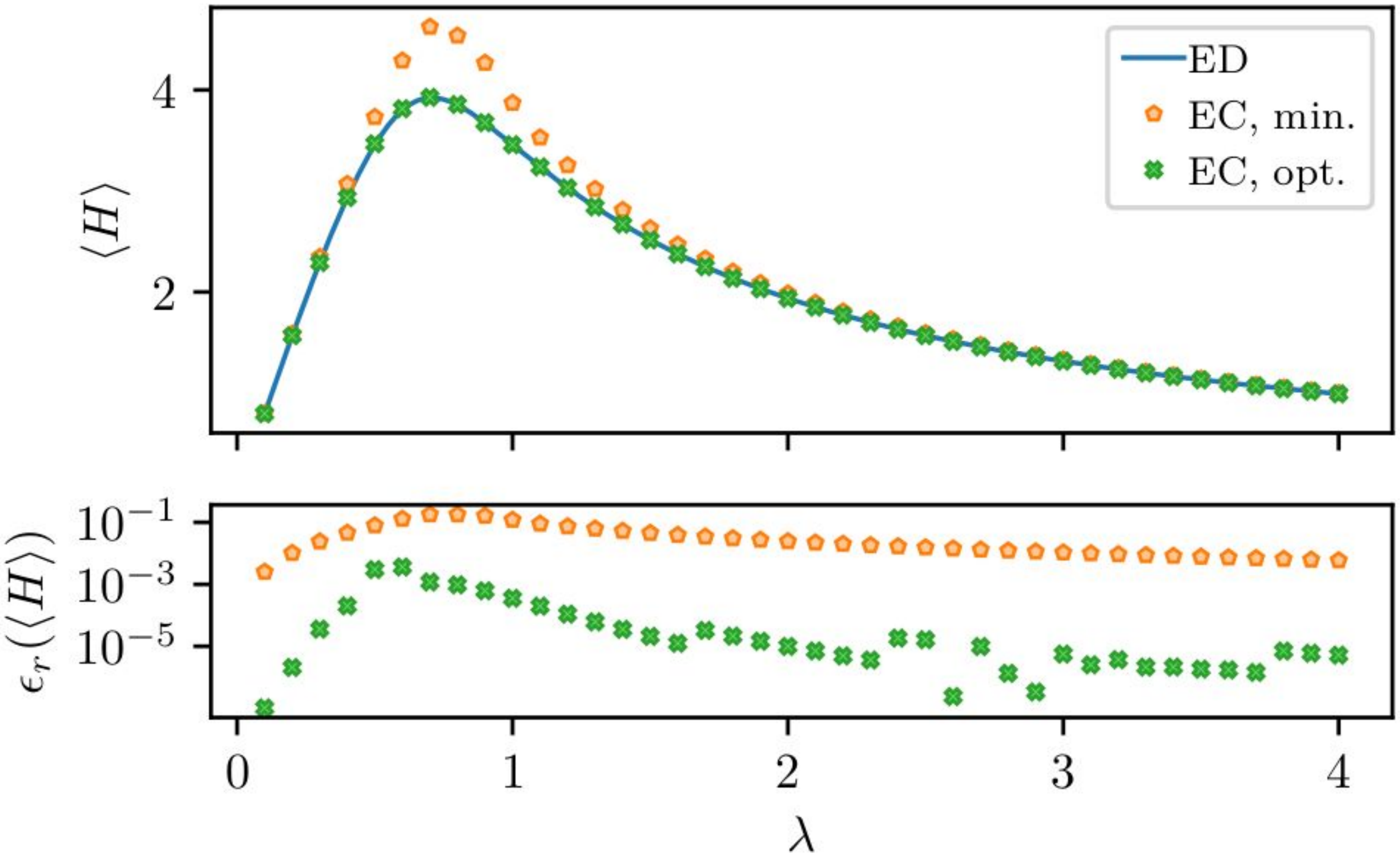
Kogut Susskind Hamiltonian

$$H = H_E + H_B$$
$$= \lambda \sum_{\ell} [1 - \sigma_\ell^z] + \frac{1}{\lambda} \sum_p \left[1 - \sigma_{p_1}^x \sigma_{p_2}^x \sigma_{p_3}^x \sigma_{p_4}^x\right]$$



John Kogut and Leonard Susskind (1975) Phys. Rev. D 11 pp. 395–408
D. Horn, M. Weinstein, and S. Yankielowicz (1979) Phys. Rev. D 19 pp. 3715–3731

Checking the Numerics



Minimal setting

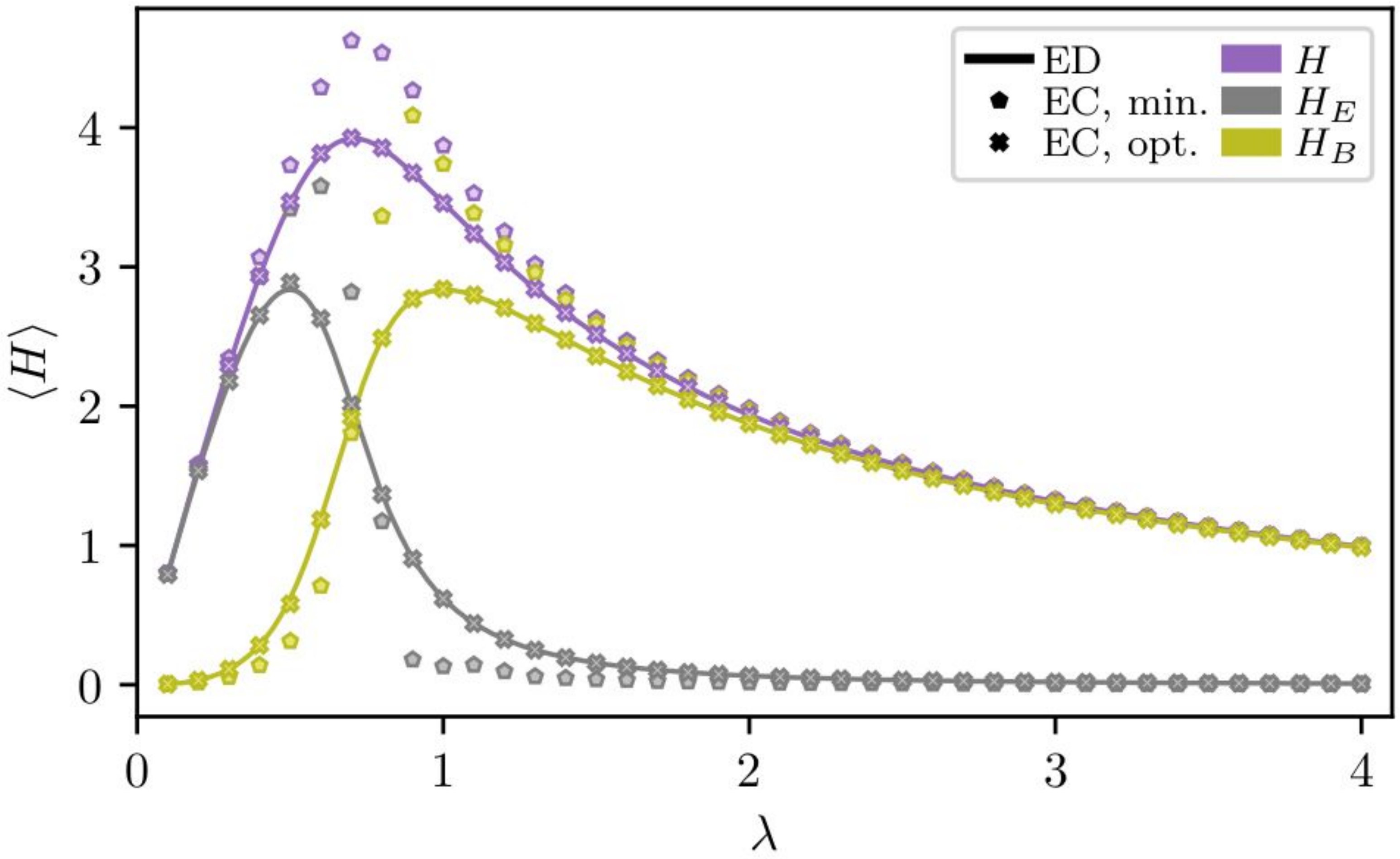
One virtual fermion per link

Optimized setting

Two virtual fermions per link

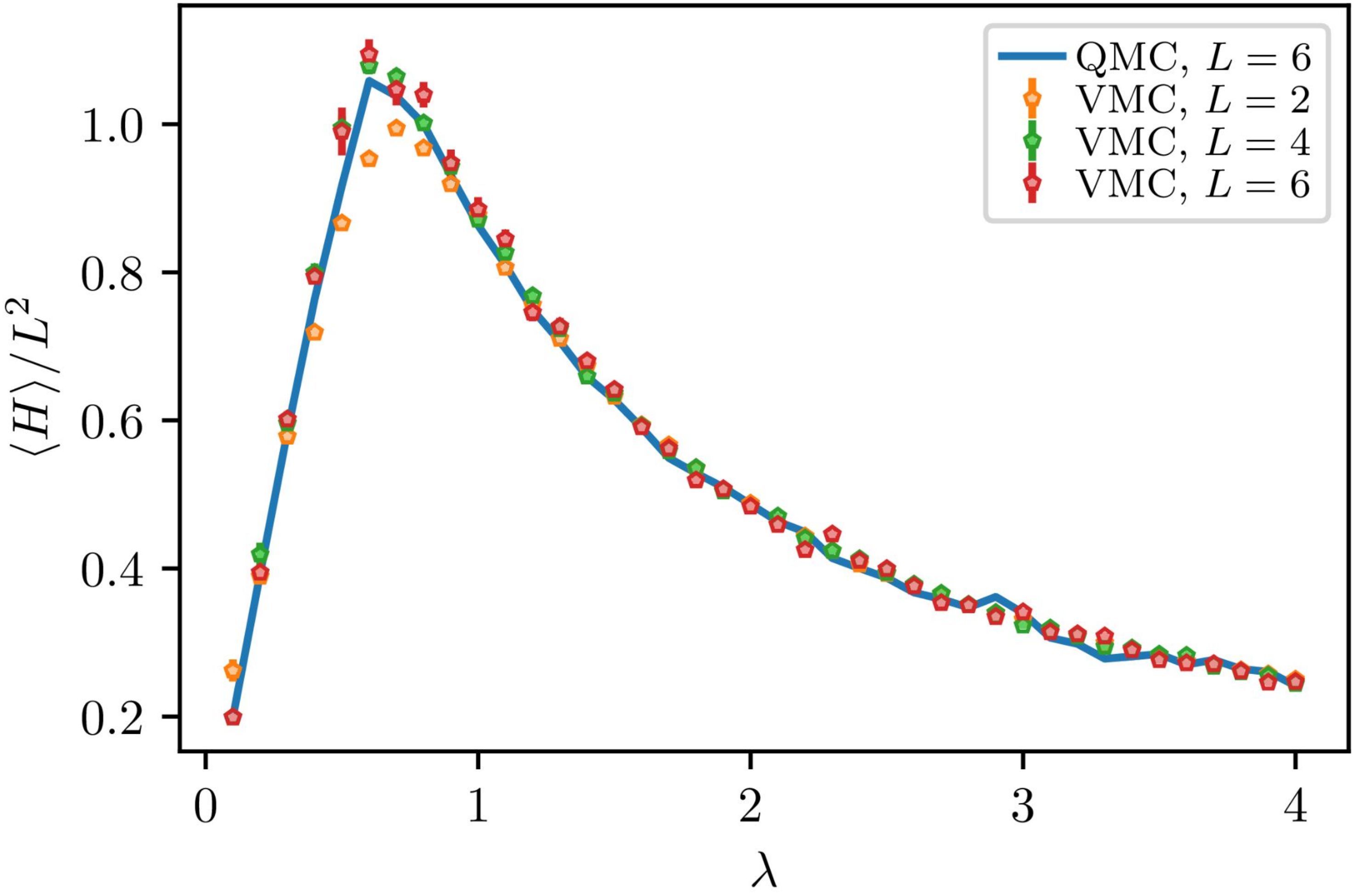
Emonts, P. et al. Phys. Rev. D 107, 014505 (2023).

Electric and Magnetic Energy



Emonts, P. et al. Phys. Rev. D 107, 014505 (2023).

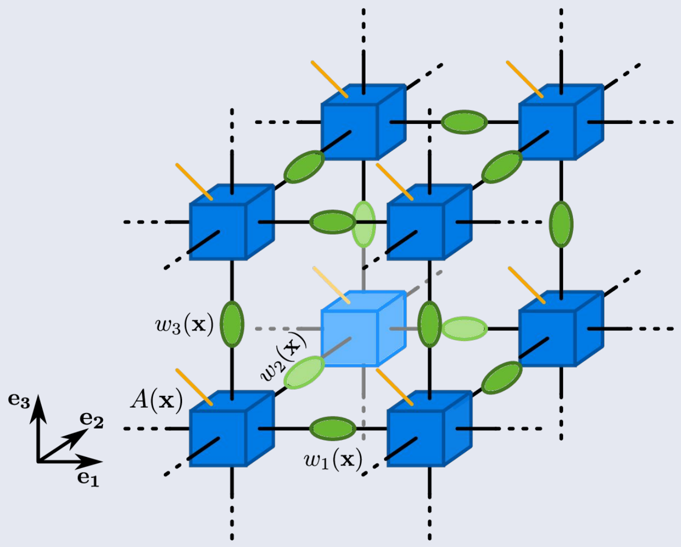
Monte Carlo for larger systems



Emonts, P. et al. Phys. Rev. D 107, 014505 (2023).

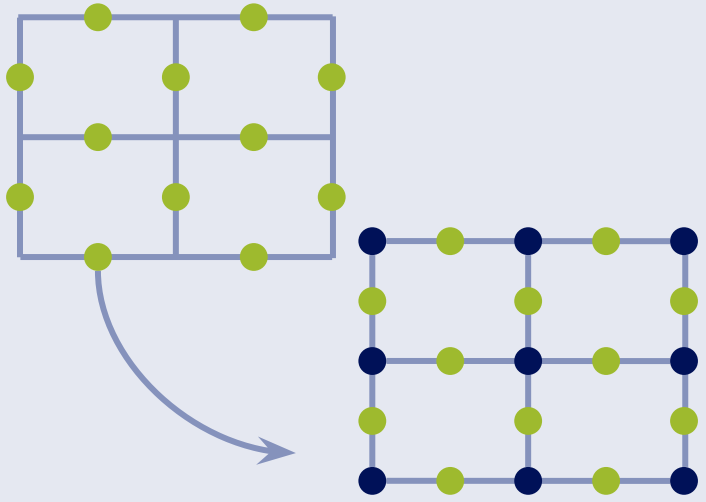
What are the next steps?

Three spatial dimensions?



P. Emonts and E. Zohar,. Phys. Rev. D 108, 014514 (2023).

Dynamic fermionic matter?



A. Kelman, U. Borla, P. Emonts, E. Zohar, in preparation (2024)

Non-Abelian Gauge Groups?

$$SU(2)$$

$$SU(3)$$

E. Zohar and J. I. Cirac, Phys. Rev. D 97, 034510 (2018).

Science is a team effort


Ariel Kelman


Umberto Borla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Snir Gazit


Ignacio Cirac


Erez Zohar


Mari Carmen Banuls

Summary and Outlook

Tensor networks are an interesting class of states

Tensor networks give analytical and numerical control

Tensor networks can complement lattice MC computations

GGPEPS are one option to contract high-dimensional networks efficiently

Tensor Networks for Lattice Gauge Theories

Patrick Emonts | HHIQCD 2024 | 01.11.2024 | Kyoto

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