Emergent Symmetry from Entanglement Suppression



- Ian Low
- Argonne/Northwestern
- Hadron and Hadrons Interactions in QCD 2024, Yukawa Institute
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Acknowledgement:



Office of Nuclear Physics

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- Thomas Mehen (Duke), Carlos Wagner (Argonne/Chicago), Marcela Carena (Fermilab/Chicago)

Entanglement is quantum world's most prominent feature:

- It refers to the situation where a measurement on a subsystem will improve our knowledge on the rest of the system.
- A quantum state of a system is entangled if it cannot be written as a tensor-product state of its subsystems.
- Consider a bipartite system $\mathcal{H}_{12} = \mathcal{H}_1 \otimes \mathcal{H}_2$, a state vector $|\psi\rangle \in \mathcal{H}_{12}$ is *entangled* if there is NO $|\psi_1\rangle \in \mathcal{H}_1$ and $|\psi_2\rangle \in \mathcal{H}_2$ such that

$$|\psi
angle = |\psi_1
angle \otimes |\psi_2
angle$$

Consider a system of two spin-1/2 particles.

• $|\uparrow\downarrow\rangle \equiv |\uparrow\rangle \otimes |\downarrow\rangle$ is an unentangled state:

Measurement of one spin would not change the outcome of the other.

(|↑↓⟩+ |↓↑⟩)/V2 is an entangled state:
 Measurement of the first spin would collapse the state into |↑↓⟩ or
 |↓↑⟩, which consequently determines the second spin.

Erwin Schrodinger coined the phrase "entanglement":



DISCUSSION OF PROBABILITY RELATIONS BETWEEN SEPARATED SYSTEMS

By E. SCHRÖDINGER

[Communicated by Mr M. BORN]

[Received 14 August, read 28 October 1935]

1. When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or ψ -functions) have become <u>entangled</u>. To disentangle them we must

Einstein famously attacked "entanglement" as spooky action at a distance:

MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, Institute for Advanced Study, Princeton, New Jersey (Received March 25, 1935)



A. Einstein



B. Podolsky



N. Rosen

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

The Nobel Prize in Physics 2022



The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science" On the other hand, symmetry is among the most fundamental principles in physics:

Powerful characterization of nature based on invariance under a specified group of transformations.

Symmetries give rise to conserved quantities: energy, momentum, angular momentum, etc.

Combining with quantum mechanics, there is a subtle realization of symmetry – spontaneous symmetry breaking.

All known fundamental interactions are based on symmetry principles.

Chen-Ning Yang famously coined the phrase:

Symmetry dictates Interaction!

- Lorentz invariance →
 Special Relativity
- General coordinate invariance →
 General Relativity
- Gauge invariance →
 QCD and Electroweak theory.



But what is the origin of symmetry?

There are two historical perspectives:

Beauty In, Garbage Out –

As we explore higher and higher energy regimes, we discover more and more symmetries. The symmetry is usually hidden or broken in low energies. But what is the origin of symmetry?

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Garbage In, Beauty Out – At high energy level there is no symmetry. Rather symmetry emerges only at large distances, in the infrared. These are emergent symmetries.

But neither explain whether symmetry can be the natural outgrowth of more fundamental principles.

John Wheeler famously coined the phrase:

It from bit : "All things physical are information-theoretic in origin"

INFORMATION, PHYSICS, QUANTUM: THE SEARCH FOR LINKS

John Archibald Wheeler * [†]

Abstract

This report reviews what quantum physics and information theory have to tell us about the age-old question, How come existence? No escape is evident from four



winnowing: It from bit. Otherwise put, every it — every particle, every field of force, even the spacetime continuum itself — derives its function, its meaning, its very existence entirely — even if in some contexts indirectly — from the apparatus-elicited answers to yes or no questions, binary choices [52], bits.

Indeed, we have seen remarkable connections between fundamental physics and information science in the past decade.

It is natural to ask:

0



0

In 2018 a group from Seattle made a fascinating observation regarding emergent symmetries and entanglement suppression in low-energy QCD:

Entanglement Suppression and Emergent Symmetries of Strong Interactions

Silas R. Beane,¹ David B. Kaplan,² Natalie Klco,^{1,2} and Martin J. Savage²

¹Department of Physics, University of Washington, Seattle, WA 98195-1560, USA ²Institute for Nuclear Theory, University of Washington, Seattle, WA 98195-1550, USA (Dated: December 10, 2018 - 1:30)

Entanglement suppression in the strong interaction S-matrix is shown to be correlated with approximate spin-flavor symmetries that are observed in low-energy baryon interactions, the Wigner SU(4) symmetry for two flavors and an SU(16) symmetry for three flavors. We conjecture that dynamical entanglement suppression is a property of the strong interactions in the infrared, giving rise to these emergent symmetries and providing powerful constraints on the nature of nuclear and hypernuclear forces in dense matter.

This raises the intriguing possibility of understanding symmetry from quantum entanglement!

In this talk, we will use low-energy QCD as the primary playground to study:

- Unexpected, emerging (approximate) global symmetries in low-energy hadronic physics.
- Connection between symmetry and entanglement suppression in nonrelativistic 2-to-2 scattering of fermions.
- Elucidate the connection from an information-theoretic viewpoint.

However, the lessons we learned seem to be quite general – will see examples ranging from two-Higgs-doublet-model to flavor physics.

Emergent symmetries in low-energy QCD:

• Schrodinger symmetry (non-relativistic conformal invariance)

The largest symmetry group preserved by the Schrodinger equation, which includes Galilean boosts, scale and special conformal transformations.

• Spin-flavor symmetries

Symmetries mixing flavor (internal) with spin (spacetime). Possible only in non-relativistic systems. Examples: SU(2N_f) quark spin-flavor symmetries; Wigner's "supermultiplet" SU(4) spin-flavor symmetry. In low-energy nuclear physics, Wigner observed the SU(4) spin-flavor symmetry in 1936:

 JANUARY 15, 1937
 PHYSICAL REVIEW
 VOLUME 51

 On the Consequences of the Symmetry of the Nuclear Hamiltonian on the Spectroscopy of Nuclei
 E. WIGNER*

 E. WIGNER*
 Princeton University, Princeton, New Jersey (Received October 23, 1936)

 The structure of the multiplets of nuclear terms is investigated, using as first approximation a Hamiltonian which does not involve the ordinary spin and corresponds to equal forces between all nuclear constituents, protons and neutrons. The multiplets turn out to have a

In this case the neutron and proton fill out a "supermultiplet":

$$N = \begin{pmatrix} p_{\uparrow} \\ p_{\downarrow} \\ n_{\uparrow} \\ n_{\downarrow} \end{pmatrix} \qquad N \to \mathcal{U}N \ , \quad \mathcal{U} \in SU(4)$$

Schrodinger Symmetry (NR conformal symmetry)

• Unnaturally large scattering lengths in low-energy NN scattering in the s-wave, which include ${}^{1}S_{0}$ and ${}^{3}S_{1}$ channels.

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In a non-relativistic QFT, the S-matrix is

$$S = e^{2i\delta(p)} = 1 + i \frac{Mp}{2\pi} \mathcal{A} , \qquad \qquad \mathcal{A} = \frac{4\pi}{M} \frac{1}{p \cot \delta - ip}$$

It is long known that it's $p \cot \delta$ which admits an expansion in 1/p, the Effective Range Expansion (ERE):

$$p \cot \delta = -\frac{1}{a} + \frac{1}{2} r_0 p^2 + \dots = -\frac{1}{a} + \frac{1}{2} \Lambda^2 \sum_{n=0}^{\infty} r_n \left(\frac{p^2}{\Lambda^2}\right)^n$$

The scattering length

In NN scattering:

- ${}^{1}S_{0}$: a₀ = -23.7 fm
- ${}^{3}S_{1}$: a_{1} = 5.4 fm \rightarrow Deuteron, which is a shallow, near-threshold bound state!
- $1/m_{\pi} = 1.4 \text{ fm}$



In the limit the scattering length *a* diverges, the system has no scale and exhibits Schrodinger symmetry, also known as the non-relativistic conformal invariance. Mehen, Stewart, Wise (1999)

At the infinitesimal level,

boosts:	$\vec{x}' = \vec{x} + \vec{v}t \;\;,$	t'=t ,
scale:	$\vec{x}' = \vec{x} + s\vec{x}$,	$t'=t+2st \ ,$
conformal:	$\vec{x}' = \vec{x} - ct\vec{x} \ ,$	$t' = t - ct^2 \; ,$

So NN scattering has approximate Schrodinger symmetry.

WHO ORDERED THAT??!

• Nucleons are part of spin-1/2 octet baryons (three-quark bound states):



QCD Lagrangian has SU(3) quark-flavor symmetry in limit $m_u = m_d = m_s$.

Under this SU(3), the spin-1/2 baryons form an eight-dimensional irreducible representation -- the octet representation.

• Nucleons are part of spin-1/2 octet baryons:

In the SU(3) flavor-symmetric limit :

$$B=egin{pmatrix} \Sigma^0/\sqrt{2}+\Lambda/\sqrt{6} & \Sigma^+ & p \ \Sigma^- & -\Sigma^0/\sqrt{2}+\Lambda/\sqrt{6} & n \ \Xi^- & \Xi^0 & -\sqrt{rac{2}{3}}\Lambda \end{pmatrix}$$

A low-energy effective field theory:

$$\begin{split} \langle \cdot \rangle &\equiv \mathrm{Tr}(\ \cdot \) \\ \mathcal{L}_{\mathrm{LO}}^{n_f=3} &= -\frac{c_1}{f^2} \langle B_i^{\dagger} B_i B_j^{\dagger} B_j \rangle - \frac{c_2}{f^2} \langle B_i^{\dagger} B_j B_j^{\dagger} B_i \rangle - \frac{c_3}{f^2} \langle B_i^{\dagger} B_j^{\dagger} B_i B_j \rangle - \frac{c_4}{f^2} \langle B_i^{\dagger} B_j^{\dagger} B_j B_i \rangle \\ &- \frac{c_5}{f^2} \langle B_i^{\dagger} B_i \rangle \langle B_j^{\dagger} B_j \rangle - \frac{c_6}{f^2} \langle B_i^{\dagger} B_j \rangle \langle B_j^{\dagger} B_i \rangle, \quad i, j = \uparrow, \downarrow \qquad \text{Savage, Wise (1995)} \end{split}$$

Lattice QCD could compute the six Wilson coefficients under some special circumstances:



INT-PUB-17-017, MIT-CTP-4912, NSF-ITP-17-076

Baryon-Baryon Interactions and Spin-Flavor Symmetry from Lattice Quantum Chromodynamics

Michael L. Wagman,^{1,2} Frank Winter,³ Emmanuel Chang, Zohreh Davoudi,⁴ William Detmold,⁴ Kostas Orginos,^{5,3} Martin J. Savage,^{1,2} and Phiala E. Shanahan⁴ (NPLQCD Collaboration)



m_π = 804 MeV

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ICCUB-20-020, UMD-PP-020-7, MIT-CTP/5238, INT-PUB-20-038 FERMILAB-PUB-20-498-T

Low-energy Scattering and Effective Interactions of Two Baryons at $m_\pi\sim 450$ MeV from Lattice Quantum Chromodynamics

Marc Illa,¹ Silas R. Beane,² Emmanuel Chang, Zohreh Davoudi,^{3,4} William Detmold,⁵ David J. Murphy,⁵ Kostas Orginos,^{6,7} Assumpta Parreño,¹ Martin J. Savage,⁸ Phiala E. Shanahan,⁵ Michael L. Wagman,⁹ and Frank Winter⁷ (NPLQCD Collaboration)



 m_{π} = 450 MeV

 m_{π} = 150 MeV in reality

In the limit where all coefficients but c_5 are vanishing:

$$\begin{aligned} \mathcal{L}_{\mathrm{LO}}^{n_{f}=3} &= -\frac{c_{1}}{f^{2}} \langle B_{i}^{\dagger} B_{j} B_{j}^{\dagger} B_{j} \rangle - \frac{c_{2}}{f^{2}} \langle B_{i}^{\dagger} B_{j} B_{j}^{\dagger} B_{i} \rangle - \frac{c_{3}}{f^{2}} \langle B_{i}^{\dagger} B_{j}^{\dagger} B_{i} B_{j} \rangle - \frac{c_{4}}{f^{2}} \langle B_{i}^{\dagger} B_{j}^{\dagger} B_{j} \rangle \\ &- \frac{c_{5}}{f^{2}} \langle B_{i}^{\dagger} B_{i} \rangle \langle B_{j}^{\dagger} B_{j} \rangle - \frac{c_{6}}{f^{2}} \langle B_{i}^{\dagger} B_{j} \rangle \langle B_{j}^{\dagger} B_{i} \rangle, \qquad i, j = \uparrow, \downarrow \end{aligned}$$

The remaining operator can be re-written,

$$\mathcal{B} = (n_{\uparrow}, n_{\downarrow}, p_{\uparrow}, p_{\downarrow}, \dots) \;, \qquad \mathcal{L} = -c_5 \left(\mathcal{B}^{\dagger} \mathcal{B}
ight)^2$$

which is invariant under an SU(16) spin-flavor symmetry $U = 16 \times 16$ unitary matrix!

There is no large N_c explanation!

To summarize, in low-energy QCD there exist several emergent global symmetries that are not symmetries of the fundamental QCD Lagrangian:

1. Wigner's SU(4) symmetry and approximate SU(16) symmetry as indicated by lattice simulations.

2. Approximate Schrodinger symmetry in NN scattering.

These are emergent symmetries in low-energy QCD. Our goal is to understand 3 and 4 from a quantum information-theoretic perspective.

To discuss entanglement suppression, we need to quantify the amount of entanglement \rightarrow Entanglement Measure!

Many possibilities for Entanglement Measure. For bipartite systems:

von Neumann entropy:
$$E(
ho) = -\mathrm{Tr}(
ho_1 \ln
ho_1) = -\mathrm{Tr}(
ho_2 \ln
ho_2)$$

Linear entropy:

$$E(\rho) = -\operatorname{Tr}(\rho_1(\rho_1 - 1)) = 1 - \operatorname{Tr}\rho_1^2$$

$$ho = |\psi
angle\langle\psi| \qquad \qquad
ho_{1/2} = {
m Tr}_{2/1}(
ho)$$

The common property is that the entanglement measure vanishes for a product state $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$, but attains the maximum for maximally entangled states (such as the Bell states.)

For a system with two spin-1/2 particles, let's define the "computational basis:"

 $\{ |\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle \}_{1}$

Then for a general normalized state,

 $|\psi\rangle = \alpha \left|\uparrow\uparrow\right\rangle + \beta \left|\uparrow\downarrow\right\rangle + \gamma \left|\downarrow\uparrow\right\rangle + \delta \left|\downarrow\downarrow\right\rangle, \qquad |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1 \ ,$

The reduced density matrix and linear entropy are

$$\rho_1 = \operatorname{Tr}_2 |\psi\rangle \langle \psi| = \begin{pmatrix} |\alpha|^2 + |\beta|^2 & \alpha\gamma^* + \beta\delta^* \\ \alpha^*\gamma + \beta^*\delta & |\gamma|^2 + |\delta|^2 \end{pmatrix},$$

Easy to check that $E(|\psi\rangle) = 1 - \text{Tr}_1 \rho_1^2 = 2|\alpha \delta - \beta \gamma|^2$.

- 1. It vanishes for a product state.
- 2. Maximal entanglement is 1/2, which is the case for the Bell states:

$$(|\uparrow\uparrow\rangle\pm|\downarrow\downarrow\rangle)/\sqrt{2}$$
 $(|\uparrow\downarrow\rangle\pm|\downarrow\uparrow\rangle)/\sqrt{2}$

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$$(|\uparrow\uparrow\rangle\pm|\downarrow\downarrow\rangle)/\sqrt{2}$$
 $(|\uparrow\downarrow\rangle\pm|\downarrow\uparrow\rangle)/\sqrt{2}$

 $2|\alpha\delta - \beta\gamma|$ is the "concurrence" in QIS literature.

Entanglement is a property of the quantum state.

But we are more interested in the ability of a *quantum-mechanical operator* (i.e. the S-matrix) to entangle.

However, there is a subtlety here, as the amount of entanglement generated by an operator could depend on the initial state.

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But we are more interested in the ability of a *quantum-mechanical operator* (i.e. the S-matrix) to entangle.

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Consider the CNOT (controlled NOT) gate in the computational basis: $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

CNOT does not entangle any of the basis state. However,

$$\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} \otimes |\uparrow\rangle \xrightarrow{\text{CNOT}} \frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}}$$

The "entanglement power" deals with this issue is by averaging over the initial states:

$$E(U) = \overline{E(U |\psi_1\rangle \otimes |\psi_2\rangle)},$$

For quibts, the average is over the Bloch sphere.

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A minimally entangling operator has E(U) = 0, i.e.,

$$| \rangle \otimes | \rangle \xrightarrow{U} | \rangle \otimes | \rangle$$

There is, however, a notion of equivalent classes in this definition:

$$U \sim U'$$
 if $U = (U_1 \otimes U_2)U'(V_1 \otimes V_2)$

LOCC does not change entanglement!

Modulo the equivalent class, there are two and only two minimally entangling operators, which in the computational basis,

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad \text{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Identity gate: do nothing. SWAP gate: interchange the qubits.

$$SWAP \sim -1$$
 as $[SWAP]^2 = 1$

In terms of Pauli matrices,

SWAP =
$$(1 + \boldsymbol{\sigma} \cdot \boldsymbol{\sigma})/2$$
, $\boldsymbol{\sigma} \cdot \boldsymbol{\sigma} \equiv \sum_{a} \boldsymbol{\sigma}^{a} \otimes \boldsymbol{\sigma}^{a}$.

Low, Mehen: 2104.10835
In the scattering process the S-matrix acts on the IN-state:

$$|\mathrm{out}\rangle = S |\mathrm{in}\rangle$$

For 2-to-2 scattering of spin-1/2 fermions, the S-matrix can be viewed as a quantum logic gate acting on the spin-space:



The distinction between "entanglement in a state" and "entanglement power in an operator" cannot be over-emphasized.

A lot of literature computed the entanglement in the final state as a function of scattering angle, instead of entanglement power:









2306.03239

Consider the scattering of two qubits, Alice and Bob, in the low-energy:

- Only the s-wave channel dominates.
- The S-matrix can be decomposed into ${}^{1}S_{0}$ and ${}^{3}S_{1}$ channels \rightarrow there are two phase shifts: δ_{0} and δ_{1} , respectively.
- Rotational invariance and Unitarity then uniquely fix the S-matrix:



Low, Mehen: 2104.10835

In terms of quantum logic gates,

$$S = \frac{1}{2} \left(e^{2i\delta_1} + e^{2i\delta_0} \right) \mathbf{1} + \frac{1}{2} \left(e^{2i\delta_1} - e^{2i\delta_0} \right) \text{ SWAP},$$

Conditions for the S-matrix to minimize entanglement: 1. S = 1 if $\delta_0 = \delta_1 \implies SU(4)$ or SU(16) spin-flavor sym. 2. S = SWAP if $|\delta_0 - \delta_1| = \pi/2 \implies Schrodinger sym.$

Low, Mehen: 2104.10835

This is precisely the observation of the Seattle group:



Slide by D.B. Kaplan

Let's extend the analysis to other spin-1/2 baryons, which have a rich theoretical structure and phenomenology:

-- Scattering of two baryons in general will change flavors, unlike in the nucleon scattering.

-- In the limit of exact SU(3) flavor symmetry, Pauli exclusion principle forces the two-baryon wave function to be totally anti-symmetric \rightarrow an interesting interplay between flavor and spin.

We will consider the spin-1/2 octet baryons:



2-to-2 scattering contains 64 channels, but group theory says:



We will focus on the spin-1/2 octet baryons:



2-to-2 scattering contains 64 channels, but group theory says:



	0	<u> </u>		0	<u> </u>	_		0	<u>a</u>
	Q	S		Q	S	_		Q	S
nn	0	0	$\Sigma^{-}\Sigma^{-}$	-2	-2		$\Sigma^{-}\Xi^{-}$	-2	-3
np	1	0	$\Sigma^{-}\Lambda$			_	$\Sigma^{-}\Xi^{0}$		
pp	2	0	$\Sigma^{-}\Sigma^{0}$	-1	-2		$\Xi^{-}\Sigma^{0}$	-1	-3
$n\Sigma^{-}$	-1	-1	$n \Xi^-$			_	$\Xi^-\Lambda$		
$n\Lambda$			$\Sigma^+\Sigma^-$				$\Xi^{-}\Sigma^{+}$		
$n\Sigma^0$	0	-1	$\Sigma^0 \Sigma^0$				$\Xi^0\Lambda$	0	-3
$p\Sigma^-$			$\Lambda\Sigma^0$				$\Xi^0 \Sigma^0$		
$p\Lambda$			$\Lambda\Lambda$	0	-2	_	$\Xi^0\Sigma^+$	1	-3
$p\Sigma^0$	1	-1	$n\Xi^0$				$\Xi^{-}\Xi^{-}$	-2	-4
$n\Sigma^+$			$p \Xi^-$			_	$\Xi^-\Xi_0$	-1	-4
$p\Sigma^+$	2	-1	$\Sigma^+\Lambda$			-	$\Xi_0 \Xi_0$	0	-4
			$\Sigma^+\Sigma^0$	1	-2	_			
			$p\Xi^0$						
			$\Sigma^+\Sigma^+$	2	-2				

The S-matrix is block-diagonal among different (Q,S) sectors.



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The S-matrix is block-diagonal among different (Q,S) sectors.

6-d	sector
0-u	Sector

	Q	S		Q	S		Q	S
nn	0	0	$\Sigma^{-}\Sigma^{-}$	-2	-2	$\Sigma^- \Xi^-$	-2	-3
np	1	0	$\Sigma^{-}\Lambda$			$\Sigma^- \Xi^0$		
pp	2	0	$\Sigma^{-}\Sigma^{0}$	-1	-2	$\Xi^{-}\Sigma^{0}$	-1	-3
$n\Sigma^-$	-1	-1	$n\Xi^-$			$\Xi^-\Lambda$		
$n\Lambda$			$\Sigma^+\Sigma^-$			$\Xi^{-}\Sigma^{+}$		
$n\Sigma^0$	0	-1	$\Sigma^0 \Sigma^0$			$\Xi^0\Lambda$	0	-3
$p\Sigma^-$			$\Lambda\Sigma^0$			$\Xi^0\Sigma^0$		
$p\Lambda$			Γ ΛΛ	0	-2	$\Xi^0\Sigma^+$	1	-3
$\bar{p}\Sigma^0$	1	-1	$n\Xi^0$			$\Xi^-\Xi^-$	-2	-4
$n\Sigma^+$			${igsim} p\Xi^-$			$\Xi^{-}\Xi^{0}$	-1	-4
$p\Sigma^+$	2	-1	$\Sigma^+\Lambda$			$\Xi_0\Xi_0$	0	-4
			$\Sigma^+\Sigma^0$	1	-2		1	
			$p\Xi^0$					
			$\Sigma^+\Sigma^+$	2	-2			
	$egin{array}{c c} nn \ np \ pp \ n\Sigma^- \ n\Lambda \ n\Sigma^0 \ p\Sigma^- \ p\Lambda \ p\Sigma^0 \ n\Sigma^+ \ p\Sigma^+ \ p\Sigma^+ \end{array}$	$egin{array}{c c c c c c } & Q & & & & & & & & & & & & & & & & & $	$\begin{array}{c c c c c c c c c c } & Q & S \\ \hline nn & 0 & 0 \\ \hline np & 1 & 0 \\ \hline np & 1 & 0 \\ \hline pp & 2 & 0 \\ \hline n\Sigma^- & -1 & -1 \\ \hline n\Lambda & & & \\ n\Sigma^0 & 0 & -1 \\ \hline p\Sigma^- & & & \\ \hline p\Sigma^- & & & \\ \hline p\Sigma^- & & & \\ \hline p\Sigma^+ & 1 & -1 \\ \hline n\Sigma^+ & & & \\ \hline p\Sigma^+ & 2 & -1 \\ \hline \end{array}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $

The S-matrix is block-diagonal among different (Q,S) sectors.

A summary table on possible emerging symmetries:

Flavor Subspace	Symmetry of Lagrangian
np	
$\Sigma^{-}\Xi^{-}$	SU(6) spin-flavor symmetry
$\Sigma^+ \Xi^0$	or conformal symmetry in 27 and $\overline{10}$ irrep channels
$n\Sigma^-$	
$p \Sigma^+$	conjugate of $SU(6)$ spin-flavor symmetry
$\Xi^- \Xi^0$	or conformal symmetry in 27 and 10 irrep channels
$(p\Lambda,p\Sigma^0,n\Sigma^+)$	
$(n\Lambda,n\Sigma^0,p\Sigma^-)$	
$(\Sigma^{-}\Lambda, \Sigma^{-}\Sigma^{0}, n \Xi^{-})$	SO(8) flavor symmetry
$(\Sigma^+\Lambda, \Sigma^+\Sigma^0, p\Xi^0)$	or conformal symmetry in $27, 8_{\mathbf{S}}, 8_{\mathbf{A}}, 10$ and $\overline{10}$
$(\Sigma^- \Xi^0, \Xi^- \Sigma^0, \Xi^- \Sigma^0)$	irrep channels
$\left(\Xi^{-}\Sigma^{+},\Xi^{0}\Lambda,\Xi^{0}\Sigma^{0} ight)$	
$\overline{(\Sigma^+\Sigma^-,\Sigma^0\Sigma^0,\Lambda\Sigma^0,\Xi^-p,\Xi^0n,\Lambda\Lambda)}$	SU(16) symmetry
	or $SU(8)$ and conformal symmetry

TABLE V. Symmetries predicted by entanglement minimization in each flavor sector.

A summary table on possible emerging symmetries:

Flavor Subspace	Symmetry of Lagrangian
np	
$\Sigma^- \Xi^-$	SU(6) spin-flavor symmetry
$\Sigma^+ \Xi^0$	or conformal symmetry in 27 and $\overline{10}$ irrep channels
$n\Sigma^-$	
$p \Sigma^+$	conjugate of $SU(6)$ spin-flavor symmetry
$\Xi^- \Xi^0$	or conformal symmetry in 27 and 10 irrep channels
$(p\Lambda,p\Sigma^0,n\Sigma^+)$	
$(n\Lambda,n\Sigma^0,p\Sigma^-)$	
$(\Sigma^{-}\Lambda, \Sigma^{-}\Sigma^{0}, n \Xi^{-})$	SO(8) flavor symmetry
$(\Sigma^+\Lambda, \Sigma^+\Sigma^0, p\Xi^0)$	or conformal symmetry in $27, 8_{\mathbf{S}}, 8_{\mathbf{A}}, 10$ and $\overline{10}$
$(\Sigma^- \Xi^0, \Xi^- \Sigma^0, \Xi^- \Sigma^0)$	irrep channels
$(\Xi^-\Sigma^+,\Xi^0\Lambda,\Xi^0\Sigma^0)$	
$\overline{(\Sigma^+\Sigma^-,\Sigma^0\Sigma^0,\Lambda\Sigma^0,\Xi^-p,\Xi^0n,\Lambda\Lambda)}$	SU(16) symmetry
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TABLE V. Symmetries predicted by entanglement minimization in each flavor sector.

What does the data say? Let's look at hyperon-nucleon interactions! Understanding YN interactions has broad implications:

- Understand hypernuclear structures and hyperon matters
- NN and YN interactions together give a unified understanding of baryonbaryon interactions.
- The formation of heavy neutron star is not supported by current theory/modeling of the core → The hyperon puzzle

- It turns out there are global fits of scattering phases using YN data, based on the meson-exchange potential models and xEFT.
- E40 collaboration at J-PARC also fitted the scattering phases in (Sigma+, p) scattering:



Fig. 28. Obtained phase shifts $\delta_{{}^{3}S_{1}}$ and $\delta_{{}^{1}P_{1}}$ as a function of the incident momentum. The black dashed, green solid, and blue dotted lines represent the calculated phase shifts of ESC16 [16], NSC97f [8], and fss2 [6], respectively.

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Data do not yet have the discriminating power to break the sign degeneracy in 3S1 channel!

We considered the S=-1 hyperons:

Q	-1	0	1	2
Flavor	$\Sigma^{-}n$	$ig \Lambda n, \Sigma^0 n, \Sigma^- p$	$ig \Lambda p, \Sigma^0 p, \Sigma^+ n$	$\Sigma^+ p$
Total	2137	$\Lambda n:2055$	$\Lambda p:2054$	2128
Mass (MeV)		$\Sigma^0 n:2132$	$\Sigma^+ n: 2129$	
		$\Sigma^- p:2136$	$\Sigma^0 p:2131$	

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We stay below the pion production Threshold:

Pion production process	p_{CM} (MeV/c)	$p_{lab}~({\rm MeV/c})$
$\Lambda n o \Lambda p \pi^-$	382.8	893.9
$\Sigma^+ p ightarrow \Sigma^+ n \pi^+$	390.3	943.4

Recall (Lambda, p) and (Lambda, n) are related by isospin invariance. They share similar features.

Q. Liu and IL: 2312.02289

One outlier is (Sigma+, p) channel, where differing global fits give different results:



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One outlier is (Sigma+, p) channel, where differing global fits give different results:



For (Lambda+, p), we proposed a "quantum observable" which could break the degeneracy among different global fits:



FIG. 4. Predicted polarizations of the recoiling Σ^+ (recoil) and the recoiling p (target) in Σ^+ p scattering, assuming an unpolarized proton target and a 25% polarized hyperon beam. Very recently there's a study on entanglement suppression in the scattering of charmed mesons and the associated exotic mesons X(3872) and $T_{cc}(3875)^+$:

 X(3872) can be described as a shallow bound state (hadronic molecule) of D and Dbar* mesons.

$$M_X - M_{D^0} - M_{\bar{D}^{*0}} = 0.00^{+0.09}_{-0.15} \,\mathrm{MeV}$$

• $T_{cc}(3875)^+$ is conjectured to be a bound state of D^{*}D mesons.

$$M_{T_{cc}^+} - M_{D^{*+}} - M_{D^0} = (-0.36 \pm 0.04) \,\mathrm{MeV}$$

 Entanglement suppression predicts the existence of a new symmetry – the light-quark spin symmetry.

This symmetry predicts 5 and 1 isoscalar partners of X(3872) and $T_{cc}(3875)^+$, respectively. (HQSS predicts 3 and 1 partners.)

TABLE II. Partners of the X(3872) predicted by HQSS or the two solutions of entanglement suppression given in Eqs. (53) and (54). The symbol " \odot " denotes the input X(3872), " \otimes " represents its predicted partners, " \oslash " indicates no near-threshold state is allowed, and "-" signifies that no prediction can be made without further inputs. Moreover, " \oplus " means that the corresponding meson pair needs to be mixed with another one to get a spin partner of X(3872), see Eqs. (57) and (58).

Channel	HQSS		Eq. (53)	predictions	Eq. (54) predictions	
	I = 0	I = 1	I = 0	I = 1	I = 0	I = 1
$Dar{D}(0^{++})$	\oplus	_	\otimes	\oslash	\otimes	\otimes
$Dar{D}^*(1^{++})$	\odot	_	\odot	\oslash	\odot	\otimes
$Dar{D}^*(1^{+-})$	\oplus	_	\otimes	\oslash	\otimes	\otimes
$D^*ar{D}^*(0^{++})$	\oplus	_	\otimes	\oslash	\otimes	\otimes
$D^* ar{D}^* (1^{+-})$	\oplus	_	\otimes	\oslash	\otimes	\otimes
$D^* \bar{D}^* (2^{++})$	\otimes	_	\otimes	\oslash	\otimes	\otimes

Predictions of entanglement suppression!

Next we will consider a very different physical system...

The Great Success of the Higgs Boson!



H⁰

J = 0

Mass $m = 125.09 \pm 0.24$ GeV Full width $\Gamma < 1.7$ GeV, CL = 95%

H^0 Signal Strengths in Different Channels

See Listings for the latest unpublished results.

Combined Final States = 1.10 ± 0.11 $W W^* = 1.08^{+0.18}_{-0.16}$ $Z Z^* = 1.29^{+0.26}_{-0.23}$ $\gamma \gamma = 1.16 \pm 0.18$ $b \overline{b} = 0.82 \pm 0.30$ (S = 1.1) $\mu^+ \mu^- < 7.0$, CL = 95% $\tau^+ \tau^- = 1.12 \pm 0.23$ $Z \gamma < 9.5$, CL = 95% $t \overline{t} H^0$ Production = $2.3^{+0.7}_{-0.6}$

The LHC data favors a SM-like Higgs boson



Nature 607, no. 7917, 52-59; 60-68 (2022) [arXiv:2207.00092]; [arXiv:2207.00043]

The SM does not explain the origins of electroweak symmetry breaking

We put by hand the condition for EWSB

$$V(\phi) = -m^2 |\phi|^2 + \lambda |\phi|^4$$

Non-perturbativity

200

150

Higgs mass Mh in GeV

- The SM does not explain how the Higgs mass parameter and self-coupling are determined
- Furthermore, once you include the effects of the Higgs coupling to fermions (especially to the top quark), the Higgs potential shows an instability



What is behind the EWSB mechanism?

A prototype of models for electroweak symmetry breaking is the two-Higgs-doublet model:

$$\begin{split} \mathcal{V} &= m_{11}^2 \Phi_1^{\dagger} \Phi_1 + m_{22}^2 \Phi_2^{\dagger} \Phi_2 - [m_{12}^2 \Phi_1^{\dagger} \Phi_2 + \text{h.c.}] \\ &+ \frac{1}{2} \lambda_1 (\Phi_1^{\dagger} \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^{\dagger} \Phi_2)^2 + \lambda_3 (\Phi_1^{\dagger} \Phi_1) (\Phi_2^{\dagger} \Phi_2) + \lambda_4 (\Phi_1^{\dagger} \Phi_2) (\Phi_2^{\dagger} \Phi_1) \\ &+ \left\{ \frac{1}{2} \lambda_5 (\Phi_1^{\dagger} \Phi_2)^2 + \left[\lambda_6 (\Phi_1^{\dagger} \Phi_1) + \lambda_7 (\Phi_2^{\dagger} \Phi_2) \right] \Phi_1^{\dagger} \Phi_2 + \text{h.c.} \right\} \,. \end{split}$$

After minimization of the potential: $\langle \Phi_i \rangle = v_i / \sqrt{2}$, with $v^2 = v_1^2 + v_2^2 = 246 \text{ GeV}^2$ $\tan \beta = v_2 / v_1$ $0 \le \beta \le \pi/2$, $c_\beta \equiv \cos \beta = v_1 / v$ $s_\beta \equiv \sin \beta = v_2 / v$.

We will study quantum entanglement in the "flavor space" in 2-to-2 scattering:



• Demanding the flavor entanglement is minimized, the scalar potential must have the following form:

$$\mathcal{V} = Y(H_1^{\dagger}H_1 + H_2^{\dagger}H_2) + \frac{Z}{2}(H_1^{\dagger}H_1 + H_2^{\dagger}H_2)^2$$
$$= \frac{Z}{2} \left(\sum_{i=1,2} |H_i^0|^2 + G^+G^- + H^+H^- - \frac{v^2}{2} \right)^2$$

- This potential has a maximal SO(8) symmetry, broken down to SO(7) by the Higgs VEV.
- More importantly, a SM-like Higgs boson follows from this scalar potential automatically!

Last but not least, there are efforts to explain the flavor pattern of the SM:

- Using a specific limit of tree-level 2-to-2 quark scattering mediated by gauge bosons, requiring entanglement suppression recovers the structure of the CKM matrix qualitatively.
- Applying the same logic to the lepton sector, the PMNS matrix is recovered qualitatively.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \mathsf{PMNS} \\ \mathsf{matrix} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

J. Thaler and S. Trifinopoulos, to appear

<u>Outlook</u>

In pursuit of a new paradigm:

Can symmetry be the outgrowth of more fundamental principles?

The answer appears to be a tantalizing YES!

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 - Can we understand (un)naturalness through the lens of quantum information?

Understanding these issues might help us devise more efficient quantum algorithms for simulating systems exhibiting a particular type of symmetry.

Back up Slides

An example of Wigner's SU(4) in A=18 isobar β decays:



An example of Wigner's SU(4) in A=18 isobar β decays:



In the EFT language, Wigner's SU(4) is accidental in that, after imposing the SU(4) quark spin-flavor symmetry, the only remaining operator has this symmetry. To investigate what emerging symmetries appear, it's most convenient to use the EFT Lagrangian, where the symmetry is manifest:

$$\begin{aligned} \mathcal{L}_{\mathrm{LO}}^{n_f=3} &= -\frac{c_1}{f^2} \langle B_i^{\dagger} B_i B_j^{\dagger} B_j \rangle - \frac{c_2}{f^2} \langle B_i^{\dagger} B_j B_j^{\dagger} B_i \rangle - \frac{c_3}{f^2} \langle B_i^{\dagger} B_j^{\dagger} B_i B_j \rangle - \frac{c_4}{f^2} \langle B_i^{\dagger} B_j^{\dagger} B_j B_j \rangle \\ &- \frac{c_5}{f^2} \langle B_i^{\dagger} B_i \rangle \langle B_j^{\dagger} B_j \rangle - \frac{c_6}{f^2} \langle B_i^{\dagger} B_j \rangle \langle B_j^{\dagger} B_i \rangle, \end{aligned}$$

These Wilson coefficients can be projected into SU(3)-symmetric Wilson coefficients:

Relation between scattering phase and Wilson coefficient:

$$p \cot \delta_i = -\left(\mu + \frac{4\pi}{MC_i}\right)$$

For natural scattering length, set $\mu = 0$.

$$\begin{split} C_{27} &= c_1 - c_2 + c_5 - c_6 \ , \\ C_{8_S} &= -\frac{2}{3}c_1 + \frac{2}{3}c_2 - \frac{5}{6}c_3 + \frac{5}{6}c_4 + c_5 - c_6 \ , \\ C_1 &= -\frac{1}{3}c_1 + \frac{1}{3}c_2 - \frac{8}{3}c_3 + \frac{8}{3}c_4 + c_5 - c_6 \ , \\ C_{\overline{10}} &= c_1 + c_2 + c_5 + c_6 \ , \\ C_{10} &= -c_1 - c_2 + c_5 + c_6 \ , \\ C_{8_A} &= \frac{3}{2}c_3 + \frac{3}{2}c_4 + c_5 + c_6 \ . \end{split}$$

Wagman et. al.: 1706.06550