Reconciling constraints from the supernova remnant HESS J1731-347 with the parity doublet model

B. Gao, Y. Yan, M. Harada *Phys.Rev.C* 109 (2024) 6, 065807
B. Gao, M. Harada Arxive: 2410.16649

Outline

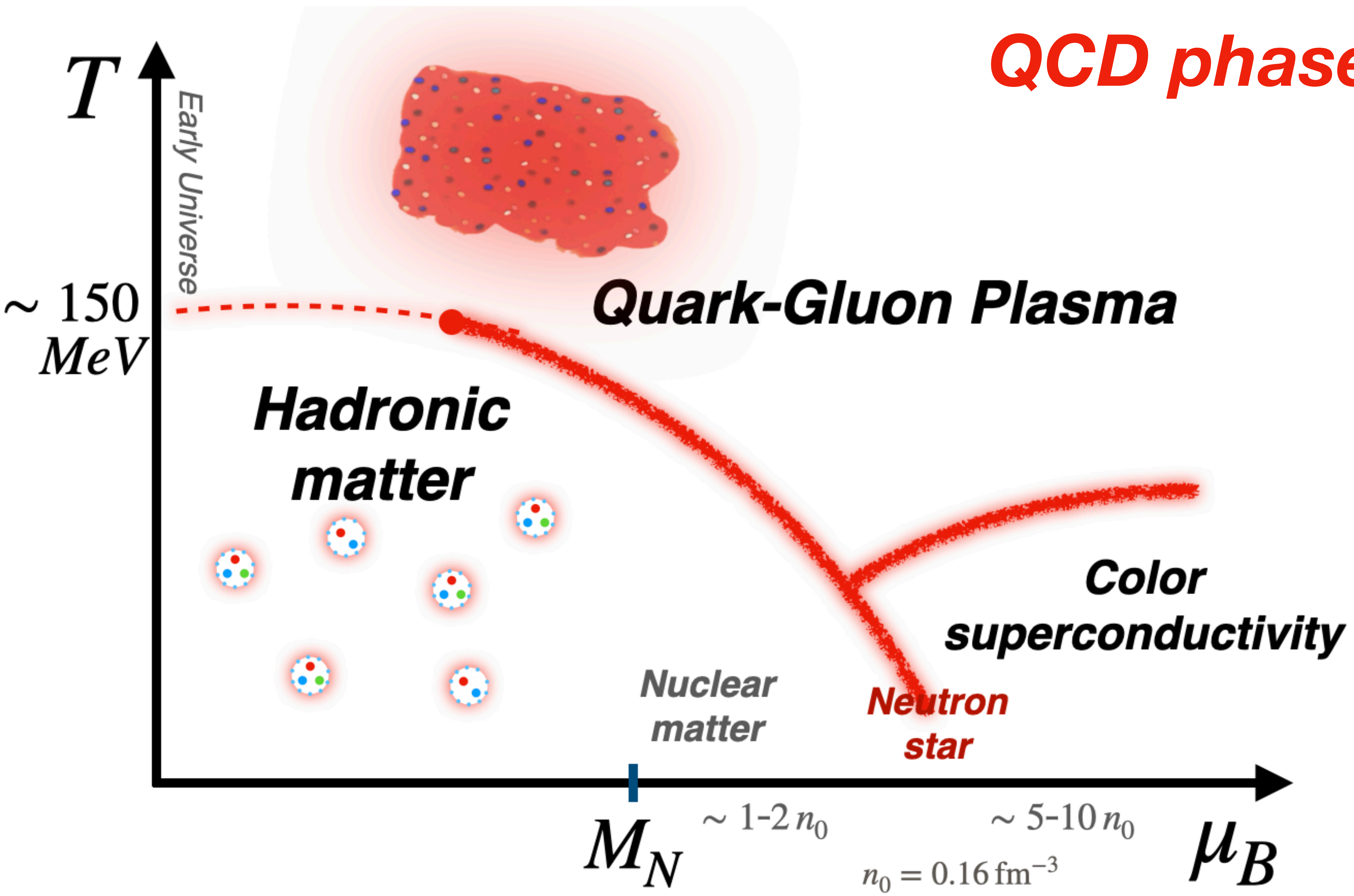
1. **Introduction**

2. **Unified Equation of State & Analysis**(*Phys.Rev.C* 109 (2024) 6, 065807)
   Parity doublet model
   NJL-type quark model

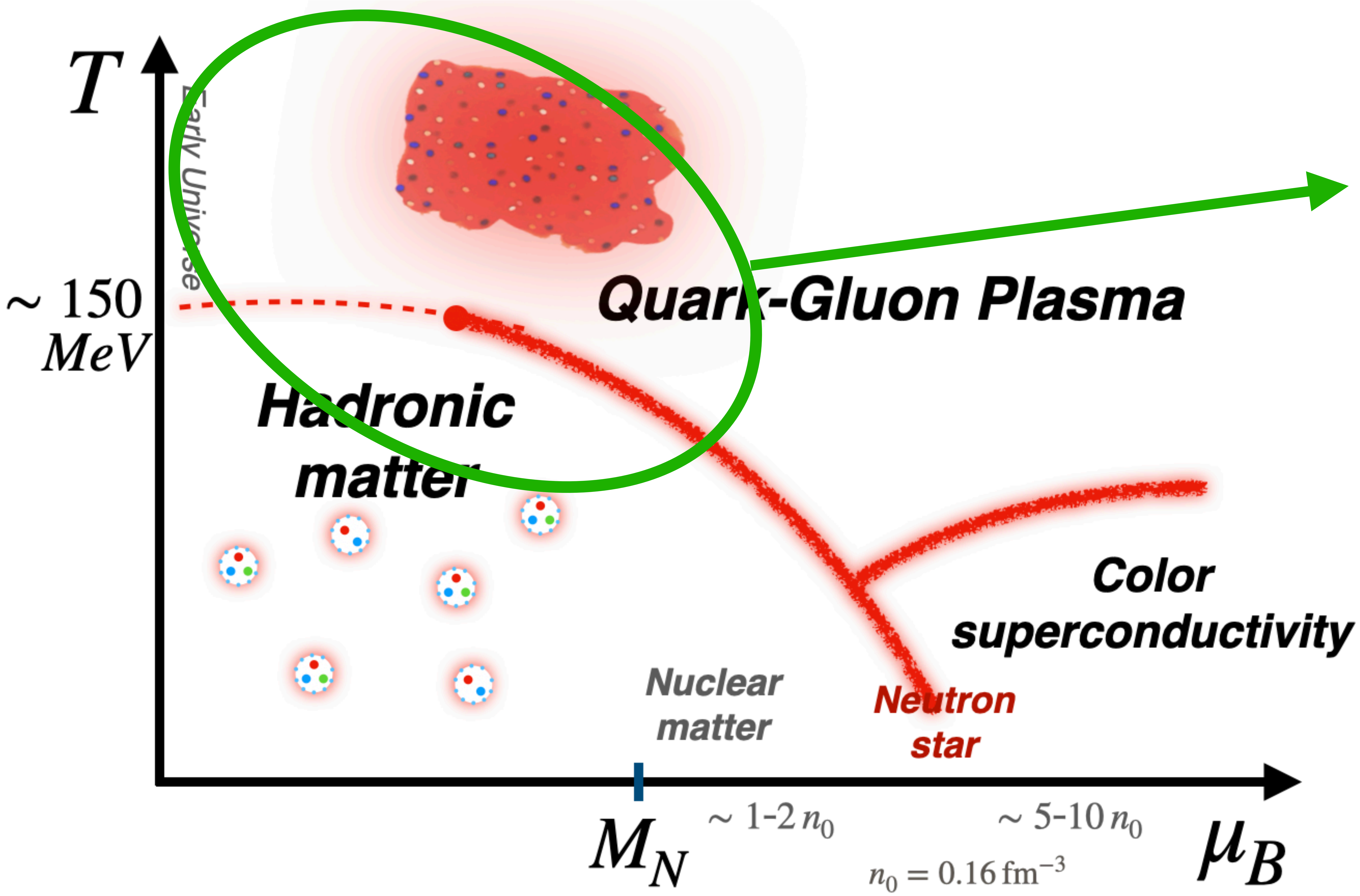
3. **Quarkyonic matter with parity doublet**(Arxive: 2410.16649)

Introduction

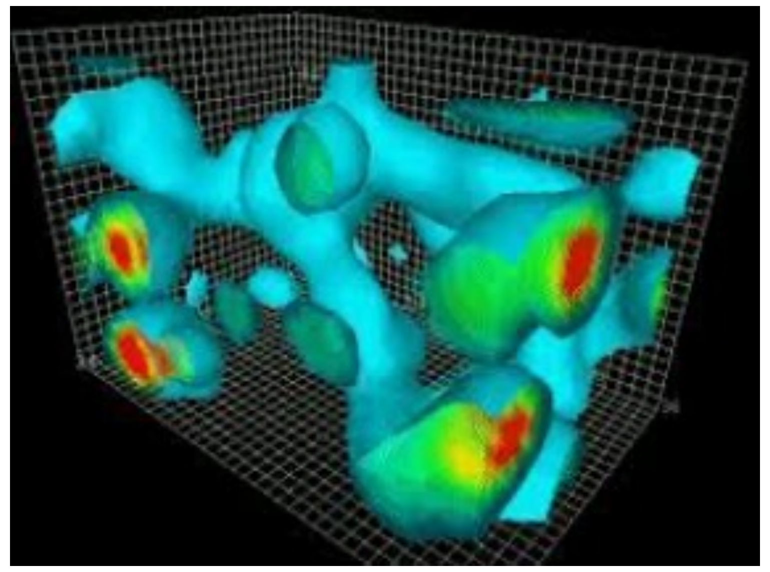
QCD phase diagram



High temperature region



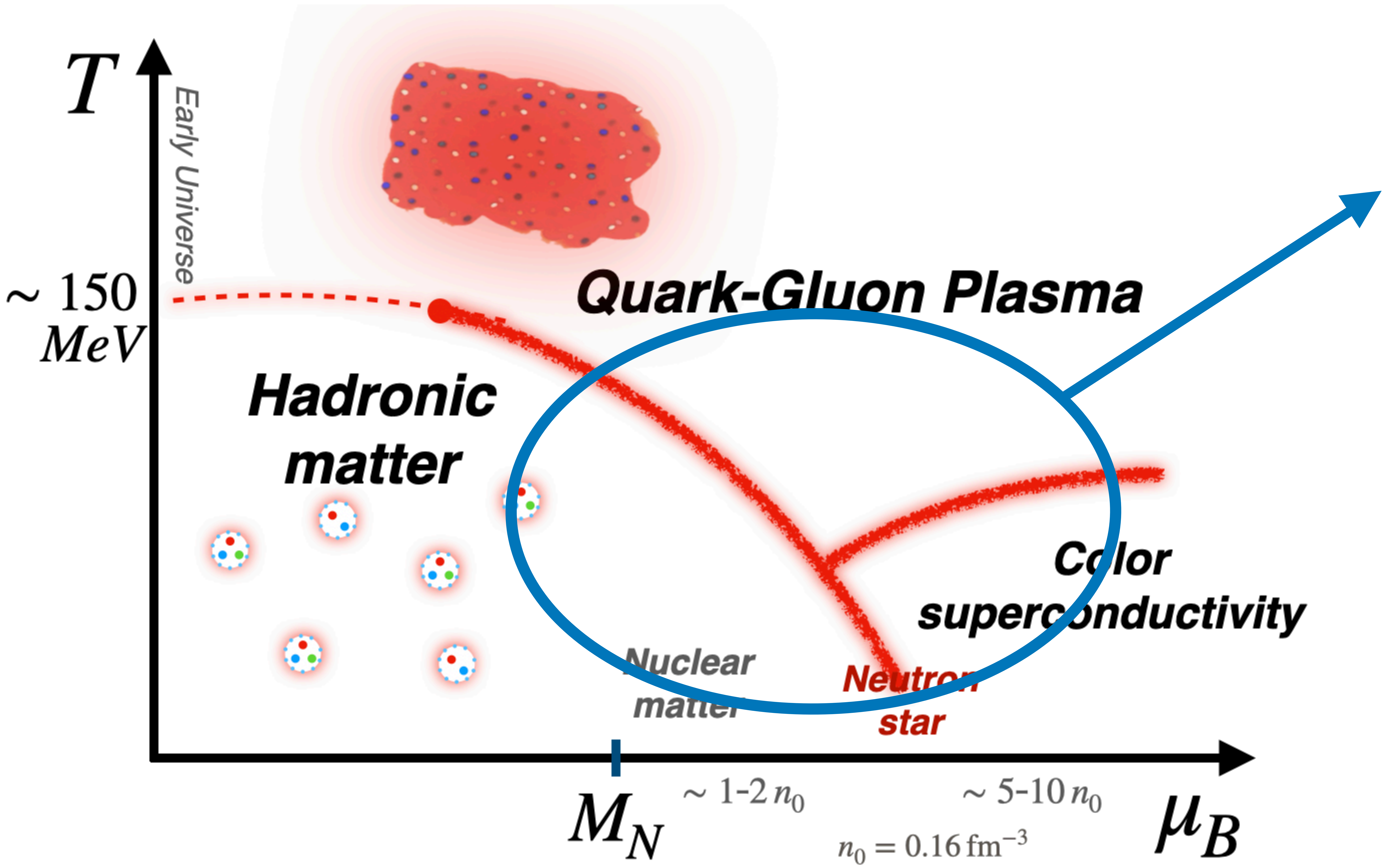
Lattice QCD;



Large Hadron Collider;

Heavy ion collision

Difficulties in high dense matter



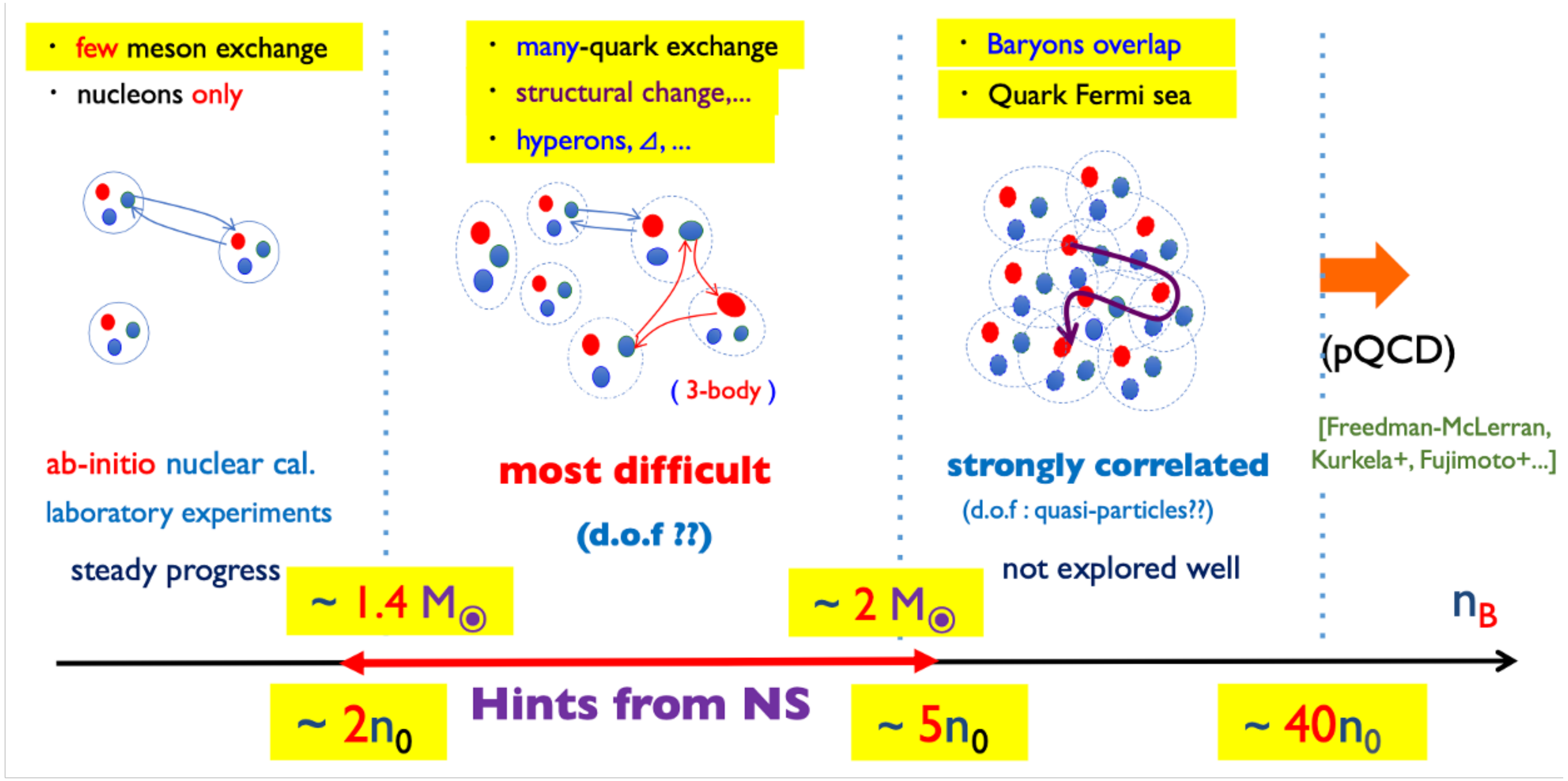
Lattice Monte-Carlo simulation Not possible(sign problem)

Cannot design laboratories, have to wait for signals (unlike heavy ion collision)

……

Fundamental questions in dense QCD

Masuda et al.

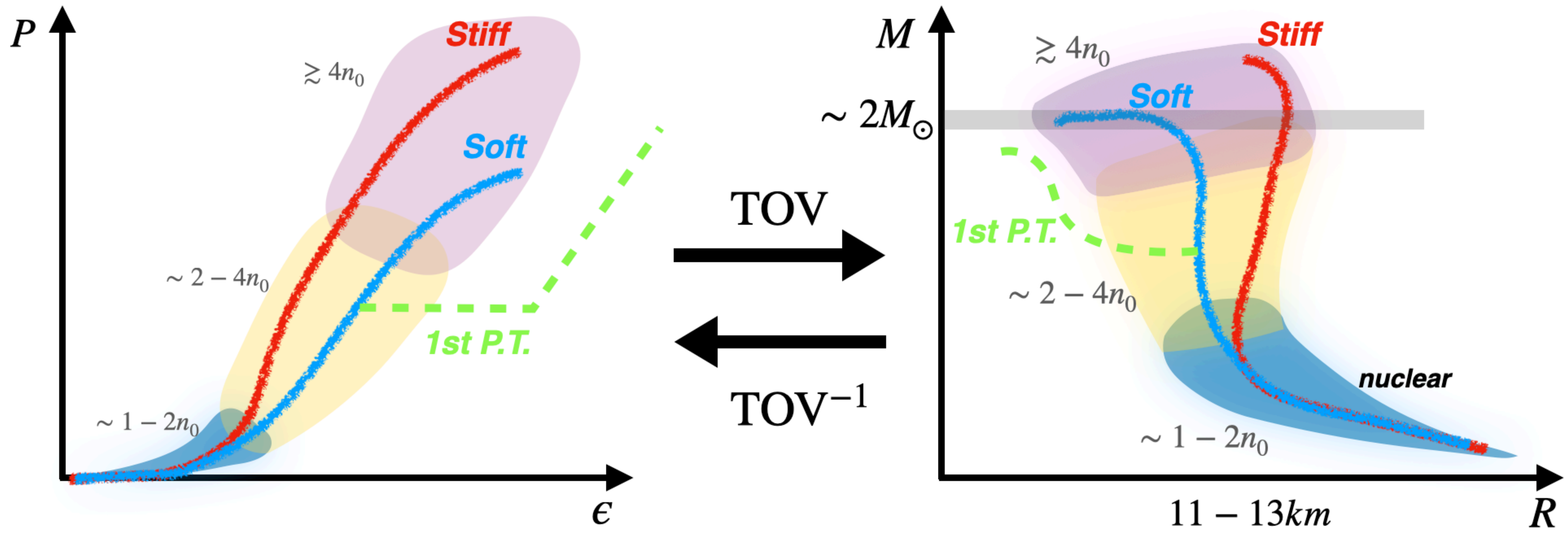


How does dense matter respond to compression, the EOS?

How hadronic matter dissolves into quark matter?

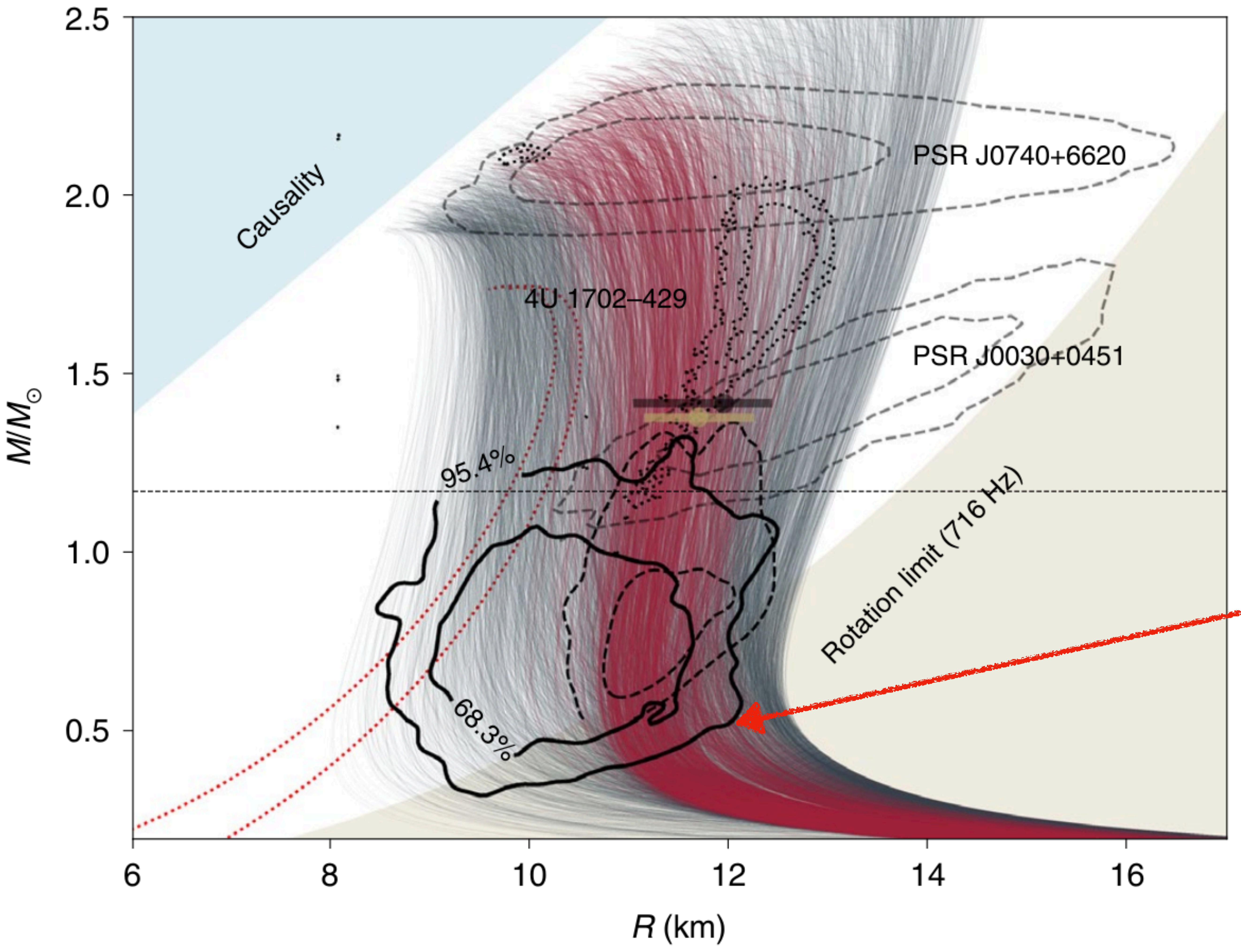
.....

Correlation between EoS and M-R



Neutron Star	Mass (M⊙)	Radius (km)	Source
J0740+6620	2.14 ± 0.10	12.35 ± 0.75	NICER
J0030+0451	1.44 ± 0.15	12.45± 0.65	NICER
GW170817	1.33-1.60	11.9 ± 1.4	LIGO/Virgo

Strange CCO HESS J1731-347



Nature Astronomy volume **6**, 1444–1451 (2022)

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HESS J1731-347

A Strange light central compact object supernova remnant

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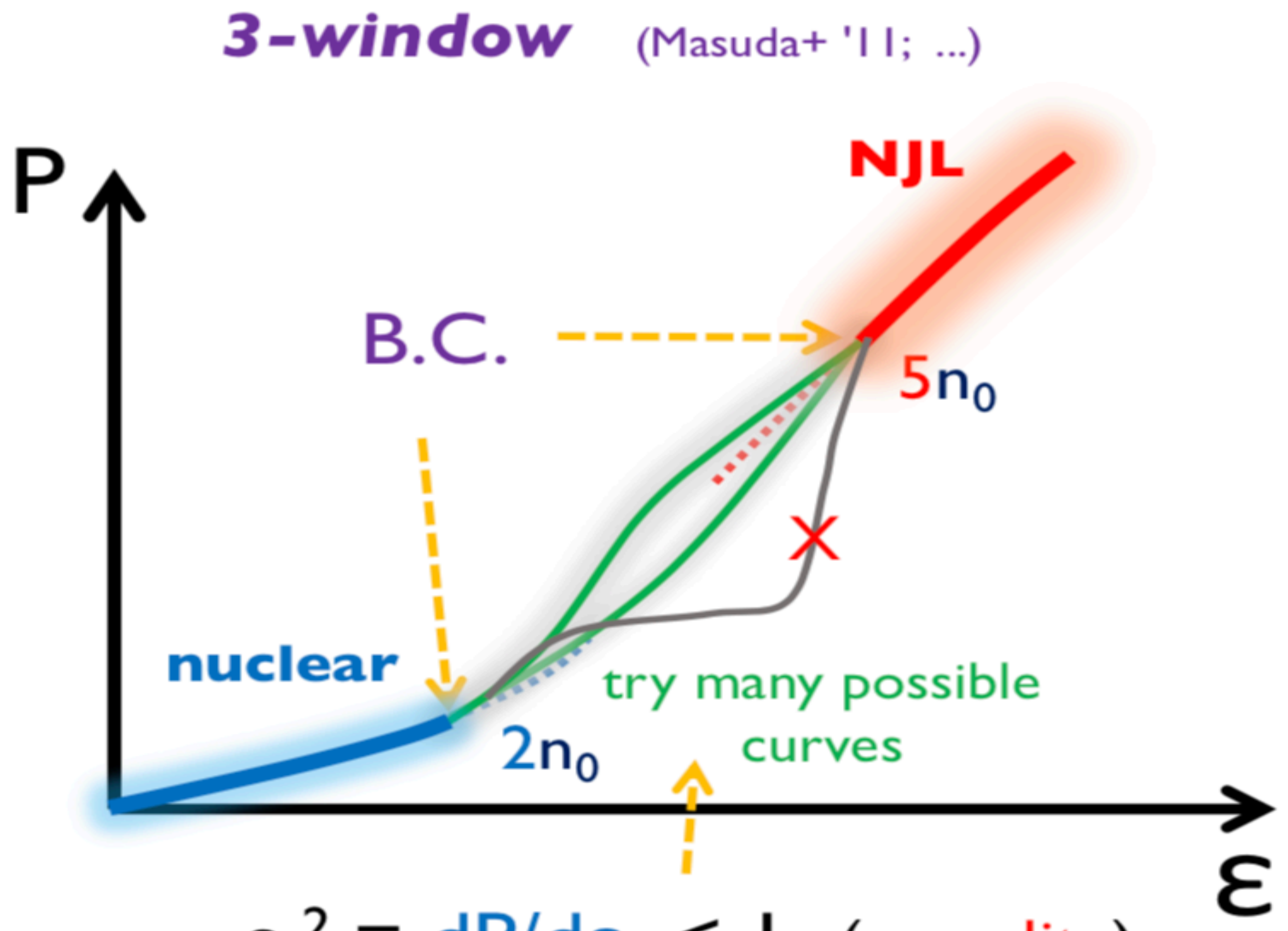
Unified Equation of State



$c_s^2 = dP/d\varepsilon < 1$ (causality)

→ removes unphysical curves

An effective hadron model

(Parity doublet model) (n_B<=$2n_0$, blue curve)

Two baryons with positive and negative-parity are introduced. They have a **degenerate chiral invariant mass** when the chiral symmetry is restored.

Interpolated(red curve)

interpolate w/ polynomial: $P = \sum_{n=0}^{5} c_n \mu_B^n$

An effective quark model

(Nambu–Jona-Lasinio(NJL)-type model)

(n_B>=$5n_0$, green curve)

Parity doublet model

DeTar, Kunihiro, 1989; Jido, Oka, Hosaka, 2001

PDM: chiral symmetric nucleon-meson effective model

$$\mathscr{L}_{\text{PDM}} = \mathscr{L}_{\text{Nucleon}}(\psi_1, \psi_2, \ldots) + \mathscr{L}_{\text{Meson}}(\sigma, \pi, \underline{\omega, \rho, .}\ldots)$$

vector mesons, with HLS

ordinal dirac mass term:

$$m\bar{\psi}\psi = m(\bar{\psi}^L\psi^R + \bar{\psi}^R\psi^L)$$

$$\rightarrow \underline{m(\bar{\psi}^L L^\dagger R \psi^R + \bar{\psi}^R R^\dagger L \psi^L)}$$

chiral variant

	$L \in \mathrm{SU}(N_\mathrm{f})_\mathrm{L}$ left-handed	$R \in \mathrm{SU}(N_\mathrm{f})_\mathrm{R}$ right-handed
nucleon ψ_1	$\psi_1^L \rightarrow L\psi_1^L$	$\psi_1^R \rightarrow R\psi_1^R$
nucleon ψ_2	$\psi_2^L \rightarrow R\psi_2^L$	$\psi_2^R \rightarrow L\psi_2^R$

in PDM:

$$m_0(\bar{\psi}_1\gamma_5\psi_2 - \bar{\psi}_2\gamma_5\psi_1) = m_0(\bar{\psi}_1^L\psi_2^R + \bar{\psi}_1^R\psi_2^L + \mathrm{h.c.})$$

$$\rightarrow \underline{m_0(\bar{\psi}_1^L L^\dagger L \psi_2^R + \bar{\psi}_1^R R^\dagger R \psi_2^L + \mathrm{h.c.})}$$

chiral invariant

Parity Doublet Model

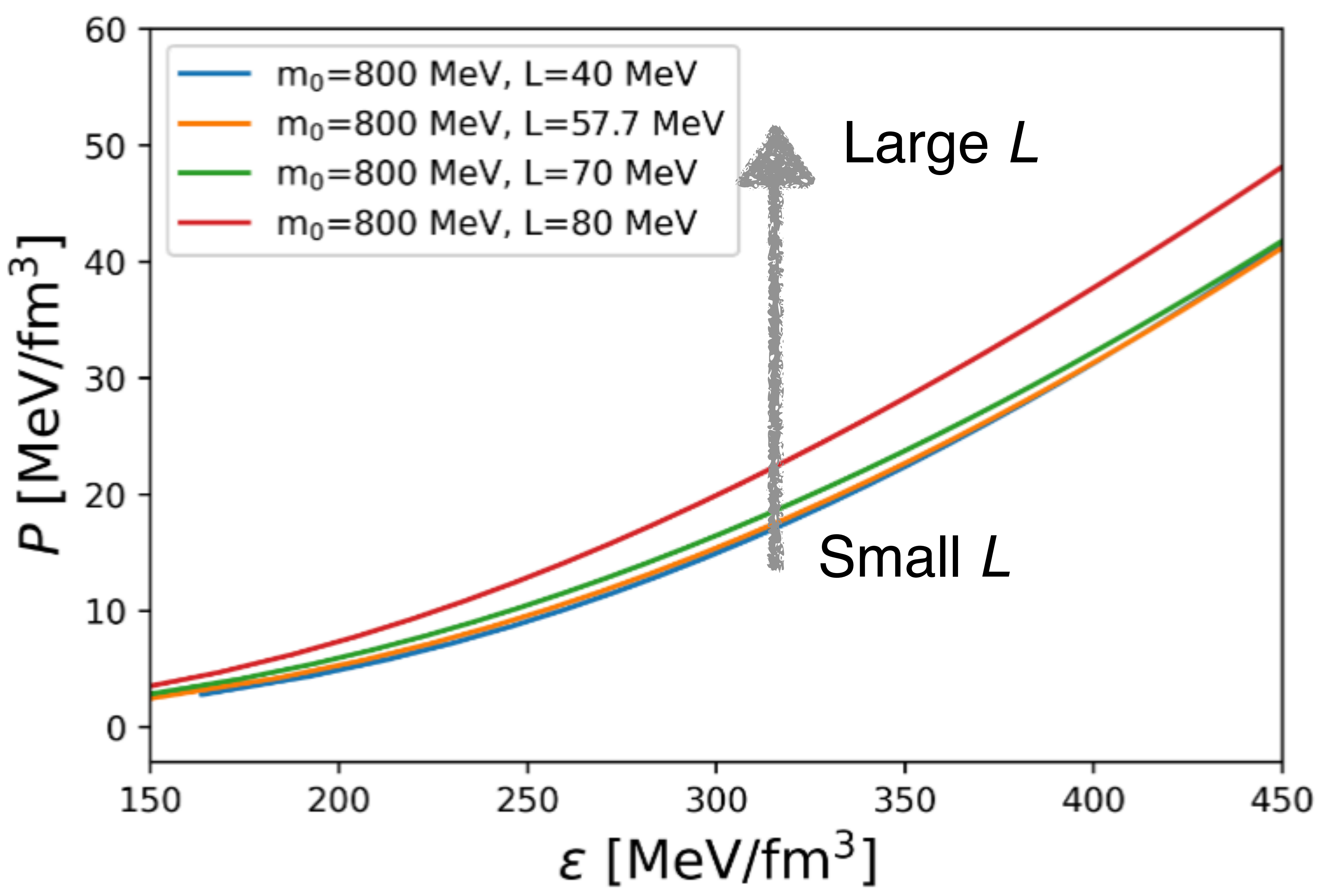
mass formula of nucleons N(939) and N*(1535)

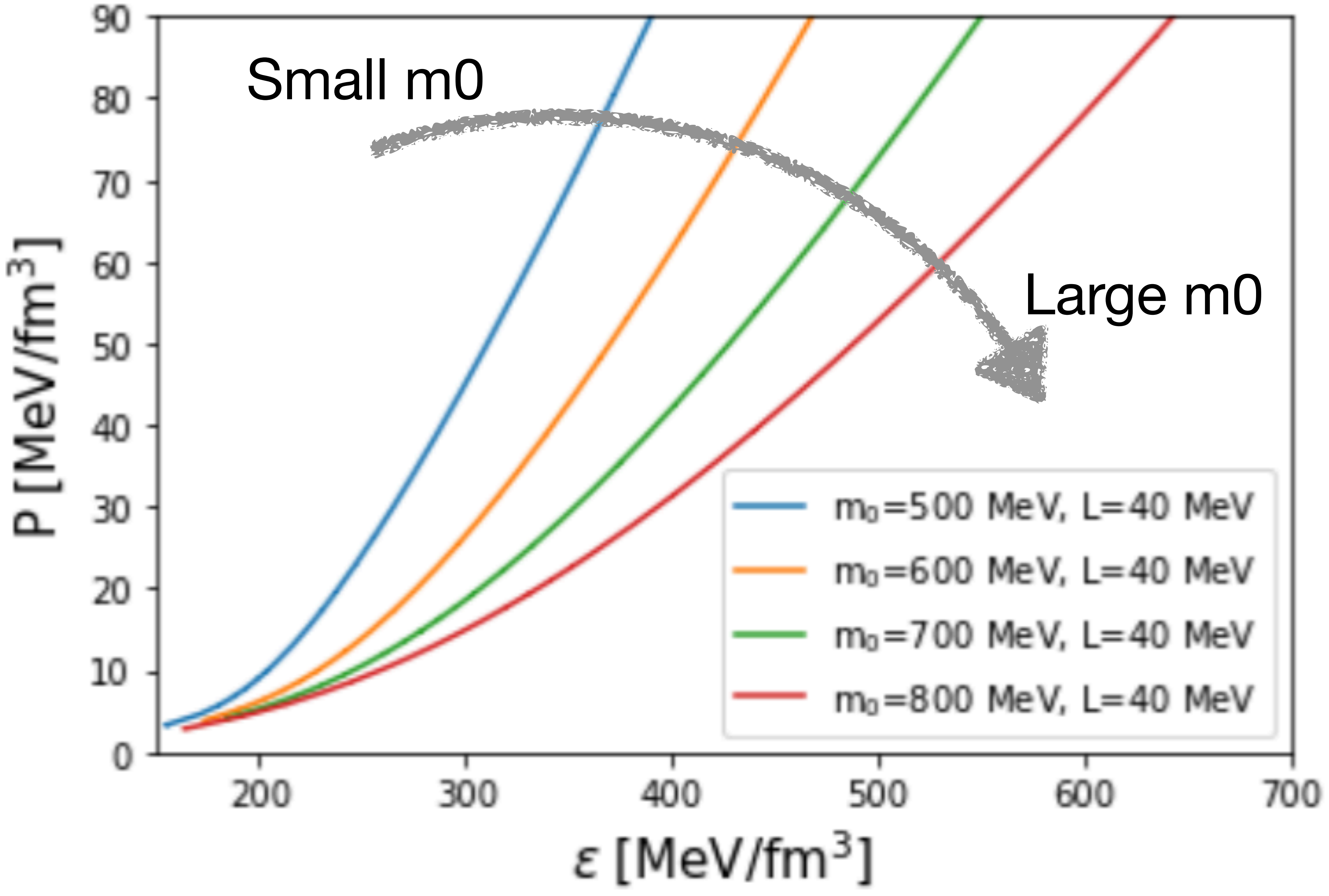
$$M_{N\pm} = \sqrt{m_0^2 + g_+^2\sigma^2} \mp g_-\sigma \quad \overset{\sigma \to 0}{\to} \quad m_0$$

Parameters in the model are determined by the saturation properties

n_0 [fm^{-3}]	B_0 [MeV]	K_0 [MeV]	S_0 [MeV]
0.16	16	240	31

Two parameters m_0 , L (density dependence of the nuclear symmetry energy around the saturation density)





NJL-type quark model

$$\mathscr{L} = \mathscr{L}_{\text{NJL}} - H(q^T\Gamma_A q)(\bar{q}\Gamma^A \bar{q}^T) + g_V(\bar{q}\gamma^0 q)^2 \quad + \sum_i \mu_i Q_i$$

- Original NJL-type model(Hatsuda and Kunihiro) includes four point interaction $+G(\bar{\psi}\psi)^2$
- U(1) axial anomaly $-K\det(\bar{\psi}\psi)$

HK parameters: $G\Lambda^2 = 1.835, \quad K\Lambda^5 = 9.29$

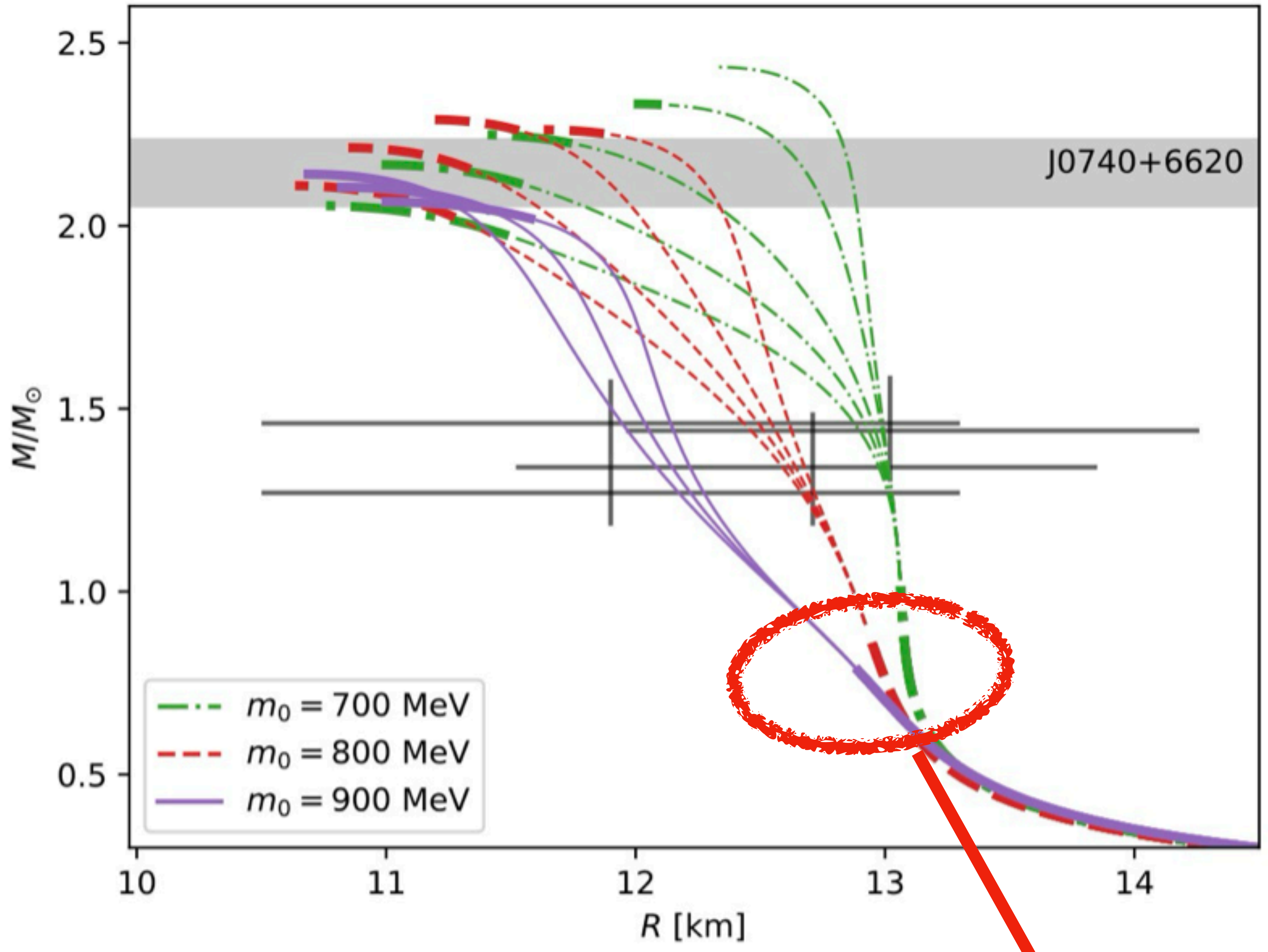
$\Lambda = 631.4\text{MeV}$

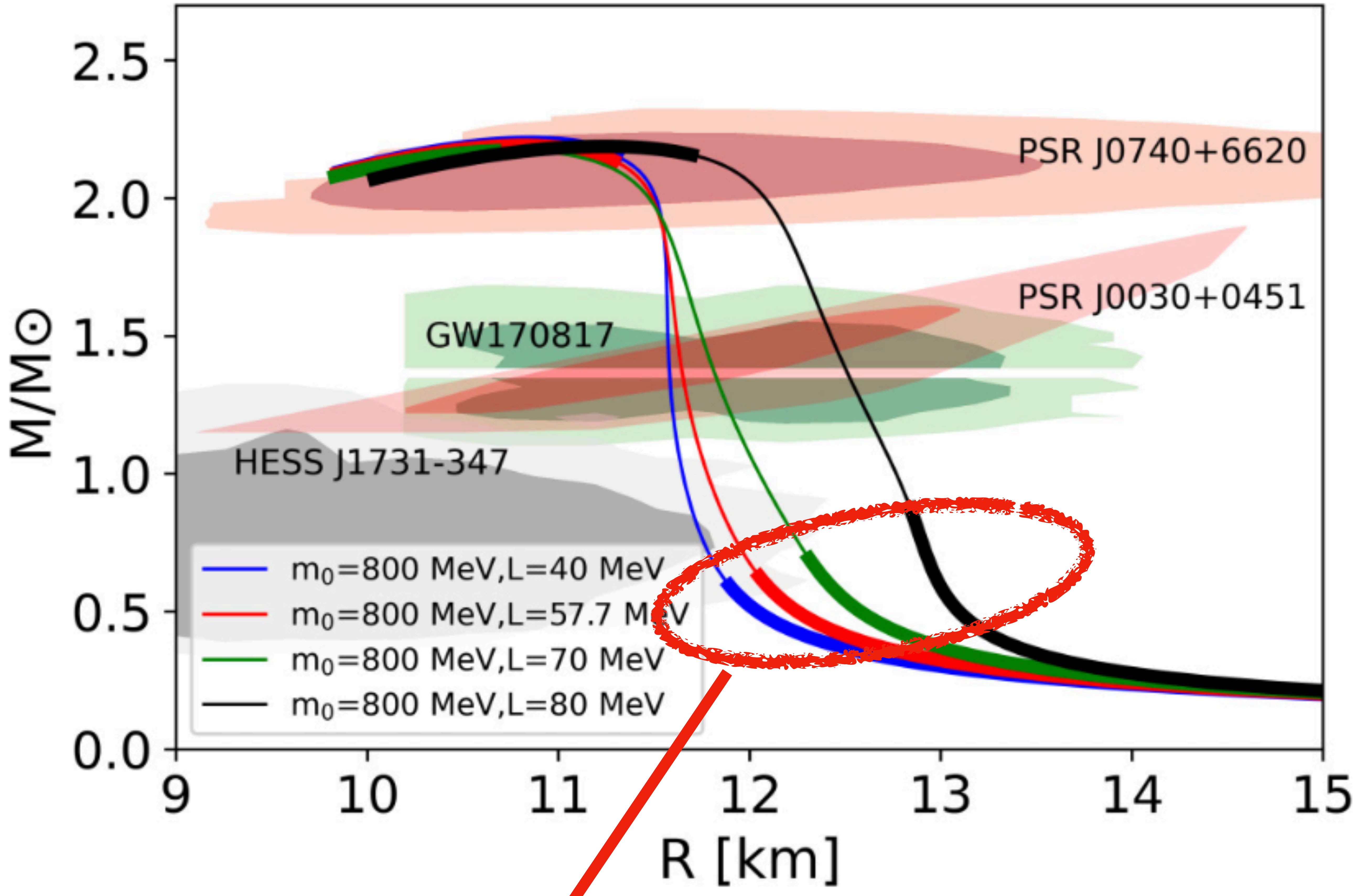
H: coupling for diquark condensates
gV: coupling for vector (repulsive) interaction

(H,gV): not well-constrained before
→ survey wide range for given nuclear EOS + NS constraints

Results

For fixed slope parameter *L=80* MeV





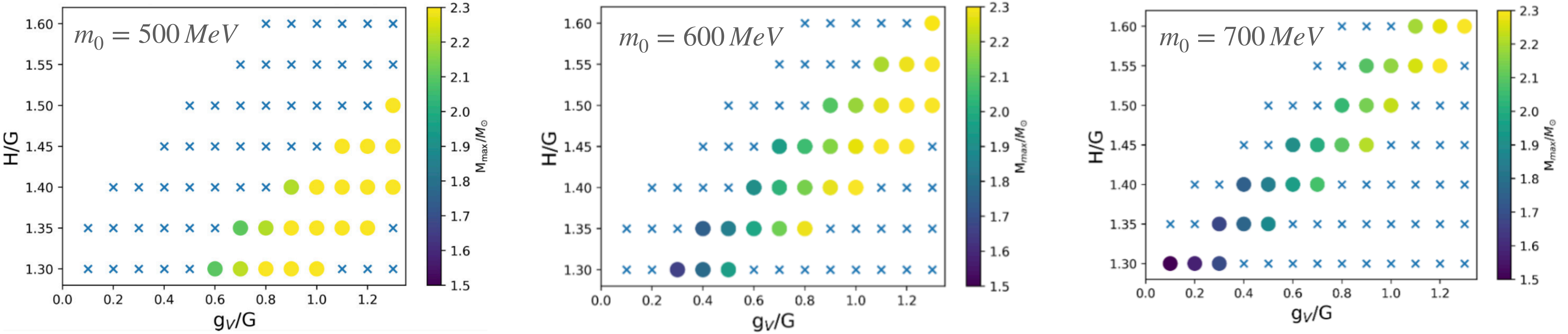
The hadronic matter EoS is crucial to determine the radius of a NS.
(From soft to stiff)

Results

H: coupling for diquark condensates

gV: coupling for vector (repulsive) interaction

Slope parameter *L* =40 MeV



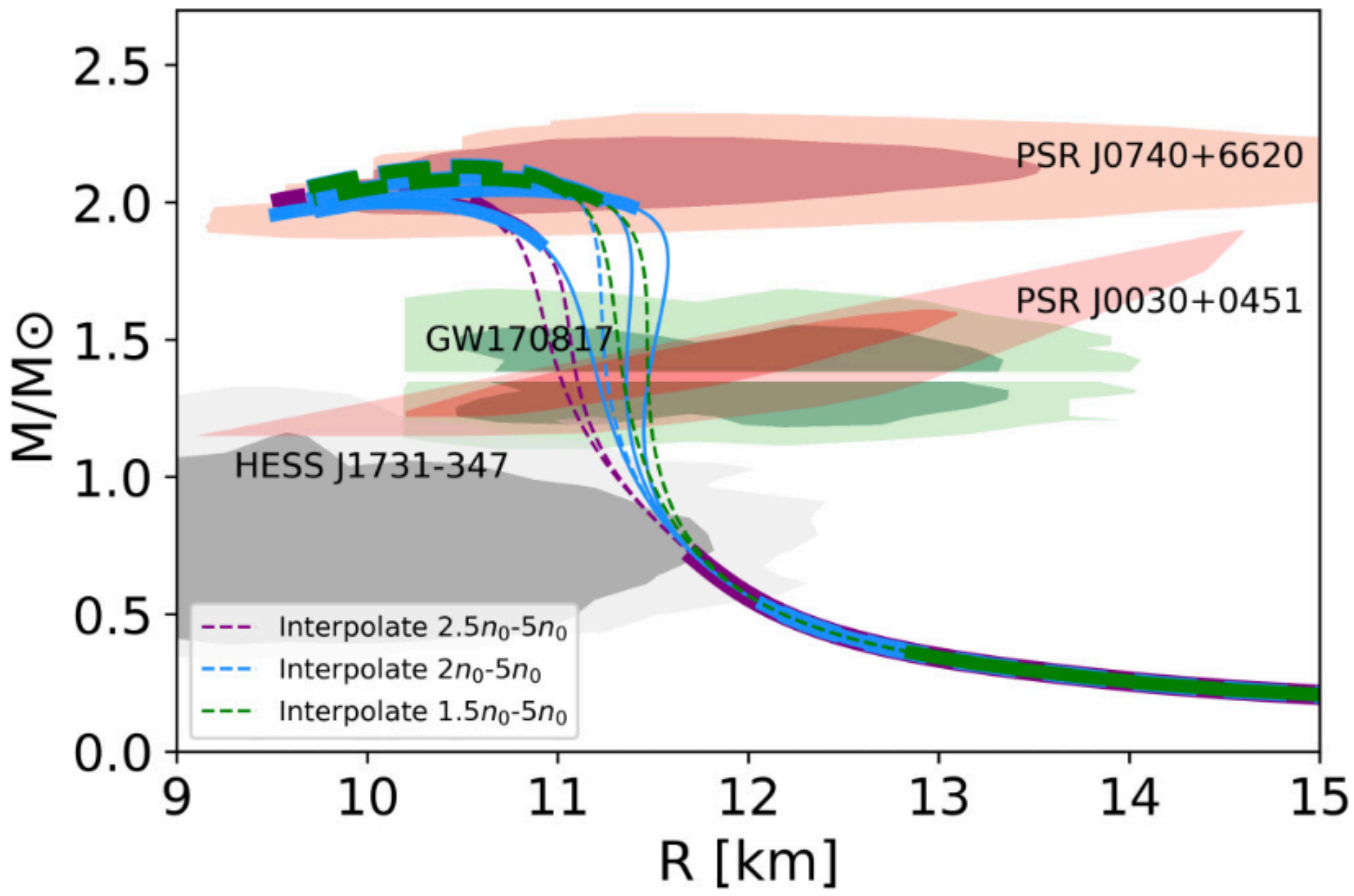
(m_0, L) ⟷ (H,gV)

constrain each other

Causality + Mmax

Results

$m_0 = 850\ \mathrm{MeV}, L = 40\ \mathrm{MeV}$



Check for the ambiguity from the interpolation range:

At $M \sim 1M_\odot$

Radius only change around 0.3 km

At $M \sim 1.4M_\odot$

Radius only change around 0.6 km

Our approach is robust!

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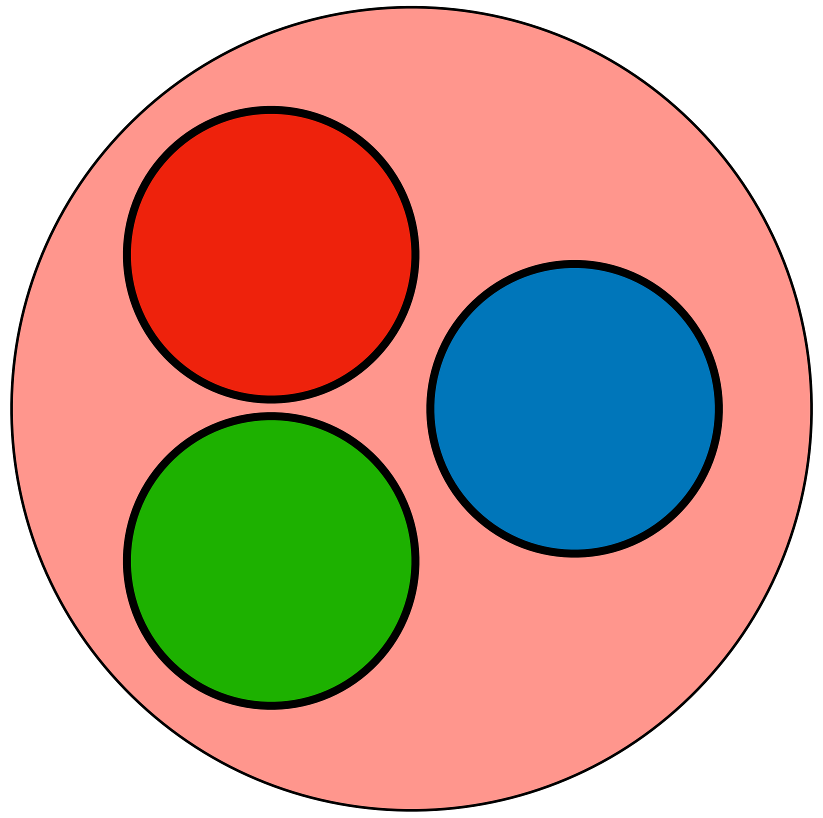
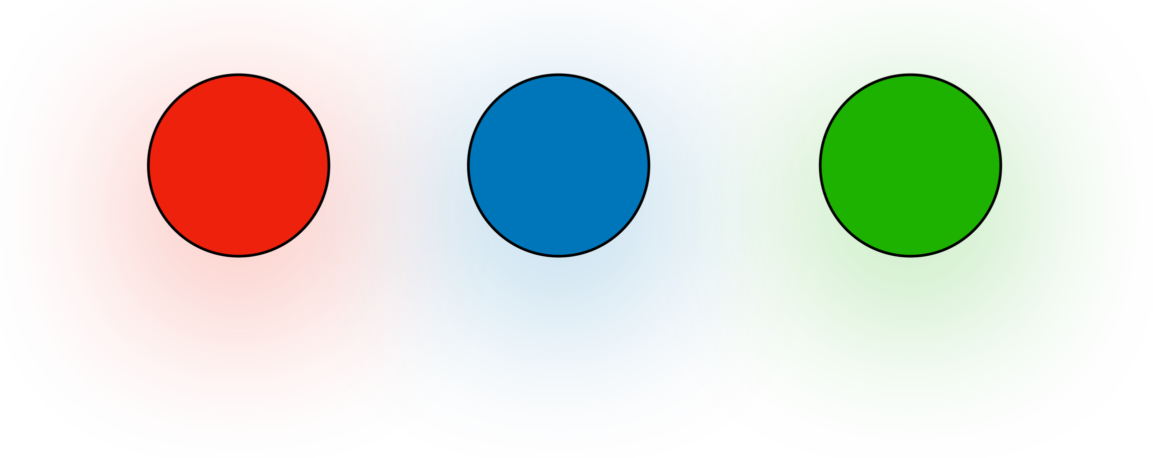
Chiral invariant mass in the constituent quark

Nucleon



Constituent Quarks



Constituent quarks also retain a non-zero mass even if the chiral symmetry is restored

Gluon condensates; topological structure?

mass of a nucleon
~1000 MeV

mass of constituent quarks
~300 MeV

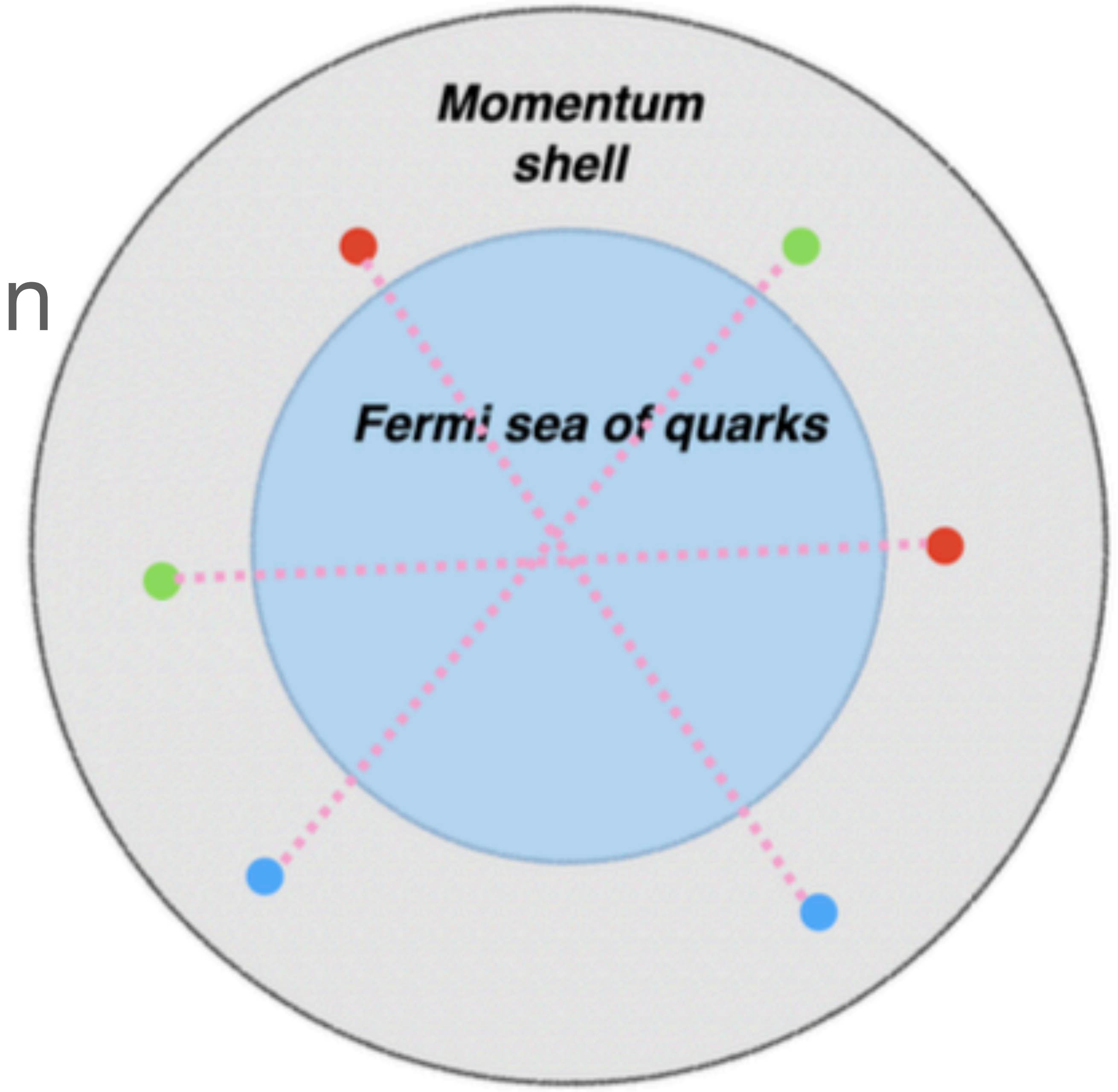
In constituent quark model, hadrons are composed of constituent quarks (quasi-particles)

Quarkyonic matter

At sufficiently high baryon chemical potential, **the degrees of freedom inside the Fermi sea can be treated as quarks; Confining forces remain important only near the Fermi surface**

Two-particle correlation

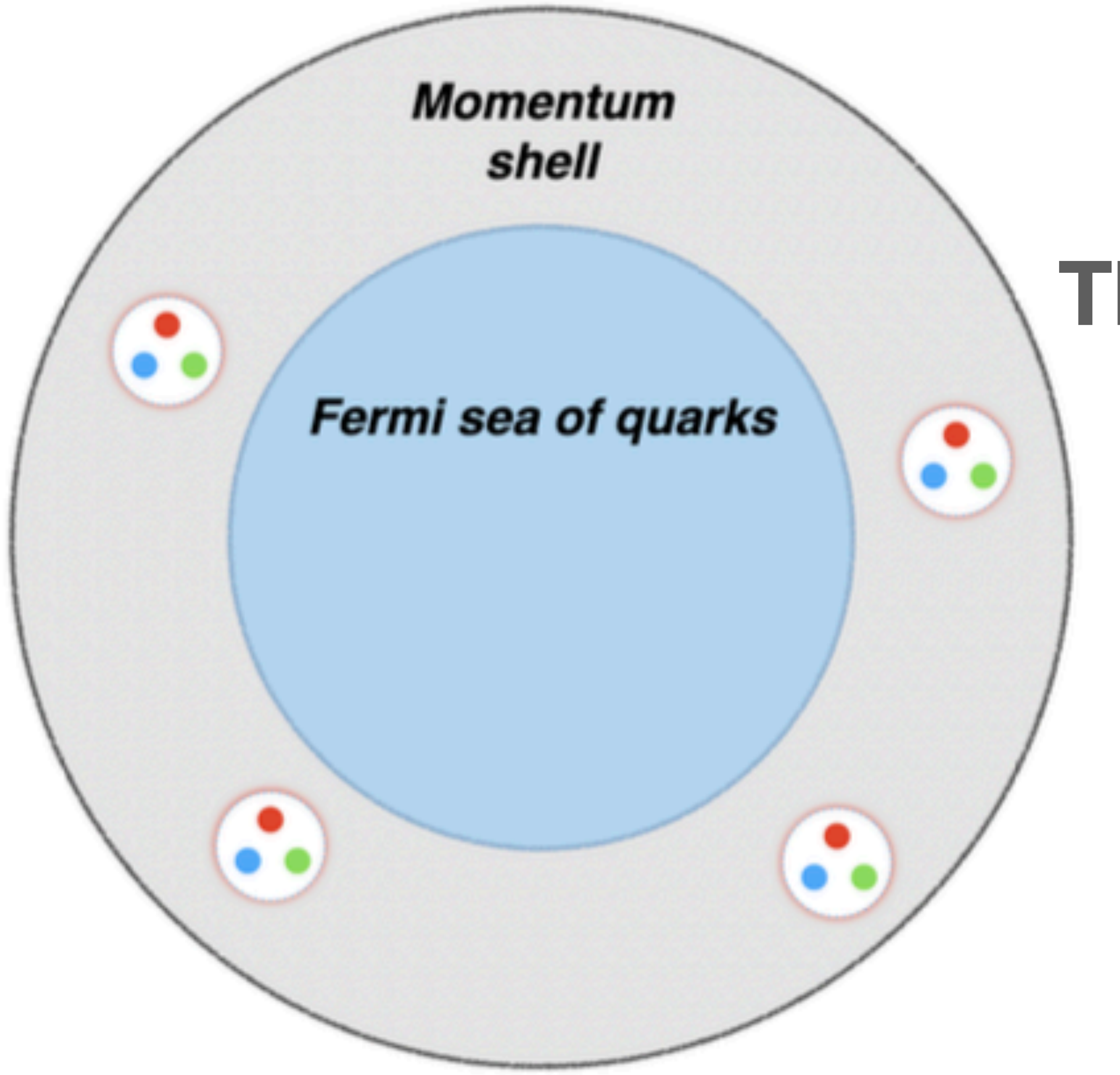
Diquarks have color



Color-superconductivity

Three-particle correlation

Nucleons are colorless



Quarkyonic matter (McLerran-Pisarski 07; Hidaka, Toru Kojo et)

<u>**Motivation: Investigate the impacts of the invariant mass in the constituent quark**</u>

Model construction

The thermodynamic potential in PDM with Nf = 2 is

$$\Omega_{\mathrm{PDM}} = V(\sigma) - V_0 - \frac{1}{2}m_\omega^2\omega^2 - \frac{1}{2}m_\rho^2\rho^2 - \lambda_{\omega\rho}\left(g_\omega\omega\right)^2\left(g_\rho\rho\right)^2 + \Omega_F$$

$$\Omega_F = -2\sum_{i=+,-}\sum_{\alpha=p,n}\int^{k_f}\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\mu_\alpha^* - E_\mathrm{p}^i\right).$$

Parity of nucleons

$f_\pi = 92.4\,\mathrm{MeV}$ $\quad E_p^i = \sqrt{p^2 + m_i^2}$

With

$$V(\sigma) = -\frac{1}{2}\bar{\mu}^2\sigma^2 + \frac{1}{4}\lambda_4\sigma^4 - \frac{1}{6}\lambda_6\sigma^6 - m_\pi^2 f_\pi\sigma\,,$$

$$V_0 = -\frac{1}{2}\bar{\mu}^2 f_\pi^2 + \frac{1}{4}\lambda_4 f_\pi^4 - \frac{1}{6}\lambda_6 f_\pi^6 - m_\pi^2 f_\pi^2\,.$$

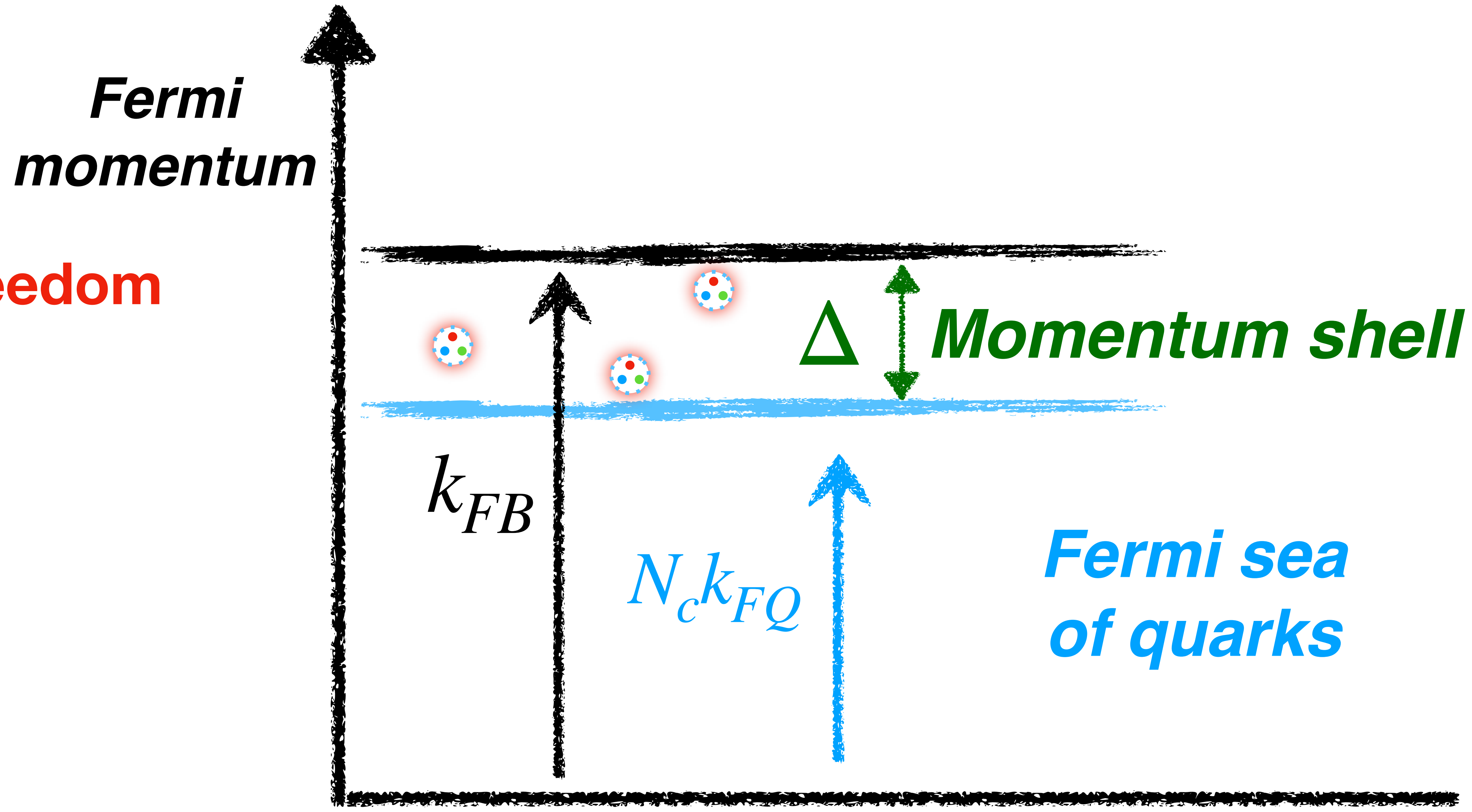
$\bar{\mu}^2, \lambda_4, \lambda_6$ are parameters to be determined

Conventionally, new degrees of freedom enter when chemical potential surpasses their mass threshold

Quark state saturation shifts the onset of heavier degrees of freedom due to the Pauli blocking of quarks

Validity of quarkyonic picture: $\Lambda_{QCD} < \mu_q < \sqrt{N_c}\Lambda_{QCD}$

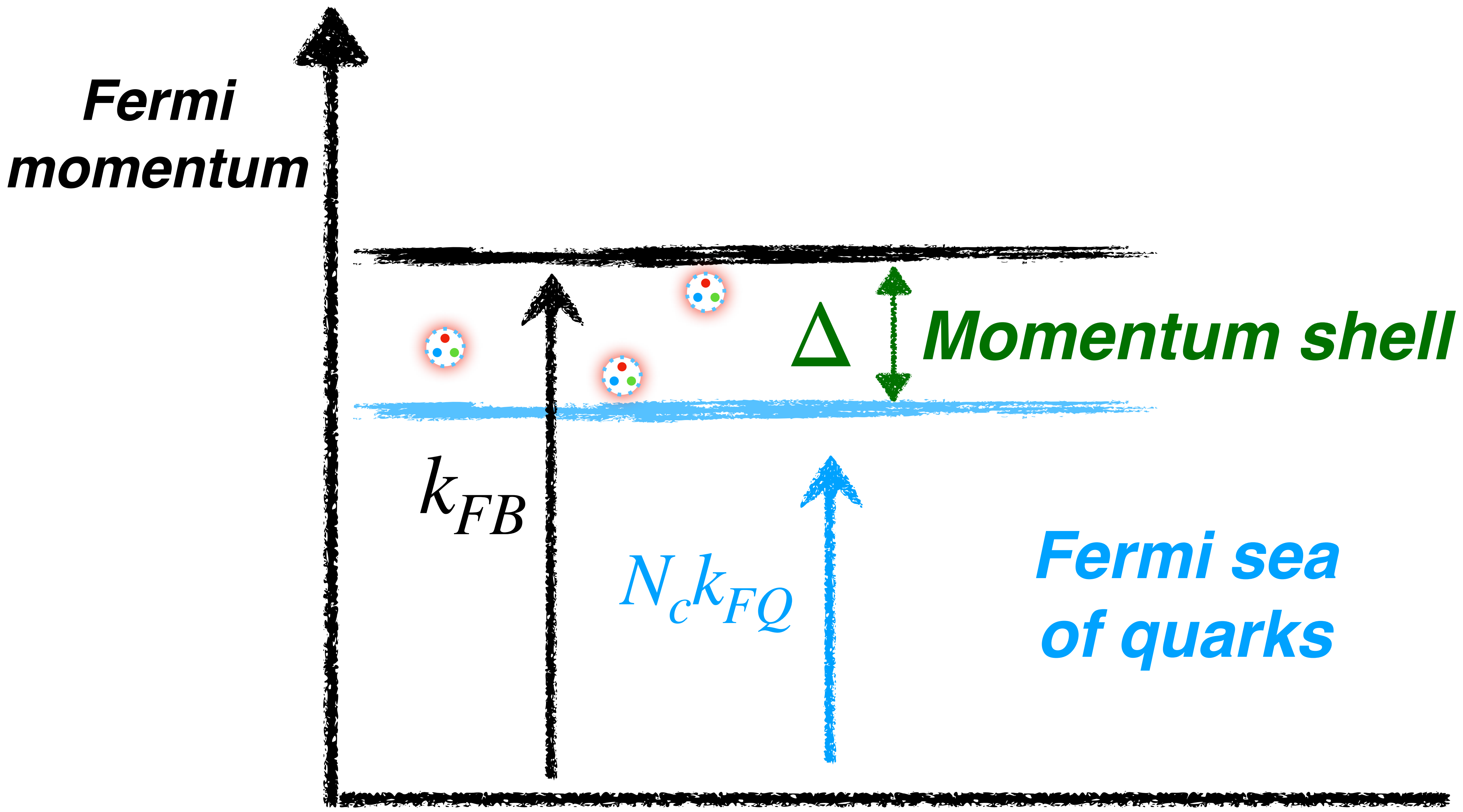
$\mu_B = 3\mu_q \approx 1558\,\mathrm{MeV}$ **No N(1535) !!**



Model construction

Confining forces remain only near the Fermi surface and nucleons appear in the **momentum shell** defined as

$$\Delta = \frac{\Lambda_{\rm QCD}^3}{k_{FB}^2},$$



Since $P = -\Omega$

$$P_F = P_H + P_Q,$$

$$P_H = 2 \sum_{\alpha=p,n} \int_{N_c k_{FQ}}^{k_{FB}} \frac{{\rm d}^3\mathbf{p}}{(2\pi)^3} \left(\mu_\alpha^* - E_{\mathbf{p}}^i\right),$$

$$P_Q = 4N_c \int_0^{k_{FQ}} \frac{{\rm d}^3\mathbf{q}}{(2\pi)^3} (\mu_q^* - E_{\mathbf{q}}),$$

with

$$k_{FQ} = \frac{k_{FB} - \Delta}{N_c} \Theta(k_{FB} - \Delta),$$

$$E_{\mathbf{q}} = \sqrt{\mathbf{q}^2 + M_Q^2}.$$

Constituent quark mass

The quarkyonic phase is signaled by the non-zero quark Fermi momentum

Model construction

From the thermodynamic relation: $n_B = \partial P / \partial \mu_B$

Baryon number density $n_B = \frac{1}{3\pi^2} \sum \left[k_{FB}^3 - (N_c k_{FQ})^3\right] + \frac{2k_{FQ}^3}{3\pi^2}.$ $\xrightarrow{k_{FQ}=0}$ $n_B = \frac{1}{3\pi^2} \sum k_{FB}^3$

Quarkyonic phase **_Hadronic phase_**

$$k_{\rm FQ} = \frac{(k_{\rm FB} - \Delta)}{N_c} \Theta(k_{\rm FB} - \Delta) .$$

The contribution from the quarks relative to nucleons is suppressed by $\frac{1}{N_c^3}$

For simplicity, in this work we consider the **symmetric matter**

As a first step, we define

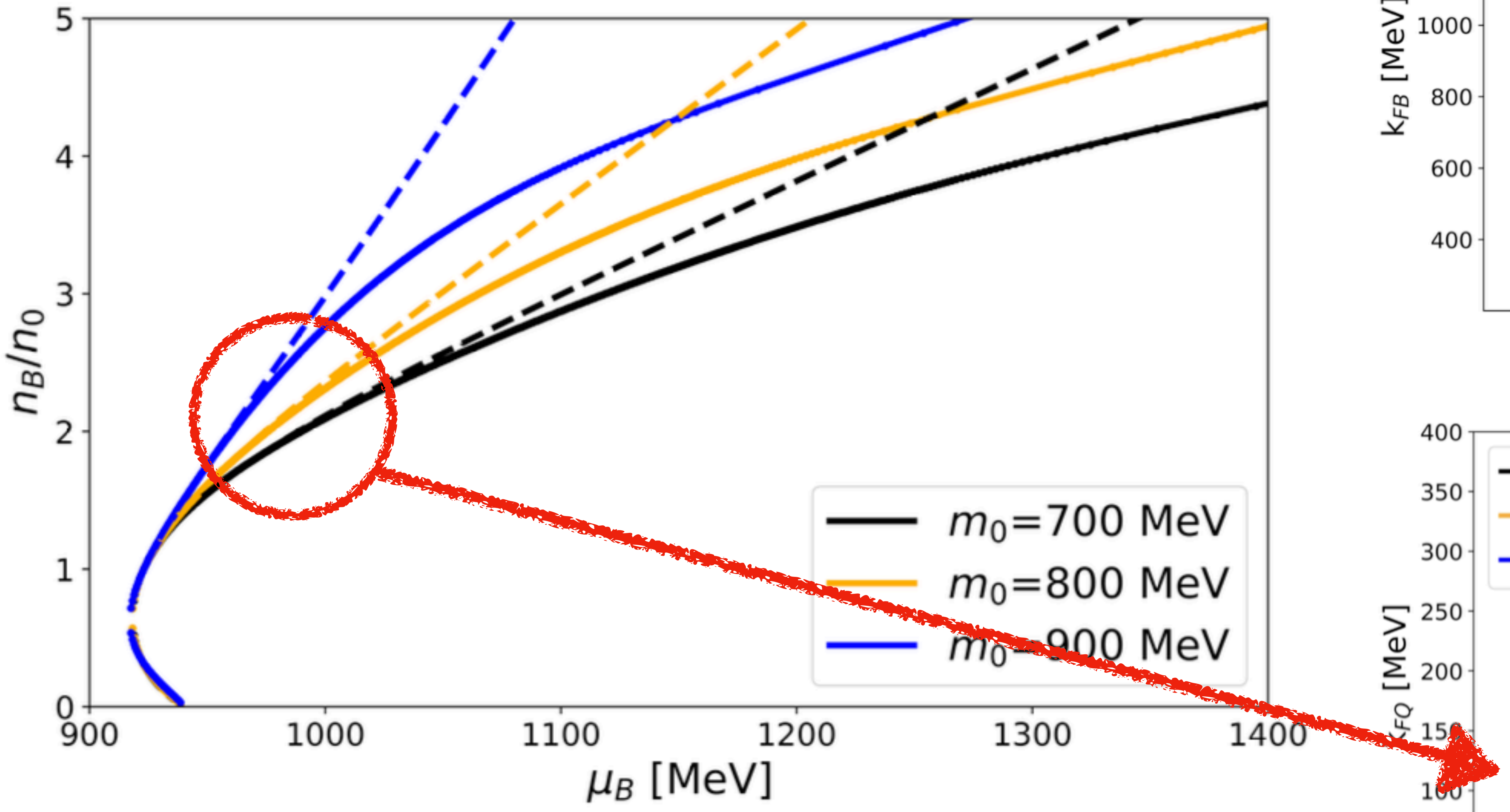
$$M_Q = \frac{m_+}{3},$$

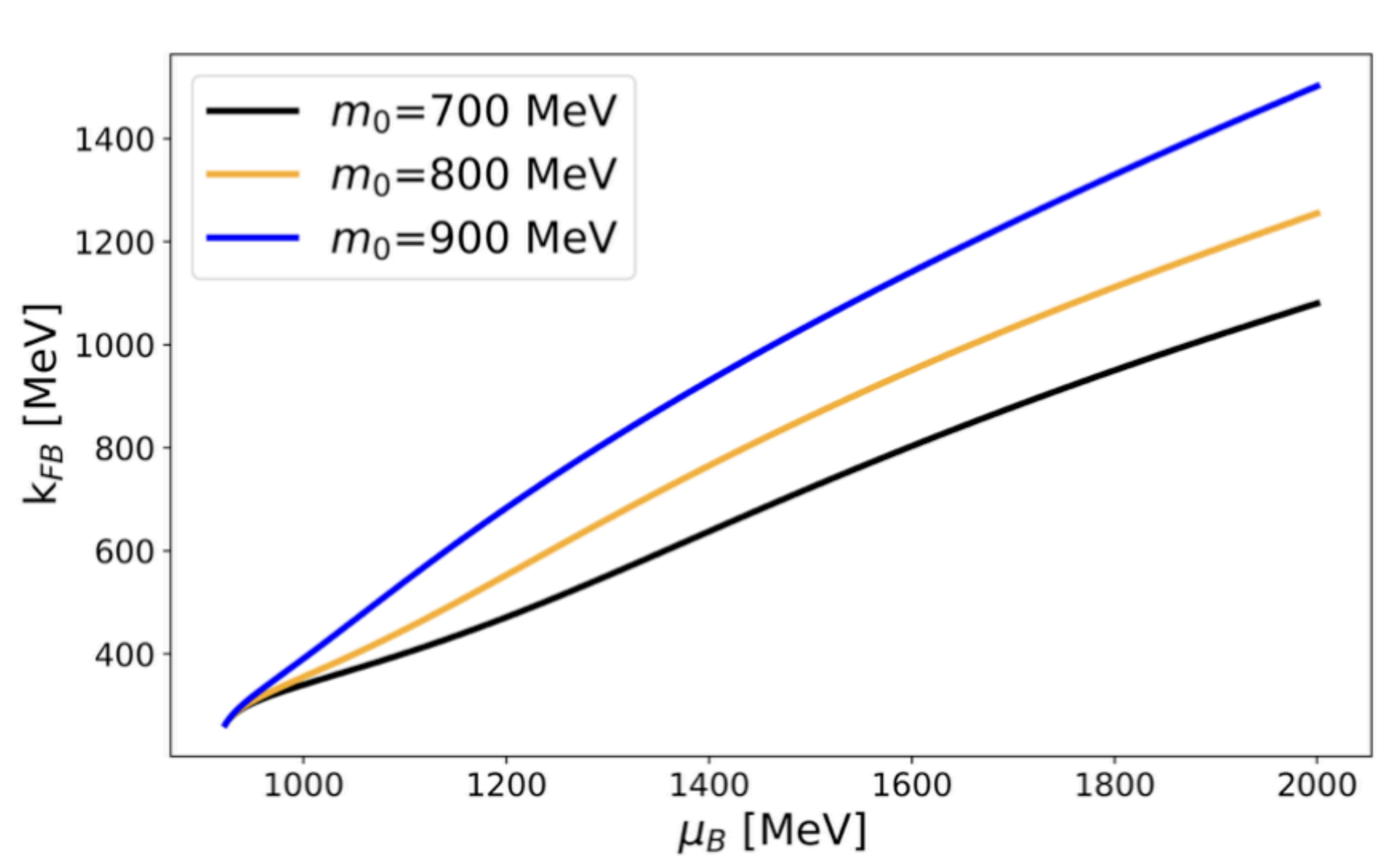
$$m_\pm = \sqrt{m_0^2 + \left(\frac{g_1 + g_2}{2}\right)^2} \mp \frac{g_1 - g_2}{2}\sigma.$$

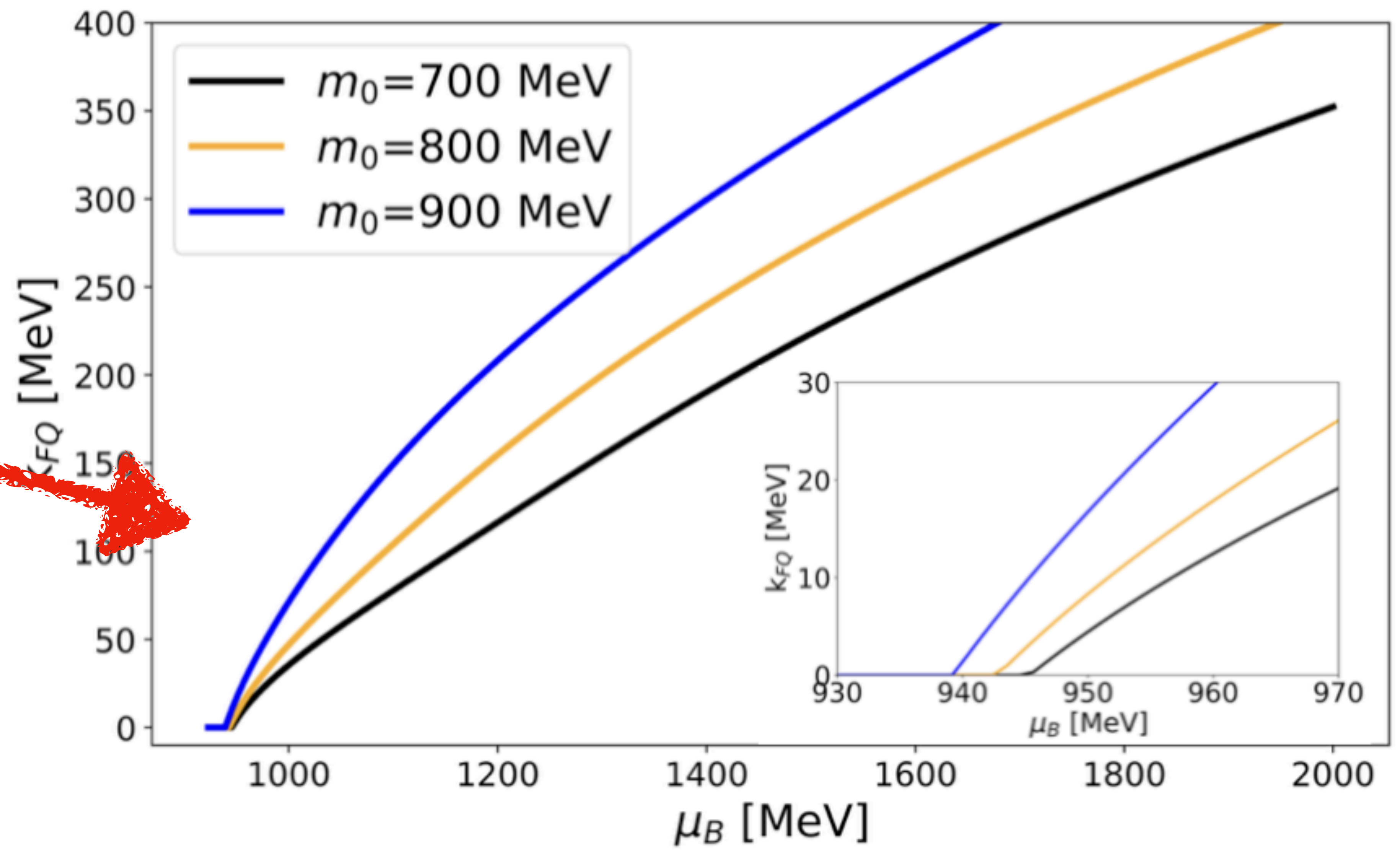
Parameters in the model are determined by the saturation properties

n_0 [fm^{-3}]	B_0 [MeV]	K_0 [MeV]	S_0 [MeV]
0.16	16	240	31

Results





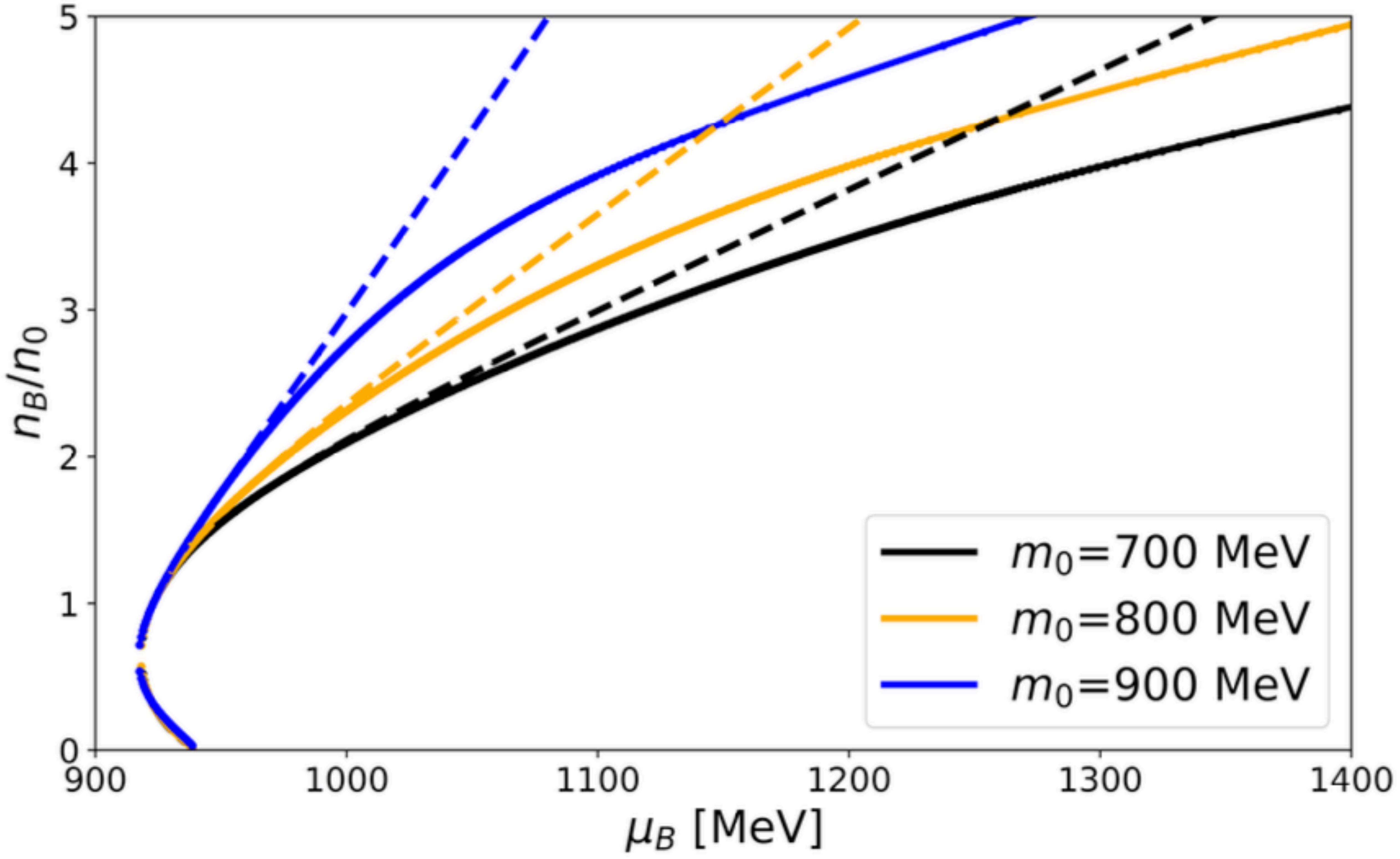


Baryon number density vs. baryon chemical potential

(Dashed: ordinary PDM, Solid: Quarkyonic PDM)

The contribution from the quarks relative to nucleons is suppressed

Results



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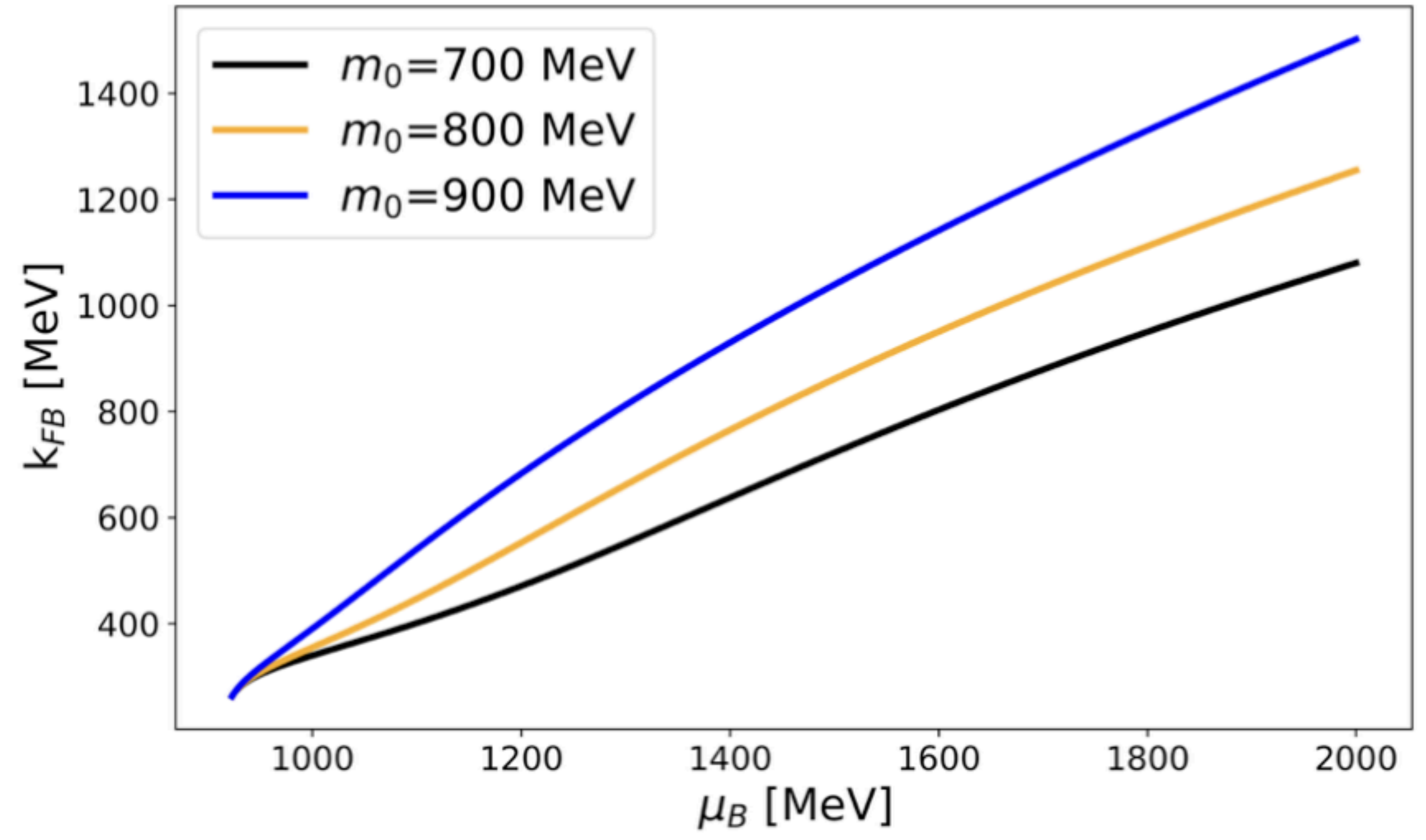
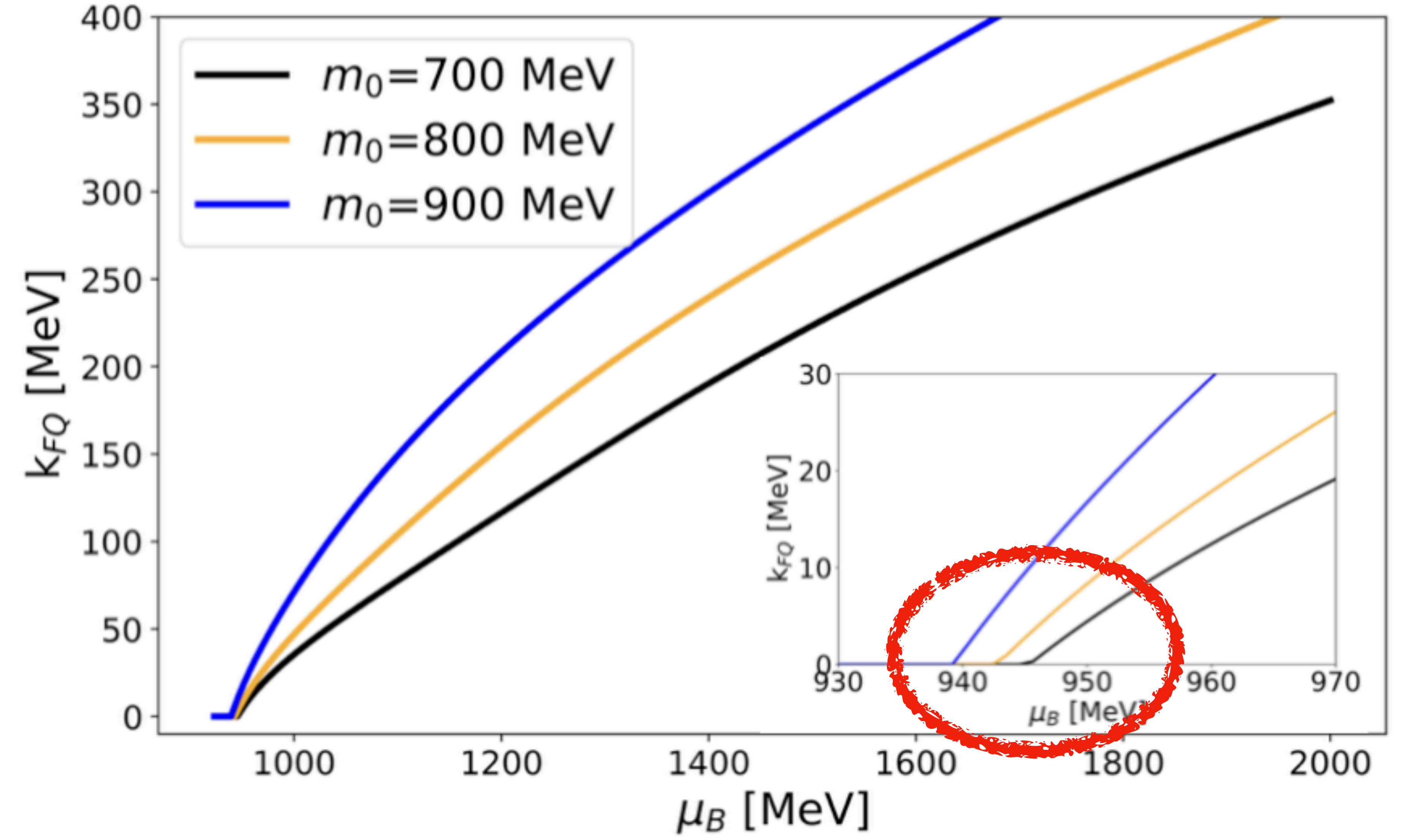
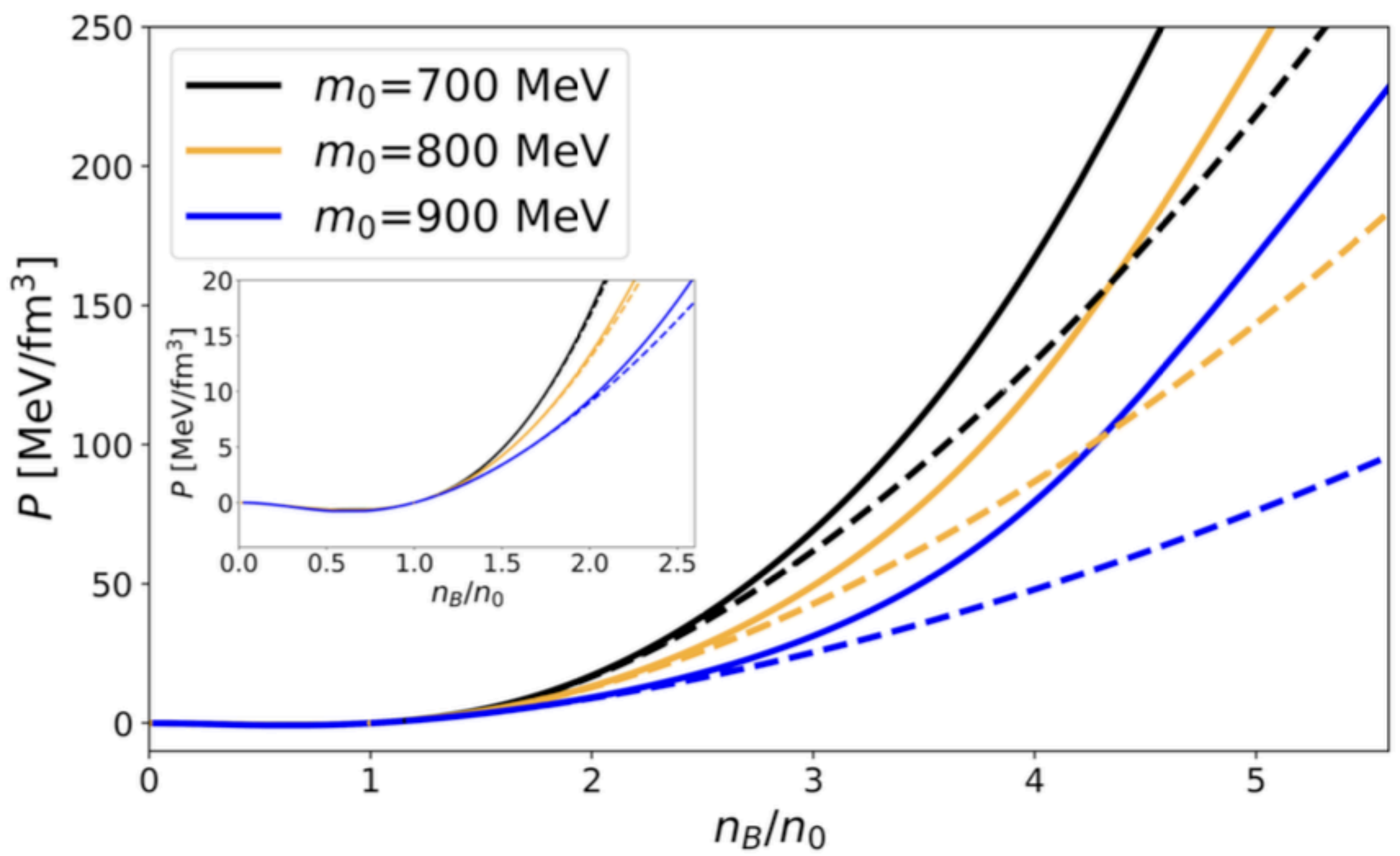


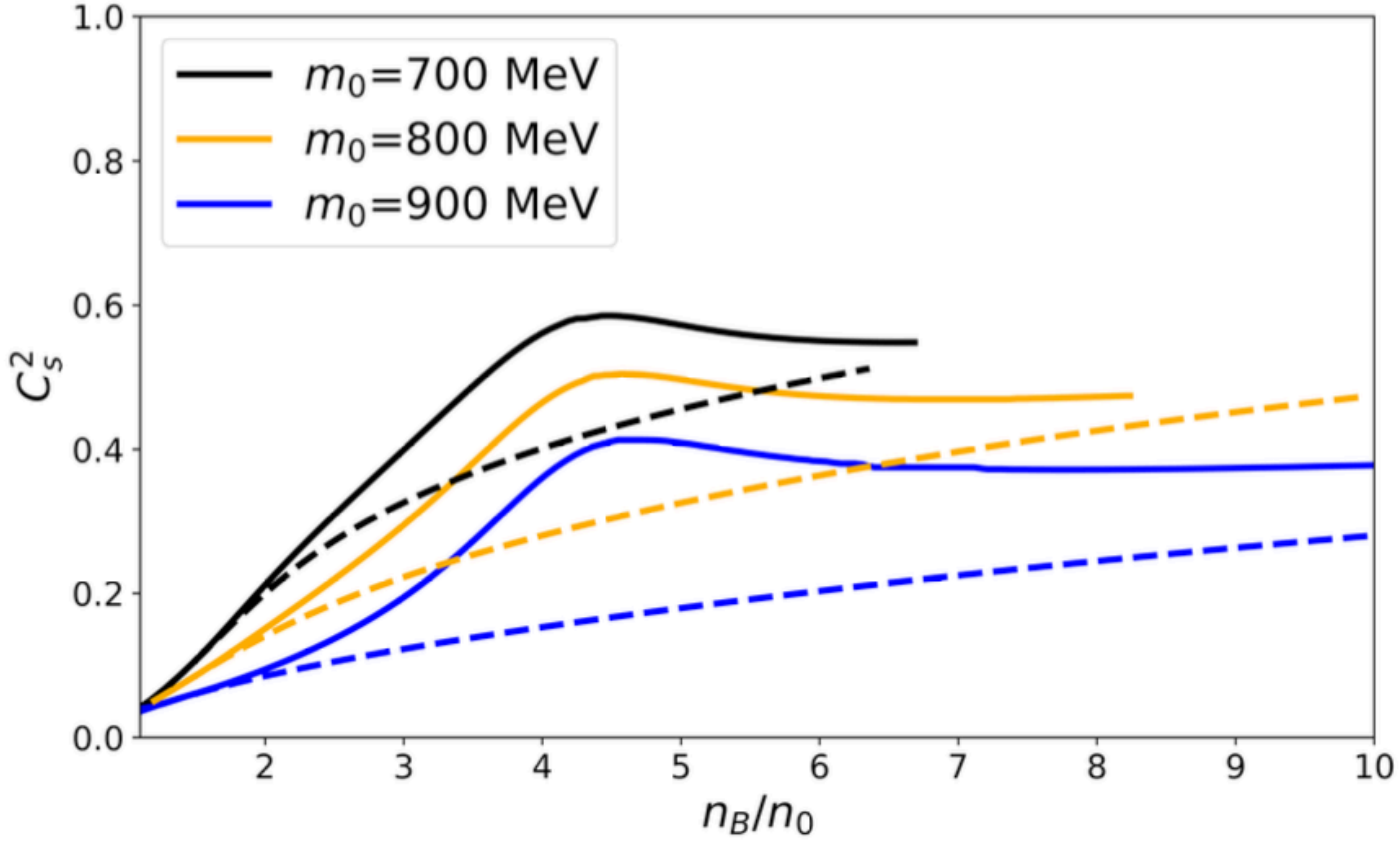


$$n_B = \sum k_{FB}^3 / 3\pi^2 \qquad \Delta = \frac{\Lambda_{\rm QCD}^3}{k_{FB}^2},$$

For larger values of m0, the quarkyonic matter appears at lower μB

Results





Stiffening of the EOS after entering the quarkyonic phase

$$4N_c \int_0^{k_{FQ}} \frac{d^3\mathbf{q}}{(2\pi)^3} (\mu_q^* - E_\mathbf{q})$$
$$= 4N_c^4 \int_0^{N_c k_{FQ}} \frac{d^3\mathbf{q}'}{(2\pi)^3} \left(\mu_q^* - N_c \sqrt{(q')^2 + (\frac{M_Q}{N_c})^2} \right)$$

Enhanced by a factor of approximately Nc^3

Rapid increase of the pressure $c_s^2 = \frac{dP}{d\varepsilon}$

Non-monotonic behavior of sound velocity

Invariant mass in the constituent quark

To examine the impact of including an invariant mass component in the constituent quark

$$M_Q = m_+/w(\sigma),$$
$$w(\sigma) = w_0 - (w_0 - 3)\frac{\sigma}{f_\pi}.$$

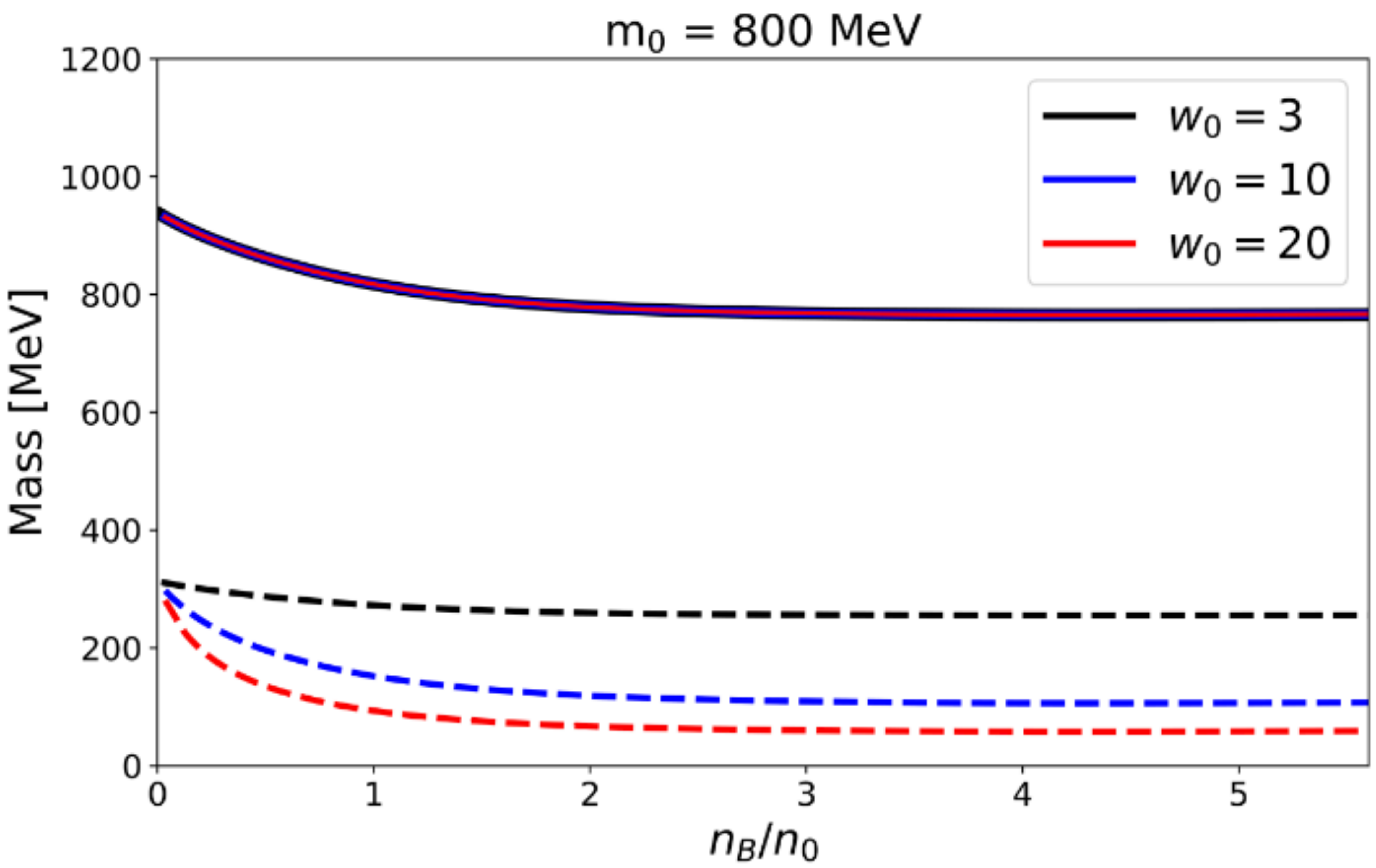
ω_0 is a constant parameter

In vacuum $\sigma = f_\pi$ Conventional quark model

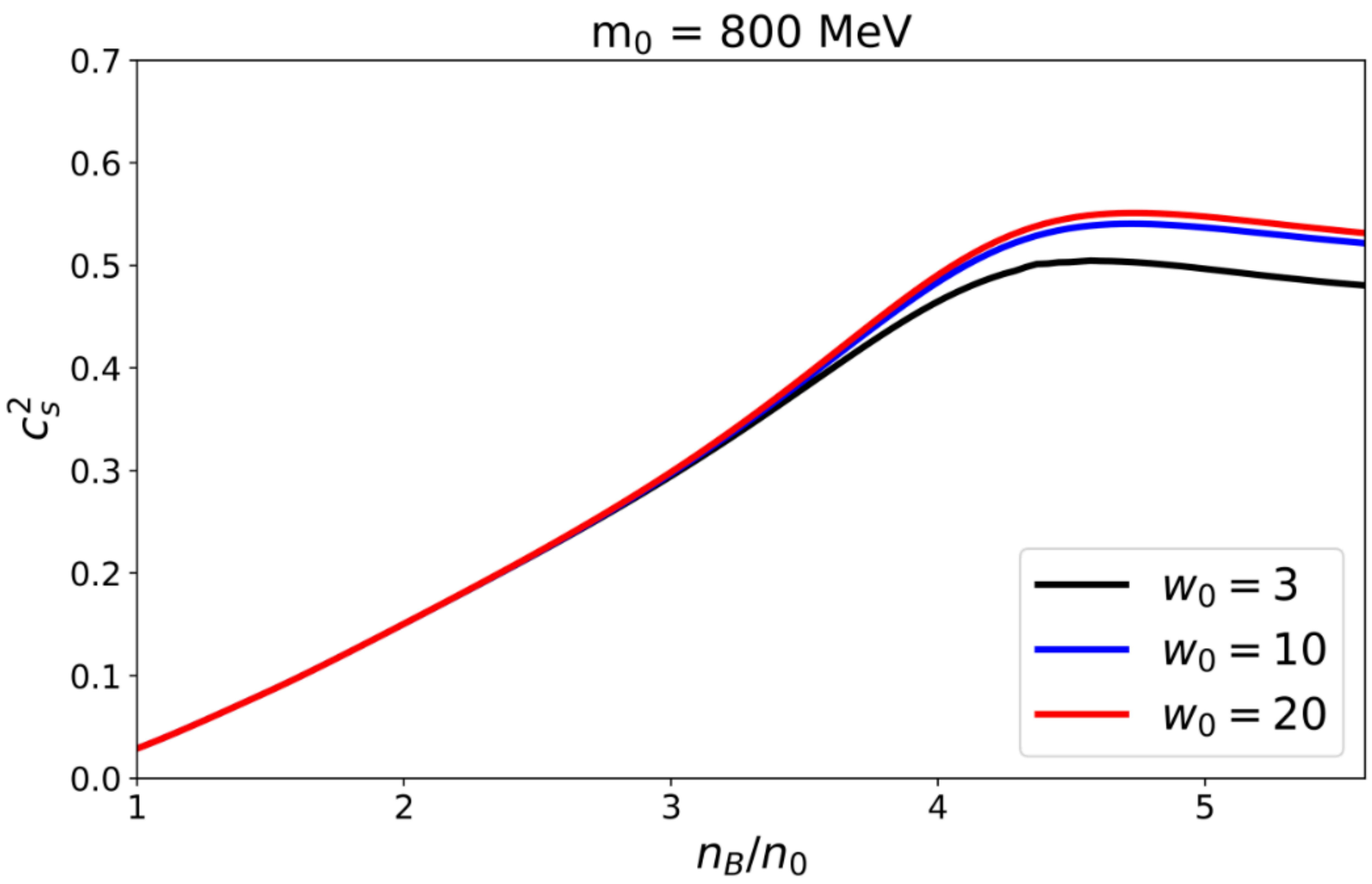
$M_Q = m_+/3$

Chiral symmetry restored $\sigma \to 0$

$M_Q = m_+/\omega_0$



Invariant mass in the constituent quark



Smaller invariant mass component in the constituent quark leads to **larger values in the sound velocity**

Yukawa interaction of σ to the constituent quark becomes weaker!

The reduced interaction strength manifests as a smaller maximum value in the sound velocity.

Summary & Future

- We use the parity double model together with the NJL-type quark model to construct the unified EoS.
- We successfully reconcile with the multi-messenger constraints at the same time
- We have presented a novel approach to describe dense nuclear matter by integrating the quarkyonic matter framework with the PDM
- We introduced **chiral invariant mass for both baryons and quarks**, allowing for a smooth transition between hadronic and quark degrees of freedom

Non-monotonic behavior in the sound velocity

Future work:

Extend the model to neutron star matter;
Compare with the recent neutron star observations

Thank you for your attention!