OSAKA INSTITUTE OF TECHNOLOGY

Four-dimensional equation of state of QCD matter with multiple chemical potentials

Akihiko Monnai (Osaka Institute of Technology)

with Grégoire Pihan (Wayne State U.), Björn Schenke (BNL), Chun Shen (Wayne State U./RBRC)

AM, G. Pihan, B. Schenke, C. Shen, Phys. Rev. C **110**, 044905 (2024)

Hadrons and Hadron Interactions in QCD 2024

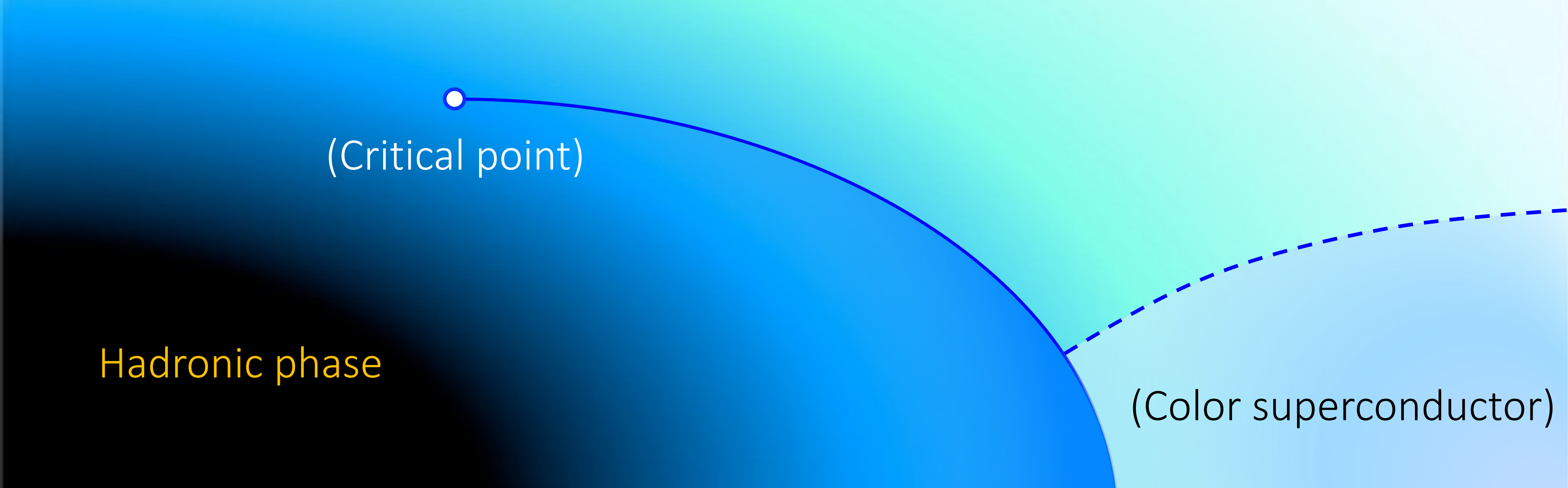
7th November 2024, YITP, Kyoto, Japan

Introduction

- Exploring the QCD Phase diagram

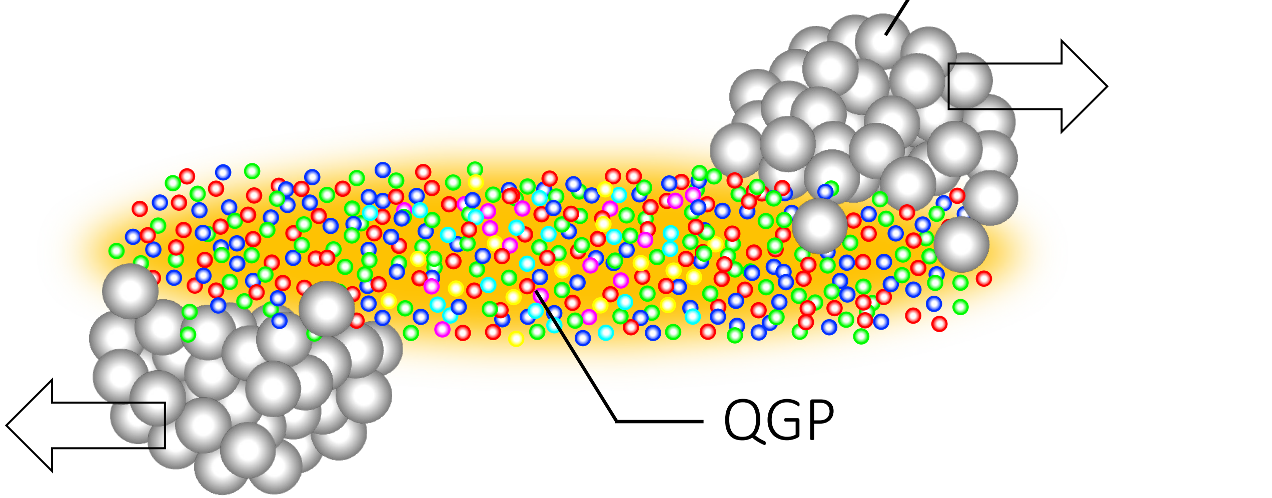
QCD has a rich phase structure depending on the temperature and chemical potentials

Quark-gluon plasma (QGP) phase



Introduction

- How to make the quark-gluon plasma (QGP)

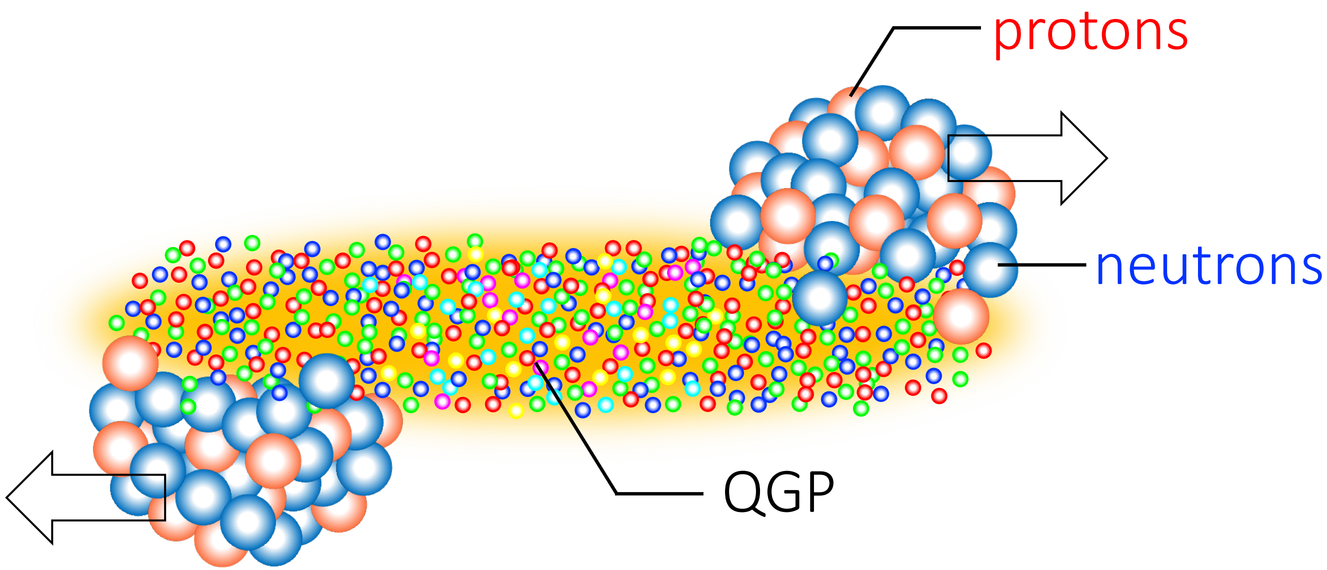


The QGP can be created in nuclear collisions at relativistic energies

BNL Relativistic Heavy Ion Collider (RHIC)
CERN Large Hadron Collider (LHC)

Introduction

- A more precise view of nuclear collisions



Protons and neutrons should be distinguished for precision analyses

BNL Relativistic Heavy Ion Collider (RHIC)

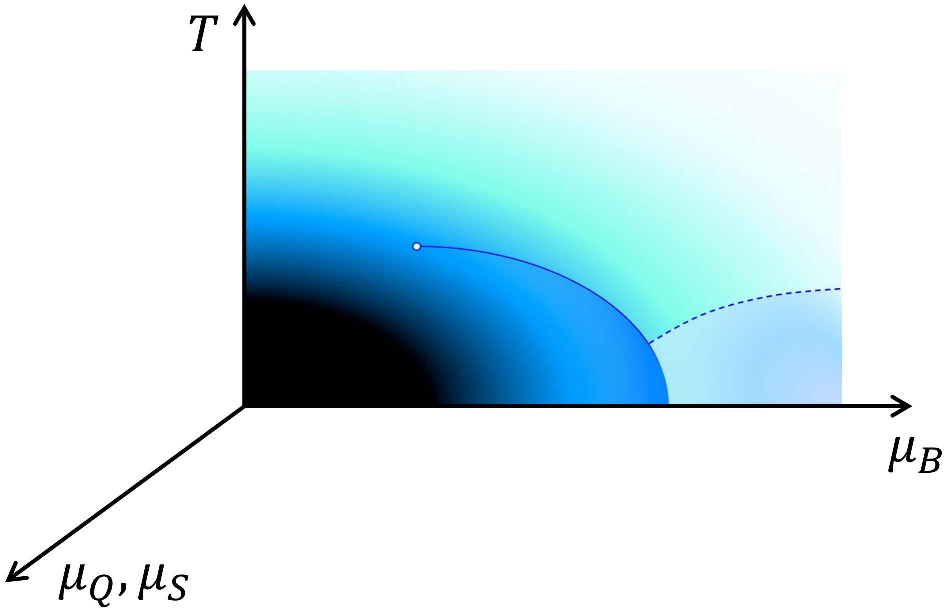
CERN Large Hadron Collider (LHC)

Nuclear collisions

- Conserved charges

The QGP in nuclear collisions are made of light quarks (u, d, s) ($T \sim 200$ MeV)

Baryon (B) | Electric charge (Q) | Strangeness (S) are conserved

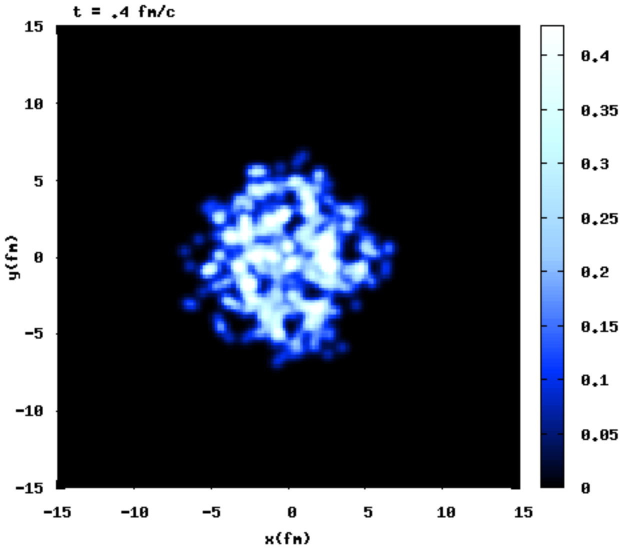


The QCD phase diagram has to be extended to 4 dimensions

- T: Temeperature
- μ_B: Baryon chemical potential
- μ_Q: Charge chemical potential
- μ_S: Strangeness chemical potential

Nuclear collisions

- Relativistic hydrodynamic model



Equation of state

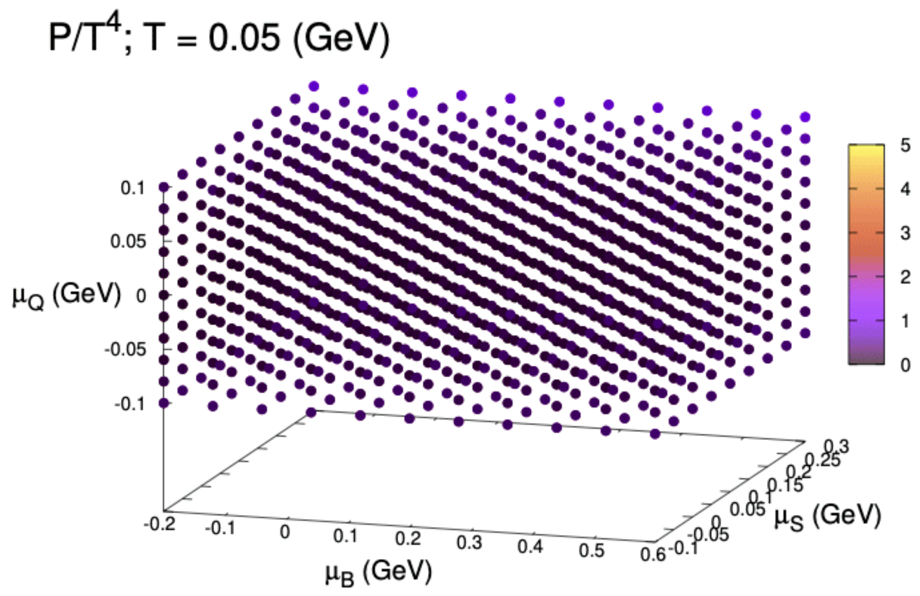
*A relation among state variables such as pressure, temperature, etc.

Initial condition → Hydro evolution → Hadronic transport → Observa-bles

We construct a 4-dimensional QCD equation of state at finite chemical potentials for nuclear collisions

NEOS-4D

- A lattice QCD-based equation of state model



- It has B, Q, S charges without constraints, *i.e.*, it is fully 4-dimensional

- Generalization of NEOS BQS, that is tuned to $n_Q = 0.4\, n_B, n_S = 0$ for heavy nuclei (^{197}Au, ^{208}Pb, etc.)
  AM, B. Schenke, C. Shen, Phys. Rev. C **100**, 024907 (2019)

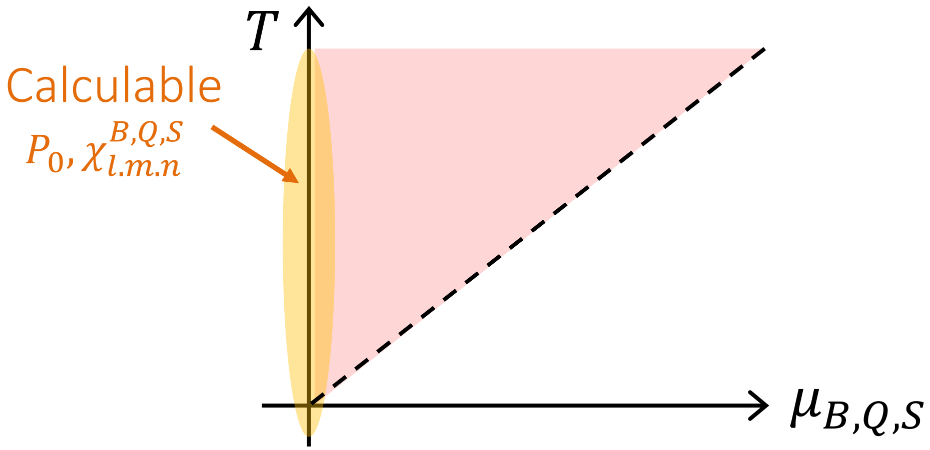
- Applicable to systems with various nuclei and with fluctuations and diffusion

Construction

- QGP phase: Taylor expansion method of lattice QCD

$$\frac{P_{\mathrm{lat}}}{T^4} = \frac{P_0}{T^4} + \sum_{l,m,n} \frac{\chi_{l.m.n}^{B,Q,S}}{l!\,m!\,n!} \left(\frac{\mu_B}{T}\right)^l \left(\frac{\mu_Q}{T}\right)^m \left(\frac{\mu_S}{T}\right)^n$$

HotQCD Collaboration, PRD 86, 034509 (2012);
PRD 90, 094503 (2014); PRD 92, 074043 (2015);
PRD 95, 054504 (2017)



Pro: Ab initio calculation

Con: not reliable when $\frac{\mu}{T}$ is too large

- Susceptibilities up to the 4th order from lattice QCD
- $\chi_6^B, \chi_{5,1}^{B,Q}, \chi_{5,1}^{B,S}$ parametrized as required by thermodynamic conditions

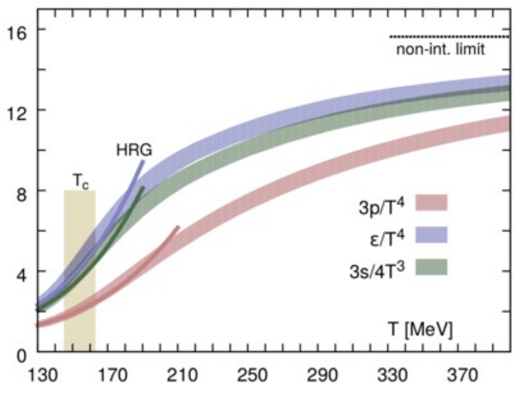
Construction

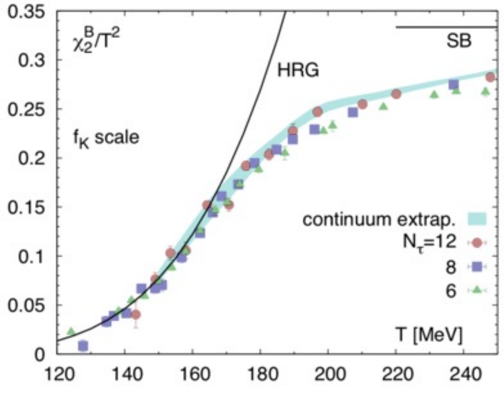
■ Hadronic phase: Hadron resonance gas model

$$P_{\mathrm{had}} = \pm T \sum_i \frac{g_i d^3 p}{(2\pi)^3} \ln[1 \pm e^{-(E_i - \mu_i)/T}]$$

Particle Data Group: PRD 98, 030001 (2018)

- Hadrons and resonances with u, d, s components with the mass below 2 GeV are used





Pro: Consistent with lattice QCD

Con: Describes only the hadronic phase

Construction

- The crossover-type EoS is obtained by smoothly connect the two EoS

$$P = \frac{1}{2}\left(1 - \tanh\frac{T - T_c}{\Delta T_c}\right) P_{\mathrm{had}} + \frac{1}{2}\left(1 + \tanh\frac{T - T_c}{\Delta T_c}\right) P_{\mathrm{lat}}$$

Hadron resonance gas model

$$P_{\mathrm{had}} = \pm T \sum_i \frac{g_i d^3 p}{(2\pi)^3} \ln[1 \pm e^{-(E_i - \mu_i)/T}]$$

Lattice QCD with Taylor expansion

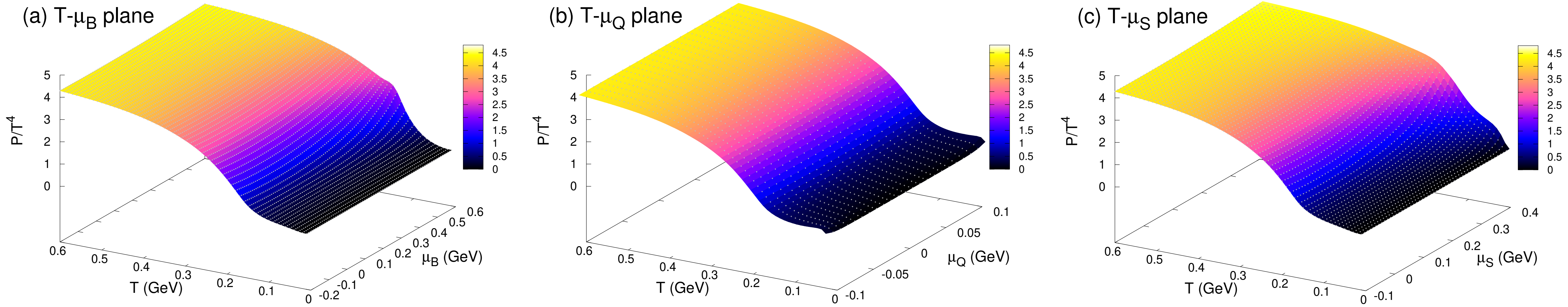
$$\frac{P_{\mathrm{lat}}}{T^4} = \frac{P_0}{T^4} + \sum_{l,m,n} \frac{\chi_{l.m.n}^{B,Q,S}}{l!\,m!\,n!} \left(\frac{\mu_B}{T}\right)^l \left(\frac{\mu_Q}{T}\right)^m \left(\frac{\mu_S}{T}\right)^n$$

($T_c(\mu_B) = 0.16 - 0.4(0.139\mu_B^2 + 0.053\mu_B^4)$ GeV, $\Delta T_c = 0.1 T_c(0)$ J. Cleymans et. al., PRC 73, 034905 (2006))

Numerical results

Results

■ Pressure



The dimensionless pressure on the 2D slices of temperature and chemical potentials in the 4D phase space

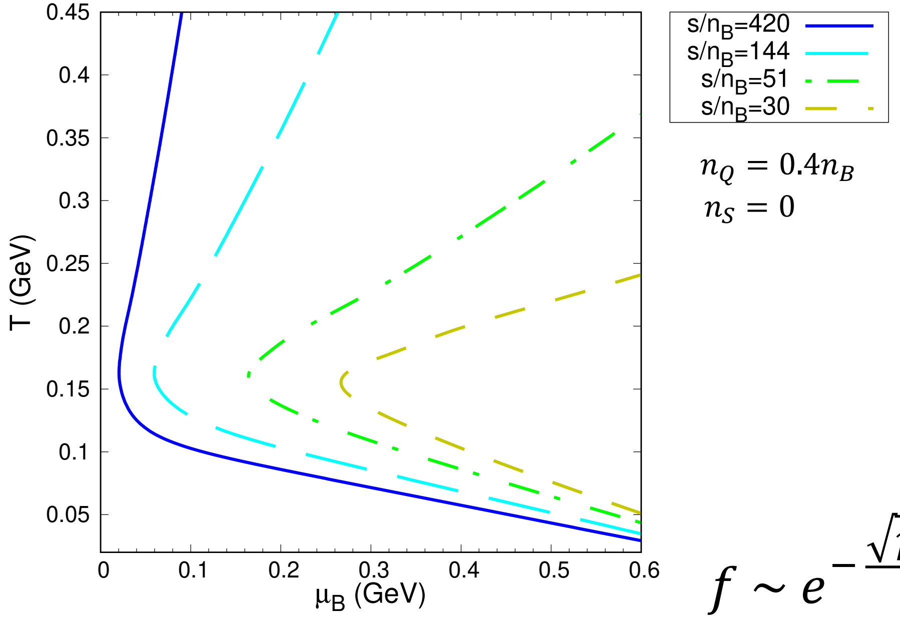
Phase diagram

- Regions explored in nuclear collisions

s/n_B is constant when entropy and net baryon number are conserved

$s/n_B = 420$	$\sqrt{s_{NN}} = 200$ GeV
$s/n_B = 144$	$\sqrt{s_{NN}} = 62.4$ GeV
$s/n_B = 51$	$\sqrt{s_{NN}} = 19.6$ GeV
$s/n_B = 30$	$\sqrt{s_{NN}} = 14.5$ GeV

J. Gunther et. al., Nucl. Phys. A 967, 720 (2017)



$n_Q = 0.4 n_B$

$n_S = 0$

$$f \sim e^{-\frac{\sqrt{p^2+m^2}-\mu}{T}}$$

The QGP phase has straight lines because $s/n_B \approx T/\mu_B$

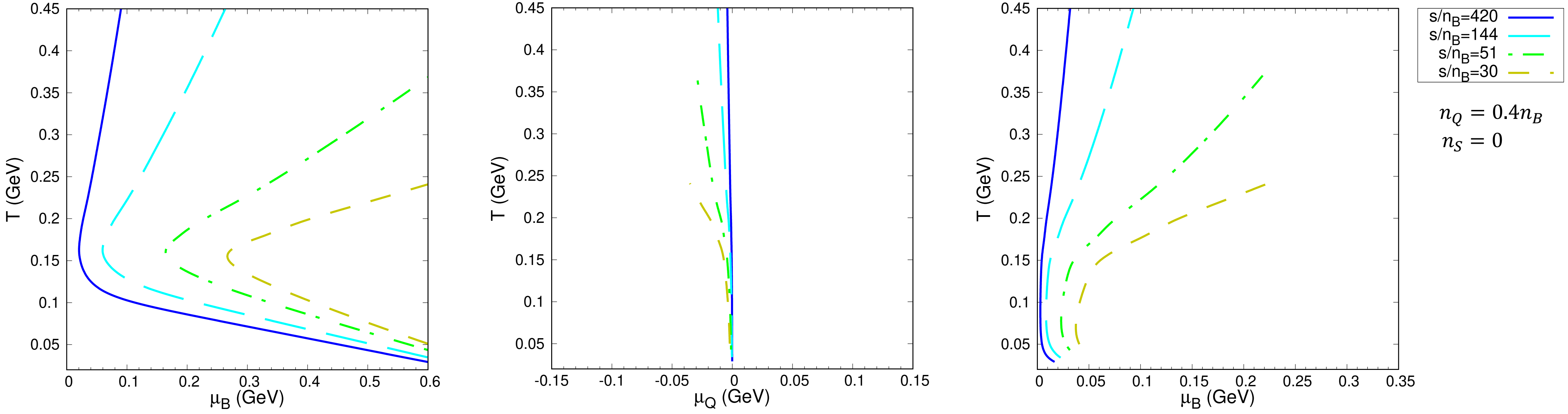
Larger μ_B is required in hadronic phase because protons are heavy

Results

$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q > \mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q$$

$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S = 0$$

- Trajectories in the phase diagram



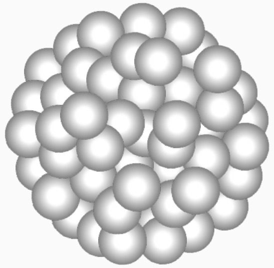
$n_Q = 0.4 n_B$

$n_S = 0$

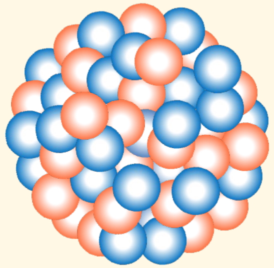
The estimated region explored in nuclear collisions is narrow in μ_Q with the "nucleon" approximation of $n_Q/n_B = 0.4$

Nuclear collisions

- The charge-to-baryon ratio in nuclear collisions



$\frac{n_Q}{n_B} \approx 0.4$ on average for heavy nuclei (^{197}Au, ^{208}Pb) and $n_S = 0$

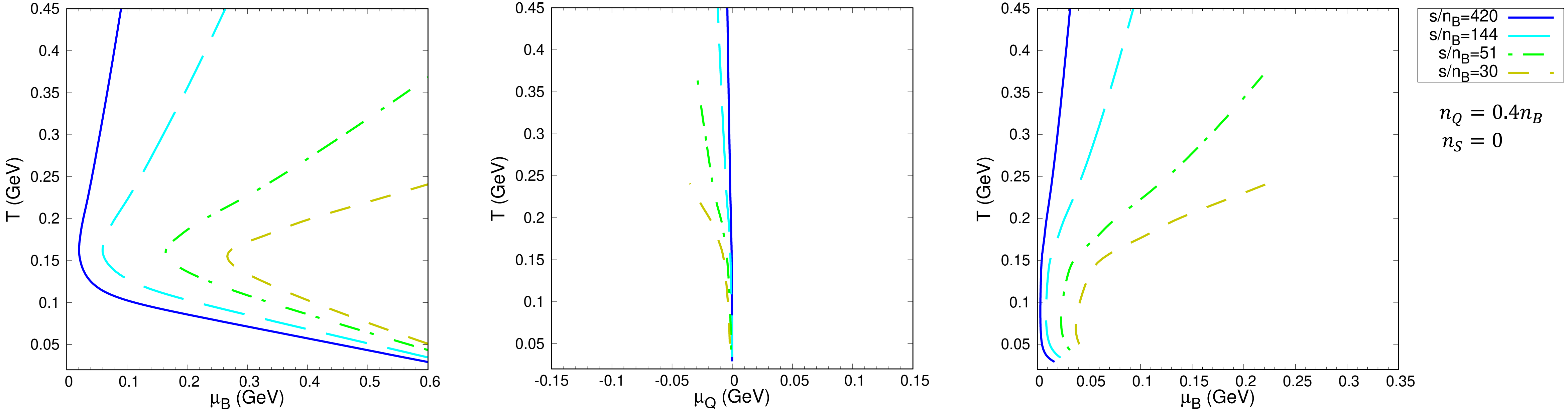


$\frac{n_Q}{n_B} = 1$ in proton-rich regions and $\frac{n_Q}{n_B} = 0$ in neutron-rich regions

⚠ Additional dynamics (e.g. fluctuation, diffusion) can lead to $\frac{n_Q}{n_B} > 1$ or $\frac{n_Q}{n_B} < 0$

Results

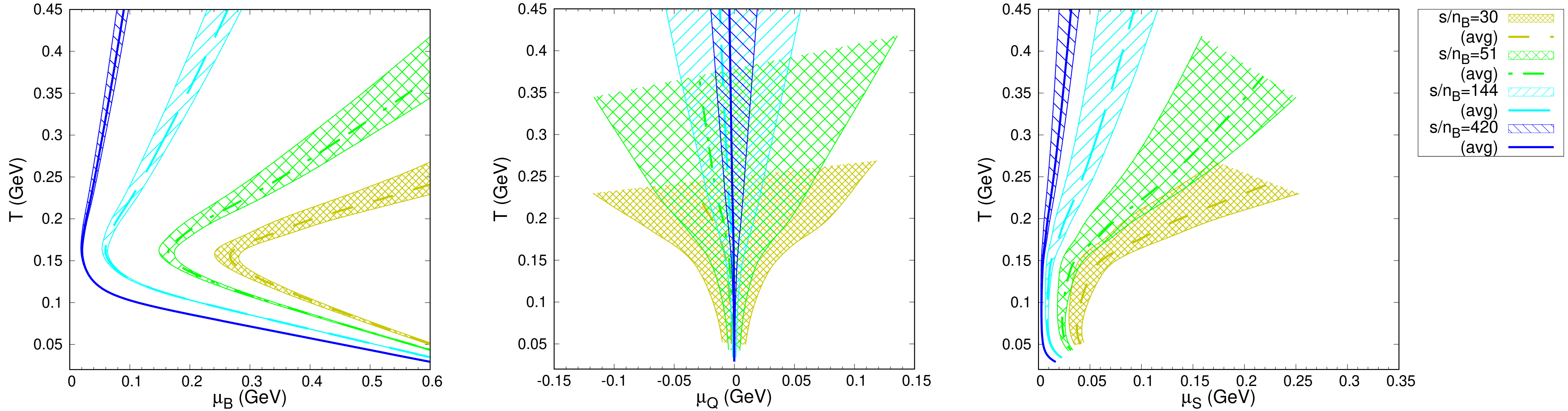
- Trajectories in the phase diagram



$n_Q = 0.4 n_B$
$n_S = 0$

Results

- Trajectories in the phase diagram



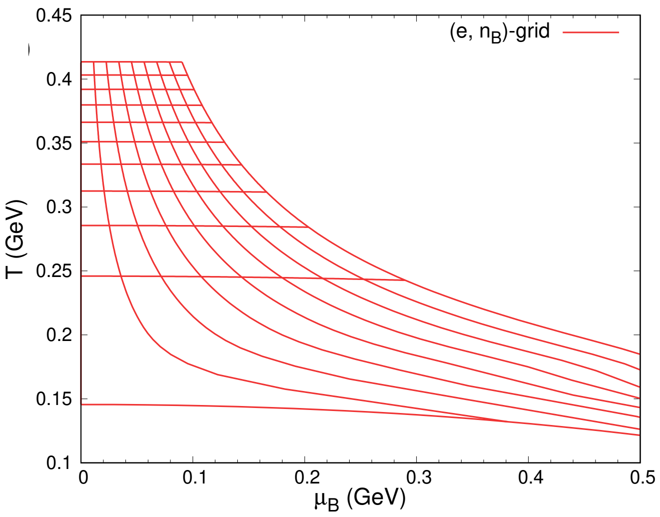
Bands denote the regions between $n_Q/n_B = 1$ and 0; Wide regions of the phase diagram will be explored in colliders

Application to hydrodynamic model

- Hydrodynamic model require $P, T, \mu_B, \mu_Q, \mu_S$ as functions of e, n_B, n_Q, n_S

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N_B^\mu = 0, \quad \partial_\mu N_Q^\mu = 0, \quad \partial_\mu N_S^\mu = 0$$

⇨ One often prepares pre-calculated tables of the EoS for efficient numerical simulations



However, a grid with equal spacing in e, n_B, n_Q, n_S results in a warped grid in T, μ_B, μ_Q, μ_S

Covering it leads to a huge redundancy in the 4D case, making hydro simulations difficult

Application

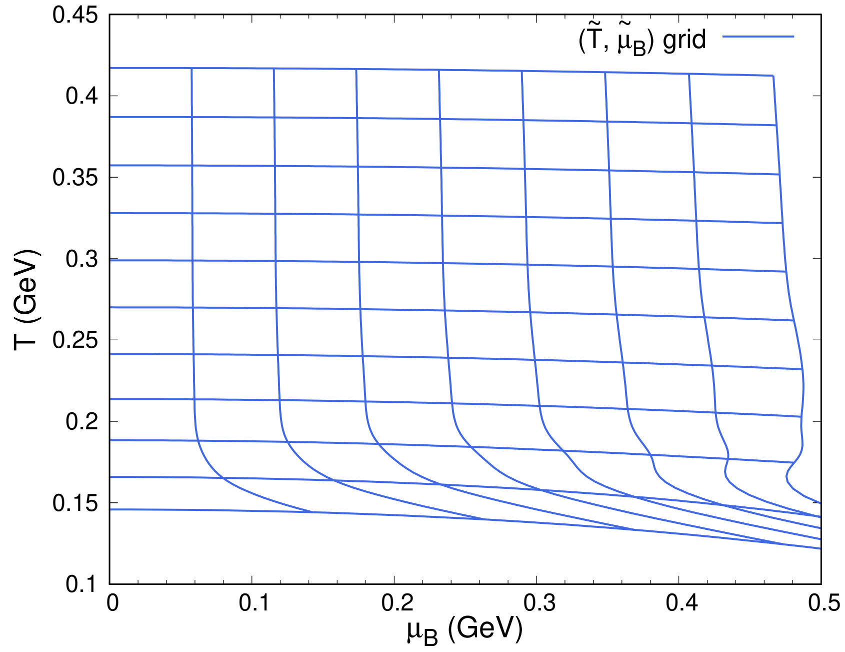
- We introduce $\tilde{T}$, $\tilde{\mu}_B$, $\tilde{\mu}_Q$, $\tilde{\mu}_S$, defined as the temperature and chemical potentials of a parton gas with the given e, n_B, n_Q, n_S, for tabulation

$$\tilde{T}(e,n_B,n_Q,n_S) = \left(\frac{12}{19\pi^2}e\right)^{1/4}$$

$$\tilde{\mu}_B(e,n_B,n_Q,n_S) = \frac{5n_B - n_Q + 2n_S}{\tilde{T}^2}$$

$$\tilde{\mu}_Q(e,n_B,n_Q,n_S) = \frac{-n_B + 2n_Q - n_S}{\tilde{T}^2}$$

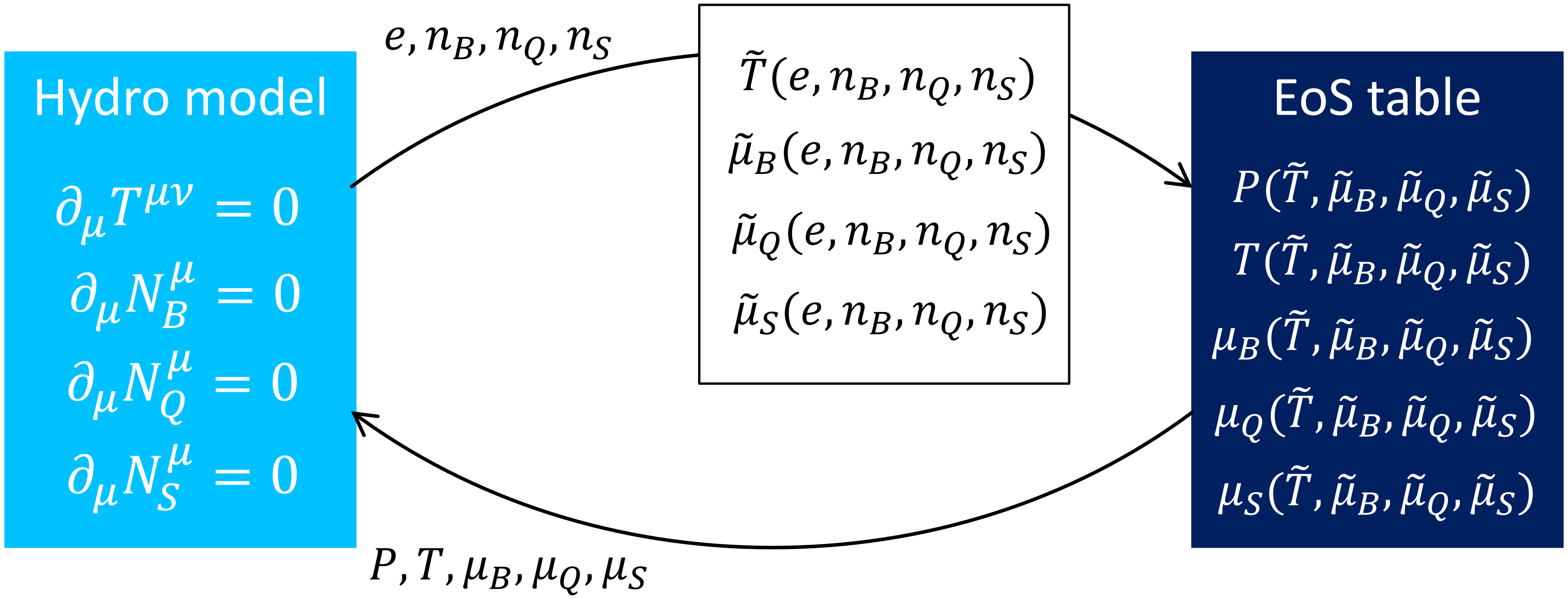
$$\tilde{\mu}_S(e,n_B,n_Q,n_S) = \frac{2n_B - n_Q + 2n_S}{\tilde{T}^2}$$



A grids with equal spacing in $\tilde{T}$, $\tilde{\mu}_B$, $\tilde{\mu}_Q$, $\tilde{\mu}_S$ is relatively straight in T, μ_B, μ_Q, μ_S

Application

- Schematic of EoS implementation to hydrodynamic model of nuclear collisions

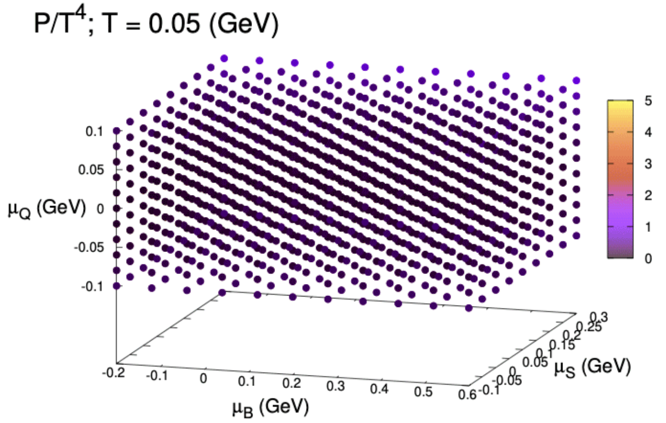


Calculations become efficient; see our recent analyses of isobar collisions for successful applications G. Pihan, AM, B. Schenke, C. Shen, Phys. Rev. Lett. **133**, 182301 (2024)

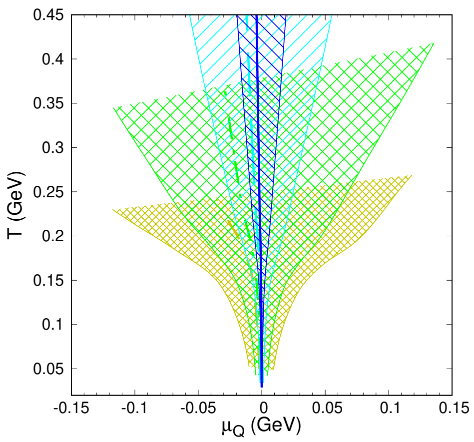
Summary and outlook

- We have constructed a crossover-type QCD EoS model, NEOS-4D, with net baryon (B), electric charge (Q) and strangeness (S)

  - Lattice QCD results from Taylor expansion method is utilized
  - It is smoothly matched to the hadron resonance gas model at lower temperatures

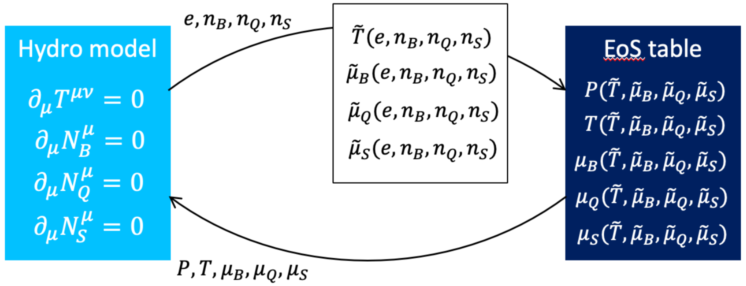


  - One can distinguish protons and neutrons; wide ranges in the T-μ_B-μ_Q-μ_S space are explored



Summary and outlook

- An efficient method of numerical implementation of the 4D EoS to the hydrodynamic model is developed using $\tilde{T}$, $\tilde{\mu}_B$, $\tilde{\mu}_Q$, $\tilde{\mu}_S$ variables



Outlook

- Introduction of higher order susceptibilities from Lattice QCD
- Application to the hydrodynamic analyses of nuclear collisions at beam energy scan energies and of different nuclear species
- Estimation of the effects of fluctuations and diffusions

Summary and outlook

- The results of our equation of state model NEOS-4D are publicly available:

  https://sites.google.com/view/qcdneos4d/home

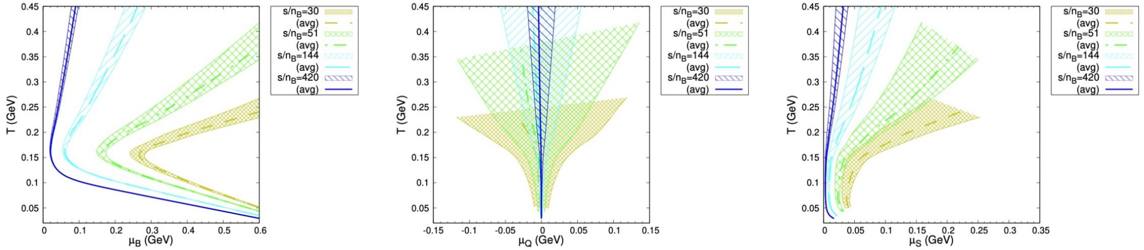
QCD equation of state

QCD equation of state

NEOS-4D

Reference: A. Monnai, G. Pihan, B. Schenke, C. Shen, arXiv:2406.11610 [nucl-th]

NEOS-4D is a four-dimensional extension of the QCD equation of state (EoS) model model NEOS constructed to cover a wide range in the QCD phase diagram with the chemical potentials of net baryon, electric charge, and strangeness that could be explored in relativistic nuclear collisions.

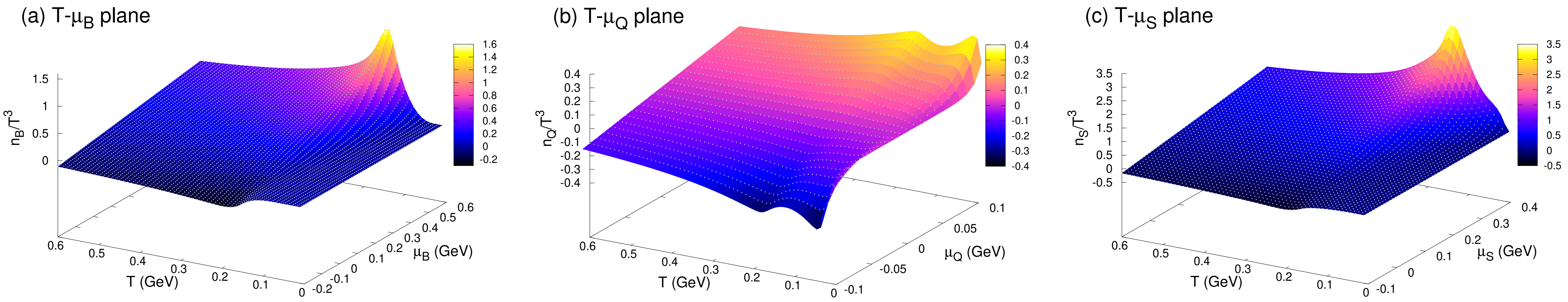


Thank you for listening!

Backup slides

Results

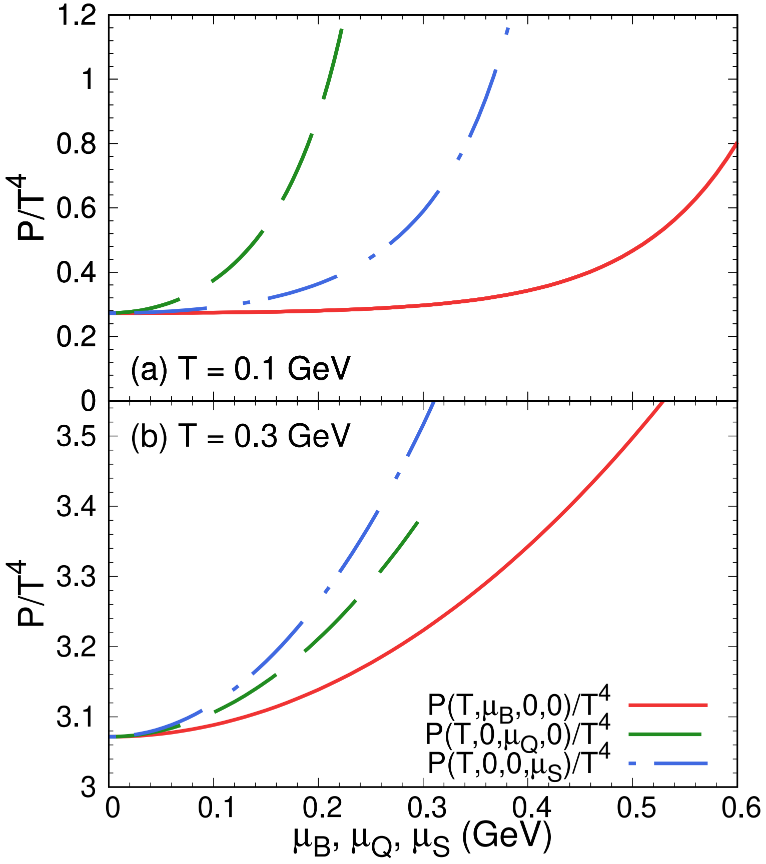
- Net baryon, charge, and strangeness densities



The dimensionless conserved charges on 2D slices of the temperature and chemical potentials in the 4D phase space

Results

- Effects of chemical potentials on the pressure



Hadronic phase

μ_Q has the largest effect followed by μ_S and μ_B because the lightest hadrons to carry the charges are ordered in mass as $m_p > m_K > m_\pi$

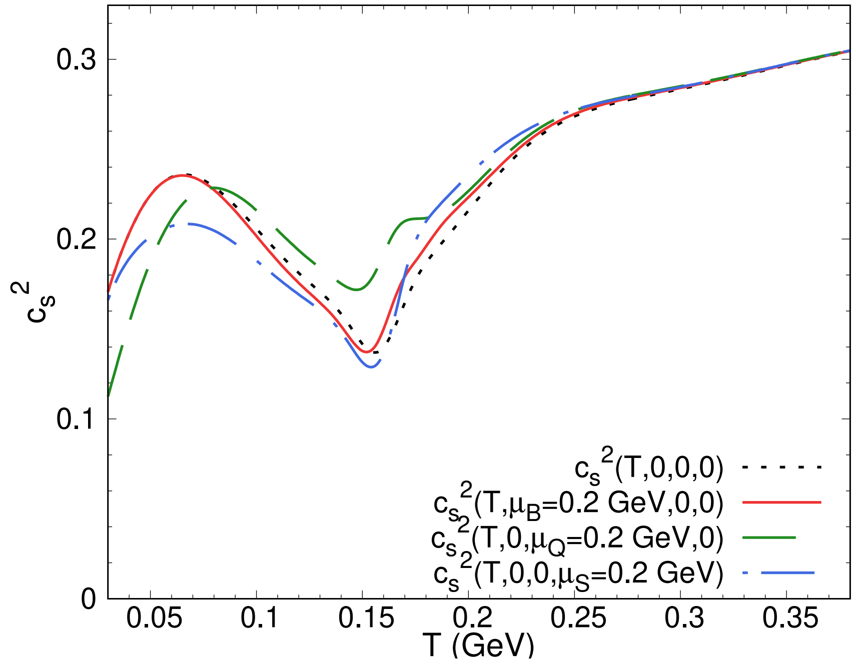
QGP phase

μ_S has the largest effect followed by μ_Q and μ_B

Can be interpreted in the parton picture as $\chi_2^B = 1/3,\ \chi_2^Q = 2/3,\ \chi_2^S = 1$ hold

Results

- Sound velocity



The chemical potentials have non-trivial effects on the sound velocity

μ_Q has the largest effect in the hadronic phase and μ_S has the largest effect in the QGP phase

$$c_S^2 = \left.\frac{\partial P}{\partial e}\right|_{n_B,n_Q,n_S} + \left.\frac{n_B}{e+P}\frac{\partial P}{\partial n_B}\right|_{e,n_Q,n_S} + \left.\frac{n_Q}{e+P}\frac{\partial P}{\partial n_Q}\right|_{e,n_B,n_S} + \left.\frac{n_S}{e+P}\frac{\partial P}{\partial n_S}\right|_{e,n_B,n_Q}$$

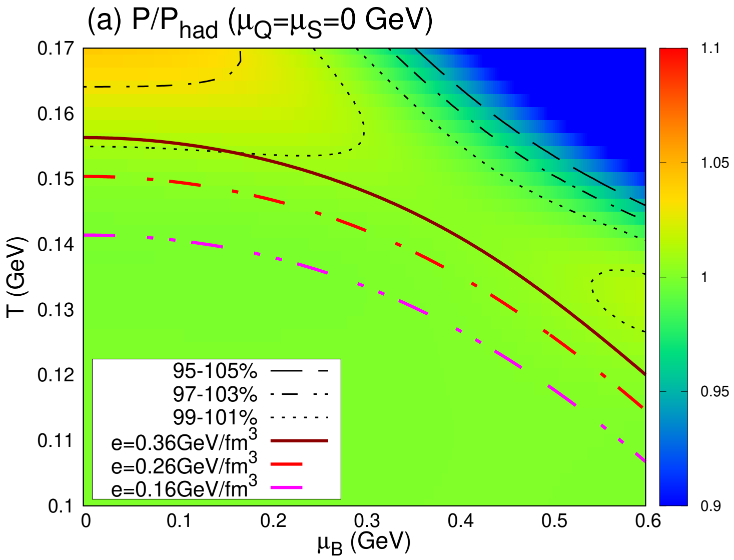
Particlization

- Hydrodynamic flow needs to be converted into particles using kinetic theory

$$E\frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3}\int_\Sigma f_i\, p^\mu d\sigma_\mu$$

Cooper and Frye, Phys. Rev. D 10, 186 (1974)

⇨ The EoS of hydrodynamic model and kinetic theory should match at particlizaition for energy-momentum/charge conservation



Dependence on the particlization energy density e

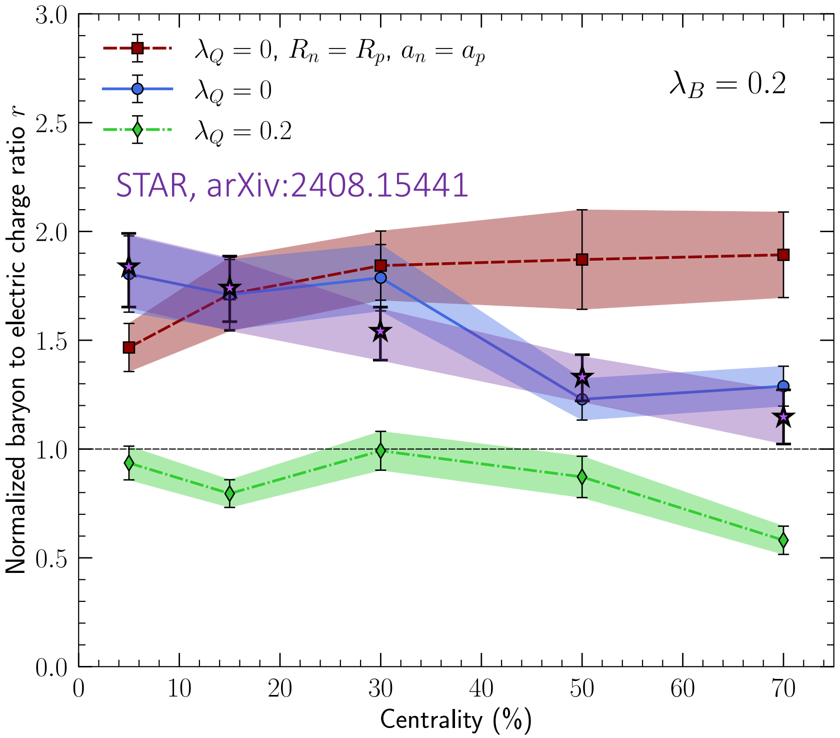
$$\left|1-\frac{P}{P_{\rm had}}\right| < 1\% \text{ for } e = 0.16,\ 0.26 \text{ GeV/fm}^3$$

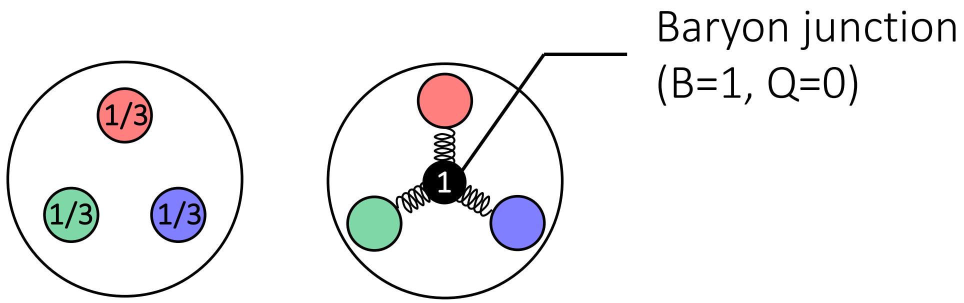
$$\left|1-\frac{P}{P_{\rm had}}\right| < 3\% \text{ for } e = 0.36 \text{ GeV/fm}^3$$

Application

G. Pihan, AM, B. Schenke, C. Shen, Phys. Rev. Lett. **133**, 182301 (2024)

- Effects of neutron skin and baryon junction in isobar collisions ($^{96}_{44}$Ru, $^{96}_{40}$Zr)





$r > 1$ if baryon junctions carry B

r decreases in peripheral collisions if Zr has neurton skin

$$r = \frac{N_B^{\rm Ru} + N_B^{\rm Zr}}{2\left(N_Q^{\rm Ru} - N_Q^{\rm Zr}\right)} \times \frac{Z_{\rm Ru} - Z_{\rm Zr}}{A}$$

$$\sim \frac{N_B}{N_Q^{\rm Ru} - N_Q^{\rm Zr}}$$

$Z_{\rm Ru} = 44, Z_{\rm Zr} = 40,$
$A = 96$

Application

- The equation of state for Nf = 3 parton gas model and the derivation of the expressions of $\tilde{T}$, $\tilde{\mu}_B$, $\tilde{\mu}_Q$, $\tilde{\mu}_S$

$$e = \frac{19\pi^2}{12}\tilde{T}^4$$
$$n_B = \frac{1}{3}\tilde{\mu}_B\tilde{T}^2 - \frac{1}{3}\tilde{\mu}_S\tilde{T}^2$$
$$n_Q = \frac{2}{3}\tilde{\mu}_Q\tilde{T}^2 - \frac{1}{3}\tilde{\mu}_S\tilde{T}^2$$
$$n_S = -\frac{1}{3}\tilde{\mu}_B\tilde{T}^2 + \frac{1}{3}\tilde{\mu}_Q\tilde{T}^2 + \tilde{\mu}_S\tilde{T}^2$$

$\Leftrightarrow$

$$\tilde{T}(e, n_B, n_Q, n_S) = \left(\frac{12}{19\pi^2}e\right)^{1/4}$$
$$\tilde{\mu}_B(e, n_B, n_Q, n_S) = \frac{5n_B - n_Q + 2n_S}{\tilde{T}^2}$$
$$\tilde{\mu}_Q(e, n_B, n_Q, n_S) = \frac{-n_B + 2n_Q - n_S}{\tilde{T}^2}$$
$$\tilde{\mu}_S(e, n_B, n_Q, n_S) = \frac{2n_B - n_Q + 2n_S}{\tilde{T}^2}$$