Study of Chiral Symmetry and $U(1)_A$ using Spatial Correlatiors for $N_f = 2$ QCD at finite temperature with Domain Wall Fermions

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Motivation

This talk focuses on symmetries of $N_f = 2$ QCD at temperatures around the critical point through screening mass differences of the mesonic spatial correlators.

- JLQCD simulates $N_f = 2$ QCD with Möbius domain-wall fermions with $m_{res} < 1\text{MeV}$. This approach is theoretically clean with well defined chiral $SU(2)_L \times SU(2)_R$ and axial $U(1)_A$ symmetries.
- Previous work done by JLQCD [Rohrhofer 2020] focused on temperatures above $1.1\,T_c$.
- This talk will focus on work with $0.9\,T_c$ and T_c added.

Symmetries of QCD and Chiral phase transition

$$S \xleftrightarrow[\exp(i\pi\gamma_5/2)]{U(1)_A} PS$$

$$V_x \xleftrightarrow[\exp(i\pi\gamma_5\tau^a/2)]{SU(2)_L \times SU(2)_R} A_x$$

$$T_t \longleftrightarrow X_t$$

- Below T_c, $\langle \bar{q}q(x) \rangle \neq 0$ indicating a broken $SU(2)_L \times SU(2)_R$ symmetry. While $U(1)_A$ is broken by anomaly.
- Above T_c, $\langle \bar{q}q(x) \rangle = 0$ "chiral" symmetry is restored.

Overlap Operator and Domain Wall Fermions

- The Wilson operator removes doublers but explicitly breaks chiral symmetry.

$$S_W = -\frac{1}{2}\sum_{x,\mu}\bar{\psi}(x)(r-\gamma_\mu)U_\mu(x)\psi(x+\hat{\mu}) + \bar{\psi}(x+\hat{\mu})(r+\gamma_\mu)U_\mu^\dagger(x)\psi($$

$$+ (\hat{M}_0 + 4r)\sum_{x}\bar{\psi}(x)\psi(x)$$

- The matrix form of the Wilson operator can be used to create the overlap operator

$$D_{OV} = m\left(1 + \frac{D_W(-M)}{\sqrt{D^\dagger D}}\right)$$

  When $m \to 0$ the overlap operator describes a chiral symmetry in the continuum limit.

- However, we want to have chiral symmetries on lattice and the overlap operator only accomplishes an approximate symmetry.

Introduce a fifth dimensional operator

$$S_{DW} = \sum_x \bar{\psi}(x) D^5_{DW} \psi(x)$$

Where D_{DW} is an $L_s \times L_s$ matrix, where L_s is the extent of the fifth dimension.

$$D_{DW} = \begin{bmatrix} D^1_+ & D^1_- P_- & 0 & & \dots & -mD^1_- P_+ \\ D^2_- P_+ & D^2_+ & D^2_- P_- & 0 & \dots & 0 \\ 0 & D^3_- P_+ & D^3_+ & D^3_- P_- & & \vdots \\ \vdots & 0 & \ddots & D^4_+ & \ddots & \vdots \\ \vdots & 0 & \ddots & \ddots & \ddots & D^{L_s-1}_- P_- \\ -mD^{L_s}_- P_- & 0 & \dots & 0 & D^{L_s}_- P_+ & D^{L_s}_+ \end{bmatrix}$$

$$D^i_+ = 1 + b_i D_w, \qquad D^i_- = c_i D_w - 1$$

Decomposition of this operator leads us to an effective 4D operator which is simply the overlap operator[Brower, et al., 2005].

Möbius Domain Wall Fermions

- Möbius Domain Wall fermions are defined by the kernel operator $H = \frac{\gamma_5 D_w}{2+\gamma_5 D_w}$ in approximation $\epsilon(H)$ of the $\mathrm{sgn}(H)$ in the overlap operator

$$D_{OV} = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \mathrm{sgn}(H).$$

- Where the sign function is approximated to be tanh-like

$$\epsilon(H) = \frac{(H+1)^{L_s} - (H-1)^{L_s}}{(H+1)^{L_s} + (H-1)^{L_s}} = \tanh[L_s \tanh^{-1}(H)].$$

  L_s is the extent of the fifth dimension.

- Approximation of the sign function combined using the MWDF operator increases suppression of the lattice artifact with $m_{res} \sim 1\mathrm{MeV}$.

Mesonic Correlators

- We consider the flavor triplet bilinear quark operators:

$$O(x) = \bar{q}(x)(\Gamma \otimes \frac{\vec{\tau}}{2})q(x).$$

Here τ^a is an element of the generators of $SU(2)$.

- We measure the spatial correlator through:

$$C_\Gamma(z) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_0^\beta d\tau \, \langle O_\Gamma(z,x,y,\tau) O_\Gamma^\dagger(0) \rangle$$

On the lattice this becomes

$$C_\Gamma(n_z) = \sum_{n_y,n_x,n_t} \langle O_\Gamma(n_z,n_x,n_y,n_t) O_\Gamma^\dagger(0,0,0,0) \rangle \, .$$

Mesonic Spatial Correlators and Operators

Γ	Reference Name	Abbr.	Symmetry Correspondences	
$\mathbb{I}$	Scalar	S	$U(1)_A$	
γ_5	Psuedo Scalar	PS		
γ_k	Vector	**V**	$SU(2)_L \times SU(2)_R$	
$\gamma_k\gamma_5$	Axial Vector	**A**		$SU(2)_{CS}$?
$\gamma_k\gamma_3$	Tensor	**T**	$U(1)_A$	
$\gamma_k\gamma_3\gamma_5$	Axial Tensor	**X**		

- $O(x) = \bar{q}(x)(\Gamma \otimes \frac{\tau}{2})q(x)$
- For our purpose we will fix to spatial mesonic correlation functions along the z-axis and study the screening masses.

$$\langle O(t)O(0)\rangle \to \langle O(z)O(0)\rangle$$

Updates to $N_f = 2$ simulations

- Two new temperatures $T = 146\text{MeV} \approx 0.9T_c$ and $T = 165\text{MeV} \approx T_c$ have been added to the set of temperatures studied previously [JLQCD 2019].
- We simulate different volumes for each temperature: $32(N_s/N_\tau = 2)$, $40(N_s/N_\tau = 2.5)$ at $T = 165MeV$ and $36(N_s/N_\tau = 2)$, $48(N_s/N_\tau = 2.6)$ at $T = 146MeV$.
- Spatial screening masses are determined from effective mass and fits.

Simulation Parameters

- $N_f = 2$ QCD with Möbius domain wall quarks with $m_{res} < 1$MeV and Symanzick gauge action.
- $L_s = 16$
- $a^{-1} = 2.640$GeV
- $L = 32 - 48$ $(2.40 - 3.60fm)$
- m_{ud} from ~ 2.6MeV to 13.2MeV(covering $m_{phys} \sim 4$MeV)
- Temperature ranges from $T = 146MeV - 330$MeV.
- psuedo $T_c \sim 165$MeV estimated by chiral susceptibility

$L^3 \times L_t$	β	$T[MeV]$	am	$m[MeV]$
$36^3 \times 18$	4.30	146	0.0010	2.6
			0.0050	13.2
$48^3 \times 18$	4.30	146	0.0010	2.6
			0.0050	13.2
$32^3 \times 16$	4.30	165	0.0010	2.6
			0.0050	13.2
$40^3 \times 16$	4.30	165	0.0010	2.6
			0.0050	13.2
$32^3 \times 14$	4.30	190	0.0010	2.6
			0.0050	13.2
$24^3 \times 12$	4.30	220	0.0010	2.6
			0.0100	26.4
$32^3 \times 12$	4.30	220	0.0010	2.6
			0.0100	26.4
$40^3 \times 12$	4.30	220	0.0050	13.2
			0.0100	26.4
$48^3 \times 12$	4.30	220	0.0010	2.6
			0.0050	13.2
$32^3 \times 10$	4.30	264	0.0050	13.2
			0.0150	39.6
$32^3 \times 8$	4.30	330	0.0010	2.6
			0.0400	106

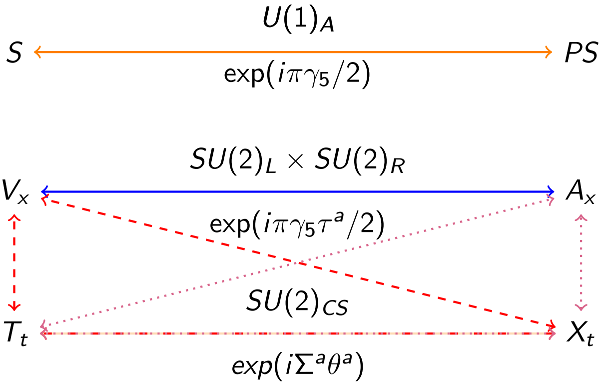
Effective Mass and Fit

- We use the $\cosh(z)$ fitting ansatz.
- Previous work on $N_f = 2$ included the $\exp(z)/z$ fitting ansatz for $T > 1.1T_c$, at T_c and below we found this ansatz was no longer a good approximation.
- Symmetries examined from the difference in the screening masses between channels related by associated transformations. i.e.

$$\text{For } SU(2)_L \times SU(2)_R : \quad \Delta M = |m_{fit}^{Ax} - m_{fit}^{Vx}|$$

$$\text{For } U(1)_A : \quad \begin{aligned} \Delta M &= |m_{fit}^{PS} - m_{fit}^{S}| \\ \Delta M &= |m_{fit}^{Xt} - m_{fit}^{Tt}| \end{aligned}$$

$$\text{For } SU(2)_{CS} : \quad \Delta M = |m_{fit}^{Vx} - m_{fit}^{Xt}|$$



- Dashed and dotted lines represent respective isospin triplets related by $SU(2)_L \times SU(2)_R$ transformations.
- $SU(2)_{CS} \supset U(1)_A$[Glozman 2015, Glozman and Pak 2015]

Emergence of $SU(2)_{CS}$ for heavy Matsubara frequency

For $T \to \infty$ we expect the emergence of an additional symmetry.

- Beginning with the free quark Lagrangian:

$$\mathcal{L} = \bar{q}(x)(i\not{\partial} + m)q(x).$$

- The associated propagator in the z-direction with fixed p_2 and p_1:

$$\begin{aligned} \langle \bar{q}(z)q(0)\rangle (p_1, p_2) &= \sum_{p_0} \int_{-\infty}^{\infty} \frac{dp_z}{(2\pi)} \frac{m - (i\gamma_0 p_0 + i\gamma_i p_i)}{p_0^2 + \delta_{ij} p_i p_j + m^2} e^{ip_3 z} \\ &= \sum_{p_0} \frac{m + \gamma_3 E - i\gamma_0 p_0 - i\gamma_1 p_1 - i\gamma_2 p_2}{2E} e^{-Ez} \end{aligned}$$

where $E = \sqrt{p_0^2 + m^2 + p_1^2 + p_2^2}$.

Emergence of $SU(2)_{CS}$ for heavy Matsubara frequency

For lattices with $T \gg m^2 + p_1^2 + p_2^2$ we can expand the propagator in terms of $1/T$:

$$\langle \bar{q}(z) q(0) \rangle = \gamma_3 \frac{1 + i\text{sgn}(p_0)\gamma_0\gamma_3}{2} e^{-\pi T z} + \mathcal{O}(1/T)$$

This quark propagator is invariant under the set of transformations:

$$\begin{aligned} q(x) &\rightarrow e^{i\Sigma^a \theta^a} q(x) \\ \bar{q}(x) &\rightarrow \bar{q}(x) \gamma_0 e^{i\Sigma^a \theta^a} \gamma_0 \end{aligned}$$

where

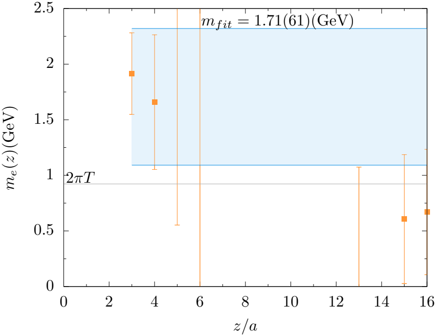
$$\Sigma = \begin{bmatrix} \gamma_5 \\ \gamma_1 \\ \gamma_2 \end{bmatrix}$$

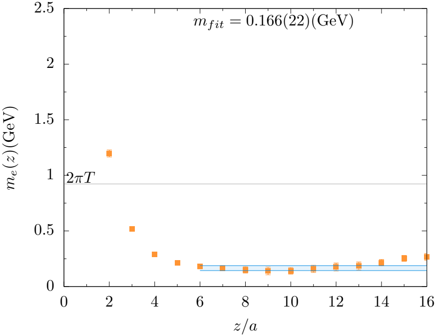
forms the so-called chiral spin $SU(2)_{CS}$ group [Glozman 2015, Glozman and Pak 2015, 2017, Rohrhofer et al. 2017,2019, 2020, Lattice 2019].

Effective Mass and Fit Range – $U(1)_A$

S

PS

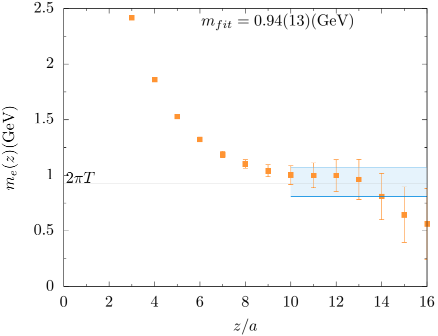




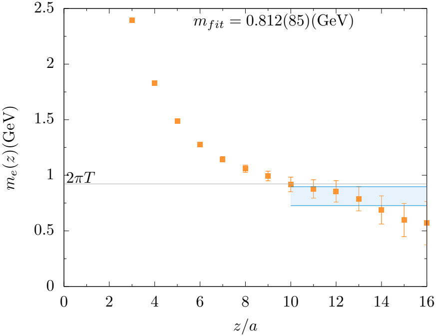
$T = 146$MeV at $m_{ud} = 2.6$MeV

Effective Mass and Fit Range – $SU(2)_L \times SU(2)_R$

Ax



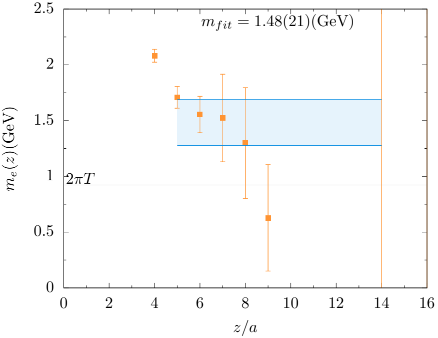
Vx



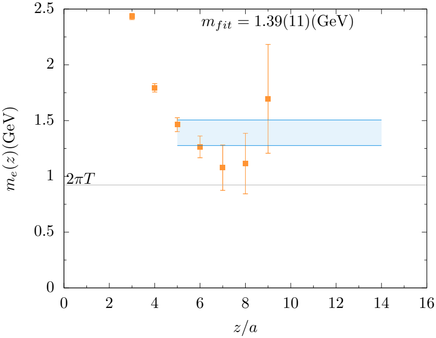
$T = 146$MeV at $m_{ud} = 2.6$MeV

Effective Mass and Fit Range – $U(1)_A$

Xt



Tt



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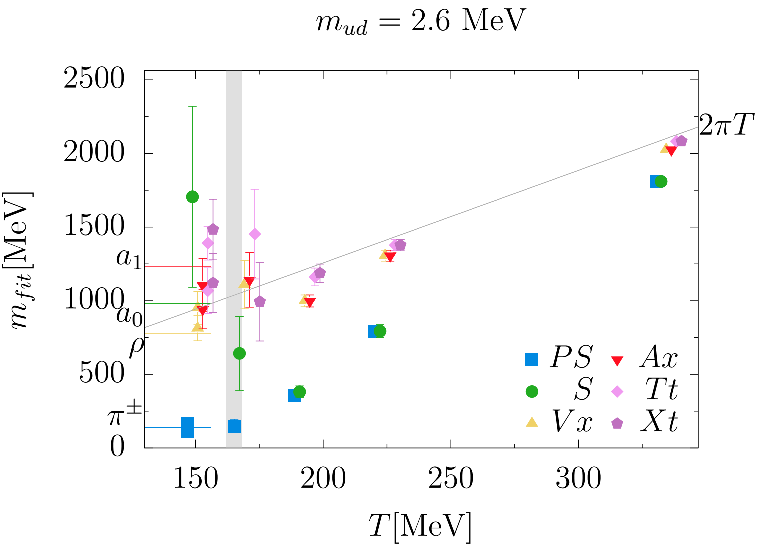
Correlator Channel Temperature Spectrum



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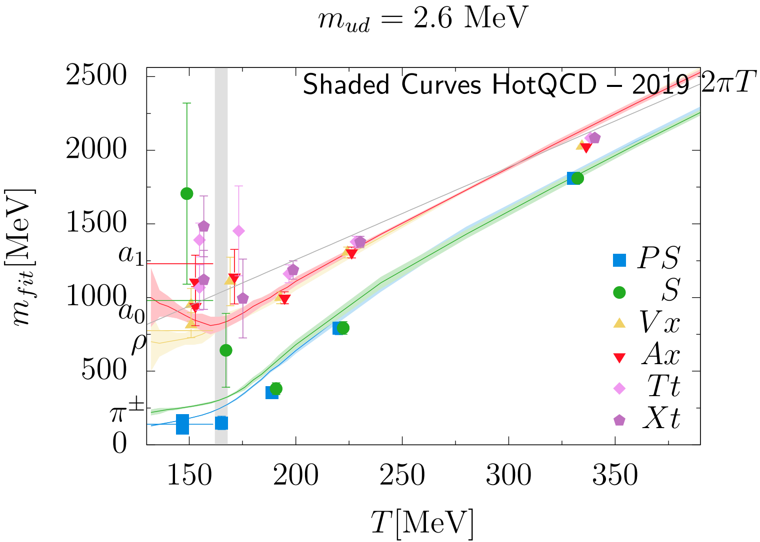
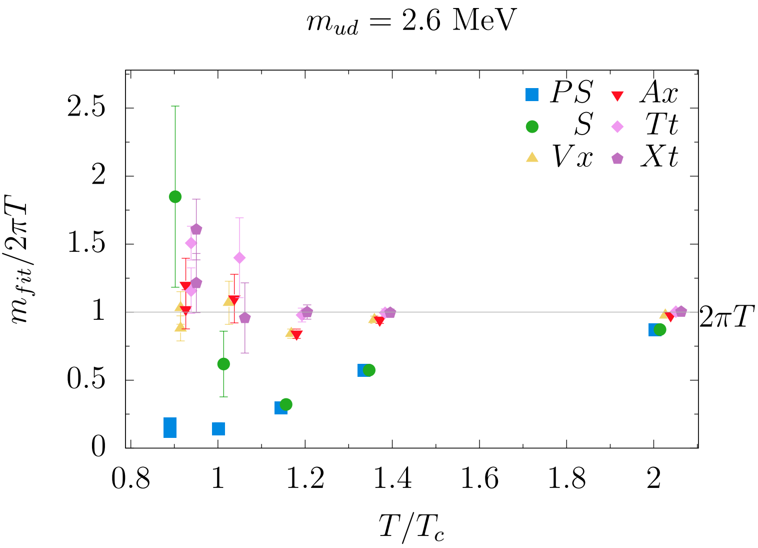


Normalized Correlator Channel Temperature Spectrum



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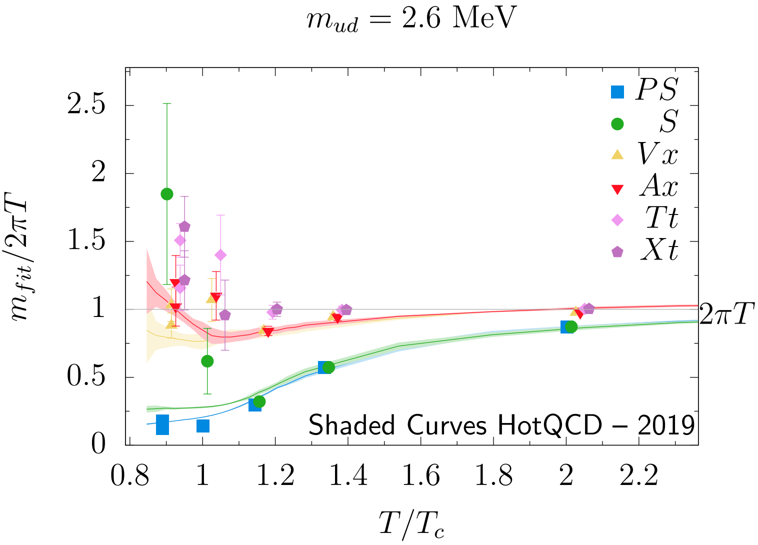


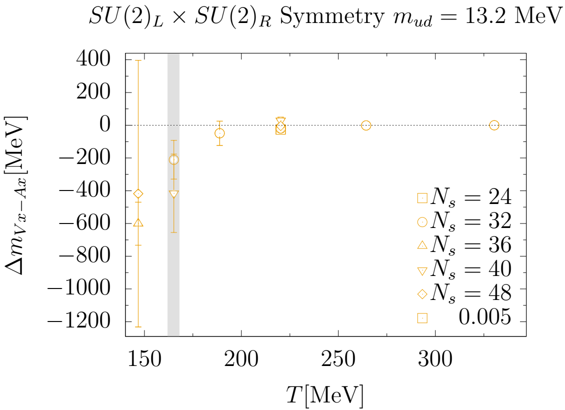
2πT convergence

- For the temperature dependent screening mass it is predicted that the high temperature limit of the spectrum tends toward an effective field theory correction[Laine et al. 2004].

$$\frac{m_{screen}}{2\pi T} = 1 + g^2 \frac{1}{3\pi^2}(1/2 + E_0) \approx 1 + 0.02980106477 g^2$$

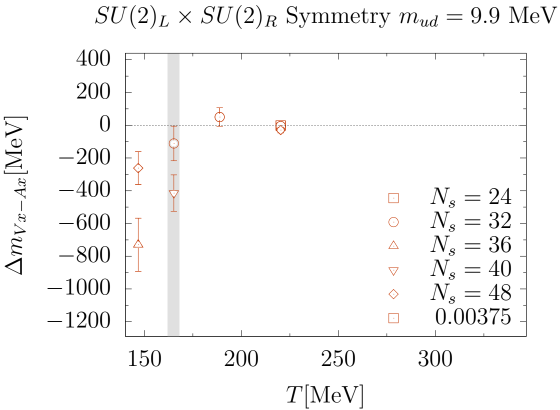
- Previous work done by HotQCD has shown that temperatures $T \gtrsim 2$GeV do not converge to the predicted perturbative correction[HotQCD 2019].
- An improvement in the effective mass and fits at very high temperatures $\mathcal{O} \gtrsim 1$GeV may be a version of the free two quark function $C(z) \sim \exp(-mz)/z$.

$SU(2)_L \times SU(2)_R$ symmetry



- At $T \sim 165$MeV $SU(2)_L \times SU(2)_R$ is restored for the lightest mass 2.6MeV.
- Almost no fluctuations above T_c for all quark masses.
- For larger quark masses psuedo T_c appears to increase.

$SU(2)_L \times SU(2)_R$ symmetry



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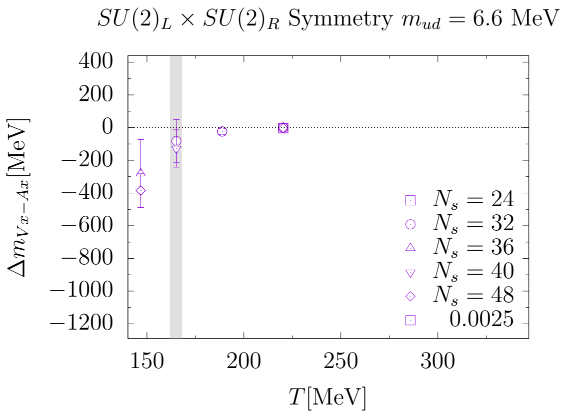

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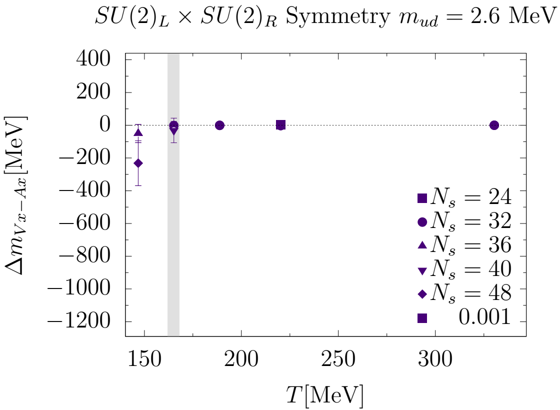

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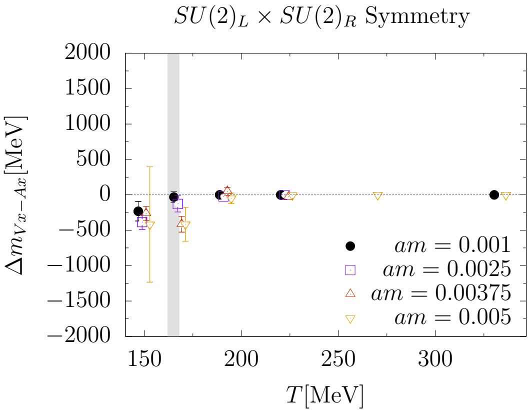

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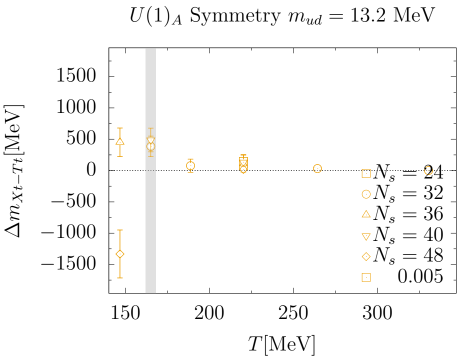
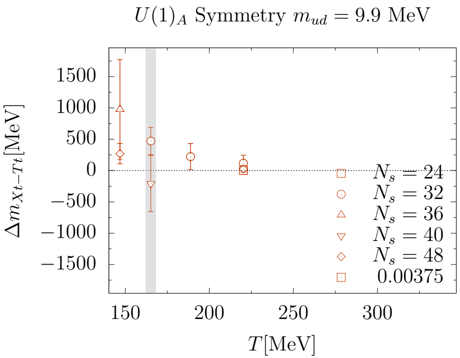
$U(1)_A$ symmetry through $Xt - Tt$



- Similar to $SU(2)_L \times SU(2)_R$ at $am = 0.0010$. $U(1)_A$ appears to be "restored" for $T \sim T_c$.
- $Xt - Tt$ also has no fluctuations for quarks which have undergone transition.
- As with $SU(2)_L \times SU(2)_R$ there is an increase in the critical temperature corresponding to increased mass.

$U(1)_A$ symmetry through $Xt - Tt$



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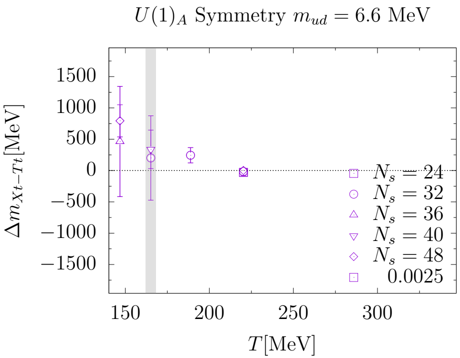

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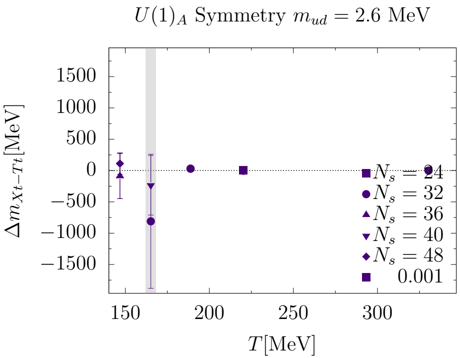

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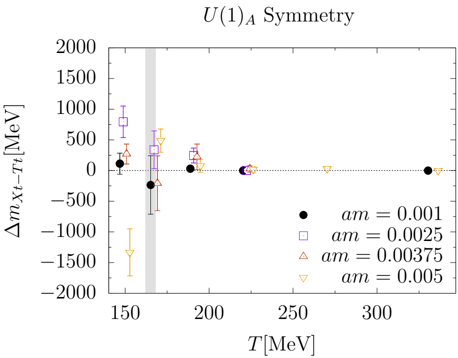
$U(1)_A$ symmetry through $Xt - Tt$



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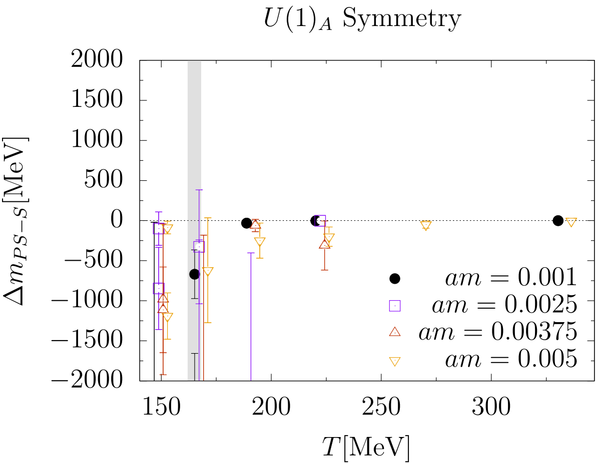
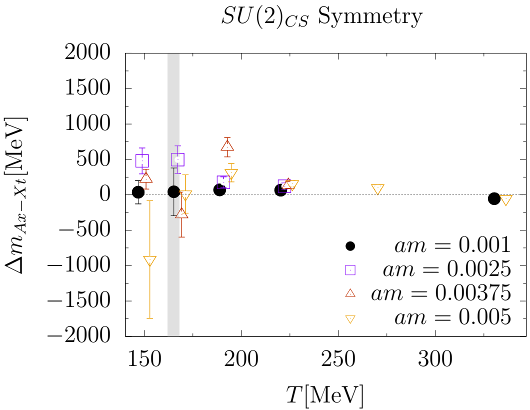
- $Xt - Tt$ also has no fluctuations for quarks which have undergone transition.
- As with $SU(2)_L \times SU(2)_R$ there is an increase in the critical temperature corresponding to increased mass.

$U(1)_A$ symmetry through $PS - S$



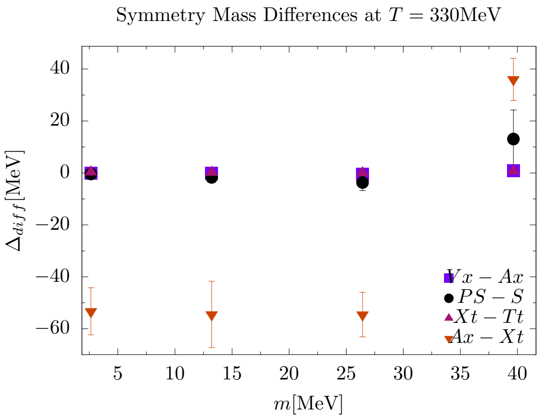
- $PS - S$ shows the same behaviors as $Xt - Tt$ and mirrors behaviors in $SU(2)_L \times SU(2)_R$ but is significantly more noisy due to scalar channel noise.

$SU(2)_{CS}$ symmetry



- Noise reduces greatly upon cross over of T_c, however, mass difference for both $Vx - Xt$ and $Ax - Xt$ remain nonzero.
- Potentially at higher temperatures an emergence of $SU(2)_{CS}$ may occur.

$SU(2)_{CS}$ symmetry



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Checking Systematics

- At temperatures around T_c we measure several lattices with difference spatial volumes to eliminate finite size effects.
- Simulations done with Möbius domain wall fermions introduce an automatic $\mathcal{O}(a)$ improvement in measured values.
- For symmetries such as $SU(2)_L \times SU(2)_R$ and $U(1)_A$ we consider dynamical fermions with a mass range from 2.6MeV $\sim$ 6.6MeV.

Conclusions

- From our $N = 2$ lattice QCD simulations with Möbius domain wall fermions, we can see that screening masses are consistent with the $T = 0$ meson spectrum already at $0.9T_c$.
- Likewise, at high temperatures the screening mass approaches $2\pi T$.
- At $T_c \sim 165$MeV we observe restoration of both $SU(2)_L \times SU(2)_R$ as well as $U(1)_A$.
- $SU(2)_{CS}$ is quite clean of noise at high temperatures but appears to remain "broken" up to $330MeV$.