Reaching Conformality in Neutron Stars

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MM, McLerran, Redlich, Sasaki, PRC 107 (2023) 2, 025802
MM, Redlich, Sasaki, PRD 109 (2024) 4, L041302
MM, PRC 110 (2024) 4, 045811

Hadrons and Hadron Interactions in QCD 2024

YITP, Kyoto University, 07.11.2024

Uniwersytet Wrocławski

RESEARCH UNIVERSITY
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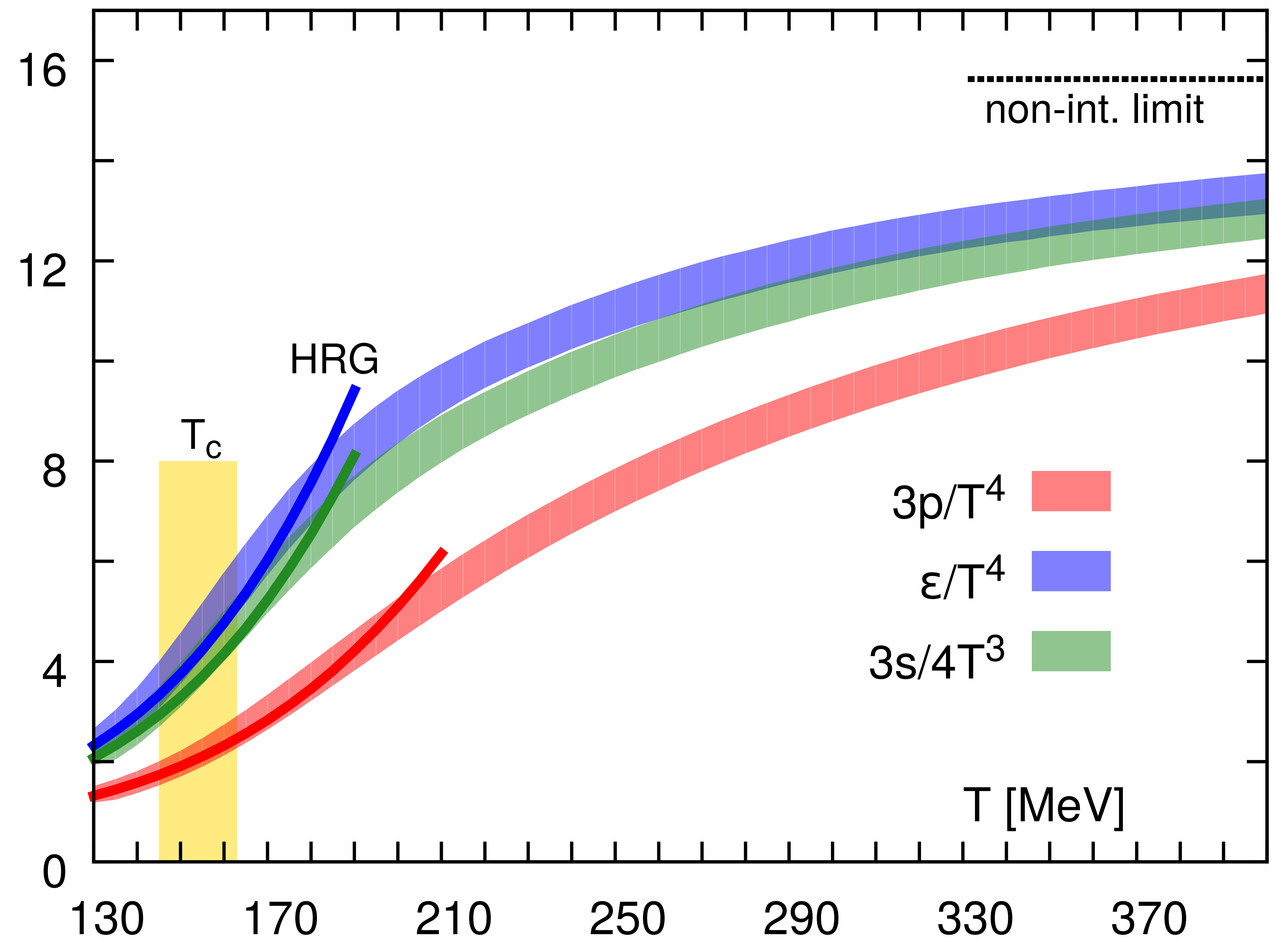
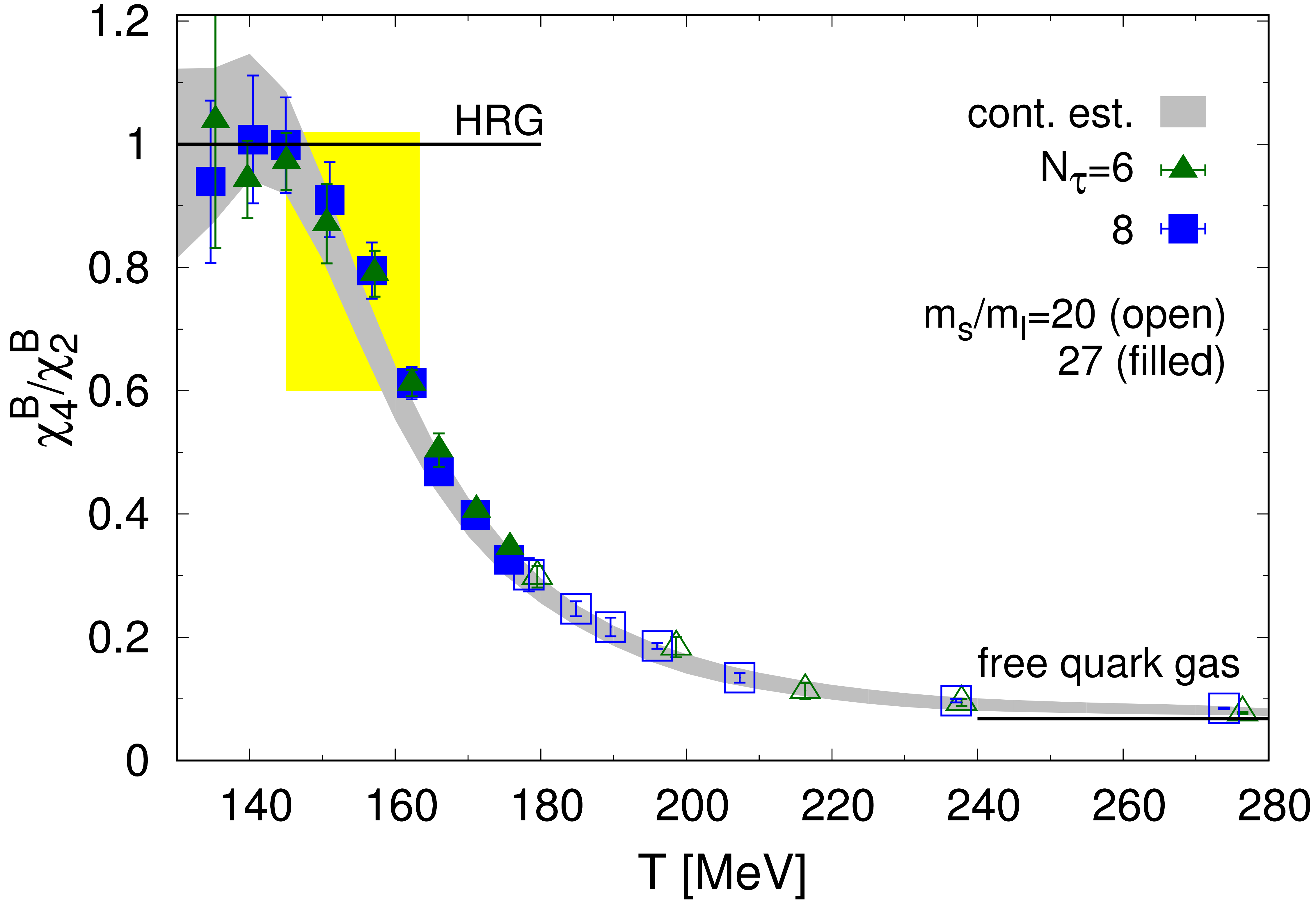
Lattice Quantum Chromodynamics

HotQCD, 2014



HotQCD, 2017



Pressure in the HRG model

$$P^{\mathrm{HRG}} = \sum_{i \in \mathrm{had}} P^{\mathrm{id}}\left(T, \mu_i; m_i\right)$$

Agreement with LQCD EoS up to $\simeq T_c$

Taylor expansion of LQCD EoS

$$\frac{P}{T^4} = \sum_{k=0}^{\infty} \left(\frac{\mu_B}{T}\right)^k \frac{\chi_k^B}{k!}, \text{ where } \chi_k^B = \frac{\partial^k P/T^4}{\partial\left(\mu_B/T\right)^k}$$

Kurtosis: $\frac{\chi_4^B}{\chi_2^B} \sim B^2$: breakdown $\sim T_c$: changeover to QGP

Quark Matter in Neutron Stars?

Solid Constraints

- Low density: χEFT $(n \lesssim 1.1 n_0)$ Tews et al, 2013
- High density pQCD $(n \gtrsim 40 n_0)$ Gorda et al, 2018

Interpolation methods

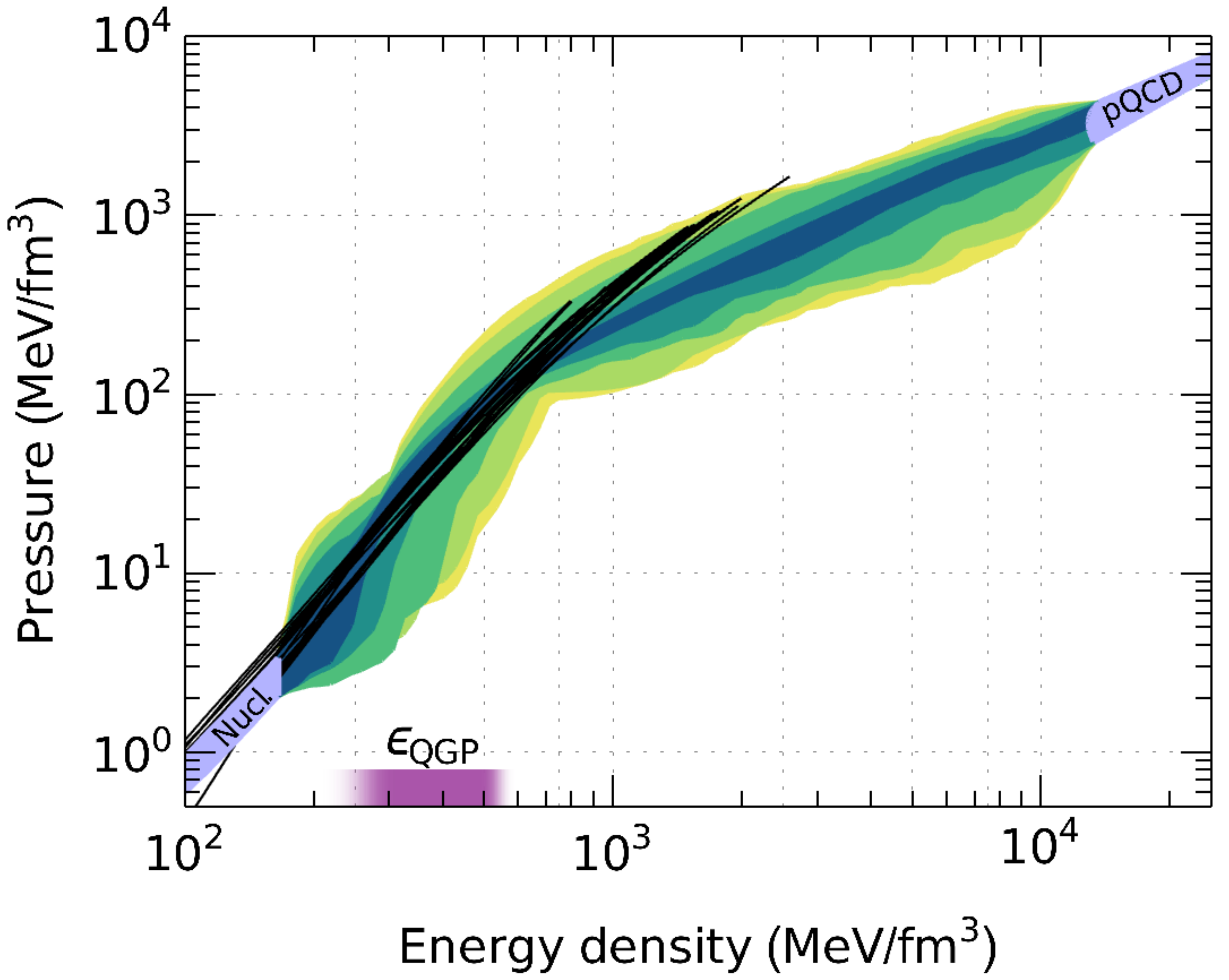
- Polytropes, CSS, Linear Speed of Sound

eg. Annala et al, 2018, 2020; Alford et al 2013, 2017, Li et al 2021

Deconfinement by polytropic index

$$\gamma = \frac{d \log p}{d \log \epsilon} \rightarrow \begin{cases} \gamma > 1.75 \rightarrow \text{Hadrons} \\ \gamma < 1.75 \rightarrow \text{Quarks} \end{cases}$$

Annala et al 2020



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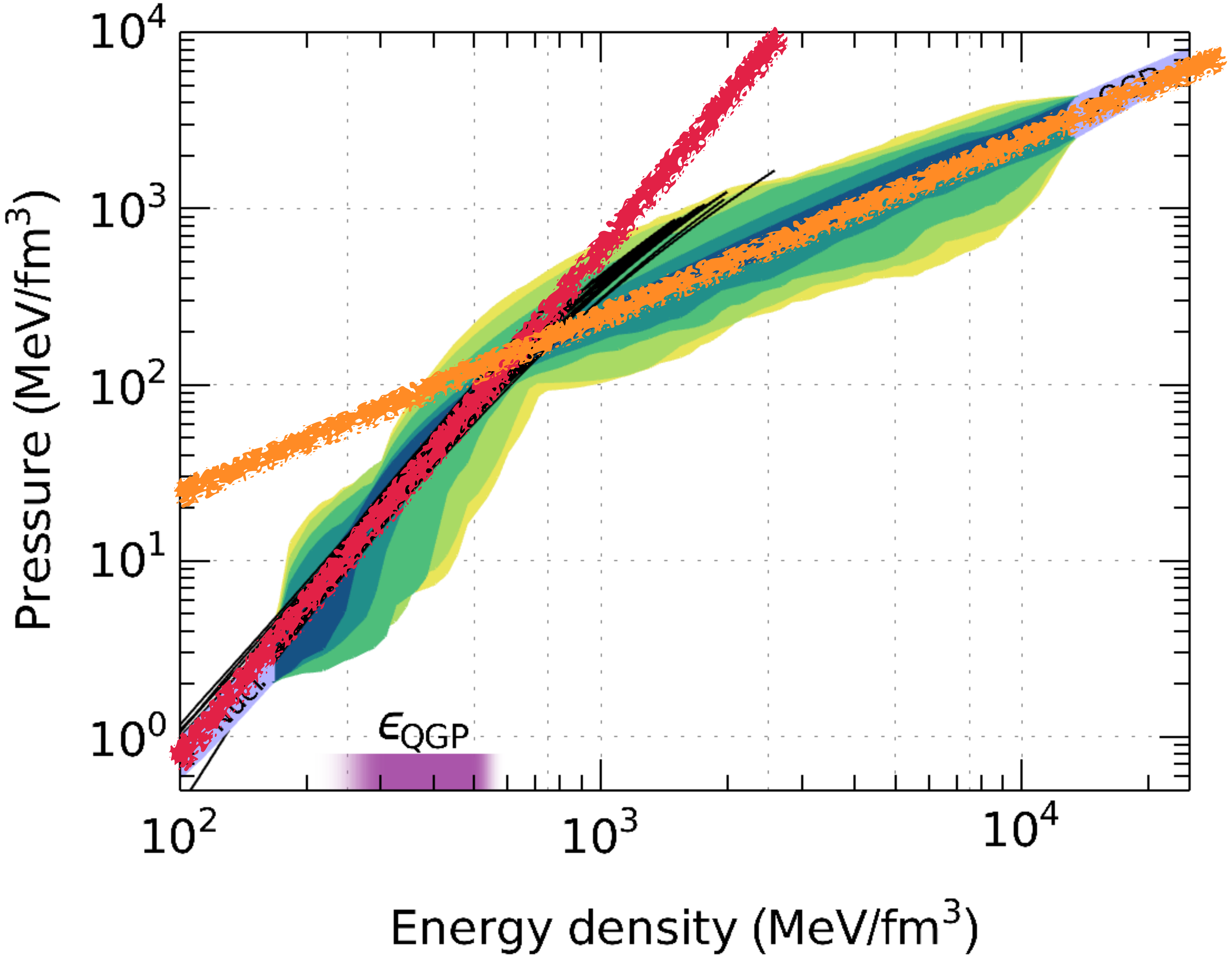


Methodology: Piecewise-linear speed of sound

Annala et al 2020

$$c_s^2 = \frac{n}{\mu}\frac{d\mu}{dn} \qquad c_{s,i}^2 = \frac{(\mu_i - \mu)c_{s,i}^2 + (\mu - \mu_i)c_{s,i+1}^2}{\mu_{i+1} - \mu_i}$$

+

χEFT + pQCD

+

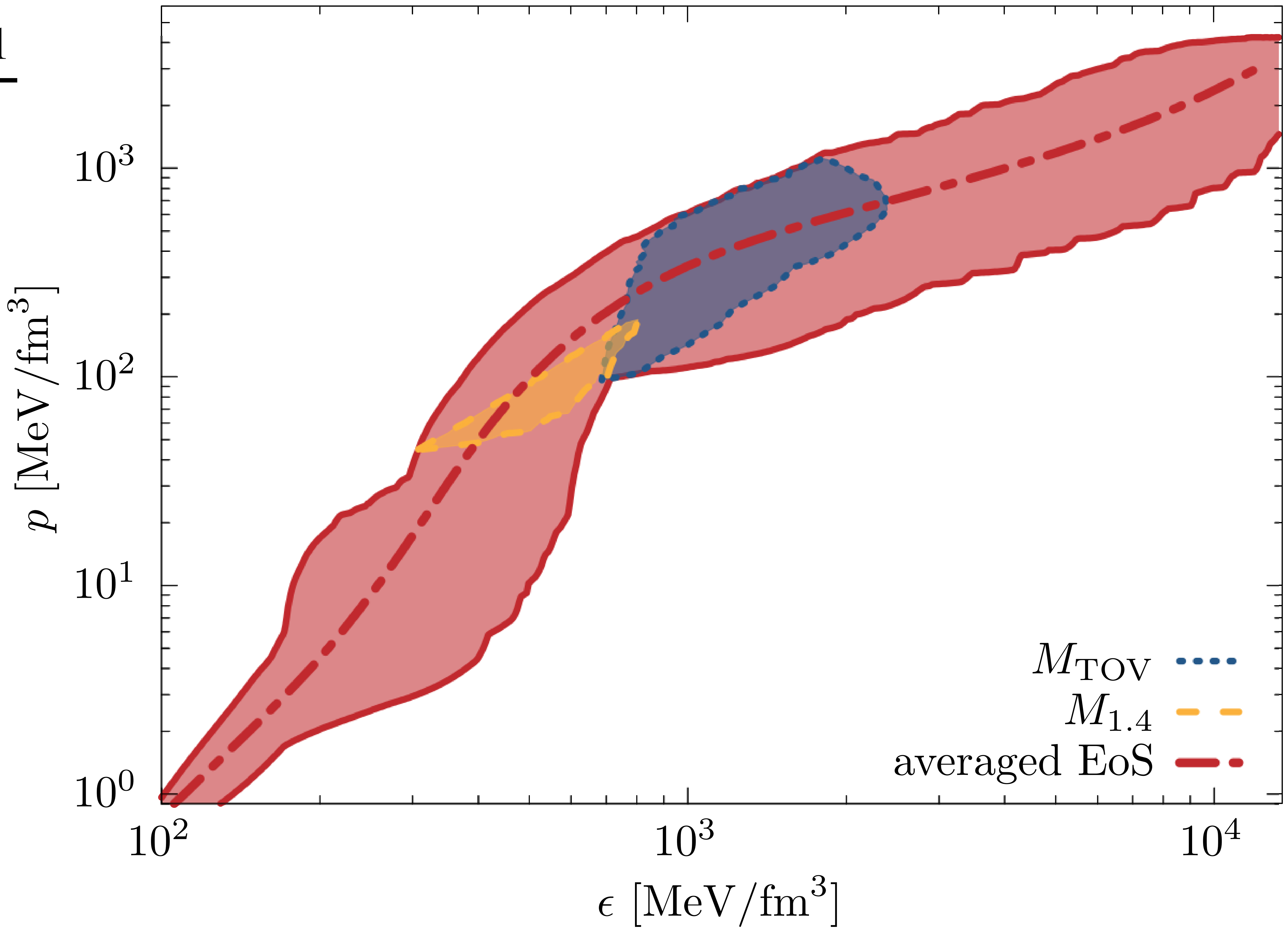
Mass measurement of J0740+6620

$M_{\rm TOV} \geq (2.08 \pm 0.07)\ M_\odot$ Fonseca et al 2021

+

Tidal Deformability from GW170817

$\Lambda_{1.4M_\odot} = 190^{+380}_{-120}$ Abbott et al 2018



6×10^5 viable Equations of State

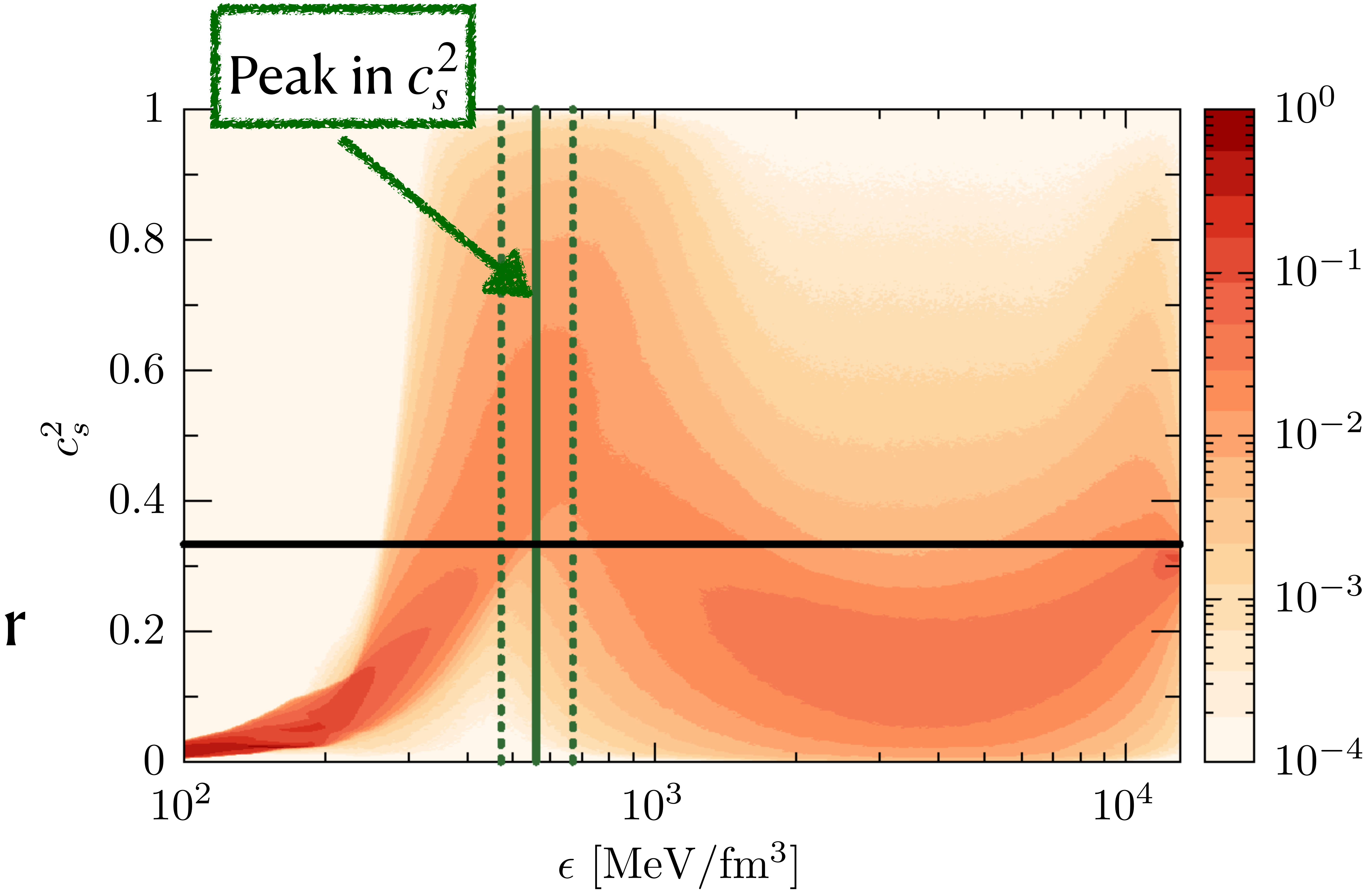
General Structure of Speed of Sound

- General peak-dip structure

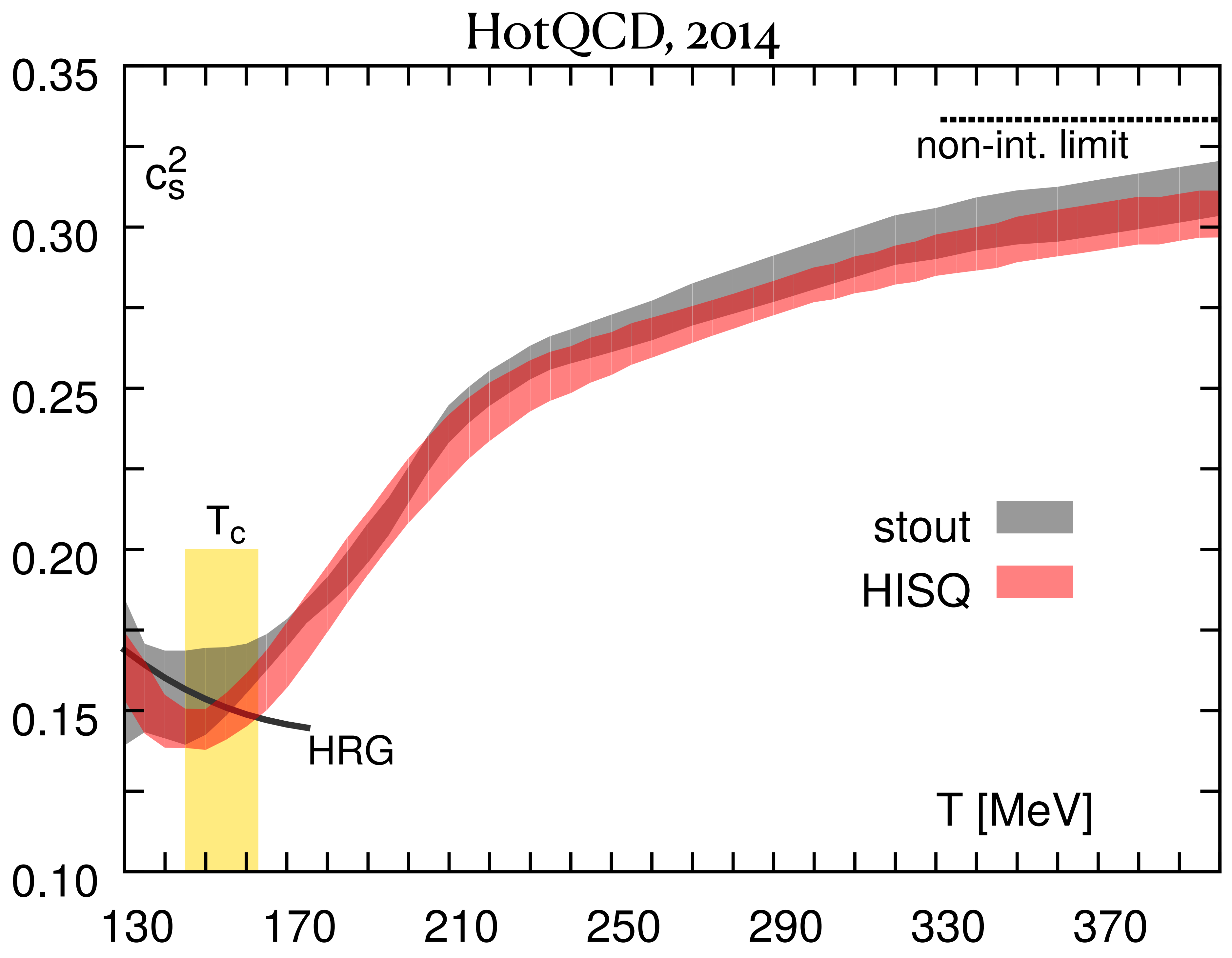
  Altiparmak et al, 2022

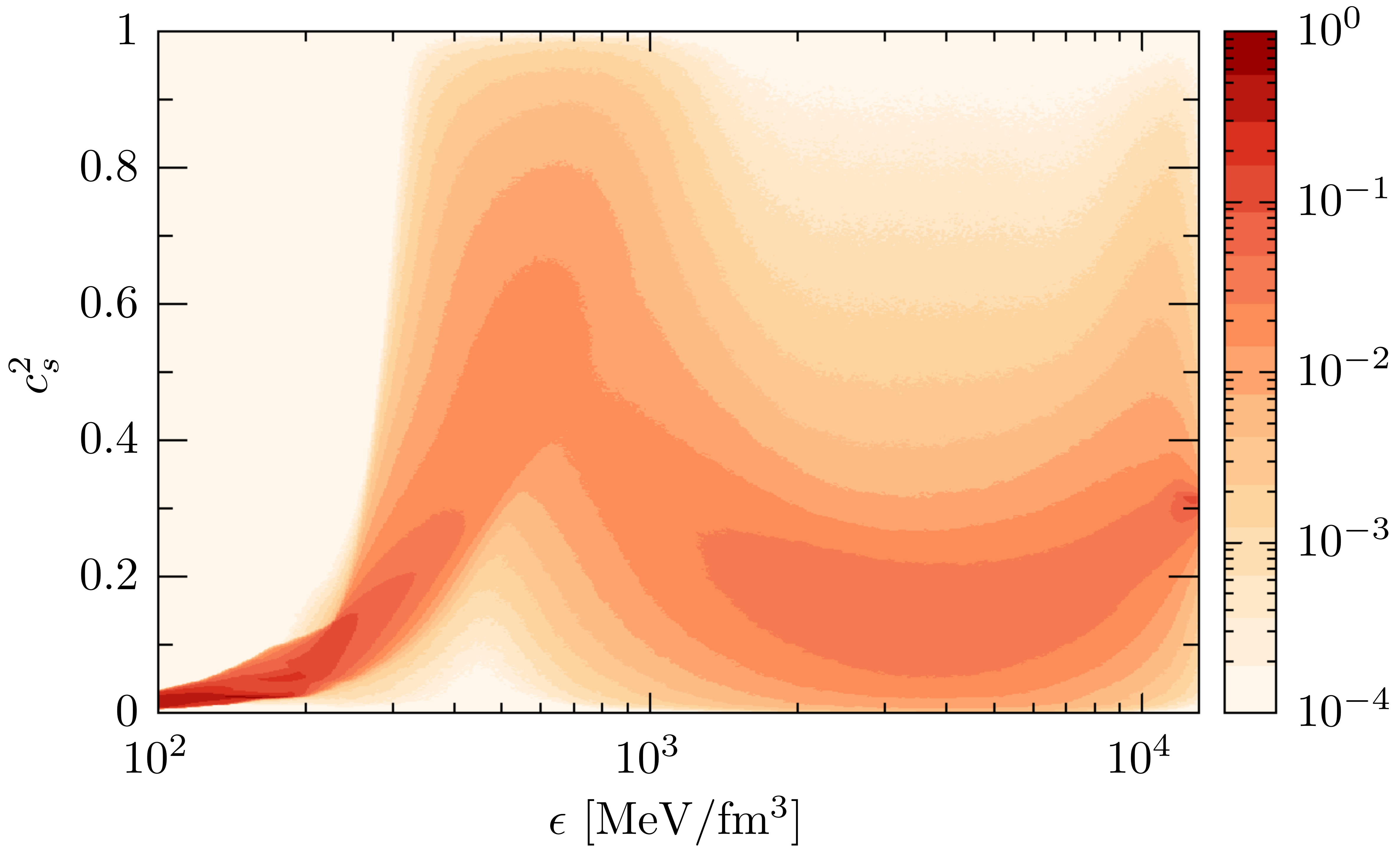
- Peak similar to quarkyonic matter

  McLerran, Reddy, 2019; Pang et al, 2023



Local maximum at $\epsilon_{\text{peak}} = 0.56^{+0.11}_{-0.09}$ GeV/fm^3 with $c_s^2 = 0.82 \pm 0.08$





$$c_s^2 = \frac{S}{T}\frac{dT}{dS} < \frac{1}{3}$$

$$c_s^2 = \frac{n}{\mu}\frac{d\mu}{dn} > \frac{1}{3}$$

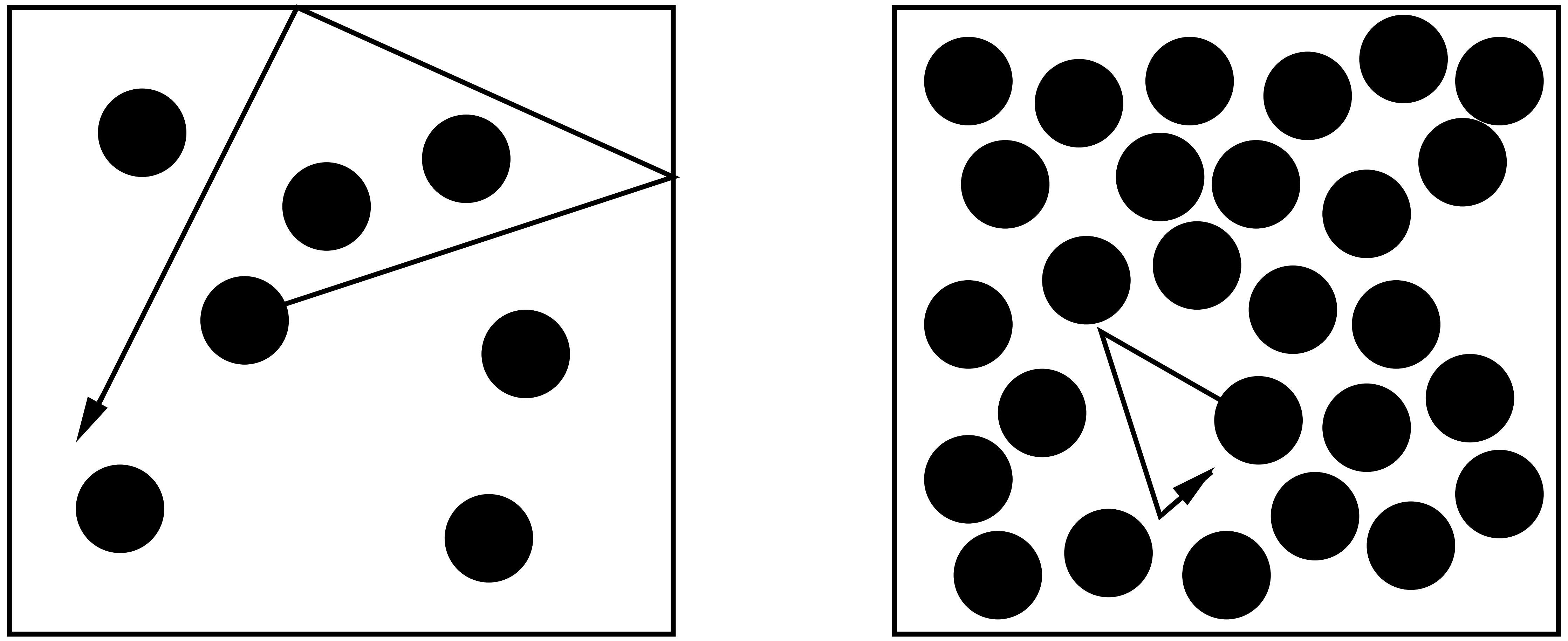
- Attractive interactions with resonance formation
- Chiral symmetry restoration and deconfinement

- Dominance of repulsive interactions
- Onset of quark or quarkyonic, or baryquark matter?

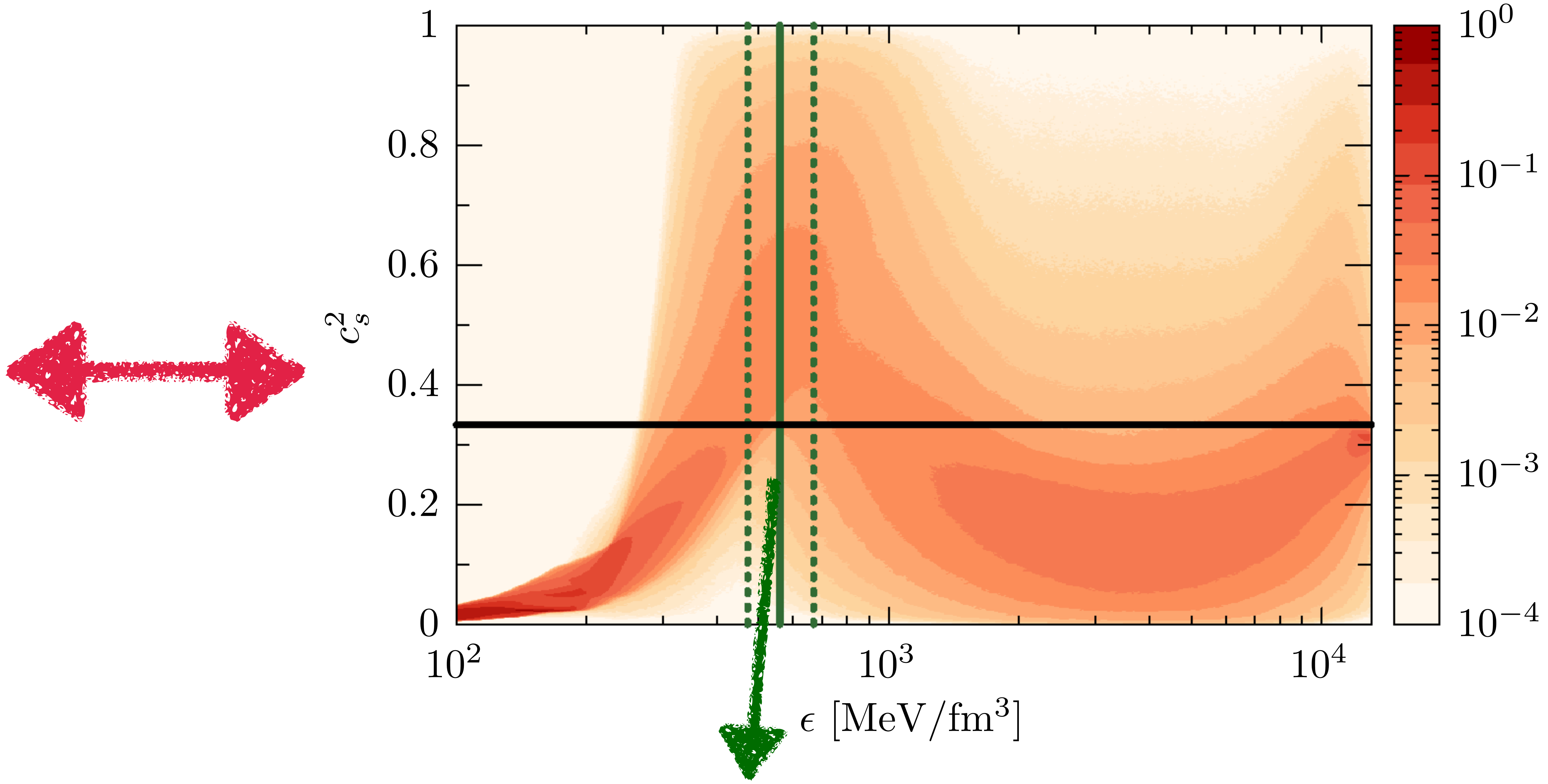
Non-monotonicity ⟷ Change of medium composition

Percolation theory vs speed of sound

see e.g. Satz, 1998; Castorina et al, 2009; Fukushima, 2020



Percolation theory: $n_c = 1.22/V_0$



$$n_{\text{peak}} = 0.54^{+0.09}_{-0.07}\ \text{fm}^{-3}$$

Avg. proton radius: $R_0 = 0.80 \pm 0.05$ fm
Wang et al 2022

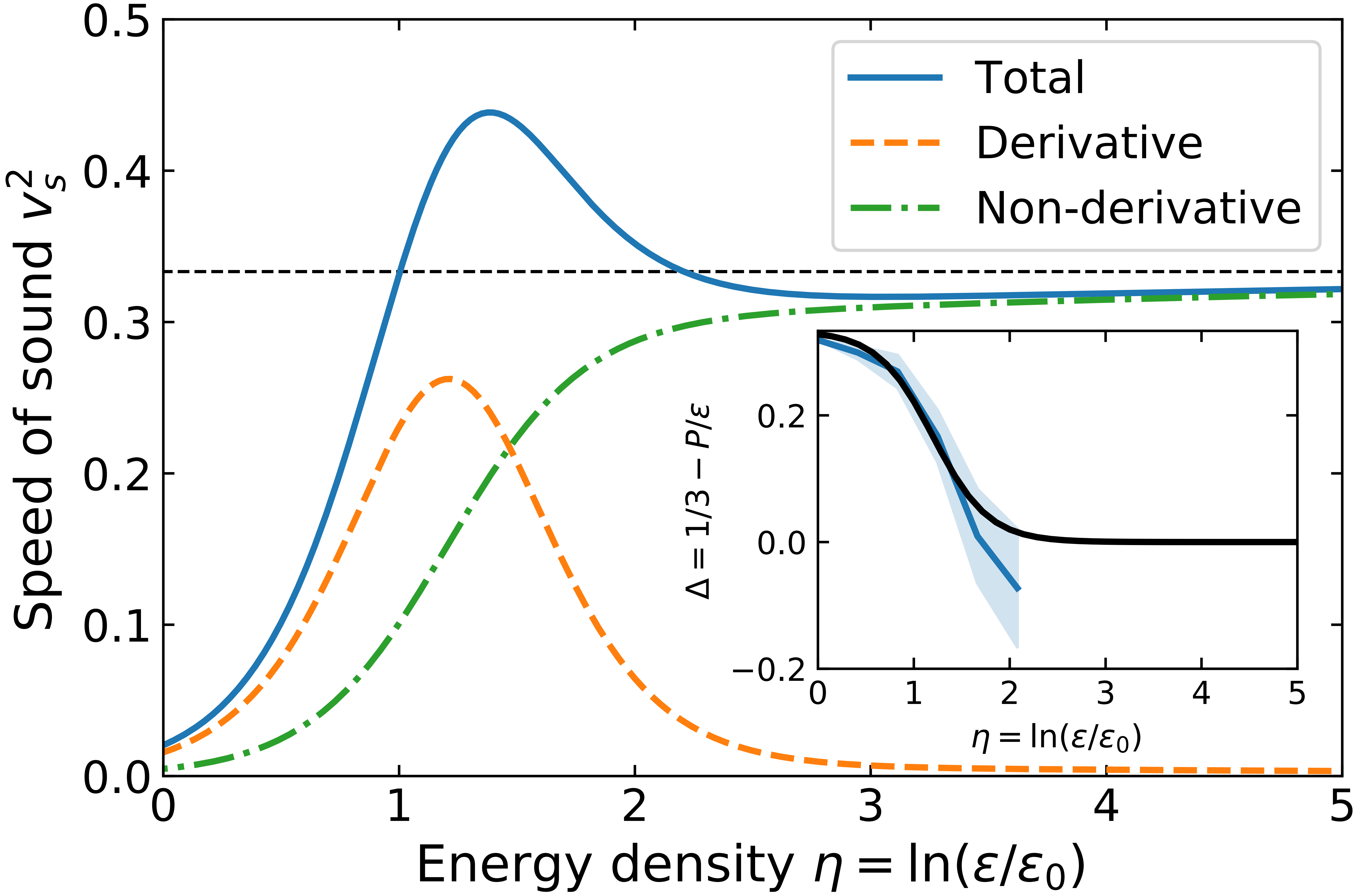
$$n_c = 0.57^{+0.12}_{-0.09}\ \text{fm}^{-3}$$

Pb-Pb collisions at $\sqrt{s} = 2.76$ TeV
Andronic et al 2018

$$n_c = 0.60 \pm 0.07\ \text{fm}^{-3}$$

Speed of Sound as Trace Anomaly

Fujimoto et al 2022



Trace Anomaly measure

$$\Delta = \frac{1}{3} - \frac{p}{\epsilon}$$

$$c_s^2 = \frac{d\left(\epsilon \frac{p}{\epsilon}\right)}{d\epsilon} = \frac{1}{3} - \Delta - \epsilon\frac{d\Delta}{d\epsilon}$$

Trace anomaly more informative than speed of sound

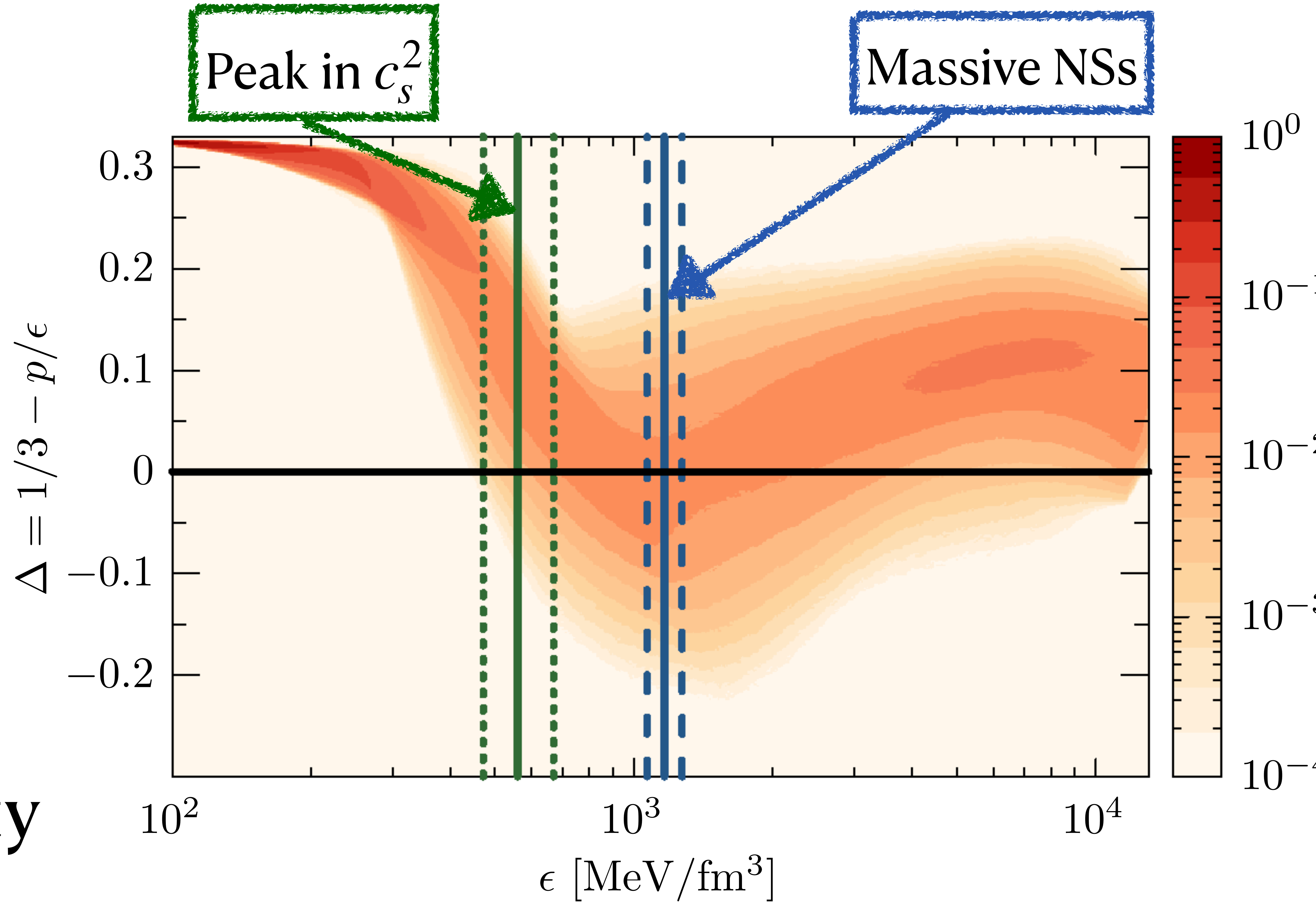
Measure of conformality

Δ monotonic up to $\simeq \epsilon_{\mathrm{TOV}}$

$$c_s^2 = \frac{1}{3} - \Delta - \epsilon \frac{d\Delta}{d\epsilon}$$

Maximum in c_s^2

Fast approach to comformality



$\Delta \simeq 0$ at $\epsilon \simeq 1$ GeV/fm^3

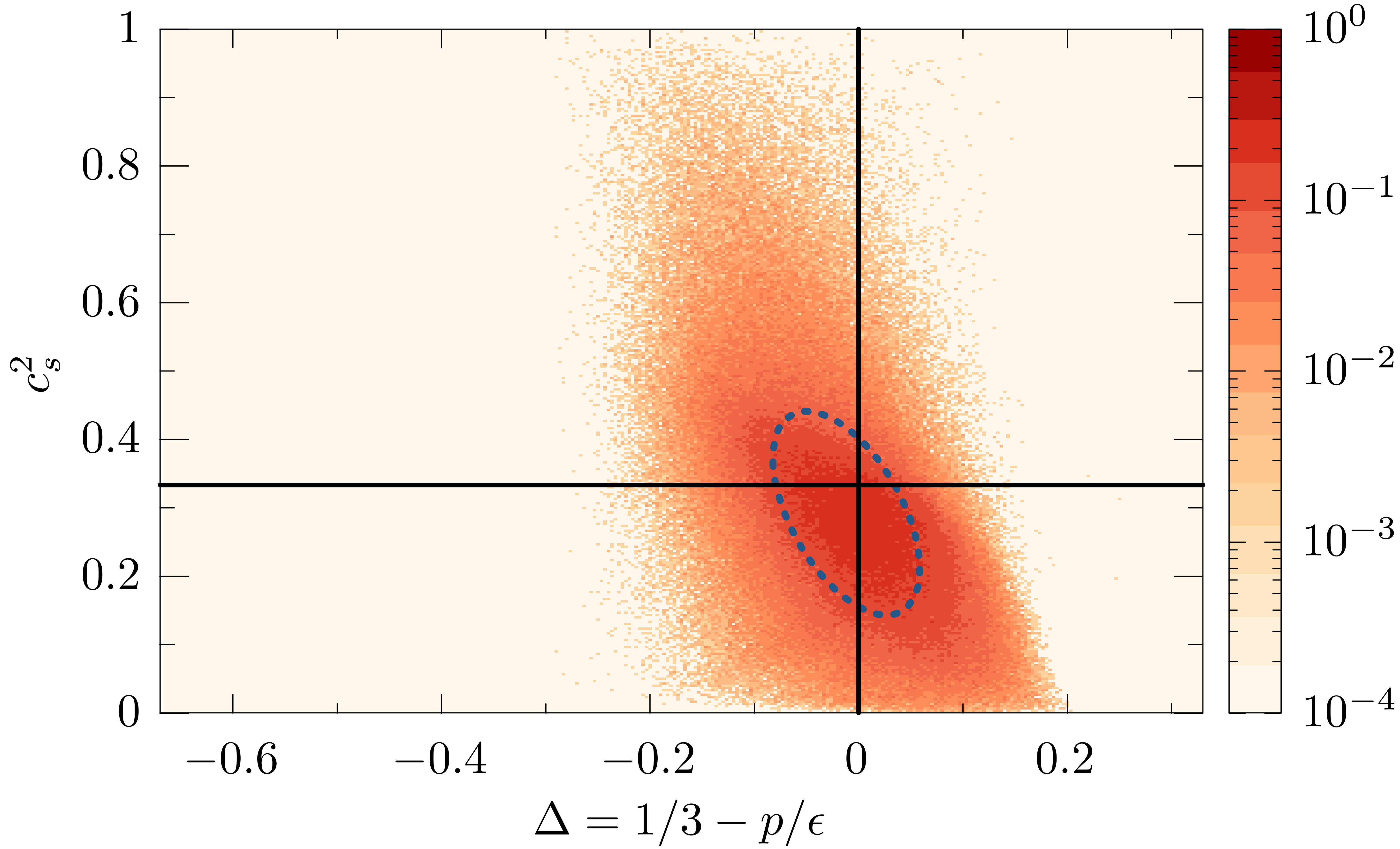
c_s^2 and Δ in Heavy Neutron Stars

Conformality:

$c_s^2 = 1/3$ and $\Delta = 0$

$c_{s,\,\mathrm{TOV}}^2 = 0.28 \pm 0.16 \simeq 1/3$

$\Delta_{\mathrm{TOV}} = -0.01 \pm 0.03 \simeq 0$



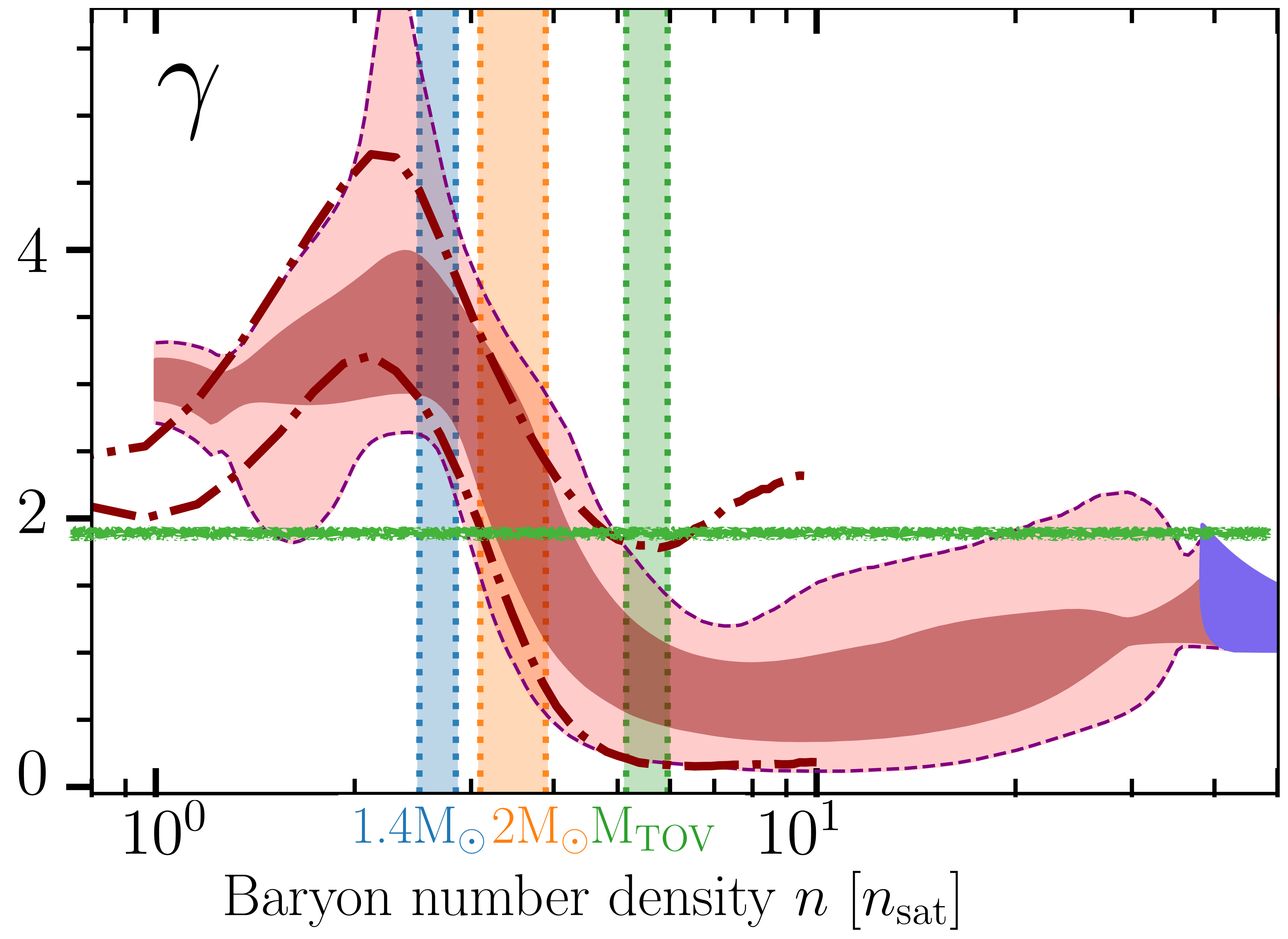
Matter almost conformal in the cores of maximally massive NSs

Changeover to nearly-conformal regime

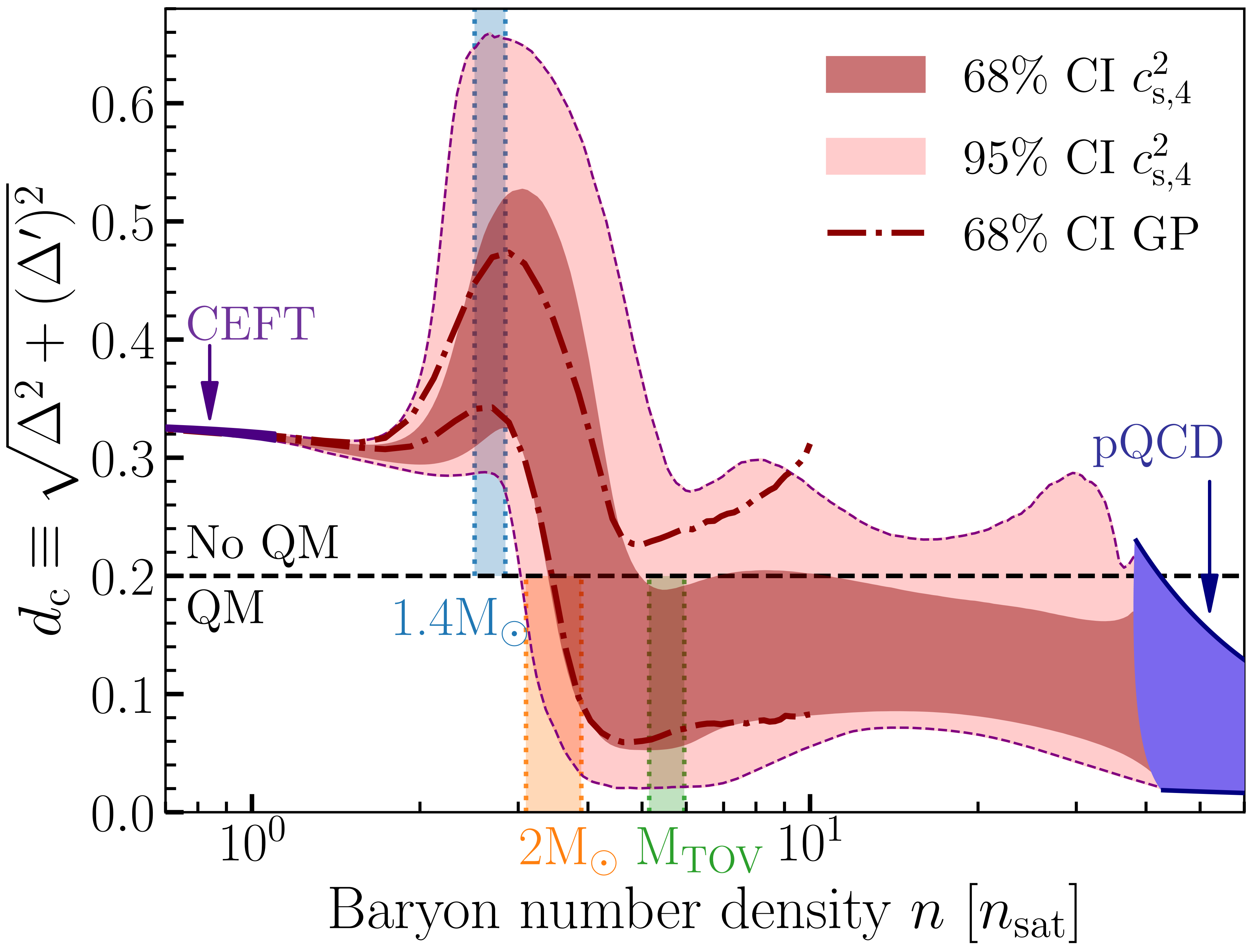
$$\gamma = \frac{\epsilon}{p} c_s^2 = \frac{c_s^2}{1/3 - \Delta} \lesssim 1.75$$

$$d_c = \sqrt{\Delta^2 + (\epsilon \Delta')^2} \lesssim 0.2$$

Annala et al 2020



Annala et al 2023



Curvature of the energy per particle

Pressure from $\frac{\epsilon}{n} \to p = n^2 \frac{d\epsilon/n}{dn}$

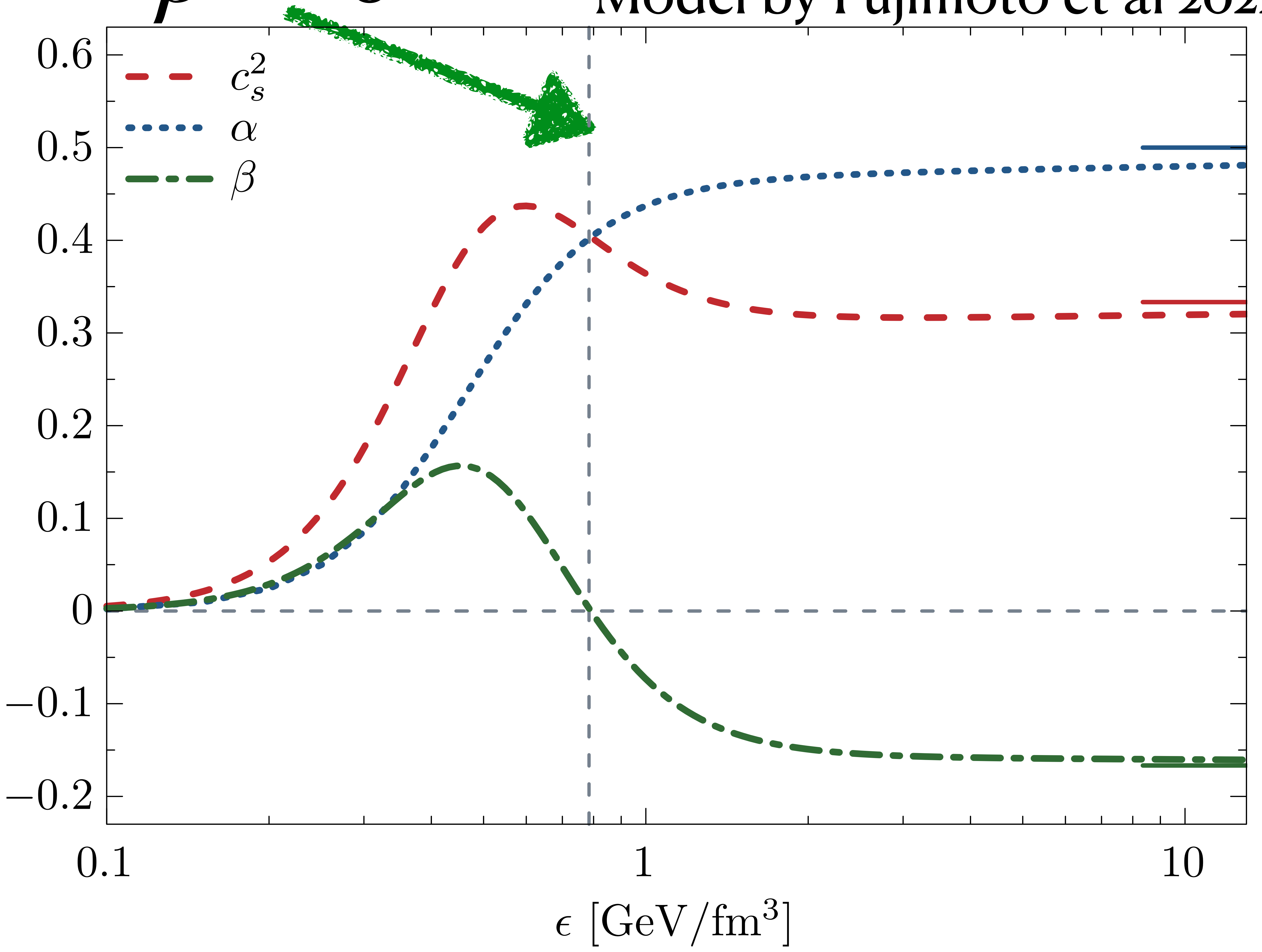
$$c_s^2 = \frac{1}{\mu}\frac{dp}{dn} = \alpha + \beta$$

$$\alpha = 2\frac{n}{\mu}\frac{d\epsilon/n}{dn} = 2\frac{1/3 - \Delta}{4/3 - \Delta}$$

$$\beta = \frac{n^2}{\mu}\frac{d^2\epsilon/n}{dn^2} = c_s^2 - \alpha$$

$\beta = 0$

Model by Fujimoto et al 2022

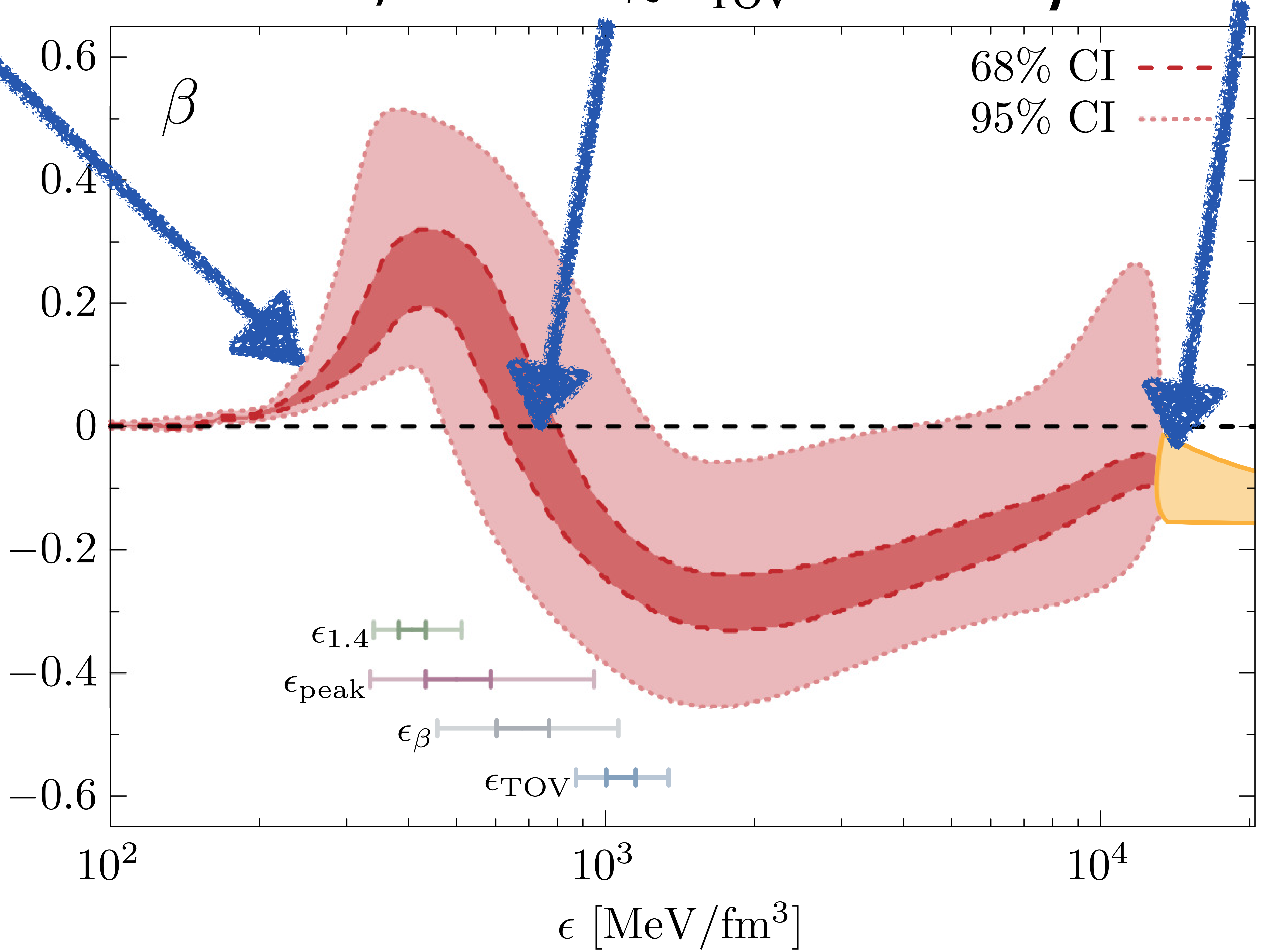


$\beta = 0 \to$ changeover to conformal regime

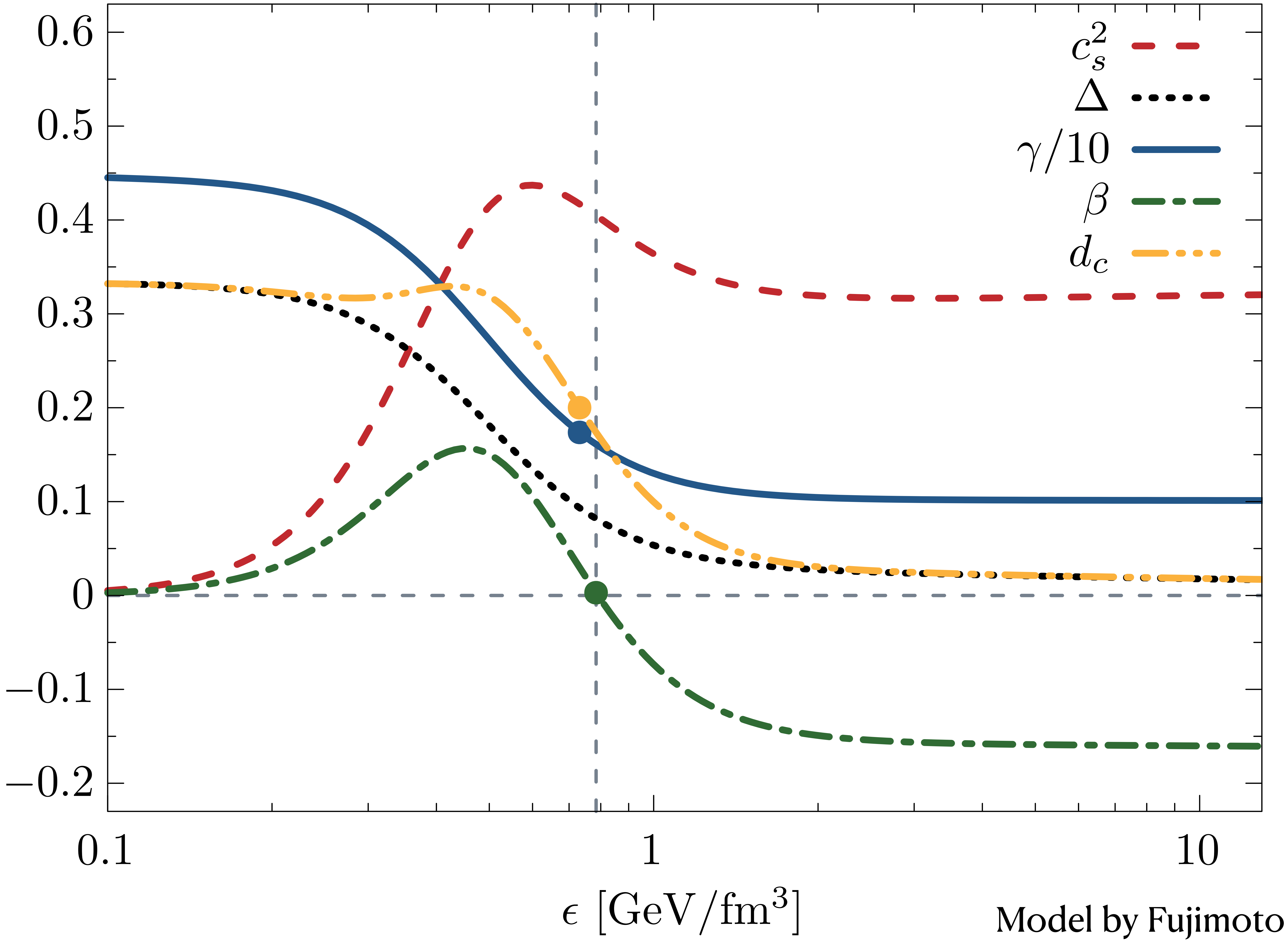
$\beta \simeq c_s^2 > 0$

$\beta < 0$ at $\epsilon \lesssim \epsilon_{\mathrm{TOV}}$

$\beta \simeq -1/6$



Changeover consistent other measures



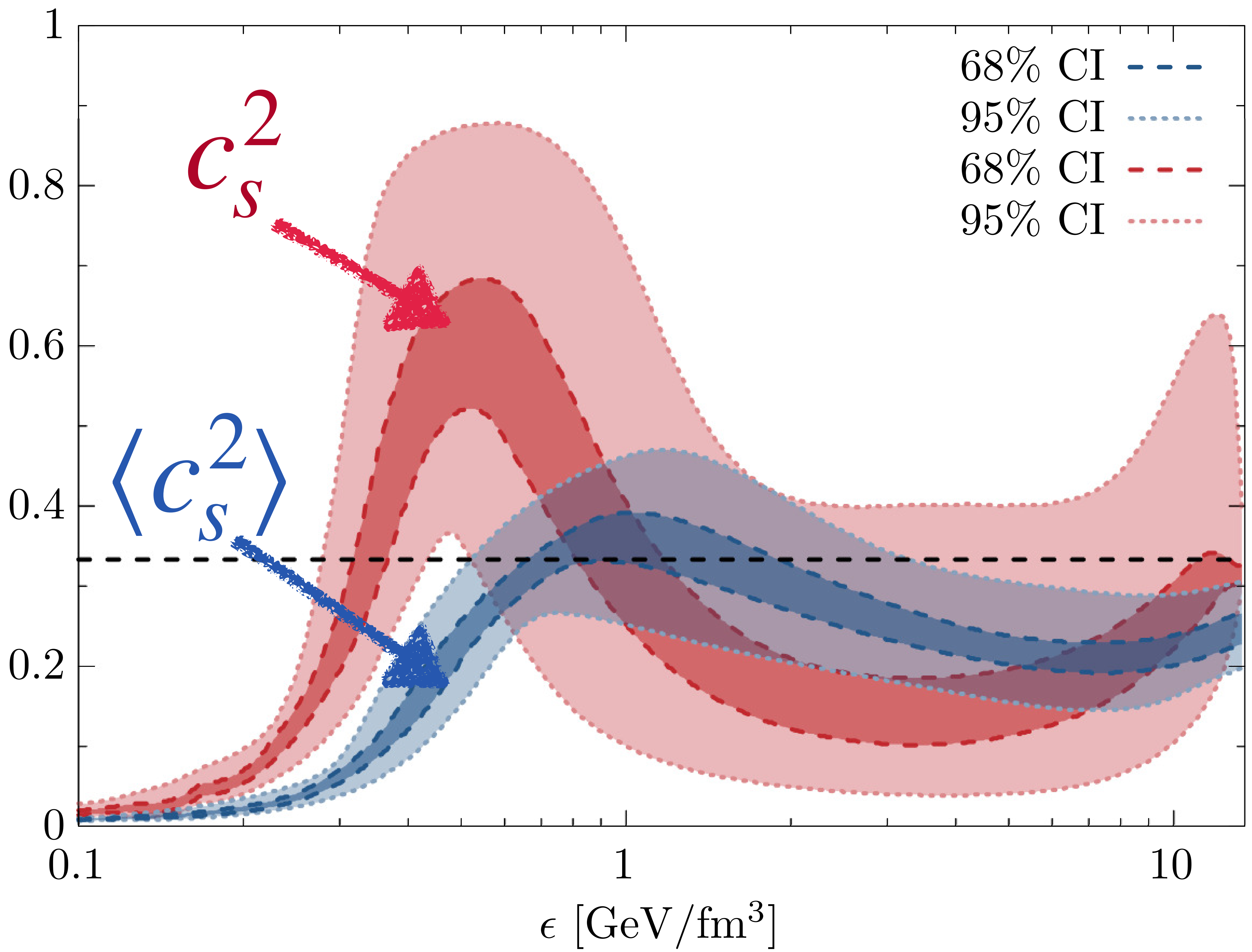
Model by Fujimoto et al 2022

Average Speed of Sound in NSs

$$\frac{p}{\epsilon} = \frac{1}{\epsilon}\int_0^{\epsilon} d\epsilon' \frac{dp}{d\epsilon'} = \langle c_s^2 \rangle$$

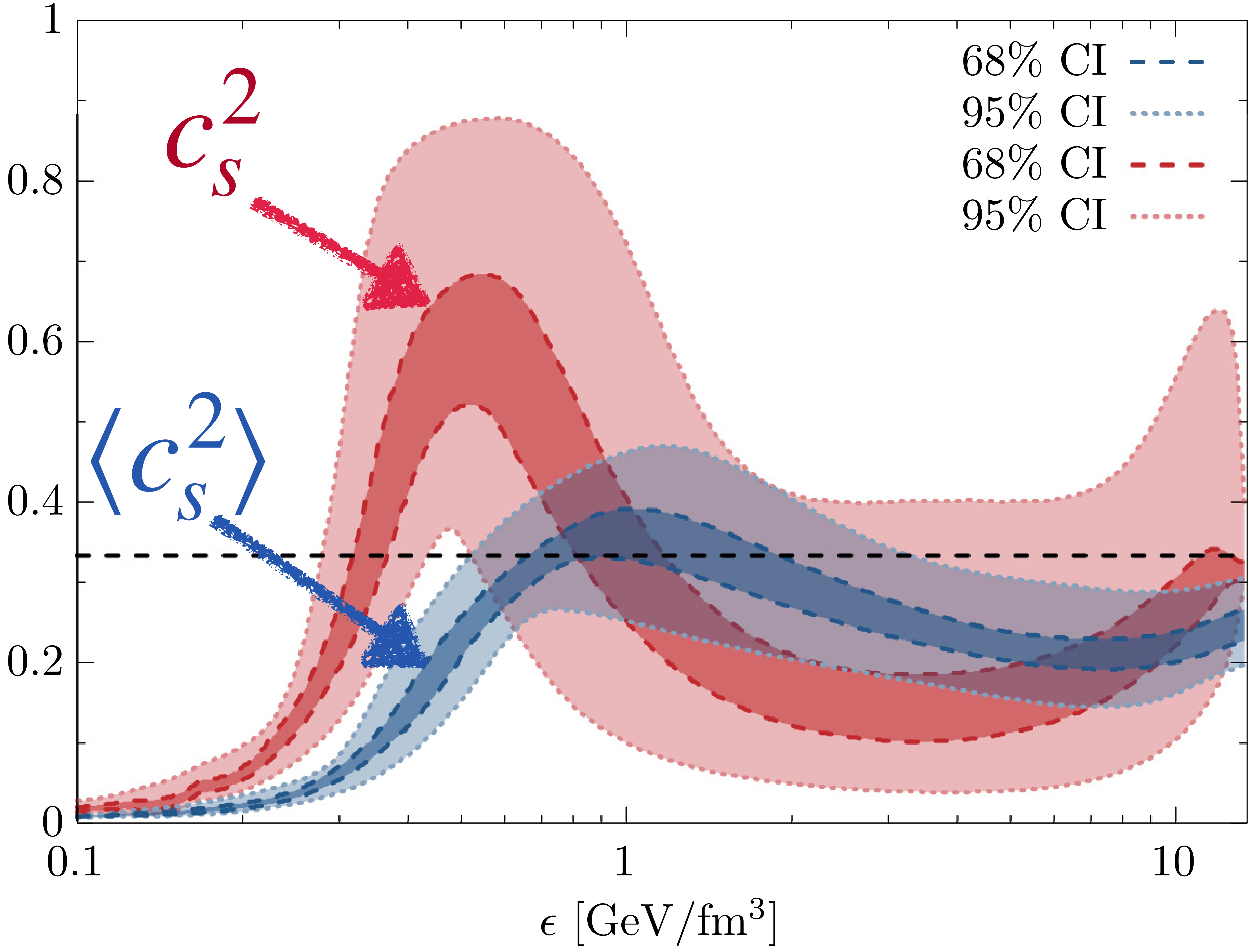
$$\Delta = \frac{1}{3} - \langle c_s^2 \rangle \qquad \epsilon\Delta' = \langle c_s^2 \rangle - c_s^2$$

$$\gamma = c_s^2 / \langle c_s^2 \rangle$$



Conformality : $\Delta \simeq 0 \Leftrightarrow \langle c_s^2 \rangle \simeq 1/3$ and $\epsilon\Delta' \simeq 0 \Leftrightarrow c_s^2 \simeq \langle c_s^2 \rangle$

Implications of vanishing trace anomaly

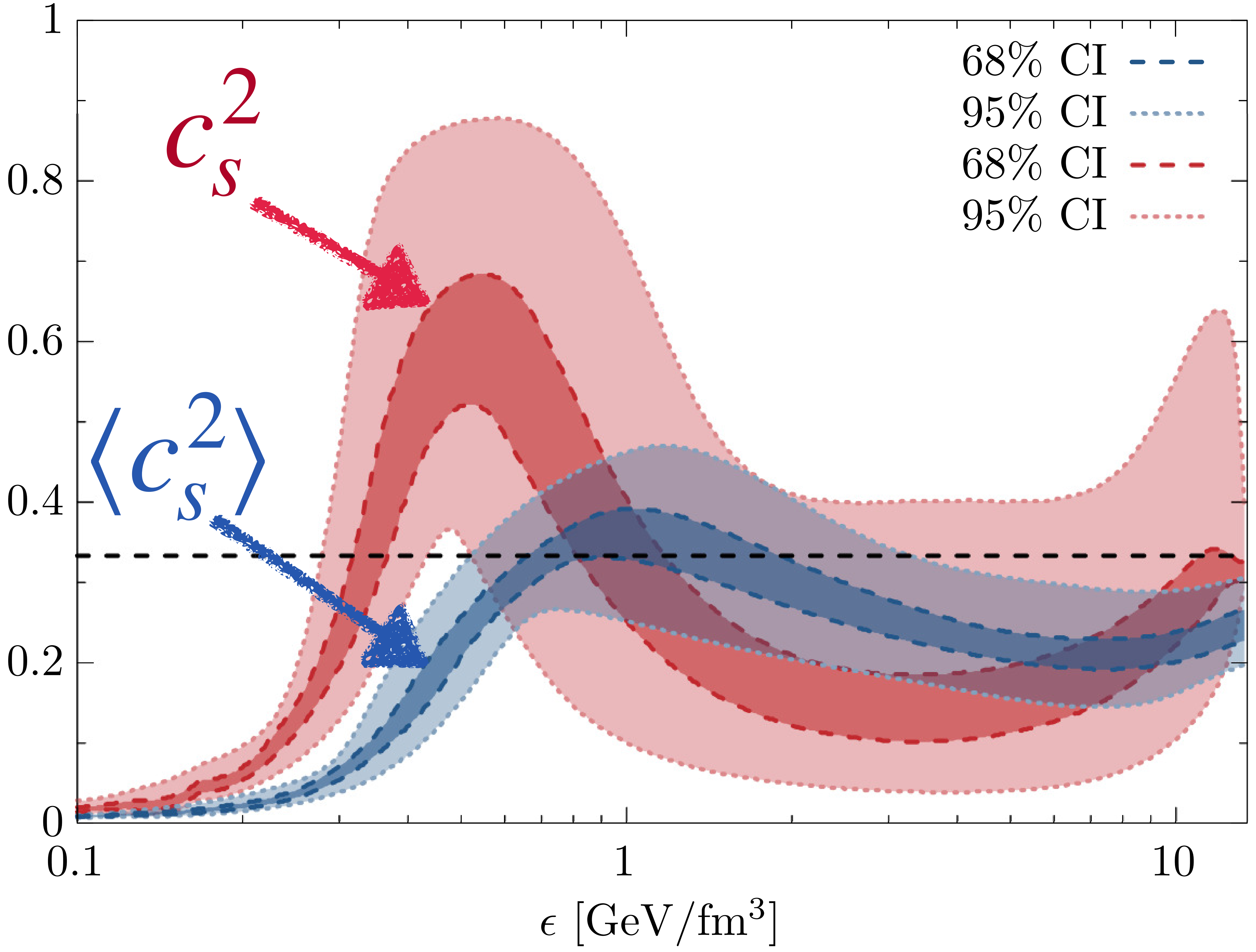


$\Delta_{\mathrm{TOV}} = -0.01 \pm 0.03 \simeq 0$

Ansatz 1: $\Delta_{\mathrm{TOV}} = 0 \Leftrightarrow \langle c_s^2 \rangle_{\mathrm{TOV}} = 1/3$

- c_s^2 must exceed 1/3
- c_s^2 features maximum

Implications of vanishing trace anomaly



$$\Delta_{\mathrm{TOV}} = -0.01 \pm 0.03 \simeq 0$$

Ansatz 1: $\Delta_{\mathrm{TOV}} = 0 \Leftrightarrow \langle c_s^2 \rangle_{\mathrm{TOV}} = 1/3$

Ansatz 2: $\Delta \geq 0 \Leftrightarrow \langle c_s^2 \rangle \leq 1/3$

- c_s^2 must exceed 1/3
- c_s^2 features maximum at $\epsilon \leq \epsilon_{\mathrm{TOV}}$
- consequences for NS phenomenology

Implications of vanishing trace anomaly 2

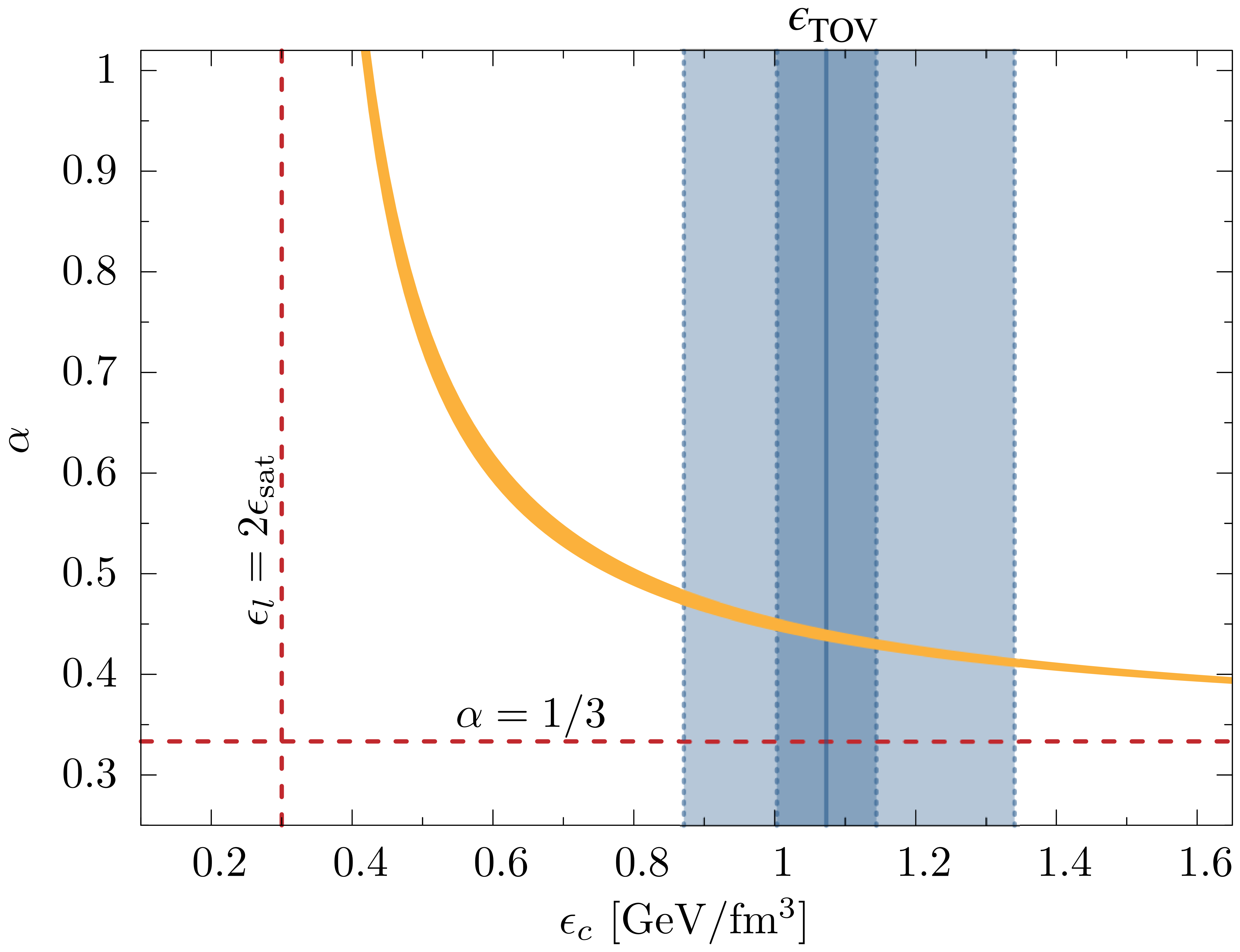
$$\langle c_s^2 \rangle_c = \frac{1}{\epsilon_c}\int_0^{\epsilon_l} d\epsilon\, c_s^2 + \frac{1}{\epsilon_c}\int_{\epsilon_l}^{\epsilon_c} d\epsilon\, c_s^2 = 1/3$$

$$\langle c_s^2 \rangle_c = \langle c_s^2 \rangle_l \frac{\epsilon_l}{\epsilon_c} + \alpha \left(1 - \frac{\epsilon_l}{\epsilon_c}\right) = 1/3$$

Average c_s^2 at $\langle \epsilon_l, \epsilon_c \rangle$

$\langle c_s^2 \rangle_l$ from χEFT at $\epsilon_l = 2\epsilon_{\rm sat}$

Drischler et al (2021)



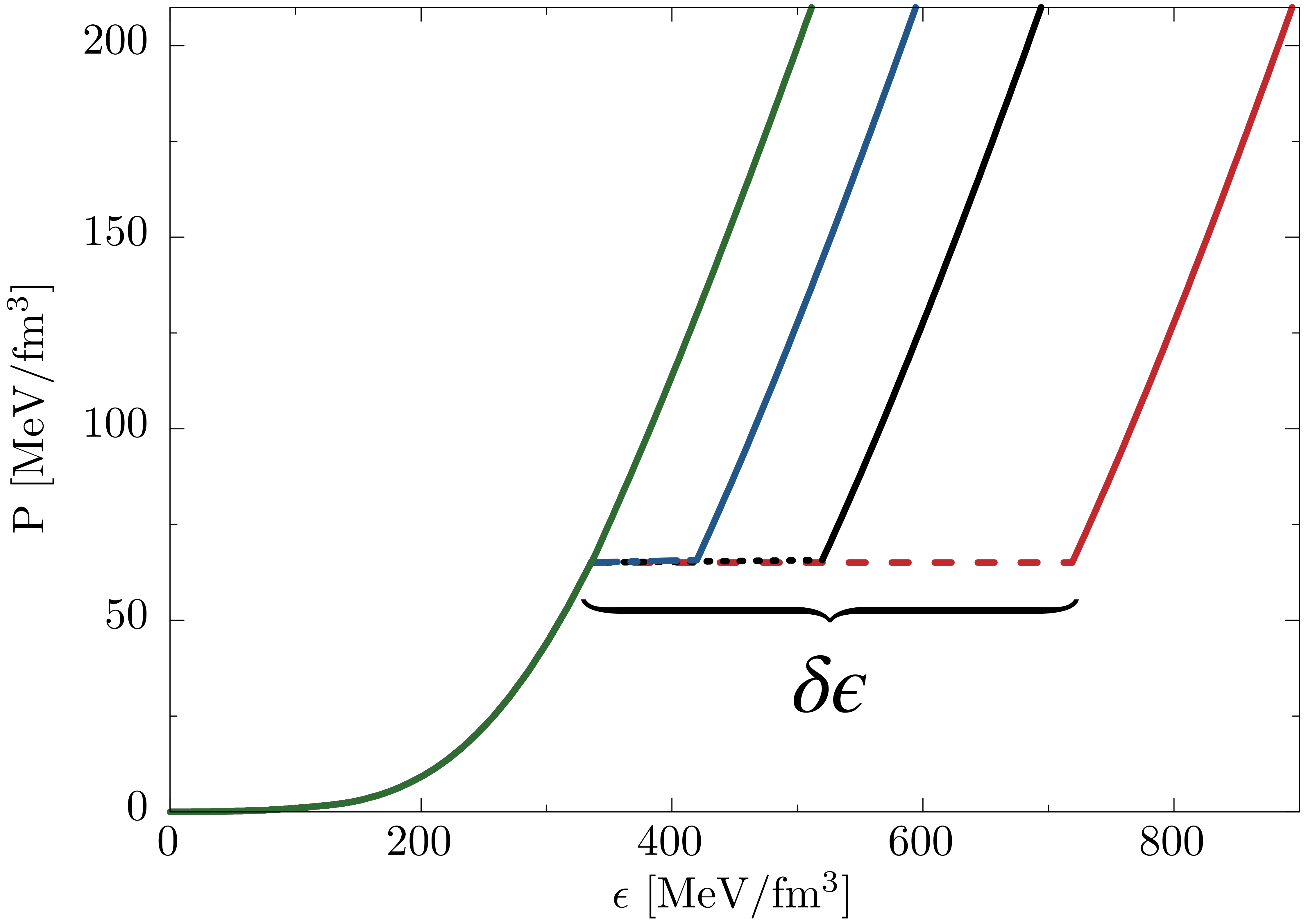
$\epsilon_c = \epsilon_{\rm TOV} \simeq 1$ GeV/fm^3 → $\alpha \simeq 0.4 - 0.5$ → $c_{s,\max}^2 > \alpha$

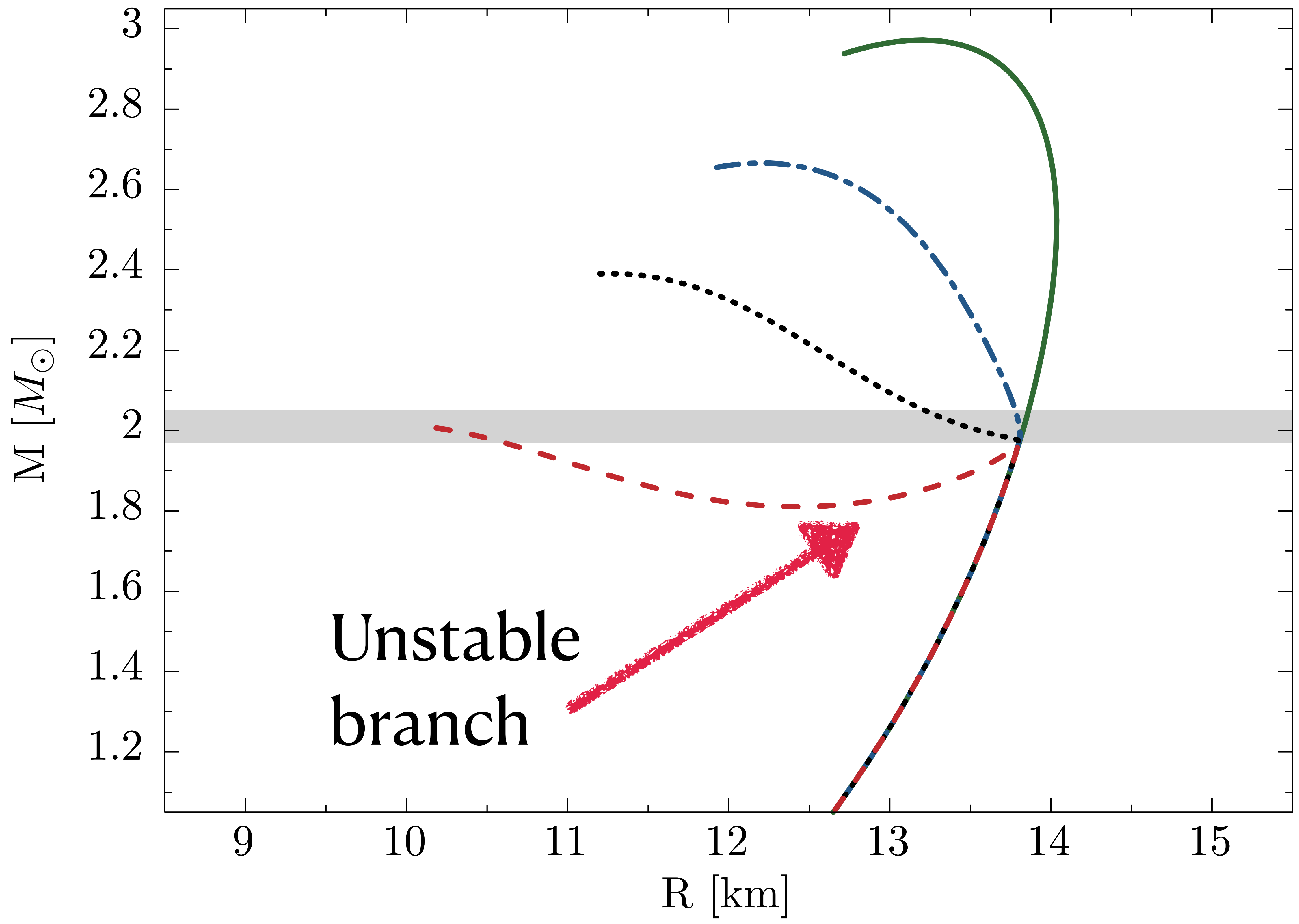
Conditions for (Un)stable Hybrid Stars (Seidov 1971)

- FOPT at ϵ_t with $\delta\epsilon = \epsilon_0 - \epsilon_t$
- Seidov instability condition: $\frac{3}{2}\left(\epsilon_t + p_t\right) < \epsilon_0$

At transition

After transition



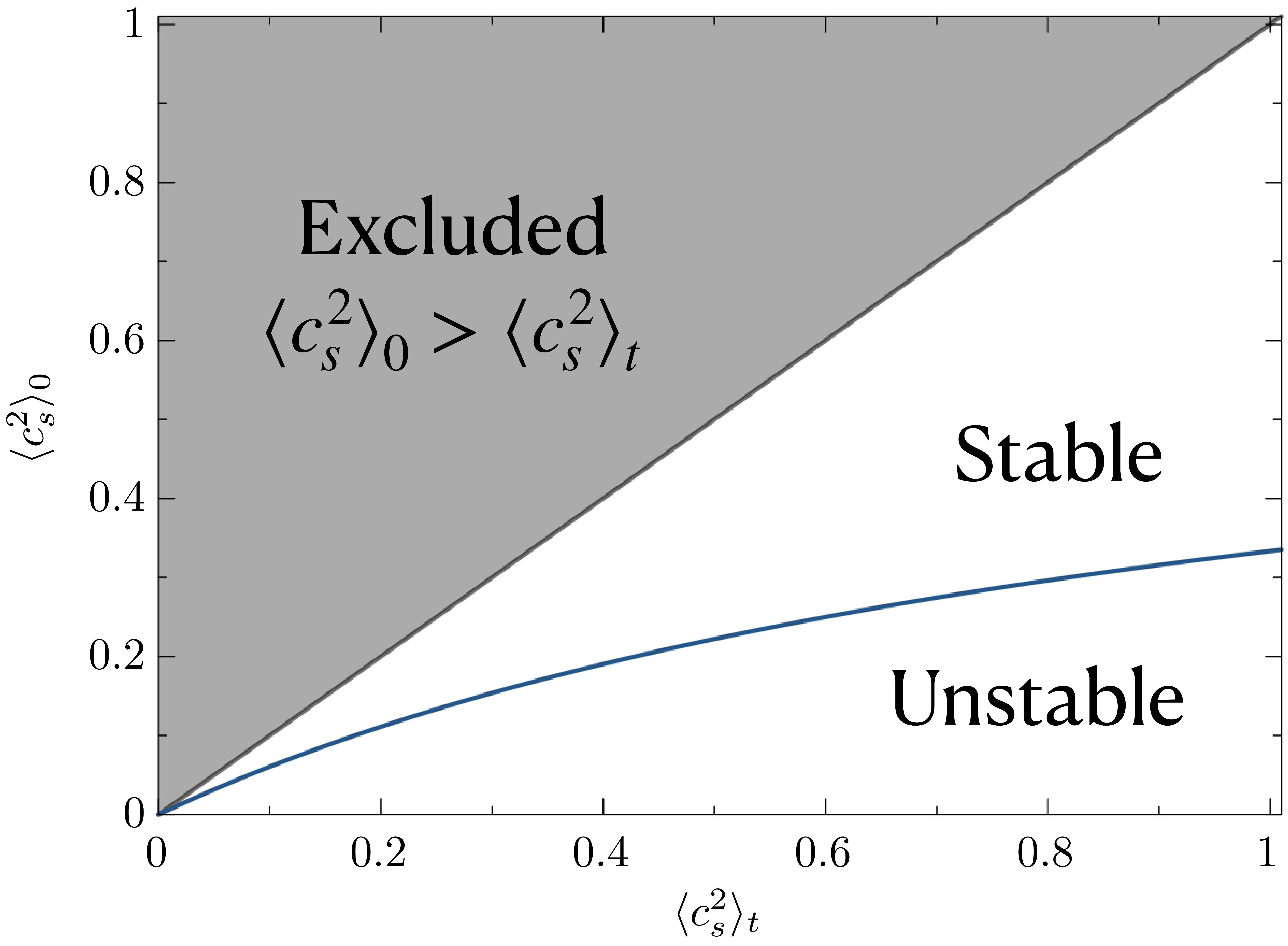


Phase Diagram of Hybrid Stars

$$\epsilon_0 > \frac{3}{2}\left(\epsilon_t + p_t\right)$$

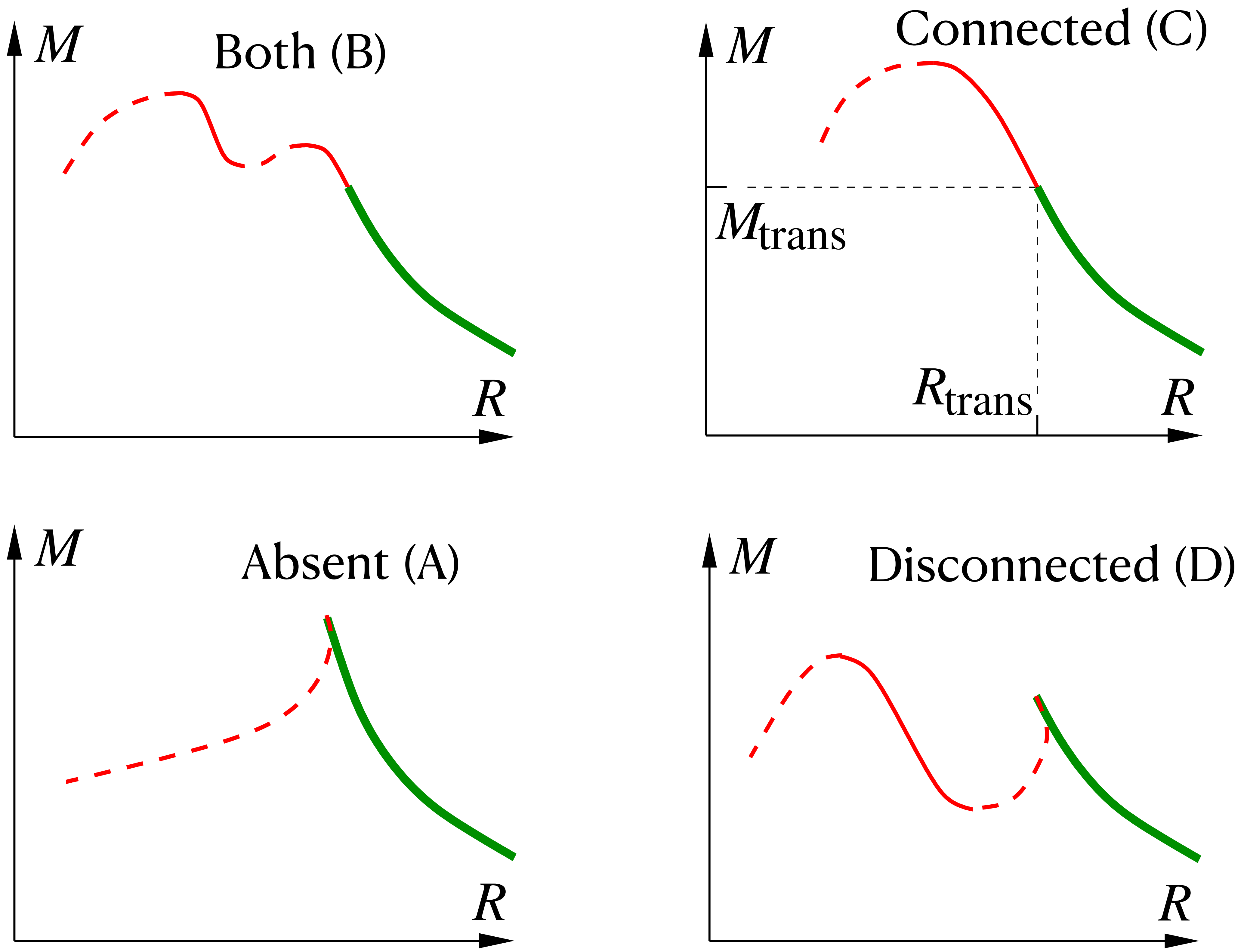
$$\Updownarrow$$

$$\langle c_s^2 \rangle_0 < \frac{2}{3}\frac{\langle c_s^2 \rangle_t}{1 + \langle c_s^2 \rangle_t}$$



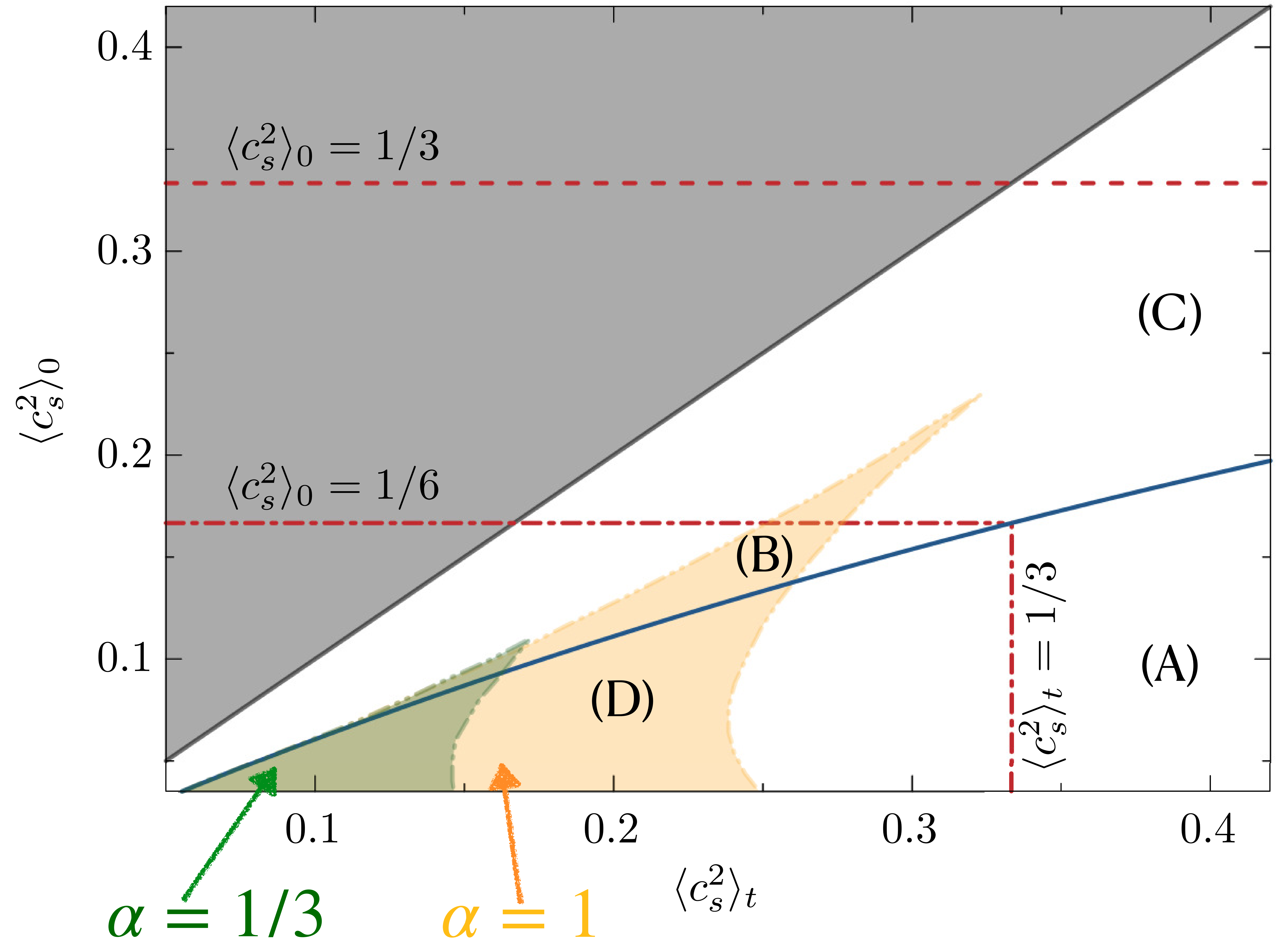
Phase Diagram of Hybrid Stars

M(R) classified based on stability (Alford et al 2013)



- Insensitive to low-density EOS
- Sensitive to high-density EOS

Hadronic EOS + CSS: $\epsilon(p > p_t) = \epsilon_0 + \alpha^{-1}(p - p_t)$



Summary

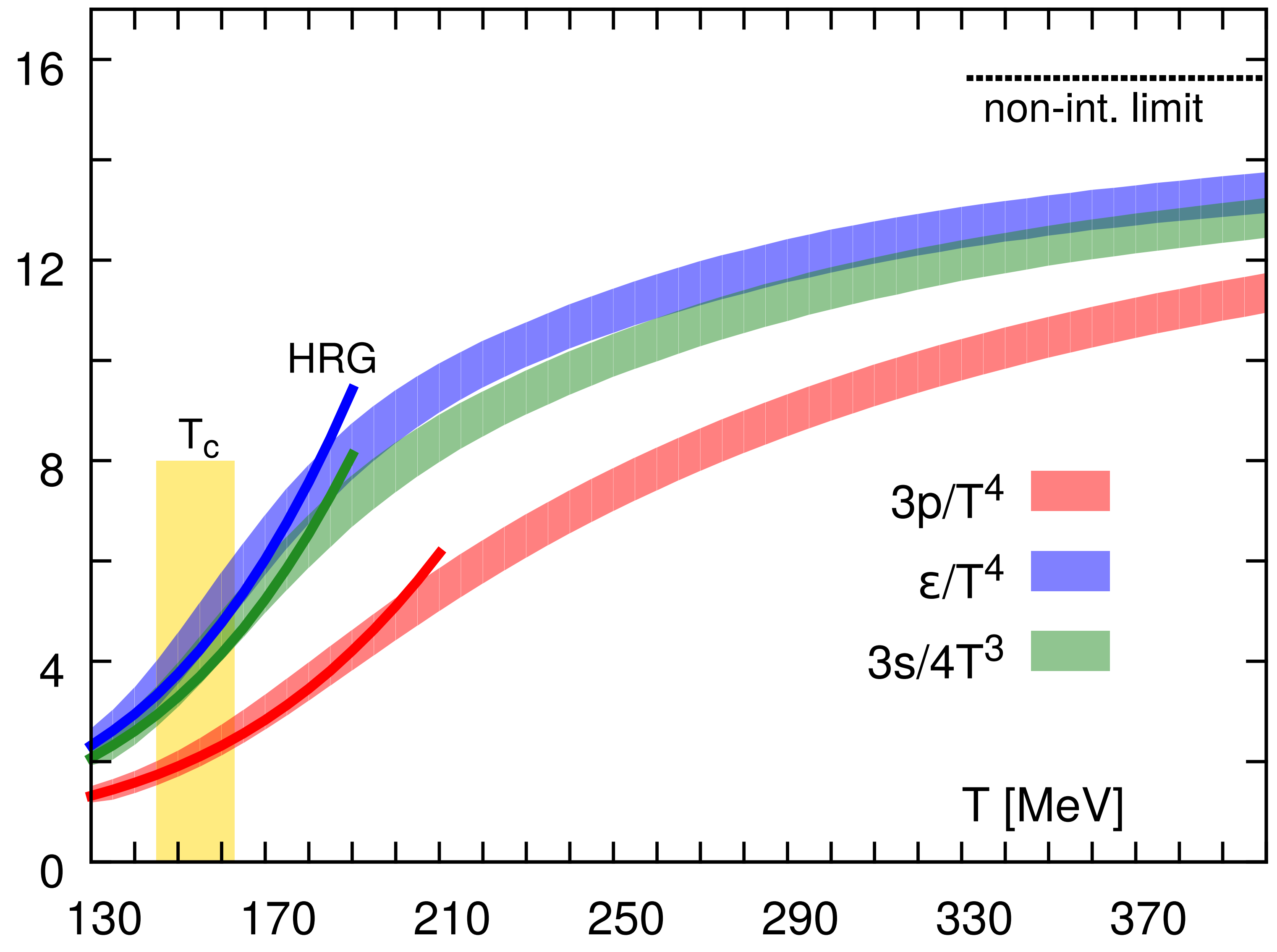
Maximum of c_s^2 consistent with percolation threshold

Matter seems to be conformal in the cores of massive NSs

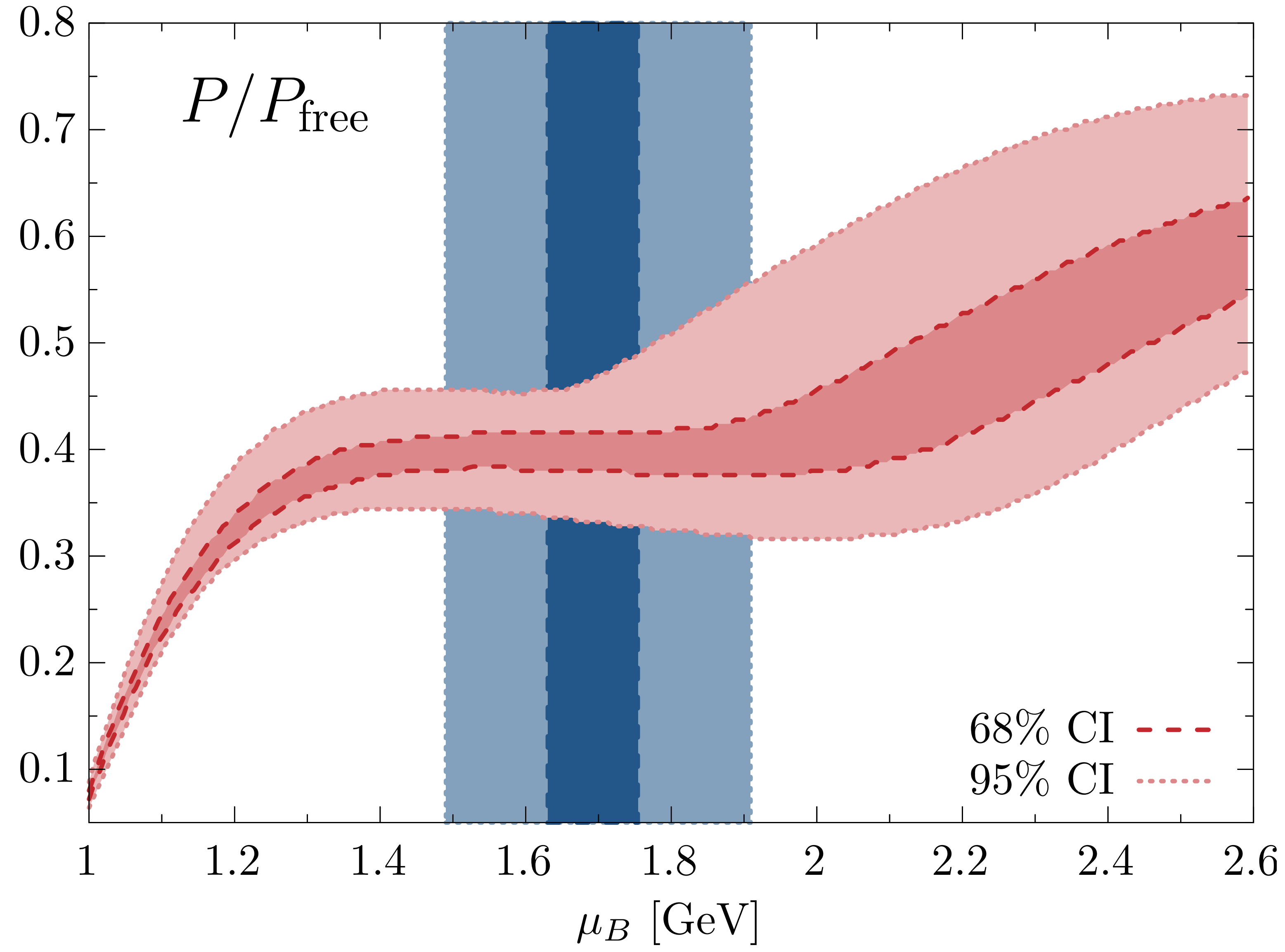
Curvature of ϵ/n can quantify restoration of conformal symmetry

Thank You

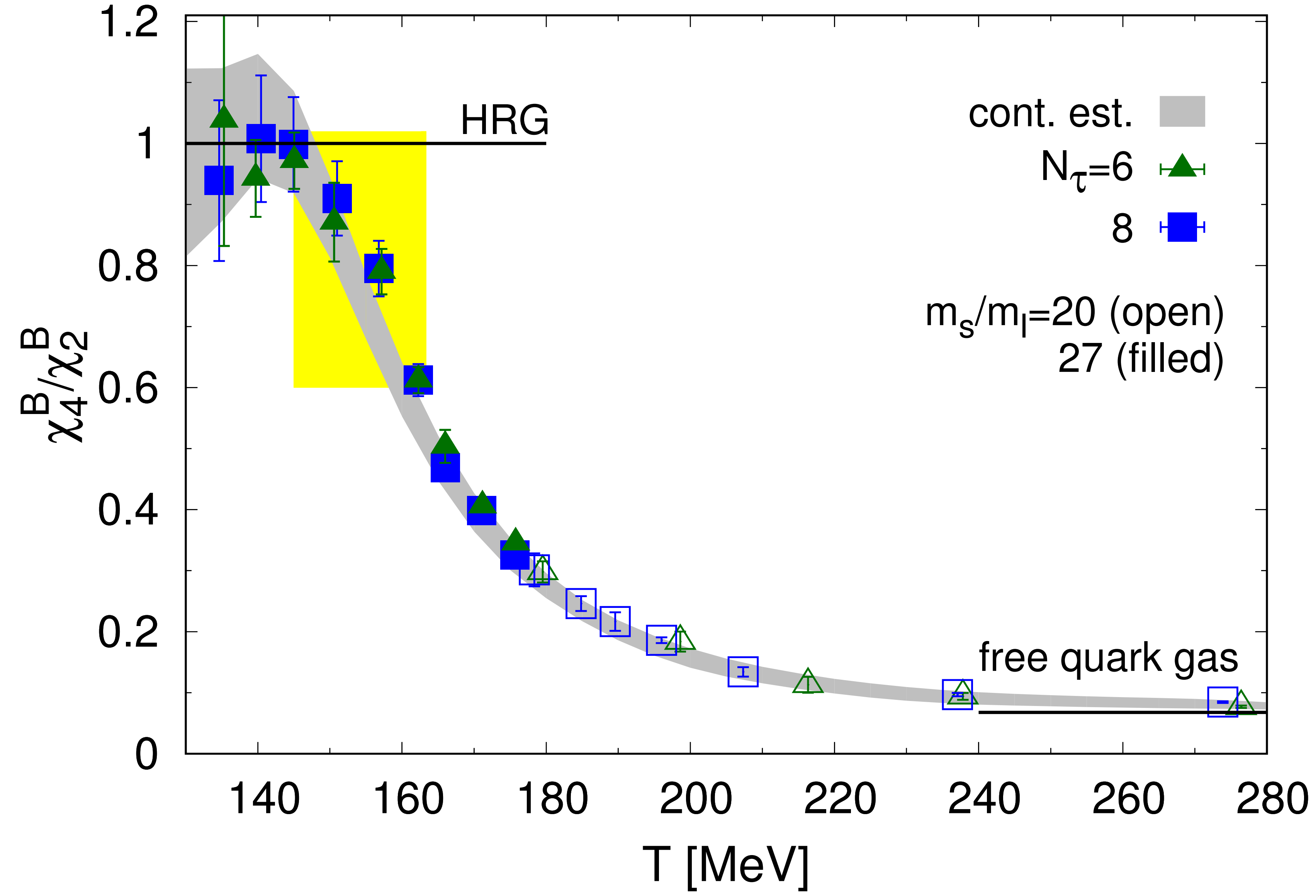
Summary



16
12
8
4
0
130
170
210
250
290
330
370
non-int. limit
HRG
T_c
$3p/T^4$
ε/T^4
$3s/4T^3$
T [MeV]

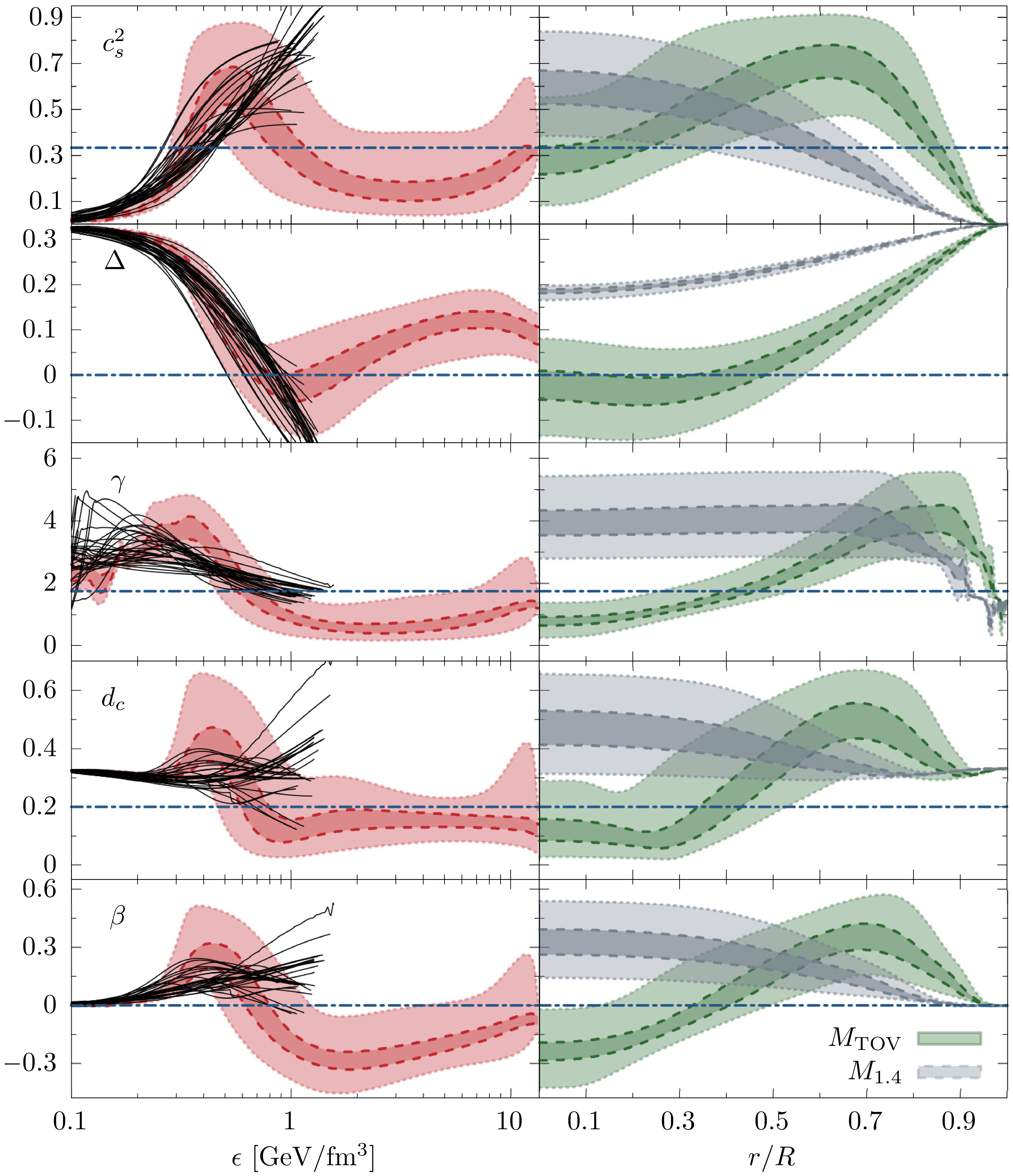


P/P_{free}
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
μ_B [GeV]
68% CI
95% CI



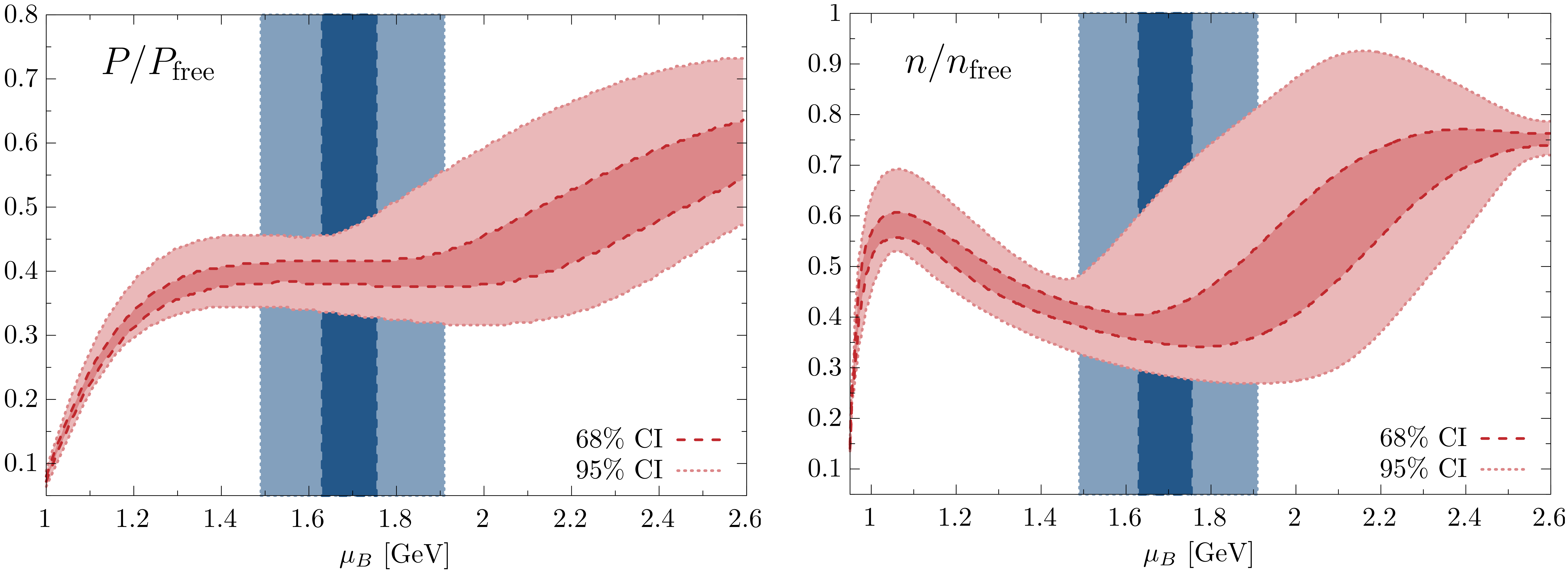
χ_4^B/χ_2^B
1.2
1
0.8
0.6
0.4
0.2
0
140
160
180
200
220
240
260
280
T [MeV]
HRG
cont. est.
N_τ=6
8
m_s/m_l=20 (open)
27 (filled)
free quark gas

???



c_s^2
Δ
γ
d_c
β
M_{TOV}
$M_{1.4}$
ϵ [GeV/fm^3]
r/R

Equation of State



Net-baryon number susceptibility