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The effect of Isovector Scalar Meson on Neutron Star Matter Based on a Parity Doublet Model

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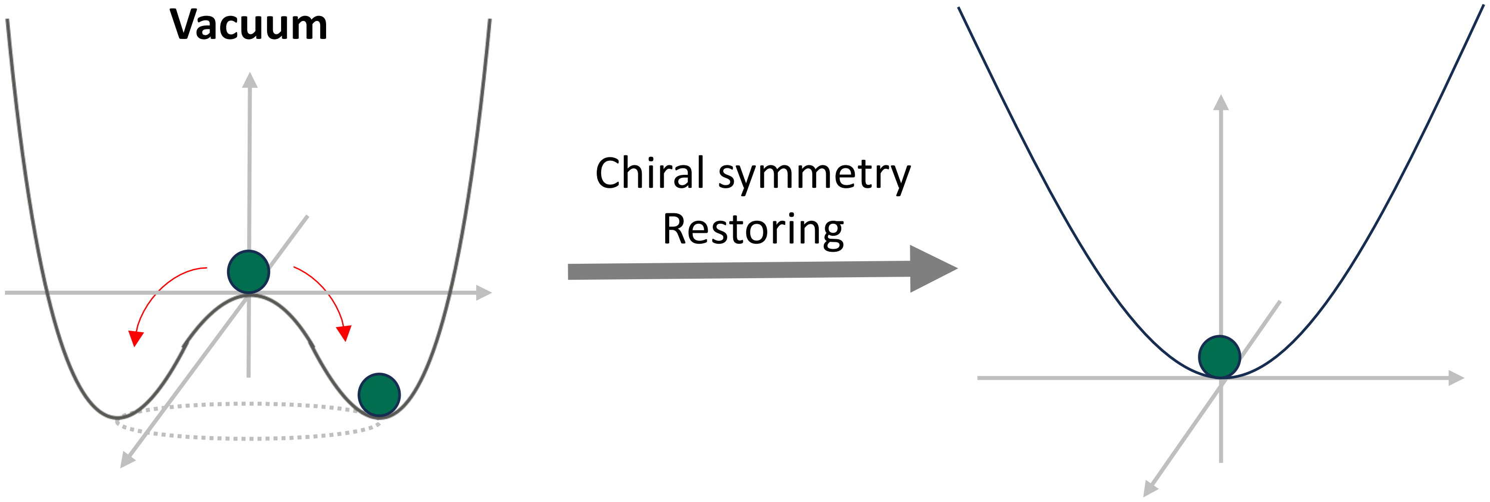
Chiral symmetry restoration and the origin of hadron mass

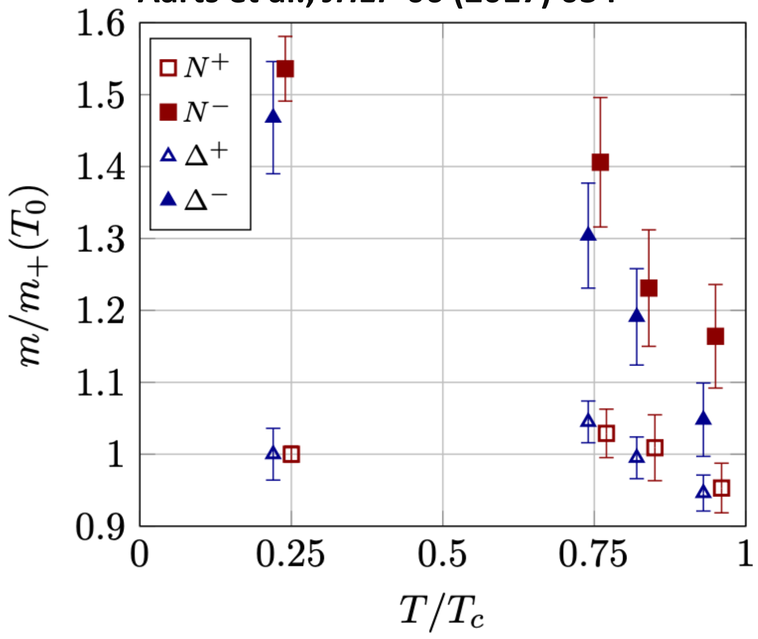


$< \bar{q}q > \neq 0$ $\qquad$ $< \bar{q}q > = 0$

Chiral symmetry restoration happens at high T and/or density

e.g. Finite-T lattice calculation

Aarts et al., *JHEP* 06 (2017) 034

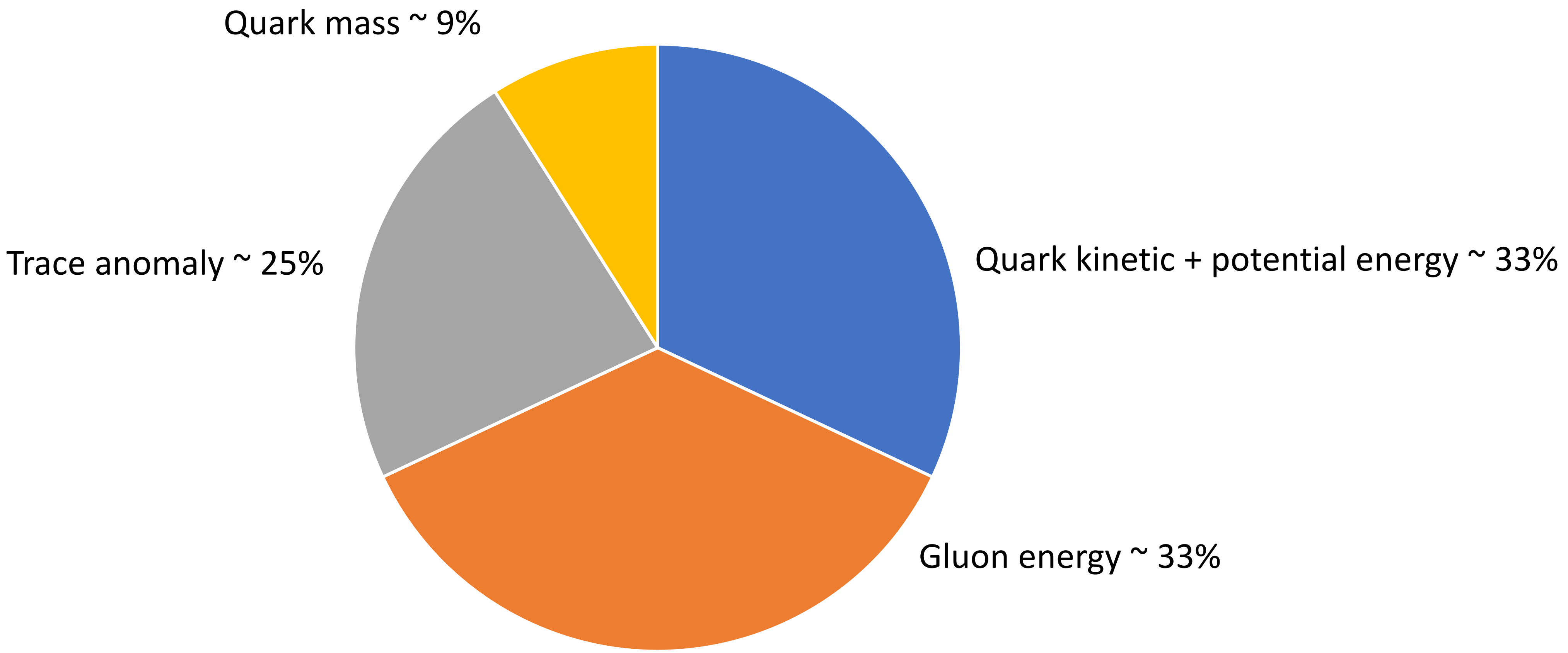


Spontaneous breaking of chiral symmetry generates the mass of hadrons:
In traditional models such as linear sigma model,

$$m_N \propto < \bar{q}q > \longrightarrow 0 \ ?$$

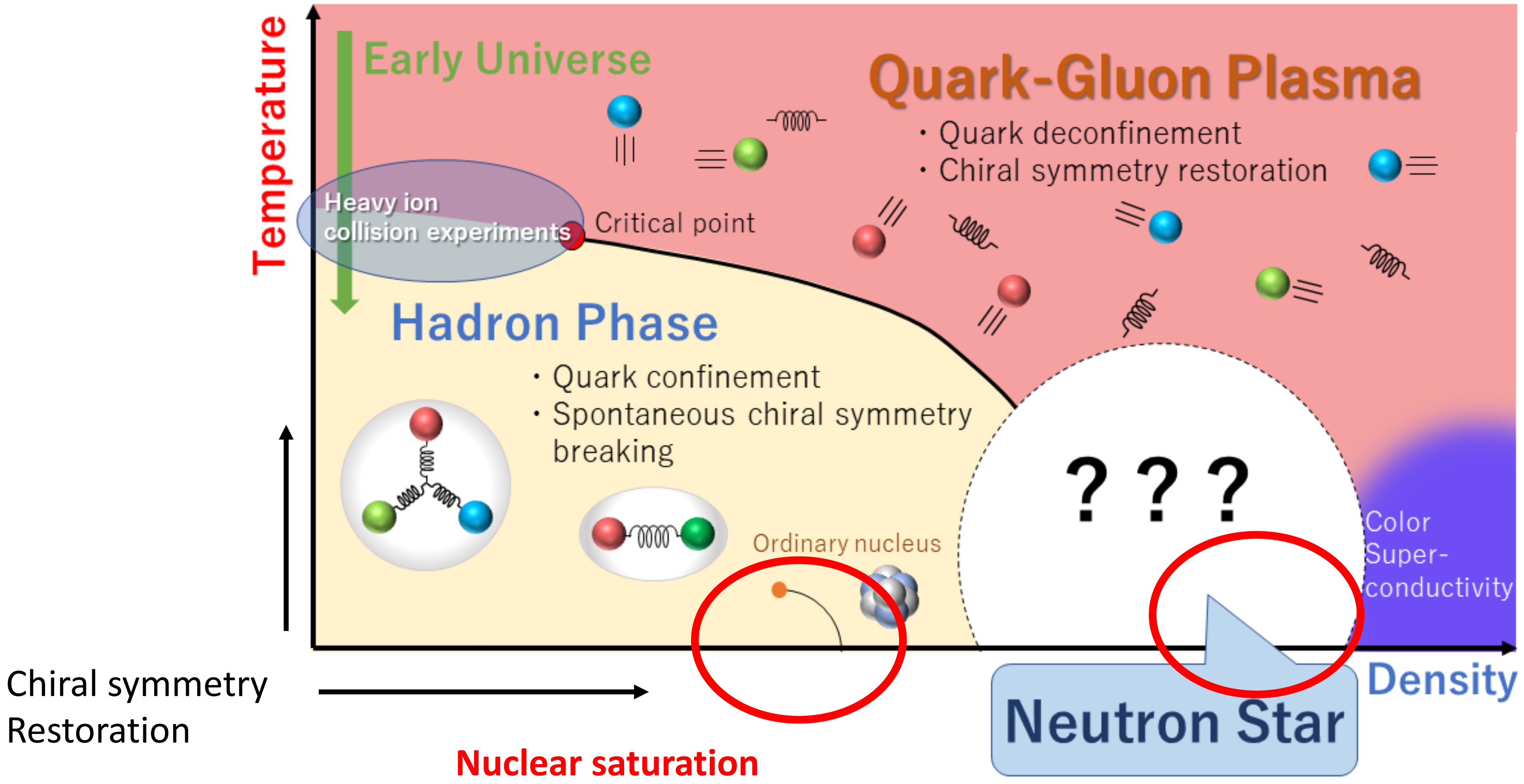
What is inside the mass of nucleon?

- From the Proton Mass Decomposition (Ji, *Phys. Rev. Lett.* **74** (1995) 1071.)



Neutron star: laboratory for highly dense matter

Neutron star = highly (iso-spin) asymmetric matter



Implication to the chiral invariant mass of nucleon?

Parity doublet model (PDM)

Parity doublet model is a **linear sigma model with parity doubling of nucleon** (DeTar and Kunihiro, *Phys. Rev. D* **39** (1989) 2805.)

A term between positive parity and negative parity nucleon is introduced

$$\Delta\mathcal{L}_N = -m_0[\bar{N}_1\gamma_5 N_2 - \bar{N}_2\gamma_5 N_1]$$

The mass of nucleon in PDM is given by

$$m_N \sim m_{\bar{q}q} + m_0 \rightarrow m_0$$

The m_0 is called chiral invariant mass of the nucleon

PDM with isovector scalar meson a_0(980)

- To study the asymmetric matter like neutron star, it maybe important to consider the isovector scalar meson which mediate the attractive force in the isovector channel

	Isoscaler	Isovector
Scaler	σ	$\vec{a}_0$
Pseudoscaler	η	$\vec{\pi}$

a_0(980) : 0^{++}
Lightest isovector scalar

- Its effect is usually ignored due to its heavy mass and absence in symmetric matter

- **We construct a PDM with a_0(980) meson => $U(2)_L x U(2)_R$ PDM**

$$M = (\sigma + i\vec{\tau}\cdot\vec{\pi}) - (\vec{\tau}\cdot\vec{a}_0 + i\eta)$$

In asymmetric matter,

$$\vec{a}_0 \sim \bar{q}\vec{\tau}q \rightarrow \bar{q}\tau_3 q \sim \bar{u}u - \bar{d}d \neq 0$$

a0 meson does not appear in symmetry matter

Motivation of this work

- In this work, we study the effect of $a_0(980)$ to:

  (1) the symmetry properties of nuclear matter

  (2) the neutron star properties

Finally, we constrain the value of chiral invariant mass of nucleon and asset how important is the effect of $a_0(980)$ meson to matter properties

Parity Doublet Model (PDM)

- **Parity doublet model (PDM) models considers the parity doubling of nucleons using linear**

$$m_{\pm j} = \frac{1}{2}\left[\sqrt{(g_1+g_2)^2(\sigma - ja)^2 + 4m_0^2} + \pm(g_1-g_2)(\sigma - ja)\right]$$

$$\mathcal{L} = \sum_{\alpha j} \bar{N}_{\alpha j}(i\partial\!\!\!/ - m_{\alpha j})N_{\alpha j}$$

$$(p, n) : j = (+, -)$$

$$+\mathcal{L}^{L\sigma M}(M) + \mathcal{L}^{HLS}(\omega, \rho)$$

- **The nucleon mass in this model is given by**

$$m_N \sim m_{\bar{q}q} + m_0$$

$$N_{ir} = \frac{1+\gamma_5}{2}N_i$$

$$N_{il} = \frac{1-\gamma_5}{2}N_i,$$
$$(i = 1, 2)$$

Parity Doublet Model (PDM)

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The mesonic Lagrangian is based on the extended linear sigma model

$$
\begin{aligned}
\mathcal{L}^{L\sigma M}(M) = & \frac{1}{4}\mathrm{tr}\left[\partial_\mu M \partial^\mu M^\dagger\right] \\
& + \frac{\bar{\mu}^2}{4}\mathrm{tr}[M^\dagger M] \\
& - \frac{\lambda_{41}}{8}\mathrm{tr}[(M^\dagger M)^2] + \frac{\lambda_{42}}{16}\{\mathrm{tr}[M^\dagger M]\}^2 \\
& + \frac{\lambda_{61}}{12}\mathrm{tr}[(M^\dagger M)^3] + \frac{\lambda_{62}}{24}\mathrm{tr}[(M^\dagger M)^2]\mathrm{tr}[M^\dagger M] + \frac{\lambda_{63}}{48}\{\mathrm{tr}[M^\dagger M]\}^3 \\
& + \frac{m_\pi^2 f_\pi}{4}\mathrm{tr}[M + M^\dagger] \\
& + \frac{K}{8}\{\det M + \det M^\dagger\}
\end{aligned}
$$

$$M = (\sigma + i\vec{\tau}\cdot\vec{\pi}) - (\vec{\tau}\cdot\vec{a}_0 + i\eta)$$

Taking mean field approximation,

$$\sigma(x) \to \sigma, \qquad \pi(x) \to 0, \qquad \eta(x) \to 0.$$

$$\vec{a_0}(x) \to a\delta_{i3}, \qquad a_0^{i=3} \equiv a.$$

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Hidden Local Symmetry (HLS)

The vector meson is included basing on Hidden Local Symmetry (HLS) to account for the repulsive interaction in the matter:

$$
\begin{aligned}
\mathcal{L}^{HLS} = {} & a_{VNN}\left[\bar{N}_{1l}\gamma^{\mu}\xi_L^{\dagger}\hat{\alpha}_{\parallel\mu}\xi_L N_{1l} + \bar{N}_{1r}\gamma^{\mu}\xi_R^{\dagger}\hat{\alpha}_{\parallel\mu}\xi_R N_{1r}\right] \\
& + a_{VNN} \ldots \\
& + a_{0NN} \ldots \\
& + \frac{m_\rho{}^2}{g_\rho{}^2}\mathrm{tr}[\hat{\alpha}_{\parallel}^{\mu}\hat{\alpha}_{\parallel\mu}] + \left(\frac{m_\omega{}^2}{8g_\omega{}^2} - \frac{m_\rho{}^2}{2g_\rho{}^2}\right)\mathrm{tr}[\hat{\alpha}_{\parallel}^{\mu}]\mathrm{tr}[\hat{\alpha}_{\parallel\mu}] - \frac{1}{8g_\omega{}^2}\mathrm{tr}[\omega^{\mu\nu}\omega_{\mu\nu}] - \frac{1}{2g_\rho{}^2}\mathrm{tr}[\rho^{\mu\nu}\rho_{\mu\nu}] \\
& + \lambda_{\omega\rho}(a_{VNN} + a_{0NN})^2 a_{VNN}^2\left[\frac{1}{2}\mathrm{tr}[\hat{\alpha}_{\parallel}^{\mu}\hat{\alpha}_{\parallel\mu}]\mathrm{tr}[\hat{\alpha}_{\parallel}^{\nu}]\mathrm{tr}[\hat{\alpha}_{\parallel\nu}] - \frac{1}{4}\left\{\mathrm{tr}[\hat{\alpha}_{\parallel}^{\mu}]\mathrm{tr}[\hat{\alpha}_{\parallel\mu}]\right\}^2\right],
\end{aligned}
$$

Complicated! ☹

$$M = \xi_L^{\dagger} S \xi_R$$

$$\xi_R = \xi_L^{\dagger} = \exp\left(iP/f_\pi\right)$$

$$D^\mu \xi_R = \partial^\mu \xi_R - i g_\rho \rho^\mu \xi_R - i g_\omega \omega^\mu \xi_R + i\xi_R \mathcal{R}^\mu + i\xi_R \mathcal{A}^\mu$$

$$D^\mu \xi_L = \partial^\mu \xi_L - i g_\rho \rho^\mu \xi_L - i g_\omega \omega^\mu \xi_L + i\xi_L \mathcal{L}^\mu - i\xi_L \mathcal{A}^\mu$$

$$\hat{\alpha}_{\perp}^{\mu} \equiv \frac{1}{2i}[D^\mu \xi_R \xi_R^{\dagger} - D^\mu \xi_L \xi_L^{\dagger}]$$

$$\hat{\alpha}_{\parallel}^{\mu} \equiv \frac{1}{2i}[D^\mu \xi_R \xi_R^{\dagger} + D^\mu \xi_L \xi_L^{\dagger}]$$

Parity Doublet Model (PDM)

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Under mean field approximation, the vector meson Lagrangian is written in a more familiar form:

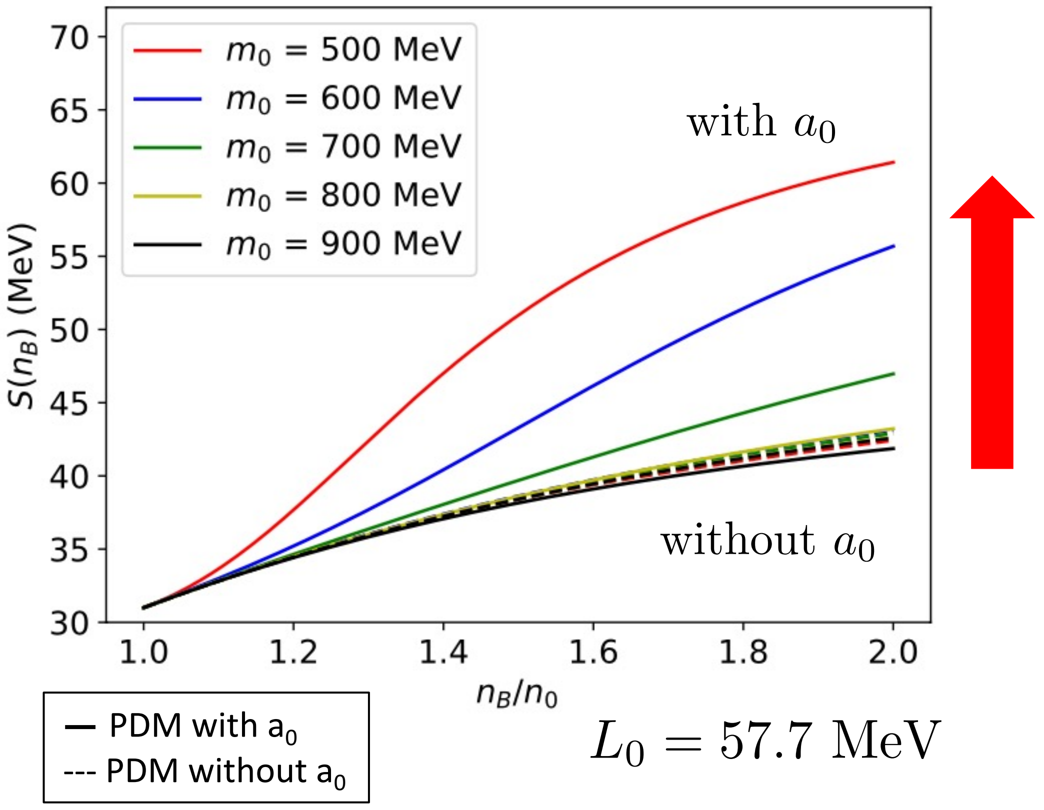
$$\mathcal{L}^{HLS} = -\, g_{\omega NN} \sum_{\alpha j} \bar{N}_{\alpha j}\gamma^0 \omega N_{\alpha j} - g_{\rho NN} \sum_{\alpha j} \bar{N}_{\alpha j}\gamma^0 \frac{\tau_3}{2} \rho N_{\alpha j} + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2 + \underline{\lambda_{\omega\rho} g_{\omega NN}^2 g_{\rho NN}^2 \omega^2 \rho^2}$$

Vector mixing interaction is included to control the slope parameter

$$\omega_\mu(x) \to \omega\delta_{\mu 0}, \qquad \rho_\mu^i(x) \to \rho\delta_{\mu 0}\delta_{i3},$$

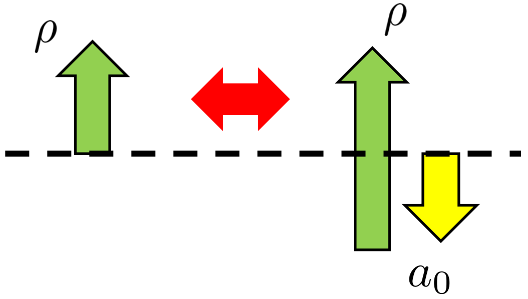
Symmetry energy $S(n_B)$

$S_0 = 31$ MeV



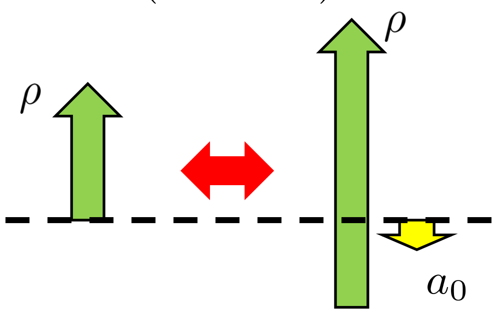
$L_0 = 57.7$ MeV

We determine the ρ coupling by **fitting saturation properties**, in the a_0 model the ρ coupling is stronger to fit S_0=31 MeV



$S(n_B > n_0)$

When $n_B > n_0$, the repulsive force of ρ become larger and attractive force of a_0 become smaller



$$m_{\pm j} = \frac{1}{2}\left[\sqrt{(g_1+g_2)^2(\sigma - ja)^2 + 4m_0^2} + \pm(g_1 - g_2)(\sigma - ja)\right]$$

Symmetry incompressibility K_{sym}

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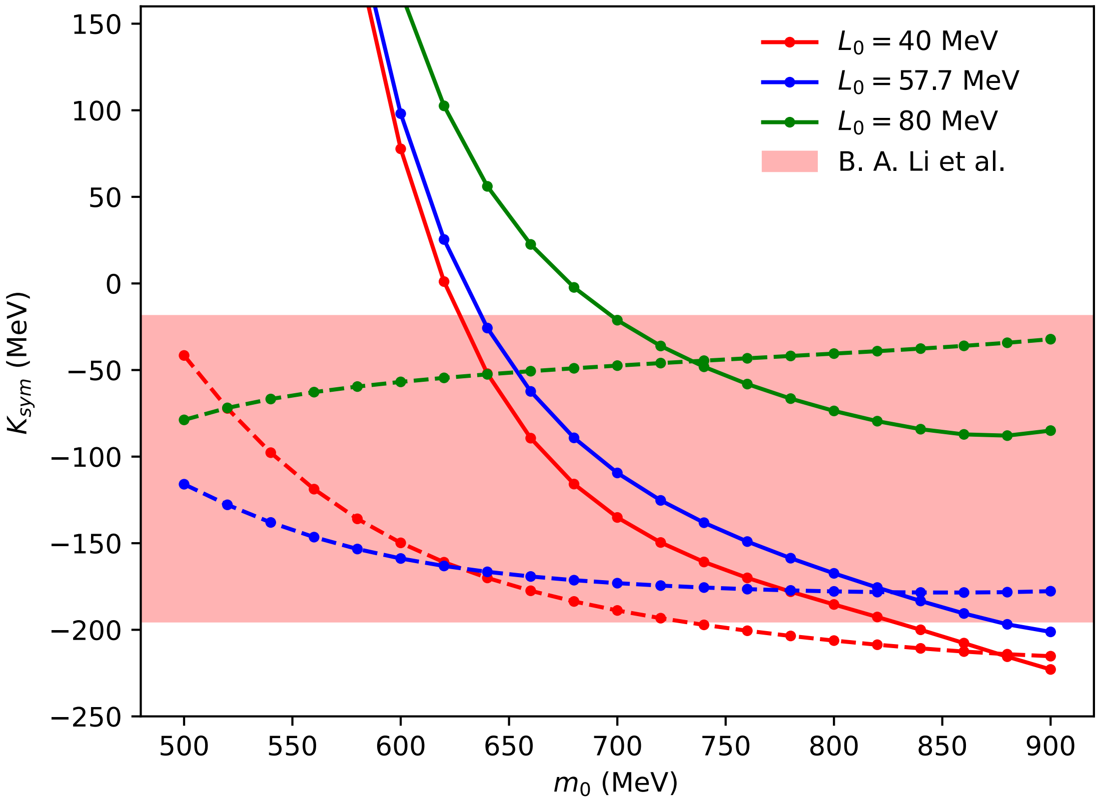
Recent accepted value of K_{sym} = −107±88 MeV

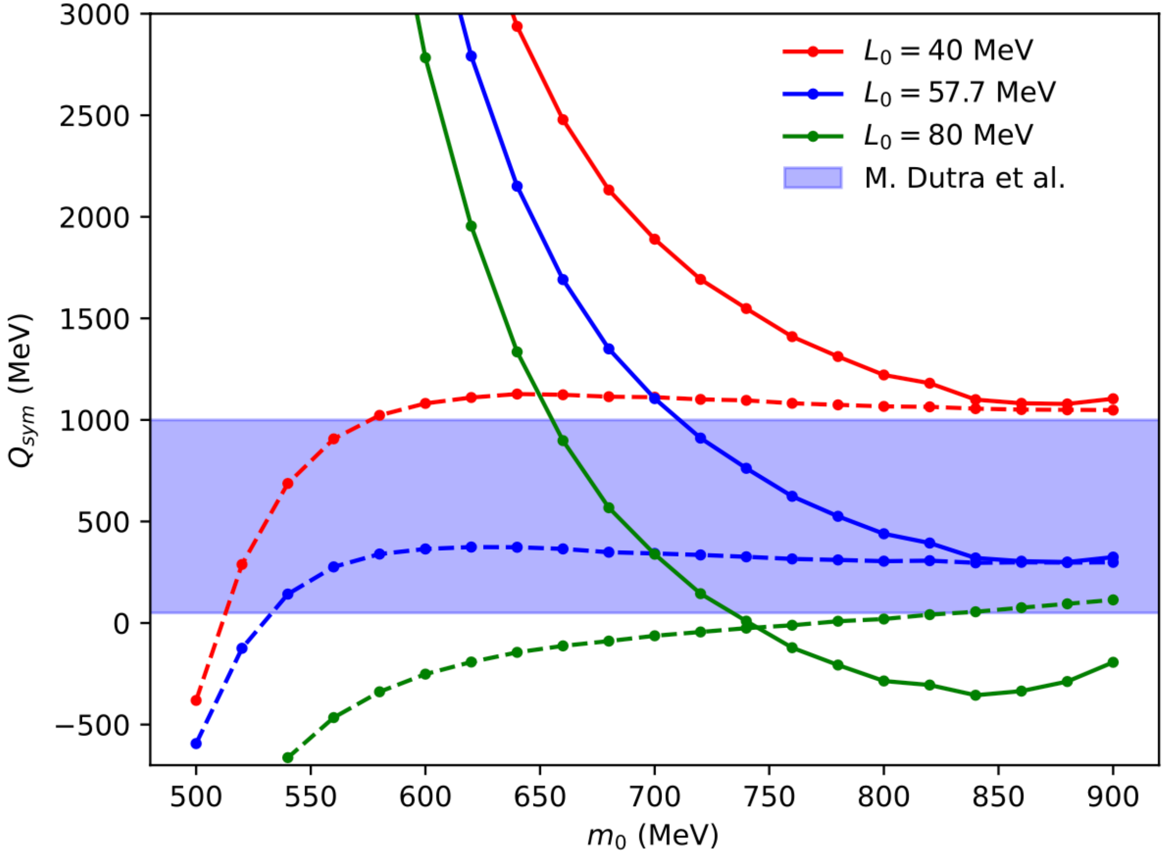
B. A. Li et al. Universe, 2021, 7(6).

$$K_{sym} = 9n_0^2 \frac{\partial^2 S}{\partial n_B^2}\bigg|_{n_0}$$



Solid line: with a0 meson
Dash line: without a0 meson

Symmetry skewness Q_{sym}

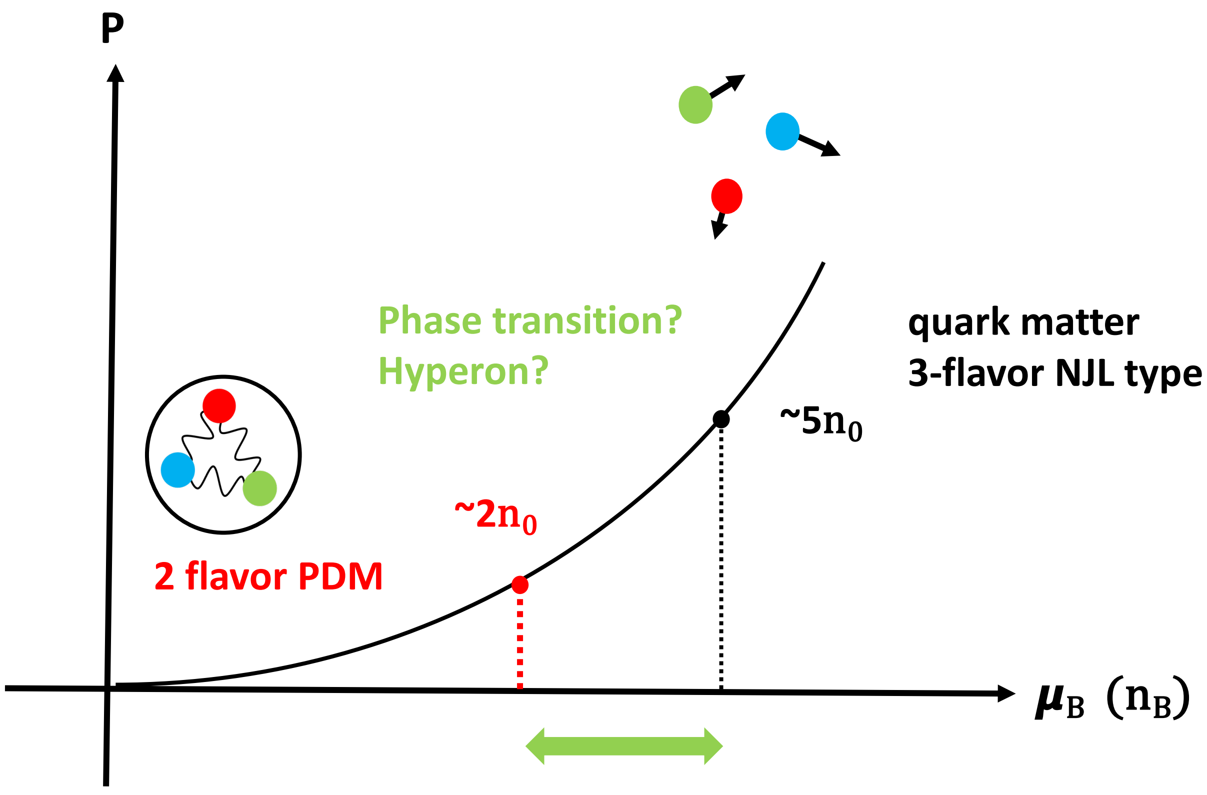


$$Q_{sym} = 27 n_0^3 \frac{\partial^3 S}{\partial n_B^3}\bigg|_{n_0}$$

Solid line: with a0 meson
Dash line: without a0 meson

Unified equation of state of NS: crossover model

$$P_I(\mu_B) \equiv \sum_{i=0}^{5} c_i \mu_B^i$$



Parity doublet model (PDM) is considered in low density

PDM consider **two opposite parity baryons** which degenerate to non-zero mass m_0 when chiral symmetry is restored

Interpolation
(crossover phase)
Masuda et al. (2011)
G. Baym et al. (2017)

Nambu-Jona-Lasinio (NJL)-type model to reproduce quark matter in the high density

Neutron star M-R relation

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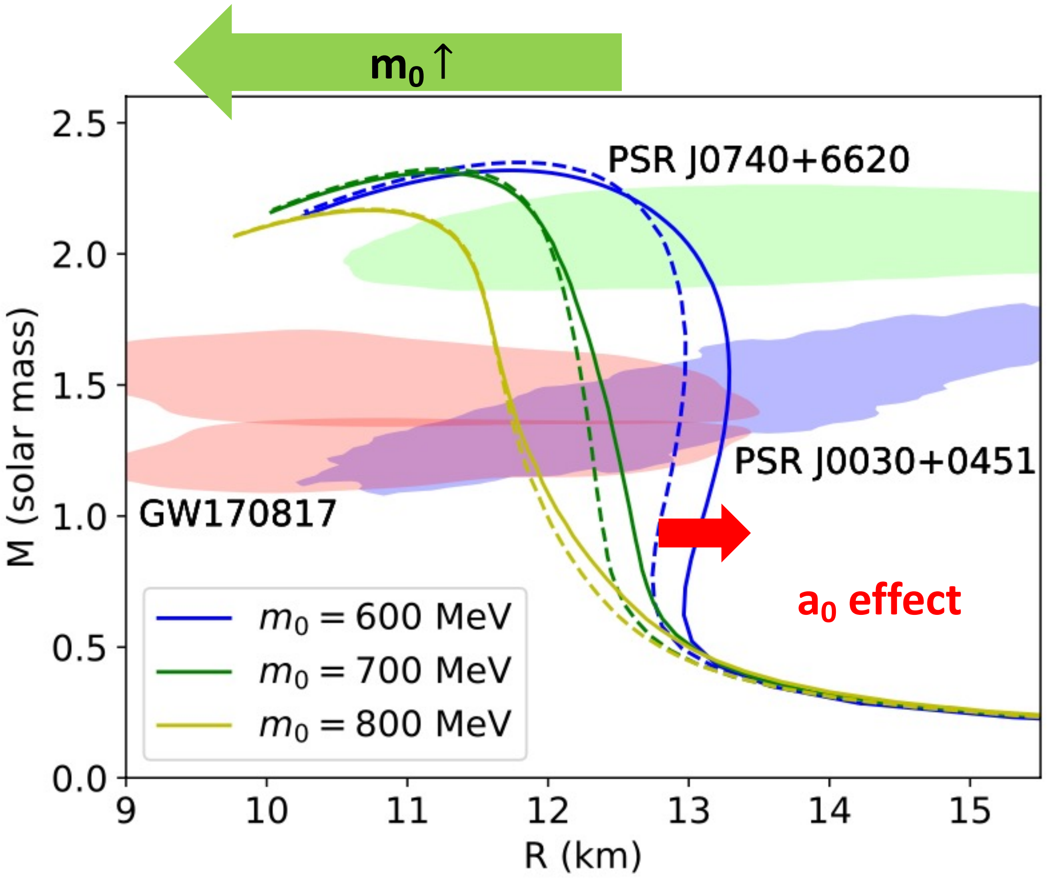
- We compute the M-R relation by solving the TOV equation
- $a_0(980)$ **increase the radius of intermediate mass NS**

$$540 \text{ MeV} \lesssim m_0 \lesssim 870 \text{ MeV} \ (\text{without } a_0)$$

$a_0(980)$

$$\boxed{580 \ \text{MeV} \lesssim m_0 \lesssim 860 \ \text{MeV}}$$

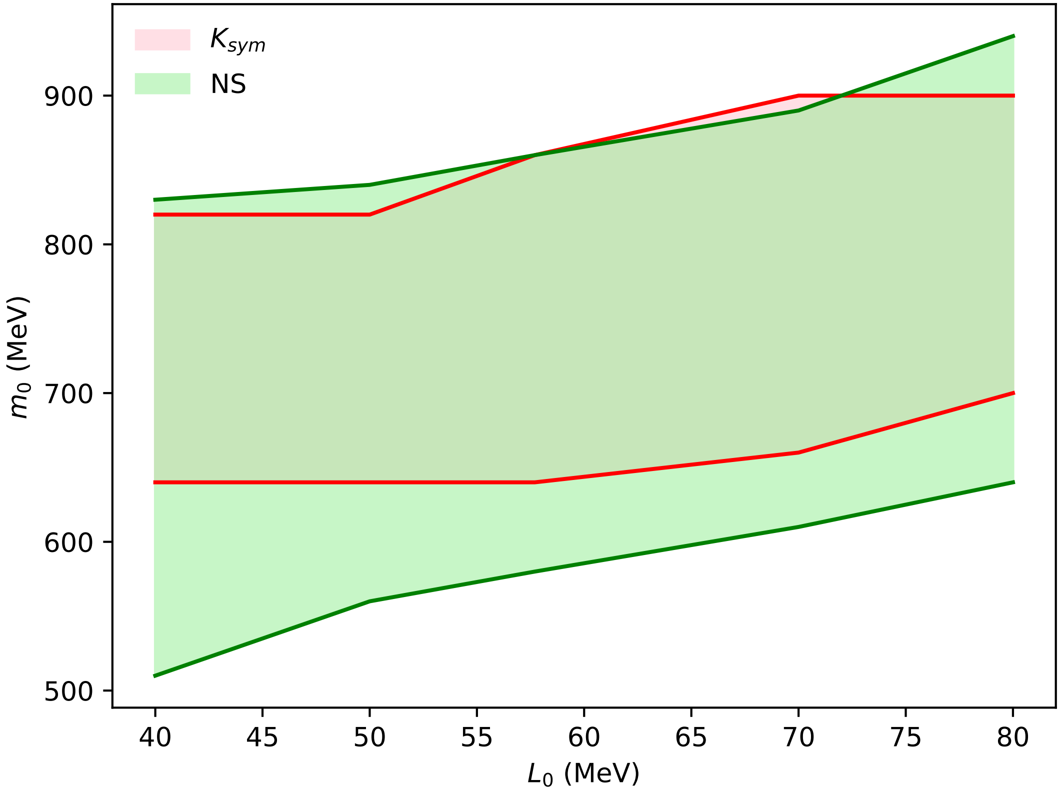


— PDM with a_0
--- PDM without a_0

$$L_0 = 57.7 \text{ MeV}$$

$$\boxed{\Delta R \lesssim 1 \text{ km for } 0.5 \lesssim M/M_\odot \lesssim 2}$$

Constraint to m_0 in the a0-PDM model



$$640 \text{ MeV} \lesssim m_0 \lesssim 860 \text{ MeV for } L_0 = 57.7 \text{ MeV}$$

The constraint from NS (high density) and K_{sym} (low density) consistent with each other

Quark core?

Quark matter appear from $n_B \geq 5n_0$

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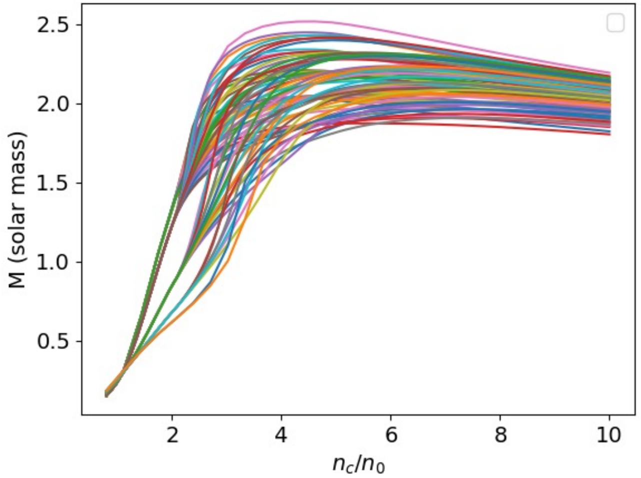
NS constraints only: $580 \text{ MeV} \lesssim m_0 \lesssim 860 \text{ MeV for } L_0 = 57.7 \text{ MeV}$

$$n_c(1.4M_\odot) \approx 2 - 3.6\ n_0$$

$$n_c(2.1M_\odot) \gtrsim 2.6\ n_0$$

$$n_c(2.3M_\odot) \approx 2.9 - 8.4\ n_0$$

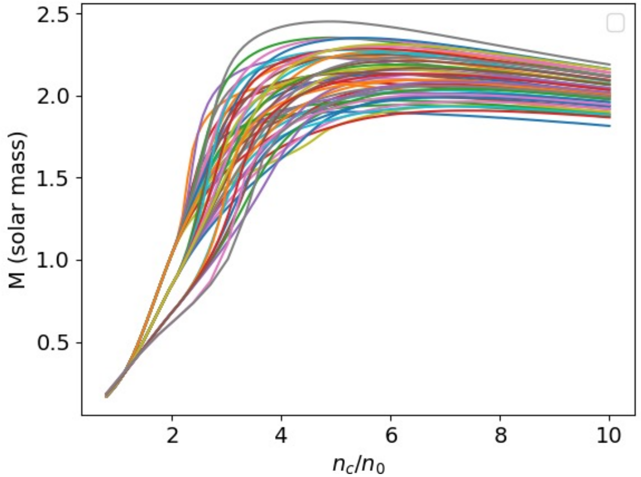


NS + K_{sym} constraints: $640 \text{ MeV} \lesssim m_0 \lesssim 860 \text{ MeV for } L_0 = 57.7 \text{ MeV}$

$$n_c(1.4M_\odot) \approx 2.2 - 3.6\ n_0$$

$$n_c(2.1M_\odot) \gtrsim 2.9\ n_0$$

$$n_c(2.3M_\odot) \approx 3.3 - 8.2\ n_0$$



Summary & future work

- **We find that the existence of a_0(980) stiffens the matter:**
  - **increase the symmetry energy at $n_B > n_0$**
  - **increase the radius of intermediate mass NS**

- **a_0(980) meson increase the radius of NS less than ~ 1km for large m0**

- **Our model consistent with the constraints from low density (symmetry properties at n_0) and high density (NS)**

- **We constrains the chiral invariant mass m_0 in a_0 model to**

640 – 860 MeV

- **Given the constraint of m_0, the effect of a_0(980) meson seems to be weak even in highly-asymmetric matter**

- **We are now studying the effect of a_0(980) to finite nuclei using RCHB theory with PDM**

Thank you!

Backup

Vector meson mixing interaction

In our a_0 PDM without vector meson mixing interaction, we can compute the slope parameter L_0 :

	m_0 [MeV]	600	700	800	900
$K_0 = 215\,\mathrm{MeV}$	$g_{\rho NN}$	12.52	11.20	9.94	8.90
	L_0 [MeV]	120.14	105.21	97.05	87.65
$K_0 = 240\,\mathrm{MeV}$	$g_{\rho NN}$	12.47	11.16	9.90	8.86
	L_0 [MeV]	126.58	108.78	98.67	87.75
$K_0 = 260\,\mathrm{MeV}$	$g_{\rho NN}$	12.43	11.13	9.86	8.83
	L_0 [MeV]	131.19	111.45	99.86	87.75

Recent accepted L0 = 57.7 ± 19 MeV

Li, B.A. et al., Universe 2021, 7

Reduce the stiffness of the matter with vector meson mixing interaction

Parity Doublet Model (PDM)

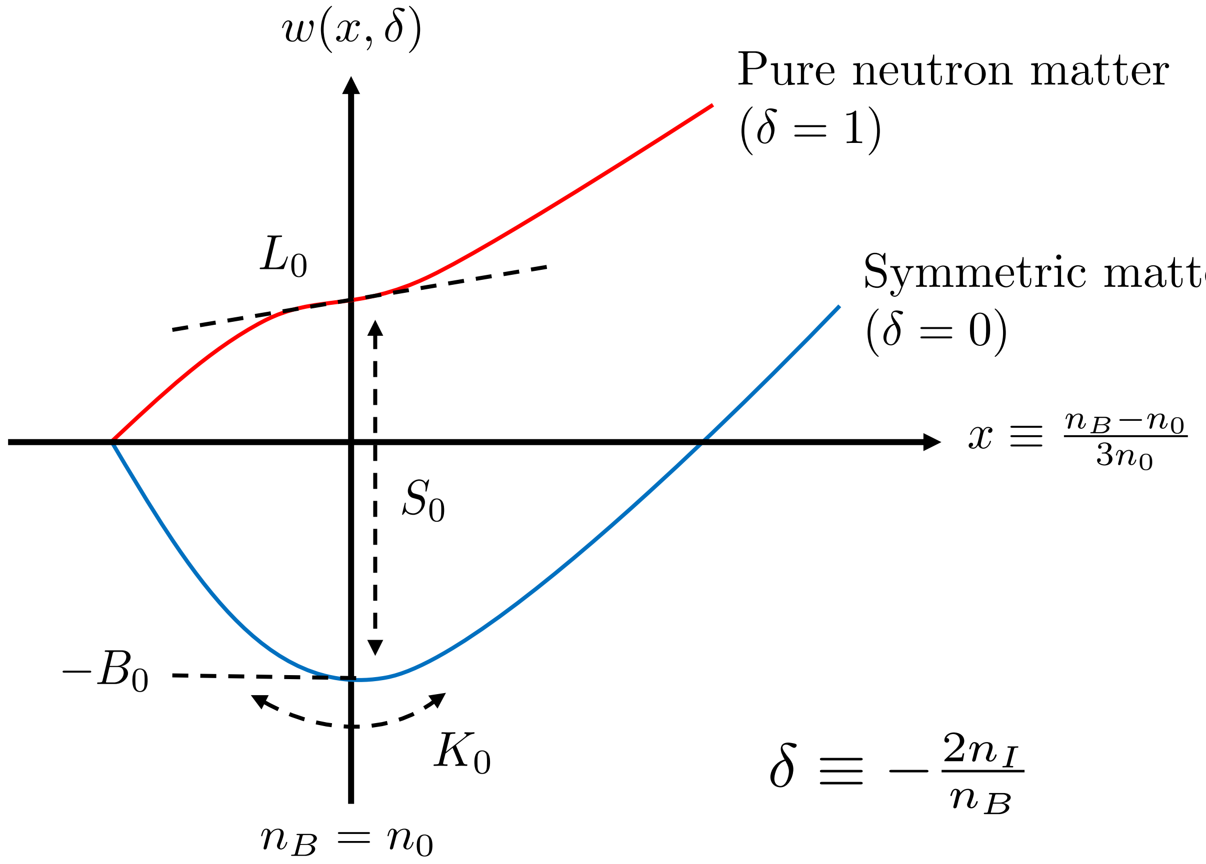
The mean field Lagrangian is then given by

$$\mathcal{L}^{L\sigma M} = \frac{\bar{\mu}_\sigma^2}{2}\sigma^2 + \frac{\bar{\mu}_a^2}{2}a^2 - \frac{\lambda_4}{4}(\sigma^4 + a^4) - \frac{\gamma_4}{2}\sigma^2 a^2$$
$$+ \frac{\lambda_6}{6}(\sigma^6 + 15\sigma^2 a^4 + 15\sigma^4 a^2 + a^6) - \lambda_6^{'}(\sigma^2 a^4 + \sigma^4 a^2)$$
$$+ m_\pi^2 f_\pi \sigma$$

$$\bar{\mu}_\sigma^2 \equiv \bar{\mu}^2 + \frac{1}{2}K \ ,$$
$$\bar{\mu}_a^2 \equiv \bar{\mu}^2 - \frac{1}{2}K = \bar{\mu}_\sigma^2 - K \ ,$$
$$\lambda_4 \equiv \lambda_{41} - \lambda_{42} \ ,$$

$$\gamma_4 \equiv 3\lambda_{41} - \lambda_{42} \ ,$$
$$\lambda_6 \equiv \lambda_{61} + \lambda_{62} + \lambda_{63} \ ,$$
$$\lambda_6^{'} \equiv \frac{4}{3}\lambda_{62} + 2\lambda_{63}$$

Properties of nuclear matter



$$\delta \equiv -\frac{2n_I}{n_B}$$

$n_0 = 0.16\ \mathrm{fm}^{-3}$ (Saturation density)
$K_0 = 215$ MeV (Incompressibility)
$B_0 = 16$ MeV (Binding energy)
$S_0 = 31$ MeV (Symmetry energy)
$L_0 = 57.7$ MeV (Slope parameter)

Determination of model parameters

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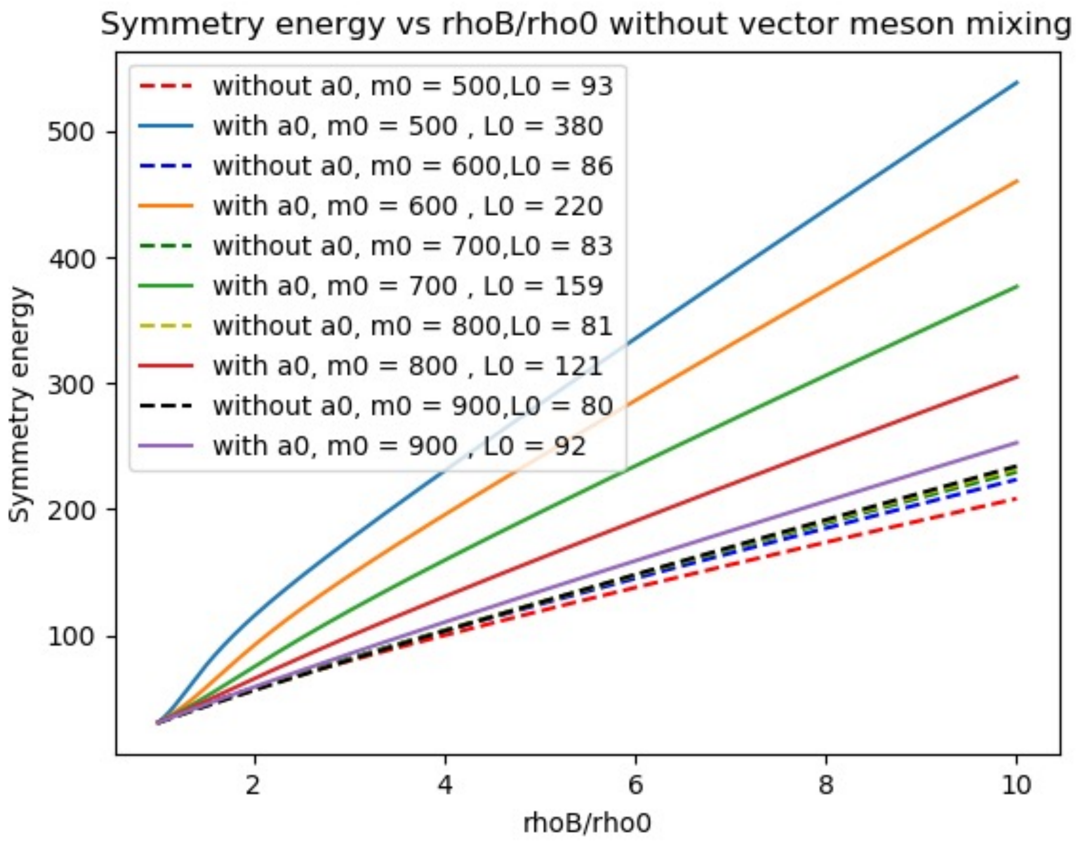
- **The physical input we used are as follows:**

$$
\begin{aligned}
m_\pi &= 140MeV \\
m_a &= 980MeV \\
m_\eta &= 550MeV \\
m_\omega &= 783MeV \\
m_\rho &= 776MeV \\
m_{N-} &= 1535MeV \\
m_{N+} &= 939MeV \\
m_e &= 0.511MeV \\
m_\mu &= 105MeV
\end{aligned}
$$

$$
\begin{aligned}
m_0 &= 500 - 900MeV \\
L_0 &= 50MeV \\
f_\pi &= 92.4MeV \\
K_0 &= 240MeV \\
S_0 &= 31MeV
\end{aligned}
$$

a0-PDM model without vector meson mixing



- **The coupling of a0 meson (g1,g2) is smaller for larger m0**

m0 (MeV)	500	600	700	800	900
g_1	9.02	8.48	7.81	6.99	5.96
g_2	15.47	14.93	14.26	13.44	12.41

$$m_{\pm j} = \frac{1}{2}\left[\sqrt{(g_1+g_2)^2(\sigma - ja)^2 + 4m_0^2} + \pm(g_1 - g_2)(\sigma - ja)\right]$$

$$(p, n) : j = (+, -)$$

ρ meson coupling constant

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With a0

	M0 (MeV)	500	600	700	800	900
L0 = 40 (MeV)	gρ	19.430 1	15.522 4	13.889 6	12.641 6	11.399 1
L0 = 50 (MeV)	gρ	18.751 0	15.034 9	13.352 3	12.000 9	10.686 5
L0 = 60 (MeV)	gρ	18.138 5	14.590 7	12.872 8	11.448 6	10.092 7
L0 = 70 (MeV)	gρ	17.582 3	14.183 6	12.441 6	10.966 2	9.5880
L0 = 80 (MeV)	gρ	17.074 3	13.808 8	12.051 0	10.540 1	9.1522

Without a0

	M0 (MeV)	500	600	700	800	900
L0 = 40 (MeV)	gρ	12.475 1	10.993 2	10.724 1	10.642 4	10.610 2
L0 = 50 (MeV)	gρ	10.717 5	10.005 0	9.9065	9.893	9.9148
L0 = 60 (MeV)	gρ	9.5406	9.2430	9.2512	9.2912	9.3404
L0 = 70 (MeV)	gρ	8.6820	8.6321	8.7109	8.7836	8.8555
L0 = 80 (MeV)	gρ	8.0201	8.1283	8.2554	8.3511	8.4391

ρ meson coupling constant

With a0

	M0 (MeV)	500	600	700	800	900
L0 = 60 (MeV)	gρ	18.138 5	14.590 7	12.872 8	11.448 6	10.092 7

Without a0

	M0 (MeV)	500	600	700	800	900
L0 = 60 (MeV)	gρ	9.5406	9.2430	9.2512	9.2912	9.3404