Phase Transition Scenarios in the Core of Neutron Star

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HHIQCD at YITP, Oct 31, 2024

N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries
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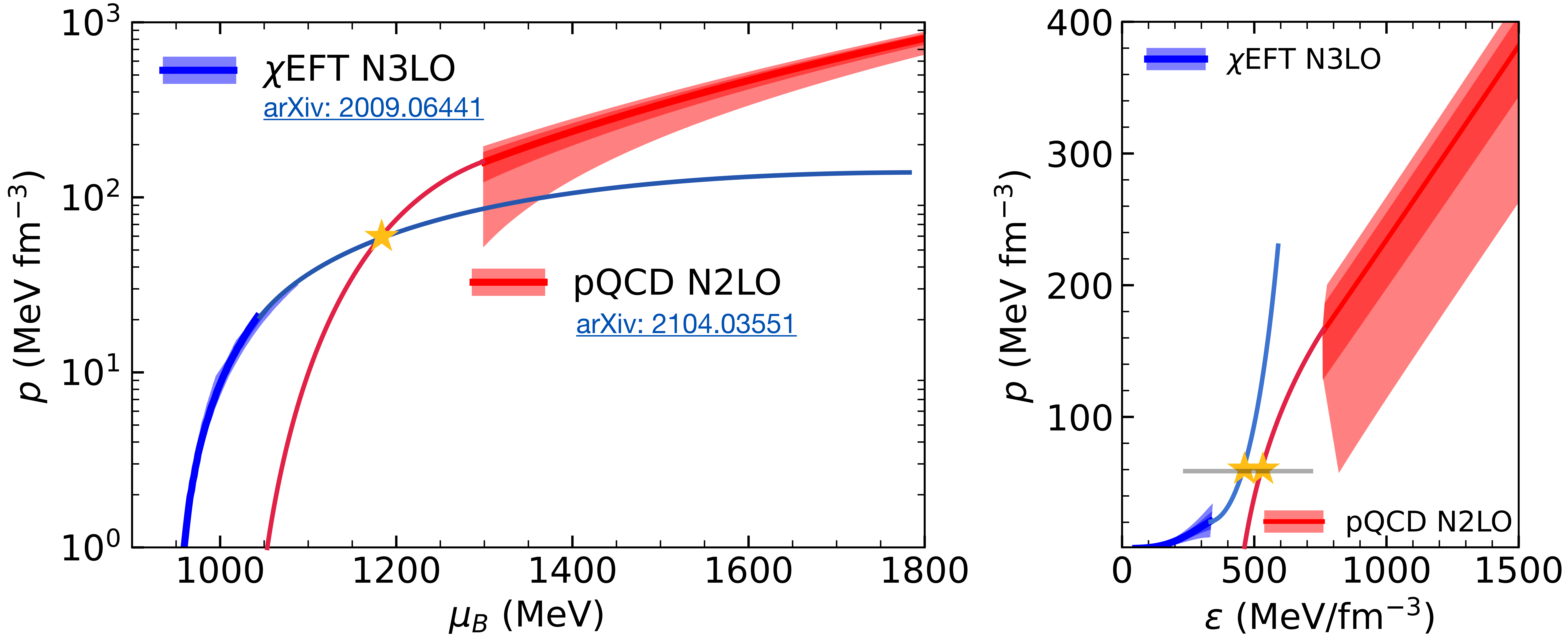
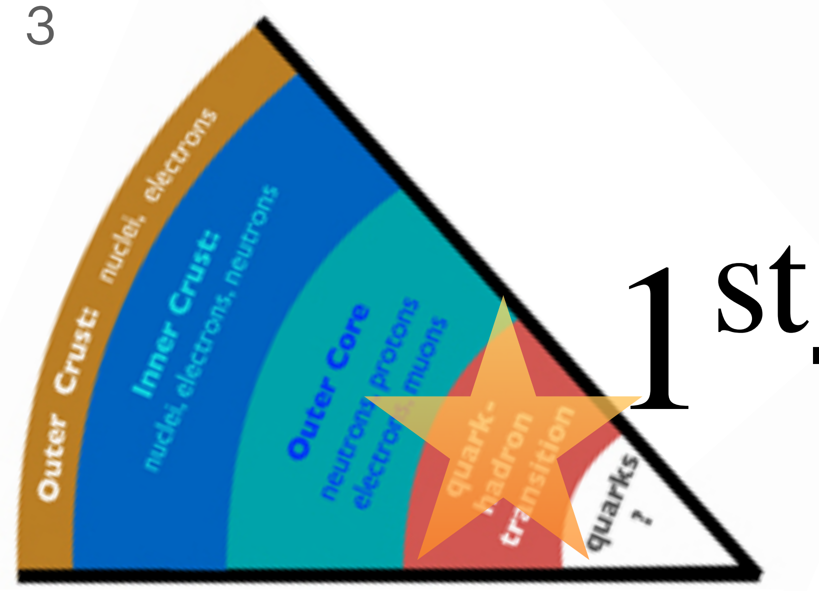
Maxwell Construction

Hybrid Neutron Stars





1^{st}-order Transition

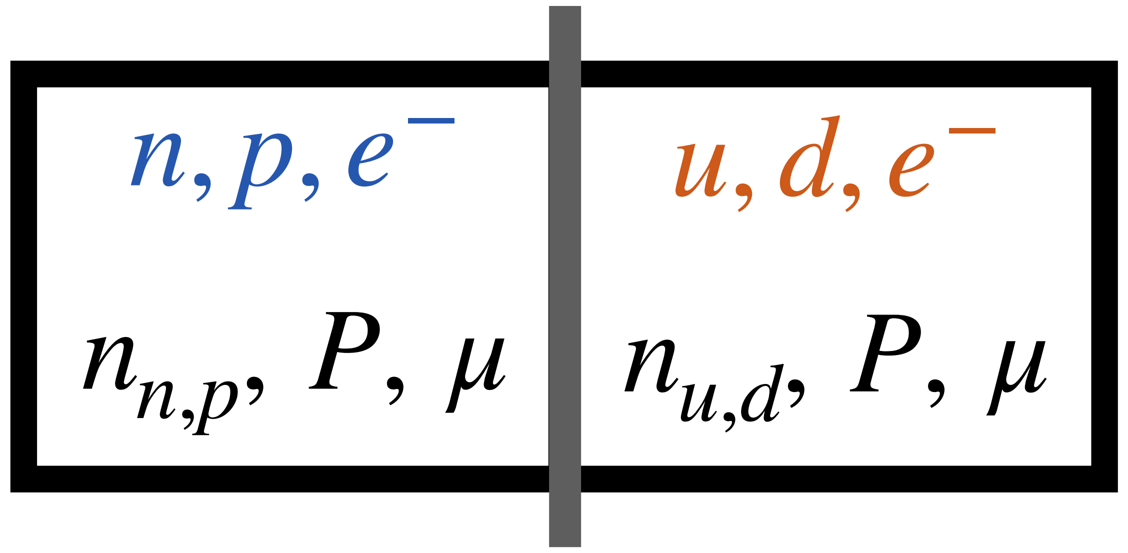
Maxwell construction

- Mechanical equilibrium

  $P_{npe} = P_{ude} = P$

- Strong equilibrium

  $\mu_n = \mu_u + 2\mu_d = \mu$



- (Modified) Urca process

  $\bar{\nu} + u + e^- \leftarrow d \quad (+N)$

  $(N+) \; u + e^- \rightarrow d + \nu$

  leads to $\mu_u + \mu_e = \mu_d = \mu_s$

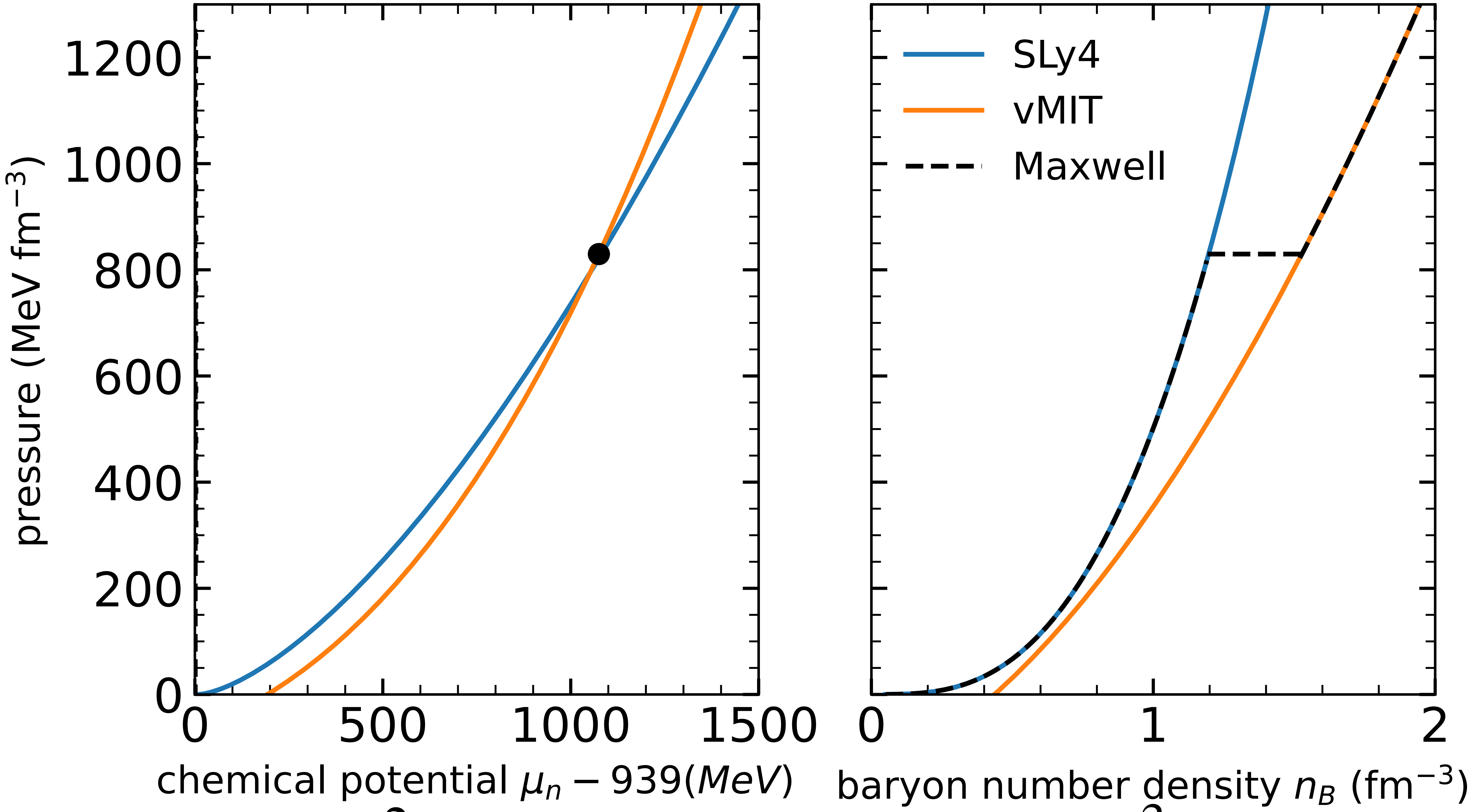
- Baryon number conservation:

  $n_u + n_d + n_s = n_Q = n_B/3$

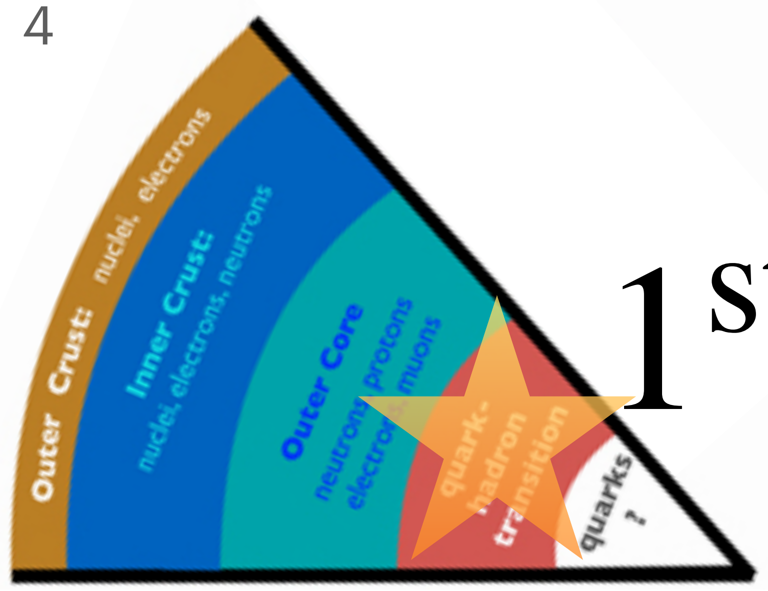
- Local charge neutrality:

  $n_{e,Q} = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s$

  $n_{e,N} = n_p$

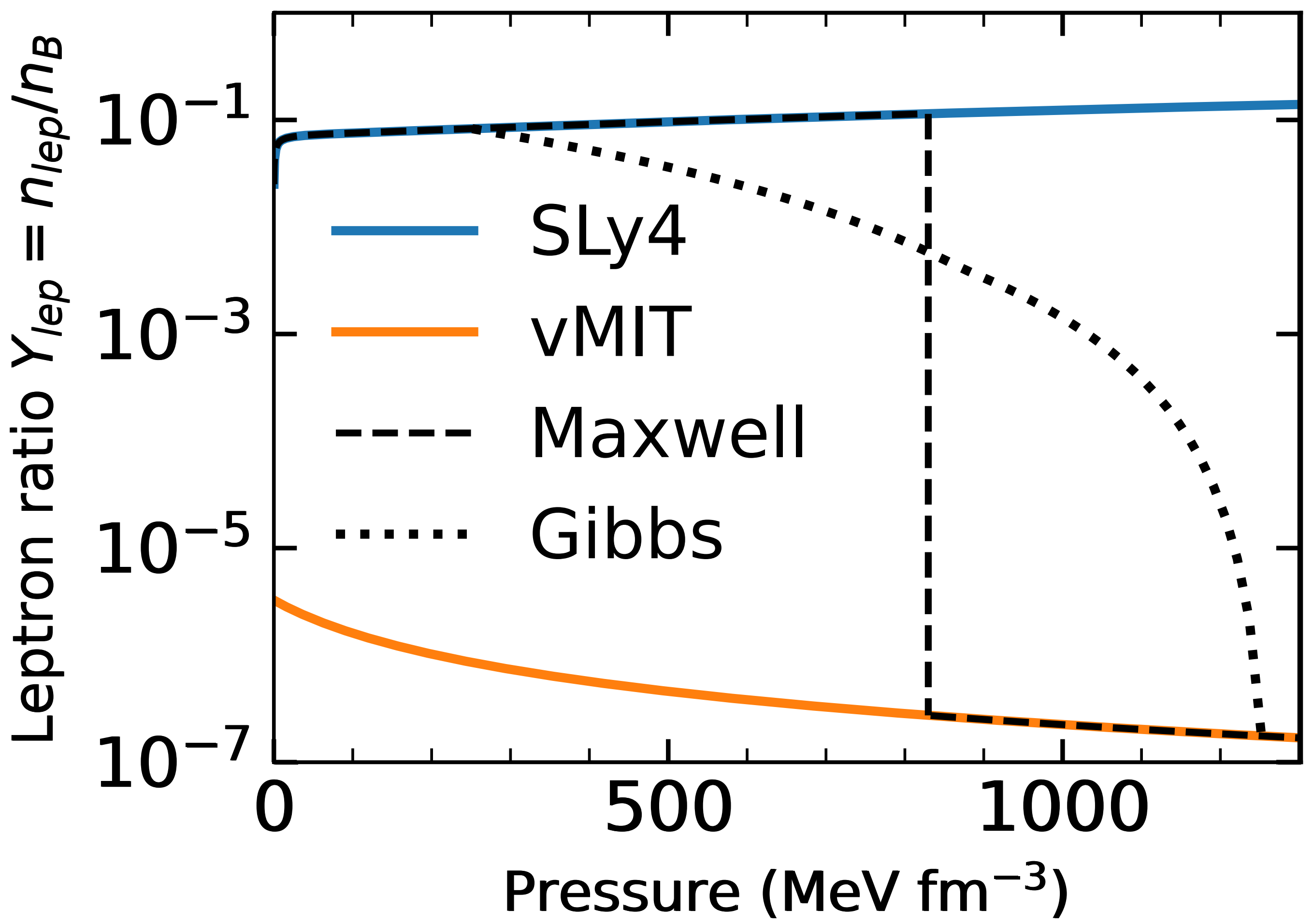


$\mu_u + 2\mu_d = \mu_n$ $\qquad$ $3n_Q = n_B$



1^{st}-order Transition

Maxwell or Gibbs construction



Gibbs construction

n, p e^-	u, d
$n_{n,p}$, P, μ	$n_{u,d}$, P, μ
Hadronic fraction	Quark fraction
f	$1-f$

- Local charge neutrality (Maxwell):

$$n_{e,Q} = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s$$

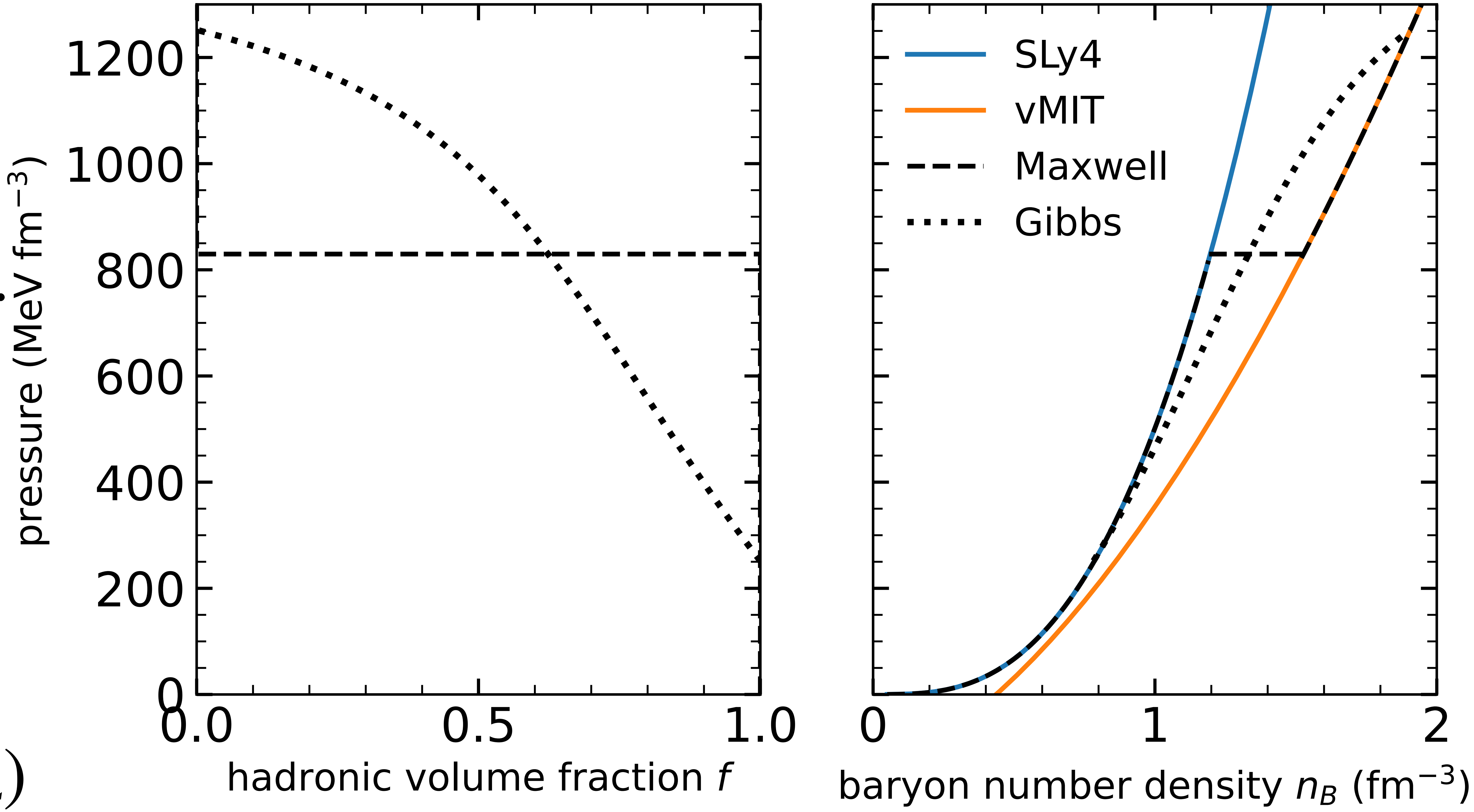
$$n_{e,N} = n_p$$

- *Leptons **aren't** balanced at the interface.*
- *Energy **isn't** minimized!*

- Global charge neutrality (Gibbs):

$$n_e = fn_{e,N} + (1-f)n_{e,Q}$$

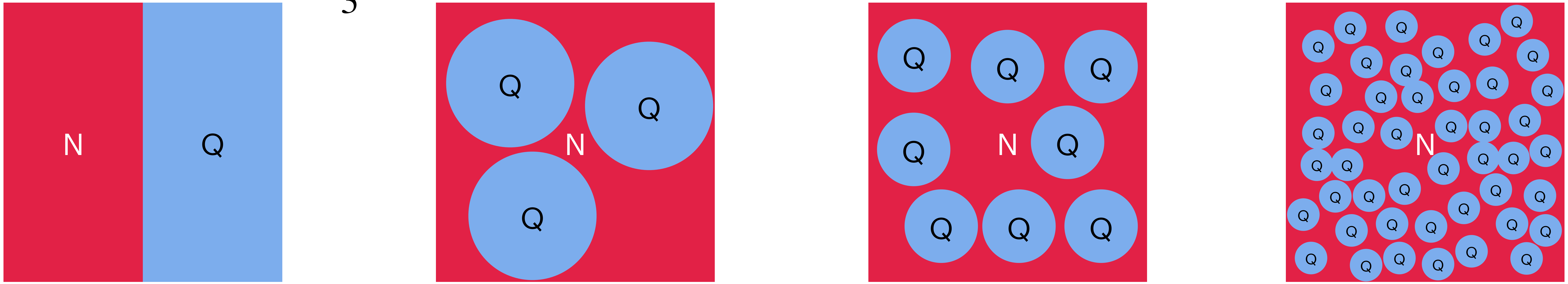
$$n_B = f(n_p + n_n) + \frac{1-f}{3}(n_u + n_d + n_s)$$



Problem of Gibbs Construction

- e.g. volume fraction $f = 0.5$:

$n_{e,N} = n_p$ $\quad n_{e,Q} = \dfrac{2n_u - n_d - n_s}{3}$ $\qquad n_e = f n_{e,N} + (1-f) n_{e,Q}$

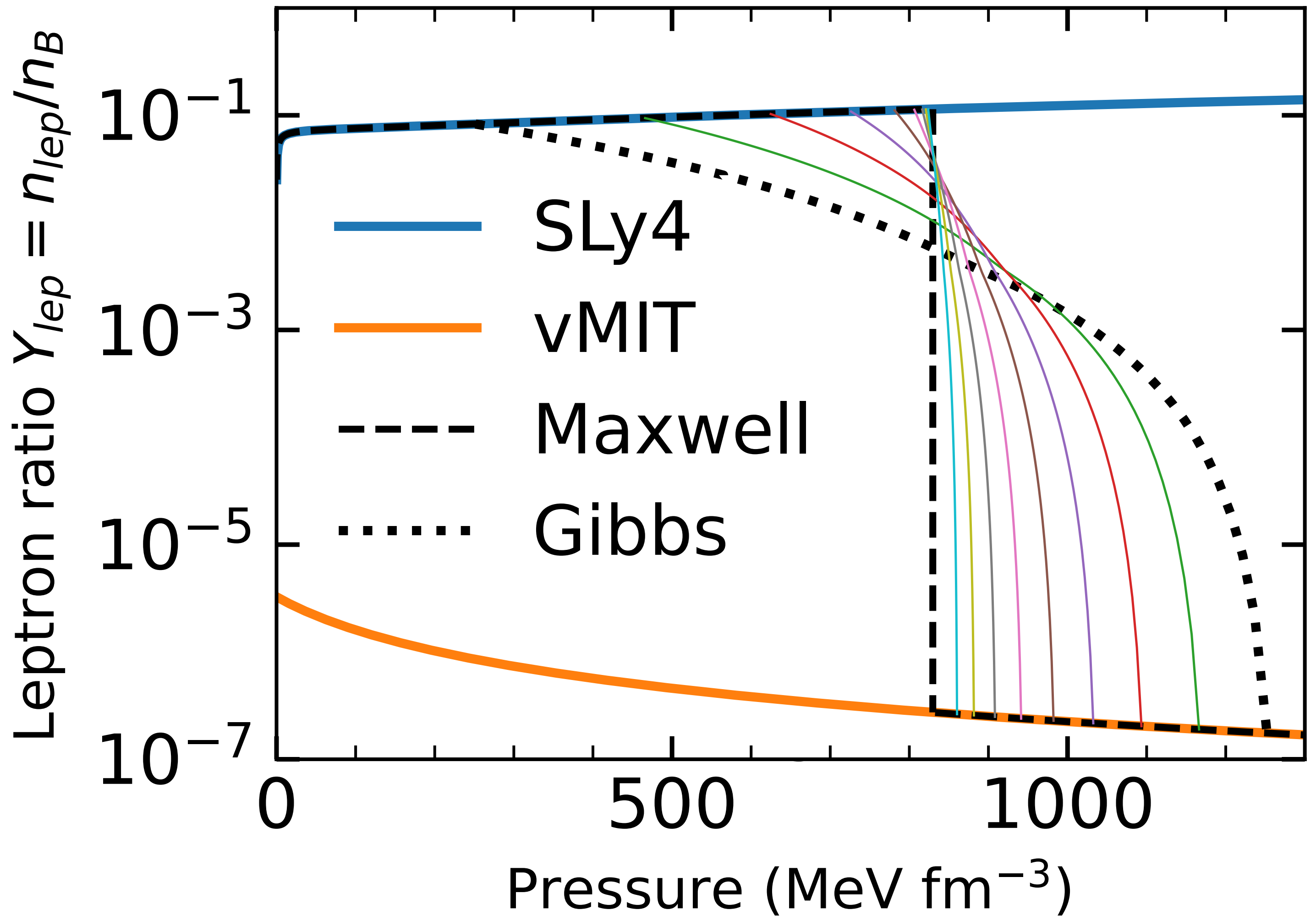


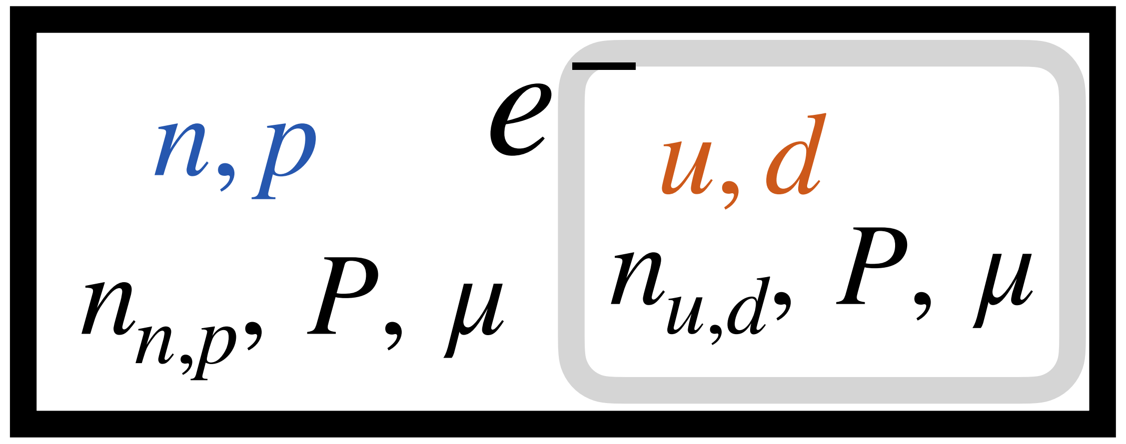
Surface energy increases ⟶ ⟵ Coulomb energy increases

- Gibbs construction assumes infinite mixing leading to infinite boundary.
- Gibbs construction is realistic only when surface tension is negligibly small.
- (local or global) charge neutrality condition determines the amount of boundary.

Between Maxwell & Gibbs

Partially local & partially global





Global lepton fraction
$1 - g$

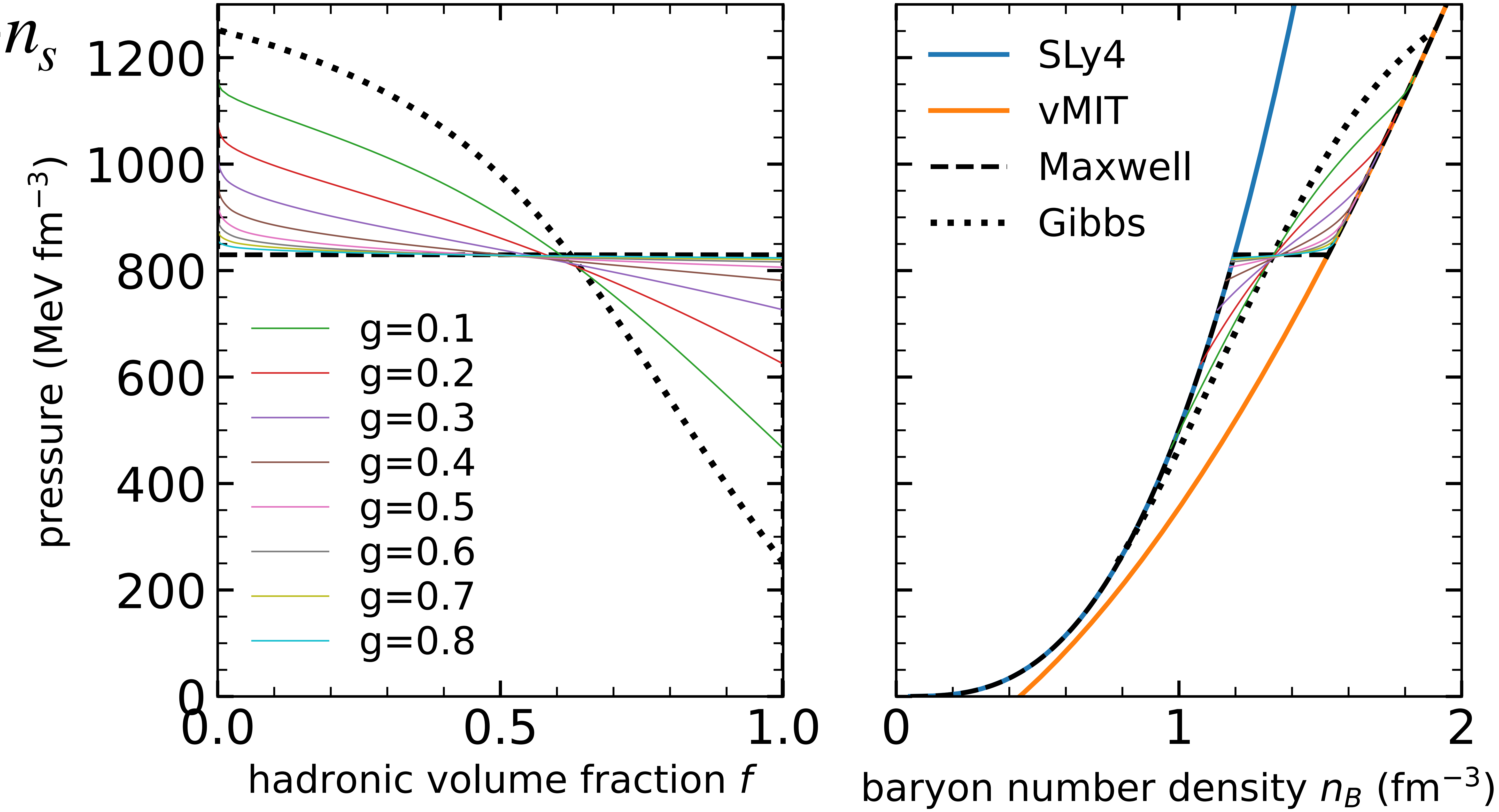
- Locally neutral lepton densities:

$$n_{e,N} = n_p,\ n_{e,Q} = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s$$

- Global lepton density, $n_{e,G}$
- Total lepton density:

$$n_e = g(fn_{e,N} + (1-f)n_{e,Q}) + (1-g)n_{e,G}$$

- $g = 0$ → Gibbs transition
  $g = 1$ → Maxwell transition
- g could be determined by Surface & Coulomb energy.



arXiv: 2302.04289

Between Maxwell & Gibbs

Extend to finite temperature:

- Introduce anti-particles as,

  $\mu_{e^-} = -\mu_{e^+}$

  $\mu_{\mu^-} = -\mu_{\mu^+}$

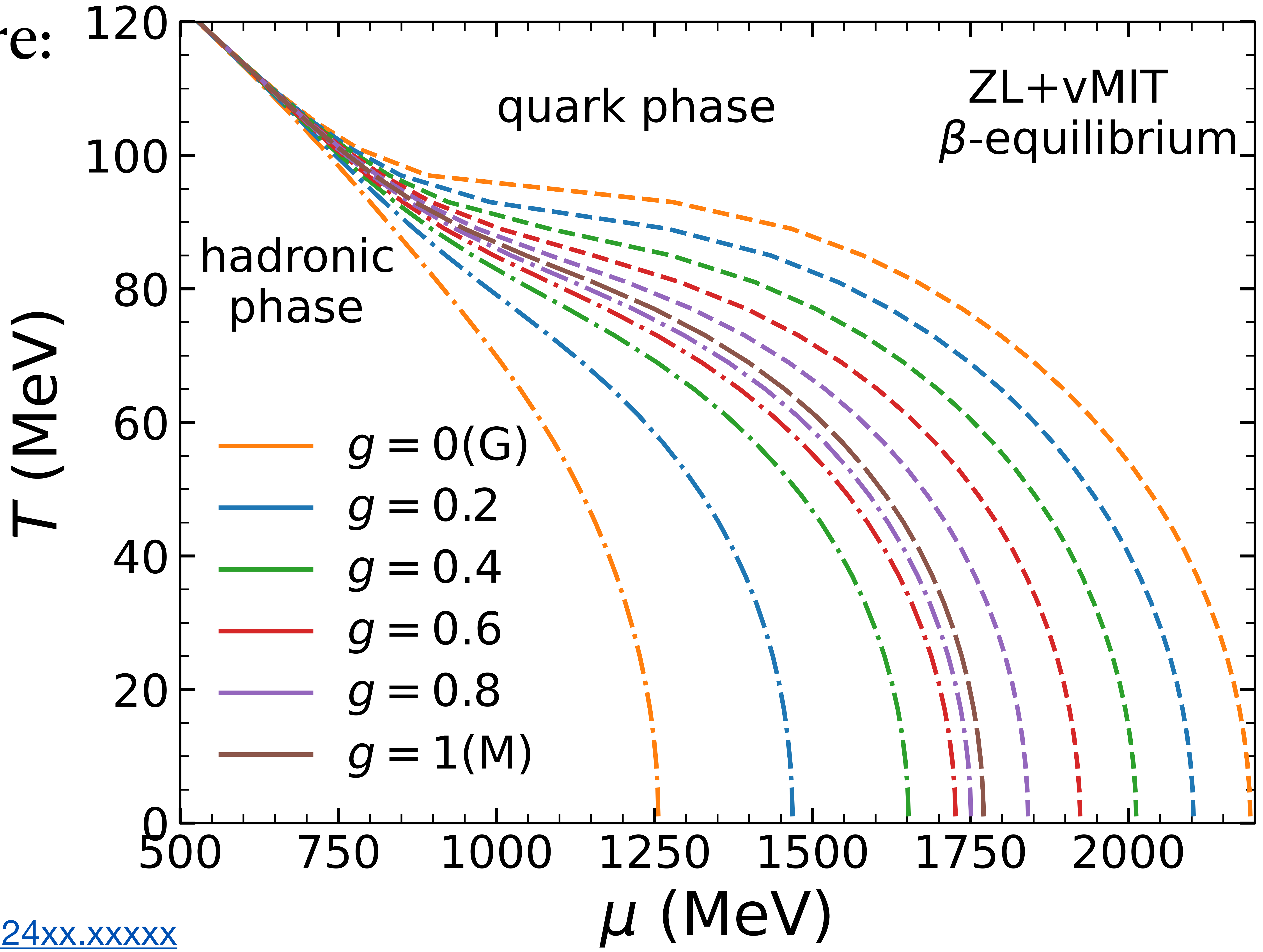
  $\mu_{u^-} = -\mu_{u^+}$

  $\mu_{d^-} = -\mu_{d^+}$

  $\mu_{s^-} = -\mu_{s^+}$

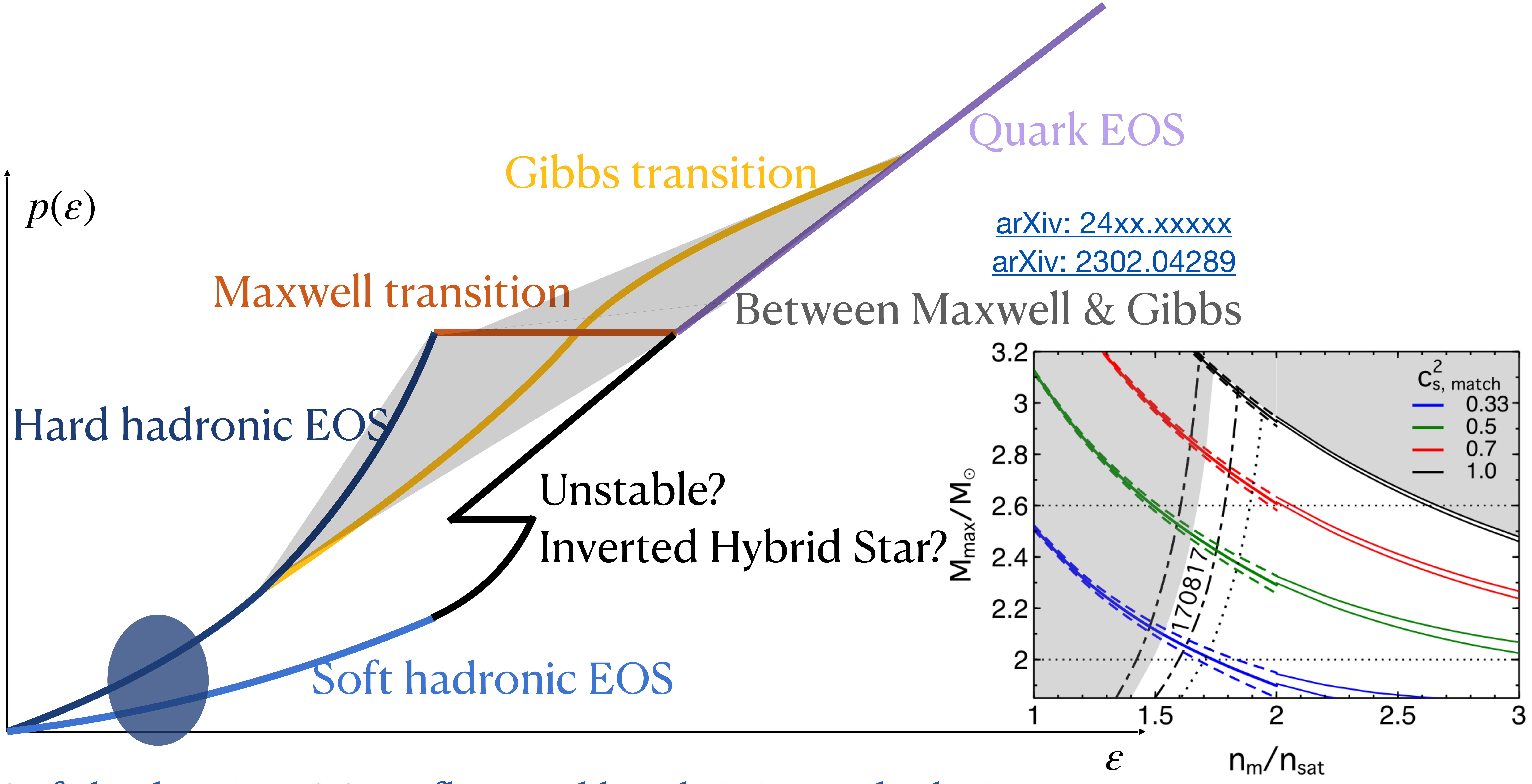
- Add photon contribution,

  $\varepsilon_{photon} \propto T^4$



arXiv: 24xx.xxxxx

Hadron-quark Transition in Neutron Star Core

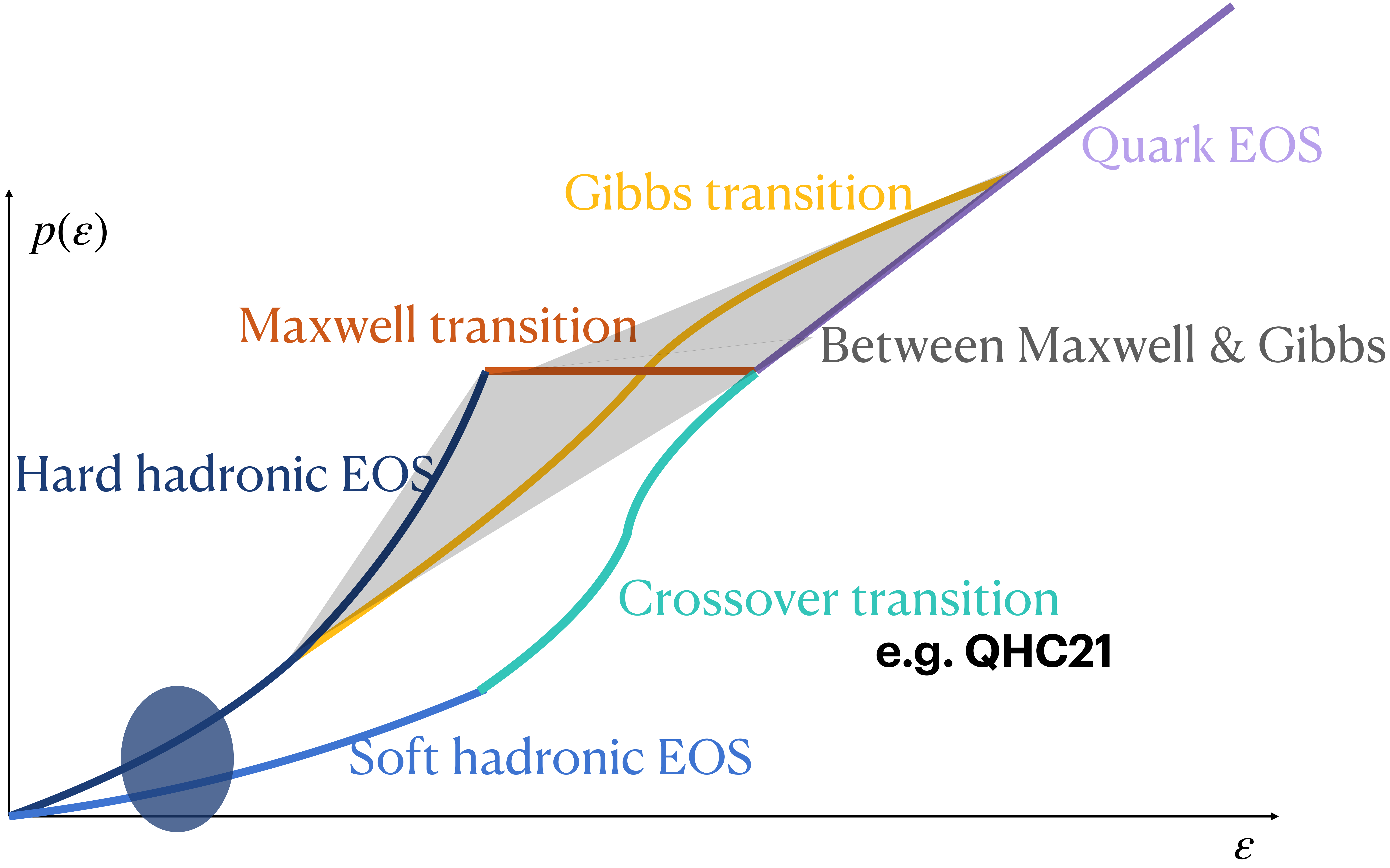


arXiv: 2009.06441

Soft hadronic EOSs is flavored by ab-initio calculation, nuclear experiments & neutron star merger observation.

arXiv: 2406.05267 arXiv: 1808.02858

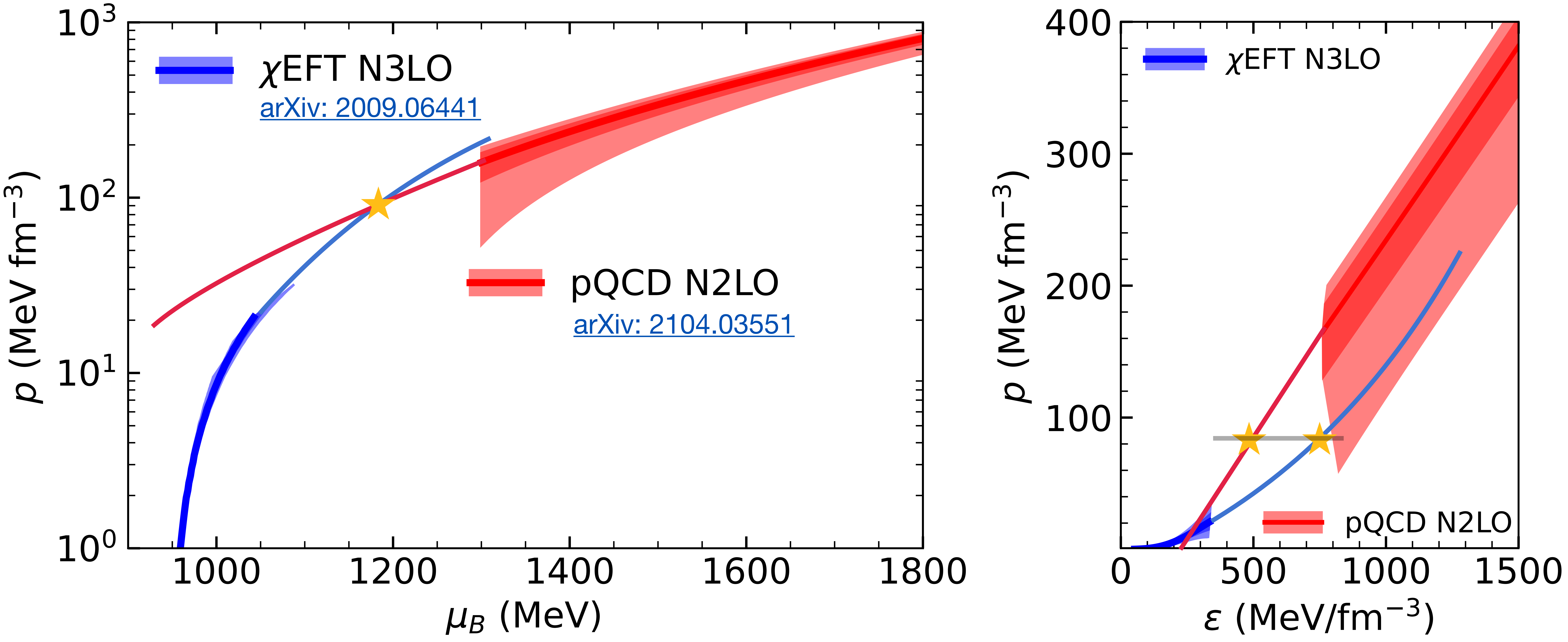
Hadron-quark Transition in Neutron Star Core



Soft hadronic EOSs is flavored by ab-initio calculation,
nuclear experiments & neutron star merger observation.

Maxwell Construction

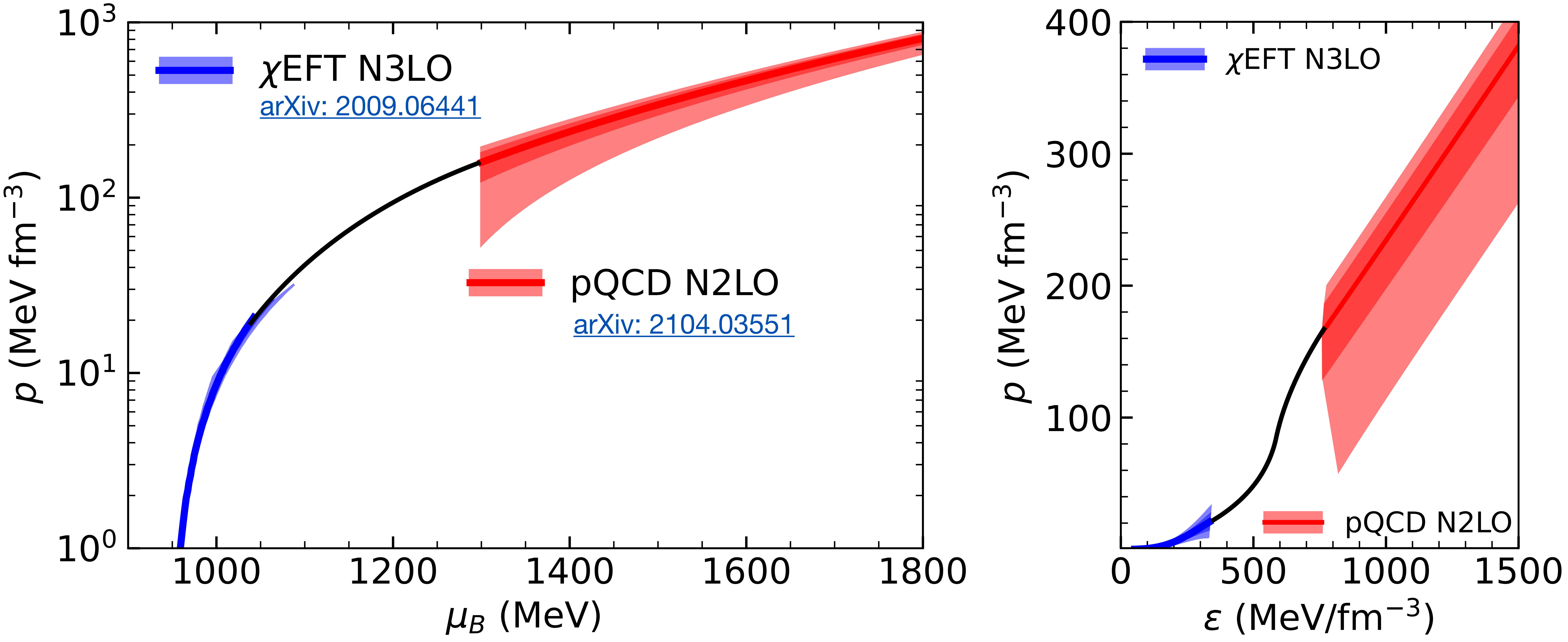
Inverted Hybrid Star **C. Zhang, J. Ren 2023**



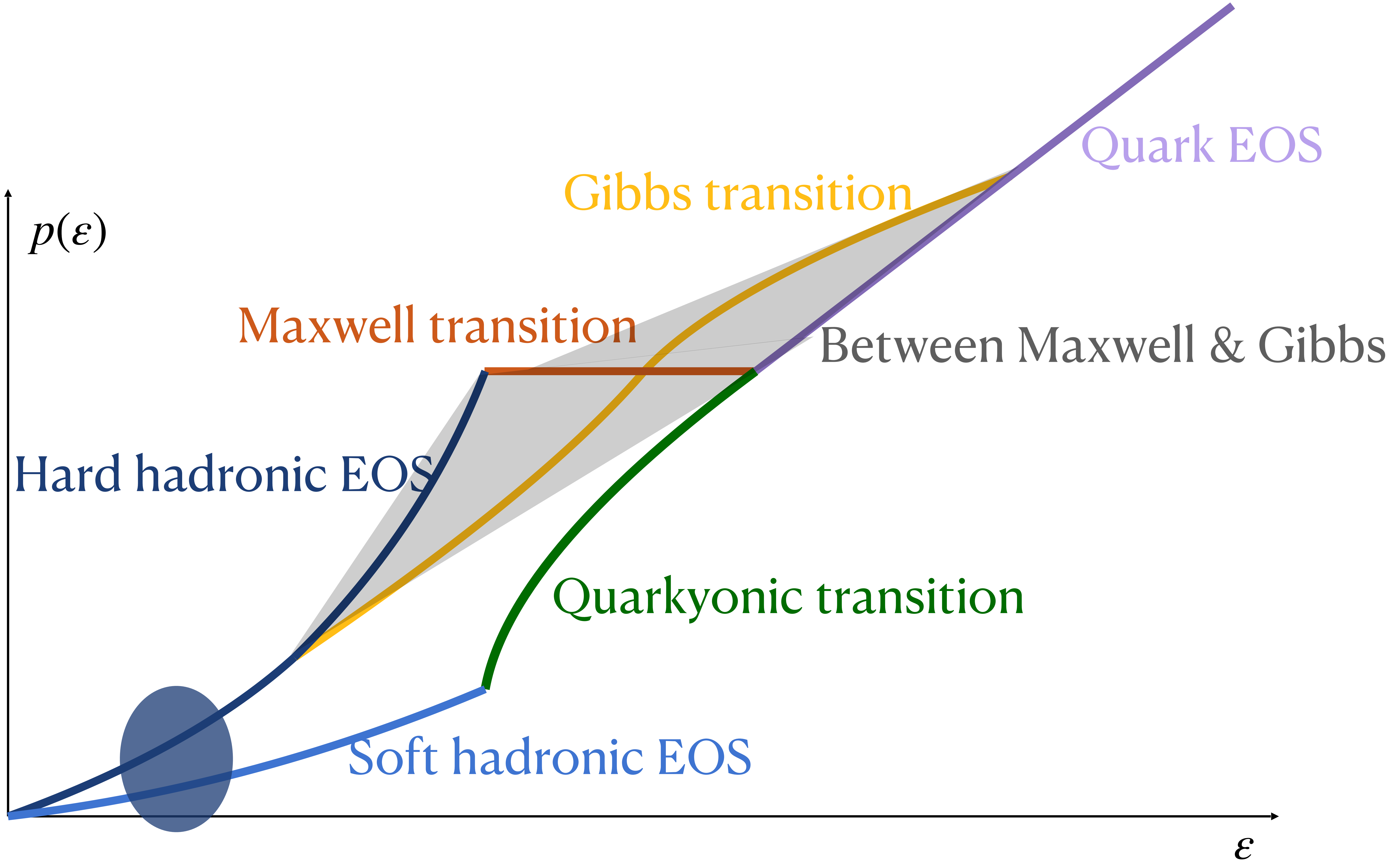
Crossover Construction

Smooth interpolation **Masuda, Hatsuda, Takatsuka 2018**

J. I. Kapusta, T. Welle 2021



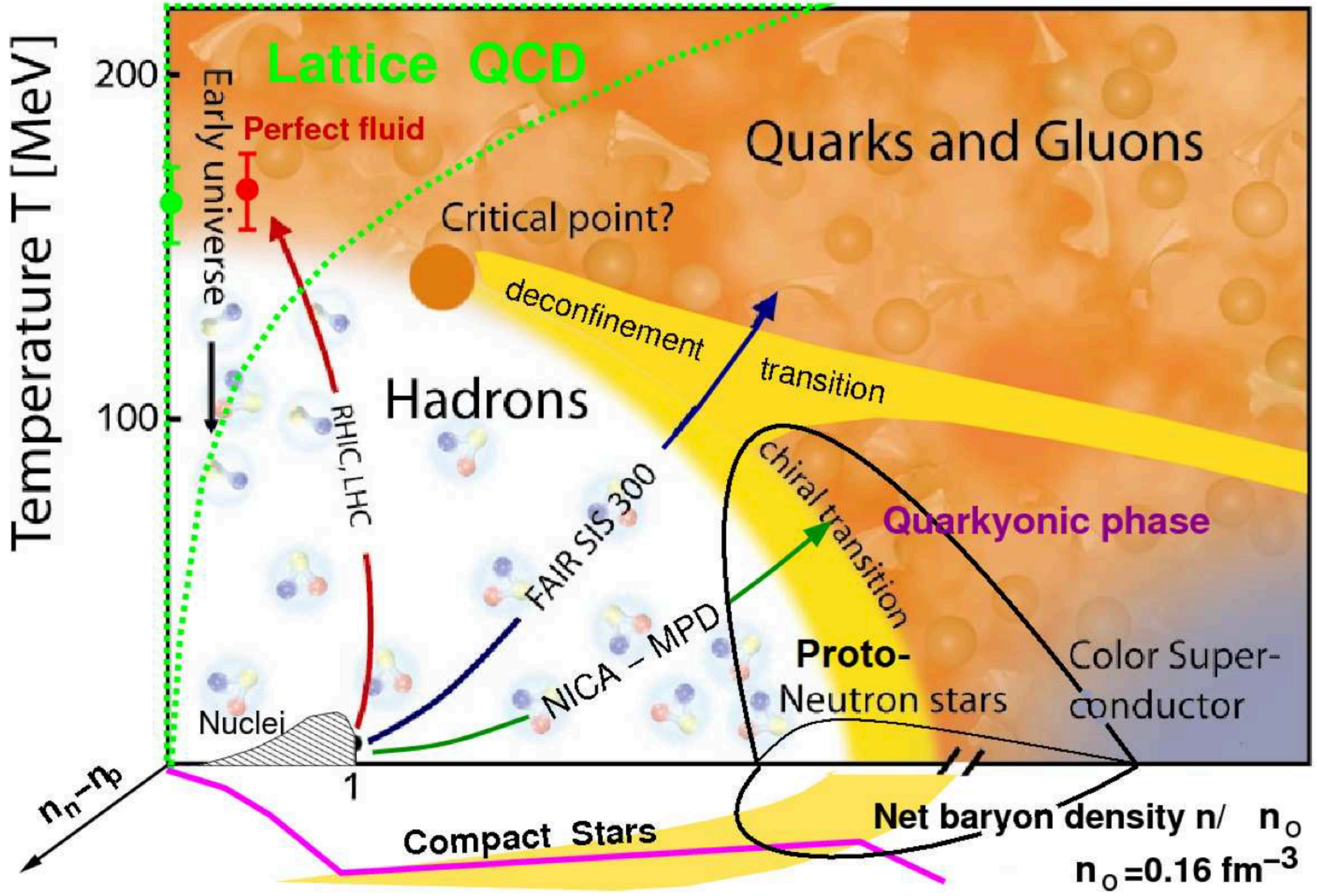
Hadron-quark Transition in Neutron Star Core



Soft hadronic EOSs is flavored by ab-initio calculation, nuclear experiments & neutron star merger observation.

Quarkyonic Matter

- The hypothetical phase between hadronic matter and deconfined quark matter, with unclear chiral symmetry.



Credit: figure from David Blaschke

Sanjay and McLerran 2018

Dynamical realization:

K. Jeong et. al. 2020
T. Kojo & D. Suenaga 2021
Y. Fujimoto et. al. 2023

Extend isospin, flavor, finite T:

Zhao & Lattimer 2020
S. Sen et. al. 2021
D. Duarte et. al. 2021
J. Margueron et. al. 2021

Include better hadronic EOS:

G. Cao et. al. 2021
A. Kumar et. al. 2022
C. Xia et. al. 2023
B. Gao & M. Harada 2024

Asymptotic Free

Gross, Wilczek and Politzer 1973

- QCD beta function:

$$\beta(\alpha_s) = q^2 \frac{\partial \alpha_s}{\partial q^2} = -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 - \cdots$$

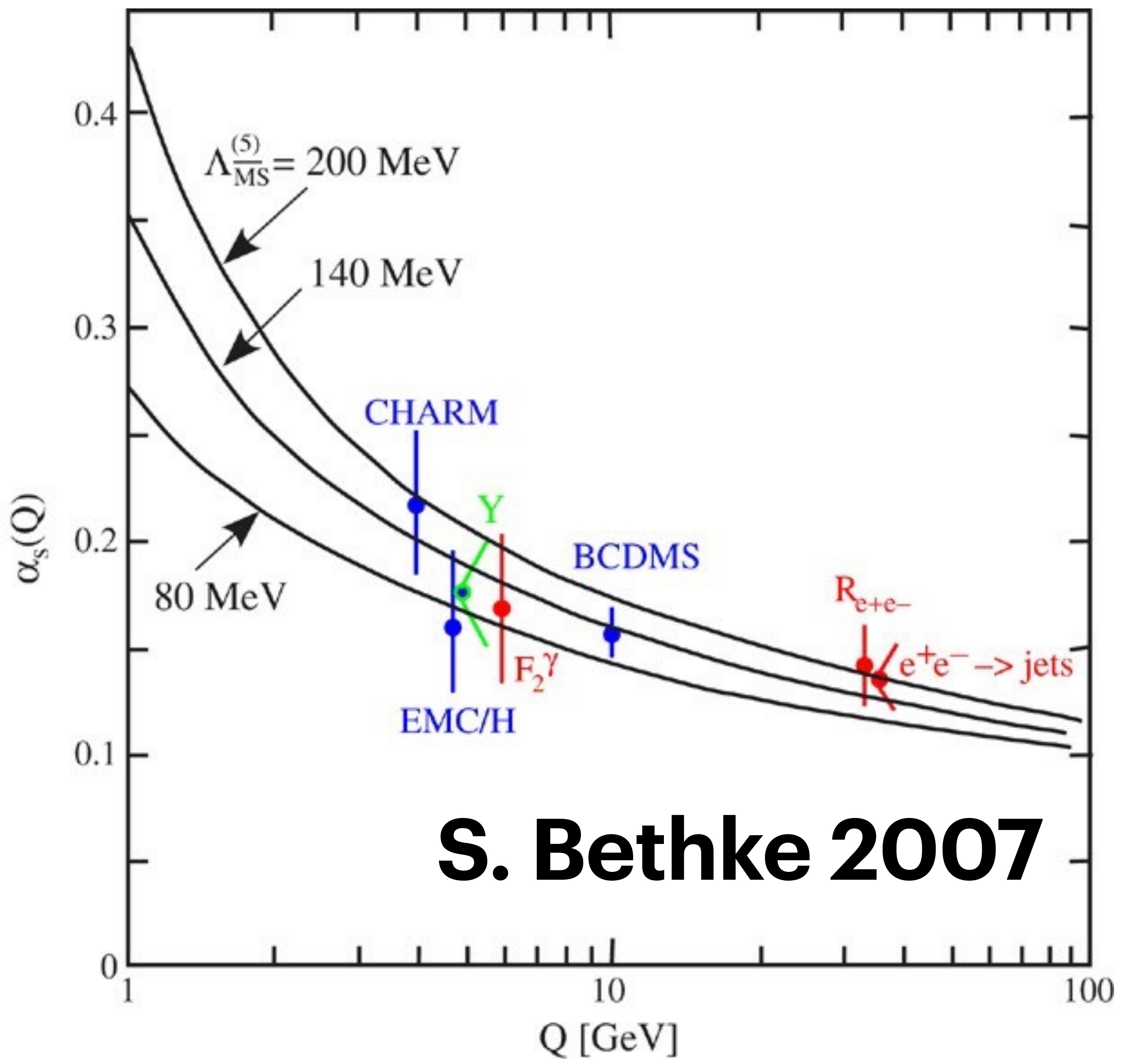
where $\alpha_s = \frac{g^2}{4\pi}$, $\beta_0 = \frac{33 - 2N_f}{12\pi} > 0$, $\beta_1 = \frac{153 - 19N_f}{24\pi^2} > 0$

- Keep only the first term on the right-hand side,

$$\alpha_s \approx \frac{1}{\beta_0 \log q^2/\Lambda_{QCD}^2}$$

therefore $\lim\limits_{q>>\Lambda_{QCD}} \alpha_s(q) \rightarrow 0$

- Perturbative QCD:
QCD Lagrangian (quark-gluon coupling)
+ Analytical method (vacuum and ring diagram)

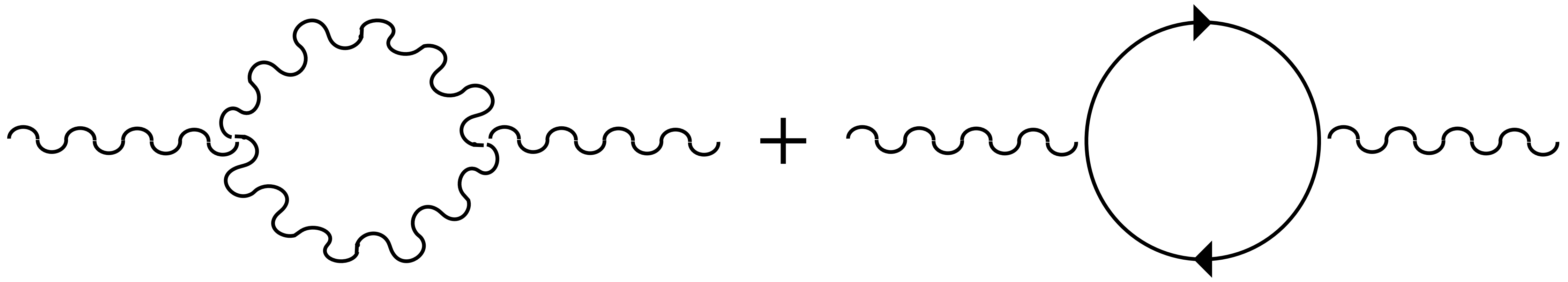


Speculation from large N_c

McLerran & Pisarski 2007

- Confinement due to screening of gluons

$$m^2_{Debye} \approx \Pi = g^2 \left[(N_c + \frac{N_f}{2})\frac{T^2}{3} + \frac{N_f \mu^2}{2\pi^2} + \cdots \right]$$



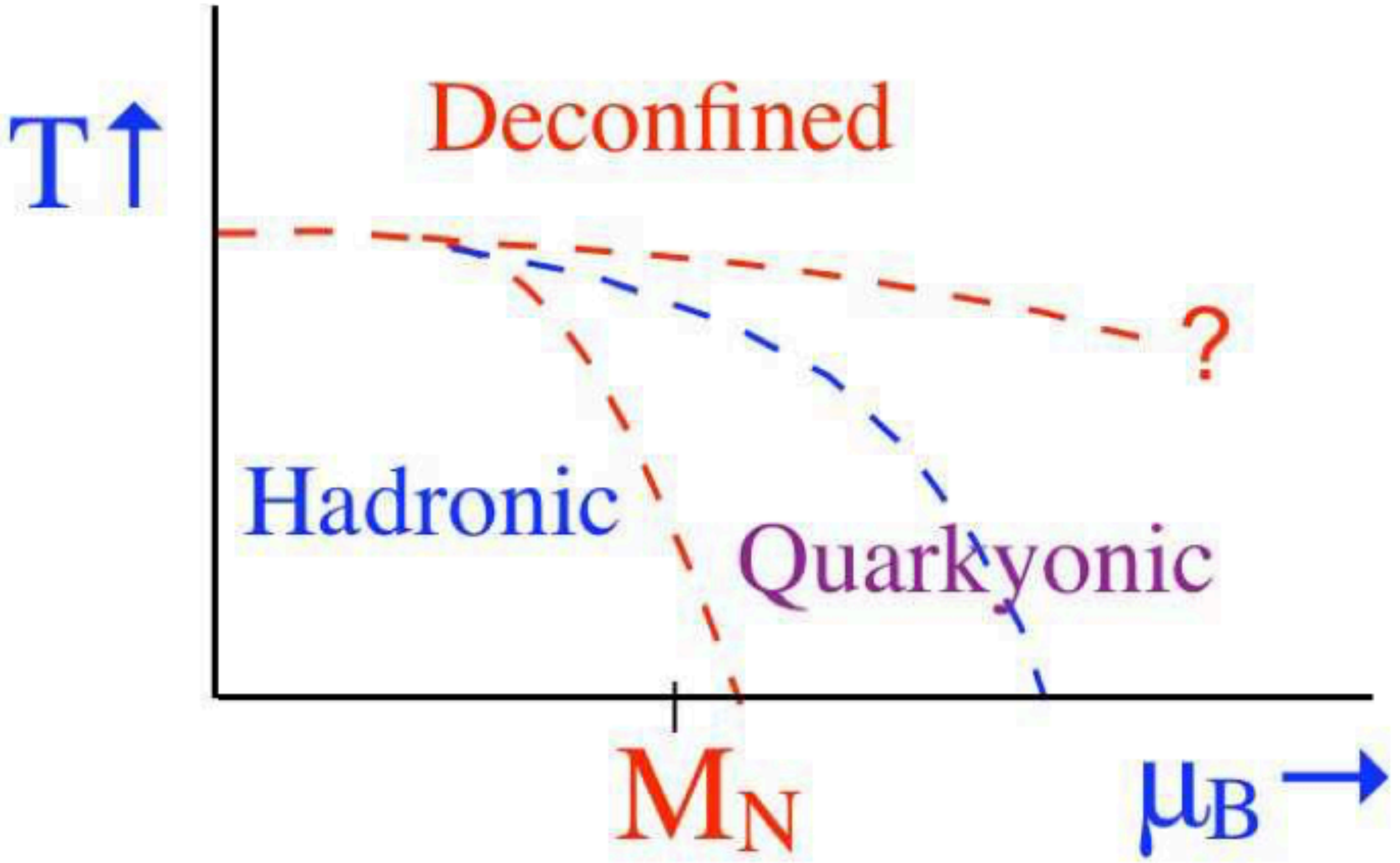
- Large $m_{Debye} \longrightarrow$ stronger screening $\longrightarrow$ weaker long-range interactions $\longrightarrow$ deconfinement
- Large N_c limit: $N_c \to \infty$ while fixing $\lambda_{'tHooft} = g^2 N_c$ and N_f:

$m^2_{Debye} \propto T^2$ for high temperature;

$m^2_{Debye} \propto \frac{\mu^2}{N_c} \to 0$ for high chemical potential.

- Asymptotic free + Confinement (at the same time) ????

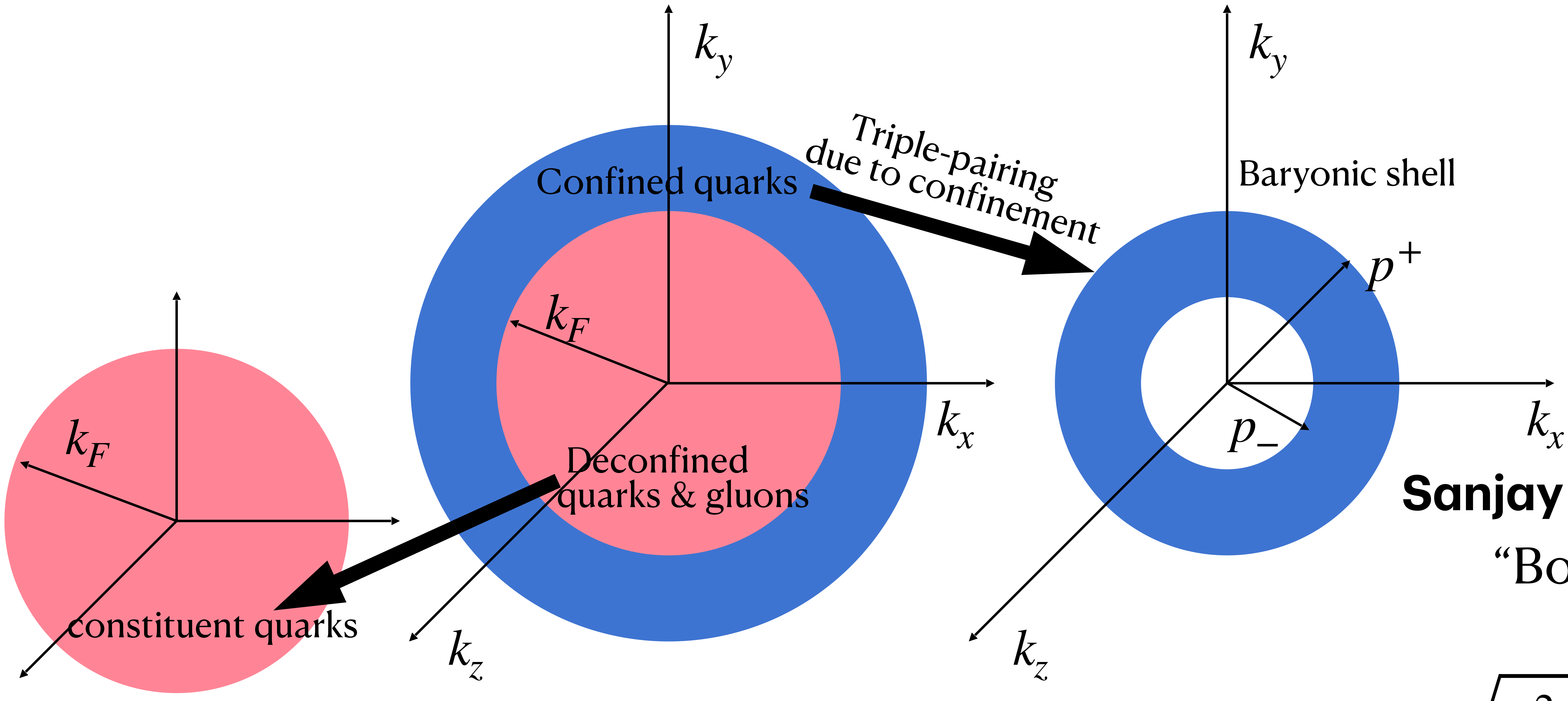
Quark + Baryon = Quarkyonic matter



Quarkyonic Matter Momentum Space

Nucleons are degenerate with quarks (quark-hadron duality)

- Perturbative quarks = quarks deep inside Fermi sphere
- Baryons = triple-pair of quarks near Fermi surface



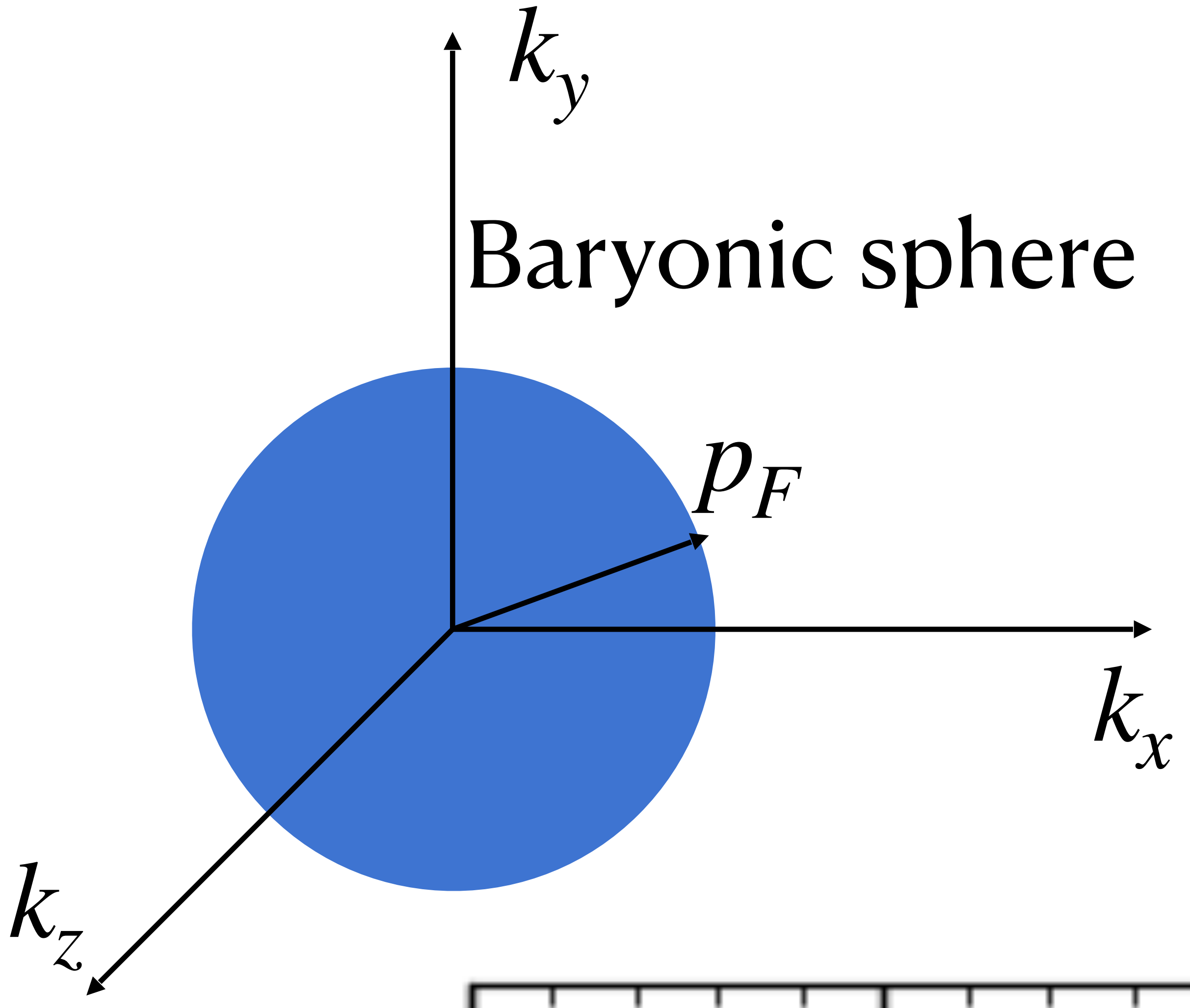
Sanjay and McLerran 2018

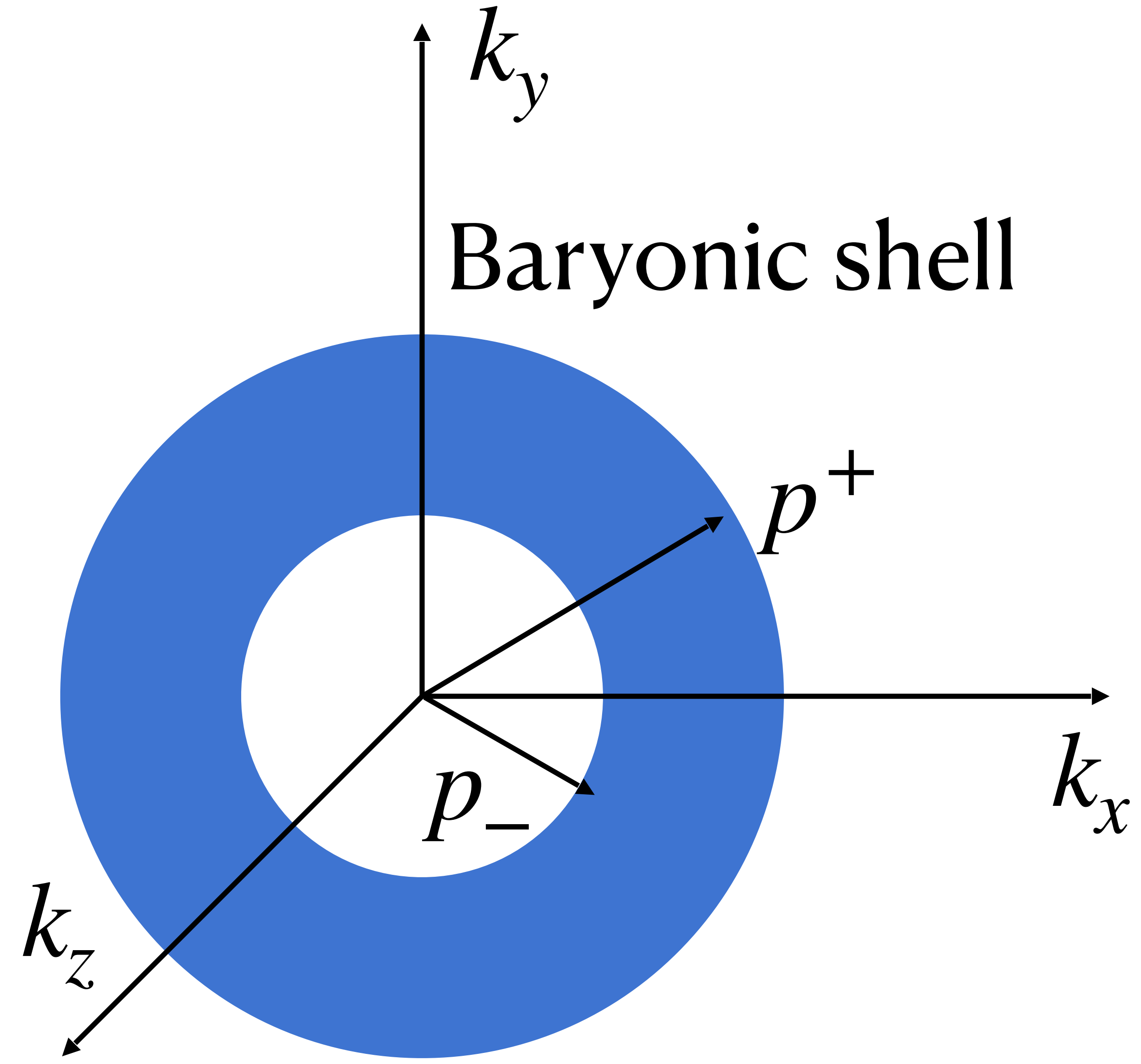
"Bold" Assumption:

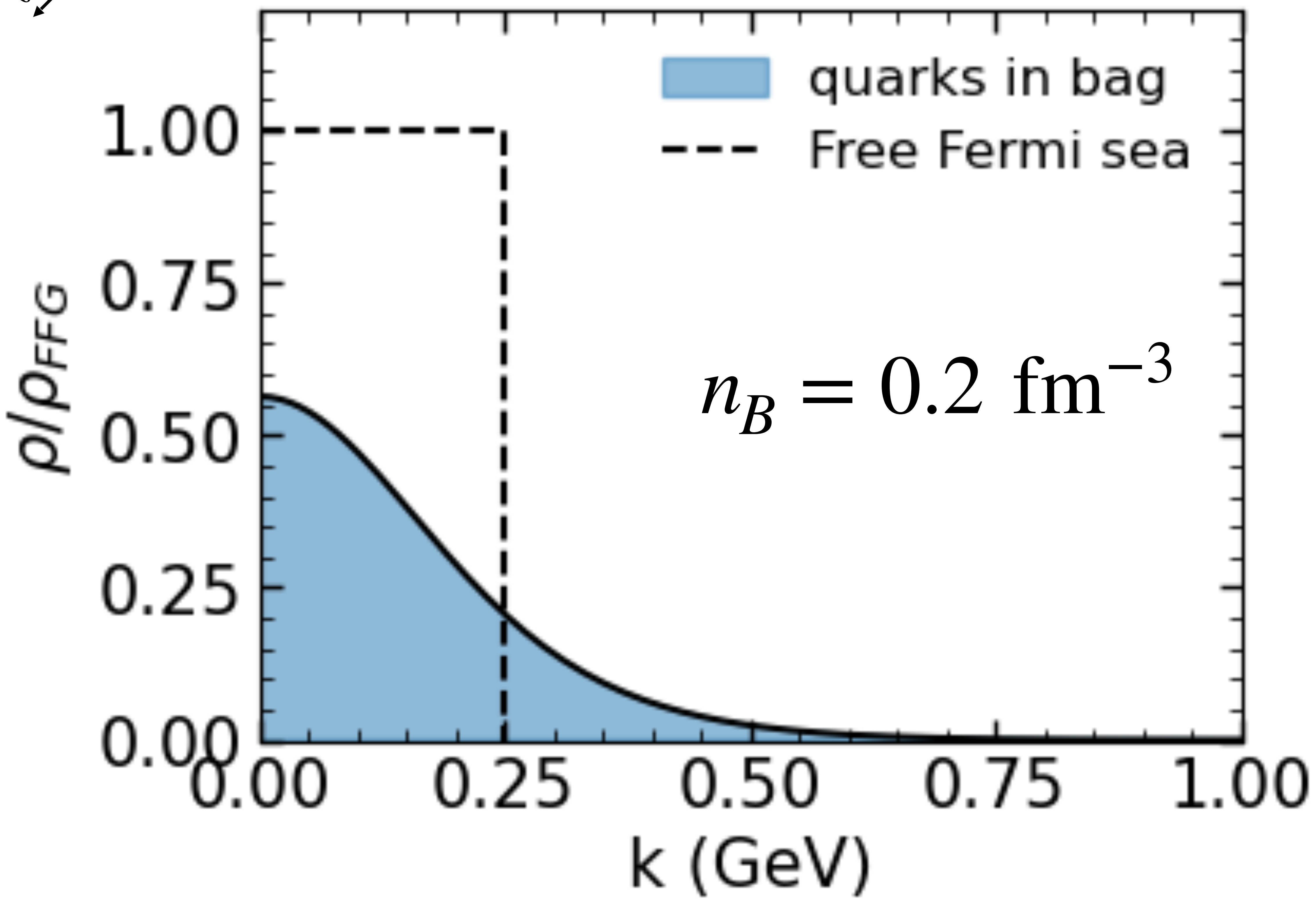
$$p_- = 3k_F$$

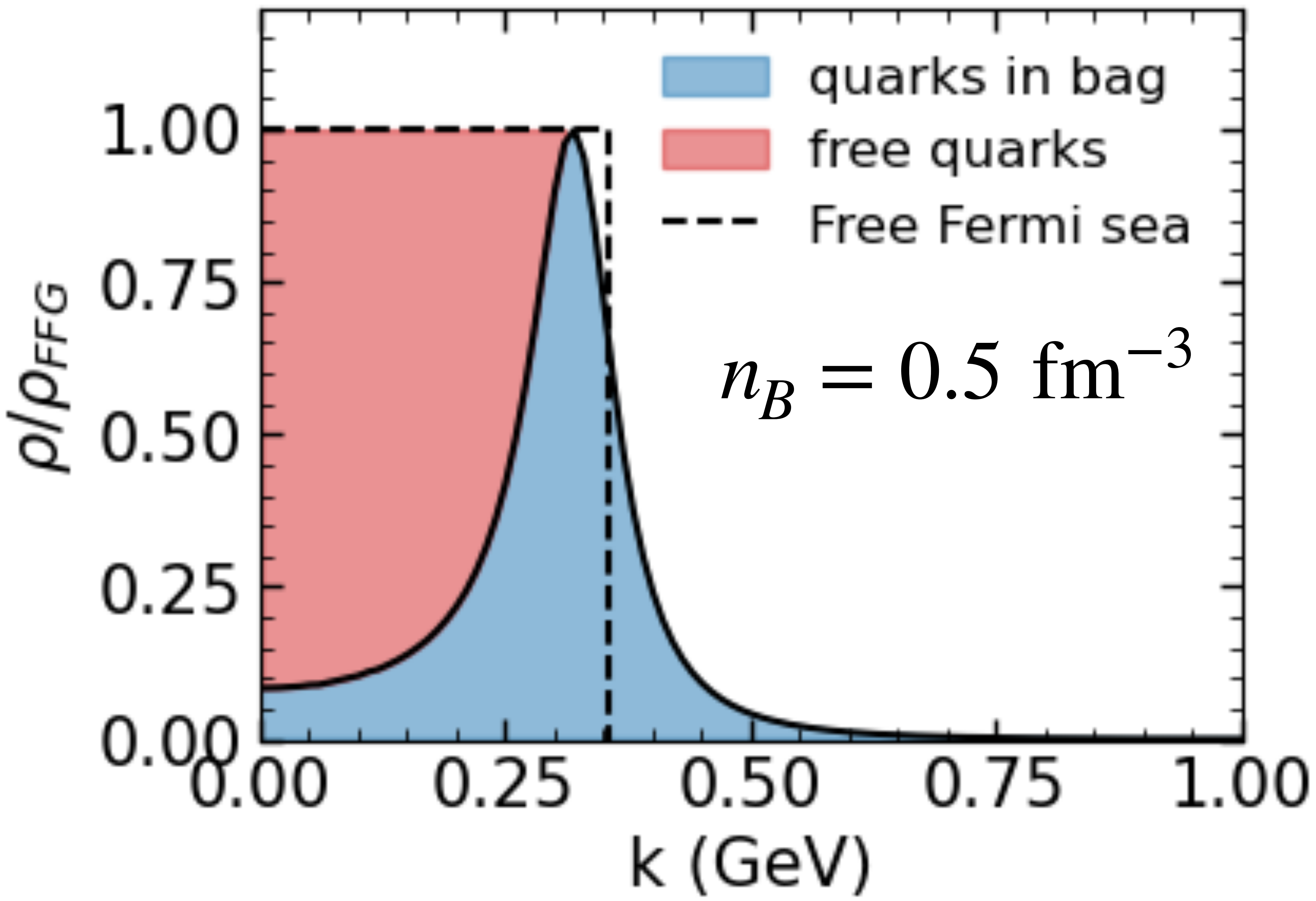
or $\sqrt{p_-^2 + m_N^2} = 3\sqrt{k_F^2 + m_Q^2}$

Hadronic to quarkyonic



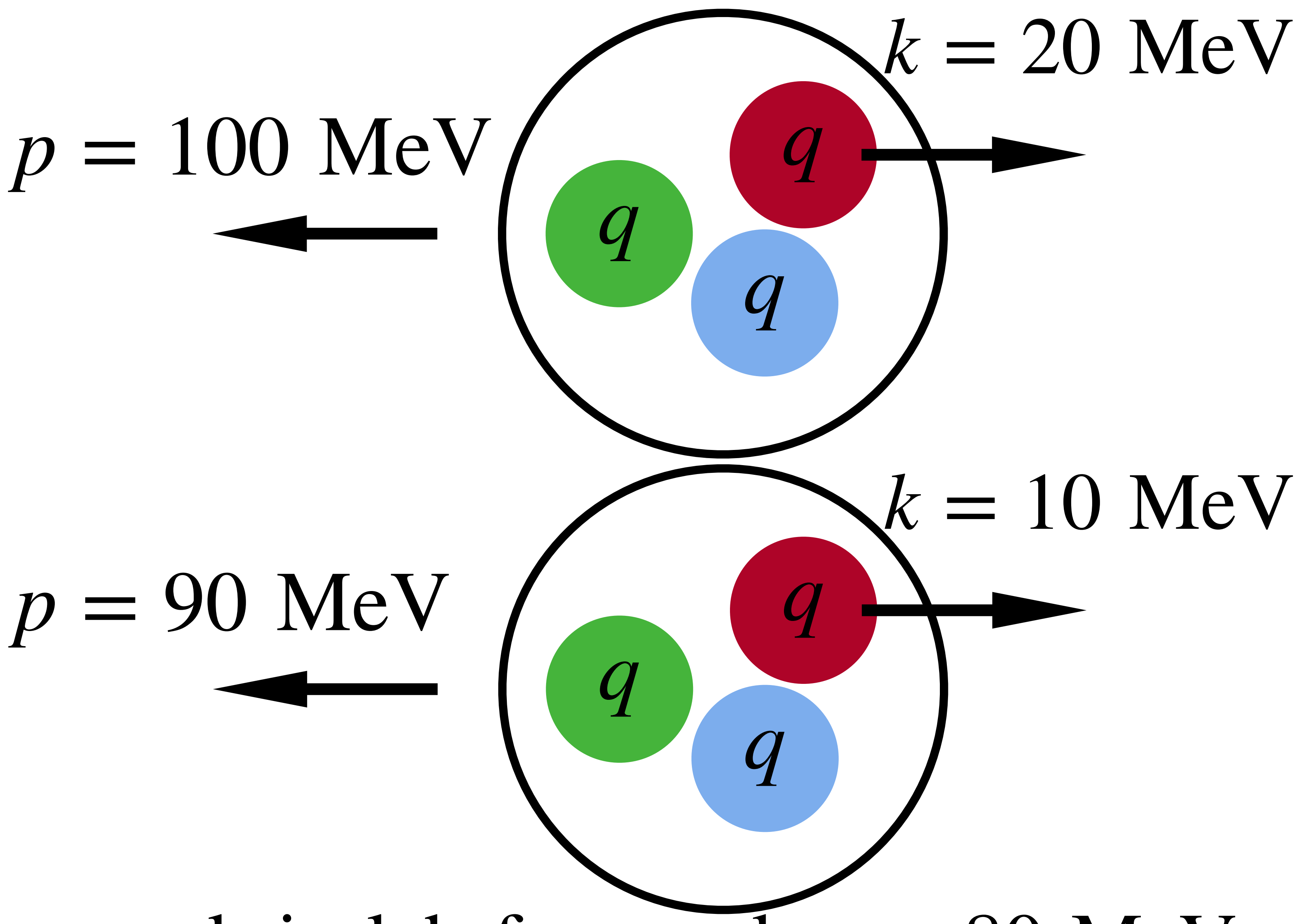






Quark Hadron Duality

Quarks from different baryon may subject to Pauli Blocking



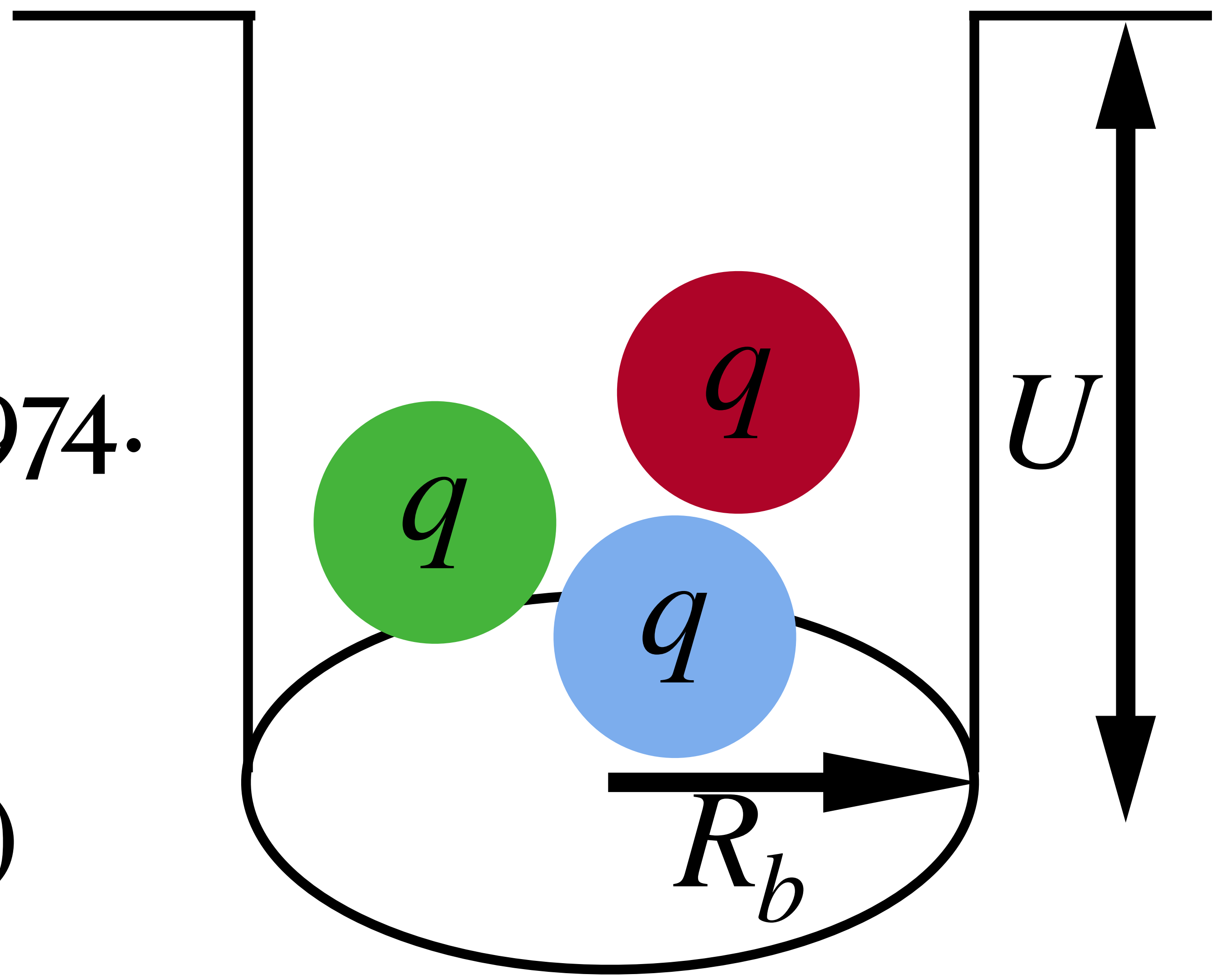
quark in lab frame : $k_{lab} = 80$ MeV

$$|\psi_Q(k_{lab})|^2 \propto \int d^3p\, |\psi_Q(k = k_{lab} - p)|^2 |\psi_B(p)|^2$$

$|\psi_Q(k_{lab} = 0)|^2$ may exceed that of free Fermi gas

- Gaussian wavepacket for quarks in baryon:
  $|\psi_Q(k)|^2 \propto e^{-k^2/\Lambda^2}$
  where $\Lambda \approx 200$ MeV for $< R^2 > \approx 0.61$ fm
- Baryons cannot follow free Fermi gas at density,
  $$n_B^{id,sat} \approx 0.09 \text{ fm}^{-3} \left(\frac{\Lambda}{200 \text{ MeV}}\right)^3$$
  T. Kojo & D. Suenaga 2021
- Modified Gaussian wavepacket:
  $|\psi_Q(k)|^2 \propto e^{-k^2/\Lambda^2}/k^2$ **Y. Fujimoto et. al. 2023**
- We apply wavefunction from the Bag model.
  K. Saito & A. W. Thomas 1994

MIT bag model



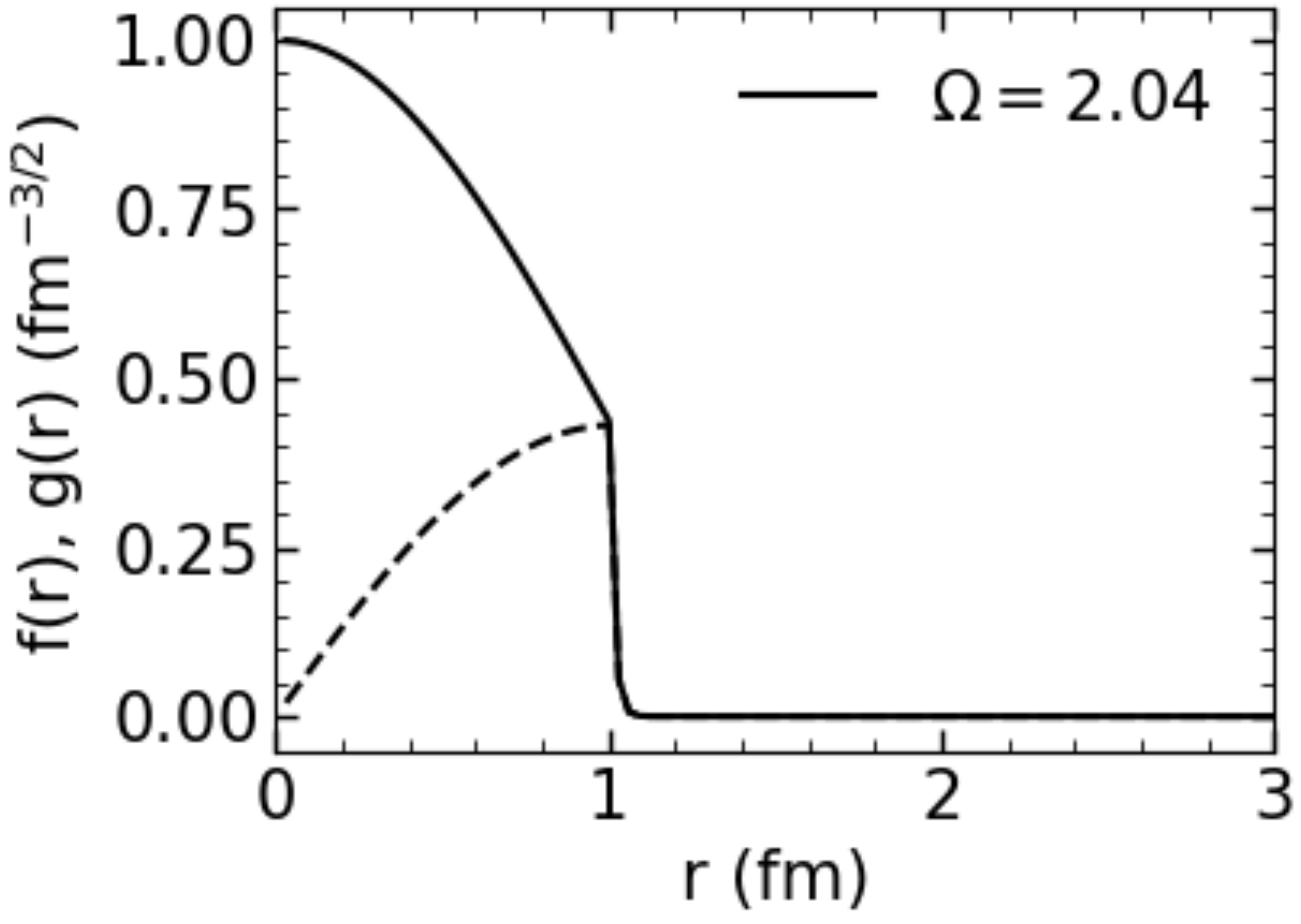
- Developed at Massachusetts Institute of Technology (MIT) in 1974.
- The total energy of the bag,

$$E_b = \frac{4\pi R_b^3}{3}B + \frac{1}{R_b}\sum_q N_q\Omega_q - \frac{Z}{R_b} \qquad (1)$$

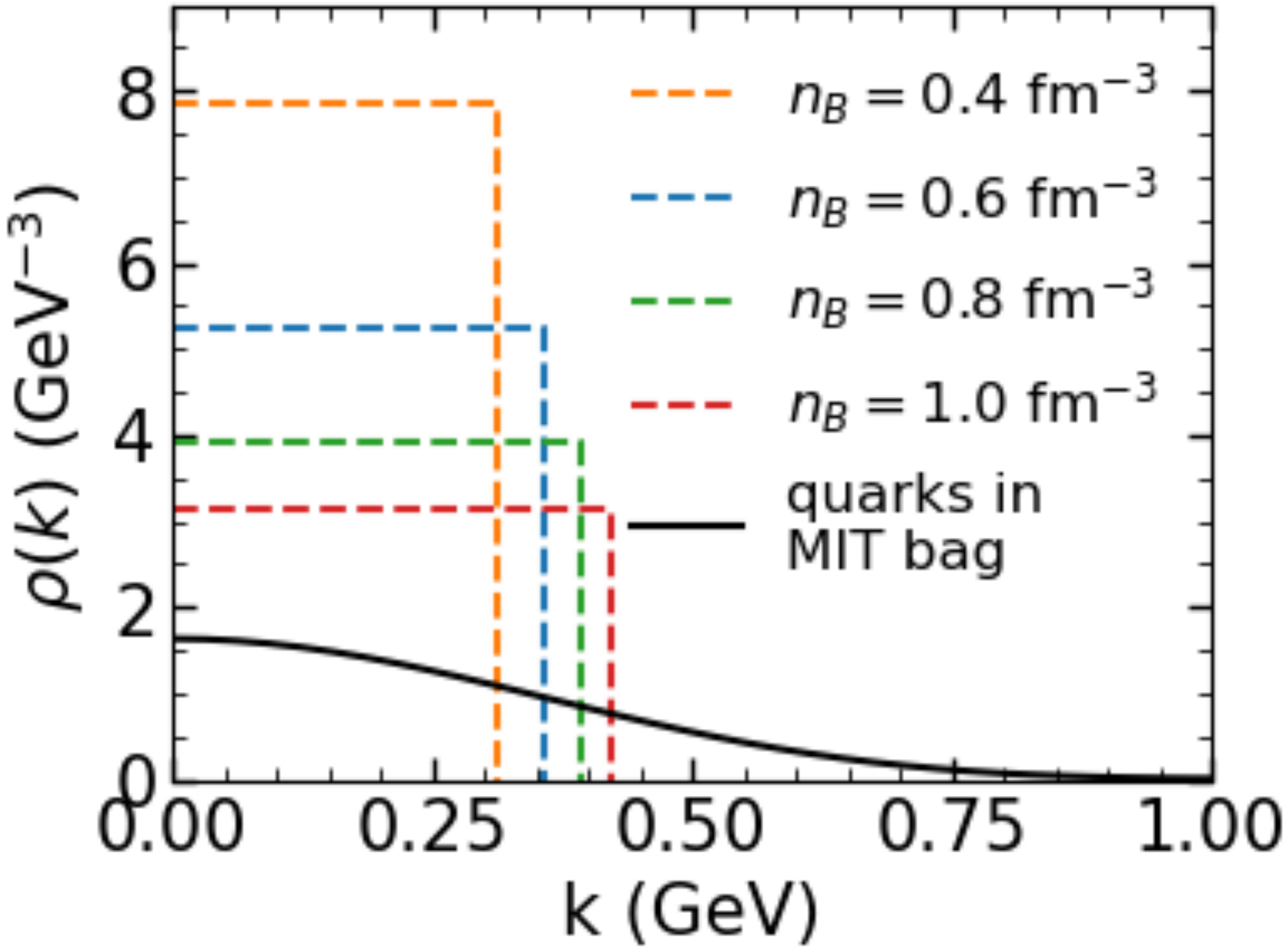
- Solving particles in a spherical infinite potential well, $\Omega_q = 2.04$ (ground state)
- The bag radius is fixed by minimization, $\frac{dE_b}{dR_b} = 0 \longrightarrow 4\pi R_b^4 B + Z = \sum_q N_q\Omega_q \quad (2)$
- Bag constant $B = 0.144\ \mathrm{GeV}^4, Z = 2.55$ fixed by $E_b = 939\ \mathrm{MeV}, R_b = 1\ \mathrm{fm}$.

Quarks in MIT bag

- Quark wave function: $f(r)$, $g(r)$

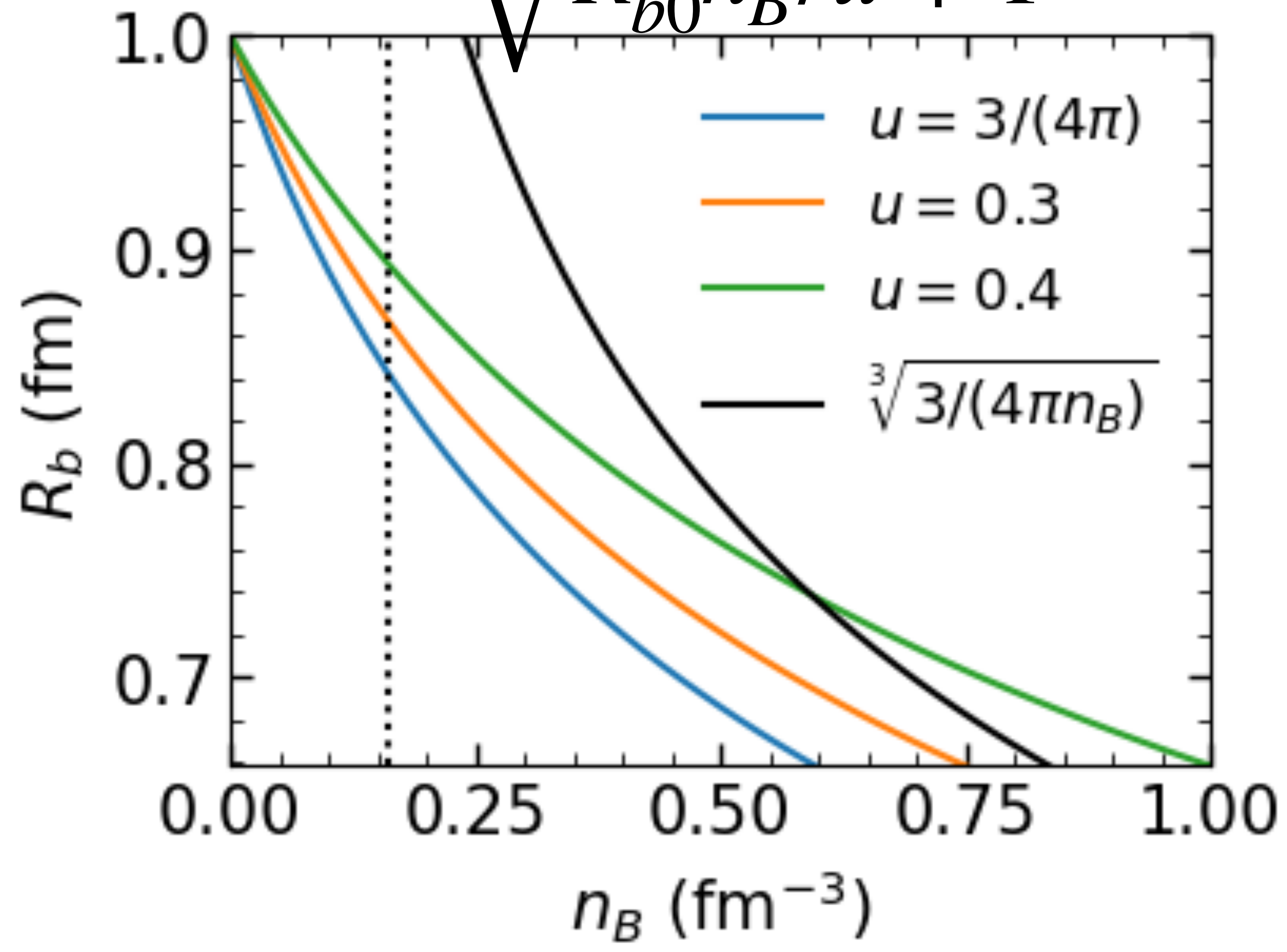


- Momentum space: $\rho(k) = (\hat{f}^2 + \hat{g}^2)/(32\pi^4)$

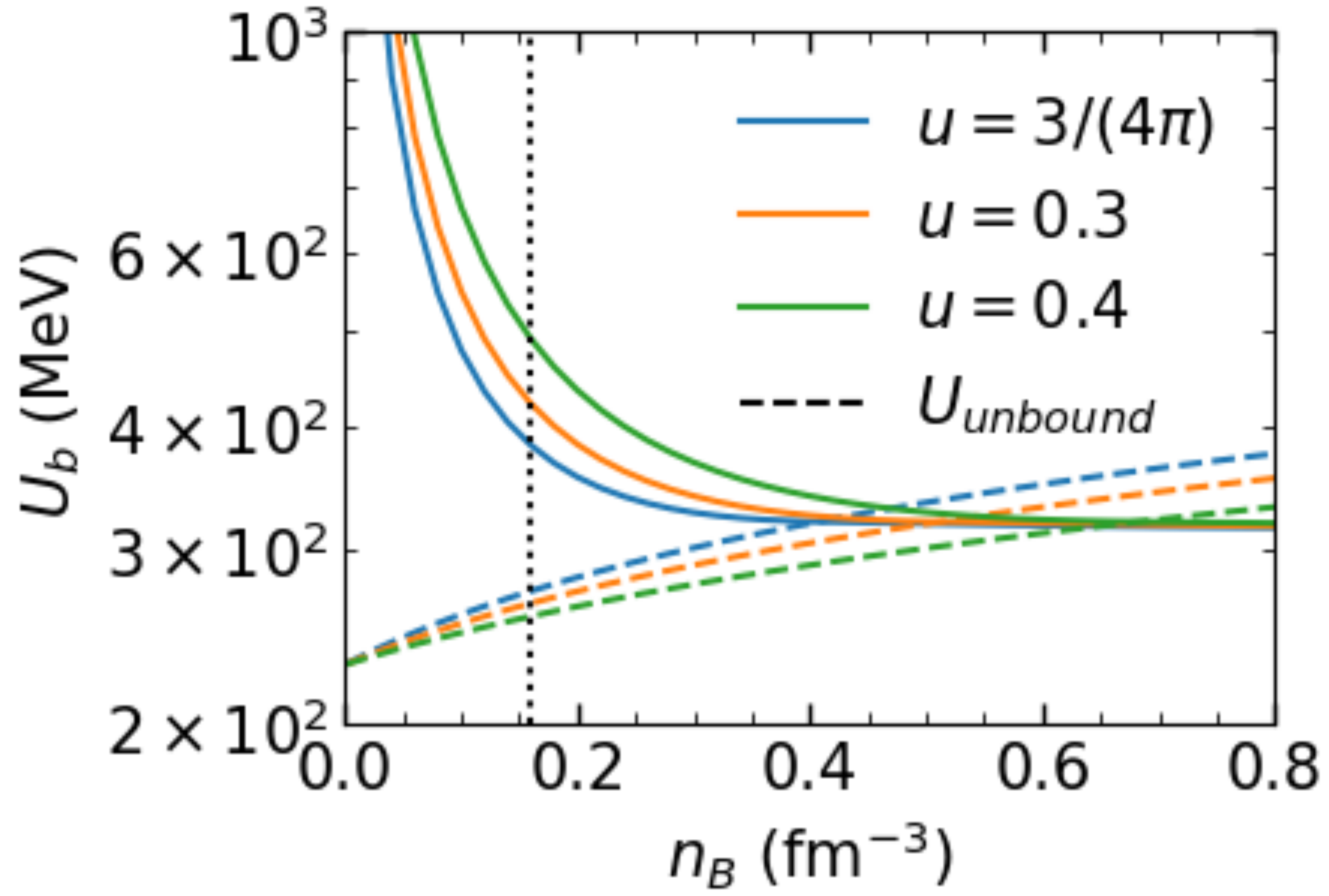


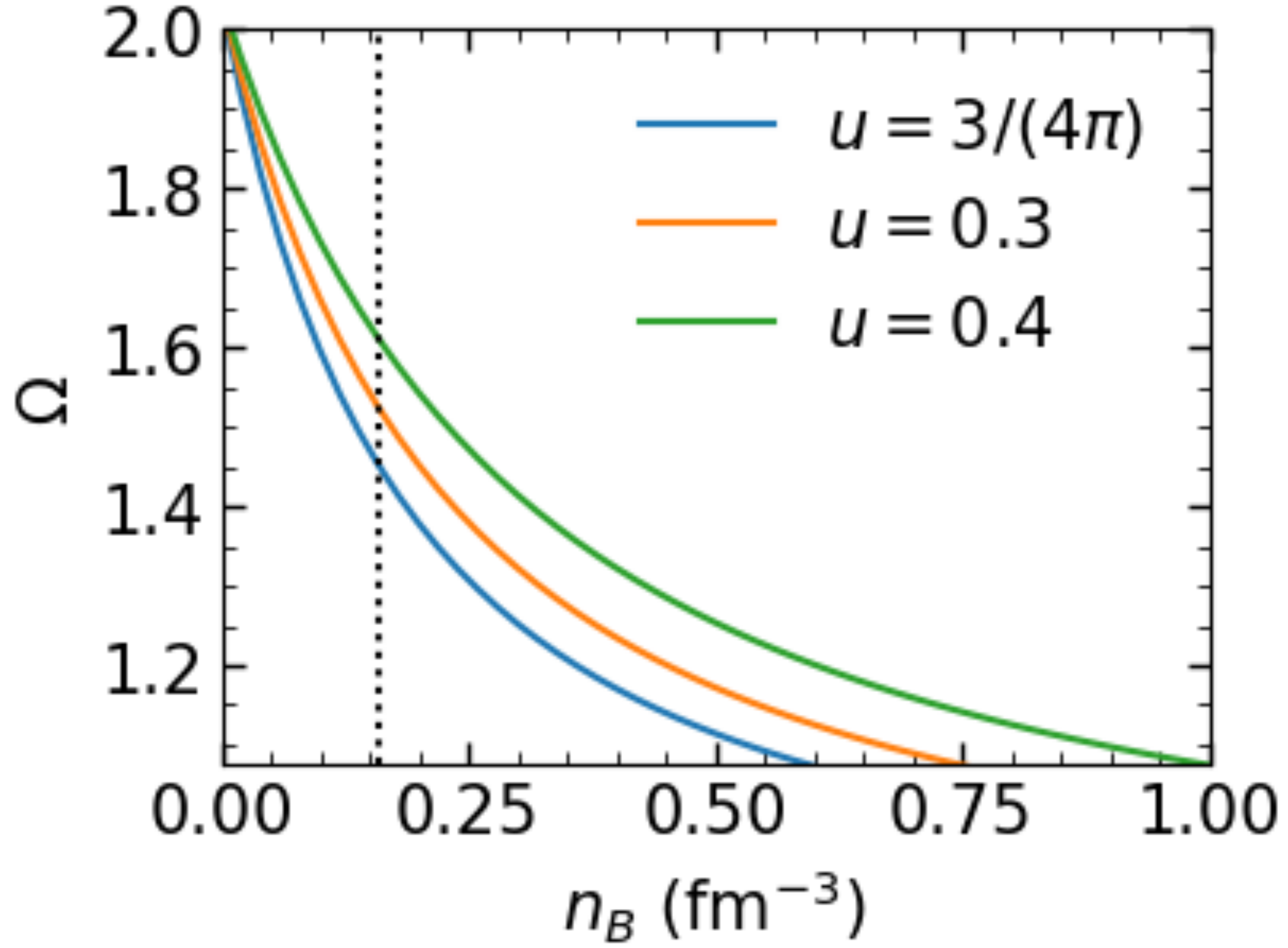
Extended bag model

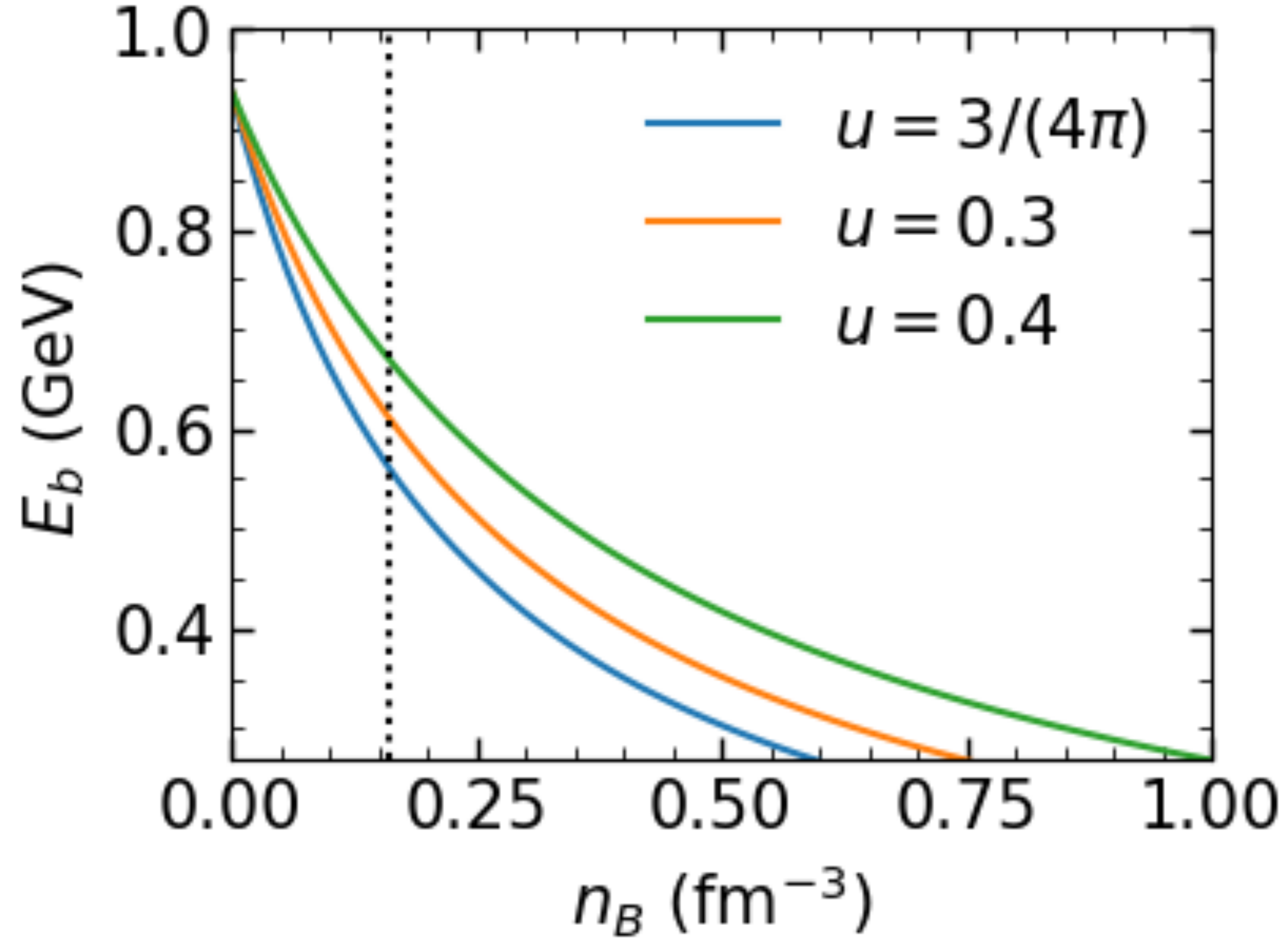
$$R_b = \frac{R_{b0}}{\sqrt[3]{R_{b0}^3 n_B/u + 1}}$$



- Non-overlapping bag, $R_b \leq \sqrt[3]{3/4\pi n_B}$
- $R_b \longleftrightarrow \Omega \longleftrightarrow E_b$
- Finite potential well U_b fixed by $\Omega \leq 2.04$



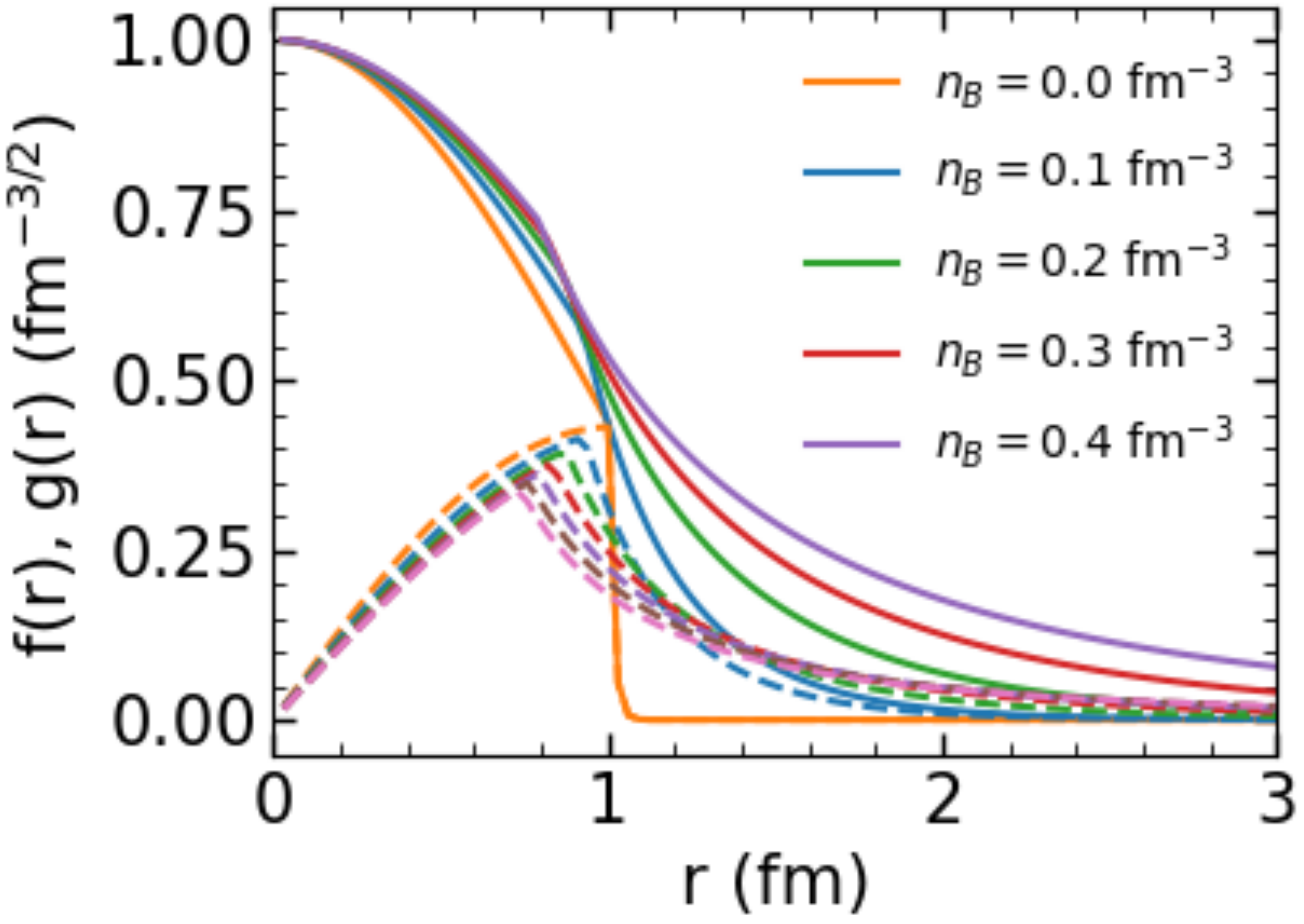




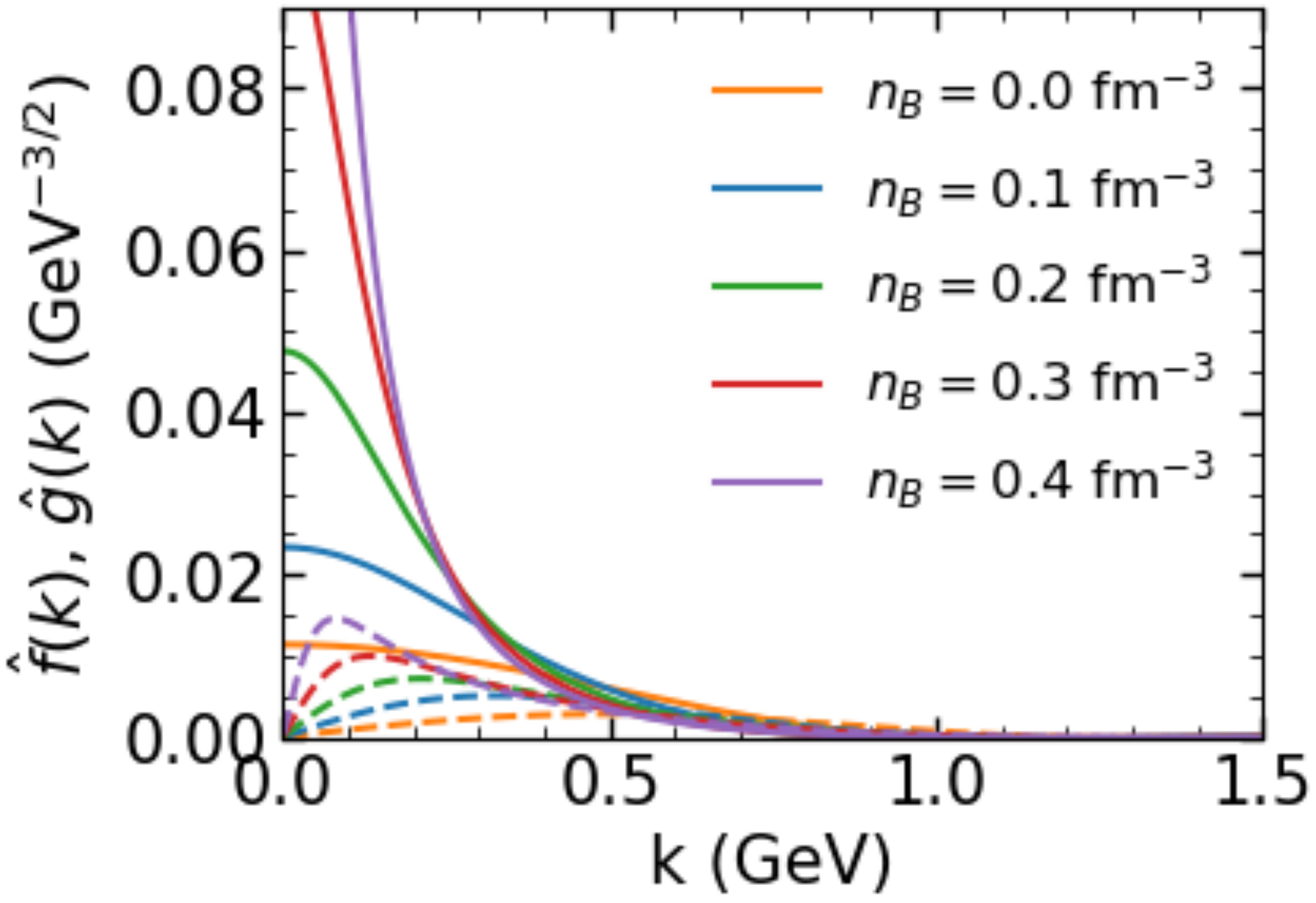
Quarks in extended MIT bag

Stationary bag (in bag frame)

- Quark wave function: $f(r), g(r)$



- Momentum space: $\hat{f}(r),\ \hat{g}(r)$



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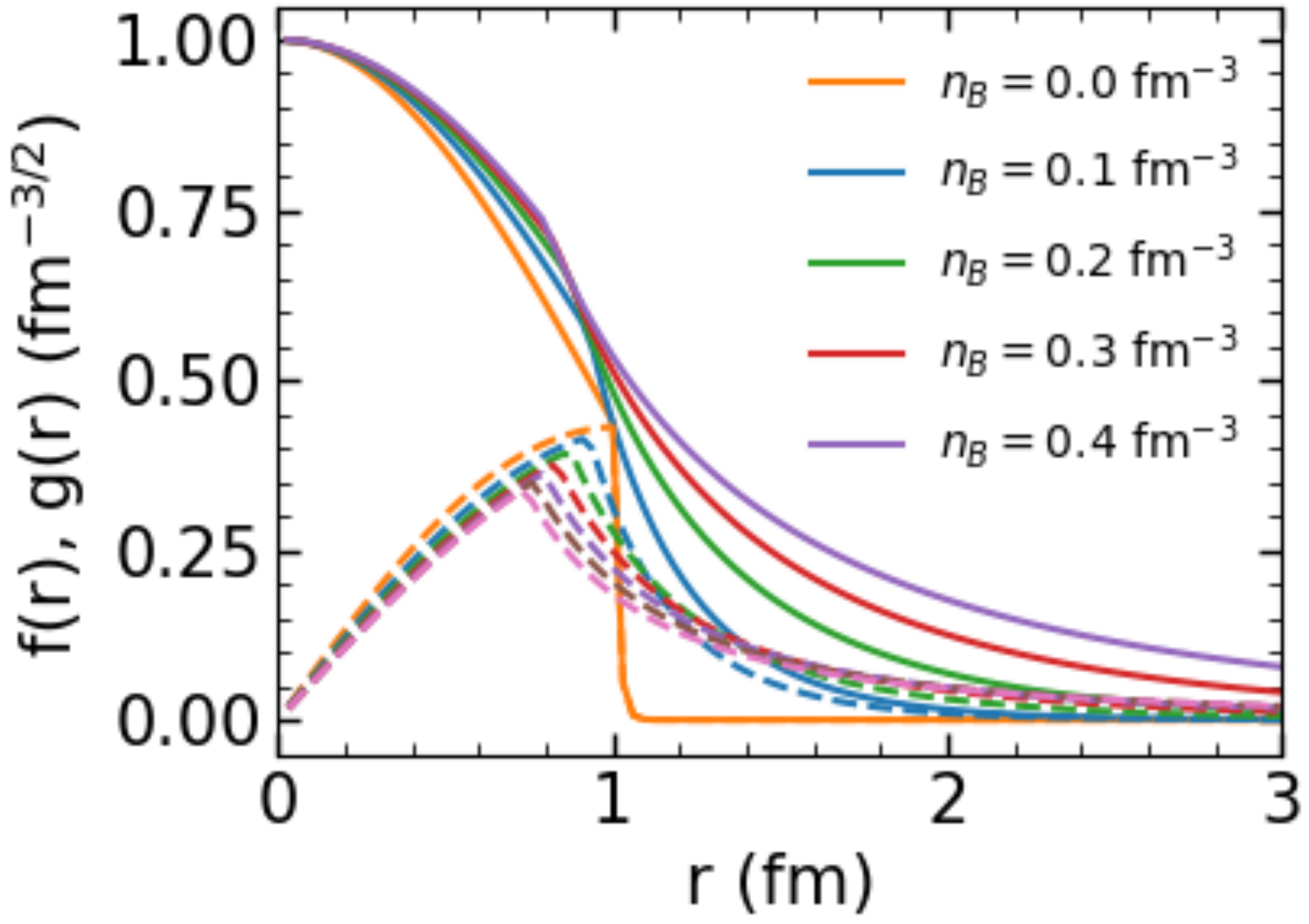
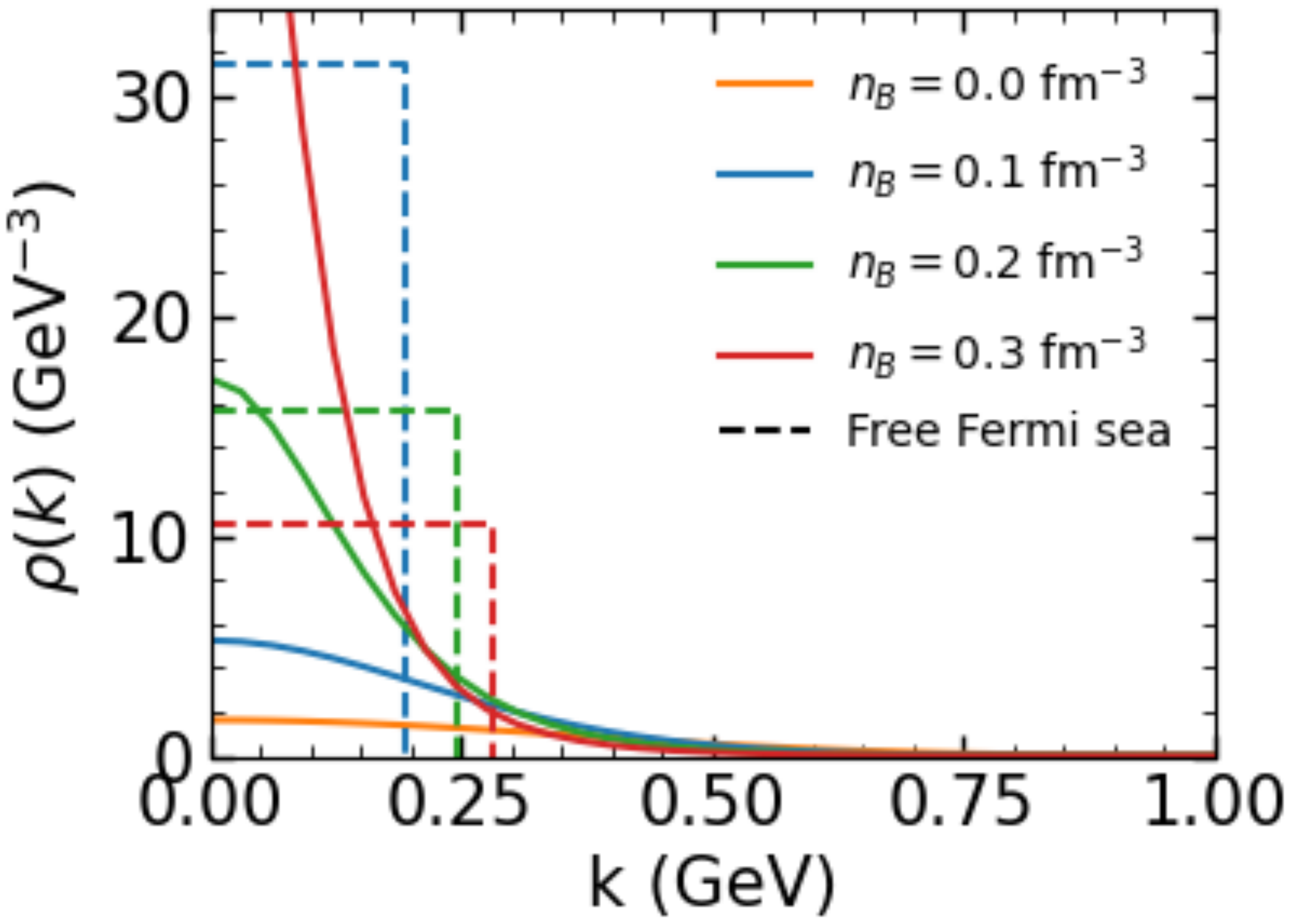


- Momentum space: $\rho(k) = (\hat{f}^2 + \hat{g}^2)/(32\pi^4)$



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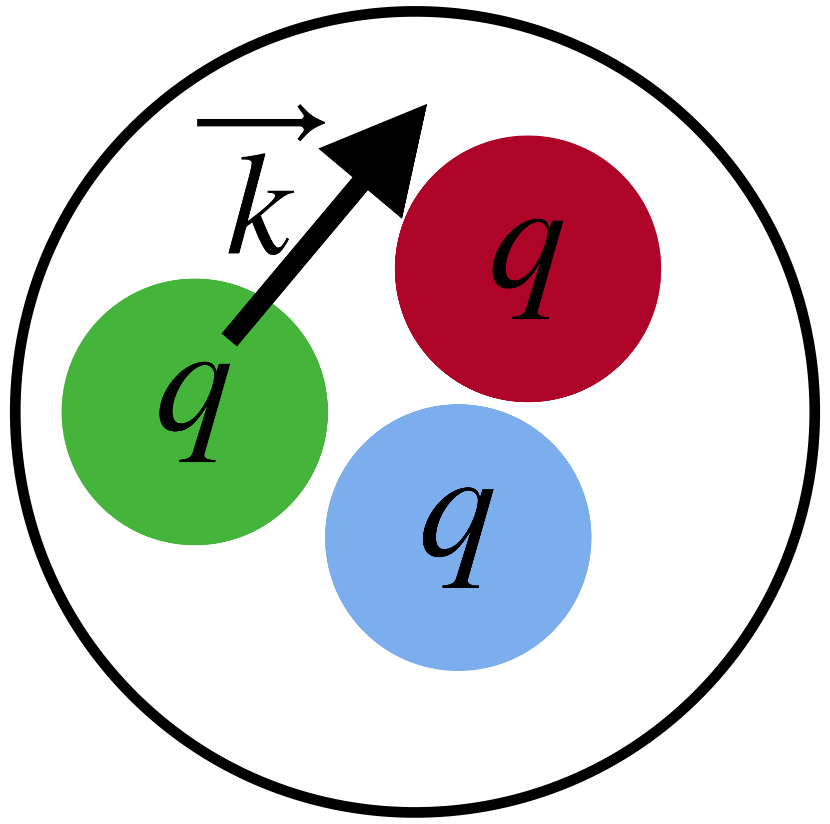

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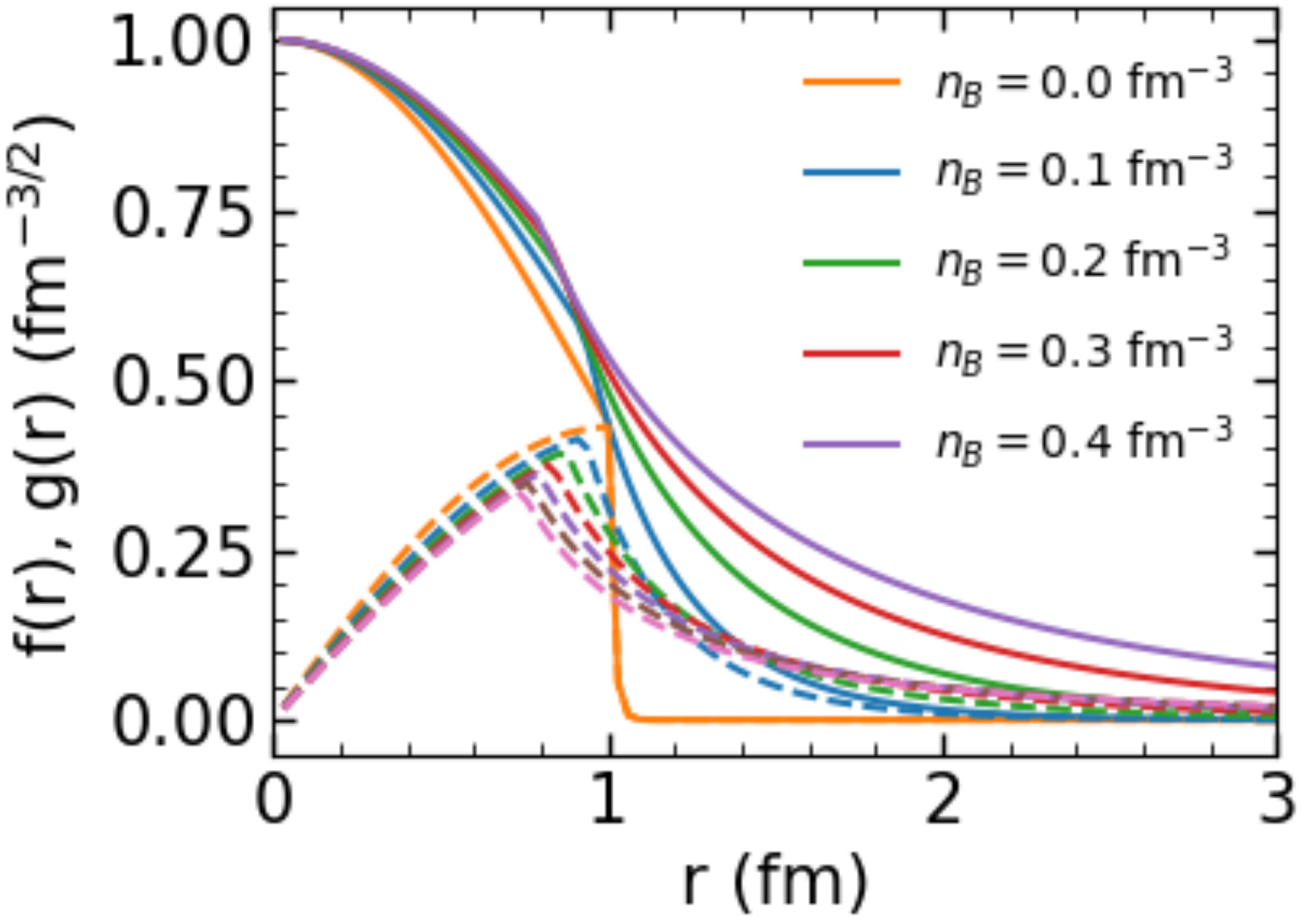
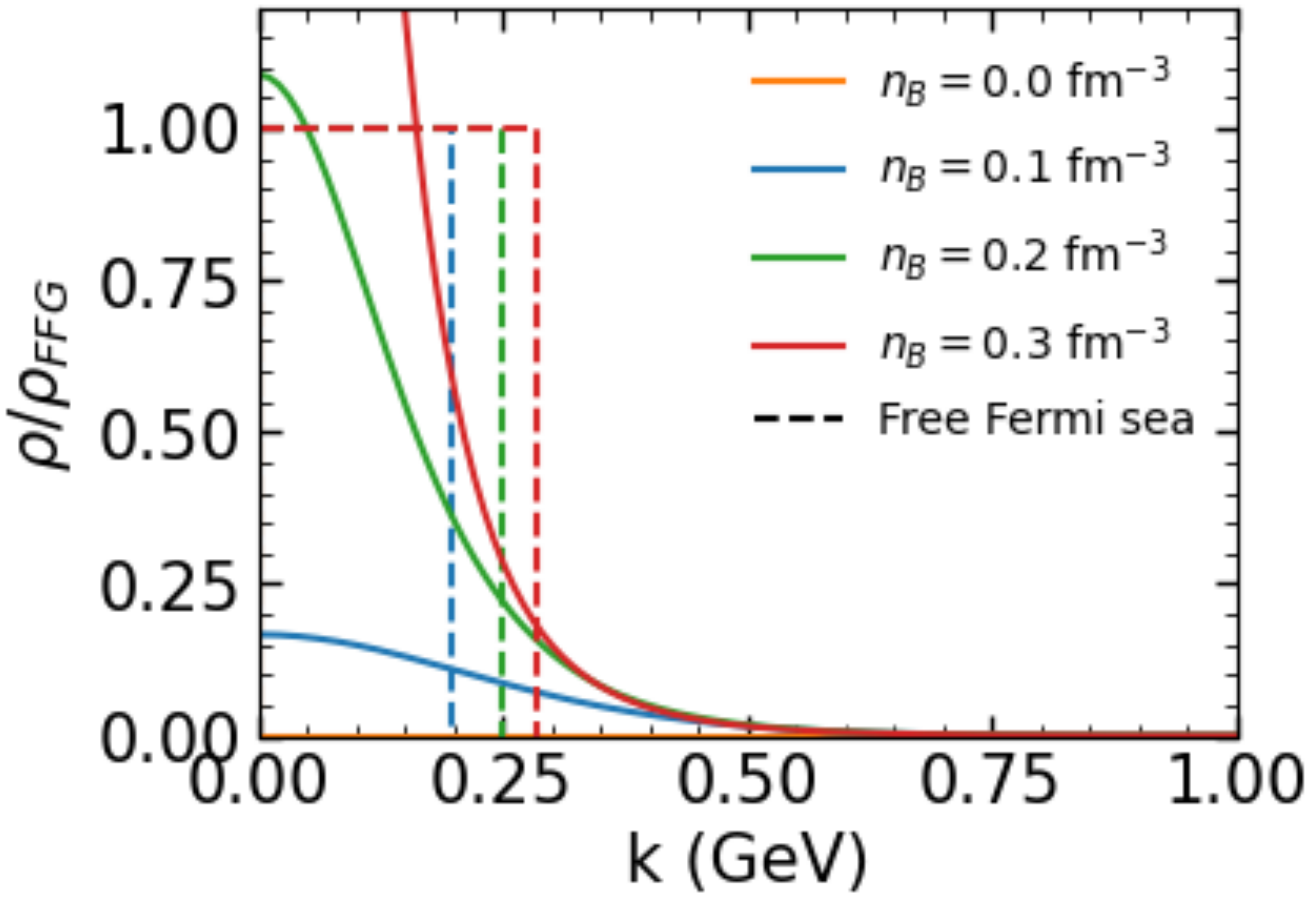
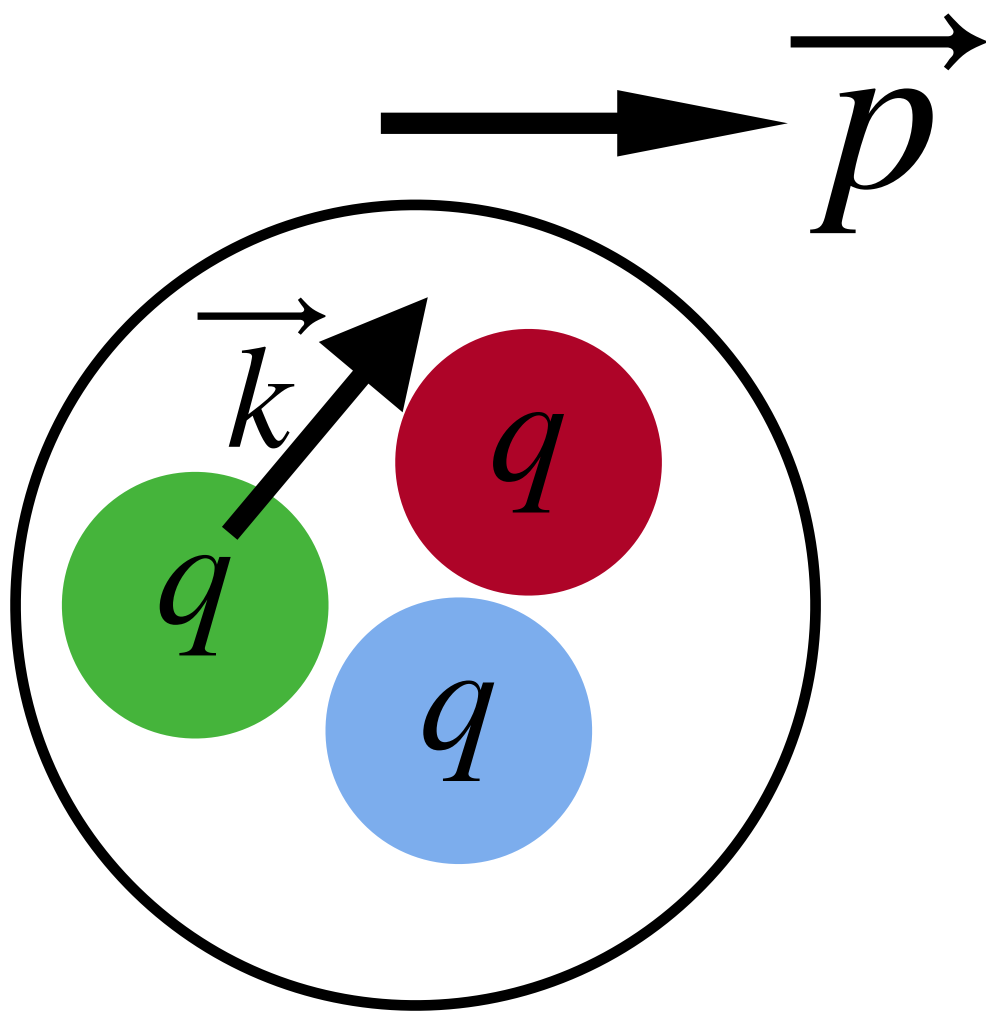


- Momentum space: $\rho_{FFG}(k) = 3/(4\pi^3 n_B)$



Quarks in extended MIT bag

Moving bag (in lab frame)

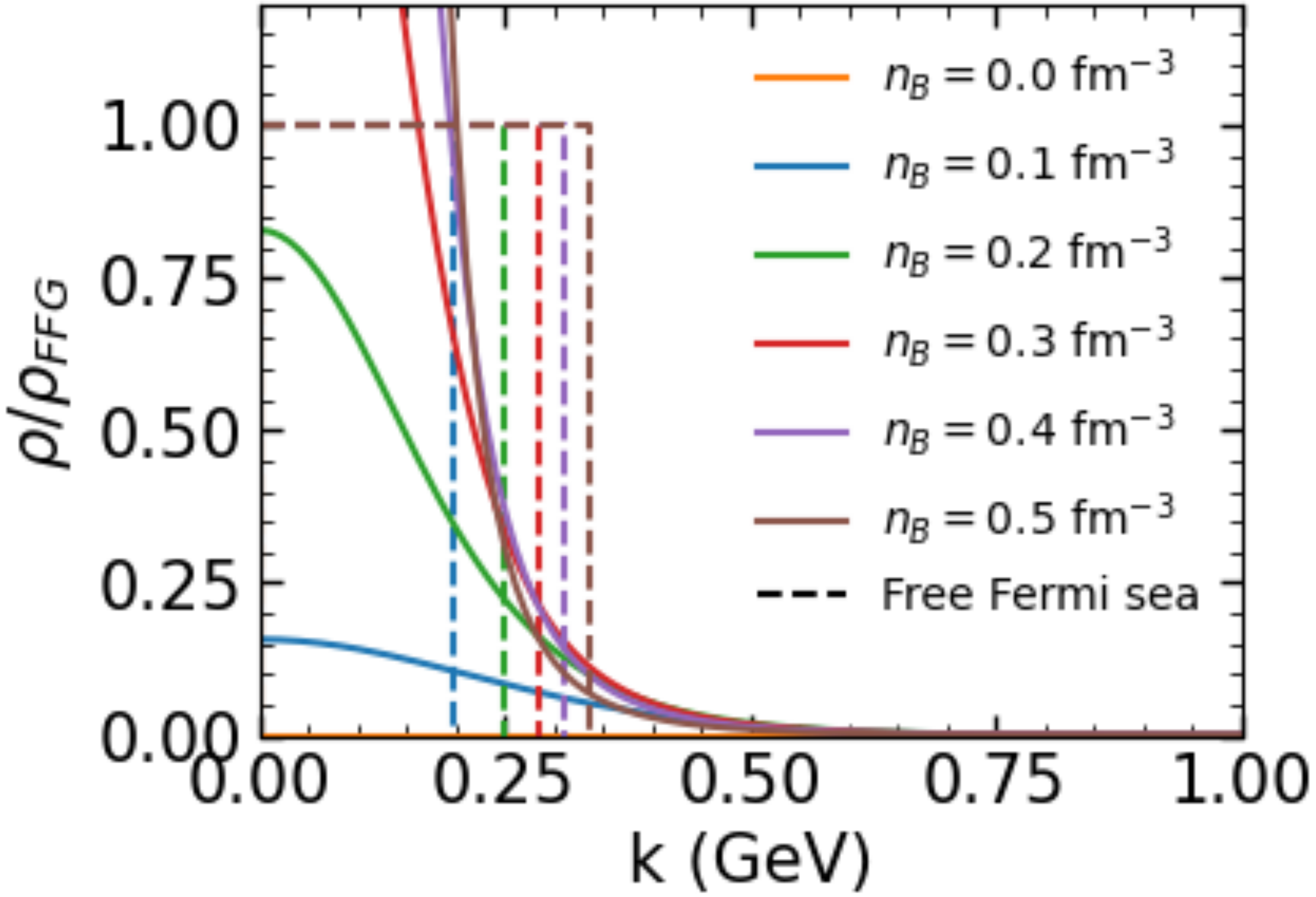


- Bag as nucleon forms its own Fermi Sea $p \in [\ 0,\ p_F]$, determined by baryon density n_B,

$$n_B = \frac{p_F^3}{3\pi^2}$$

- Quark in bag at lab frame,

$$k_{lab} = \sqrt{(p/3)^2 + k^2 - 2pk\cos(\theta)}$$



Quarks in extended MIT bag

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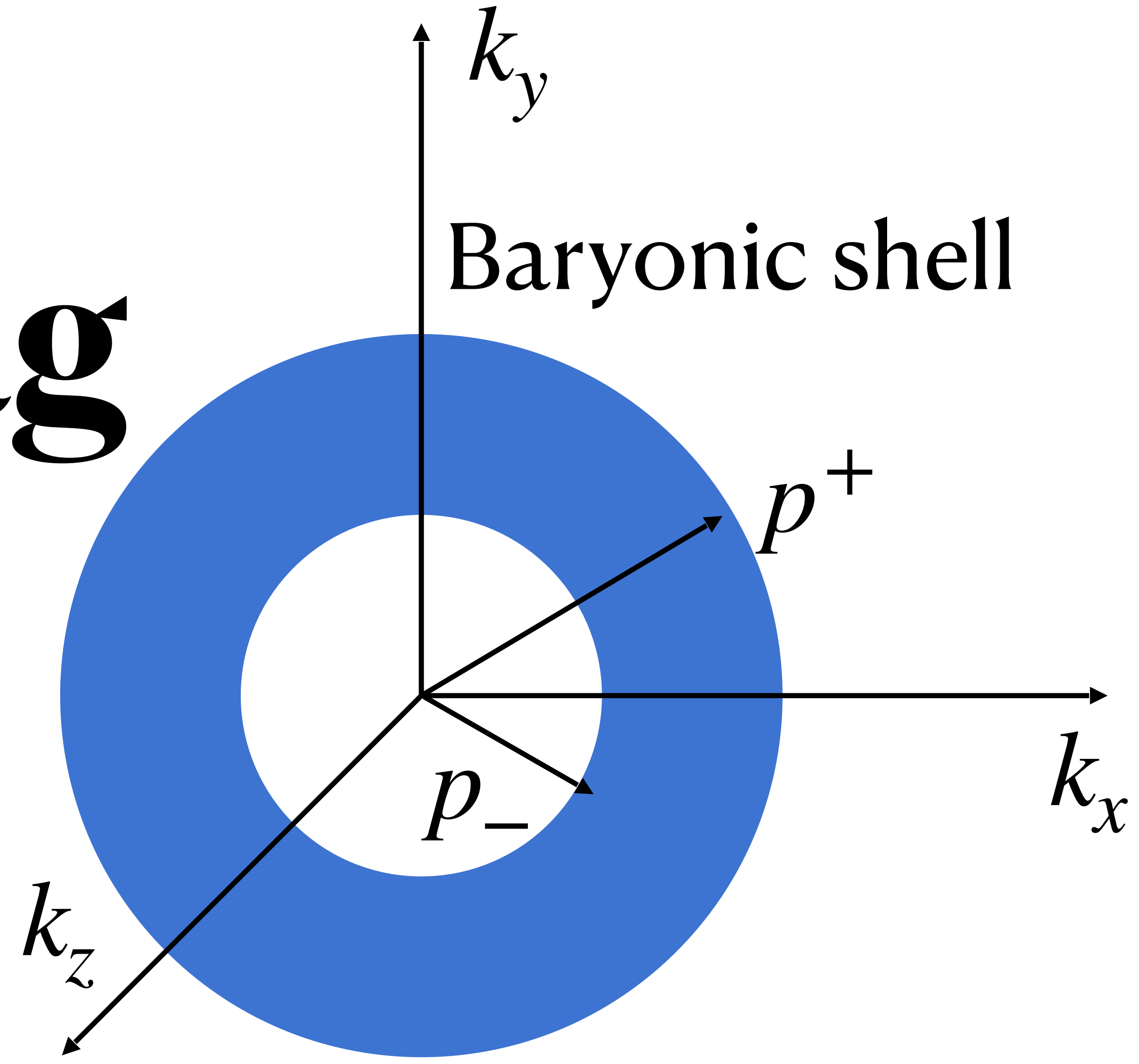
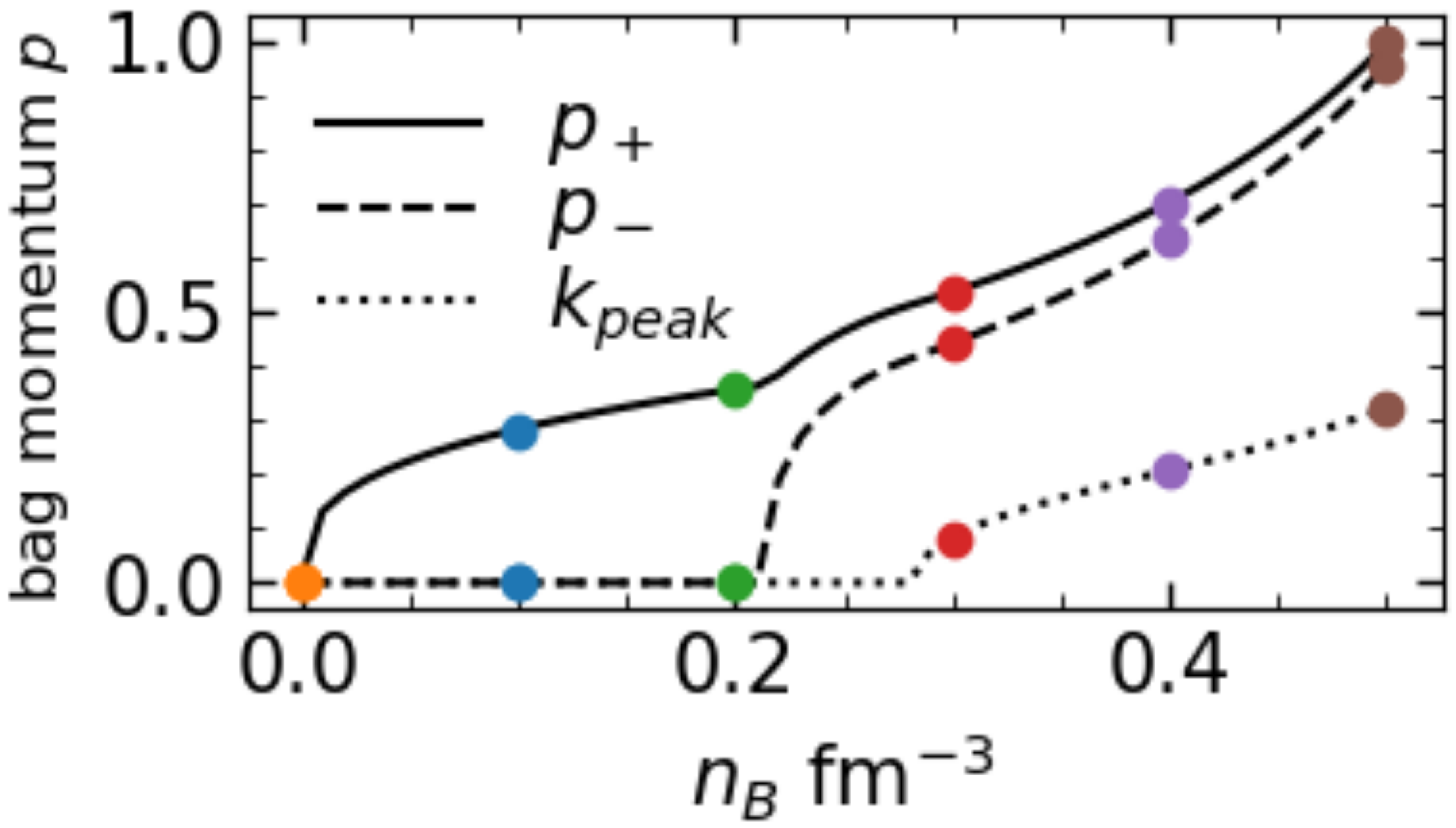
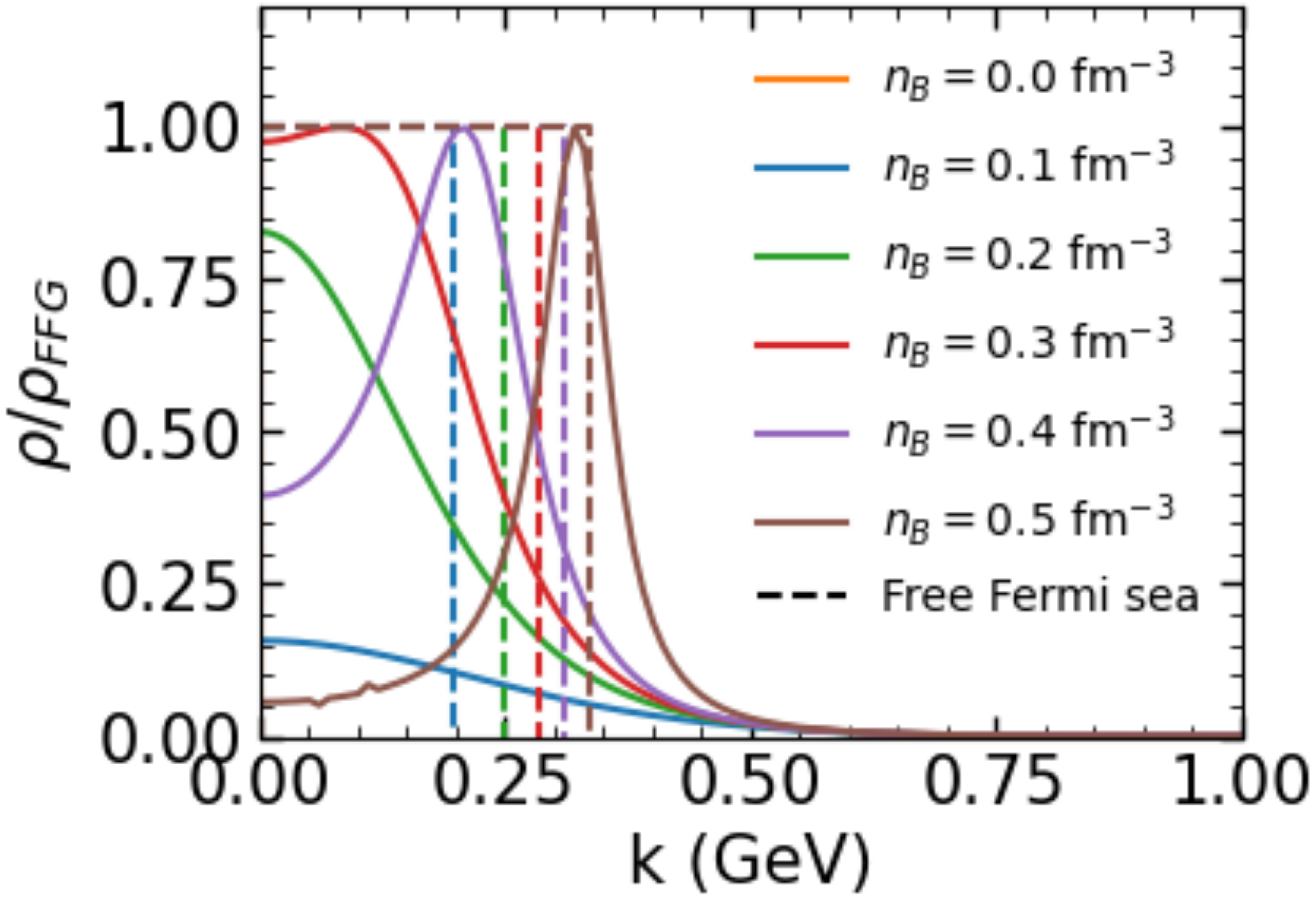


- Bag as nucleon forms its own Fermi Sea $p \in [p_-,\ p_+]$, determined by baryon density n_B,

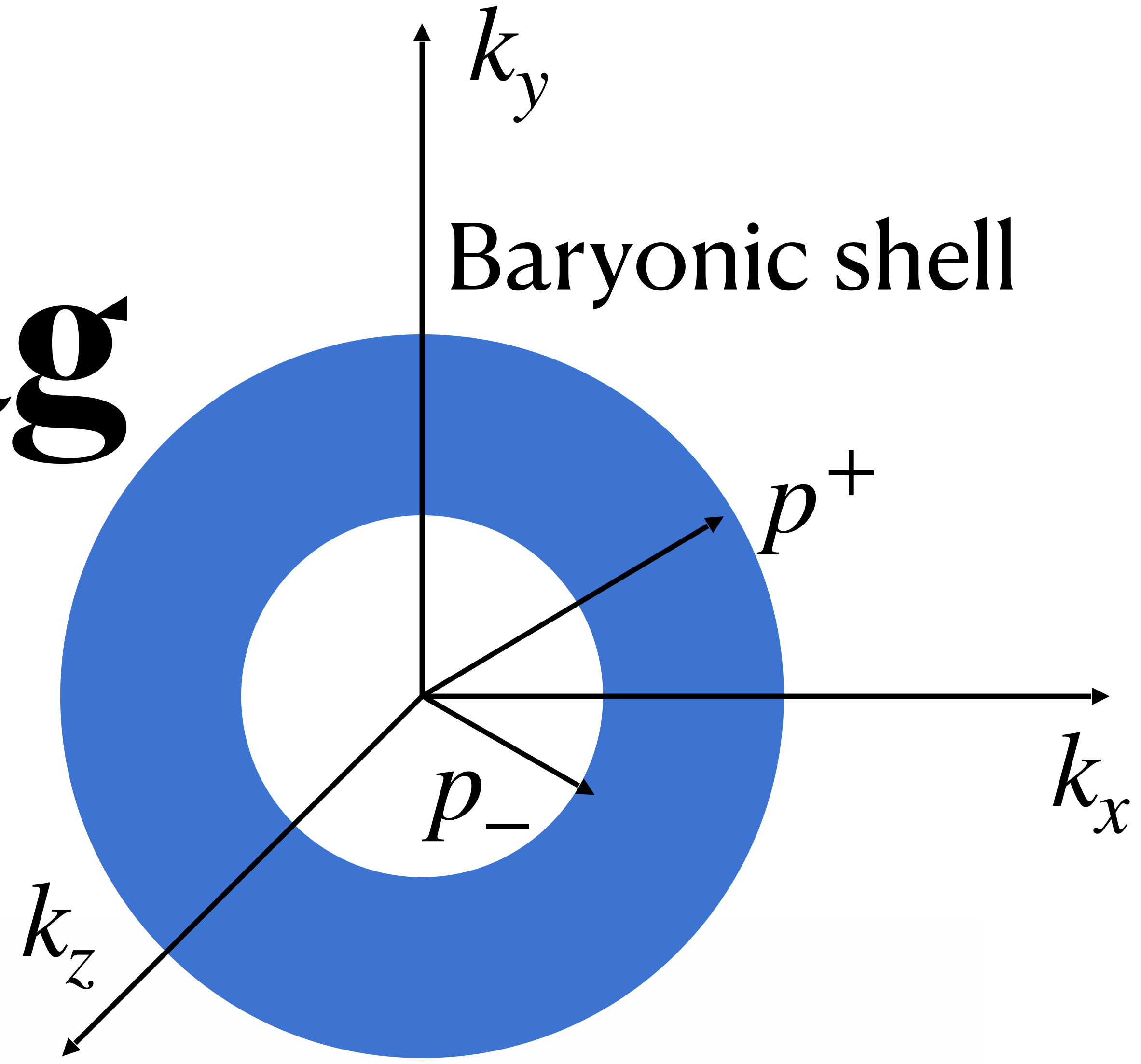
$$n_B = \frac{p_+^3}{3\pi^2} - \frac{p_-^3}{3\pi^2}$$





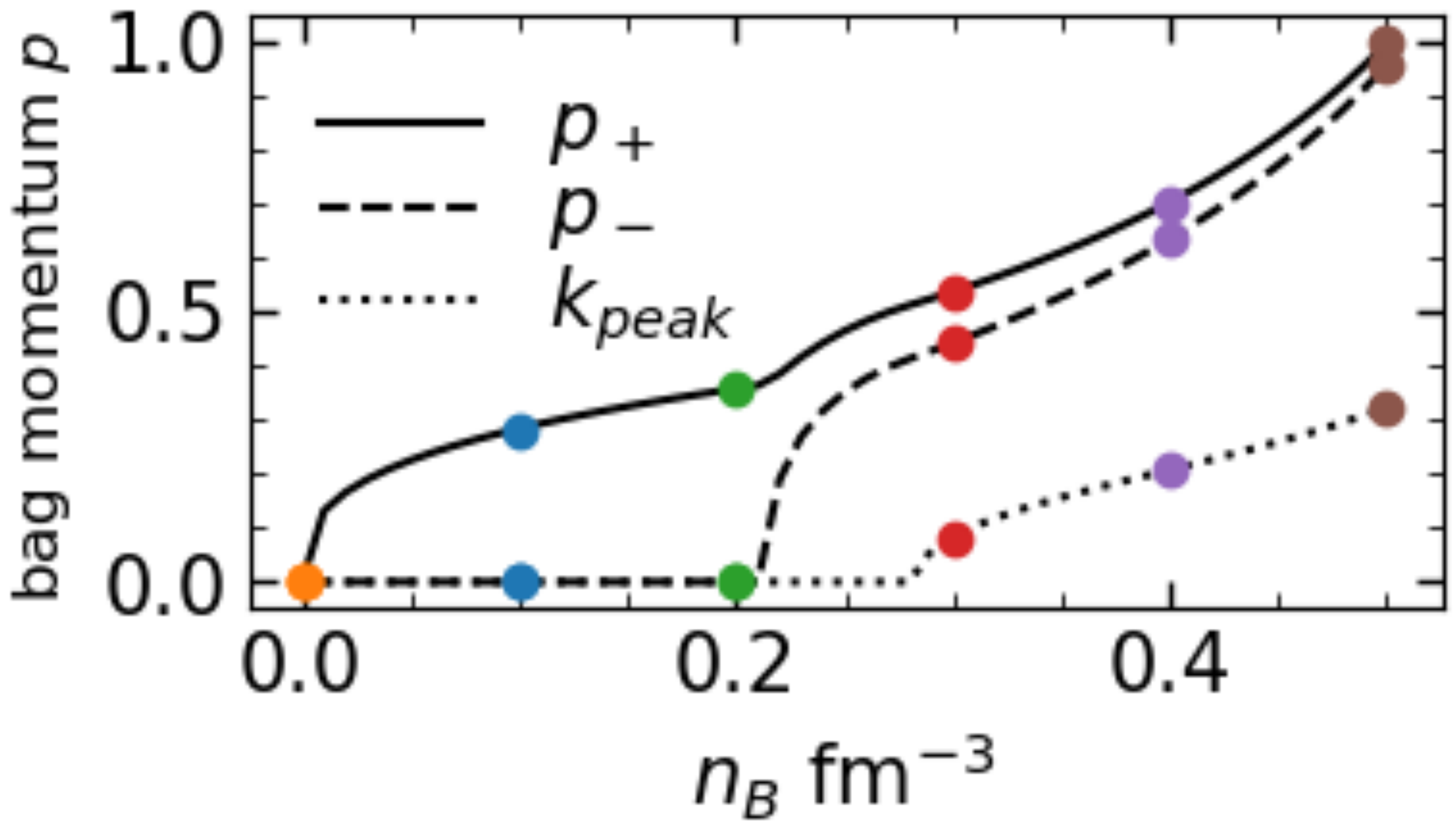
Quarks in extended MIT bag

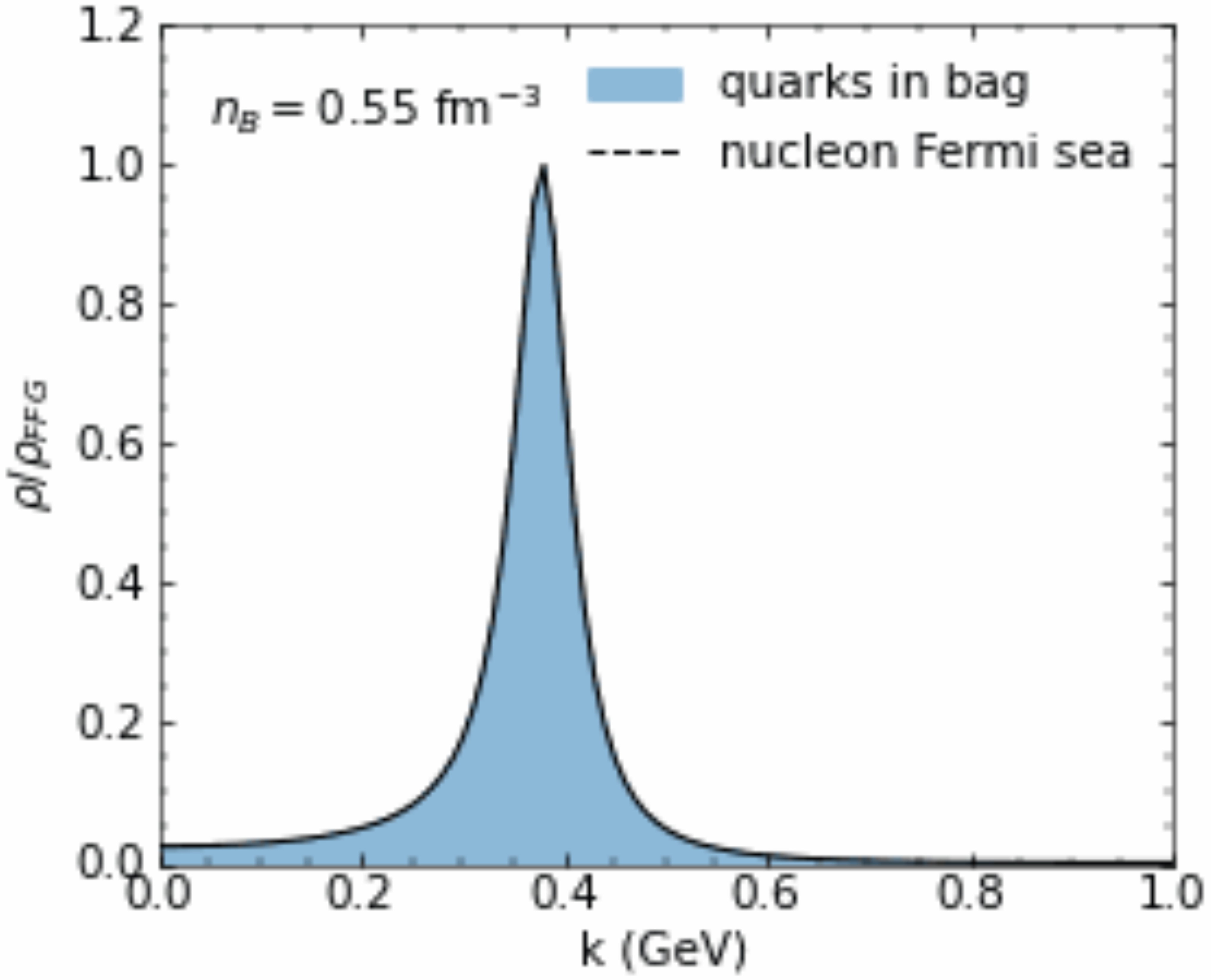
Moving bag (in lab frame)



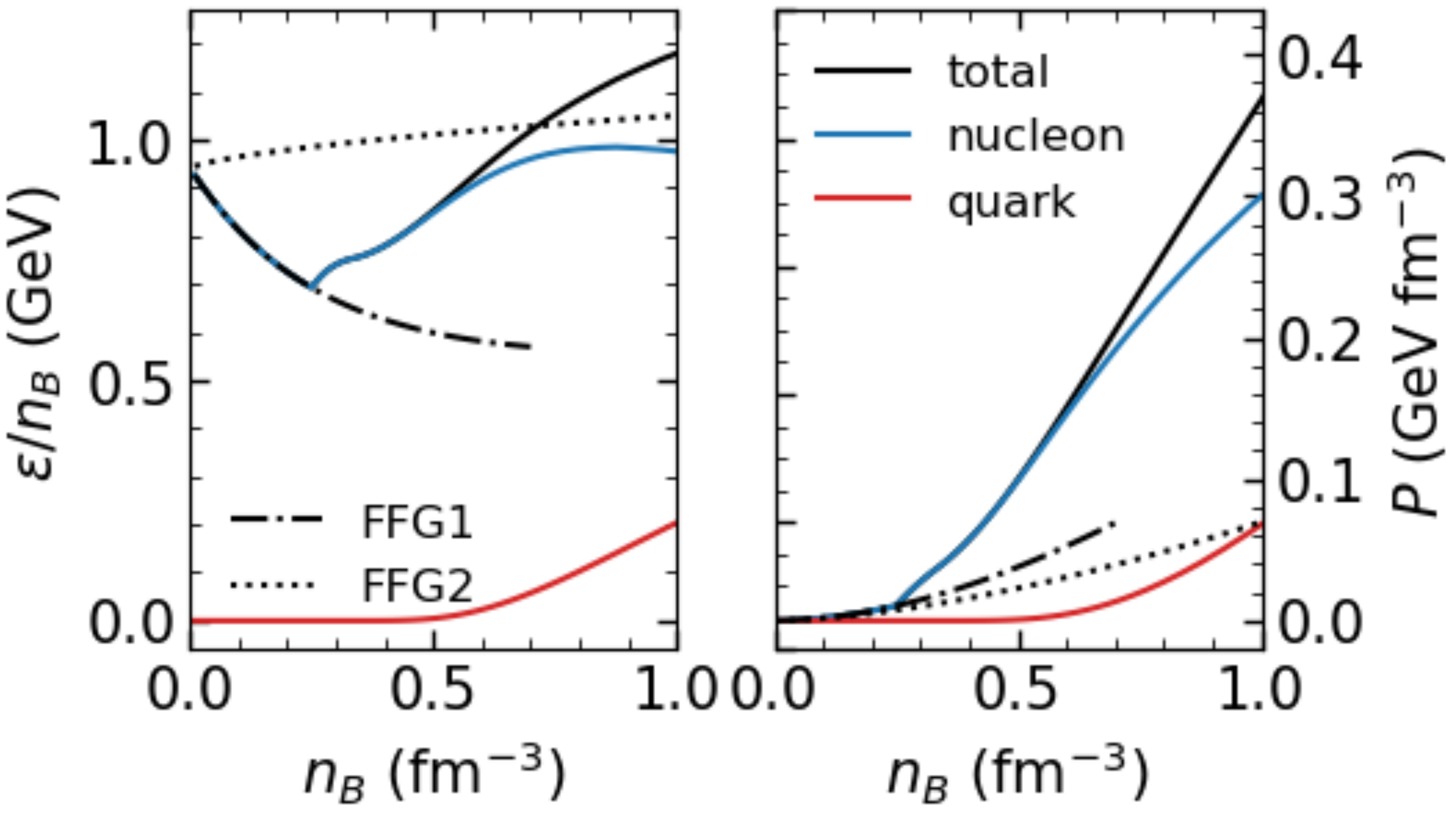
- Bag as nucleon forms its own Fermi Sea $p \in [p_-, p_+]$, determined by baryon density n_B,

$$n_B = \frac{p_+^3}{3\pi^2} - \frac{p_-^3}{3\pi^2}$$

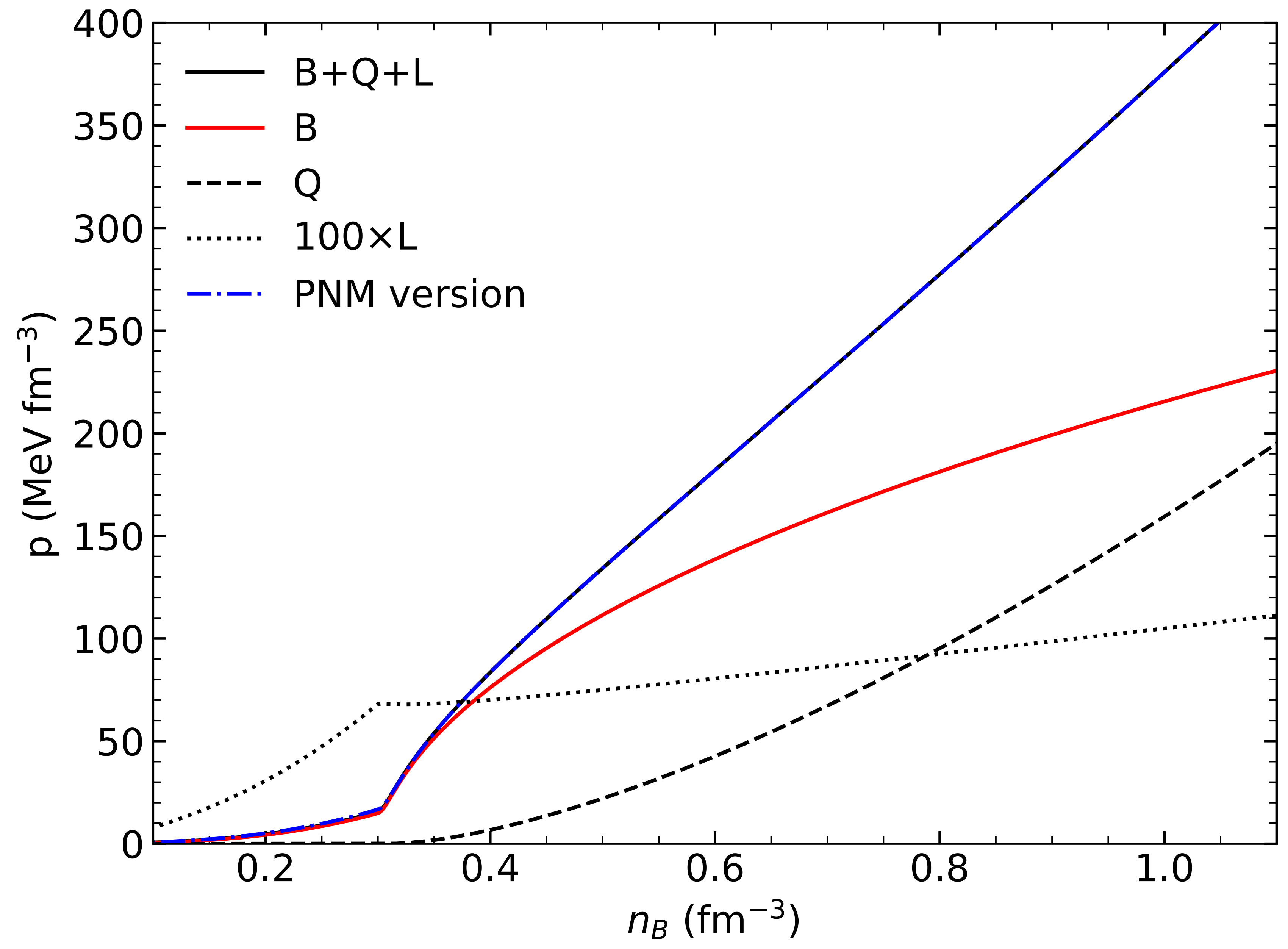




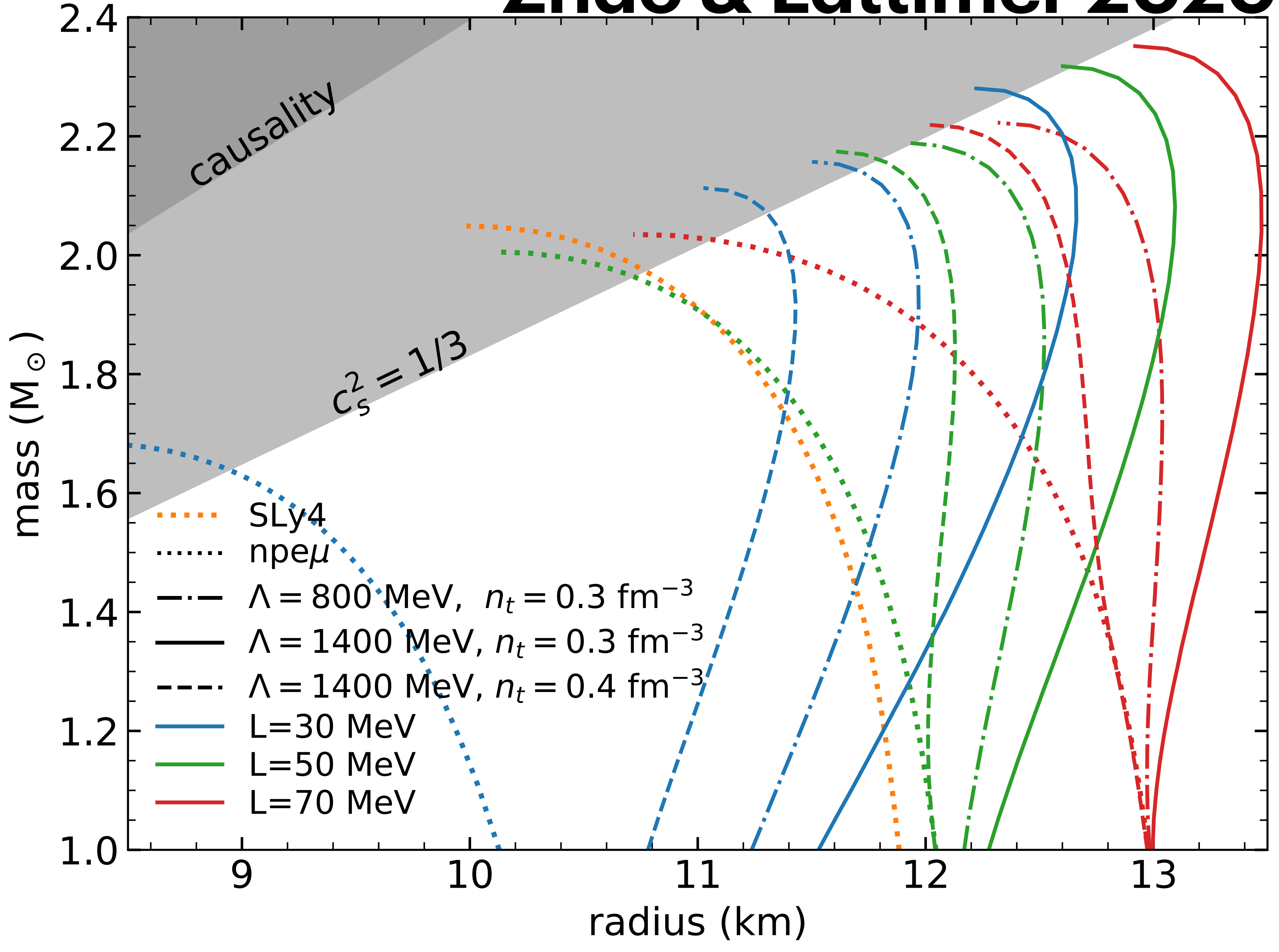
Quarkyonic EOS



[arXiv: 24xx.xxxxx]()



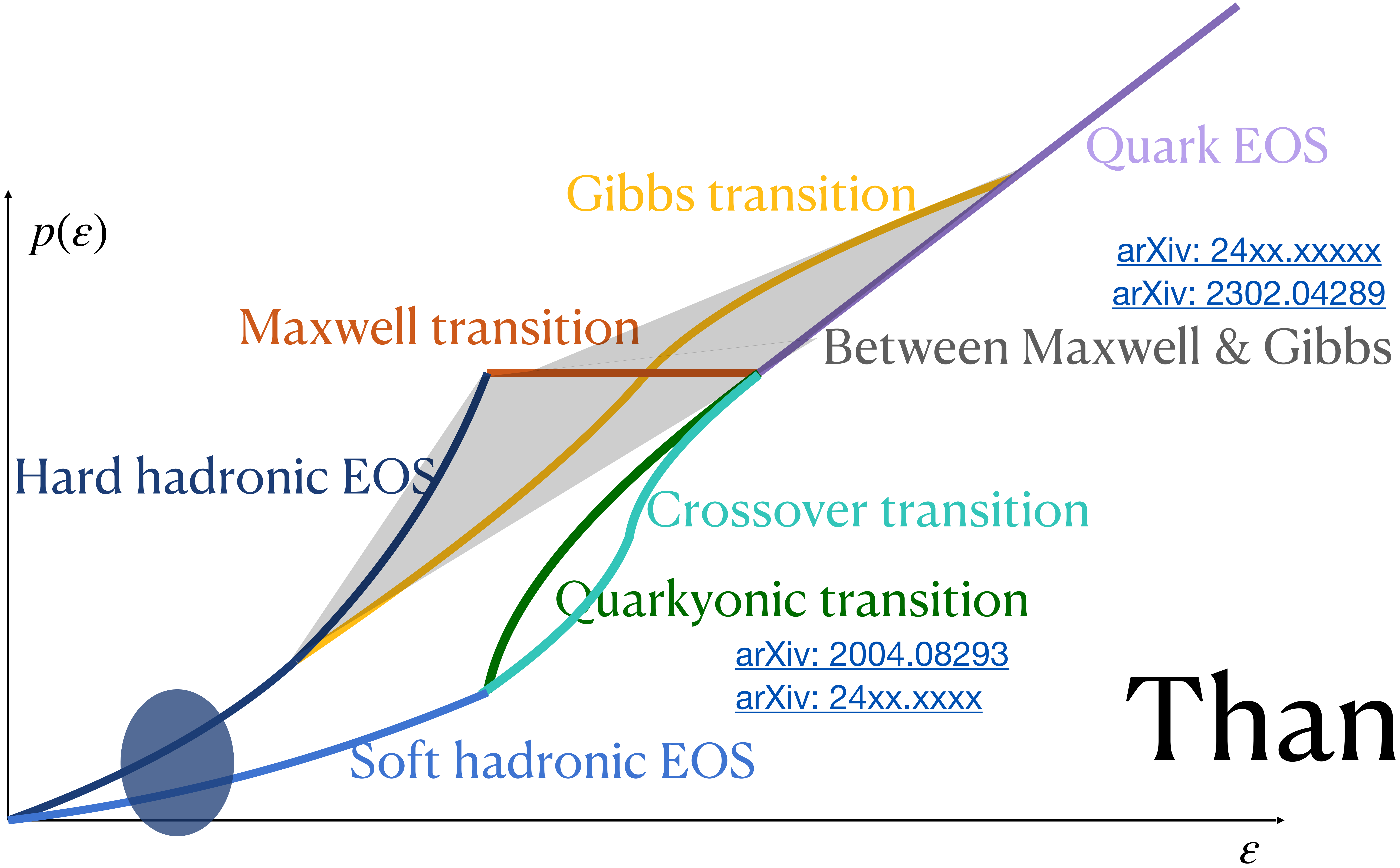
Zhao & Lattimer 2020



Summary

- The traditional MIT bag model can be extended to finite potential.
- Quarks in the extended MIT bag model have lower momentum which can saturate when hadron-to-quark transition begins.
- Due to the Pauli-exclusion of quarks in nucleons,
  the low momentum states of nucleons are excluded,
  pushing nucleons to higher energy states.
- Quarkyonic EOS can robustly stiffen a soft hadronic EOS
  without fine-turning.

Hadron-quark Transition in Neutron Star Core



Thank you!

arXiv: 2009.06441

Soft hadronic EOSs is flavored by ab-initio calculation, nuclear experiments & neutron star merger observation.

arXiv: 2406.05267 arXiv: 1808.02858