QUANTINUUM

Simulating Floquet scrambling circuits on trapped-ion quantum computers

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Introduction

Quantum advantage for artificial tasks (RCS)

Google '19;
Zuchongzhi '21;
Quantinuum '24

Quantum dynamics?
(what else?)

Fault-tolerant QC
- Prime factoring
- QFT simulation
- Many more…

time

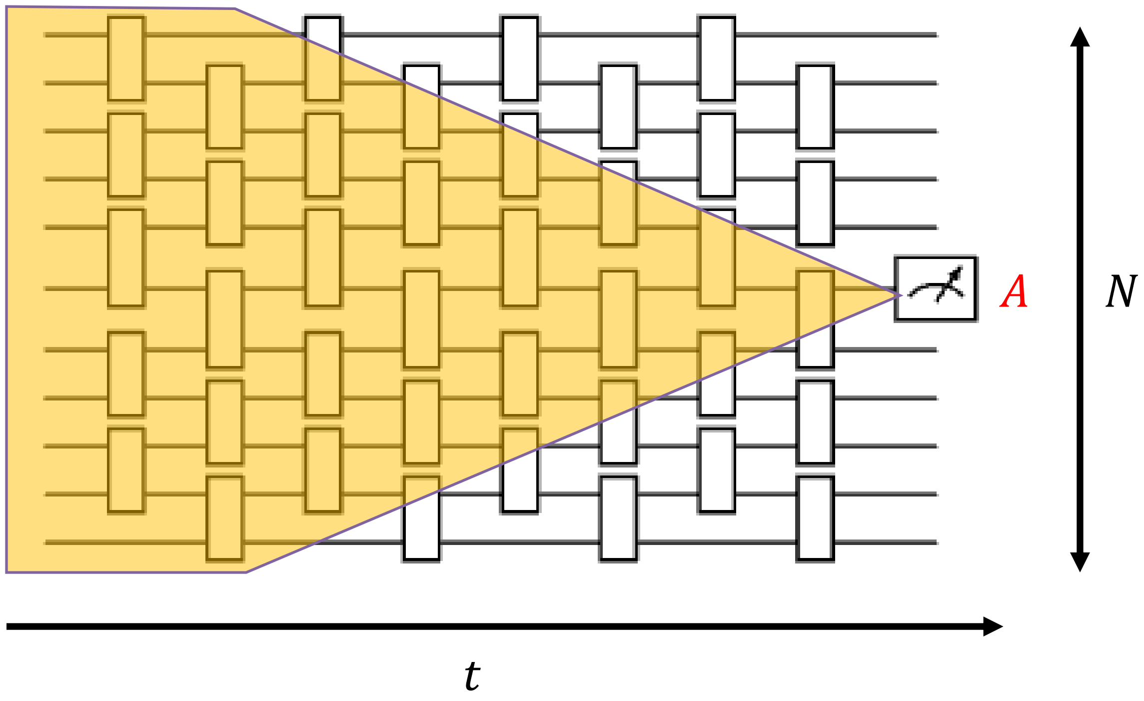
Need to cope with significant noise!!

(What can we do with **50-100 qubits** and **gate fidelity** $\mathbf{99.9\%, 99.99\%, \ldots ?}$)

Digital quantum simulation on noisy hardware

Setup

- Measure a local observable
- Circuit has geometrically local connectivity



Digital quantum simulation on noisy hardware

Setup

- Measure a local observable
- Circuit has geometrically local connectivity

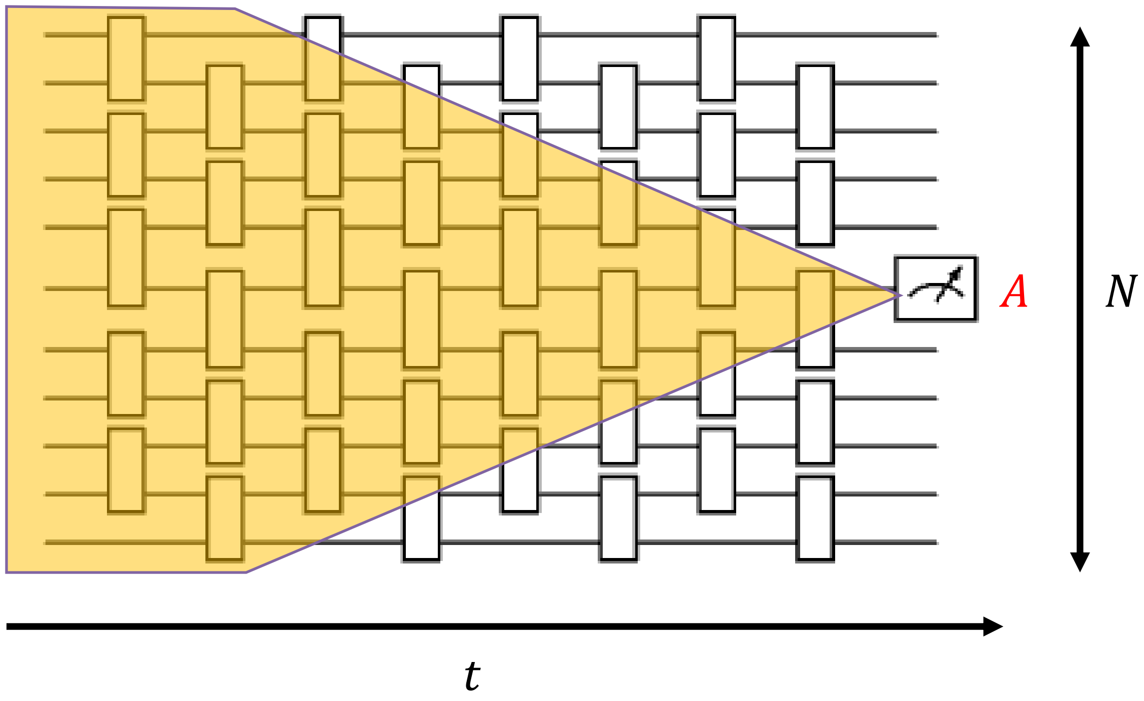
Crude estimate of error

- Dominant source of error is 2-qubit gates
- Yellow contains N_{2Q} 2-qubit gates
- Model the noisy circuit by the reduced density matrix

$$\rho_A^{\text{noisy}} = f\rho_A^{\text{ideal}} + (1-f)\frac{I^{|A|}}{2^{|A|}}, \qquad f = \left(1 - p_{2Q}\right)^{N_{2Q}}$$

- Expectation value of traceless operator A is

$$\text{Tr}_A[A\rho_A^{\text{noisy}}] = f\ \text{Tr}_A[A\rho_A^{\text{ideal}}]$$



Digital quantum simulation on noisy hardware

Hamiltonian simulation by Trotterization

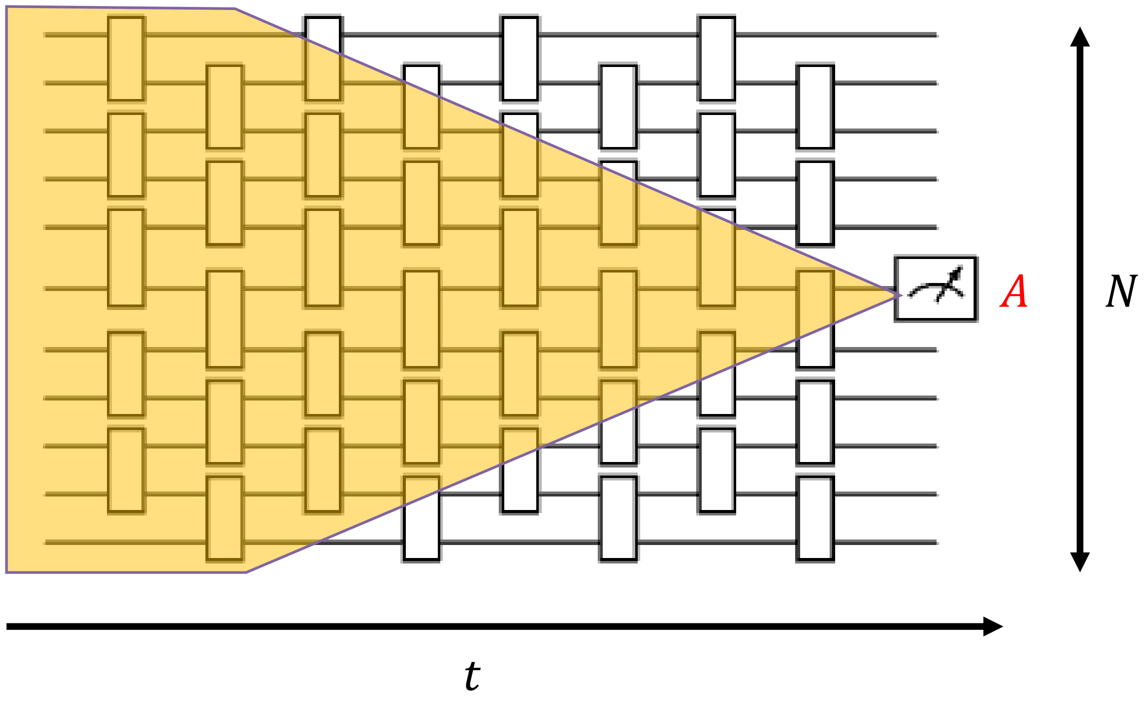
$$H = H_1 + H_2$$

$$e^{-iHt} \approx \left(e^{-iH_1 T} e^{-iH_2 T}\right)^{t/T}$$

Crude estimate of error

- Parameters: $t = 5,\ T = 0.1,\ N = 20$
- # of 2Q gates: $N_{2Q} \approx N \frac{t}{T} = 1000$
- 2Q gate error: $p_{2Q} = 2 \times 10^{-3}$
- Expectation value of traceless operator A is

$$\mathrm{Tr}_A[A\rho_A^{\mathrm{noisy}}] = f\ \mathrm{Tr}_A[A\rho_A^{\mathrm{ideal}}], \quad f = (1 - p_{2Q})^{N_{2Q}} = 0.135$$



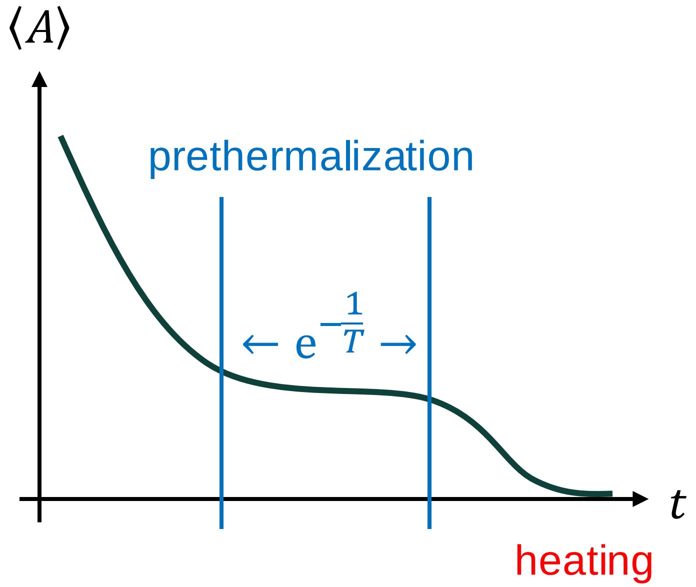
Floquet dynamics

Floquet dynamics:

- Periodically driven dynamics described by Hamiltonian $H(t+T) = H(t)$
- Floquet systems eventually heat up to infinite temperature by acquiring energy from the driving force (Floquet heating)

$$\langle\psi(t)|A|\psi(t)\rangle \approx \mathrm{Tr}[A(I^N/2^N)] \quad \text{for} \quad |\psi(t)\rangle = \mathcal{T}\mathrm{e}^{-\mathrm{i}\int_0^t \mathrm{d}t' H(t')}|\psi(0)\rangle$$

Lanzarides, Das & Moessner '14;
D'Alessio & Rigol '14;
Abanin, Roeck & Huveneers '15;
Mori, Kuwahara & Saito '16



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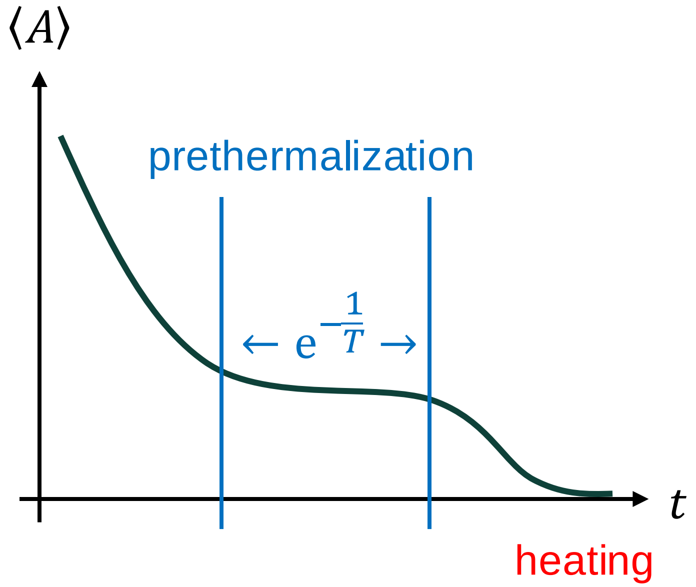
Trotterization

- Trotter dynamics for time mT, $\left(e^{-iH_1T/2}e^{-iH_2T/2}\right)^{2m}$, is a Floquet dynamics of m cycles
- Qualitative change happens as the period T is increased

→ Floquet heating happens during Trotter dynamics

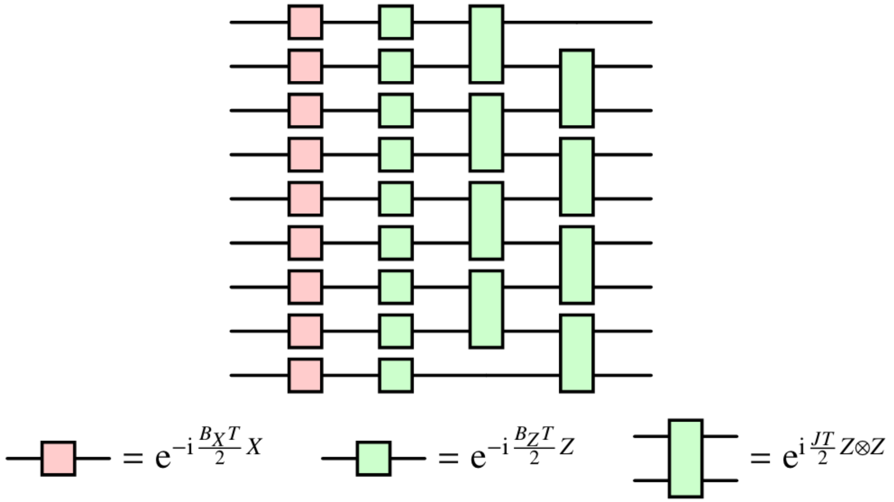
Heyl, Hauke & Zoller ‘18;
Varnier, Bertini, Giudici & Piroli ‘23



Information scrambling by Floquet circuits

Kicked-Ising model Prosen ’02, ‘07

- $U_{\mathrm{F}} = \mathrm{e}^{-\mathrm{i}H_Z\frac{T}{2}}\mathrm{e}^{-\mathrm{i}H_X\frac{T}{2}},$
- $H_Z = -J\sum_i Z_i Z_{i+1} + B_Z \sum_i Z_i, \quad H_X = B_X \sum_i X_i$
- Maximally chaotic point at $|JT| = |B_X T| = \frac{\pi}{2}$



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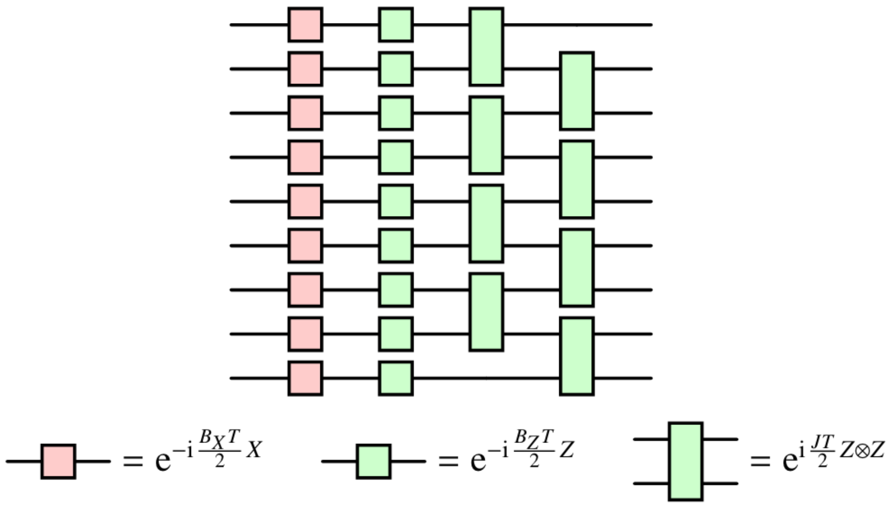


Information scrambling:

- A process of the lost information spreading across the system
- Scrambling dynamics makes it hard to recover the initial information

→ Diagnose the complexity of dynamics

Experiments on trapped-ion quantum computers

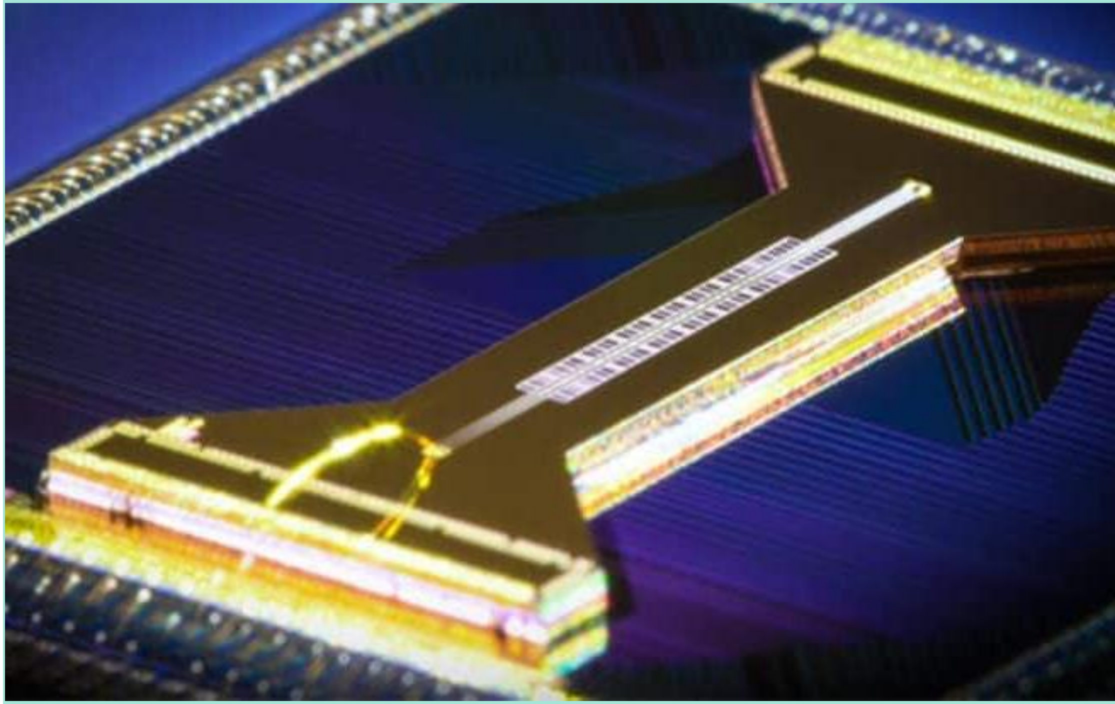
System Model H1
Available on various platforms

GOAL:

Access the feasibility of scrambling simulation on the current hardware

Seki, YK, Hayata, Yunoki ‘24

- ❑ Hayden-Preskill recovery protocol
- ❑ Interferometric protocol for OTOCs
- ❑ Thermal expectation value with microcanonical TPQ states



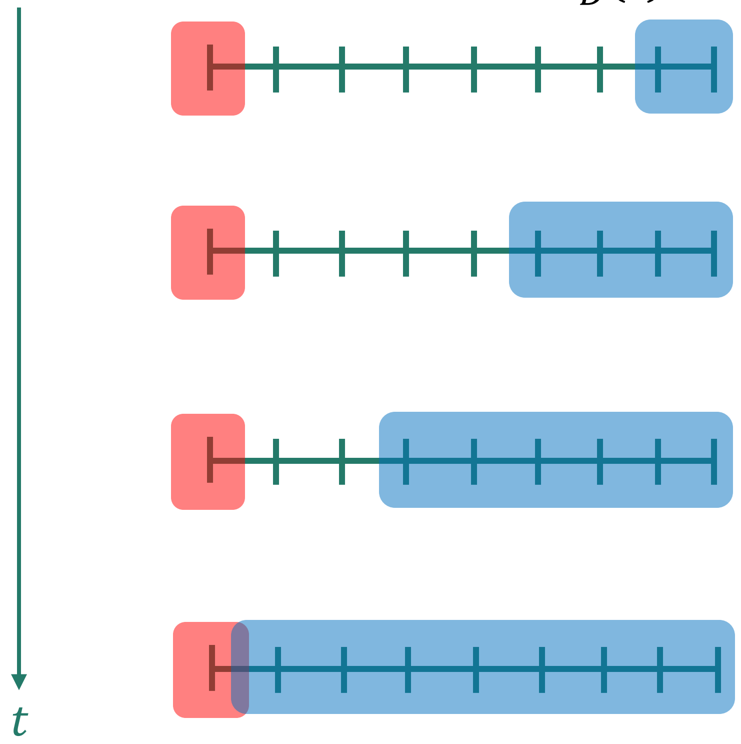
- 99.91% Two-qubit gate fidelity (arbitrary angle)
- 20 qubits
- 99.998% Single-qubit gate fidelity
- Measurement cross talk error < 0.01%
- All-to-all-connectivity
- SPAM fidelity > 99.7%

OTOCs

Larkin & Ovchinikov 1969;
Shenker & Stanford 2014

Operator growth

O_A $\quad O_D(t) = U^\dagger O_D U$



$[O_A, O_D(t)] = 0$

$[O_A, O_D(t)] \neq 0$

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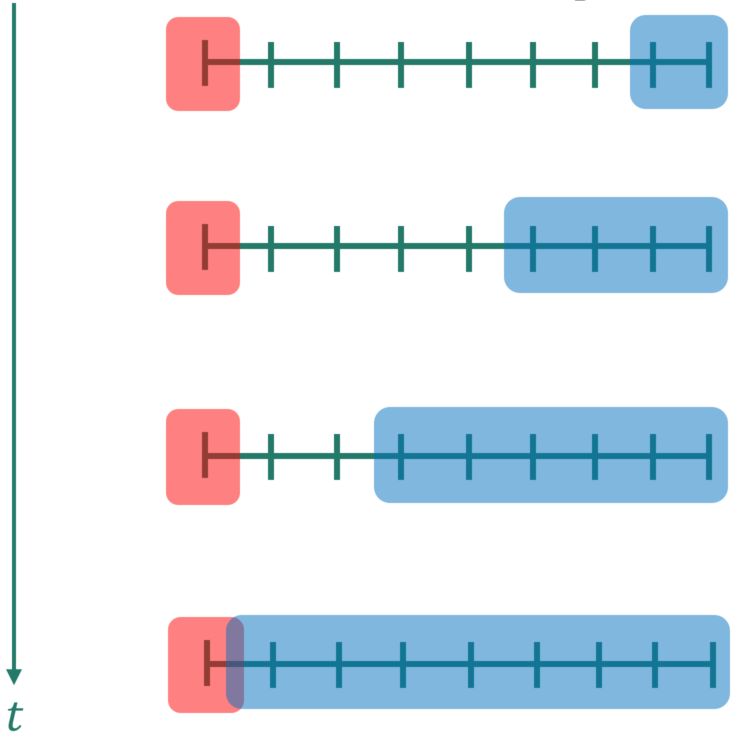

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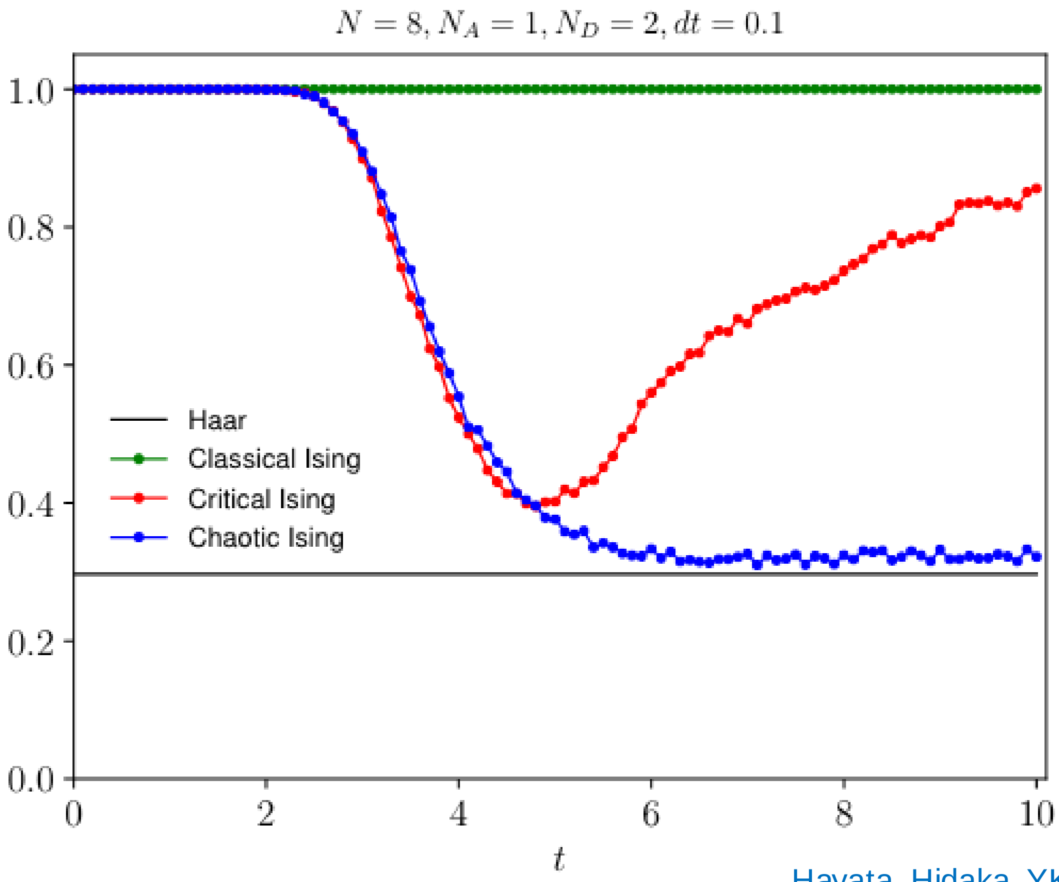
$$\mathbf{OTOC} = \langle O_A O_D(t) O_A^{\dagger} \; O_D(t)^{\dagger} \rangle$$



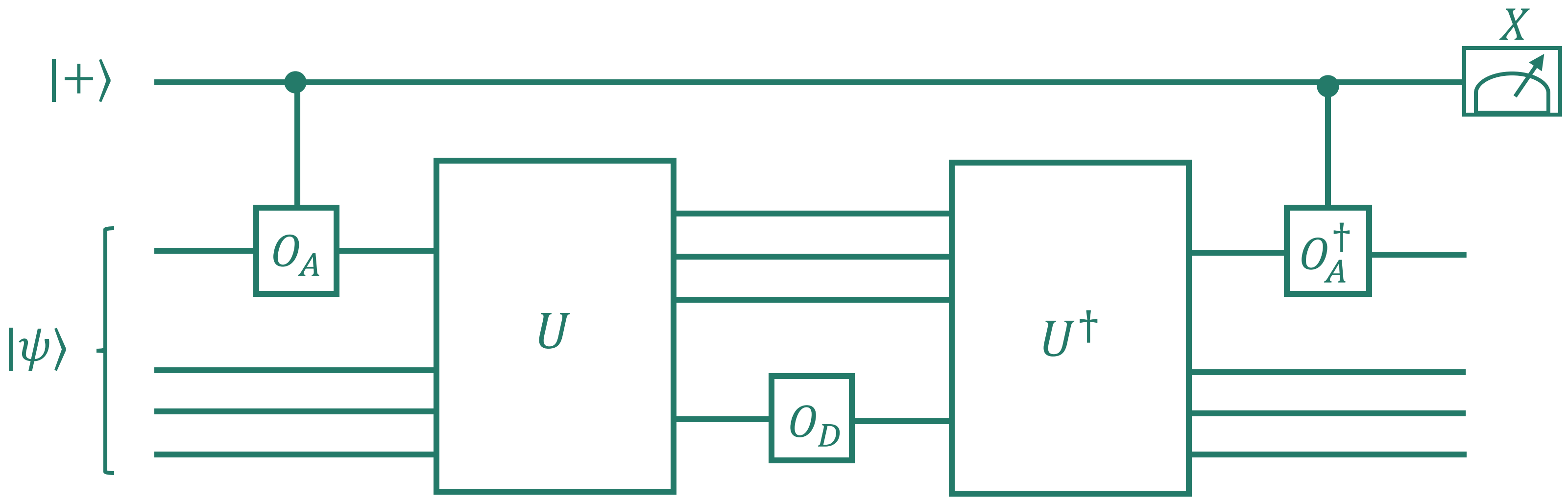
Hayata, Hidaka, YK '21

$$H = -J \sum_i Z_i Z_{i+1} + B_Z \sum_i Z_i + B_X \sum_i X_i$$

OTOCs

Interferometric protocol

$$\mathrm{OTOC} = \langle O_A O_D(t) O_A^\dagger \; O_D(t)^\dagger \rangle$$



Swingle, Bentsen, Schleier-Smith & Hayden '16
Swingle & Halpern '18
Mi et.al. '21

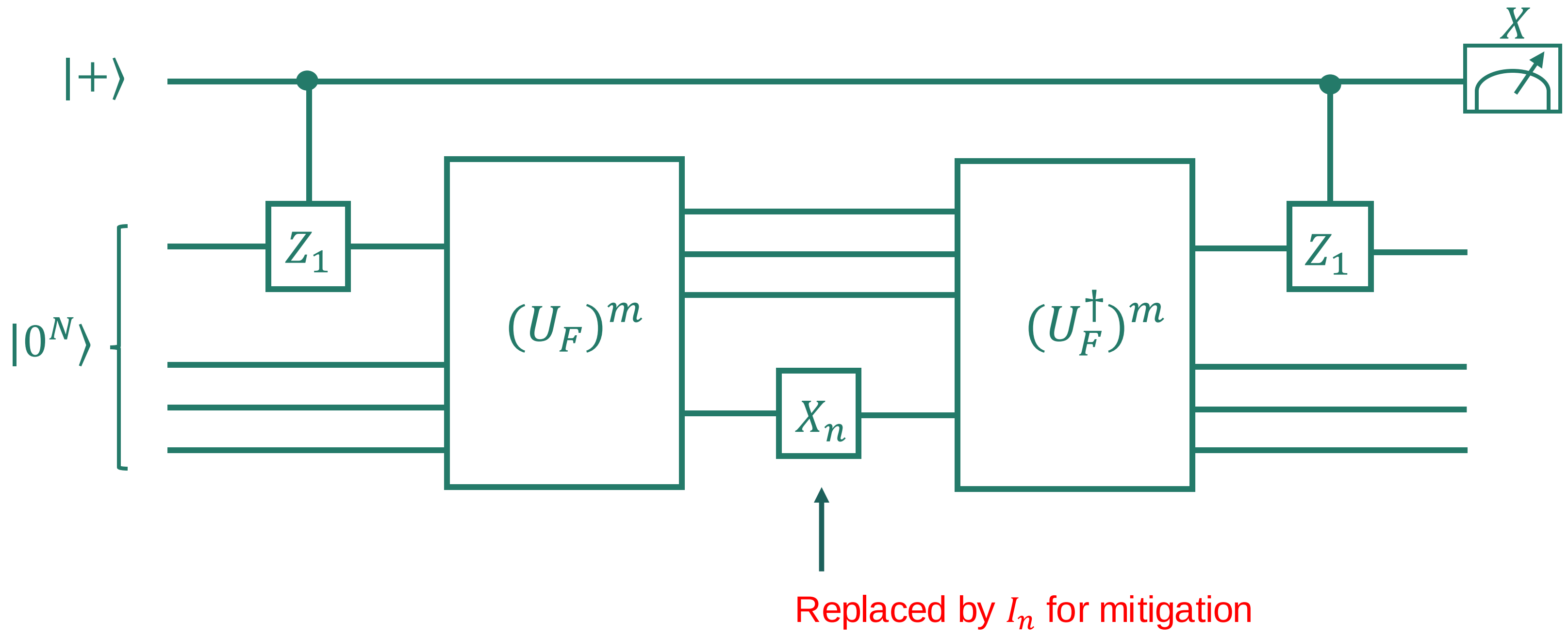
OTOCs

Interferometric protocol

$$\text{OTOC} = \langle Z_1\, X_n(t) Z_1\, X_n(t) \rangle$$

$$\text{OTOC}' = \frac{\langle Z_1\, X_n(t) Z_1\, X_n(t) \rangle}{\langle Z_1\, I_n(t) Z_1\, I_n(t) \rangle}$$: mitigates incoherent errors

$\langle Z_1\, I_n(t) Z_1\, I_n(t) \rangle = 1$ w/o noise



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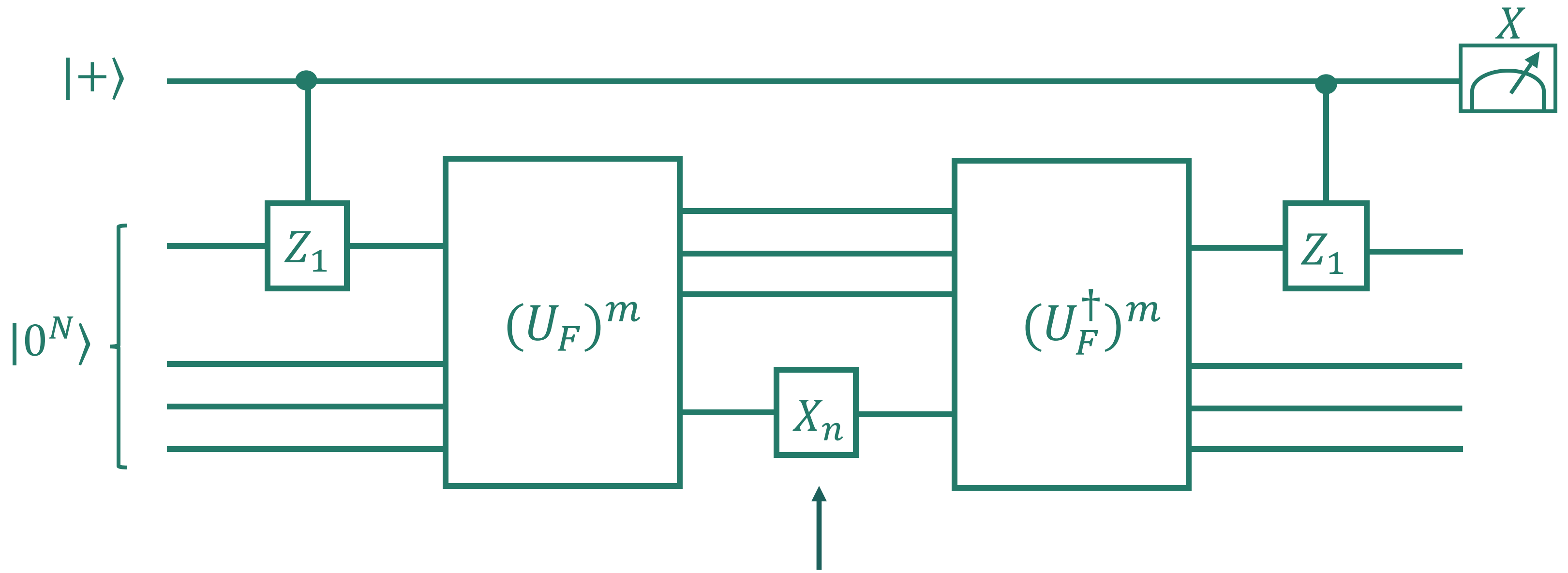


Replaced by I_n for mitigation

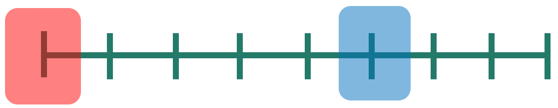
Setup

- 19-qubit spin chain + 1 ancilla qubit
- $U_F = \mathrm{e}^{-\mathrm{i}H_Z \frac{T}{2}}\, \mathrm{e}^{-\mathrm{i}H_X \frac{T}{2}}$
- 367 two-qubit gates inside the causal cone at maximum

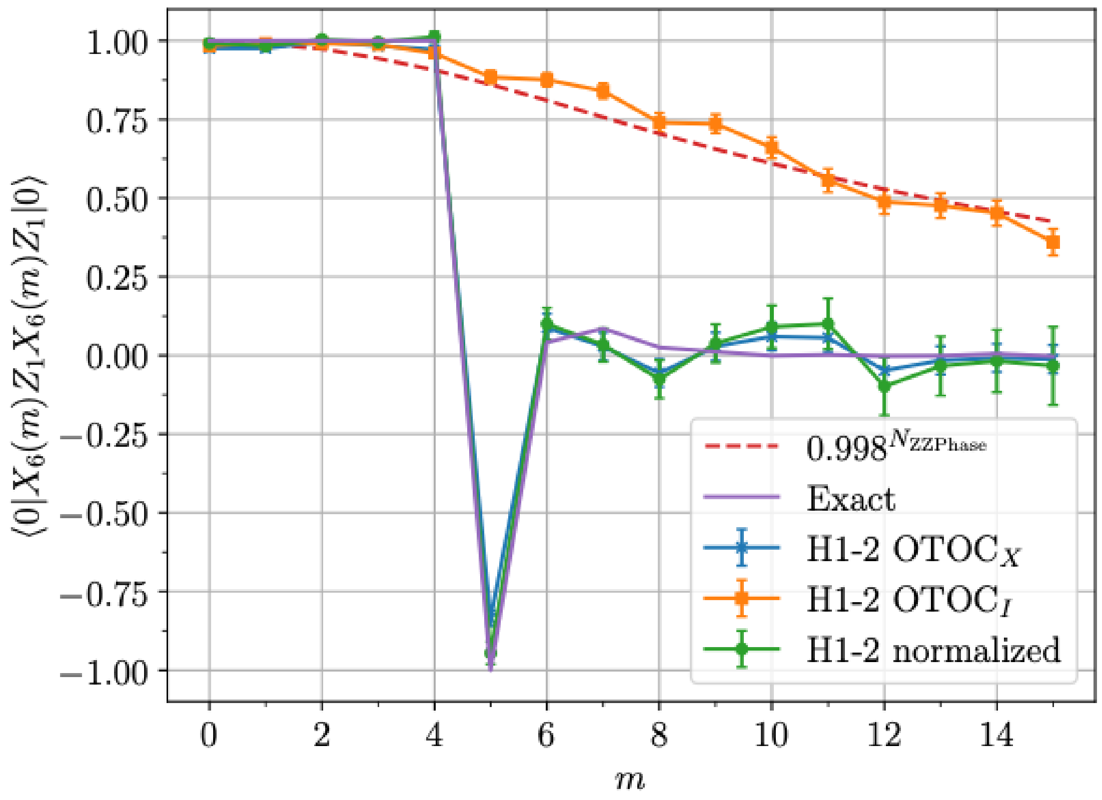
Swingle, Bentsen, Schleier-Smith & Hayden '16
Swingle & Halpern '18
Mi et.al. '21

OTOCs [Experiment]

Seki, YK, Hayata, Yunoki '24



$n = 6$



number of Trotter steps

$$\mathrm{OTOC_X} = \langle Z_1\, X_n(t) Z_1\, X_n(t)\rangle$$

$$\mathrm{OTOC_I} = \langle Z_1\, I_n(t) Z_1\, I_n(t)\rangle \approx (1 - p_{2Q})^{N_{2Q}} \approx 0.998^{N_{2Q}}$$

Observations

- OTOC_I decays due to hardware noise
- $(1 - p_{2Q})^{N_{2Q}}$ approximates OTOC_I very well

OTOCs [Experiment]

Seki, YK, Hayata, Yunoki ‘24

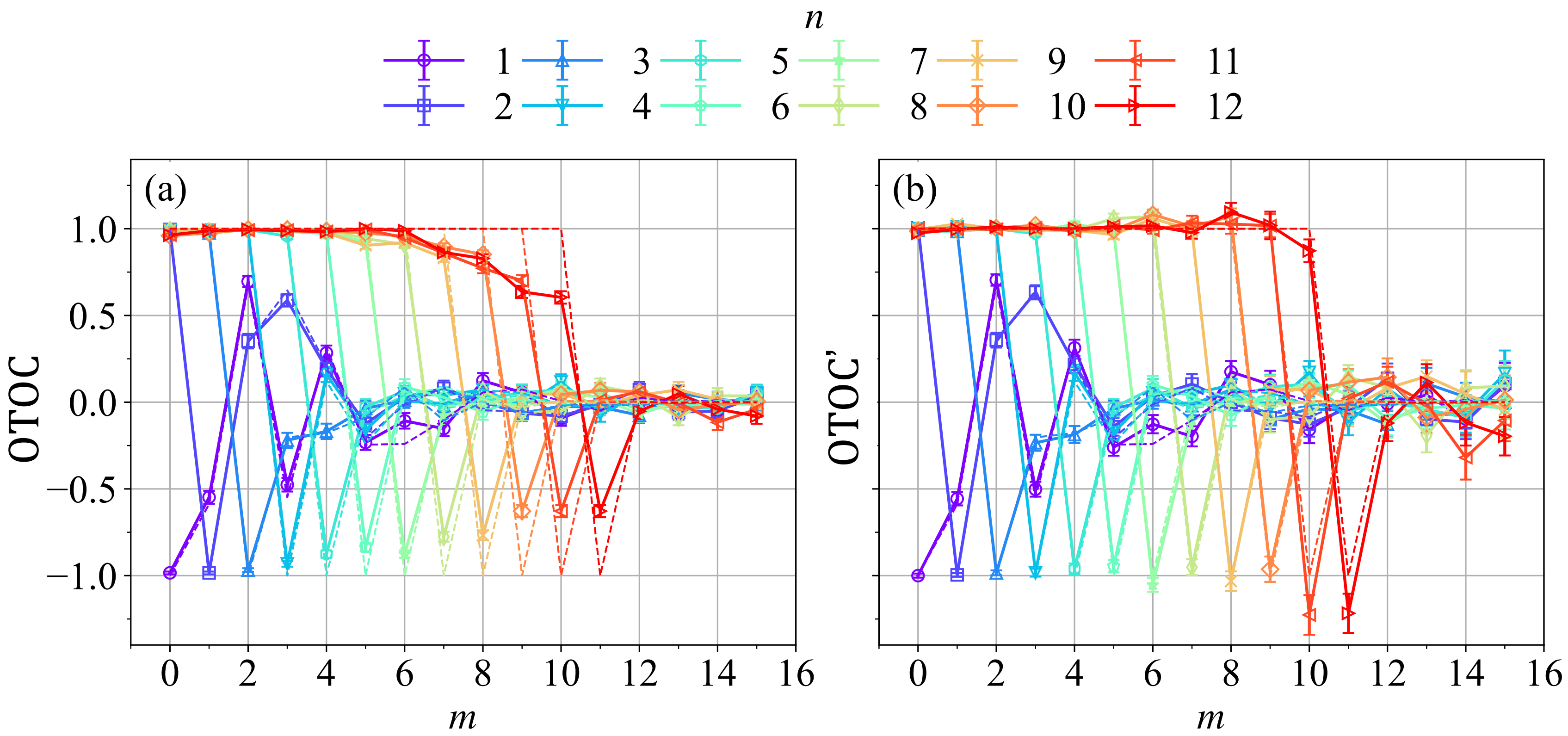
Dashed: exact

Left: $\mathrm{OTOC} = \langle Z_1\, X_n(t) Z_1\, X_n(t) \rangle$

Right: $\mathrm{OTOC}' = \dfrac{\langle Z_1\, X_n(t) Z_1\, X_n(t) \rangle}{\langle Z_1\, I_n(t) Z_1\, I_n(t) \rangle}$

Observations

- OTOC' suffers less from noise at early times
- Statistical error of OTOC' is amplified at late times



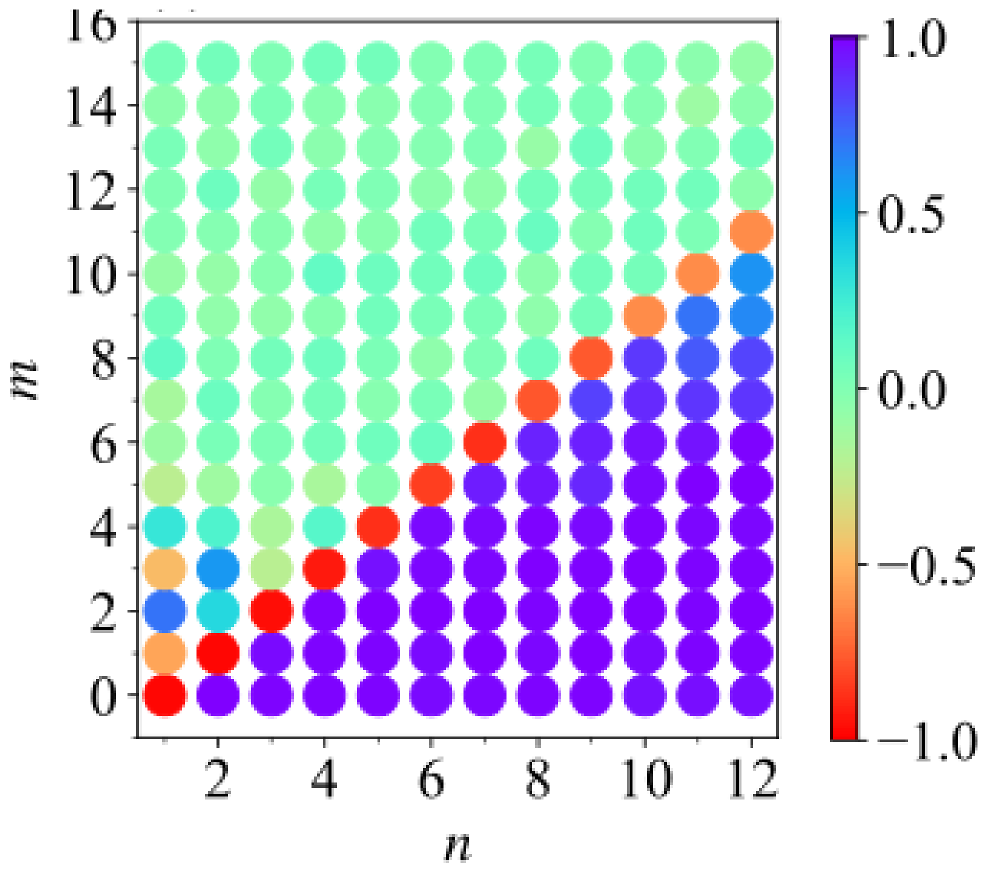
OTOCs [Experiment]

Seki, YK, Hayata, Yunoki '24

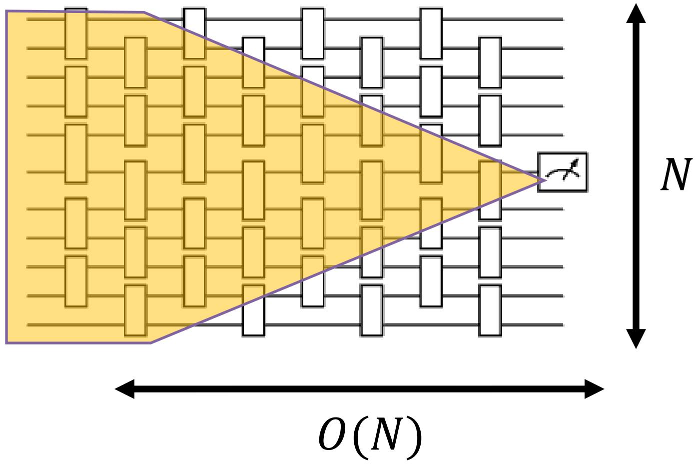
$$\mathrm{OTOC}' = \frac{\langle Z_1\, X_n(t) Z_1\, X_n(t)\rangle}{\langle Z_1\, I_n(t) Z_1\, I_n(t)\rangle}$$

Observations

- Ballistic growth of entanglement
- Statistical error of OTOC' is amplified at late times



Discussion



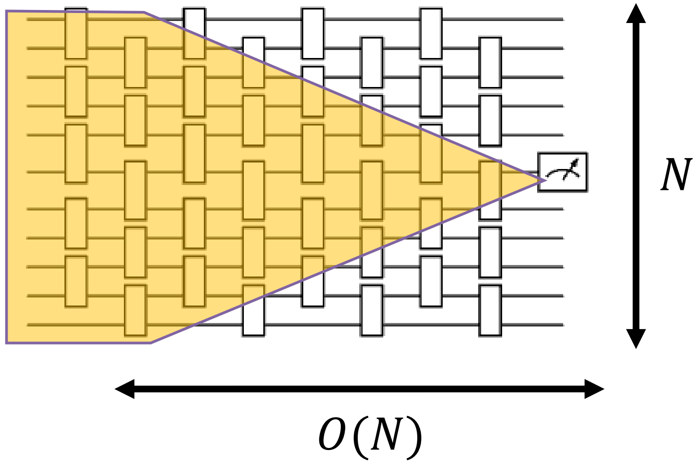
Circuit fidelity is crudely (sometimes accurately) estimated by

$$f = (1 - p_{2Q})^{N_{2Q}}$$

Geometrically local models requires $\mathrm{poly}(N)$ gates to scramble information

- 1D geometrically circuit requires $t \approx O(N)$ for the entire system to get involved: $N_{\mathrm{gate}} \sim O(N^2)$
- E.g. we used 400 2Q gates for the 20-qubit 1D system
  $\Longrightarrow$ a 40-qubit system requires 4 times larger gate counts ~ 1600 2Q gates : $f = 0.999^{1600} \approx 0.2$

Discussion



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What if hardware result deviates from the estimate?

- Memory error (on idling qubits, during ion shuttling)
- SPAM error (bias between $0 \rightarrow 1$ and $1 \rightarrow 0$)
- Gate counting analysis overestimates the error

Schiffer, Rubio Trivedi & Cirac ‘24
Granet & Dreyer ‘24
Chertkov, Chen, Lubasch, Hayes, Foss-Feig ‘24