The Physics of Parity-Doubled Nucleons at High Density

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Outline

Clarify the role of $N^*(\frac{1}{2}^-)$ near chiral restoration

cf. $\sigma(0^+)$ forming a parity doublet with pions

I. Net-baryon number fluctuations near LG & chiral phase boundaries

- DeTar-Kunihiro model (aka parity doublet model)

[DeTar & Kunihiro (89)]

II. Emergent chiral symmetry in neutron matter

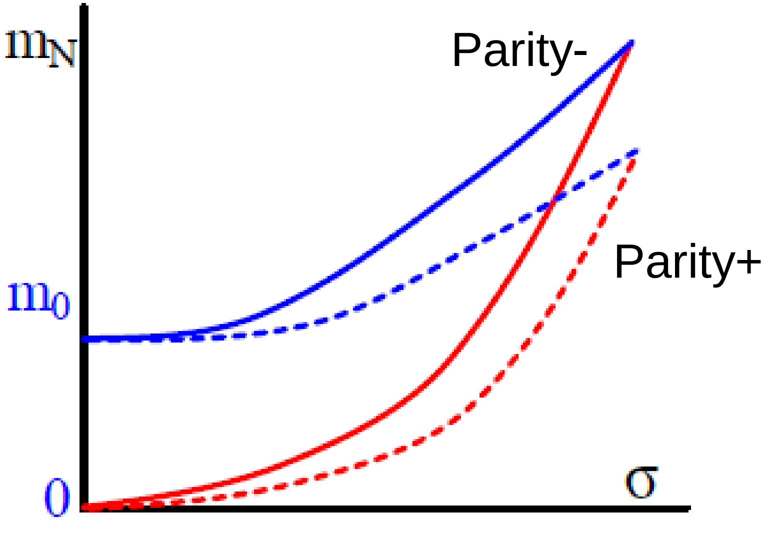
- P-wave neutron superfluidity

[Tamagaki (70); Takatsuka & Tamagaki (71)]

DeTar-Kunihiro/Parity doublet model

- SU(2) chiral transformation of 2 nucleons
  → how to assign 2 indep. rotation to them?

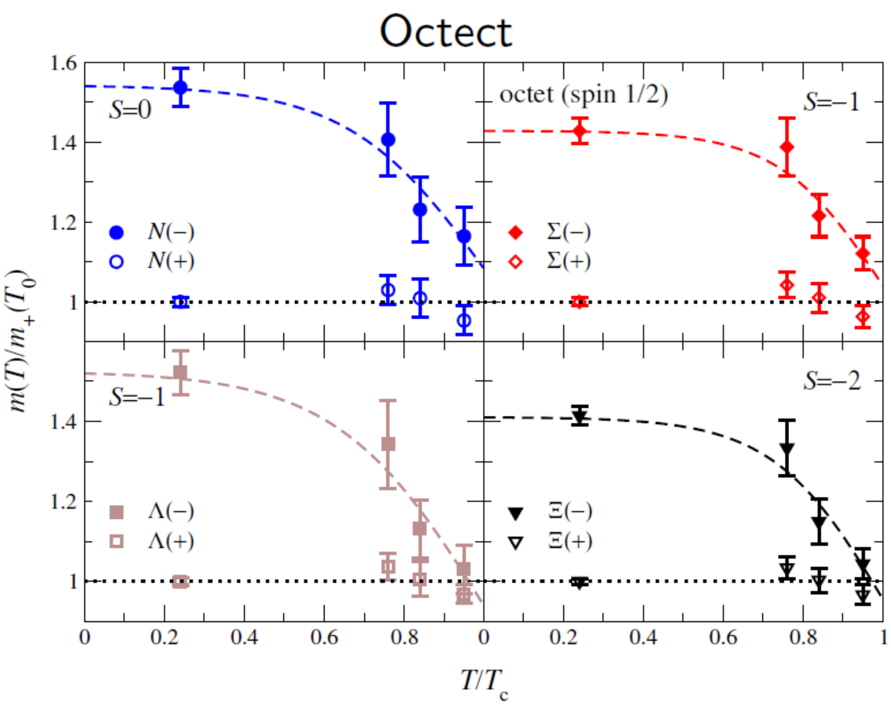
$$\mathcal{L}_m = m_0 \left(\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2 \right) \Rightarrow m_{N_\pm} = \frac{1}{2} \left[\sqrt{c_1 \sigma^2 + 4 m_0^2} \mp c_2 \sigma \right]$$

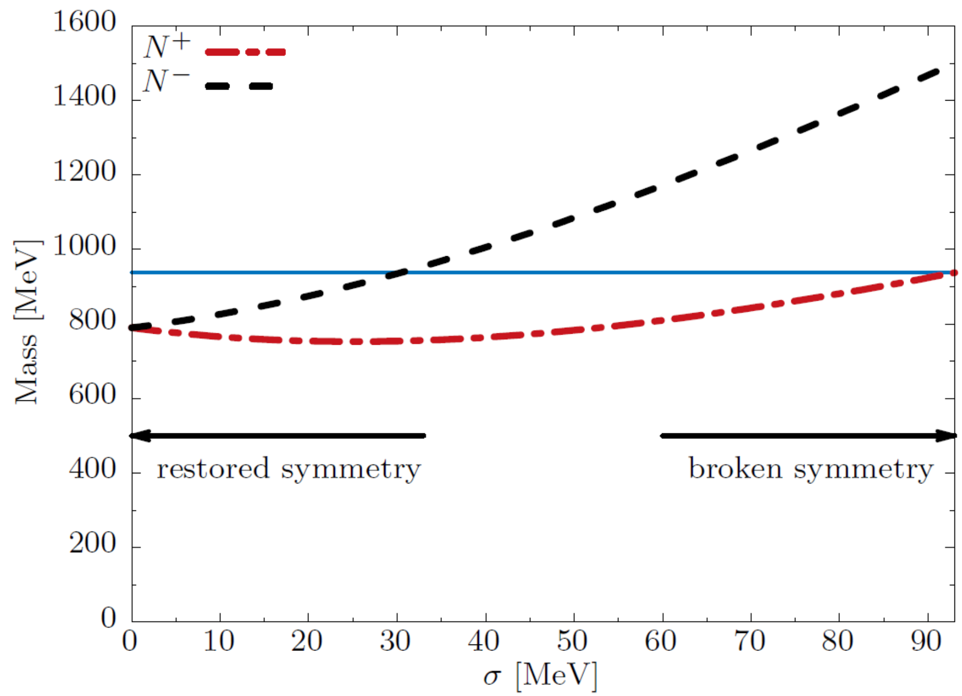


[DeTar and Kunihiro, 1989]

Red: Naive
Blue: Mirror

Parity doubling of baryons





- Lattice QCD at zero μ

[Aarts et al., 2016-2019]

- Survival mass $m_N \approx m_0 \neq 0$

$$M_{\pm} = \sqrt{m_0^2 + c_1^2\sigma^2} \mp c_2\sigma \xrightarrow{\sigma\to 0} m_0$$

Caution

- ❑ N*(1535): the lowest-lying resonance
  - πN & ηN interactions in a dynamical approach
  - cf. the nature of $f_0(500)$ vs. σ meson in LSM
- ❑ Hadronic picture at high density
  - A consistent description for CS restoration
  - CS realized linearly → parity (chiral) doublet

❖ $\mathcal{L}_{\rm eff}$ near CSR; Symmetries & universality

V. Koch, M. Marczenko, K. Redlich and C. Sasaki, Phys.Rev.D 109 (2024);
M. Marczenko, K. Redlich and C. Sasaki, arXiv:2410.21746 [nucl-th]

FLUCTUATIONS AND CORRELATIONS OF BARYONIC CHIRAL PARTNERS

Net proton vs. baryon number fluct.

χ_2^B sensitive to the QCD phase transition

→Net proton fluctuations as a good proxy for net baryon fluctuations: folklore

✓Nucleon parity doublet: N(939) & N*(1535)

- Mean: $\langle N_B \rangle \equiv \kappa_1^B = \kappa_1^+ + \kappa_1^-$
- Variance: $\langle \delta N_B \delta N_B \rangle \equiv \kappa_2^B = \kappa_2^{++} + \kappa_2^{--} + 2\kappa_2^{+-}$
- Cumulants → susceptibilities:

$$\kappa_n^B = VT^3 \chi_n^B \qquad \chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$$

- Sign and strength of χ_2^{+-}?

Thermodynamics of parity doubler

Linear sigma model for (σ,π), ω, (N,N*) & MF

- New chemical potentials $\mu_{+,-}$ for N,N*
- Set at the end $\mu_{\pm} = \mu_N = \mu_B - g_\omega \omega$
- Susceptibilities from thermodynamics pot.

$$\Omega = \Omega_+ + \Omega_- + V_\sigma + V_\omega$$

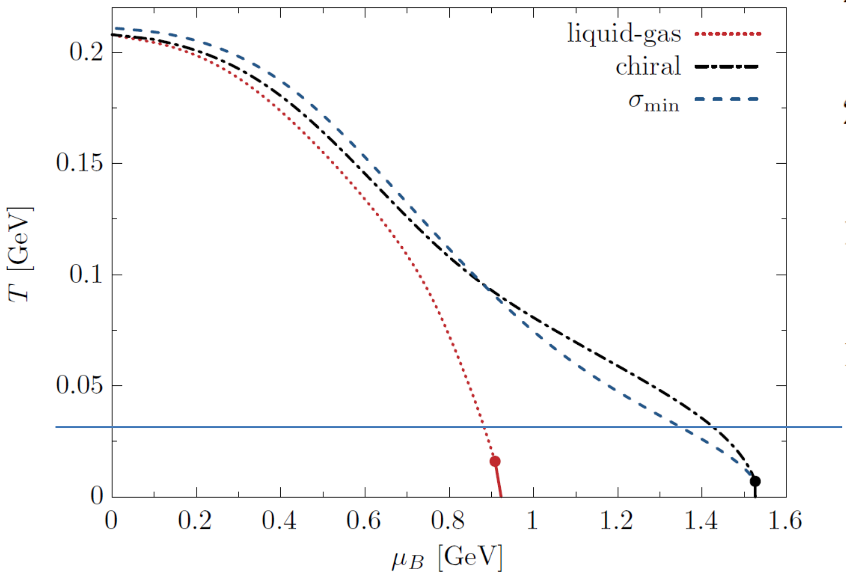
$$0 = \frac{\partial \Omega}{\partial \sigma}$$

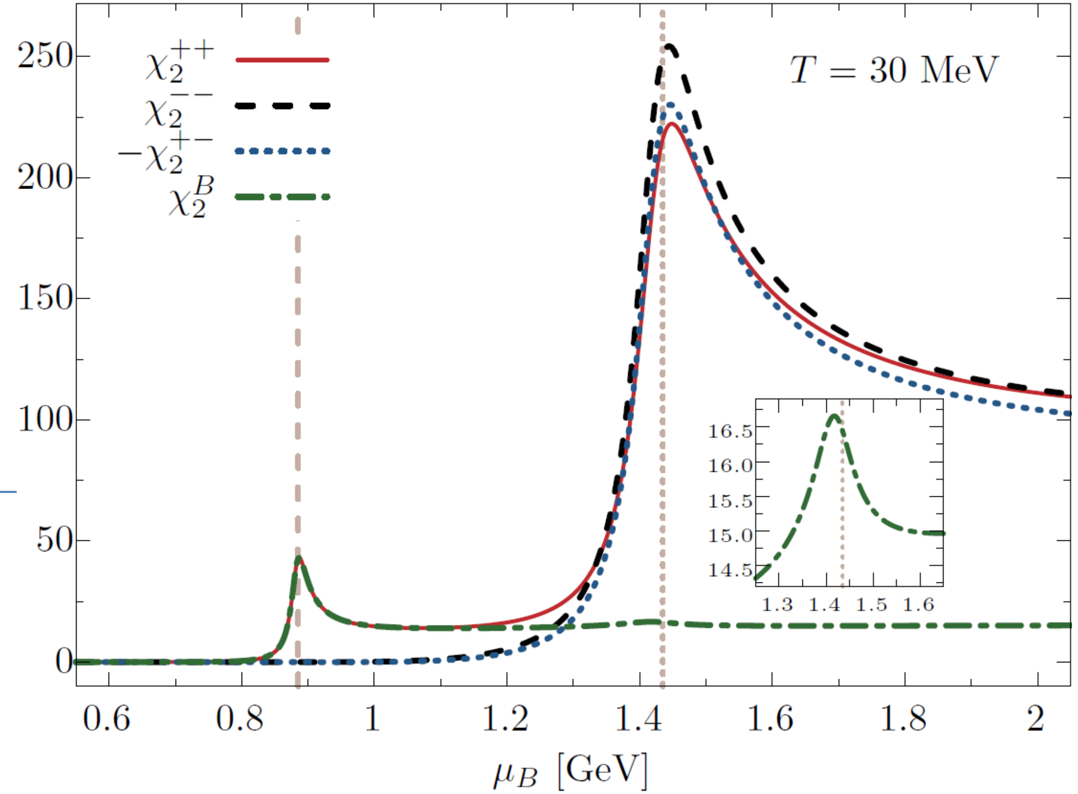
$$0 = \frac{\partial \Omega}{\partial \omega}$$

$$\chi_2^{\alpha\beta} = \frac{1}{VT^3}\kappa_2^{\alpha\beta} = -\frac{\mathrm{d}^2\hat{\Omega}}{\mathrm{d}\hat{\mu}_\alpha \mathrm{d}\hat{\mu}_\beta}\Bigg|_{T,\mu_\alpha=\mu_\beta=\mu_N}$$

$$\chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$$

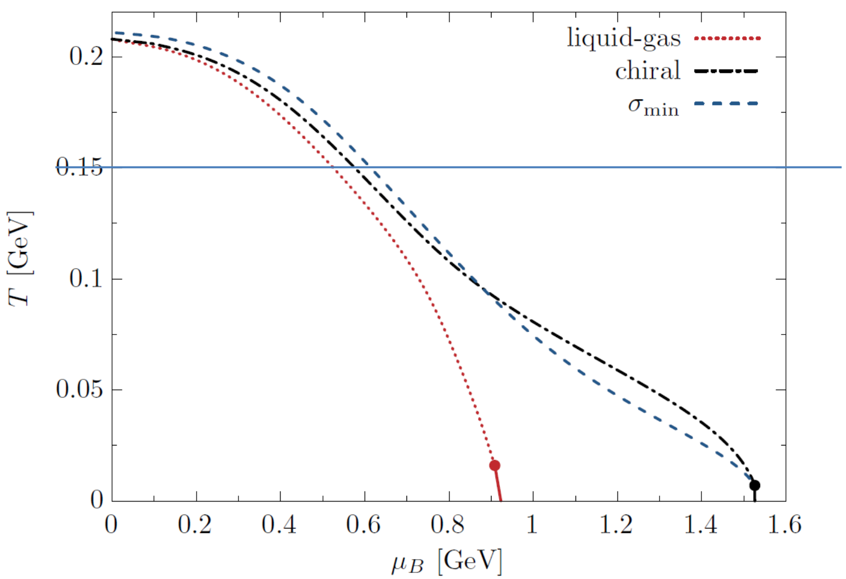
Liquid-gas vs. chiral

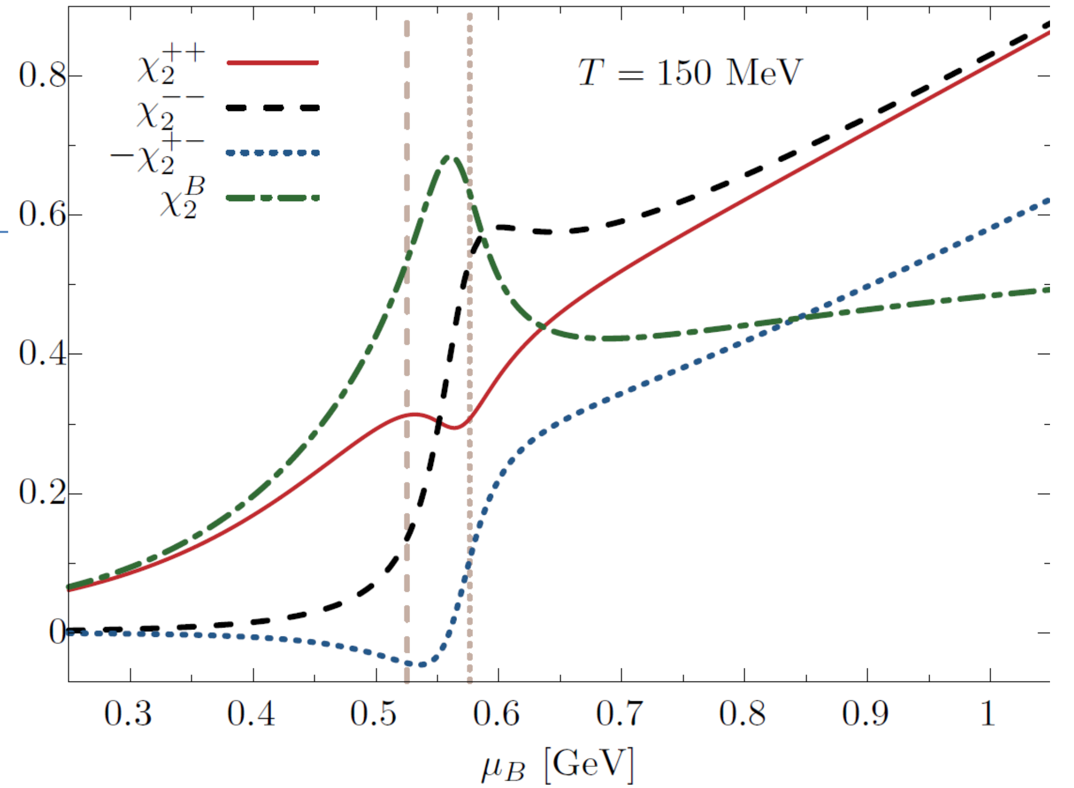




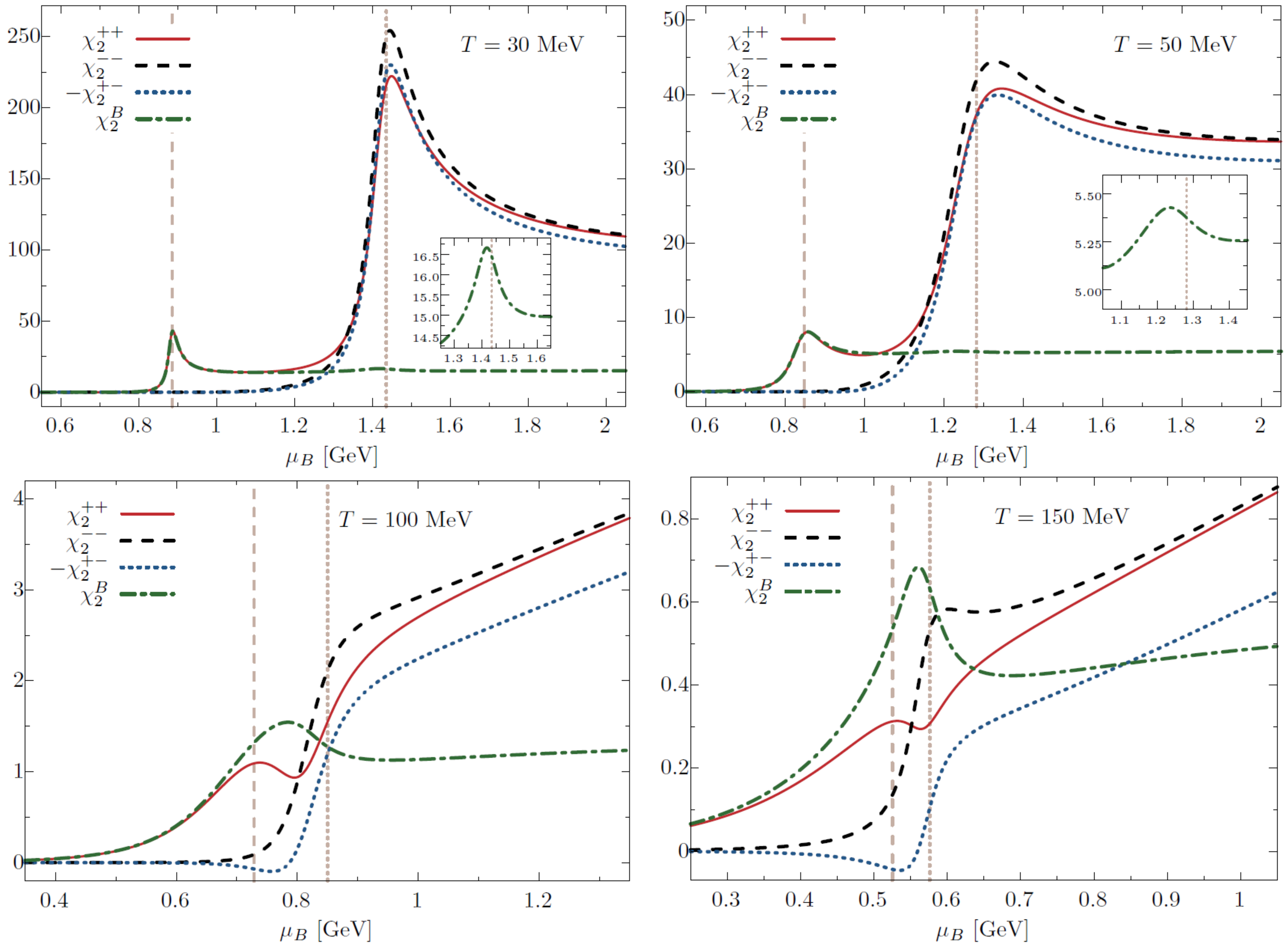
- LG dominated by χ_2^{++}
- Chiral dominated by both, but $\chi_2^{--} > \chi_2^{++}$
- Peaks diminished by χ_2^{+-} ➔ weak signal in χ_2^B

Liquid-gas vs. chiral





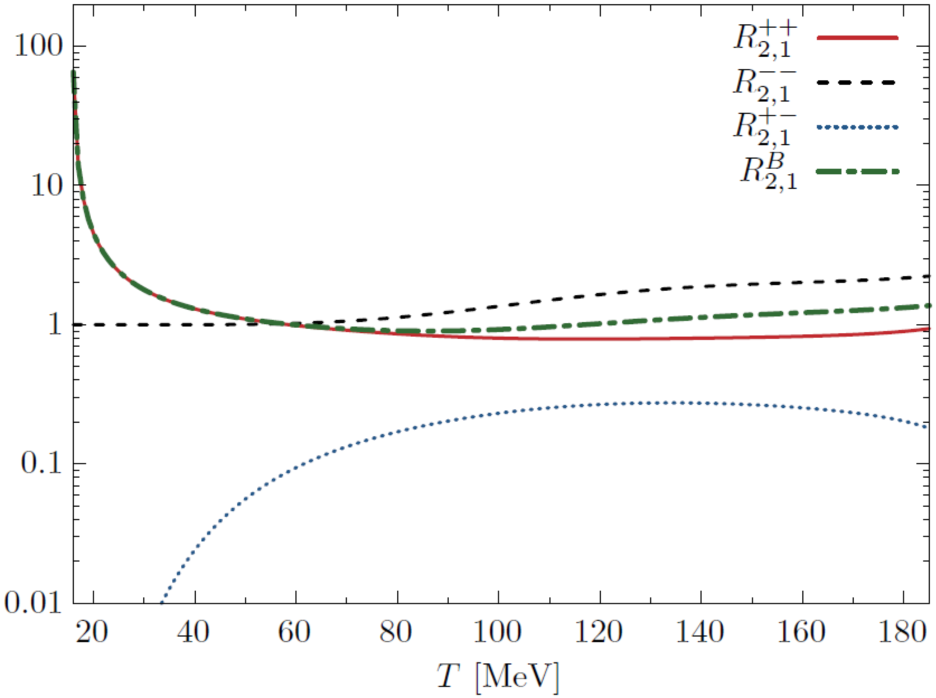
- ❑ Increasing T ➔ 2 peaks getting closer
- ❑ Qualitative difference of χ_2^{++} from χ_2^{--}
- ❑ Stronger signal left in χ_2^B



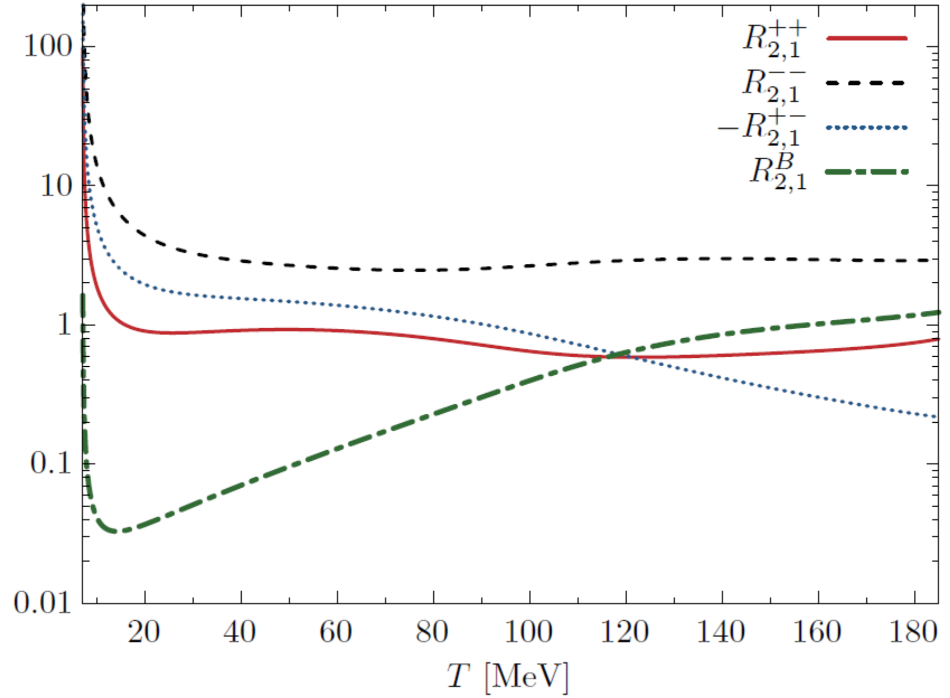
$T = 30$ MeV
$T = 50$ MeV
$T = 100$ MeV
$T = 150$ MeV
χ_2^{++}
χ_2^{--}
$-\chi_2^{+-}$
χ_2^{B}
μ_B [GeV]

χ_2/χ_1 along the phase boundary

↓ CP of LG

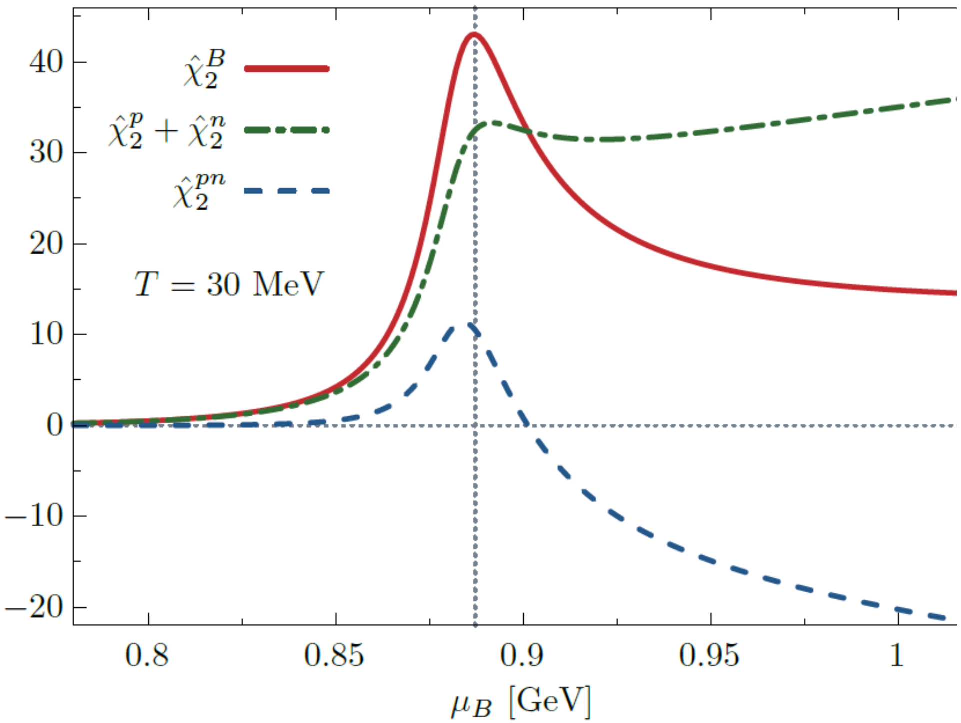


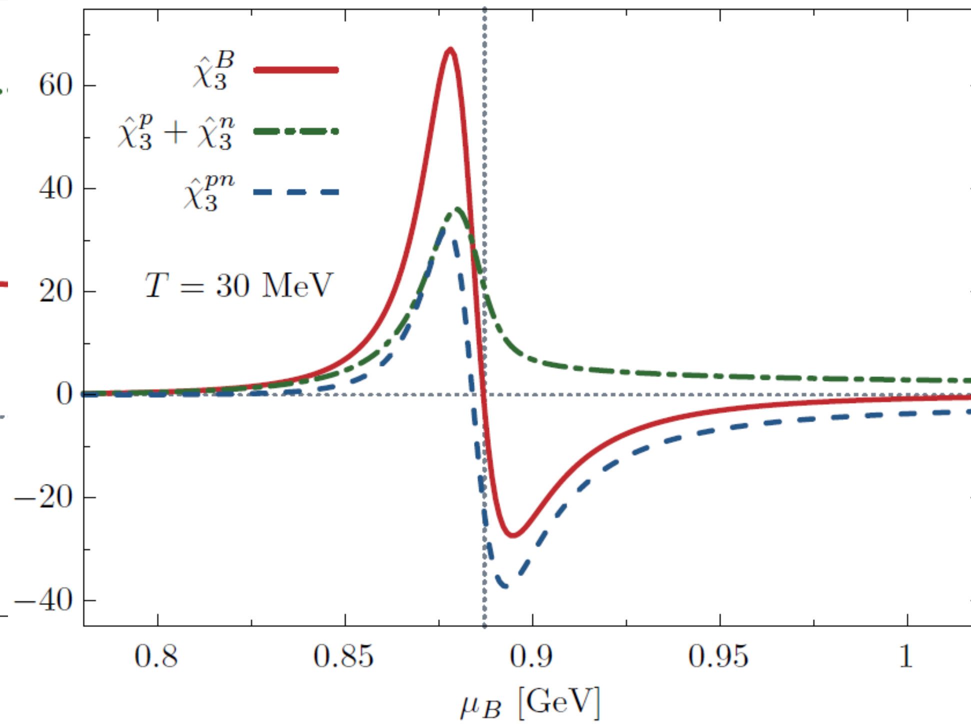
↓ QCD CP



- The net-proton fluctuations do not necessarily reflect the net-baryon fluctuations at the chiral phase boundary.

Isospin correlations near LG





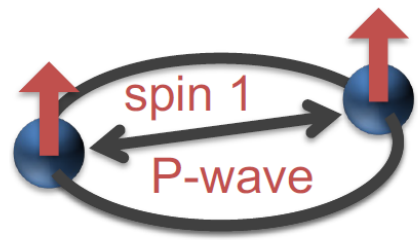
$$\chi_n^B \neq \chi_n^p$$

S. Yasui, M. Nitta and C. Sasaki, arXiv:2409.05670

SUPERFLUIDITY IN NEUTRON STARS

Superfluidity in neutron stars

- s-wave superfluid by $^1S_0$ [Migdal, ‘60]
- p-wave superfluid by $^3P_2$ at $\rho/\rho_0 > ½$ [Tabakin, ‘68]
  - ✓ Pulser glitches
  - ✓ Rapid cooling



$^{2S+1}L_J$

S: spin
L: angular momentum
J: spin+angular momentum

- This study: Cooper pairing of parity-doubled neutrons at high density → the role of N*
  - Generalized χ-sym G such that G ⊃ naïve&mirror
    $G = U(1)_{1L} \times U(1)_{1R} \times U(1)_{2L} \times U(1)_{2R}$
  - Common operators to the naïve & mirror assign.

Symmetries

- $U(1)_L \times U(1)_R$ chiral symmetry

$$(n, n^*) \Leftrightarrow (\psi_1, \psi_2),\ \ \psi_i = \psi_{iL} + \psi_{iR}$$

  - Naïve assignment

$$\psi_{1\mathrm{L}} \to U_{\mathrm{L}}\psi_{1\mathrm{L}}, \quad \psi_{2\mathrm{L}} \to U_{\mathrm{L}}\psi_{2\mathrm{L}}, \quad \psi_{1\mathrm{R}} \to U_{\mathrm{R}}\psi_{1\mathrm{R}}, \quad \psi_{2\mathrm{R}} \to U_{\mathrm{R}}\psi_{2\mathrm{R}}$$

  - Mirror assignment

$$\psi_{1\mathrm{L}} \to U_{\mathrm{L}}\psi_{1\mathrm{L}}, \quad \psi_{2\mathrm{L}} \to U_{\mathrm{R}}\psi_{2\mathrm{L}}, \quad \psi_{1\mathrm{R}} \to U_{\mathrm{R}}\psi_{1\mathrm{R}}, \quad \psi_{2\mathrm{R}} \to U_{\mathrm{L}}\psi_{2\mathrm{R}}$$

- Generalized chiral symmetry

$$G = U(1)_{1L} \times U(1)_{2L} \times U(1)_{1R} \times U(1)_{2R}$$

$$\psi_{1\mathrm{L}} \to U_{1\mathrm{L}}\psi_{1\mathrm{L}}, \quad \psi_{2\mathrm{L}} \to U_{2\mathrm{L}}\psi_{2\mathrm{L}}, \quad \psi_{1\mathrm{R}} \to U_{1\mathrm{R}}\psi_{1\mathrm{R}}, \quad \psi_{2\mathrm{R}} \to U_{2\mathrm{R}}\psi_{2\mathrm{R}}$$

Naïve: $U_{1L} = U_{2L}, U_{1R} = U_{2R}$ Mirror: $U_{1L} = U_{2R}, U_{1R} = U_{2L}$

Symmetries

Define 2 symmetries as $[\psi_L^t = (\psi_{1L}, \psi_{2L})^t]$

$$\psi_{\mathrm{L}} \to e^{i\theta_{\mathrm{L}}}\psi_{\mathrm{L}}, \quad \psi_{\mathrm{R}} \to e^{i\theta_{\mathrm{R}}}\psi_{\mathrm{R}}, \quad \text{with} \quad (e^{i\theta_{\mathrm{L}}}, e^{i\theta_{\mathrm{R}}}) \in \mathrm{U}(1)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{R}}$$

$$\psi_{\mathrm{L}} \to e^{i\tau_3\theta_{\mathrm{L}}}\psi_{\mathrm{L}}, \quad \psi_{\mathrm{R}} \to e^{i\tau_3\theta_{\mathrm{R}}}\psi_{\mathrm{L}}, \quad \text{with} \quad (e^{i\tau_3\theta_{\mathrm{L}}}, e^{i\tau_3\theta_{\mathrm{R}}}) \in \mathrm{U}(1)_{(1-2)\mathrm{L}} \times \mathrm{U}(1)_{(1-2)\mathrm{R}}$$

$$\mathrm{U}(1)_{1\mathrm{L}} \times \mathrm{U}(1)_{2\mathrm{L}} = \frac{\mathrm{U}(1)_{\mathrm{L}} \times \mathrm{U}(1)_{(1-2)\mathrm{L}}}{\mathbb{Z}'_{2\mathrm{L}}}, \quad \mathrm{U}(1)_{1\mathrm{R}} \times \mathrm{U}(1)_{2\mathrm{R}} = \frac{\mathrm{U}(1)_{\mathrm{R}} \times \mathrm{U}(1)_{(1-2)\mathrm{R}}}{\mathbb{Z}'_{2\mathrm{R}}}$$

- Global sym G and its subgroups

$$\mathrm{U}(1)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{R}} \subset \frac{\mathrm{U}(1)_{\mathrm{L}} \times \mathrm{U}(1)_{(1-2)\mathrm{L}}}{\mathbb{Z}'_{2\mathrm{L}}} \times \frac{\mathrm{U}(1)_{\mathrm{R}} \times \mathrm{U}(1)_{(1-2)\mathrm{R}}}{\mathbb{Z}'_{2\mathrm{R}}} = \mathrm{U}(1)_{1\mathrm{L}} \times \mathrm{U}(1)_{2\mathrm{L}} \times \mathrm{U}(1)_{1\mathrm{R}} \times \mathrm{U}(1)_{2\mathrm{R}}$$

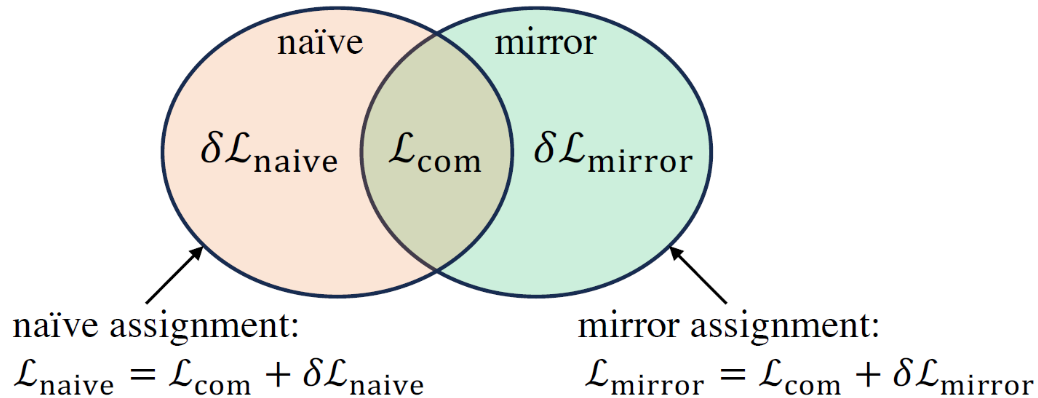
- ***Emergent chiral symmetry*** for $(\psi_1, \psi_2)^t$

$$\mathrm{U}(1)_{(1-2)\mathrm{L}} \times \mathrm{U}(1)_{(1-2)\mathrm{R}}$$

- Both naïve & mirror as subgroups of ECS

The Lagrangian w/ ECS

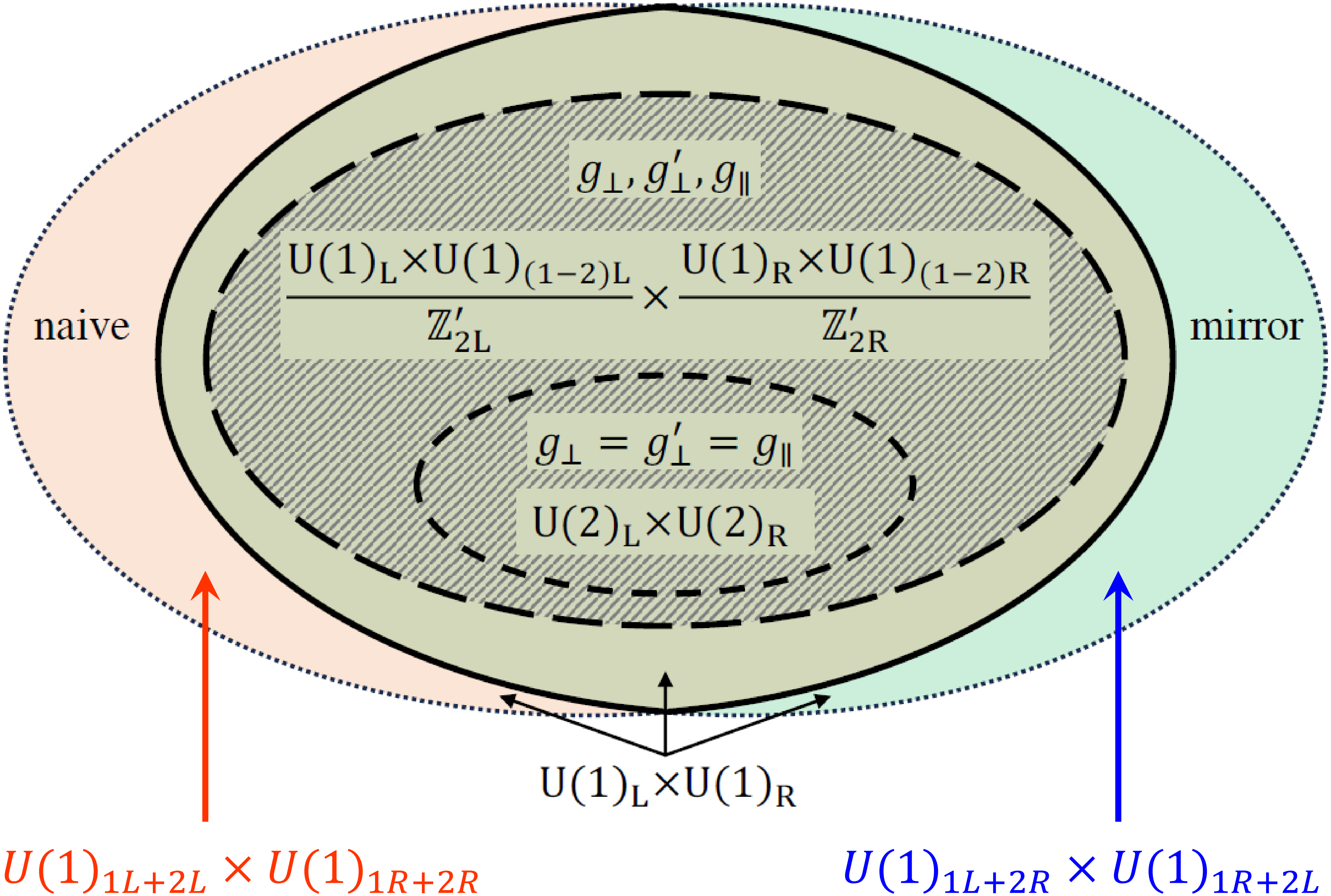
$$\mathcal{L} = \mathcal{L}_{\text{com}} + \delta\mathcal{L}_{\text{naive}} + \delta\mathcal{L}_{\text{mirror}}$$



- Pairing formation → 4-point interactions

$$\begin{aligned}\mathcal{L}_{\text{com}} &= \bar{\psi}_1 i\gamma\partial\psi_1 + \bar{\psi}_2 i\gamma\partial\psi_2 \\ &- 4g_\perp\left((\bar{\psi}_1\psi_1)^2 + (\bar{\psi}_1 i\gamma_5\psi_1)^2\right) - 4g'_\perp\left((\bar{\psi}_2\psi_2)^2 + (\bar{\psi}_2 i\gamma_5\psi_2)^2\right) \\ &- 8g_{||}\left((\bar{\psi}_1\psi_2)(\bar{\psi}_2\psi_1) + (\bar{\psi}_1 i\gamma_5\psi_2)(\bar{\psi}_2 i\gamma_5\psi_1)\right).\end{aligned}$$

- Special case: 3 equal coupling constants
  → $SU(2)_L \times SU(2)_R$ emergent chiral sym.



$g_{\perp}, g'_{\perp}, g_{\parallel}$
$\frac{U(1)_L \times U(1)_{(1-2)L}}{\mathbb{Z}'_{2L}} \times \frac{U(1)_R \times U(1)_{(1-2)R}}{\mathbb{Z}'_{2R}}$
naive
mirror
$g_{\perp} = g'_{\perp} = g_{\parallel}$
$U(2)_L \times U(2)_R$
$U(1)_L \times U(1)_R$
$U(1)_{1L+2L} \times U(1)_{1R+2R}$
$U(1)_{1L+2R} \times U(1)_{1R+2L}$

Mean-field analyses

- ❑ A simplified Lagrangian assuming $g_\perp = g'_\perp$

$$\mathcal{L}_{\rm com} = \bar\psi i\gamma\partial\psi - 2g_\perp\left((\bar\psi\tau_0\psi)^2 + (\bar\psi\tau_3\psi)^2 + (\bar\psi i\gamma_5\tau_0\psi)^2 + (\bar\psi i\gamma_5\tau_3\psi)^2\right)$$
$$- 2g_{||}\left((\bar\psi\tau_1\psi)^2 + (\bar\psi\tau_2\psi)^2 + (\bar\psi i\gamma_5\tau_1\psi)^2 + (\bar\psi i\gamma_5\tau_2\psi)^2\right),$$

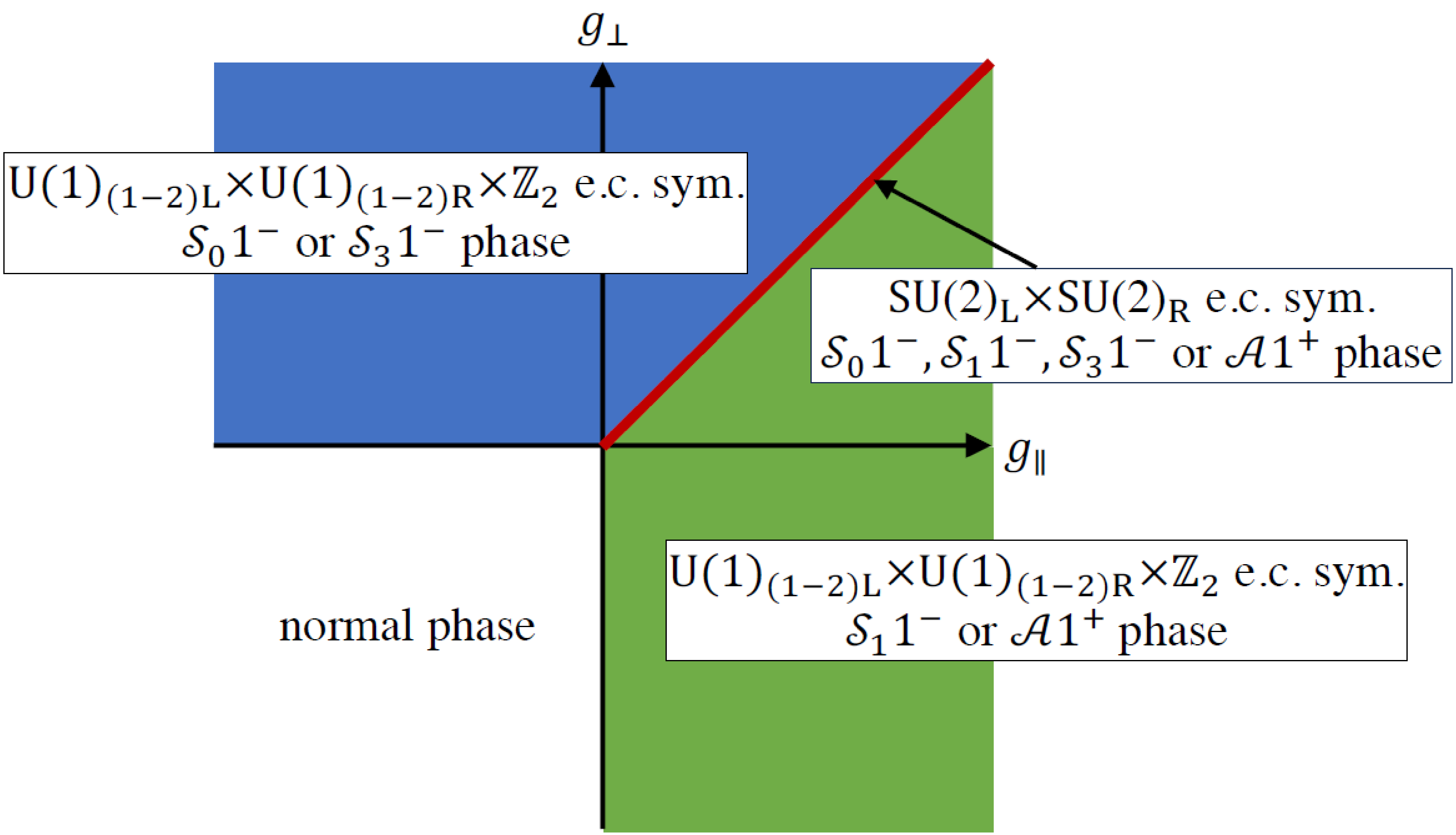
- ❑ Nambu-Gor'kov formalism, mean-field approx. to get the thermodynamic potential

❖ Pairings [note: $\psi_C = C\gamma^0\psi^*, C = i\gamma^2\gamma^0$]

- $\bar\psi_C\vec\gamma\gamma_5\tau_a\psi$: vector (a=0,1,3), symmetric → $\mathcal{S}_a 1^-$
- $\bar\psi_C\vec\gamma\tau_2\psi$: axial-vector, anti-symmetric → $\mathcal{A}1^+$

Phase diagram

- Cooper pairs: exp. values of $\mathcal{S}_a 1^-$ and $\mathcal{A}1^+$



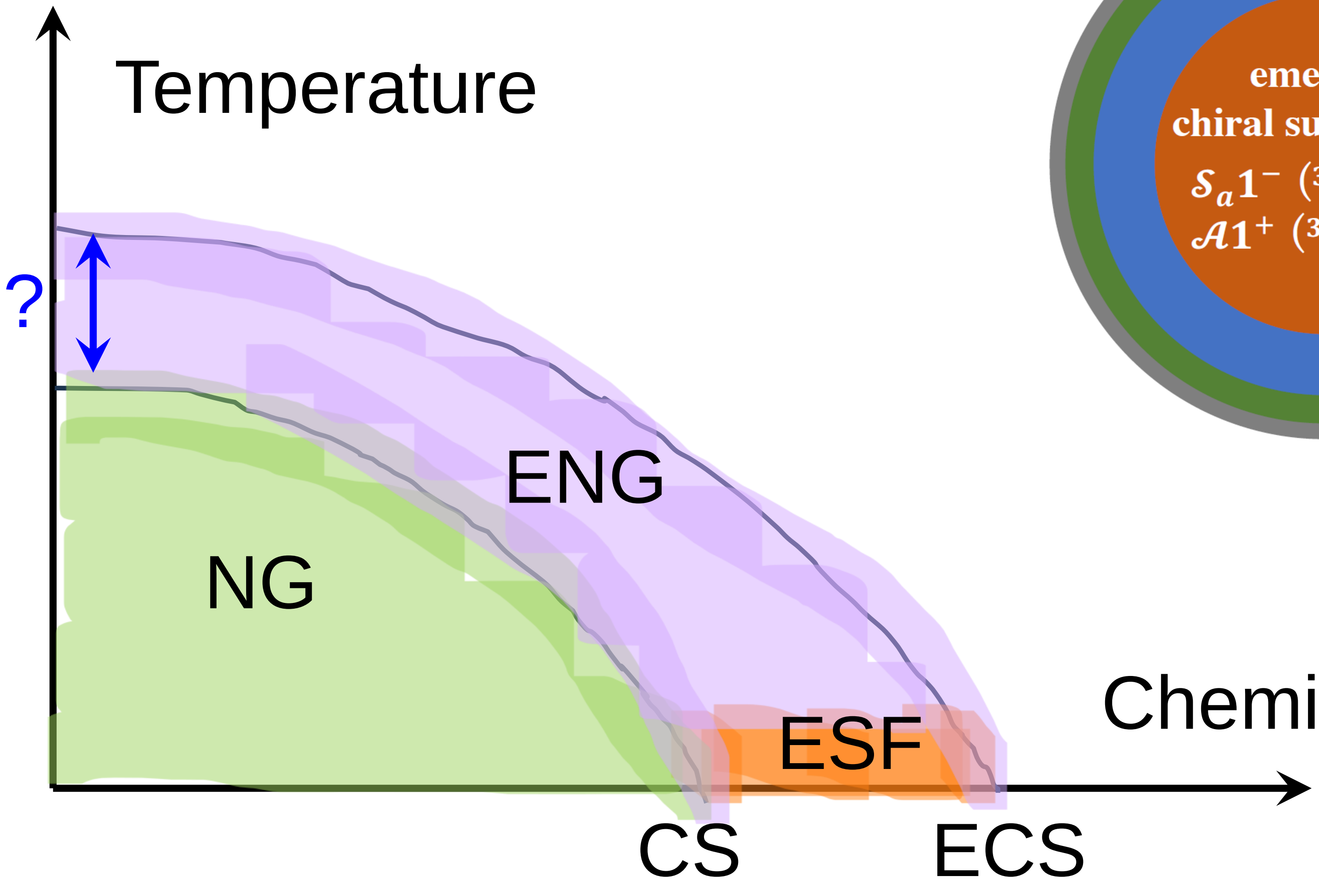
Dynamical symmetry breaking

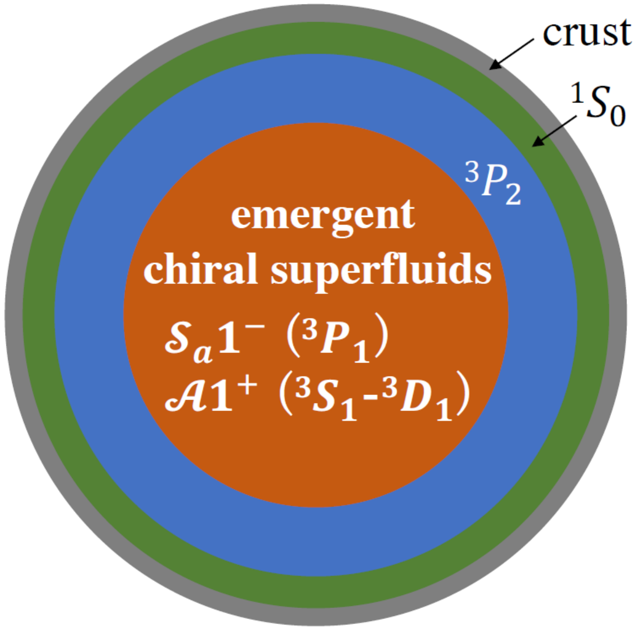
$$\frac{\mathrm{U}(1)_{\mathrm{L}} \times \mathrm{U}(1)_{(1-2)\mathrm{L}}}{\mathbb{Z}'_{2\mathrm{L}}} \times \frac{\mathrm{U}(1)_{\mathrm{R}} \times \mathrm{U}(1)_{(1-2)\mathrm{R}}}{\mathbb{Z}'_{2\mathrm{R}}} \times \mathrm{SO}(3)_{\mathrm{S}}$$

- ❑ Vectorial symmetry $U(1)_{L+R}$ broken
  → superfluid phonons
- ❑ Axial symmetry $U(1)_{L-R}$ unbroken
- ❑ Emergent chiral symmetry broken to $U(1)_{(1-2)(L+R)}$ → emergent pions
- ❑ Spatial rotation symmetry broken to $SO(2)_S$
  → magnons

❖ NG bosons as sexaquark states w/ B=2: exotic

Anticipated phase diagram





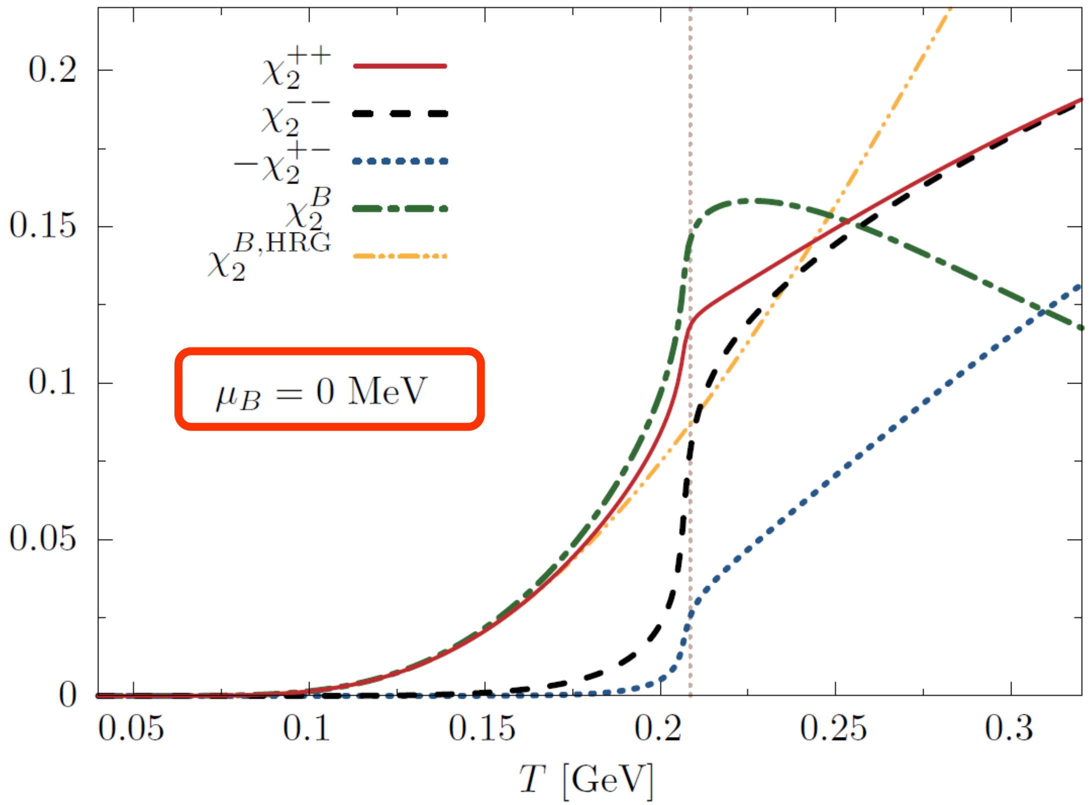
SUMMARY

Conclusions

- Negative correlations between N and N*
  - $\chi_2^{++} \approx \text{proton}$ may not reflect χ_2^B at the chiral phase boundary.
  - $\chi_2^{++,--,+-}$ in other non-perturbative approaches
- Emergent chiral symmetry at high density
  - New superfluidity in NSs, strong anisotropy
  - Toward understanding of multi-quark states in dense QCD
  - Specific in mirror model? Vortices? Cooling? Interface to QM?

BACKUP

Correlations between N & N*



- ❑ χ_2^B dominated by positive-parity fluct.
- ❑ N* being relevant only near Tc
- ❑ χ_2^{+-} sets in only near Tc, and it's **negative**.
- ❑ χ_2^{+-} becomes more negative with repulsive int.

Dirac points

- Single-particle energy with a gap

$$\vec{\delta} = (0,0,\delta)$$

Dirac points (massless) at $p_z = \pm\sqrt{\mu^2 + \delta^2}$

$$\varepsilon_q \cong \sqrt{\frac{q_x^2 + q_y^2}{1 + \frac{\mu^2}{\delta^2}} + q_z^2}$$

- Propagation along x&y directions in $v \ll 1$
- Propagation along z direction in $v = c = 1$

→ Anisotropy in transport phenomena, NS cooling