HHIQCD 2024, YITP

QED in 2 spatial dimensions, A study of confinement with quantum computers

Enrico Rinaldi – 2024/11/13 – Lead R&D Scientist at QUANTINUUM

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Work in collaboration with Arianna Crippa and Karl Jansen (DESY) → arxiv:2411.05628

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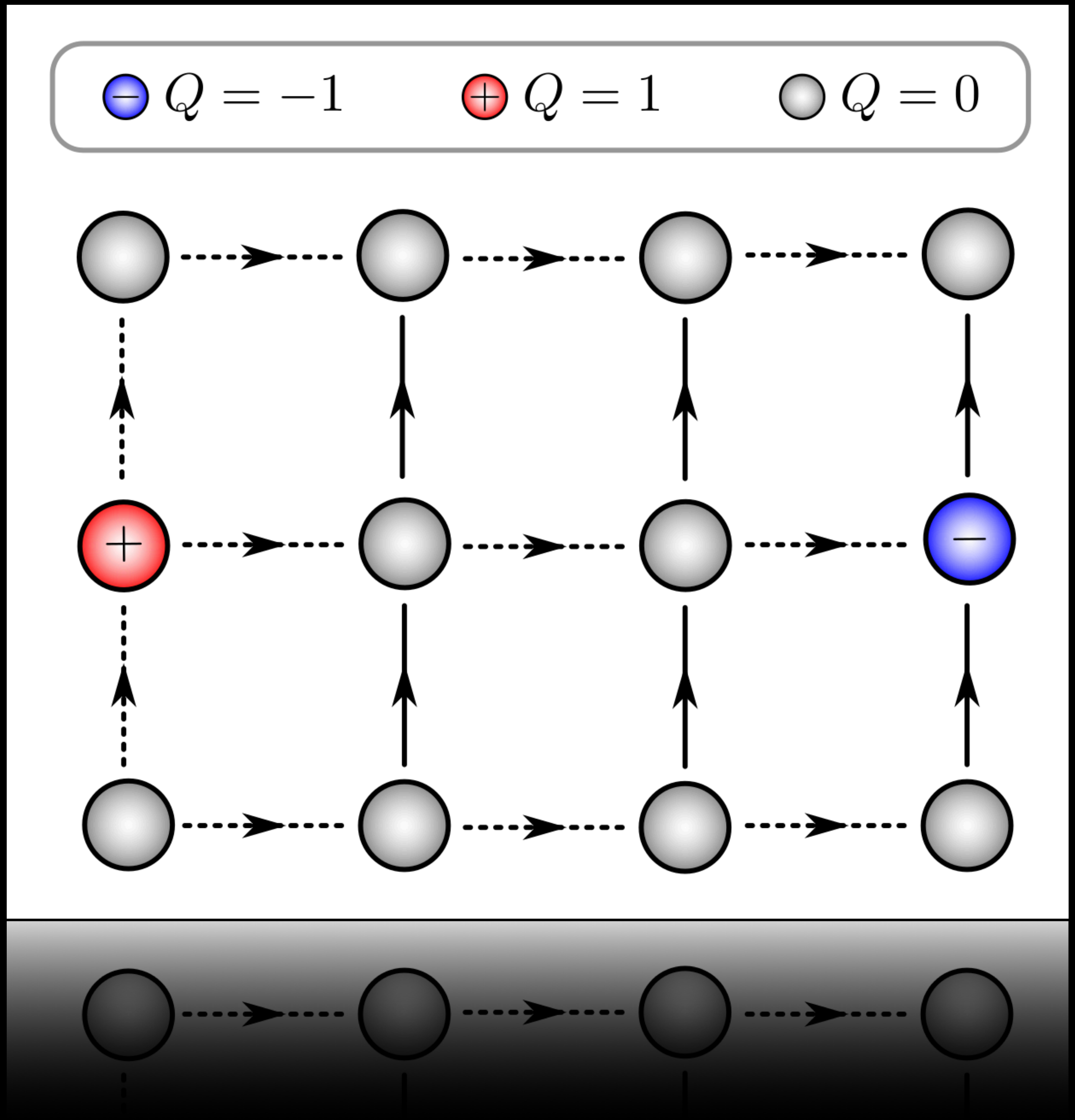
QED in 2 spatial dimensions …

Hamiltonian Lattice QED

In 2 spatial dimensions

- We use the Kogut-Susskind Hamiltonian formalism of lattice gauge theory. Time is continuous.
- The Hilbert space is defined as the tensor product of the local Hilbert spaces of each degree of freedom on the lattice
- A state is a superposition of amplitudes for each possible configuration of degrees of freedom on the lattice



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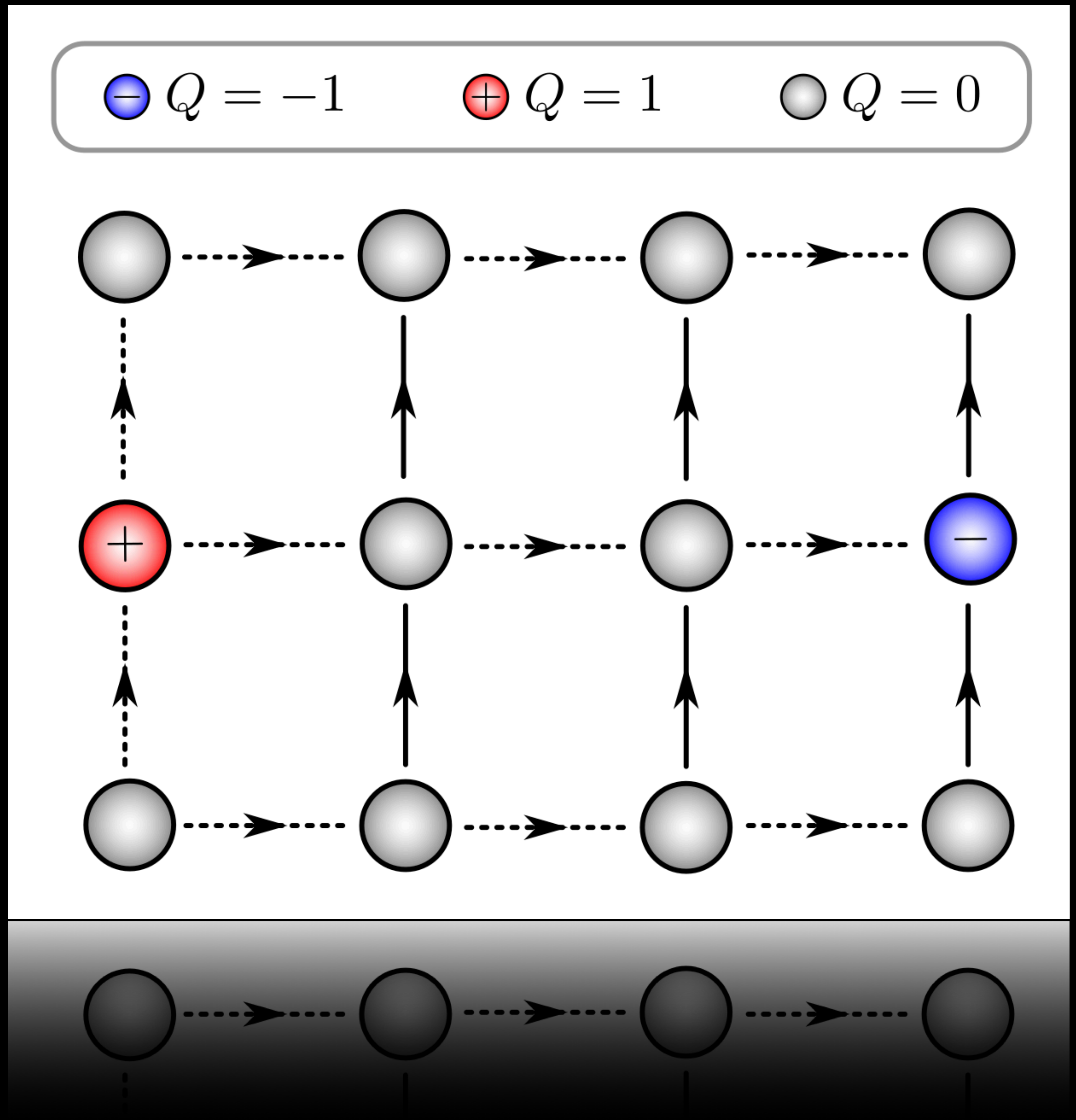

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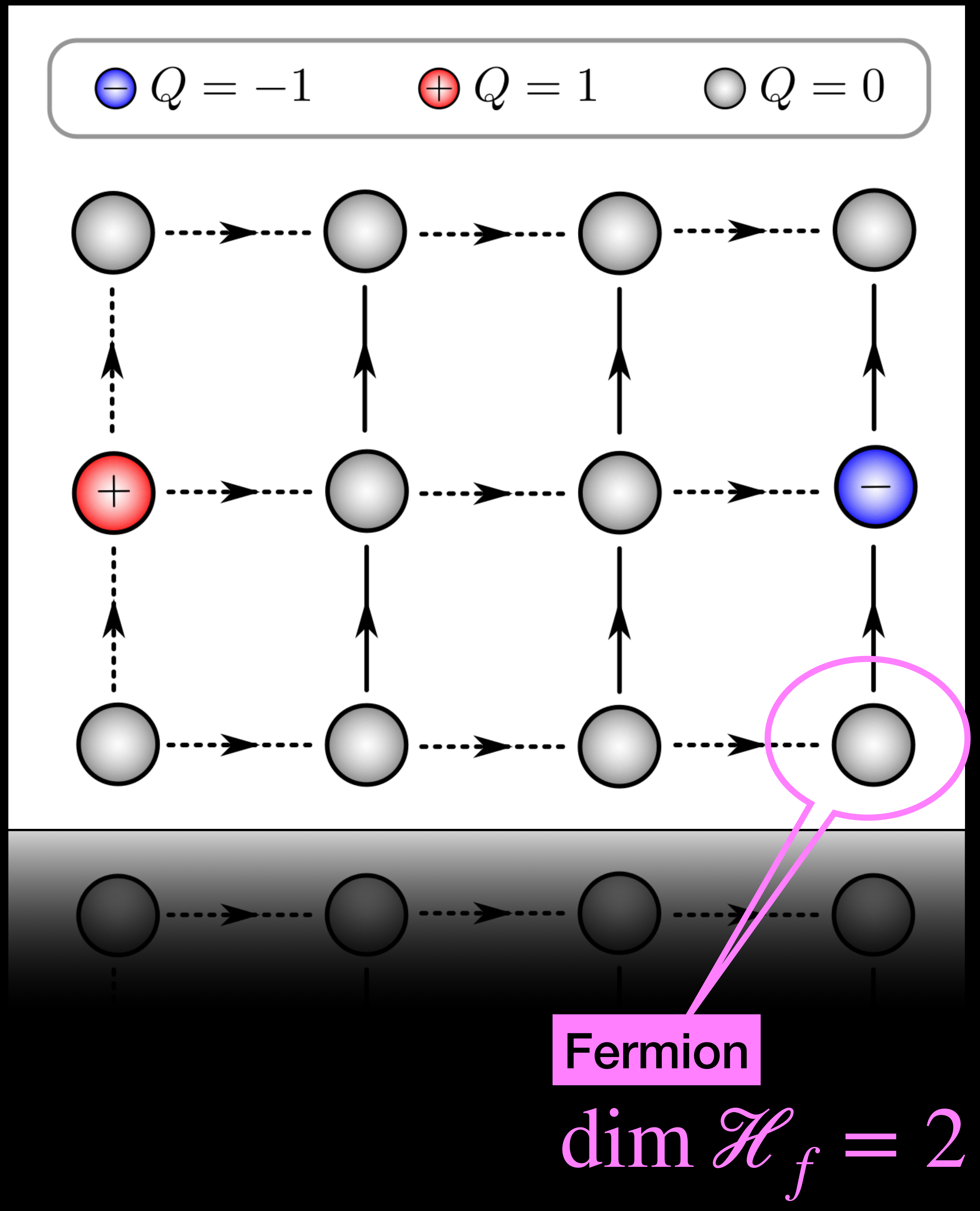

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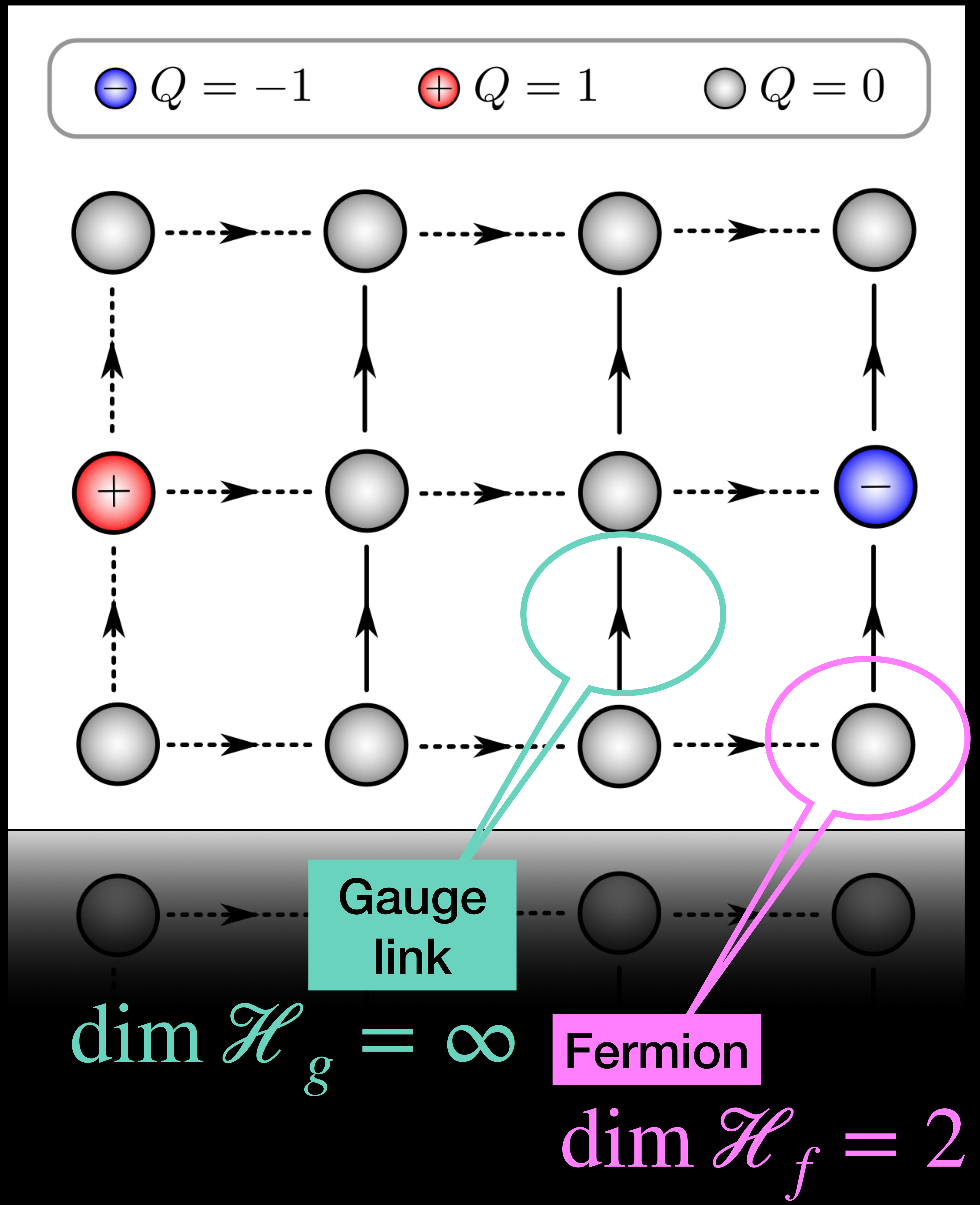

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QED on qubits

Electric and Magnetic terms

$$\hat{H}_E = \frac{g^2}{2} \sum_{\vec{n}} \left(\hat{E}^2_{\vec{n},x} + \hat{E}^2_{\vec{n},y} \right)$$

$$\hat{E}_{\vec{n},\mu} \left| e_{\vec{n}} \right\rangle = e_{\vec{n}} \left| e_{\vec{n}} \right\rangle$$

$$\hat{H}_B = -\frac{1}{2g^2} \sum_{\vec{n}} \left(\hat{U}_{\vec{n},x} \hat{U}_{\vec{n}+x,y} \hat{U}^\dagger_{\vec{n}+y,x} \hat{U}^\dagger_{\vec{n},y} + \ldots \right)$$

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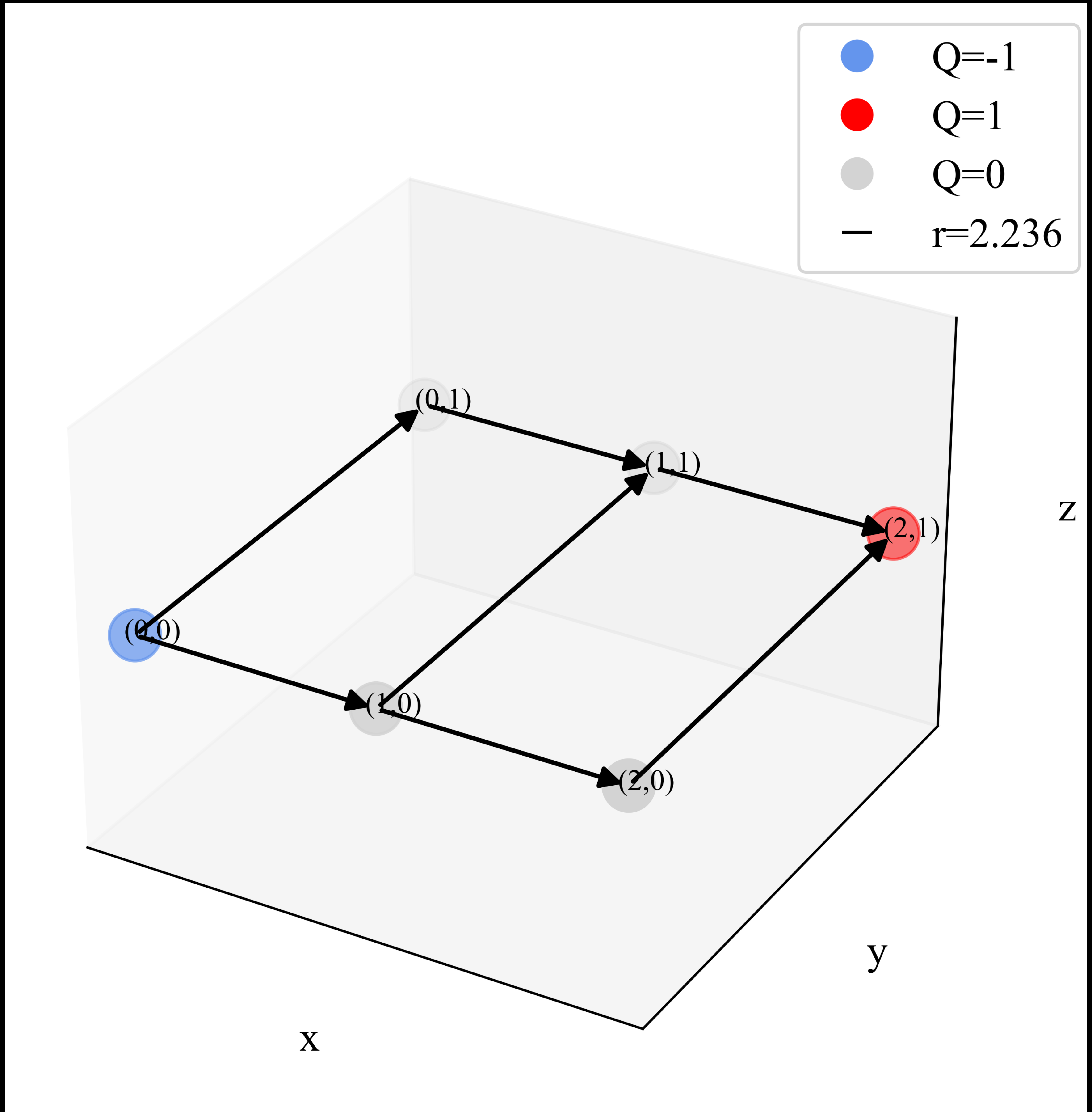

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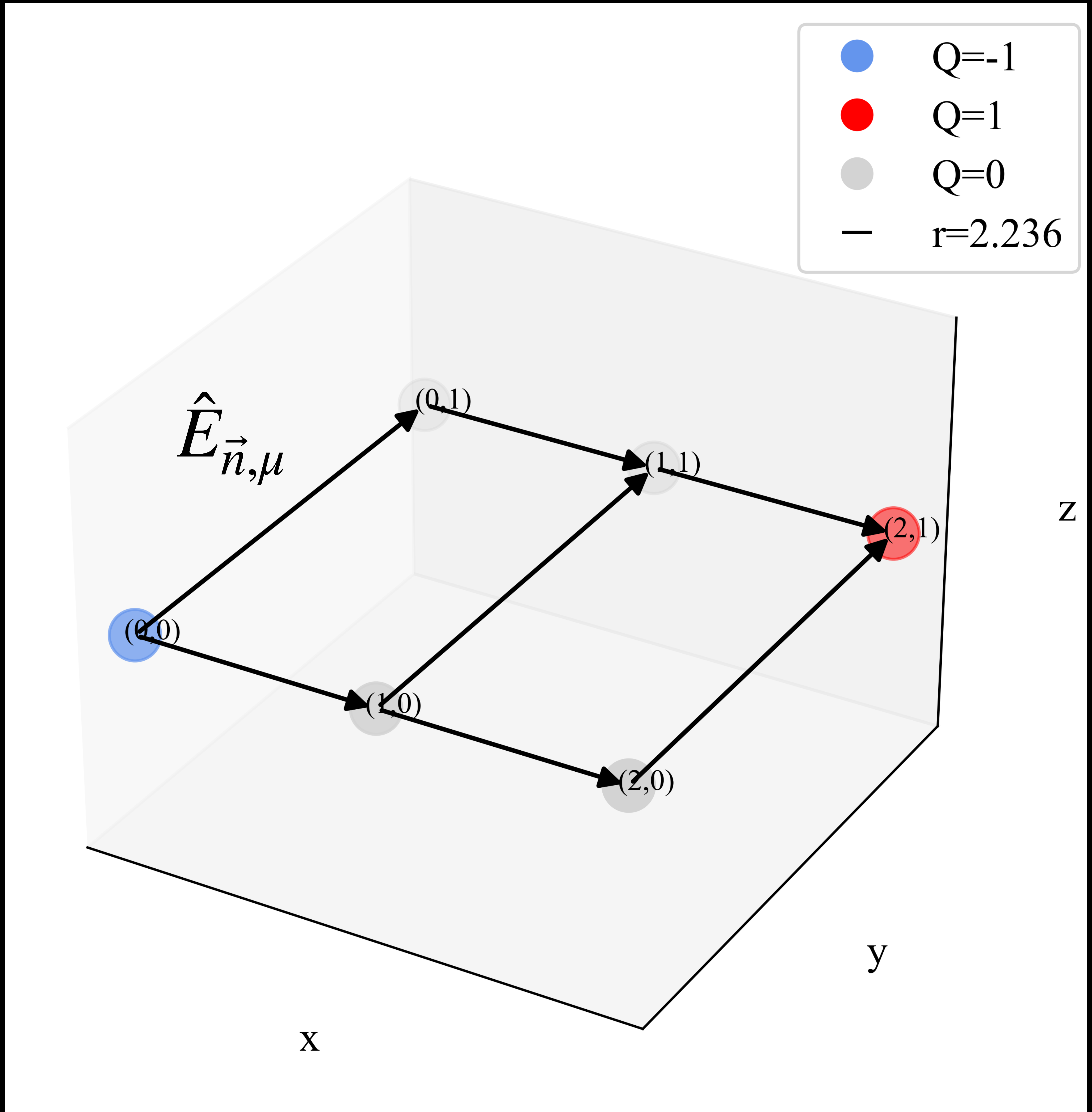

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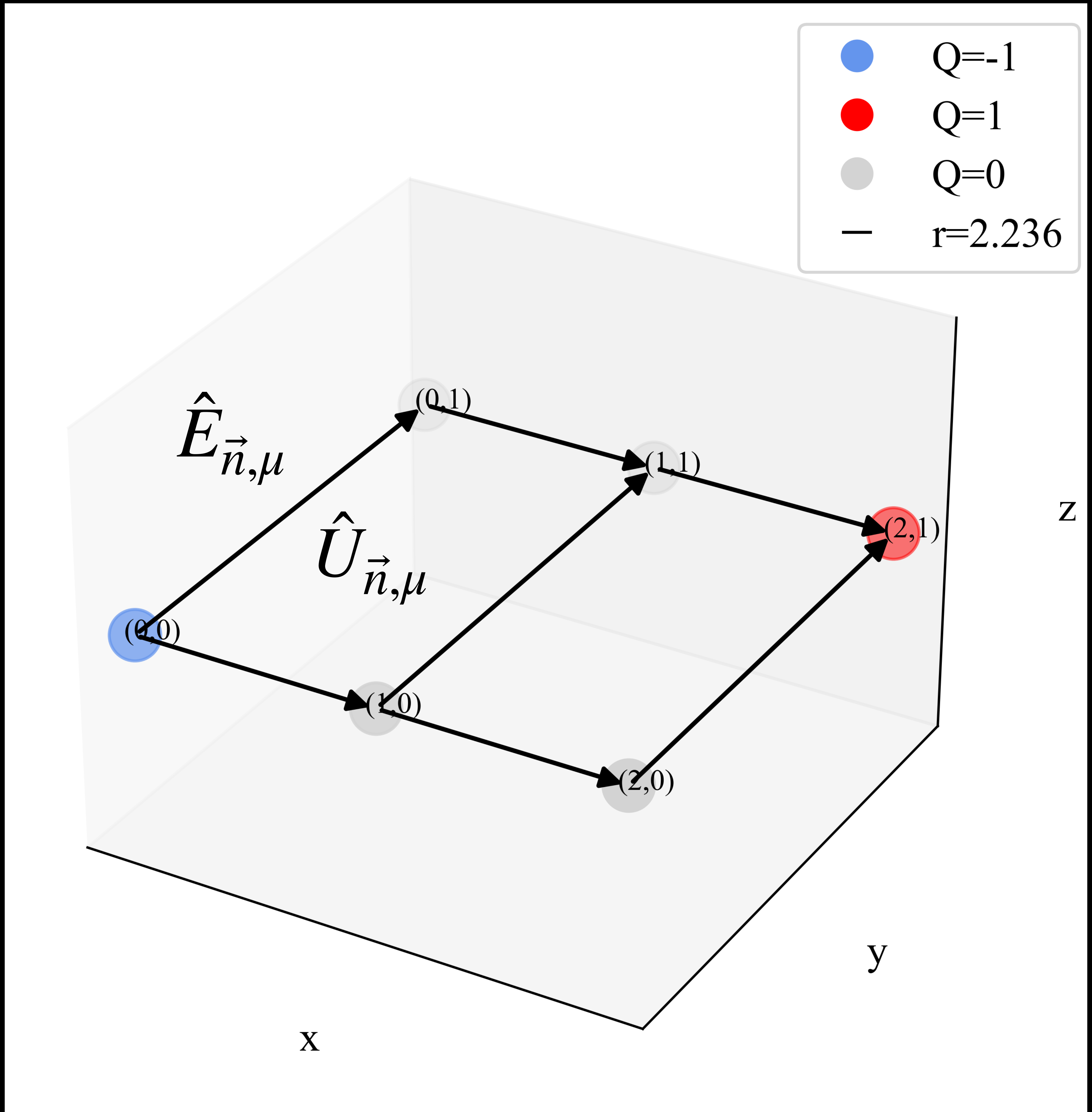
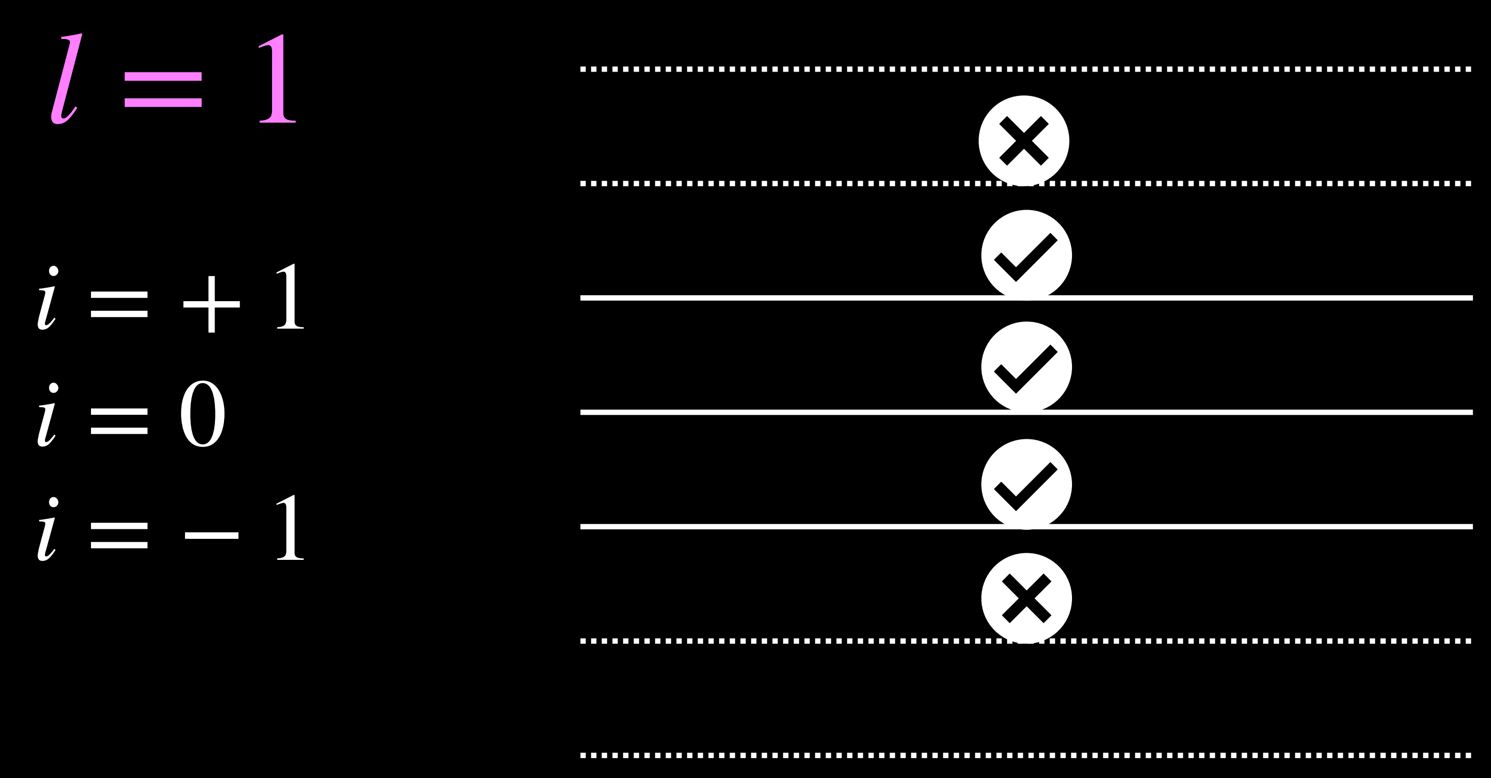


QED on qubits

Discretize: $U(1) \rightarrow \mathbb{Z}_{2l+1}$



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Encoding to qubits:
We need 2 qubits to represent 4 states. 1 state is "unphysical"

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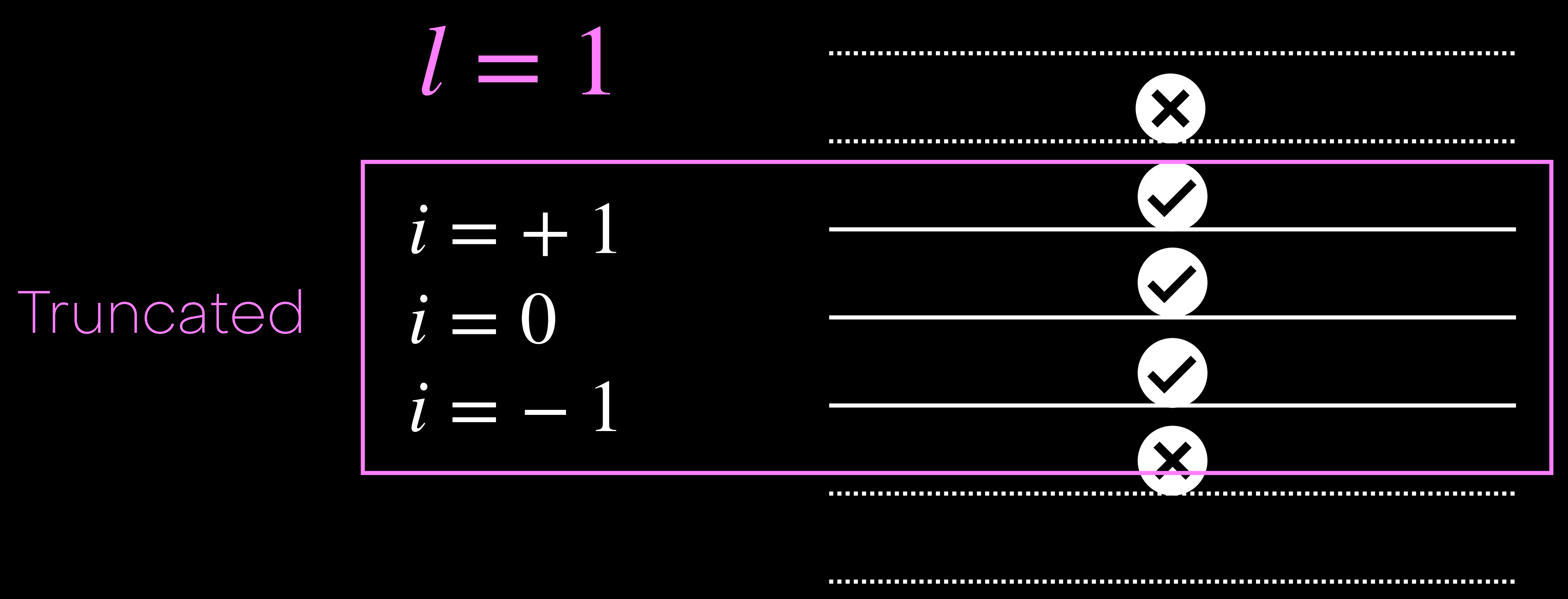

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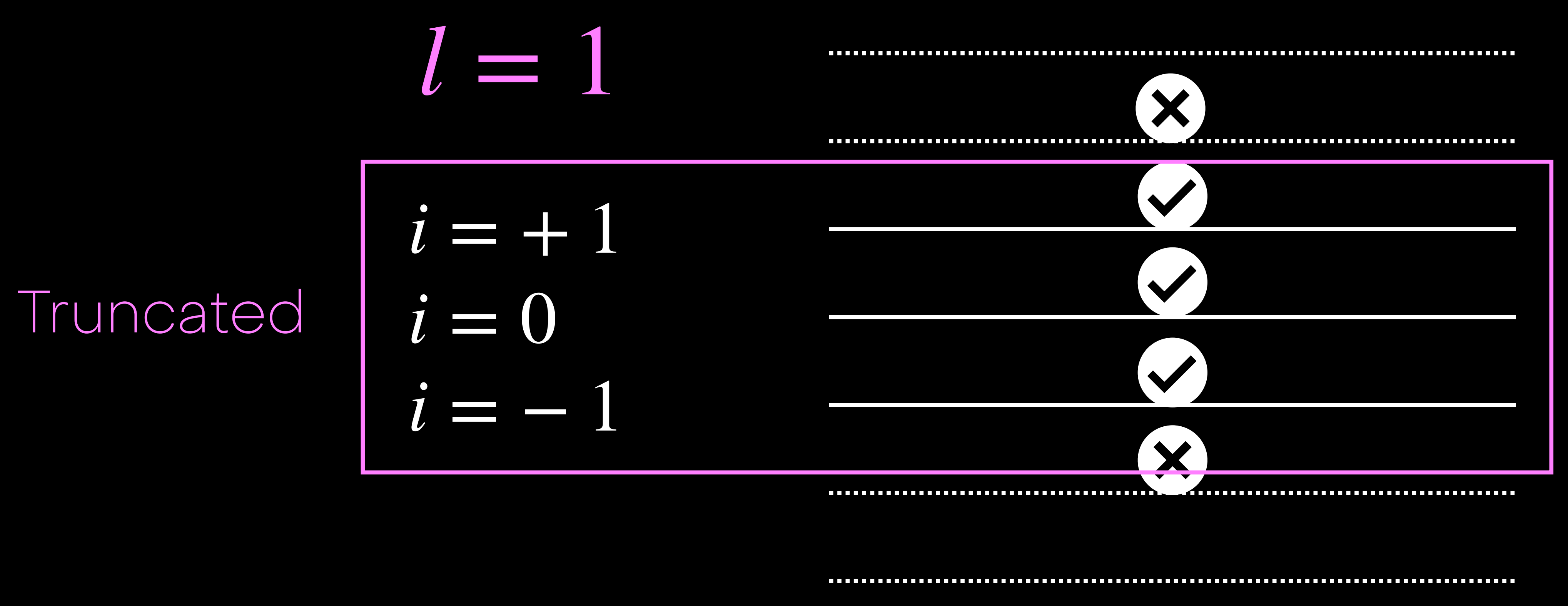

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Gray Encoding

$\lvert i \rangle_{\text{phys}}$	$\lvert i \rangle$
$\lvert -1 \rangle_{\text{phys}}$	$\lvert 00 \rangle$
$\lvert 0 \rangle_{\text{phys}}$	$\lvert 01 \rangle$
$\lvert +1 \rangle_{\text{phys}}$	$\lvert 11 \rangle$
Unphysical	$\lvert 10 \rangle$

$l = 1$

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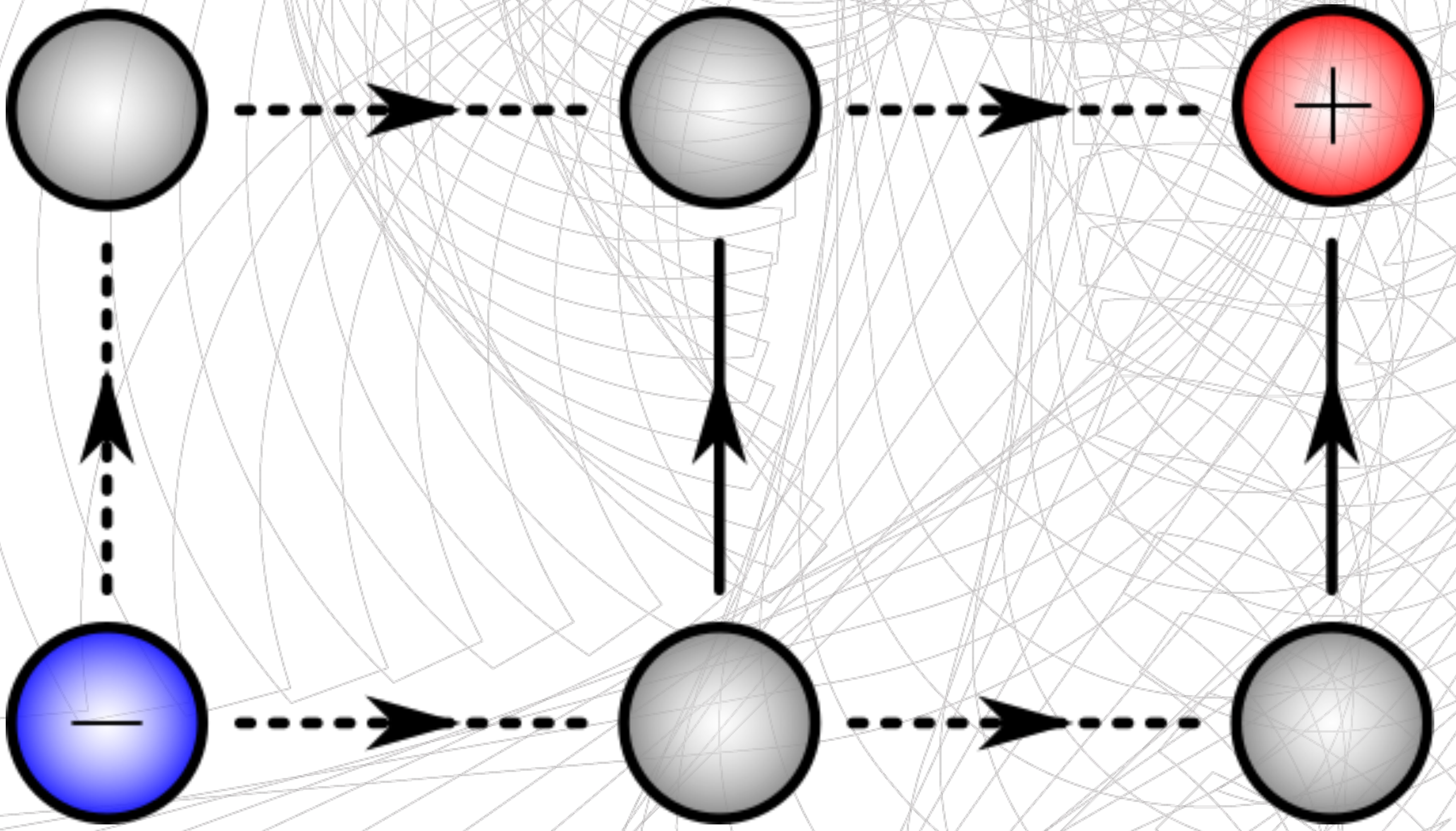
Example on a 3×2 lattice

Gauss' law $\left[\sum_{\mu=x,y}\left(\hat{E}_{\vec{r},\mu} - \hat{E}_{\vec{r}-\mu,\mu}\right) - \hat{q}_{\vec{r}} - Q_{\vec{r}}\right]|\Phi\rangle = 0$

- Using Gauss' law we reduce the number of dynamical degrees of freedom: **6 sites**, **2 links**
- We use **1 qubit for each site**
- We use **2 qubits for each link**
- Any state of this lattice QED theory is defined on **10 qubits**
- This classically requires manipulating a vector with $2^{10} = 1024$ complex components



$$\left|\Psi\right\rangle = \left|\psi_{\text{sites}}\right\rangle \otimes \left|\psi_{links}\right\rangle$$

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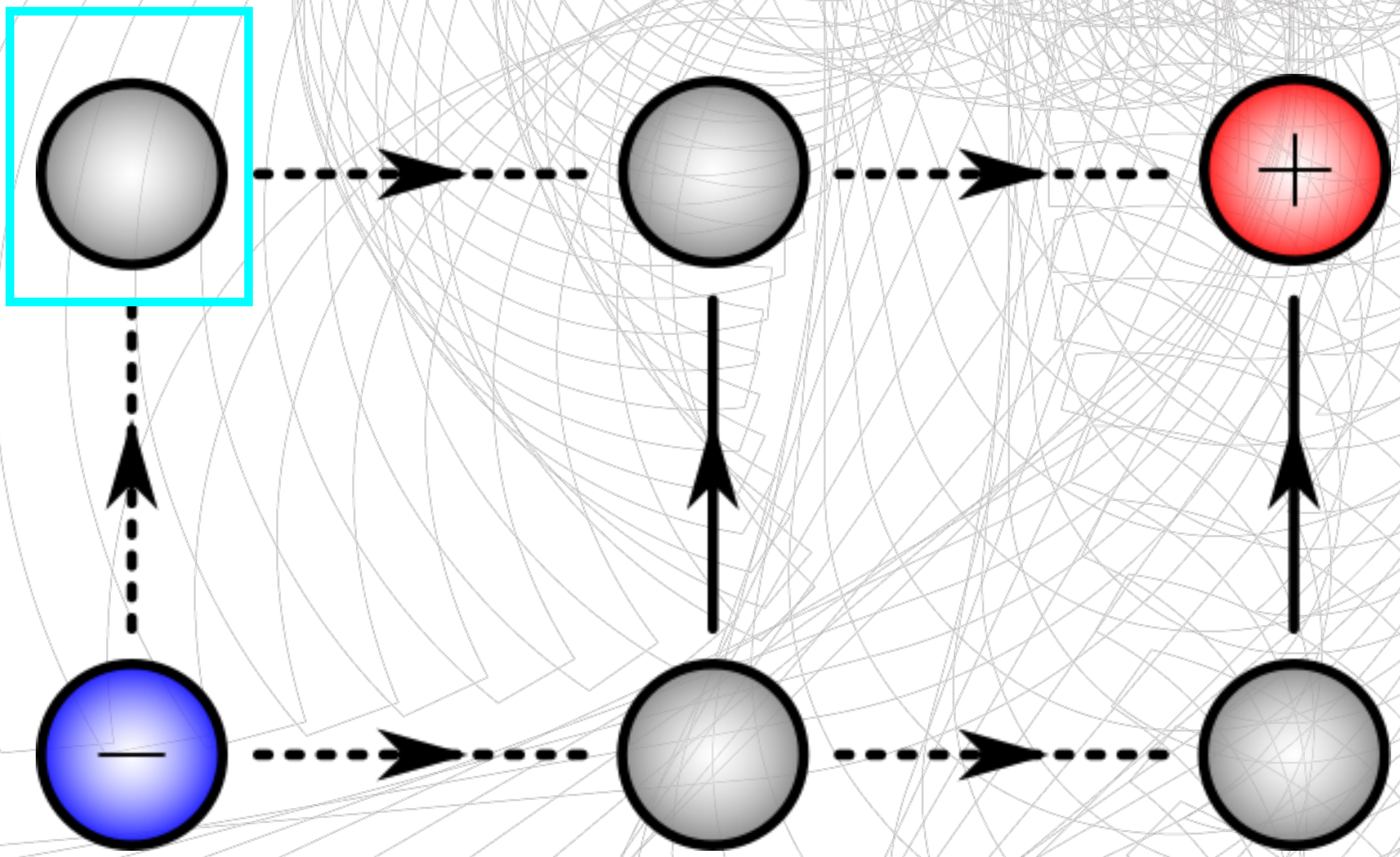

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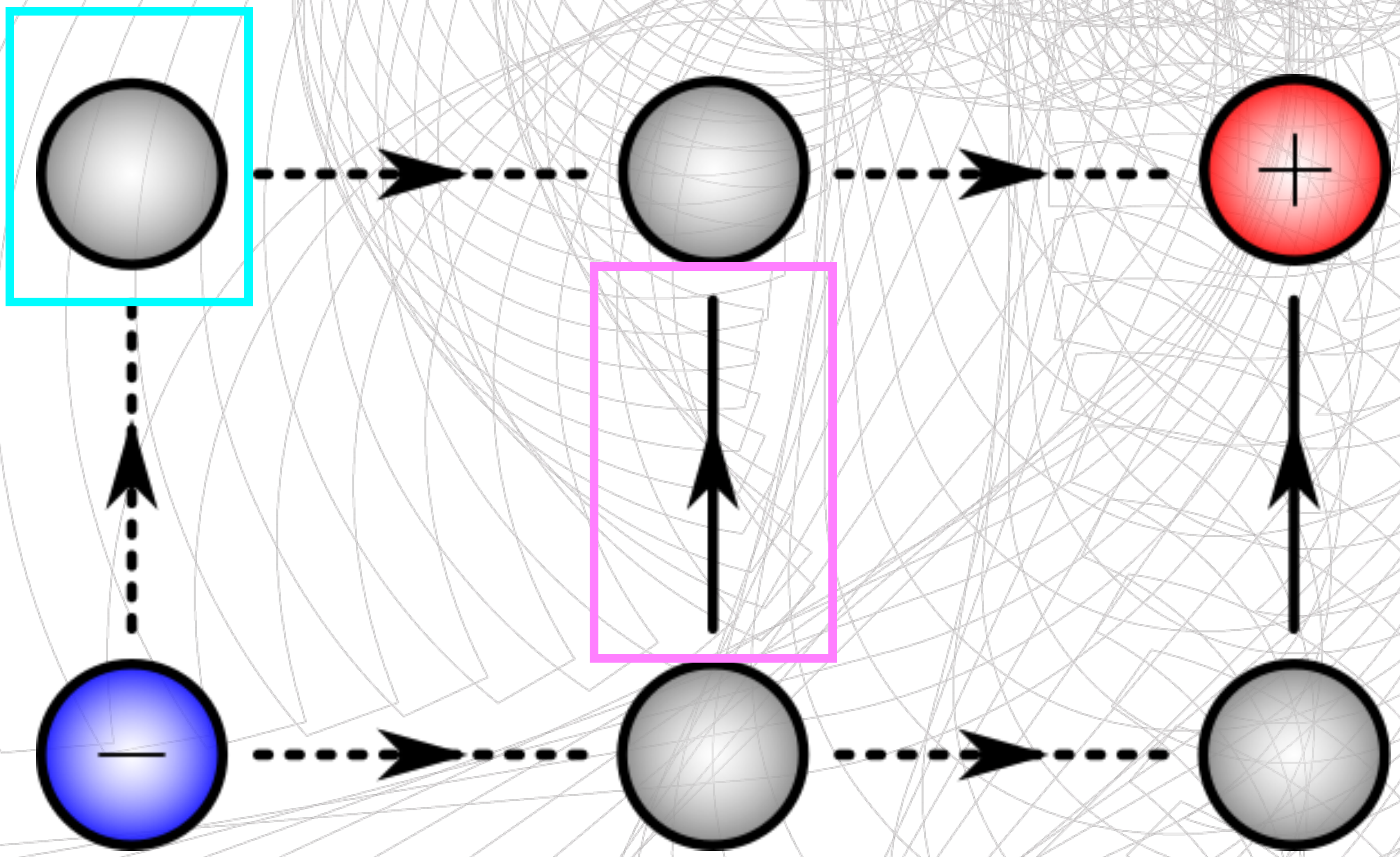

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A study of confinement …

QUANTINUUM

The potential energy

Between two static charges

- In Path Integral Monte Carlo we extract this by computing Wilson Loops of various lengths

- In quantum computing we have direct access to the Hamiltonian and the states of the system!

- The potential $V(r)$ is the energy of the ground state with 2 opposite static charges

- By changing the distance between static charges we can study the force between them

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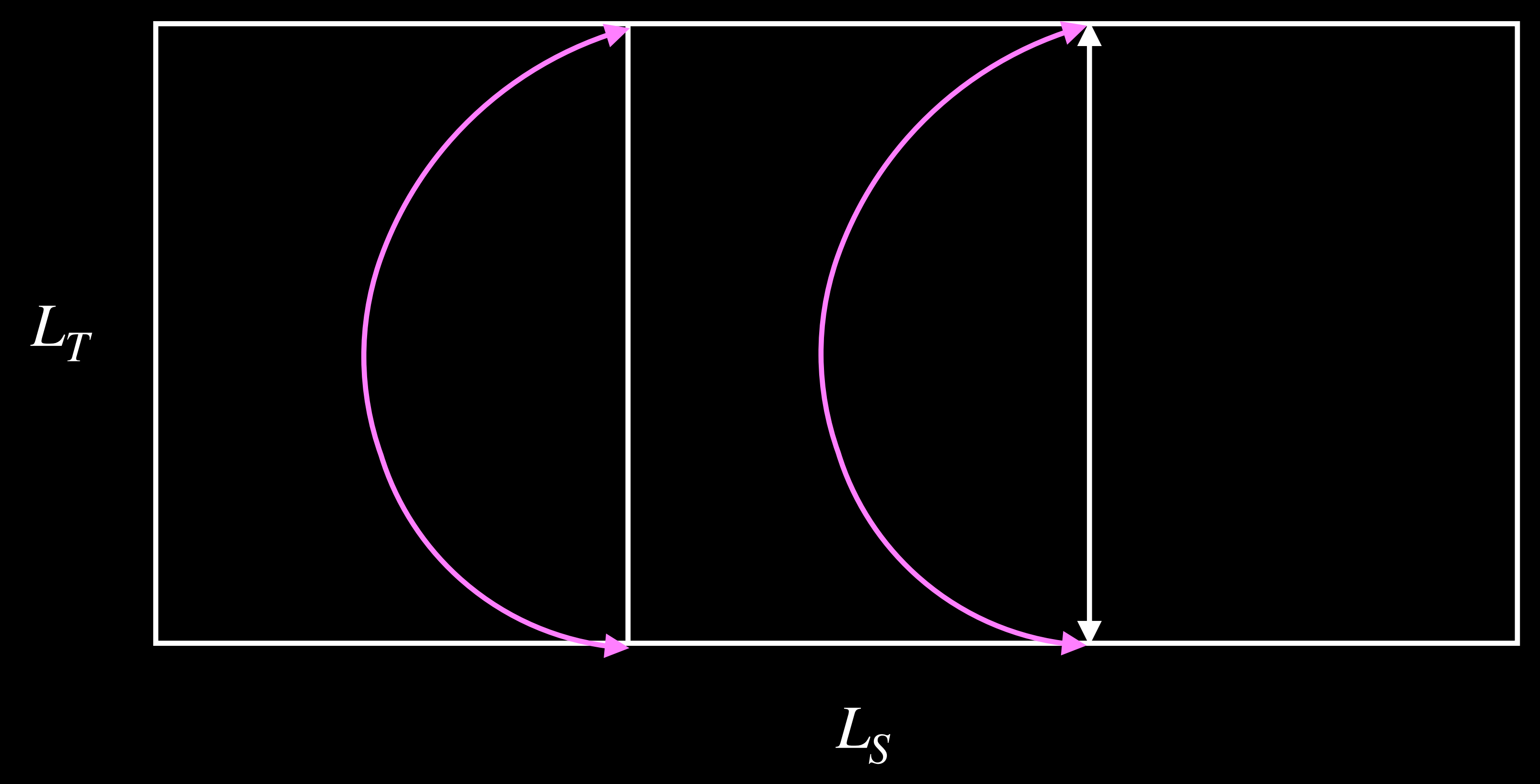

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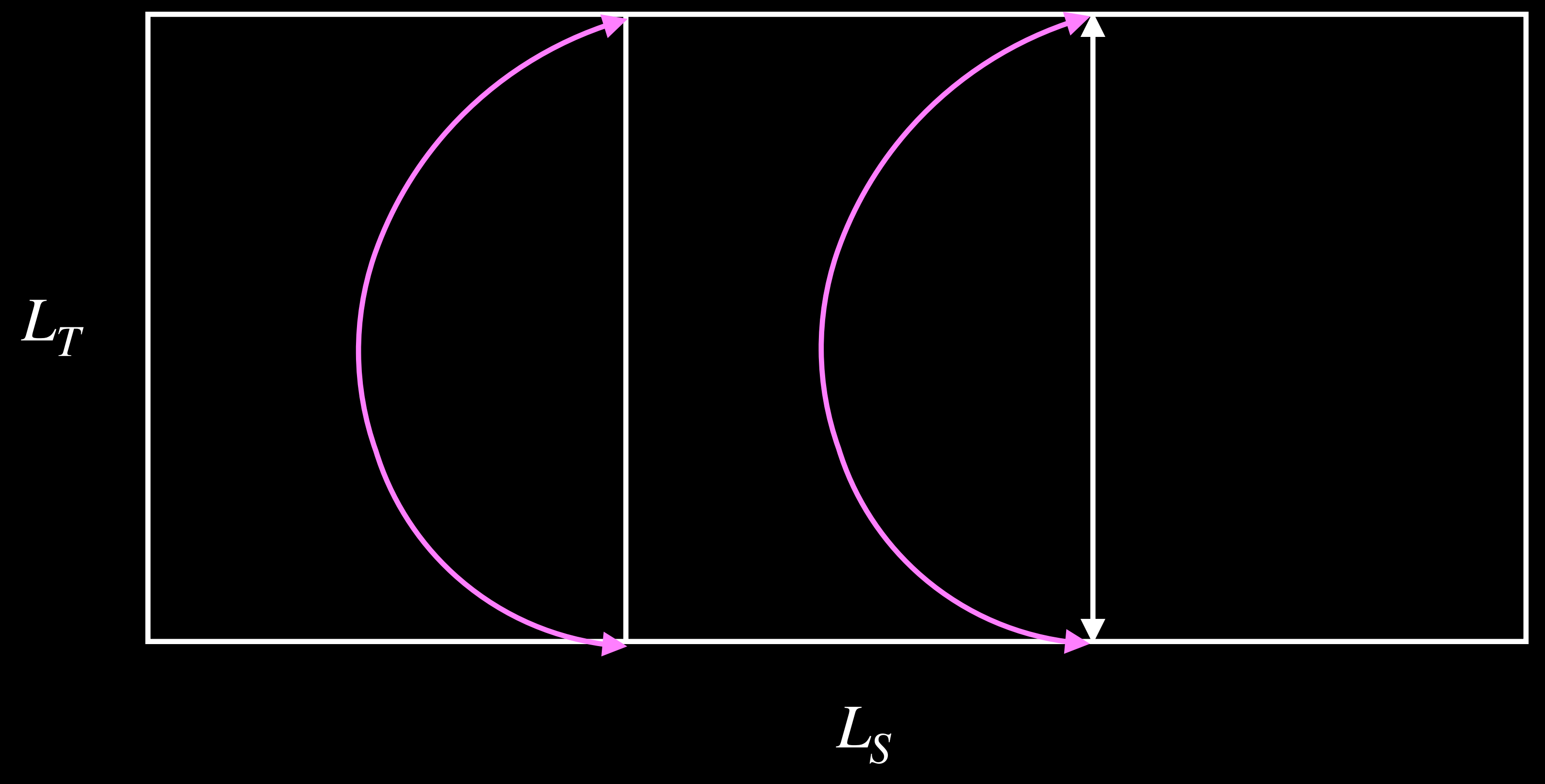

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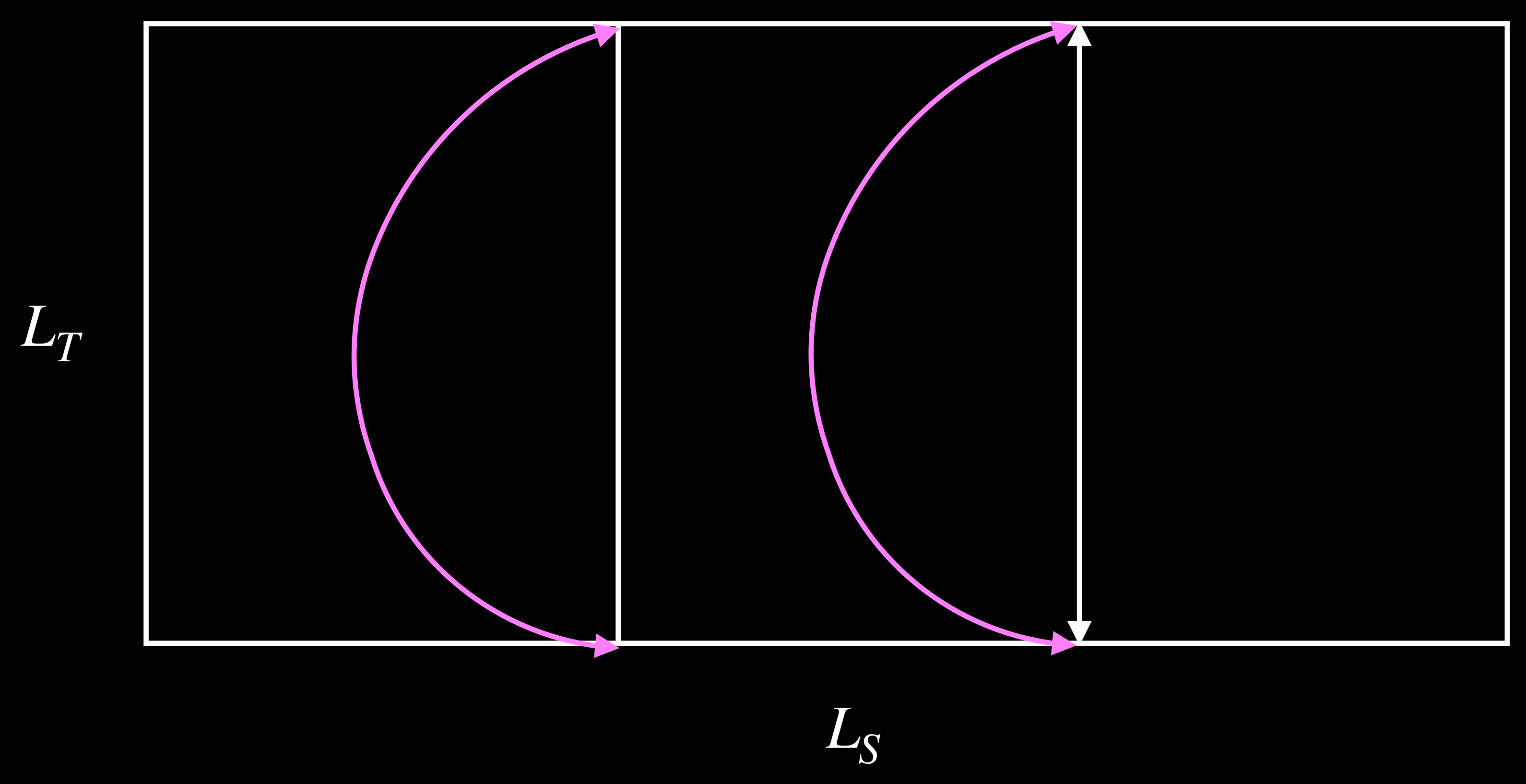

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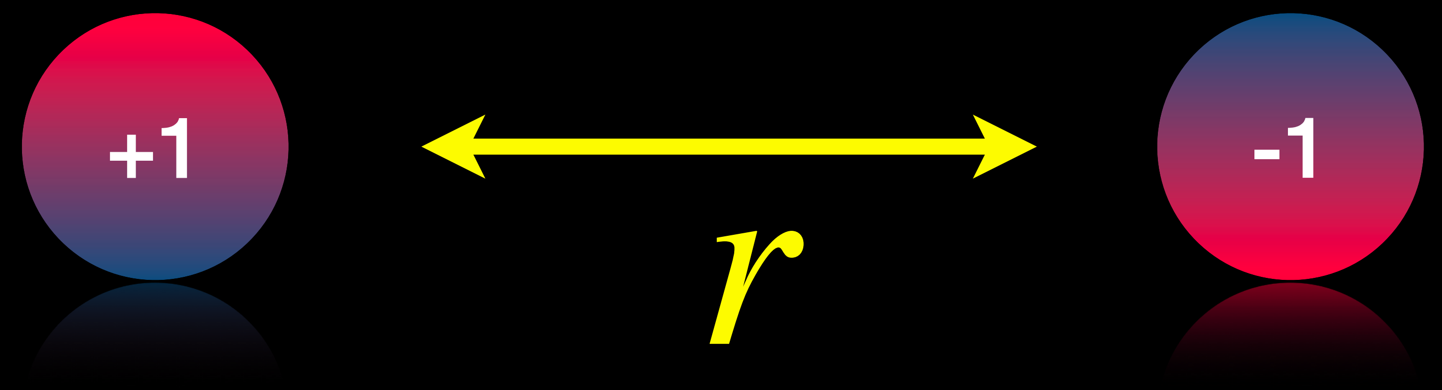


Static Potential

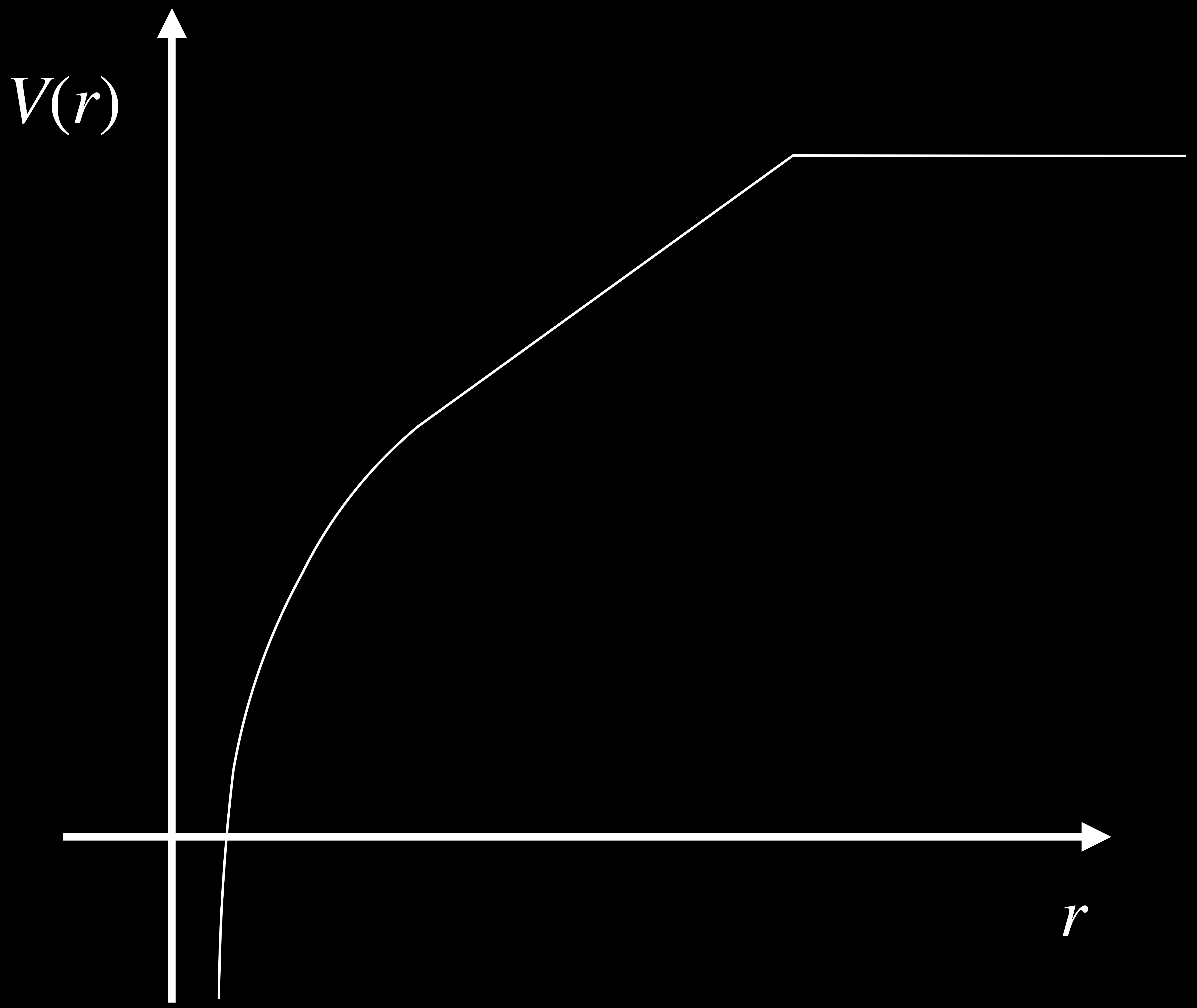
In (2+1)D QED

- $V(r) = V_0 + \alpha \log r + \sigma r$

- On the lattice we can change r by changing the lattice spacing a

- The lattice spacing a depends non-perturbatively on the coupling constant g

  - $V(r) \rightarrow V(g)$



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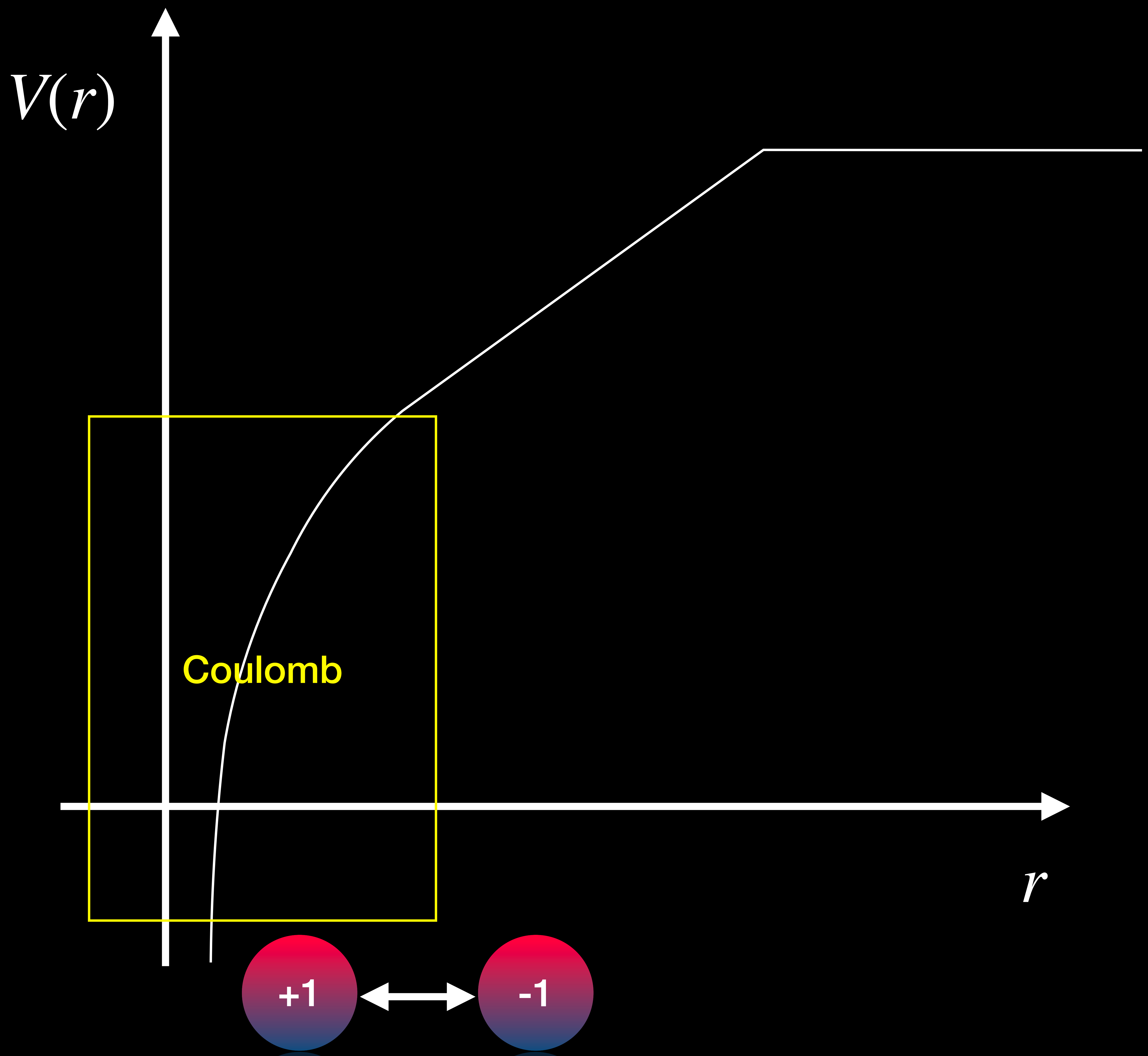

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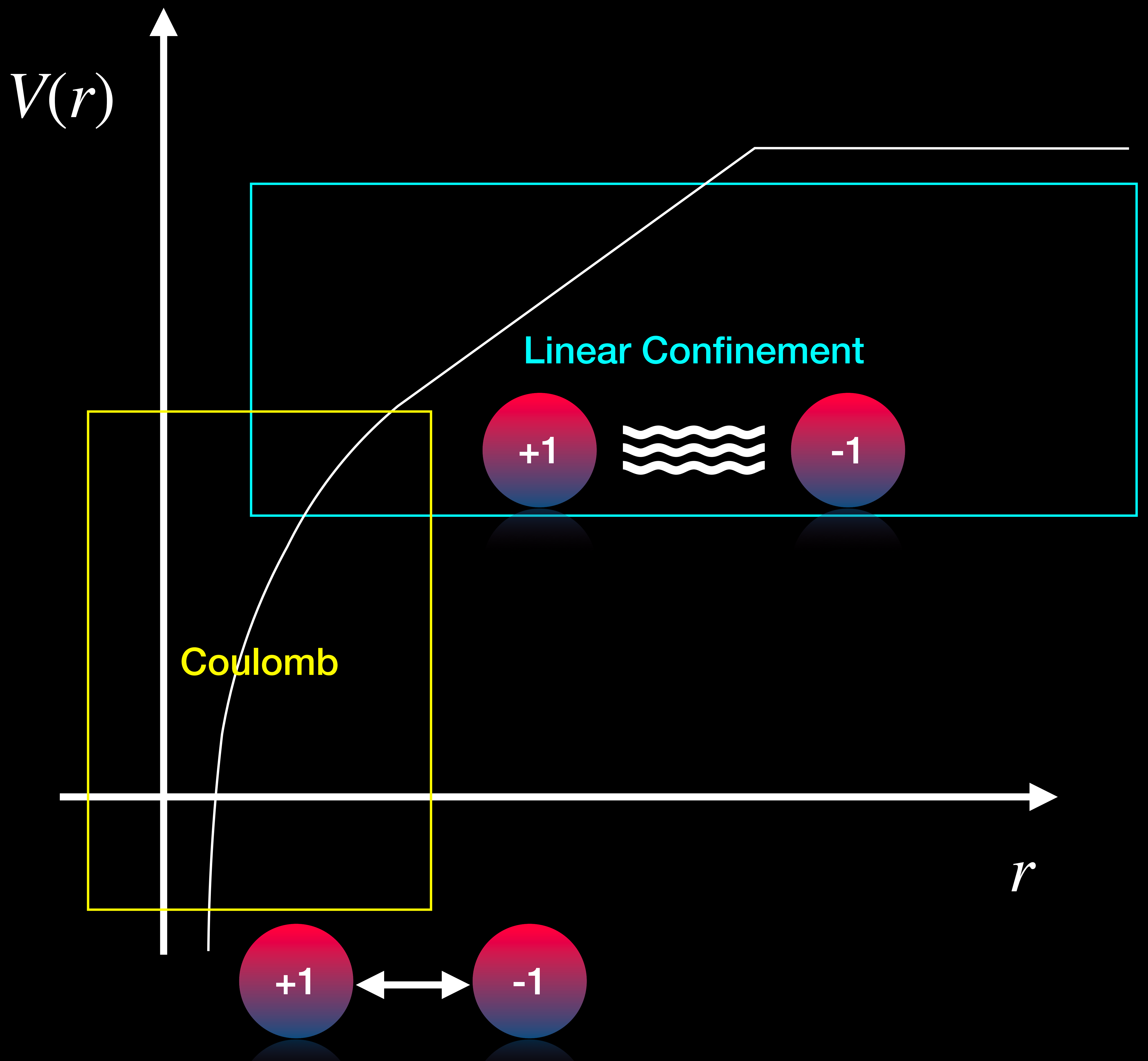

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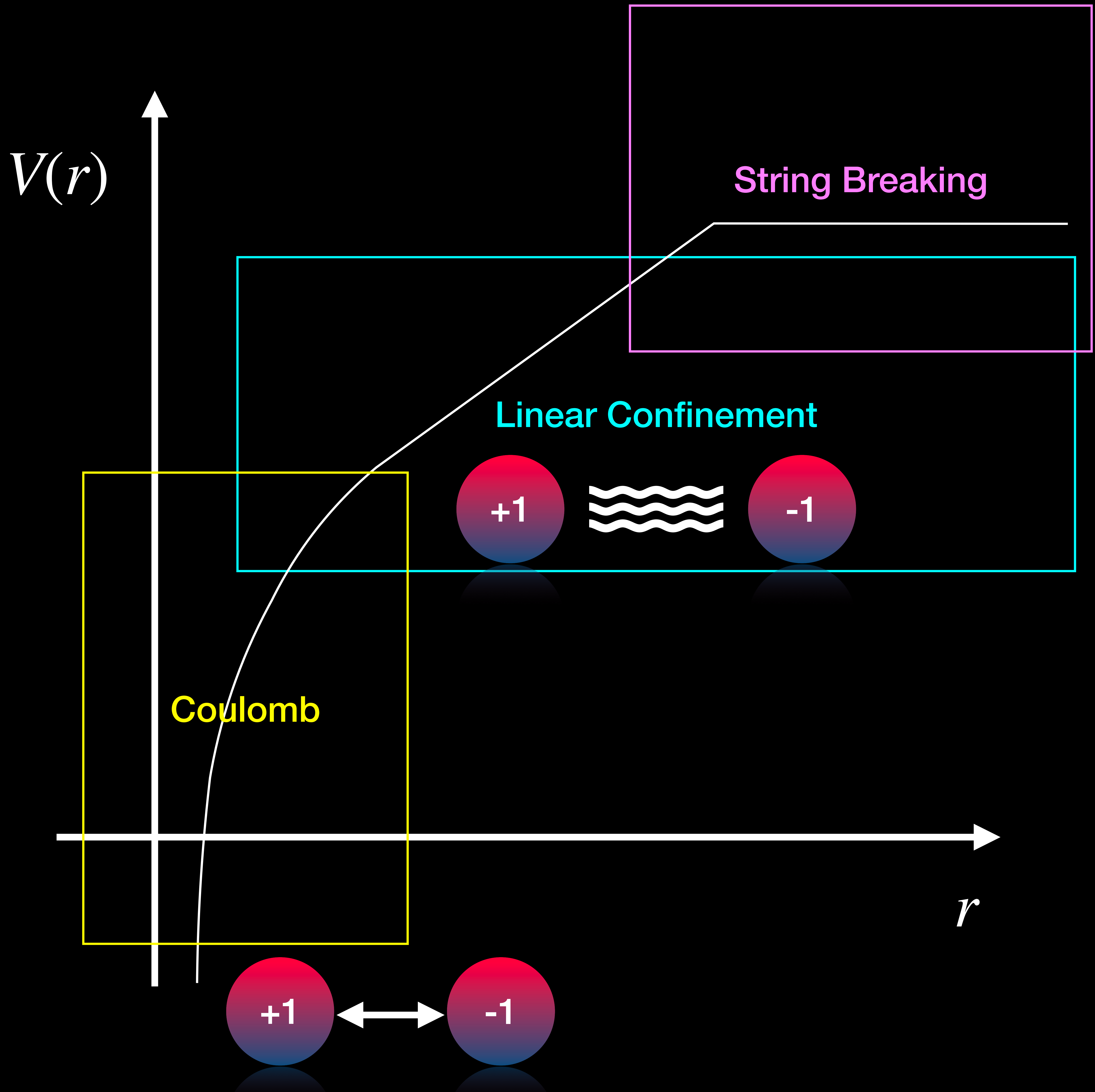

… with quantum computers

QUANTINUUM

Quantum computing

For Lattice Gauge Theory

- A new paradigm for scientific computing (many beautiful reviews online)
- Quantum algorithms work by manipulating quantum states in Hilbert space and measuring them
- Represent the full wavefunction of a quantum many-body system
- Can do unitary time evolution of such wavefunction
- Digital quantum computing implements unitary operators as sequence of gates

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$$U = e^{-iHt} \equiv \prod_i U_i$$

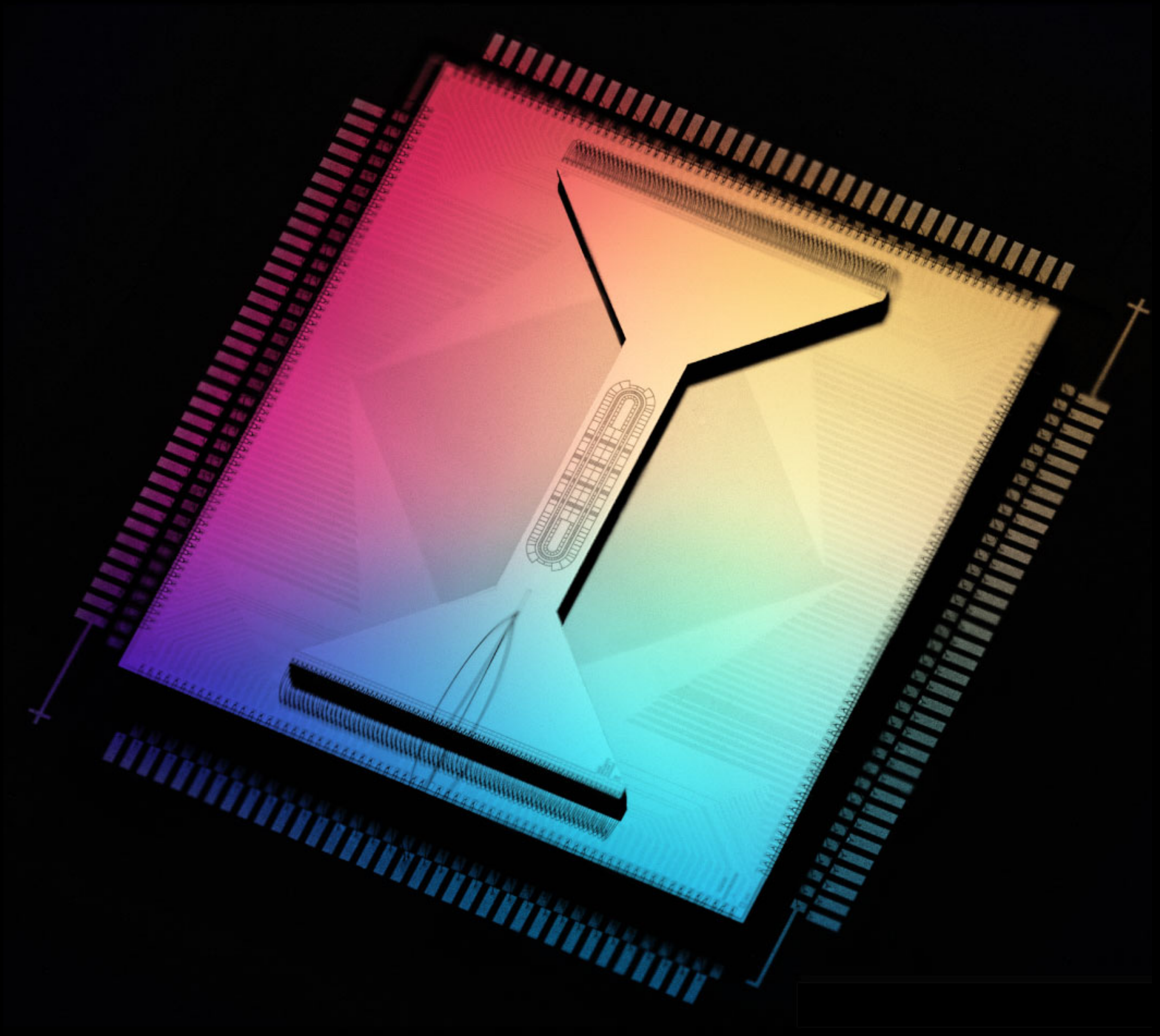
Quantinuum

H-series Quantum Hardware

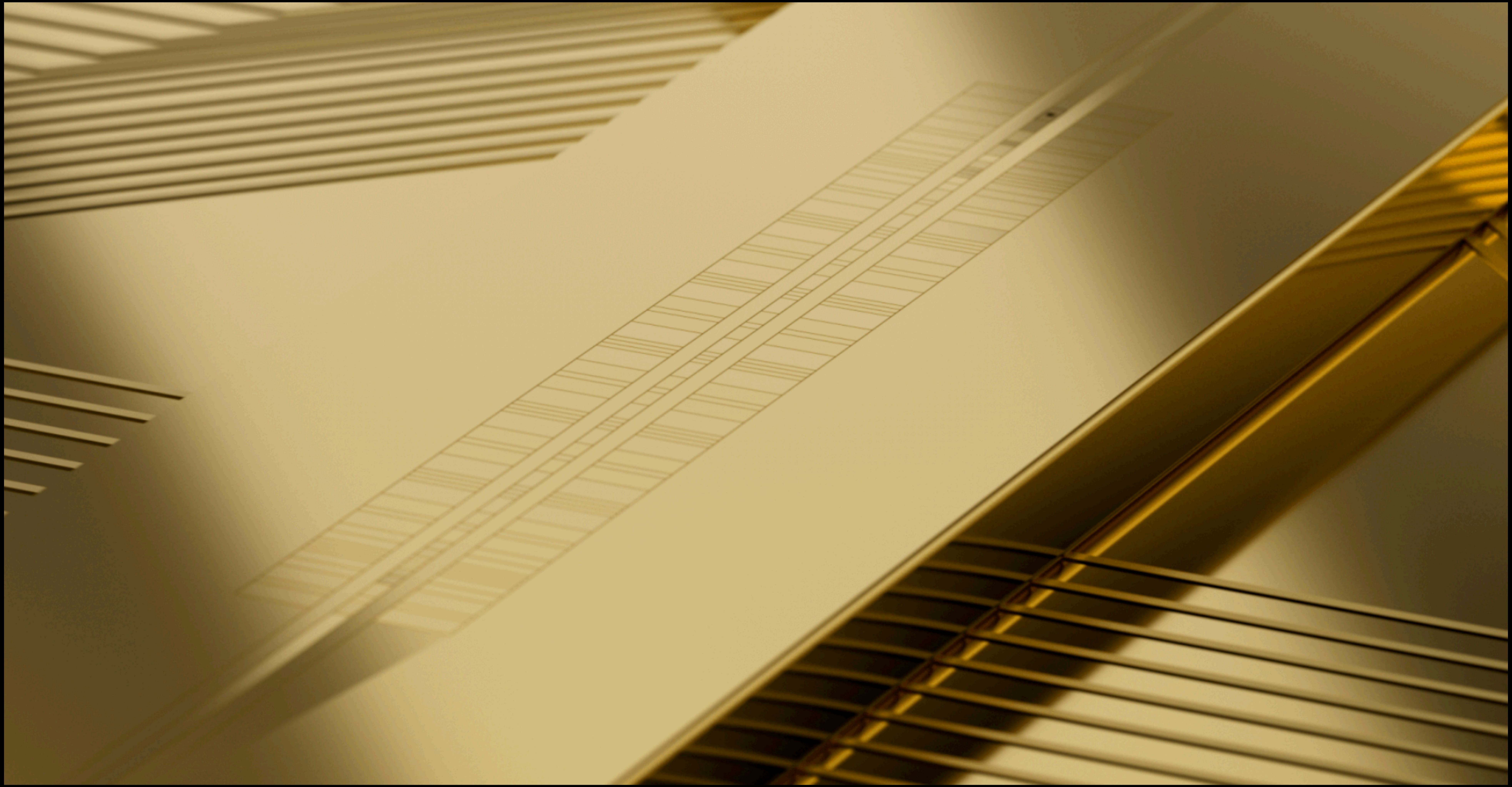
Most benchmarked quantum computer

Lowest-error commercial quantum device

20 and 56 qubits on trapped ions



H1-1



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FULLY-CONNECTED QUBITS	QUANTUM VOLUME	SINGLE-QUBIT GATE FIDELITY	TWO-QUBIT GATE FIDELITY

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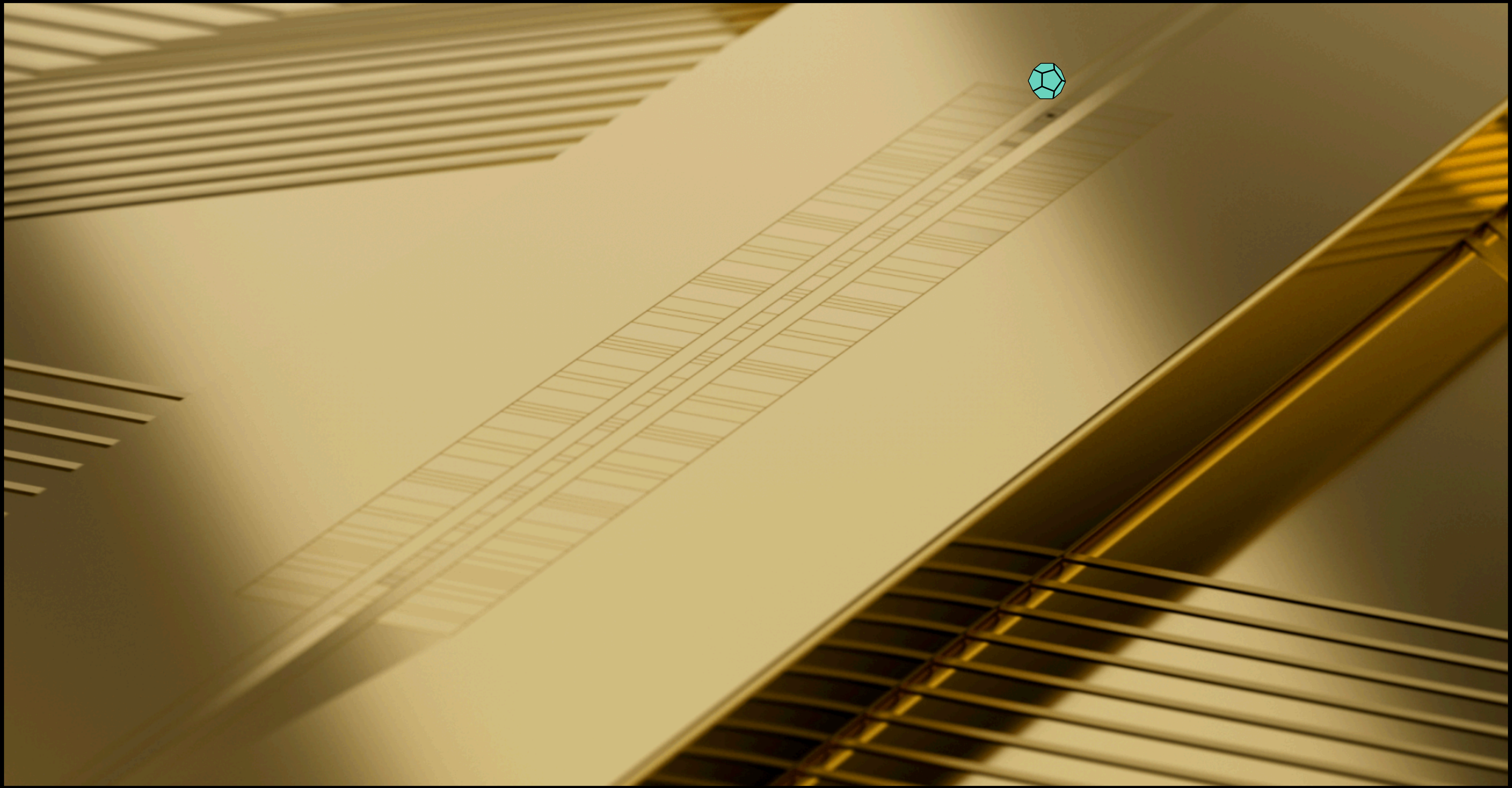

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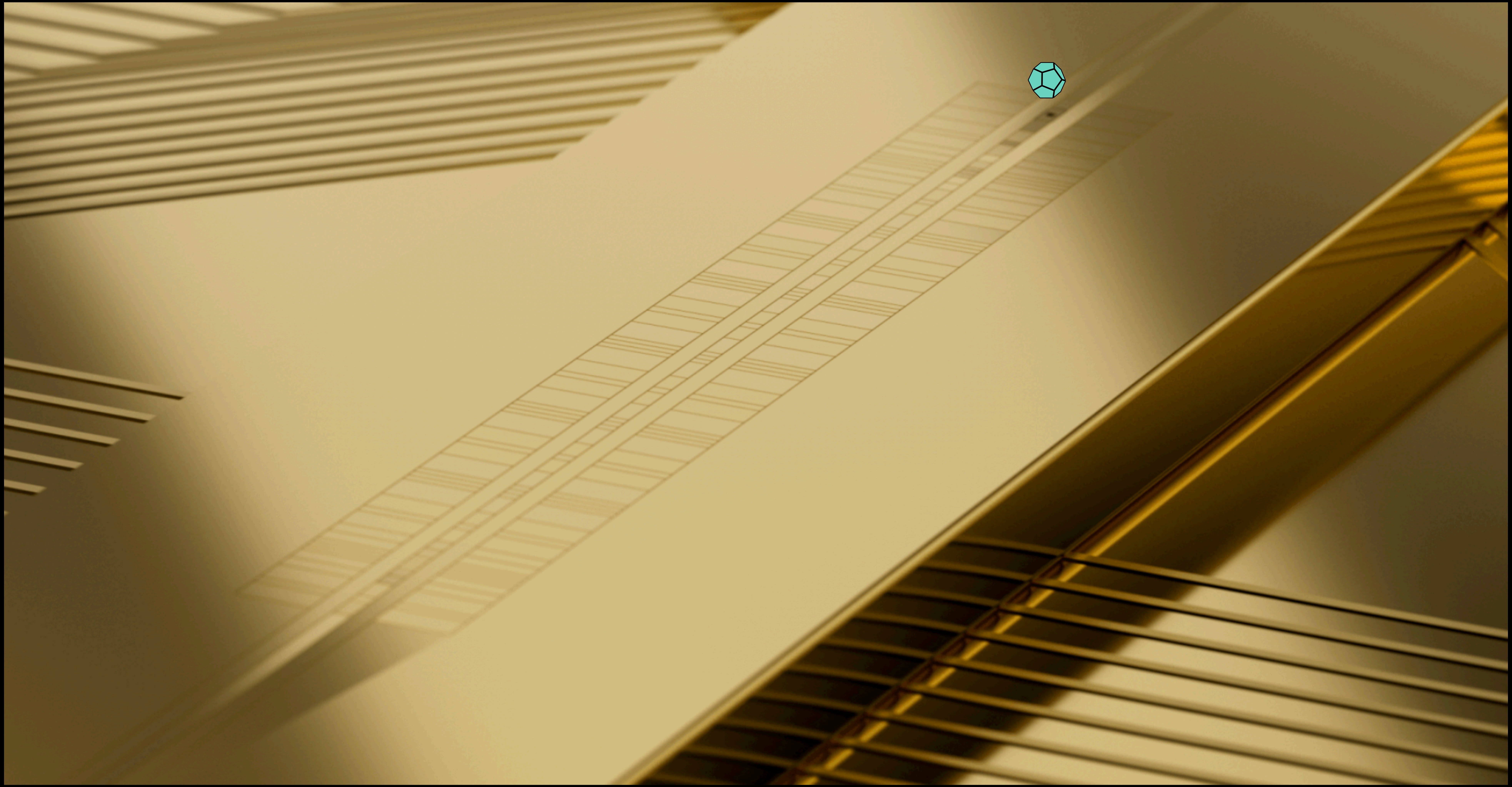

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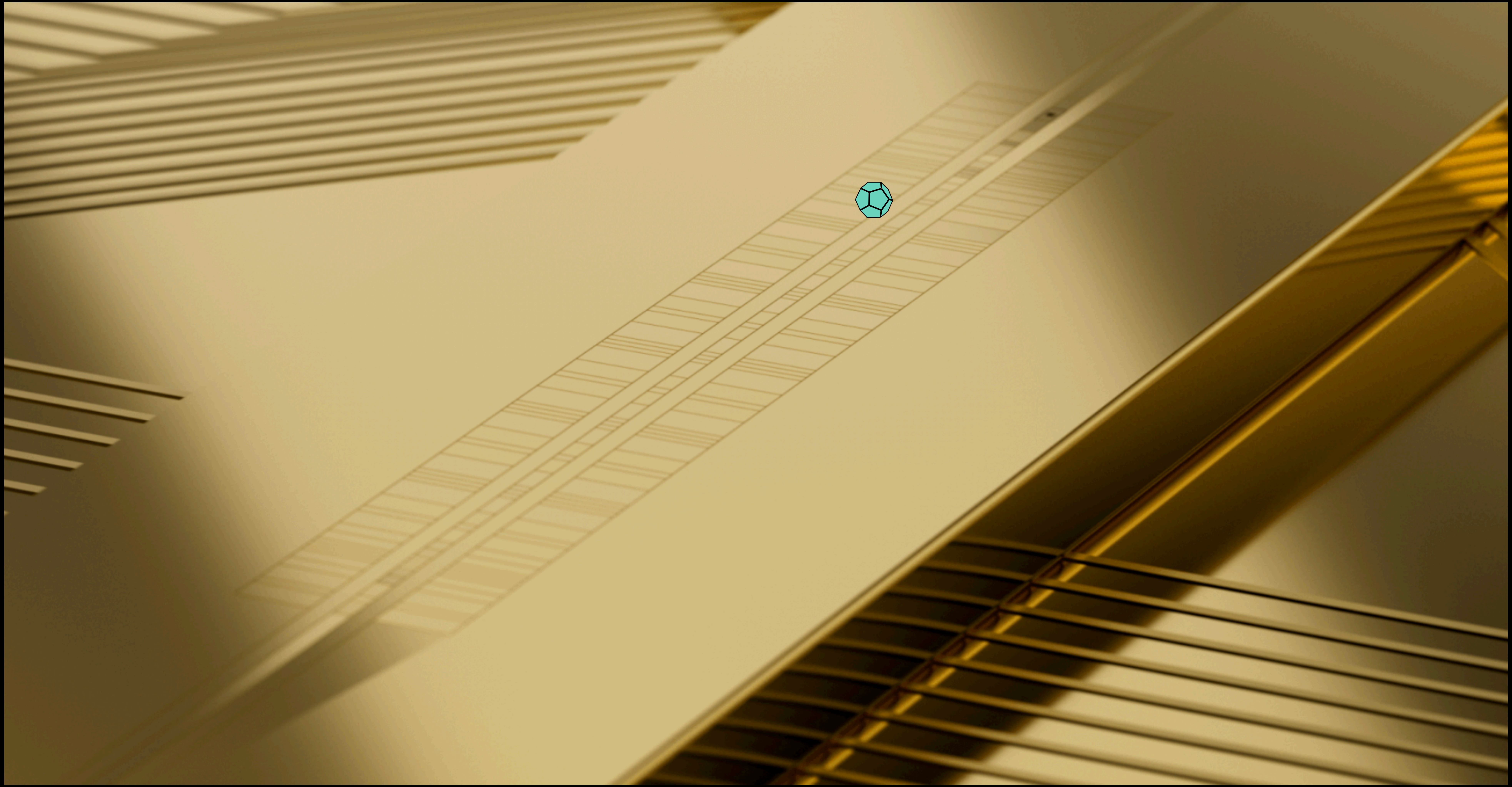

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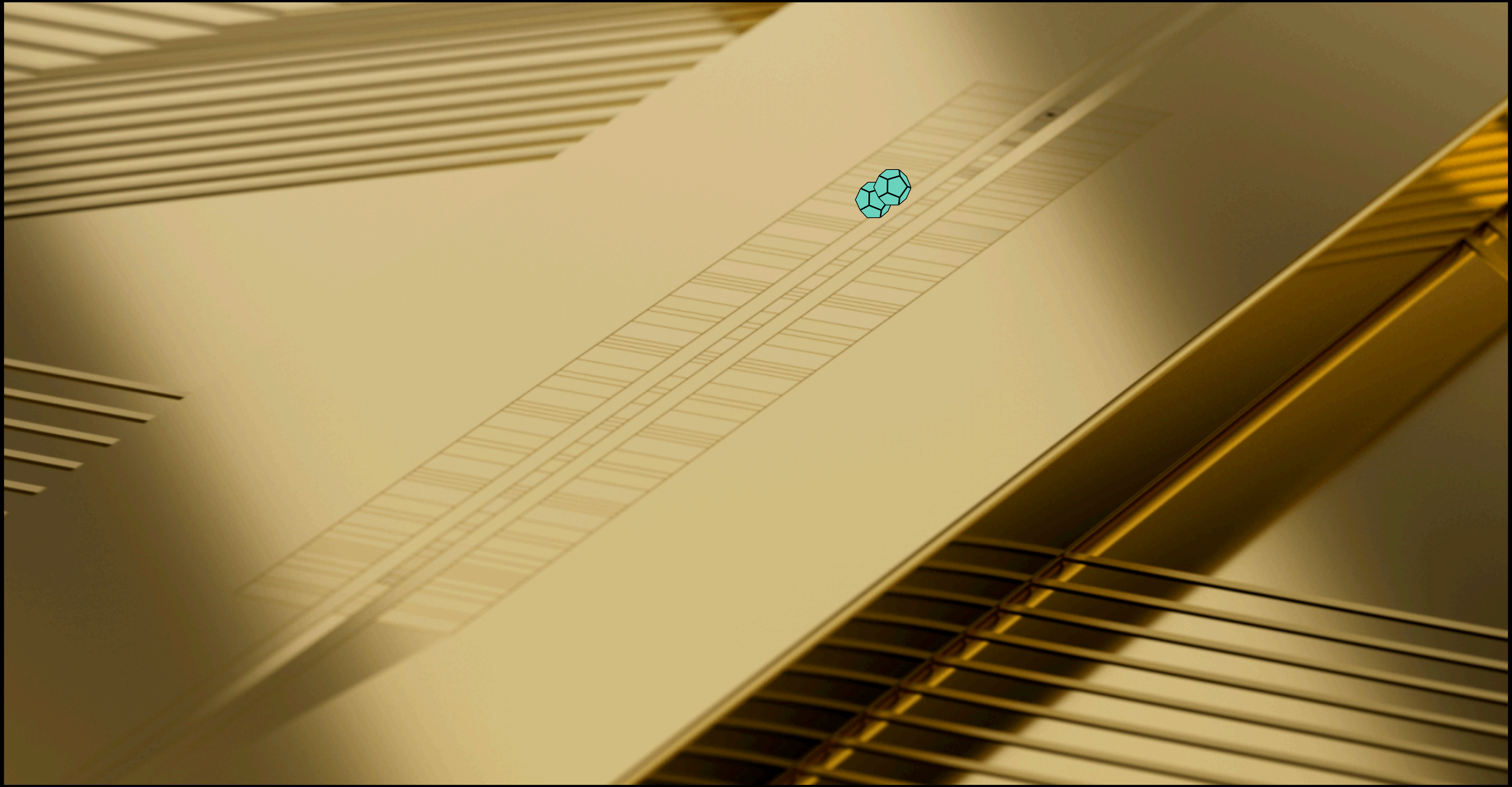

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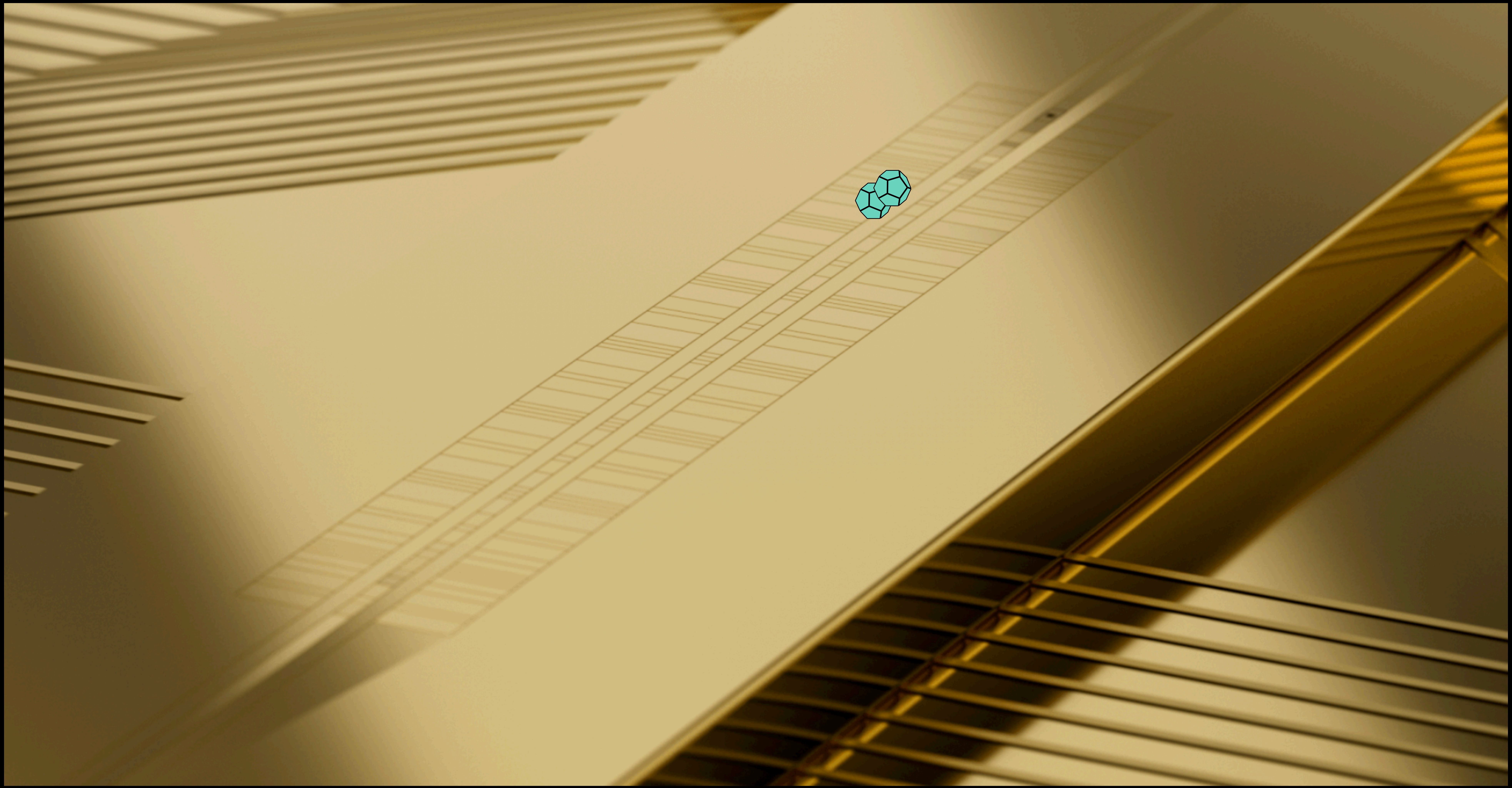

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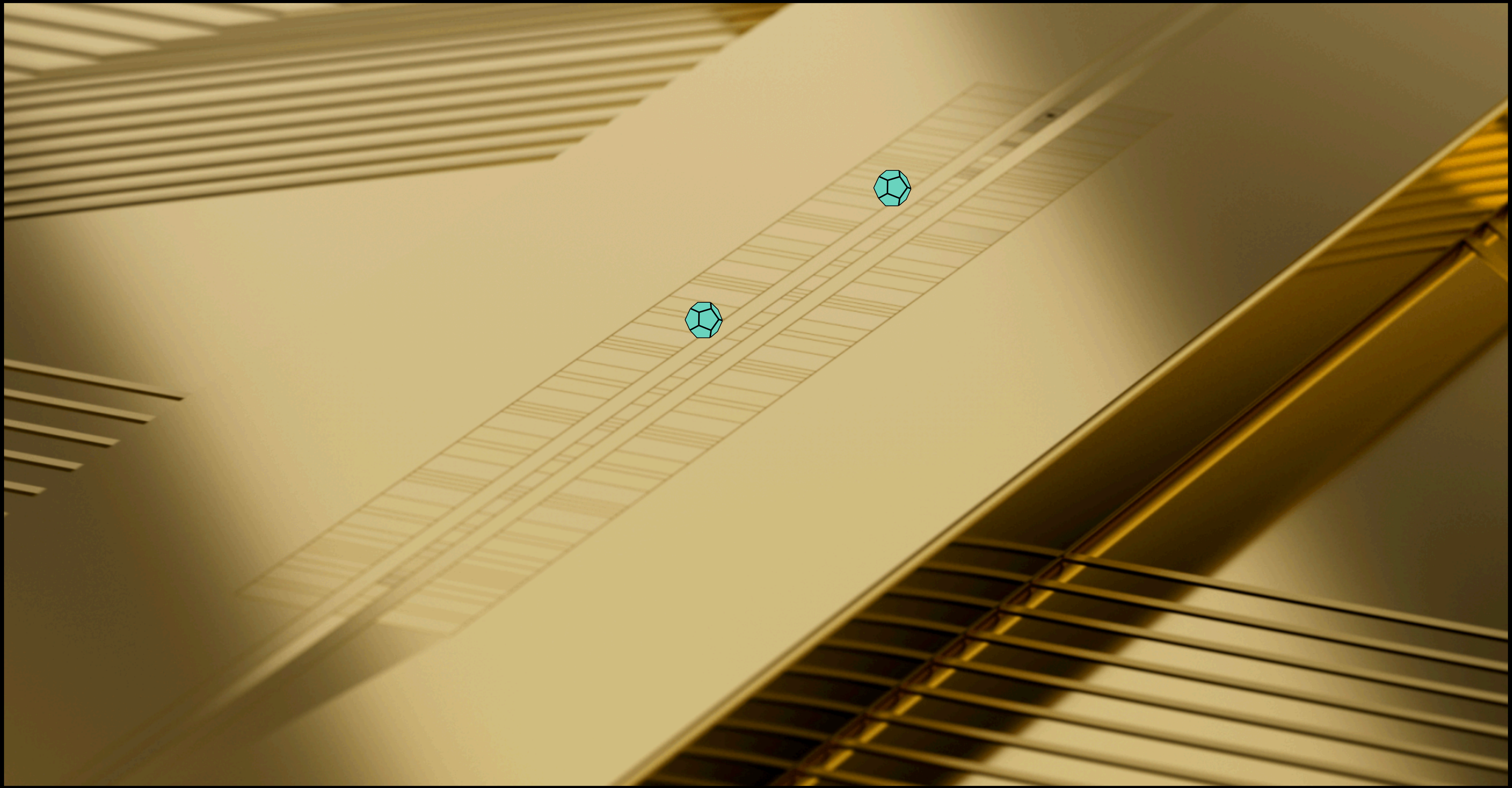

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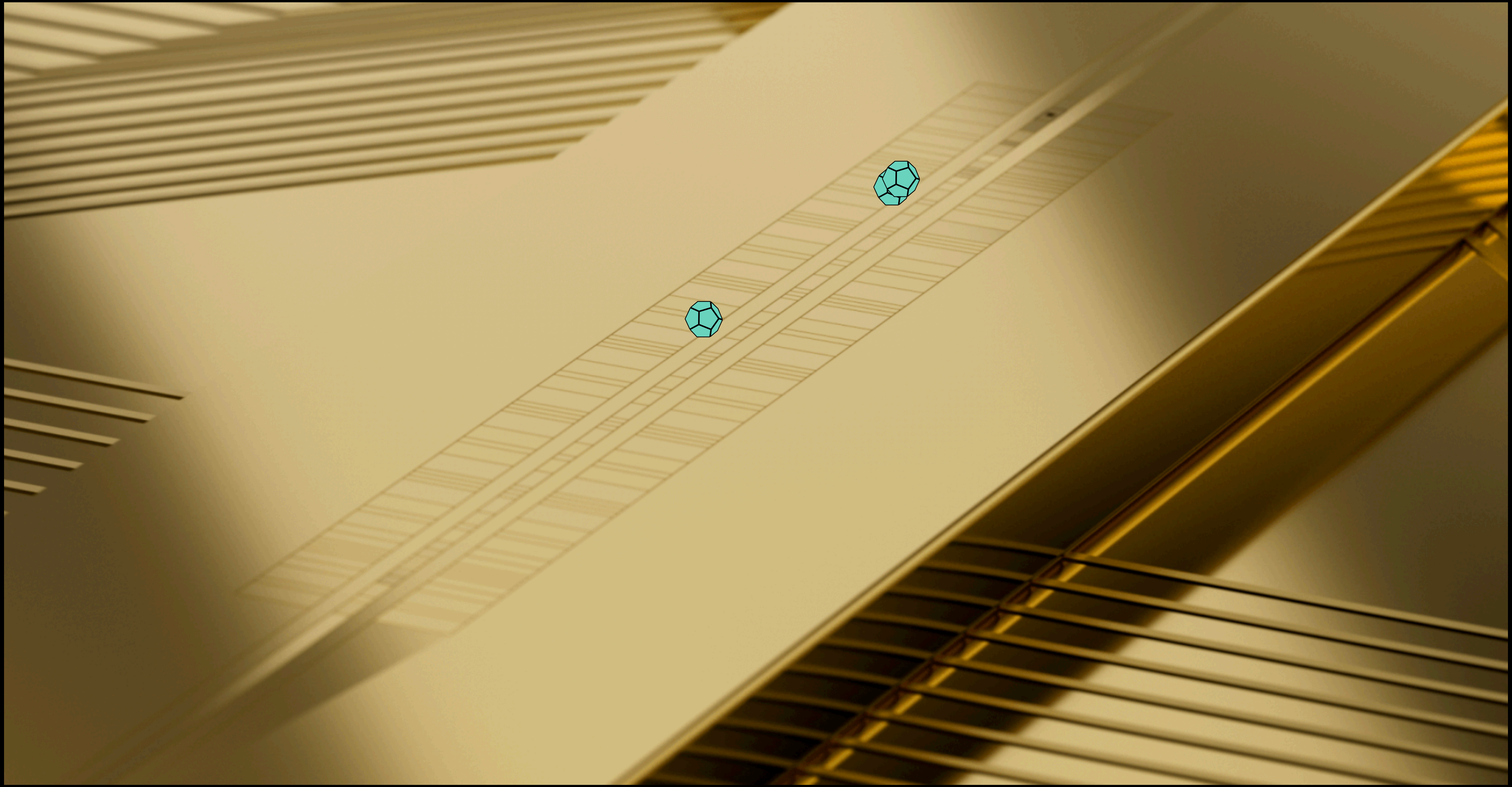

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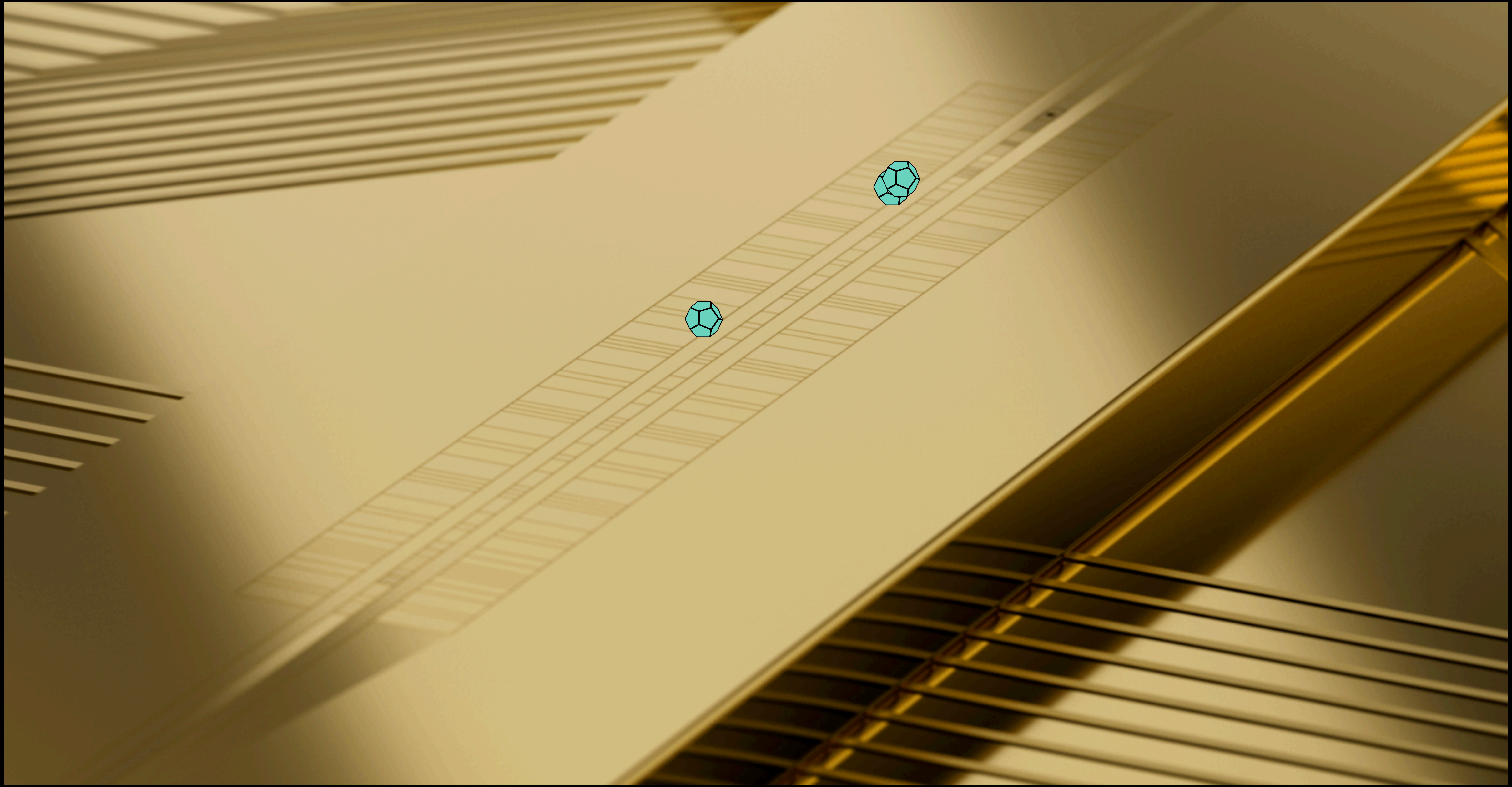

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State Preparation

With variational methods

- The ground state is prepared using the variational quantum eigensolver (VQE)
- A trial state is obtained using a parametrized quantum circuit $C(\theta)$ acting on some initial state
  - $|\Psi(\theta)\rangle = C(\theta)|\Psi_0\rangle$
- The expectation value of the Hamiltonian is measured
- An optimizer updates the parameters towards the minimum of the energy landscape

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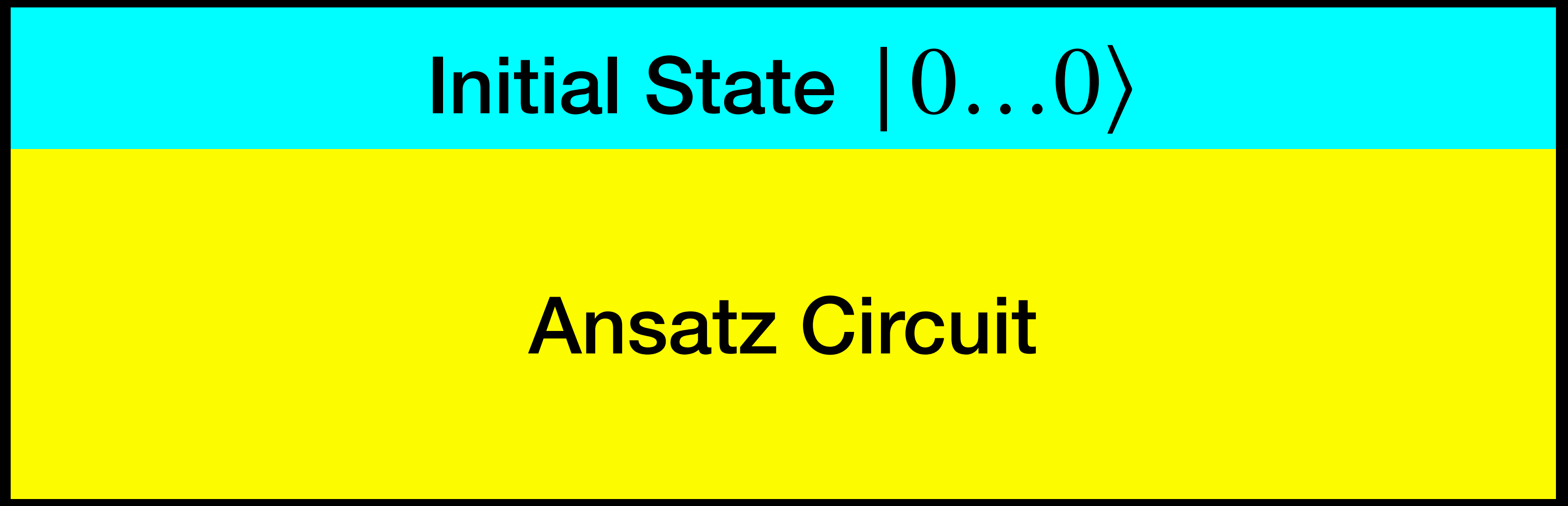

State Preparation

With variational methods

- The ground state is prepared using the variational quantum eigensolver (VQE)
- A trial state is obtained using a parametrized quantum circuit $C(\theta)$ acting on some initial state
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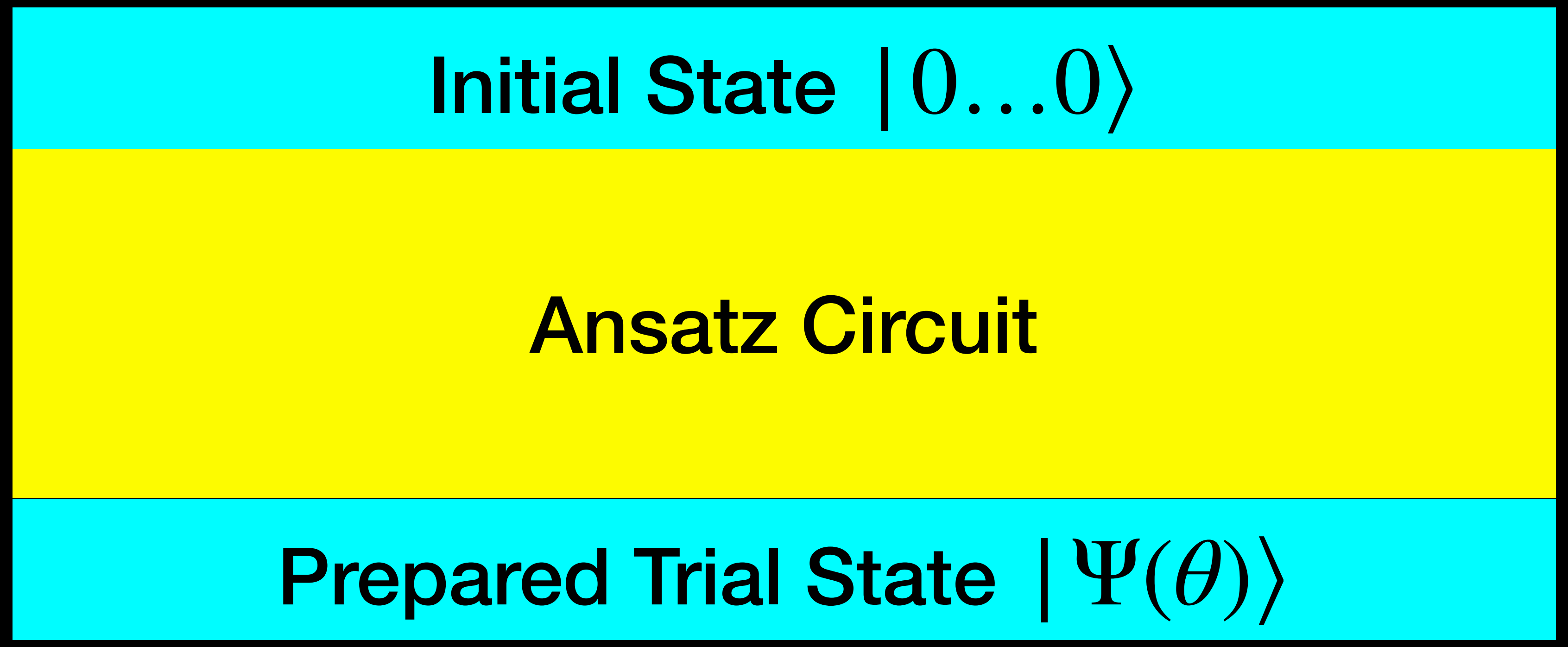

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Initial State $|0\ldots0\rangle$

Ansatz Circuit

Prepared Trial State $|\Psi(\theta)\rangle$

Cost function $E(\theta) = \langle\psi|\hat{H}|\psi\rangle$

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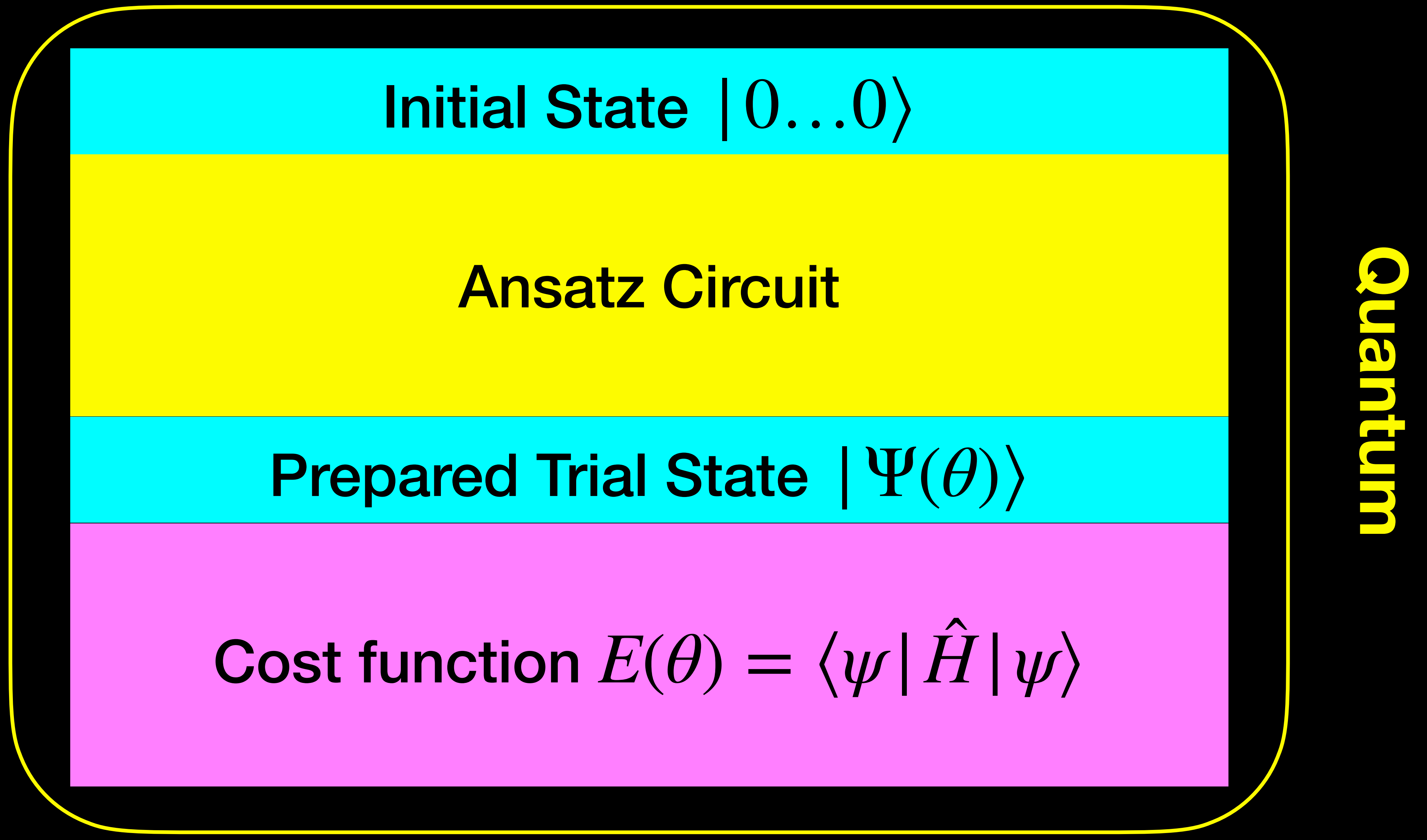

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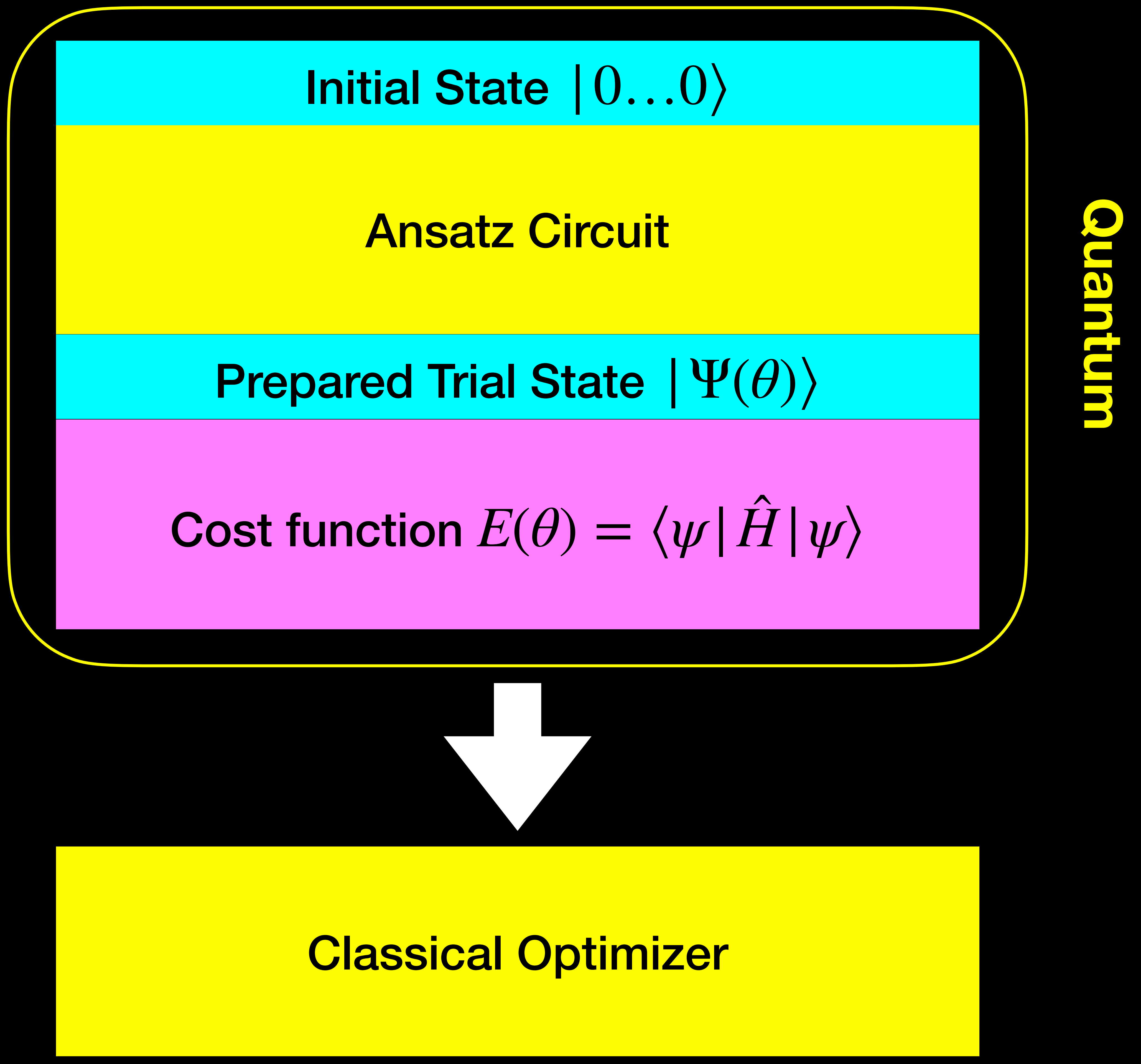

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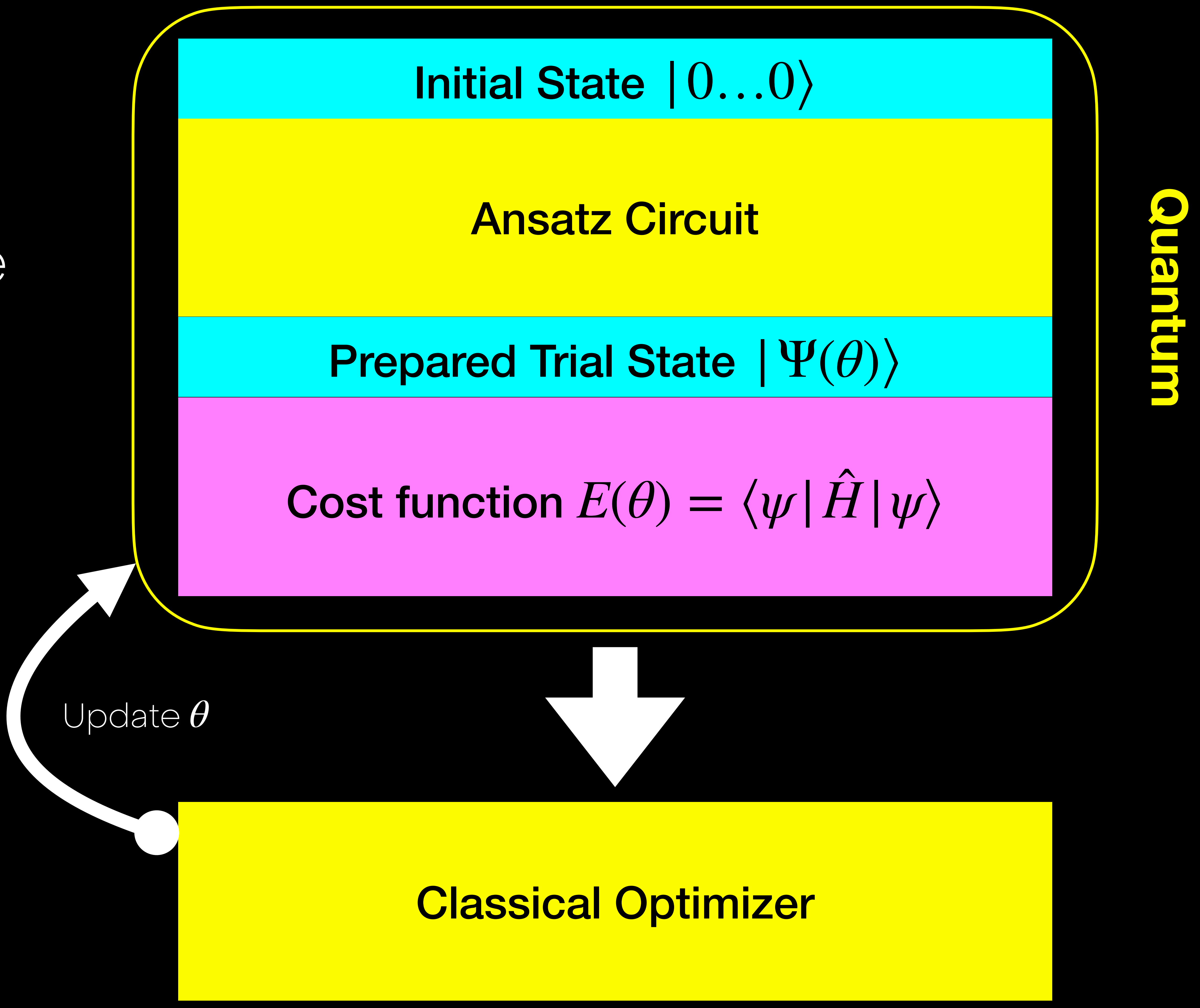
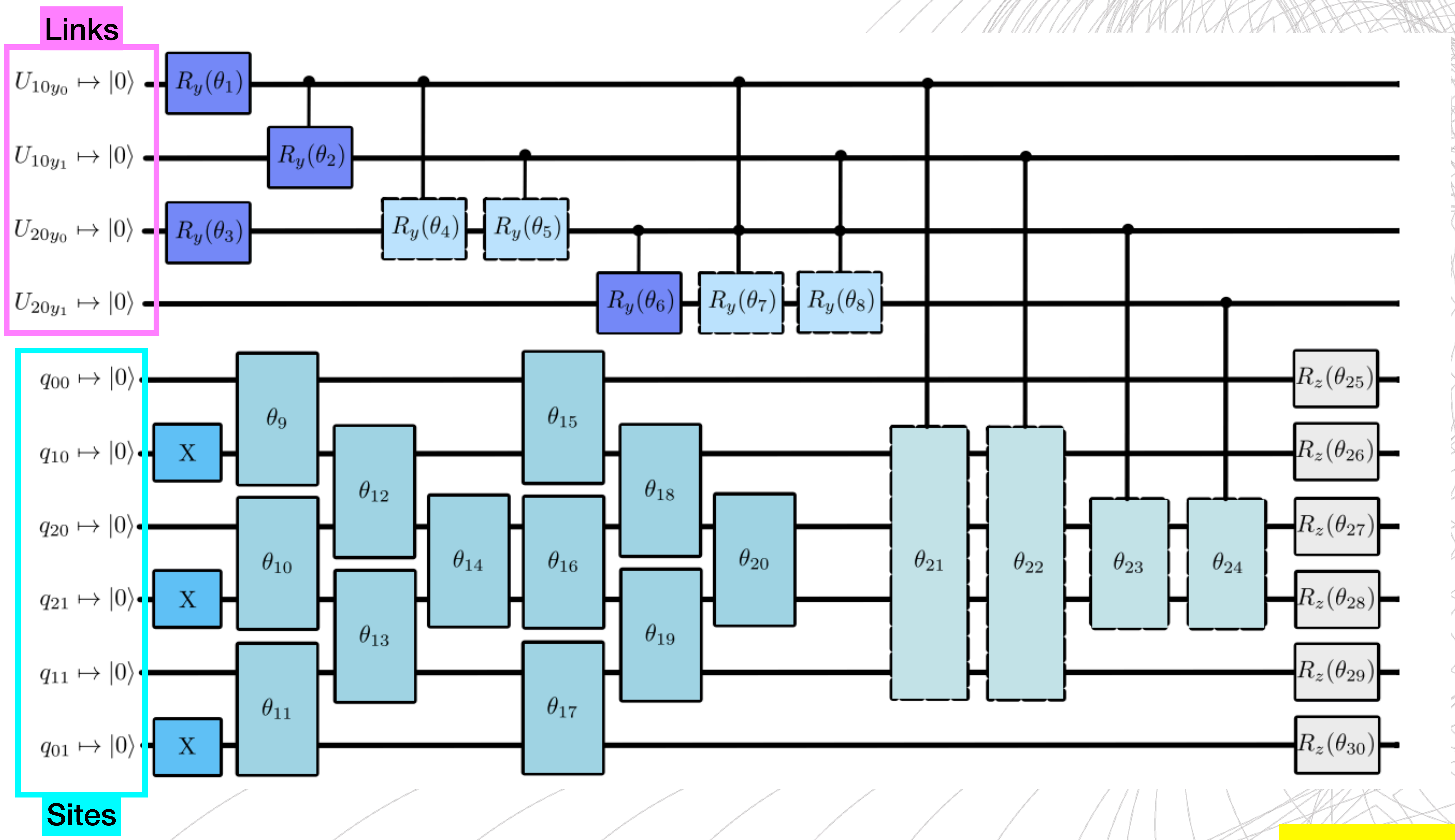
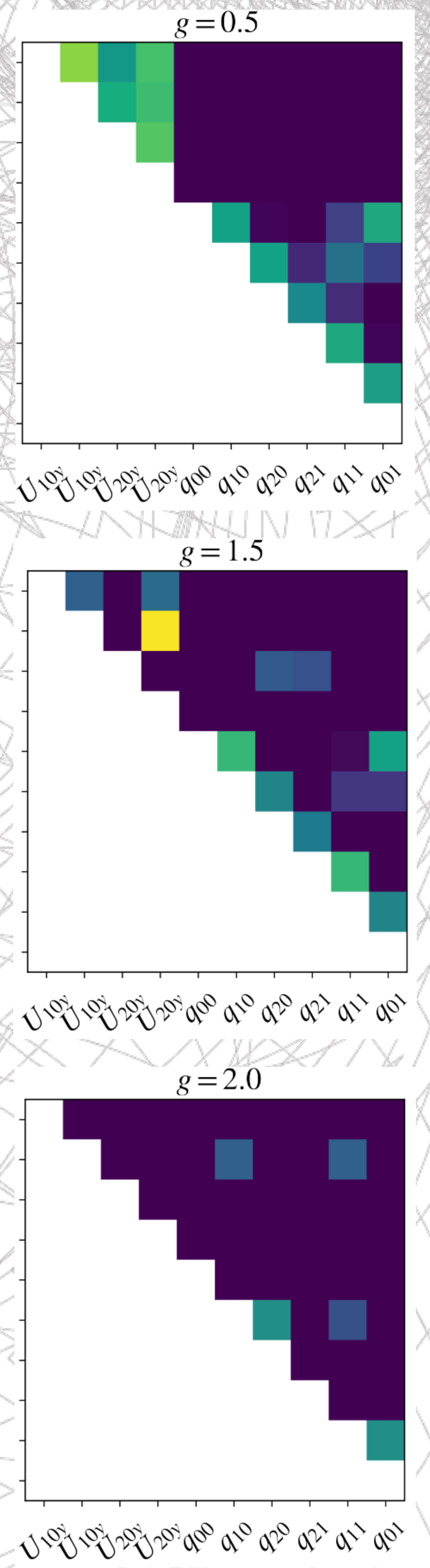


Mutual Information Ansatz





QUANTINUUM

Example ansatz circuit

10 qubits, 30 parameters

$$\left|\Psi(\theta)\right\rangle = \left|\psi_{\text{sites}}\right\rangle \otimes \left|\psi_{links}\right\rangle = C(\theta)\left|\Psi_0\right\rangle$$

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$|1010100101\rangle$
$|1010100100\rangle$
$|1000110100\rangle$
$|1000110111\rangle$
…

QUANTINUUM

Example of gate decomposition

{H, X, Rz, Rx, Ry, CNOT}: ≈ 115 2-qubit gates

H-series Native Gates: ≈ 80 2-qubit gates

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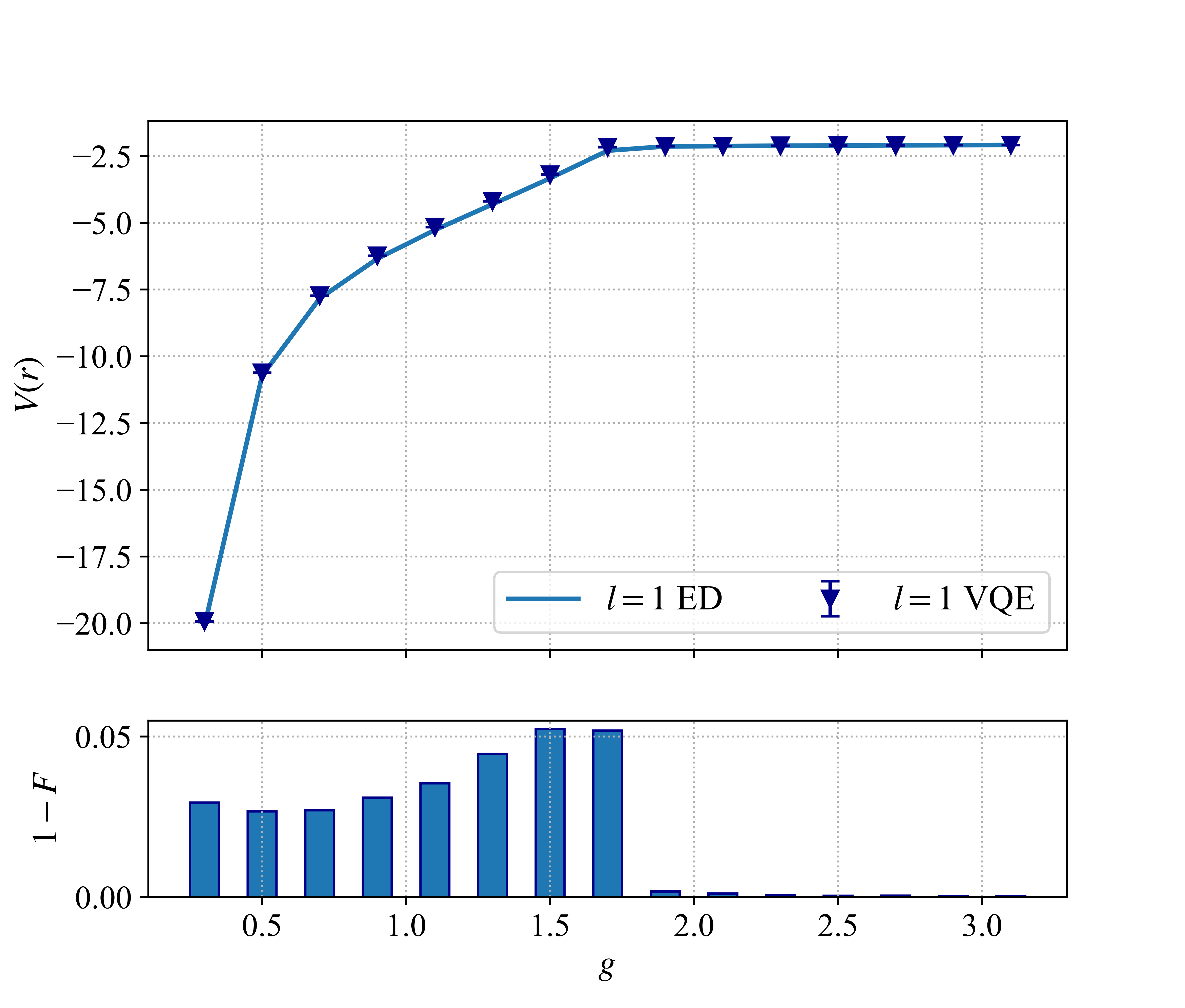
Results

Consistency checks

Benchmark *quantum* results against *classical* methods

- A quantum state of 10 qubits can be represented with classical memory on a laptop and the Hamiltonian can be easily written as a matrix
- Use exact diagonalization (ED) to find the ground state and its energy
- Use VQE to find the optimal parameters for the ground state circuit: check fidelity (F)



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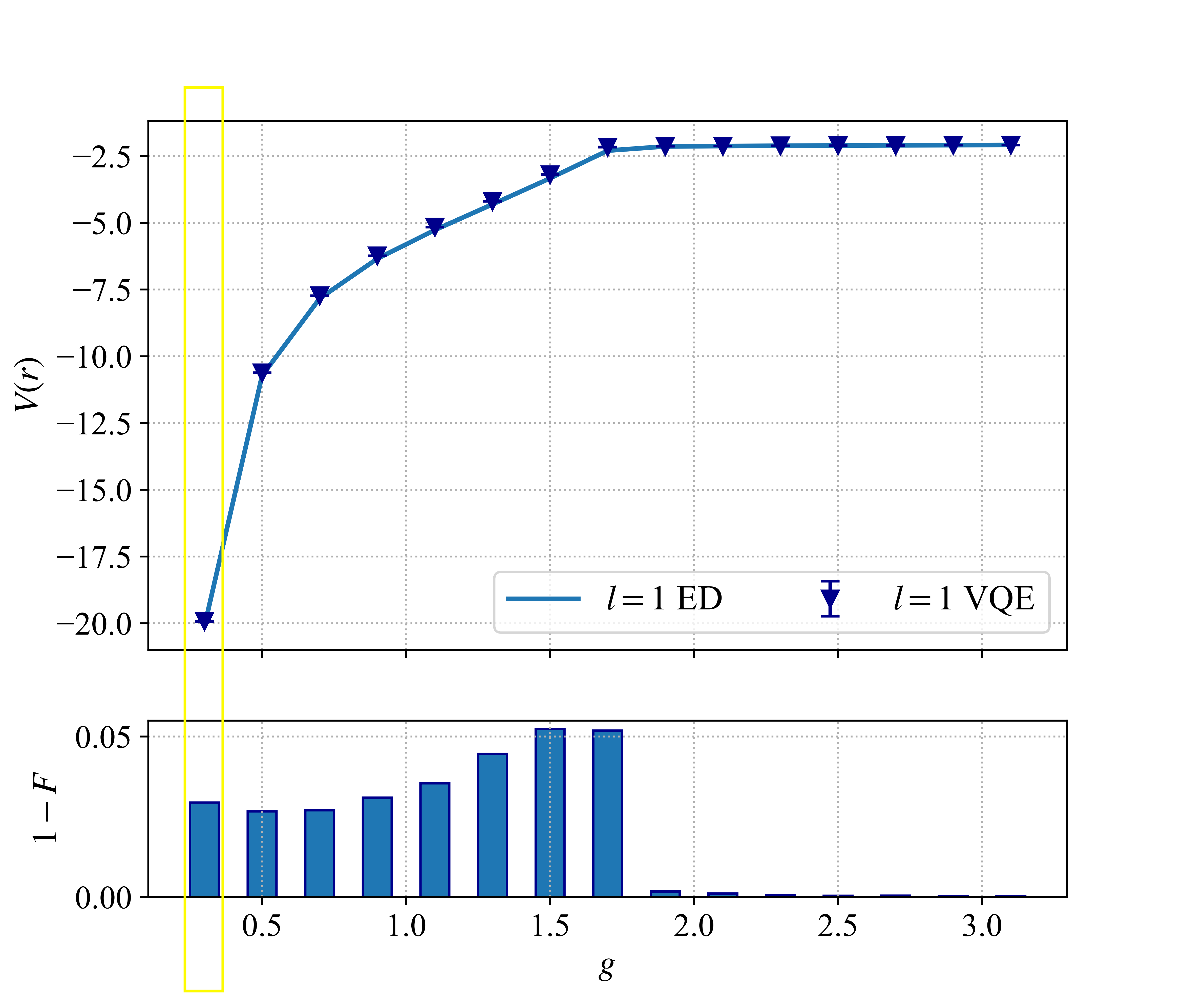

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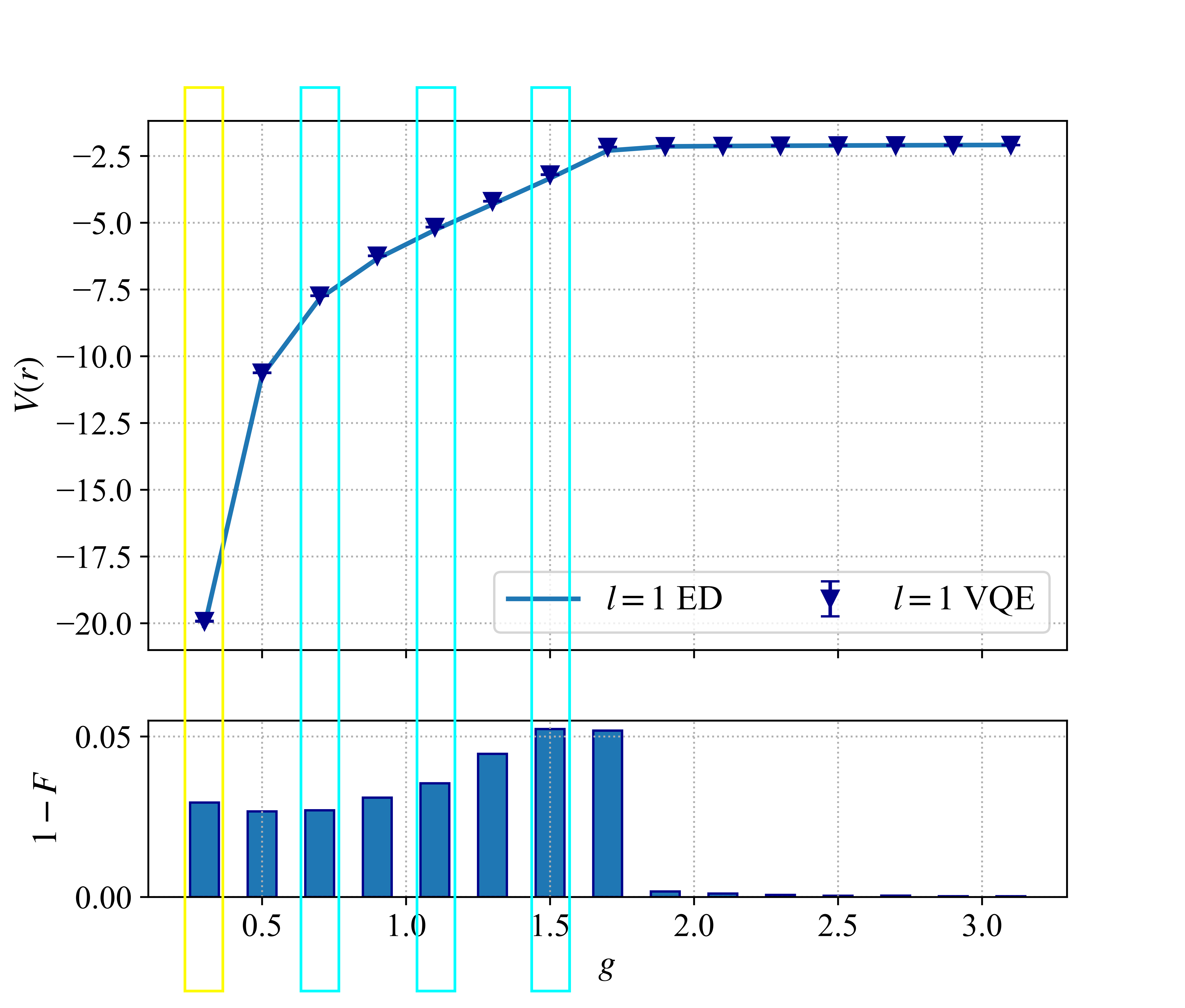

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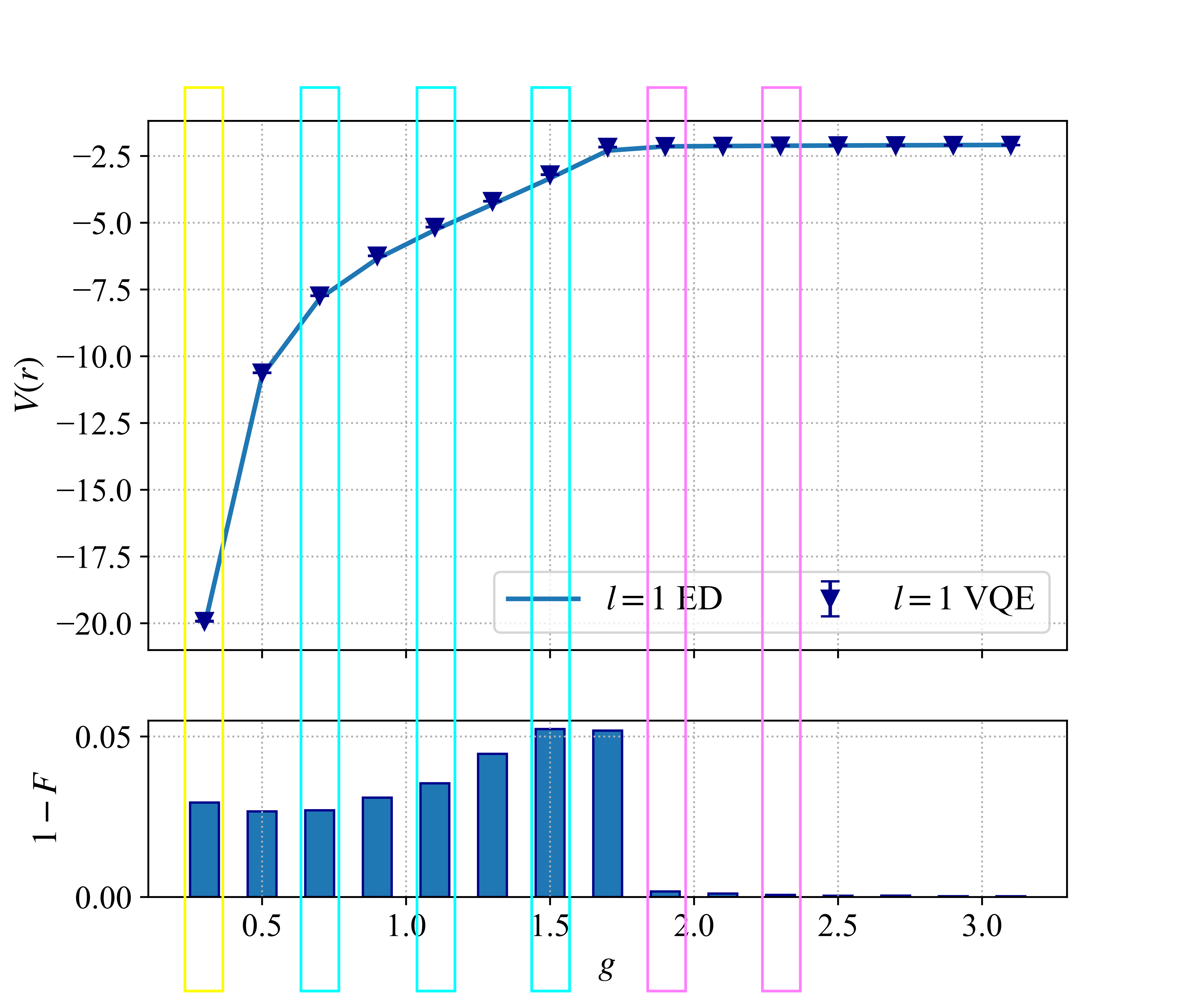

Simulation and Emulation

And real hardware experiments

- Given the optimal parameters for the ansatz circuit at each coupling we can:
  - Simulate the circuit classically without measuring
  - Simulate the circuit classically with measurements
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H1-1E ↔ H1-1

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H1-1E ↔ H1-1

$$|\Psi_{GS}\rangle = \sum_{i}^{2^N} c_i |i\rangle \quad \Rightarrow \quad \text{prob}_\text{i} = |c_i|^2 = |\langle i|\Psi_{GS}\rangle|^2$$

Simulations on laptops

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$$\text{prob}_\text{i} = |c_i|^2 = |\langle i|\Psi_\text{GS}\rangle|^2$$

Probabilities of the different states for each g value:AerStateBackend shots=None

$\text{prob}_{\text{i}} = |c_i|^2 = |\langle i | \Psi_{\text{GS}} \rangle|^2$ Probabilities of the different states for each g value:AerStateBackend shots=None

QUANTINUUM

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$|\Psi_{GS}\rangle = c_0|1010100101\rangle + c_1|1010101101\rangle + c_2|1010100111\rangle$

$|\Psi_{GS}\rangle = c_0|1000110101\rangle + \ldots$

Probabilities of the different states for each g value:AerBackend shots=100000

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Small Noise Levels

Large Noise Levels

Large Noise Levels

Large Noise Levels

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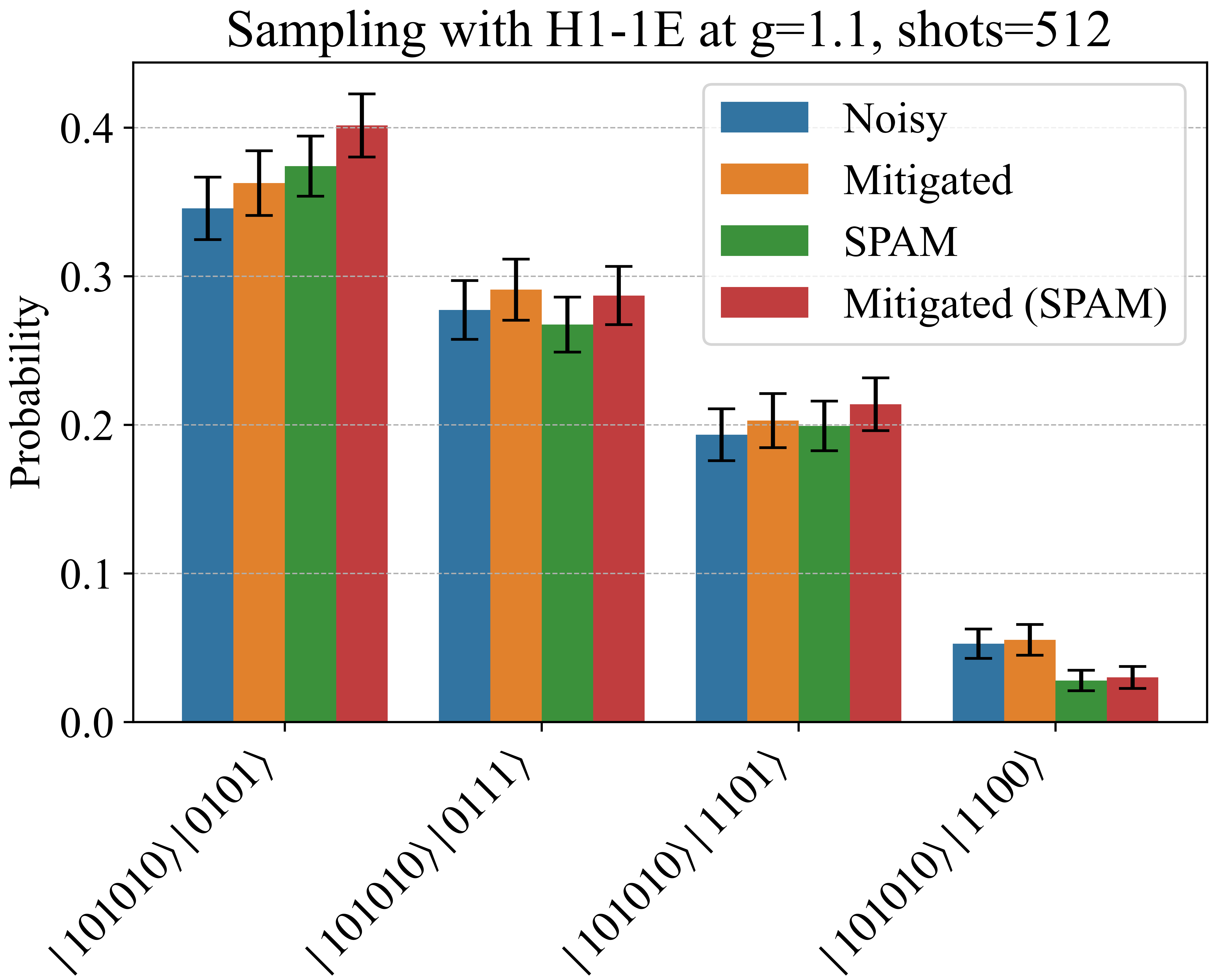
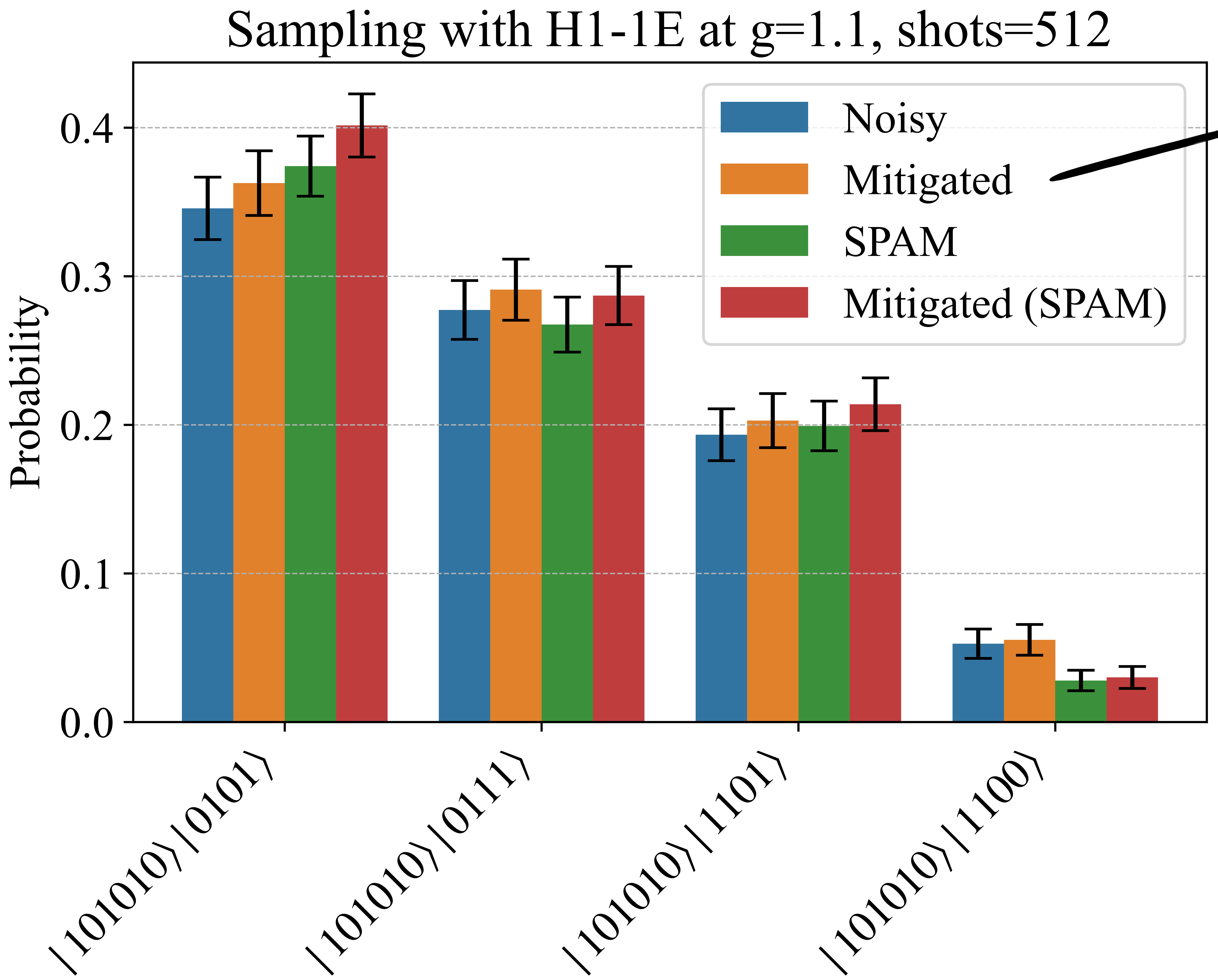
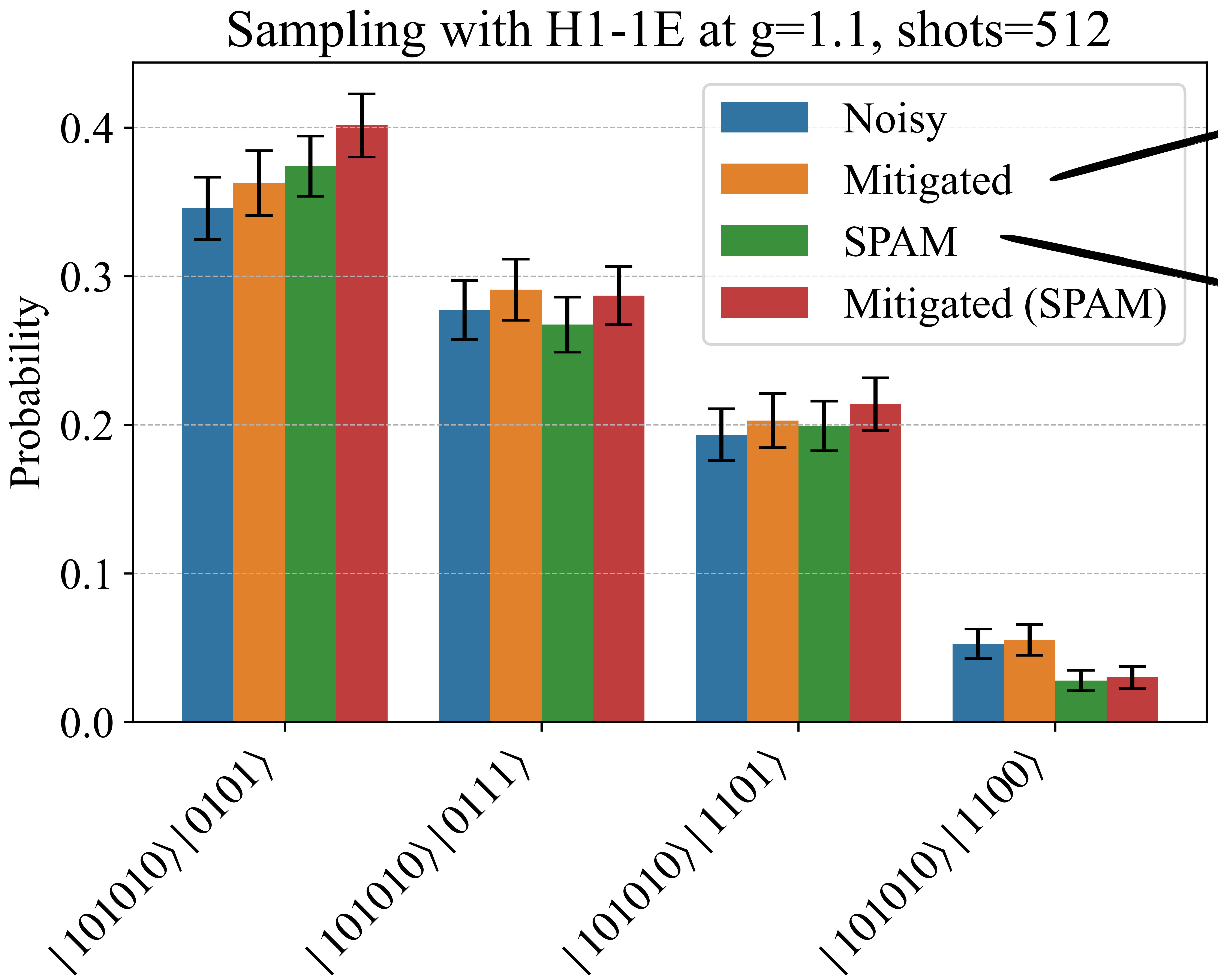
Emulations on H1-1E


Sampling with H1-1E at g=1.1, shots=512
Probability
0.0
0.1
0.2
0.3
0.4
Noisy
Mitigated
SPAM
Mitigated (SPAM)
|101010⟩|0101⟩
|101010⟩|0111⟩
|101010⟩|1101⟩
|101010⟩|1100⟩



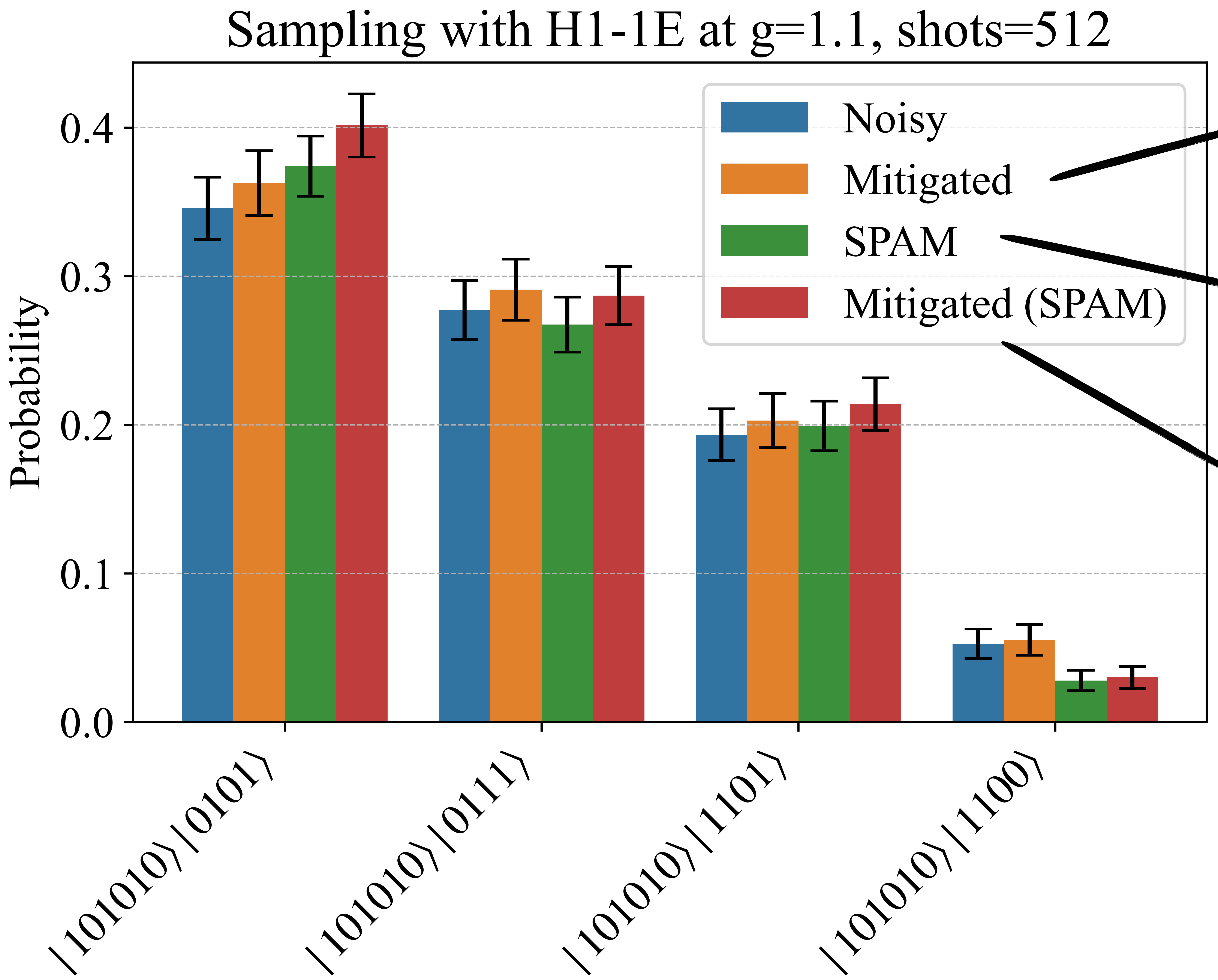
Remove bit strings that do not correspond to physical configurations

QUANTINUUM



Remove bit strings that do not correspond to physical configurations

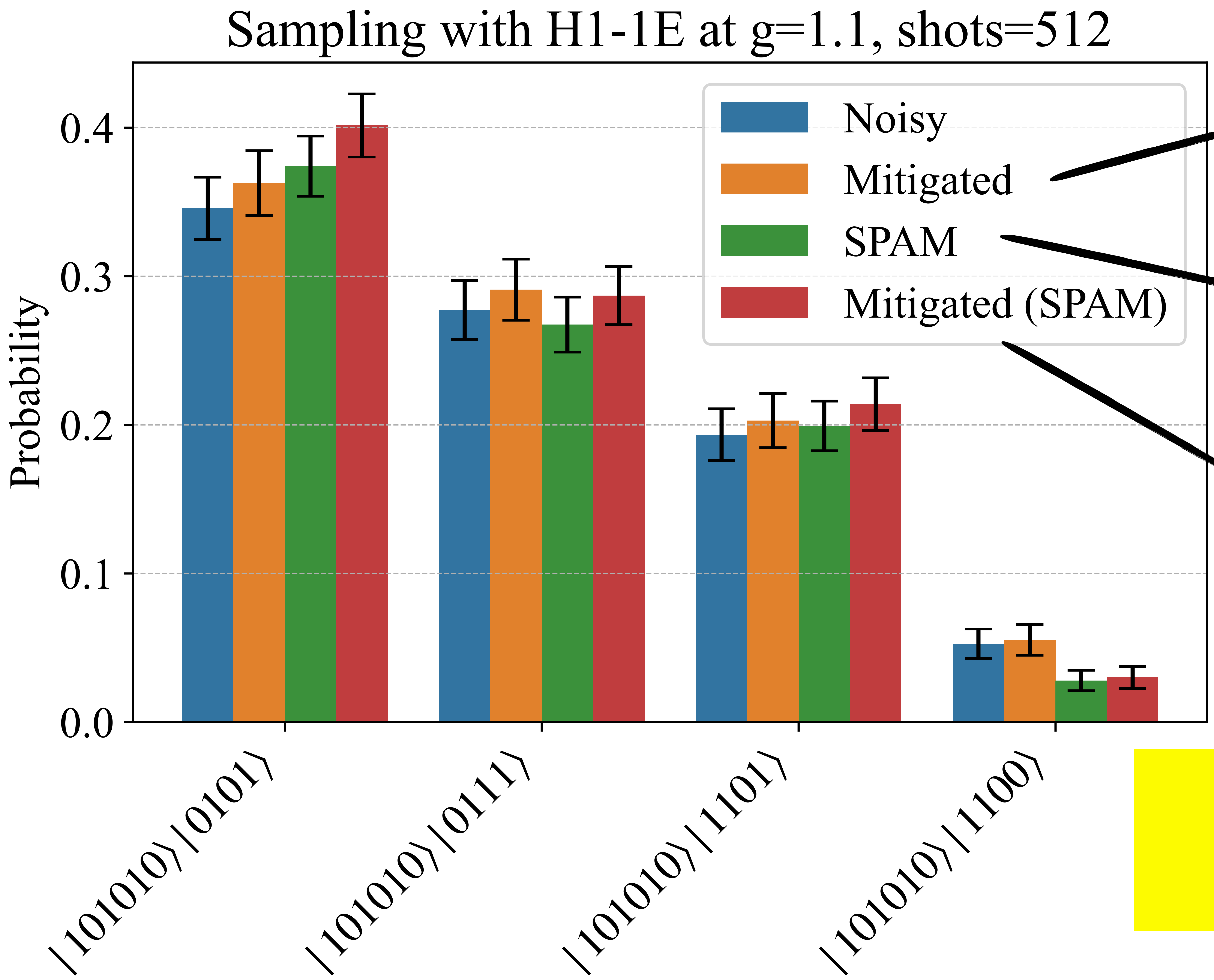
State Preparation and Measurement correction



Remove bit strings that do not correspond to physical configurations

State Preparation and Measurement correction

Filter out unphysical bit strings after correcting for SPAM



Remove bit strings that do not correspond to physical configurations

State Preparation and Measurement correction

Filter out unphysical bit strings after correcting for SPAM

Very little difference raw results and mitigated ones

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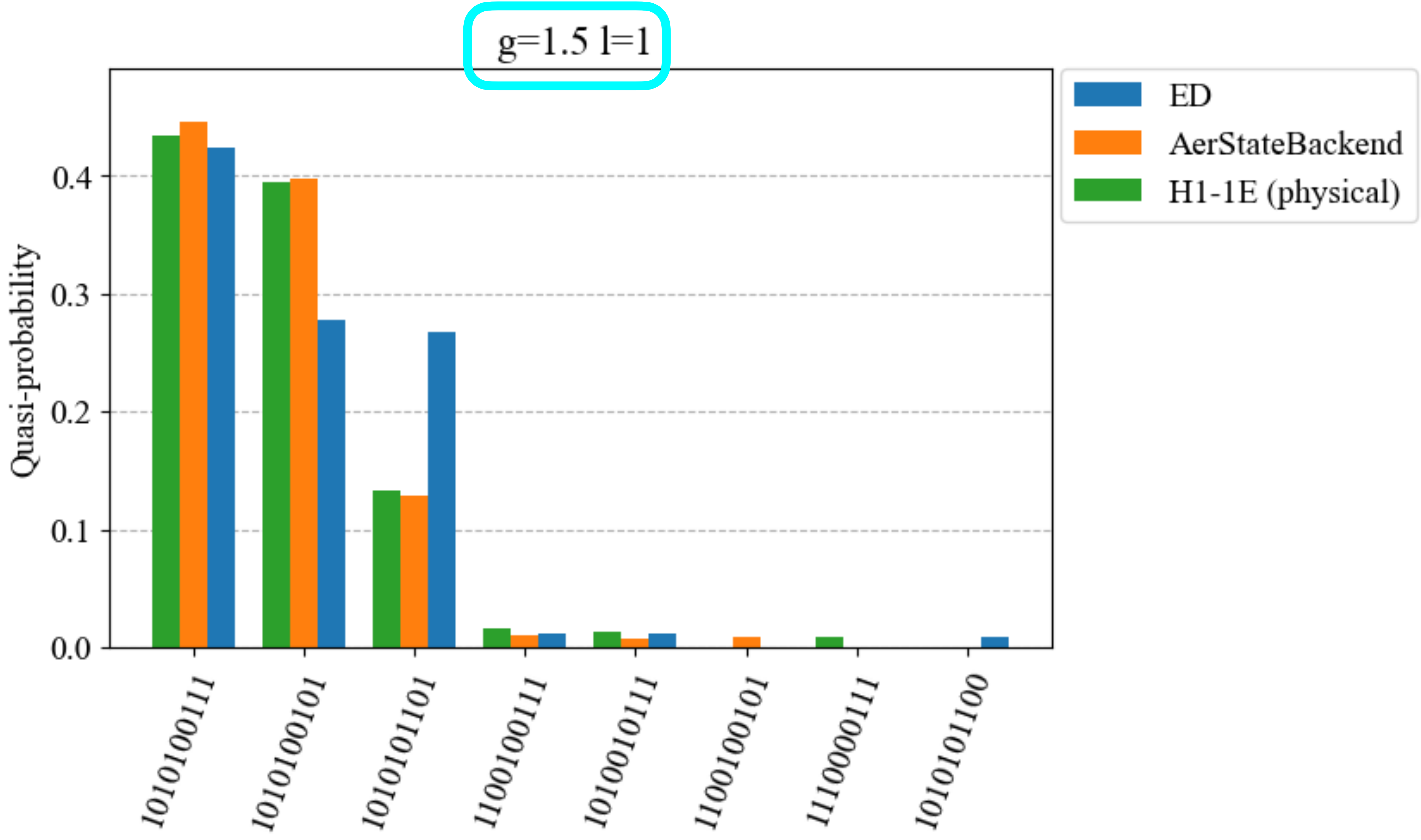
H1-1E

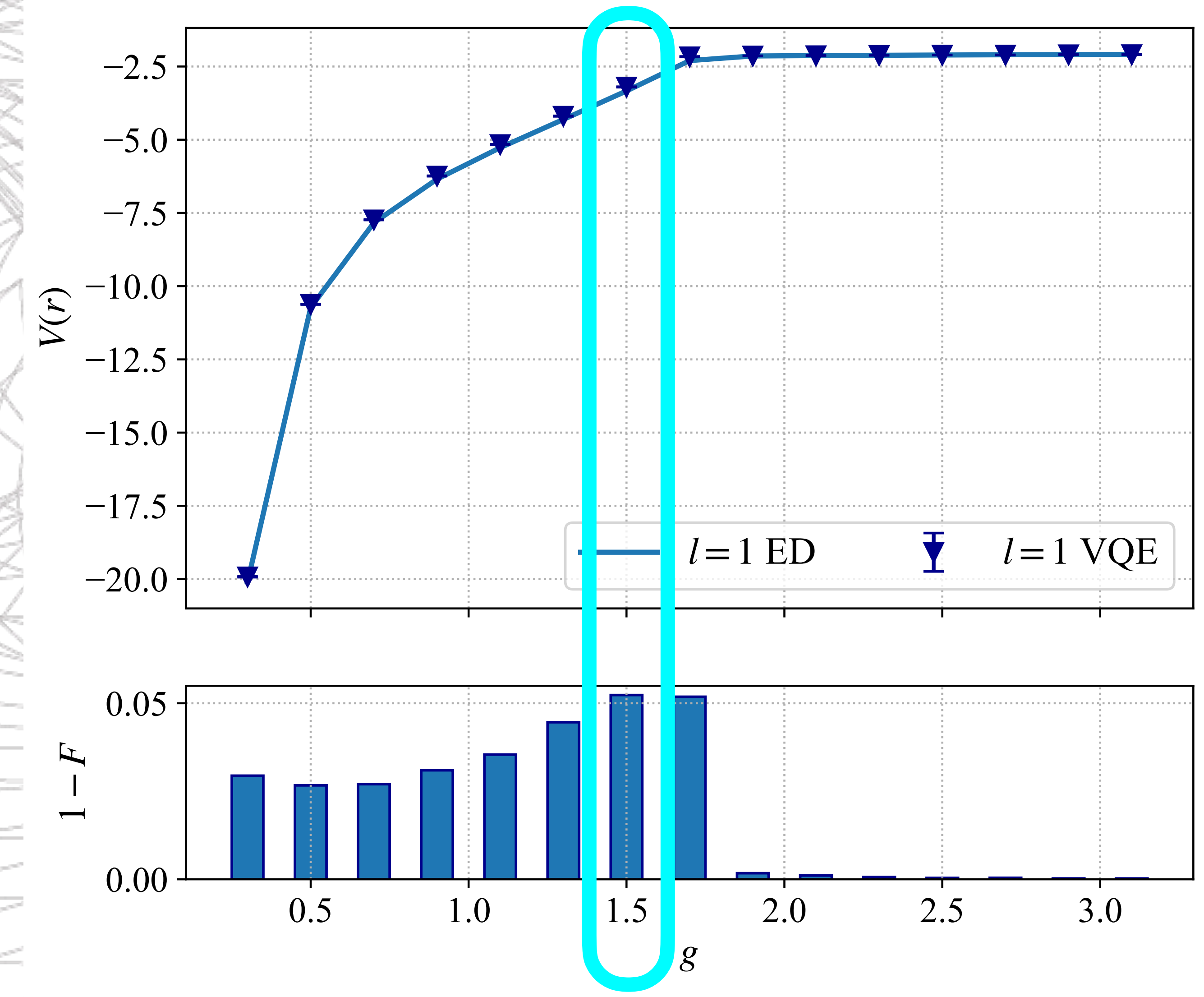
QUANTINUUM

H1-1E

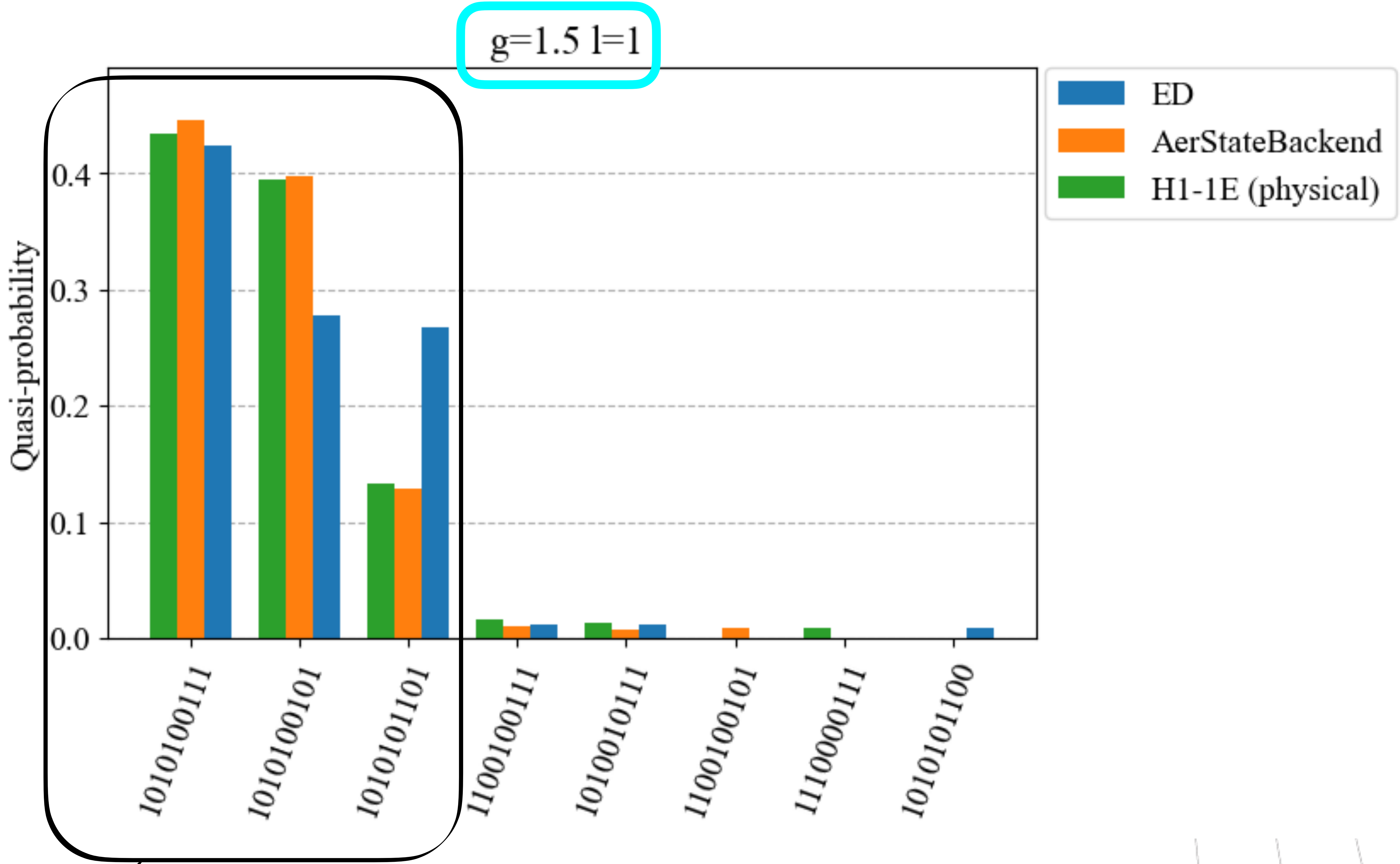
Remove unphysical states/bitstrings

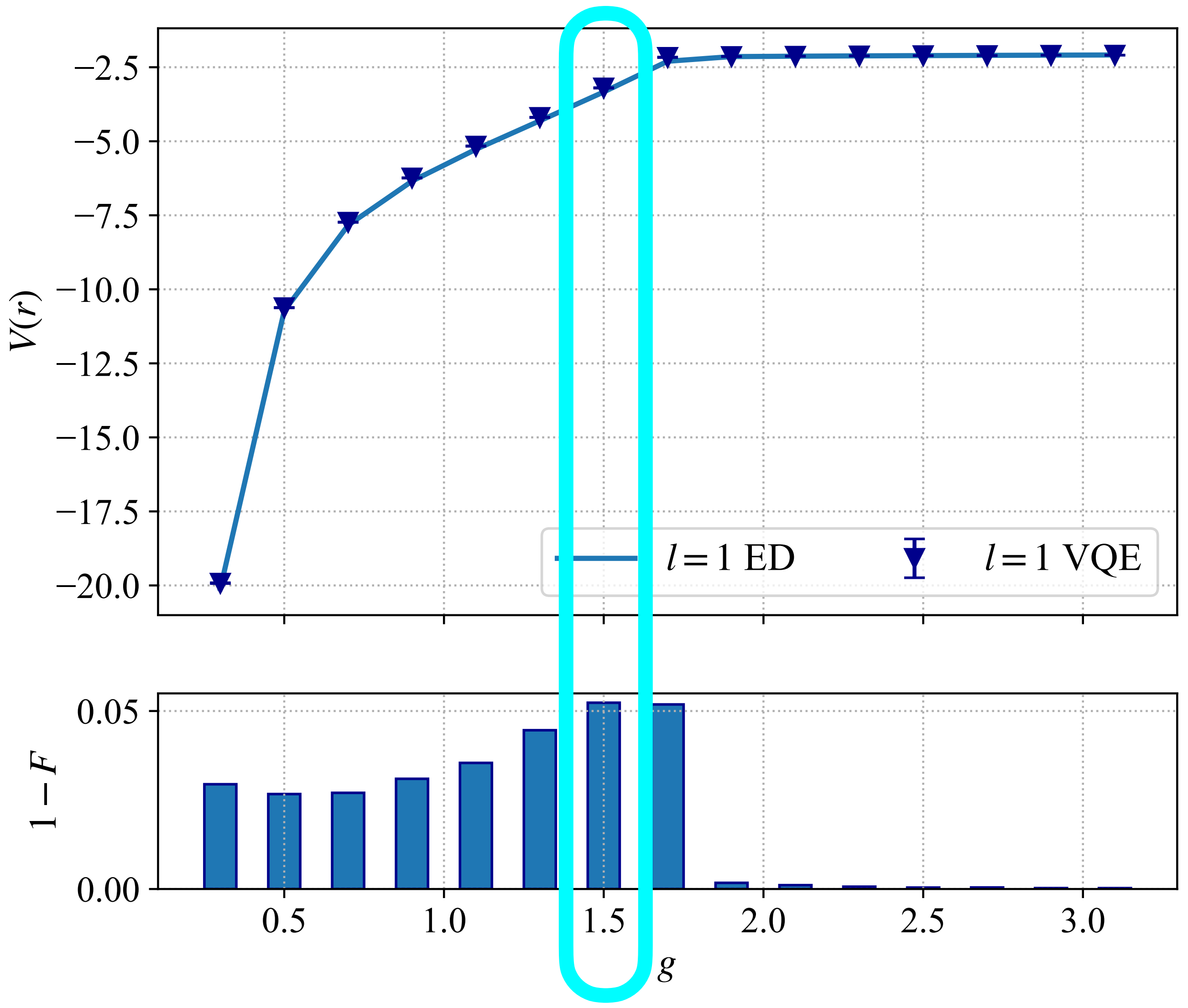


g=1.5 l=1
ED
AerStateBackend
H1-1E (physical)
Quasi-probability
0.0
0.1
0.2
0.3
0.4
1010100111
1010100101
1010101101
1100100111
1010010111
1100100101
1110000111
1010101100

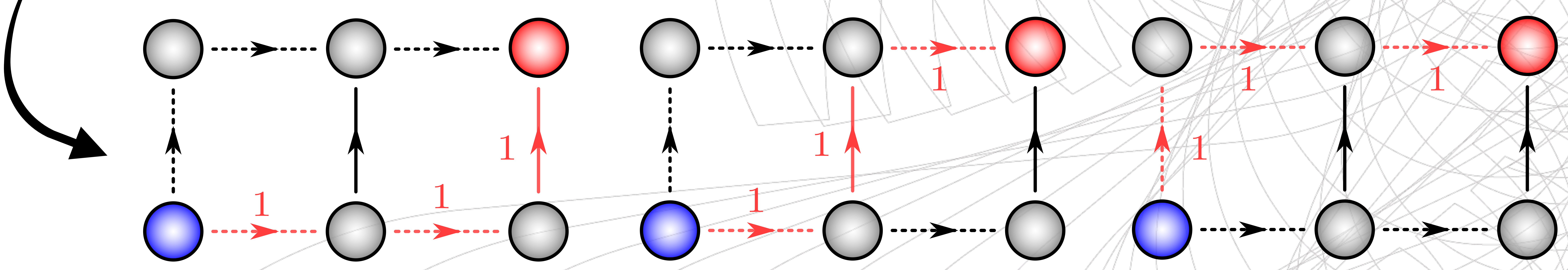


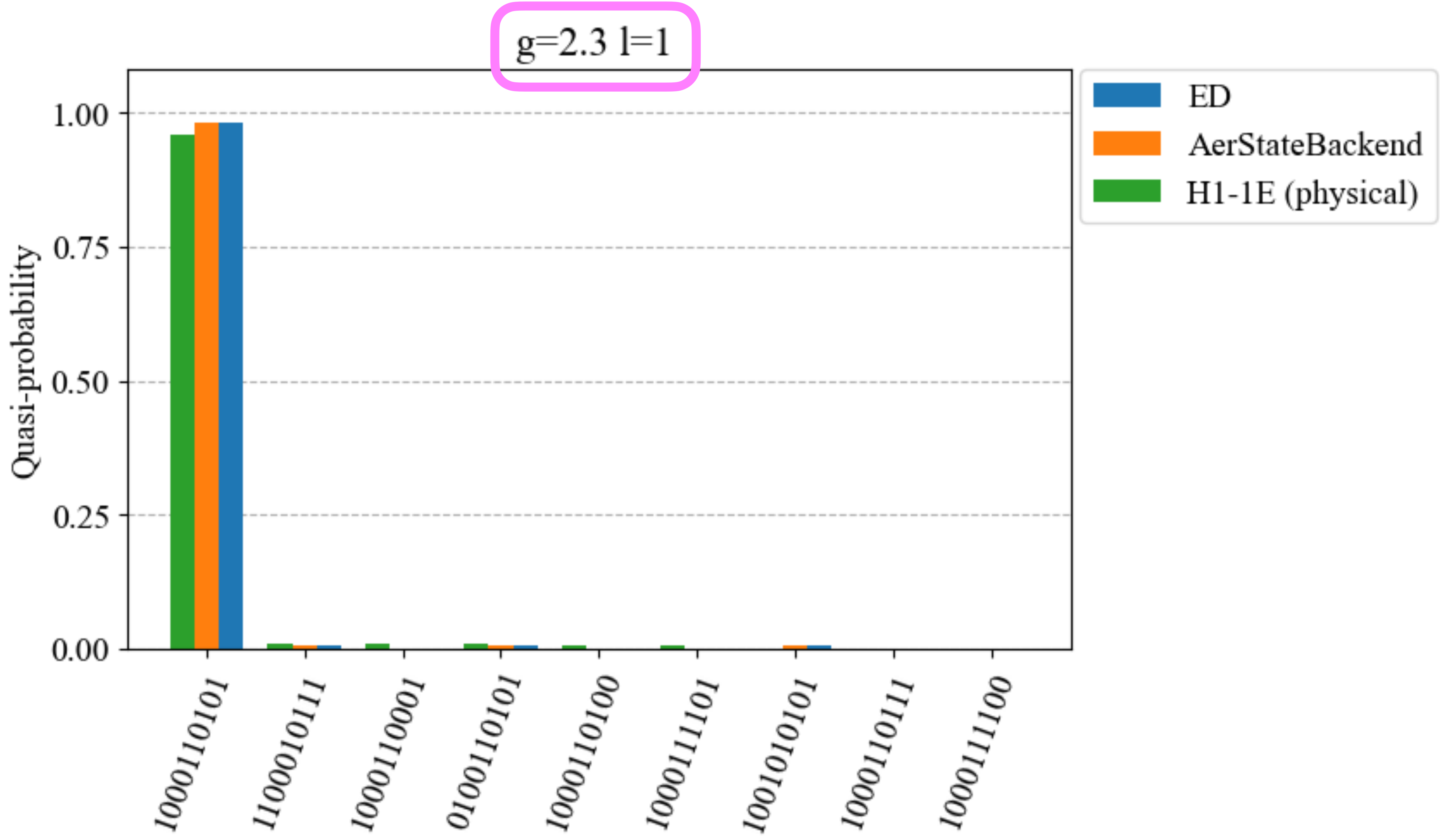
V(r)
−2.5
−5.0
−7.5
−10.0
−12.5
−15.0
−17.5
−20.0
l = 1 ED
l = 1 VQE
1 − F
0.05
0.00
0.5
1.0
1.5
2.0
2.5
3.0
g

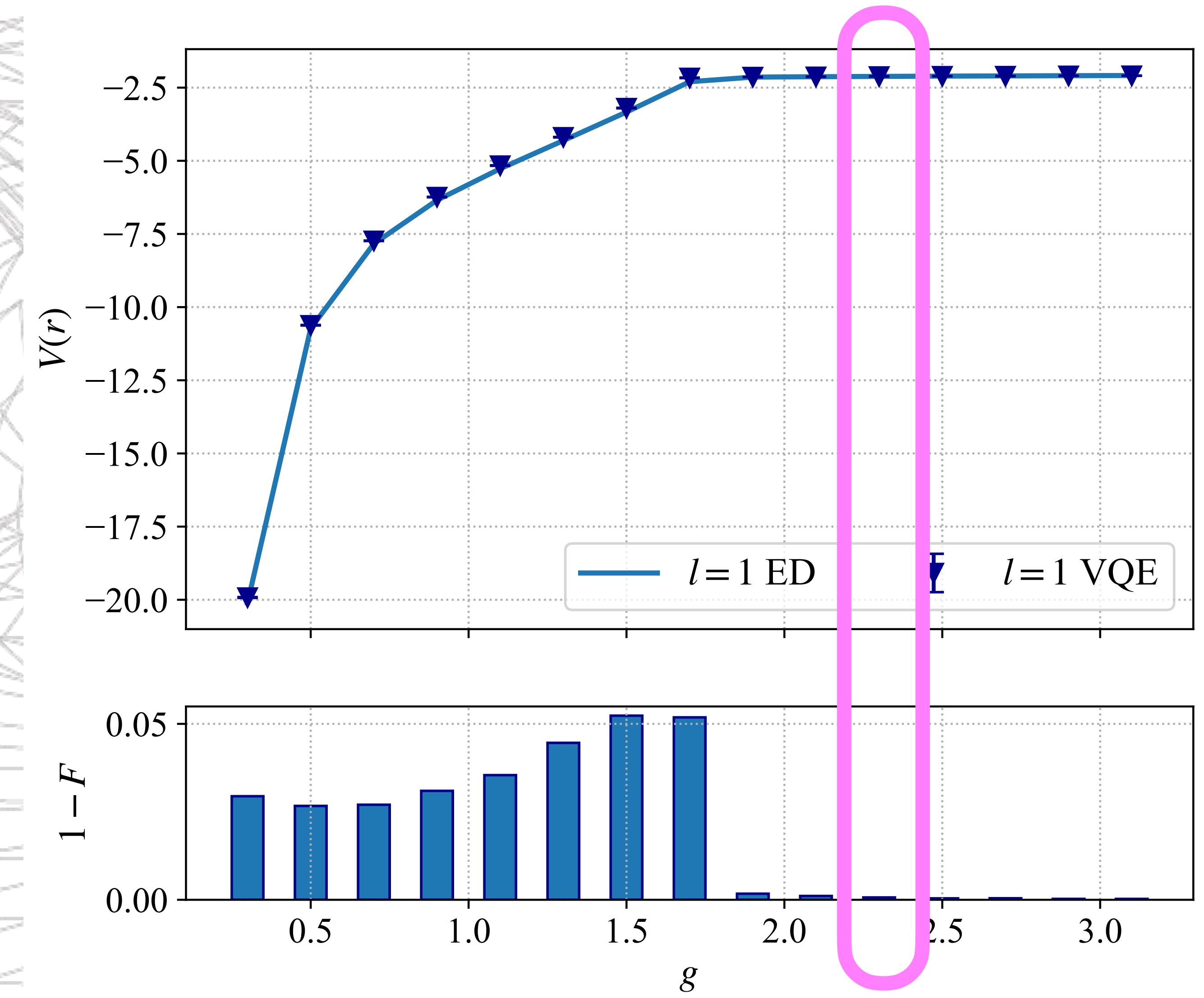




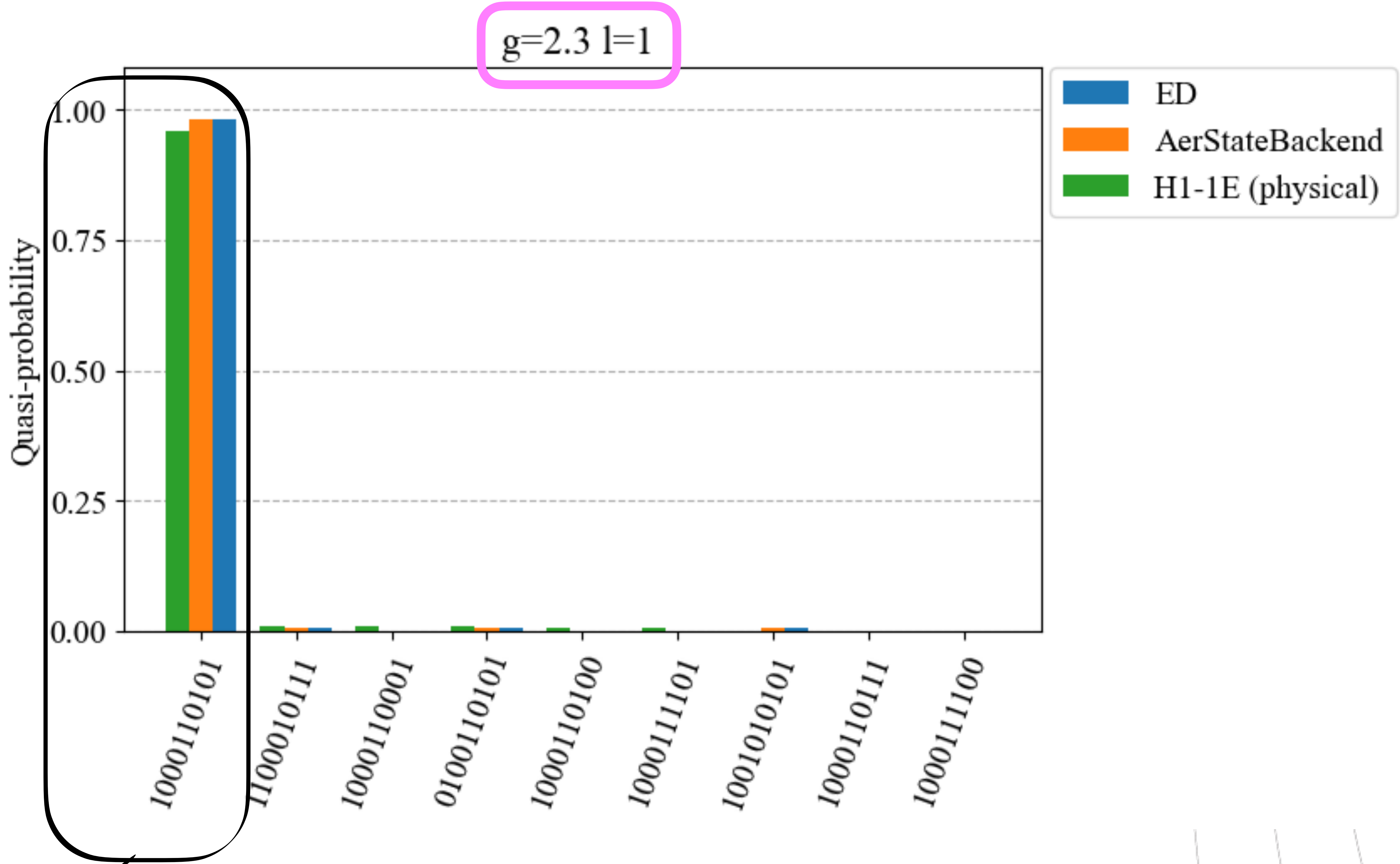
Configurations with electric fluxes between static charges

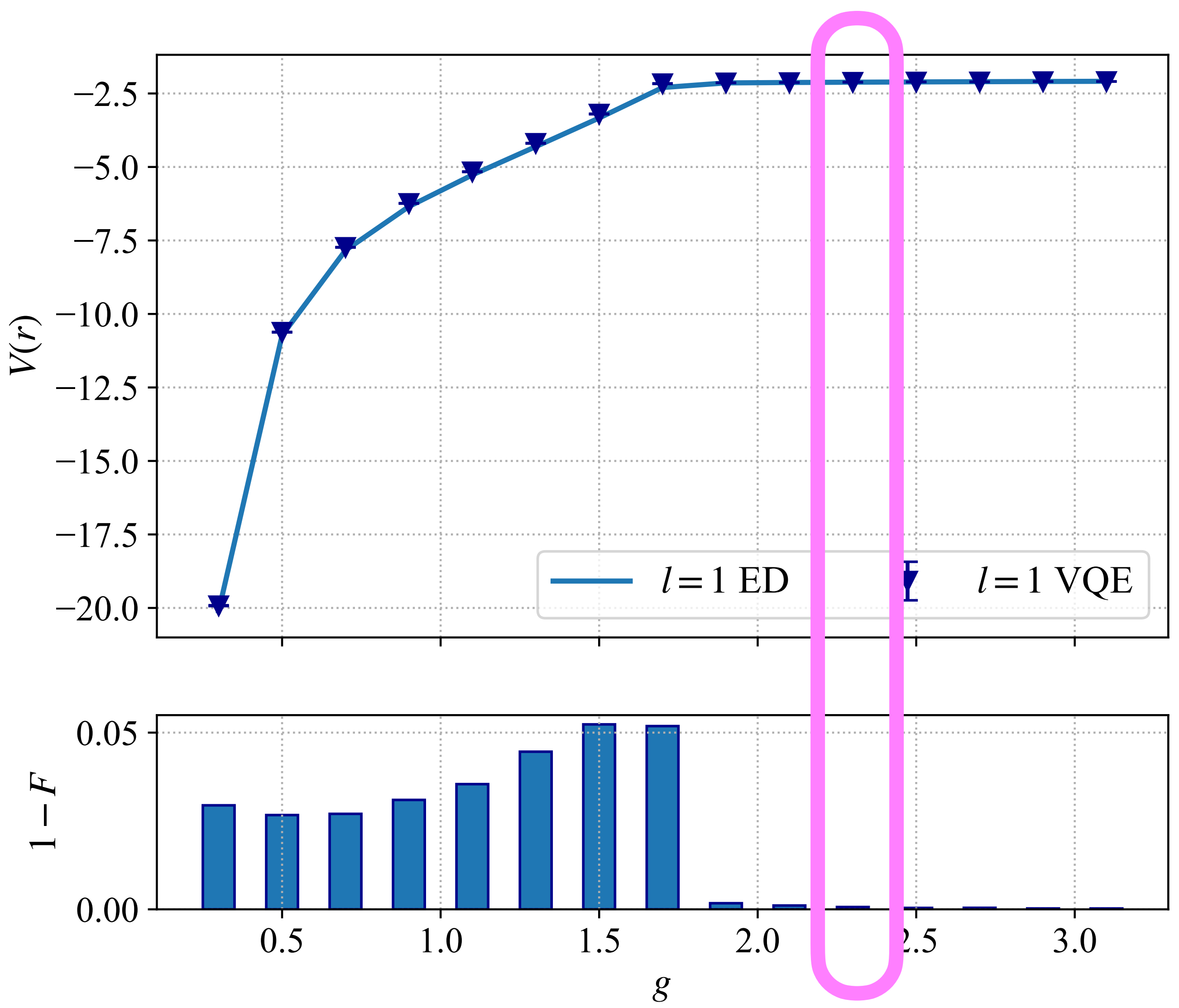



g=2.3 l=1
Quasi-probability
1.00
0.75
0.50
0.25
0.00
ED
AerStateBackend
H1-1E (physical)
1000110101
1100010111
1000110001
0100110101
1000110100
1000111101
1001010101
1000110111
1000111100

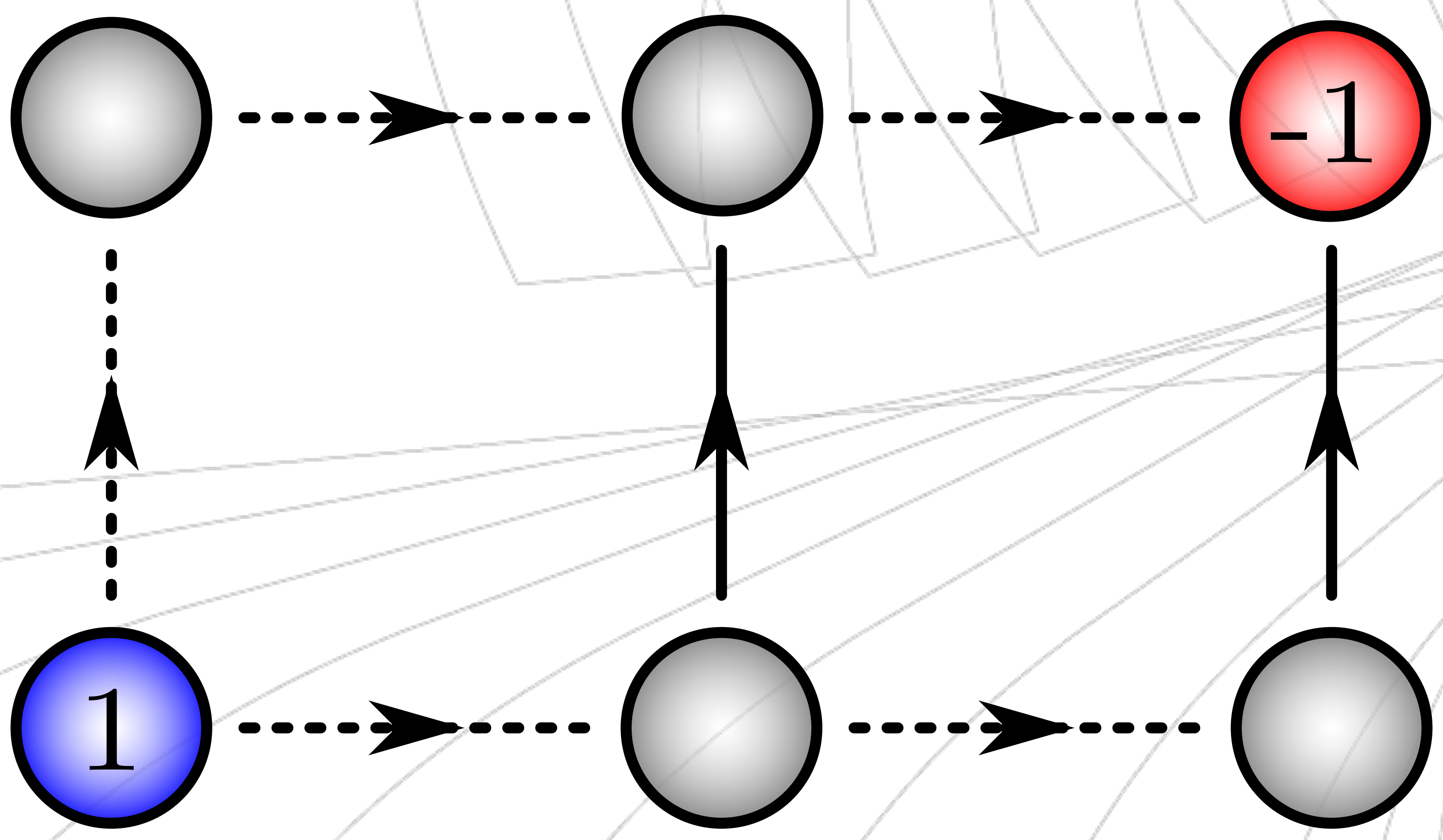

V(r)
−2.5
−5.0
−7.5
−10.0
−12.5
−15.0
−17.5
−20.0
l = 1 ED
l = 1 VQE
1 − F
0.05
0.00
0.5
1.0
1.5
2.0
2.5
3.0
g

QUANTINUUM





Configurations with string breaking: no flux and mesons

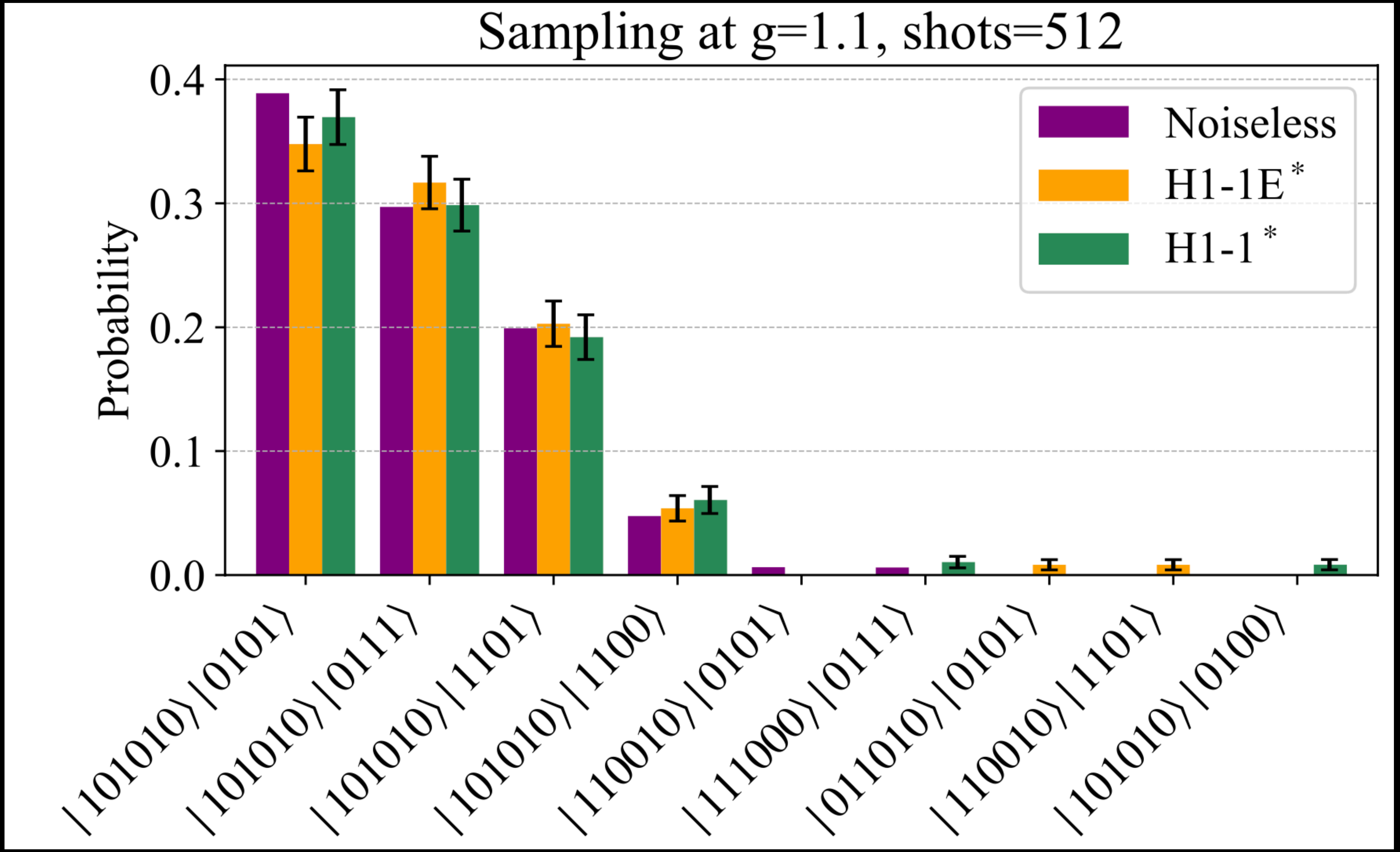


Experiments on H1-1

Real Hardware

Results compared to Emulator

- 512 shots on H1-1
- No error mitigation! (post processing)



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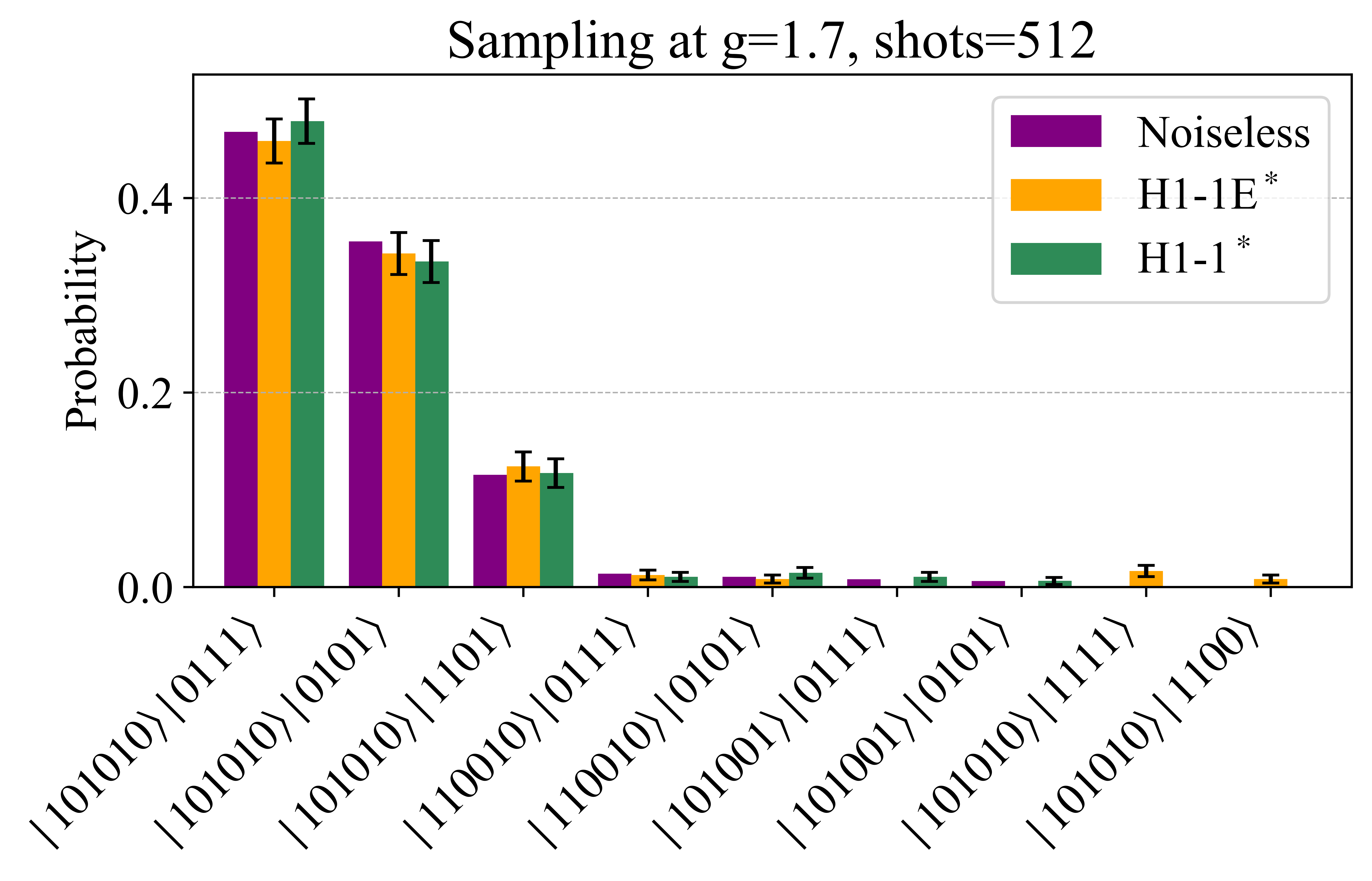
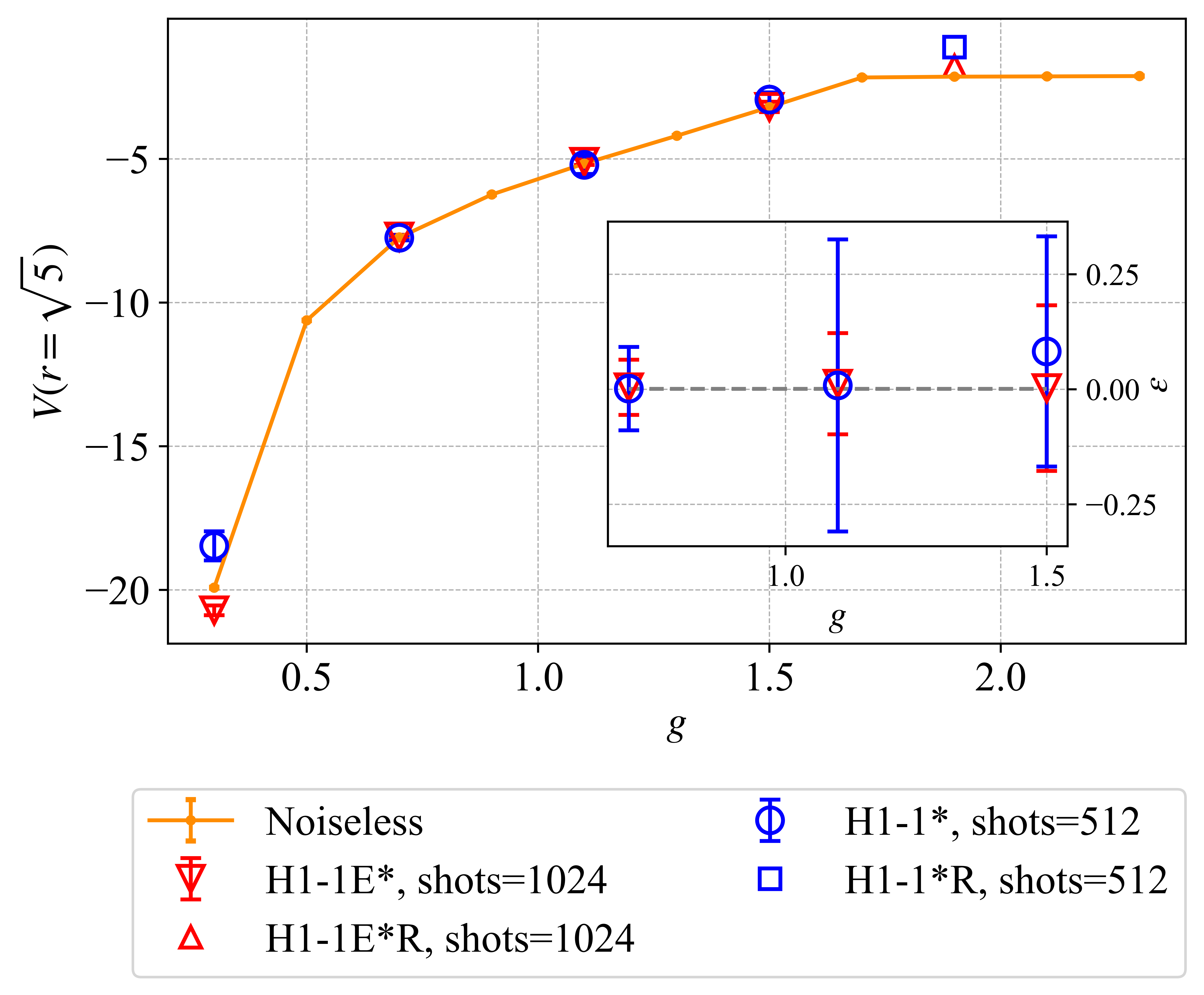


Real Hardware

Results compared to Emulator

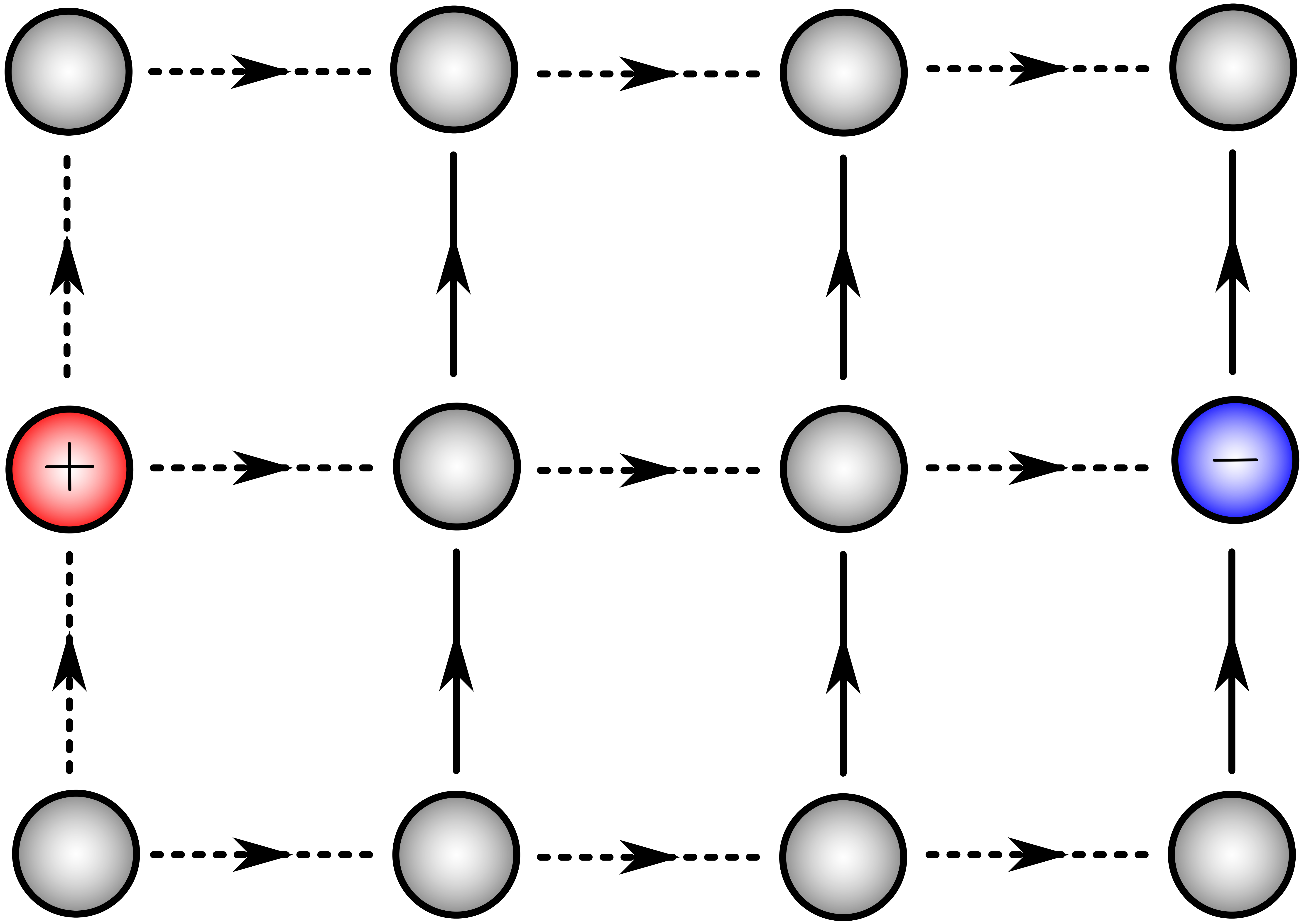
- 512 shots on H1-1
- Simple Error Mitigation: (PMSV, SPAM)



Conclusions

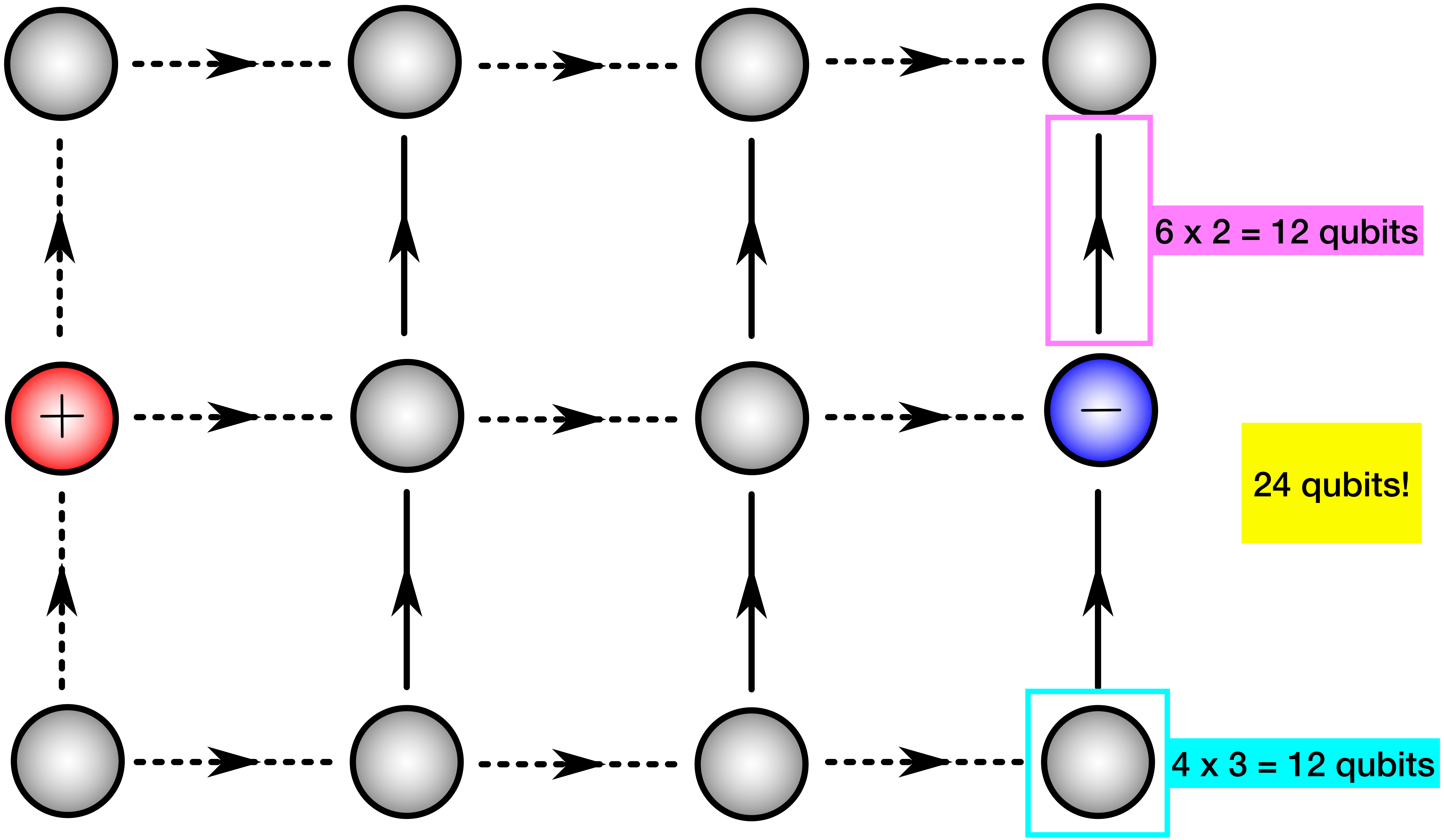
And future directions

- We demonstrated on real quantum hardware a calculation of the confining potential of (2+1)D QED in the Hamiltonian formulation
- Access to the ground state, even in a variational sense, allows us to visualize the confining fluxes between static charges
- String breaking and the formation of "mesons" is observed
- Scaling up the quantum state preparation step is important to fully leverage the computational power of quantum hardware
- Formulations of Hamiltonian lattice gauge theories that are different from Kogut-Susskind might help scaling to non-Abelian theories in (3+1)D



Can we do larger lattices?

Even if we have 20 qubits?



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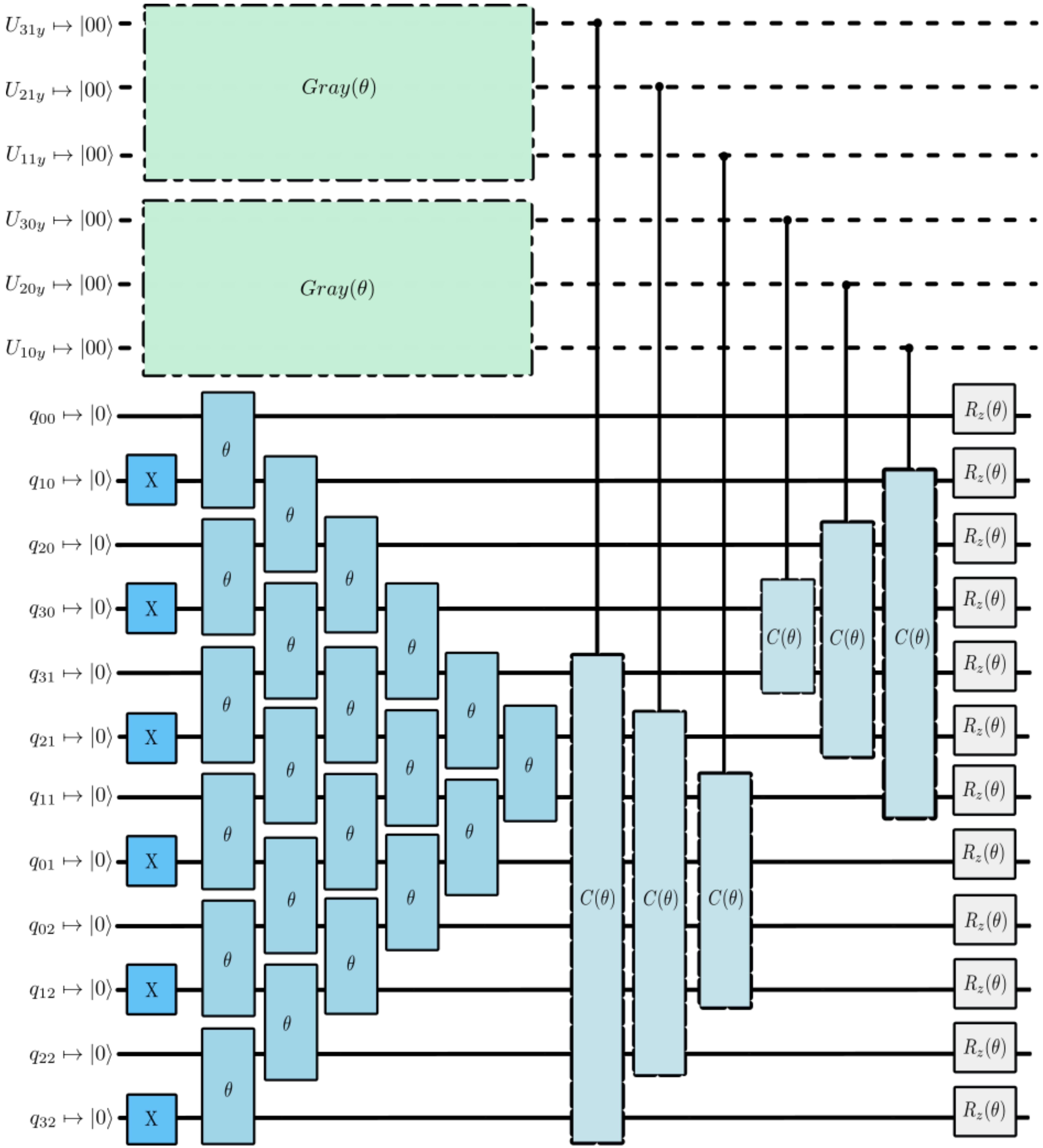
Even if we have 20 qubits?

Mutual Information Ansatz

- More qubits: 10 → 24
- More parameters: 30 → 81
- More 2qb gates: 152 → 450



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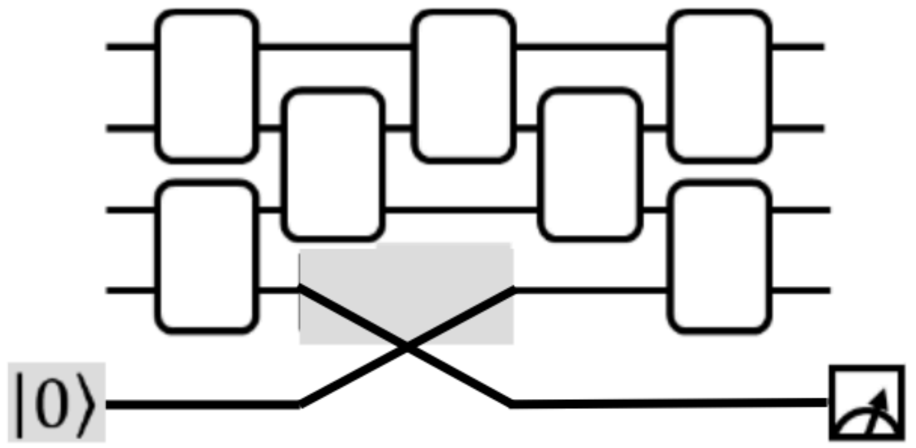
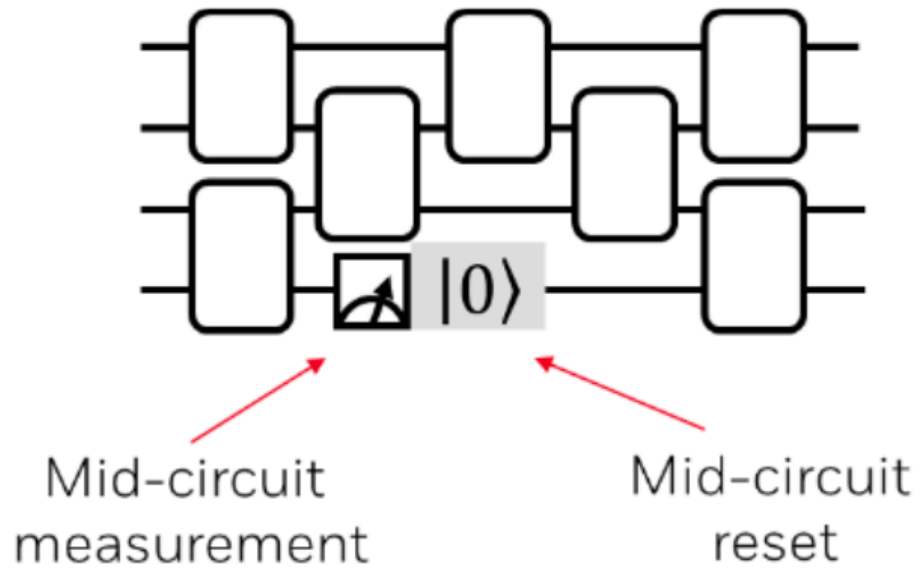
Qubit Reuse

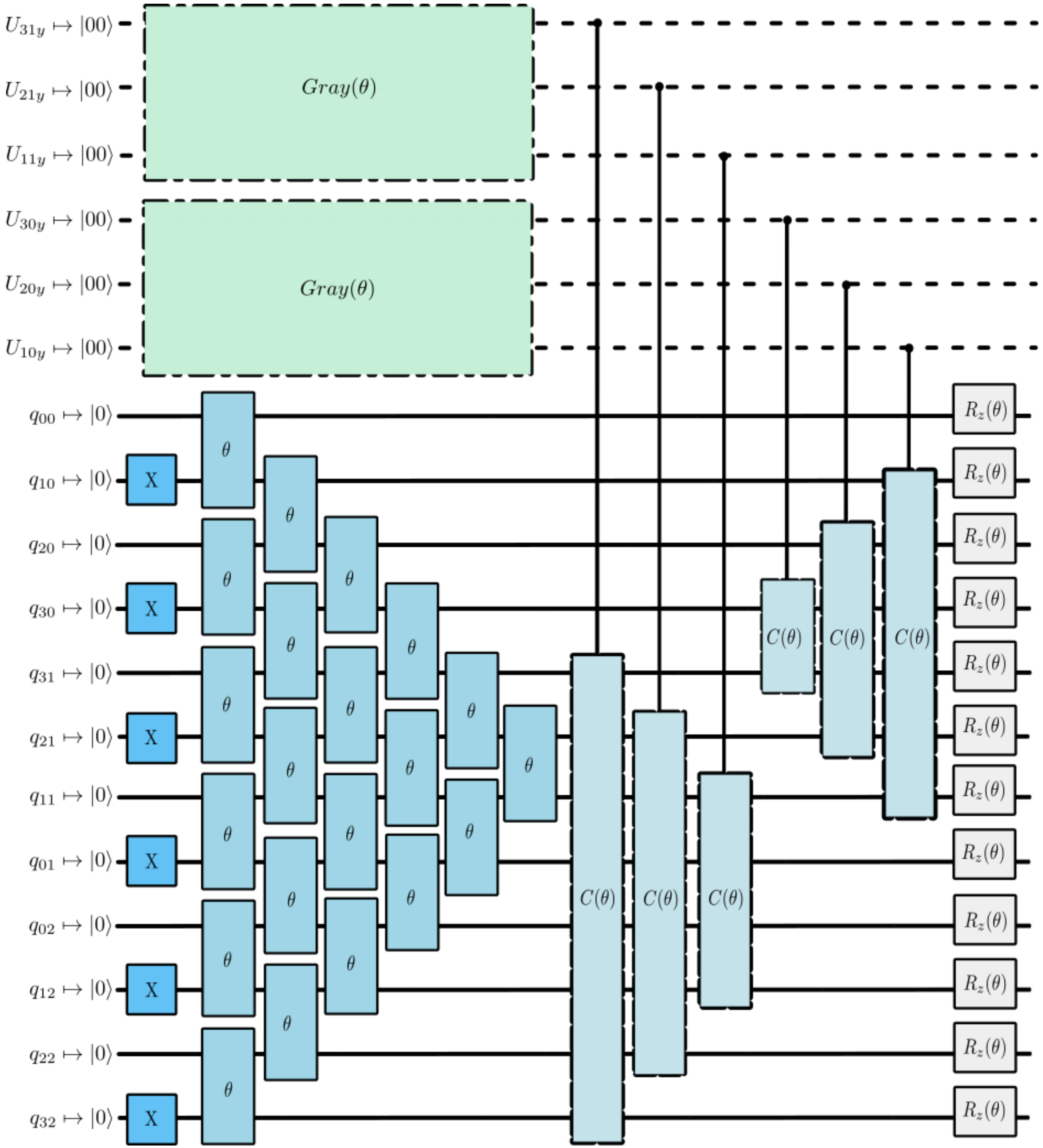
De Cross et al, PRX 13, 041057 (2023)

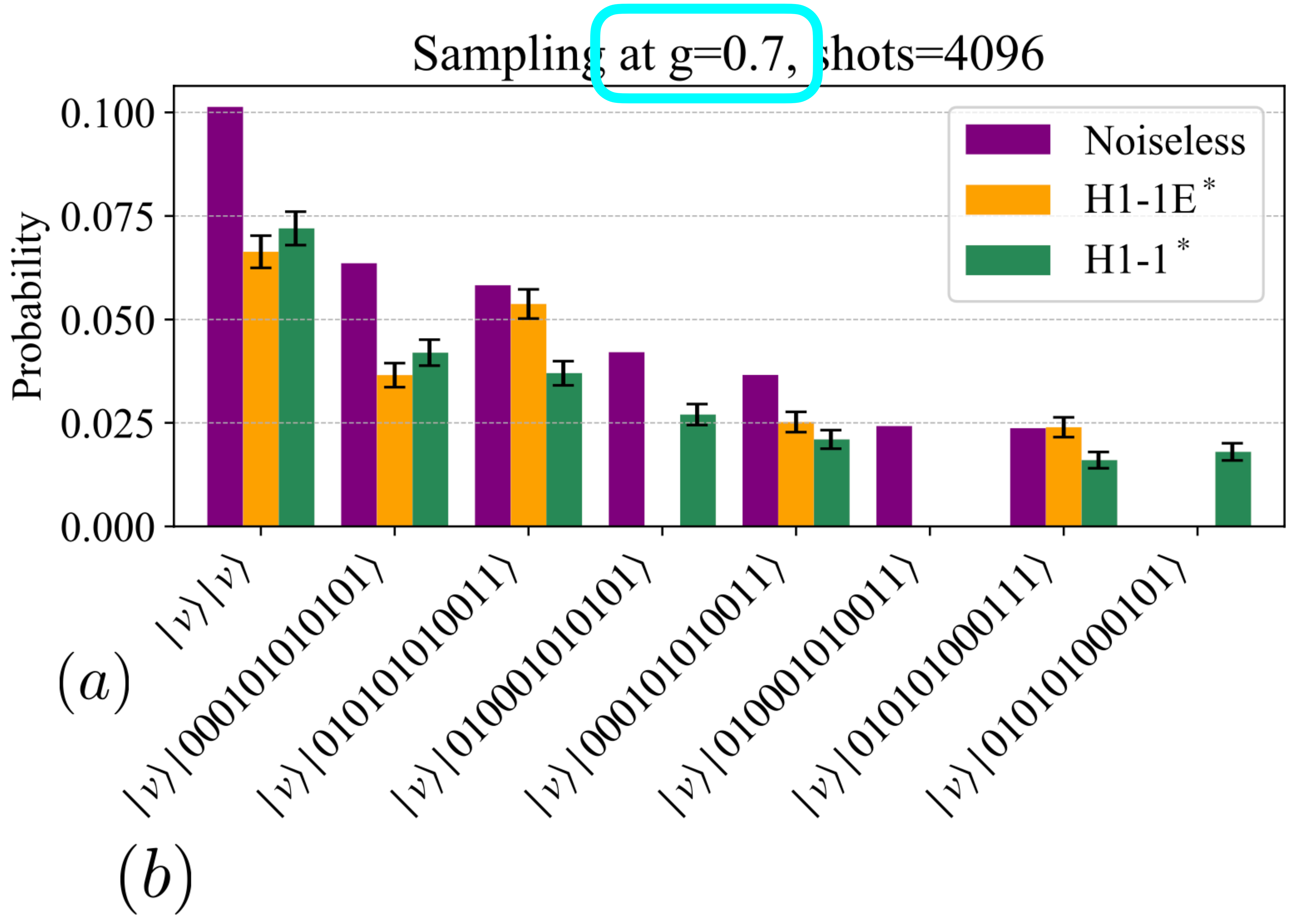
Without qubit-reuse

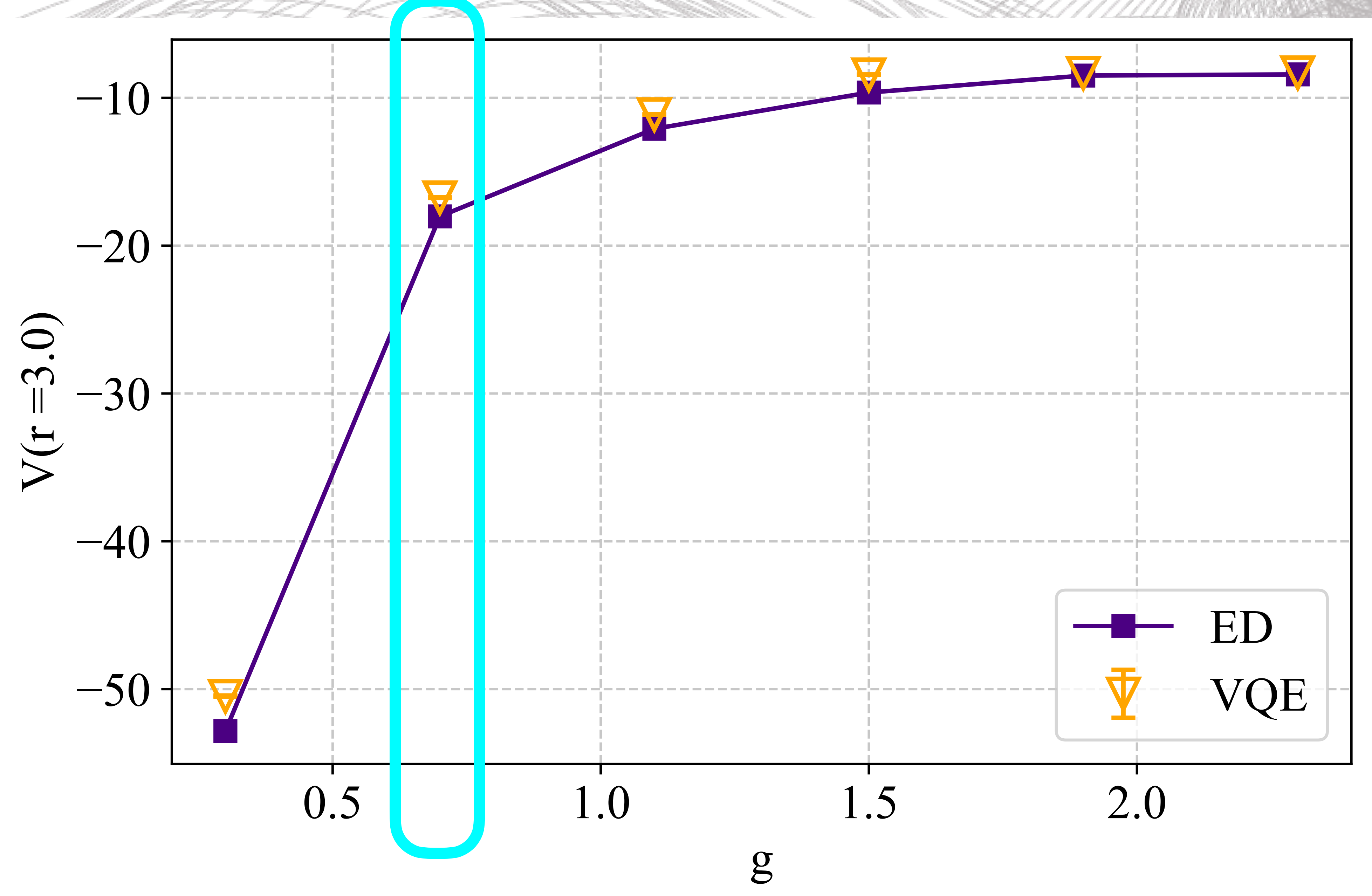


With qubit-reuse

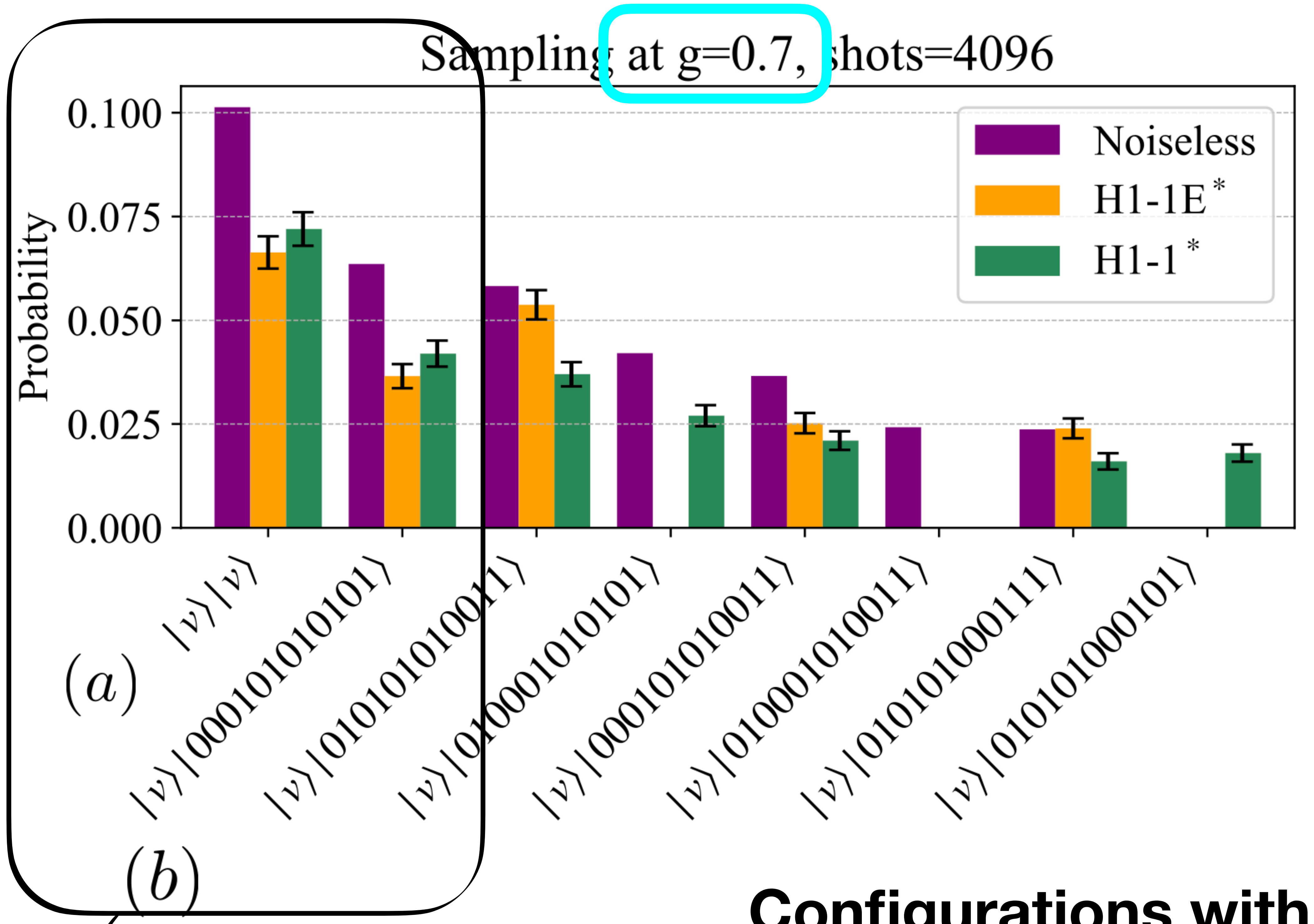


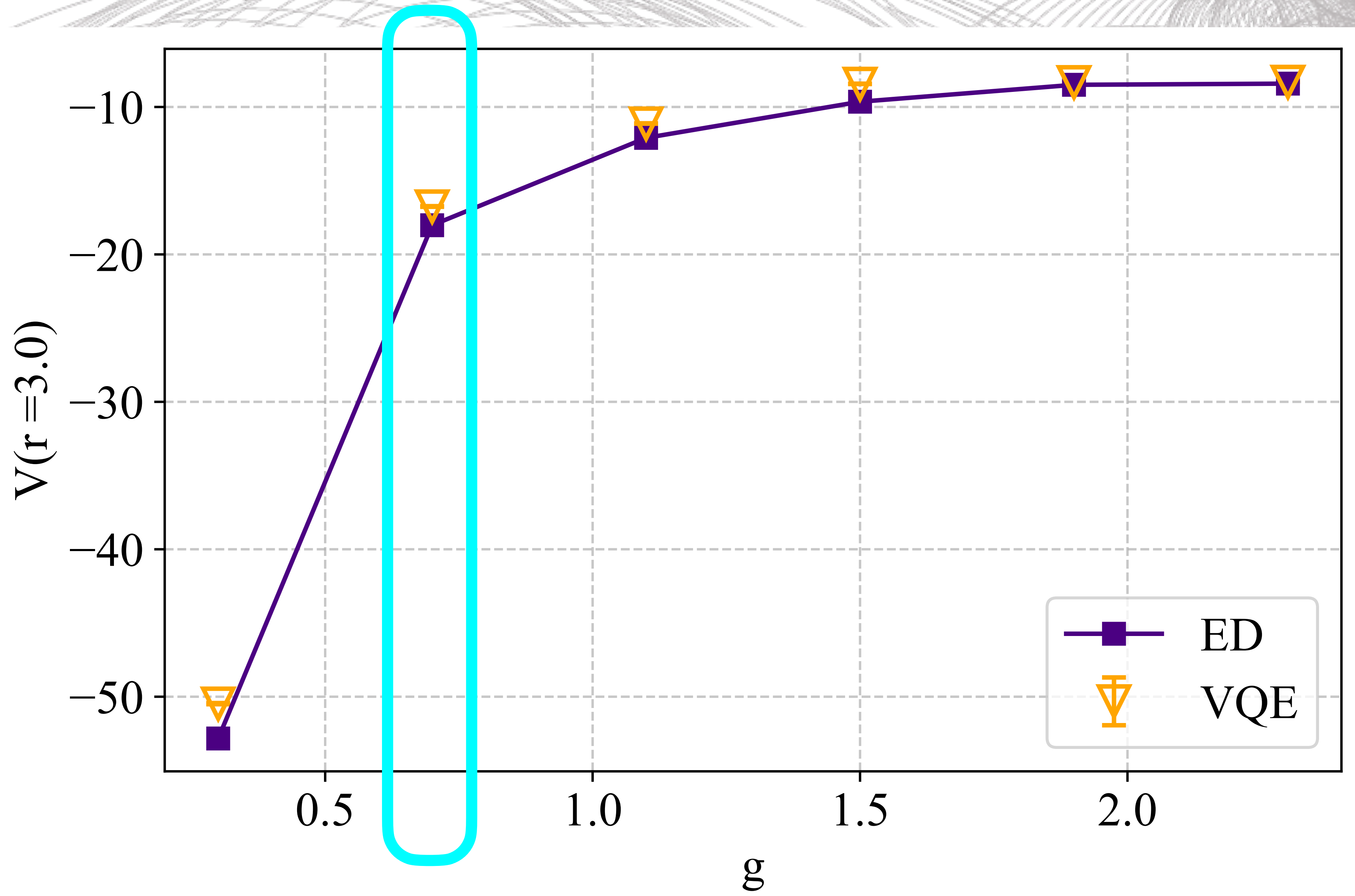



Sampling at g=0.7, shots=4096
Probability
0.100
0.075
0.050
0.025
0.000
Noiseless
H1-1E*
H1-1*
|ν⟩|ν⟩
|ν⟩|000101010101⟩
|ν⟩|010101010011⟩
|ν⟩|010001010101⟩
|ν⟩|000101010011⟩
|ν⟩|010001010011⟩
|ν⟩|010101000111⟩
|ν⟩|010101000101⟩
(a)
(b)

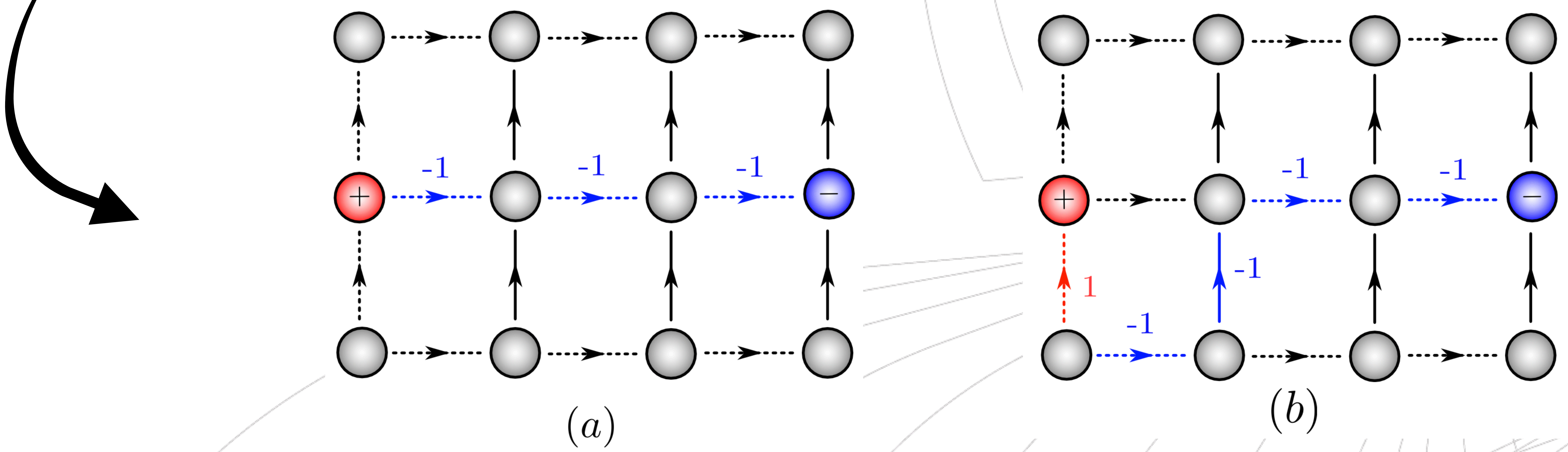

V(r =3.0)
−10
−20
−30
−40
−50
0.5
1.0
1.5
2.0
g
ED
VQE

QUANTINUUM

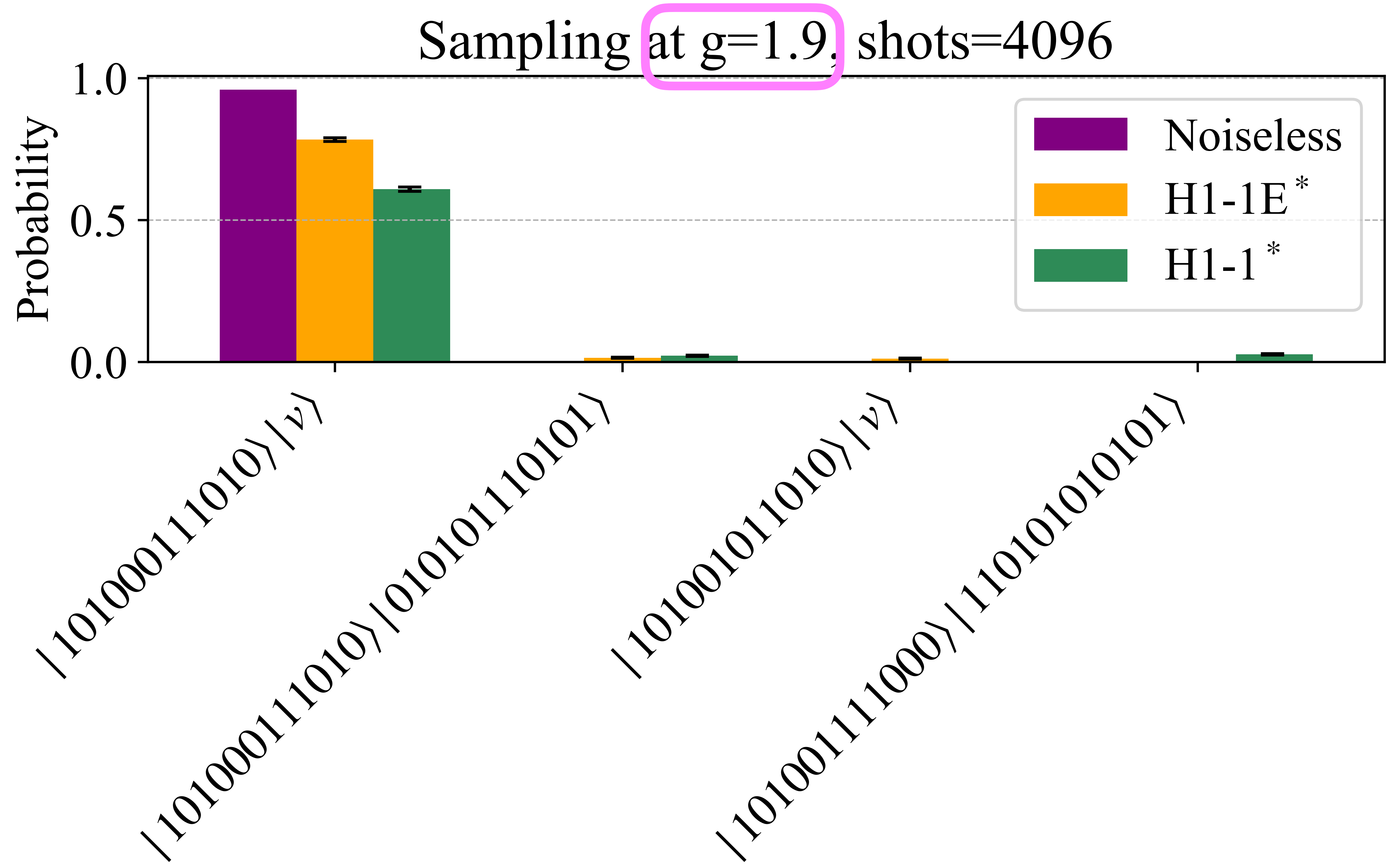




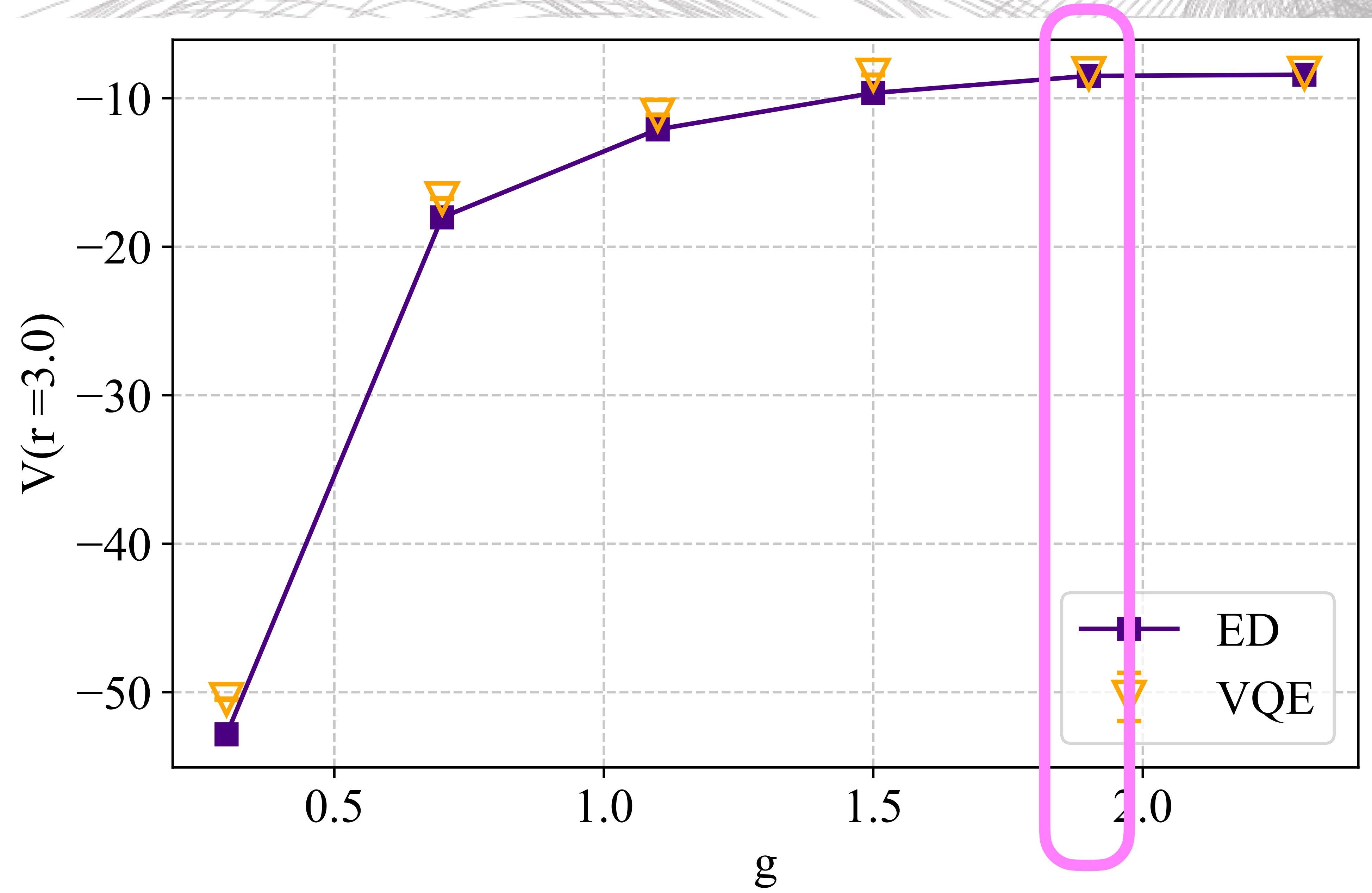
Configurations with electric fluxes between static charges



QUANTINUUM

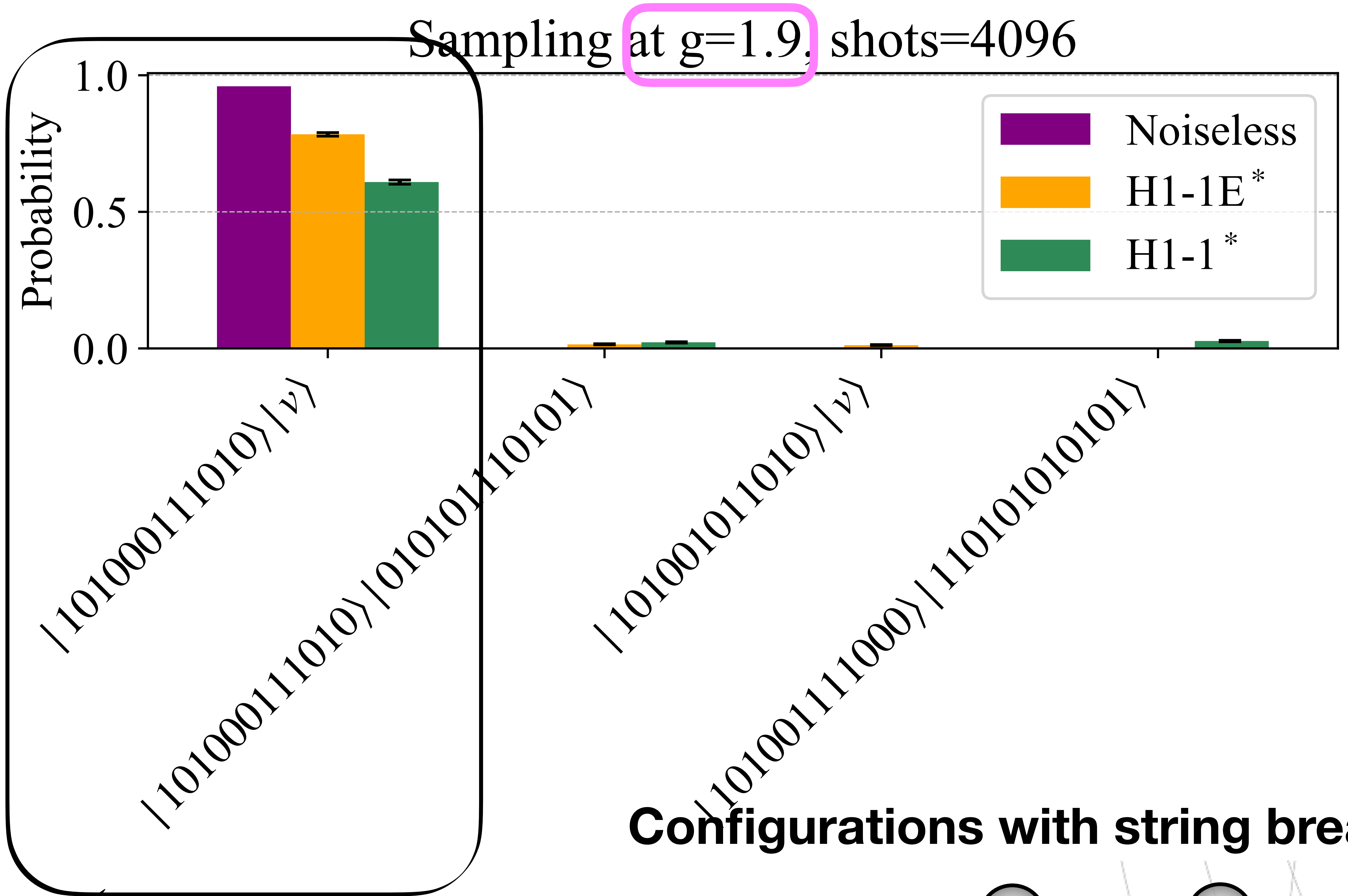


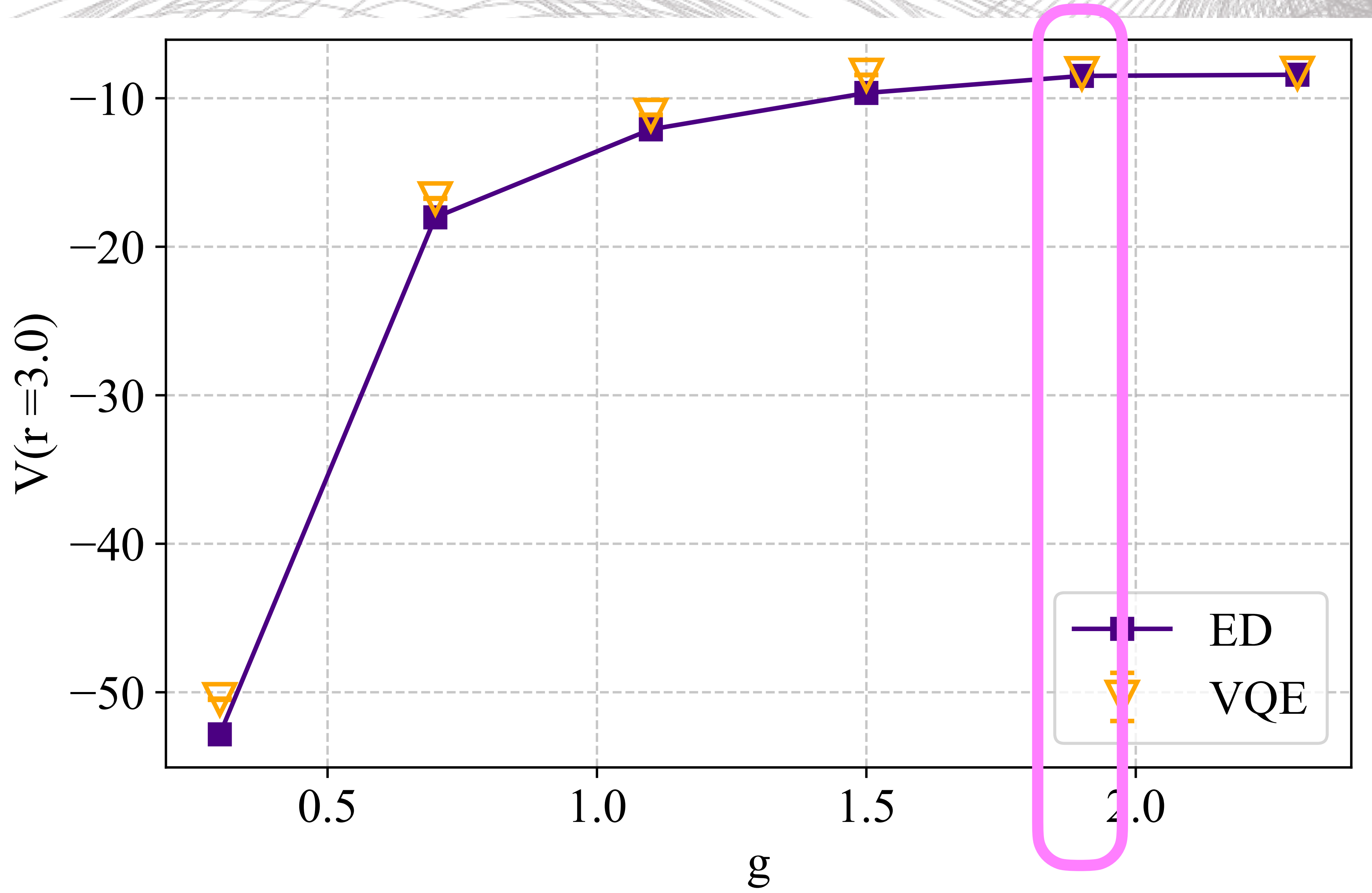
Sampling at g=1.9, shots=4096
Probability
1.0
0.5
0.0
Noiseless
H1-1E*
H1-1*
|101000111010⟩|ν⟩
|1010001110l0⟩|0101011101010l⟩
|101001011010⟩|ν⟩
|101001111000⟩|110101010101⟩



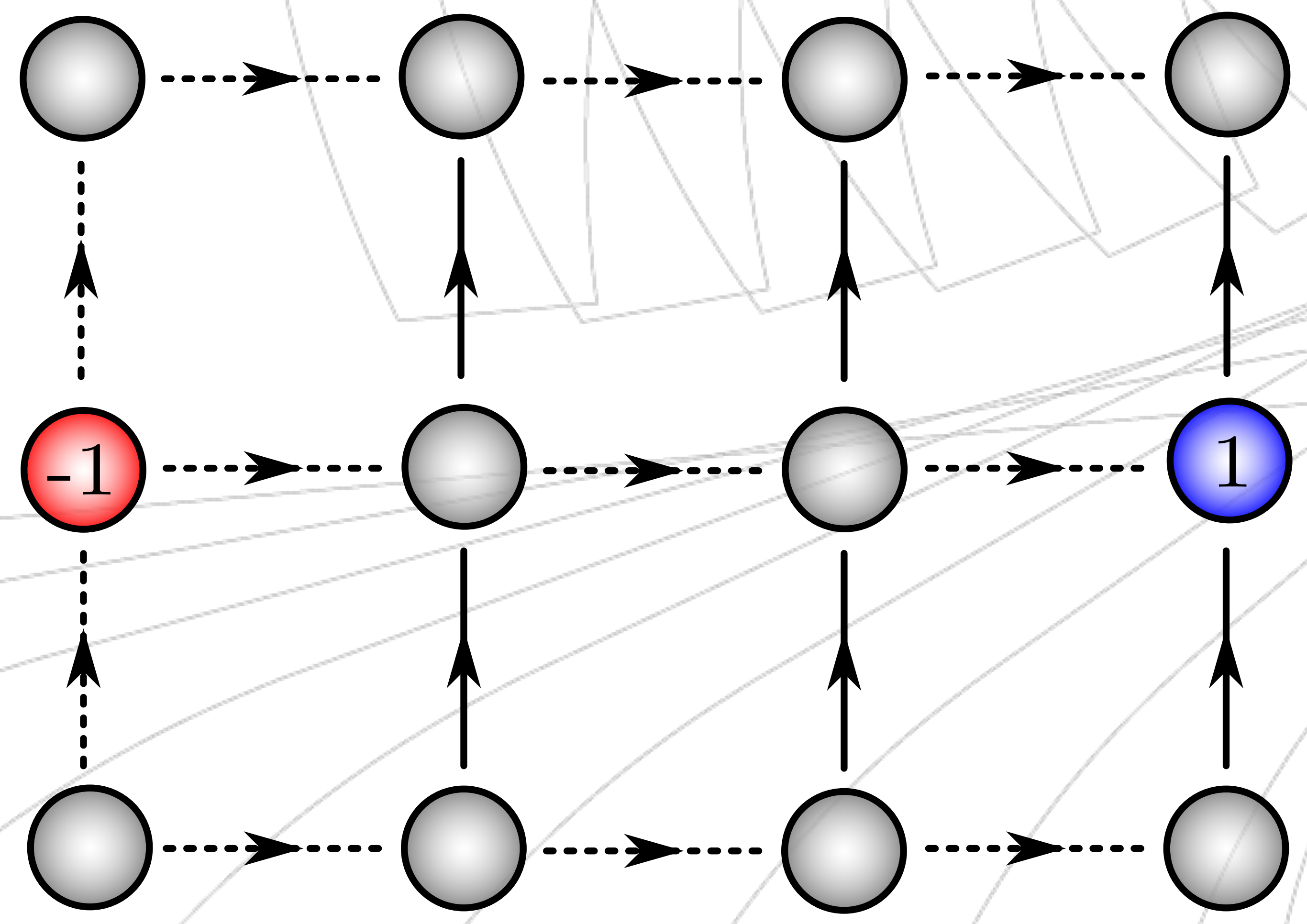
V(r=3.0)
−10
−20
−30
−40
−50
0.5
1.0
1.5
2.0
g
ED
VQE

QUANTINUUM





Configurations with string breaking: no flux and mesons



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