Floquet prethermalization of lattice gauge theories on superconducting qubits

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References:

Kazuhiro Seki, Yuta Kikuchi, Tomoya Hayata, Seiji Yunoki, arXiv: 2405.07613 [quant-ph]
Tomoya Hayata, Kazuhiro Seki, Arata Yamamoto, arXiv:2408.10079 [hep-lat]
Tomoya Hayata, Yoshimasa Hidaka, arXiv: 2409.20263 [hep-lat]

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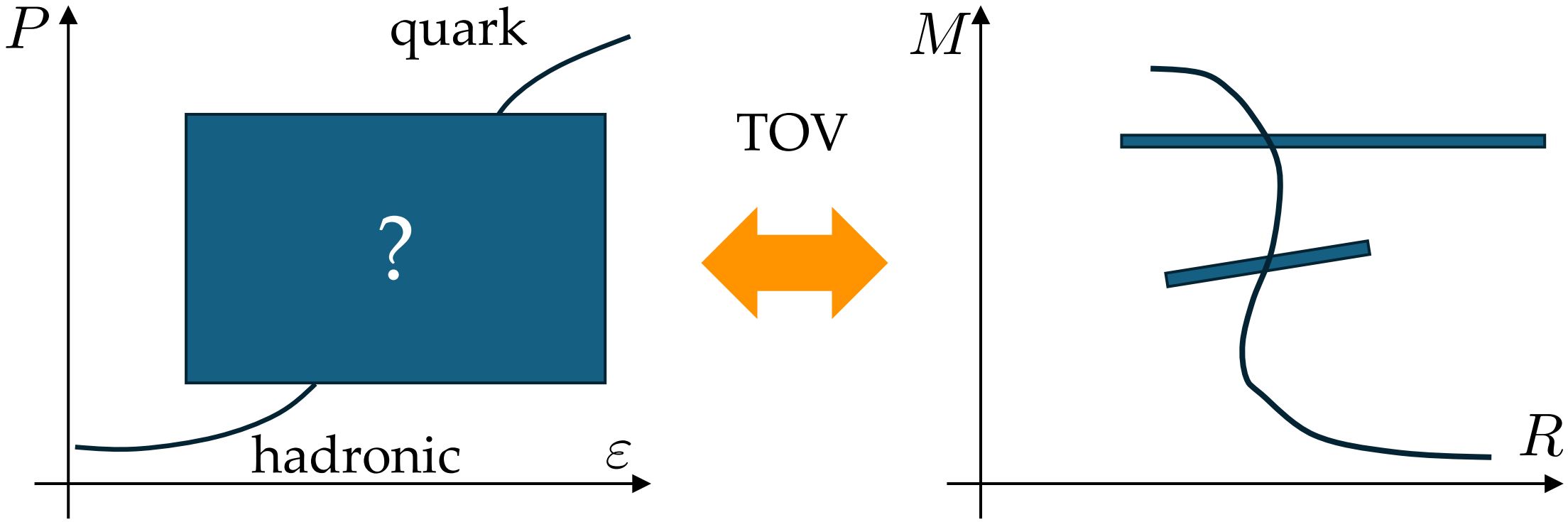
Dense QCD and Neutron stars

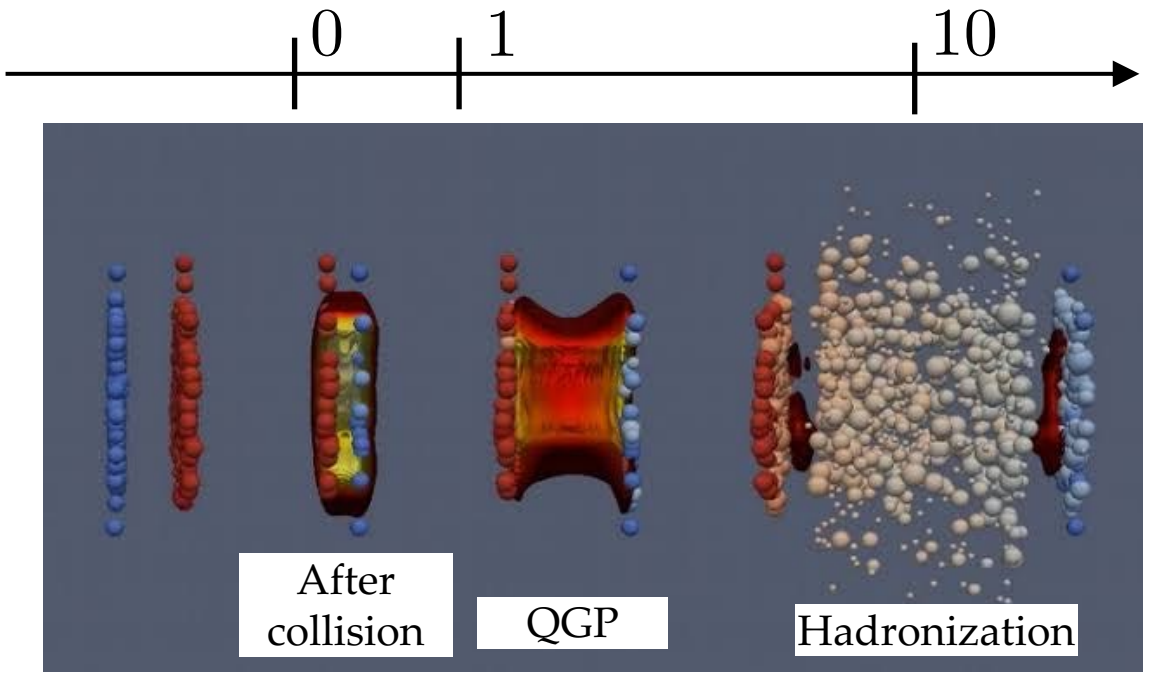
○ Equation of state of QCD is relevant to astrophysics



Ground state “energy” of QCD as a function of the baryon number density

Nonequilibrium QCD and Little bang

○ Understand the dynamics of quark-gluon-plasma (QGP) created in heavy-ion collision experiments



Ref: MADAI collaboration

Ab initio calculations of real-time QCD

Two directions

○ Implementing QCD on quantum devices is never trivial

➡ Develop quantum algorithms and wait fault-tolerant QCs

➡ Simulating toy models of LGTs on NISQ devices

e.g., Schwinger model (QED in 1+1 dimensions)

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- Why QC? → QC of QFT
- q-deformed Yang-Mills theory (Easy examples: $SU(2)_1$ and $SU(3)_1$)
- Why Floquet systems?
- Experimental results
- Summary

$SU(N)_k$ and graph representation

○ Wilson line (generates computational basis)

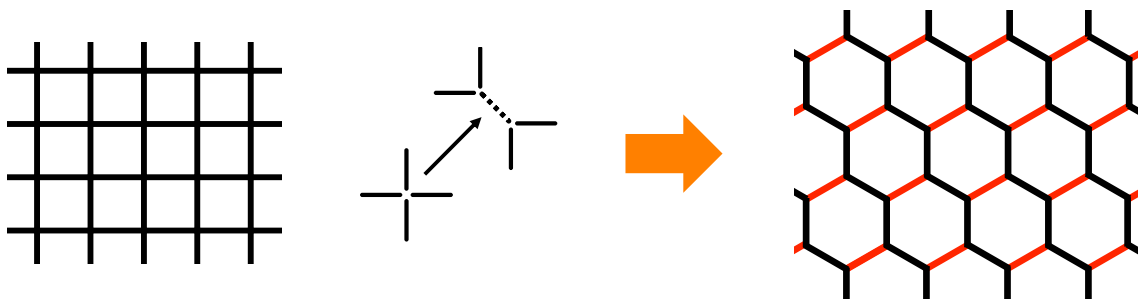
d_a quantum dimension

$$\sqrt{d_a}U_a = \quad a$$

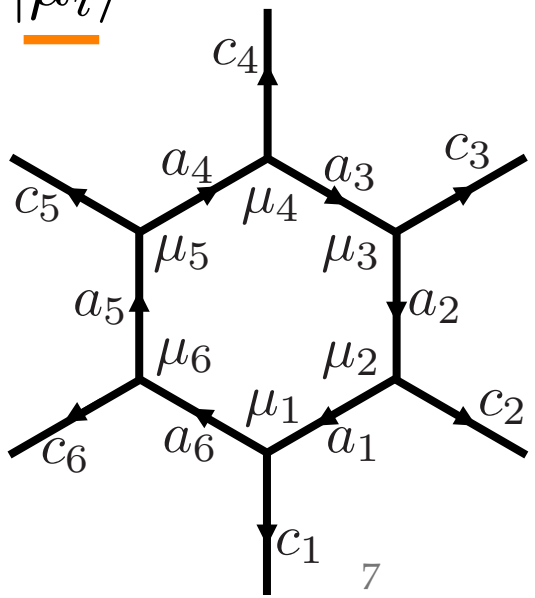
$a = (p_a, q_a, \cdots)$ Dynkin index

multiplicity

○ 2d square lattice to state (2+1d Yang-Mills)



$$\prod_{i=1}^{6} |a_i\rangle|c_i\rangle|\mu_i\rangle =$$



・Gauss's laws on vertices → Fusion rules

$$a \times b = \sum_c N_{ab}^c c$$

$SU(N)_k$ and graph representation

○ Wilson line (generates computational basis)

d_a quantum dimension

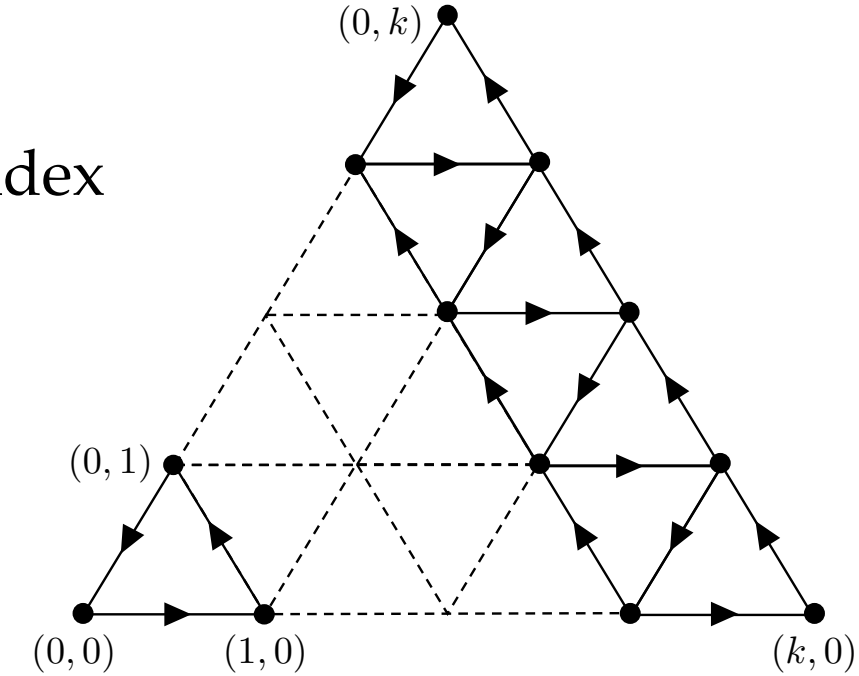
$$\sqrt{d_a}U_a = \quad \Big| \; a$$

$a = (p_a, q_a, \cdots)$ Dynkin index

○ $SU(2)_k$, $SU(3)_k$,…

(d+1)-$SU(2)_k$ = (d+1+1)-spin$\frac{1}{2}$

= (d+1)-qudit



(d+1)-$SU(3)_k$ = (d+1+2)-spin$\frac{1}{2}$

k is the length of "extra" dimensions

$SU(N)_k$ and graph representation

○ Wilson line (generates computational basis)

d_a quantum dimension

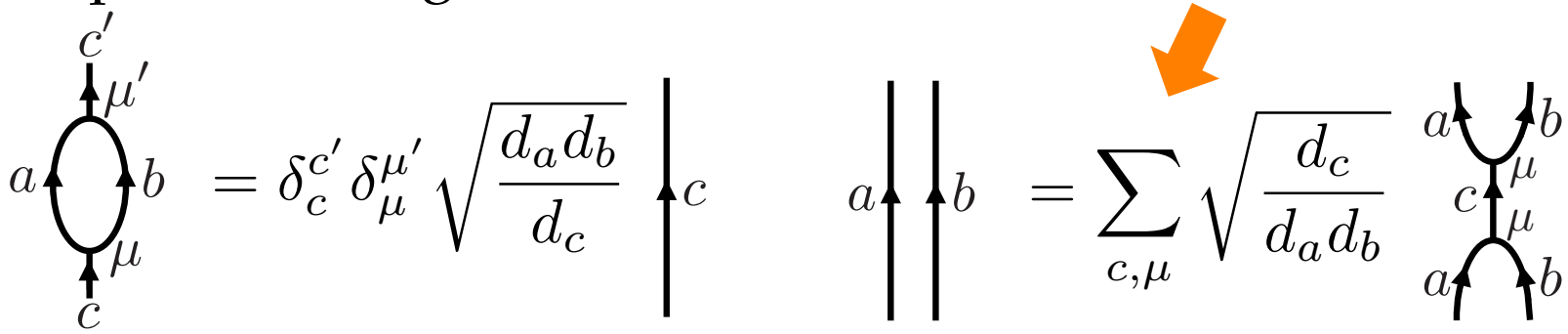
$$\sqrt{d_a}U_a = \quad a$$

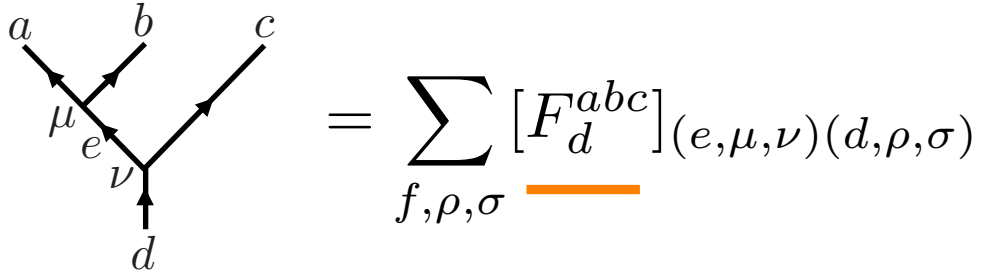
$a = (p_a, q_a, \cdots)$ Dynkin index

Fusion rules

$$a \times b = \sum_c N_{ab}^c c$$

○ Three important things





q-deformed Yang-Mills theory

Zache, González-Cuadra, Zoller, PRL 131, 171902 (2023)
TH, Hidaka, JHEP 2023, 126 (2023); JHEP 2023, 123 (2023)

○ Kogut-Susskind Hamiltonian

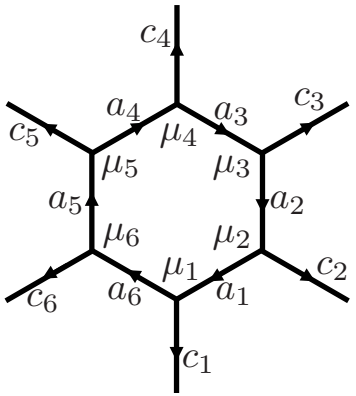
$$H = \sum_{\boldsymbol{x},\mu} \frac{1}{2} E^{a2}(\boldsymbol{x},\mu) - \frac{K}{2} \sum_{p\in P} (\mathrm{tr}U_p + \mathrm{tr}U_p^\dagger)$$

$$K = 1/g^4$$

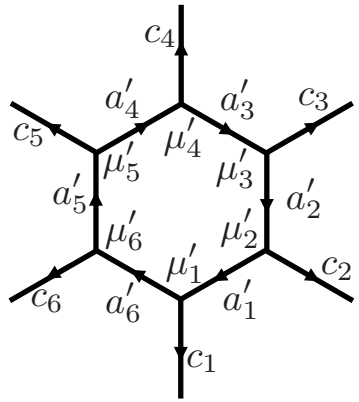
○ q-deformed representation

$$(E^a(x,\mu))^2 \uparrow a \;=\; C_2(a) \uparrow a$$

$$\mathrm{tr}U_d$$

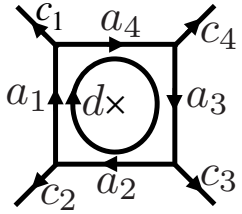


$$= \sum_{\{a_i',\mu_i'\}} \prod_{i=1}^{6} [F_{a_i'}^{c_i a_{i-1} d}]_{(a_i,\mu_i,\mu_{i+1}'),(a'_{i-1},\mu_{i-1}',\mu_i')}$$

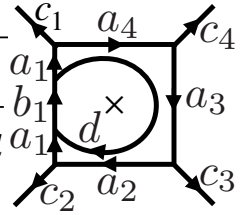


Wilson loop (matrix elements)

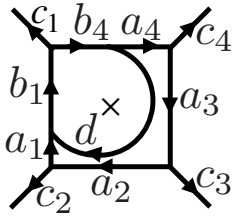
Robson, Webber, Z. Phys. C 15, 199 (1982)
Levin, Wen, PRB 71 045110 (2005)



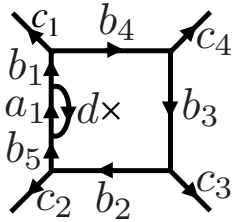
$$= \sum_{b_1} \sqrt{\frac{d_{b_1}}{d_{a_1} d_d}}$$



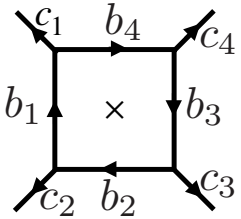
$$= \sum_{b_1, b_4} \sqrt{\frac{d_{b_1}}{d_{a_1} d_d}} [F^{c_1 a_4 d}_{b_1}]_{a_1 b_4}$$



$$= \sum_{b_1, b_2, b_3, b_4, b_5} \sqrt{\frac{d_{b_1}}{d_{a_1} d_d}} [F^{c_1 a_4 d}_{b_1}]_{a_1 b_4} [F^{c_4 a_3 d}_{b_4}]_{a_4 b_3} [F^{c_3 a_2 d}_{b_3}]_{a_3 b_2} [F^{c_2 a_1 d}_{b_2}]_{a_2 b_5}$$



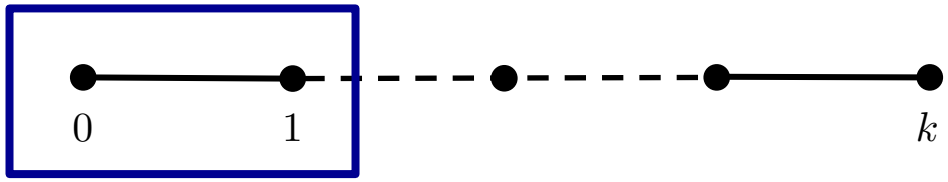
$$= \prod_{i=1}^{4} \sum_{b_i} [F^{c_i a_{i-1} d}_{b_i}]_{a_i b_{i-1}}$$



$$[F^{abc}_d]_{ef} = (-1)^{j_a + j_b + j_c + j_d} \sqrt{d_e d_f} \begin{Bmatrix} j_a & j_b & j_e \\ j_c & j_d & j_f \end{Bmatrix}$$

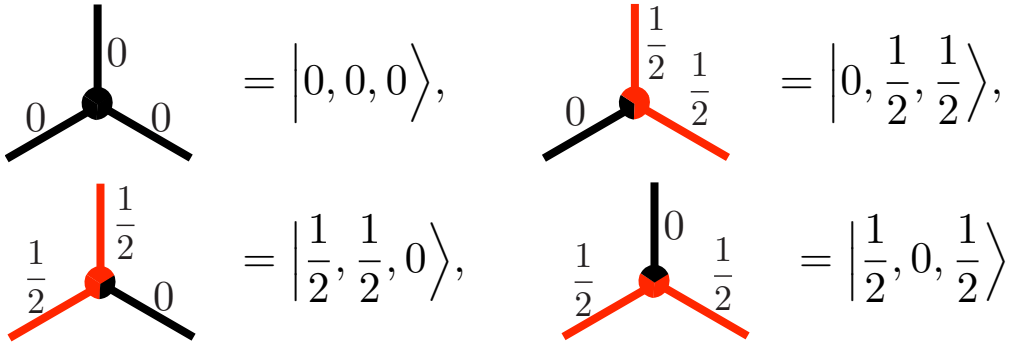
Wigner 6j symbol

$SU(2)_1$ Yang-Mills model on a two-leg ladder

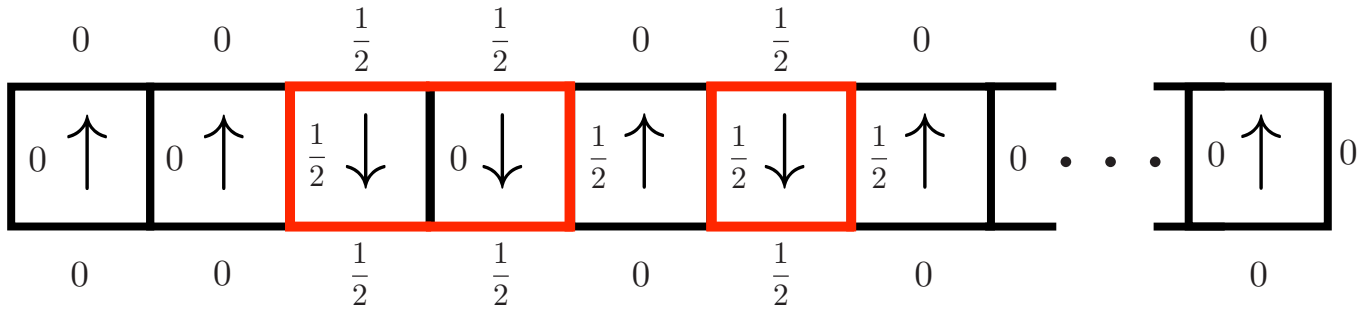


=qubit

Fusion rules



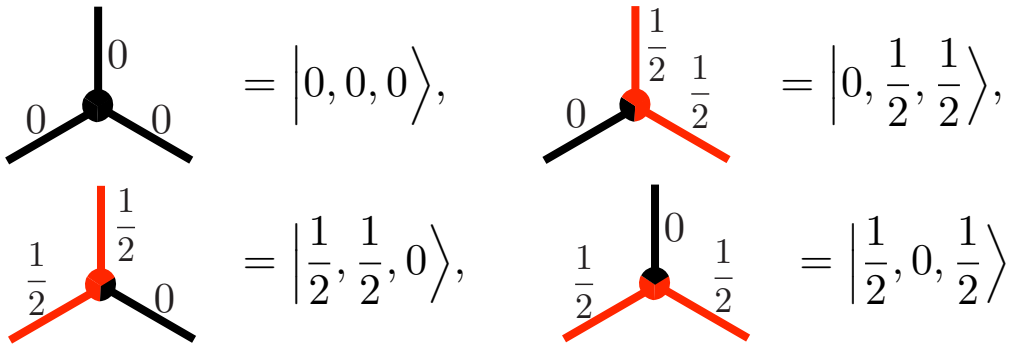
Plaquette chain = Qubit-chain



$N \times 1$ square lattice/ chain of plaquette

$SU(2)_1$ Yang-Mills model on a two-leg ladder

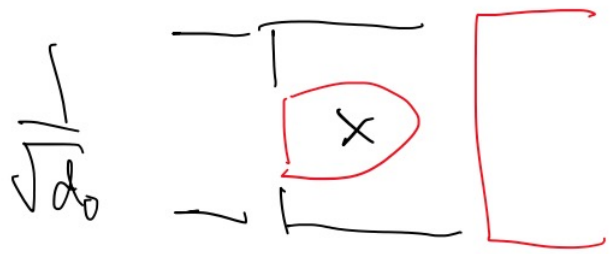
Fusion rules



F moves

$$\mathrm{tr}U_{\frac{1}{2}}|\uparrow\uparrow\downarrow\rangle =$$

I



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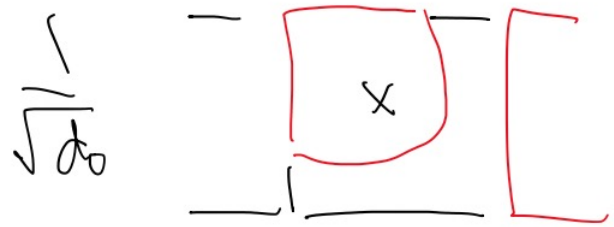
II



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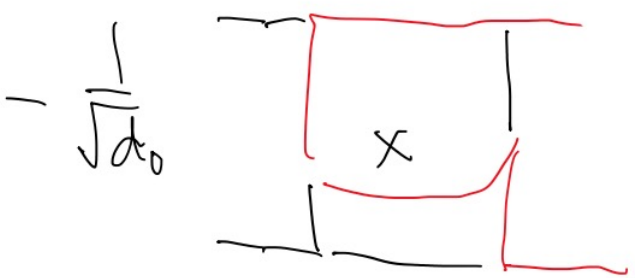

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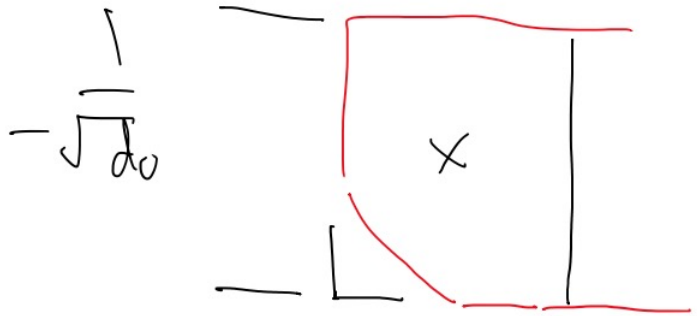
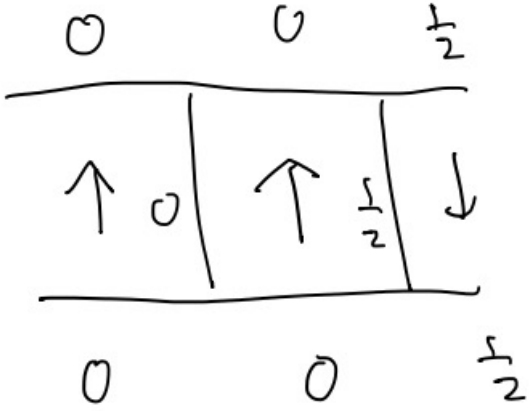
IV





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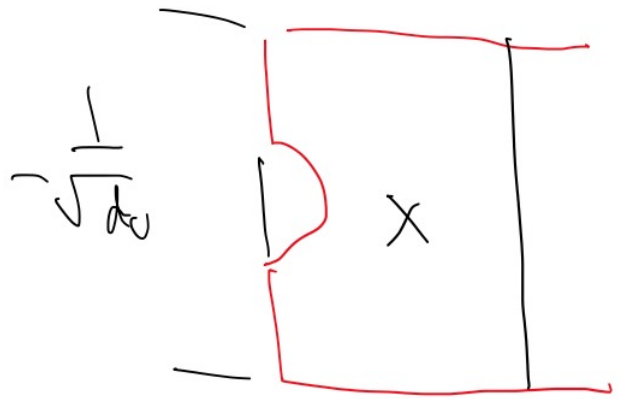
V



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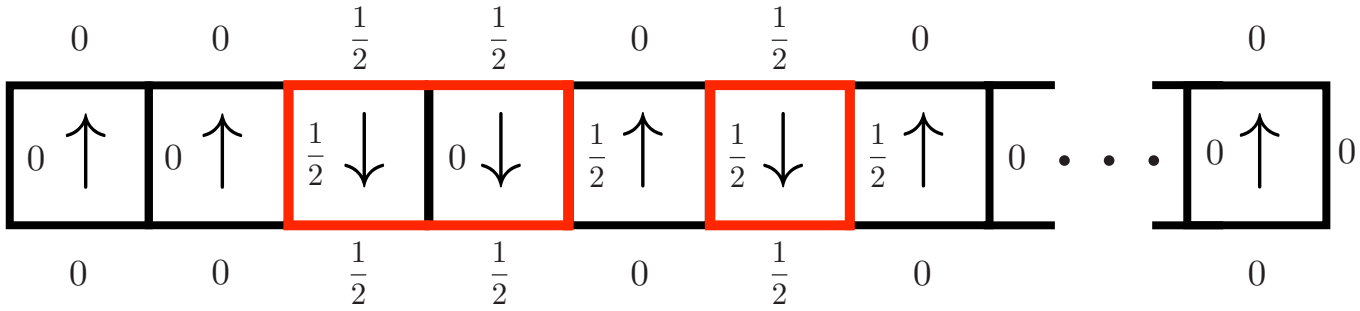
VI

$$= -|\uparrow\downarrow\downarrow\rangle$$

$SU(2)_1$ Yang-Mills model on a two-leg ladder



$$\mathrm{tr}U_{\square}(i)|\uparrow\uparrow\uparrow\rangle = |\uparrow\downarrow\uparrow\rangle \qquad \mathrm{tr}U_{\square}(i)|\downarrow\downarrow\uparrow\rangle = -|\downarrow\uparrow\uparrow\rangle$$

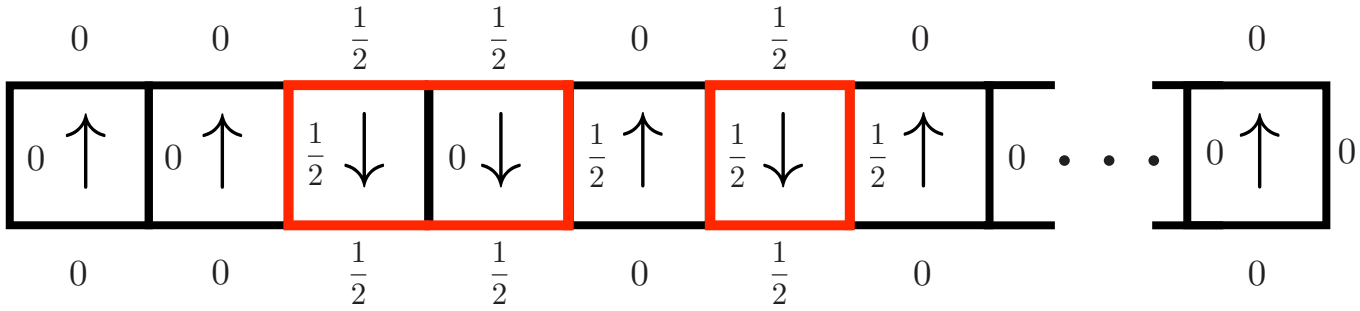
$$\mathrm{tr}U_{\square}(i)|\uparrow\downarrow\uparrow\rangle = |\uparrow\uparrow\uparrow\rangle \qquad \mathrm{tr}U_{\square}(i)|\downarrow\uparrow\uparrow\rangle = -|\downarrow\downarrow\uparrow\rangle$$

$$\mathrm{tr}U_{\square}(i)|\uparrow\uparrow\downarrow\rangle = -|\uparrow\downarrow\downarrow\rangle \qquad \mathrm{tr}U_{\square}(i)|\downarrow\uparrow\downarrow\rangle = |\downarrow\downarrow\downarrow\rangle$$

$$\mathrm{tr}U_{\square}(i)|\uparrow\downarrow\downarrow\rangle = -|\uparrow\uparrow\downarrow\rangle \qquad \mathrm{tr}U_{\square}(i)|\downarrow\downarrow\downarrow\rangle = |\downarrow\uparrow\downarrow\rangle$$

$$\mathrm{tr}U_{\square}(i) = Z_{i-1}X_iZ_{i+1}$$

$SU(2)_1$ Yang-Mills model on a two-leg ladder



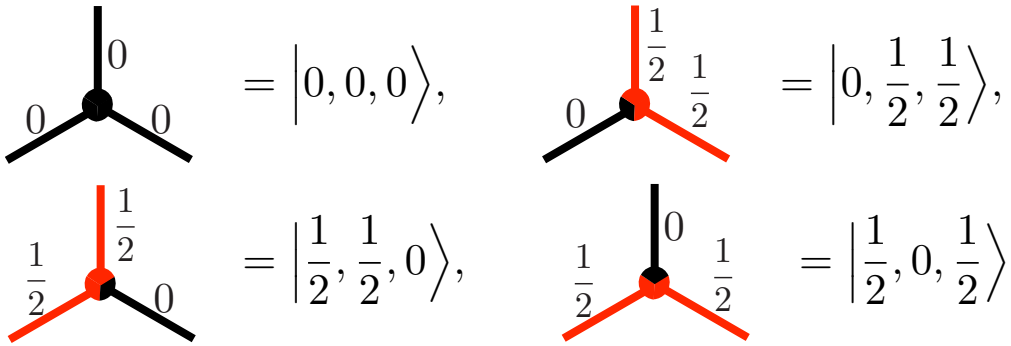
$$H = \frac{c}{2}\sum_{n=1}^{N}(1 - Z_n) + \frac{c}{4}\sum_{n=1}^{N-1}(1 - Z_n Z_{n+1}) - K\sum_{n=1}^{N} Z_{n-1} X_n Z_{n+1}$$

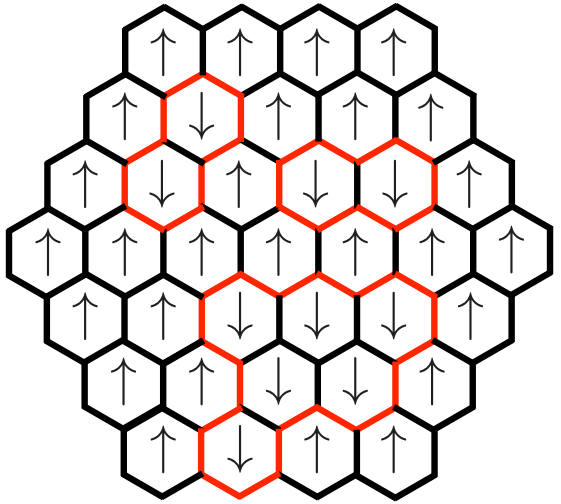
This model is unitary equivalent to the transverse-field Ising (TFI) model

$$U = \mathrm{e}^{-\mathrm{i}\frac{\pi}{4}\sum_n (-1)^n Z_n Z_{n+1}}$$

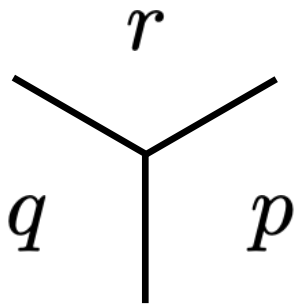
$$H = \frac{c}{2}\sum_{n=1}^{N}(1 - Z_n) + \frac{c}{4}\sum_{n=1}^{N-1}(1 - Z_n Z_{n+1}) - K\sum_{n=1}^{N} X_n$$

$SU(2)_1$ Yang-Mills model on a honeycomb lattice





$$H = \frac{c}{4}\sum_{(n,m)}(1 - Z_n Z_m) - K\sum_p X_p \prod_{(q,r)} \mathrm{i}^{\frac{1-Z_q Z_r}{2}}$$



This model is unitary equivalent to the TFI model

$$U = \prod_{<pqr>} \mathrm{e}^{-\mathrm{i}\frac{\pi}{24}(3Z_p Z_q Z_r - Z_p - Z_q - Z_r)}$$

$$H = \frac{c}{4}\sum_{(n,m)}(1 - Z_n Z_m) - K\sum_p X_p$$

Levin, Gu PRB 86 115109 (2012)

What is the simplest LGT more complex than the TFI model?

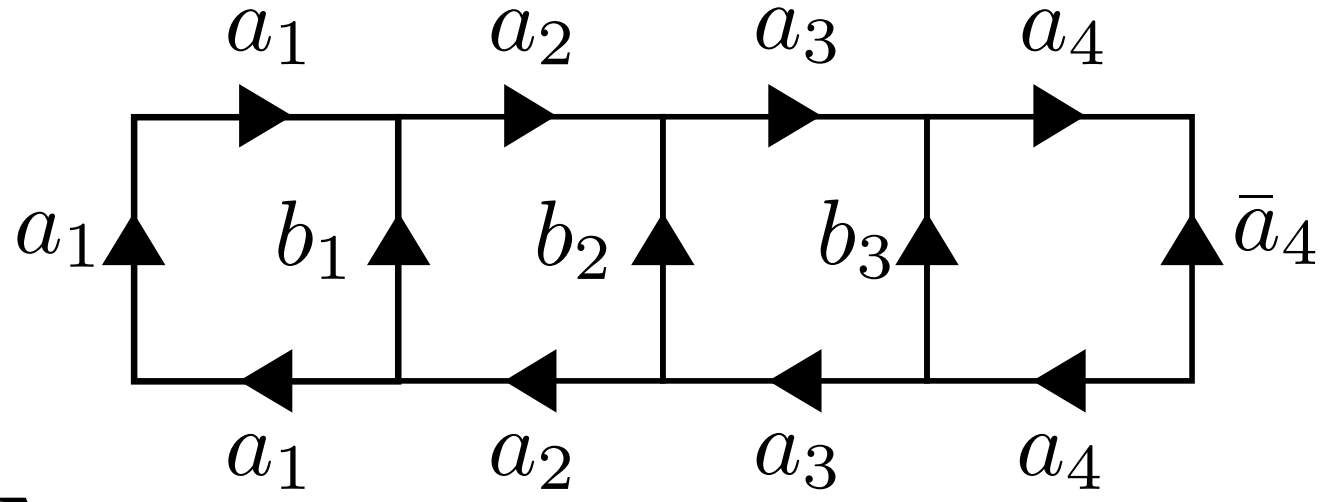
$SU(3)_1$ on a two-leg ladder

$SU(3)_1$ Yang-Mills model on a two-leg ladder

Gauss law constraints

$$1 \times 1 = 1, \;\; 1 \times 3 = 3 \times 1 = 3, \;\; 1 \times \bar{3} = \bar{3} \times 1 = \bar{3},$$
$$3 \times 3 = \bar{3}, \;\; 3 \times \bar{3} = 1, \;\; \bar{3} \times \bar{3} = 3$$

Hilbert space

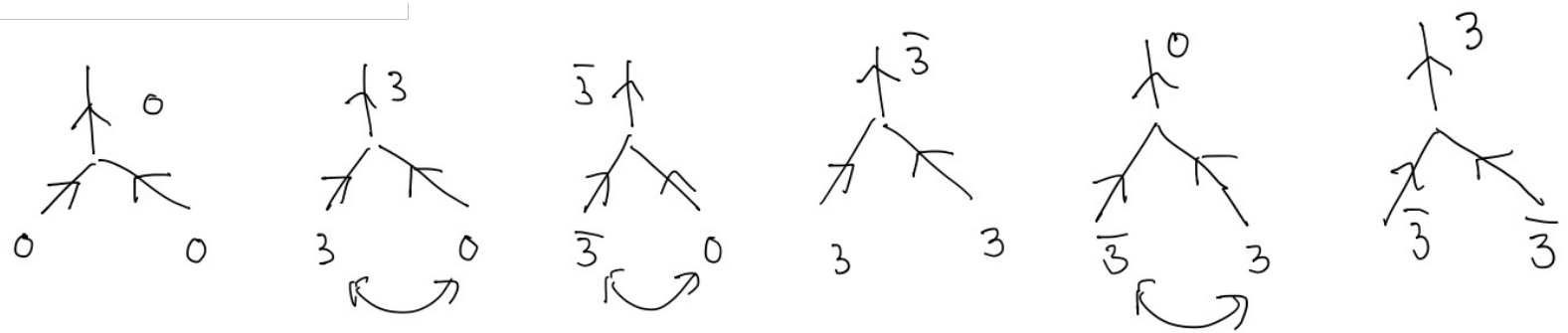


$$|\psi\rangle = \sum_{a_1,a_2,a_3,a_4 \in \{1,3,\bar{3}\}} \psi(a_1, a_2, a_3, a_4) |a_1, a_2, a_3, a_4\rangle$$

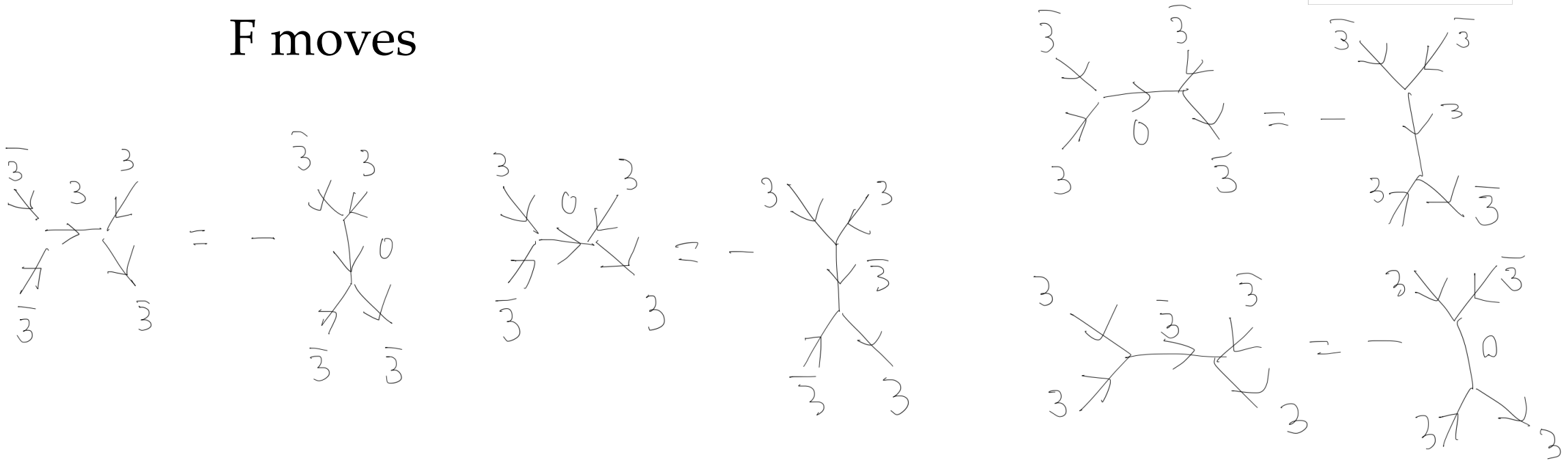
Qutrit chain

$SU(3)_1$ Yang-Mills model on a two-leg ladder

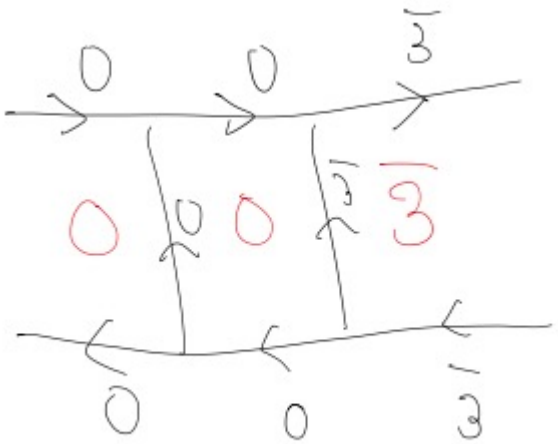
Fusion rules



F moves



$$\mathrm{tr}U_3|00\bar{3}\rangle =$$



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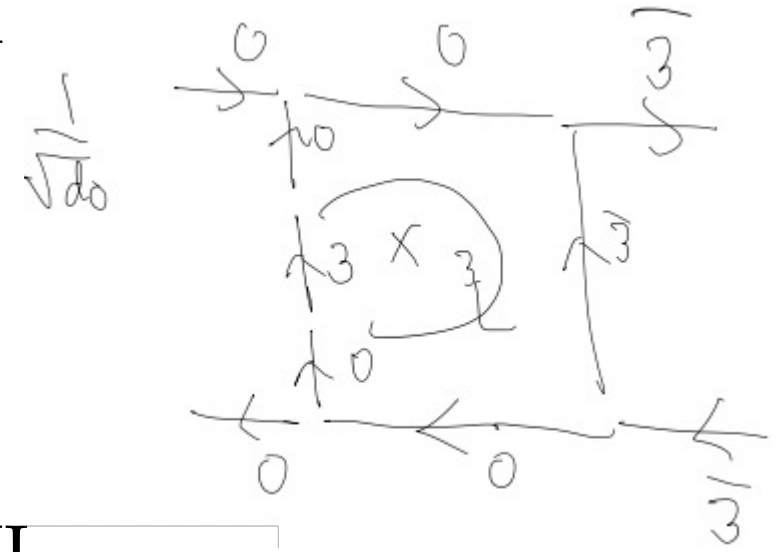
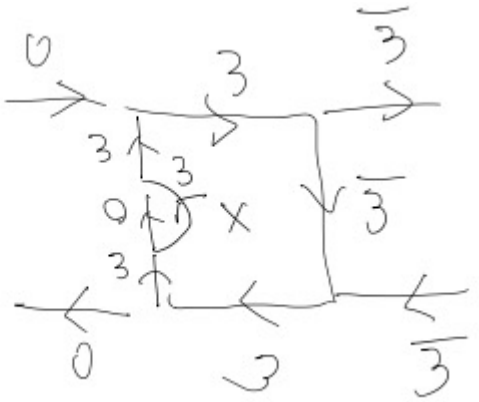


II

$= -\frac{1}{\sqrt{d_0}}$



III

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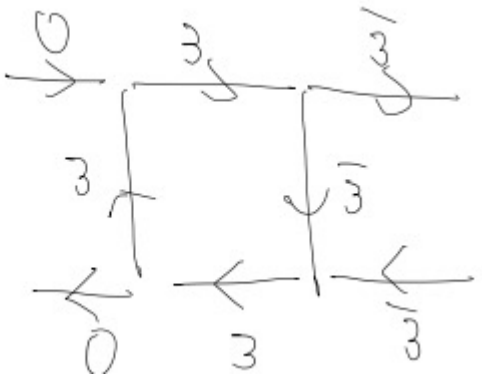


IV

$= -|03\bar{3}\rangle$

$3 \times 3 \times 3$ patterns → unitary transformation → spin model

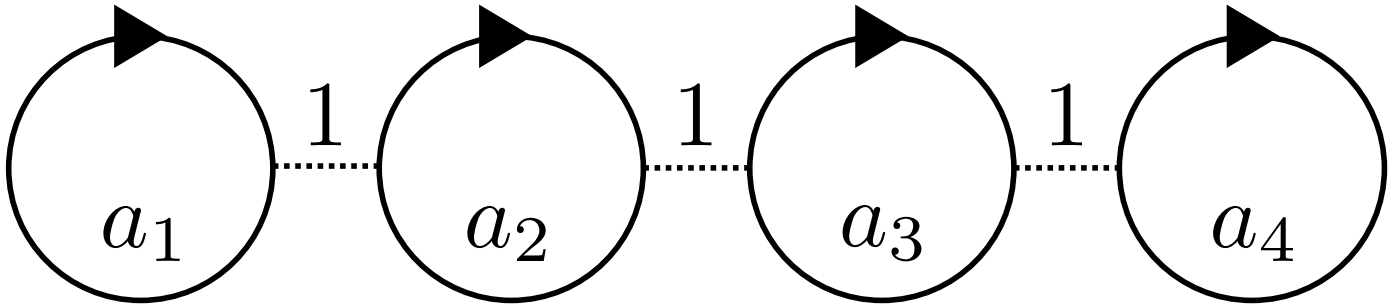
$SU(3)_1$ Yang-Mills model on a two-leg ladder

Gauss law constraints

$$1 \times 1 = 1, \;\; 1 \times 3 = 3 \times 1 = 3, \;\; 1 \times \bar{3} = \bar{3} \times 1 = \bar{3},$$
$$3 \times 3 = \bar{3}, \;\; 3 \times \bar{3} = 1, \;\; \bar{3} \times \bar{3} = 3$$

Hilbert space

Dual basis = Plaquette basis



$$|\psi\rangle = \sum_{a_1,a_2,a_3,a_4 \in \{1,3,\bar{3}\}} \psi(a_1,a_2,a_3,a_4)|a_1,a_2,a_3,a_4\rangle$$

Qutrit chain

$SU(3)_1$ Yang-Mills model on a two-leg ladder

○ Hamiltonian of a qutrit chain

$$H_E = \frac{c}{2}\sum_{x=1}^{N} M(x) + \frac{c}{2}\sum_{x=1}^{N-1} M^r(x, x+1)$$

$$M = 2\sum_{a=3,\bar{3}} |a\rangle\langle a| \qquad M^r = \sum_{a=3,\bar{3}} \left(|a,1\rangle\langle a,1| + |1,a\rangle\langle 1,a| + |a,\bar{a}\rangle\langle a,\bar{a}|\right)$$

$$H_B = -\frac{K}{2}\sum_{x=1}^{N}\left[W(x) + W^\dagger(x)\right] \qquad W = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

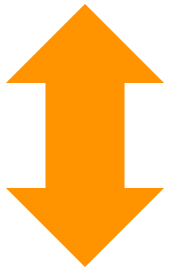
$SU(3)_1$ Yang-Mills model on a two-leg ladder

○ Mapping to qubit chain

$$|1\rangle \to |00\rangle \quad |3\rangle \to |01\rangle \quad |\bar{3}\rangle \to |10\rangle$$

$$\begin{aligned} M^{r(q)} &= \frac{1}{8}(3 + Z_1 + Z_2 + Z_3 + Z_4) \\ &+ \frac{1}{8}(-Z_1Z_2 + Z_2Z_3 - Z_3Z_4 + Z_1Z_4 - Z_1Z_3 - Z_2Z_4) \\ &+ \frac{1}{8}(-Z_1Z_2Z_3 - Z_2Z_3Z_4 - Z_1Z_2Z_4 - Z_1Z_3Z_4) \\ &- \frac{1}{8}Z_1Z_2Z_3Z_4 \end{aligned}$$

$$M_\phi = |11\rangle\langle 11|$$



Equivalent on logical space

$$\begin{aligned} \bar{M}^{r(q)} &= M^{r(q)} - 2M_\phi \otimes M_\phi + 2M_1 \otimes M_\phi + 2M_\phi \otimes M_1 \\ &= \frac{1}{2} + \frac{1}{4}(Z_1 + Z_2 + Z_3 + Z_4) - \frac{1}{2}(Z_1Z_3 + Z_2Z_4) - \frac{1}{4}(Z_2Z_3 + Z_1Z_4). \end{aligned}$$

$SU(3)_1$ Yang-Mills model on a two-leg ladder

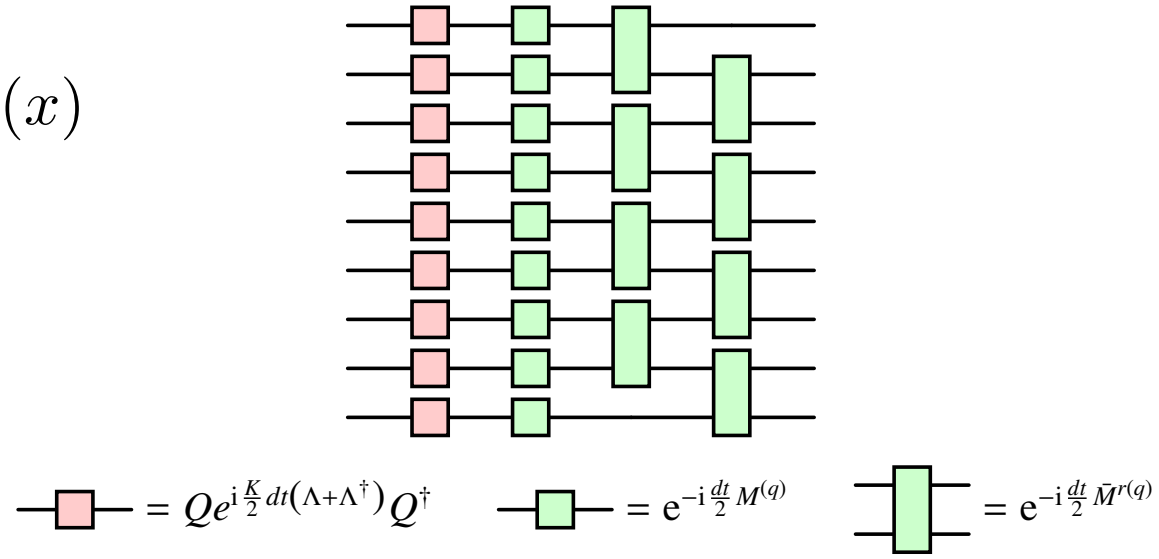
○ 1st order Suzuki-Trotter decomposition

$$\mathcal{U}(T) = \left(e^{-\mathrm{i}dtH_E} e^{-\mathrm{i}dtH_B}\right)^M$$

$$W^{(q)} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad W^{(q)} = Q\Lambda Q^\dagger$$

$$e^{-\mathrm{i}dtH_E} = \prod_{x:\mathrm{odd}} e^{-\mathrm{i}\frac{1}{2}dt\bar{M}^{r(q)}(x,x+1)} \prod_{x:\mathrm{even}} e^{-\mathrm{i}\frac{1}{2}dt\bar{M}^{r(q)}(x,x+1)} \prod_x e^{-\mathrm{i}\frac{1}{2}dtM^{(q)}(x)}$$

$$e^{-\mathrm{i}dtH_B} = \prod_x Q(x) e^{\mathrm{i}\frac{K}{2}dt\left(\Lambda(x)+\Lambda^\dagger(x)\right)} Q^\dagger(x)$$



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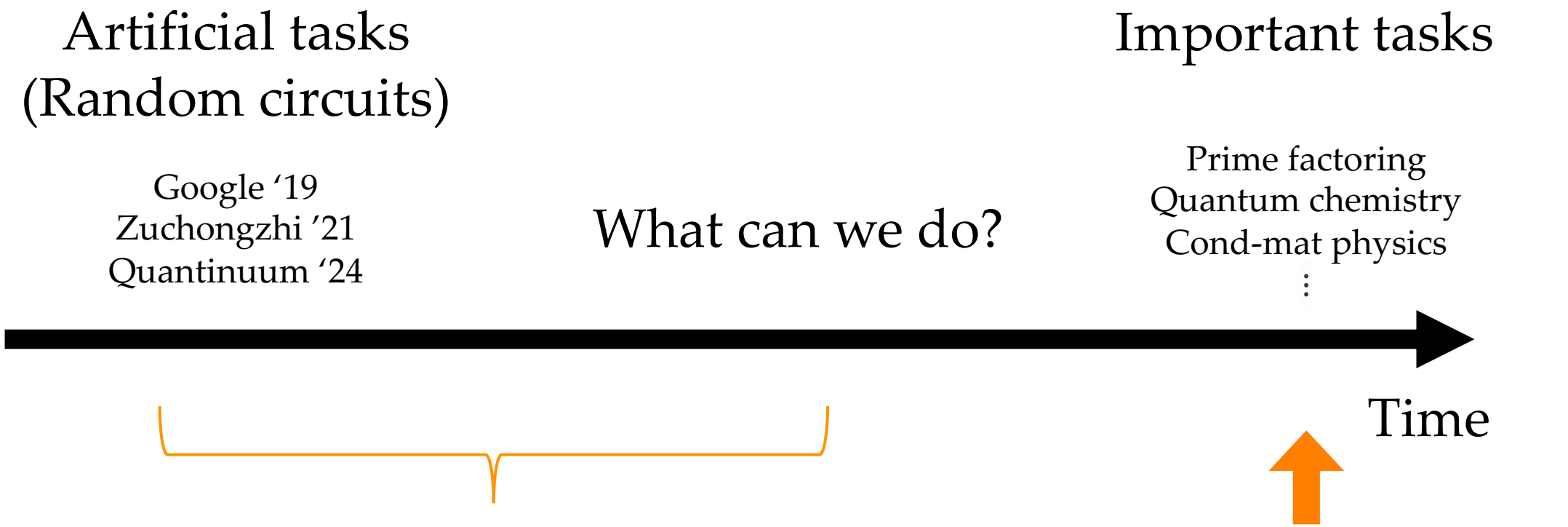
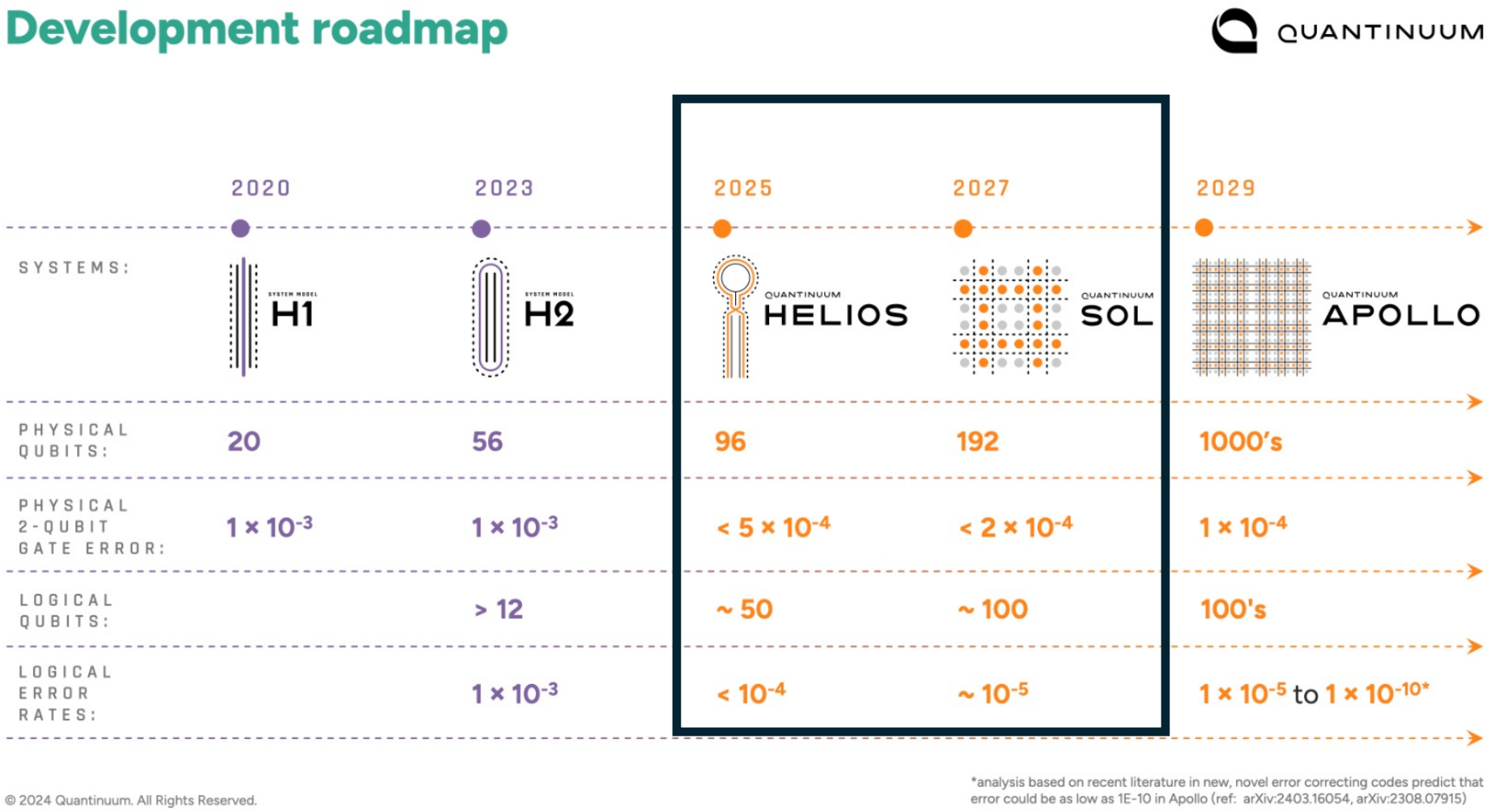
- Why QC? → QC of QFT
- q-deformed Yang-Mills theory (Easy examples: $SU(2)_1$ and $SU(3)_1$)
- Why Floquet systems?
- Experimental results
- Summary


Artificial tasks
(Random circuits)
Google ‘19
Zuchongzhi ’21
Quantinuum ‘24
What can we do?
Important tasks
Prime factoring
Quantum chemistry
Cond-mat physics
Time
Deal with noises
Fault-tolerant QC

What can we do?



physical qubits are more useful than logical qubits

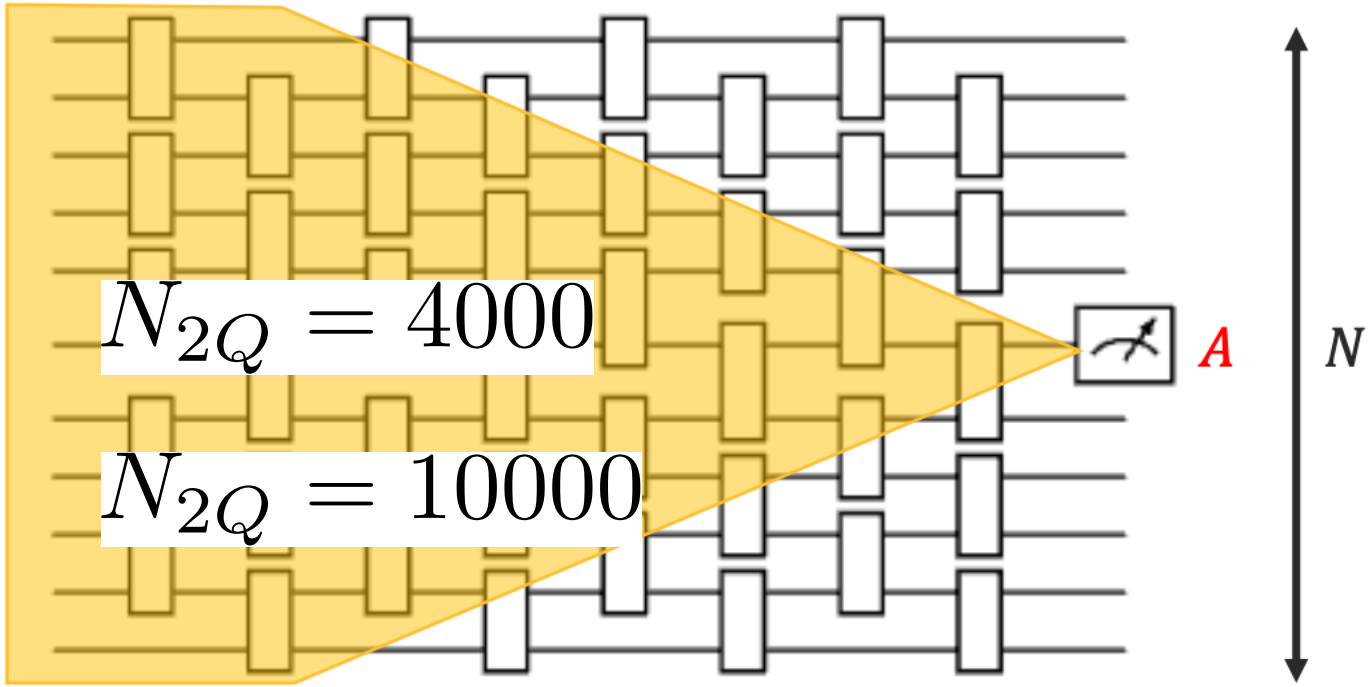
What can we do?

○ global depolarizing model $\langle \hat{O} \rangle_{\text{noisy}} = f \langle \hat{O} \rangle_{\text{ideal}}$

$$f = (1-p)^{N_{2Q}} = 0.135$$

$$p = 5 \times 10^{-4}$$

$$p = 2 \times 10^{-4}$$



$$N = 100, \quad dt = 0.1, \quad t = 10, \quad N_{2Q} \sim 10000$$

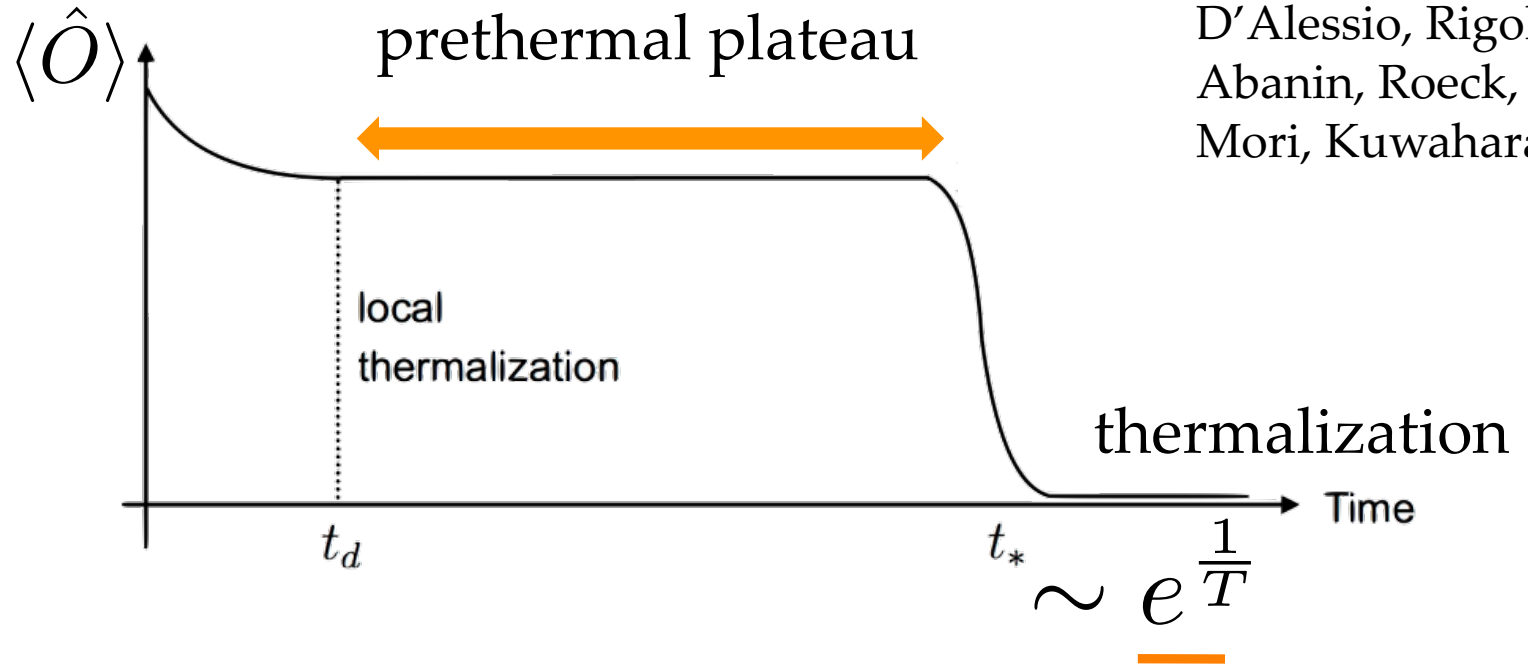
Hamiltonian simulation of LGTs is challenging in the NISQ era

Floquet dynamics

○ Time evolution with a time-dependent Hamiltonian

$$i\partial_t|\Psi\rangle = H|\Psi\rangle \qquad\qquad H(t+T) = H(t)$$

driven system → heat up to infinite temperature



Lanzarides, Das, Moessner '14
D'Alessio, Rigol '14
Abanin, Roeck, Huveneers '15
Mori, Kuwahara, Saito '16

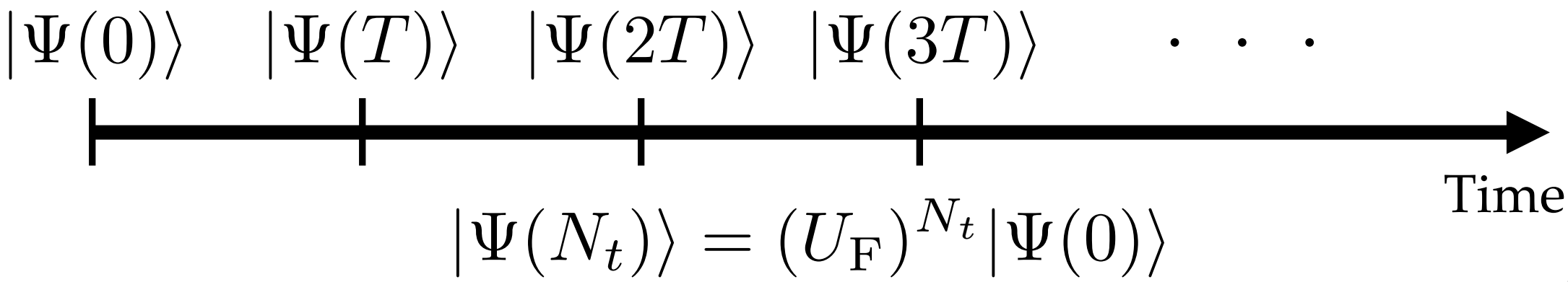
Trotter dynamics

- Trotter dynamics can be understood as Floquet dynamics

$$U_{\mathrm{F}} = \mathrm{e}^{-\mathrm{i}H_1 dt}\mathrm{e}^{-\mathrm{i}H_2 dt} \qquad H(t) = \begin{cases} H_1 & t \in [0, T/2), \\ H_2 & t \in [T/2, T) \end{cases}$$

$$dt = \frac{T}{2}$$

$$i\partial_t|\Psi\rangle = H|\Psi\rangle \qquad H(t+T) = H(t)$$



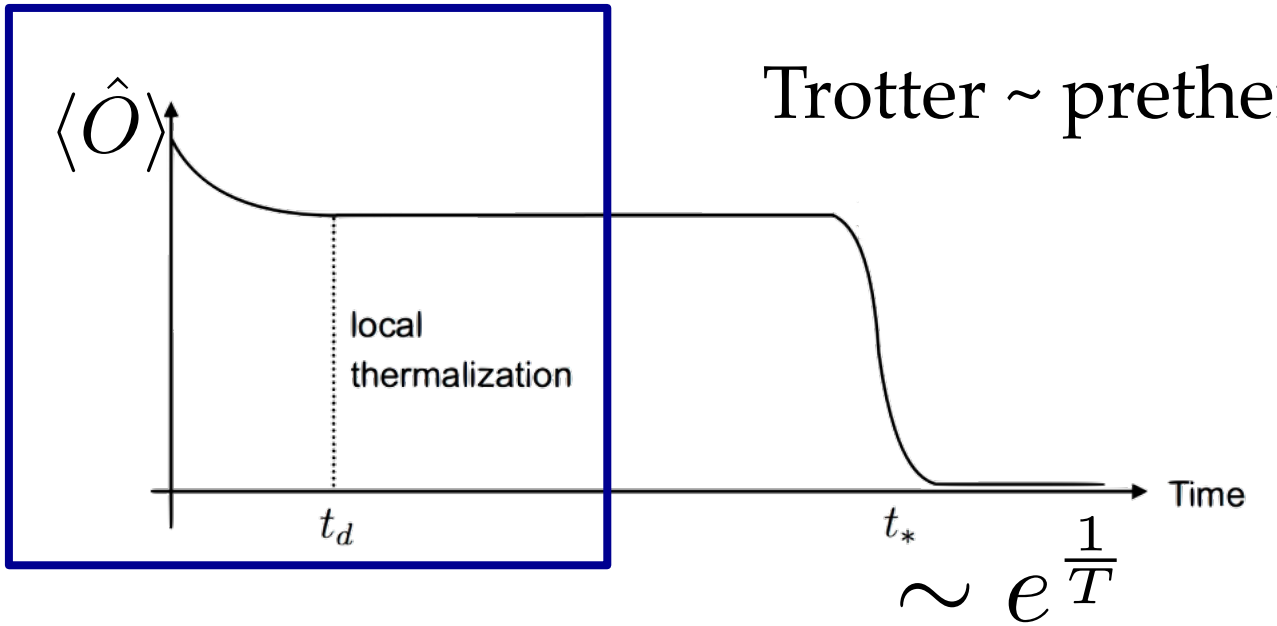
$$|\Psi(N_t)\rangle = (U_{\mathrm{F}})^{N_t}|\Psi(0)\rangle$$

Trotter dynamics

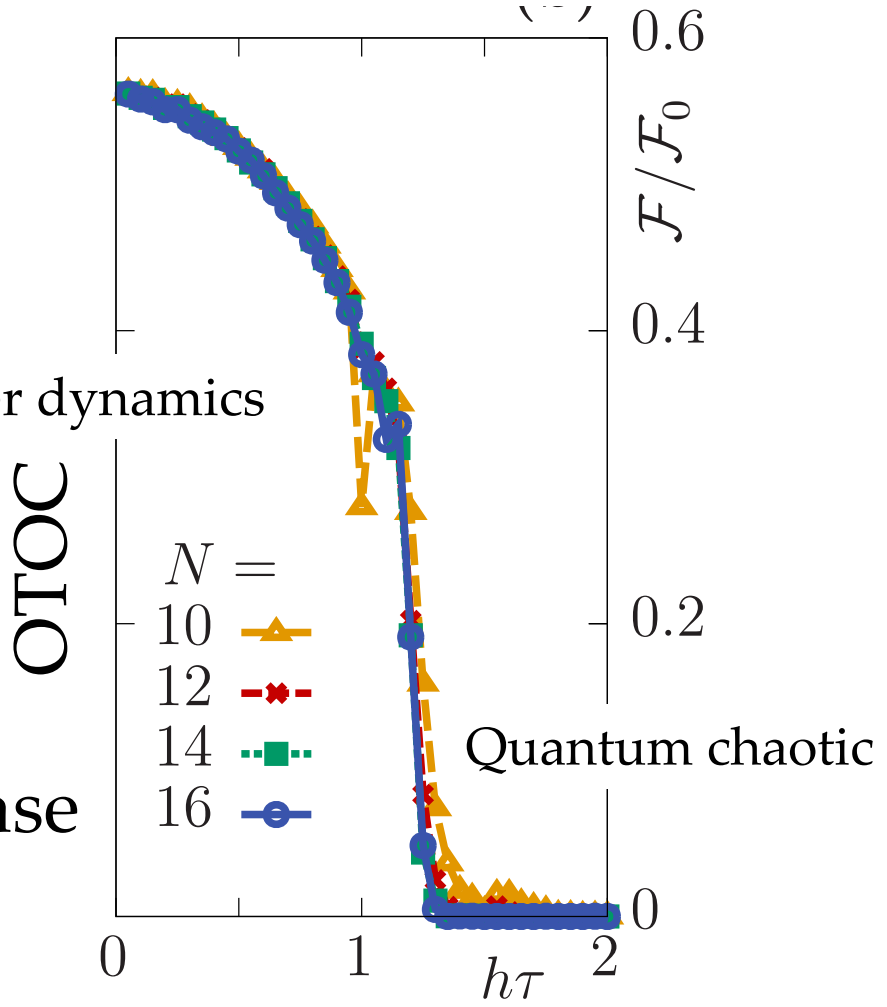
○ Trotter transition

Heyl, Hauke, Zoller '18
Varnier, Bertini, Giudici, Piroli '23

Trotter error is under control if $dt < dt_c$



Trotter ~ prethermal phase



prethermalizaion dynamics
is feasible to NISQ devices

Trotter dynamics as Floquet dynamics

○ Feasibility of information scrambling on the present best fidelity hardware

Next talk!

Ion traps: Kazuhiro Seki, Yuta Kikuchi, Tomoya Hayata, Seiji Yunoki, 2405.07613 [quant-ph]

○ Prethermalization dynamics with many qubits

Superconducting qubits:
1+1d Z_2 : Tomoya Hayata, Kazuhiro Seki, Arata Yamamoto, arXiv:2408.10079 [hep-lat]
$SU(3)_1$: Tomoya Hayata, Yoshimasa Hidaka, 2409.20263 [hep-lat]

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$SU(3)_1$ Yang-Mills model on a two-leg ladder

○ 1st order Suzuki-Trotter decomposition

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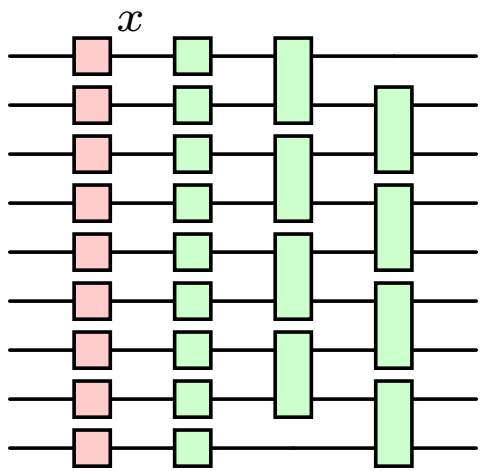
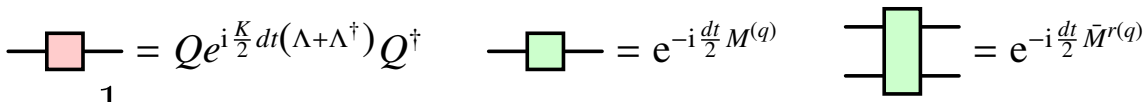
$$e^{-\mathrm{i}dtH_B} = \prod_x Q(x) e^{\mathrm{i}\frac{K}{2}dt\left(\Lambda(x)+\Lambda^\dagger(x)\right)} Q^\dagger(x)$$

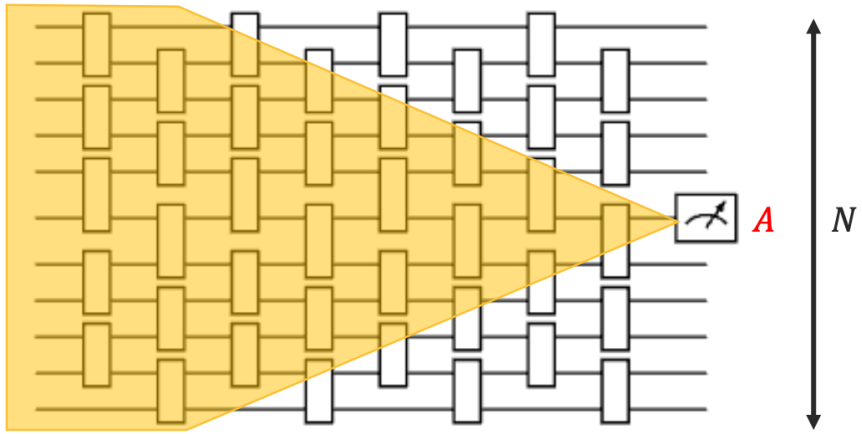




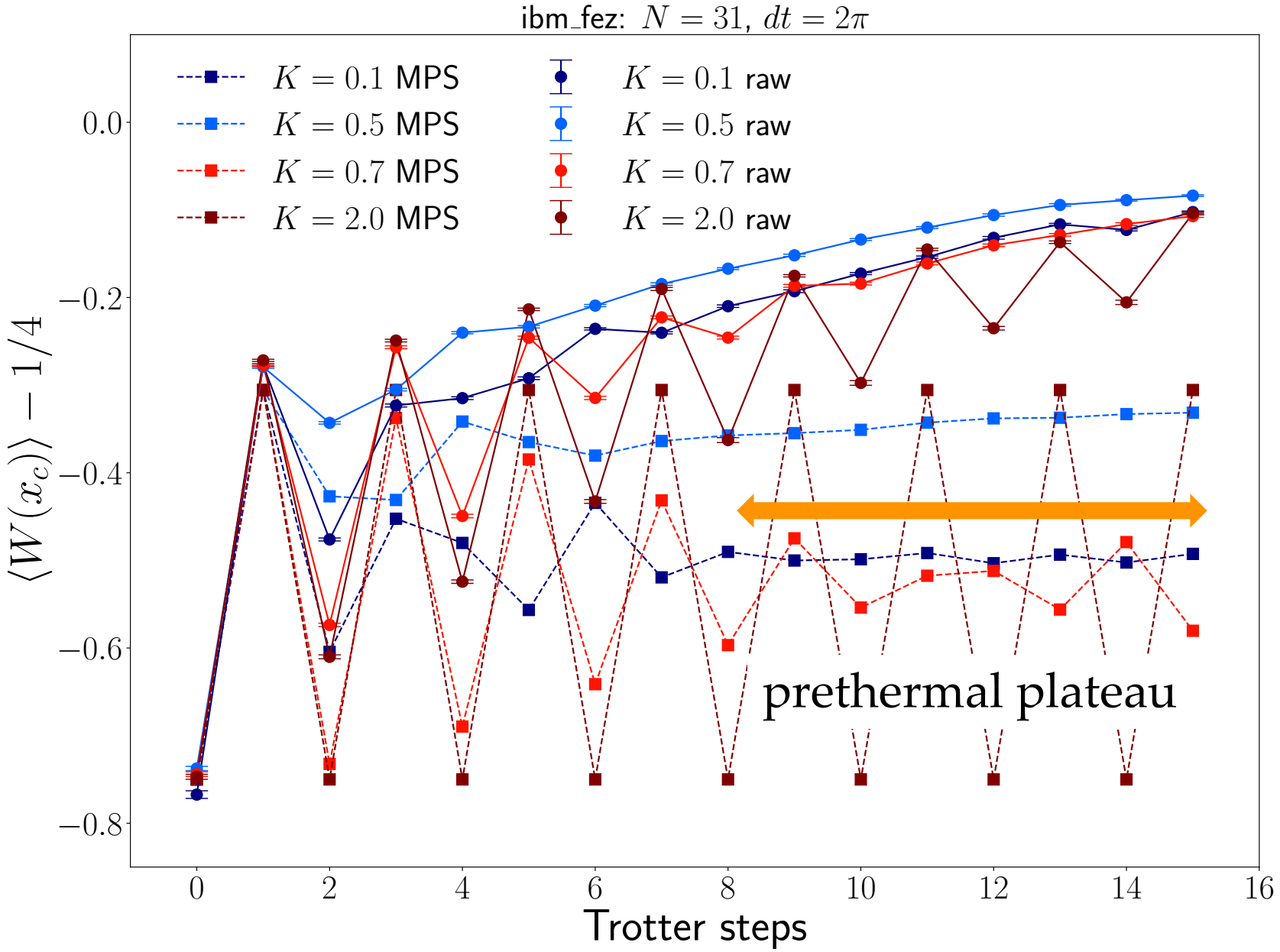
$$\bar{M}^{r(q)} = \frac{1}{2} + \frac{1}{4}(Z_1 + Z_2 + Z_3 + Z_4) - \frac{1}{2}(Z_1Z_3 + Z_2Z_4) - \frac{1}{4}(Z_2Z_3 + Z_1Z_4)$$

QC summary

- $dt = 2\pi$ (We can reduce 2Q gates)/ Change K
- ibm_fez is used
- 10,000 shot is used
- Pauli twirling is enabled/ DD is disabled
- Unnecessary gates outside of the causal cone are removed



Experimental results (raw data)



Error mitigation is necessary to do quantitative analysis

Quantum error mitigation ("Gauss' laws")

- global depolarizing channel

$$\langle \hat{O} \rangle_{\text{raw}} = f \langle \hat{O} \rangle_{\text{ideal}} + \frac{1-f}{2^{3N-1}} \text{Tr} \hat{O}$$

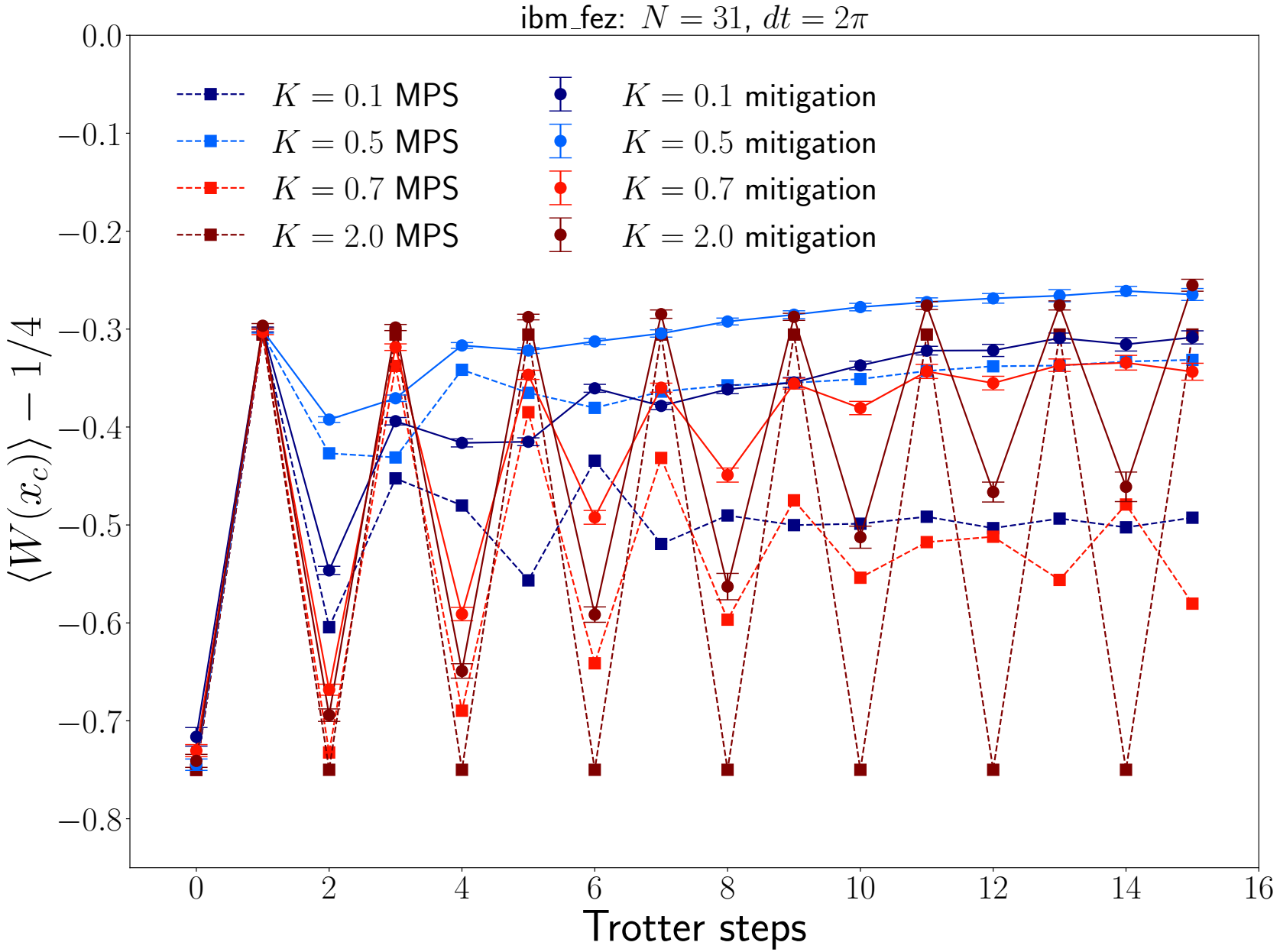
- estimate f by measuring $M_\phi = |11\rangle\langle 11|$

$$\langle M_\phi \rangle_{\text{ideal}} = 0 \quad \Rightarrow \quad \langle M_\phi \rangle_{\text{raw}} = \frac{1-f}{4}$$

- rescale observables by

$$\langle O(x) \rangle_{\text{mit}} = \frac{\langle O(x) \rangle_{\text{raw}}}{1 - 4 \langle M_\phi(x) \rangle_{\text{raw}}}$$

Experimental results ("Gauss's law")



Improved but not better than we expected

Quantum error mitigation (ODR)

- global depolarizing channel

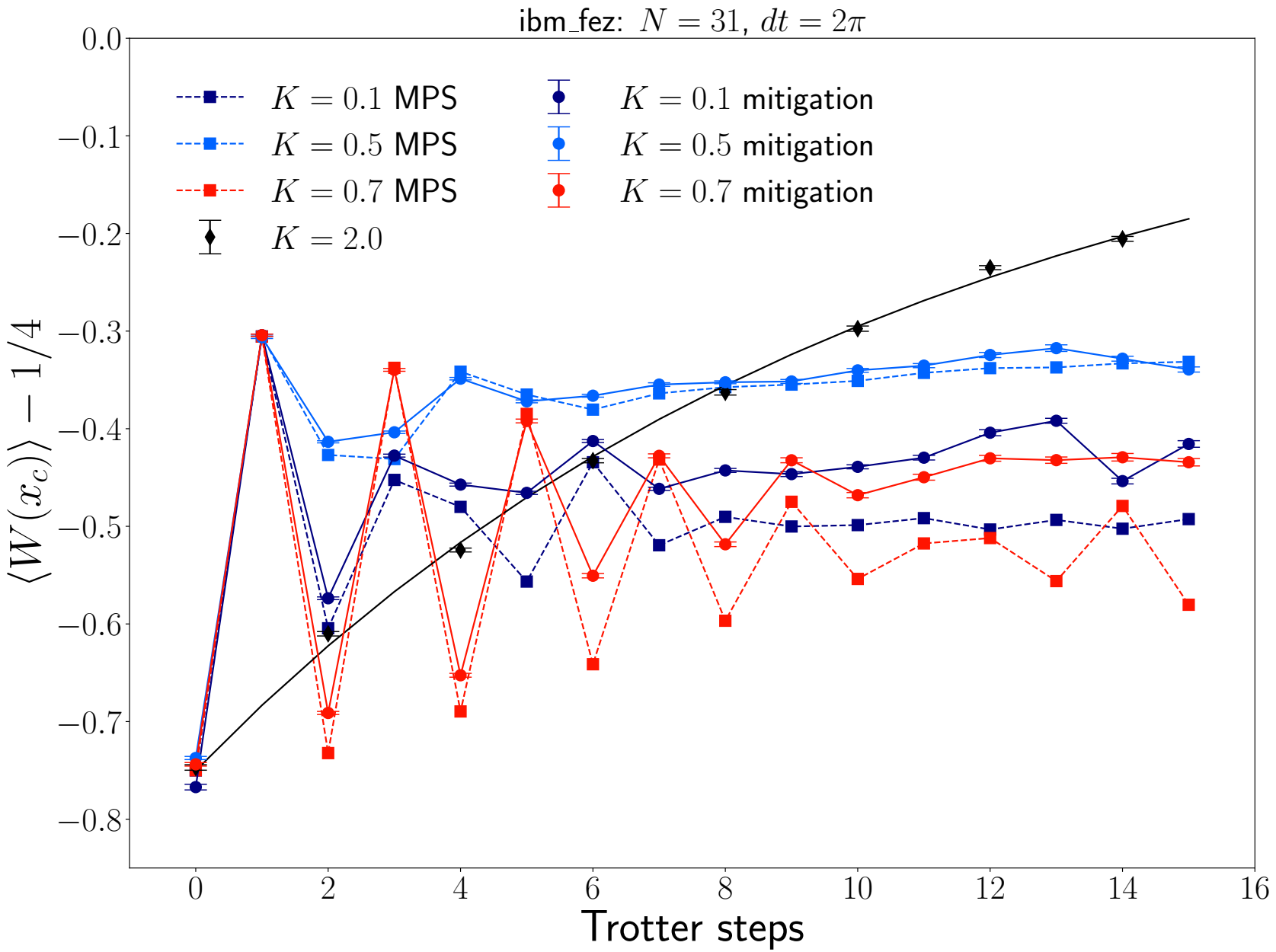
$$\langle \hat{O} \rangle_{\text{raw}} = f \langle \hat{O} \rangle_{\text{ideal}} + \frac{1-f}{2^{3N-1}} \text{Tr} \hat{O}$$

- run the same circuit with the “solvable” parameter

- rescale observables by

$$\langle \hat{O} \rangle_{\text{mit}} = \frac{\langle \hat{O} \rangle_{\text{raw}}}{f}$$

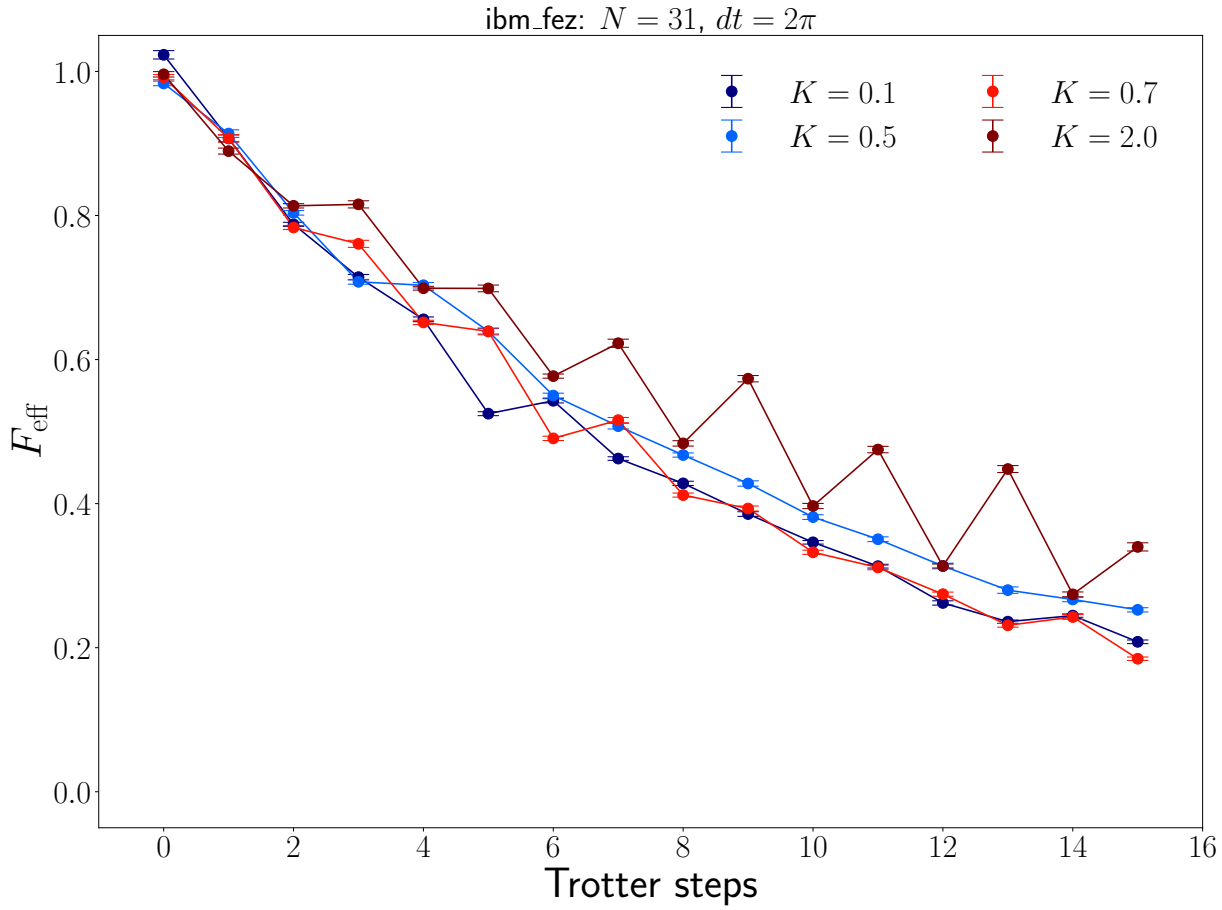
Experimental results (ODR)

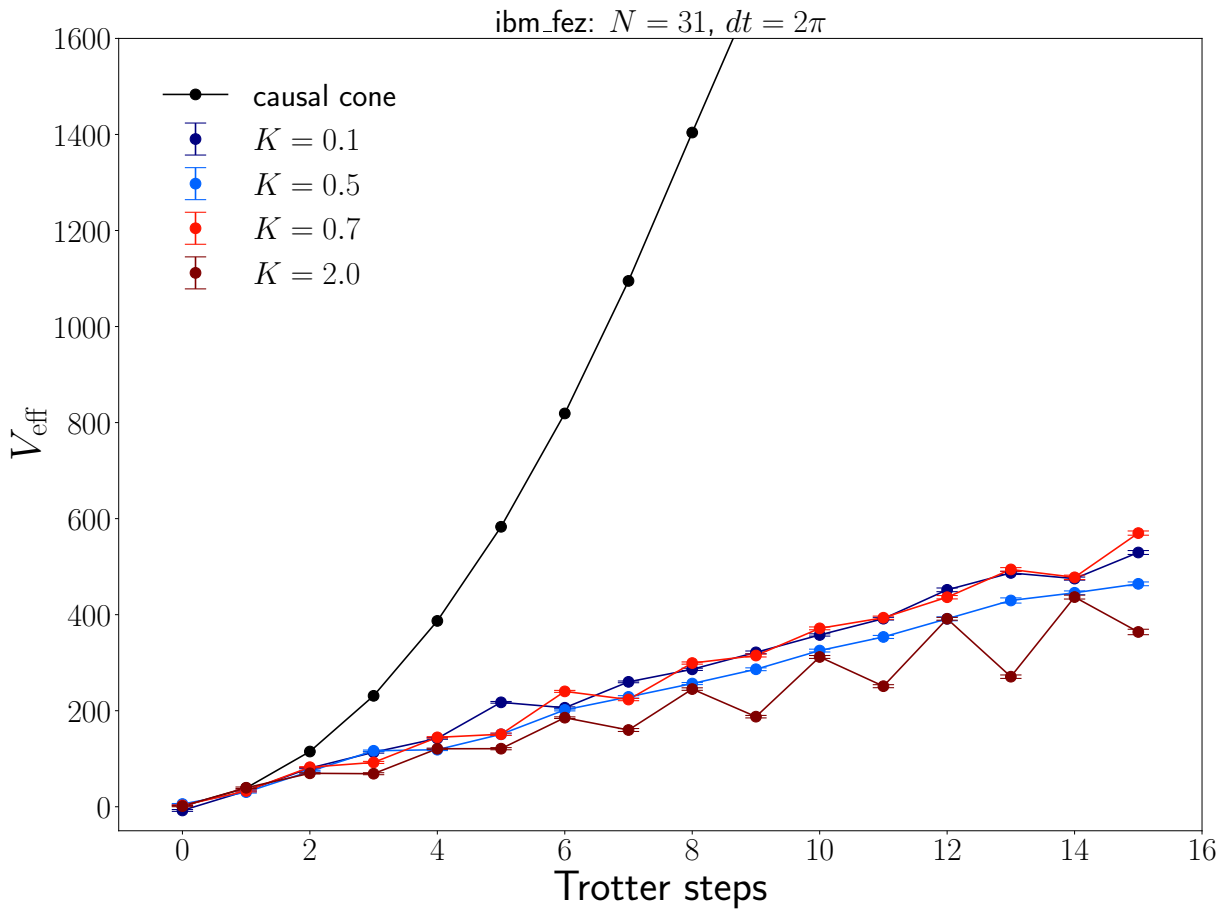


Successful in running Floquet circuit with $N_t > 10$

ODR may be the first choice if ZNE is not applicable

Fidelity and circuit volume





Estimate 2Q gate involved to compute the expectation value from the fidelity decay

Summary

○ QC for QCD is anticipated but challenging

○ Quantum simulation of Floquet circuits in near future devices is interesting and may be useful for showing quantum advantage

○ Lattice gauge theories have complex Hamiltonians and may provide good playgrounds for testing the capability of QCs

Thank you!