Conserved charge fluctuations in (2+1)-flavor QCD with Möbius Domain Wall fermions

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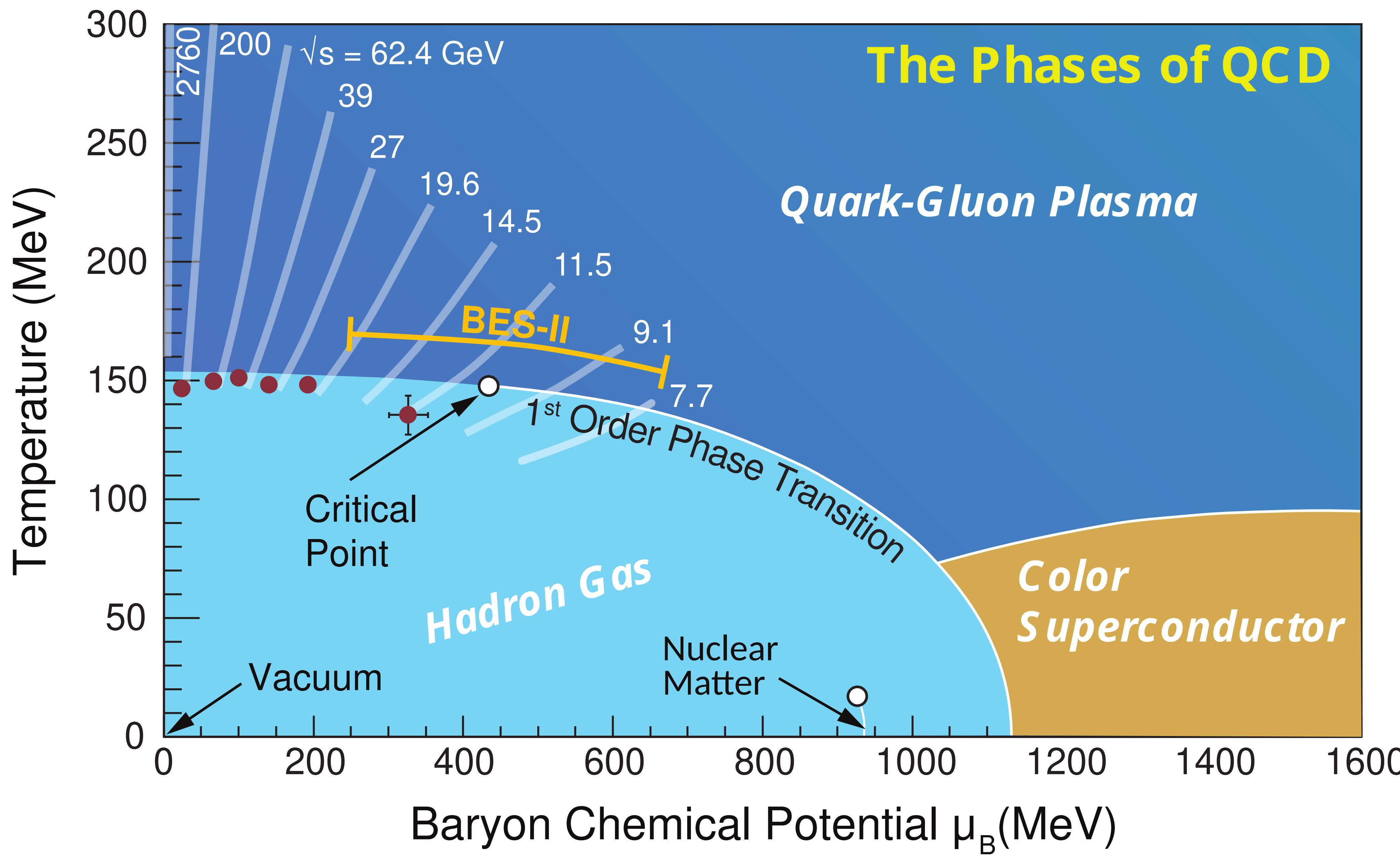
Hadrons and Hadron Interactions in QCD
-- Experiments, Effective theories, and
Lattice -- (HHIQCD2024):2024

QCD phase diagram : fluctuation of conserved charges

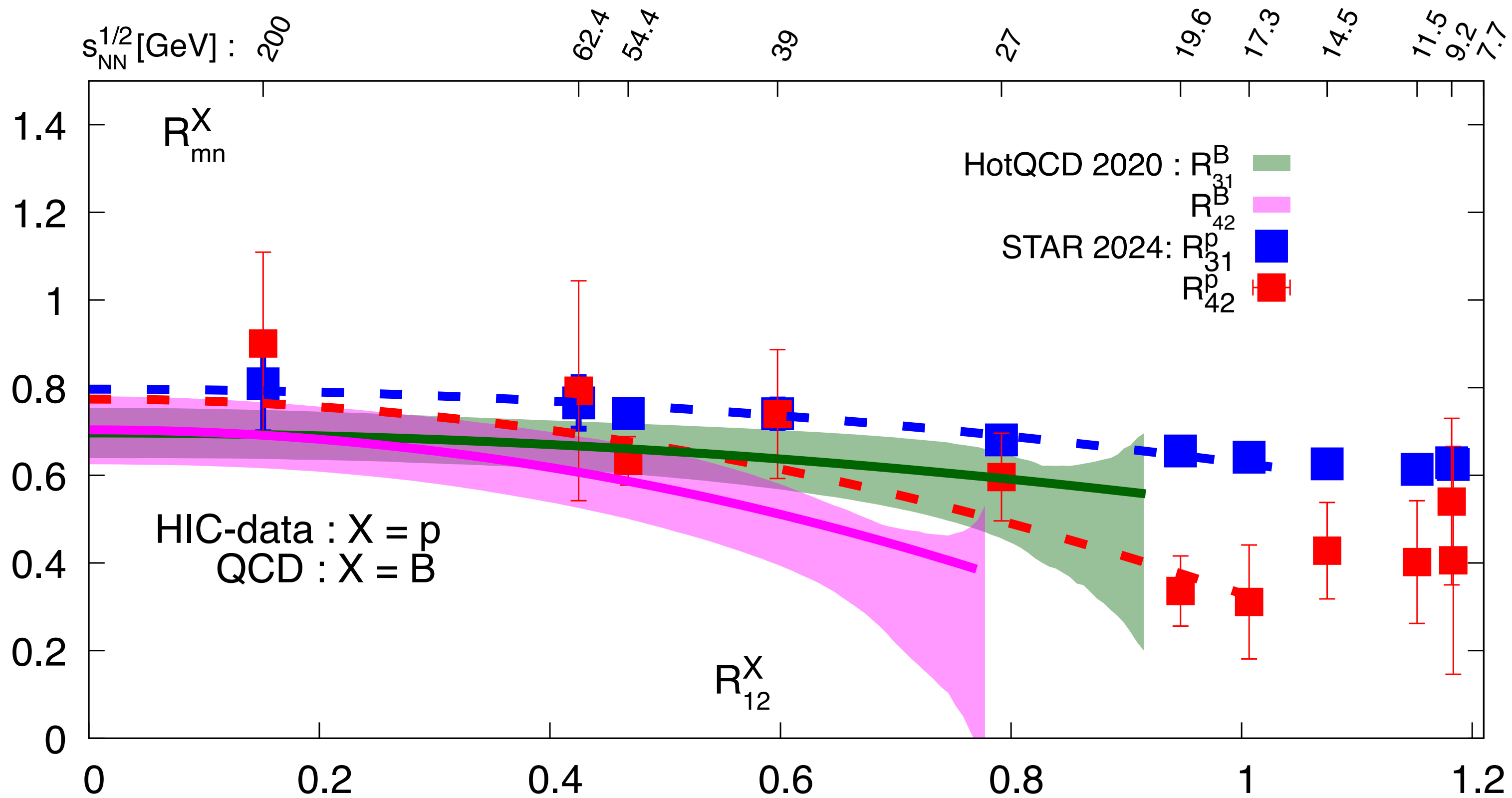
"Mapping the Phases of Quantum Chromodynamics with Beam Energy Scan", Bzdaket al., Phys. Rept. '20



Conserve charge in Heavy Ion Collision experiment : Baryon number (B), Electric charge (Q) and Strangeness number (S).

A detailed review : Masayuki Asakawa, Masakiyo Kitazawa, Prog.Part.Nucl.Phys. 90 (2016) 299-342

Higher order baryon number fluctuation



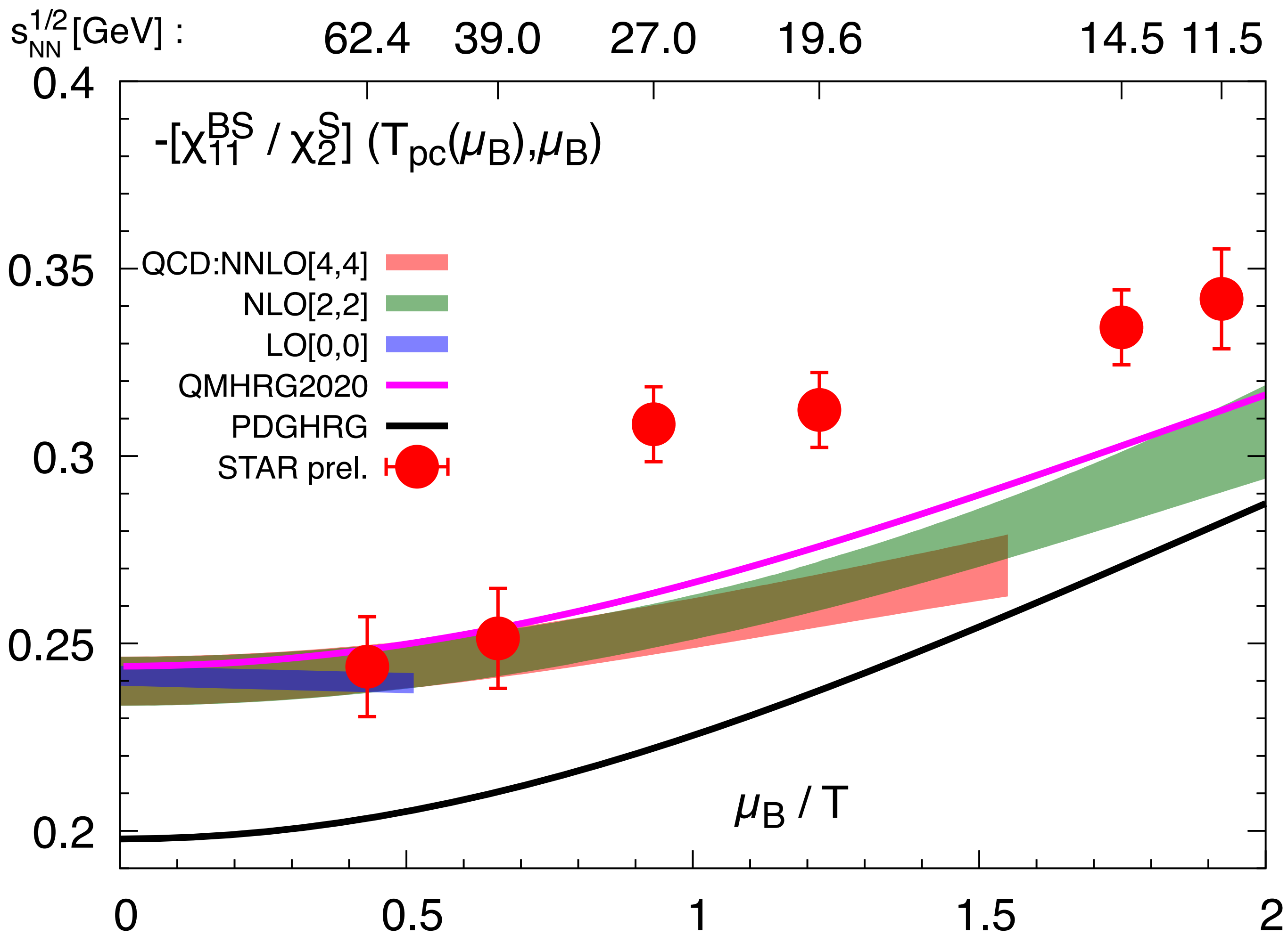
QCD CEP most likely doesn't exist in STAR BESII in collidor mode.

$$R_{12}^B = M_B/\sigma_B^2 = \frac{\chi_1^B}{\chi_2^B} \quad R_{31}^B = S_B\sigma_B^3 = \frac{\chi_3^B}{\chi_1^B} \quad R_{42}^B = \kappa_B\sigma_B^2 = \frac{\chi_4^B}{\chi_2^B}$$

HotQCD 2017, 2020 : PRD; Goswami, Karsch , XQCD 2024

STAR results : CPOD2024

Baryon-strangeness correlations



Baryon strangeness correlations.

QCD and STAR results are in good agreement for $\sqrt{s_{NN}} \geq 39$ GeV.

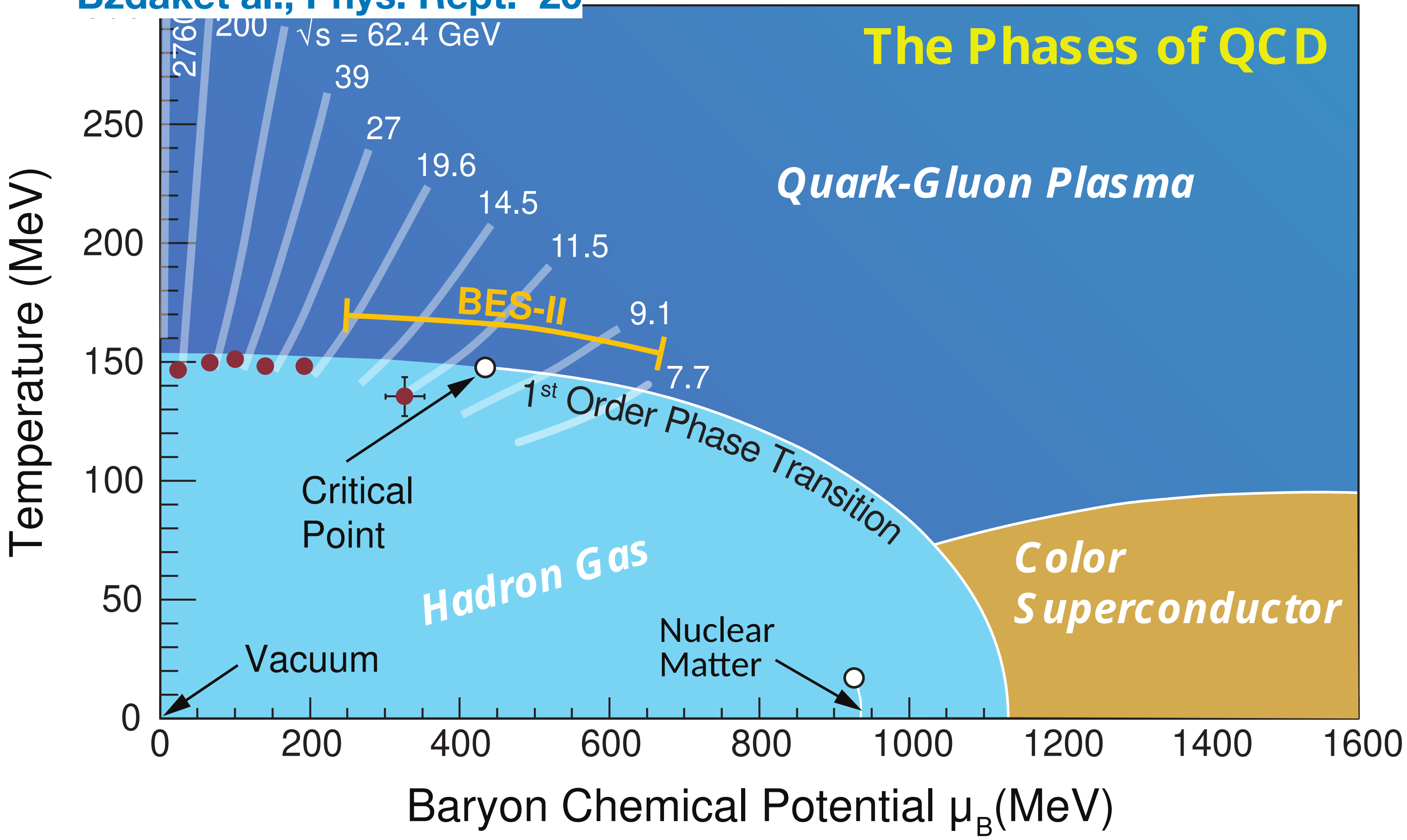
Significant differences between QCD and STAR results for $\sqrt{s_{NN}} \leq 27$ GeV.

HotQCD 2024 : *Phys.Rev.D* 110 (2024);

STAR results : CPOD2024

QCD phase diagram

“Mapping the Phases of Quantum Chromodynamics with Beam Energy Scan”,
Bzdaket al., Phys. Rept. ‘20



Bigger Picture: Understand the thermodynamics at the QCD chiral transition and exploration of the QCD phase diagram with lattice chiral fermions i.e. Möbius Domain Wall fermions.

Studies with the similar lattice action and setup:

1) Chiral phase transtion and topological susc. : Talk by Y. Aoki on lattice2024, I. Kanamori on JPS2024,

2) Symmteries at finite temperature: D. Ward (last week on this workshop)

3) chiral phase transition with 3 degenerate flavors, Y. Zhang lattice 2023, 2024.

Acknowledgments

1. Computational resource:

- Supercomputer Fugaku (hp230207, hp220174, hp210165, hp200130, ra000001).

2. Funding sources :

- MEXT as "Program for Promoting Researches on the Supercomputer Fugaku",*Simulation for basic science: from fundamental laws of particles to creation of nuclei*, JPMXP1020200105; "Simulation for basic science: approaching the quantum era" (JPMXP1020230411).
- JICFuS.
- JPS KAKENHI(JP20K0396, I. Kanamori).

And to all the JLQCD members for regular meetings and discussions.

Code bases

Configuration generation: Grid (https://github.com/paboyle/Grid)

Measurements : (i) Hadrons (https://github.com/aportelli/Hadrons)

(ii) Bridge++ (https://bridge.kek.jp/Lattice-code/)

Data Analysis : https://github.com/LatticeQCD/AnalysisToolbox

Möbius Domain wall fermions : parameter set up

Möbius Domain wall fermions (MDWF):

Discretization of the Dirac operator in 5 space-time dimensions. We denote the fifth dimension as L_s. Gauge Fields in 4d and Fermion Fields in 5d.

Better control on the chiral symmetry at finite lattice spacing.

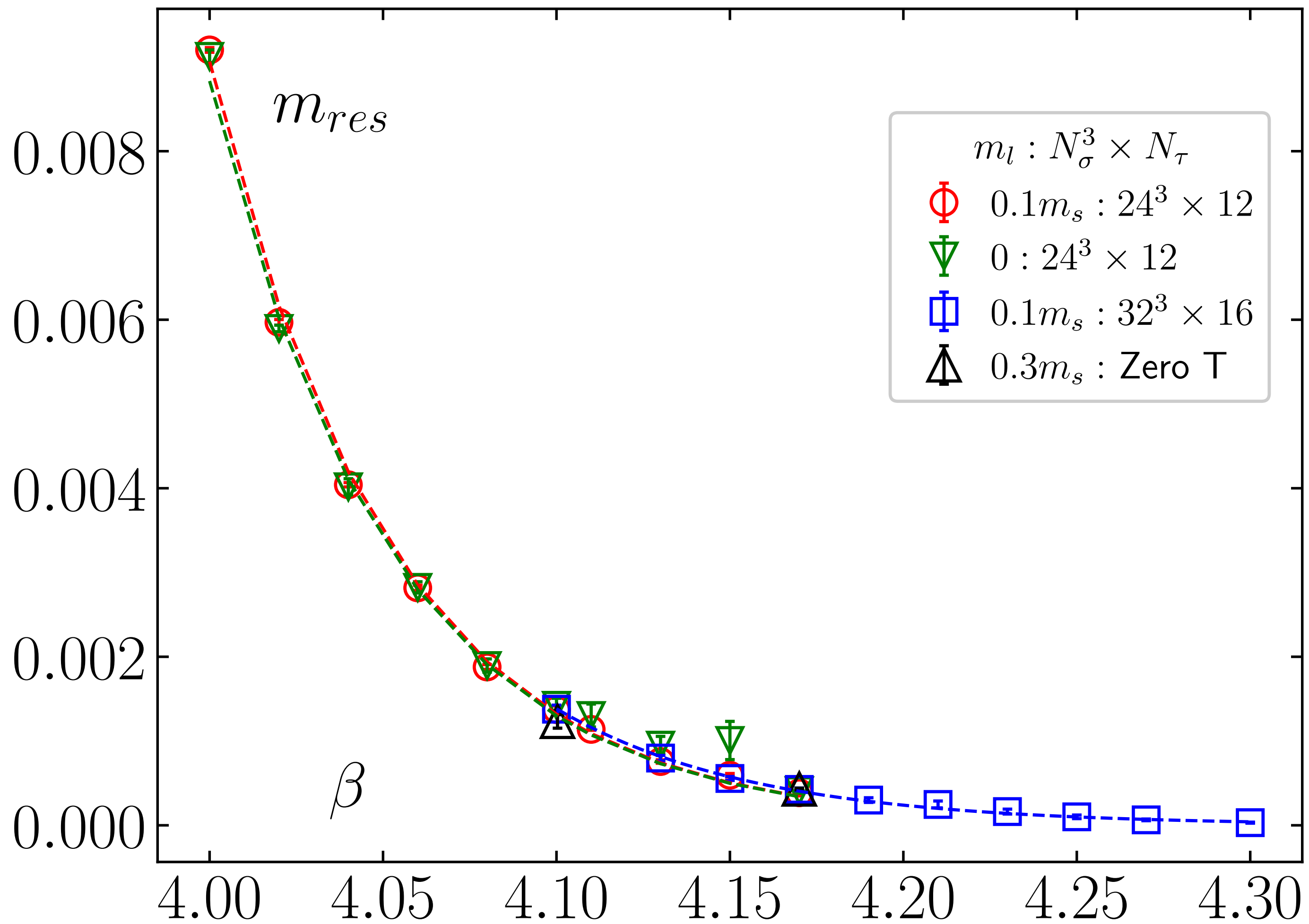
Gauge configuration generation : Symanzik with stout smear + Möbius Domainwall fermion.

Specific parameter choice : b=1.5 , c=0.5 and $L_s = 12$ with 3 levels of stout-link smearing minimizes chiral symmetry violations.

S. Hashimoto et al, "Residual mass in five-dimensional fermion formulations", *PoS* LATTICE2013 (2014) 431.

B. Colquhoun et al (JLQCD collaboration), Phys. Rev. D 106, 054502

m_{res} correction for $L_S = 12$



Tuning of input light quark masses for measurement : $m_f = m_f^{latt} - m_{res}\ ;\ f = \{u, s\}$

- Performed calculations on the Line of constant physics (LCP) (m_l/m_s fixed).
- m_{res} is almost independent of m_l.
- We use m_{res} calculated on the LCP, $m_l/m_s = 0.1$ to tune the input quark masses for configuration generation and measurements for $m_l/m_s = 0.036$

A “Recap” on Chemical potential on the lattice

Continuum prescription, Divergence for the free fermion case in QNS: $\chi_2 \sim 1/a^2$

The prescription for chemical potential on the lattice,

$$(1 \pm \gamma_4)U_{\pm 4}(x) \rightarrow (1 \pm \gamma_4)e^{\pm\hat{\mu}}U_{\pm 4}(x)$$

P. Hasenfratz, F. Karsch, Phys.Lett.B 125 (1983) 308-310
R. V. Gavai,Phys. Rev. D 32, 519

No additional divergences appear in the interacting theory.

Steven Gottlieb, W. Liu, D. Toussaint, R. L. Renken, and R. L. Sugar, Phys. Rev. Lett. 59, 2247.
Rajiv V. Gavai, Sayantan Sharma, *Phys.Lett.B* 749 (2015) 8-13

For Domain Wall fermions :

$$(1 \pm \gamma_4)U_{\pm 4}(x)((1 \pm \gamma_4)U_{\pm 4}(x) \rightarrow (1 \pm \gamma_4)e^{\pm\hat{\mu}}U_{\pm 4}(x)$$

J. C. R. Bloch and T. Wettig, Phys. Rev. D 76, 114511

$$Z = \int DU \prod_{f=u,d,s} detM(m_f)\exp[-S_g], \quad \det M(m_f, \hat{\mu}_f) = \left[\frac{\det D(m_f, \hat{\mu}_f)^{DWF}}{\det D(m_{PV}, \hat{\mu}_f)^{DWF}}\right]$$

Sign problem for, $\mu_f \neq 0$. We use Taylor expansions.

Quark number susceptibility and conserved charge fluctuations in (2+1)-flavor QCD

In QCD with two light (u, d) and one strange flavor (s), pressure is expressed via a Taylor expansion in quark chemical potentials (μ_f).

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \vec{\mu}) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{uds}}{i!j!k!} \hat{\mu}_u^i \hat{\mu}_d^j \hat{\mu}_s^k$$

In the context of heavy ion collision experiments there are 3 **conserved charges**, **B, Q and S** that couples to μ_B, μ_Q, μ_S,

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q, \quad \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q, \quad \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S .$$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{uds}}{i!j!k!} \hat{\mu}_u^i \hat{\mu}_d^j \hat{\mu}_s^k = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k .$$

Quark number susceptibility with Domain wall fermions

The diagonal and off-diagonal quark number susceptibilities can be written as,

$$\chi_2^f = \frac{N_\tau}{N_\sigma^3} \frac{\partial^2 \ln Z}{\partial \hat{\mu}_f^2}\bigg|_{\hat{\mu}_f=0} = \frac{N_\tau}{N_\sigma^3}\left[\left\langle \frac{\partial^2}{\partial \hat{\mu}_f^2} \ln \det M \right\rangle + \left\langle \left(\frac{\partial}{\partial \hat{\mu}_f} \ln \det M \right)^2 \right\rangle\right]$$

$$= \frac{N_\tau}{N_\sigma^3} \langle D_2^f \rangle + \langle (D_1^f)^2 \rangle,\ f = \{u, d, s\}$$

M. Cheng et al, Phys.Rev.D81:054510,2010 ; P. Hegde et al, PoS LATTICE2008:187,2008

$$\chi_{11}^{fg} = \frac{N_\tau}{N_\sigma^3} \frac{\partial^2 \ln Z}{\partial \hat{\mu}_f \partial \hat{\mu}_g}\bigg|_{\hat{\mu}_f=0} = \frac{N_\tau}{N_\sigma^3} \langle D_1^f D_1^g \rangle,\ f \neq g,\ f, g = \{u, d, s\}$$

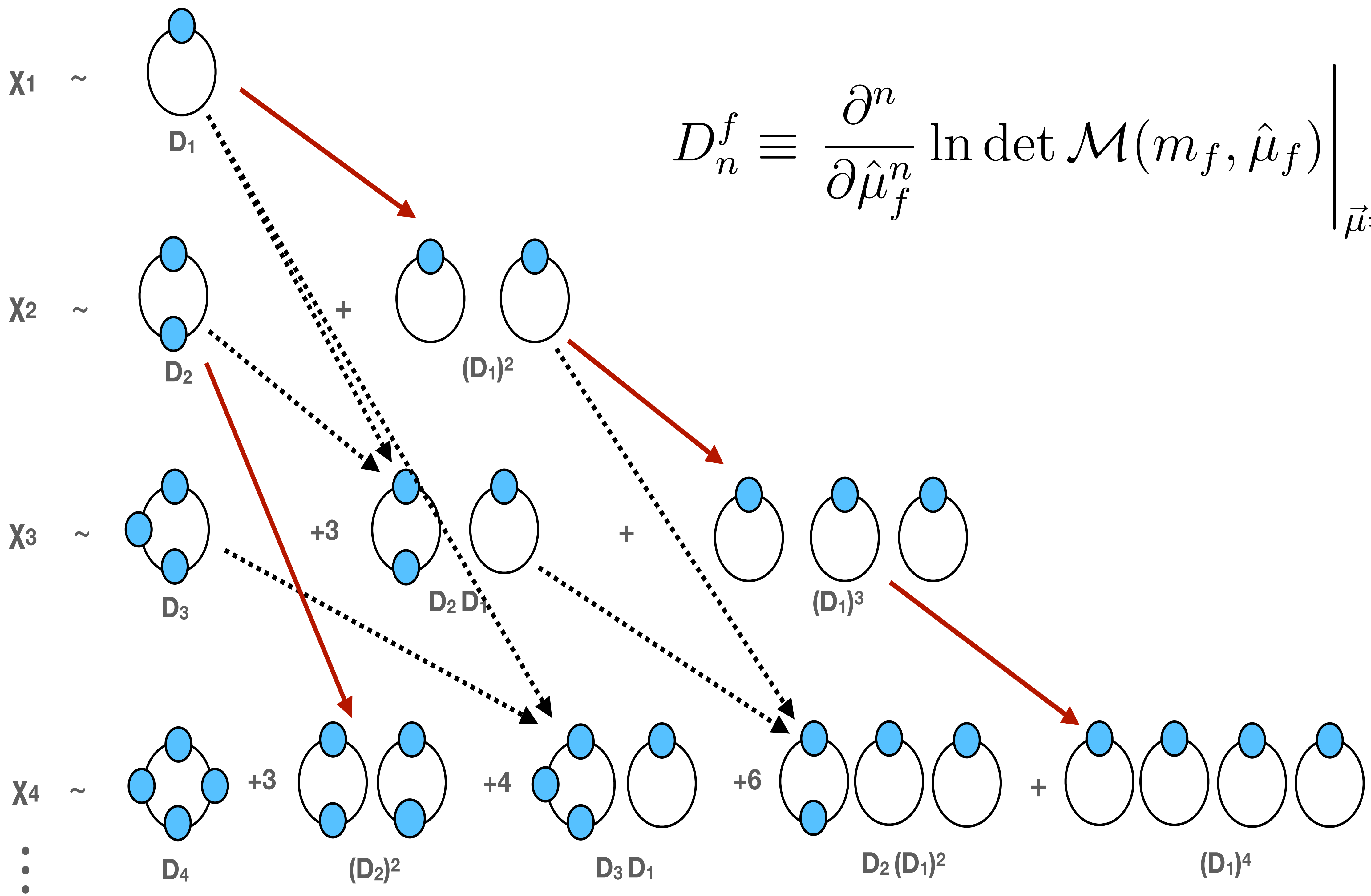
$$D_1^f = \mathrm{Tr}\left[D(m_f)^{-1} \frac{dD}{d\mu_f} \right] - \mathrm{Tr}\left[D(m_{PV})^{-1} \frac{dD}{d\mu_f} \right]$$

$$= \frac{1}{N_n} \sum_{j=1}^{N_n} \left[\eta_j^\dagger D(m_f)^{-1} \frac{dD}{d\mu_f} \eta_j - \eta_j^\dagger D(m_{PV})^{-1} \frac{dD}{d\mu_f} \eta_j \right]$$

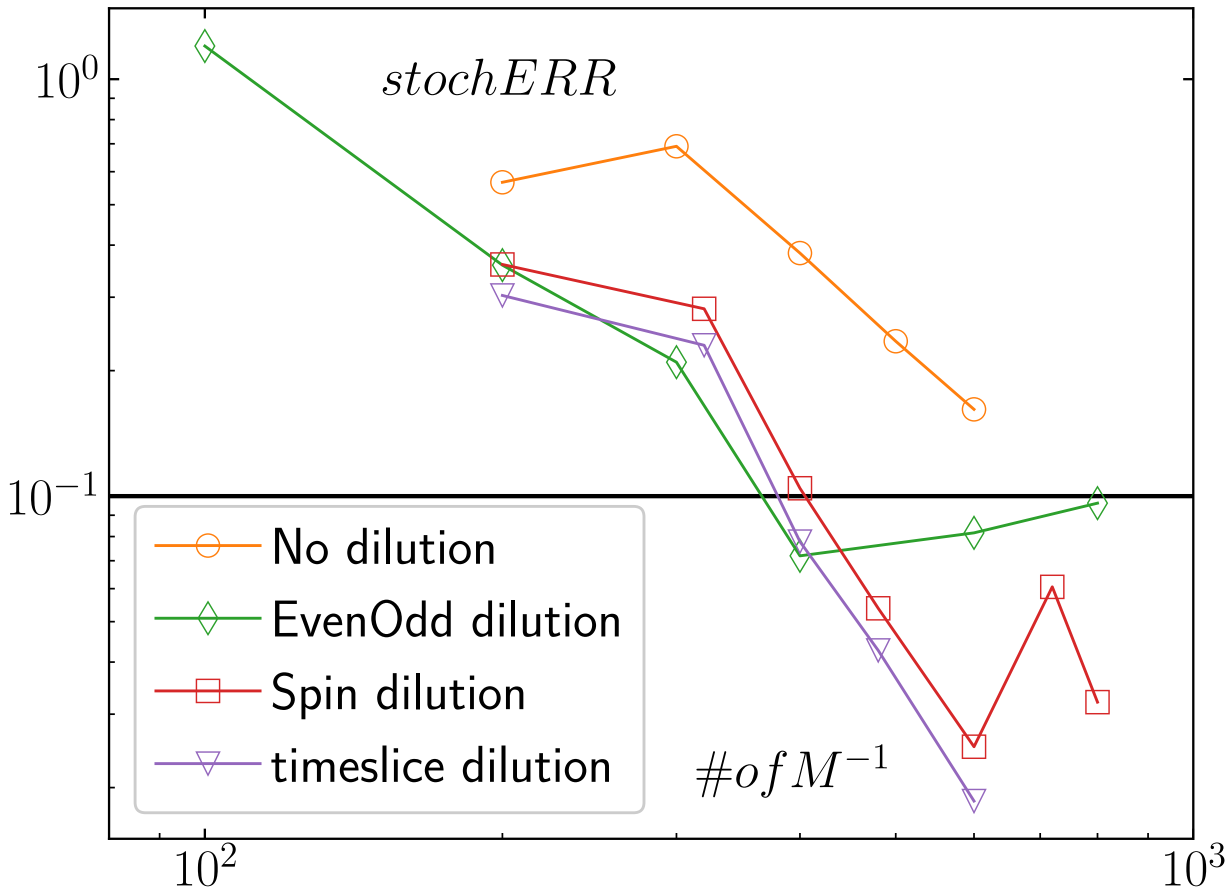
η_j is a Gaussian random number source, N_n is the total number of random noise.

Quark number susceptibilities for Domain Wall fermions

$$D_n^f \equiv \frac{\partial^n}{\partial \hat{\mu}_f^n} \ln \det \mathcal{M}(m_f, \hat{\mu}_f) \Big|_{\vec{\mu}=0} ,$$



Stochastic error reduction



$$D_1^f = \mathrm{Tr}\left[D(m_f)^{-1}\frac{dD}{d\mu_f}\right] - \mathrm{Tr}\left[D(m_{\mathrm{PV}})^{-1}\frac{dD}{d\mu_f}\right]$$
$$\simeq \frac{1}{N_n}\sum_{j=1}^{N_n}\left[\eta_j^\dagger D(m_f)^{-1}\frac{dD}{d\mu_f}\eta_j - \eta_j^\dagger D(m_{\mathrm{PV}})^{-1}\frac{dD}{d\mu_f}\eta_j\right]$$

Diluted noise vector,

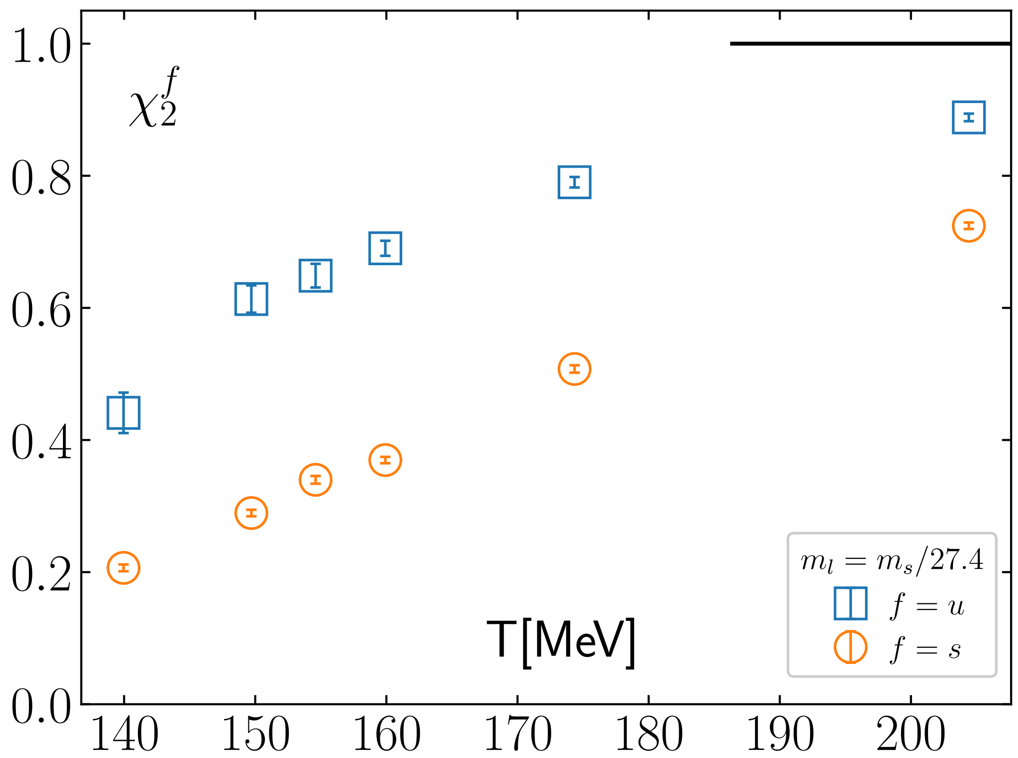
$$D_1^f \simeq \frac{1}{N_n}\sum_{j=1}^{N_n}\left[\sum_{a=1}^{N}\eta_{aj}^\dagger D(m_f)^{-1}\frac{dD(m_f)}{d\mu_f}\eta_{aj} - \sum_{a=1}^{N}\eta_{aj}^\dagger D(m_{\mathrm{PV}})^{-1}\frac{dD(m_{\mathrm{PV}})}{d\mu_f}\eta_{aj}\right]$$

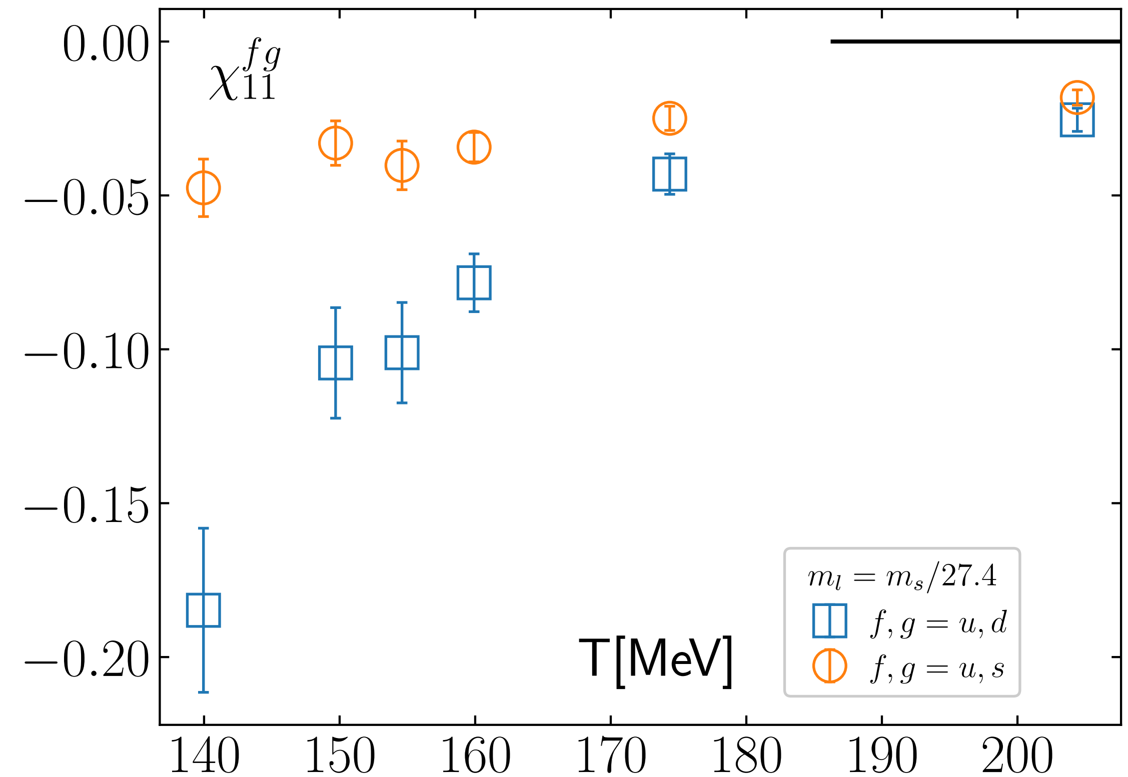
We see 2-3 times error reduction using Spin and time slice dilution.

Lattice setup and results

- m_{res} and tuning of the input quark masses.
- Quark number susceptibilities and conserved charge fluctuations for $m_l = 0.0036 m_s (m_\pi \sim 135 \text{ MeV})$ for $36^3 \times 12$.
- Sensitivity of the fluctuations on the pion masses.
- Fourth-order conserved charge fluctuations for physical quark masses.

Quark number susceptibility with Möbius Domain Wall Fermions in (2+1)-flavor QCD





χ_2^f's rise rapidly in the vicinity of the T_{pc}.

At high T: χ_2^f's are smaller than the Ideal gas limit.

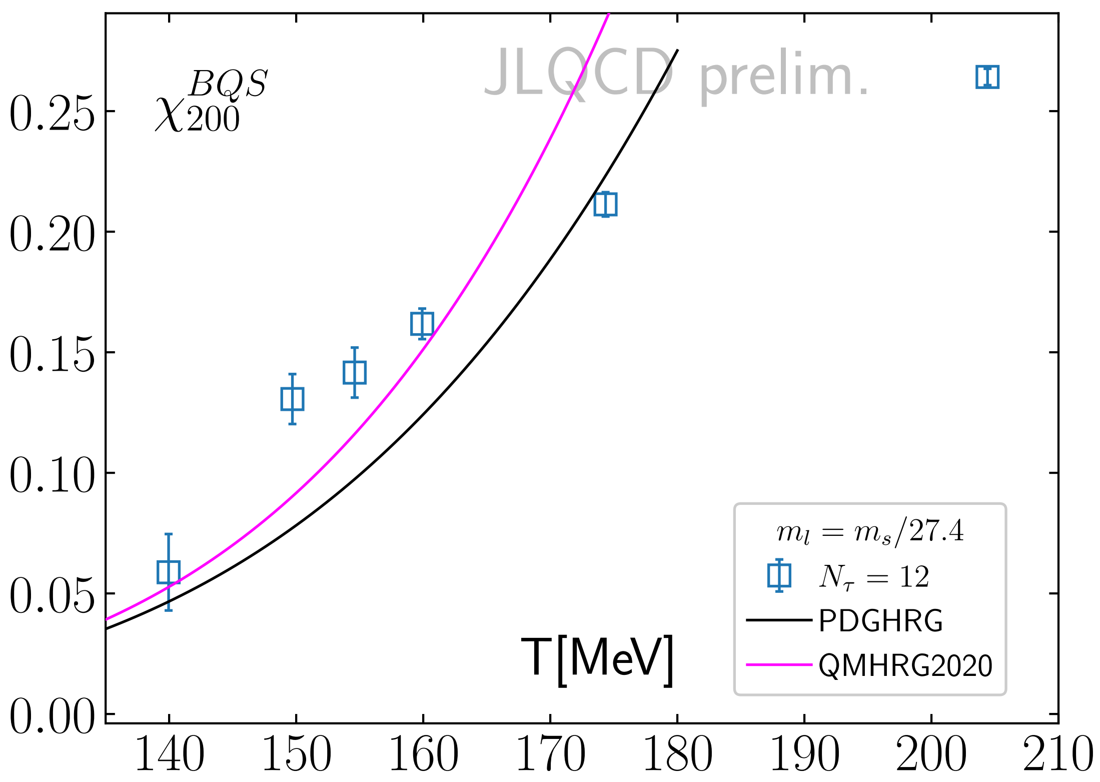
χ_{11}^{fg} reaches closer to Ideal gas limit.

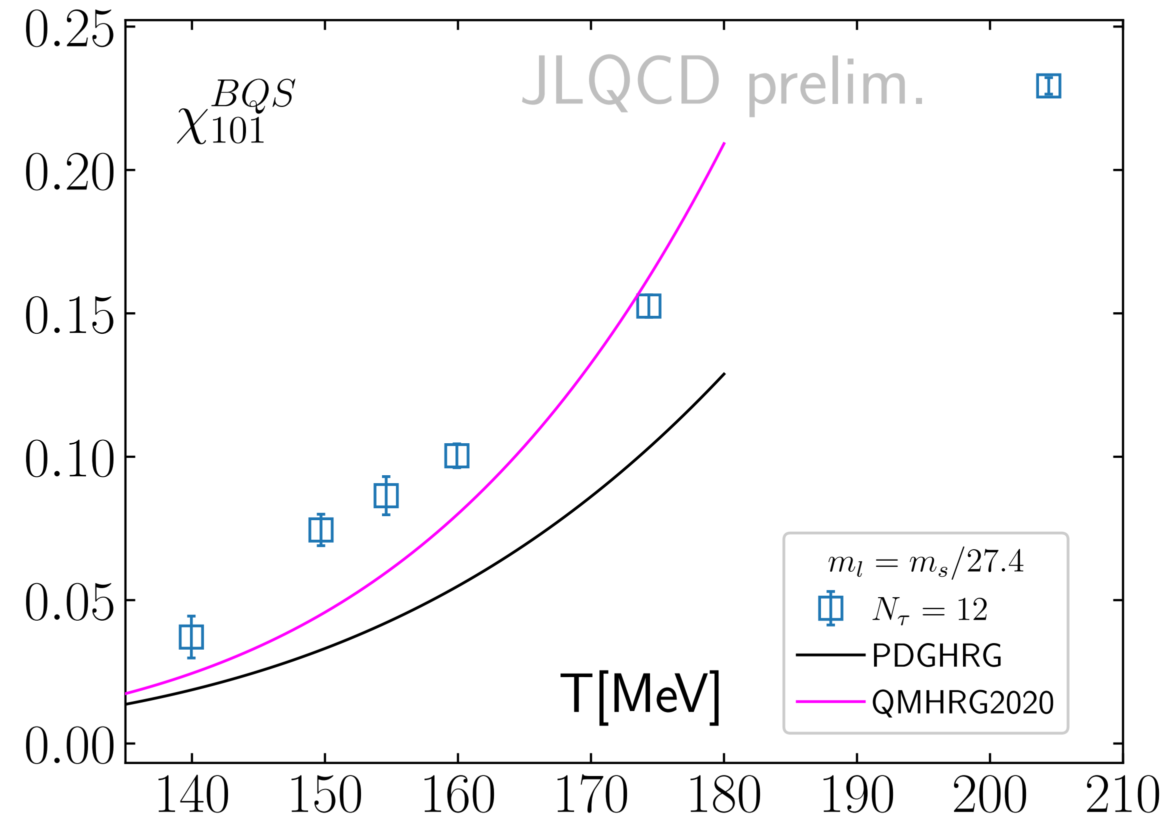
In high T PT : $\chi_2^f \sim \chi_2^{f,ideal} + O(g^2)$, $\chi_{11}^{fg} \sim O(g^6 lng)$ **A. Vuorinen, PRD68, 054017 (2003)**

Second order conserved charge calculations with Möbius Domain Wall Fermions

$$\chi_2^B = \frac{1}{9}(2\chi_2^u + \chi_2^s + 2\chi_{11}^{ud} + 4\chi_{11}^{us})$$

$$\chi_{11}^{BS} = -\frac{1}{3}\left(\chi_2^s + 2\chi_{11}^{us}\right)$$

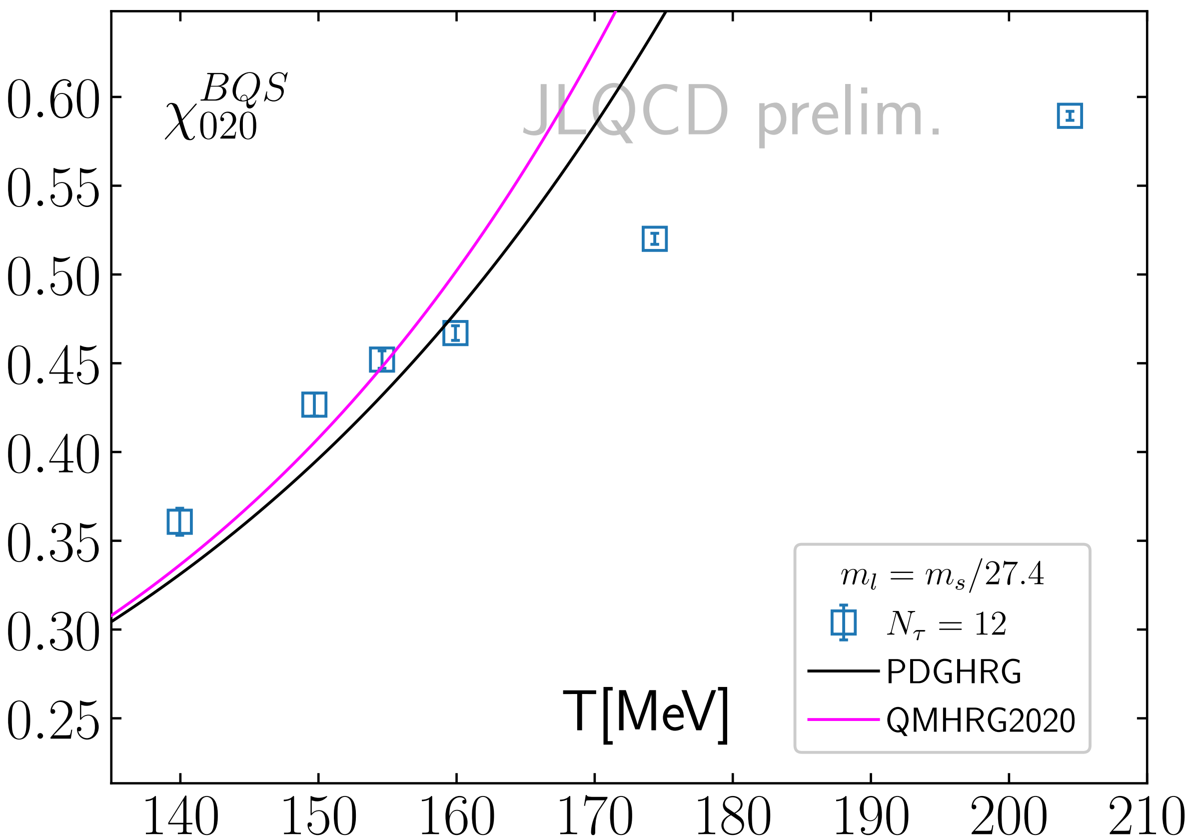


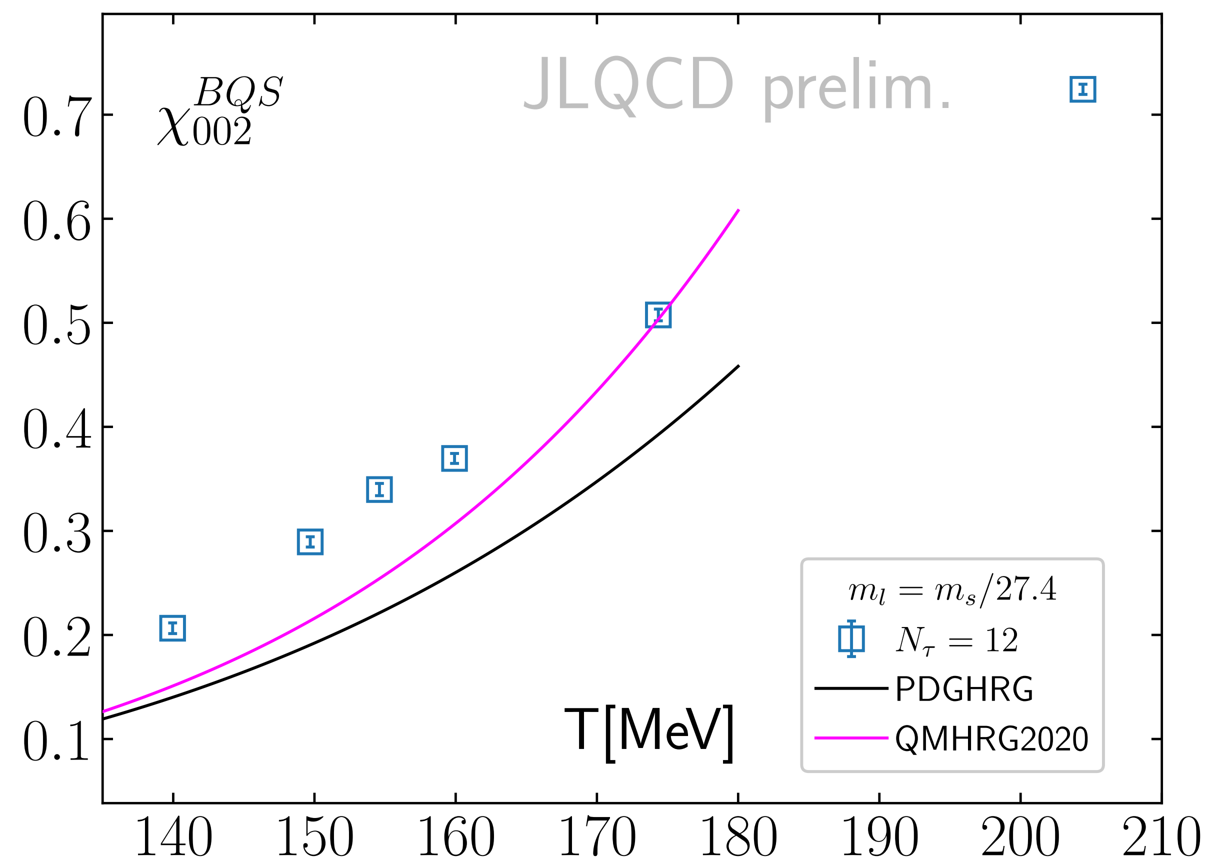


Second order conserved charge calculations with Möbius Domain Wall Fermions

$$\chi_2^Q = \frac{1}{9}\left(5\chi_2^u + \chi_2^s - 4\chi_{11}^{ud} - 2\chi_{11}^{us}\right)$$

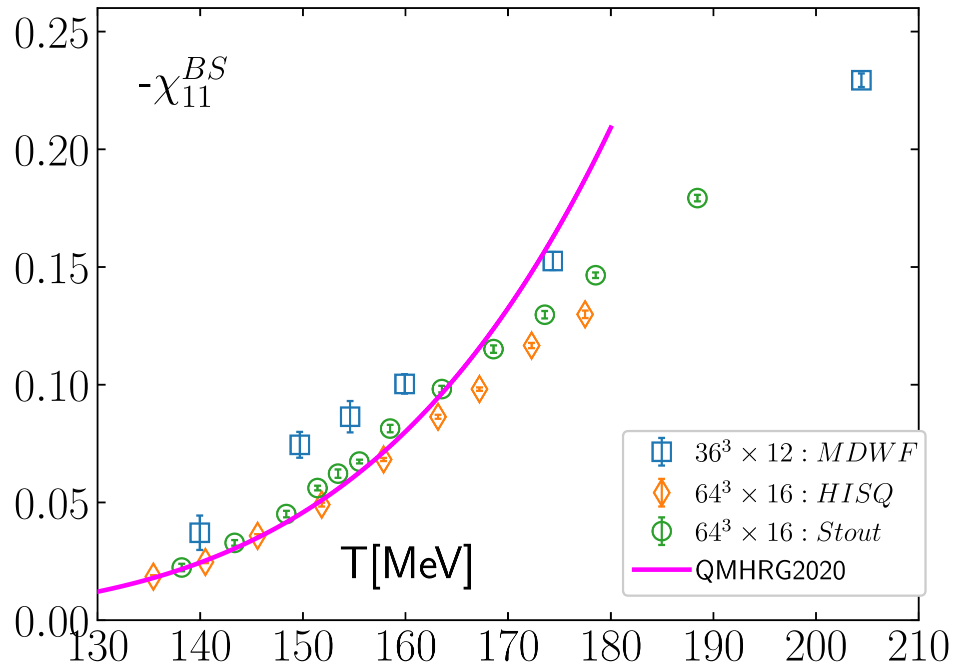
$$\chi_2^S = \chi_2^s$$

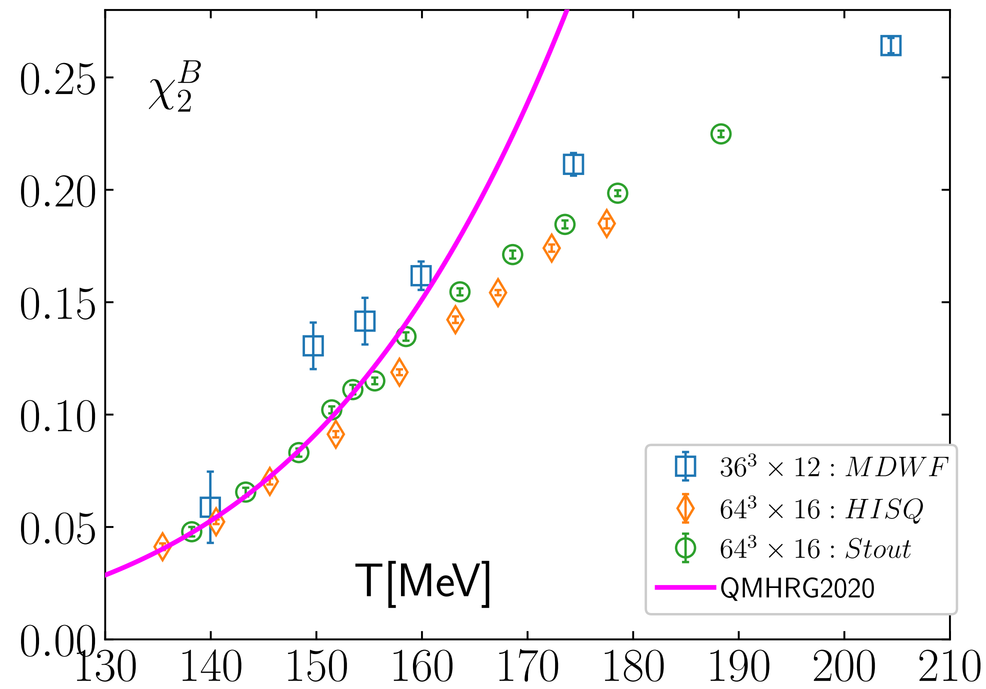




In lattice QCD, $m_u = m_d$, some of the disconnected contributions get cancelled for χ_2^Q

Comparison of χ_2^B calculations with Möbius Domain Wall Fermions and Staggered fermions

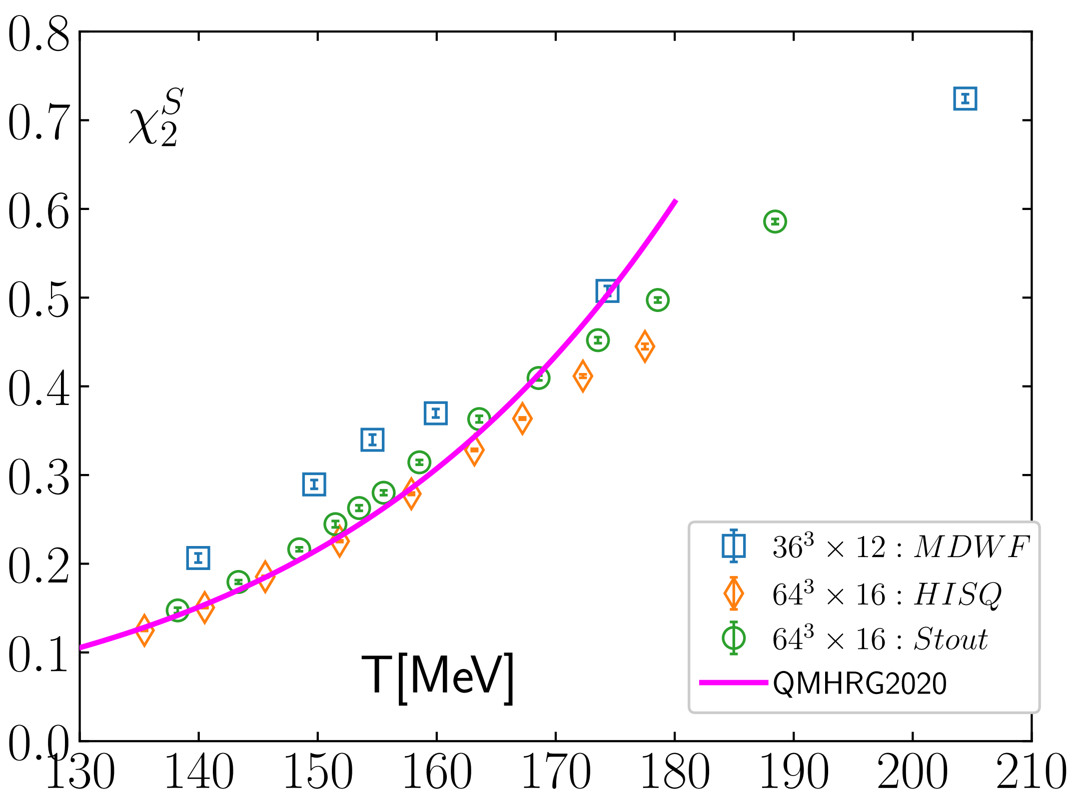


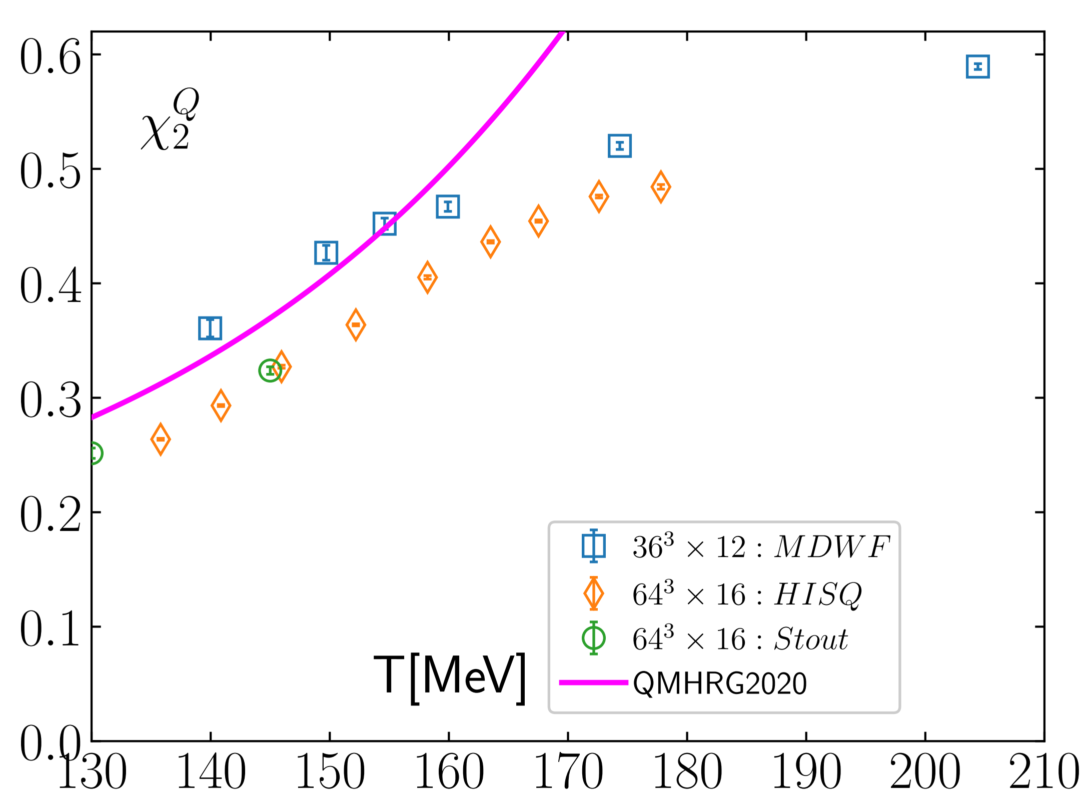


- **Data Comparison**: Our lattice data are systematically higher than those from HISQ and stout smeared staggered quarks near the pseudo-critical temperature.
- **Measurements**: Performed on 150 gauge configurations per temperature, with 100 trajectory separations.
- **Further Analysis**: Additional lattice spacing and additional volume is required to better understand this discrepancy.

Refs: HiSQ : (HotQCD) D. Bollweg et al, arXiv:2107.10011 [hep-lat].
Stout : (WB) R. Bellwied et al, arXiv:1910.14592 [hep-lat]

Comparison of χ_2^Q, χ_2^S calculations with Möbius Domain Wall Fermions and Staggered fermions

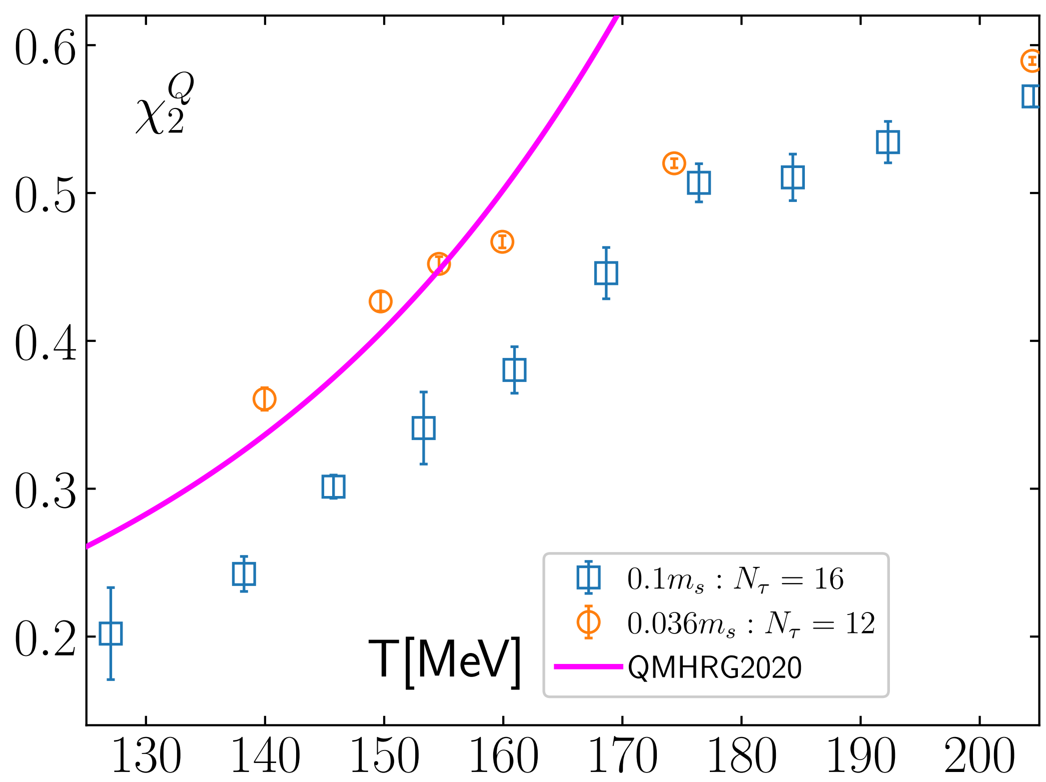


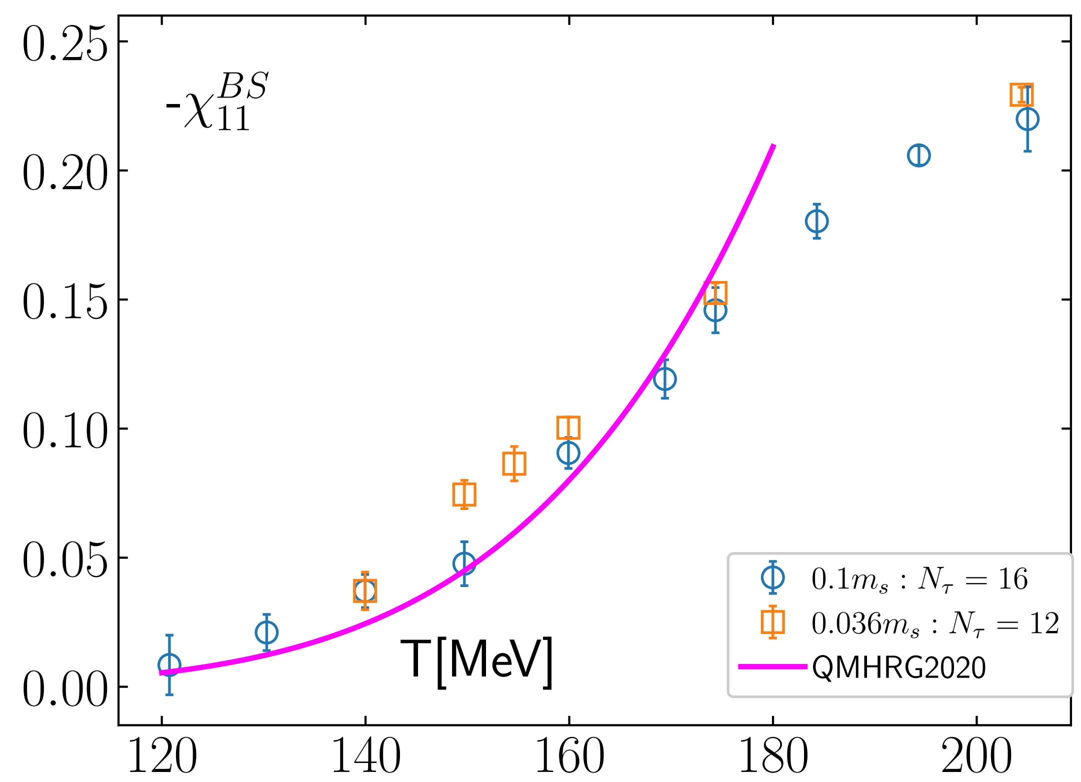


- We saw larger value in the χ_2^Q in the hadronic phase, compared to the HISQ and stout smeared staggered quarks calculations at finite lattice spacing.
- In a non-interacting HRG : χ_2^Q is dominated by pions and χ_2^S is dominated by kaons.

Refs: HISQ : D. Bollweg et al, arXiv:2107.10011 [hep-lat].
Stout : R. Bellwied et al, arXiv:1507.04627 [hep-lat]

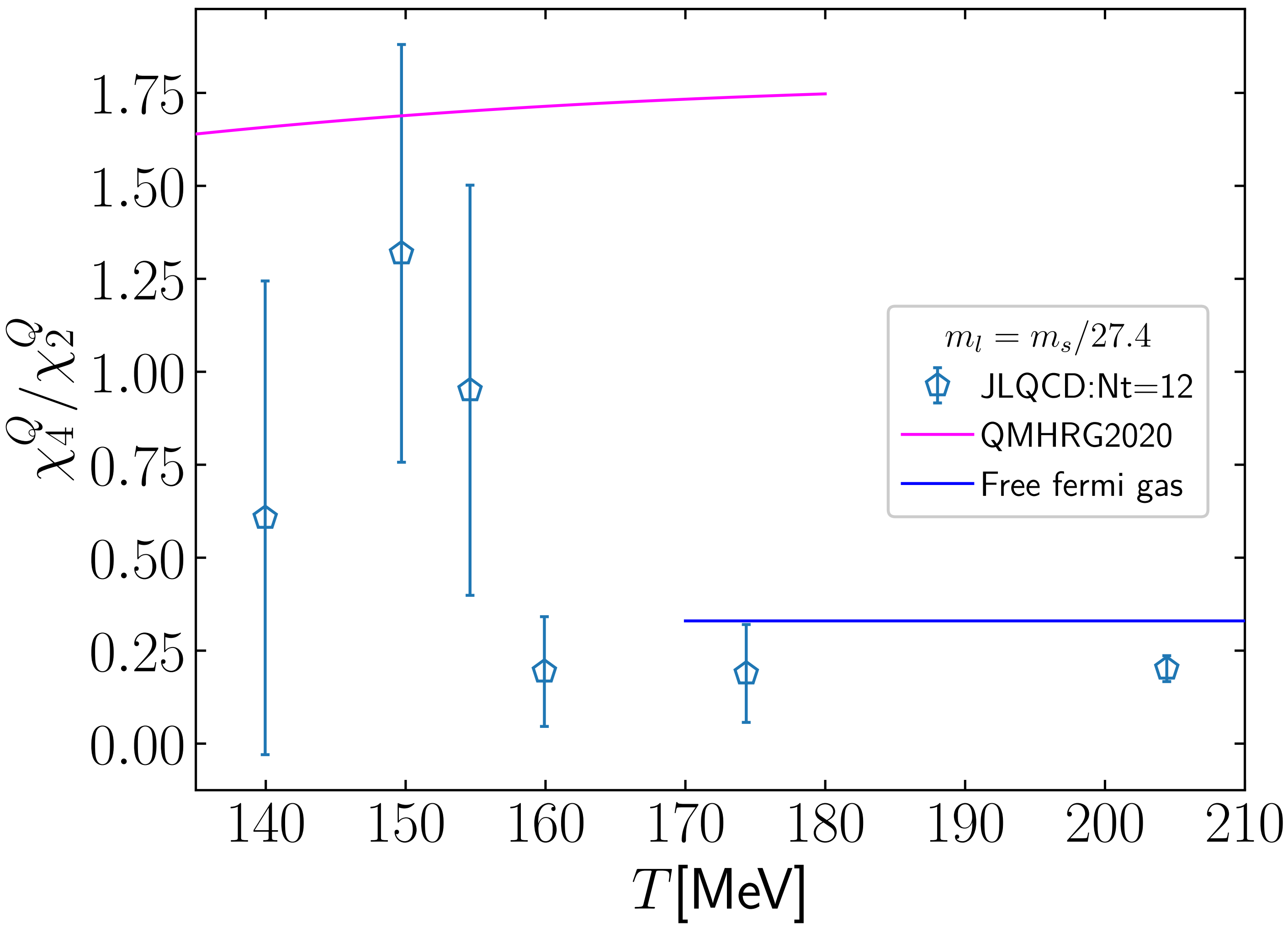
Sensitivity of χ_{11}^{BS}, χ_2^Q on different light quark masses





- In hadronic phase , $\chi_2^X \sim \exp(-m_H/T)$
- In a non-interacting HRG : χ_2^Q is dominated by pions and χ_{11}^{BS} is dominated by lambda baryons.
- We see that χ_2^Q is sensitive to the pion mass in the temperature, $T_{pc} \leq 160$ MeV however χ_{11}^{BS} is not that sensitive to the hadron masses.

Leading order kurtosis of electric charge cumulants



$\overrightarrow{\mu} = \{\mu_B, \mu_Q, \mu_S\}$

$R_{42}^Q = \chi_4^Q/\chi_2^Q + O(\overrightarrow{\mu}^2)$

Leading order kurtosis value close to the Pseudo-critical temperature,

- $R_{42}^Q = 1.3(5)$, $T = 150$ MeV
- $R_{42}^Q = 0.9(5)$, $T = 155$ MeV

Summary and Conclusions

- We present results of conserved charge fluctuations using (2+1)-flavor QCD with a chiral fermion formalism, specifically Möbius Domain Wall Fermions.
- We compare our calculations of second order fluctuations with the staggered fermion formalism calculations at finite lattice spacing.
- We also present fourth order conserved charge fluctuations for the physical value of the quark masses.
- In future, we will extend our calculations to smaller lattice spacings to study the cut-off dependence of conserved charge fluctuations.

Thank you for your attention !!

Quark number susceptibility with Domain wall fermions

$\hat{\mu}_f = \mu_f/T$, where , μ_f is the quark chemical potential for flavor f.

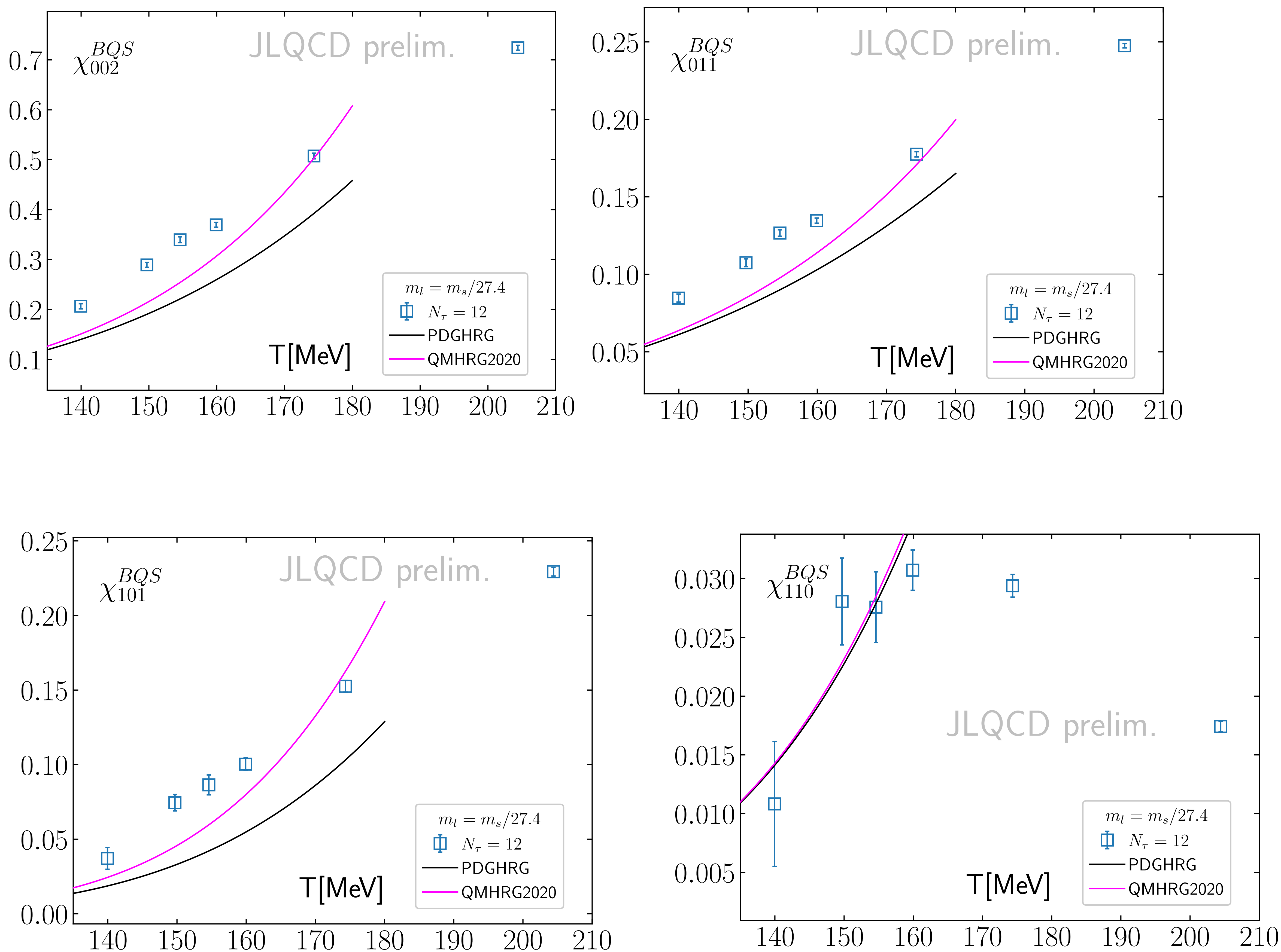
The diagonal and off-diagonal quark number susceptibilities can be written as,

$$\chi_2^f = \frac{N_\tau}{N_\sigma^3}\frac{\partial^2 \ln Z}{\partial \hat{\mu}_f^2}\bigg|_{\hat{\mu}_f=0} = \frac{N_\tau}{N_\sigma^3}\left[\left\langle \frac{\partial^2}{\partial \hat{\mu}_f^2}\ln\det M\right\rangle + \left\langle\left(\frac{\partial}{\partial \hat{\mu}_f}\ln\det M\right)^2\right\rangle\right]$$

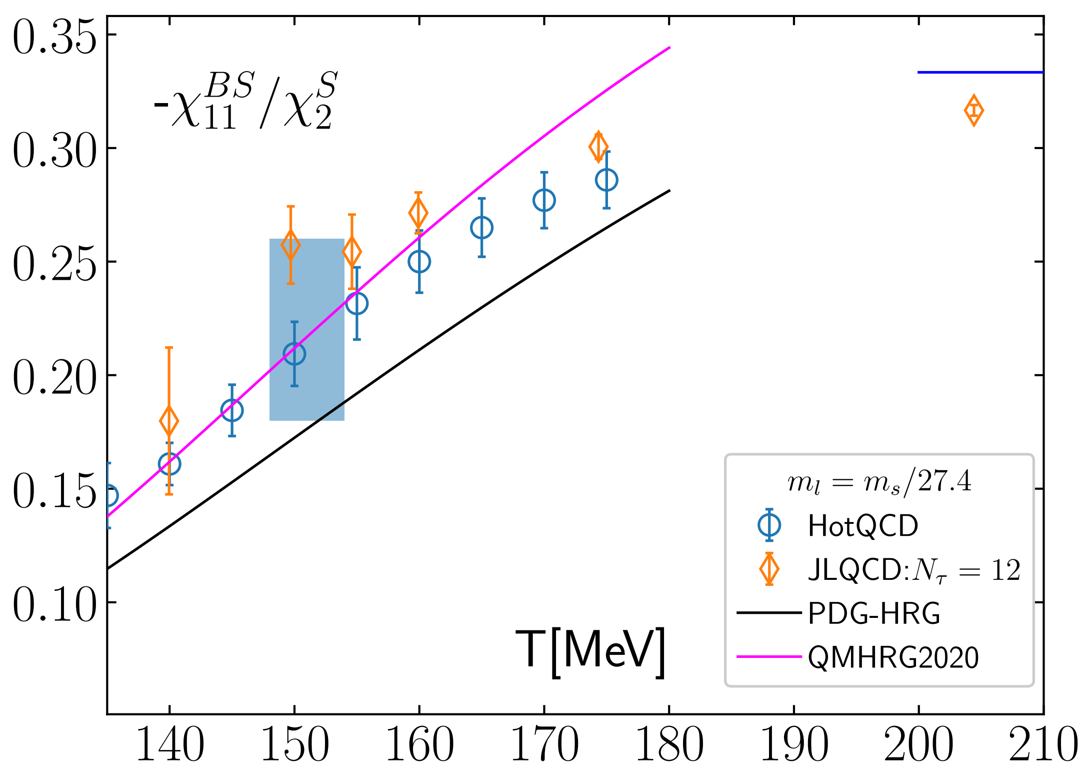
$$= \frac{N_\tau}{N_\sigma^3}\langle D_2^f\rangle + \langle (D_1^f)^2\rangle,\ f = \{u, d, s\}$$

$$\chi_{11}^{fg} = \frac{N_\tau}{N_\sigma^3}\frac{\partial^2 \ln Z}{\partial \hat{\mu}_f \partial \hat{\mu}_g}\bigg|_{\hat{\mu}_f=0} = \frac{N_\tau}{N_\sigma^3}\langle D_1^f D_1^g\rangle,\ f \neq g,\ f, g = \{u, d, s\}$$

$(D_1^f)^2$ and $D_1^f D_1^g$ **are the most noisy part in our calculation**



χ_{002}^{BQS}
JLQCD prelim.
$m_l = m_s/27.4$
$N_\tau = 12$
PDGHRG
QMHRG2020
T[MeV]
χ_{011}^{BQS}
JLQCD prelim.
$m_l = m_s/27.4$
$N_\tau = 12$
PDGHRG
QMHRG2020
T[MeV]
χ_{101}^{BQS}
JLQCD prelim.
$m_l = m_s/27.4$
$N_\tau = 12$
PDGHRG
QMHRG2020
T[MeV]
χ_{110}^{BQS}
JLQCD prelim.
$m_l = m_s/27.4$
$N_\tau = 12$
PDGHRG
QMHRG2020
T[MeV]



$-\chi_{11}^{BS}/\chi_2^S$
0.35
0.30
0.25
0.20
0.15
0.10
140
150
160
170
180
190
200
210
T[MeV]
$m_l = m_s/27.4$
HotQCD
JLQCD:$N_\tau = 12$
PDG-HRG
QMHRG2020