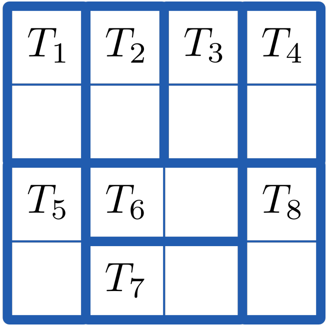
ETH zürich

ETH, Institute for Theoretical Physics
High Performance Computational Physics group

Quantum Many-Body Scars in 2+1D Gauge Theories

Joao C. Pinto Barros
Thea Budde, Marina Krstić Marinković

Lattice 2024
15th of November | Kyoto



What Happens to an Isolated Quantum System When Left Alone?

Prepare Quantum State

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Go away

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Come back

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Measure

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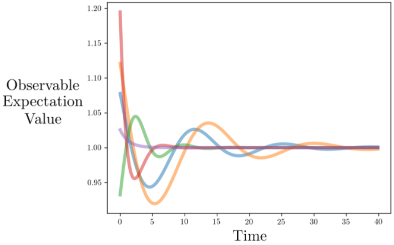
How Do Quantum Systems Thermalize?



Thermalization: observable converge to values independent of the initial details.

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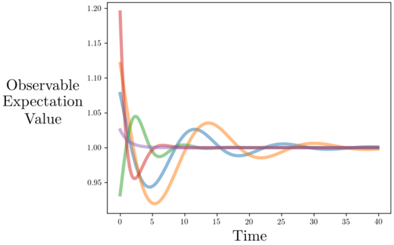

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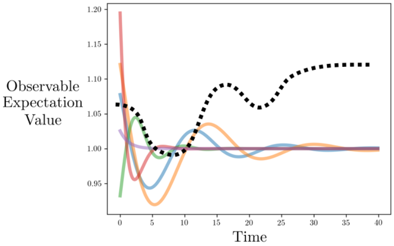

Scar: special initial conditions avoid thermalization.
(For a review: S. Moudgalya, B. A. Bernevig, N. Regnault. RPP (2022))

How Quantum Many-Body Systems Thermalize?

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Long-time dynamics in lattice gauge theories

- Fundamental question in quantum many-body physics;
- Real-time dynamics usually hard (e.g. sign problem);
- Scars challenge foundational aspects of thermalization;
- Playground for quantum simulators;

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This Talk

Pure gauge, $U(1)$, 2+1D.

Outline

1. The Eigenstate Thermalization Hypothesis
2. $U(1)$ Pure Gauge Theories
3. Low Entropy Zero-Energy States
4. Conclusions and Outlook

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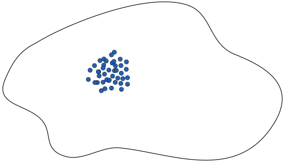
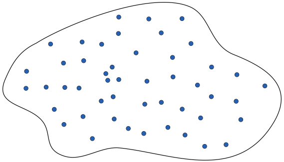
No assumption on initial states

Highly non-local observable

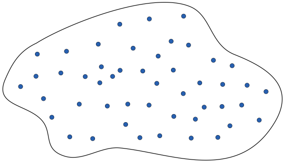
The Intuition: Large system can serve as a thermal bath for its small subsystems



Initial state (at $t = 0$)



State at $t = t_1 > 0$



State at $t = t_2 > t_1$

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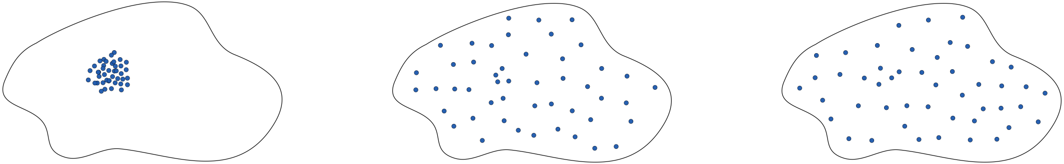
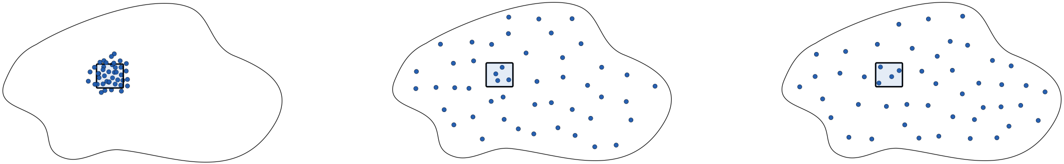


Initial state (at $t = 0$) State at $t = t_1 > 0$ State at $t = t_2 > t_1$



Real Time-Evolution of Local Observables

Initial state $\left|\psi\right\rangle = \sum_n c_n \left|\psi_n\right\rangle$

$$\left\langle\psi\left(t\right)\right| O \left|\psi\left(t\right)\right\rangle = \sum_n \left|c_n\right|^2 O_{nn} + \sum_{n\neq m} c_n c_l^* e^{i(E_l - E_n)t} O_{ln}$$

Three ingredients towards thermalization:

- O_{nn} varies smoothly with the energy;
- O_{ln} $(l \neq m)$ is very small;
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$$\left\langle\psi\left(t\right)\right| O \left|\psi\left(t\right)\right\rangle \to O(E), \quad \text{where } E = \left\langle\psi\right| H \left|\psi\right\rangle$$

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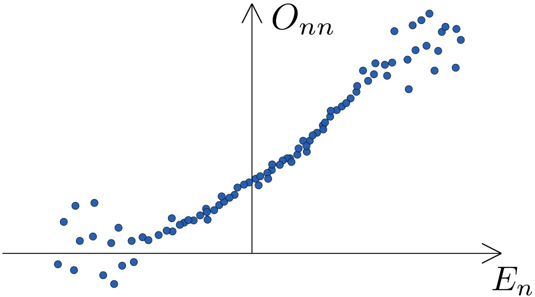
$O_{nn} = \langle E_n|\,O\,|E_n\rangle$

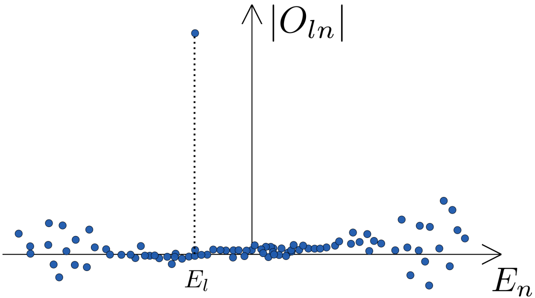


We describe this continuum function as $O_{nn} = O(E_n)$

O_{ln} $(l \neq m)$ is very small

$$\langle \psi(t)| O |\psi(t)\rangle \rightarrow \sum_n |c_n|^2 O(E_n) + \sum_{n \neq m} c_n c_l^* e^{i(E_l - E_n)t} O_{ln}$$

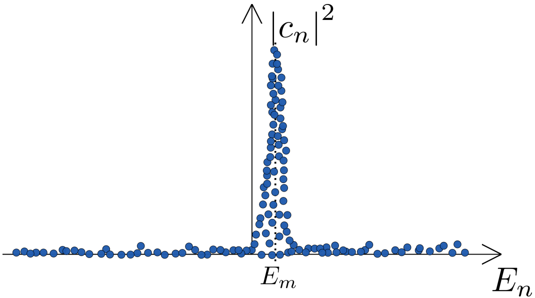
$O_{ln} = \langle E_l| O |E_n\rangle$



We can drop those terms for sufficiently large times.

$|\psi\rangle$ has high energy and has "small" variance

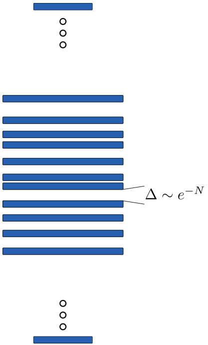
$$\langle\psi(t)|\,O\,|\psi(t)\rangle = \sum_n |c_n|^2\, O(E_n)$$



We can approximate the sum by the average $\langle\psi(t)|\,O\,|\psi(t)\rangle \rightarrow O(E_m)$

Motivating the Eigenstate Thermalization Hypothesis

What happens to the spectrum of a Hamiltonian under small changes of parameters $H \to H + \delta H$?



Mid-spectrum states have
exponential small gaps

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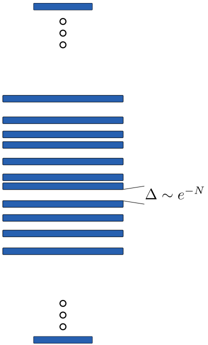

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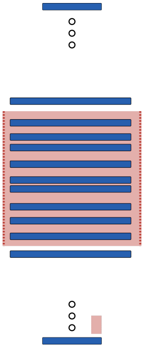

Small changes mix many of these states

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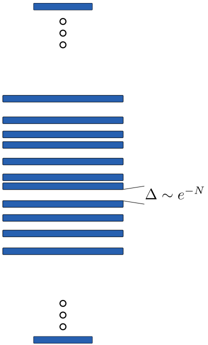

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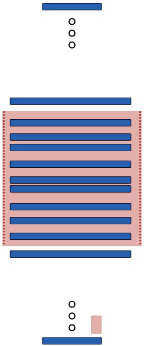

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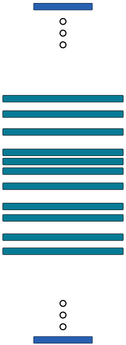
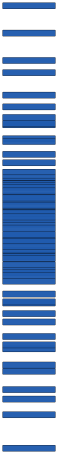
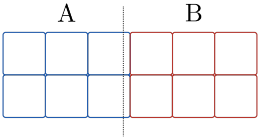


Highly entangled seemingly "random" mid-spectrum states.

Entanglement Entropy Across the Spectrum

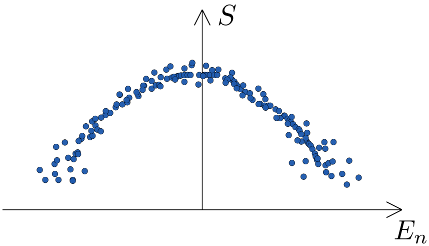


Entanglement Entropy



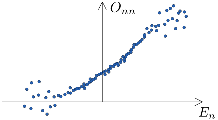
Trace out subsystem B: $\rho_A = \mathrm{tr}_B \rho$

Compute entropy of entanglement: $S = -\mathrm{tr}\rho_A \log \rho_A$

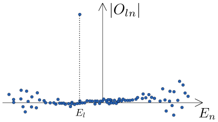


Quick Summary of Expectations

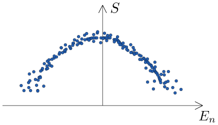
Local operators have continuous diagonal matrix elements



Local operators are ineffective in transforming one eigenstate into another



Mid-spectrum states have small gaps and are highly entangled.



The Eigenstate Thermalization Hypothesis

$$O_{mn} = \underbrace{O\left(E_m\right)}_{\text{continuous}} \delta_{mn} + \underbrace{e^{-\frac{1}{2}S\left(\frac{E_m+E_n}{2}\right)}}_{\text{exponentially small}} \underbrace{f_O\left(E_m - E_n, E_m + E_n\right)}_{\text{continuous}} \underbrace{R_{mn}}_{\text{random}}$$

$$O\left(E_m\right) = O\left(\langle\psi(0)|\, H \,|\psi(0)\rangle\right) = \frac{1}{\operatorname{tr}\left(e^{-\beta H}\right)} \operatorname{tr}\left(O e^{-\beta H}\right)$$

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Strong breaking of ETH:

- Integrable models;
- Disordered systems.

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NEXT: Weak breaking of ETH with Quantum Many-Body Scars

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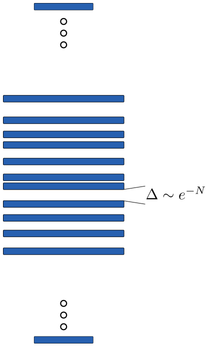

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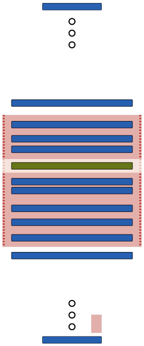
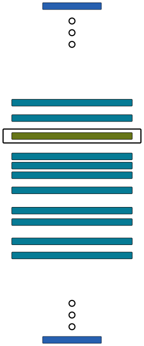


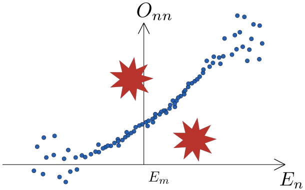
Special scar state with low entropy



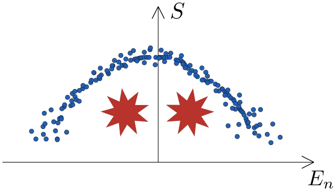
Scar is not significantly mixing with other states

Signatures of Quantum Many-Body Scars

Scarred systems violate this picture for a *few* eigenstates



Some observables are no longer continuous



There are exceptionally low entanglement entropy

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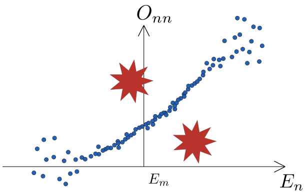

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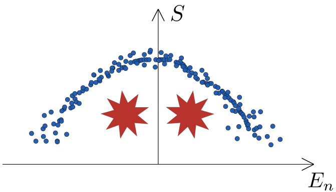

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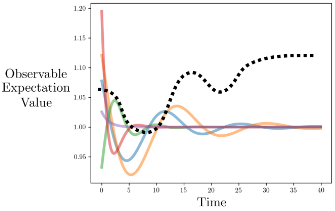

Quantum Many-Body Scars and Gauge Theories

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  H. Bernien et al. Nature (2017)

- PXP maps exactly to a $U(1)$ gauge theory in 1+1D;

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- See also the non-Abelian case;

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HERE: Quantum Many-Body Scars for arbitrary truncation in 2+1D pure gauge theories

MAIN FOCUS: T. Budde, M. K. Marinkovic, JPB; PRD 110 (2024) 9, 094506, arXiv:2403.08892

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2. $U(1)$ Pure Gauge Theories

3. Low Entropy Zero-Energy States

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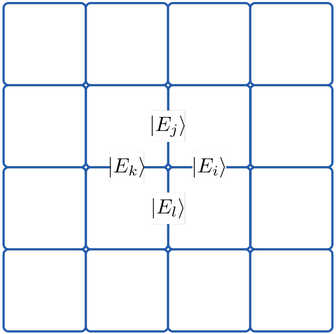
Hamiltonian for $U(1)$ Pure Gauge Theories

$$H = -t \sum_n \underbrace{U^\dagger_{n1} U^\dagger_{n+\hat{1}2} U_{n2} U_{n+\hat{2}1}}_{U_\square} + \text{h.c.} + \kappa \sum_n E_n^2$$



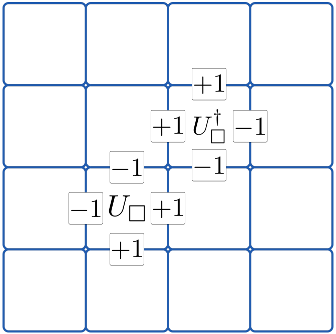
$E_n \in \mathbb{Z}$

U_n unitary raising operator

$U_i |E_i\rangle = |E_i + 1\rangle$

Gauss' Law

$E_i - E_k + E_j - E_l = 0$



Symmetries

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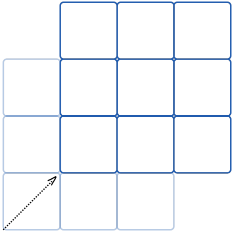
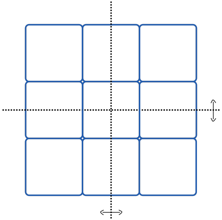
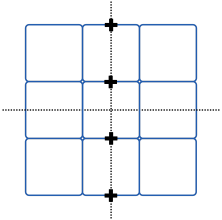
Translations



Reflections

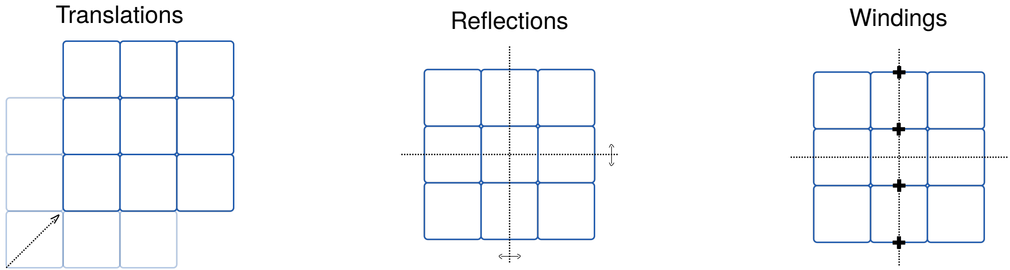


Windings



Symmetries

$$H = -t \sum_n \underbrace{U^\dagger_{n1} U^\dagger_{n+\hat{1}2} U_{n2} U_{n+\hat{2}1}}_{U_\square} + \text{h.c.} + \kappa \sum_n E_n^2$$

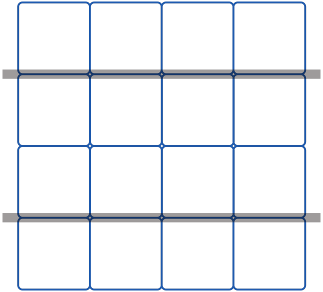


Winding symmetry: $\left[H, \sum_a \sum E_{n+a\hat{2}1}\right] = 0.$

Most of this talk: zero winding sector $\sum_a E_{n+a\hat{2}1} = 0, \quad \sum_a E_{n+a\hat{1}2} = 0$

Spectral Symmetry for $\kappa = 0$

$$H = -t \sum_n \underbrace{U^\dagger_{n1} U^\dagger_{n+\hat{1}2} U_{n2} U_{n+\hat{2}1}}_{U_\square} + \text{h.c.} + \kappa \sum_n E_n^2$$



Define

$$\zeta \left|\varepsilon\right\rangle = (-1)^\varepsilon \left|\varepsilon\right\rangle$$

$\mathcal{C} = \prod_{a=0}^{L_1-1} \prod_{n=0}^{L_2/2-1} \zeta_{(n,2a)1}$ then $\{\mathcal{C}, H_\square\} = 0$.

Then

$$\mathcal{C} \left|E\right\rangle \to \left|-E\right\rangle$$

An Index Theorem

$$H = -t \sum_n \underbrace{U^\dagger_{n1} U^\dagger_{n+\hat{1}2} U_{n2} U_{n+\hat{2}1}}_{U_\square} + \text{h.c.} + \kappa \sum_n E_n^2$$

There are an exponentially large number of zero-energy states

- Consider $\mathrm{tr}\,(R_x \mathcal{C})$ (R_x reflection with respect to x axis);
- Only zero modes contribute;
- Grows exponentially with the volume.

M. Schecter and T. Iadecola PRB (2018)

T. Budde, M. Marinkovic, **JPB** - arXiv:2403.08892

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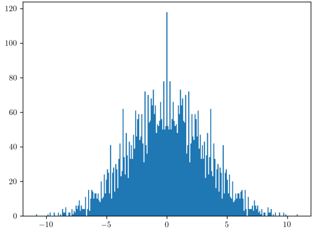
Density of States



An exponential number of zero modes

Can we use the exponential number of zero-energy states to build low-entropy states?

Non-Integrability and Level Statistics

Integrable systems leave their imprint on spectrum statistics. **Is this model integrable?**

- Resolve symmetries of the model;
- Compute the spectrum for one sector $\{E_m\}_m$;
- Compute consecutive level ratios $r_n = \min\left\{\frac{E_n - E_{n-1}}{E_{n+1} - E_n}\right\}$;
- Compute the distribution $p\left(r\right)$ with $r \in [0, 1]$.

Integrable: $p(r) = \frac{2}{(1+r)^2}$

Non-Integrable: $p(r) = \frac{27}{4}\frac{r+r^2}{(1+r+r^2)^{5/2}}$

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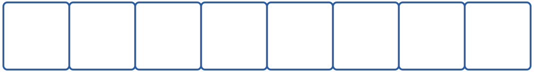
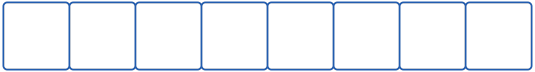
We will analyze this in detail for a ladder system



Non-Integrability of the Ladder - Numerical Results

$$H = \sum_n \left(U_\square + U_\square^\dagger \right) + \lambda \sum_{l \text{ top row}} E_l$$



- Break translations using open boundaries;
- Include an electric field term $\lambda \sum_{l \text{ in top row}} E_l$ breaks vertical reflections;
- Resolve horizontal reflection.

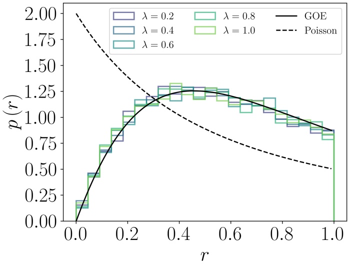
Non-Integrability of the Ladder - Numerical Results

Address integrability:

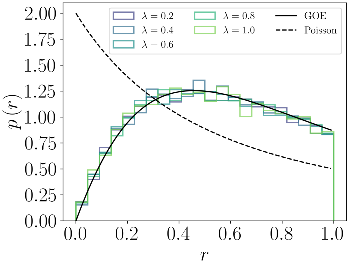
- Resolve symmetries;
- Compute level space distribution.

$$p(r)\,,\ r_n = \min\left\{\frac{E_{n+1}-E_n}{E_n-E_{n-1}}, \frac{E_n-E_{n-1}}{E_{n+1}-E_n}\right\}$$

$-$ Parity Symmetry Sector



$+$ Parity Symmetry Sector



Integrable systems: expected Poisson

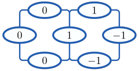
Non-integrable: expected Gaussian Orthogonal Ensemble (GOE)

Outline

Spin-1 QLM: Two Plaquettes Zero Mode

We construct a two-plaquette state (7 spins)

$$|\mathrm{Blue}\rangle = \frac{1}{\sqrt{2}}$$



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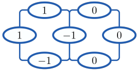

ZERO-ENERGY

$$\sum_n \left(U^\dagger_{n1} U^\dagger_{n+\hat{1}2} U_{n2} U_{n+\hat{2}1} + \mathrm{h.c.}\right) |\mathrm{Blue}\rangle = 0$$

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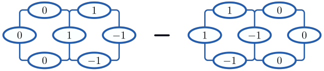
$$|\mathrm{Blue}\rangle = \frac{1}{\sqrt{2}}$$ 

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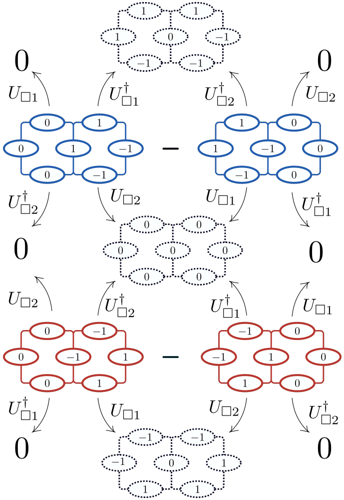
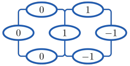
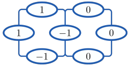
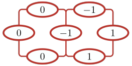


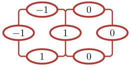
Spin-1 QLM: Two Plaquettes Zero Mode

Two-plaquette states

$|\mathrm{Blue}\rangle = \frac{1}{\sqrt{2}}$ 

- 

$|\mathrm{Red}\rangle = \frac{1}{\sqrt{2}}$ 

− 

$$\sum_n \left(U_\square + U_\square^\dagger\right)|\mathrm{Blue}\rangle = \sum_n \left(U_\square + U_\square^\dagger\right)|\mathrm{Red}\rangle = 0$$

Spin-1 QLM: Two Plaquettes Zero Mode

Two-plaquette states

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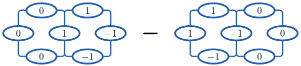

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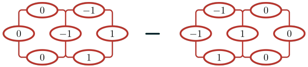

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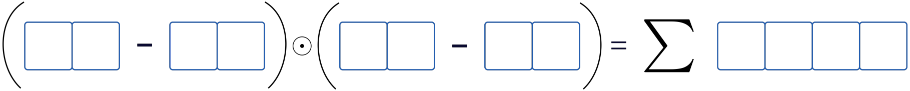
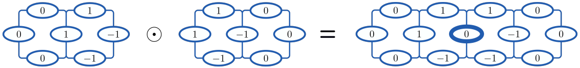
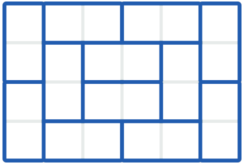
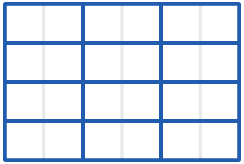
Larger Volumes: tiling





An Interlude on Tatami





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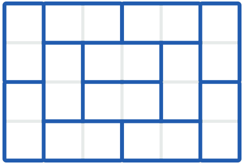
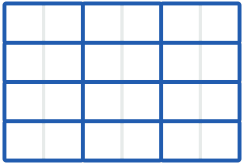


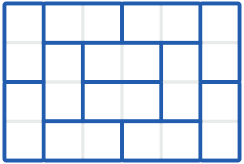
Auspicious



Inauspicious

Determining whether a large room has an auspicious arrangement using only full mats is NP-complete.

An Interlude on Tatami



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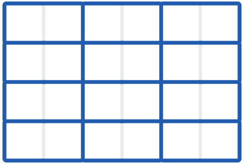


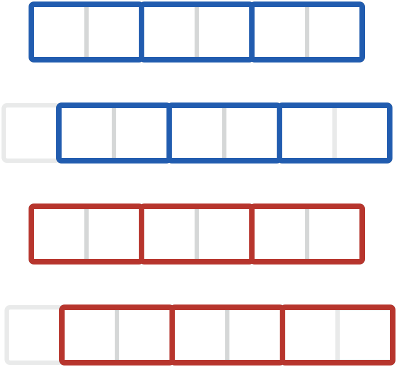
Inauspicious

Determining whether a large room has an auspicious arrangement using only full mats is NP-complete.

This is a NP-complete problem we don't have to worry about here!

The Periodic Ladder

$$H = \sum_n \left(U_\square + U_\square^\dagger \right) + \lambda \sum_{l \text{ top row}} E_l$$



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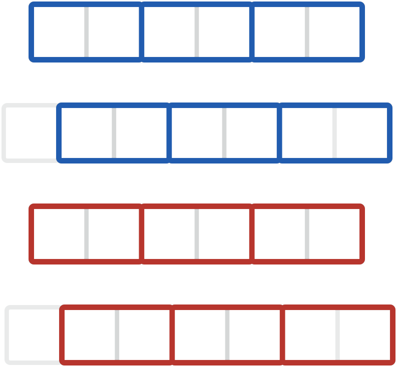
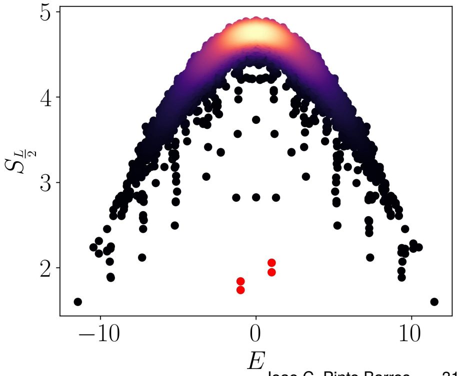


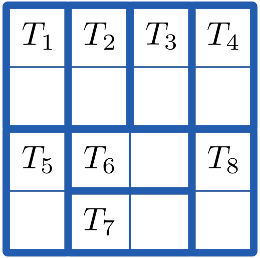
Bipartite Entanglement Entropy $\lambda = 0.2$



Scars for Arbitrary Volumes and Truncations

$$|\psi_s^{(i,T)}\rangle = \frac{1}{(S+1)^{|T|/2}} \prod_{(n,n')\in T} \left(\sum_{k=0}^{S} (-1)^k (U_{\Box n})^{i-S+k} (U_{\Box n'})^{i-k} \right) |\mathbf{0}\rangle$$

$|\mathbf{0}\rangle \equiv$ State where all links are zero



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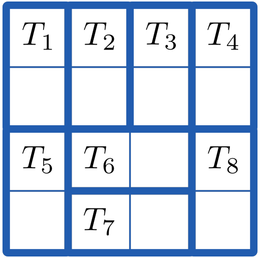
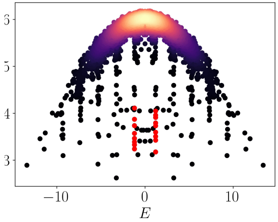
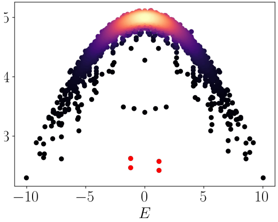


Entanglement entropy for $S = 1$ and 6×2 volume



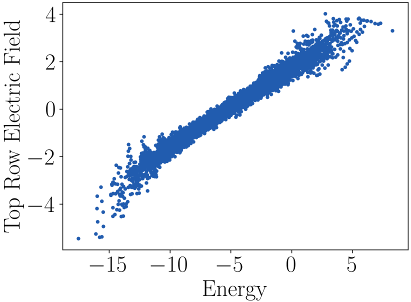
Entanglement entropy for a $S = 2$ ladder



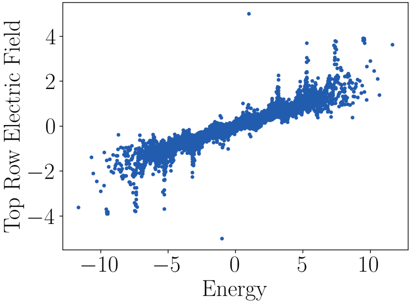
For details and other types of scars see T. Budde, M. Marinkovic, **JPB** - arXiv:2403.08892

Effect of Scars in Observables

Non-scarred Ladder



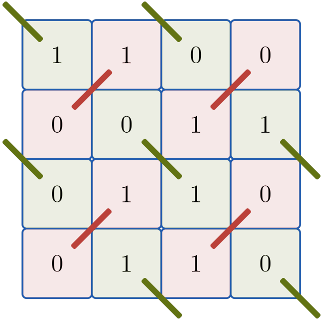
Scarred Ladder



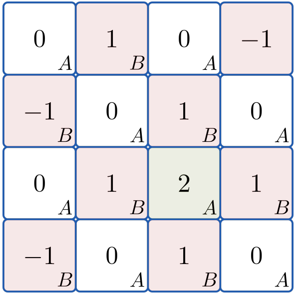
The expectation value of the magnetization of the top row is not continuous with respect to the energy

Scars with the E^2 Term ($S = 1$)

$$H = \sum_n \left(U_\square + U_\square^\dagger\right) + \kappa \sum_l E_l^2$$



Half of the links are 0 and the other half ± 1



All links are ± 1

Outline

1. The Eigenstate Thermalization Hypothesis

2. $U(1)$ Pure Gauge Theories

3. Low Entropy Zero-Energy States

4. Conclusions and Outlook

Are These States Relevant?

NO

For ground state or finite temperature physics

Are These States Relevant?

NO

For ground state or finite temperature physics

YES

For long-time dynamics starting from physically relevant initial states

- Scar states are ground-states of different local Hamiltonians;
- State preparation does not need to be perfect (at least at finite volume).

Conclusions

Scars violate ETH and can spoil thermalization

Scars appear in pure $U(1)$ gauge theories for arbitrary volumes

T. Budde, M. K. Marinkovic, JPB; PRD 110 (2024) 9, 094506, arXiv:2403.08892

- For $S = 1$ Quantum Link Models;
- For truncated models, with arbitrary truncation.

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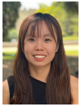
Ongoing work:

- Other winding sectors;
- Connections to integrability;
- Non-zero-mode scars.

K. Keršič



J. Dong

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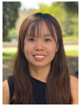
J. Dong

Some other interesting directions:

- Parent Hamiltonians: scars as ground states;
- Connection between gauge symmetry and scarring?
- Any consequence in the continuum limit?