Bridging two semiclassical confinement mechanisms: monopole and center vortex

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Hadrons and Hadron Interactions in QCD 2024 (HHIQCD 2024)

November 12, 2024

Based on:
PRL **133**, 171902 (2024) [arXiv:2405.12402 [hep-th]] with Yuya Tanizaki (YITP)
also [arXiv:2410.21392 [hep-th]] with Tatsuhiro Misumi (Kindai U.) and Yuya Tanizaki (YITP)
(special thanks to Mithat Ünsal(NCSU))

Confinement mechanisms

Two scenarios for quark confinement: monopole and center vortex

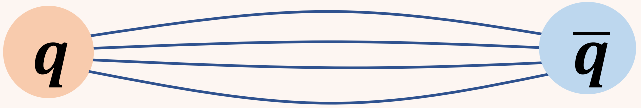
Dual superconductor picture (monopole condensation)

[Nambu ’74, ‘t Hooft ‘75, Mandelstam ‘76,…]

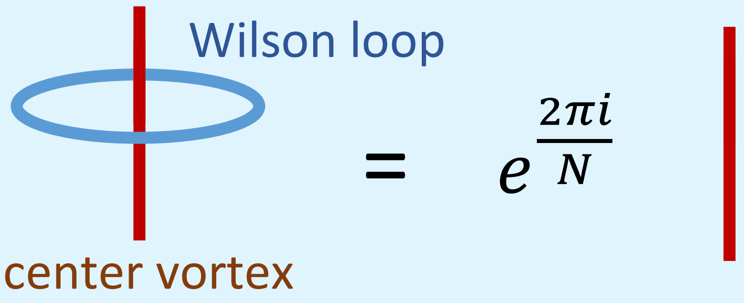
monopole condensation

⇒ dual Meissner effect

⇒ linear $q\bar{q}$-potential



Center-vortex proliferation [‘t Hooft ‘78, …]



Center vortex: rotating Wilson loop by $e^{\frac{2\pi i}{N}}$.

Proliferation ⇒ $\langle W(C)\rangle \sim e^{-\sigma\,(\mathrm{Area})}$

cf.) restoration of $\mathbb{Z}_N^{[1]}$: proliferation of co-dim-2 defects

Connection between them? [Ambjørn-Giedt-Greensite ‘99, Engelhardt-Reinhardt ‘99, Cornwall ’99, …]

“monopole as junction of center vortices”

Summary

Quark confiners: monopole and center vortex

Weak-coupling semiclassical realizations:

3d monopole semiclassics

[Ünsal ’07, Ünsal-Yaffe ’08,…]

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S}^1$ with “center-stabilizing deformation”

⇒ **confinement by 3d monopole gas**

2d center-vortex semiclassics

[Tanizaki-Ünsal ’22, …]

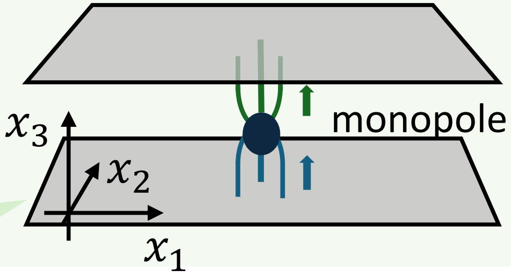
SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T}^2$ with ‘t Hooft flux

⇒ **confinement by 2d center-vortex gas**

This work: Consider an interpolating setup on $(\mathbb{R}^2 \times \boldsymbol{S}^1) \times \boldsymbol{S}^1$

Monopole in $\mathbb{R}^2 \times \boldsymbol{S}^1$

“monopole as junction of center vortices”



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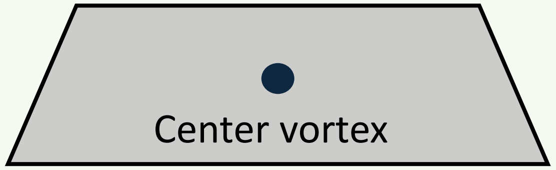
Center vortex in 2d



Outline

1. Introduction (2 slides)
2. Monopole semiclassics and center-vortex semiclassics (7 slides)
3. Monopole-vortex continuity (9 slides)
4. Summary

Semiclassical approaches to confinement

Motto: deforming SU(N) YM to **weakly-coupled** theory with **keeping confinement.**

compactification

center-stabilizing deformation
(to avoid deconfinement transition)

3d monopole semiclassics

[Ünsal '07, Ünsal-Yaffe '08,...]

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S}^1$ with "center-stabilizing deformation"

$\Rightarrow$ **confinement by 3d monopole gas**

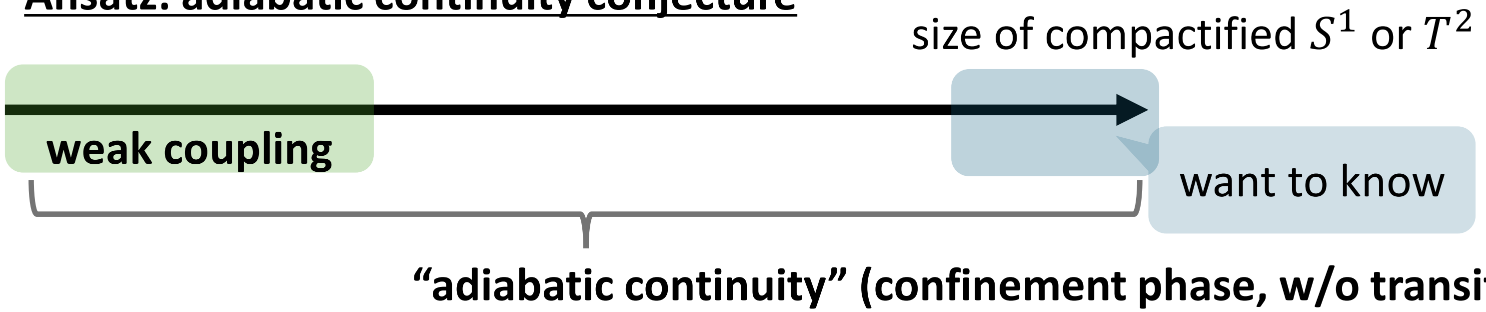
2d center-vortex semiclassics

[Tanizaki-Ünsal '22, ...]

SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T}^2$ with 't Hooft flux ($\mathbb{Z}_N^{[1]}$ background)

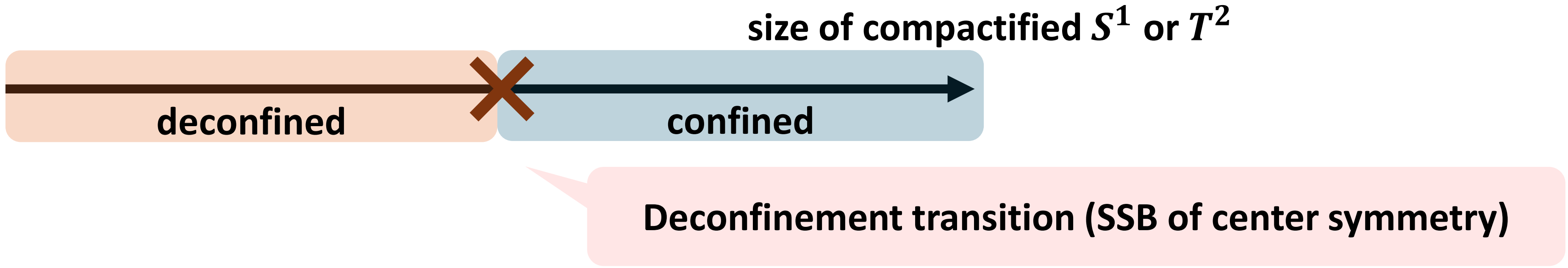
$\Rightarrow$ **confinement by 2d center-vortex gas**

Ansatz: adiabatic continuity conjecture

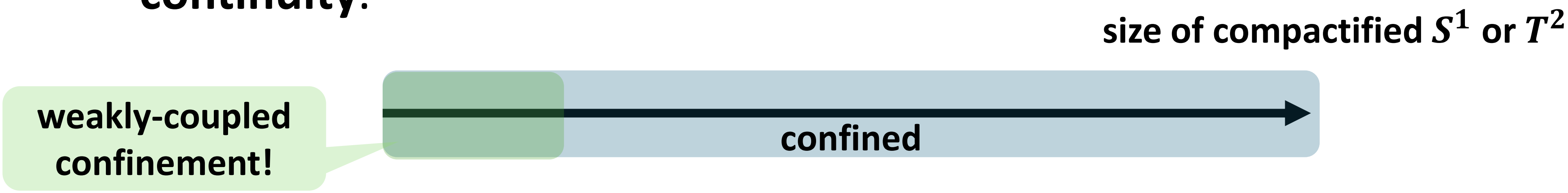


Adiabatic continuity

- With the naïve compactification, there is a deconfinement transition somewhere



- By adding "**center-stabilizing deformation**" (adding Polyakov-loop potential in 3d semiclassics; inserting 't Hooft flux in 2d semiclassics), we expect the **adiabatic continuity**.



3d monopole semiclassics

[Ünsal '07, Ünsal-Yaffe '08,…] (cf. [Davies-Hollowood-Khoze-Mattis '99,…])

- **SU(N) Yang-Mills on $\mathbb{R}^3 \times S^1$ with "center-stabilizing deformation"** [Ünsal-Yaffe '08]:

$$S = S_{YM} + \int d^3x \sum_{n=1}^{[N/2]} a_n \,|\mathrm{tr}\,(P^n)|^2$$

Add a potential for Polyakov loop (by hand) to keep center symmetry

⇒ Center symmetry is kept for **small S^1** (, realizing weak-coupling confinement)

- **3d effective theory on $\mathbb{R}^3$**

The Polyakov loop behaves as an adjoint scalar field.

At the center symmetric vacuum, "$\langle P \rangle \sim C$" (up to gauge)

⇒ adjoint higgsing $SU(N) \to U(1)^{N-1}$

e.g.) clock matrix for $N = 3$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$

3d effective theory = 3d $U(1)^{N-1}$ gauge theory + monopoles

- Polyakov confinement by dilute gas of monopoles (in 3d Abelian gauge theory) [Polyakov '77]

Magnetic Debye screening ⇒ area law $\langle W(C) \rangle \sim e^{-\sigma\,(\mathrm{Area})}$

3d monopole semiclassics (some details)

"compactness of adjoint higgs"
[Kraan-van Baal '98] [Lee-Lu '98]

- **N kinds of monopoles**: $Q_{top} = 1/N$ fractional instantons

(N-1) BPS monopoles + KK monopole

Magnetic charge: $\vec{\alpha}_1$ $\vec{\alpha}_2$ .... $\vec{\alpha}_{N-1}$ $\vec{\alpha}_N (= -\vec{\alpha}_1 - \cdots - \vec{\alpha}_{N-1})$

$\vec{\alpha}_1, \cdots, \vec{\alpha}_{N-1}$: simple roots
$\vec{\alpha}_N$ $(= -\vec{\alpha}_1 - \cdots - \vec{\alpha}_{N-1})$: affine root

- **3d effective theory**

3d abelian duality: $U(1)^{N-1}$ gauge field→ $U(1)^{N-1}$-valued compact boson $\vec{\sigma}$ $(d\vec{\sigma} = * \vec{f})$

In terms of $\vec{\sigma}$ (dual photon/magnetic potential), the 3d effective theory is,

$$S = \int d^3x \left[\frac{\# g^2}{L} |d\vec{\sigma}|^2 - \# \, e^{-\frac{8\pi^2}{Ng^2}} \sum_{i=1,\cdots,N} \cos(\vec{\alpha}_i \cdot \vec{\sigma} + \theta/N) \right]$$

Monopole amplitude
$[\mathcal{M}_i] \sim e^{-\frac{8\pi^2}{Ng^2}} \, \mathrm{e}^{i\vec{\alpha}_i \cdot \vec{\sigma} + i\,\theta/N}$

2d center-vortex semiclassics

[Tanizaki-Ünsal ’22, ……] (cf. [Yamazaki-Yonekura ‘17])

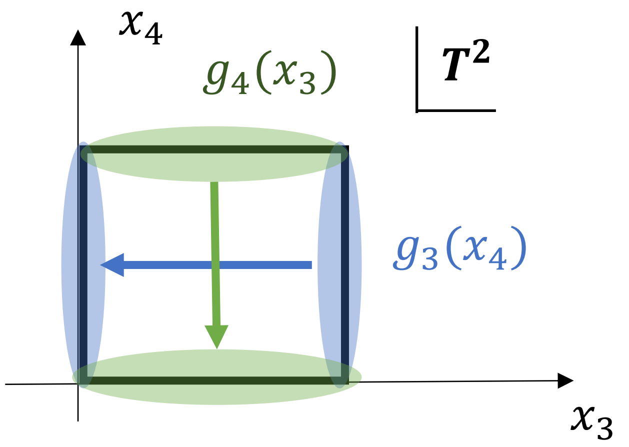
Setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times T^2$ with ‘t Hooft flux

- **‘t Hooft flux for T^2 (or $\mathbb{Z}_N^{[1]}$ background)**

A unit ‘t Hooft flux ⇔ choose $g_3(0)g_4(L)g_3^\dagger(L)g_4^\dagger(0) = e^{\frac{2\pi \mathrm{i}}{N}}$.

$(g_3(x_4), g_4(x_3))$: transition functions on T^2)

Up to gauge, we can take $g_3 = S,\ g_4 = C$ (shift and clock matrices of $SU(N)$).



$$\begin{cases} a(\vec{x}, x_3 + L, x_4) = g_3^\dagger a g_3 - \mathrm{i}\, g_3^\dagger \mathrm{d} g_3 \\ a(\vec{x}, x_3, x_4 + L) = g_4^\dagger a g_4 - \mathrm{i}\, g_4^\dagger \mathrm{d} g_4 \end{cases}$$

e.g.) $N = 3$

$$S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},\ C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$

- **Consequences from ‘t Hooft-twisted compactification**

  ✓**Center symmetry is kept at small T^2**

  Classically, $P_3 = S$ and $P_4 = C \Rightarrow \langle \mathrm{tr}\, P_3 \rangle = \langle \mathrm{tr}\, P_4 \rangle = 0$.

  ✓**Perturbatively gapped gluons: O(1/NL) KK mass**

  ✓**Numerical evidence for center vortex/fractional instantons** (as a local solution)

  [Gonzalez-Arroyo–Montero ’98, Montero ’99, ……].

exists locally,
(not globally if `regularity’ at infinity is imposed)

2d center-vortex semiclassics [Tanizaki-Ünsal '22]

- **Dilute gas of center vortices**

The center-vortex and anti-center-vortex vertices are:

$$[\mathcal{V}] = K e^{-\frac{8\pi^2}{Ng^2} + i\frac{\theta}{N}}, \qquad [\bar{\mathcal{V}}] = K e^{-\frac{8\pi^2}{Ng^2} - i\,\theta/N}$$

with a dimensionful constant K.

For calculating partition function, we compactify $\mathbb{R}^2$ without 't Hooft flux.
⇒ total topological charge is constrained $Q_{top} \in \mathbb{Z}$

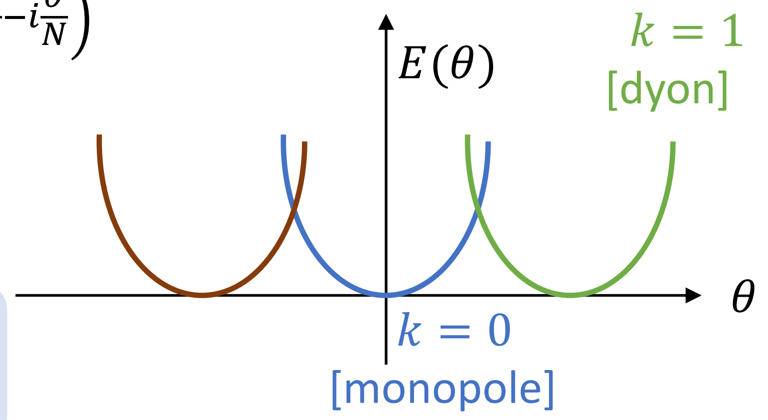
Then, the dilute gas approximation yields, (only configurations with $Q_{top} \in \mathbb{Z}$ are admitted)

$$Z_{2d} = \sum_{n,\bar{n} \geq 0} \frac{1}{n!\,\bar{n}!} \delta_{n-\bar{n} \in N\mathbb{Z}} \left(VK e^{-\frac{8\pi^2}{Ng^2} + i\frac{\theta}{N}} \right)^n \left(VK e^{-\frac{8\pi^2}{Ng^2} - i\frac{\theta}{N}} \right)^{\bar{n}}$$

$$= \sum_{k \in \mathbb{Z}_N} \exp\left[-V \left(-2K e^{-\frac{8\pi^2}{Ng^2}} \cos\left(\frac{\theta - 2\pi k}{N} \right) \right) \right]$$

N semiclassical vacua

Energy density of k-th vacuum
→multibranch structure!



✅ One can also derive area-law falloff of the Wilson loop from the dilute gas of center vortices.

Summary of Backgrounds / Question

Motto: deforming SU(N) YM to **weakly-coupled** one with **keeping confinement**.

<u>3d monopole semiclassics</u>
[Ünsal '07, Ünsal-Yaffe '08,…]
SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S}^1$ with "center-stabilizing deformation"
⇒ **3d** $U(1)^{N-1}$ **gauge theory + monopole gas**

⟺

<u>2d center vortex semiclassics</u>
[Tanizaki-Ünsal '22, …]
SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T}^2$ with 't Hooft flux
⇒ **confinement by 2d center-vortex gas**

Question: Relation between them?
How monopole transmutes to center vortex?

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1. Introduction (2 slides)
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Interpolating setup

't Hooft flux

Interpolating setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times \underbrace{\overbrace{(\boldsymbol{S^1})_3 \times (\boldsymbol{S^1})_4}^{\text{'t Hooft flux}}}$

(L_4: always small)

center-stabilizing deformation

$L_3 \to \infty$

$L_3 \to L_4$

<u>3d monopole semiclassics</u>
SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S^1}$ with center-stabilizing deformation

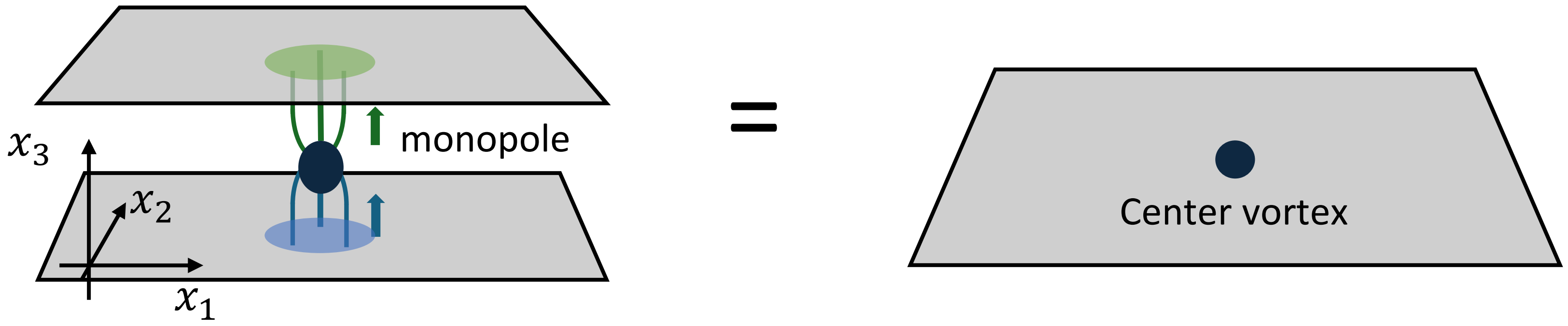
<u>2d center vortex semiclassics</u>
SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T^2}$ with 't Hooft flux

What we will see:

setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times \overbrace{(S^1)_3 \times \underbrace{(S^1)_4}_{\text{center-stabilizing deformation}}}^{\text{'t Hooft flux}}$

1. 3d effective theory on $\mathbb{R}^2 \times (S^1)_3$ $\Rightarrow$ 2d center-vortex gas on $\mathbb{R}^2$

2. BPS/KK monopole in $\mathbb{R}^2 \times (S^1)_3$ (3d monopole-instanton)
$\Rightarrow$ center vortex on $\mathbb{R}^2$ (2d center-vortex-instanton)



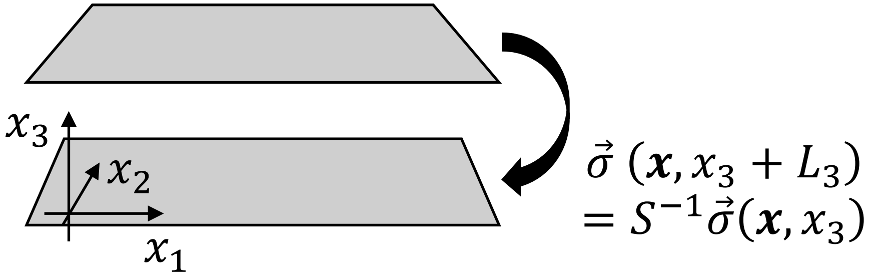
3d effective theory on $\mathbb{R}^2 \times (S^1)_3$

Interpolating setup: SU(N) Yang-Mills on $\mathbb{R}^2 \times \overbrace{(S^1)_3 \times \underbrace{(S^1)_4}_{\text{center-stabilizing deformation}}}^{\text{'t Hooft flux}}$

(L_4: always small)

small L_4, adjoint higgsing by P_4

3d $U(1)^{N-1}$ gauge theory + monopoles on $\mathbb{R}^2 \times (S^1)_3$
with "shift-twisted" boundary conditions



In the gauge $\langle P_4 \rangle = C$ (clock matrix), the transition function for $(S^1)_3$ is $g_3 = S$ (shift matrix).

or, 't Hooft flux ⇒ $\left(\mathbb{Z}_N^{[0]}\right)_{3\mathrm{d}}$-twisted boundary condition (∼ **Weyl permutation** for dual photon $\vec{\sigma}(x, x_3)$)

Example: SU(2) case

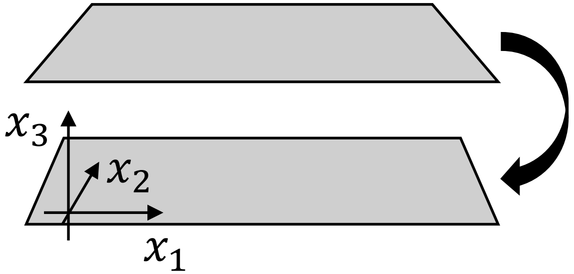
- Adjoint higgsing by $P_4(\propto \sigma_3$, up to gauge$)$: $SU(2) \to U(1)$ $\Rightarrow$ one compact scalar $\sigma \sim \sigma + 2\pi$

$$S_{3d}[\sigma] = \int d^3x \left[\frac{\# g^2}{L_4} |d\sigma|^2 - \# \, e^{-\frac{8\pi^2}{Ng^2}} \left(\cos\left(\sigma + \frac{\theta}{2}\right) + \cos\left(-\sigma + \frac{\theta}{2}\right)\right)\right]$$

- the transition function for $(\boldsymbol{S^1})_{\boldsymbol{3}}$ is $g_3 \propto \sigma_1$(shift matrix):

$\Rightarrow$ flipping the basis $P_4 \mapsto -P_4$, equivalent to $\sigma\,(\boldsymbol{x}, x_3 + L_3) = -\sigma\,(\boldsymbol{x}, x_3)$

3d $U(1)$ gauge theory + monopoles on $\mathbb{R}^2 \times (\boldsymbol{S^1})_{\boldsymbol{3}}$
with "shift-twisted" boundary conditions



$\sigma\,(\boldsymbol{x}, x_3 + L_3)$
$= -\sigma\,(\boldsymbol{x}, x_3)$

$\boldsymbol{L_3 \ll \Lambda^{-1}}$**: restricted to "zeromode":** $\boldsymbol{\sigma = -\sigma}$

$\Rightarrow$ 2 vacua: $\sigma = 0, \pi$

identical to extrema of the monopole potential
(3d-2d adiabatic continuity)

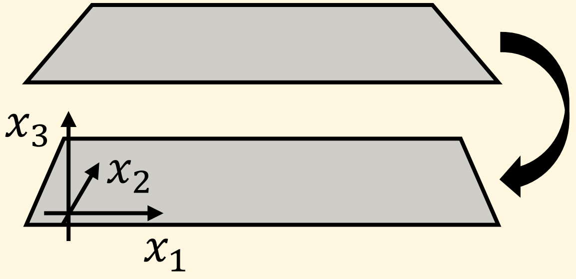
2d center-vortex gas

$$Z_{\mathbb{R}^2 \times (\boldsymbol{S^1})_{\boldsymbol{3}}} \approx \sum_{k \in \mathbb{Z}_2} \mathrm{e}^{\# V_{2d} \mathrm{e}^{-\frac{8\pi^2}{2g^2}} \cos\left(\frac{\theta + 2\pi k}{2}\right)} = Z_{\text{2d vortex gas}}$$

From 3d monopole gas to 2d center-vortex gas

3d $U(1)^{N-1}$ gauge theory + monopoles on $\mathbb{R}^2 \times (S^1)_3$
with "shift-twisted" boundary conditions

$$S_{3d}[\vec{\sigma}] = \int d^3x \left[\frac{\# g^2}{L_4} |d\vec{\sigma}|^2 - \#\, e^{-\frac{8\pi^2}{Ng^2}} \sum_{i=1,\cdots,N} \cos(\vec{\alpha}_i \cdot \vec{\sigma} + \theta/N) \right]$$



$$\vec{\sigma}\,(\boldsymbol{x}, x_3 + L_3) = S^{-1}\vec{\sigma}(\boldsymbol{x}, x_3)$$

$L_3 \ll \Lambda^{-1}$: restricted to $\vec{\sigma} = S^{-1}\vec{\sigma}$

N vacua: $\vec{\sigma} = \vec{\sigma}_k = \dfrac{2\pi k}{N}(\vec{\mu}_1 + \cdots + \vec{\mu}_{N-1})$

$(k = 0, \cdots, N-1)$

identical to extrema of the monopole potential (**3d-2d adiabatic continuity**)

2d center-vortex gas

$$Z_{\mathbb{R}^2\times(S^1)_3} = \int_{\substack{\vec{\sigma}\,(\boldsymbol{x}, x_3+L_3) \\ = S^{-1}\vec{\sigma}(\boldsymbol{x}, x_3)}} \mathcal{D}\vec{\sigma}\ \mathrm{e}^{-S_{3d}[\vec{\sigma}]} \approx \sum_{\substack{\vec{\sigma}=\vec{\sigma}_k \\ k\in\mathbb{Z}_N}} \mathrm{e}^{-S_{3d}[\vec{\sigma}]} = \sum_{k\in\mathbb{Z}_N} \mathrm{e}^{\#\, V_{2d} \mathrm{e}^{-\frac{8\pi^2}{Ng^2}} \cos\left(\frac{\theta + 2\pi k}{N}\right)} = Z_{2d}$$

How monopole looks like in $\mathbb{R}^2 \times (S^1)_3$

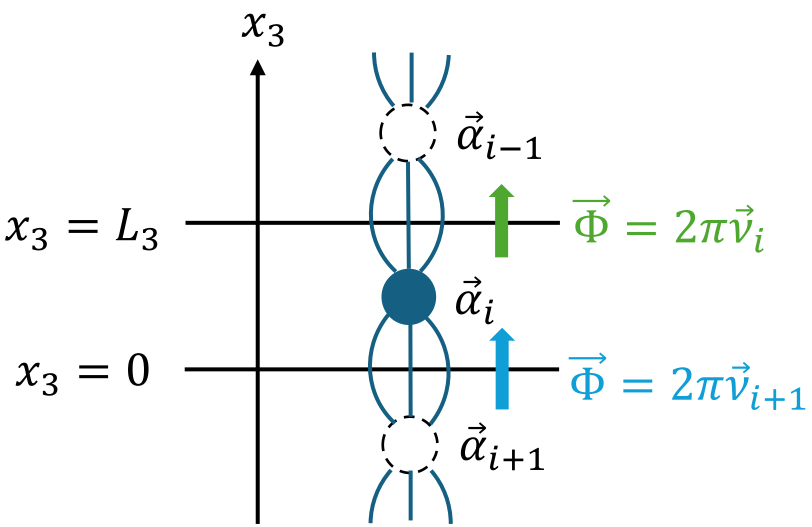
- BPS/KK monopole in 3d effective theory:

magnetic charge $\vec{\alpha}_i \Rightarrow \nabla^2 \vec{\sigma} \sim 2\pi\vec{\alpha}_i\, \delta^{(3)}(x - x_*)$

boundary condition: $\vec{\sigma}\,(\boldsymbol{x}, x_3 + L_3) = S^{-1}\vec{\sigma}(\boldsymbol{x}, x_3)$

$\Rightarrow$ "mirror image": infinite chain of BPS/KK monopoles

$$\vec{\sigma} \sim \text{"}\sum_{n\in\mathbb{Z}} \frac{\vec{\alpha}_{i-n\ (\mathrm{mod\ N})}}{|x - x_* - nL_3\,\hat{x}_3\,|}\text{"}$$



- A proper expression (with good convergence):

$$\vec{\sigma} \sim \sum_{k\in\mathbb{Z}} \left[\sum_{\ell\in\mathbb{Z}_N} \vec{v}_{i-\ell\ (\mathrm{mod\ N})} \left\{ \frac{1}{|x - x_* - (Nk+\ell)L_3\,\hat{x}_3|} - \frac{1}{|x - x_* - (Nk+\ell+1)L_3\,\hat{x}_3|} \right\} \right]$$

$\vec{v}_i$: weight vector of defining representation

$\vec{\alpha}_i = \vec{v}_i - \vec{v}_{i+1}$

outgoing magnetic flux

$\vec{\Phi} = 2\pi\vec{v}_i$

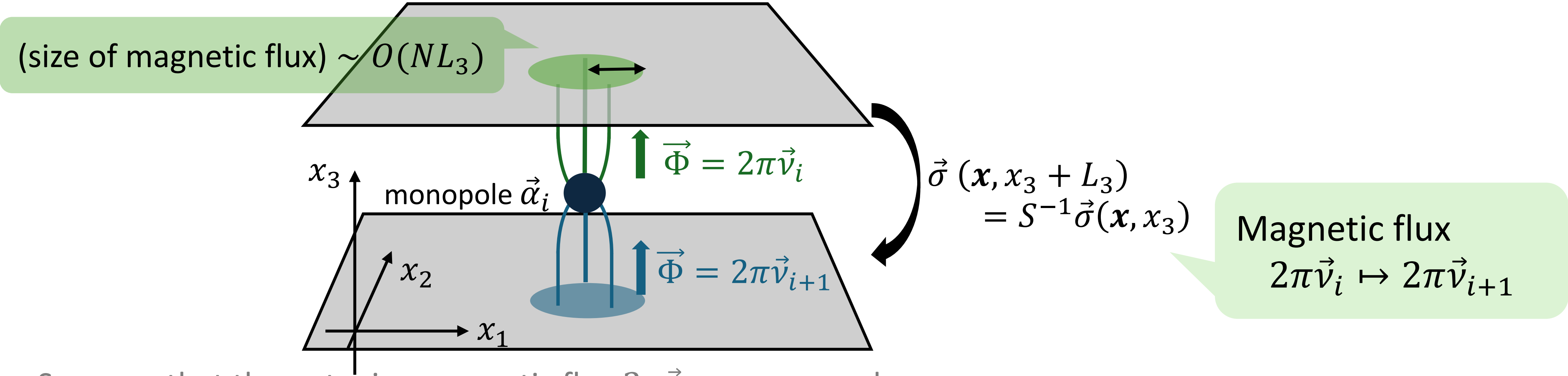
incoming magnetic flux

$\vec{\Phi} = 2\pi\vec{v}_{i+1}$

How monopole looks like in $\mathbb{R}^2 \times (S^1)_3$

This solution explains:

The $\vec{\alpha}_i$-monopole emits magnetic flux $2\pi\vec{\alpha}_i = 2\pi\vec{v}_i - 2\pi\vec{v}_{i+1}$



Suppose that the outgoing magnetic flux $2\pi\vec{v}_i$ goes upward

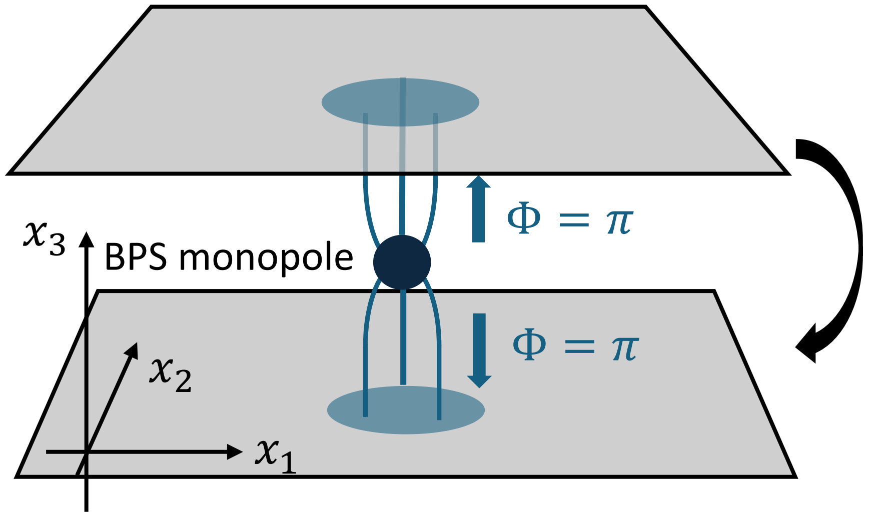
⇒ Shift-twisted boundary condition: $2\pi\vec{v}_i \mapsto 2\pi\vec{v}_{i+1}$ ⇒ The incoming flux $2\pi\vec{v}_{i+1}$ comes from bottom

The magnetic flux is localized in 2d ($\sim O(NL_3)$) .

N Species of monopole (BPS/KK) can be included in extended moduli $x_3 \in [0, NL_3)$.

Example: SU(2) case

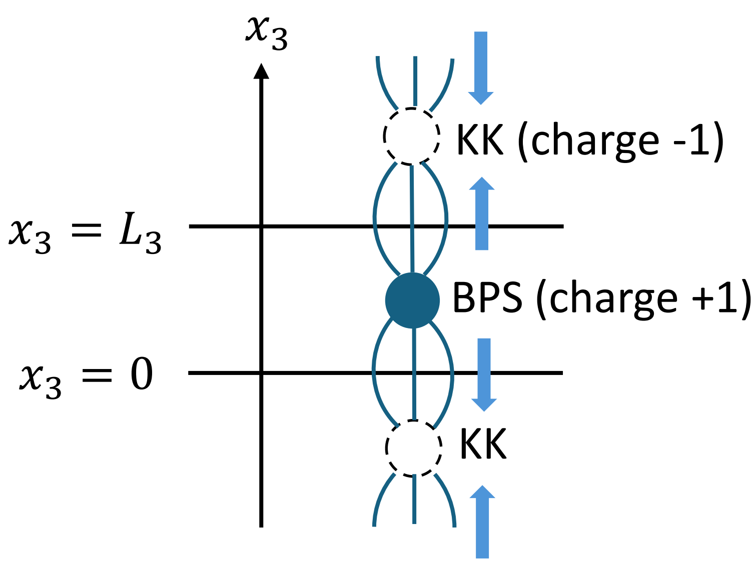
- One compact scalar $\sigma \sim \sigma + 2\pi$
- BPS monopole: magnetic charge +1, KK monopole: magnetic charge -1
- boundary condition: $\sigma\,(\boldsymbol{x}, x_3 + L_3) = -\sigma(\boldsymbol{x}, x_3)$



charge-conjugation

$$\sigma\,(\boldsymbol{x}, x_3 + L_3) = -\sigma\,(\boldsymbol{x}, x_3)$$

<u>"mirror image" solution:</u>



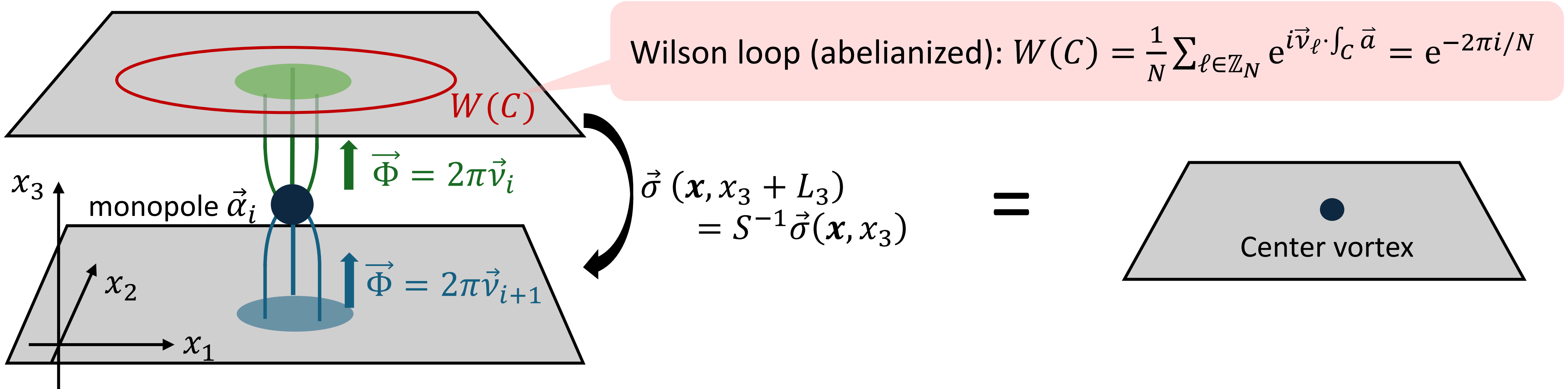
"Flux Fractionalization":
1/N fractional magnetic flux, rotating the Wilson loop by a center element (-1)

3d BPS/KK monopoles become 2d center vortex

- The magnetic flux (of size $O(NL_3)$) is indeed center vortex:
  Wilson loop acquires $e^{-2\pi i/N}$ phase.
- **3d BPS/KK monopole-instanton = 2d center-vortex-instanton**:

The 3d/2d semiclassical confinement mechanisms are essentially same!

- **"monopole as junction of center vortex" (realizing the old expectation!)**



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Summary

Quark confiners: monopole and center vortex

Weak-coupling semiclassical realizations:

3d monopole semiclassics

[Ünsal ’07, Ünsal-Yaffe ’08,…]

SU(N) Yang-Mills on $\mathbb{R}^3 \times \boldsymbol{S}^1$ with “center-stabilizing deformation”

⇒ **confinement by 3d monopole gas**

2d center vortex semiclassics

[Tanizaki-Ünsal ’22, …]

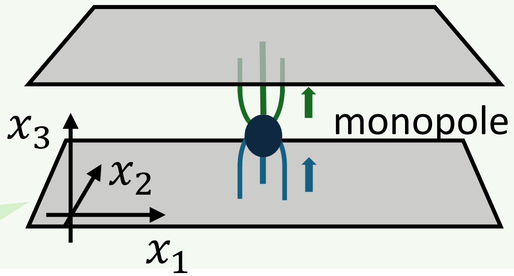
SU(N) Yang-Mills on $\mathbb{R}^2 \times \boldsymbol{T}^2$ with ‘t Hooft flux

⇒ **confinement by 2d center-vortex gas**

This work: Consider an interpolating setup on $(\mathbb{R}^2 \times \boldsymbol{S}^1) \times \boldsymbol{S}^1$

Monopole in $\mathbb{R}^2 \times \boldsymbol{S}^1$

“monopole as junction of center vortices”



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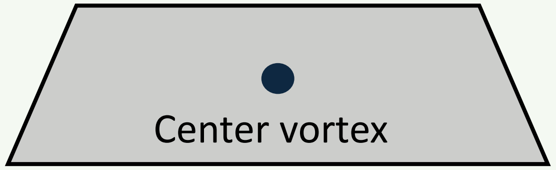
Center vortex in 2d



Further developments / future directions

- Interplay between 3d/2d semiclassics
  - $\mathcal{N} = 1$ SYM, QCD(adj) [YH-Misumi-Tanizaki '24]: 3d semiclassics is well developed, but 2d semiclassics was somewhat puzzling

    Perimeter-law in 2d (⇐ 3d double string picture), fate of bion mechanism.
  - QCD(F) (2d semiclassics unexpectedly works well, why?)
  - Other gauge groups…
  - …
- Monopole-vortex complex as soliton (in Higgs phase) [in progress, with Misumi-Nitta-Ohashi-Tanizaki]
- Resurgence?
- Analytic solution of the center vortex/fractional instanton on R2xT2?
- …….

Wilson loop transmutes to domain wall

- Let us consider SU(2) $\mathcal{N} = 1$ SYM.
- (From mixed anomaly,) Wilson loop should behave as a domain wall of discrete chiral symmetry in the 2d semiclassics. ⇒ perimeter law falloff?
- **Double string picture**: dual photon potential $\sim \cos(2\sigma)$ from magnetic bion

The Wilson loop (defect $\sigma \sim \sigma + 2\pi$) emits two kinks ($\Delta\sigma = \pi$) [Anber-Poppitz-Sulejmanpasic '15]

- Reduction from 3d to 2d

Dominant configuration for $|C| \gg L_3$

This is domain wall!