D-Theory:
Asyptotically Free Quantum Fields from the Dimensional Reduction of Discrete Variables

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Outline

Quantum Spin Models and Emergent Effective Field Theories

$SU(N)$ Quantum Spins and Emergent $\mathbb{C}P(N-1)$ Models

From Wilson's Lattice Gauge Theory to Quantum Link Models

$U(1)$ Quantum Link Model on a Triangular Lattice

Non-Abelian Quantum Link Models and Emergent QCD

Quantum Simulator for non-Abelian Gauge Theories

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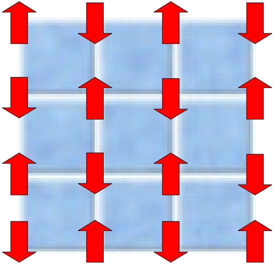
The anti-ferromagnetic spin $\frac{1}{2}$ quantum Heisenberg model



Quantum spins $[S_x^a, S_y^b] = i\delta_{xy}\varepsilon_{abc}S_x^c$ and their Hamiltonian

$$H = J\sum_{\langle xy\rangle} \vec{S}_x \cdot \vec{S}_y \ , \quad [H, \vec{S}] = 0 \ , \quad \vec{S} = \sum_x \vec{S}_x$$

Partition function at inverse temperature $\beta = 1/T$

$$Z = \mathrm{Tr}\exp(-\beta H)$$

Staggered magnetization order parameter

$$\vec{M}_s = \sum_x (-1)^{(x_1+x_2)/a} \ \vec{S}_x$$

signals spontaneous symmetry breaking $SU(2) \to U(1)$

$$\frac{a^2}{L^2} |\langle \vec{M}_s \rangle| = \mathcal{M}_s \neq 0 \text{ at } T = 0$$

Magnon (Goldstone boson) field in $SU(2)/U(1) = S^2$

$$\vec{e}(x) = (e_1(x), e_2(x), e_3(x)), \quad \vec{e}(x)^2 = 1$$

Low-energy effective action for antiferromagnetic magnons

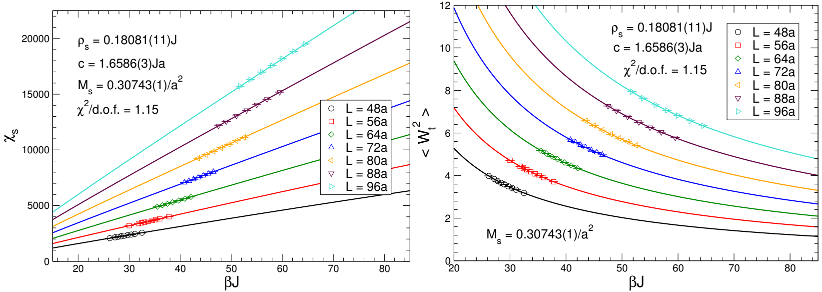
$$S[\vec{e}] = \int_0^\beta dt \int d^2x \ \frac{\rho_s}{2} \left(\partial_i \vec{e} \cdot \partial_i \vec{e} + \frac{1}{c^2} \partial_t \vec{e} \cdot \partial_t \vec{e} \right)$$

Chakravarty, Halperin, Nelson (1989)
Neuberger, Ziemann (1989)
Hasenfratz, Leutwyler (1990)
Hasenfratz, Niedermayer (1993)
Chubukov, Sentil, Sachdev (1994)

Fit to analytic predictions of effective theory

$$\chi_s = \frac{\mathcal{M}_s^2 L^2 \beta}{3} \left\{ 1 + 2\frac{c}{\rho_s L l}\beta_1(l) + \left(\frac{c}{\rho_s L l}\right)^2 \left[\beta_1(l)^2 + 3\beta_2(l)\right] \right\}$$

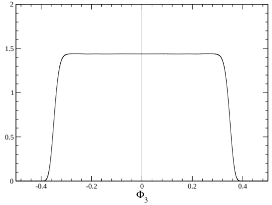
$$\chi_u = \frac{2\rho_s}{3c^2} \left\{ 1 + \frac{1}{3}\frac{c}{\rho_s L l}\widetilde{\beta}_1(l) + \frac{1}{3}\left(\frac{c}{\rho_s L l}\right)^2 \left[\widetilde{\beta}_2(l) - \frac{1}{3}\widetilde{\beta}_1(l)^2 - 6\psi(l)\right] \right\}$$

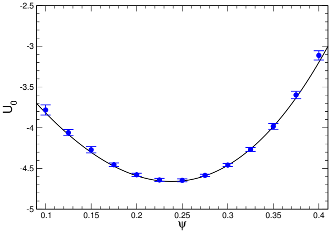


$$\mathcal{M}_s = 0.30743(1)/a^2, \quad \rho_s = 0.18081(11)J, \quad c = 1.6586(3)Ja$$

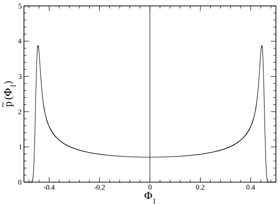
UJW, Ying (1994); Sandvik, Evertz (2010); Jiang, UJW (2010)

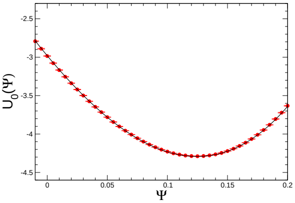
Excellent agreement with effective field theory predictions for the constraint effective potential: Göckeler, Leutwyler (1991)





Heisenberg model: Gerber, Hofmann, Jiang, Nyfeler, UJW (2009)





XY model: Gerber, Hofmann, Jiang, Palma, Stebler, UJW (2011)

Jiang (2010) $\mathcal{M} = 0.43561(1)/a^2, \ \rho = 0.26974(5)J, \ c = 1.1348(5)Ja$

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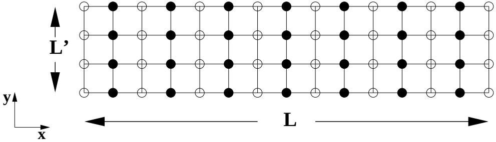
$\mathbb{C}P(N-1)$ Models from $SU(N)$ quantum spins

$$T_x^a, \quad a \in \{1, 2, ..., N^2-1\}, \quad [T_x^a, T_y^b] = i\delta_{xy} f_{abc} T_x^c$$

Spin ladder Hamiltonian

$$H = -J \sum_{x \in A} [T_x^a T_{x+\hat{1}}^{a*} + T_x^a T_{x+\hat{2}}^a] - J \sum_{x \in B} [T_x^{a*} T_{x+\hat{1}}^a + T_x^{a*} T_{x+\hat{2}}^{a*}]$$



Conserved $SU(N)$ spin

$$T^a = \sum_{x \in A} T_x^a - \sum_{x \in B} T_x^{a*}, \quad [T^a, H] = 0$$

B. B. Beard, M. Pepe, S. Riederer, UJW, PRL 94 (2005) 010603.

Goldstone boson fields in $\mathbb{C}P(N-1) = SU(N)/U(N-1)$

$$P(x)^\dagger = P(x), \quad \mathrm{Tr} P(x) = 1, \quad P(x)^2 = P(x)$$

Low-energy effective action

$$\begin{aligned} S[P] &= \int_0^\beta dt \int_0^L dx \int_0^{L'} dy \ \mathrm{Tr}\left\{\rho_s' \partial_y P \partial_y P \right. \\ &+ \left. \rho_s \left[\partial_x P \partial_x P + \frac{1}{c^2}\partial_t P \partial_t P\right] - \frac{1}{a} P \partial_x P \partial_t P \right\} \end{aligned}$$

Topological charge

$$Q[P] = \frac{1}{\pi i}\int_0^\beta dt \int_0^L dx \ \mathrm{Tr}[P \partial_x P \partial_t P] \in \Pi_2[SU(N)/U(N-1)] = Z$$

Very large correlation length

$$\xi \propto \exp(4\pi L' \rho_s / cN) \gg L'$$

Dimensional reduction to the $(1+1)$-d $\mathbb{C}P(N-1)$ model

$$S[P] = \int_0^\beta dt \int_0^L dx \, \mathrm{Tr} \left\{ \frac{1}{g^2} \left[\partial_x P \partial_x P + \frac{1}{c^2} \partial_t P \partial_t P \right] - nP \partial_x P \partial_t P \right\}$$

Emergent asymptotically free coupling and θ-vacuum angle

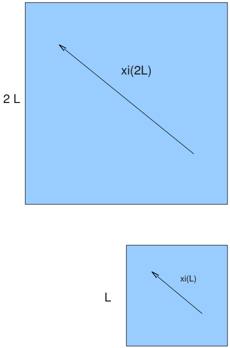
$$\frac{1}{g^2} = L' \rho_s, \quad \theta = n\pi$$

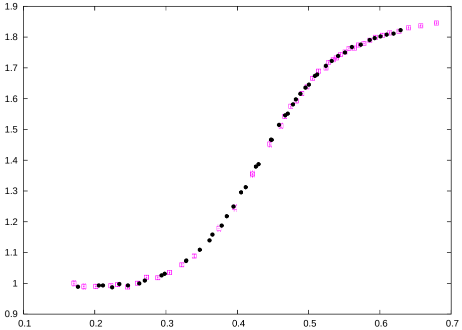
Correlation lengths in the $\mathbb{C}P(2)$ model

$$\xi(2) = 4.0(1), \quad \xi(4) = 17.6(2), \quad \xi(6) = 61(2)$$

Step-scaling approach

M. Lüscher, P. Weisz, and U. Wolff, Nucl. Phys. B359 (1991) 221



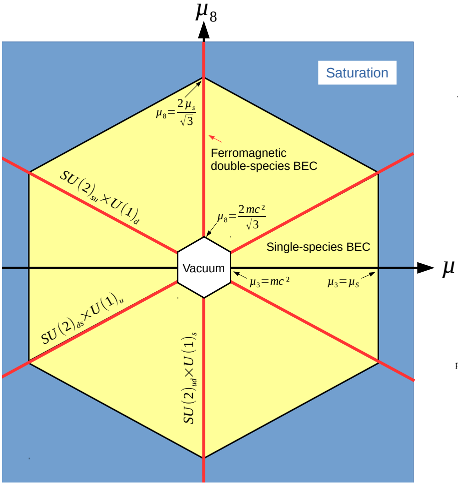


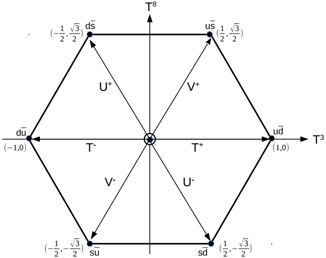
Universal step-scaling function

$$F(z) = \xi(2L)/\xi(L) \ , \quad z = \xi(L)/L$$

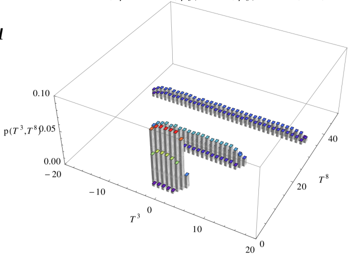
B. B. Beard, M. Pepe, S. Riederer, UJW, PRL 94 (2005) 010603.

Ferromagnetic Double-Species BEC in the $\mathbb{C}P(2)$ Model





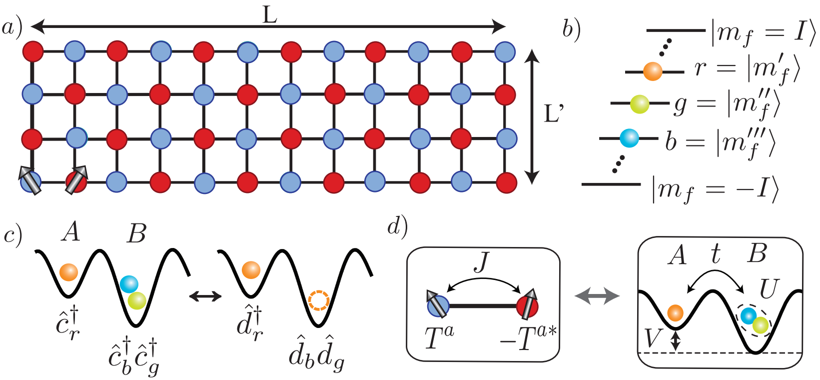
$L = 300\,a$, $\beta J = 281.5$, $\mu_3 / J = 0$, $\mu_8 / J = 0.1, 0.2, 0.3$



W. Evans, U. Gerber, M. Hornung, UJW, Annals Phys. 398 (2018) 92.

Ladder of $SU(N)$ quantum spins embodied with alkaline-earth atoms

$$H = -J \sum_{\langle xy \rangle} T_x^a T_y^{a*}, \quad [T_x^a, T_y^b] = i\delta_{xy} f_{abc} T_x^c$$



C. Laflamme, W. Evans, M. Dalmonte, U. Gerber, H. Mejia-Diaz, W. Bietenholz, UJW, and P. Zoller, Annals Phys. 360 (2016) 117.

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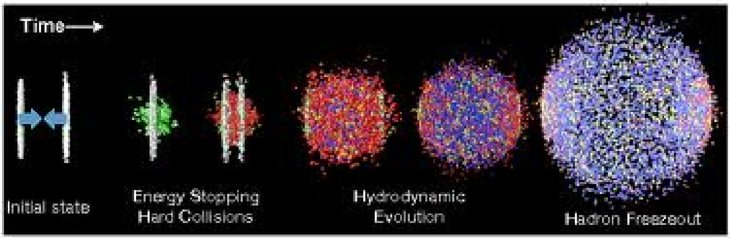
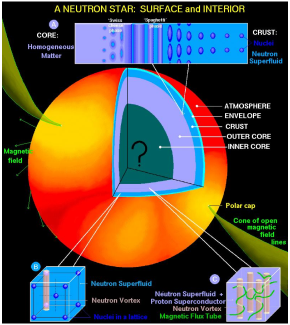
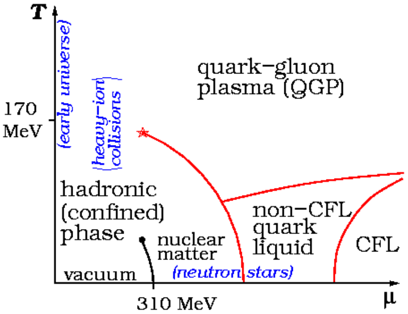
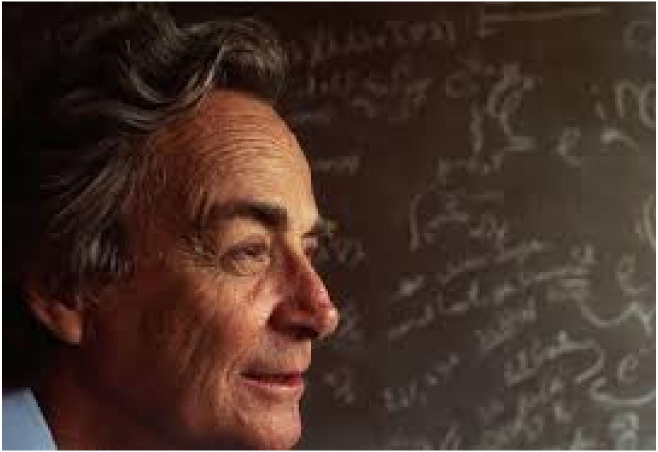
Can heavy-ion collision physics or nuclear astrophysics benefit from quantum simulations in the long run?





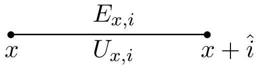


Richard Feynman's vision of 1982 (Int. J. Theor. Phys. 21)



"It does seem to be true that all the various field theories have the same kind of behavior, and can be simulated in every way, apparently, with little latticeworks of spins and other things."

Hamiltonian formulation of Wilson's $U(1)$ lattice gauge theory



$$U = \exp(i\varphi), \ U^{\dagger} = \exp(-i\varphi) \in U(1)$$

Electric field operator E

$$E = -i\partial_{\varphi}, \ [E, U] = U, \ [E, U^{\dagger}] = -U^{\dagger}, \ [U, U^{\dagger}] = 0$$

Generator of $U(1)$ gauge transformations

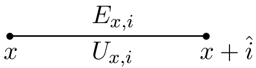
$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

$U(1)$ gauge invariant Hamiltonian

$$H = \frac{e^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2e^2} \sum_{x,i\neq j} (U_{x,i} U_{x+\hat{i},j} U^{\dagger}_{x+\hat{j},i} U^{\dagger}_{x,j} + \text{h.c.})$$

operates in an infinite-dimensional Hilbert space per link

Quantum link formulation of $U(1)$ lattice gauge theory



$$U = S^+, \ U^\dagger = S^-$$

Electric field operator E

$$E = S^3, \ [E, U] = U, \ [E, U^\dagger] = -U^\dagger, \ [U, U^\dagger] = 2E$$

Generator of $U(1)$ gauge transformations

$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

$U(1)$ gauge invariant Hamiltonian

$$H = \frac{e^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2e^2} \sum_{x,i\neq j} (U_{x,i} U_{x+\hat{i},j} U^\dagger_{x+\hat{j},i} U^\dagger_{x,j} + \text{h.c.})$$

operates in a finite-dimensional Hilbert space per link

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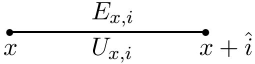
$U(1)$ quantum links from spins $\frac{1}{2}$



$U = S^1 + iS^2 = S^+, \ U^\dagger = S^1 - iS^2 = S^-$

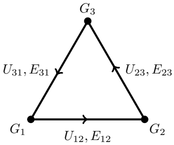
Electric flux operator E

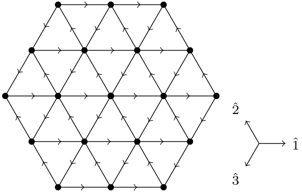
$$E = S^3, \ [E, U] = U, \ [E, U^\dagger] = -U^\dagger, \ [U, U^\dagger] = 2E$$

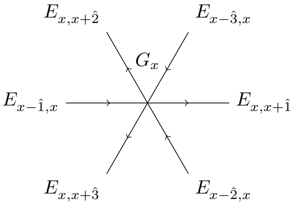
Ring-exchange plaquette Hamiltonian

$$H = -\frac{1}{2e^2} \sum_{\triangle} (U_\triangle + U_\triangle^\dagger) \ , \quad U_\triangle = U_{12} U_{23} U_{31}$$

Triangular lattice and Gauss law





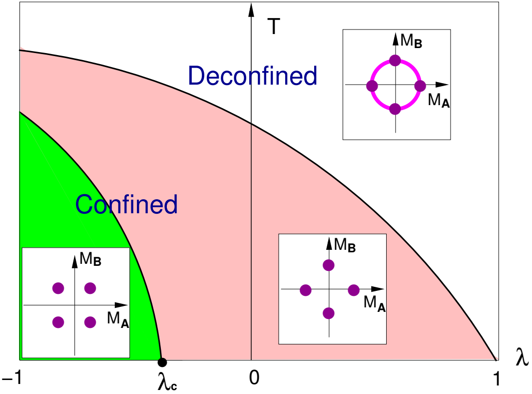


D. Horn, Phys. Lett. B100 (1981) 149
P. Orland, D. Rohrlich, Nucl. Phys. B338 (1990) 647
S. Chandrasekharan, UJW, Nucl. Phys. B492 (1997) 455

Hamiltonian with Rokhsar-Kivelson term

$$H = -\frac{1}{2e^2}\left[\sum_{\triangle}(U_{\triangle} + U_{\triangle}^{\dagger}) - \lambda \sum_{\triangle}(U_{\triangle} + U_{\triangle}^{\dagger})^2\right]$$

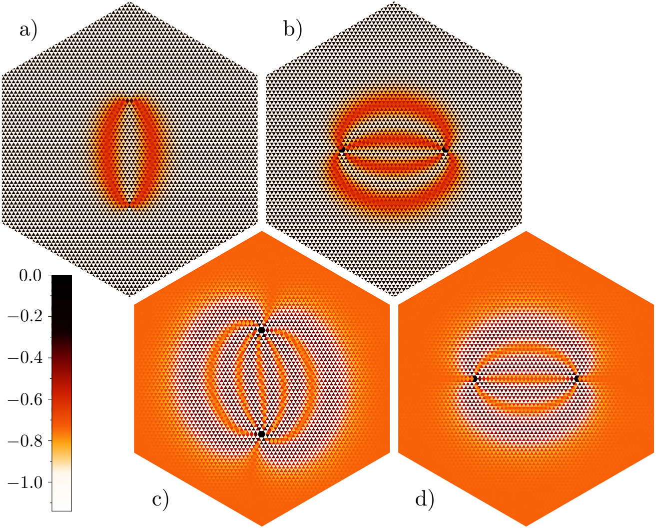
Phase diagram



D. Banerjee, S. Caspar, F.-J. Jiang, J.-H. Peng, UJW, Phys. Rev. Research 4 (2022) 023176

Energy distribution for the strings connecting external charges

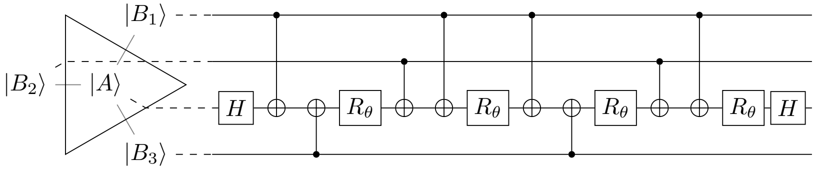
Charges ± 1 (a) and ± 2 (b) for $\lambda > \lambda_c$



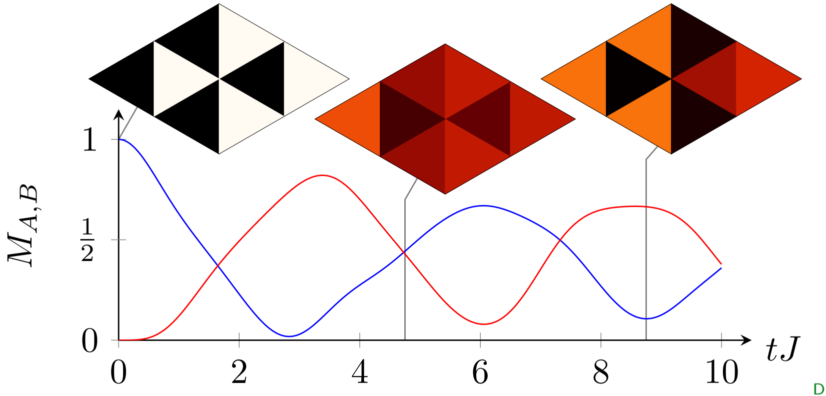
Charges ± 3 (c) and ± 2 (d) for $\lambda < \lambda_c$

D. Banerjee, S. Caspar, F.-J. Jiang, J.-H. Peng, UJW, Phys. Rev. Research 4 (2022) 023176

Circuit decomposition of the time-evolution operator



Time-evolution of the order parameters



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$U(N)$ quantum link operators

$$U^{ij} = S_1^{ij} + iS_2^{ij}, \ U^{ij\dagger} = S_1^{ij} - iS_2^{ij}, \ i,j \in \{1,2,\ldots,N\}, \ [U^{ij}, (U^\dagger)^{kl}] \neq 0$$

$SU(N)_L \times SU(N)_R$ gauge transformations of a quantum link

$$[L^a, L^b] = if_{abc}L^c, \ [R^a, R^b] = if_{abc}R^c, \ a,b,c \in \{1,2,\ldots,N^2-1\}$$

$$[L^a, R^b] = [L^a, E] = [R^a, E] = 0$$

Infinitesimal gauge transformations of a quantum link

$$[L^a, U] = -\lambda^a U, \ [R^a, U] = U\lambda^a, \ [E, U] = U$$

Algebraic structures of different quantum link models

$U(N) : U^{ij}, \ L^a, \ R^a, E, \ 2N^2 + 2(N^2-1) + 1 = 4N^2 - 1$ $SU(2N)$ generators

$SO(N) : O^{ij}, \ L^a, \ R^a, \ N^2 + 2\frac{N(N-1)}{2} = N(2N-1)$ $SO(2N)$ generators

$Sp(N) : U^{ij}, \ L^a, \ R^a, \ 4N^2 + 2N(2N+1) = 2N(4N+1)$ $Sp(2N)$ generators

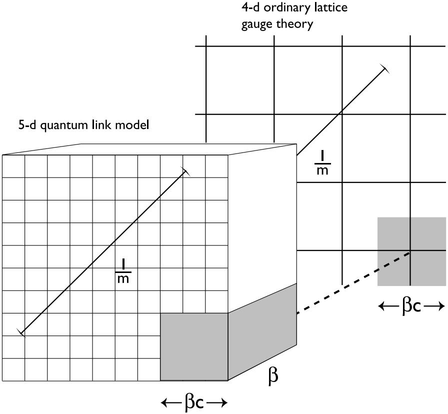
R. Brower, S. Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502

Low-energy effective action of a quantum link model

$$S[G_\mu] = \int_0^\beta dx_5 \int d^4x \, \frac{1}{2e^2} \left(\mathrm{Tr}\; G_{\mu\nu} G_{\mu\nu} + \frac{1}{c^2} \mathrm{Tr}\; \partial_5 G_\mu \partial_5 G_\mu \right), \; G_5 = 0$$

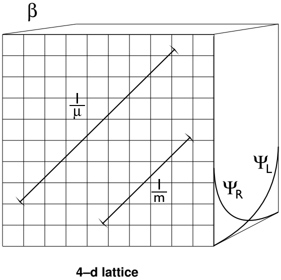
undergoes dimensional reduction from $4+1$ to 4 dimensions

$$S[G_\mu] \to \int d^4x \, \frac{1}{2g^2} \mathrm{Tr}\; G_{\mu\nu} G_{\mu\nu}, \; \frac{1}{g^2} = \frac{\beta}{e^2}, \; \frac{1}{m} \sim \exp\left(\frac{24\pi^2\beta}{11Ne^2}\right)$$



Quarks as Domain Wall Fermions

$$
\begin{aligned}
H &= J \sum_{x,\mu\neq\nu} \mathrm{Tr}[U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu}] + J' \sum_{x,\mu} [\det U_{x,\mu} + \det U^\dagger_{x,\mu}] \\
&+ \frac{1}{2} \sum_{x,\mu} [\Psi^\dagger_x \gamma_0 \gamma_\mu U_{x,\mu} \Psi_{x+\hat{\mu}} - \Psi^\dagger_{x+\hat{\mu}} \gamma_0 \gamma_\mu U^\dagger_{x,\mu} \Psi_x] + M \sum_x \Psi^\dagger_x \gamma_0 \Psi_x \\
&+ \frac{r}{2} \sum_{x,\mu} [2\Psi^\dagger_x \gamma_0 \Psi_x - \Psi^\dagger_x \gamma_0 U_{x,\mu} \Psi_{x+\hat{\mu}} - \Psi^\dagger_{x+\hat{\mu}} \gamma_0 U^\dagger_{x,\mu} \Psi_x].
\end{aligned}
$$



$$
\mu = 2M \exp(-M\beta), \ \frac{1}{m} \propto \exp(\frac{24\pi^2\beta}{(11N - 2N_f)e^2}), \ M > \frac{24\pi^2}{(11N - 2N_f)e^2}
$$

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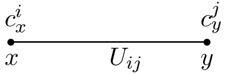
Fermionic rishons at the two ends of a link

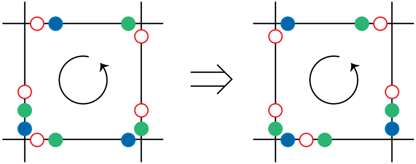
$$\{c_x^i, c_y^{j\dagger}\} = \delta_{xy}\delta_{ij}, \ \{c_x^i, c_y^j\} = \{c_x^{i\dagger}, c_y^{j\dagger}\} = 0$$

Rishon representation of link algebra

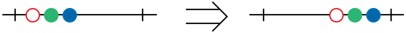


$$U_{xy}^{ij} = c_x^i c_y^{j\dagger}, \ L_{xy}^a = c_x^{i\dagger}\lambda_{ij}^a c_x^j, \ R_{xy}^a = c_y^{i\dagger}\lambda_{ij}^a c_y^j, \ E_{xy} = \frac{1}{2}(c_y^{i\dagger}c_y^i - c_x^{i\dagger}c_x^i)$$

Can a "rishon abacus" implemented with ultra-cold atoms be used as a quantum simulator?

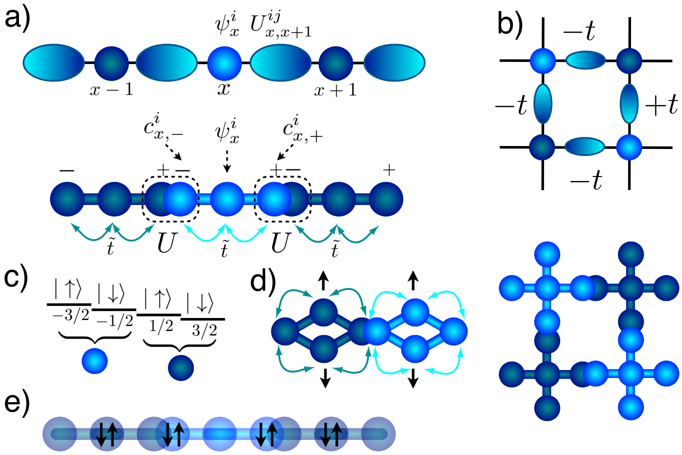


Tr Up



det $U_{x,\mu}$

Optical lattice with ultra-cold alkaline-earth atoms (^{87}Sr or ^{173}Yb) with color encoded in nuclear spin



D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303

Conclusions

• Quantum link models provide an alternative formulation of lattice gauge theory with a finite-dimensional Hilbert space per link, which allows implementations with ultra-cold atoms in optical lattices.

• Quantum simulator constructions have already been presented for Wilson's lattice gauge theory as well as for the $U(1)$ quantum link model with fermionic matter using ultra-cold Bose-Fermi mixtures. $\mathbb{C}P(N-1)$ models as well as non-Abelian $U(N)$ and $SU(N)$ quantum link models can be embodied by alkaline-earth atoms.

• This allows the quantum simulation of the real-time evolution of string breaking as well as false vacuum decay. Accessible effects also include chiral symmetry restoration at high baryon density or the expansion of a hot quark-gluon plasma.

• In quantum spin and quantum link models regularizing asymptotically free theories, including $(1+1)$-d $\mathbb{C}P(N-1)$ models and $(3+1)$-d QCD, the continuum limit is taken by dimensional reduction of discrete variables.

• The path towards quantum simulation of QCD will be a long one.

However, with a lot of interesting physics along the way.