YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

EXPLORING the MATTER in the CORE of NEUTRON STARS

Wolfram Weise
Technische Universität München
PHYSIK DEPARTMENT

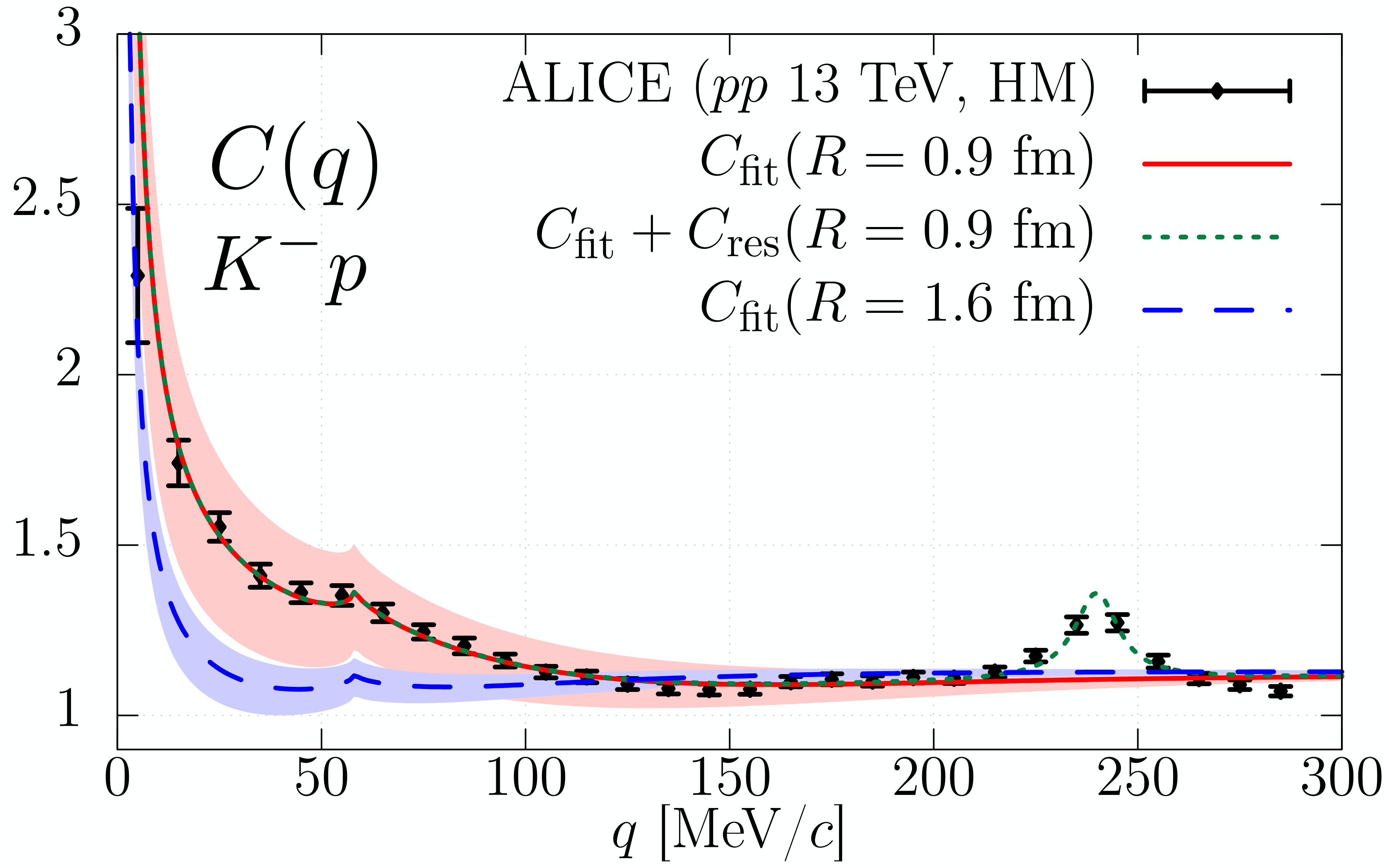
... in memory of

Akira Ohnishi

(June 14, 1964 - May 16, 2023)

Physical Review Letters
124, 132501 (2020)

K^-p correlation function from high-energy nuclear collisions and chiral SU(3) dynamics



Yuki Kamiya,[1,*] Tetsuo Hyodo,[2,3] Kenji Morita,[4,5] Akira Ohnishi,[2] and Wolfram Weise[6,7]

The two-particle momentum correlation function of a K^-p pair from high-energy nuclear collisions is evaluated in the $\bar{K}N$-$\pi\Sigma$-$\pi\Lambda$ coupled-channels framework. The effects of all coupled channels together with the Coulomb potential and the threshold energy difference between K^-p and $\bar{K}^0n$ are treated completely for the first time. Realistic potentials based on the chiral SU(3) dynamics are used which fit the available scattering data. The recently measured correlation function is found to be well reproduced by allowing variations of the source size and the relative weight of the source function of $\pi\Sigma$ with respect to that of $\bar{K}N$. The predicted K^-p correlation function from larger systems indicates that the investigation of its source size dependence is useful in providing further constraints in the study of the $\bar{K}N$ interaction.

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EXPLORING the MATTER in the CORE of NEUTRON STARS

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- Dense Matter in Neutron Stars: Speed of Sound and Equation of State
  - Empirical constraints from heavy neutron stars and binary mergers
  - Bayesian inference results and constraints on phase transitions
- Phenomenology and Models for Dense Baryonic Matter
  - Low-energy nucleon structure and a two-scales scenario
  - Hadron-quark continuity and crossover
  - Dense baryonic matter as a (relativistic) Fermi liquid

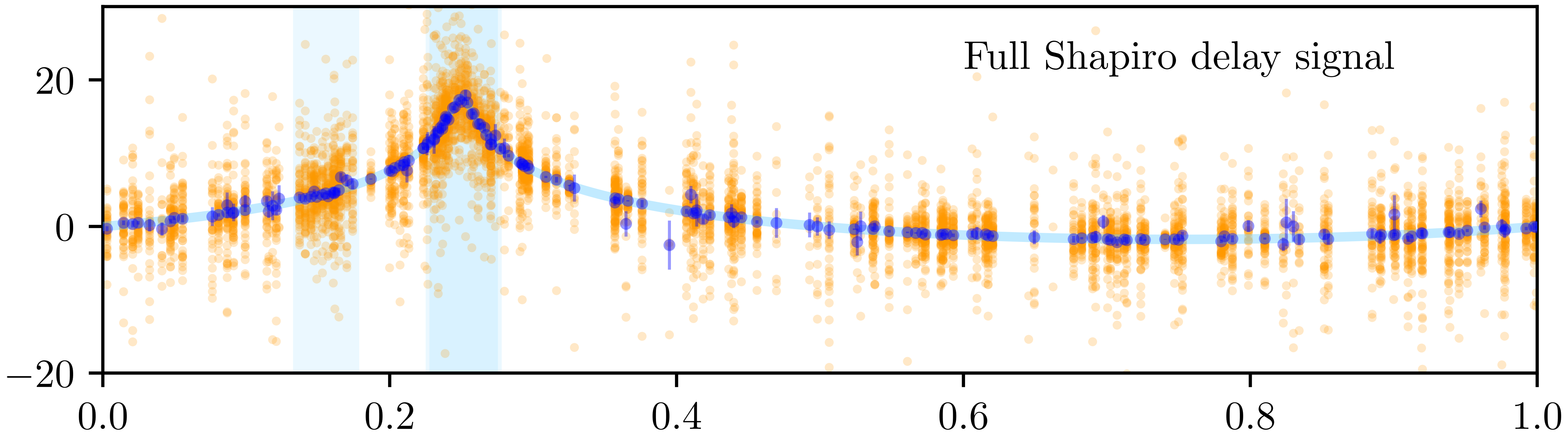
Part One

Equation-of-State of Dense Baryonic Matter :

Empirical Constraints from Neutron Star Observations

NEUTRON STARS : DATA

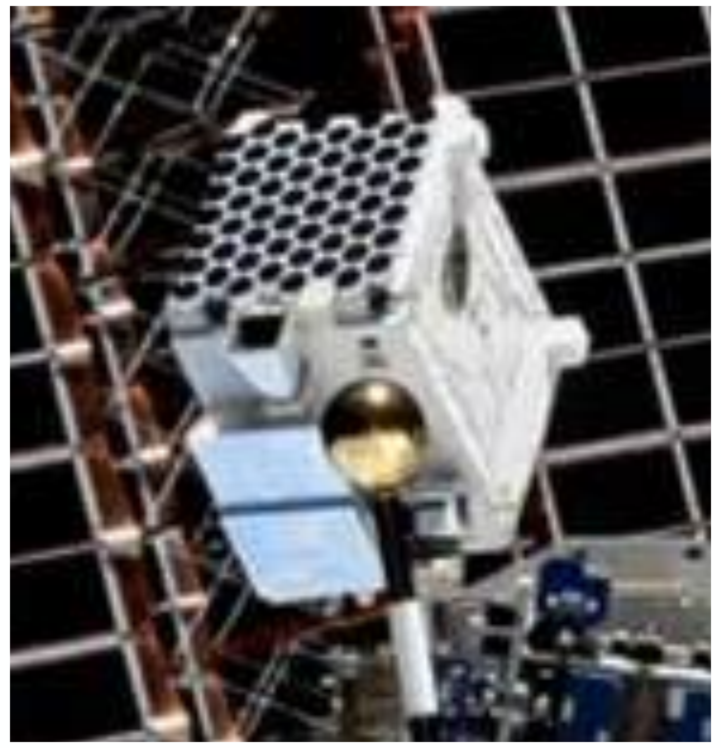
- Database for **inference of Equation-of-State** and other properties of neutron stars



- **Neutron star masses**

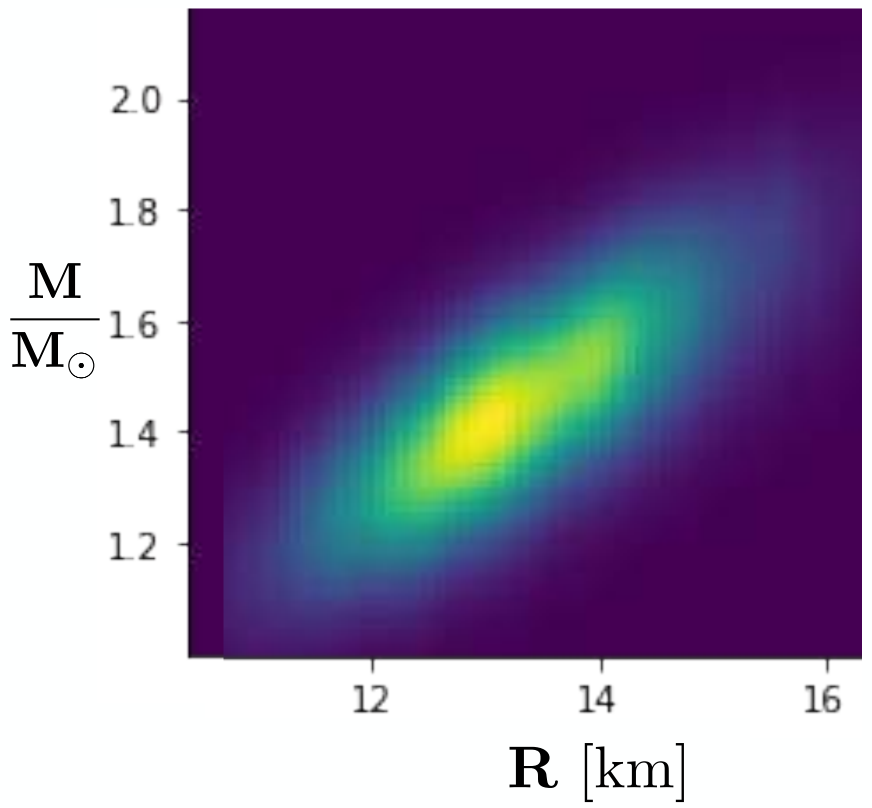
  Shapiro delay measurements
  (Green Bank Telescope)

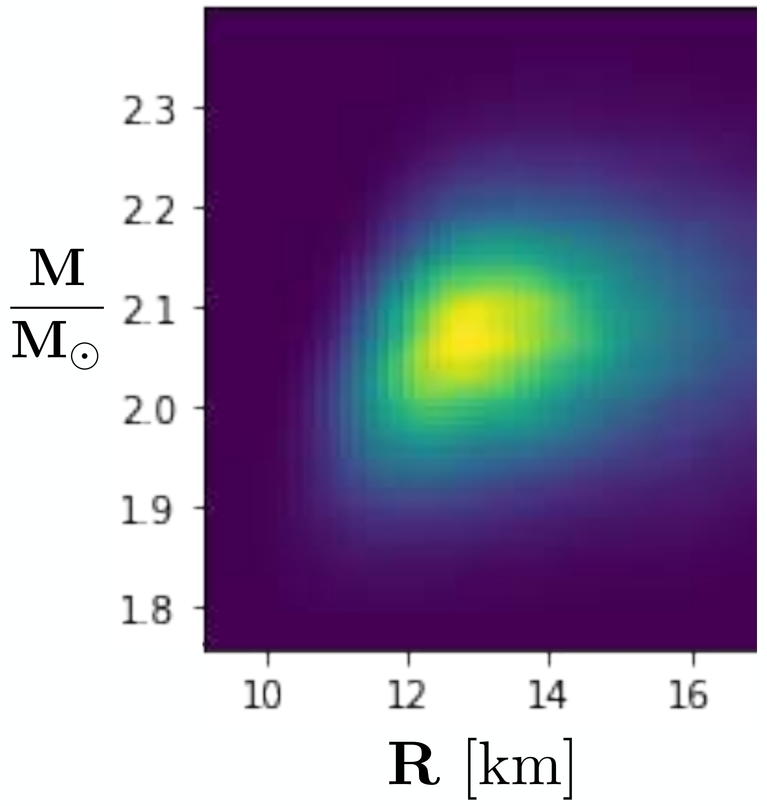
  Radio astronomy
  (Effelsberg)

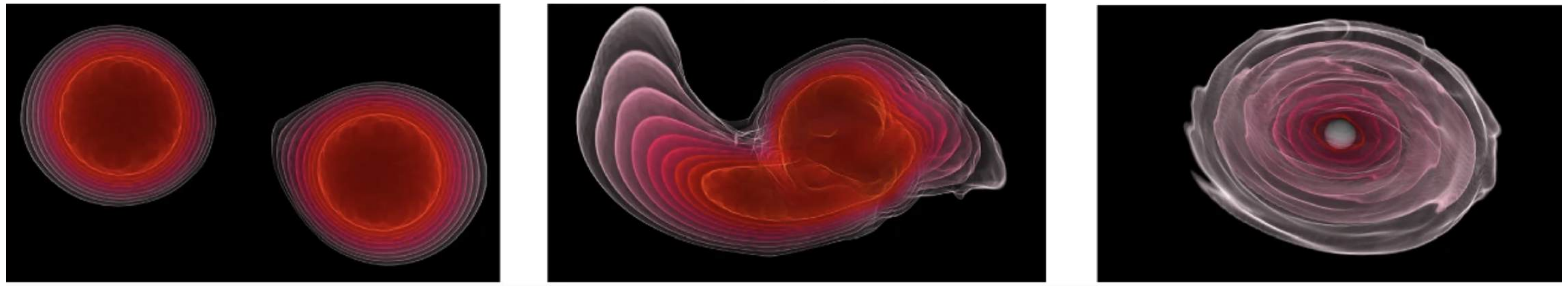


- **Masses and radii**

  X rays from hot spots on the surface of rotating neutron stars
  (NICER Telescope @ ISS)







- **Tidal deformabilities**

  Gravitational wave signals of neutron star mergers
  (LIGO and Virgo Collab.)

NEUTRON STARS : DATA BASE

- **Masses of $2\,M_\odot$ stars**
  (Shapiro delay & radio observations)

PSR J0348+0432

$$M = 2.01 \pm 0.04\ M_\odot$$

J. Antoniadis et al.: Science 340 (2013) 1233232

PSR J1614-2230

$$M = 1.908 \pm 0.016\ M_\odot$$

Z. Arzoumanian et al., Astrophys. J. Suppl. 235 (2018) 37

PSR J0740+6620

$$M = 2.08 \pm 0.07\ M_\odot$$

E. Fonseca et al., Astrophys. J. Lett. 915 (2021) L12

- **Masses and Radii** (NICER)

PSR J0030+0451

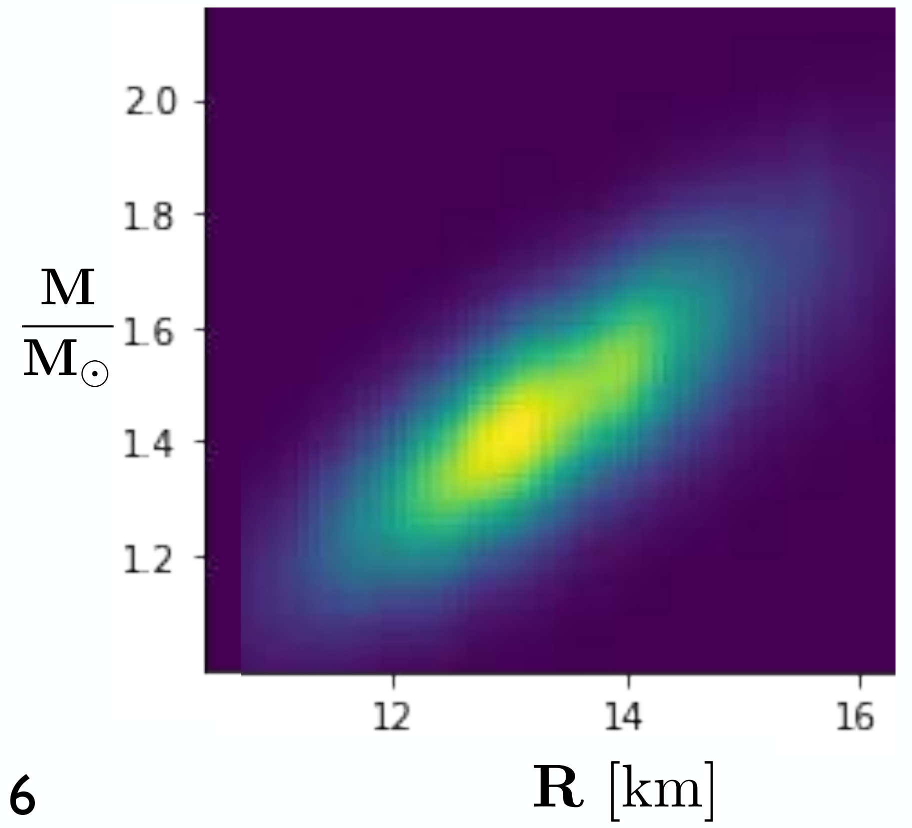
$$M = 1.34 \pm 0.16\, M_\odot \qquad R = 12.71^{+1.14}_{-1.19}\ \mathrm{km}$$

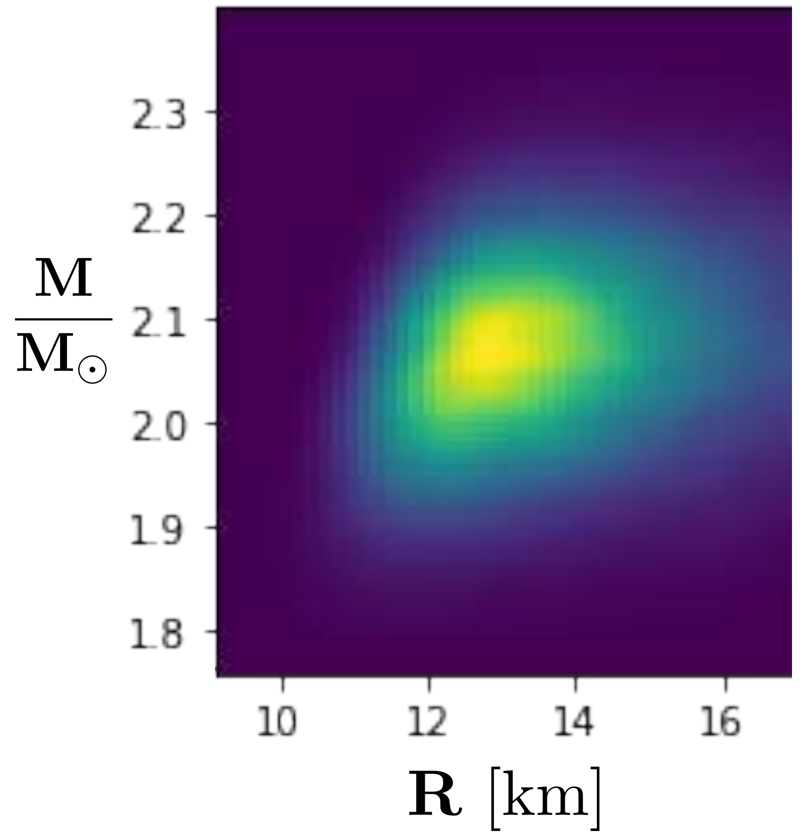
T.E. Riley et al. (NICER), Astroph. J. Lett. 887 (2019) L21

PSR J0740+6620

$$M = 2.073 \pm 0.069\, M_\odot \qquad R = 12.49^{+1.28}_{-0.88}\ \mathrm{km}$$

T.E. Riley et al. (NICER + XMM Newton), Astroph. J. Lett. 918 (2021) L27
T. Salmi et al. (NICER), arXiv:2406.14466





NEUTRON STARS : DATA (contd.)

- **Very massive and fast rotating galactic neutron star**

PSR J0952-0607

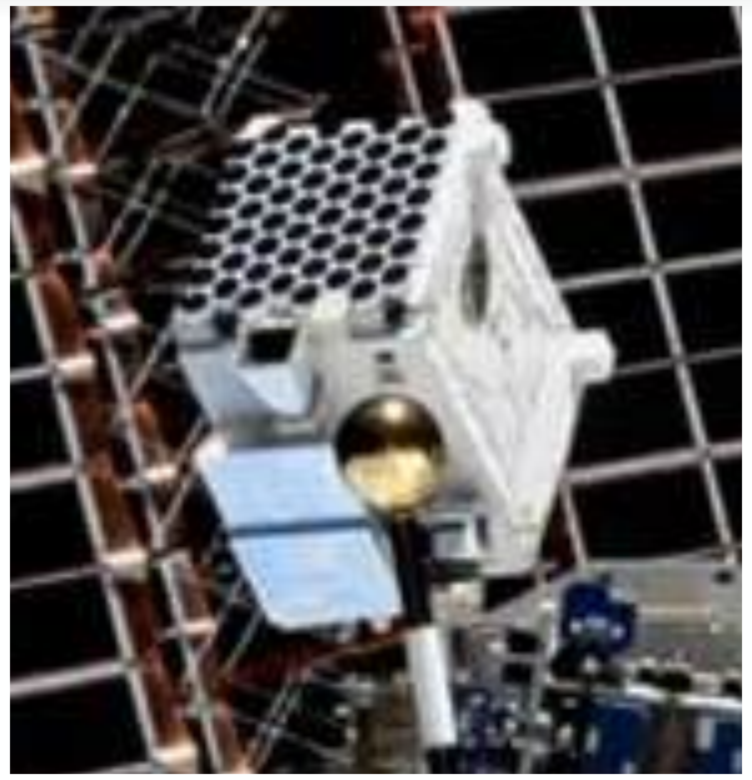
$$M = 2.35 \pm 0.17 \ M_\odot$$

R.W. Romani et al. : Astroph. J. Lett. 934 (2022) L17

(Keck Observatory)

→ **equivalent non-rotating mass after rotational correction :** $M = 2.3 \pm 0.2 \ M_\odot$

- **New accurate data from NICER**



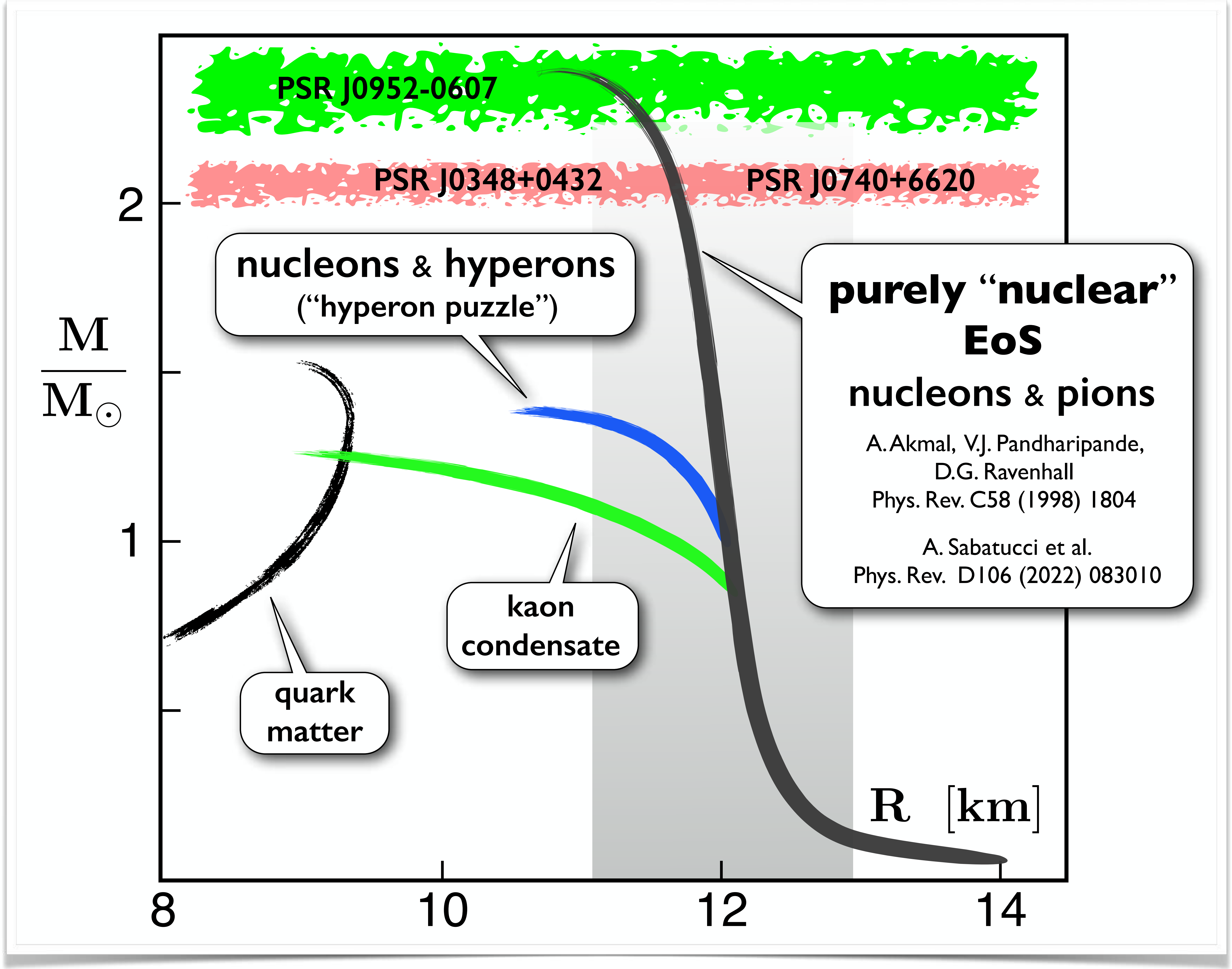
PSR J0437-4715

$$M = 1.418 \pm 0.037 \, M_\odot \qquad R = 11.36^{+0.95}_{-0.63} \ \mathrm{km}$$

D. Choudhury et al. : Astroph. J. Lett. 971 (2024) L20

CONSTRAINTS on EQUATION of STATE $P(\varepsilon)$

- from observations of massive neutron stars



Tolman - **O**ppenheimer - **V**olkov Equations

$$\frac{dP(r)}{dr} = \frac{G\,[\varepsilon(r) + P(r)]\,[m(r) + 4\pi r^3 P(r)]}{r\,[r - 2\,G\,m(r)]}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

$$M = m(R) = 4\pi \int_0^R dr\, r^2 \varepsilon(r)$$

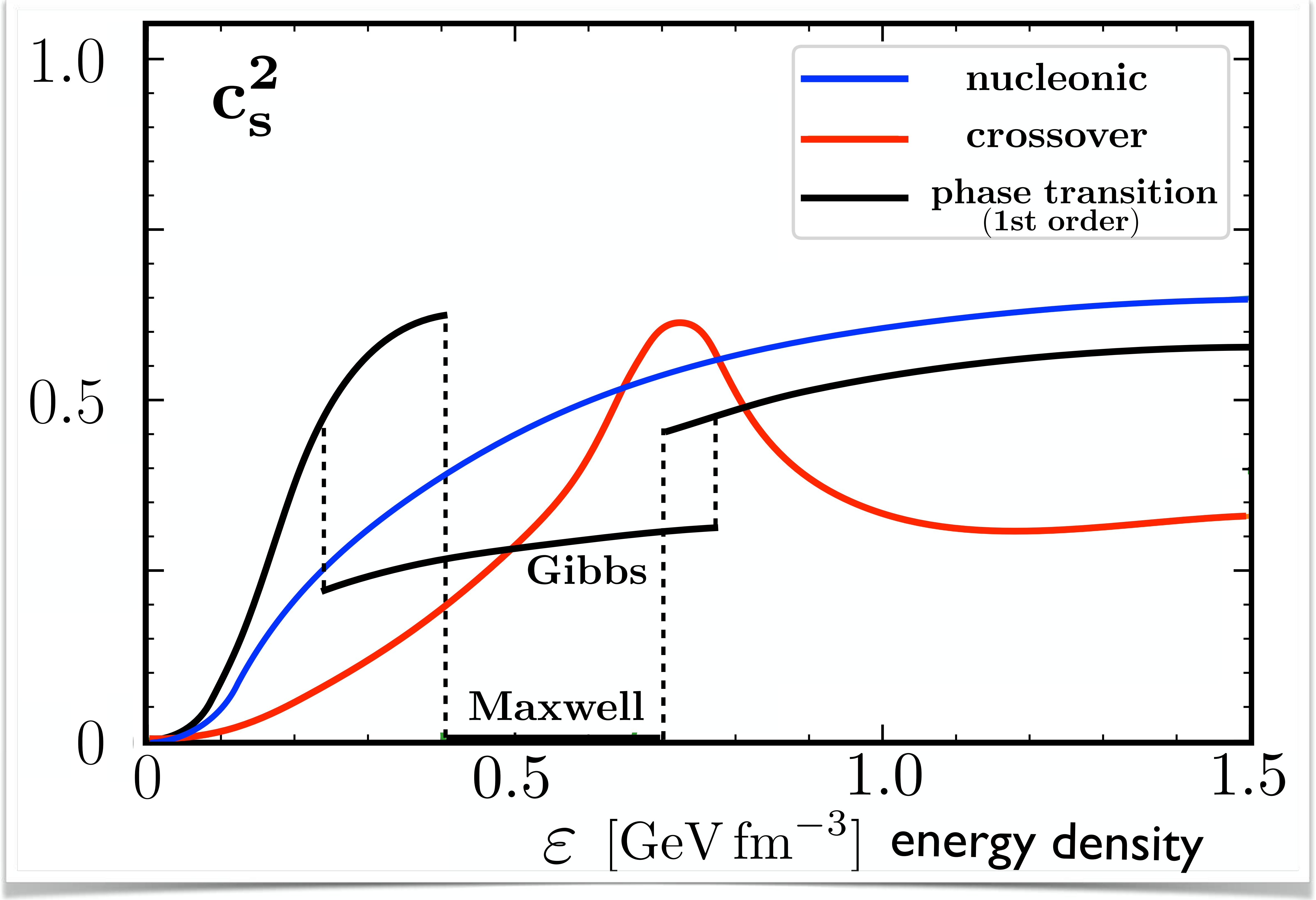
- **Stiff equation-of-state $P(\varepsilon)$ required**
- **Simplest forms of exotic matter** (kaon condensate, quark matter, …) **ruled out**

SOUND VELOCITY and EQUATION of STATE

- Key quantity : **Speed of Sound**

$$c_s^2(\varepsilon) = \frac{\partial P(\varepsilon)}{\partial \varepsilon}$$

displays characteristic signature of **phase transition** or **crossover**



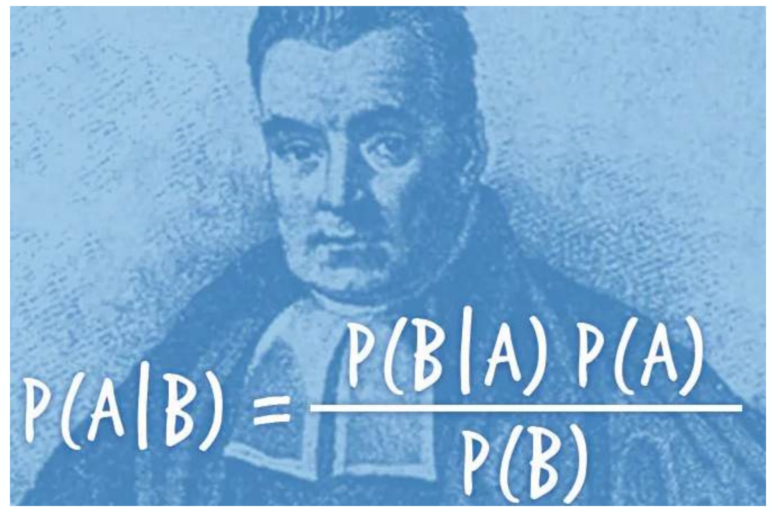
- Equation of State :

$$P(\varepsilon) = \int_0^{\varepsilon} d\varepsilon' \, c_s^2(\varepsilon')$$

- Gibbs - Duhem equation (T=0)

$$P + \varepsilon = \mu_B \, n_B = \sum_i \mu_i \, n_i$$

- Baryon density $n_B = \partial P / \partial \mu_B$
- Baryon chemical potential $\mu_B = \partial \varepsilon / \partial n_B$



INFERENCE of SOUND SPEED and RELATED PROPERTIES of NEUTRON STARS

- Introduce **general parametrization of sound velocity** : segment-wise representation

$$c_s^2(\varepsilon,\theta) = \frac{(\varepsilon_{i+1}-\varepsilon)c_{s,i}^2 + (\varepsilon-\varepsilon_i)c_{s,i+1}^2}{\varepsilon_{i+1}-\varepsilon_i}\ , \quad \text{parameter set} \quad \theta = (c_{s,i}^2, \varepsilon_i) \quad (i = 1,\dots,N)$$

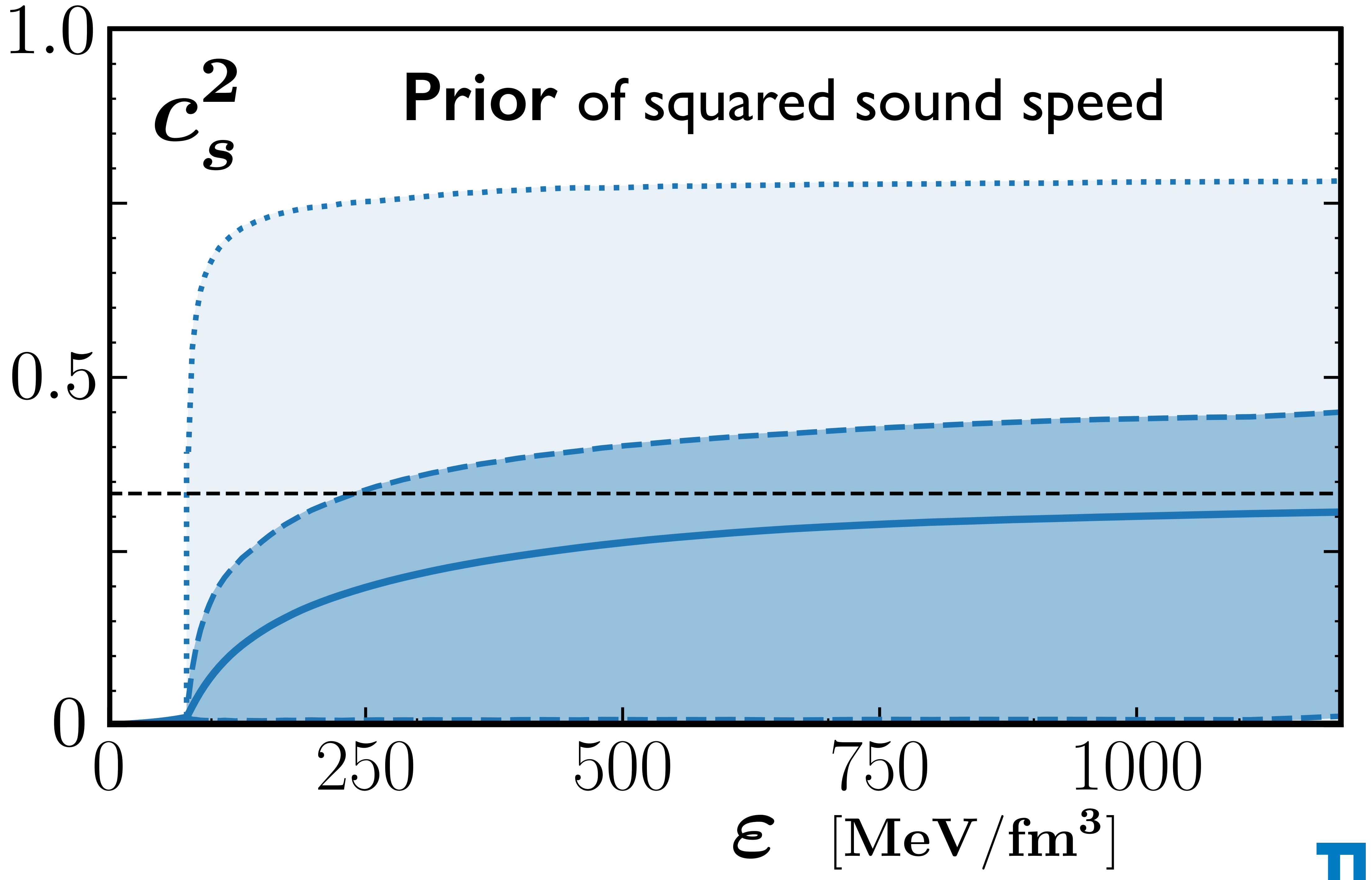
- Constrain parameters θ by **Bayesian inference** using **nuclear** and **astrophysical data** $\mathscr{D}$

$$\Pr(\theta|\mathscr{D}) \propto \Pr(\mathscr{D}|\theta)\,\Pr(\theta)$$

→ Choose **Prior** $\Pr(\theta)$

→ Compute **Posterior** $\Pr(\theta|\mathscr{D})$
from **Likelihood** $\Pr(\mathscr{D}|\theta)$

→ Quantify **Evidences** for hypotheses H_0 vs. H_1
in terms of **Bayes factors** $\mathscr{B}^{H_1}_{H_0} = \frac{\Pr(\mathscr{D}|H_1)}{\Pr(\mathscr{D}|H_0)}$



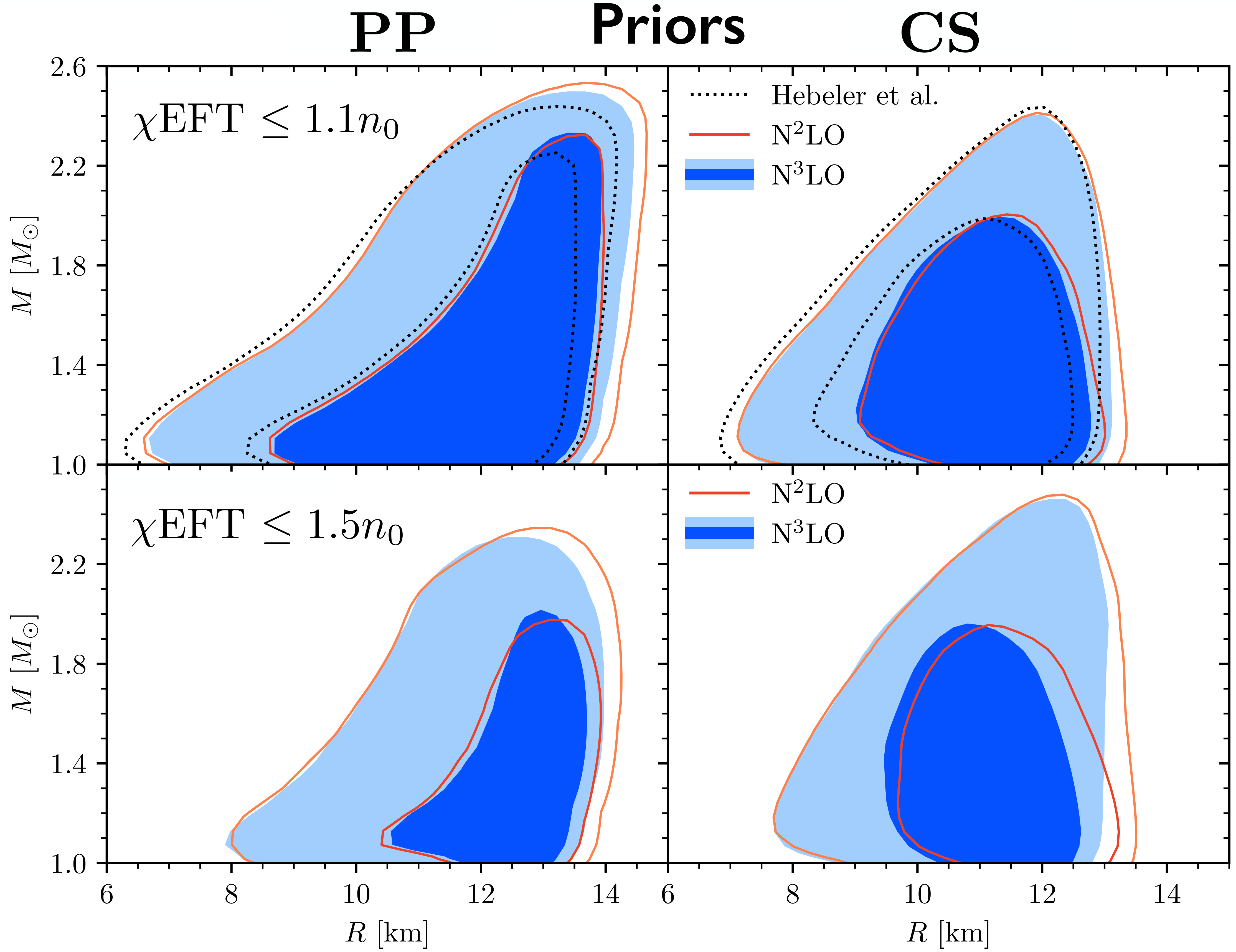
INFERENCE of EQUATION of STATE

- Important issue: Posterior output MUST be independent of chosen Prior input

Recent example where this is not the case :

N. Rutherford et al. : Astrophys. J. Lett. 971 (2024) L19

Priors



(**Chiral EFT** constraints implemented as **Priors**)

- **Prior models** (choices too restrictive)

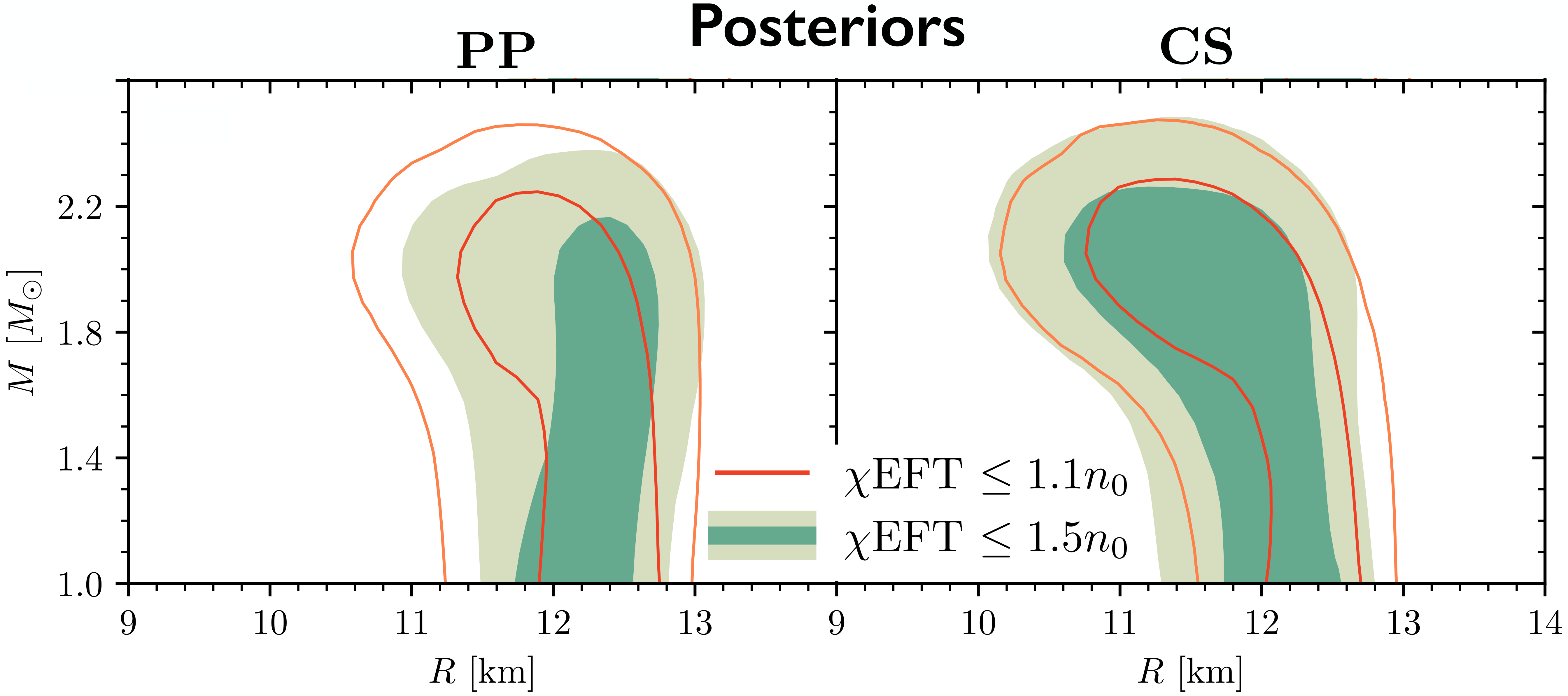
PP : Polytropic parametrization

$$P_i(n) = K_i\,(n/n_0)^{\Gamma_i}$$ (3 segments only)

CS : Gaussian parametrization of sound speed $\left(x = \dfrac{\varepsilon}{M_N n_0}\right)$

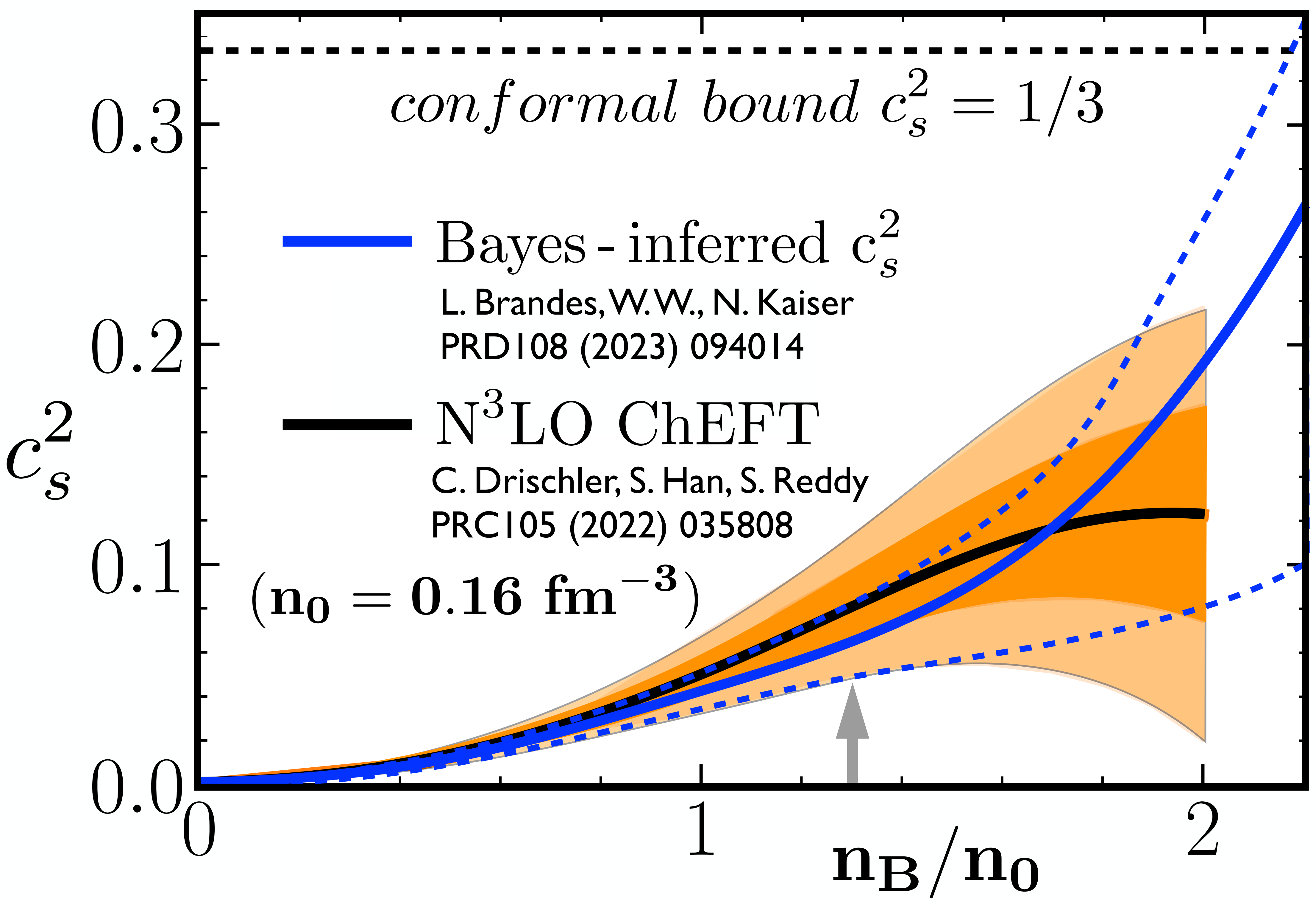
$$c_s^2(x) = a_1 \mathrm{e}^{-\frac{1}{2}(x-a_2)^2/a_3^2} + a_6 + \frac{\frac{1}{3} - a_6}{1 + \mathrm{e}^{-a_5(x-a_4)}}$$

Posteriors

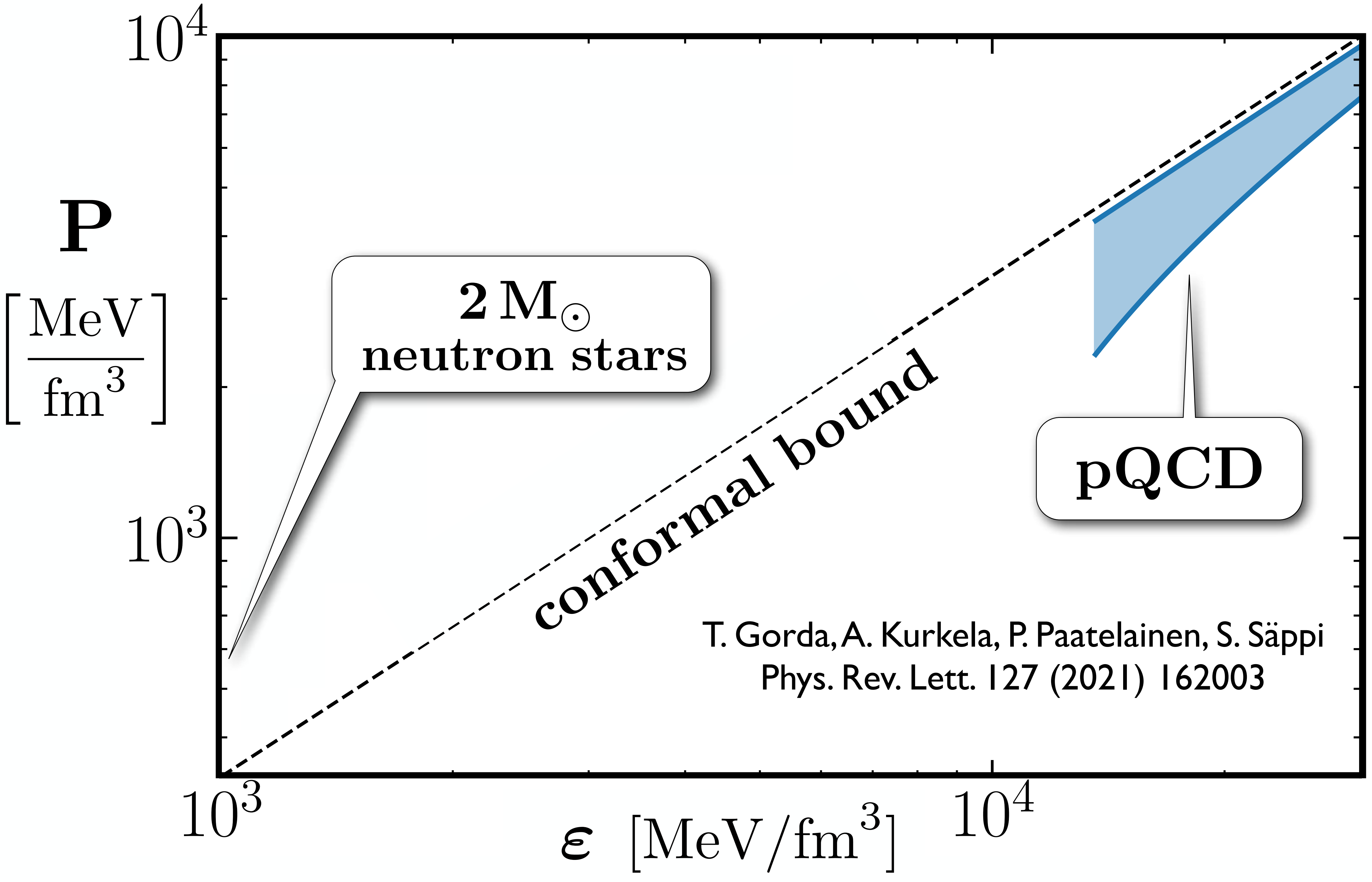


EQUATION of STATE and SOUND VELOCITY
- boundary conditions -

- **Low densities : Chiral EFT @ $n_B \lesssim 2\,n_0$**



- **Extremely high densities : $n_B >> n_c(2M_\odot)$**



- Employ ChEFT constraint at $n_B = 1.3\,n_0$ in Bayes inference as **Likelihood, NOT Prior**

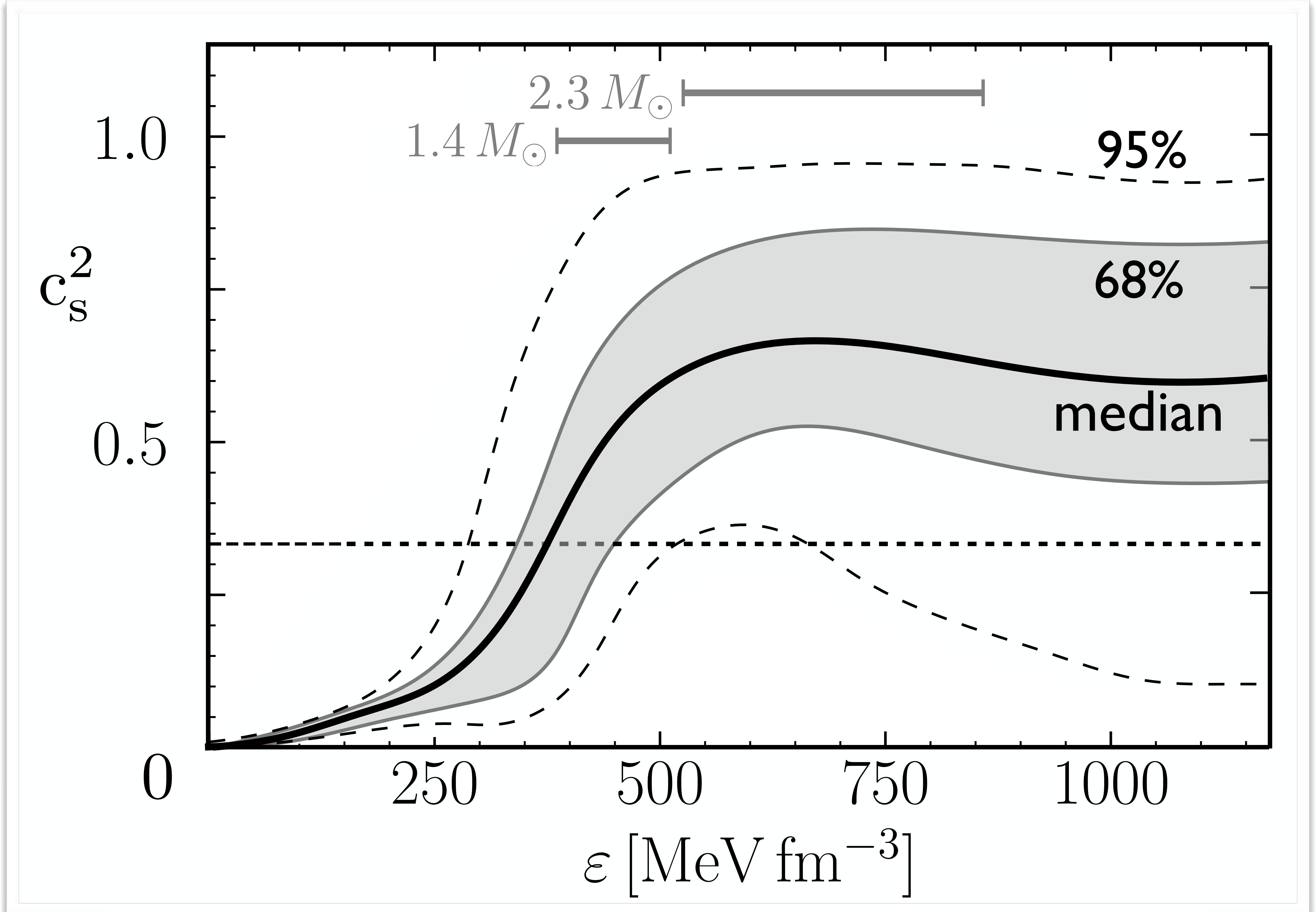
- **Conformal bound** $c_s^2 = \frac{1}{3}$ reached asymptotically

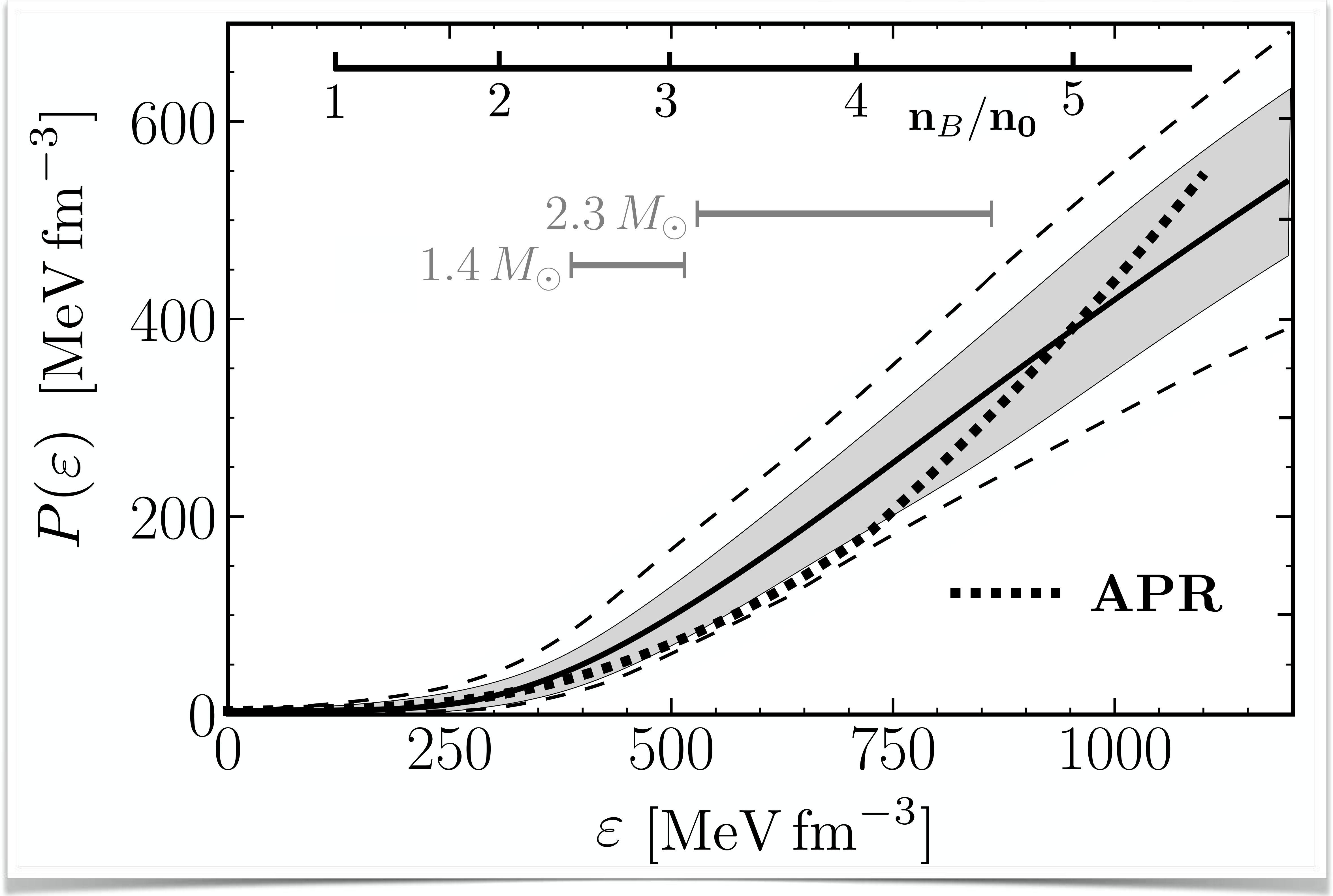
NEUTRON STAR MATTER : EQUATION of STATE

- **Bayesian inference of sound speed and EoS**

PSR masses, NICER & GW data, low-density constraints (ChEFT), **asymptotic constraints** (pQCD)

L. Brandes, W.W. , N. Kaiser : Phys. Rev. D 107 (2023) 014011 ; Phys. Rev. D 108 (2023) 094014 - L. Brandes, W.W. : Symmetry 16 (2024) 111



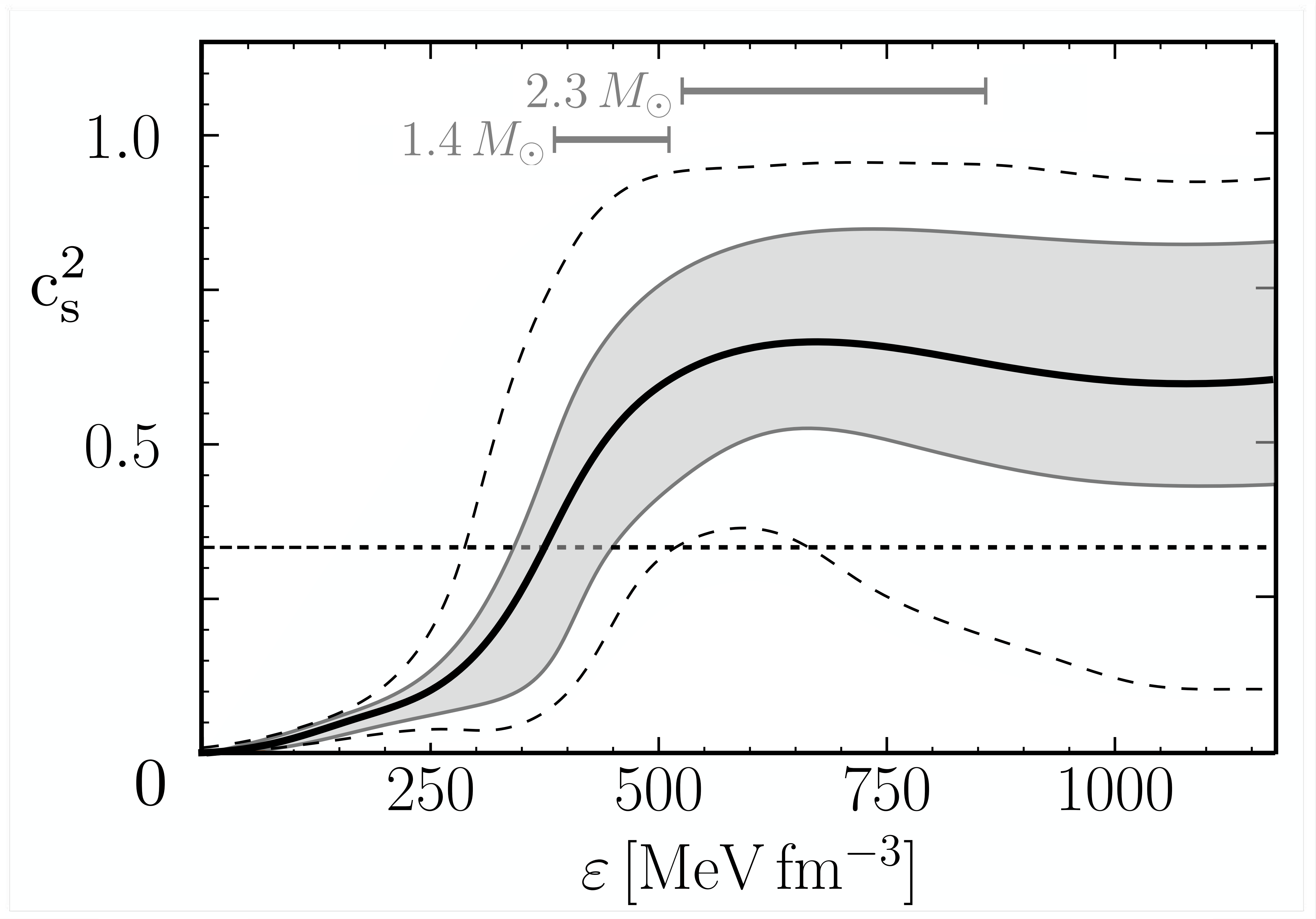


- **Speed of sound** exceeds conformal bound $c_s = 1/\sqrt{3}$ at baryon densities $n_B > 2 - 3\,n_0$
- **Strongly repulsive correlations** in dense baryonic matter

Comment : **SPEED of SOUND** exceeding **CONFORMAL BOUND**

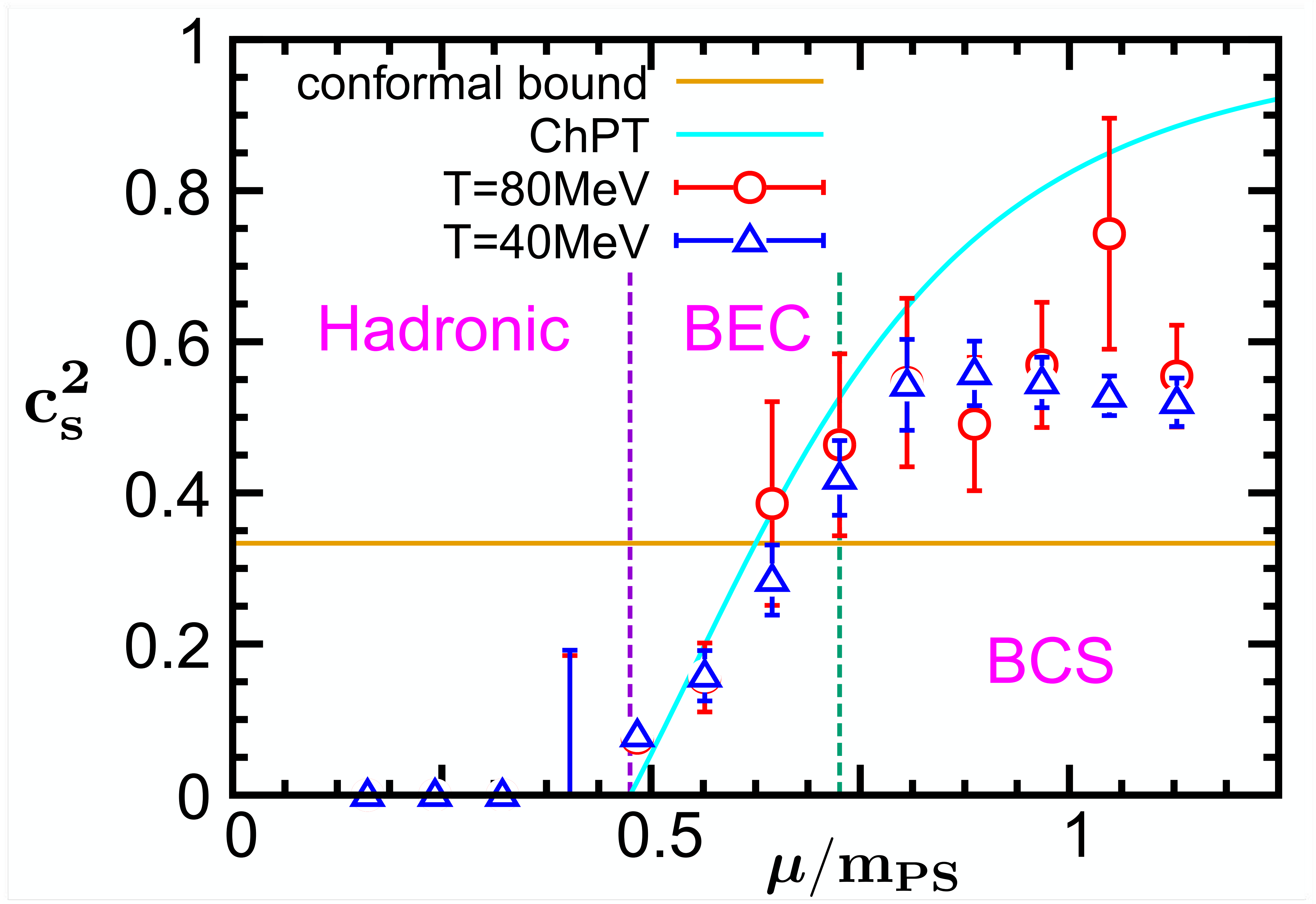
- Bayesian inference of **sound speed** in neutron star matter

L. Brandes, W.W. , N. Kaiser : Phys. Rev. D 108 (2023) 094014



- Sound speed as function of baryon chemical potential in $N_c = 2$ LQCD

K. Iida, E. Itou, K. Murakami, D. Suenaga : arXiv:2405.20566



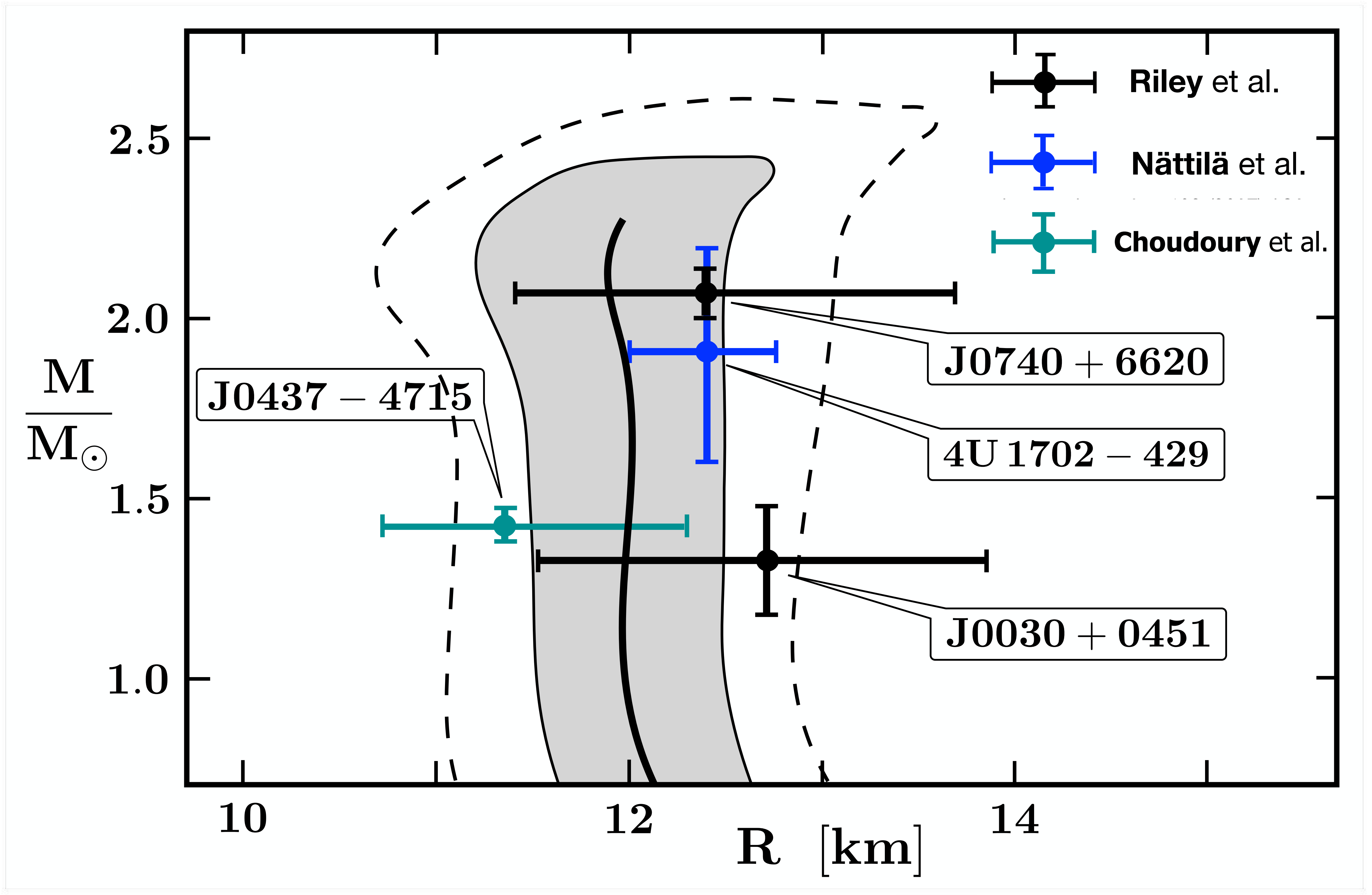
- **Speed of sound** exceeds conformal bound $c_s = 1/\sqrt{3}$ at baryon densities $n_B > 2 - 3\, n_0$

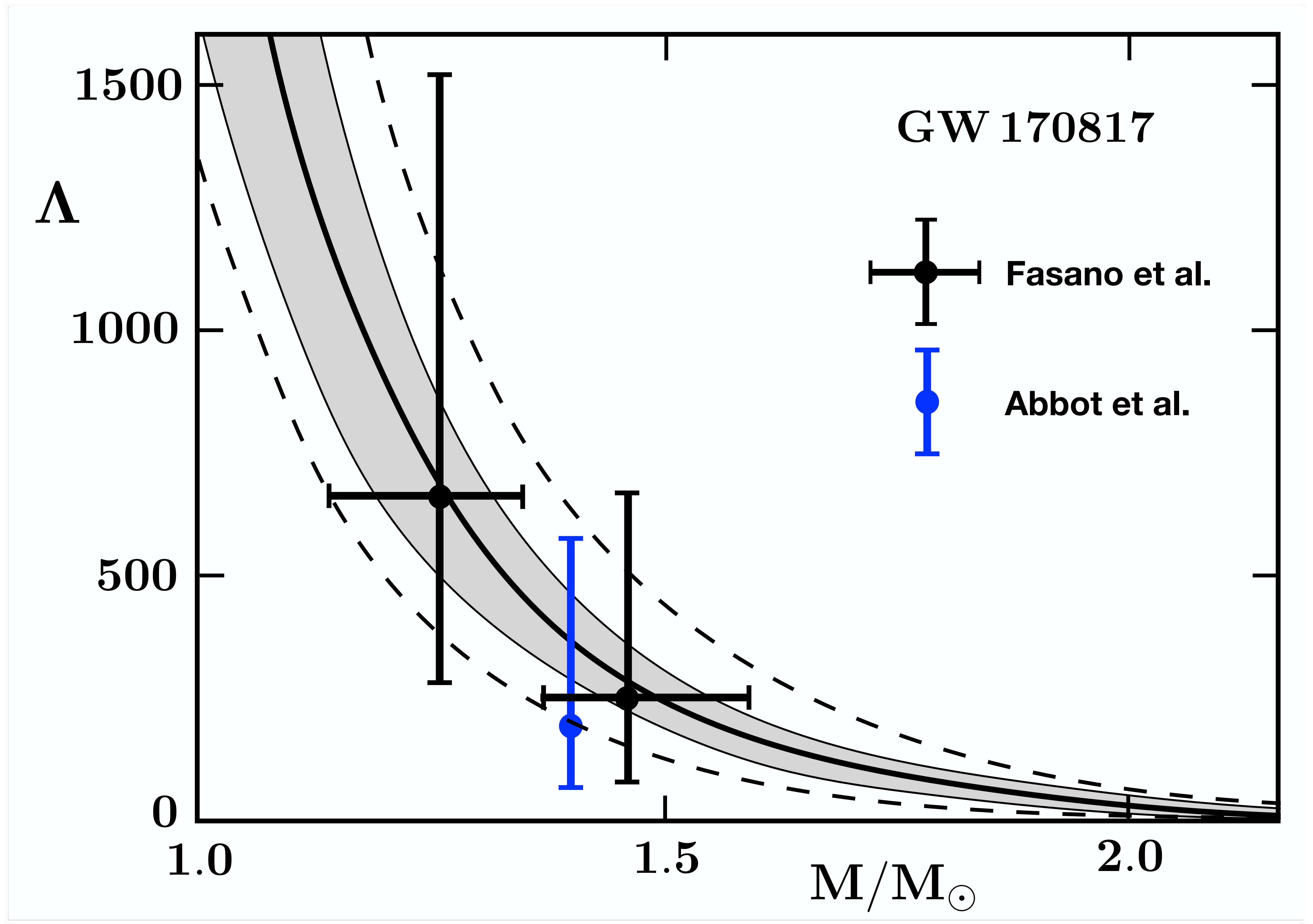
NEUTRON STAR PROPERTIES

- **Bayesian inference posterior bands** (68% and 95% c.l)

median — 68% — 95%

- **Mass - Radius relation (TOV)**
- **Tidal deformability**





L. Brandes, W. W., N. Kaiser : Phys. Rev. D 107 (2023) 014011 ; Phys. Rev. D 108 (2023) 094014 L. Brandes, W. W. (2024)

NEUTRON STAR PROPERTIES (contd.)

- **Baryon chemical potential**

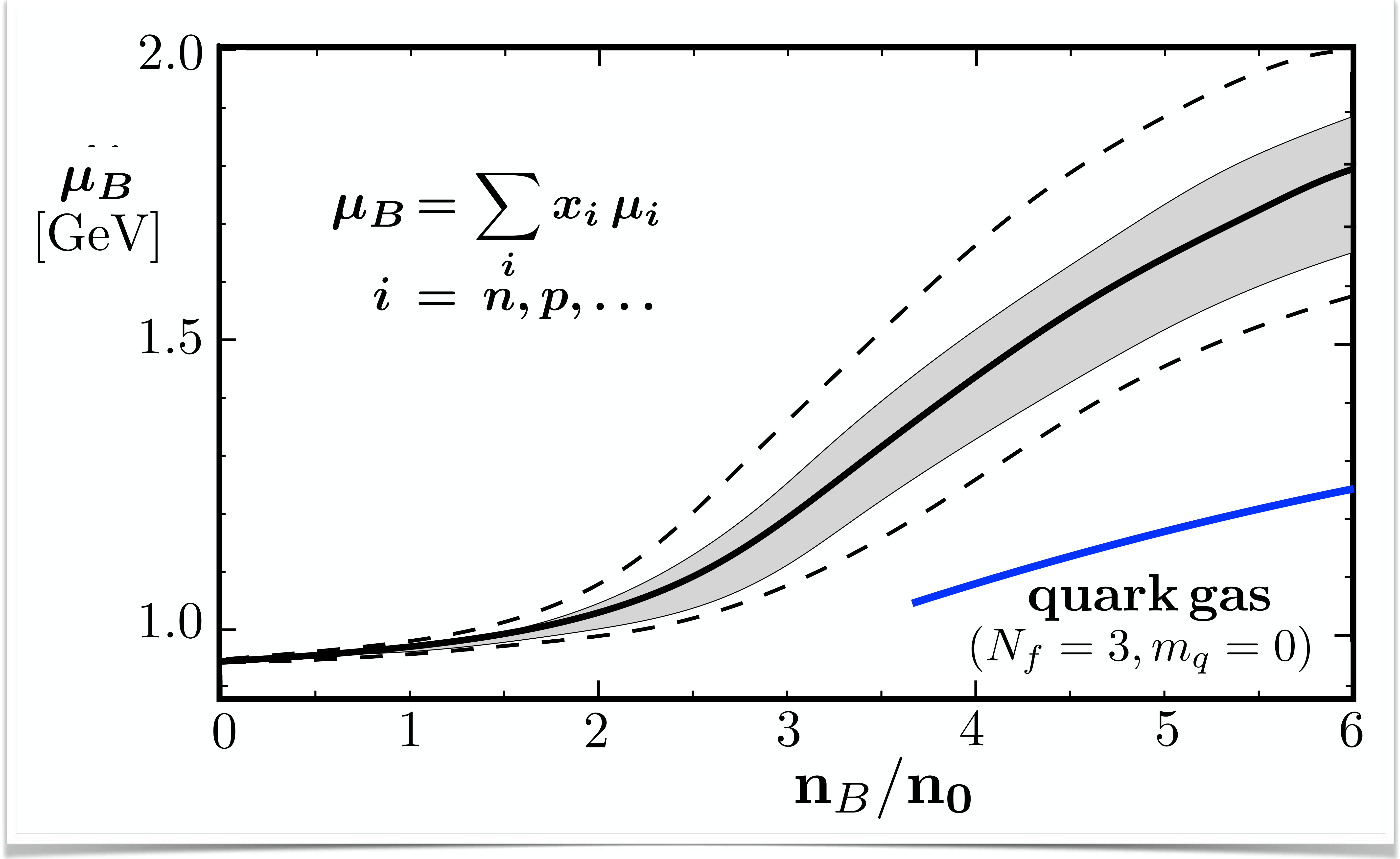
$$\mu_B = \frac{\partial \varepsilon}{\partial n_B}$$

- **Stiff equation of state**

  ↓

  strongly repulsive correlations at work between baryons / quarks

- **Quark gas ruled out at densities $n_B \sim 4 - 6\, n_0$**



L. Brandes, W. W., N. Kaiser : Phys. Rev. D 108 (2023) 094014

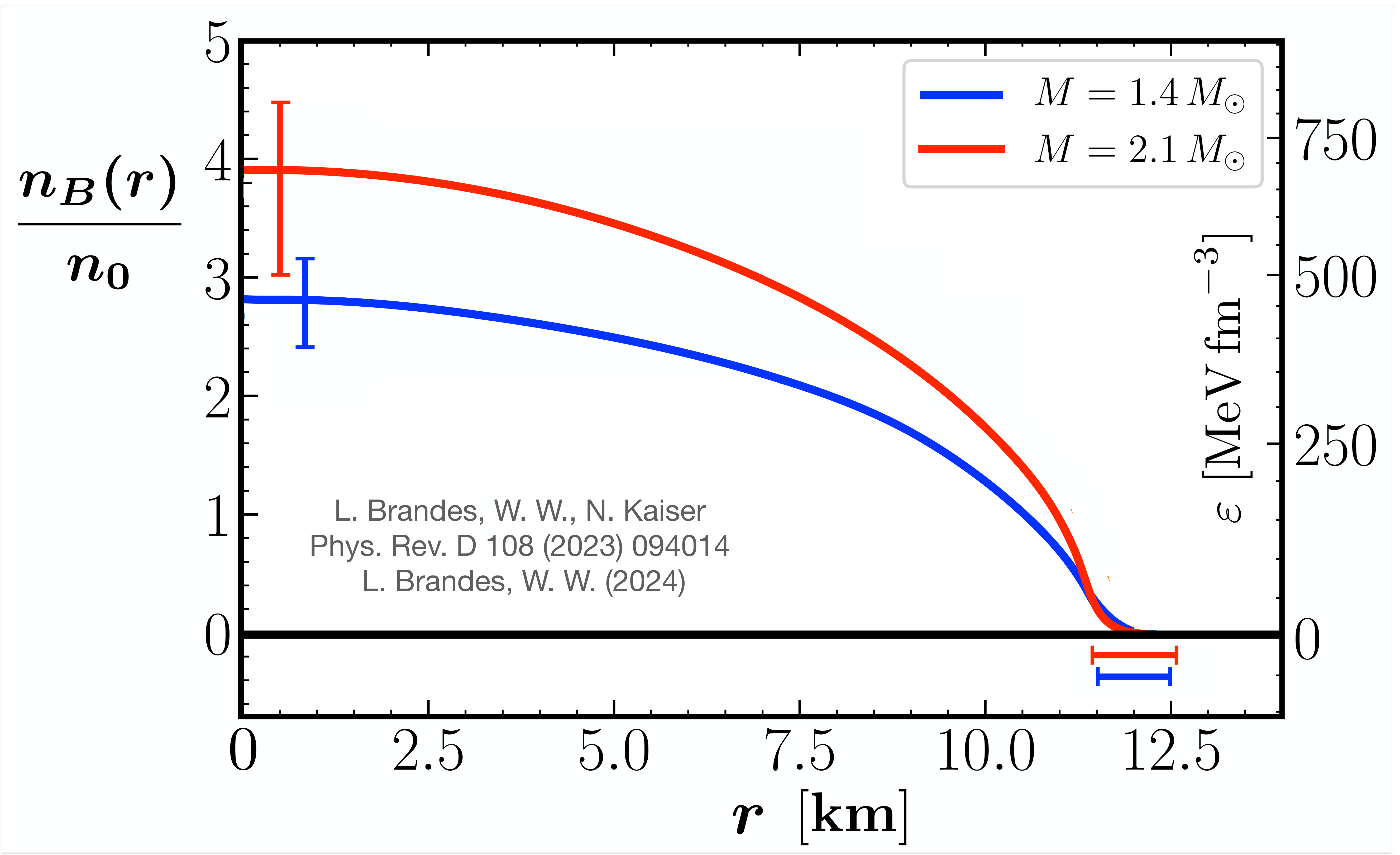
NEUTRON STAR PROPERTIES (contd.)

- **Density profiles of neutron stars**
  using inferred median of $P(\varepsilon)$

- **Central core densities in neutron stars are NOT extreme**
- **Average distance between baryons :**

  $$d \gtrsim 1\,\mathrm{fm}$$

  even for the heaviest neutron stars



$$n_c(1.4\ M_\odot) = 2.8 \pm 0.4\ n_0 \qquad n_c(2.1\ M_\odot) = 3.9^{+0.6}_{-0.9}\ n_0 \qquad n_c(2.3\ M_\odot) = 4.0^{+0.7}_{-0.8}\ n_0$$

(68% c.l. — including new NICER data and "black widow" PSR J0952-0607)

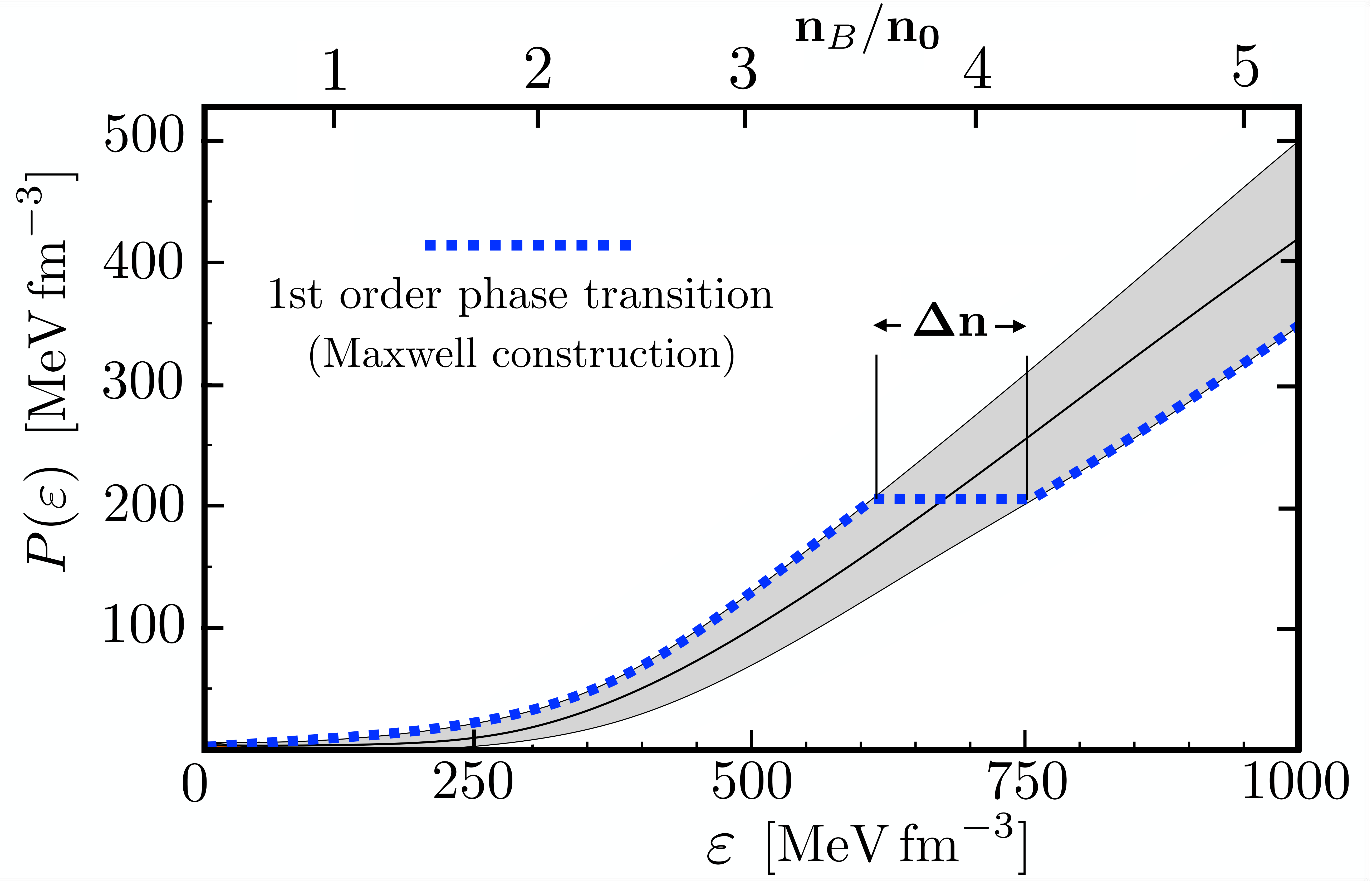
Constraints on
FIRST-ORDER PHASE TRANSITION in NEUTRON STAR MATTER

- Bayes factor analysis :

  → Extreme evidence for sound velocities $c_s > 0.5$ in cores of all neutron stars with
  $1.4 \leq M/M_\odot \leq 2.3$

- Evidence against **strong** 1st order phase transition :

→ Maximum possible extension of phase coexistence domain
$\Delta n/n_B \lesssim 0.2$ (68% c.l.)



L. Brandes, W.W., N. Kaiser : Phys. Rev. D 108 (2023) 094014 - L. Brandes, W.W. : Symmetry 16 (2024) 111

→ For comparison :
Maxwell construction for nuclear 1st order liquid-gas phase transition $(\Delta n/n_B > 1)$

Constraints on **FIRST-ORDER PHASE TRANSITION** in **NEUTRON STAR MATTER** (contd.)

- **Bayes factor analysis :**

  comparison of likelihoods for or against the occurance of small sound speeds

  ➜ quantifying evidence **against low sound velocities**

  $$c_s^2 < 0.1$$

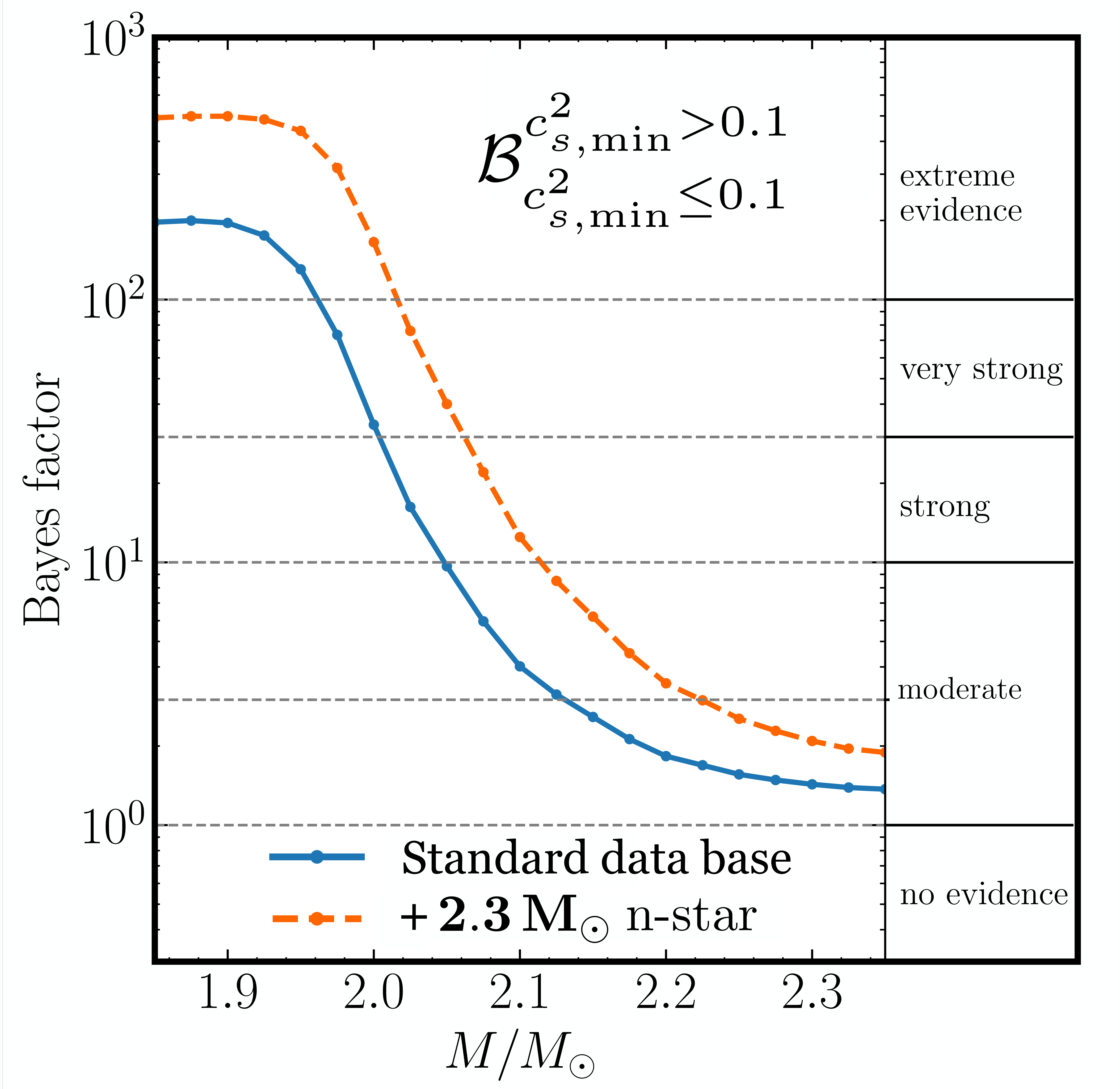
  in neutron stars

  ➜ **extreme evidence** against

  $$c_{s,min}^2 \le 0.1$$

  for all neutron stars in the mass range $M \lesssim 2\, M_\odot$

L. Brandes, W.W., N. Kaiser : Phys. Rev. D 108 (2023) 094014



QCD TRACE ANOMALY and CONFORMALITY in NEUTRON STARS

Y. Fujimoto, K. Fukushima, L.D. McLerran, M. Praszalowicz : Phys. Rev. Lett. 129 (2022) 252702

- **Trace of energy-momentum tensor :** $T^\mu_\mu = \Theta = \frac{\beta}{2g} G^a_{\mu\nu} G^{\mu\nu}_a + (1+\gamma_m) \sum_f m_f \, \bar{q}_f q_f$
- **Finite T and μ_B :**

$$\langle \Theta \rangle_{T,\mu_B} = \varepsilon - 3P$$

- **Trace anomaly measure**

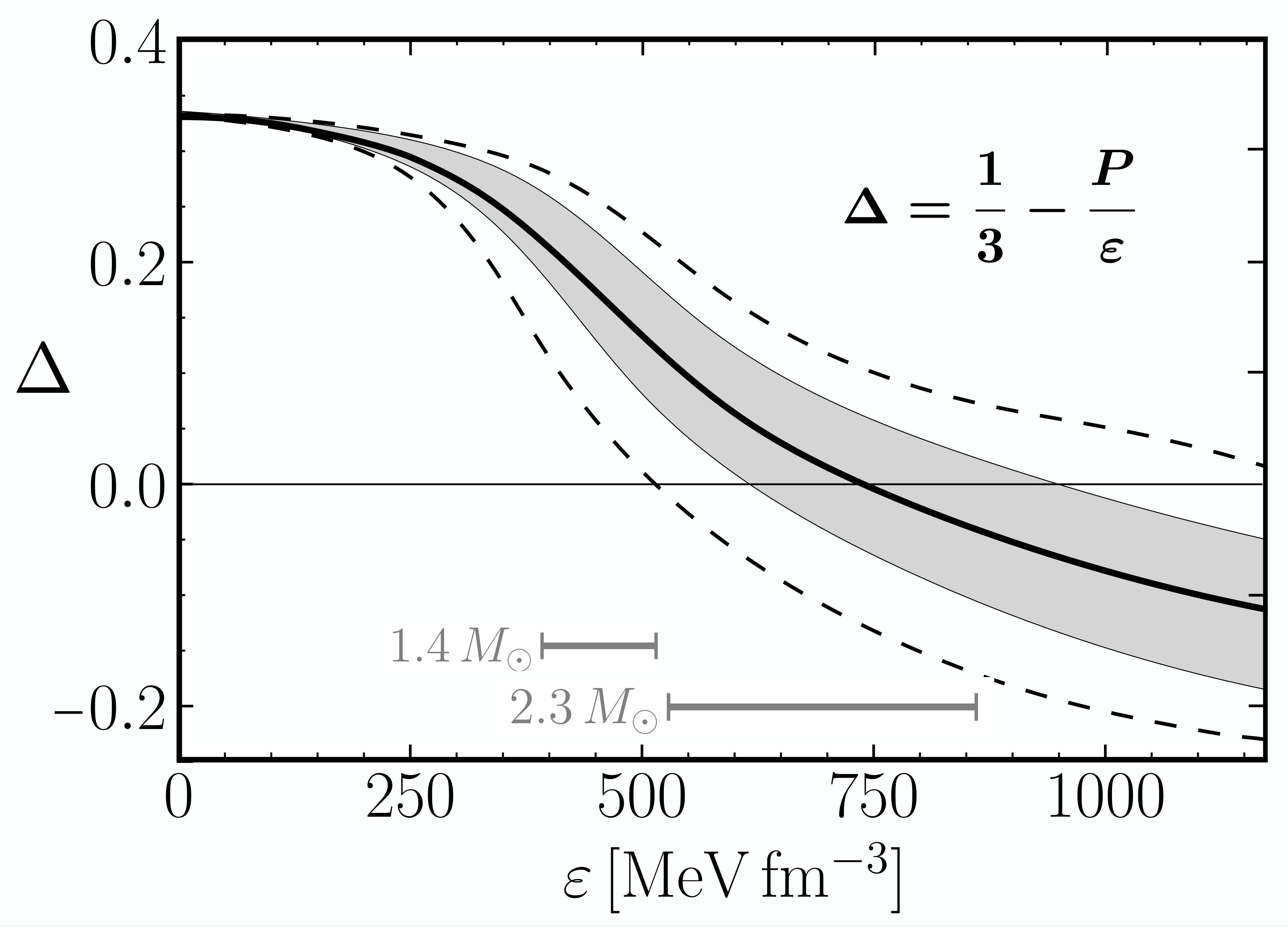
$$\Delta \equiv \frac{\langle \Theta \rangle_{T,\mu_B}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$$

- **Conformal limit :** $\Delta \to 0$
- **Bayes factor analysis:**

Strong evidence for

$$\Delta < 0 \quad (P > \varepsilon/3)$$

at densities $n_B \gtrsim 4\, n_0$



L. Brandes, W.W., N. Kaiser Phys. Rev. D 108 (2023) 094014
L. Brandes, W.W. (2024)

INTERMEDIATE SUMMARY

- **Bayesian inference analysis**
  including heavy $(M \simeq 2.3\, M_\odot)$ galactic neutron star and NICER news
  → even **stiffer equation of state** required than previously expected
  → almost **constant neutron star radii** $(\mathrm{R} \simeq 12 \pm 1\ \mathrm{km})$ for all masses

- **Extreme evidence** for sound velocities $\mathrm{c_s} > 1/\sqrt{3}$ in neutron star cores
  → **strongly repulsive correlations** at work

- **No extreme central core densities** even in the heaviest neutron stars:

  $n_B < 5\, n_0$ for $M \leq 2.3\, M_\odot$ (68% c.l.)

  → average baryon-baryon distance in the core: $\mathrm{d} \gtrsim 1\,\mathrm{fm}$

- **Evidence** against **strong 1st order phase transition** in neutron star cores
  → **not** excluded: **baryonic** matter or **hadron-quark** continuous crossover

Part Two

Phenomenology, Models

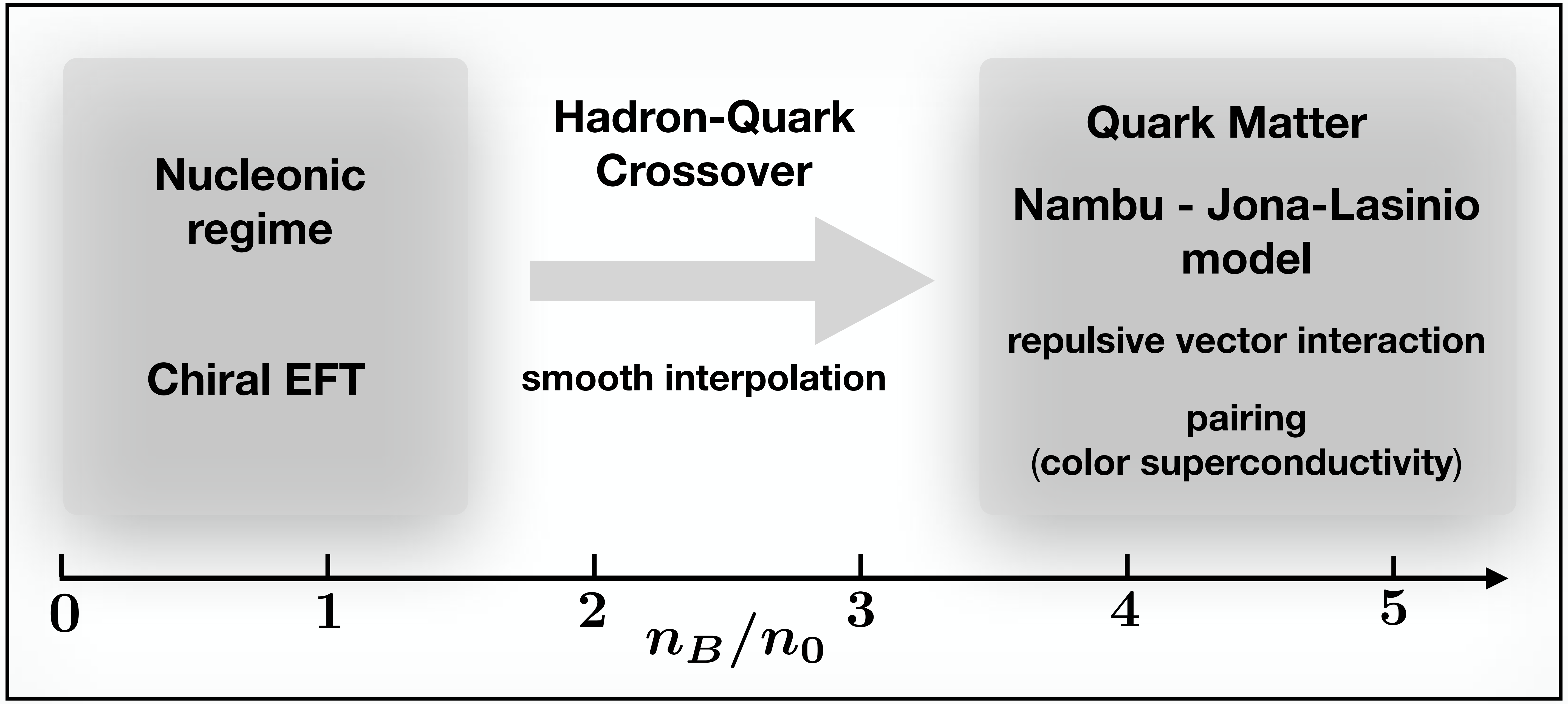
and

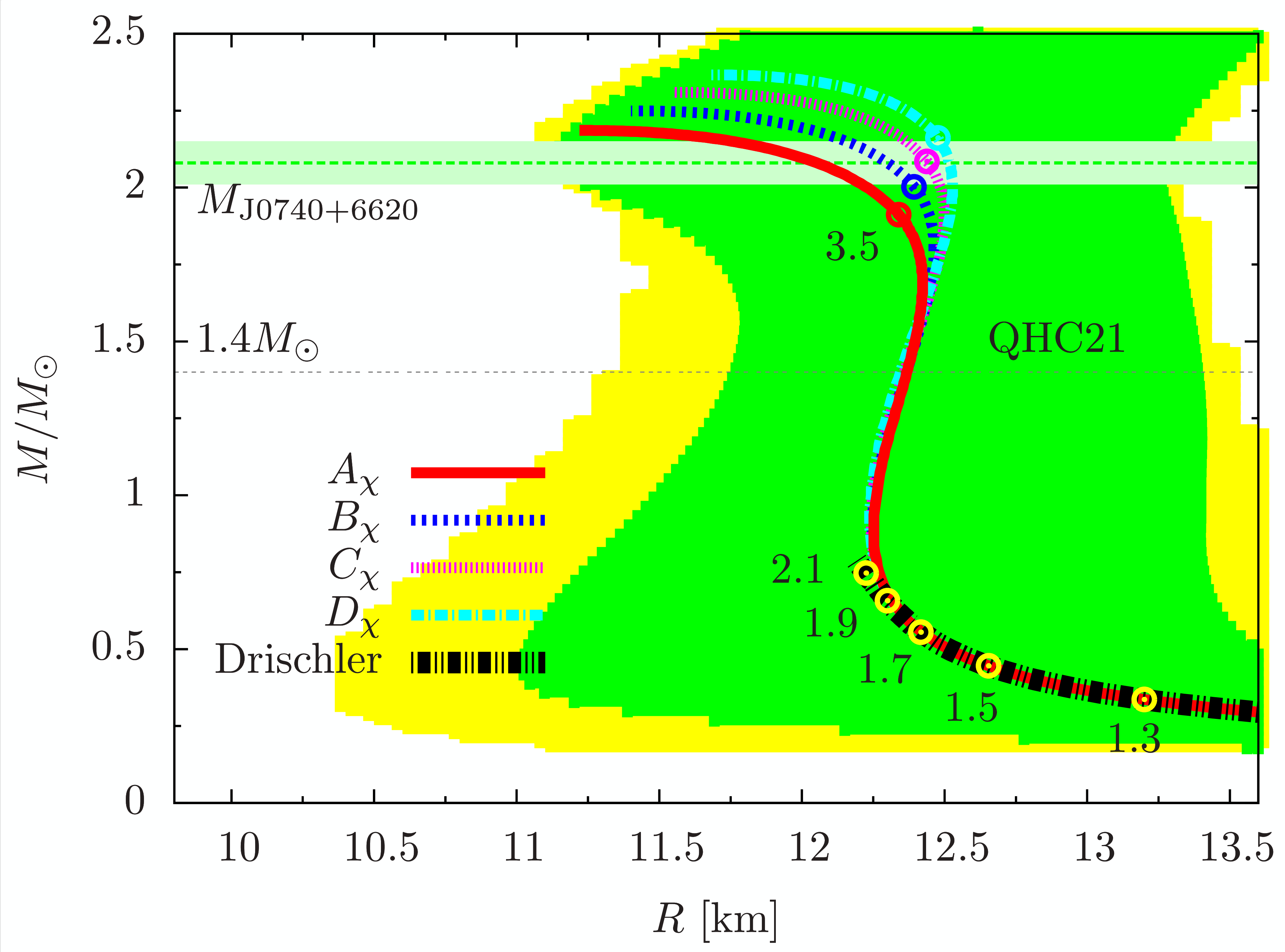
Possible Dense Matter Scenarios

COLD MATTER at EXTREME DENSITIES
Hadron - Quark Continuity

- **QHC21 Equation-of-State**

T. Kojo, G. Baym, T. Hatsuda : Astroph. J. 934 (2022) 46





- **NJL model features** : Chiral symmetry restoration   Scalar-pseudoscalar coupling G

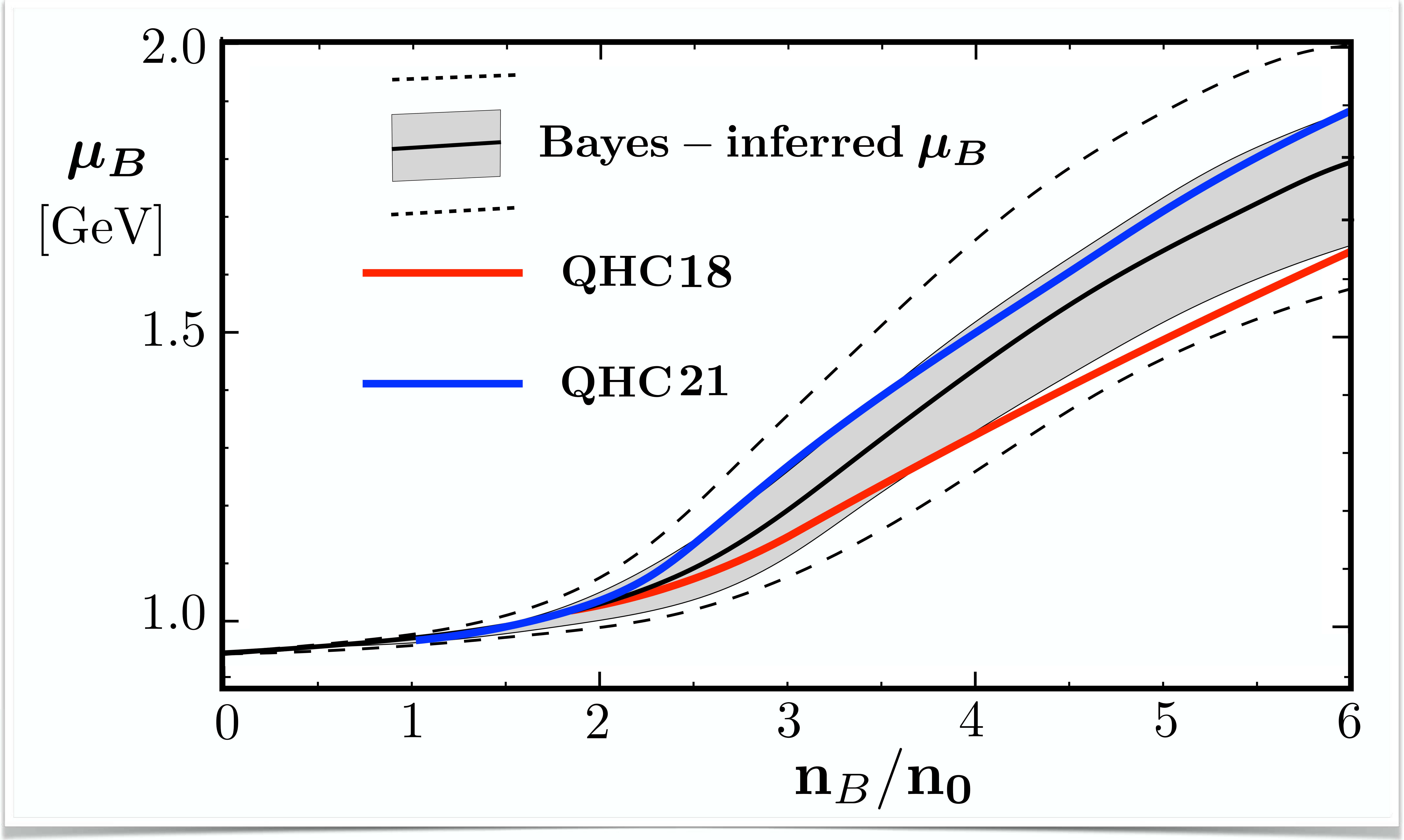
  Vector coupling $g_V/G \simeq 1.0 - 1.3$ Pairing interaction $H/G \simeq 1.5 - 1.6$

- Intermediate crossover region may involve "quarkyonic" matter

L. NcLerran, S. Reddy
Phys. Rev. Lett. 122 (2019) 122701

Outlook : **How Bayes-inferred baryon chemical potential can help improving EoS models**

- Example: QHC equation of state from **QHC18** to **QHC21**



QHC21

T. Kojo, G. Baym, T. Hatsuda
Astroph. J. 934 (2022) 46

increasing ↑ repulsion

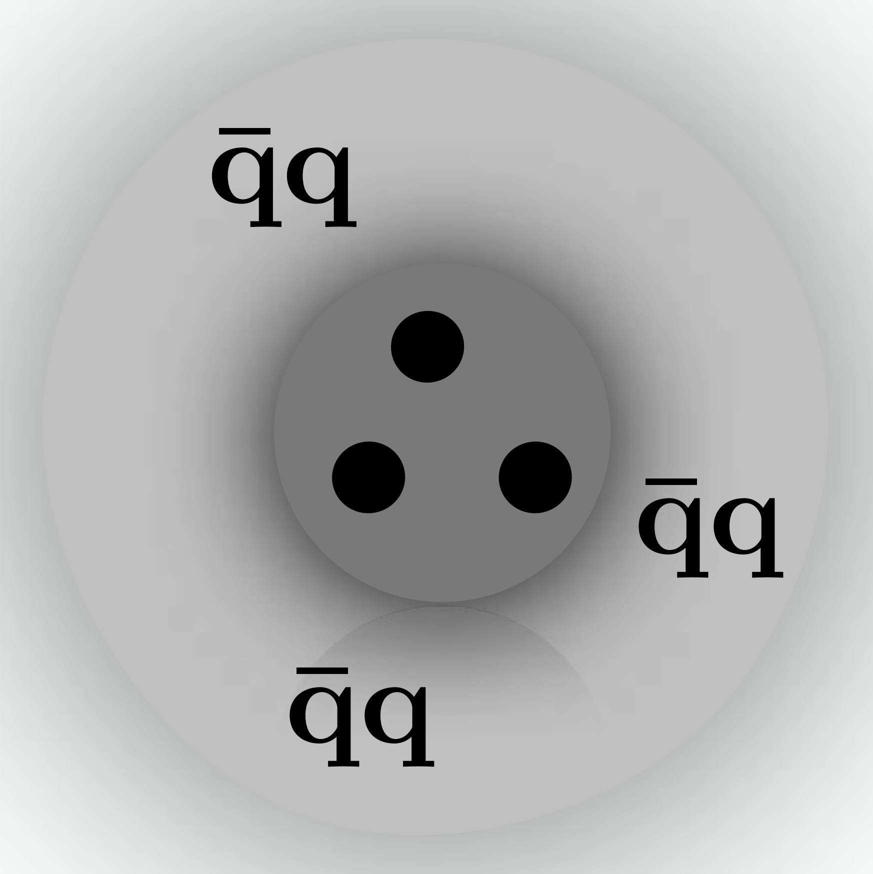
QHC18

G. Baym, T. Hatsuda, T Kojo,
P.D. Powell, Y. Song, T. Takatsuka
Rept. Prog. Phys. 81 (2018) 056902

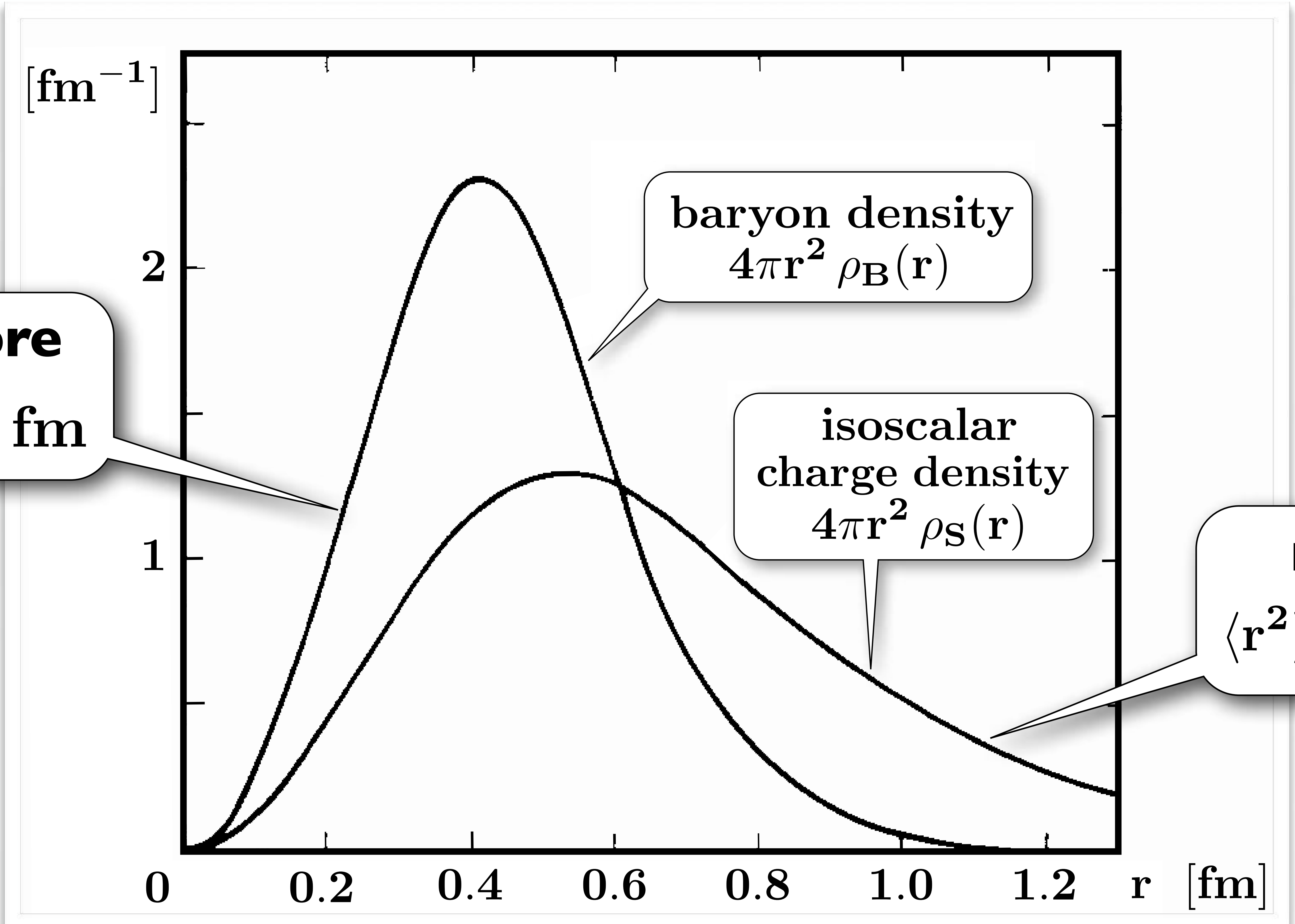
SIZES of the **NUCLEON**

Low-energy QCD: **spontaneously broken chiral symmetry + localisation (confinement)**

- **NUCLEON** : **compact valence quark core + mesonic** (multi $\bar{q}q$) **cloud**
  - Historic example: Chiral Soliton Model of the Nucleon



baryonic core
$\langle r^2 \rangle_B^{1/2} \simeq 0.5$ fm



mesonic cloud
$\langle r^2 \rangle_{E,isoscalar}^{1/2} \simeq 0.8$ fm

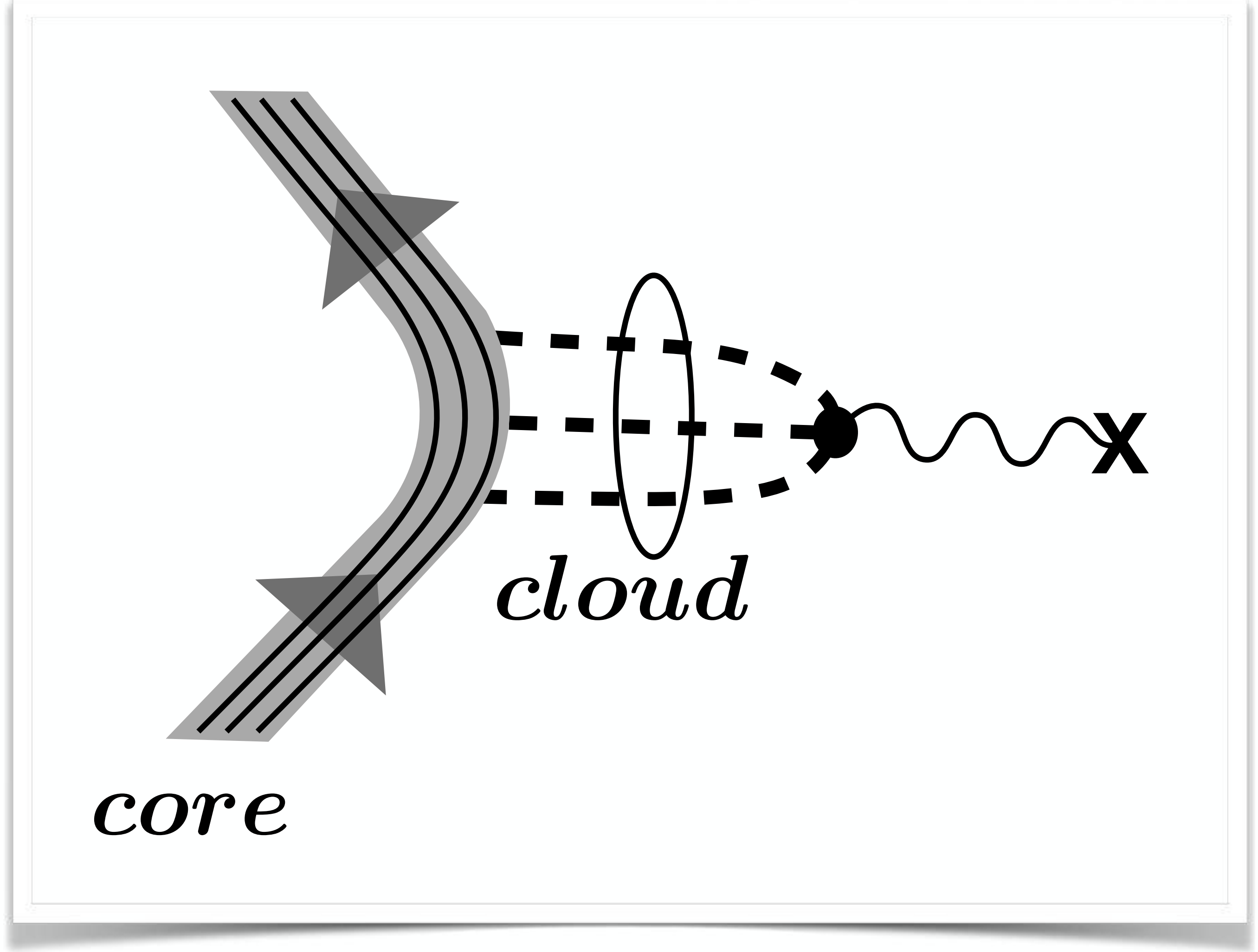
N. Kaiser,
U.-G. Meißner,
W.W.
Nucl. Phys. A466 (1987) 685

- **Separation of scales**

$$\left(\frac{R_{cloud}}{R_{core}}\right)^3 \gg 1$$

FORM FACTORS of the NUCLEON

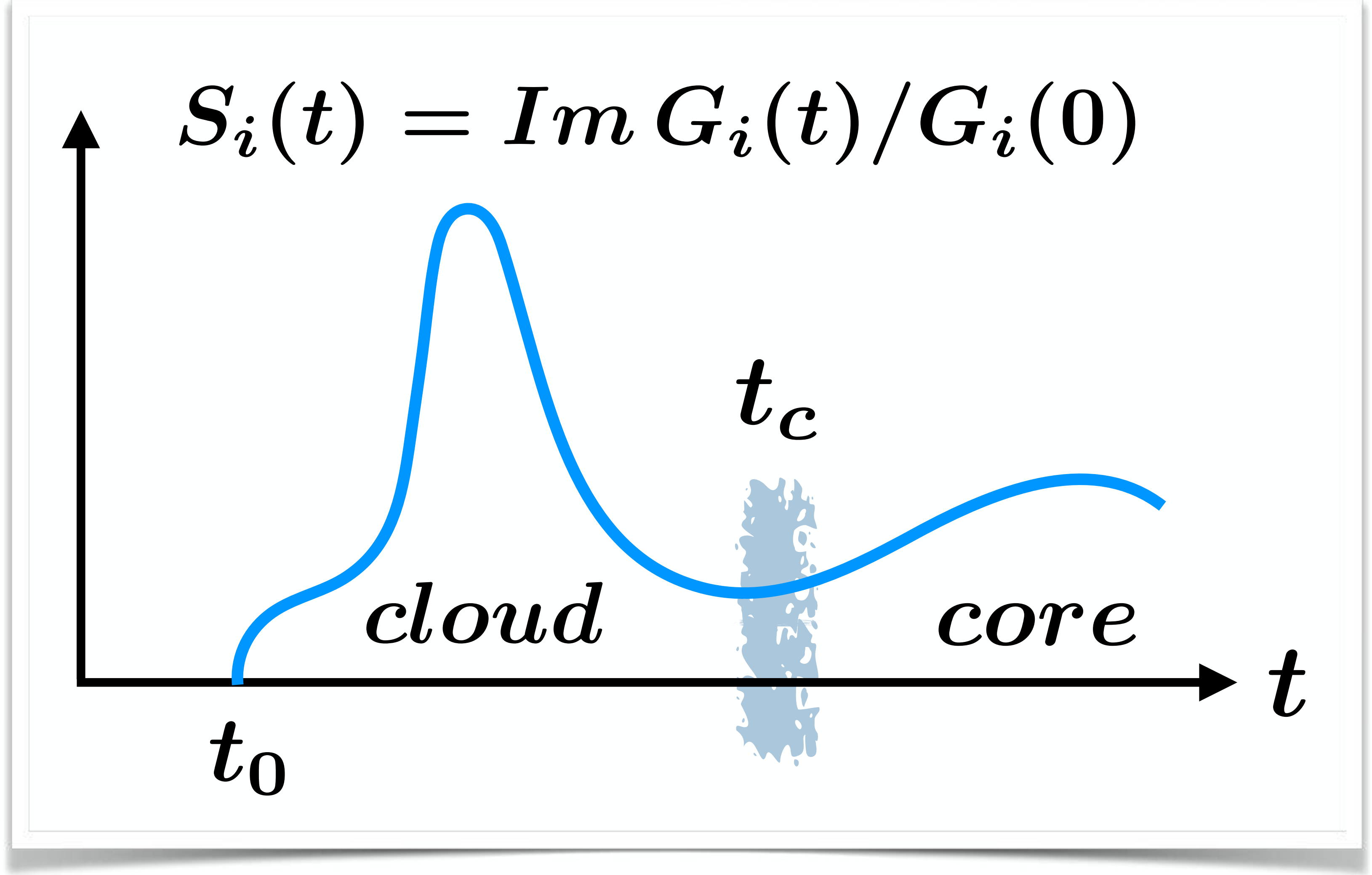
$$G_i(q^2) = G_i(0) + \frac{q^2}{\pi}\int_{t_0}^{\infty} dt\, \frac{Im\, G_i(t)}{t(t-q^2-i\epsilon)} \qquad \langle r_i^2 \rangle = \frac{6}{G_i(0)} \frac{dG_i(q^2)}{dq^2}\Big|_{q^2=0} = \frac{6}{\pi}\int_{t_0}^{\infty} \frac{dt}{t^2} S_i(t)$$



- Delineation of

valence quark (qqq) **CORE**
and
mesonic (multi $\bar{q}q$) **CLOUD**

$$t_c \simeq 1\,\mathrm{GeV}^2$$



$$\langle r_i^2 \rangle = \langle r_i^2 \rangle_{cloud} + \langle r_i^2 \rangle_{core} = \frac{6}{\pi}\left[\int_{t_0}^{t_c} \frac{dt}{t^2} S_i(t) + \int_{t_c}^{\infty} \frac{dt}{t^2} S_i(t)\right]$$

- **Detailed spectral analysis of accurately determined empirical form factors**

N. Kaiser, W.W. : Phys. Rev. C110 (2024) 015202

FORM FACTORS of the NUCLEON (contd.)

form factor		J^π(cloud)	empirical rms radii
isoscalar electric	$G_E^S(q^2)$	1^-	$\langle r_S^2\rangle^{1/2} = 0.78 \pm 0.01\,\mathrm{fm}$ Y.H. Lin, H.-W. Hammer, U.-G. Meißner PRL 128 (2022) 052002
isovector electric	$G_E^V(q^2)$	1^-	$\langle r_V^2\rangle^{1/2} = 0.90 \pm 0.01\,\mathrm{fm}$
isovector axial	$G_A(q^2)$	1^+	$\langle r_A^2\rangle^{1/2} = 0.67 \pm 0.01\,\mathrm{fm}$ $(\langle r_A^2\rangle^{1/2} = 0.68 \pm 0.11\,\mathrm{fm})$ R.J. Hill et al. : Rep. Prog. Phys. 81 (2018) 096301
mass	$G_m(q^2) = \langle p'\vert T_\mu^\mu\vert p\rangle$	0^+	$\langle r_m^2\rangle^{1/2} = 0.55 \pm 0.03\,\mathrm{fm}$ D. Kharzeev : Phys. Rev. D104 (2021) 054015 $\langle r_m^2\rangle^{1/2} = 0.53 \pm 0.04\,\mathrm{fm}$ S. Adhikari et al. : arXiv:2304.03845

extracted core radii

N. Kaiser, W.W. : Phys. Rev. C110 (2024) 015202

$$\langle r_S^2\rangle^{1/2}_{core} = 0.50 \pm 0.01\,\mathrm{fm}$$

$$\langle r_V^2\rangle_{core} \simeq 0\,(\pm 0.02)\,\mathrm{fm}^2\ !!$$

$$\langle r_A^2\rangle^{1/2}_{core} = 0.53 \pm 0.02\,\mathrm{fm}$$

$$(0.5 \pm 0.2)$$

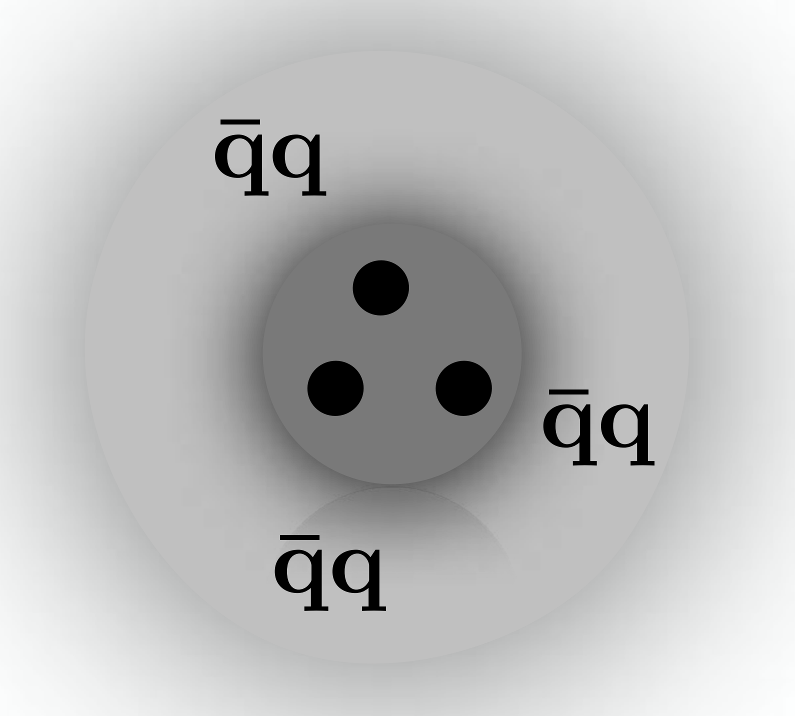
$$\langle r_m^2\rangle^{1/2}_{core} = 0.48 \pm 0.05\,\mathrm{fm}$$

TWO-SCALES Picture of the NUCLEON :
Implications for DENSE BARYONIC MATTER

$$\langle r_S^2\rangle_{core}^{1/2} \simeq \langle r_A^2\rangle_{core}^{1/2} \simeq \langle r_m^2\rangle_{core}^{1/2} \equiv R_{core} \simeq \frac{1}{2}\ \mathrm{fm}$$

$$R_{core} \sim \frac{1}{2}\ \mathrm{fm}$$

$$R_{cloud} \sim 1\ \mathrm{fm}$$



- **Separation of scales**

$$\left(\frac{R_{cloud}}{R_{core}}\right)^3 \gg 1$$

- **Soft mesonic (multi-pion) cloud**
  expected to expand with increasing baryon density along with decreasing **in-medium pion decay constant** $f_\pi^*(n_B)$

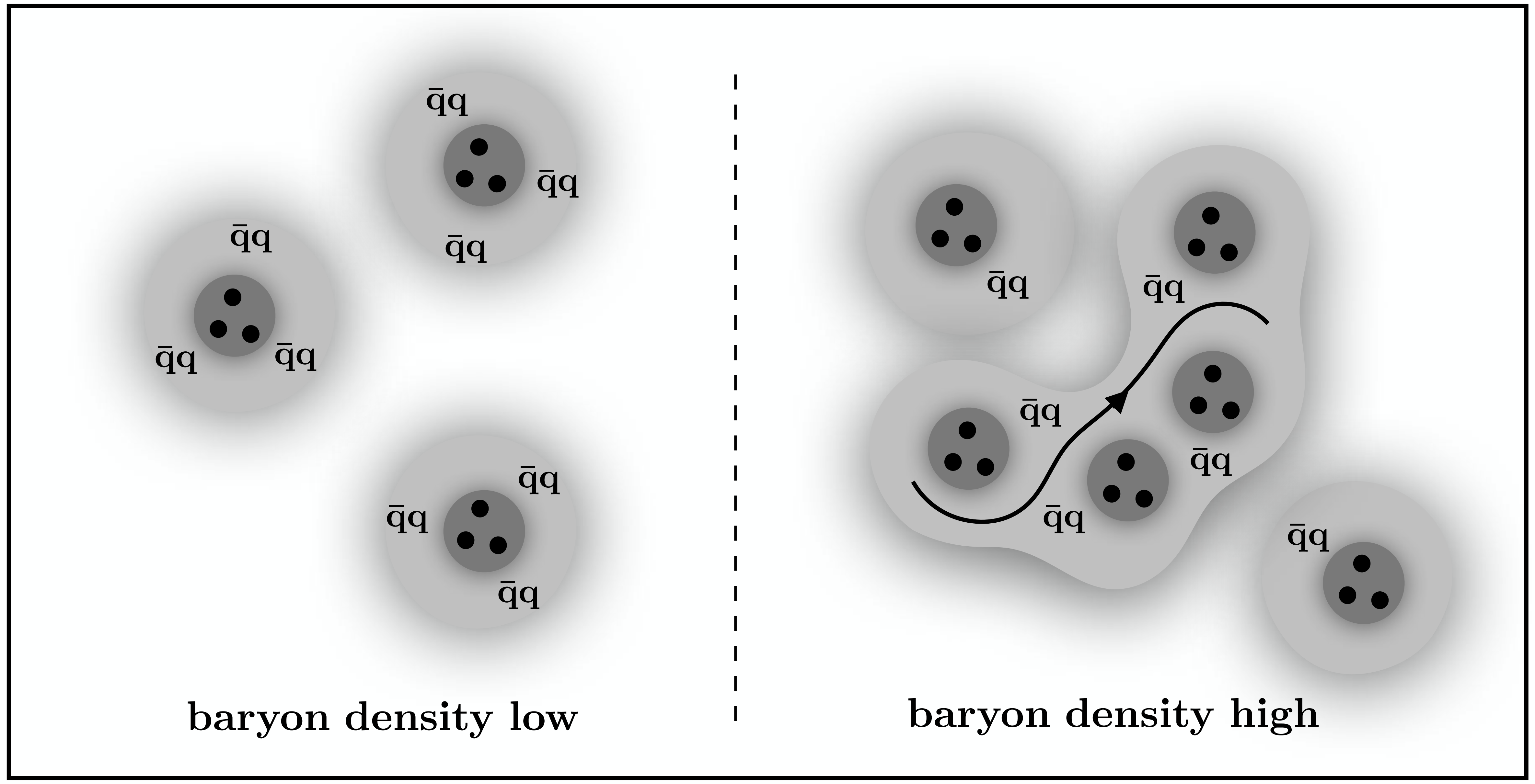
- **Hard baryonic core** governed by **gluon dynamics**
  expected to remain stable with increasing baryon density up until hard compact cores begin to touch and overlap

TWO-SCALES Scenario for DENSE BARYONIC MATTER

- Baryon densities

  $n_B \sim n_0 = 0.16\,\mathrm{fm}^{-3}$

  Tails of mesonic clouds overlap :
  two-body exchange forces
  between nucleons



- $n_B \gtrsim 2 - 3\,n_0$

  Soft $\bar{q}q$ clouds delocalize :
  percolation ➔ many-body forces
  baryonic cores still separated, but subject to increasingly strong repulsive Pauli effects

- $n_B > 5\,n_0$ (beyond central densities of neutron stars)

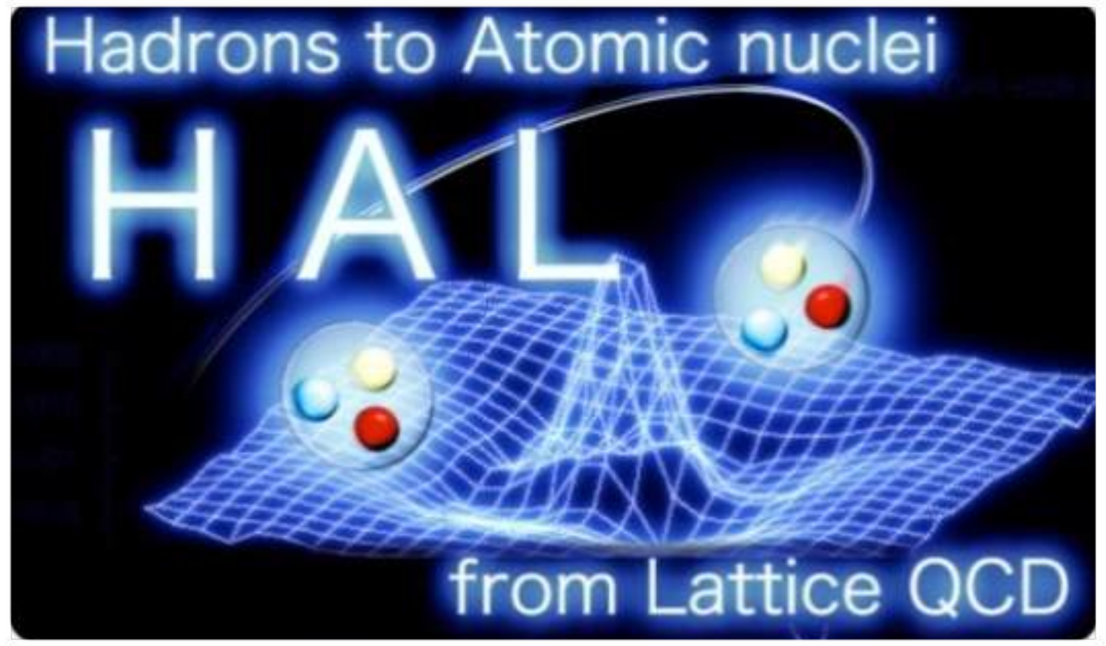
  Compact nucleon cores begin to touch and overlap at distances $d \lesssim 1\,\mathrm{fm}$
  (but still have to overcome the repulsive NN hard core)

K. Fukushima, T. Kojo, W.W.
Phys. Rev. D 102 (2020) 096017

NUCLEAR FORCES from LATTICE QCD

NN Central Potential (S = 0, I = 1)

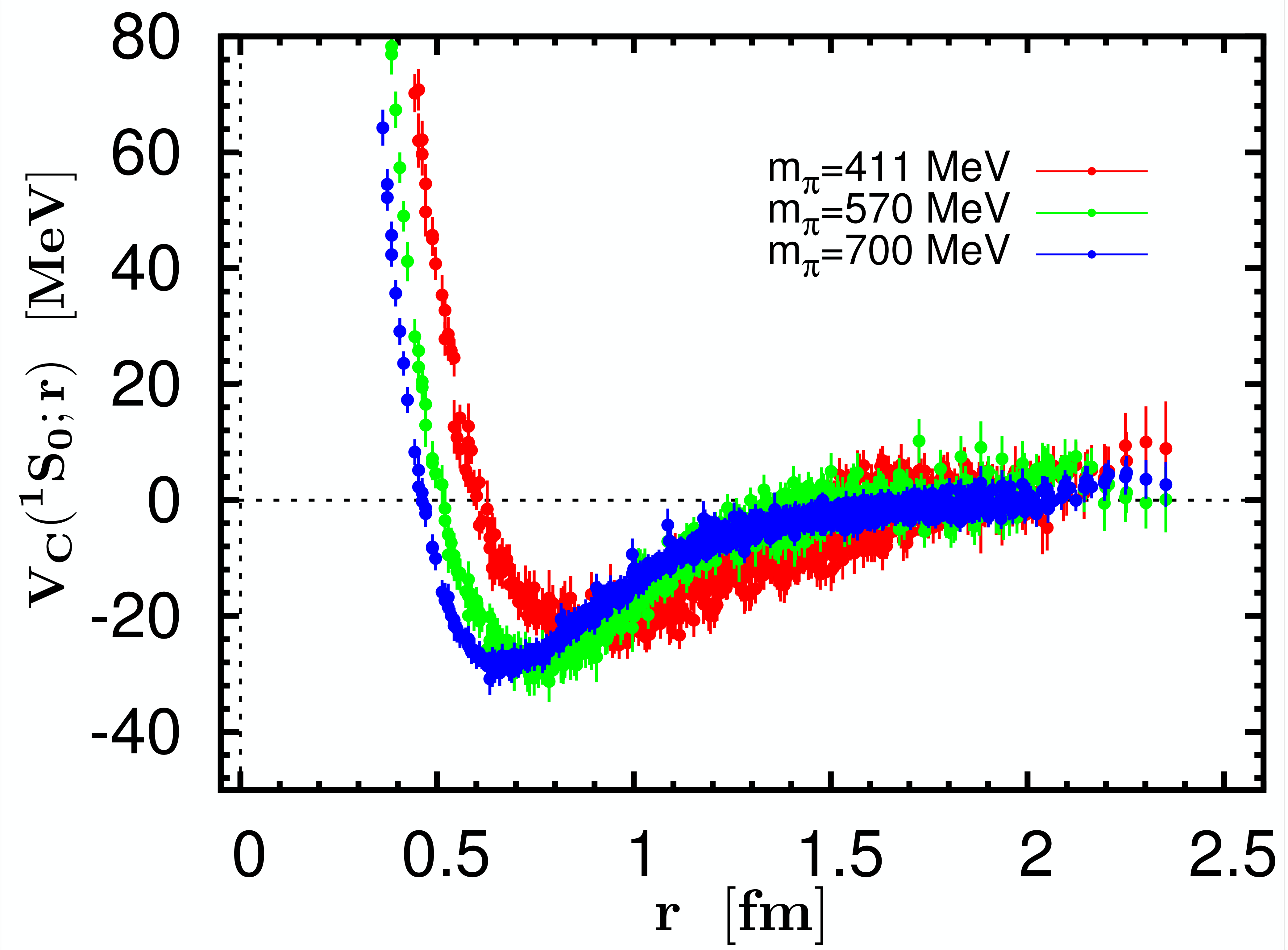
deduced from LQCD **two-nucleon (6-quark) correlation function**



S. Aoki, T. Hatsuda, N. Ishii
Prog. Theor. Phys. 123 (2010) 89

S. Aoki
Eur. Phys. J. A49 (2013) 81

S. Aoki, T. Doi
arXiv:2402.11759



Nuclear Physics Phenomenology:

Short-Range Repulsive Core

- **Compression of baryonic matter is energetically expensive**

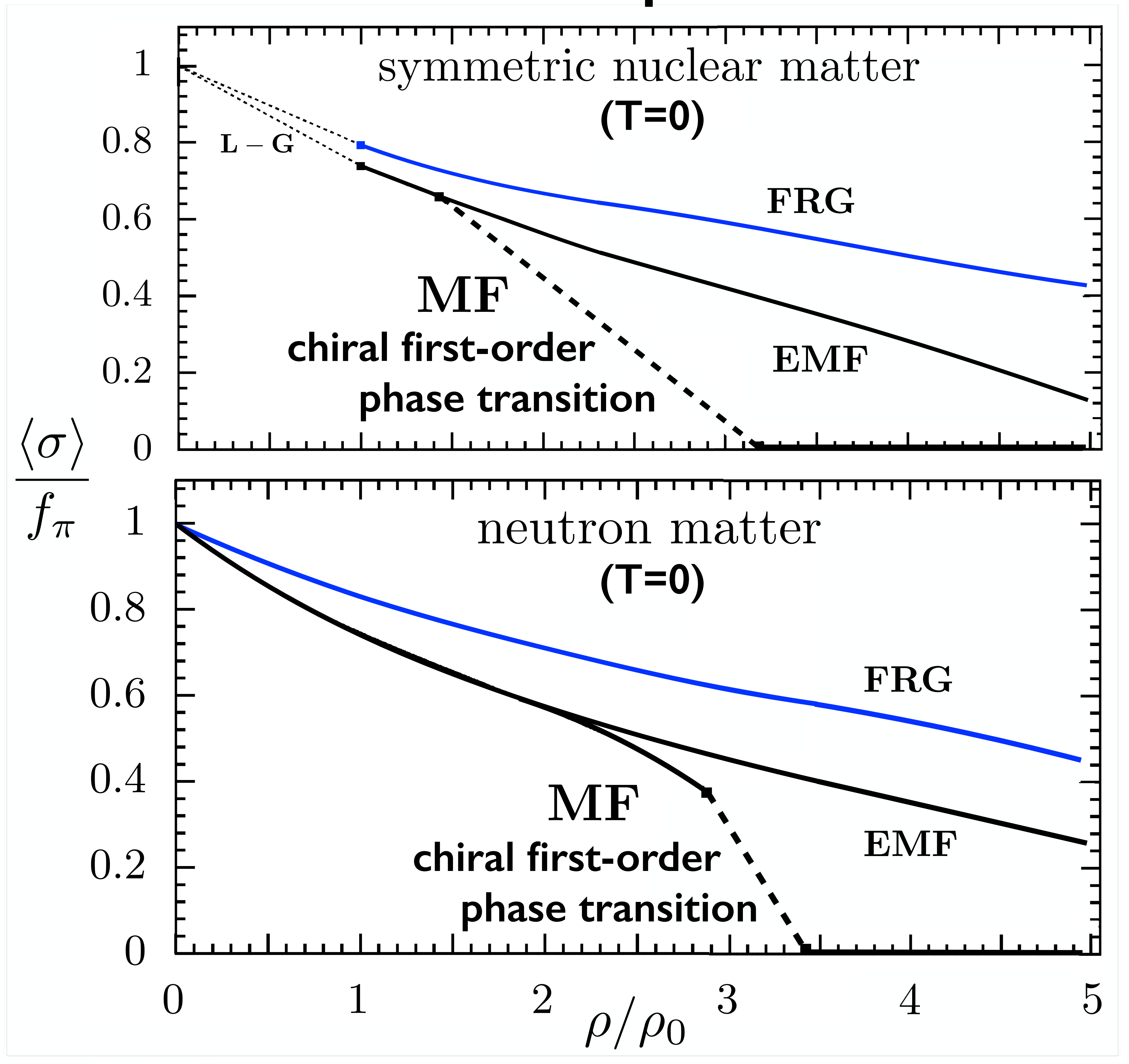
CHIRAL PHASE TRANSITION in DENSE BARYONIC MATTER ?

Studies in chiral nucleon-meson field theory

M. Drews, W.W.: Prog. Part. Nucl. Phys. 93 (2017) 69 — L. Brandes, N. Kaiser, W.W.: Eur. Phys. J. A57 (2021) 243

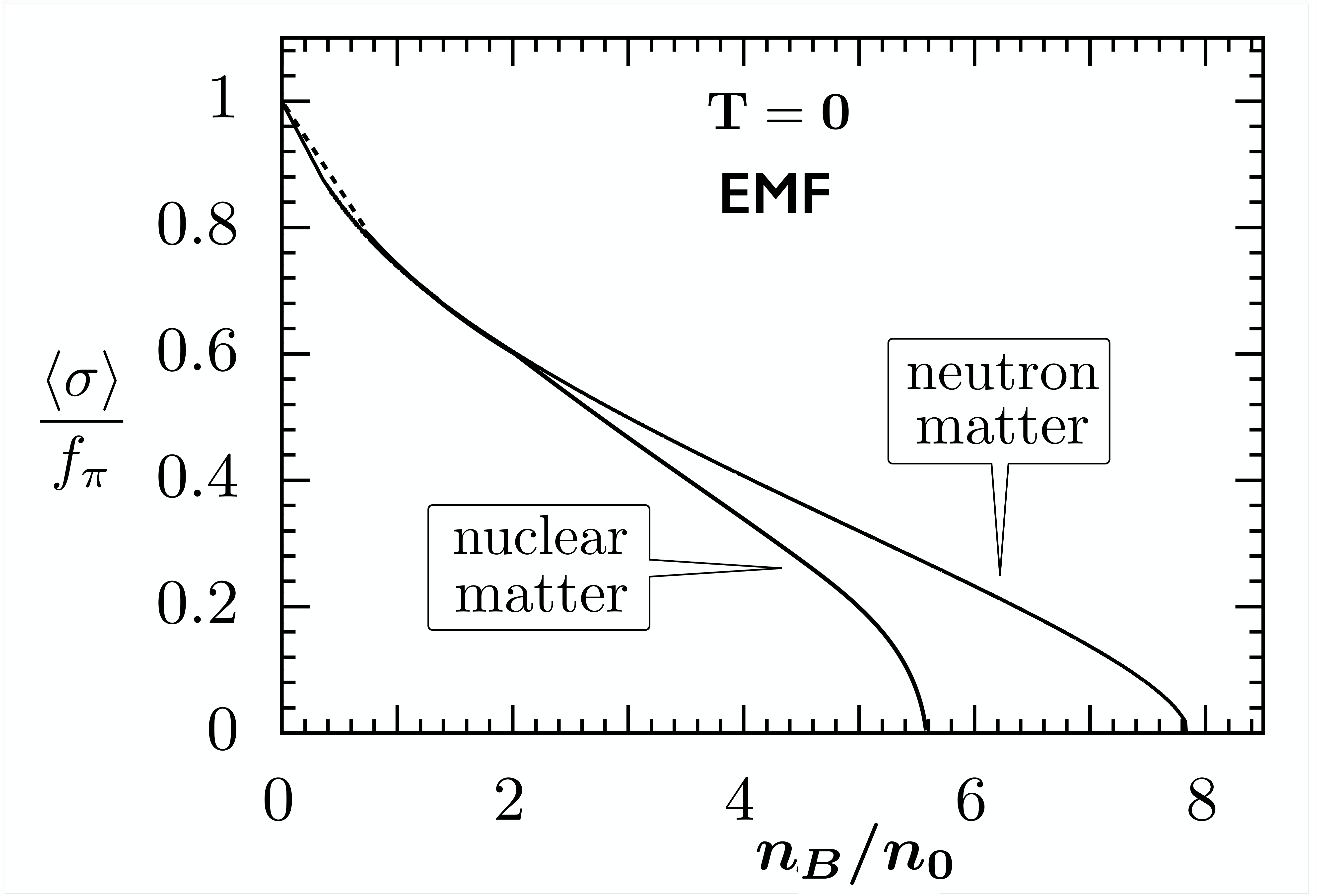
- **Mean-field** approximation **(MF)** :
  chiral first-order phase transition
  at baryon densities $n_B \sim 2 - 3\, n_0$
- **Vacuum fluctuations (EMF)** :
  shift chiral transition to **high density**
  → **smooth crossover**
- **Functional Renormalisation Group (FRG)** :
  non-perturbative loop corrections
  involving **pions** & **nucleon-hole** excitations
  → further reinforcement of **stabilising effects**

chiral order parameter



Chiral crossover transition at $n_B > 6\, n_0$
beyond core densities in neutron stars

CHIRAL LIMIT $(m_\pi \to 0)$
2nd order chiral phase transition in nuclear and neutron matter



L. Brandes, N.. Kaiser, W.W.: Eur. Phys. J. A57 (2021) 243

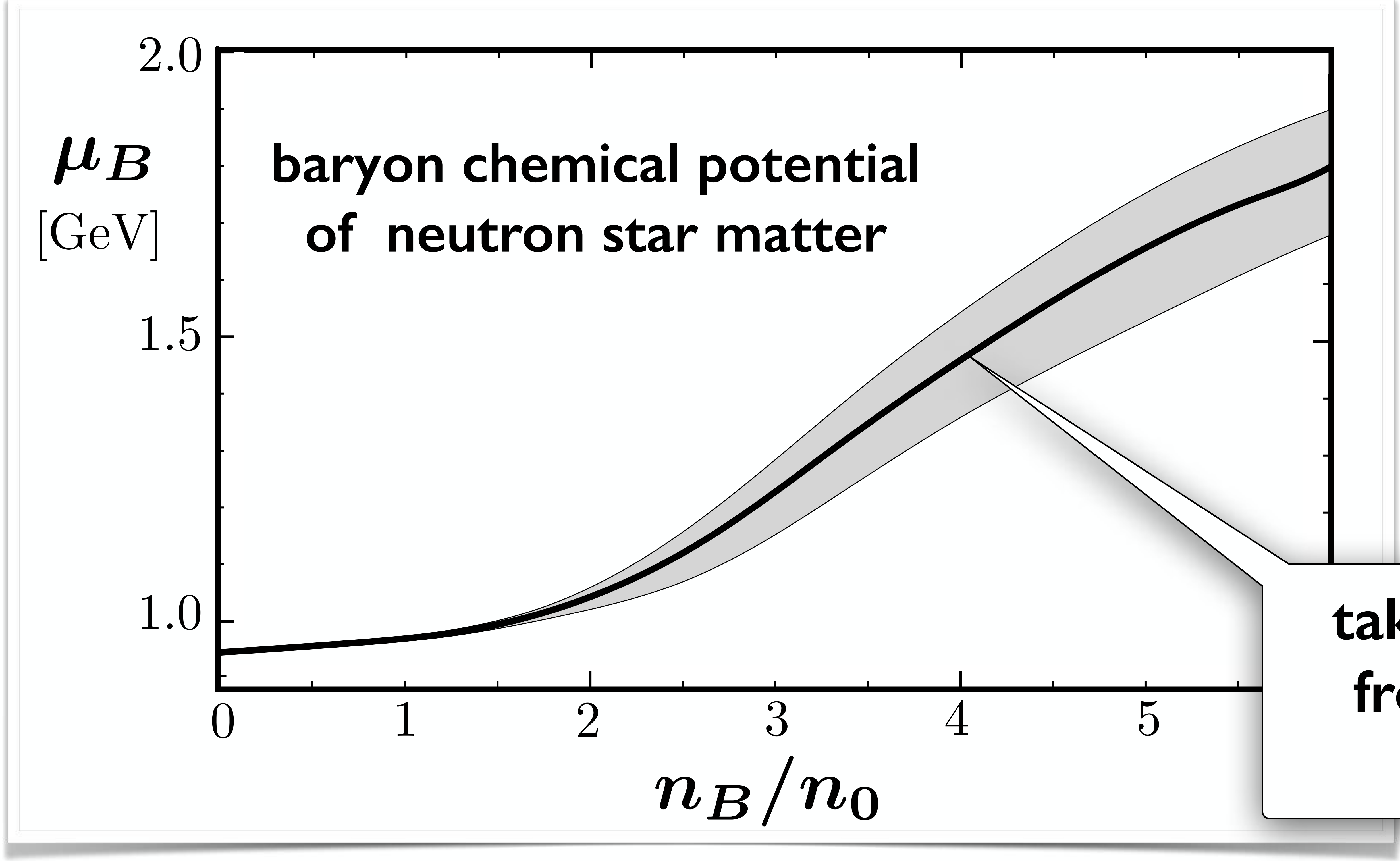
- **Chiral Nucleon-Meson Field Theory**
- **EMF calculations :**
  Extended Mean-Field including logarithmic vacuum fluctuations
- **Critical densities** (chiral limit)
  $$n_B^{cr} > 5\, n_0$$
- Alternative approach:
  Parity-doublet model
  $\{N(1/2^+) - N^*(1/2^-)\}$
  Critical densities $n_B^{cr} > 10\, n_0$

J. Eser, J.-P. Blaizot : Phys. Rev. C109 (2024) 045201
arXiv:2408.01302

DENSE BARYONIC MATTER in NEUTRON STARS as a RELAVISTIC FERMI LIQUID

B. Friman, W.W. : Rhys. Rev. C100 (2019) 065807 L. Brandes, W.W. : Symmetry 16 (2024) 111

- **Neutron Star Matter : Fermi liquid** / dominantly neutrons + ca. 5 % protons
- **Baryonic Quasiparticles** :
  baryons "dressed" by their **strong interactions** and imbedded in **mesonic (multi-pion) field**



- **Landau effective mass**

$$m_L^*(n_B) = \sqrt{p_F^2 + M_N^2(n_B)}$$

- **Baryon chemical potential**

$$\mu_B = m_L^*(n_B) + \mathcal{U}(n_B)$$

take median of $\mu_B(n_B)$ from Bayesian-inferred neutron star EoS

quasiparticle potential

Basics of (Relativistic) Fermi-Liquid Theory

G. Baym, S.A. Chin : Nucl. Phys. A262 (1976) 527

T. Matsui : Nucl. Phys. A370 (1981) 365

- **Variation of the energy** (T = 0)

$$\delta E = V\delta\mathcal{E} = \sum_p \varepsilon_p\,\delta n_p + \frac{1}{2V}\sum_{pp'}\mathcal{F}_{pp'}\delta n_p \delta n_{p'} + \ldots \qquad n_p = \Theta(\mu - \varepsilon_p)$$

quasiparticle energy

$$\varepsilon_p = \frac{\delta E}{\delta n_p}$$

quasiparticle interaction

$$\mathcal{F}_{pp'} = V\,\frac{\delta^2 E}{\delta n_p \delta n_{p'}} = f_{pp'} + g_{pp'}\,\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'$$

- **Landau effective mass**

$$m^* = \sqrt{p_F^2 + M^2(\rho)}$$

- **Density of states** at the **Fermi surface**

$$N(0) = \frac{m^*\,p_F}{\pi^2}$$

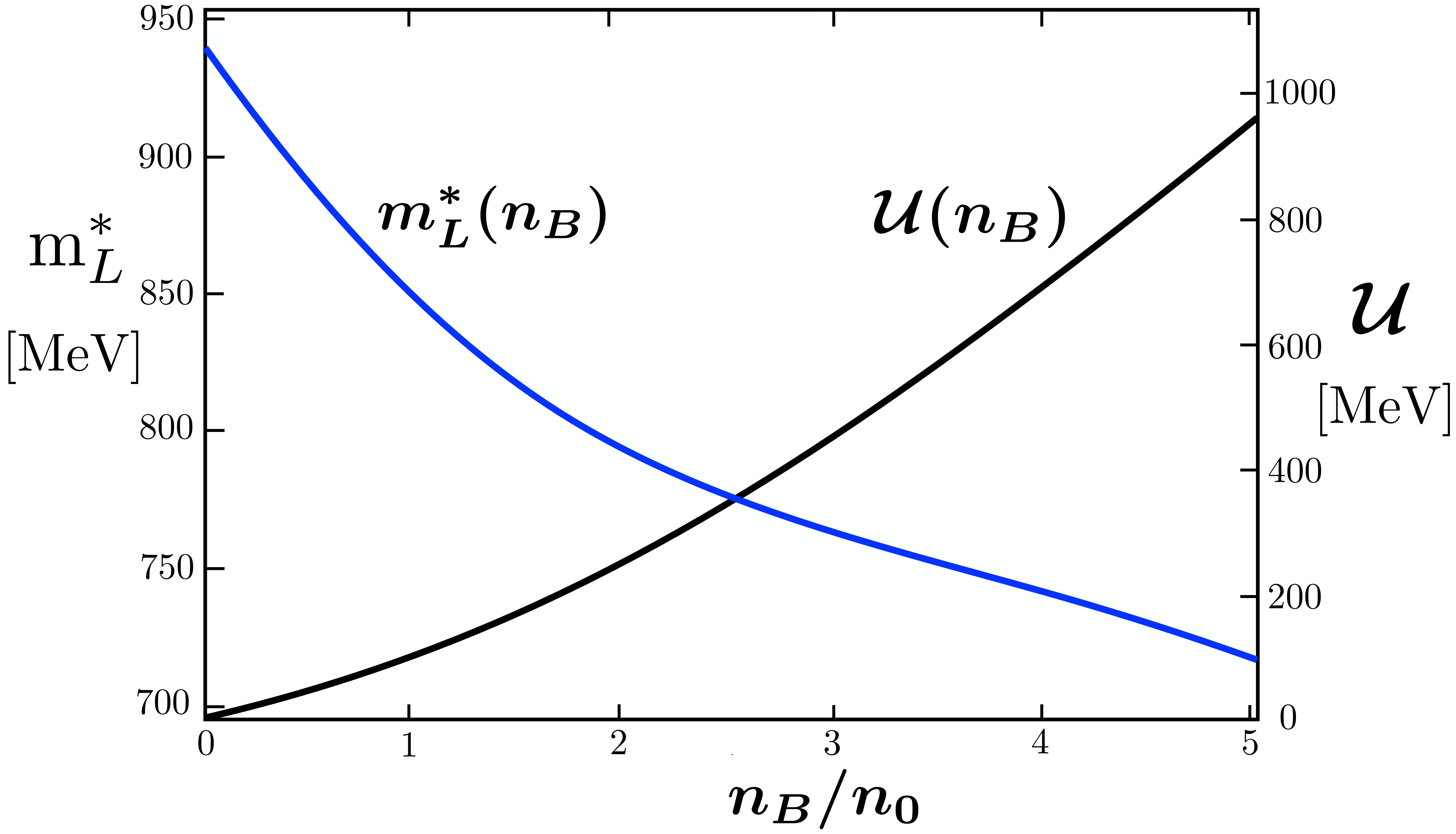
- **Landau parameters** **Quasiparticle interaction** expanded in **Legendre series**

$$f_{pp'} = \sum_{\ell=0}^{\infty} f_\ell\,P_\ell(\cos\theta_{pp'}) \qquad F_\ell = N(0)\,f_\ell$$

QUASIPARTICLE POTENTIAL and FERMI-LIQUID PARAMETERS

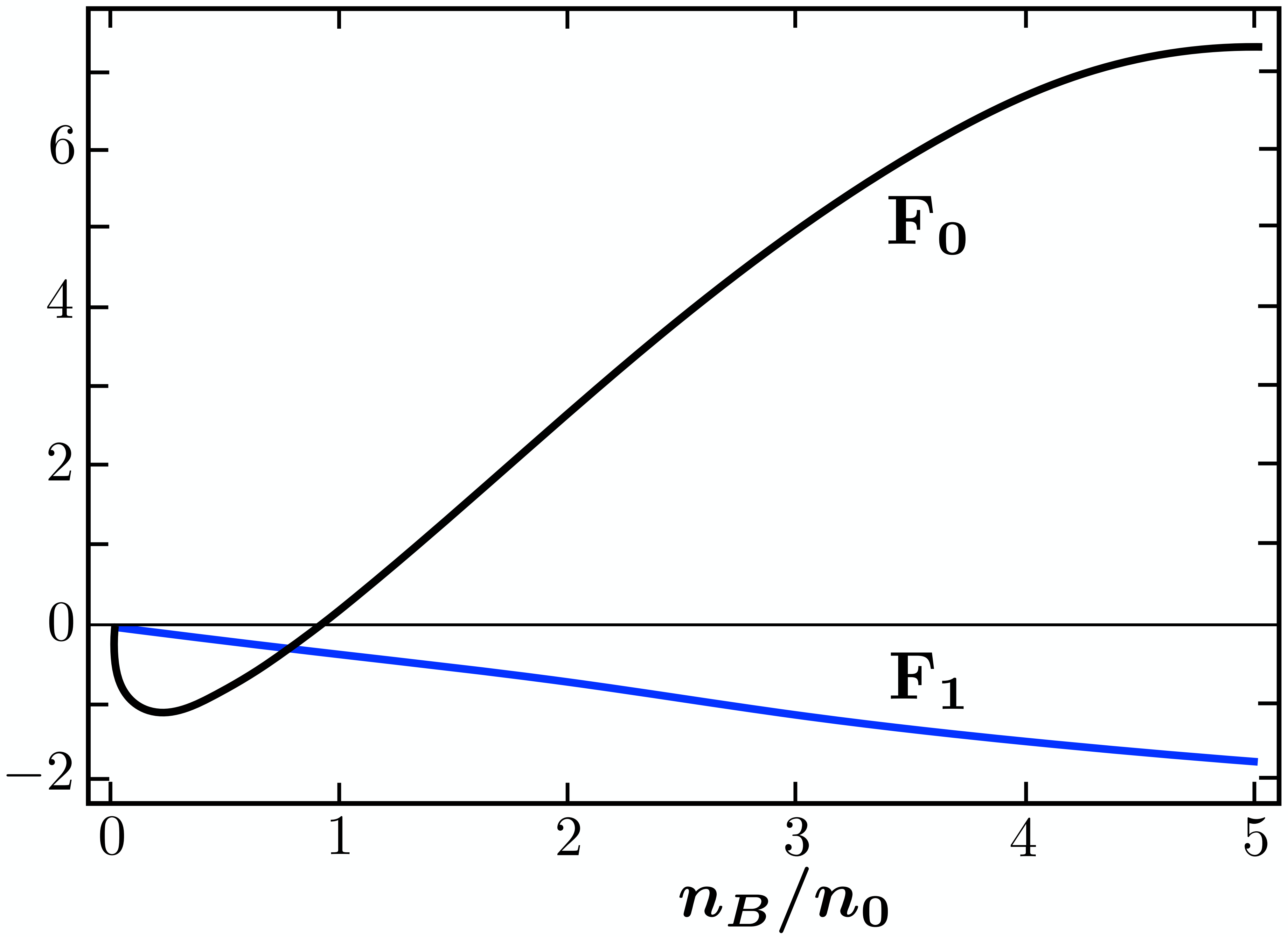
- $m_L^*(n_B)$ from **chiral nucleon-meson field theory** & Functional Renormalisation Group
- Quasiparticle effective potential

$$\mathcal{U}(n_B) = \sum_n u_n \left(\frac{n_B}{n_0}\right)^n$$



- Landau Fermi-Liquid parameters

$$F_0 = \frac{m_L^* \, p_F}{\pi^2} \frac{\partial \mu_B}{\partial n_B} - 1 \qquad F_1 = -\frac{3\mathcal{U}}{\mu_B}$$



➡ **Strongly repulsive correlations** including **many-body forces** with $n \geq 2$

LANDAU FERMI LIQUID PARAMETERS (contd.)

- **Comparison with atomic liquid helium-3 in its normal phase at low temperature** (3 K)

  G. Baym, Ch. Pethick : Landau Fermi-Liquid Theory (1991)

- Interaction between **He-3 atoms:**
  attractive van der Waals potential plus strongly **repulsive short-range core**

- **Landau Fermi Liquid parameters** of **liquid helium-3** at pressures P = (0 - 30) bar:

$$\mathbf{F_0(^3He) \sim 10 - 70} \qquad \mathbf{F_1(^3He) \sim 5 - 13}$$

D. S. Greywall, Phys. Rev. B33 (1986) 7520

… **much larger** by magnitude than Landau parameters of **neutron star matter** !

- **Neutron star matter** at central densities is a **strongly correlated Fermi system**
  … but not as extreme as one might have thought !

CONCLUSIONS

- **Constraints on phase transitions in neutron star matter**
  - **stiff equation of state** implied by Bayesian inference results
  - **strong first-order transition** unlikely in neutron star cores
  - **central baryon densities** in neutron stars : $n_c < 5\, n_0$ (68% c.l.)

- **Scenarios for cold dense matter in the core of neutron stars**
  - **hadron-quark** continuity with "core + cloud" baryons :
    two-scales scenario: soft-surface delocalisation (percolation)
    followed by hard-core deconfinement at densities around n_c
  - neutron-dominated **baryonic** matter :
    e.g. relativistic **Fermi liquid** featuring strongly repulsive
    many-body forces between **baryonic quasiparticles**

Supplementary Materials

Example 1: **ISOSCALAR ELECTRIC FORM FACTOR** of the **NUCLEON**

- **Isoscalar electric form factor** $G_E^S(q^2) = \frac{1}{2}\left[G_E^p(q^2) + G_E^n(q^2)\right]$ $\quad \langle r_S^2 \rangle = \langle r_p^2 \rangle + \langle r_n^2 \rangle$

Empirical : $\langle r_p^2 \rangle^{1/2} = 0.840 \pm 0.004\,\mathrm{fm}$

$\langle r_n^2 \rangle = -0.105 \pm 0.006\,\mathrm{fm}^2$

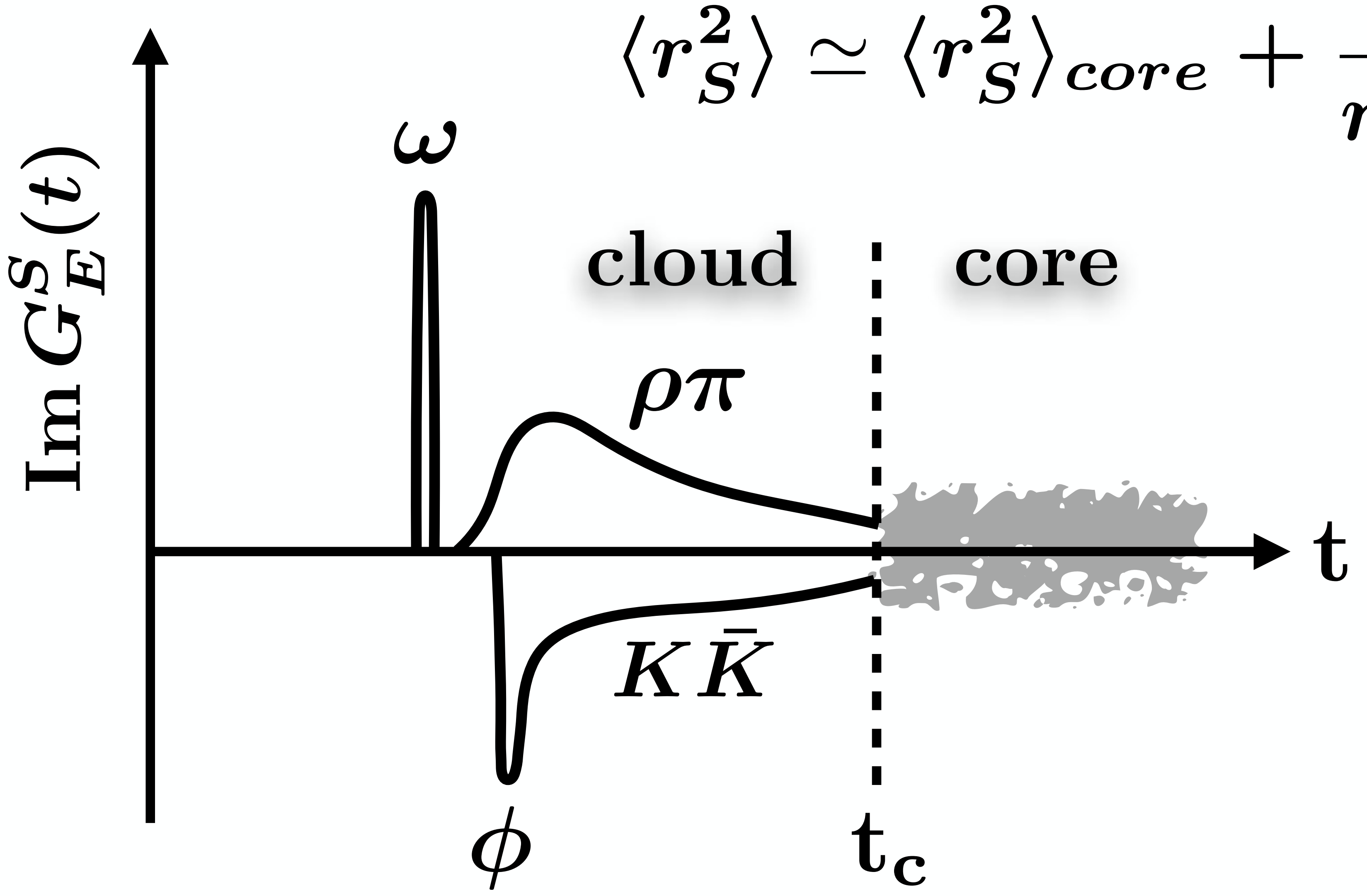
$\langle r_S^2 \rangle^{1/2} = 0.775 \pm 0.011\,\mathrm{fm}$

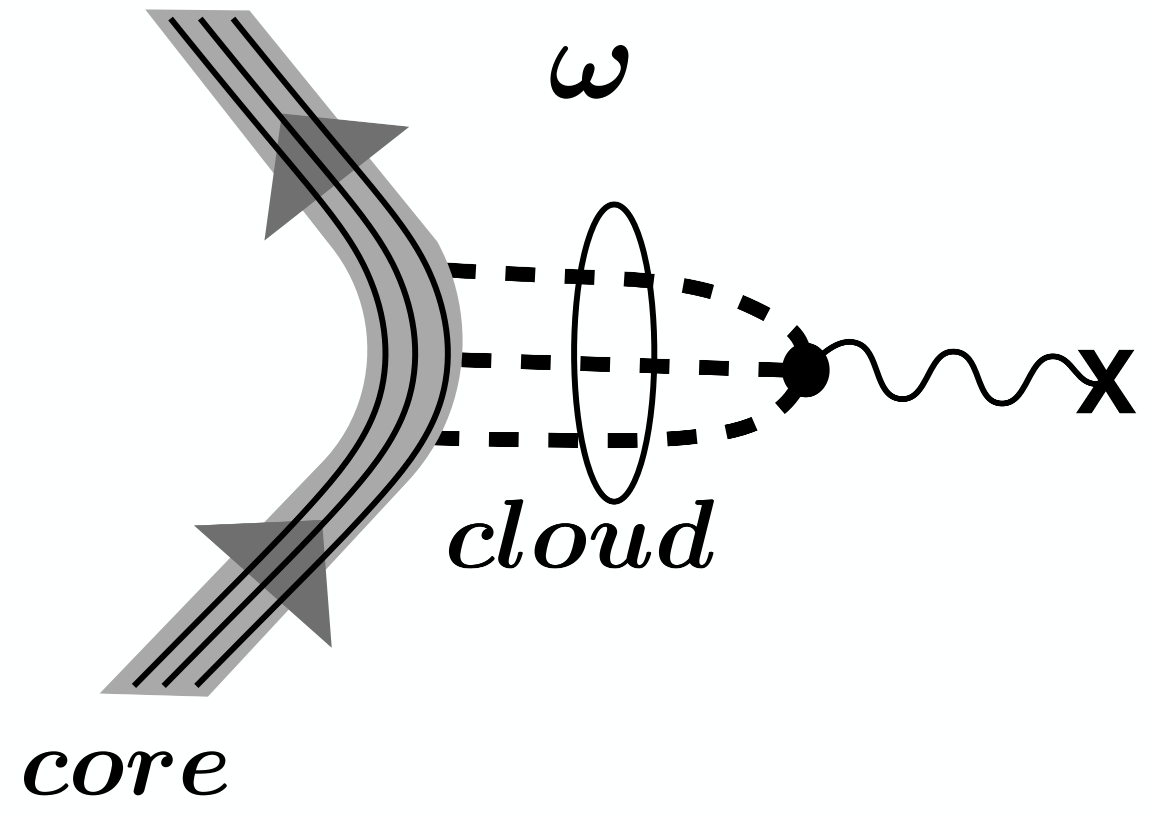
Y.H. Lin, H.-W. Hammer, U.-G. Meißner PRL 128 (2022) 052002

… based on **precision fits** to form factors at both **spacelike and timelike** q^2

- **Simplest Vector Dominance Model:** "cloud" dominated by ω meson

$$\langle r_S^2 \rangle \simeq \langle r_S^2 \rangle_{core} + \frac{6}{m_\omega^2} \quad \rightarrow \quad \langle r_S^2 \rangle_{core}^{1/2} \simeq 0.47\,\mathrm{fm}$$





- **Detailed analysis using best-fit spectral functions :**

$$\langle r_S^2 \rangle_{core}^{1/2} = \langle r_B^2 \rangle^{1/2} = 0.50 \pm 0.01\,\mathrm{fm}$$

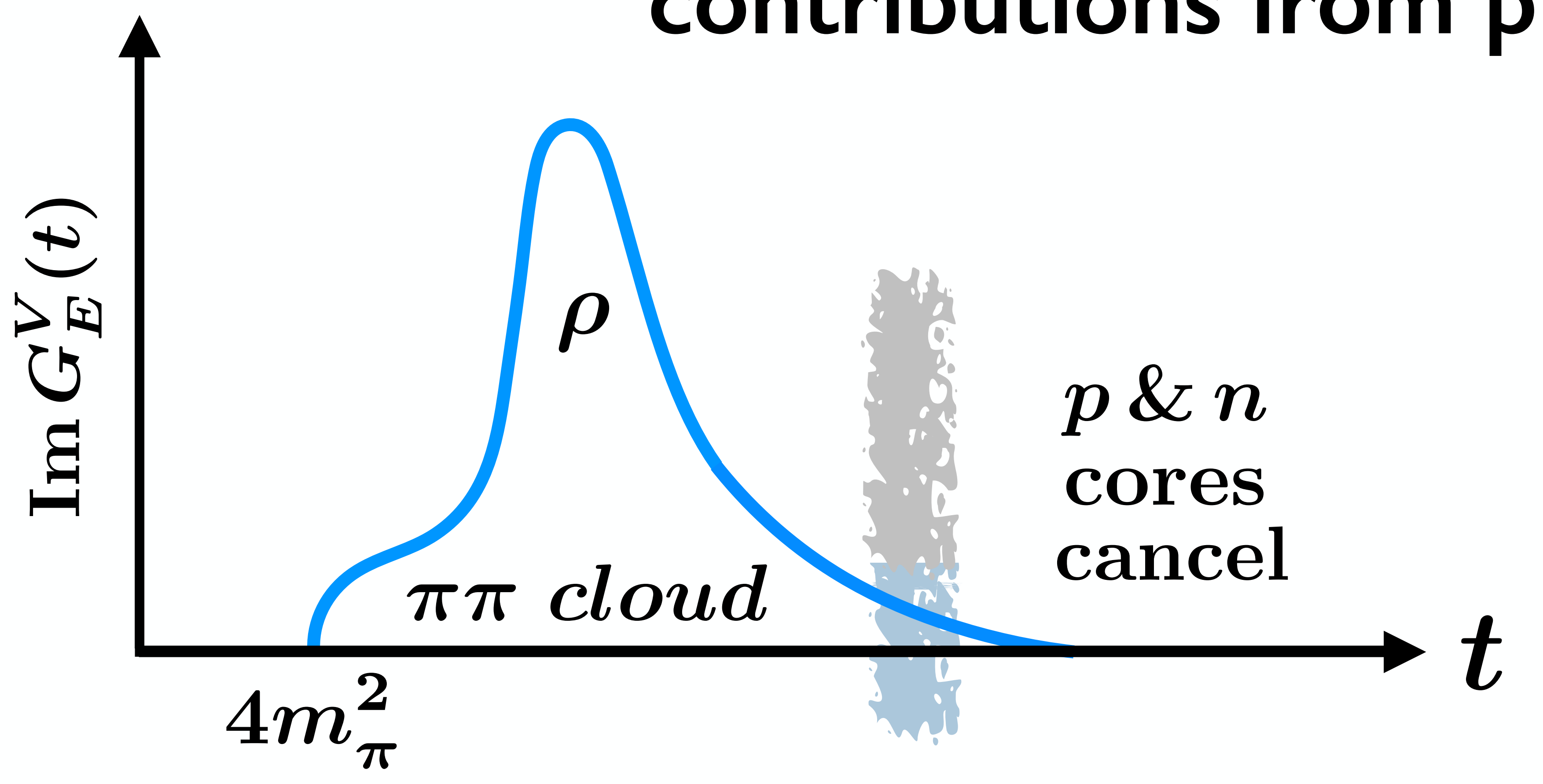
N. Kaiser, W.W. Phys. Rev. C110 (2024) 015202

Example II: **ISOVECTOR ELECTRIC FORM FACTOR** of the **NUCLEON**

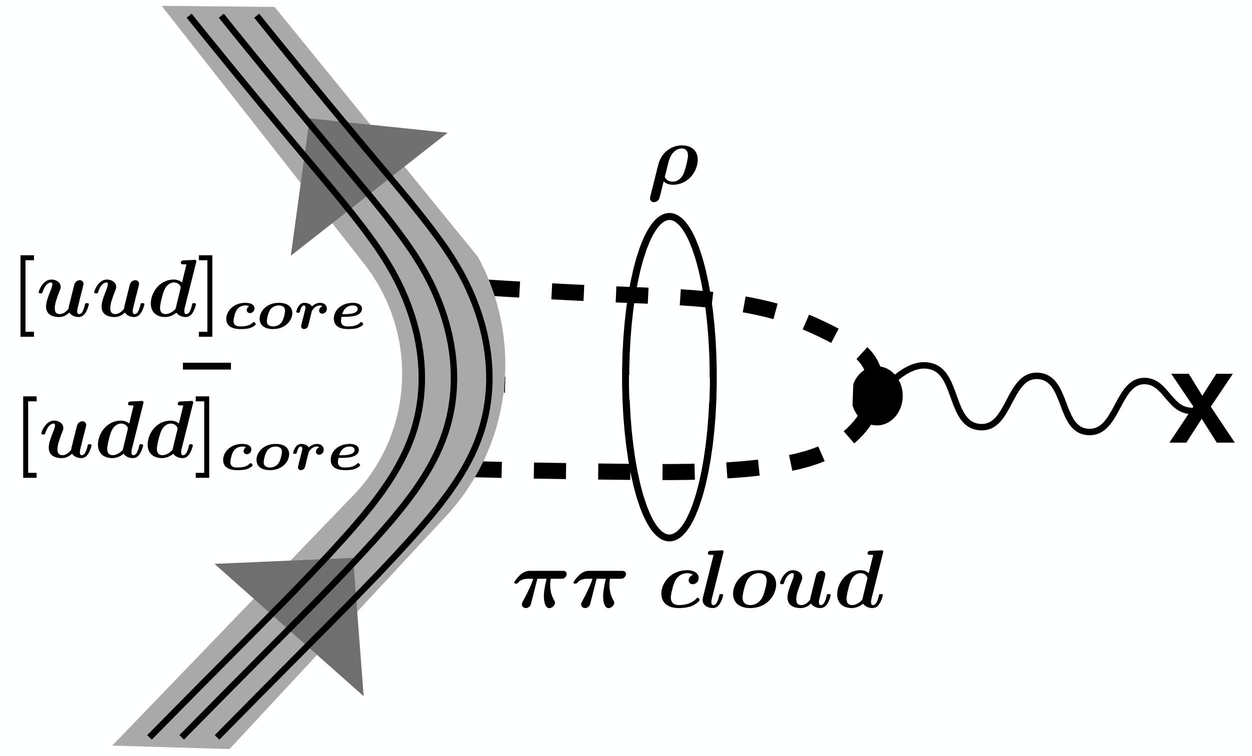
- Isovector electric form factor $G_E^V(q^2) = \frac{1}{2}\left[G_E^p(q^2) - G_E^n(q^2)\right]$ $\quad \langle r_V^2\rangle = \langle r_p^2\rangle - \langle r_n^2\rangle$

Empirical : $\langle r_V^2\rangle^{1/2} = 0.901 \pm 0.009\ \mathrm{fm}$ Y.H. Lin, H.-W. Hammer, U.-G. Meißner PRL 128 (2022) 052002

… clue and test case : in the limit of exact isospin symmetry,
contributions from proton and neutron valence quark cores CANCEL



- **Detailed analysis using best-fit spectral functions :**



$$\langle r_V^2\rangle_{core} = \langle r_p^2\rangle_{core} - \langle r_n^2\rangle_{core} = -0.025\ \mathrm{fm}^2$$ … almost vanishing

- **Isovector charge radius** almost entirely determined by **two-pion cloud**

N. Kaiser, W.W. Phys. Rev. C110 (2024) 015202

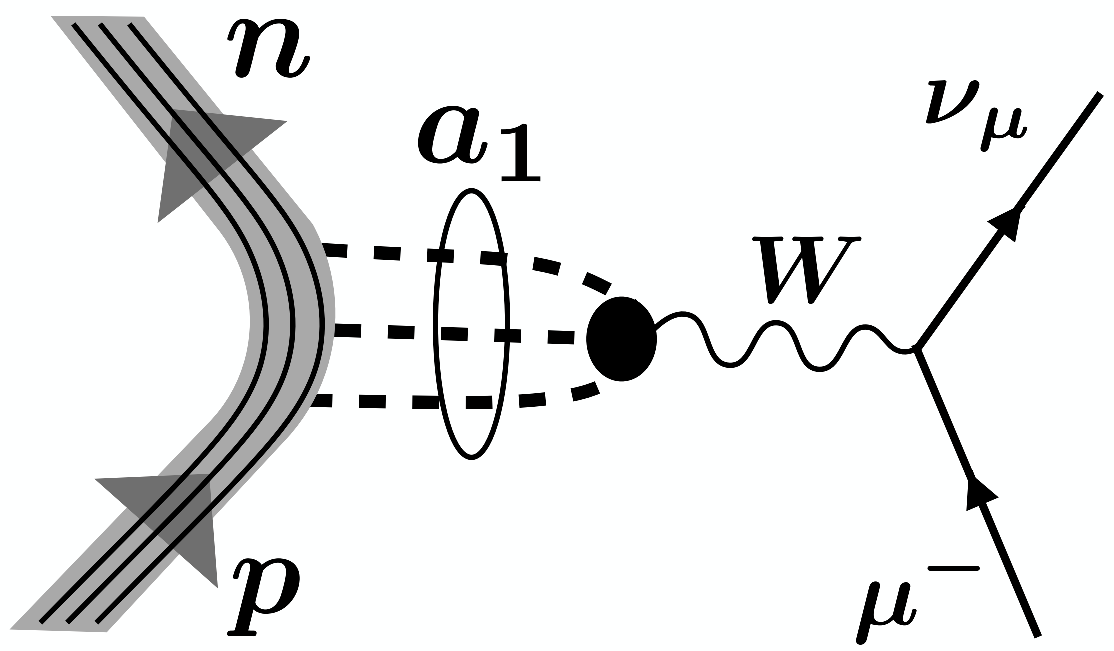
Example III: **ISOVECTOR AXIAL FORM FACTOR** of the **NUCLEON**

- **Axial form factor** $G_A(q^2) = g_A\left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \dots\right]$

  R.J. Hill, P. Kammel, W.C. Marciano, A. Sirlin
  Rep. Prog. Phys. 81 (2018) 096301

  Empirical :

  a) $\langle r_A^2\rangle = 0.454 \pm 0.013\ \mathrm{fm}^2$
  (from νd scattering and
  $e\,p \to e\,n\pi^+$ dipole fits)



  b) $\langle r_A^2\rangle = 0.46 \pm 0.16\ \mathrm{fm}^2$
  (from μp capture and
  νd scattering analysis)

  Axial radius significantly smaller than proton charge radius $\left(\langle r_p^2\rangle = 0.71 \pm 0.01\,\mathrm{fm}^2\right)$

- **Detailed analysis using three-pion spectrum dominated by broad a_1 meson :**

$$\langle r_A^2\rangle = \langle r_A^2\rangle_{core} + \frac{6}{m_a^2}(1+\delta_a) \qquad \delta_a = -\frac{m_a^3}{\pi}\int_{9m_\pi^2}^{t_{max}} dt\,\frac{\Gamma_a(t)}{t^2(t-m_a^2)}$$

→ $$\langle r_A^2\rangle_{core}^{1/2} = 0.53 \pm 0.02\ \mathrm{fm}$$

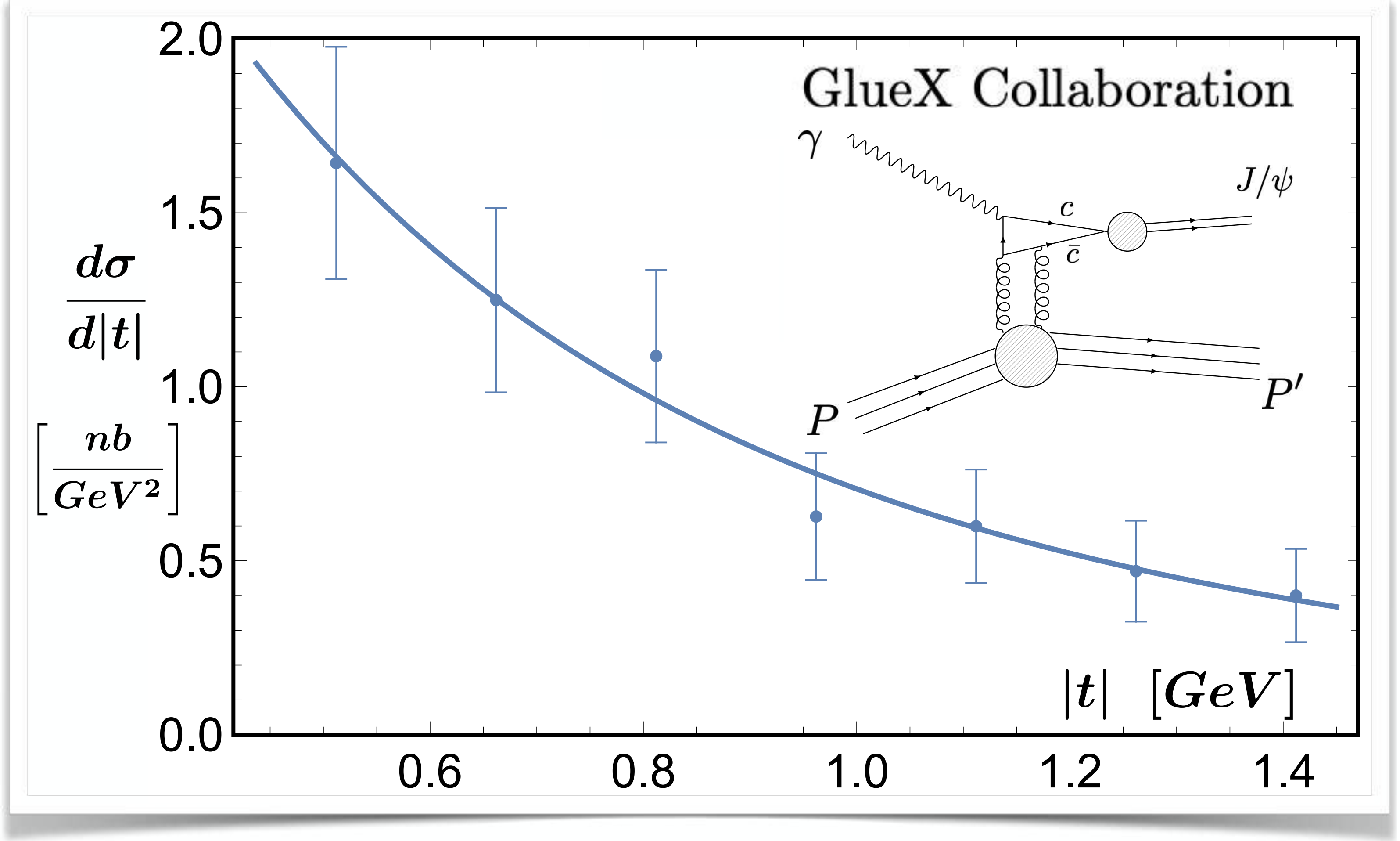
N. Kaiser, W.W.
Phys. Rev. C110 (2024) 015202

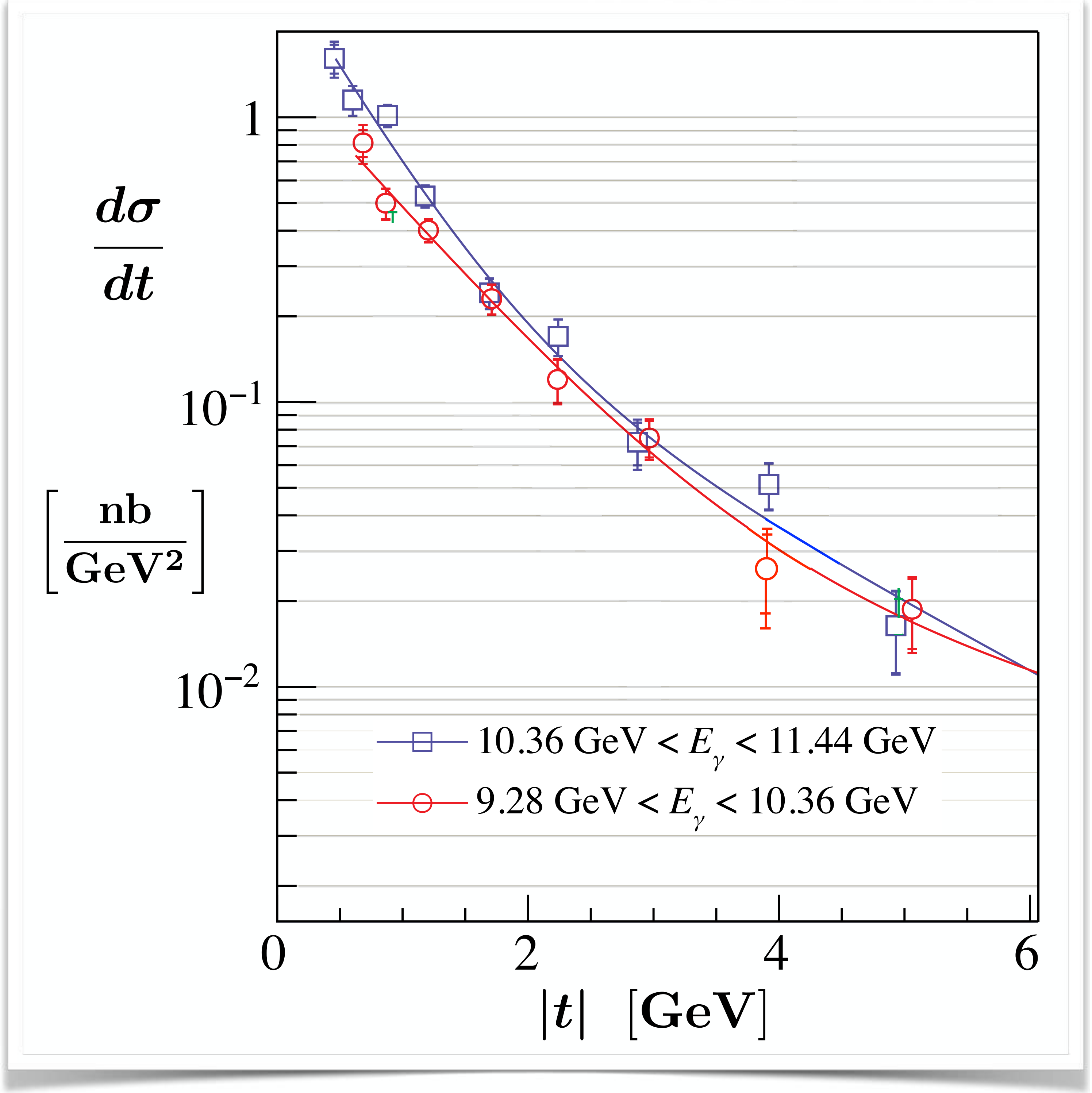
[based on **a)** ; correspondingly larger uncertainty when using **b)**]

Example IV: MASS RADIUS of the NUCLEON

- Mass ("gravitational") form factor
- Trace of QCD energy-momentum tensor

$$G_m(q^2) = \langle P'|T^\mu_\mu|P\rangle = \langle P'|\frac{\beta(g)}{2g}G^{\mu\nu}_a G^a_{\mu\nu} + m_q(\bar{u}u + \bar{d}d) + m_s\bar{s}s|P\rangle$$





$$G_m(0) = M_N \simeq 0.94\,\text{GeV}$$

$$M_N = M_0 + \sigma_N + \sigma_s$$

$$(M_0 \gtrsim 0.9\,M_N)$$

$$\langle r_m^2\rangle = \frac{6}{M_N}\frac{dG_m(q^2)}{dq^2}\Big|_{q^2=0}$$

- Empirical mass radius

$$\langle r_m^2\rangle^{1/2} = (0.55 \pm 0.03)\,\text{fm}$$

$$\langle r_m^2\rangle^{1/2} = (0.53 \pm 0.04)\,\text{fm}$$

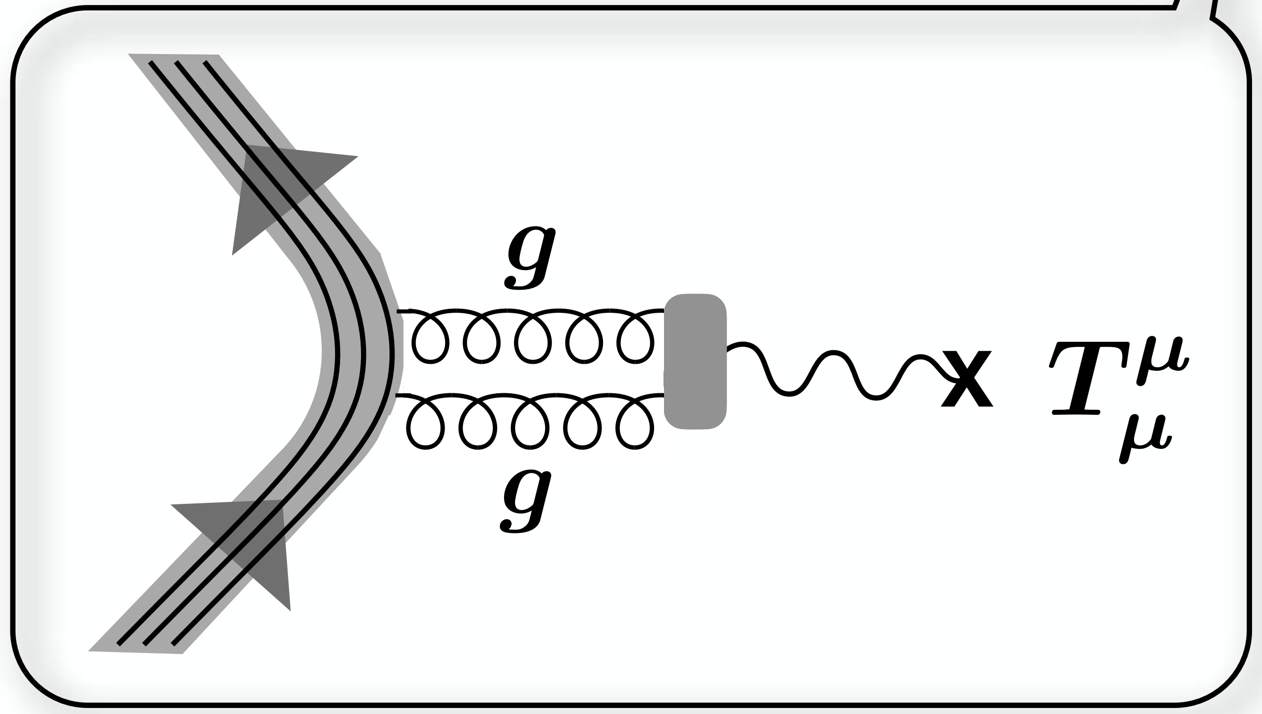
D. Kharzeev : Phys. Rev. D104 (2021) 054015

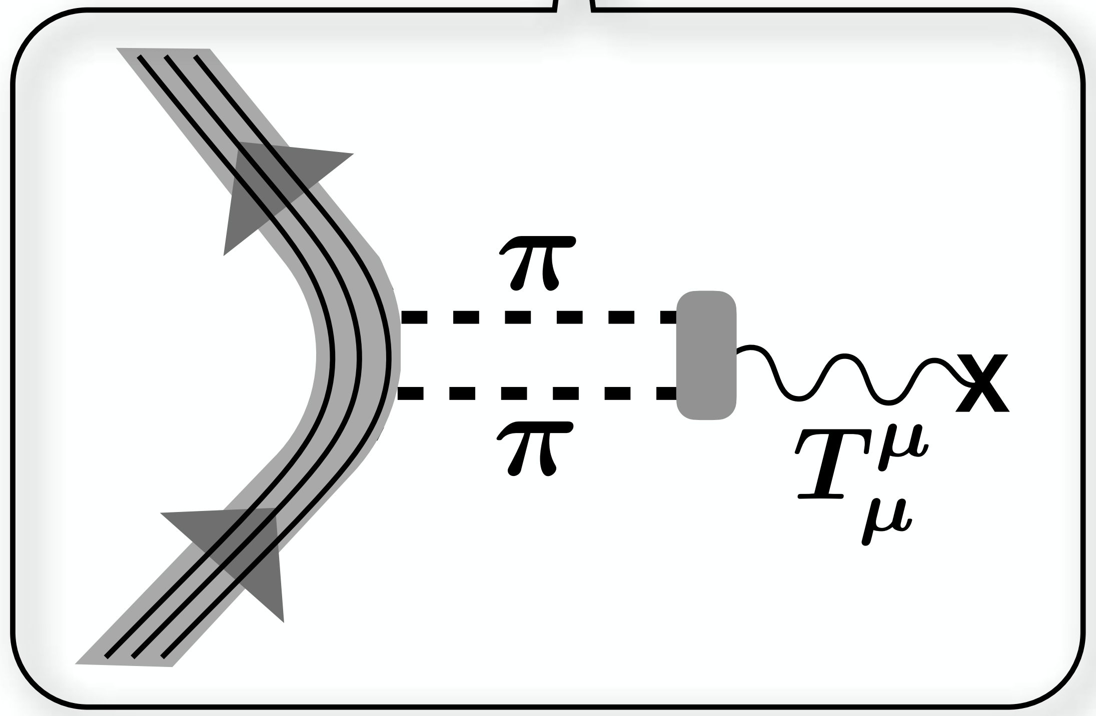
Recent GlueX update: S. Adhikari et al. ; arXiv:2304.03845

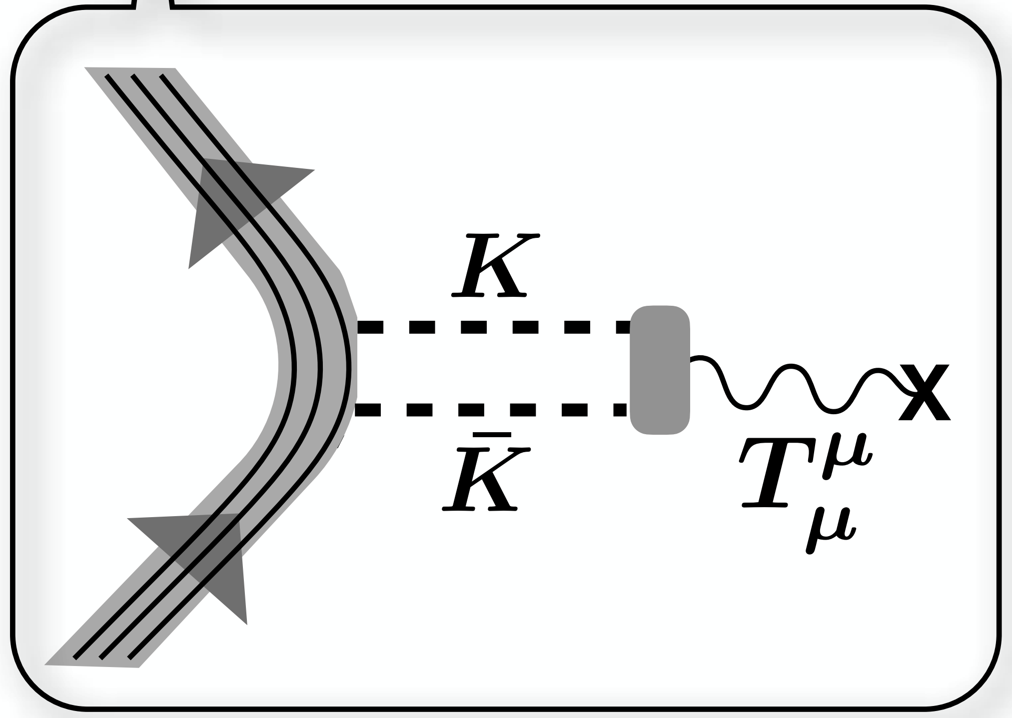
Example IV: **MASS RADIUS of the NUCLEON** (contd.)

- Core (gluon) dominance plus small corrections from sigma terms

$$\langle r_m^2 \rangle = \left[\frac{M_0}{M_N} \langle r_m^2 \rangle_{core} + \frac{\sigma_N}{M_N} \langle r_{\pi\pi}^2 \rangle + \frac{\sigma_s}{M_N} \langle r_{K\bar{K}}^2 \rangle \right]$$







- Estimates of sigma terms and associated radii from Lattice QCD and ChPT

$$\sigma_N \simeq 40 - 60 \text{ MeV} \; , \; \sigma_s \simeq 30 \text{ MeV} \qquad \langle r_{\pi\pi}^2 \rangle^{1/2} \simeq 1.3 \text{ fm} \; , \; \langle r_{K\bar{K}}^2 \rangle \sim (m_\pi / m_K)^2 \langle r_{\pi\pi}^2 \rangle$$

→ $$\langle r_m^2 \rangle_{core}^{1/2} = 0.48 \pm 0.05 \text{ fm}$$

N. Kaiser, W.W.
Phys. Rev. C110 (2024) 015202

✷ y-scaling in electron-nucleus scattering ➔ strongly correlated **NUCLEONS** at short distances corresponding to densities as high as $n_B \sim 5\, n_0$

Particles **2023**, *1*, 1–11

arXiv:2306.01367

Testing the Paradigm of Nuclear Many-Body Theory

Omar Benhar

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Abstract: Nuclear many-body theory is based on the tenet that nuclear systems can be accurately described as collections of point-like particles. This picture, while providing a remarkably accurate explanation of a wealth of measured properties of atomic nuclei, is bound to break down in the high-density regime, in which degrees of freedom other than protons and neutrons are expected to come into play. Valuable information on the validity of the description of dense nuclear matter in terms of nucleons, needed to firmly establish its limit of applicability, can be obtained from electron–nucleus scattering data at large momentum transfer and low energy transfer. The emergence of *y*-scaling in this kinematic region, unambiguously showing that the beam particles couple to high-momentum nucleons belonging to strongly correlated pairs, indicates that at densities as large as five times nuclear density—typical of the neutron star interior—nuclear matter largely behaves as a collection of nucleons.