Lectures at HHIQCD 2024 23 October 2024

Dense Baryonic Matter : From **Quarks** to **Nuclei** and **Neutron Stars**

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Phases of QCD Matter

- **High temperatures** : **Constraints from High-energy heavy-ion collisions and Lattice QCD thermodynamics**
- **High baryon densities** : **Constraints from neutron star matter**
- **Moderate temperatures and densities** : **nuclear thermodynamics**
- **Low-energy structure of the nucleon**

Dense and Cold Baryonic Matter

- **Inference of the Equation of State from neutron star observations**
- **Theoretical methods and strategies** : **Effective Field Theory Functional Renormalisation Group**
- **Neutron star matter as a Relativistic Fermi liquid**
- **Strangeness** ? **Quark Matter** ? **Hadron - Quark Continuity**

1.

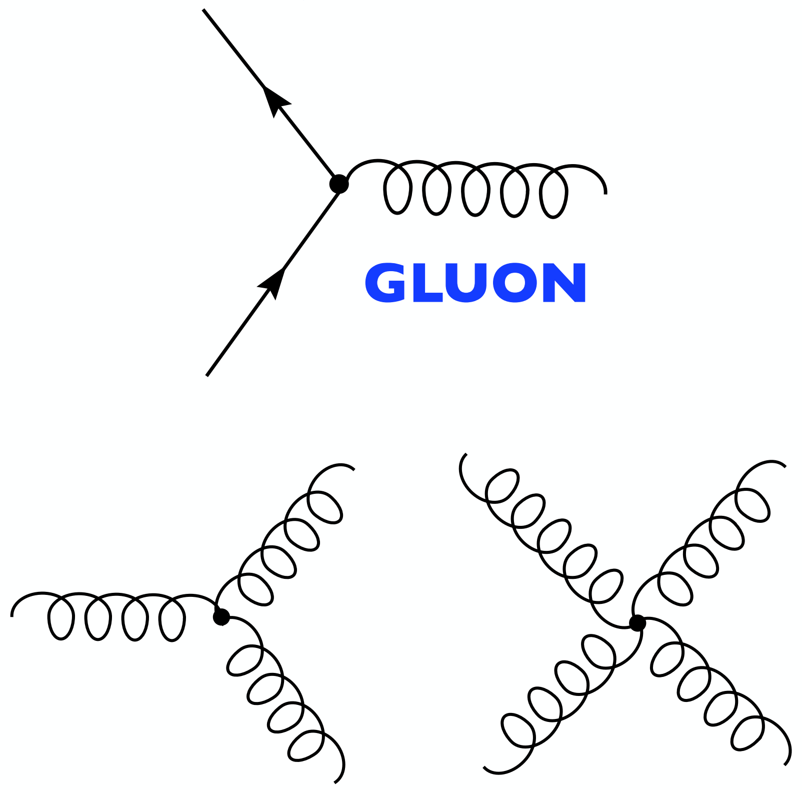
Preliminaries :

QCD and Cold Quark Matter

QUANTUM CHROMO DYNAMICS

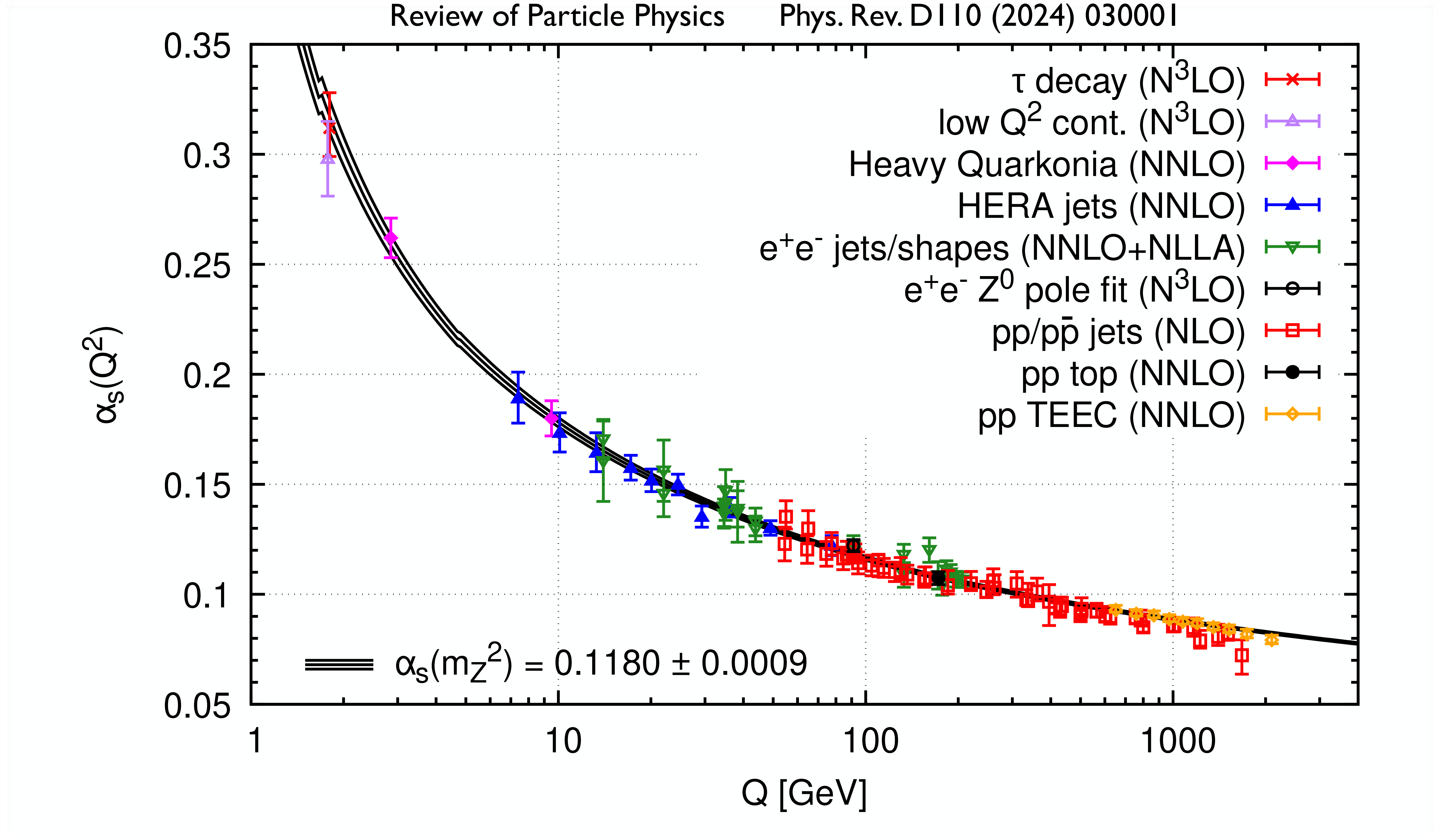
$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}\left(i\gamma_\mu \mathcal{D}^\mu - \mathbf{m}\right)\psi - \frac{1}{4}\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}$$

Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	173	2/3
b bottom	4.2	−1/3



QCD running coupling

$$\lambda_R \frac{d\alpha_s}{d\lambda_R} = \beta(\alpha_s) = -\left(\frac{33-2N_f}{12\pi^2}\alpha_s^2 + \frac{153-19N_f}{24\pi^2}\alpha_s^3 + \dots\right)$$



▶ Scale-setting example : **cold gas of massless quarks** (quark chemical potential $\mu \simeq p_F$)

Quark density : $n_q = N_f\, p_F^3/\pi^2 = 3\mu^3/\pi^2$ $(N_f = 3)$

Baryon number density : $n_B = \mu^3/\pi^2$

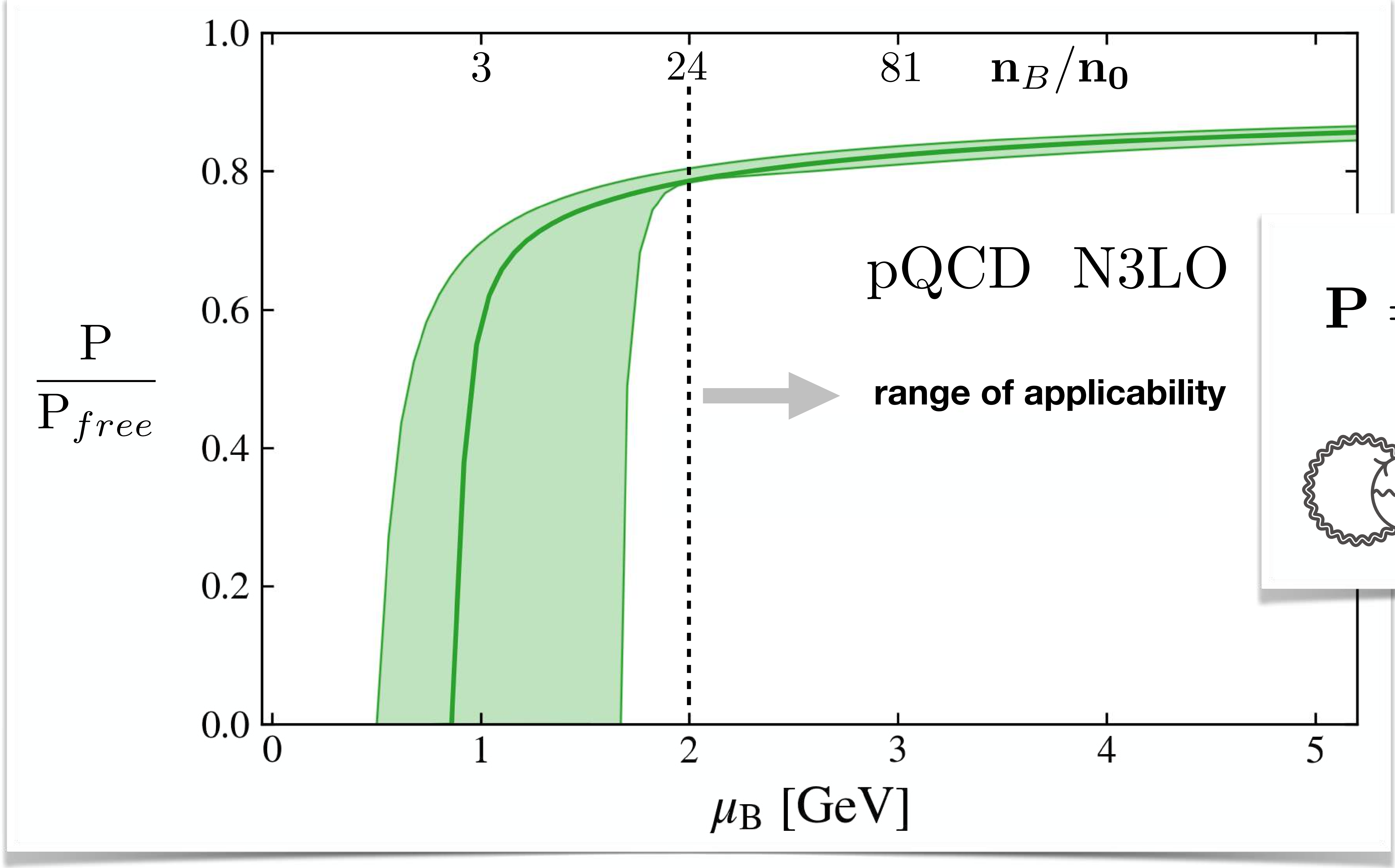
Range of applicability for perturbative QCD :

$\mu > 1\,\mathrm{GeV}$ ➔ $n_B > 80\, n_0$ **!**

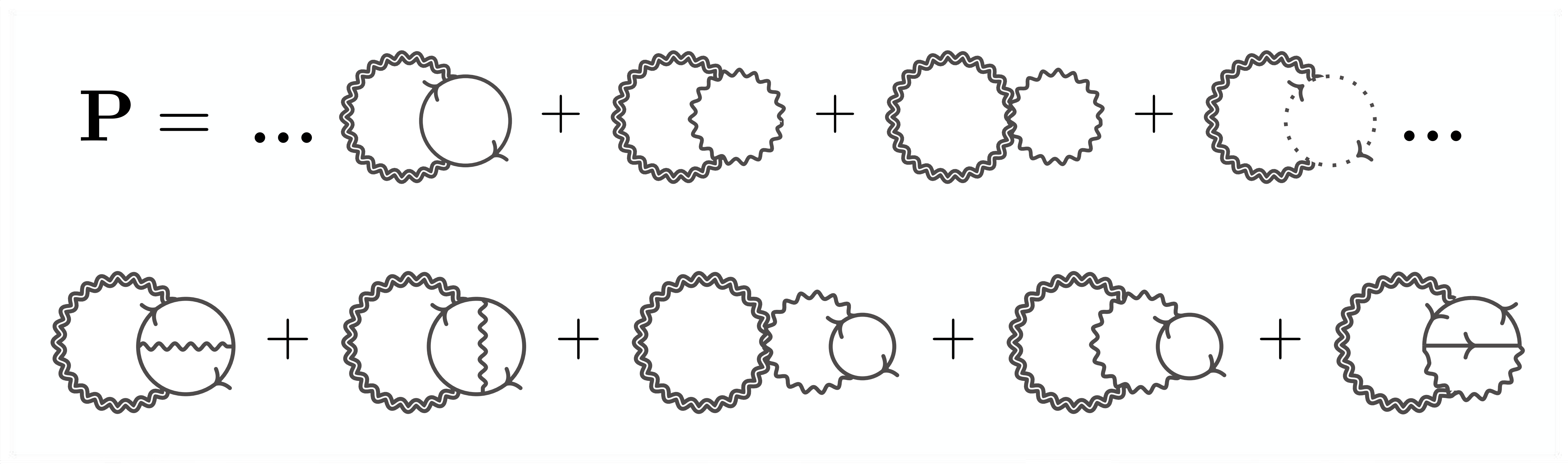
$(n_0 = 0.16\,\mathrm{fm}^{-3})$

COLD QUARK MATTER in pQCD

- **State-of-the-art : next-to-next-to-next-to leading order in α_s** ($N_f = 3$ massless quarks)
- **Pressure** $$\frac{P}{P_{free}} = 1 - \frac{\alpha_s}{\pi} - 3\left(\frac{\alpha_s}{\pi}\right)^2 f\left[\ln\left(\frac{\alpha_s}{\pi}\right), \mu\right] + 9\left(\frac{\alpha_s}{\pi}\right)^3 g\left[\ln\left(\frac{\alpha_s}{\pi}\right), \mu\right]$$



$$P_{free} = \frac{3\,\mu^4}{4\pi^2} \qquad \mu = \frac{\mu_B}{3}$$

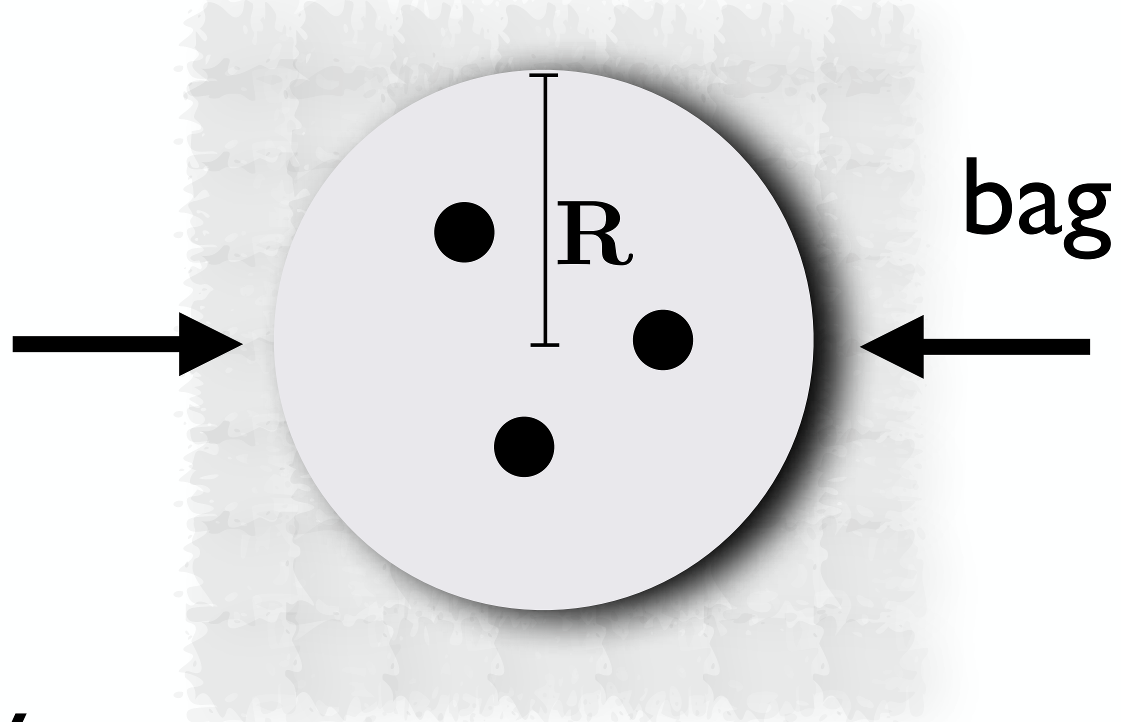


T. Gorda, R. Paatelainen, S. Säppi, K. Seppänen
Phys. Rev. Lett. 131 (2023) 181902

Basic schematic exercises:

Massless Quarks under Pressure
- Baryons vs. Quark Fermi Gas -

- Valence quarks in a confining bag



- Energy

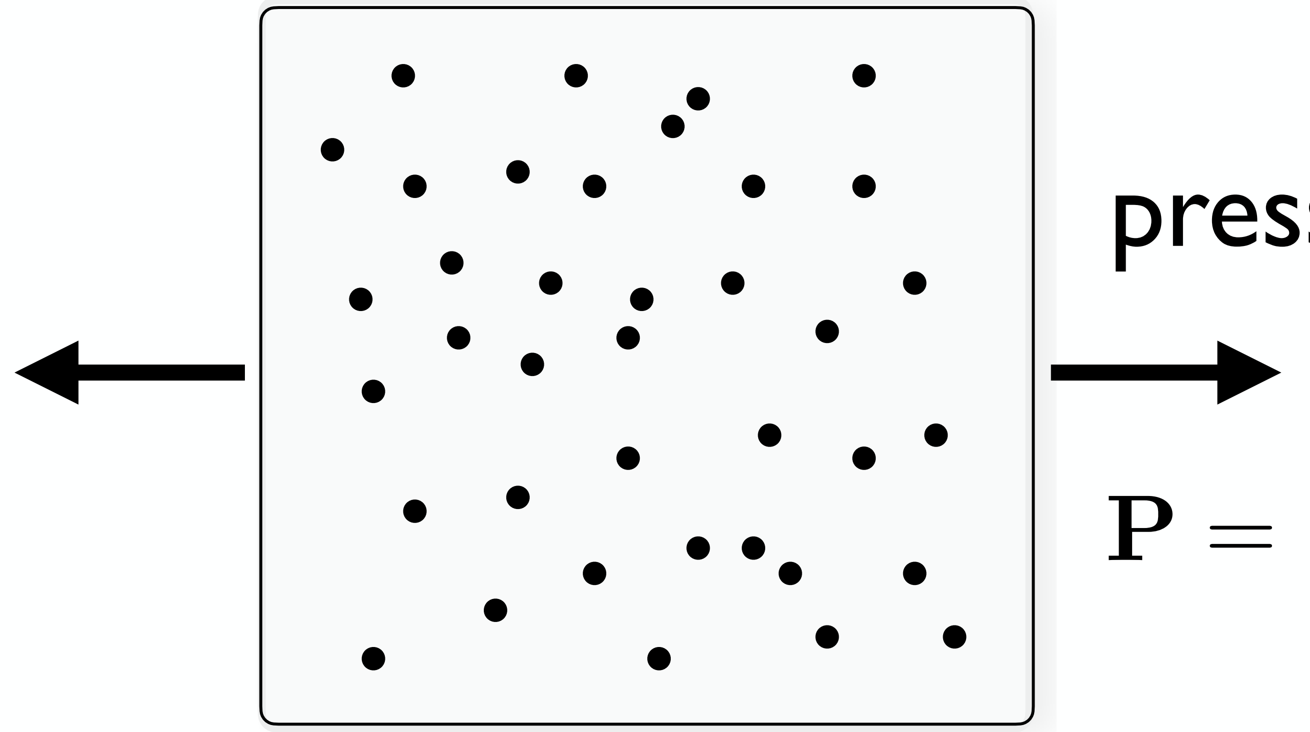
$$\mathbf{E} = \frac{\mathbf{C}}{\mathbf{R}} + \frac{4\pi}{3}\mathbf{B}\,\mathbf{R}^3 = \mathbf{M} \qquad \frac{\partial \mathbf{E}}{\partial \mathbf{R}} = 0$$

$$\mathbf{B} = \frac{3}{16\pi}\frac{\mathbf{M}}{\mathbf{R}^3} \qquad \mathbf{C} = \frac{3}{4}\mathbf{M}\,\mathbf{R}$$

- Example: $\mathbf{M} \sim 1\,\mathbf{GeV}$ $\quad \mathbf{R} \sim 0.6\,\mathbf{fm}$

bag constant: $\mathbf{B}^{1/4} \simeq 0.22\,\mathbf{GeV} \sim \Lambda_{\mathbf{QCD}}$

- Gas of ultrarelativistic quarks and gluons



$$\mathbf{P} = \frac{\mathbf{T}}{\mathbf{V}}\ln \mathcal{Z}$$

$$\mathbf{P} = \mathbf{P_Q} + \mathbf{P_G}$$

- **Fermi gas of massless quarks** $(g_Q = 2\,N_f\,N_c)$

$$\mathbf{P_Q} = \frac{g_Q}{12}T^4\left[\frac{7\pi^2}{30} + \left(\frac{\mu}{T}\right)^2 + \frac{1}{2\pi^2}\left(\frac{\mu}{T}\right)^4\right]$$

μ : chemical potential of quarks $(\mu = p_F)$

- **Gluon gas** $(g_G = 2\,(N_c^2 - 1) = 16)$

$$\mathbf{P_G} = g_G\,\frac{\pi^2}{90}\,T^4$$

Basic schematic exercise (contd.)

Massless Quarks under Pressure
- Baryons vs. Quark Fermi Gas -

- **total pressure** $(N_f = 3)$ $\mathbf{P}(T,\mu) = \frac{19}{36}\pi^2 T^4 + \frac{3}{2}\mu^2 T^2 + \frac{3}{4\pi^2}\mu^4$
- **baryon density** $\mathbf{n_B} = \frac{1}{3}\frac{\partial \mathbf{P}}{\partial \mu} = \mu T^2 + \frac{\mu^3}{\pi^2}$

- **Critical temperature** at deconfinement crossover

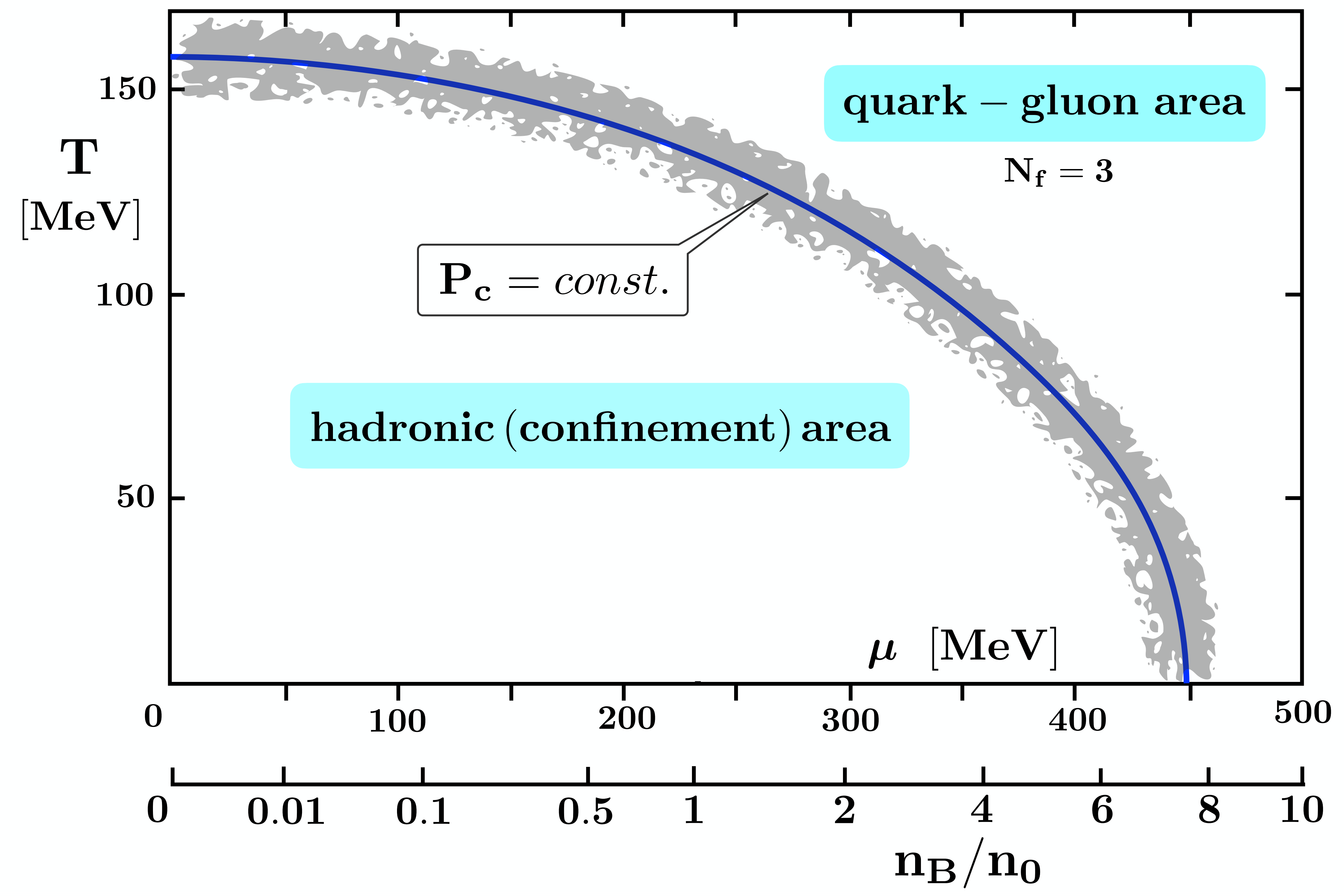
$$\mathbf{T = T_c = 156\,MeV} \;,\; \boldsymbol{\mu} = \mathbf{0}$$

- **Critical pressure**

$$\mathbf{P_c} = \mathbf{P}(T_c, \mu = 0)$$
$$\simeq \mathbf{0.4\,GeV/fm^3} \quad (N_f = 3)$$

- Note: assuming $\mathbf{P_c = B}$

→ $\mathbf{B^{1/4} \simeq 0.23\,GeV}$



Critical isobaric curve (line of constant $\mathbf{P_c}$) in the $\mathbf{T} - \mu$ diagram

Prototype of a phase diagram

quark chemical potential

baryon density

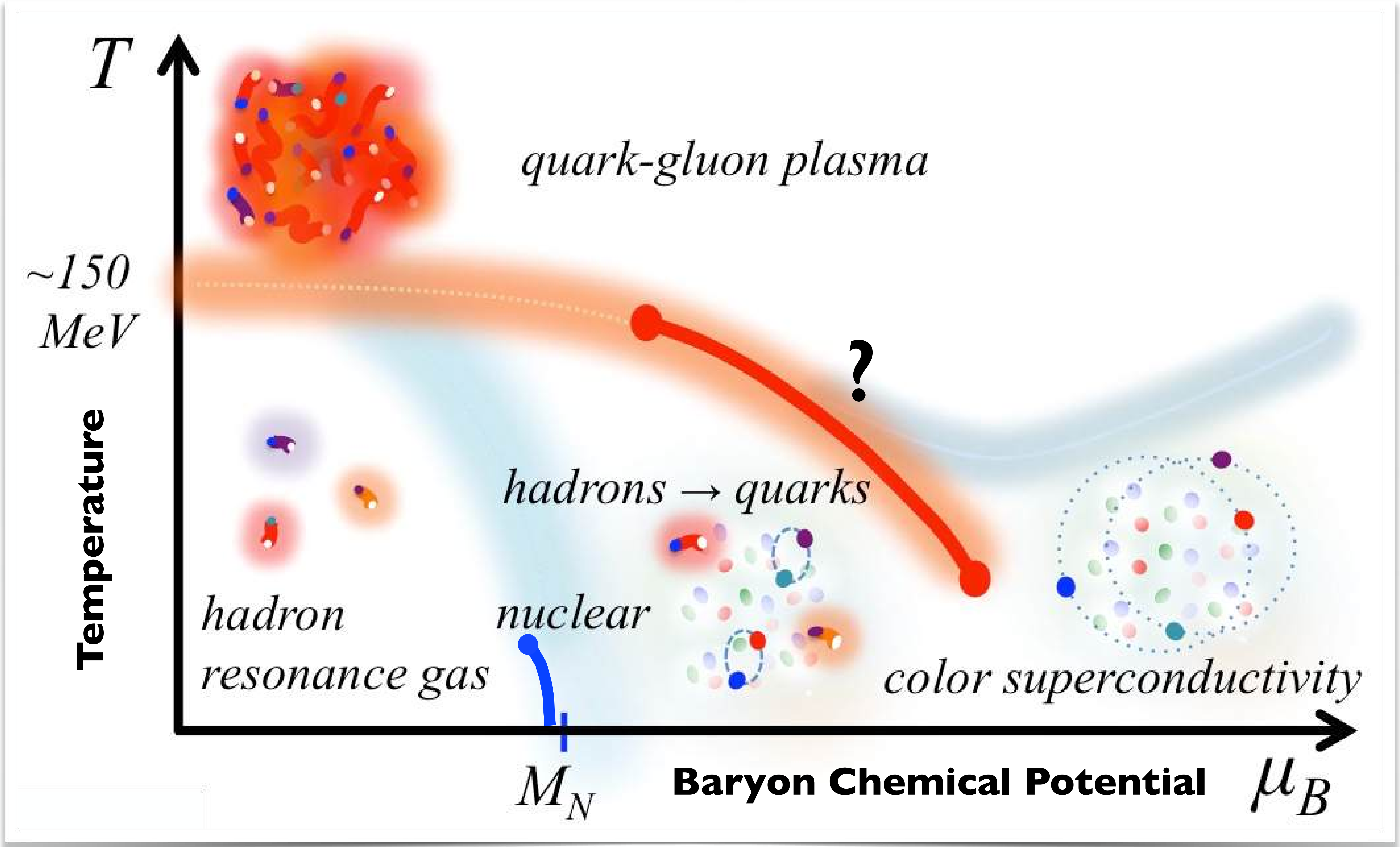
$(\mathbf{n_0 = 0.16\,fm^{-3}})$

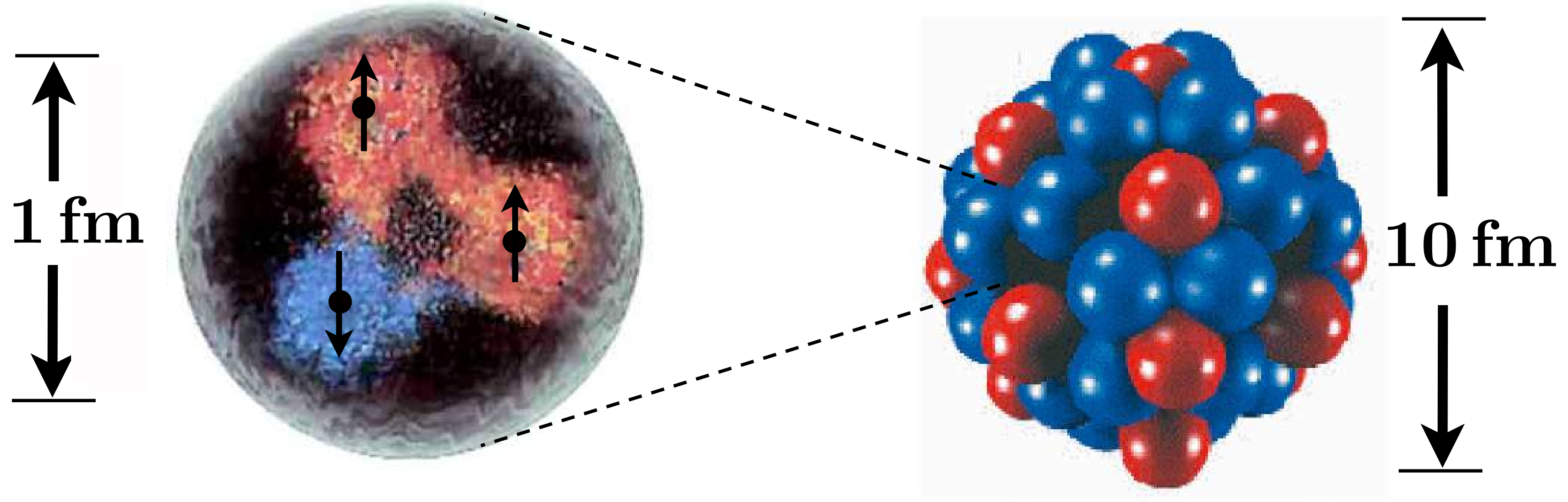
2.

Status :
What do we know about the PHASES of QCD ?

PHASES and STRUCTURES of QCD

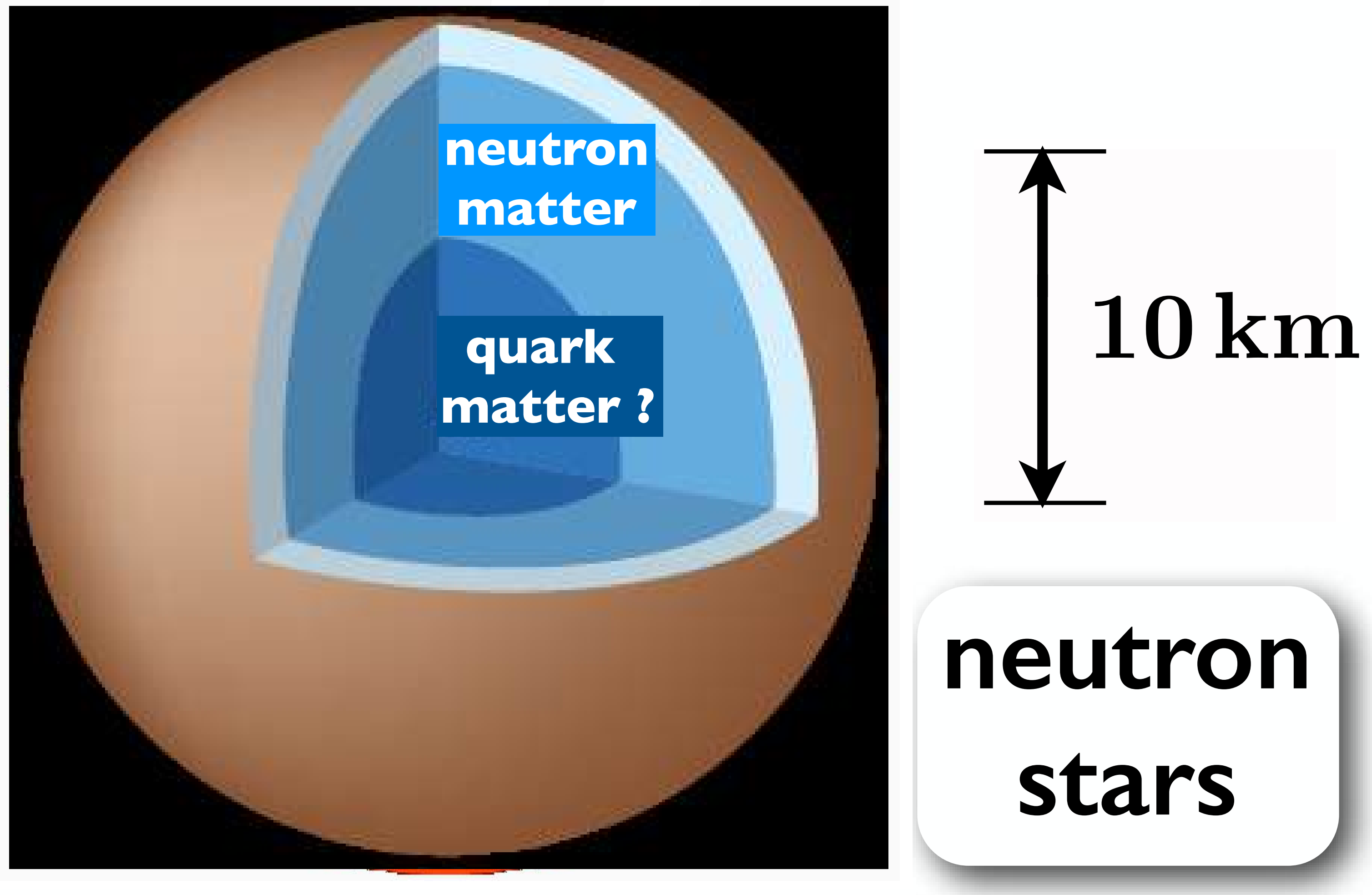
QCD PHASE DIAGRAM : Theorist's Vision





nucleons

nuclei

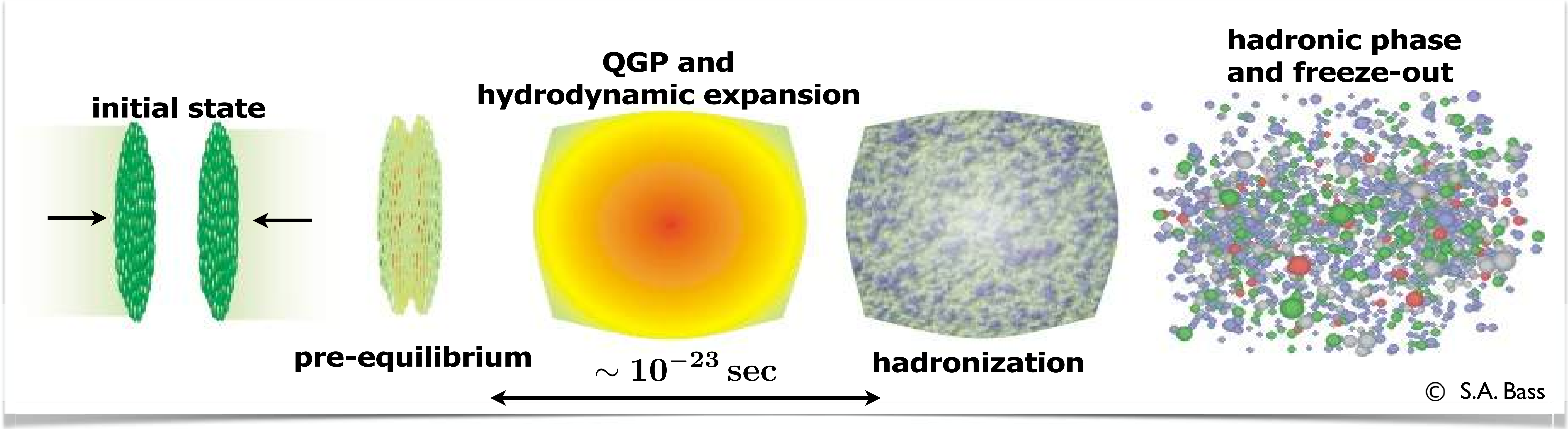


neutron stars

G. Baym, T. Hatsuda, T Kojo, P.D. Powell, Y. Song, T. Takatsuka : Rept. Prog. Phys. 81 (2018) 056902

Strategies PART I: **Heavy-Ion Collisions**

- **High Energy Nuclear Collisions** @ CERN/SPS, **RHIC**, **LHC**

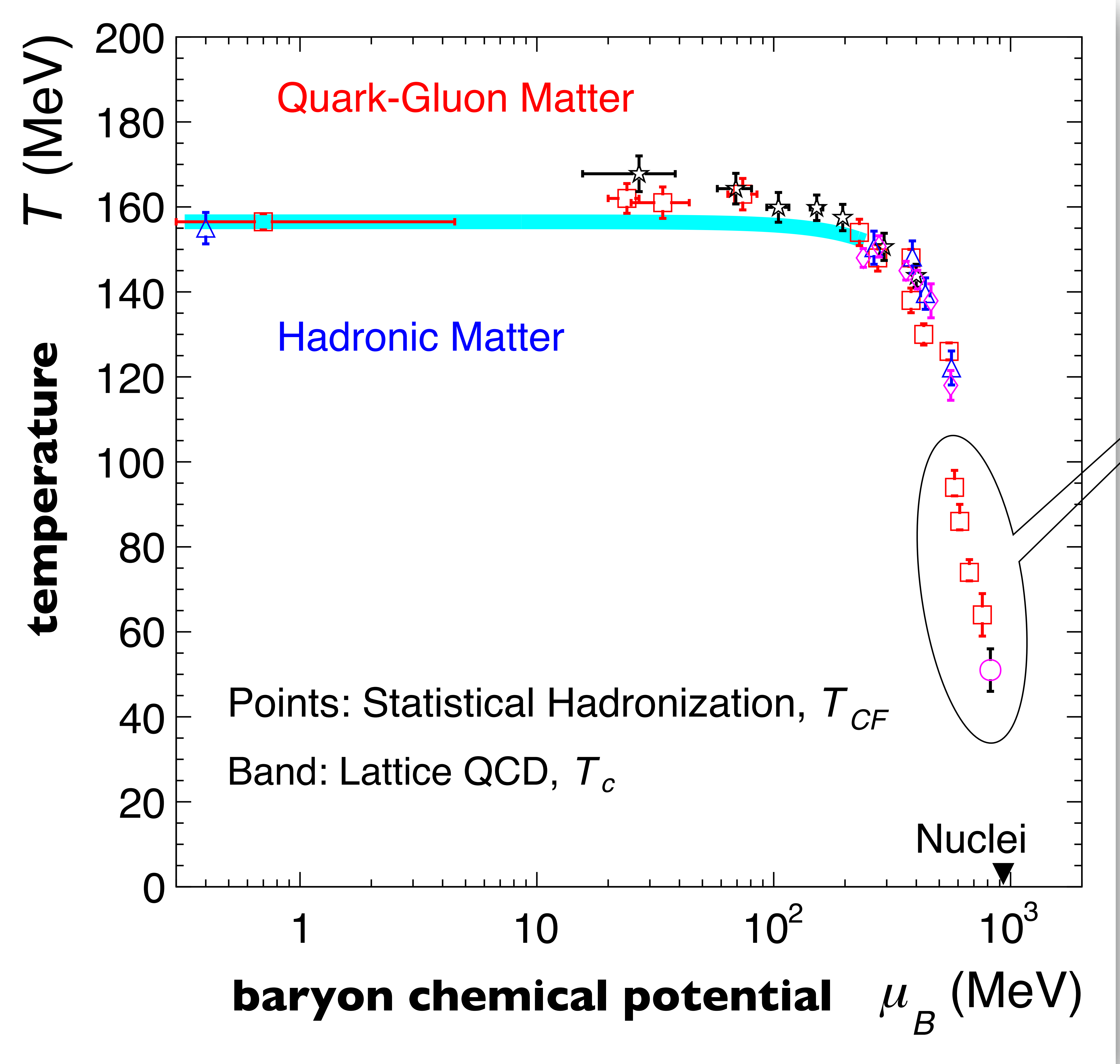


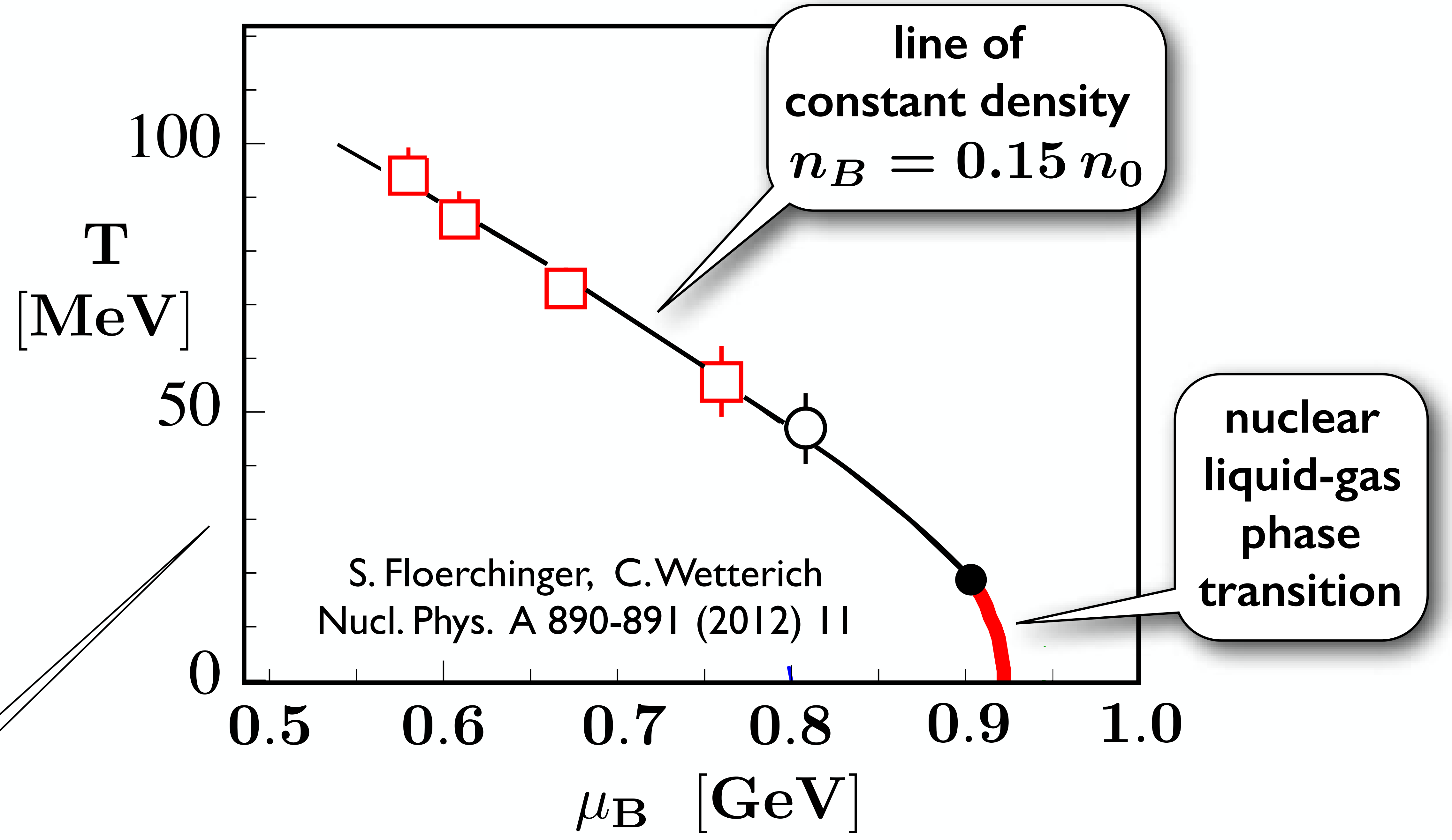
- Analysis of **RHIC** and **LHC** data:

  - **Initial temperatures 300 - 500 MeV**
  - **Fast equilibration**
  - **Strongly correlated quark-gluon matter**

CHEMICAL FREEZE-OUT

A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel
Nature 561 (2018) 321 arXiv:2101.05747





- **Freeze-out temperature**

  $T_{CF} = 156 \pm 2$ MeV at $\mu_B \simeq 0$

- **Nuclear matter in equilibrium**

  baryon density $n_0 = 0.16\, \mathrm{fm}^{-3}$

  at $T = 0$ and $\mu_B = M_N - \frac{|E_b|}{A} = 923\, \mathrm{MeV}$

- **Low-temperature freeze-out trajectory : NOT a first-order phase transition line**

Strategies PART II: **Lattice QCD**

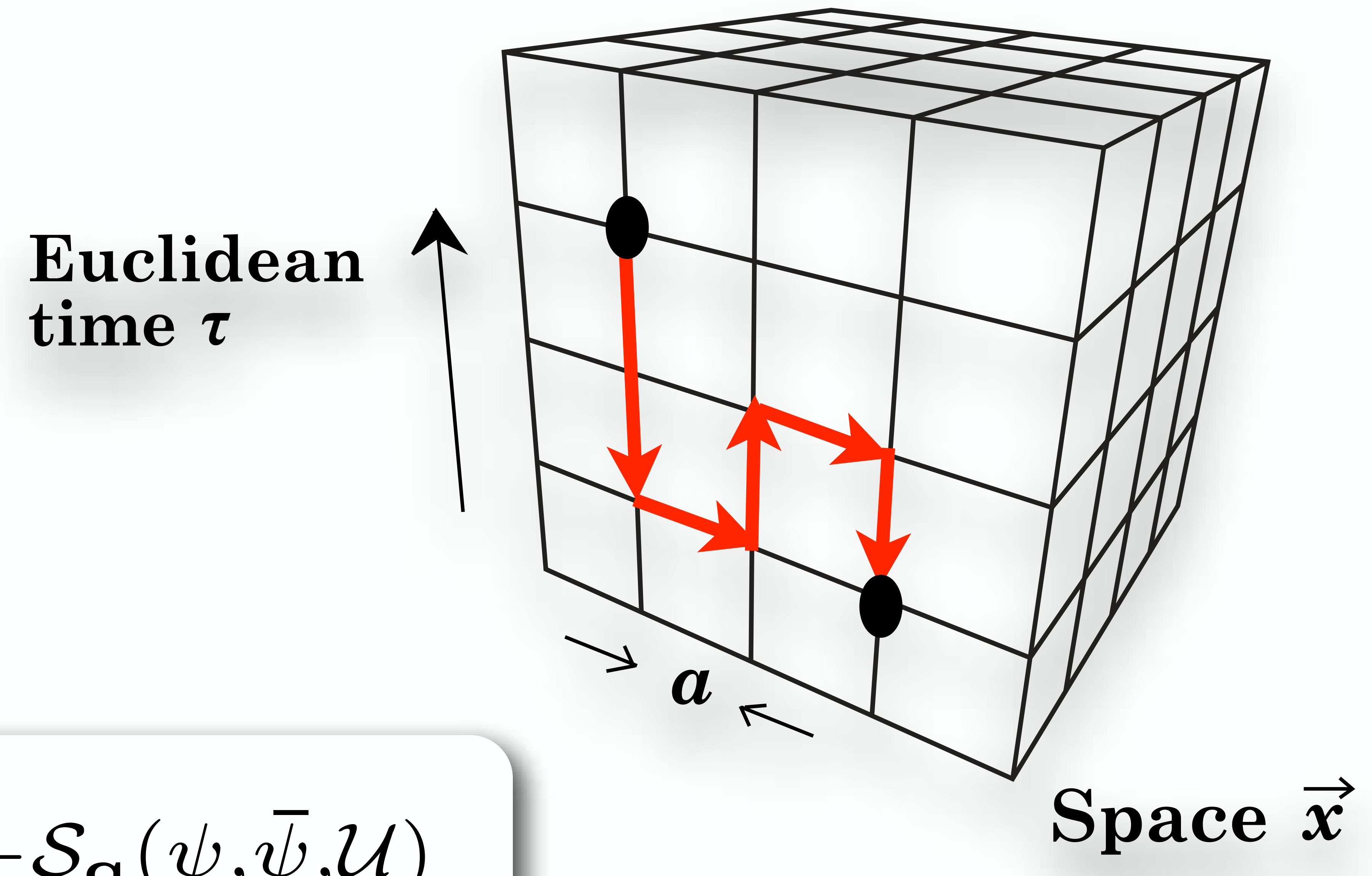
$$\mathcal{L}_{\mathbf{QCD}} = \bar{\psi}\,(i\gamma_\mu \mathcal{D}^\mu - \mathbf{m})\,\psi - \frac{1}{4}\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}$$

- Large-scale computer simulations on **EUCLIDEAN SPACE-TIME** Lattices

- Euclidean time $\hat{=}$ inverse **temperature** $\tau = 1/\mathrm{T}$

  quarks on lattice sites $\psi(n)$

  gluon fields on links $\mathcal{U}_\mu(n) = 1 + iaA_\mu(n) + \mathcal{O}(a^2)$



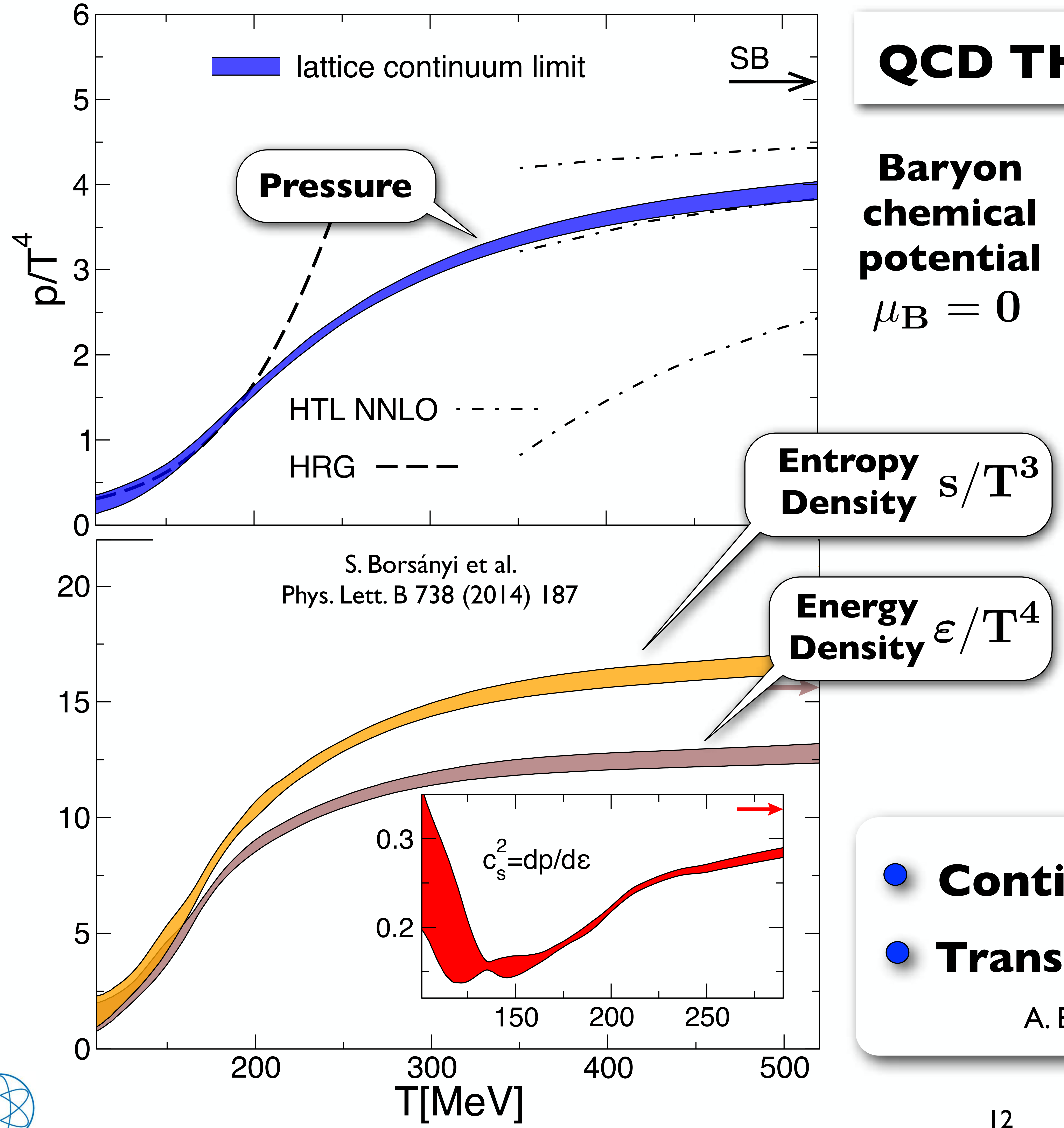
- **QCD THERMODYNAMICS**

  Partition function

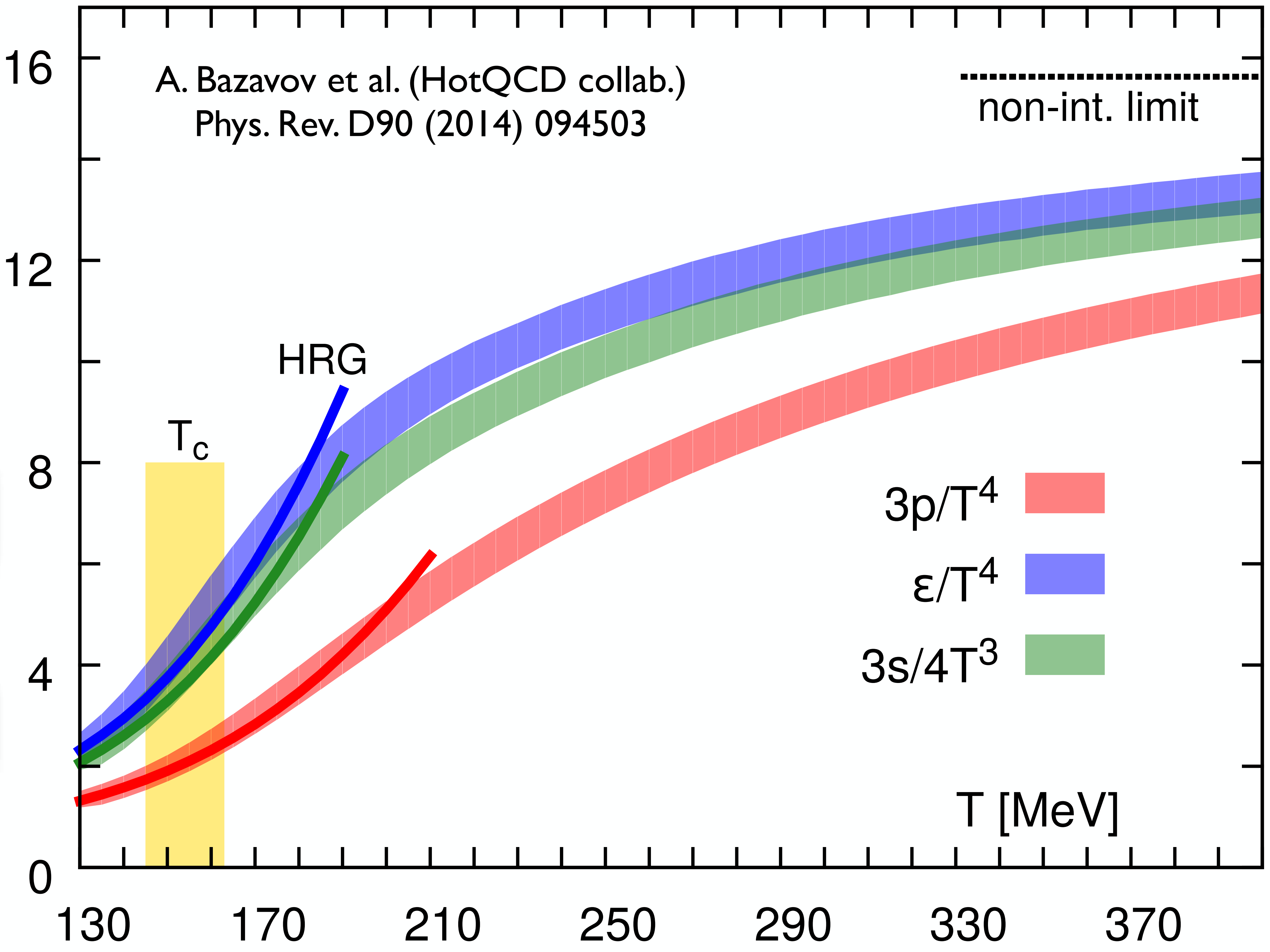
$$\mathcal{Z} = \int [\mathrm{d}\mathcal{U}\,\mathrm{d}\psi\,\mathrm{d}\bar{\psi}]\,\mathrm{e}^{-\mathcal{S}_\mathbf{G}(\mathcal{U}) - \mathcal{S}_\mathbf{q}(\psi,\bar{\psi},\mathcal{U})}$$

► **Non-perturbative "condensed matter physics"** of **QCD**

QCD THERMODYNAMICS on the LATTICE



Baryon chemical potential
$\mu_B = 0$



- **Continuous crossover** (not a phase transition)
- **Transition temperature** $T_c = 156.5 \pm 1.5$ MeV

A. Bazavov et al. (HotQCD collab.) Phys. Lett. B795 (2019) 15

QCD THERMODYNAMICS on the LATTICE

- **Extrapolations** to **non-zero baryon chemical potential** μ_B

S. Borsányi et al. : Phys. Rev. Lett. 126 (2021) 232001

- **Pressure**

$$P(T,\mu_B) = T^4 \sum_{n=0} \frac{\chi_{2n}(T,0)}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$

$$\chi_j(T,\mu_B) = \left(\frac{\partial}{\partial \mu_B/T}\right)^j \frac{P(T,\mu_B)}{T^4}$$

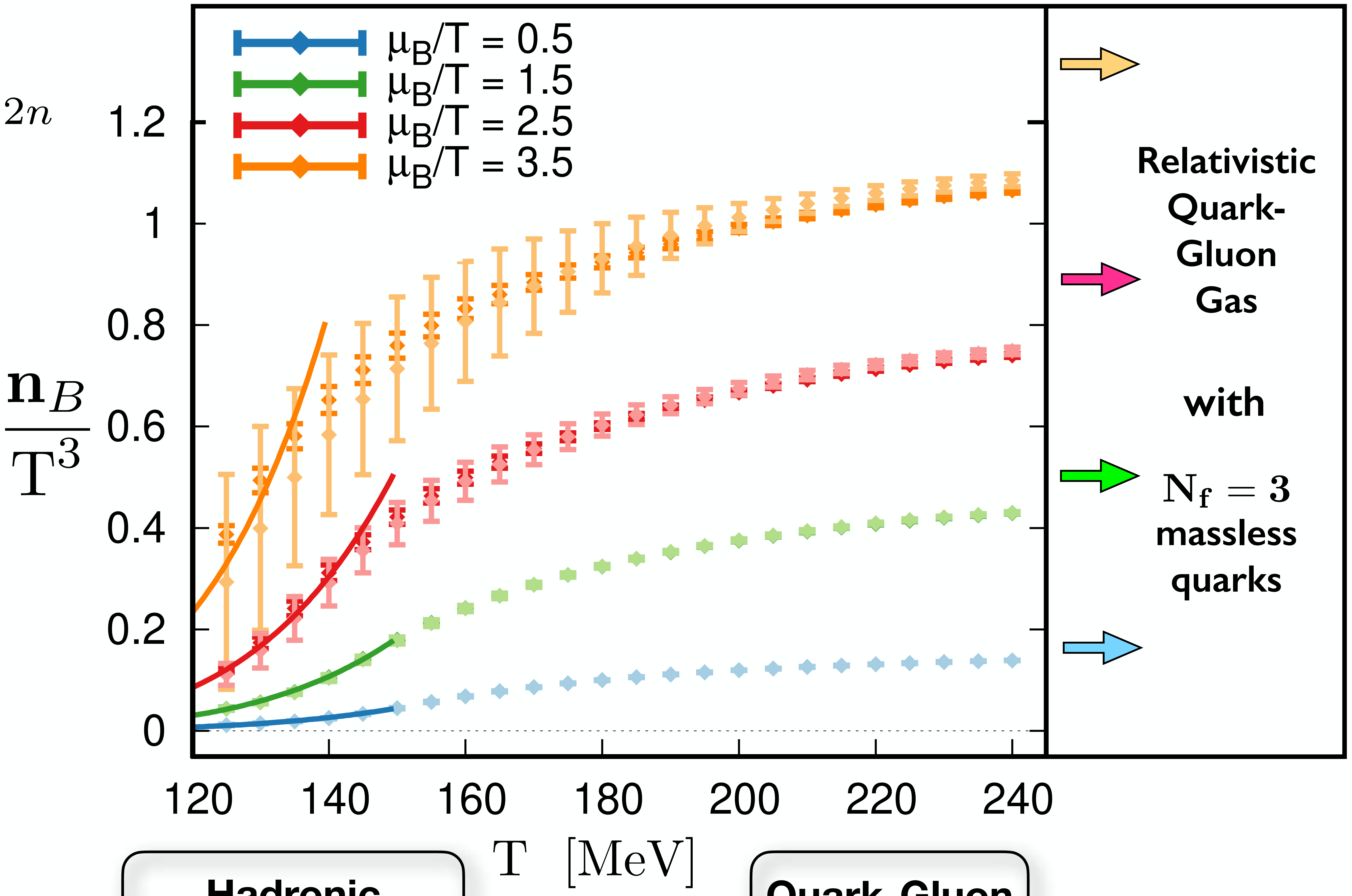
- **Baryon density**

$$\mathbf{n}_B = \frac{\partial P(T,\mu_B)}{\partial \mu_B}$$

Example:

$$\mathbf{n}_B(T,\mu_B) \lesssim 0.2\,\mathrm{fm}^{-3}$$

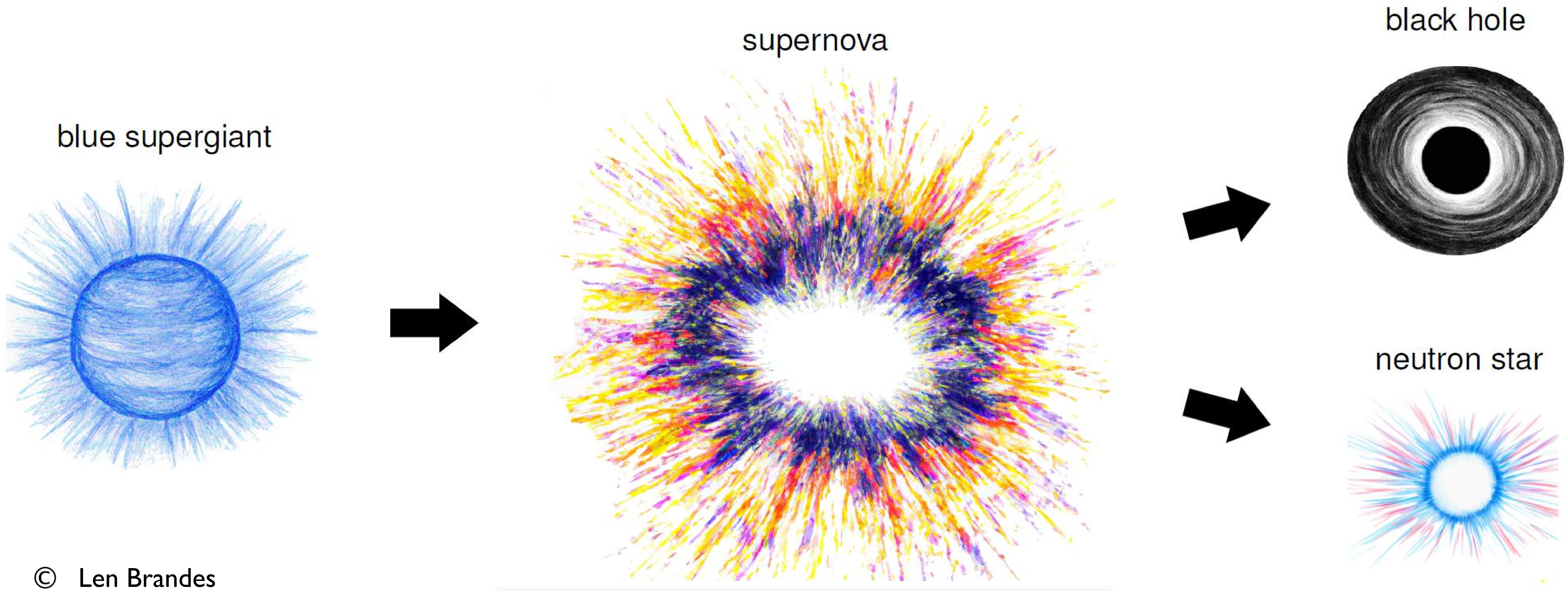
for $T \lesssim 150\,\mathrm{MeV}$ and $\mu_B/T = 2.5$



Hadronic Resonance Gas

Quark-Gluon Domain

Strategies PART III: **Astrophysical Observations**



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nature

NEWS FEATURE | 04 March 2020

The golden age of neutron-star physics has arrived

These stellar remnants are some of the Universe's most enigmatic objects – and they are finally starting to give up their secrets.

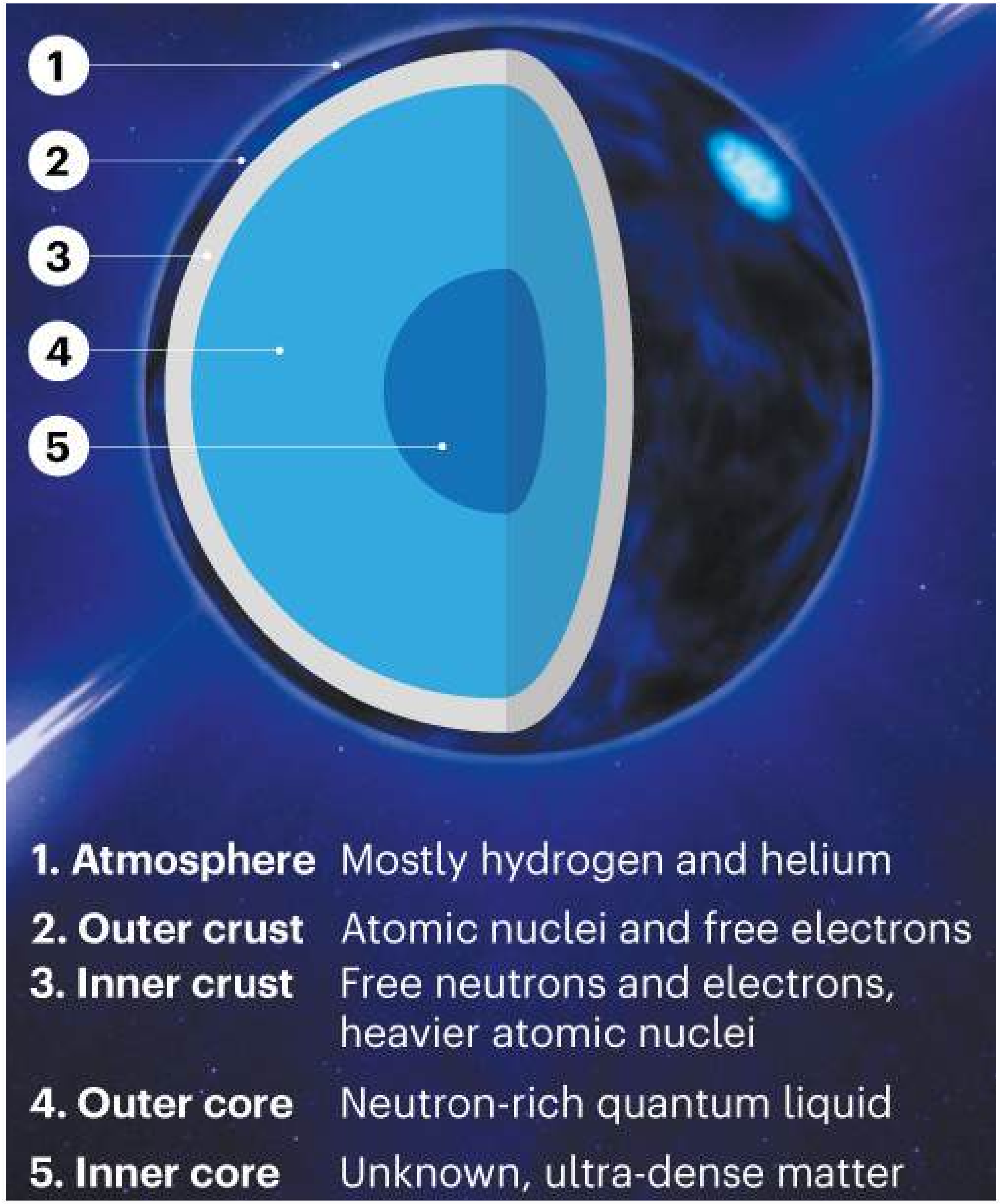
Neutron Star Matter

- Constraints on the **Equation-of-State** of **COLD** and **DENSE BARYONIC MATTER**

  - **Neutron star mass** measurements
    (Shapiro delay, Radioastronomy)

  - **Neutron star radius** measurements
    (Neutron star Interior Composition ExploreR : NICER telescope @ ISS)

  - **Gravitational wave** signals of **neutron star mergers**
    (LIGO and Virgo collaborations)

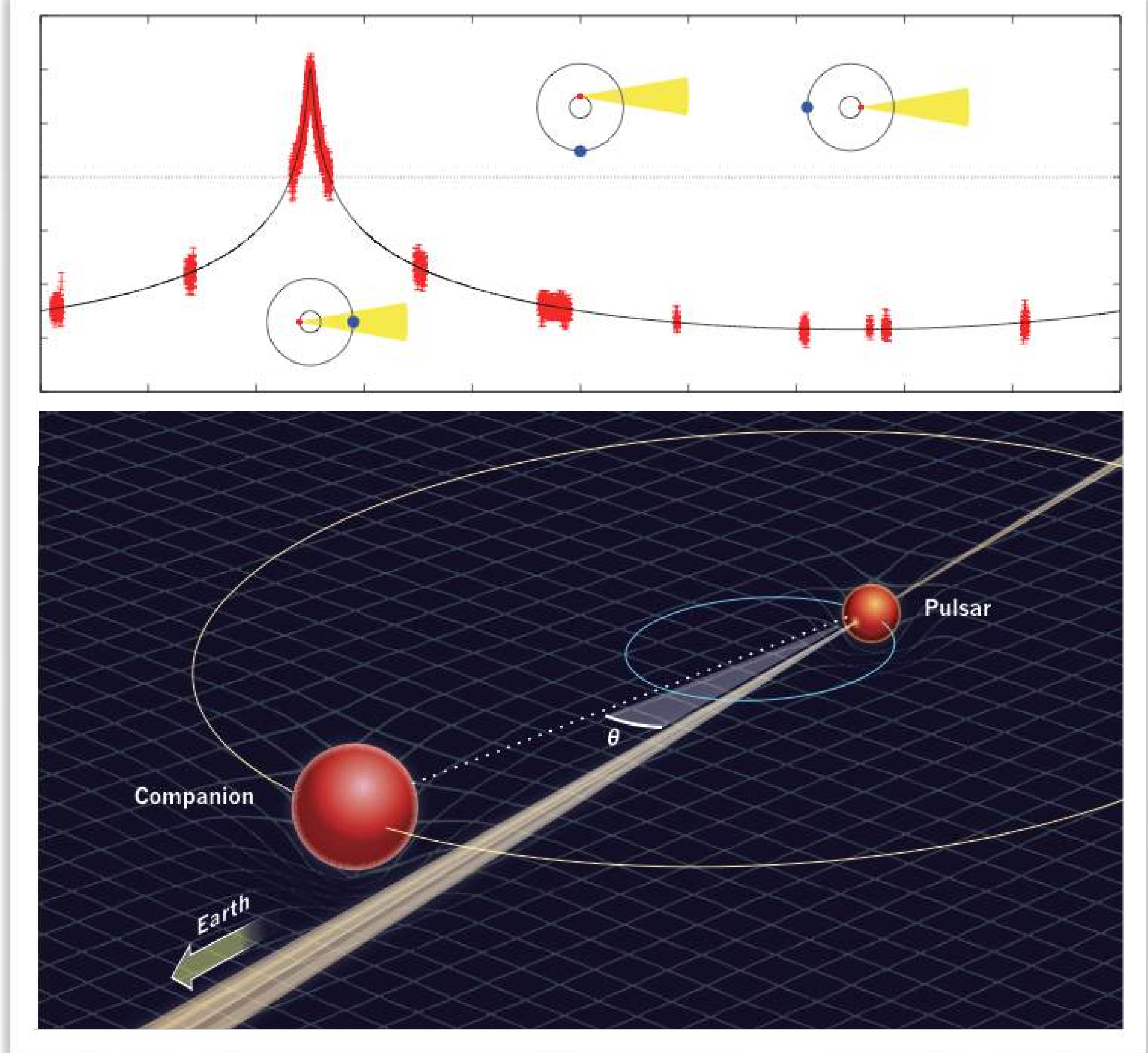


Layers of a **Neutron Star**

MASSIVE NEUTRON STARS

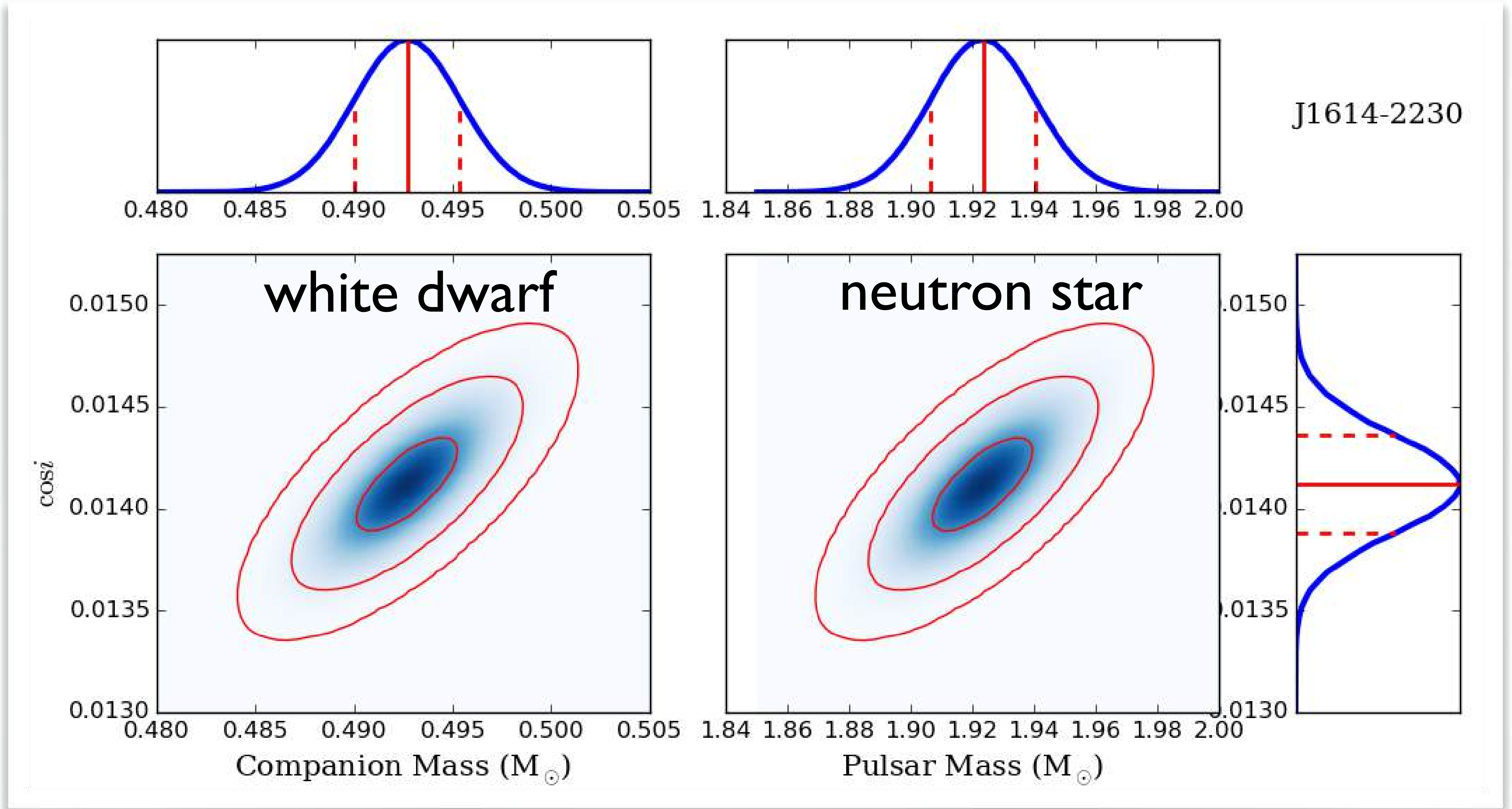
- Neutron Star - White Dwarf binaries

Shapiro delay measurements



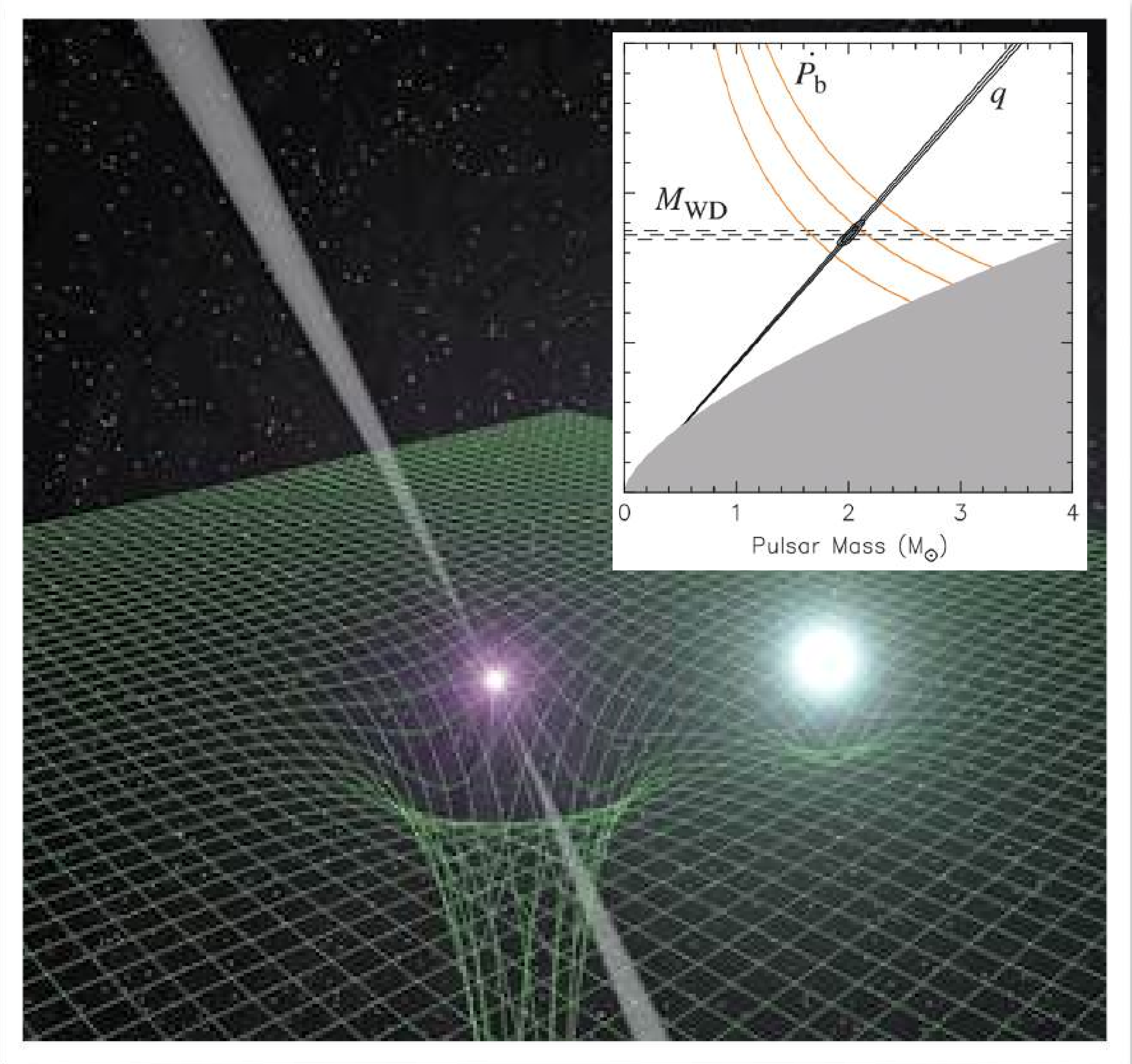
P.B. Demorest et al.
Nature 467 (2010) 1081

PSR J1614+2230



E. Fonseca et al., Astrophys. J. 832 (2016) 167

PSR J0348+0432



J. Antoniadis et al.
Science 340 (2013) 6131

PSR J1614+2230

$$M = 1.928 \pm 0.017\ M_\odot$$

PSR J0348+0432

$$M = 2.01 \pm 0.04\ M_\odot$$

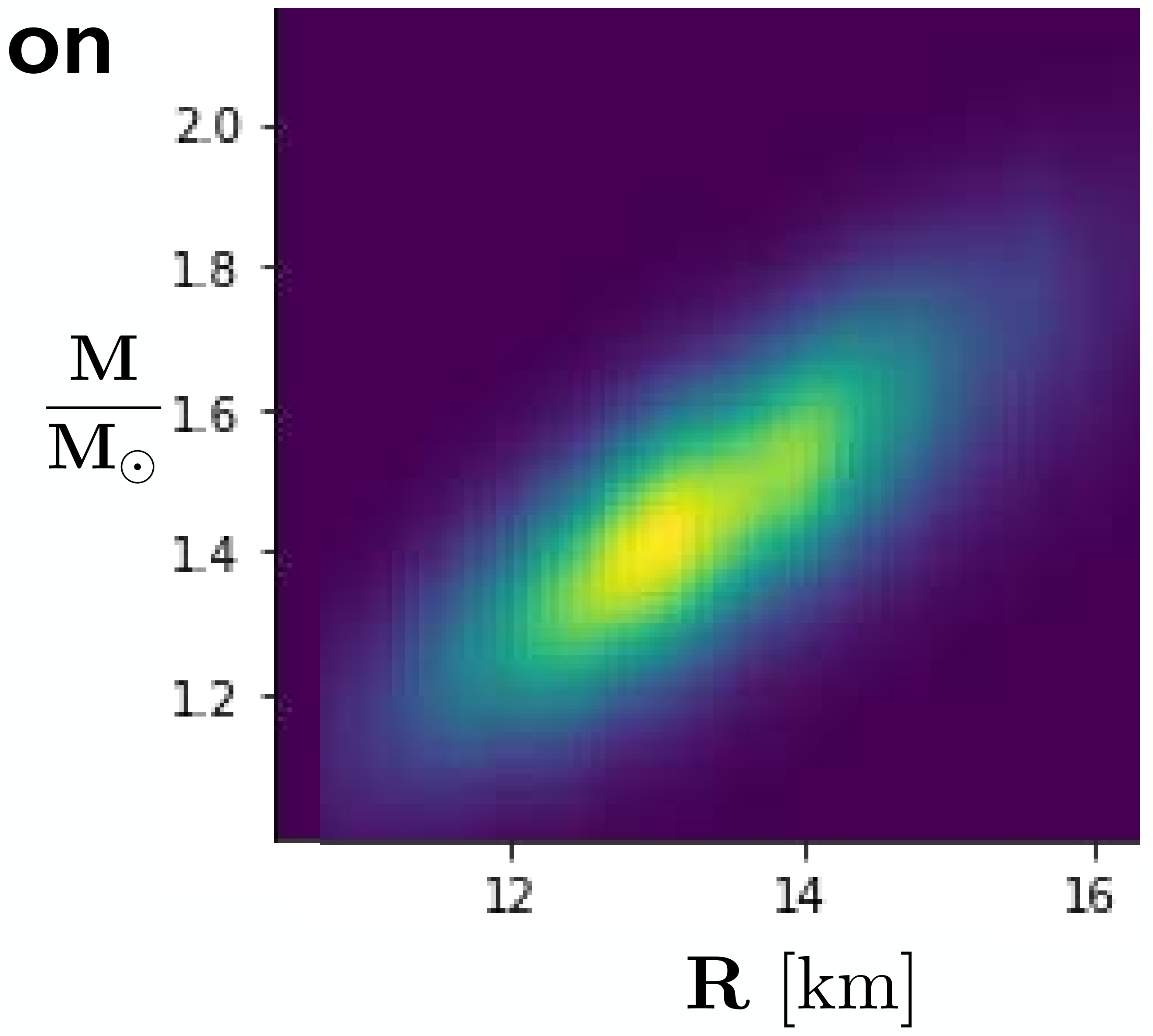
- Strong constraints for the **stiffness of the Equation-of State** of cold & dense baryonic matter

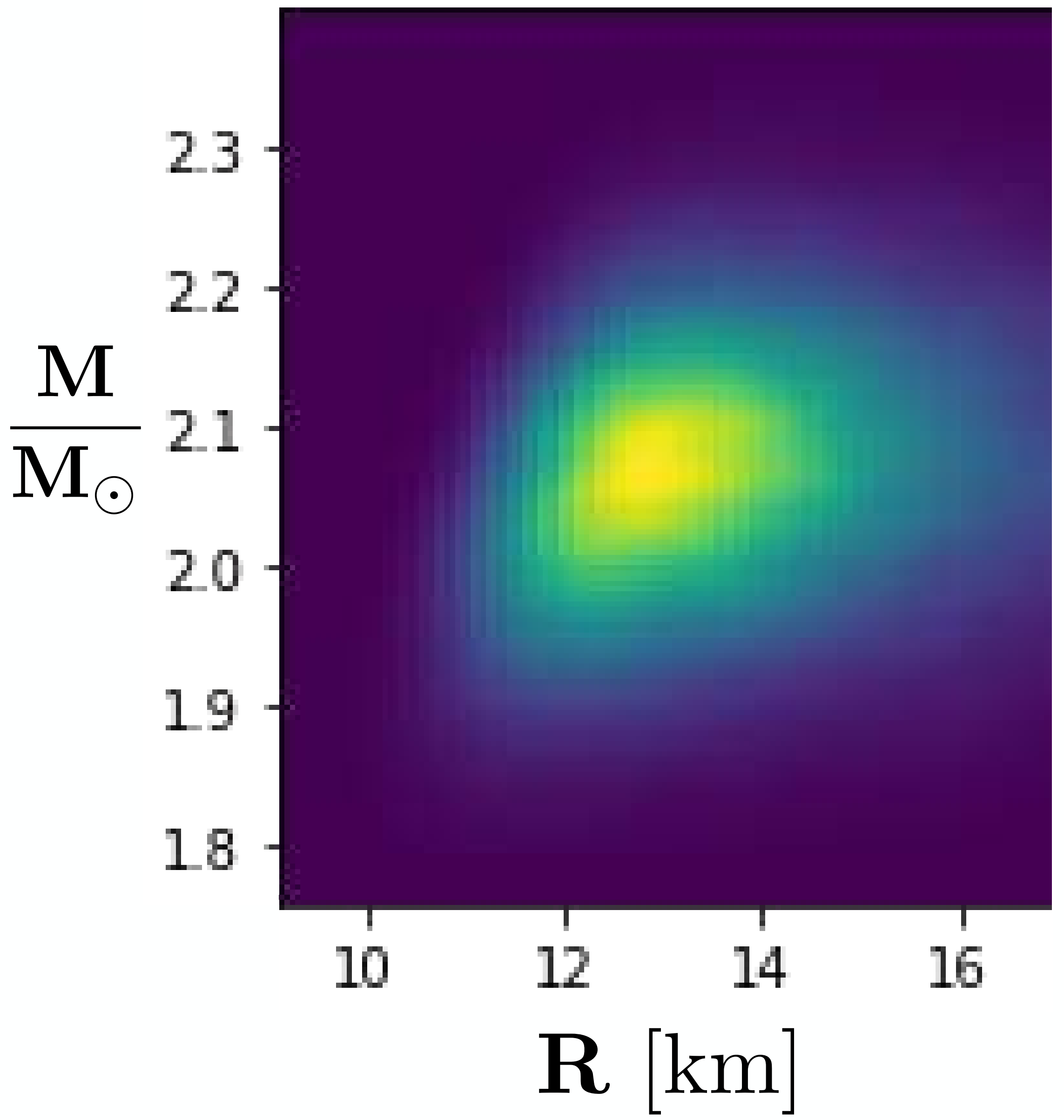
MASSES & RADII of NEUTRON STARS

- **NICER** @ International Space Station

Neutron Star
Interior Composition Explorer

- X rays from hot spots at the surface of rotating neutron stars





- $PSR\,J0030+0451$

$$M = 1.44 \pm 0.15\,M_\odot$$

$$R = 13.02^{+1.24}_{-1.06}\,\mathrm{km}$$

M.C. Miller et al. (NICER)
Astroph. J. Lett. 887 (2019) L24

- $PSR\,J0740+6620$

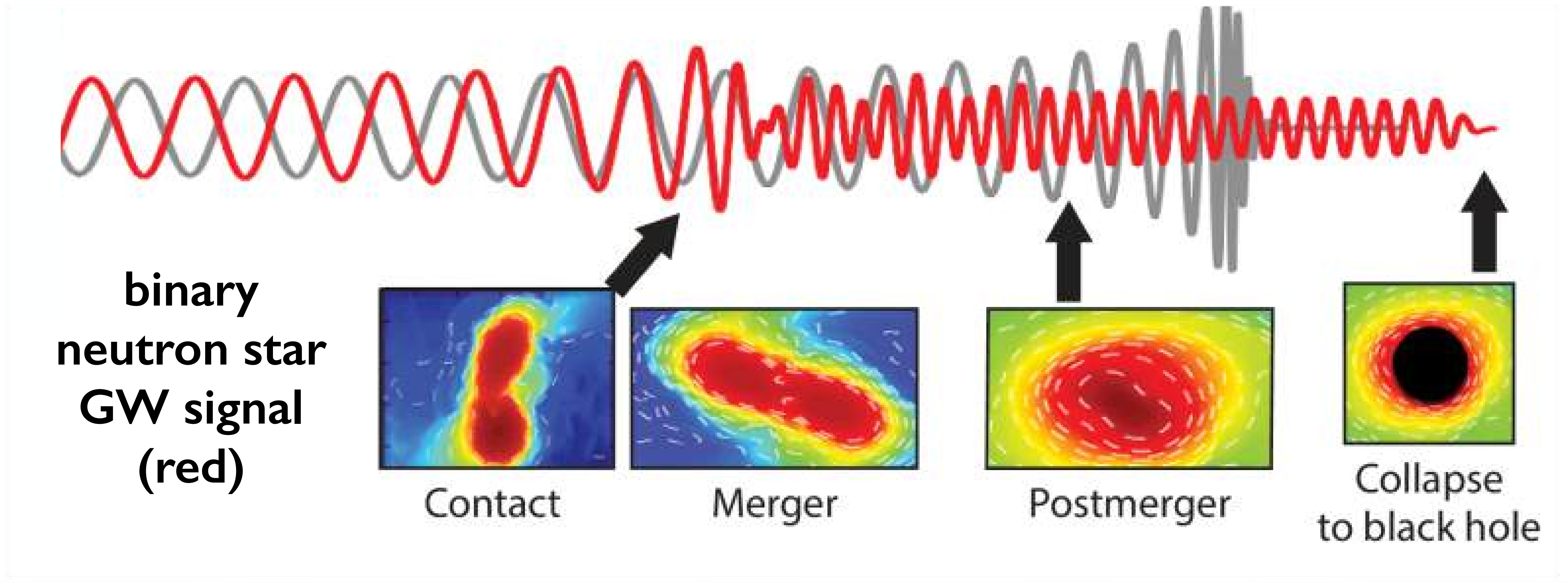
$$M = 2.08 \pm 0.07\,M_\odot$$

$$R = 13.7^{+2.6}_{-1.5}\,\mathrm{km}$$

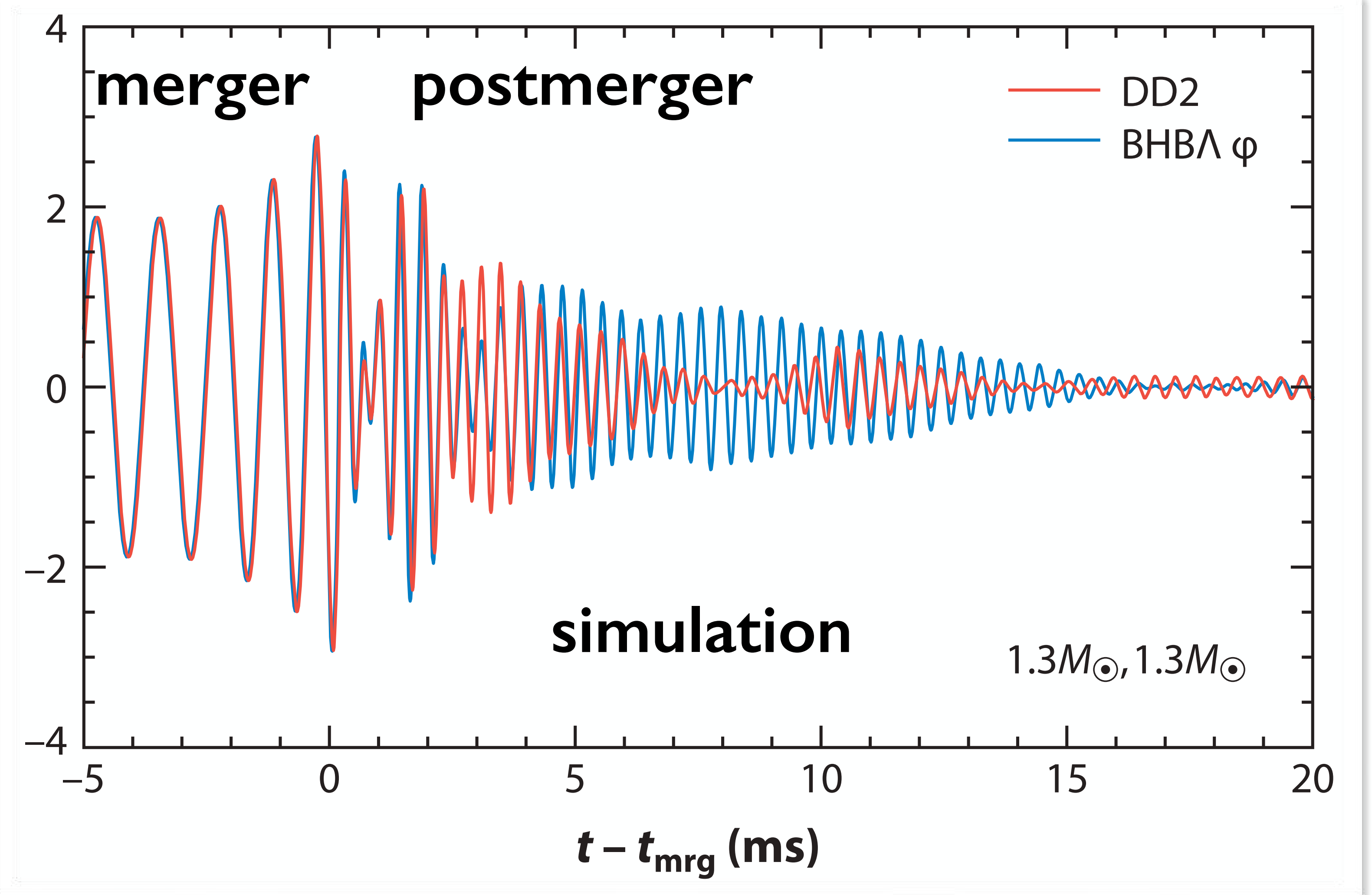
M.C. Miller et al.
(NICER + XMM Newton)
Astroph. J. Lett. 918 (2021) L28

GRAVITATIONAL WAVES from BINARY NEUTRON STAR MERGERS

LIGO and Virgo Collaborations 2017 - 2020



J.S. Lattimer : Ann. Rev. Nucl. Part. Sci. 71 (2021) 433



D. Radice, S. Bernuzzi, A. Perego : Ann. Rev. Nucl. Part. Sci. 70 (2020) 95

- Additional constraints on Equation-of-State:
  Binary tidal deformability $\tilde{\Lambda} = \frac{16}{13}\frac{(M_1 + 12\,M_2)M_1^4\Lambda_1}{(M_1+M_2)^5} + (1 \leftrightarrow 2)$ $\quad \Lambda_i \sim \left(\frac{R_i}{M_i}\right)^n$

- $\textbf{GW 170817}$: $M = M_1 + M_2 = 2.74^{+0.04}_{-0.01}\,M_\odot$
  B.P. Abbot et al. : Phys. Rev. Lett. 119 (2017) 161101 Phys. Rev. X9 (2019) 011001

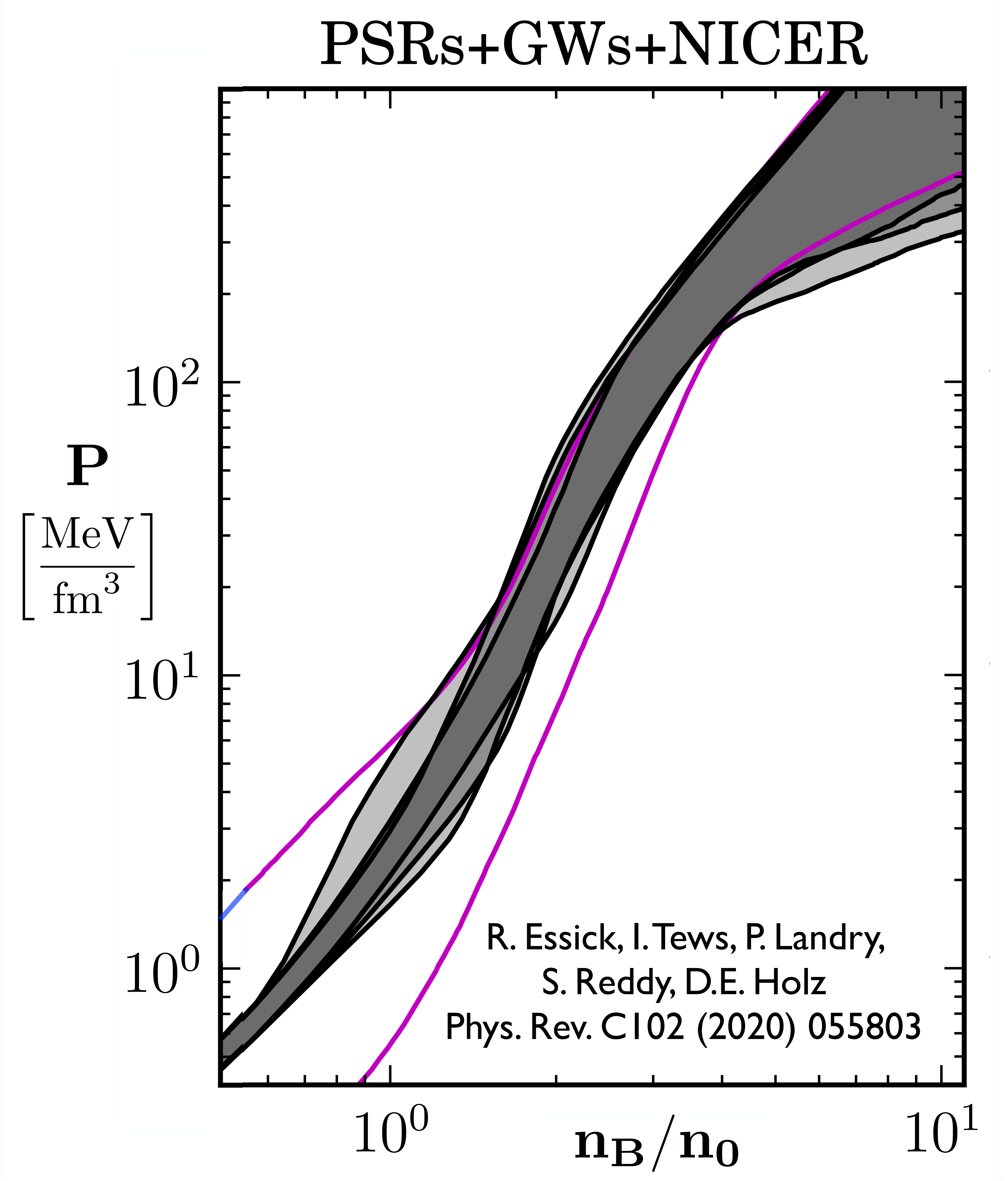
- $\textbf{GW 190425}$: $M = M_1 + M_2 = 3.3 \pm 0.1\,M_\odot$
  B.P. Abbot et al. : Astroph. J. Lett. 892 (2020) L3

$$\Lambda_{1.4} = 190^{+390}_{-120}$$

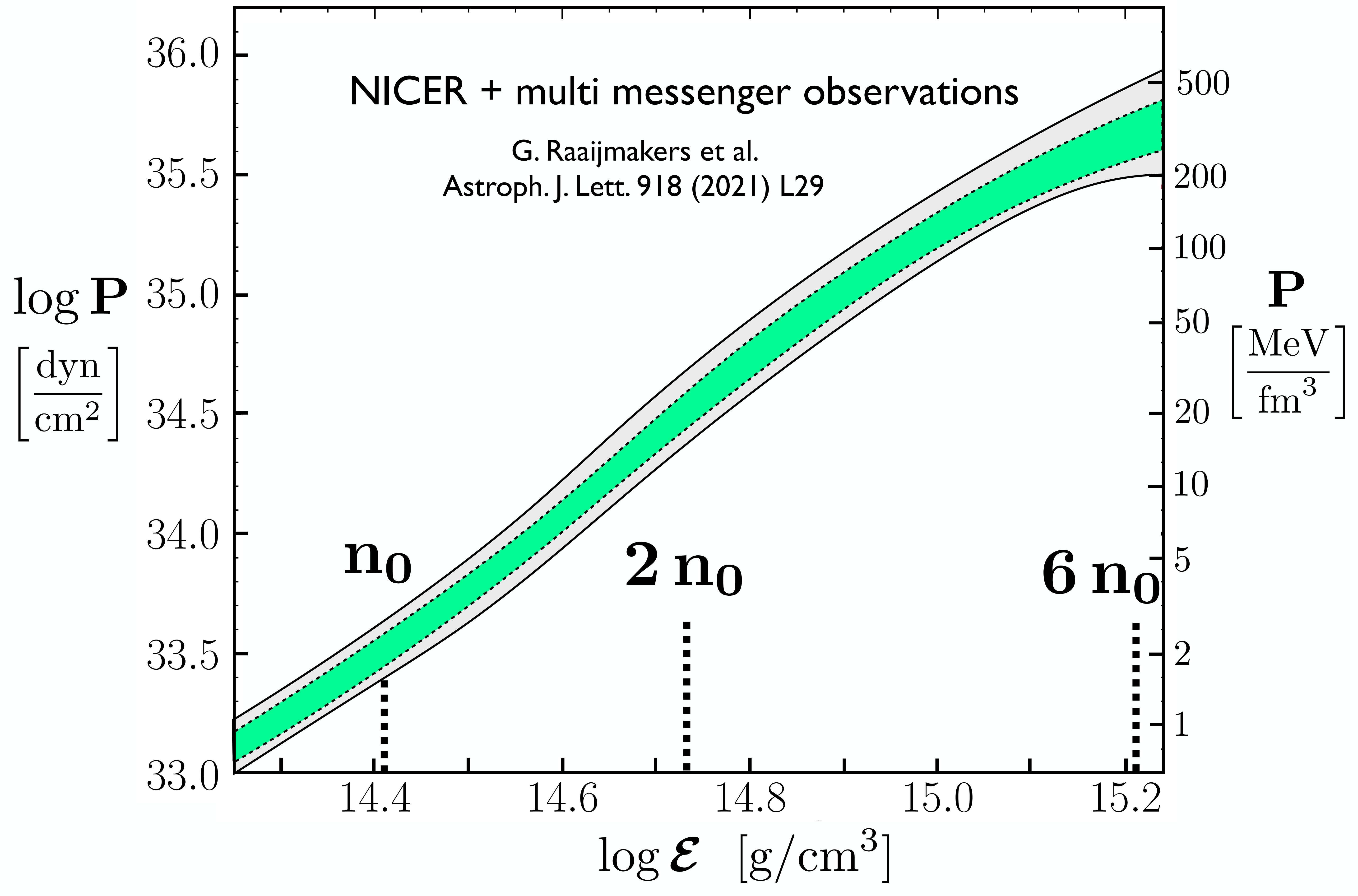
B.P. Abbot et al. : Phys. Rev. Lett. 121 (2018) 161101

NEUTRON STAR MATTER EQUATION-of-STATE

- Examples of **EoS analyses** based on multimessenger data



$(n_0 = 0.16\ \mathrm{fm}^{-3})$



3.

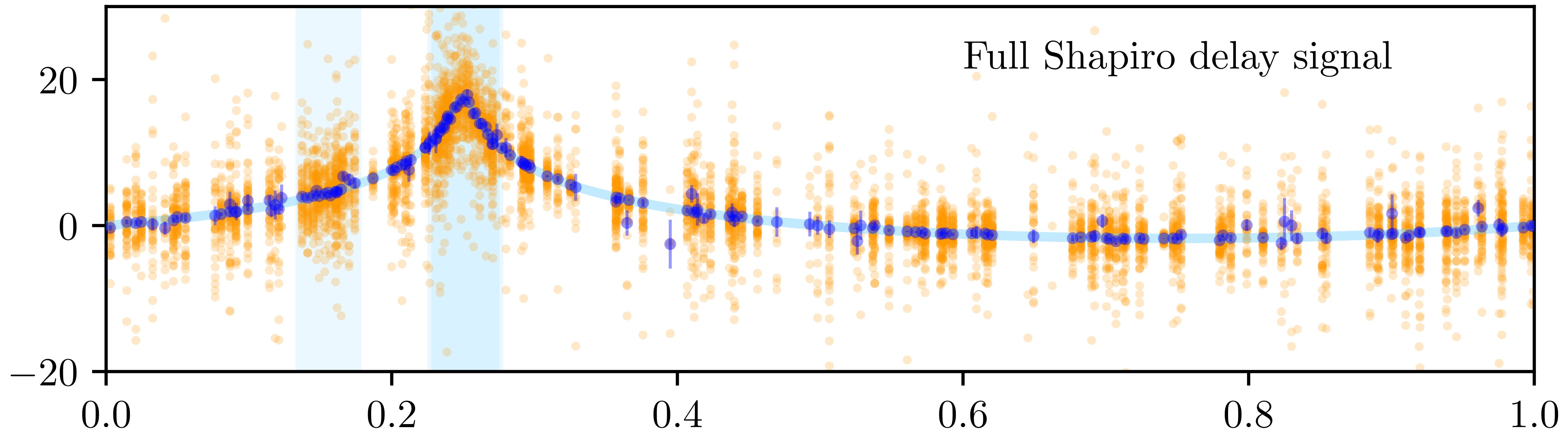
Equation-of-State of Dense Baryonic Matter :

Empirical Constraints from Neutron Stars

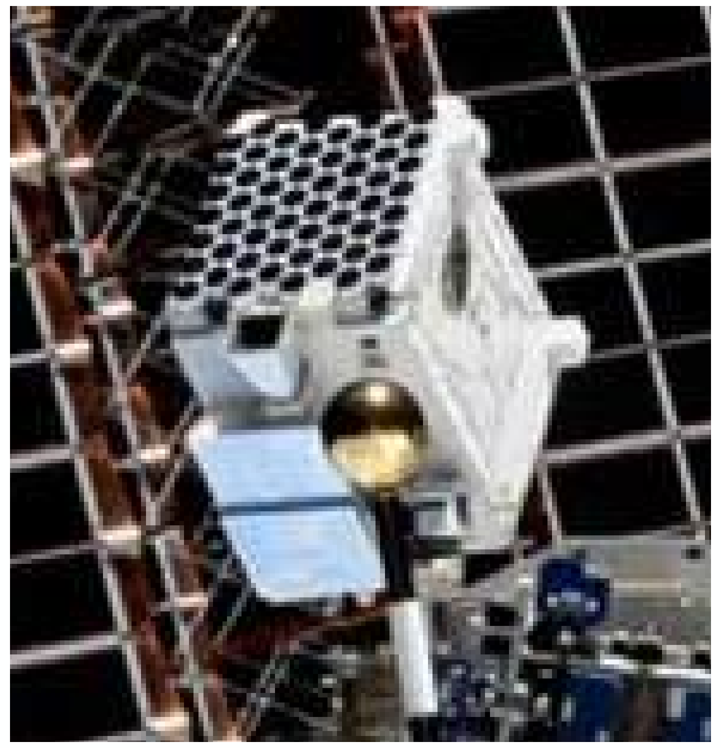
Bayesian Inference Results

NEUTRON STARS : DATA

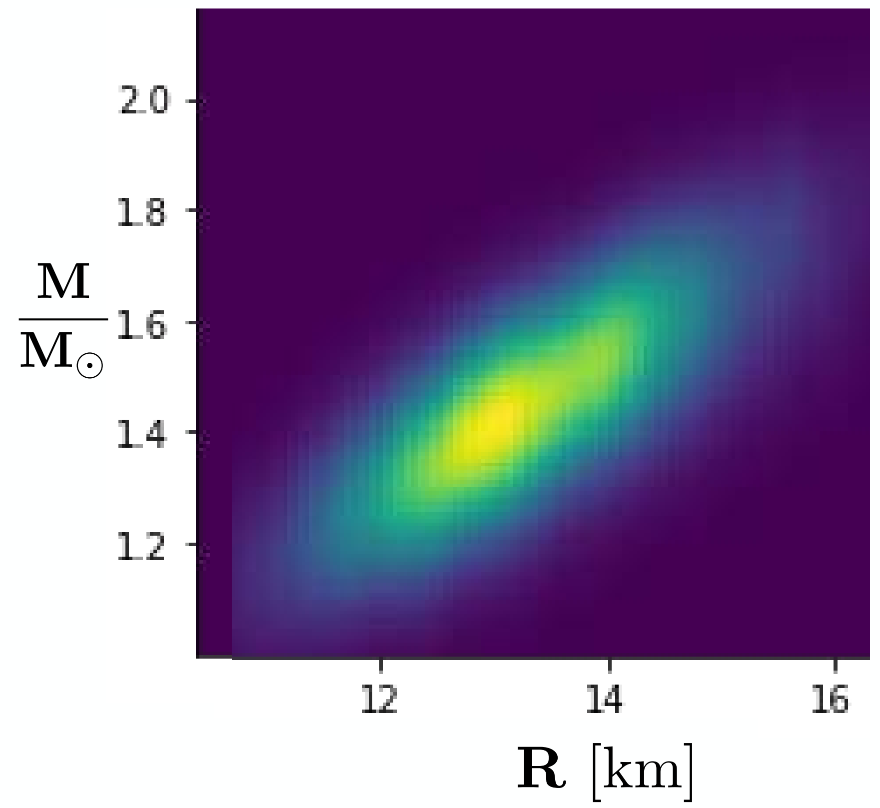
- Database for **inference of Equation-of-State** and other properties of neutron stars

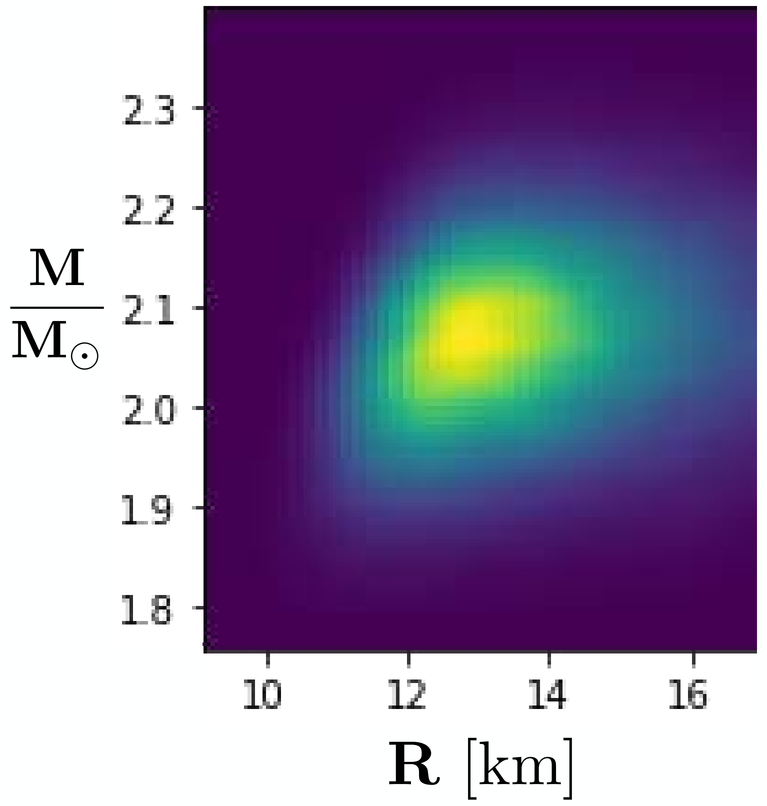


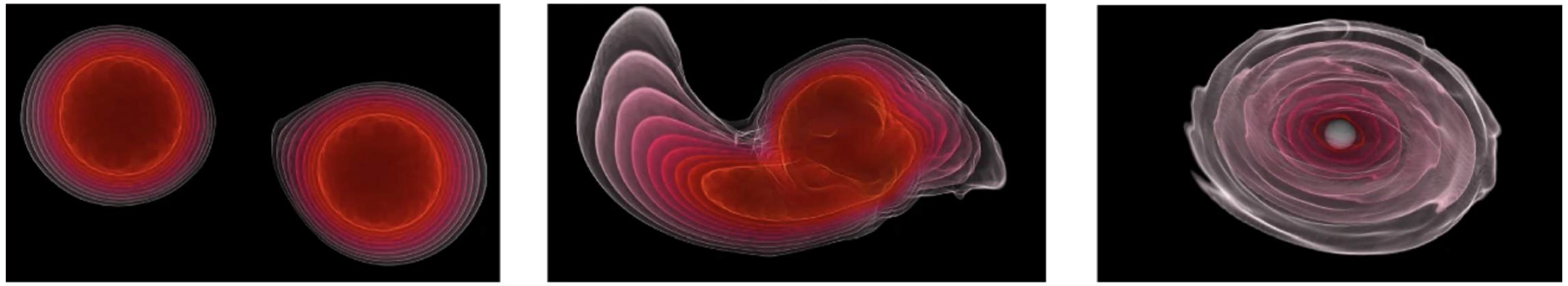
- **Neutron star masses**
  from Radio Telescopes
  (Green Bank, Effelsberg)



- **Masses and radii**
  X rays from hot spots on the surface of rotating neutron stars
  (NICER Telescope @ ISS)







- **Tidal deformabilities**
  Gravitational wave signals of neutron star mergers
  (LIGO and Virgo Collaborations)

NEUTRON STARS : DATA

- **Masses of $2\,\mathrm{M}_\odot$ n-stars**
  (Shapiro delay & radio observations)

PSR J0348+0432

$$\mathrm{M} = 2.01 \pm 0.04\ \mathrm{M}_\odot$$

J. Antoniadis et al. : Science 340 (2013) 1233232

PSR J1614-2230

$$\mathrm{M} = 1.908 \pm 0.016\ \mathrm{M}_\odot$$

Z. Arzoumanian et al., Astrophys. J. Suppl. 235 (2018) 37

PSR J0740+6620

$$\mathrm{M} = 2.08 \pm 0.07\ \mathrm{M}_\odot$$

E. Fonseca et al., Astrophys. J. Lett. 915 (2021) L12

- **Masses and Radii** (NICER)

PSR J0030+0451

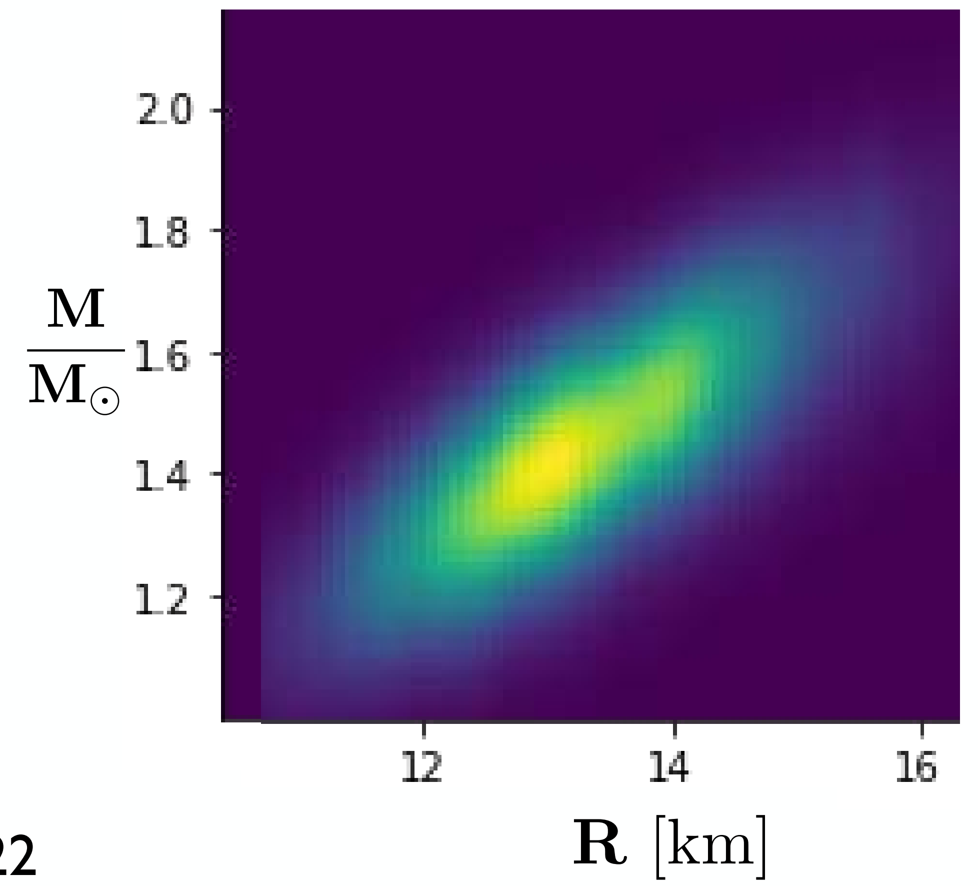
$$\mathrm{M} = 1.34 \pm 0.16\,\mathrm{M}_\odot \qquad \mathrm{R} = 12.71^{+1.14}_{-1.19}\,\mathrm{km}$$

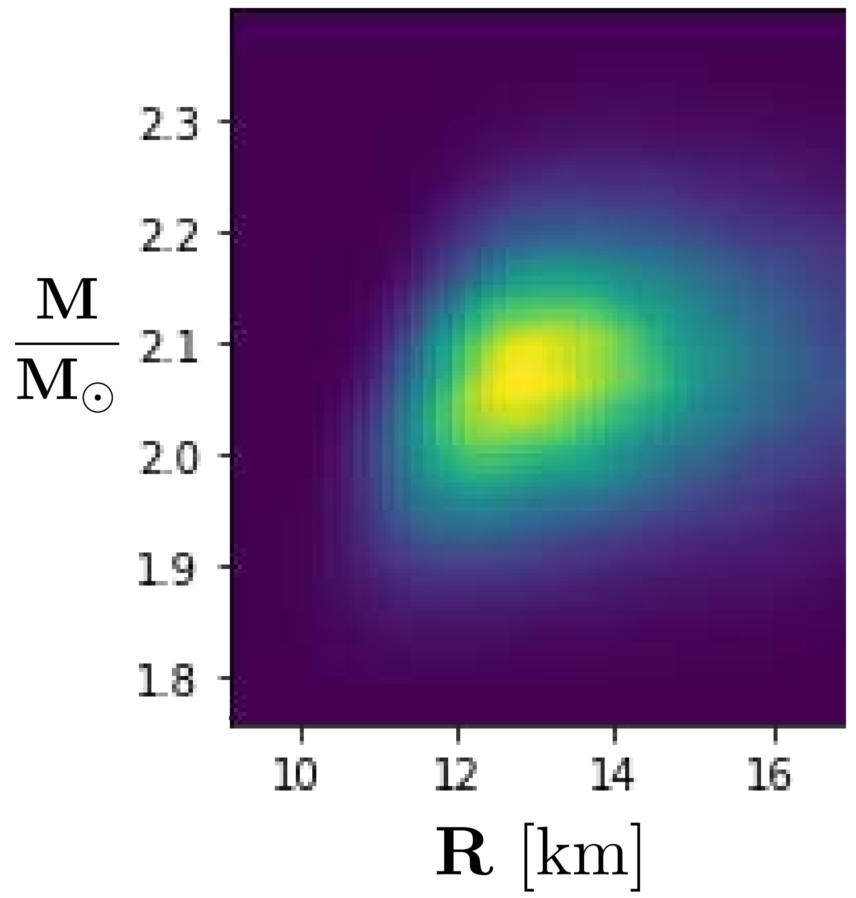
T.E. Riley et al. (NICER), Astroph. J. Lett. 887 (2019) L21

PSR J0740+6620

$$\mathrm{M} = 2.073 \pm 0.069\,\mathrm{M}_\odot \qquad \mathrm{R} = 12.49^{+1.28}_{-0.88}\,\mathrm{km}$$

T.E. Riley et al. (NICER + XMM Newton), Astroph. J. Lett. 918 (2021) L27
T. Salmi et al. (NICER), arXiv:2406.14466





NEUTRON STARS : DATA (contd.)

- **Very massive** and **fast rotating galactic neutron star**

PSR J0952-0607

$$M = 2.35 \pm 0.17\ M_\odot$$

R.W. Romani et al. : Astroph. J. Lett. 934 (2022) L17

(Keck Observatory)

→ **equivalent non-rotating mass after rotational correction :** $M = 2.3 \pm 0.2\ M_\odot$

- **New accurate data from NICER**

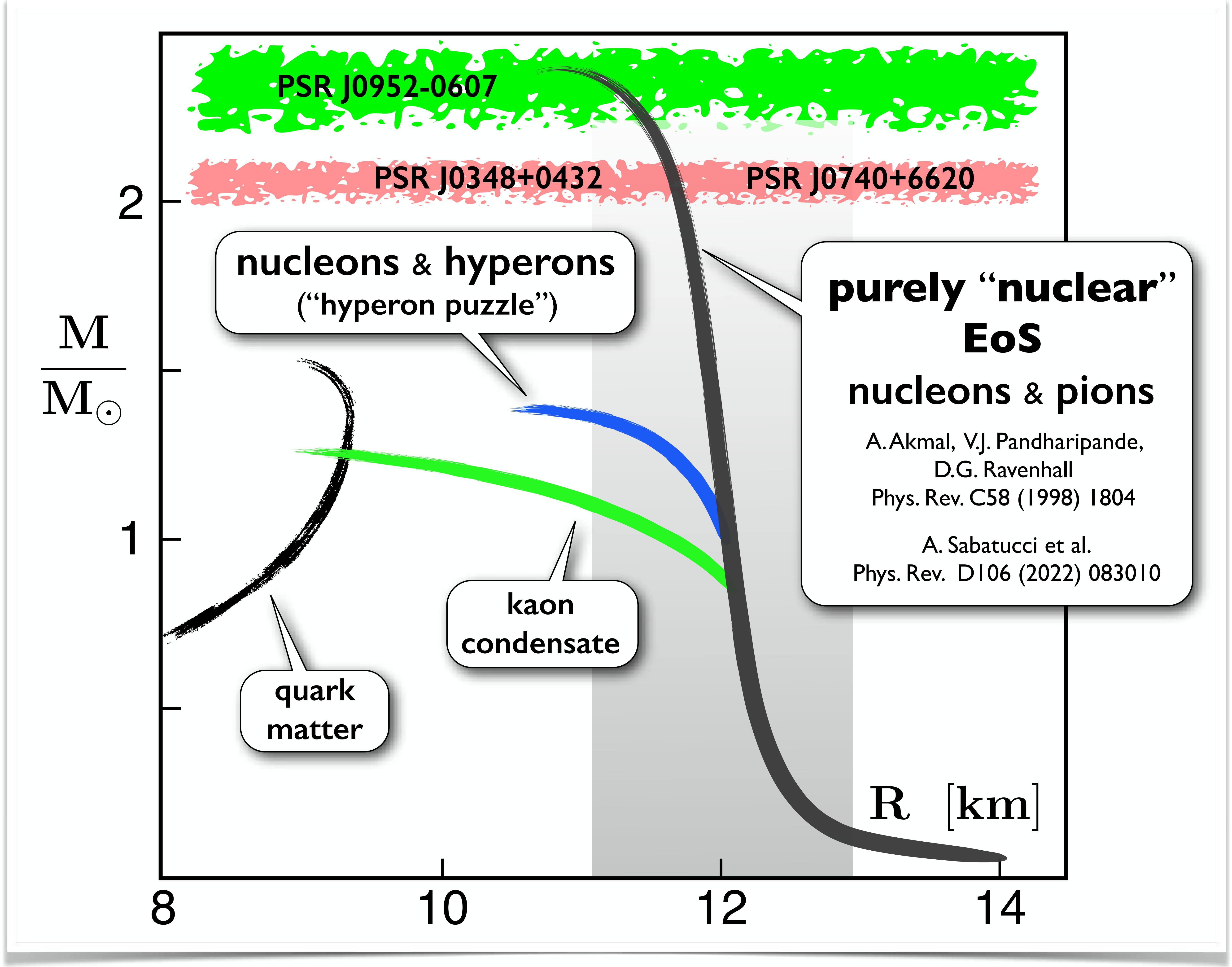
PSR J0437-4715

$$M = 1.418 \pm 0.037\, M_\odot \qquad R = 11.36^{+0.95}_{-0.63}\ \mathrm{km}$$

D. Choudhury et al. : Astroph. J. Lett. 971 (2024) L20

CONSTRAINTS on EQUATION of STATE $P(\varepsilon)$

- from observations of massive neutron stars



Tolman - **O**ppenheimer - **V**olkov Equations

$$\frac{dP(r)}{dr} = \frac{G\,[\varepsilon(r) + P(r)]\,\left[m(r) + 4\pi r^3 P(r)\right]}{r\,[r - 2\,G\,m(r)]}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \varepsilon(r)$$

$$M = m(R) = 4\pi \int_0^R dr\, r^2 \varepsilon(r)$$

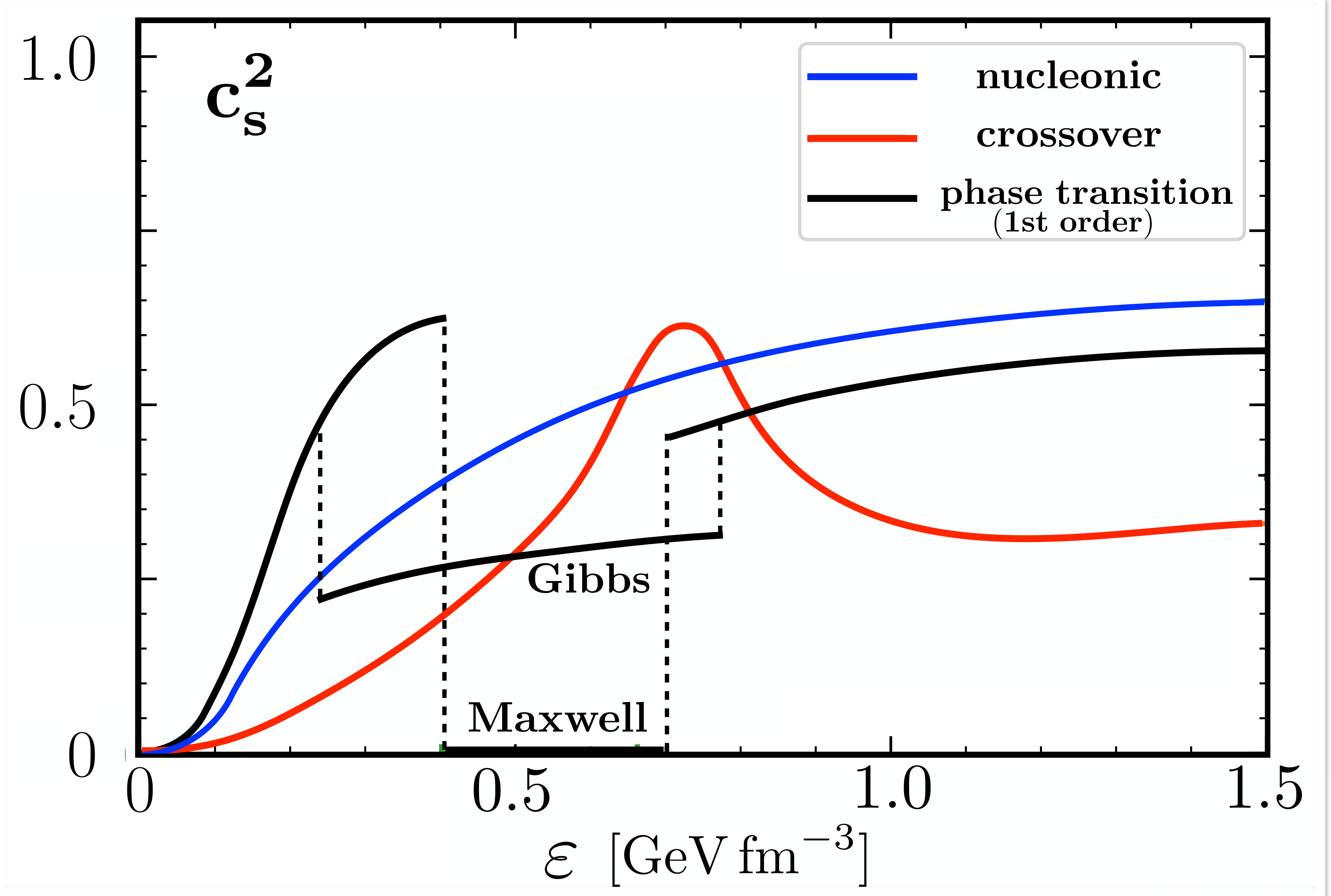
- **Stiff equation-of-state $P(\varepsilon)$ required**
- **Simplest forms of exotic matter**
  (kaon condensate, quark matter, ...)
  ruled out

SOUND VELOCITY and EQUATION of STATE

- Key quantity : **Speed of Sound**

$$c_s^2(\varepsilon) = \frac{\partial P(\varepsilon)}{\partial \varepsilon}$$

displays
characteristic signature
of
phase transition
or
crossover

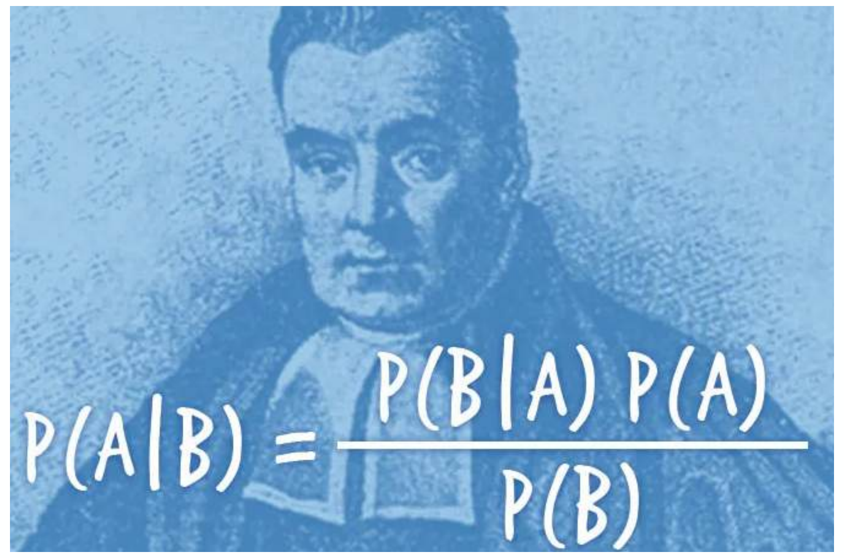


energy density

- **E**quation **of S**tate :

$$P(\varepsilon) = \int_0^{\varepsilon} d\varepsilon' \, c_s^2(\varepsilon')$$

INFERENCE of SOUND SPEED and RELATED PROPERTIES of NEUTRON STARS



- Introduce **general parametrization of sound velocity** by segment-wise representation :

$$c_s^2(\varepsilon,\theta) = \frac{(\varepsilon_{i+1}-\varepsilon)c_{s,i}^2 + (\varepsilon-\varepsilon_i)c_{s,i+1}^2}{\varepsilon_{i+1}-\varepsilon_i}\ , \quad \text{parameter set} \quad \theta = (c_{s,i}^2, \varepsilon_i) \ \ (i = 1,\ldots,N)$$

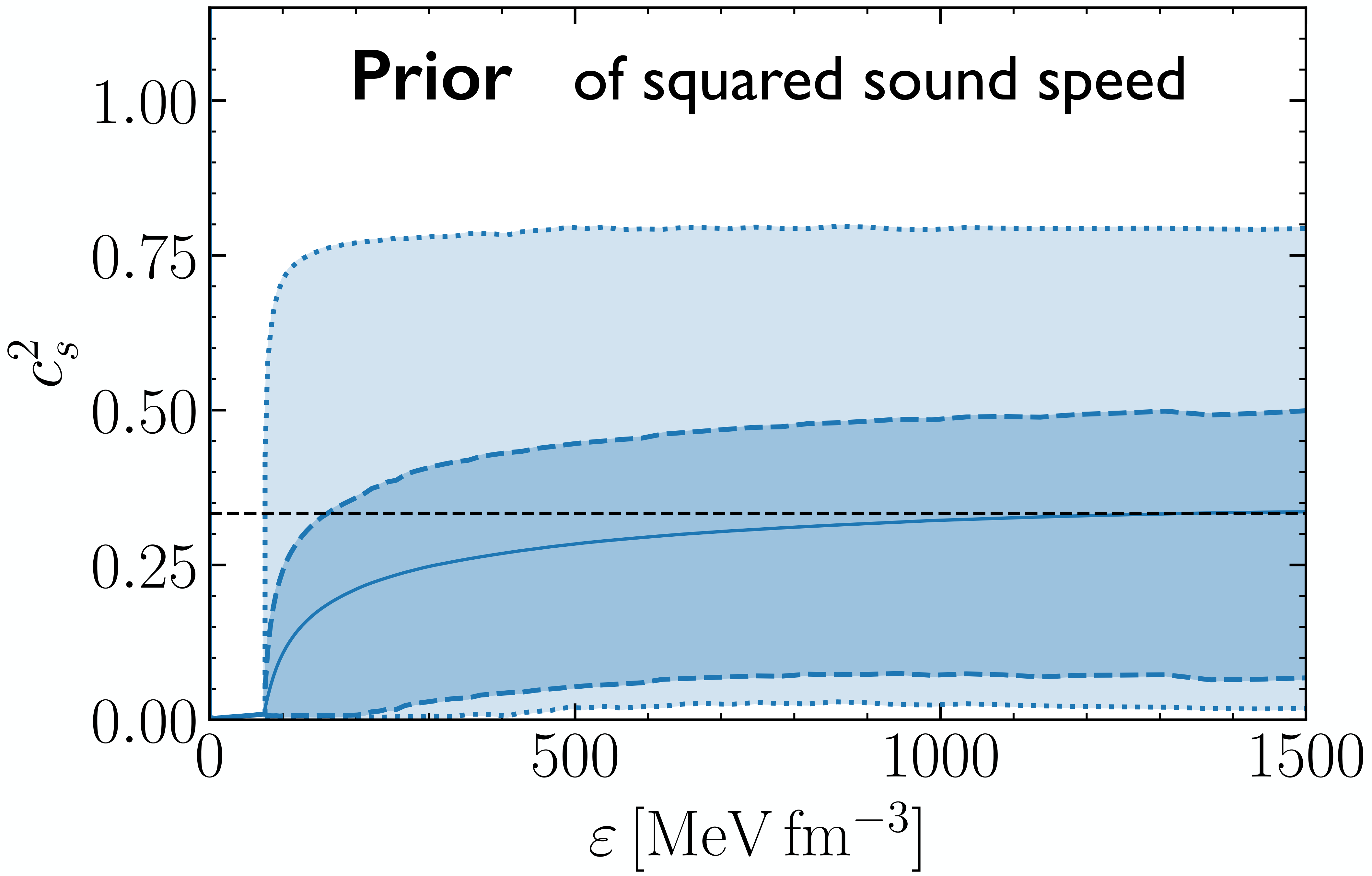
- Constrain parameters θ by **Bayesian inference** using **nuclear** and **astrophysical data** $\mathscr{D}$:

$$\Pr(\theta|\mathscr{D}) \propto \Pr(\mathscr{D}|\theta)\,\Pr(\theta)$$

→ Choose **Prior** $\Pr(\theta)$

→ Compute **Posterior** $\Pr(\theta|\mathscr{D})$ from **Likelihood** $\Pr(\mathscr{D}|\theta)$

→ Quantify **Evidences** for hypotheses H_1 vs. H_0 in terms of **Bayes factors** $\mathscr{B}^{H_1}_{H_0} = \frac{\Pr(\mathscr{D}|H_1)}{\Pr(\mathscr{D}|H_0)}$



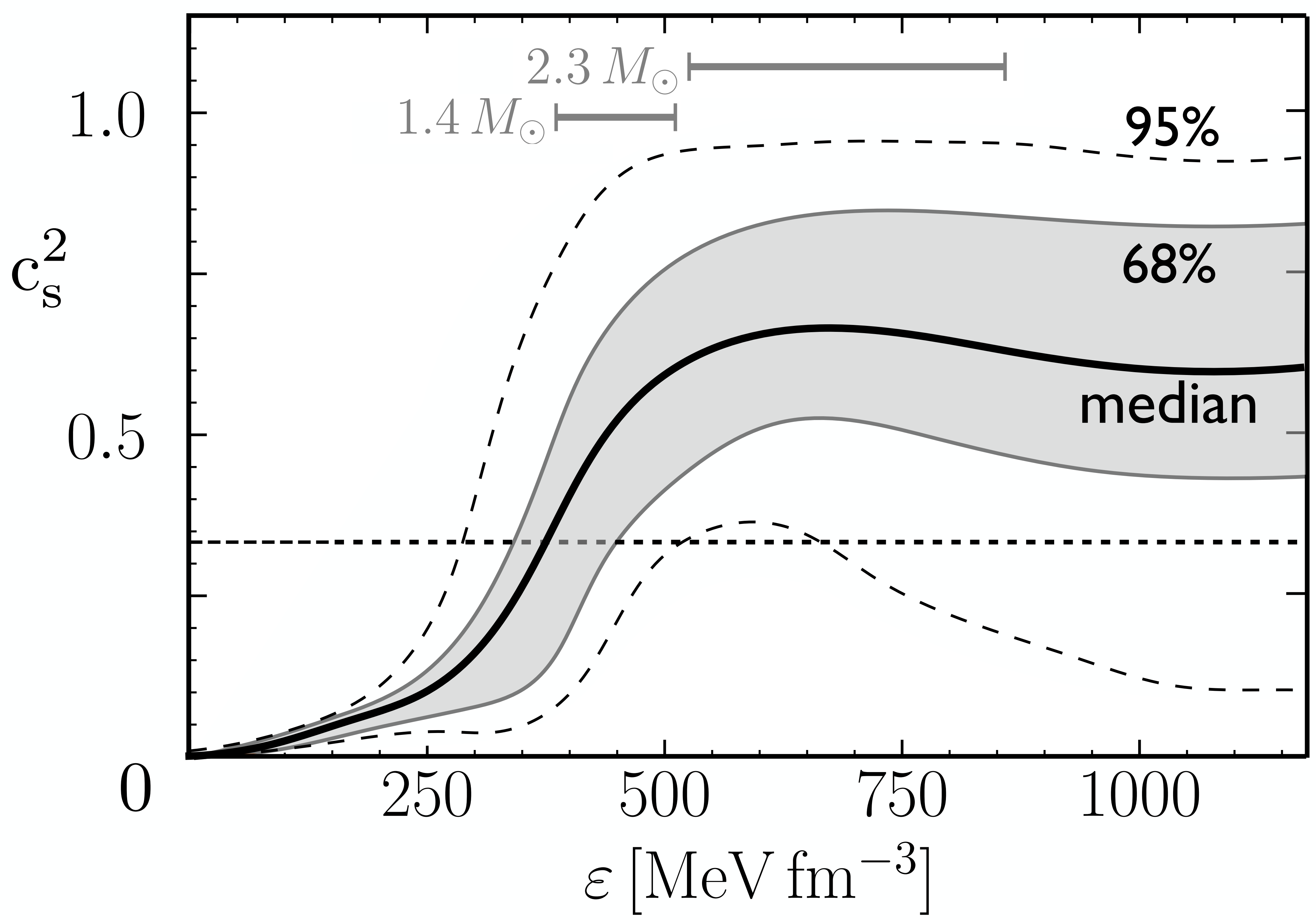
NEUTRON STAR MATTER : EQUATION of STATE

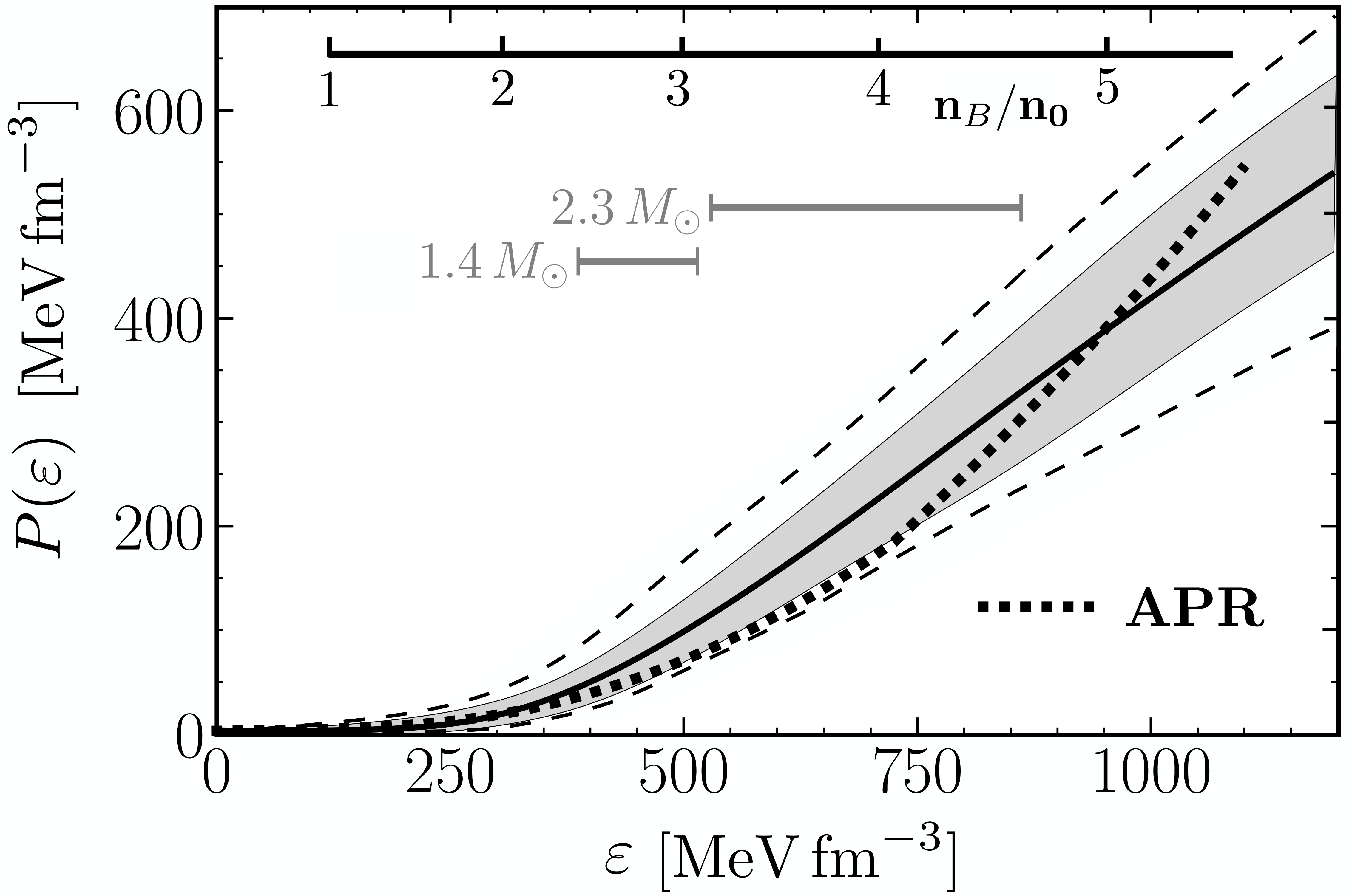
$P(\varepsilon)$

- **Bayesian inference of sound speed and EoS**

PSR masses, NICER & GW data, low-density constraints (ChEFT), **asymptotic constraints** (pQCD)

L. Brandes, W.W. , N. Kaiser : Phys. Rev. D 107 (2023) 014011 ; Phys. Rev. D 108 (2023) 094014 - L. Brandes, W.W. : Symmetry 16 (2024) 111

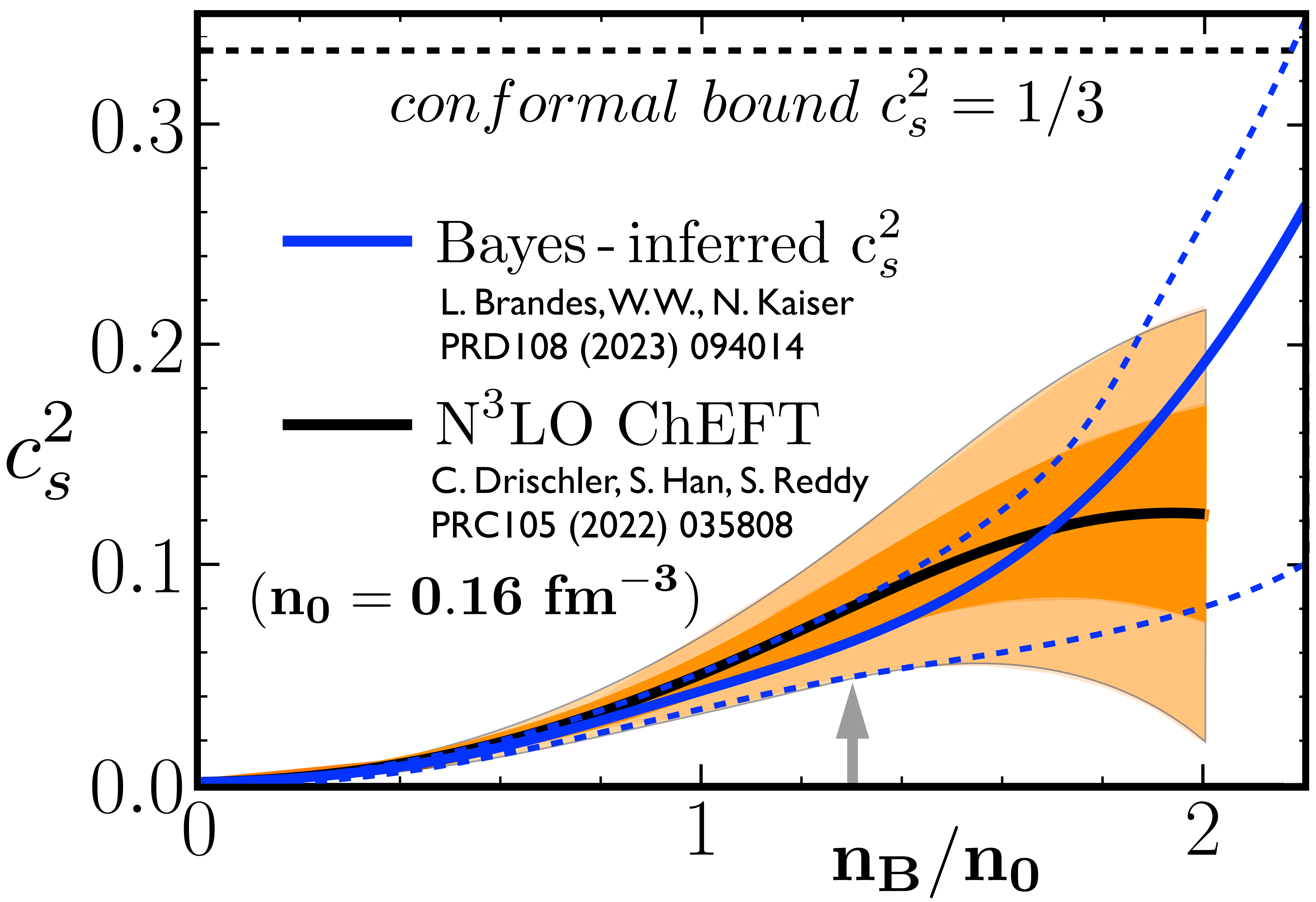




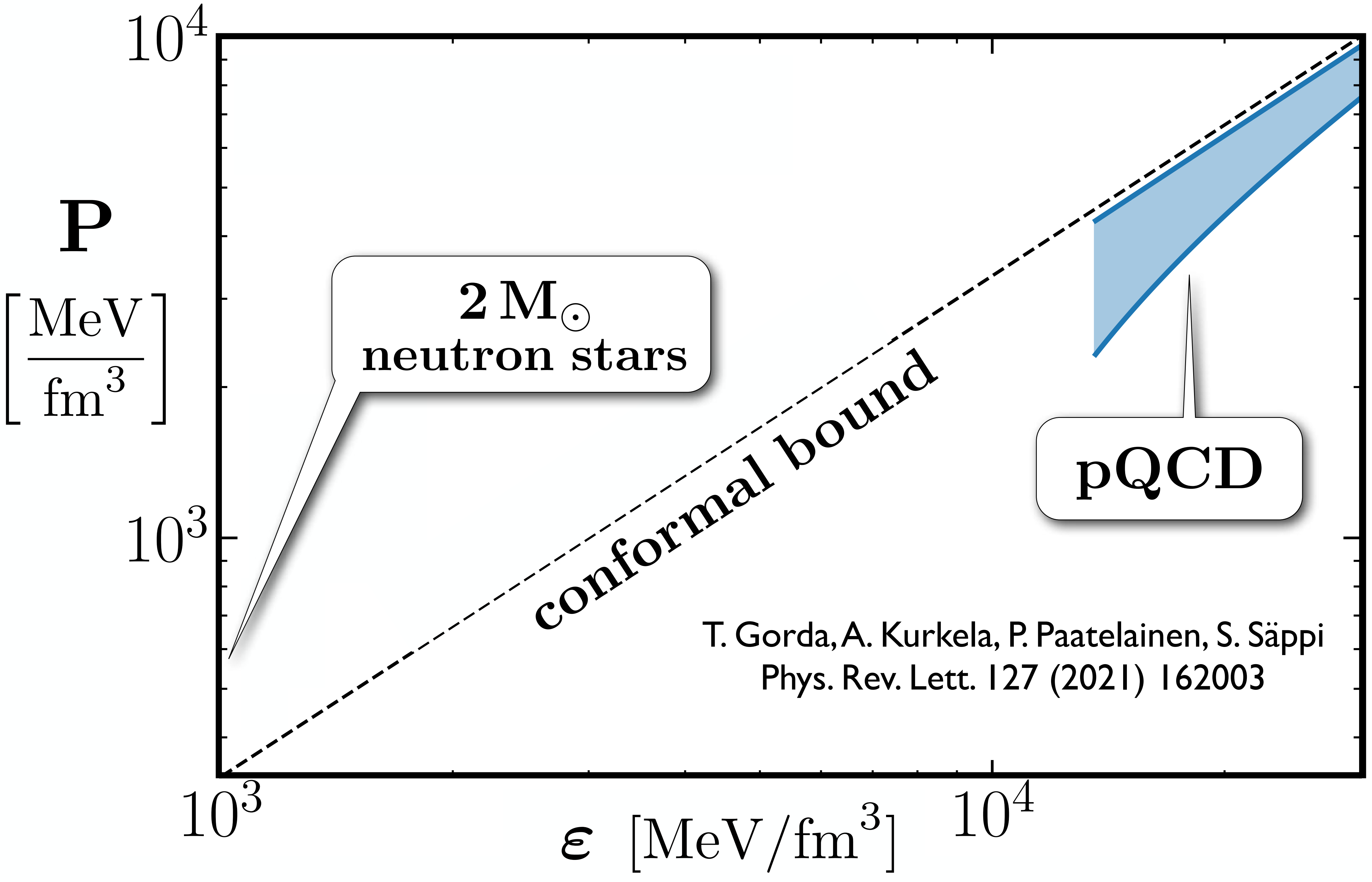
- **Speed of sound** exceeds conformal bound $c_s = 1/\sqrt{3}$ at baryon densities $n_B > 2 - 3\,n_0$
- **Strongly repulsive correlations** in dense baryonic matter

EQUATION of STATE and SOUND VELOCITY
- boundary conditions -

- **Low densities : Chiral EFT @ $\mathbf{n_B \lesssim 2\, n_0}$**



- **Extremely high densities : $\mathbf{n_B >> n_c(2M_\odot)}$**



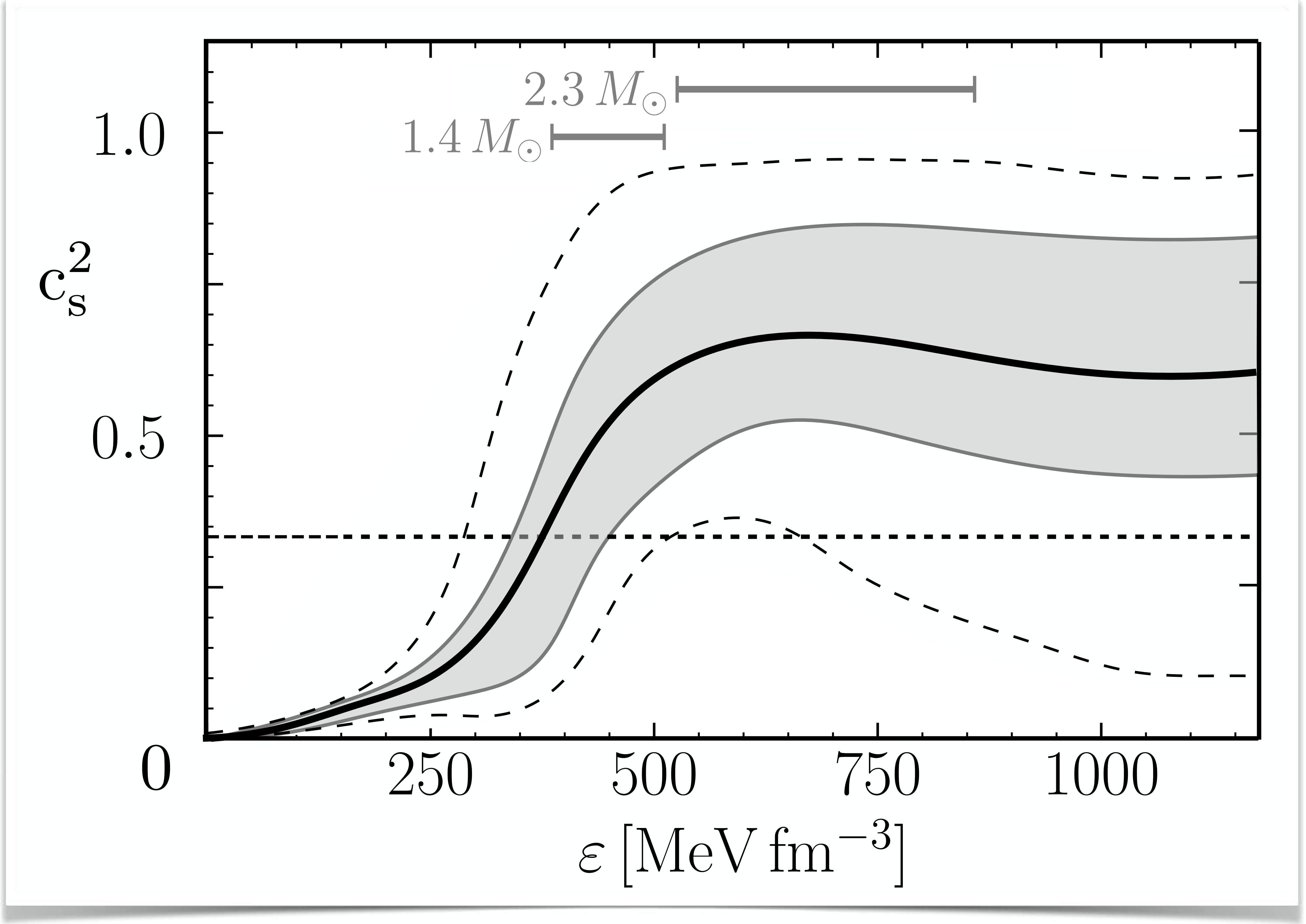
- Employ ChEFT constraint at $\mathbf{n_B = 1.3\, n_0}$ in Bayes inference as **Likelihood, NOT Prior**

- **Conformal bound** $\mathbf{c_s^2 = \frac{1}{3}}$ reached asymptotically

Comment : **SPEED of SOUND** exceeding **CONFORMAL BOUND**

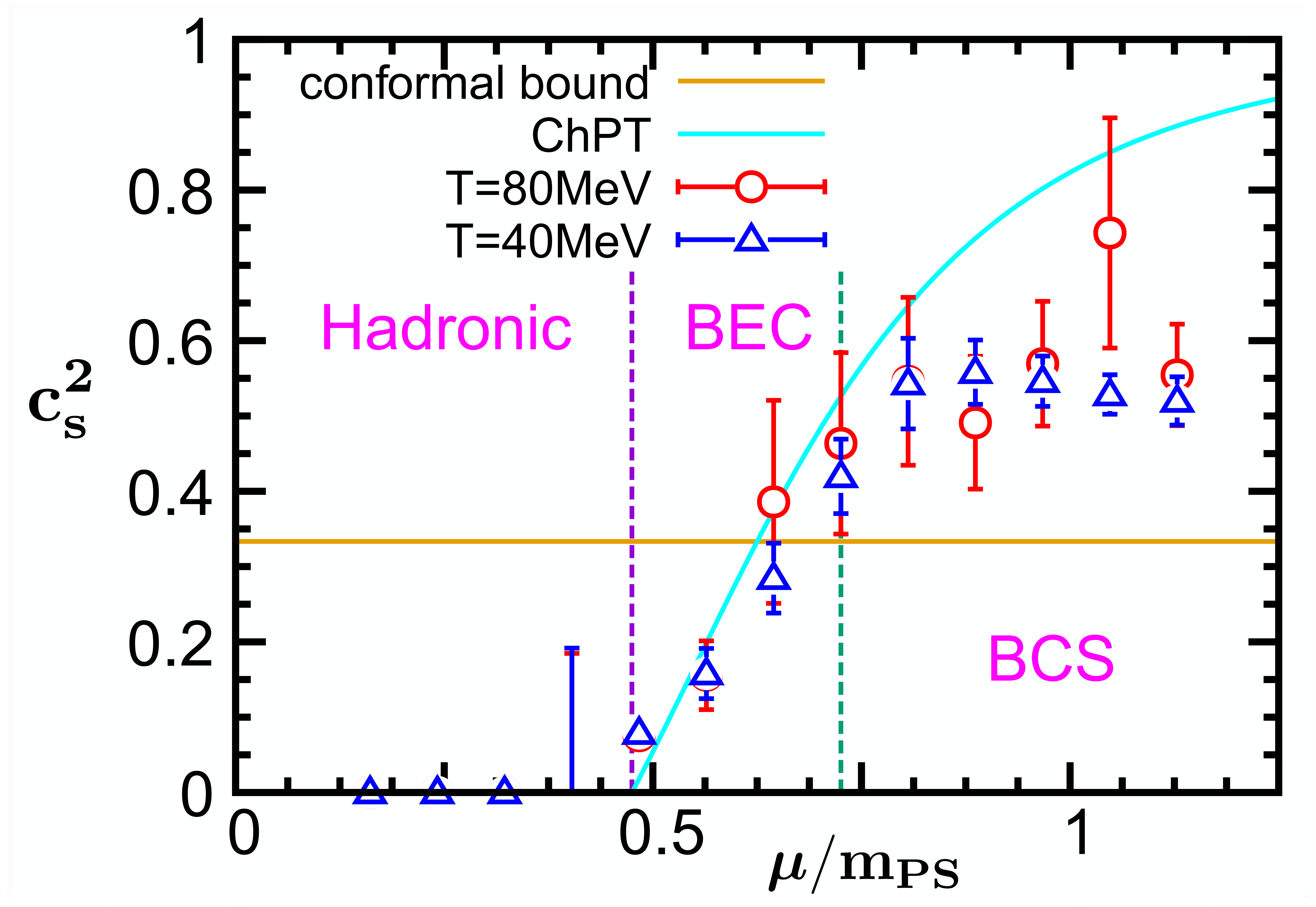
- Bayesian inference of **sound speed** in neutron star matter

L. Brandes, W.W., N. Kaiser : Phys. Rev. D 108 (2023) 094014



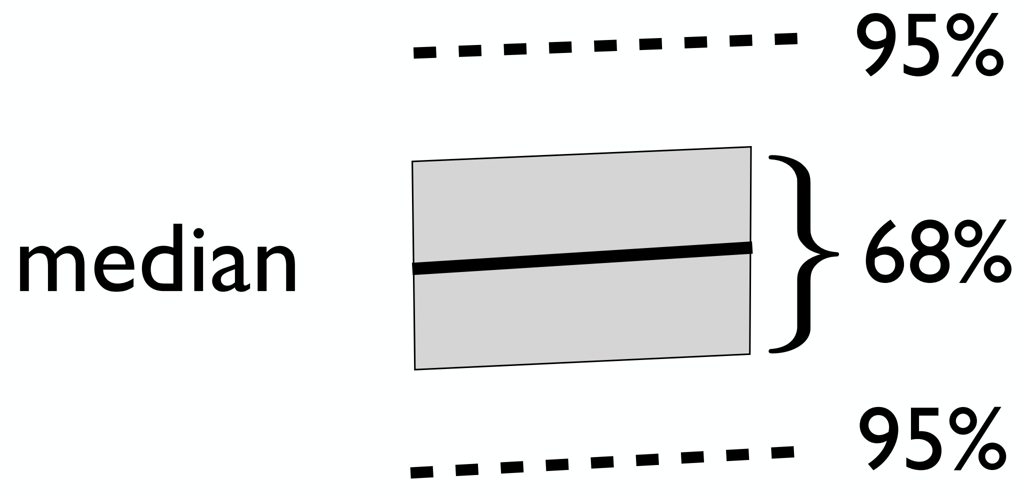
- Sound speed as function of baryon chemical potential in $N_c = 2$ LQCD

K. Iida, E. Itou, K. Murakami, D. Suenaga : arXiv:2405.20566

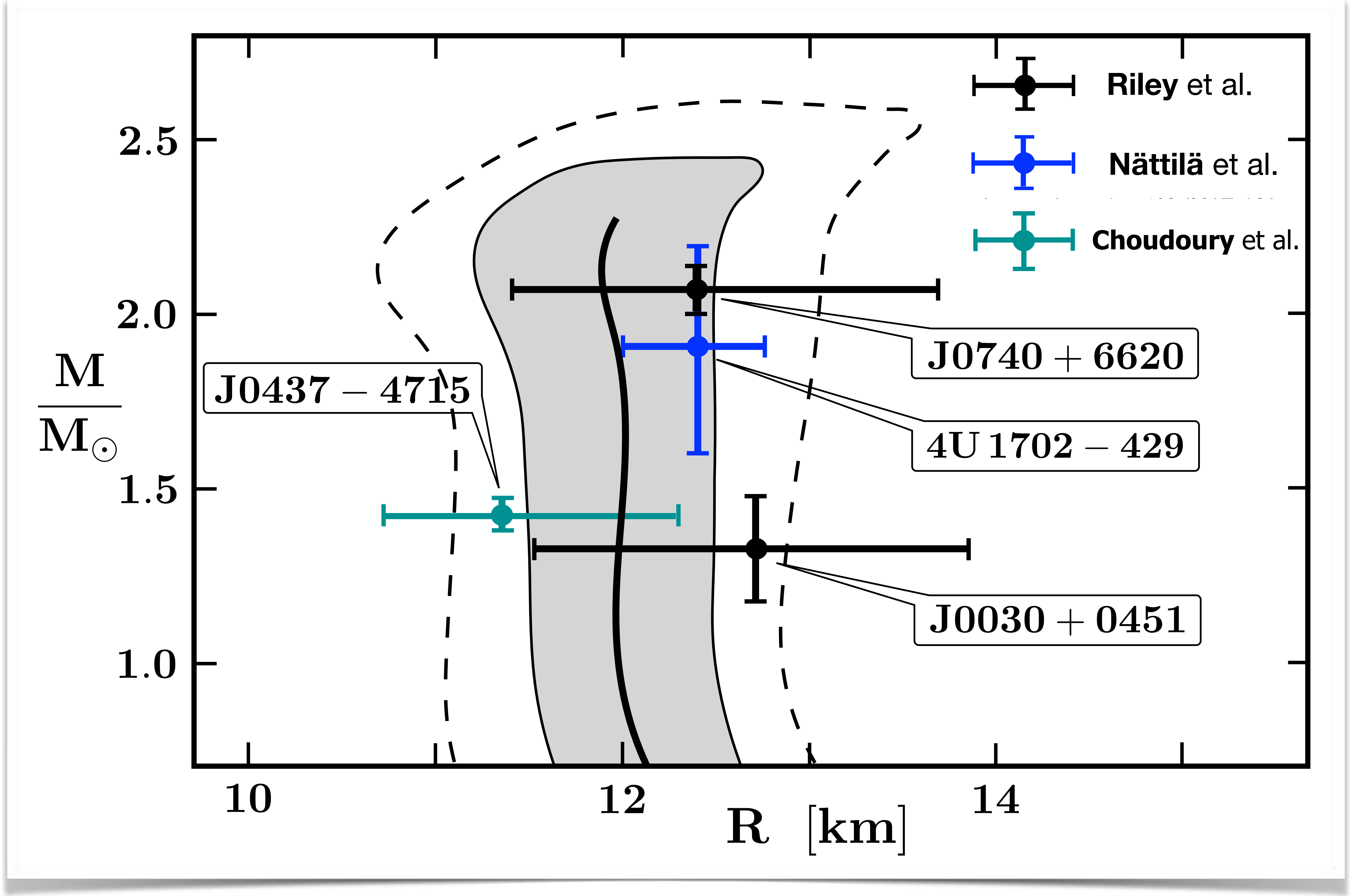


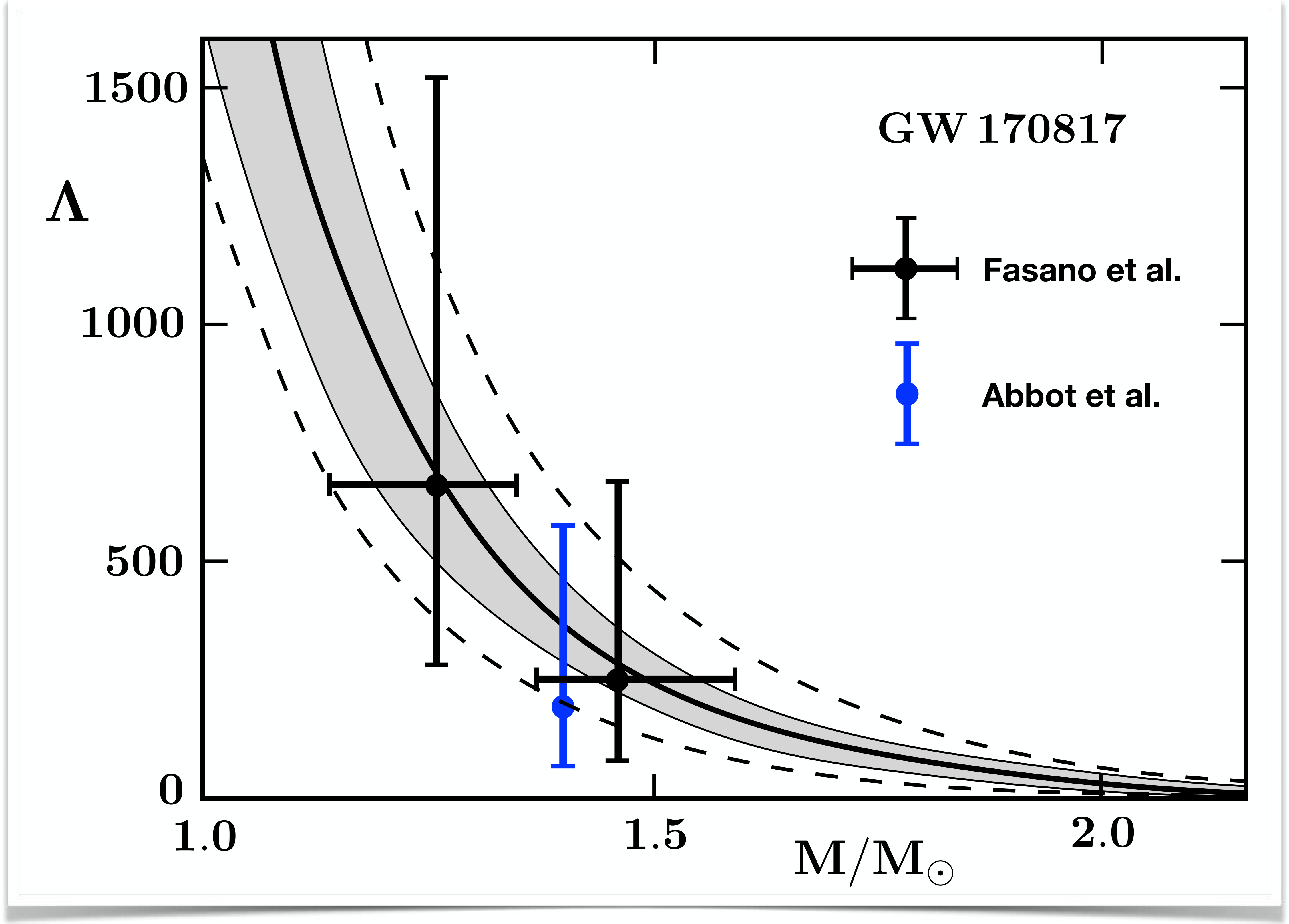
- **Speed of sound** exceeds conformal bound $c_s = 1/\sqrt{3}$ at baryon densities $n_B > 2 - 3\,n_0$

NEUTRON STAR PROPERTIES



- **Bayesian inference posterior bands** (68% and 95% c.l)
- **Mass - Radius relation (TOV)**
- **Tidal deformability**





L. Brandes, W. W., N. Kaiser : Phys. Rev. D 107 (2023) 014011 ; Phys. Rev. D 108 (2023) 094014 L. Brandes, W. W. (2024)

NEUTRON STAR PROPERTIES (contd.)

- **Baryon chemical potential**

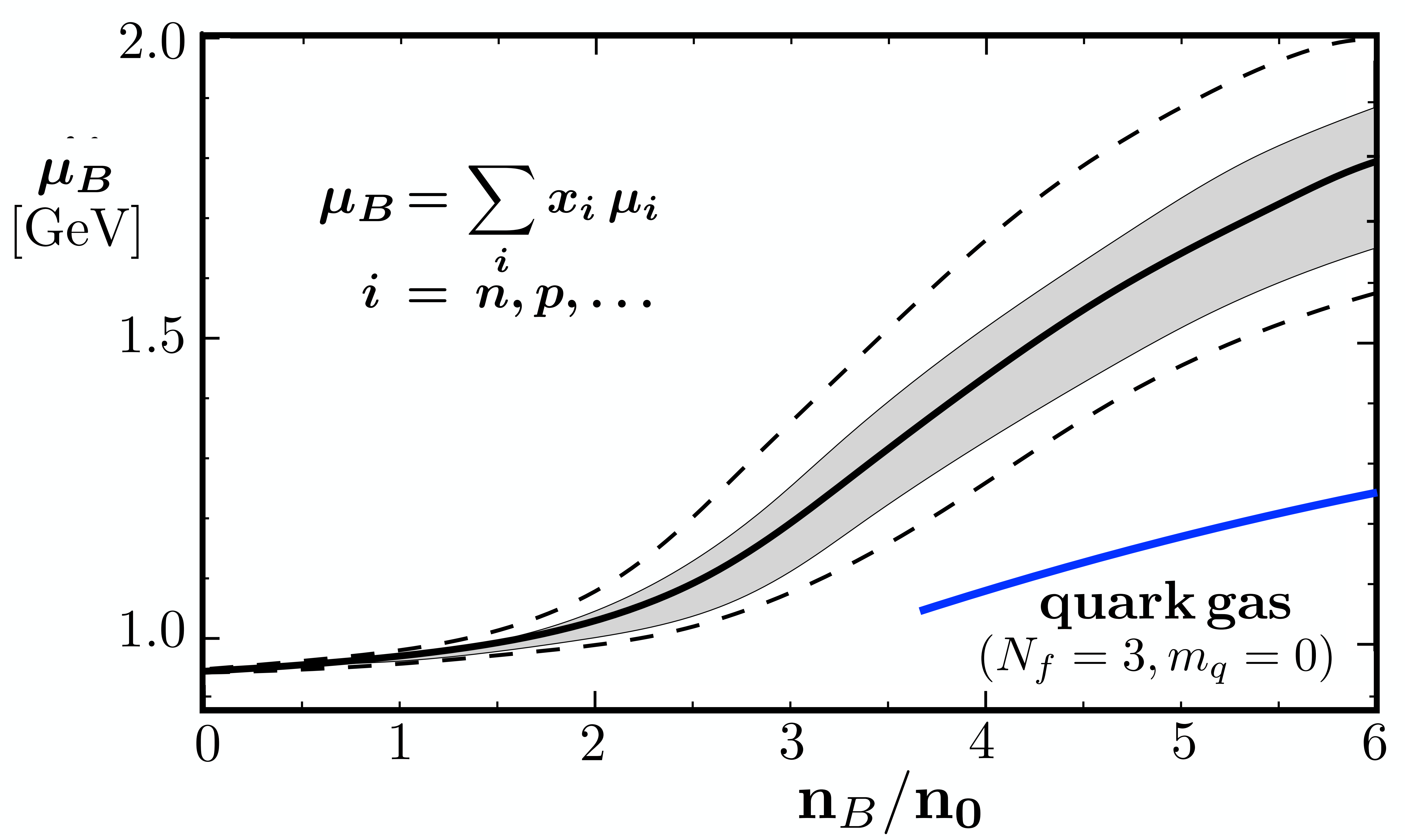
$$\mu_B = \frac{\partial \varepsilon}{\partial n_B}$$

- **Stiff equation of state**

  ↓

  strongly repulsive correlations at work between baryons / quarks

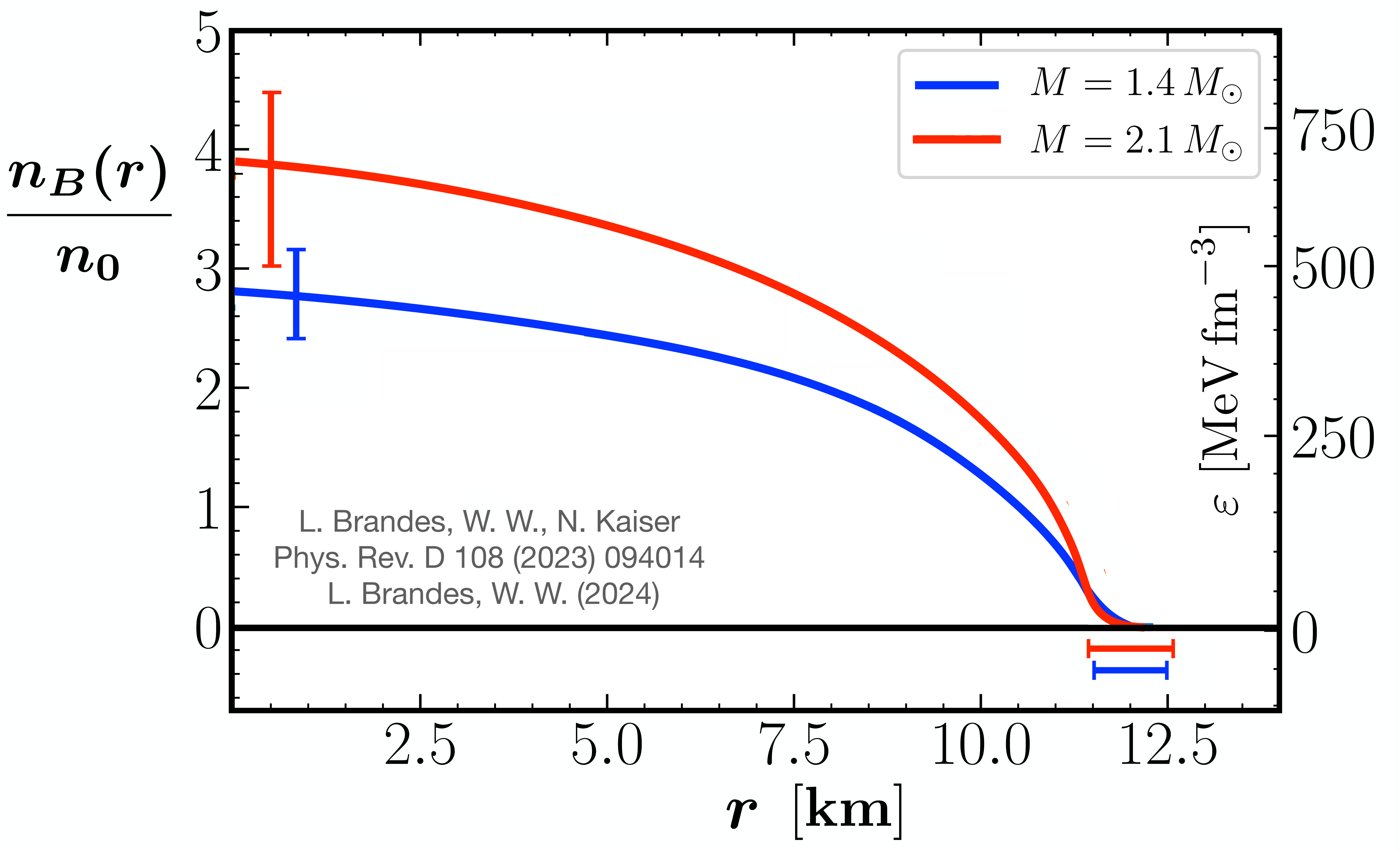
- **Quark gas ruled out at densities $n_B \sim 4 - 6\, n_0$**



L. Brandes, W. W., N. Kaiser : Phys. Rev. D 108 (2023) 094014

NEUTRON STAR PROPERTIES (contd.)

- **Density profiles of neutron stars** using inferred median of $P(\varepsilon)$
  - **Central core densities in neutron stars are NOT extreme**
  - **Average distance between baryons :** $d \gtrsim 1\,\text{fm}$ **even for the heaviest neutron stars**



$$n_c(1.4\ M_\odot) = 2.8 \pm 0.4\ n_0 \qquad n_c(2.1\ M_\odot) = 3.9^{+0.6}_{-0.9}\ n_0 \qquad n_c(2.3\ M_\odot) = 4.0^{+0.7}_{-0.8}\ n_0$$

(68% c.l. — including new NICER data and "black widow" PSR J0952-0607)

Constraints on
FIRST-ORDER PHASE TRANSITION in NEUTRON STAR MATTER

- **Bayes factor analysis :**

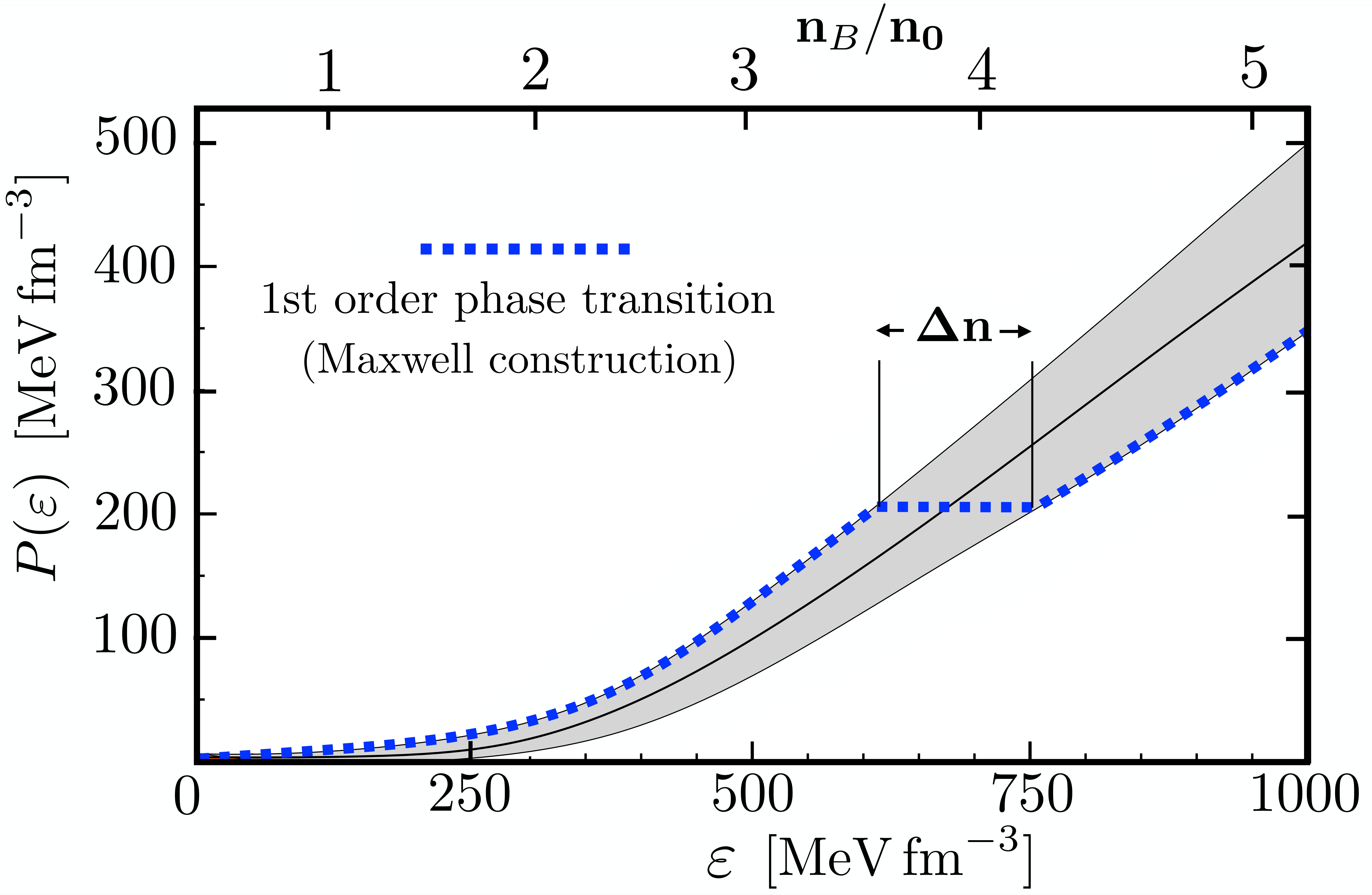
  → **Extreme evidence for sound velocities $c_s > 0.5$ in cores of all neutron stars with**

  $$1.4 \leq M/M_\odot \leq 2.3$$

- **Evidence against strong 1st order phase transition :**

  → **Maximum possible extension of phase coexistence domain**

  $$\Delta n/n_B \lesssim 0.2 \quad \text{(68\% c.l.)}$$



L. Brandes, W.W., N. Kaiser : Phys. Rev. D 108 (2023) 094014 - L. Brandes, W.W. : Symmetry 16 (2024) 111

→ **For comparison :**
Maxwell construction for nuclear 1st order liquid-gas phase transition $(\Delta n/n_B > 1)$

QCD TRACE ANOMALY and CONFORMALITY in NEUTRON STARS

Y. Fujimoto, K. Fukushima, L.D. McLerran, M. Praszalowicz : Phys. Rev. Lett. 129 (2022) 252702

- **Trace of energy-momentum tensor :** $T^{\mu}_{\mu} = \Theta = \frac{\beta}{2g} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + (1+\gamma_m) \sum_f m_f \bar{q}_f q_f$
- **Finite T and μ_B :**

$$\langle \Theta \rangle_{T,\mu_B} = \varepsilon - 3P$$

- **Trace anomaly measure**

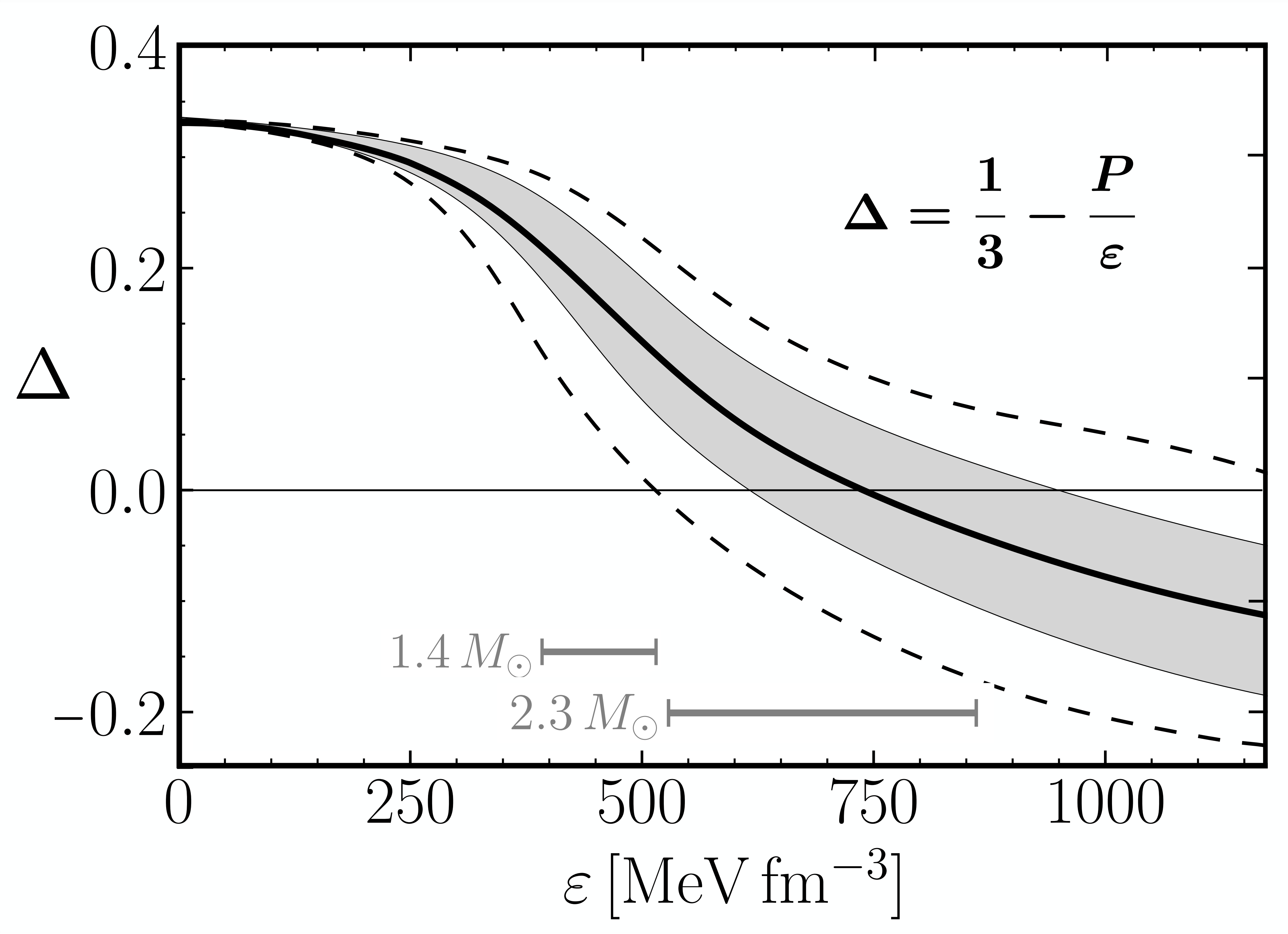
$$\Delta \equiv \frac{\langle \Theta \rangle_{T,\mu_B}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$$

- **Conformal limit :** $\Delta \to 0$
- **Bayes factor analysis:**

Strong evidence for

$$\Delta < 0 \quad (P > \varepsilon/3)$$

at densities $n_B \gtrsim 4\, n_0$



L. Brandes, W.W., N. Kaiser Phys. Rev. D 108 (2023) 094014
L. Brandes, W.W. (2024)

INTERMEDIATE SUMMARY

- **Bayesian inference analysis**
  including heavy $(\mathrm{M} = 2.35 \pm 0.17\,\mathrm{M}_\odot)$ galactic neutron star and NICER news
  - even **stiffer equation of state** required than previously expected
  - almost **constant neutron star radii** $(\mathrm{R} \simeq 12 \pm 1\ \mathrm{km})$ for all masses

- **Extreme evidence** for sound velocities $\mathrm{c_s} > 1/\sqrt{3}$ in neutron star cores
  - **strongly repulsive correlations** at work

- **Evidence** against **strong 1st order phase transition** in neutron star cores
  - **not** excluded: **baryonic** matter or **hadron-quark** continuous crossover

- **No extreme central core densities** even in the heaviest neutron stars:

  $n_B < 5\ n_0$ for $M \leq 2.3\,M_\odot$ (68% c.l.)

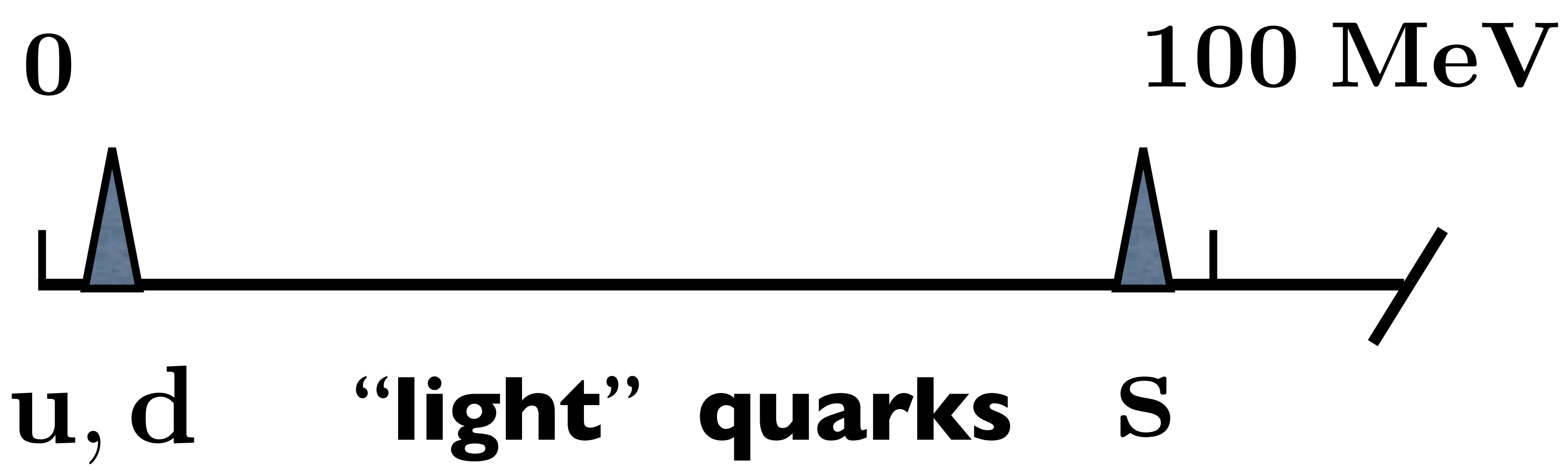
  - average baryon-baryon distance in the core: $\mathrm{d} \gtrsim 1\,\mathrm{fm}$

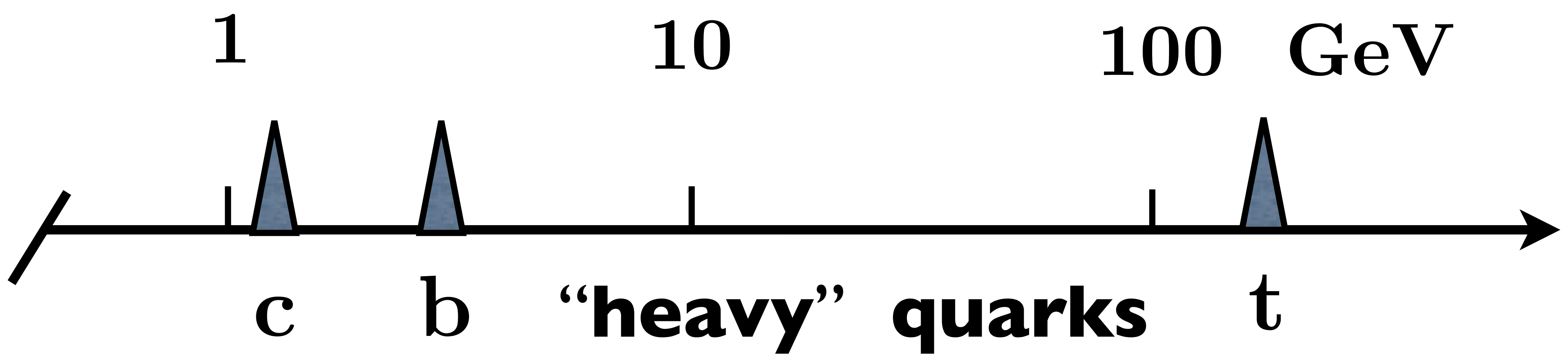
4.

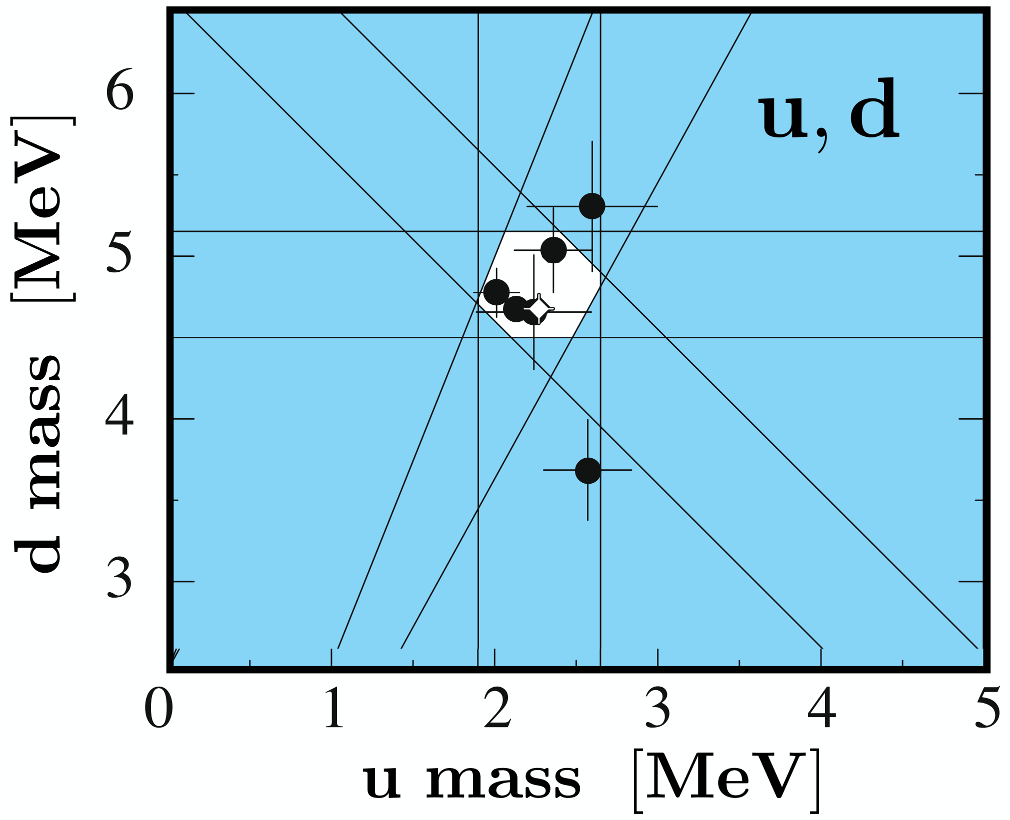
Further preparations : QCD Symmetries, Symmetry Breaking Patterns and Scales

Hierarchy of **QUARK MASSES** in **QCD**

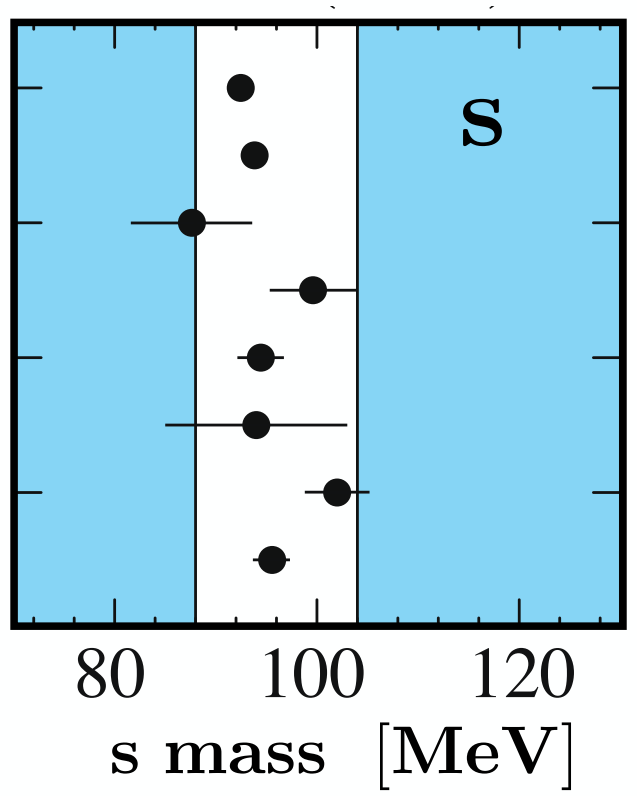
- Separation of Scales -







PDG 2020



$$m_u = 2.16 \pm 0.07 \text{ MeV}$$

$$m_d = 4.70 \pm 0.07 \text{ MeV}$$

$$m_s = 93.5 \pm 0.8 \text{ MeV}$$

$(\lambda = 2\,\text{GeV})$ **PDG 2024**

CHIRAL EFFECTIVE FIELD THEORY
of
Pseudoscalar Nambu-Goldstone Bosons
coupled to **Baryons**

- **Idealised QCD**
  $m_{u,d} \to 0 \quad (m_s \to 0)$
- **Scale Invariance** and **Trace Anomaly**
  from **massless quarks** to **massive nucleons**
- **Chiral Symmetry**
  $SU(N_f)_L \times SU(N_f)_R$
  spontaneously broken at low energy

Low-Energy QCD : CHIRAL SYMMETRY

- **QCD** with (almost) **MASSLESS u-** and **d-QUARKS**

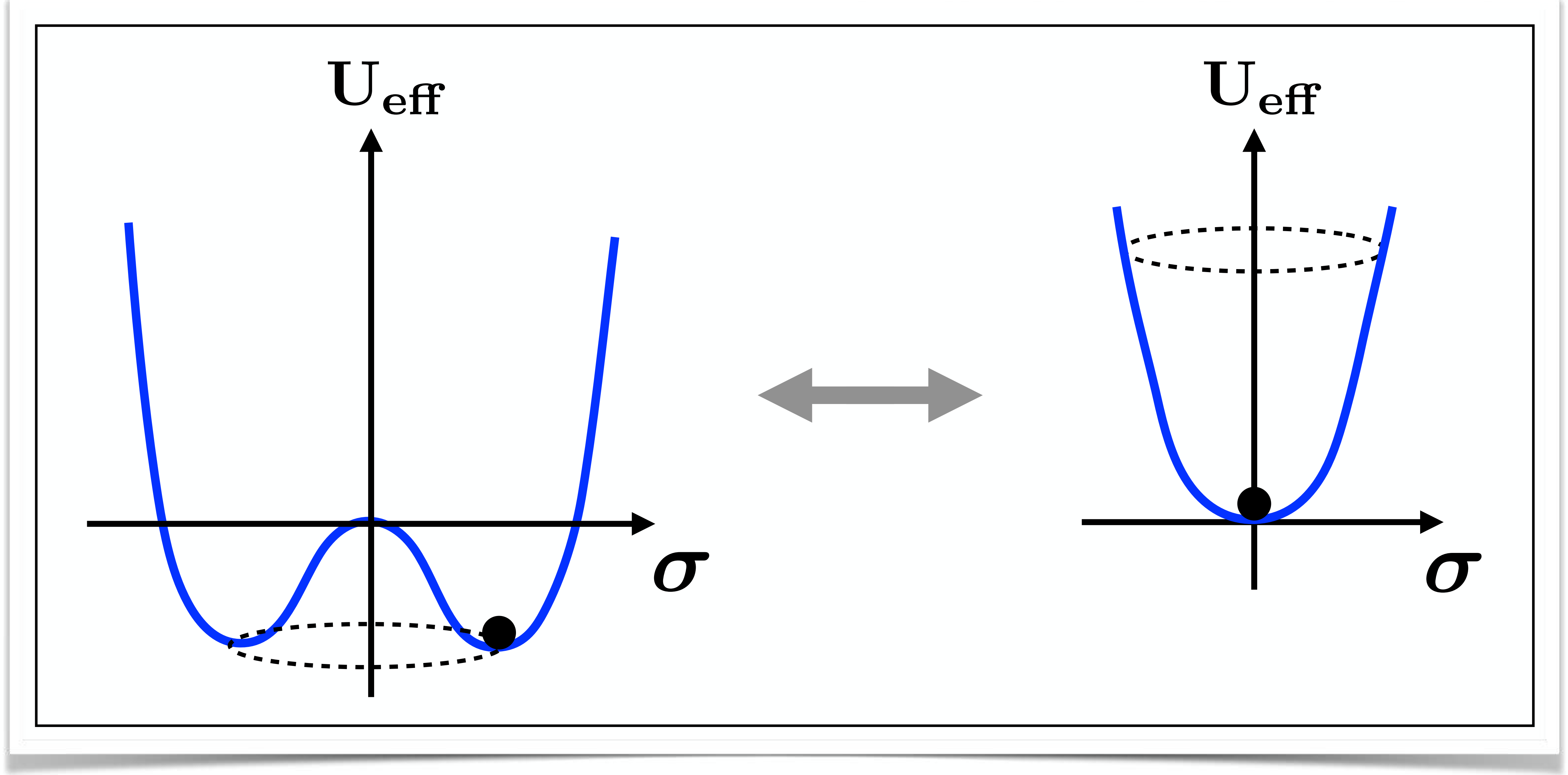
$$\mathrm{SU(2)}_L \times \mathrm{SU(2)}_R \qquad \psi = (\mathbf{u}, \mathbf{d})$$

pseudoscalar
isovector
$$\boldsymbol{\pi} \leftrightarrow \bar{\psi}\, i\gamma_5 \boldsymbol{\tau}\, \psi$$

scalar isoscalar
$$\sigma \leftrightarrow \bar{\psi}\,\psi$$

invariant :
$$\sigma^2 + \boldsymbol{\pi}^2 = f_\pi^2$$

- **Realisations** of **CHIRAL SYMMETRY**:



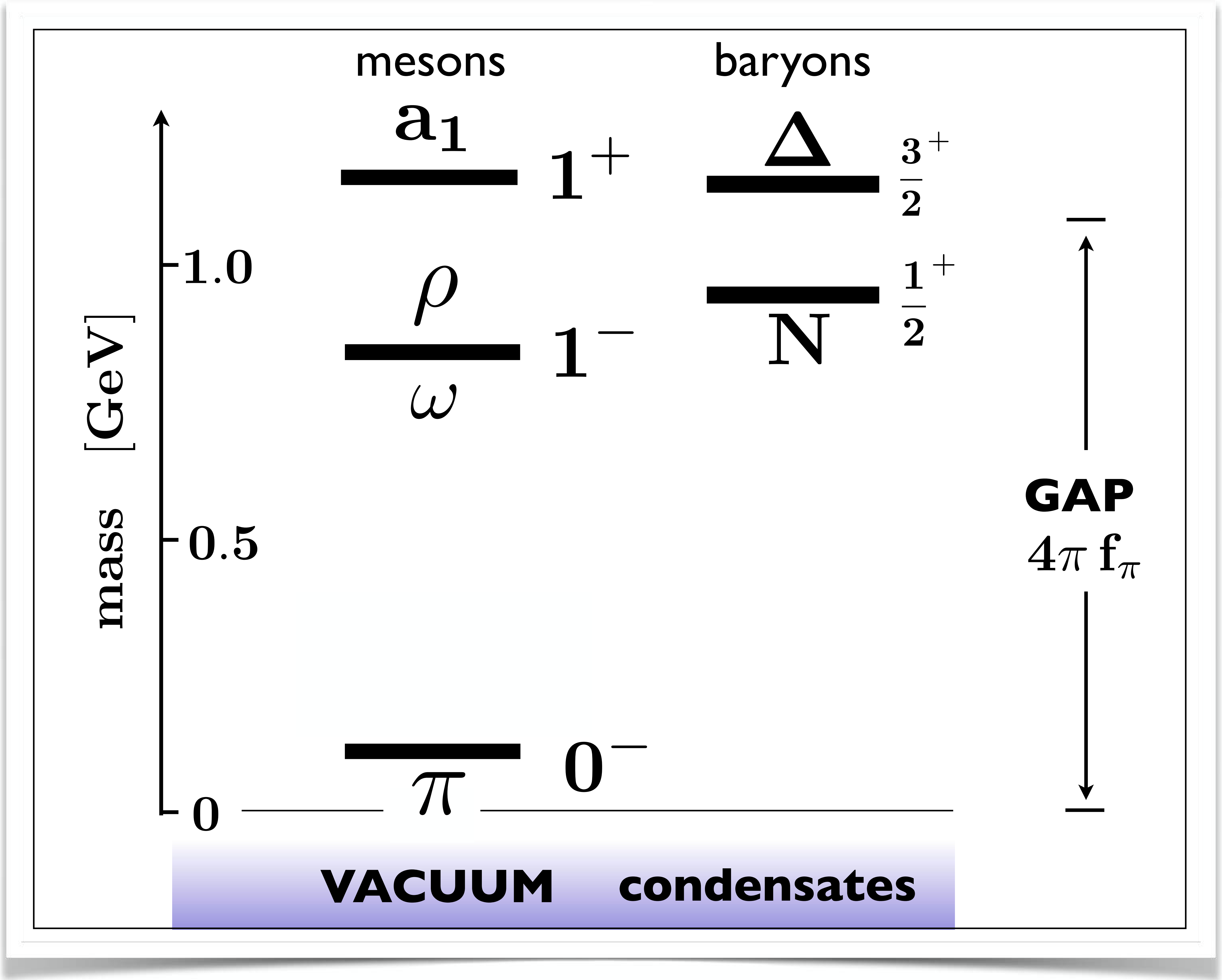
Nambu - Goldstone
$\langle \bar{\psi}\psi \rangle \neq 0$
low energy
low temperature

Wigner - Weyl
$\langle \bar{\psi}\psi \rangle = 0$
high energy
high temperature

2nd order phase transition
(for $\mathrm{m_{u,d}} = 0$)

CHIRAL SYMMETRY

- Low energy : **spontaneous chiral symmetry breaking**
- **PIONS as** (almost) **massless Nambu-Goldstone bosons**



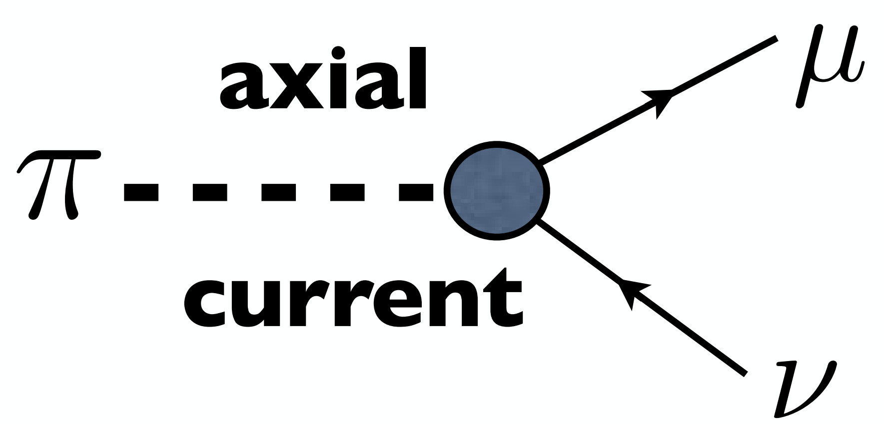
Symmetry breaking scale :

$$\Lambda_\chi = 4\pi\, f_\pi \sim 1\,\mathrm{GeV}$$

- **Pion decay constant :**

$f_\pi^{(0)} \simeq 86\,\mathrm{MeV}$ (chiral limit)

$f_\pi \simeq 92\,\mathrm{MeV}$ (empirical)



Order parameter : Pion decay constant

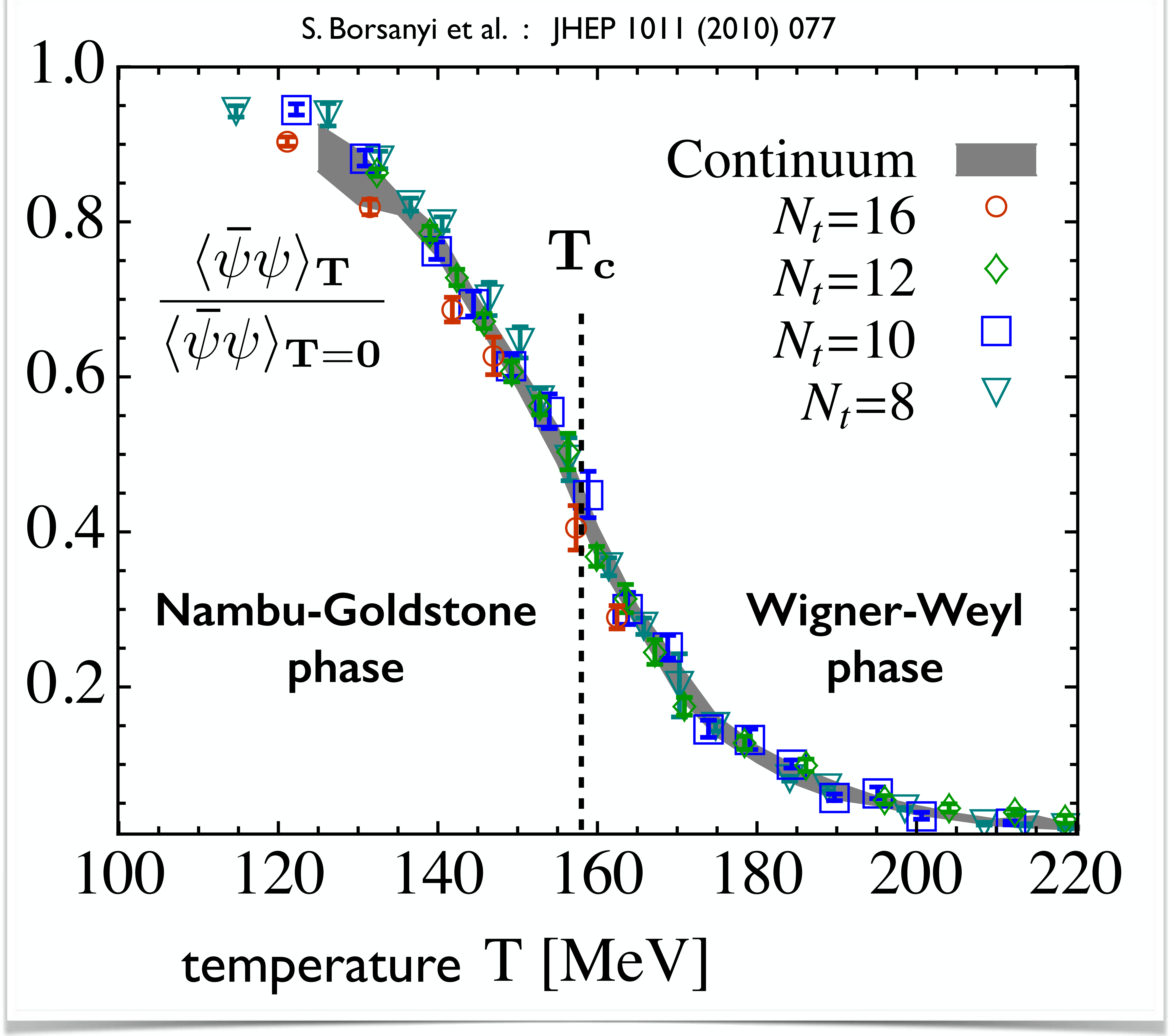
- **Chiral condensate** and **sigma field**

$$f_\pi^2 = -\frac{m_u + m_d}{2\,m_\pi^2}\langle \bar{u}u + \bar{d}d\rangle = \langle \boldsymbol{\sigma}\rangle^2$$

Gell-Mann - Oakes - Renner relation

LATTICE QCD THERMODYNAMICS: CHIRAL CROSSOVER TRANSITION

$$\mu_{\mathbf{baryon}} = 0$$



Chiral (Quark) Condensate

Crossover transition temperature

$$\mathbf{T_c} = 158.0 \pm 0.6 \text{ MeV}$$

C. Ratti et al. : J. Phys. Conf. Ser. 1602 (2020) 012011

CHIRAL and **DEFINCONFINEMENT**

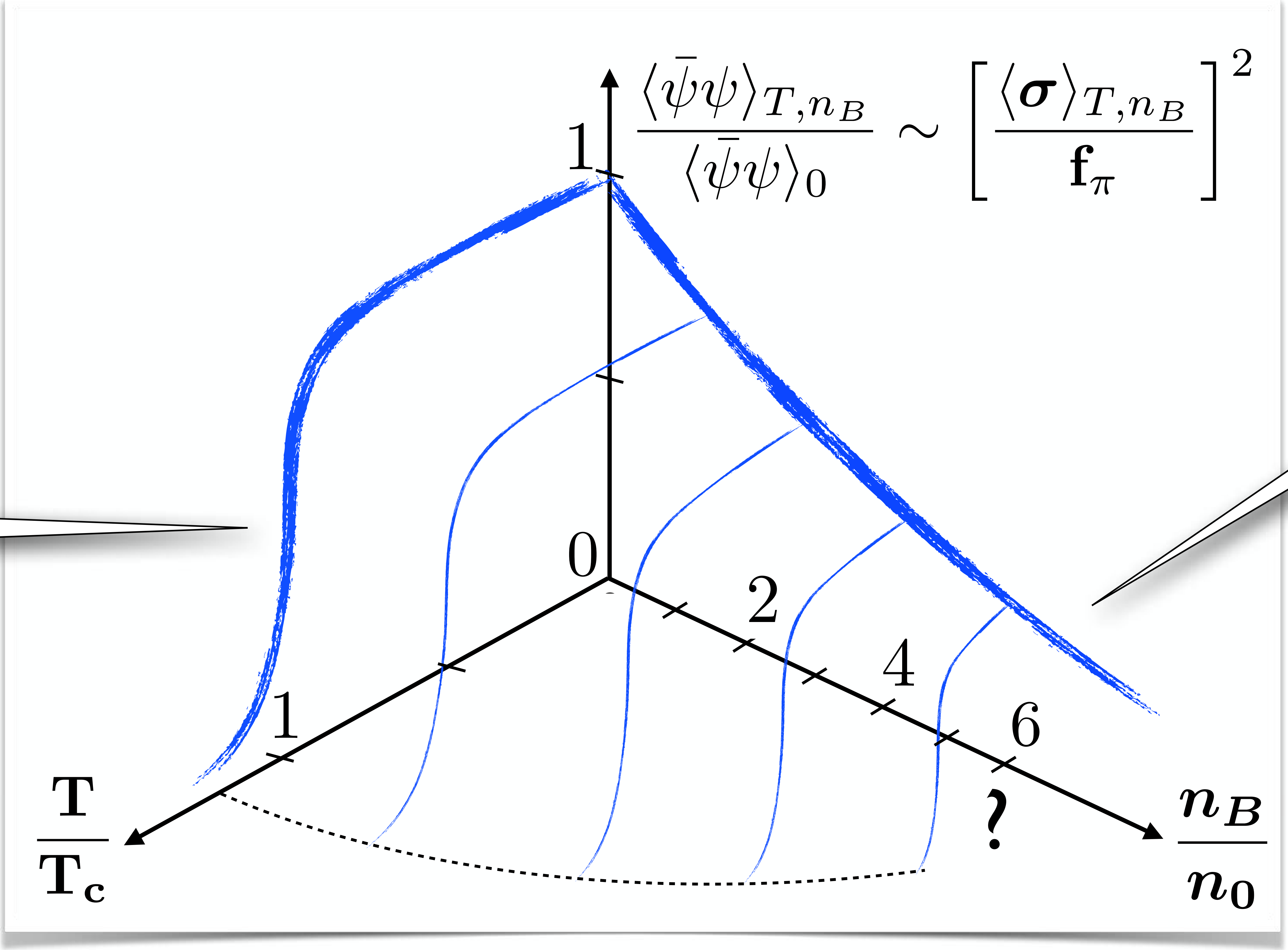
crossover transitions

appear to be closely **connected**

CHIRAL SYMMETRY RESTORATION

- From **Nambu-Goldstone** to **Wigner-Weyl** realisations of chiral symmetry
- Key issue : **Thermodynamics** of **chiral order parameter**
  its dependence on **temperature** and **baryon chemical potential / baryon density**

$\mu_B = 0$
$T_c \simeq 158\,\mathrm{MeV}$
crossover
Lattice QCD
HIC Freeze-out



$\mu_B > M_N$
$\mathrm{low}-T$
crossover ?
phase transition ?

(P)NJL type quark models,
chiral quark-meson models,
…
suggest first-order
chiral phase transition,
BUT …

5.

Low-Energy Structure of the Nucleon: Mass and Size(s)

SCALE INVARIANCE, TRACE ANOMALY and MASS of the NUCLEON

- **QCD with massless quarks : no dimensional parameter**
  - **Invariance under scale transformations** $\mathrm{x} \rightarrow \lambda \mathrm{x}$
  - **Trace of energy-momentum tensor** $\mathrm{T}^{\mu\nu}$ **vanishes classically** …
- … **but - QCD as a quantum field theory introduces renormalisation scale,** and so :

$$\mathbf{T}^{\mu}_{\mu} \propto \mathbf{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right]$$ non-vanishing (**TRACE ANOMALY**)

- **From MASSLESS QUARKS to MASSIVE NUCLEONS** :

$$\mathrm{M}_{\mathrm{N}}^{(0)} = \langle \mathrm{N}|\mathrm{T}^{\mu}_{\mu}|\mathrm{N}\rangle = \frac{9}{4}\langle \mathrm{N}|\frac{\alpha}{\pi}\left(\mathbf{B}^2 - \mathbf{E}^2\right)|\mathrm{N}\rangle \simeq 0.8\,\mathrm{GeV}$$

- **Physical nucleon mass** : $\mathrm{M}_{\mathrm{N}} = \mathrm{M}_{\mathrm{N}}^{(0)} + \sigma_{\mathrm{N}} + \sigma_{\mathrm{s}} = 0.94\,\mathrm{GeV}$
- **Sigma terms** : $\sigma_{\mathrm{N}} = \frac{1}{2}(m_u + m_d)\langle N\left|\bar{u}u + \bar{d}d\right|N\rangle$ $\quad \sigma_s = m_s\langle N|\bar{s}s|N\rangle$

NUCLEON MASS
- Aspects of Broken Symmetry -

$$\mathrm{M_N} = \mathrm{M_N^{(0)}} + \sigma_\mathrm{N} + \sigma_\mathrm{s} = 0.94\,\mathrm{GeV}$$

~ 90 %
of the nucleon mass from gluonic trace anomaly

$\mathrm{M_N}$

~ 10 %
of the nucleon mass from quark mass terms
(sigma terms)

$$\mathrm{M_N^{(0)}} \propto \langle \mathrm{N}|\mathrm{Tr}[\mathrm{G}_{\mu\nu}\mathrm{G}^{\mu\nu}]|\mathrm{N}\rangle$$

TRACE ANOMALY sum rule

broken scale invariance

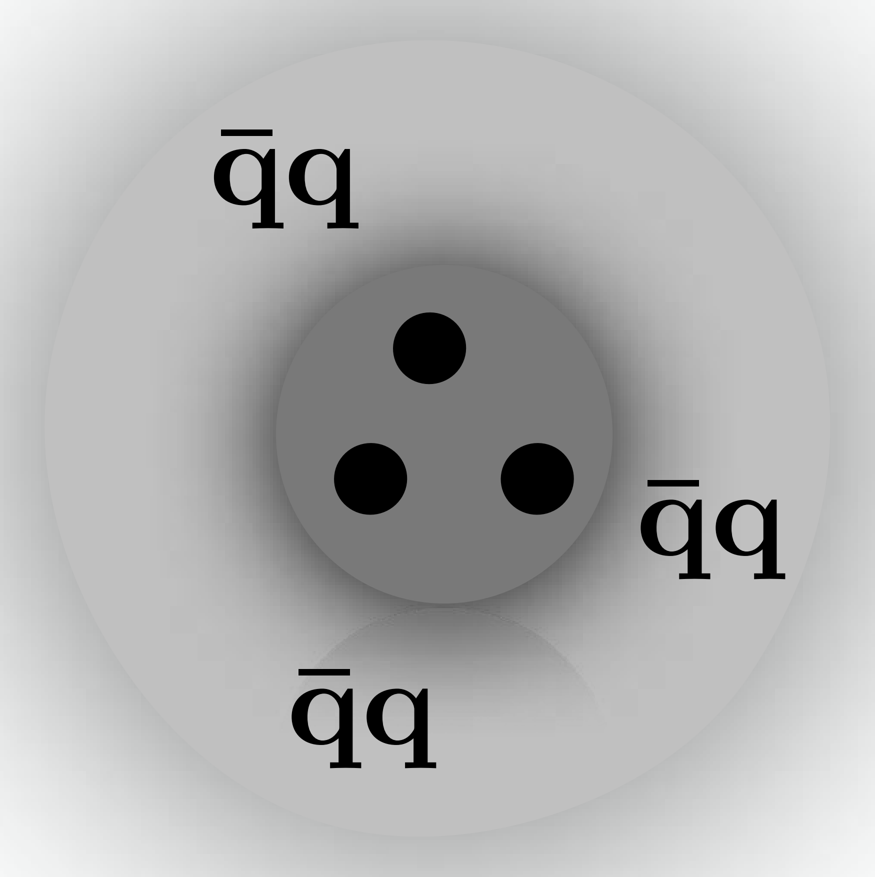
$$\mathrm{M_N} = g\,\mathrm{f}_\pi \equiv g\langle\sigma\rangle \qquad \left(g = \frac{g_{\pi N}}{g_A}\right)$$

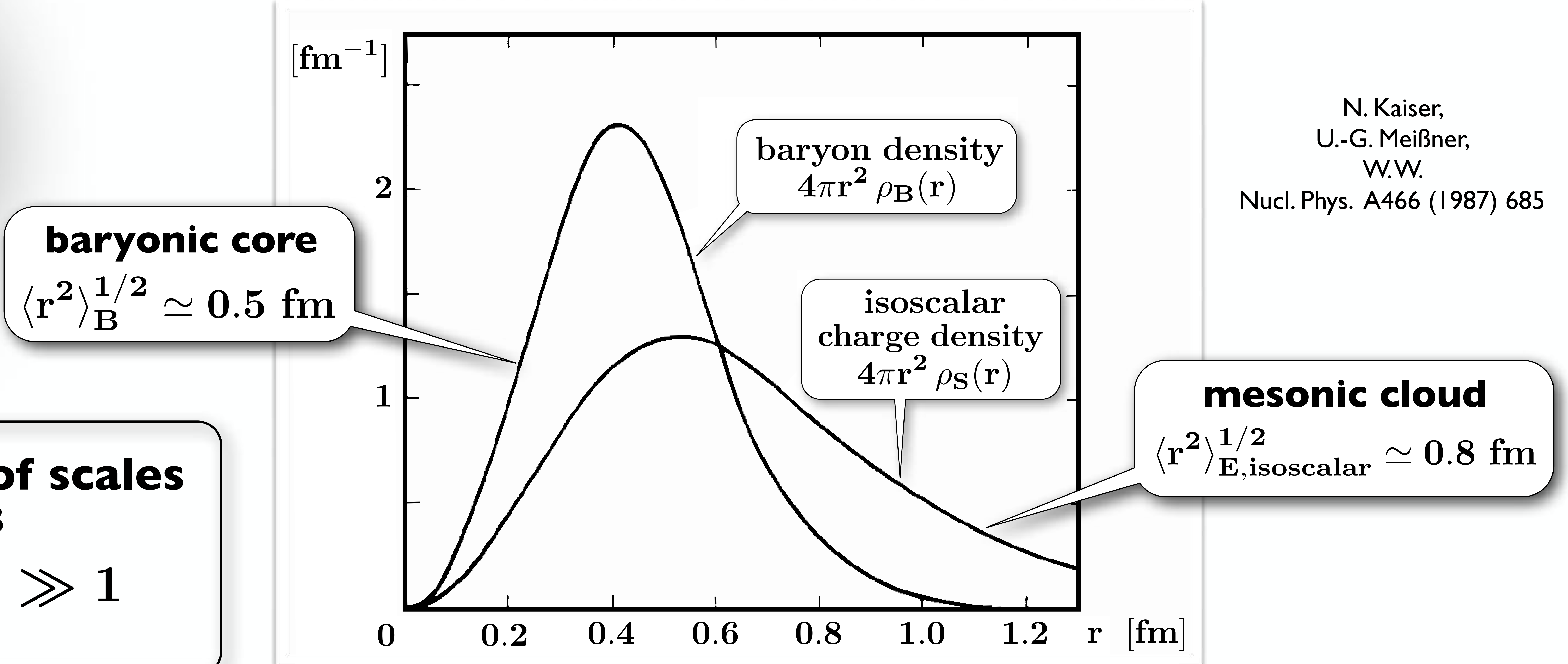
spontaneously broken
CHIRAL SYMMETRY
Goldberger - Treiman relation

SIZES of the **NUCLEON**

Low-energy QCD: **spontaneously broken chiral symmetry + localisation (confinement)**

- **NUCLEON** : **compact valence quark core + mesonic** (multi $\bar{q}q$) **cloud**
  - Historic example: Chiral Soliton Model of the Nucleon





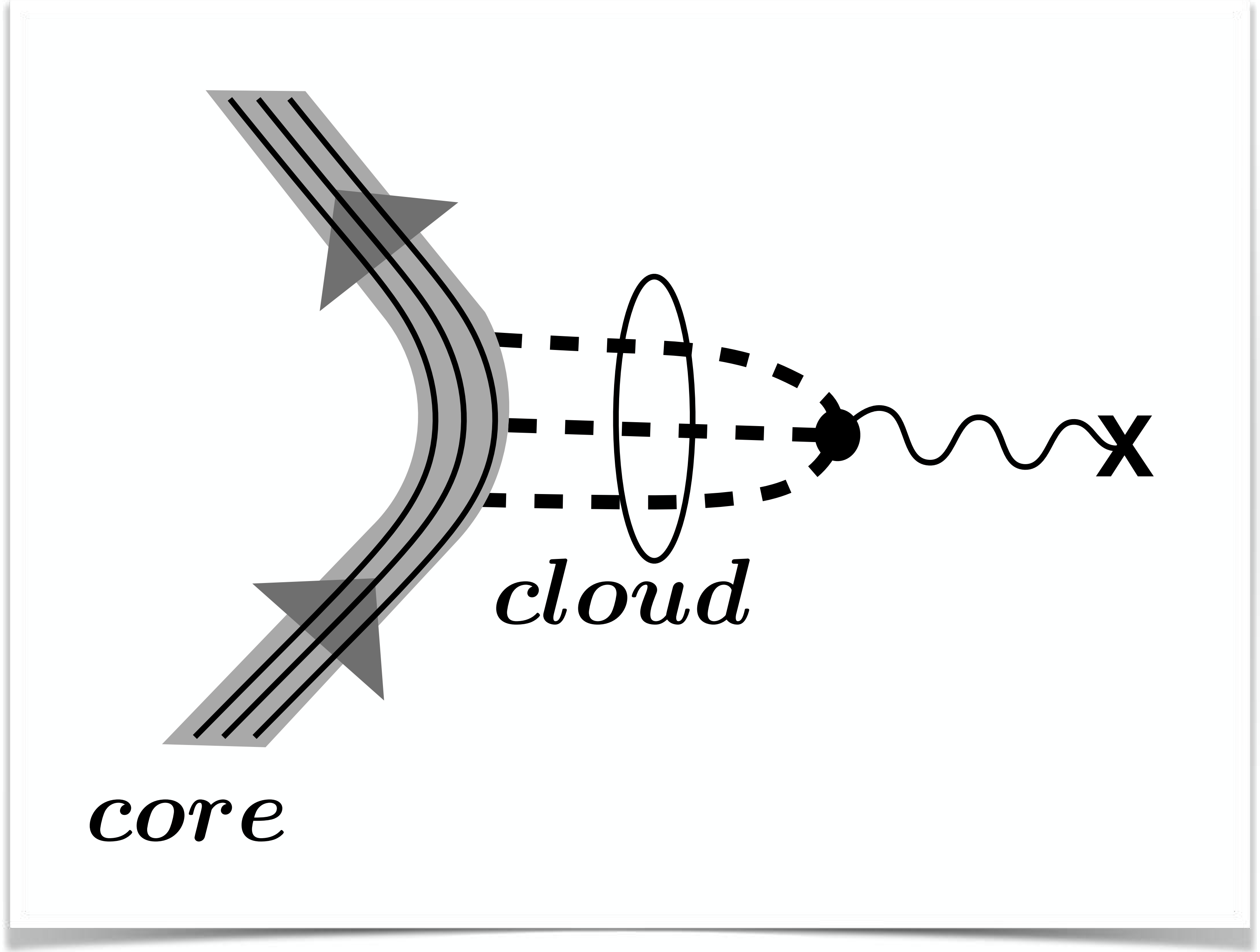
N. Kaiser,
U.-G. Meißner,
W.W.
Nucl. Phys. A466 (1987) 685

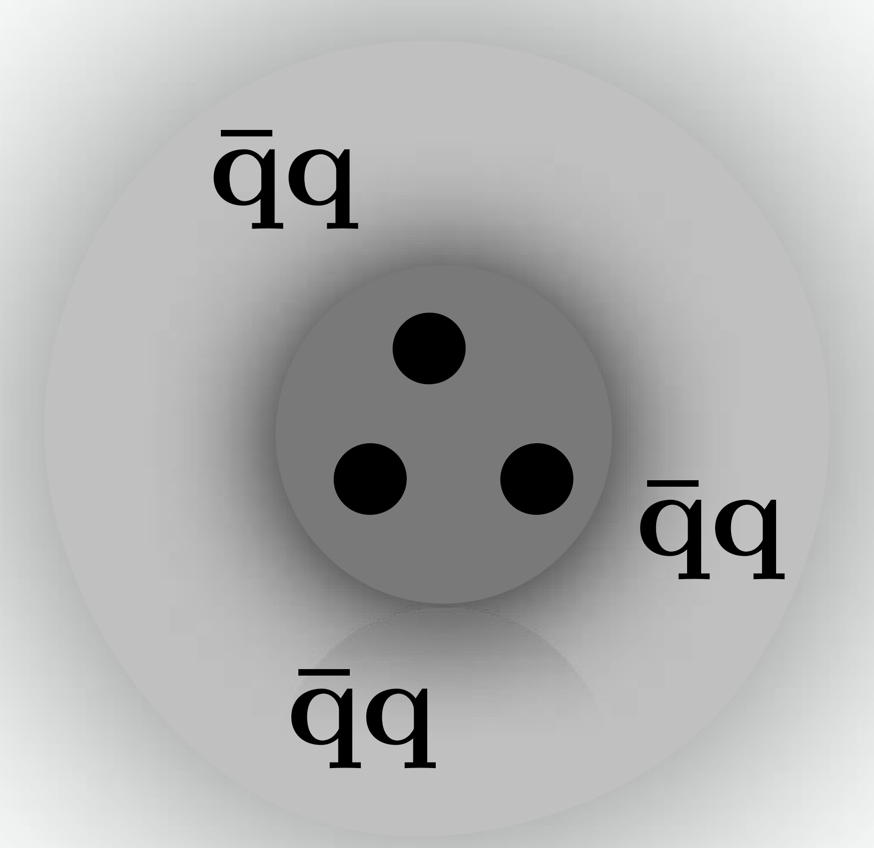
- **Separation of scales**

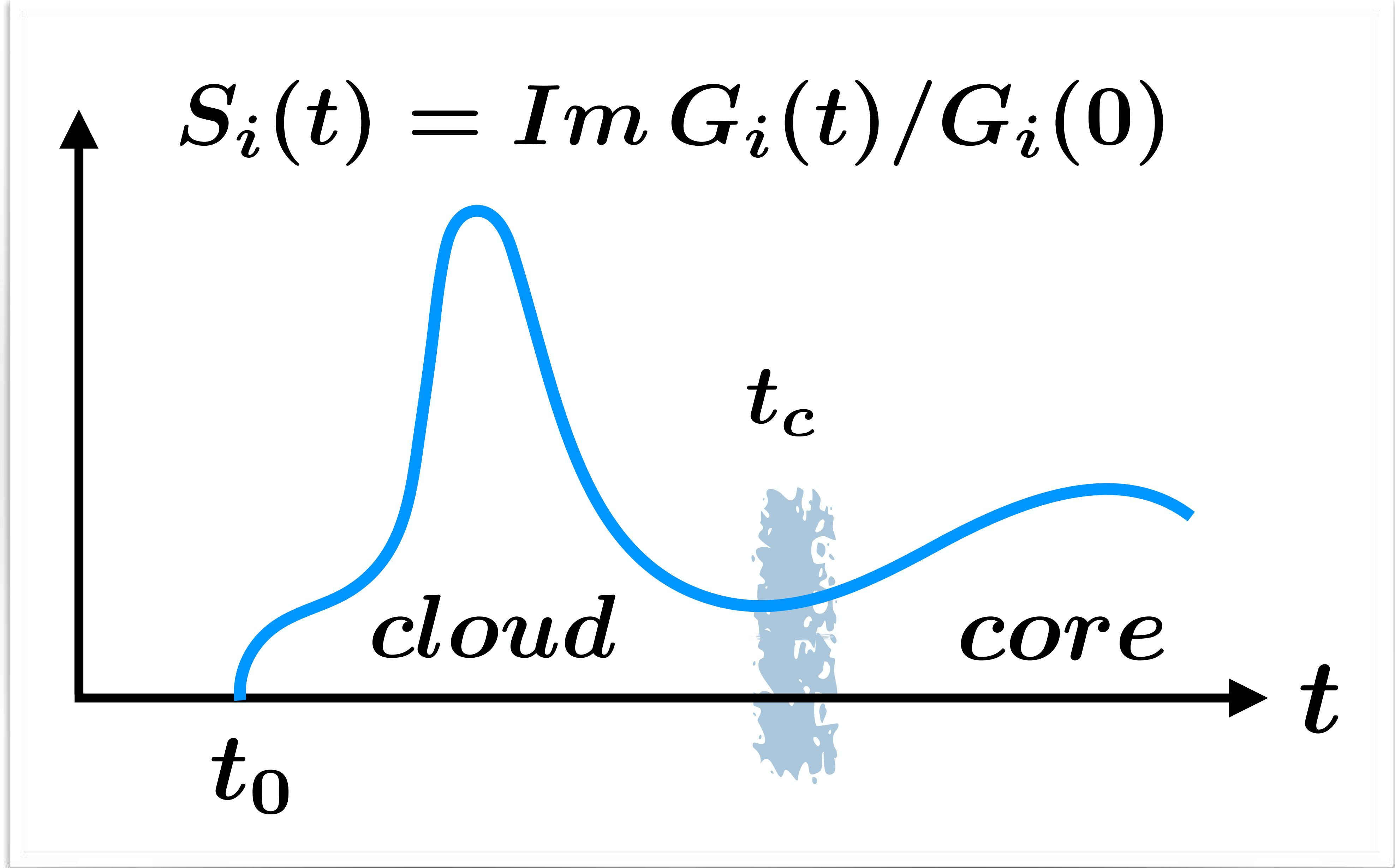
$$\left(\frac{R_{cloud}}{R_{core}}\right)^3 \gg 1$$

FORM FACTORS of the NUCLEON

$$G_i(q^2) = G_i(0) + \frac{q^2}{\pi}\int_{t_0}^{\infty} dt \frac{Im\, G_i(t)}{t(t-q^2-i\epsilon)} \qquad \langle r_i^2 \rangle = \frac{6}{G_i(0)} \frac{dG_i(q^2)}{dq^2}\Big|_{q^2=0} = \frac{6}{\pi}\int_{t_0}^{\infty} \frac{dt}{t^2} S_i(t)$$







$$\langle r_i^2 \rangle = \langle r_i^2 \rangle_{cloud} + \langle r_i^2 \rangle_{core} = \frac{6}{\pi}\left[\int_{t_0}^{t_c} \frac{dt}{t^2} S_i(t) + \int_{t_c}^{\infty} \frac{dt}{t^2} S_i(t)\right]$$

Example 1: **ISOSCALAR ELECTRIC FORM FACTOR** of the **NUCLEON**

- **Isoscalar electric form factor** $G_E^S(q^2) = \frac{1}{2}\left[G_E^p(q^2) + G_E^n(q^2)\right]$ $\quad \langle r_S^2 \rangle = \langle r_p^2 \rangle + \langle r_n^2 \rangle$

Empirical : $\langle r_p^2 \rangle^{1/2} = 0.840 \pm 0.004\,\mathrm{fm}$

$\langle r_n^2 \rangle = -0.105 \pm 0.006\,\mathrm{fm}^2$

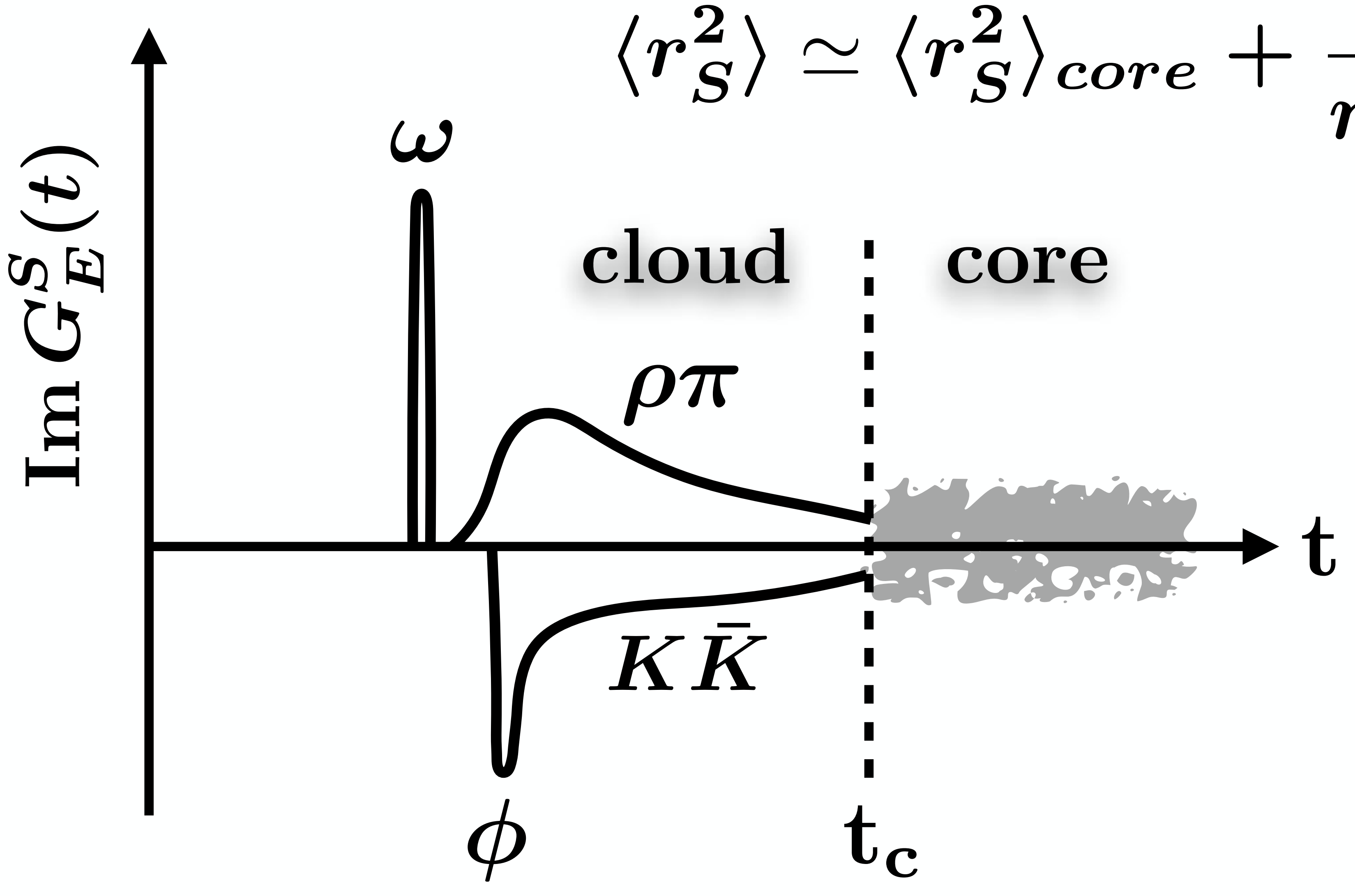
$\langle r_S^2 \rangle^{1/2} = 0.775 \pm 0.011\,\mathrm{fm}$

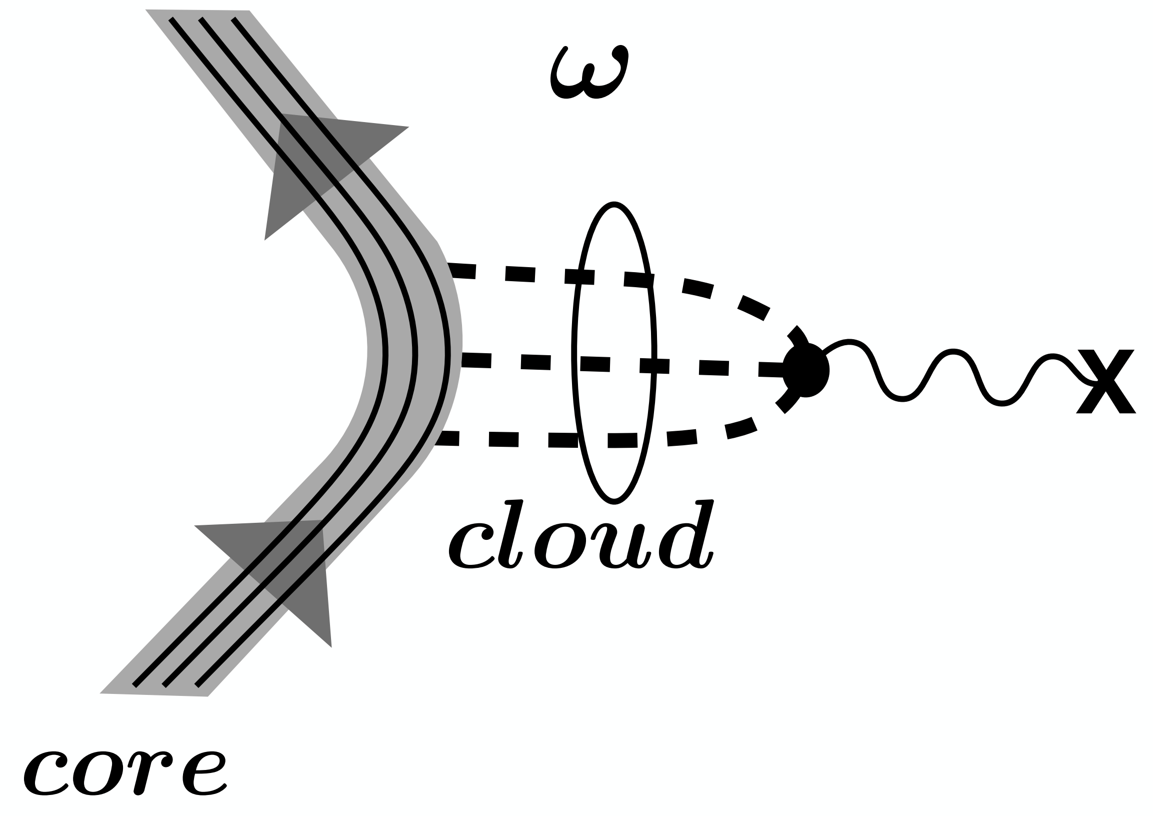
Y.H. Lin, H.-W. Hammer, U.-G. Meißner PRL 128 (2022) 052002

… based on **precision fits** to form factors at both **spacelike and timelike** q^2

- **Simplest Vector Dominance Model:** "cloud" dominated by ω meson

$$\langle r_S^2 \rangle \simeq \langle r_S^2 \rangle_{core} + \frac{6}{m_\omega^2} \quad \rightarrow \quad \langle r_S^2 \rangle_{core}^{1/2} \simeq 0.47\,\mathrm{fm}$$





- **Detailed analysis using best-fit spectral functions :**

$$\langle r_S^2 \rangle_{core}^{1/2} \equiv \langle r_B^2 \rangle^{1/2} = 0.50 \pm 0.01\,\mathrm{fm}$$

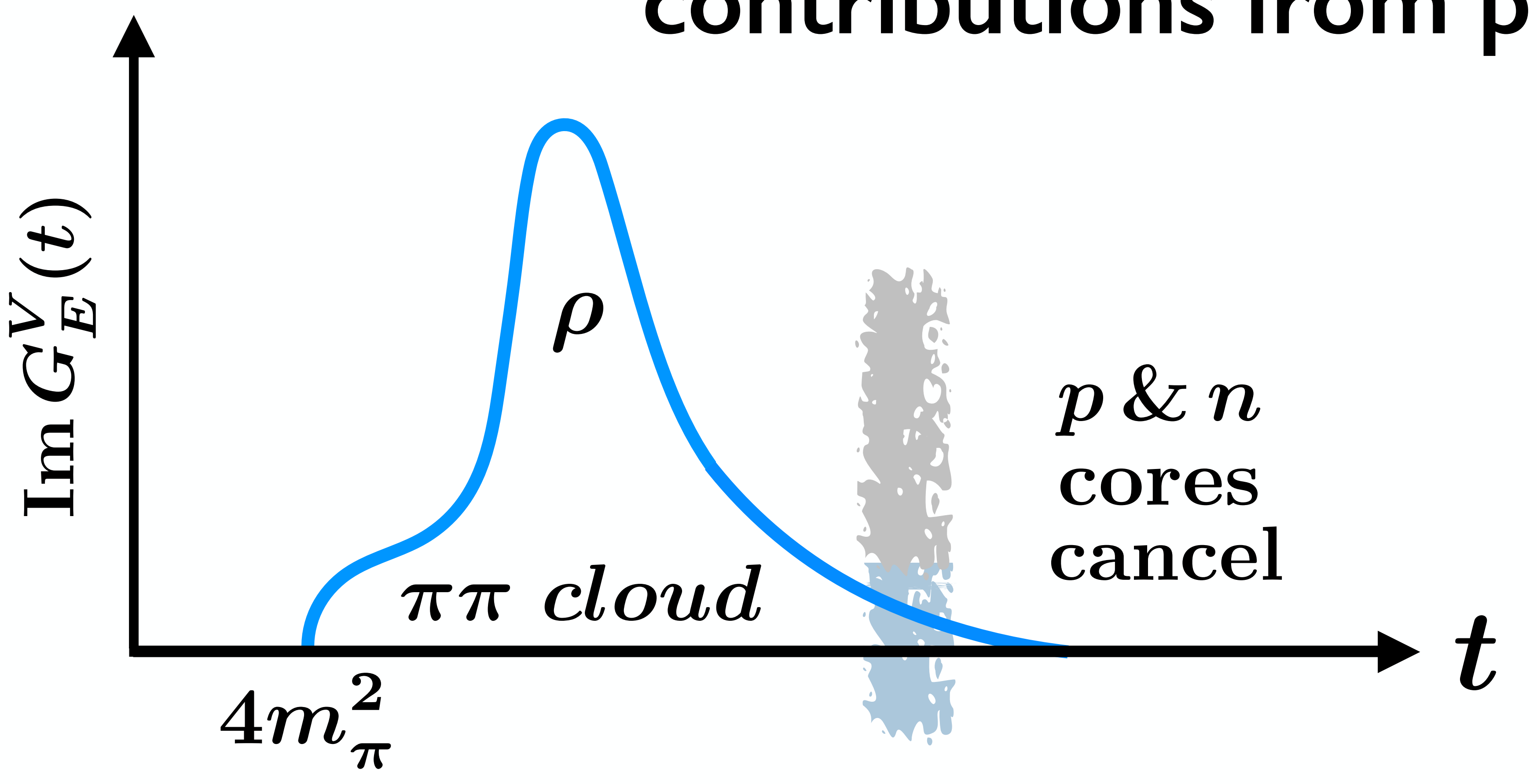
N. Kaiser, W.W. Phys. Rev. C110 (2024) 015202

Example II: **ISOVECTOR ELECTRIC FORM FACTOR** of the **NUCLEON**

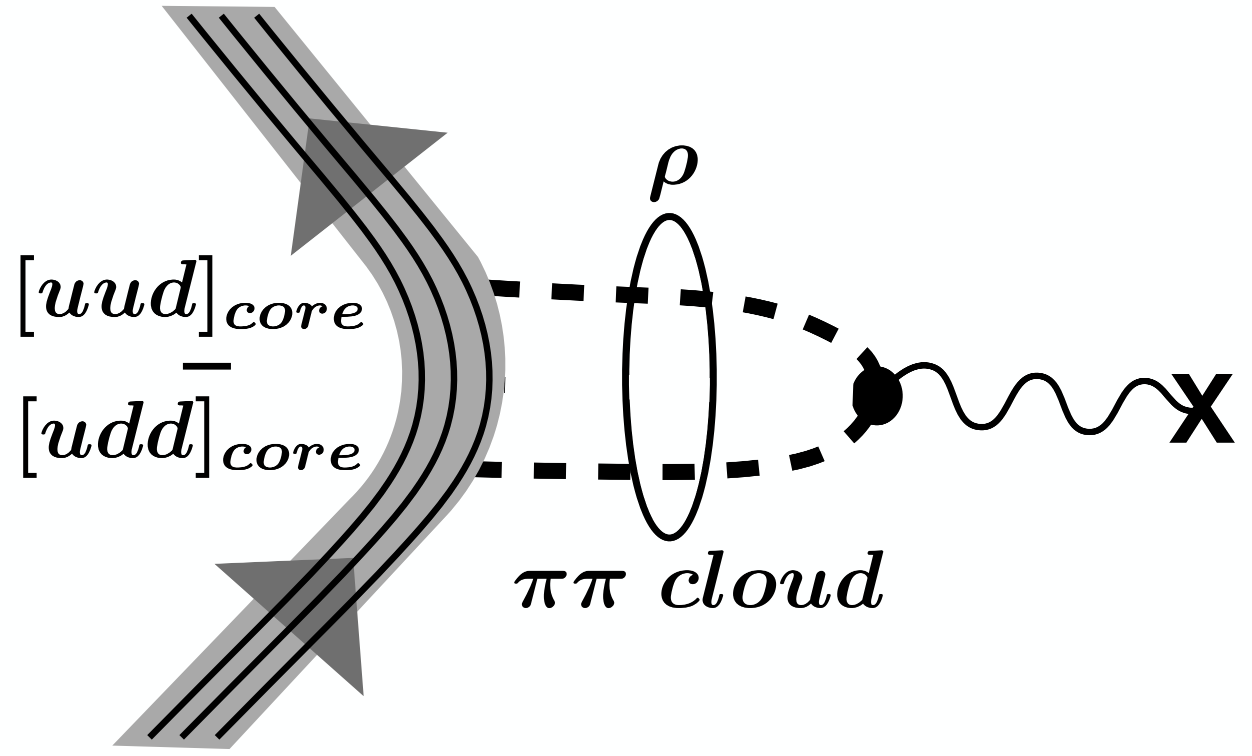
- Isovector electric form factor $G_E^V(q^2) = \frac{1}{2}\left[G_E^p(q^2) - G_E^n(q^2)\right]$ $\quad \langle r_V^2 \rangle = \langle r_p^2 \rangle - \langle r_n^2 \rangle$

Empirical : $\langle r_V^2 \rangle^{1/2} = 0.901 \pm 0.009$ fm Y.H. Lin, H.-W. Hammer, U.-G. Meißner PRL 128 (2022) 052002

… clue and test case : in the limit of exact isospin symmetry,
contributions from proton and neutron valence quark cores **CANCEL**



- Detailed analysis using best-fit spectral functions :



$$\langle r_V^2 \rangle_{core} = \langle r_p^2 \rangle_{core} - \langle r_n^2 \rangle_{core} = -0.025 \text{ fm}^2$$

… almost vanishing

N. Kaiser, W.W. Phys. Rev. C110 (2024) 015202

- **Isovector charge radius** almost entirely determined by **two-pion cloud**

Example III: **ISOVECTOR AXIAL FORM FACTOR** of the **NUCLEON**

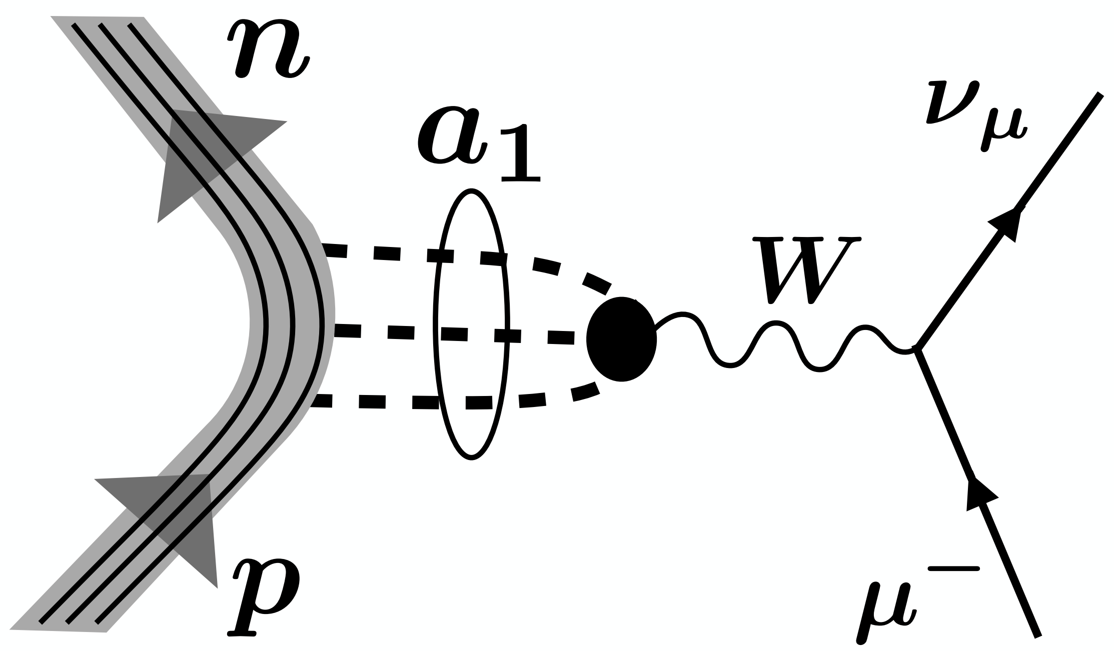
- **Axial form factor** $G_A(q^2) = g_A\left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \dots\right]$

R.J. Hill, P. Kammel, W.C. Marciano, A. Sirlin
Rep. Prog. Phys. 81 (2018) 096301

Empirical :

a) $\langle r_A^2\rangle = 0.454 \pm 0.013\ \mathrm{fm}^2$

(from νd scattering and $e\,p \to e\,n\pi^+$ dipole fits)



b) $\langle r_A^2\rangle = 0.46 \pm 0.16\ \mathrm{fm}^2$

(from μp capture and νd scattering analysis)

Axial radius significantly smaller than proton charge radius $\left(\langle r_p^2\rangle = 0.71 \pm 0.01\,\mathrm{fm}^2\right)$

- **Detailed analysis using three-pion spectrum dominated by broad a_1 meson :**

$$\langle r_A^2\rangle = \langle r_A^2\rangle_{core} + \frac{6}{m_a^2}(1+\delta_a) \qquad \delta_a = -\frac{m_a^3}{\pi}\int_{9m_\pi^2}^{t_{max}} dt \frac{\Gamma_a(t)}{t^2(t-m_a^2)}$$

$$\langle r_A^2\rangle_{core}^{1/2} = 0.53 \pm 0.02\ \mathrm{fm}$$

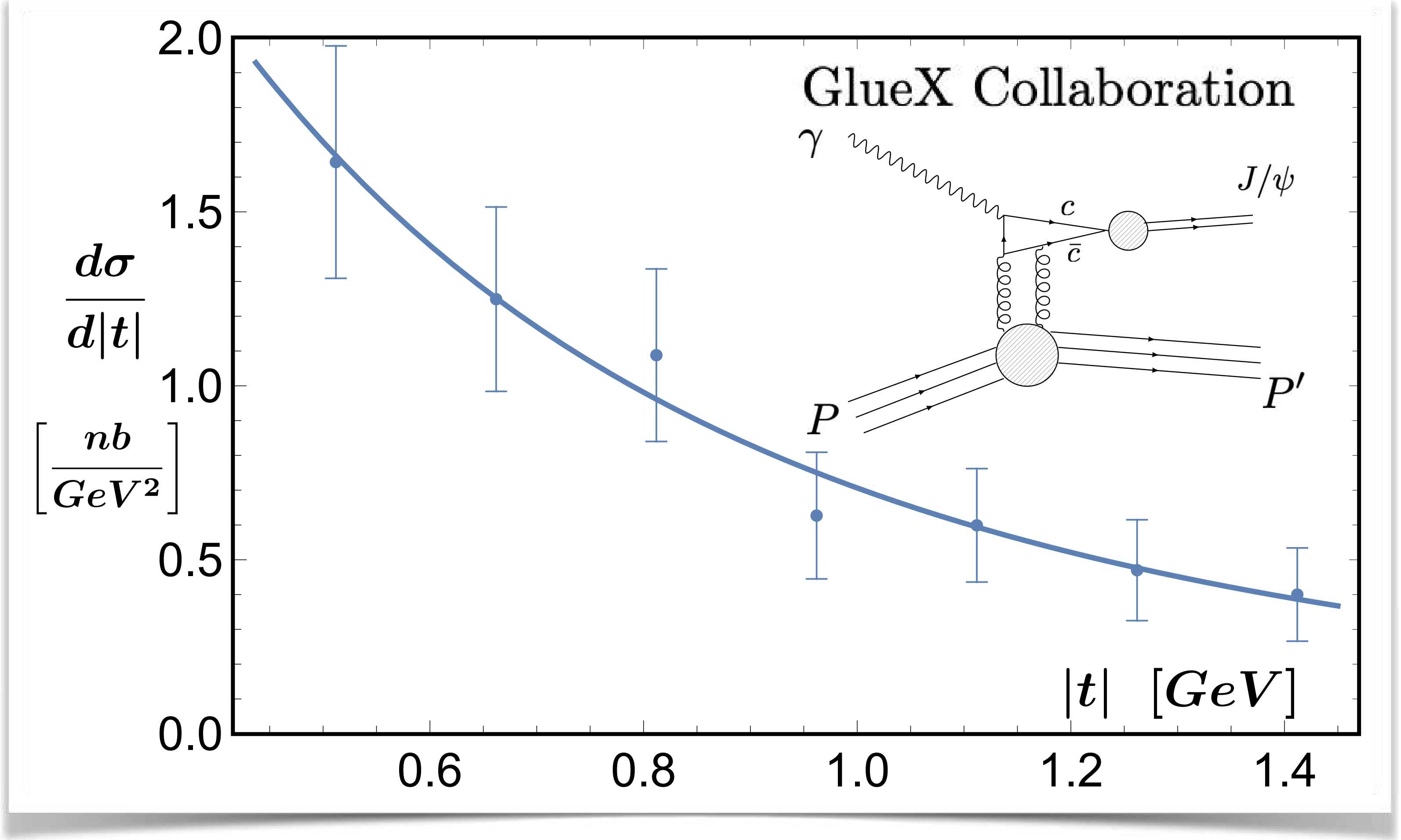
N. Kaiser, W.W.
Phys. Rev. C110 (2024) 015202

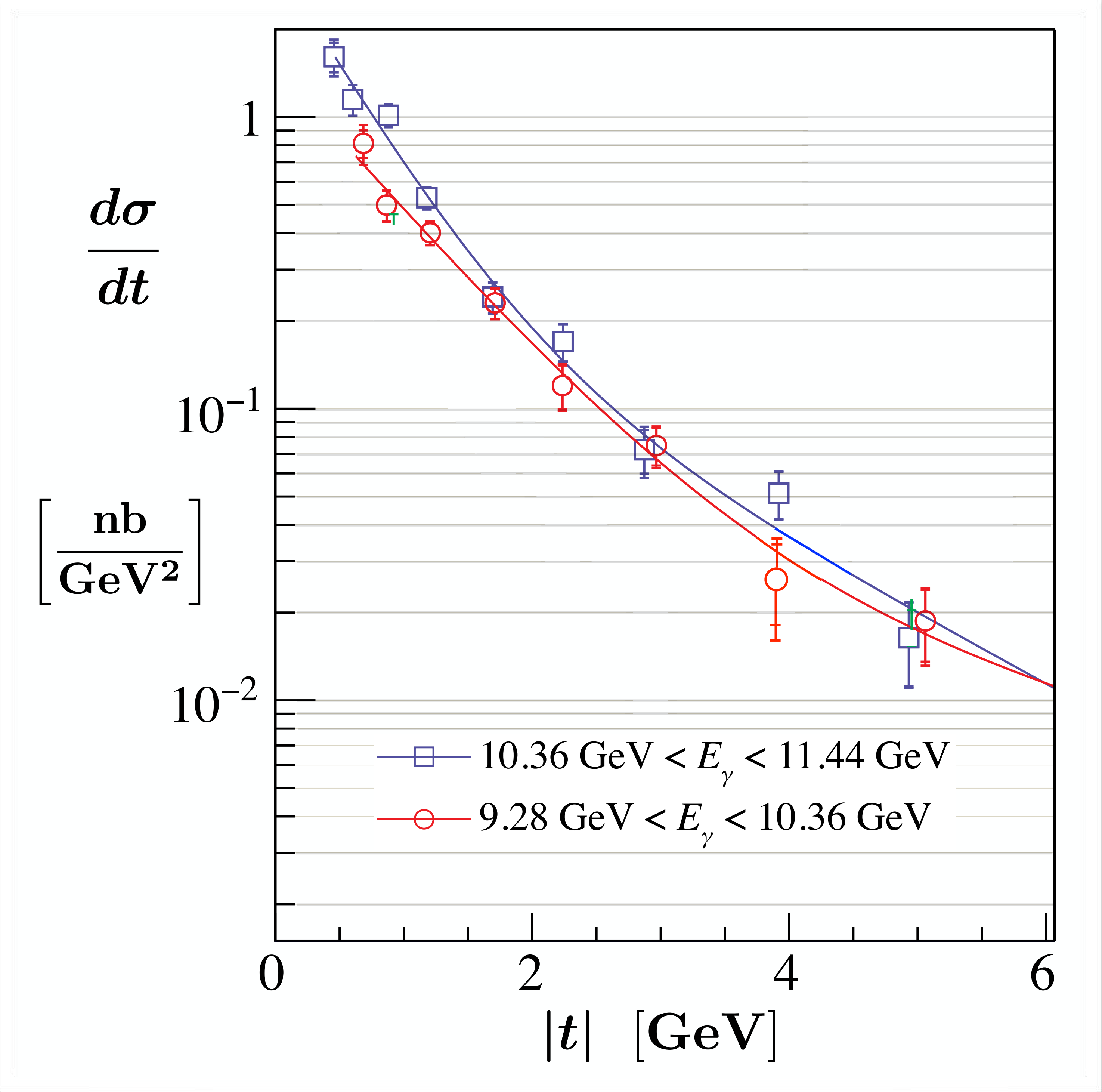
(based on a) — correspondingly larger uncertainty when using b))

Example IV: MASS RADIUS of the NUCLEON

- Mass ("gravitational") form factor
- Trace of QCD energy-momentum tensor

$$G_m(q^2) = \langle P'|T^\mu_\mu|P\rangle = \langle P'|\frac{\beta(g)}{2g}G^{\mu\nu}_a G^a_{\mu\nu} + m_q(\bar{u}u + \bar{d}d) + m_s\bar{s}s|P\rangle$$





$$G_m(0) = M_N \simeq 0.94\,\text{GeV}$$

$$M_N = M_0 + \sigma_N + \sigma_s$$

$$(M_0 \gtrsim 0.9\,M_N)$$

$$\langle r_m^2\rangle = \frac{6}{M_N}\frac{dG_m(q^2)}{dq^2}\Big|_{q^2=0}$$

$$\langle r_m^2\rangle^{1/2} = (0.53 \pm 0.04)\,\text{fm}$$

- Empirical mass radius

$$\langle r_m^2\rangle^{1/2} = (0.55 \pm 0.03)\,\text{fm}$$

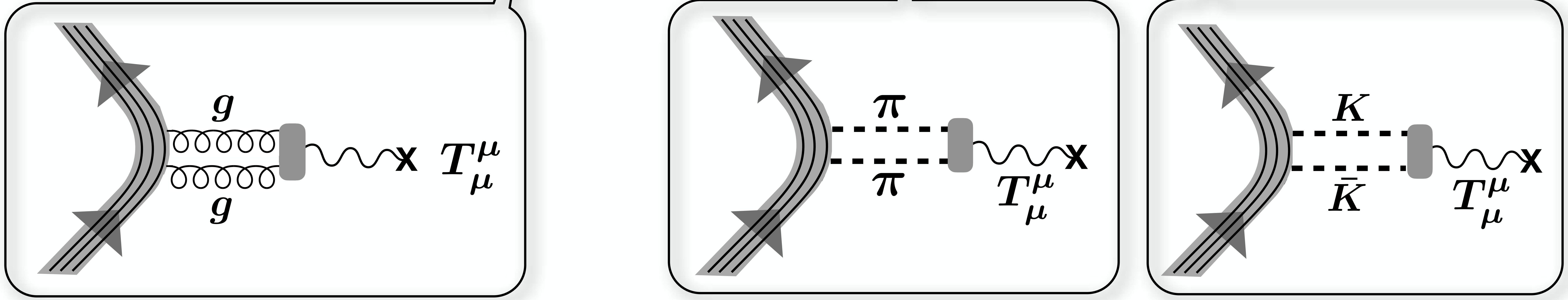
D. Kharzeev : Phys. Rev. D104 (2021) 054015

Recent GlueX update: S. Adhikari et al.; arXiv:2304.03845

Example IV: MASS RADIUS of the NUCLEON (contd.)

- Core (gluon) dominance plus small corrections from sigma terms

$$\langle r_m^2 \rangle = \left[\frac{M_0}{M_N} \langle r_m^2 \rangle_{core} + \frac{\sigma_N}{M_N} \langle r_{\pi\pi}^2 \rangle + \frac{\sigma_s}{M_N} \langle r_{K\bar{K}}^2 \rangle \right]$$



- Estimates of sigma terms and associated radii from Lattice QCD and ChPT

$\sigma_N \simeq 40 - 60\ \mathrm{MeV}\ ,\ \sigma_s \simeq 30\ \mathrm{MeV}$ $\quad \langle r_{\pi\pi}^2 \rangle^{1/2} \simeq 1.3\ \mathrm{fm}\ ,\ \langle r_{K\bar{K}}^2 \rangle \sim (m_\pi/m_K)^2 \langle r_{\pi\pi}^2 \rangle$

→ $$\langle r_m^2 \rangle_{core}^{1/2} = 0.48 \pm 0.05\ \mathrm{fm}$$

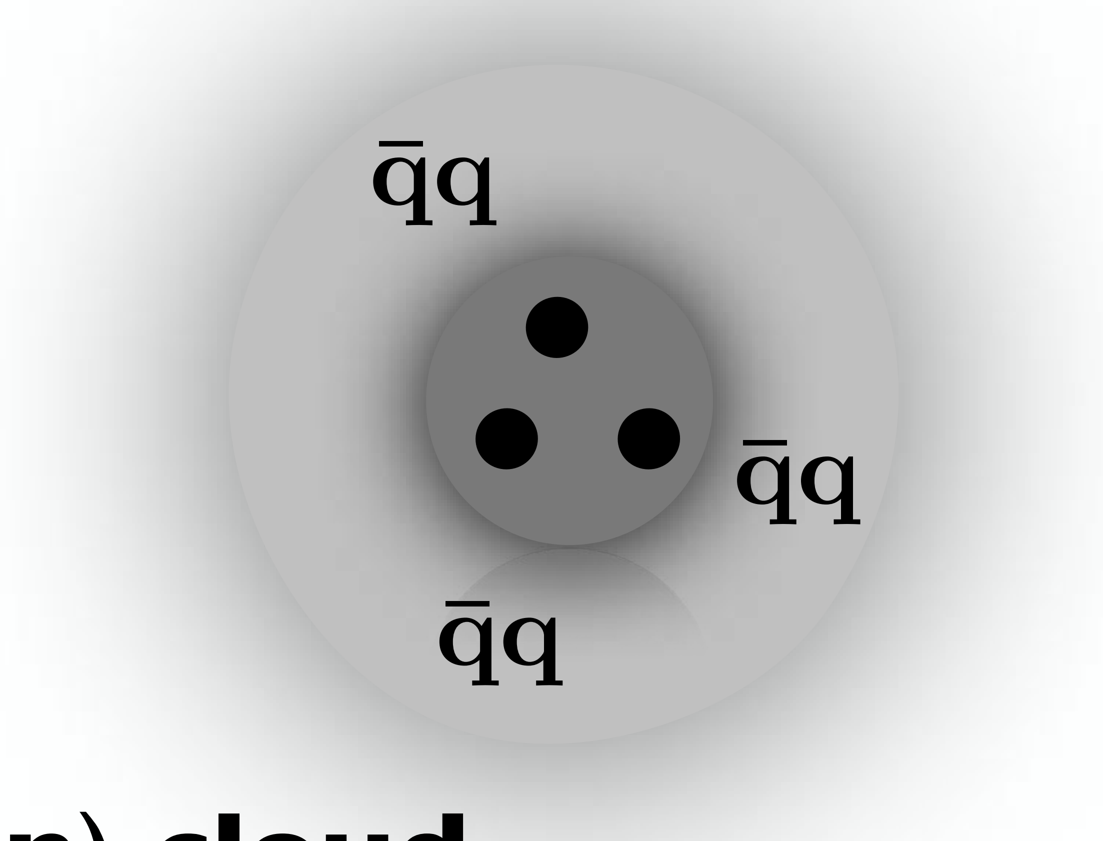
N. Kaiser, W.W.
Phys. Rev. C110 (2024) 015202

TWO-SCALES Picture of the NUCLEON :
Implications for **DENSE BARYONIC MATTER**

$$\langle r_S^2 \rangle_{core}^{1/2} \simeq \langle r_A^2 \rangle_{core}^{1/2} \simeq \langle r_m^2 \rangle_{core}^{1/2} \equiv R_{core} \simeq \frac{1}{2}\ \mathrm{fm}$$

$$R_{core} \sim \frac{1}{2}\ \mathrm{fm}$$

$$R_{cloud} \sim 1\ \mathrm{fm}$$



- **Separation of scales**

$$\left(\frac{R_{cloud}}{R_{core}}\right)^3 \gg 1$$

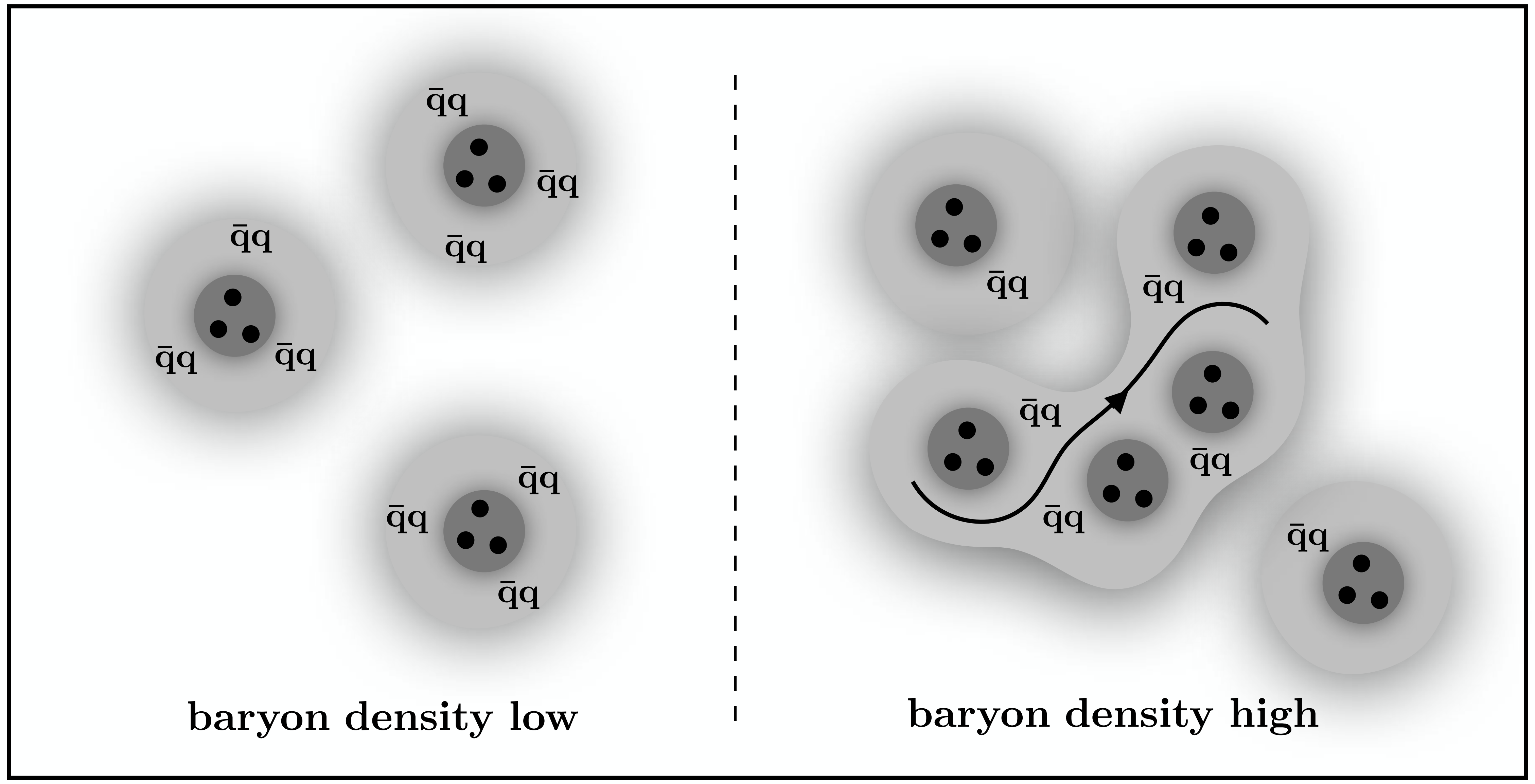
- **Soft mesonic (multi-pion) cloud**

  expected to expand with increasing baryon density along with decreasing **in-medium pion decay constant** $f_\pi^*(n_B)$

- **Hard baryonic core** governed by **gluon dynamics**

  expected to remain stable with increasing baryon density up until hard compact cores begin to touch and overlap

TWO-SCALES Scenario for DENSE BARYONIC MATTER



- Baryon densities

  $n_B \sim n_0 = 0.16\,\mathrm{fm}^{-3}$

  tails of mesonic clouds overlap :
  two-body exchange forces
  between nucleons

- $n_B \gtrsim 2 - 3\,n_0$

  Soft $\bar{q}q$ clouds delocalize:
  percolation ➜ many-body forces
  baryonic cores still separated, but subject to increasingly strong repulsive Pauli effects

- $n_B > 5\,n_0$ (beyond central densities of neutron stars)

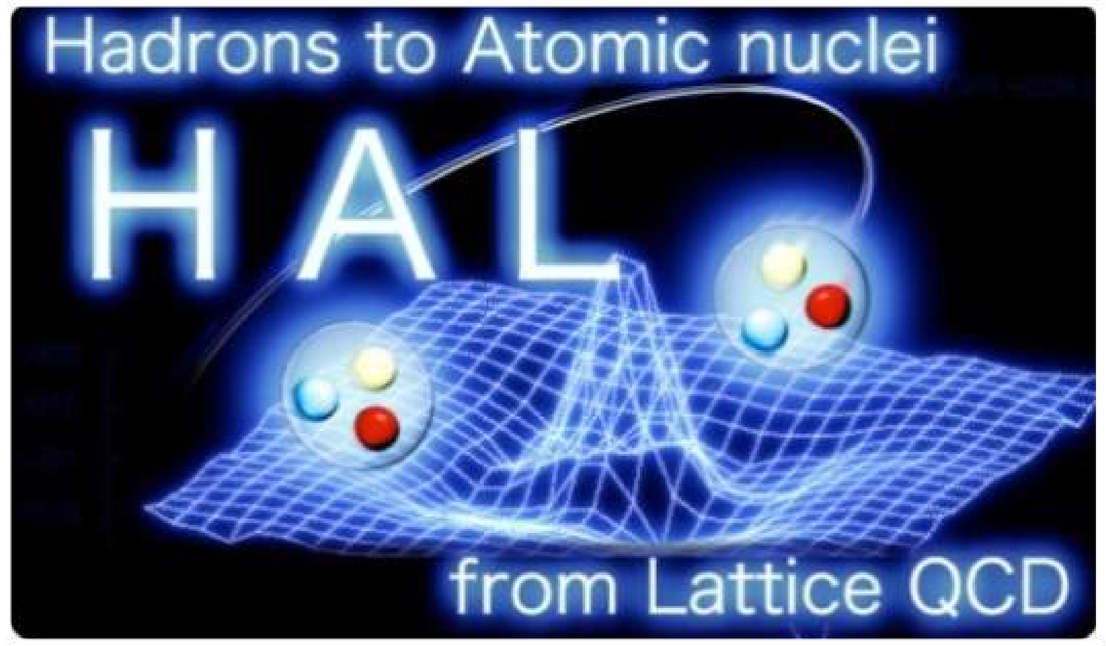
  K. Fukushima, T. Kojo, W.W.
  Phys. Rev. D 102 (2020) 096017

  compact nucleon cores begin to touch and overlap at distances $d \lesssim 1\,\mathrm{fm}$
  (but still have to overcome repulsive NN hard core)

Key words: **hadron-quark continuity** and **crossover**

NUCLEAR FORCES from LATTICE QCD

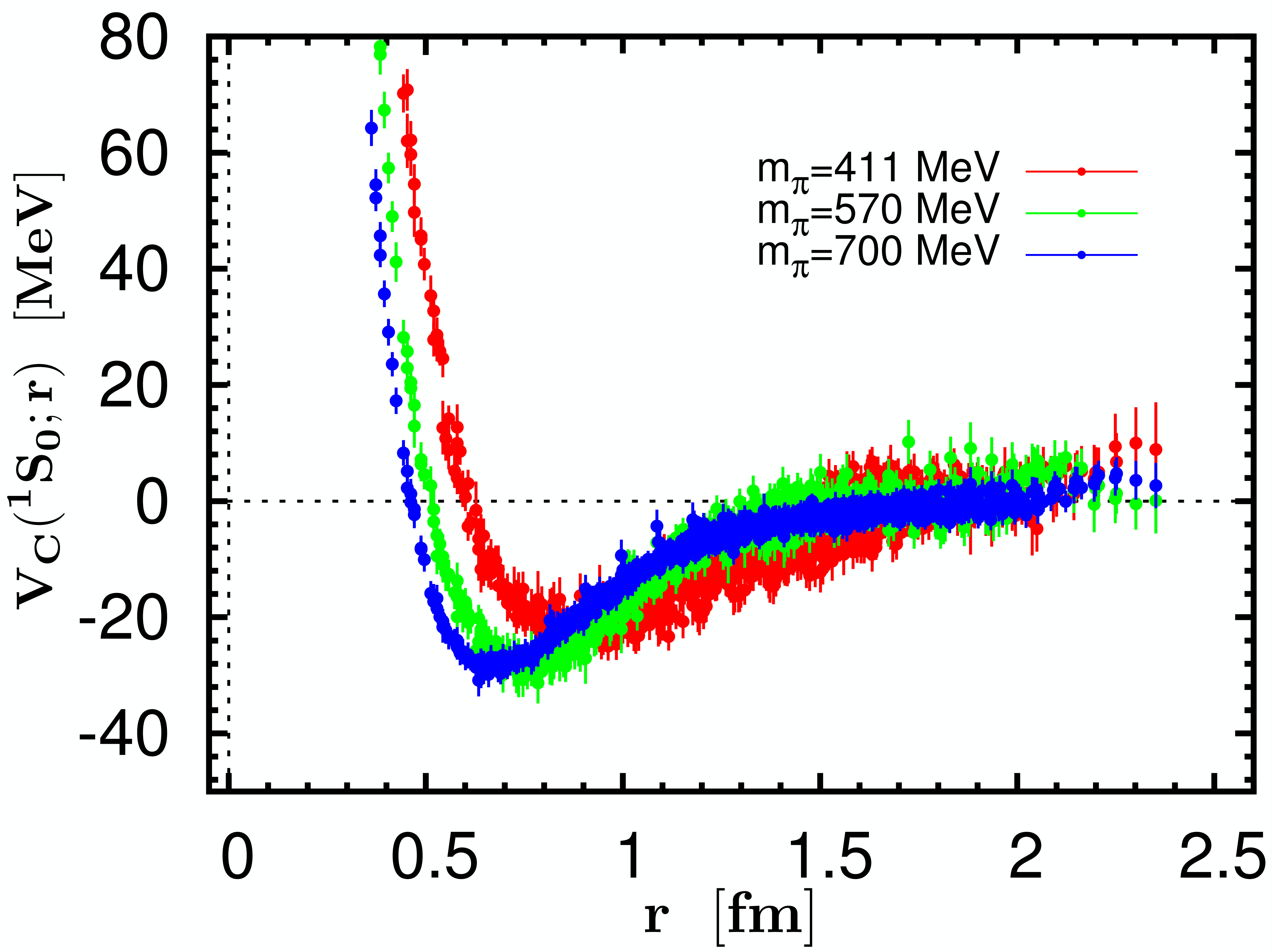
NN Central Potential (S = 0, I = 1)
deduced from LQCD **two-nucleon (6-quark) correlation function**



S. Aoki, T. Hatsuda, N. Ishii
Prog. Theor. Phys. 123 (2010) 89

S. Aoki
Eur. Phys. J. A49 (2013) 81

S. Aoki, T. Doi
arXiv:2402.11759

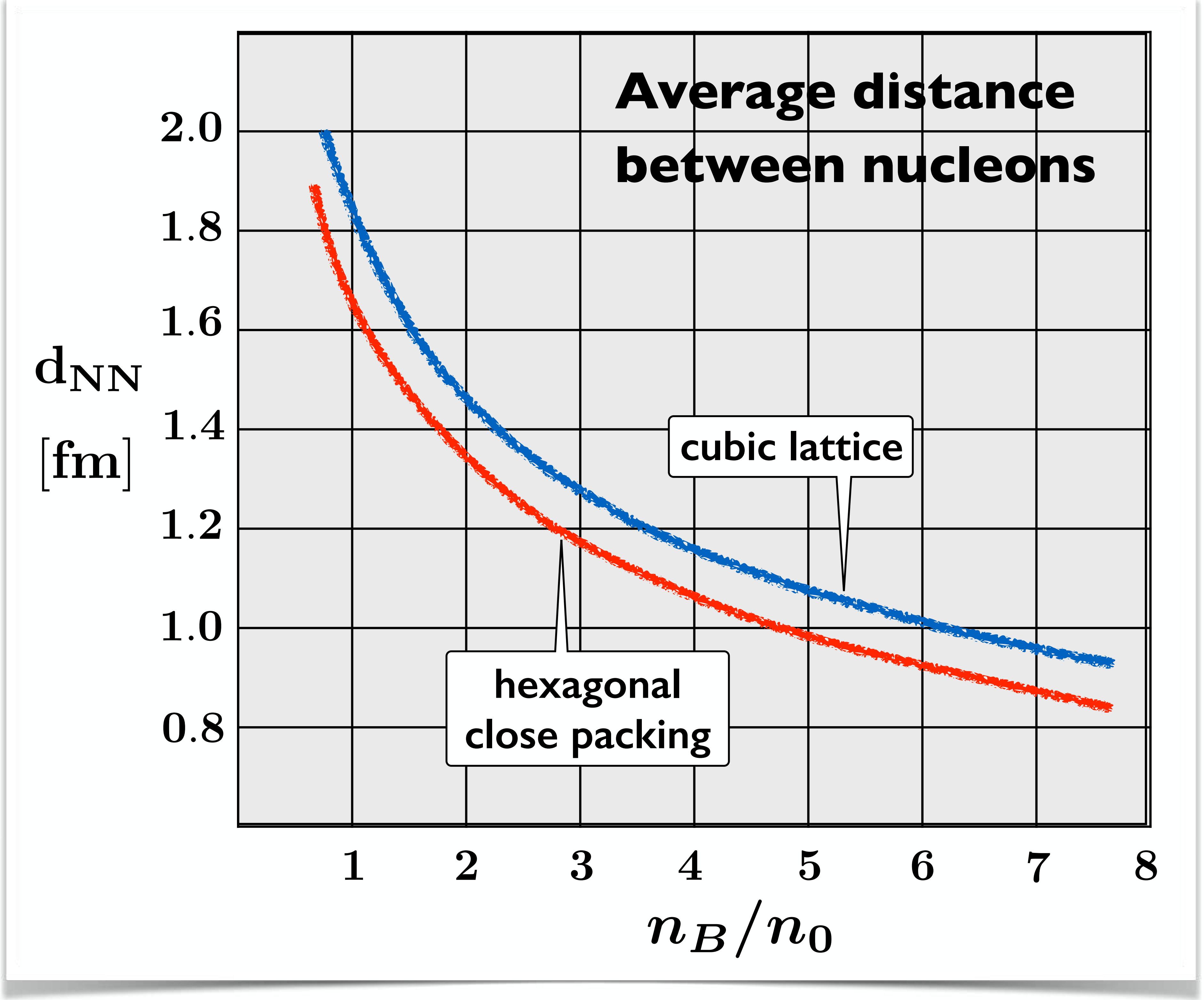


Nuclear Physics Phenomenology:

Short-Range Repulsive Core

- **Compression of baryonic matter is energetically expensive**

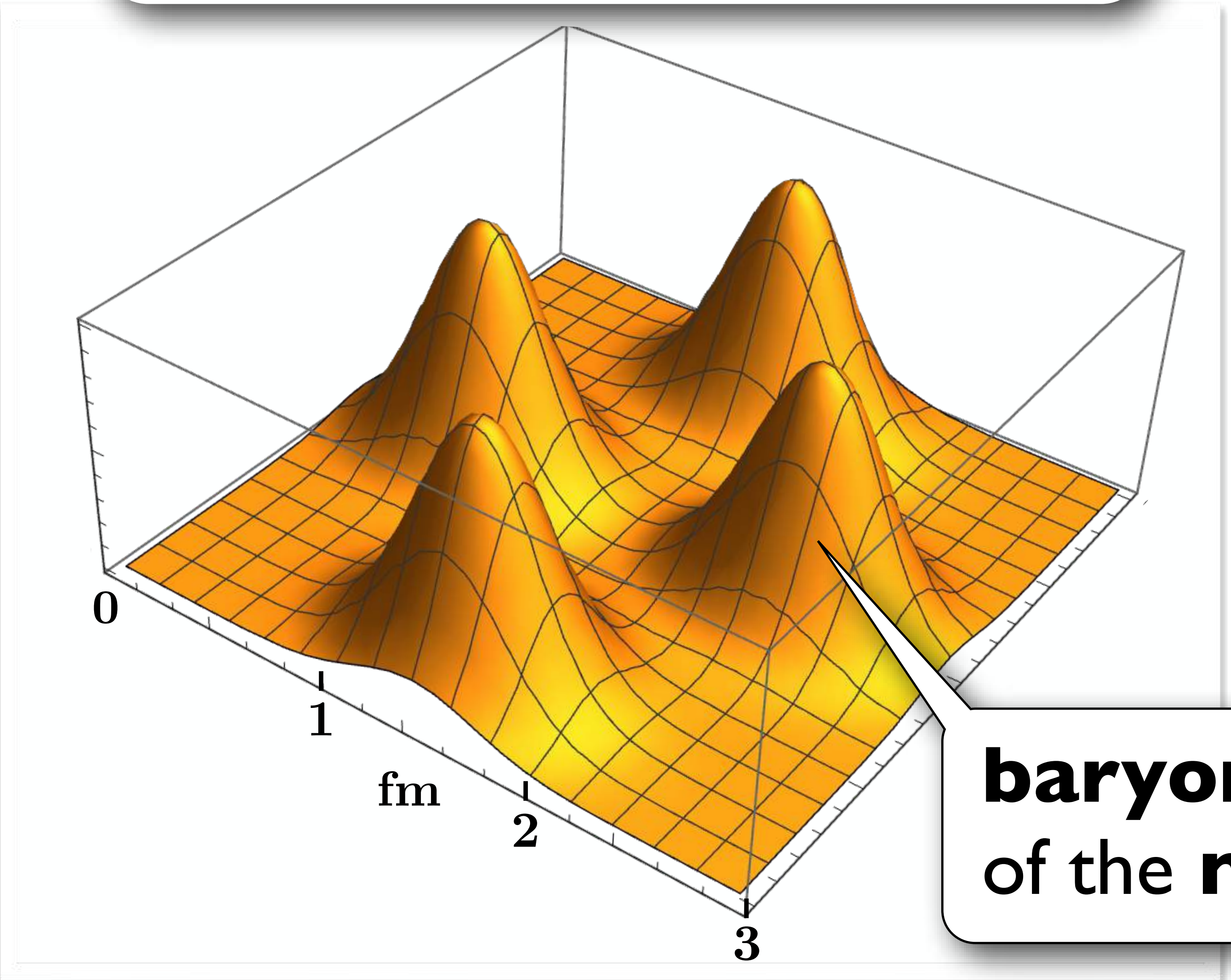
Densities and Distance Scales in Baryonic Matter



normal nuclear matter

$n_B \simeq n_0 = 0.16\,\mathrm{fm}^{-3}$

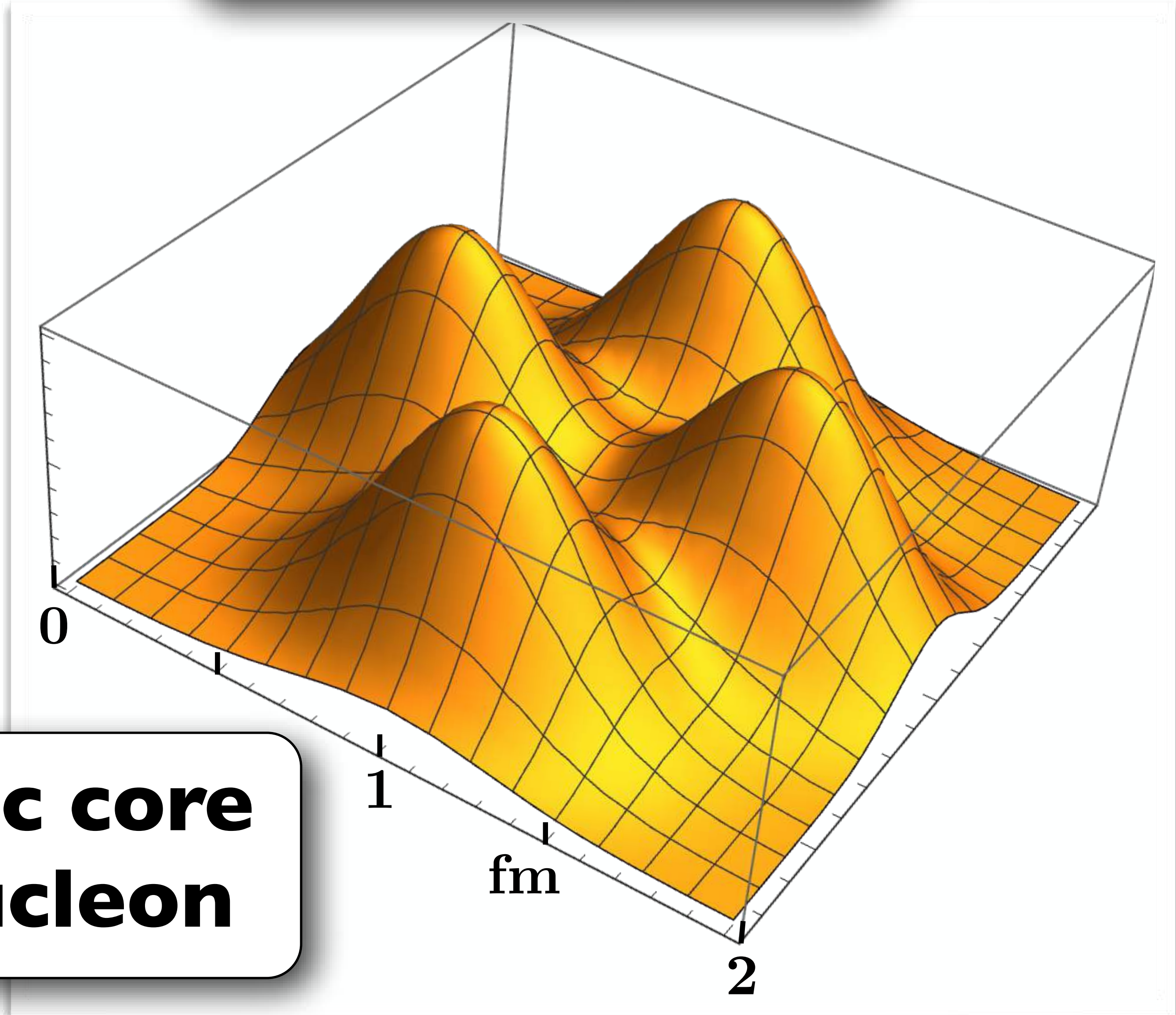
$d_{NN} \simeq 1.7\,\mathrm{fm}$



neutron star core matter

$n_B \sim 4\,n_0$

$d_{NN} \gtrsim 1\,\mathrm{fm}$



- (Multi-) pion fields populate space between baryonic sources
- **Quark cores of nucleons start overlapping at baryon densities** $n_B > 5\,n_0$
- Reminder: Density at **random close packing of spheres** (radius R) : $n_{cp} \simeq 0.16/R^3$

6.

Chiral Effective Field Theory, Nuclear Many-Body Problem and Dense Baryonic Matter

Minimal Conditions to be satisfied by theories of **Dense Baryonic Matter**

Vacuum :

- **Low-energy theorems of spontaneously broken CHIRAL SYMMETRY**
  (Gell-Mann, Oakes, Renner ; Goldberger-Treiman ; ...)
- **Low-energy pion-pion and pion-nucleon scattering**
- **Realistic nucleon-nucleon interaction**

Low density $(n_B \lesssim 1.5\, n_0)$:

- **Realistic EoS of symmetric nuclear matter and neutron matter**
- **Asymmetric nuclear matter and symmetry energy**
- **Nuclear thermodynamics : liquid-gas phase transition**

High density :

- **Neutron star maximum mass** $\mathrm{M}_{max} \gtrsim 2\,\mathrm{M}_\odot$
- **Neutron star radii (NICER)** $11\,\mathrm{km} \lesssim \mathrm{R} \lesssim 13\,\mathrm{km}$
- **Tidal deformability constraints from neutron star mergers (GW signals)**

Physics at low densities and temperatures :
PIONS and **NUCLEI** in the context of **LOW-ENERGY QCD**

- **CONFINEMENT** of quarks and gluons in hadrons
  - Spontaneously broken **CHIRAL SYMMETRY**

↓

LOW-ENERGY QCD

at (energy and momentum) scales $Q < 4\pi\, f_\pi \sim 1\,\mathrm{GeV}$ is realised as an

Effective **F**ield **T**heory

of Nambu-Goldstone Bosons (**PIONS**) coupled to
NUCLEONS as (heavy) Fermion sources

CHIRAL EFFECTIVE FIELD THEORIES and MODELS

- Underlying symmetries and symmetry breaking pattern

$$\mathbf{SU}(2)_L \times \mathbf{SU}(2)_R \to \mathbf{SU}(2)_{Isospin}$$

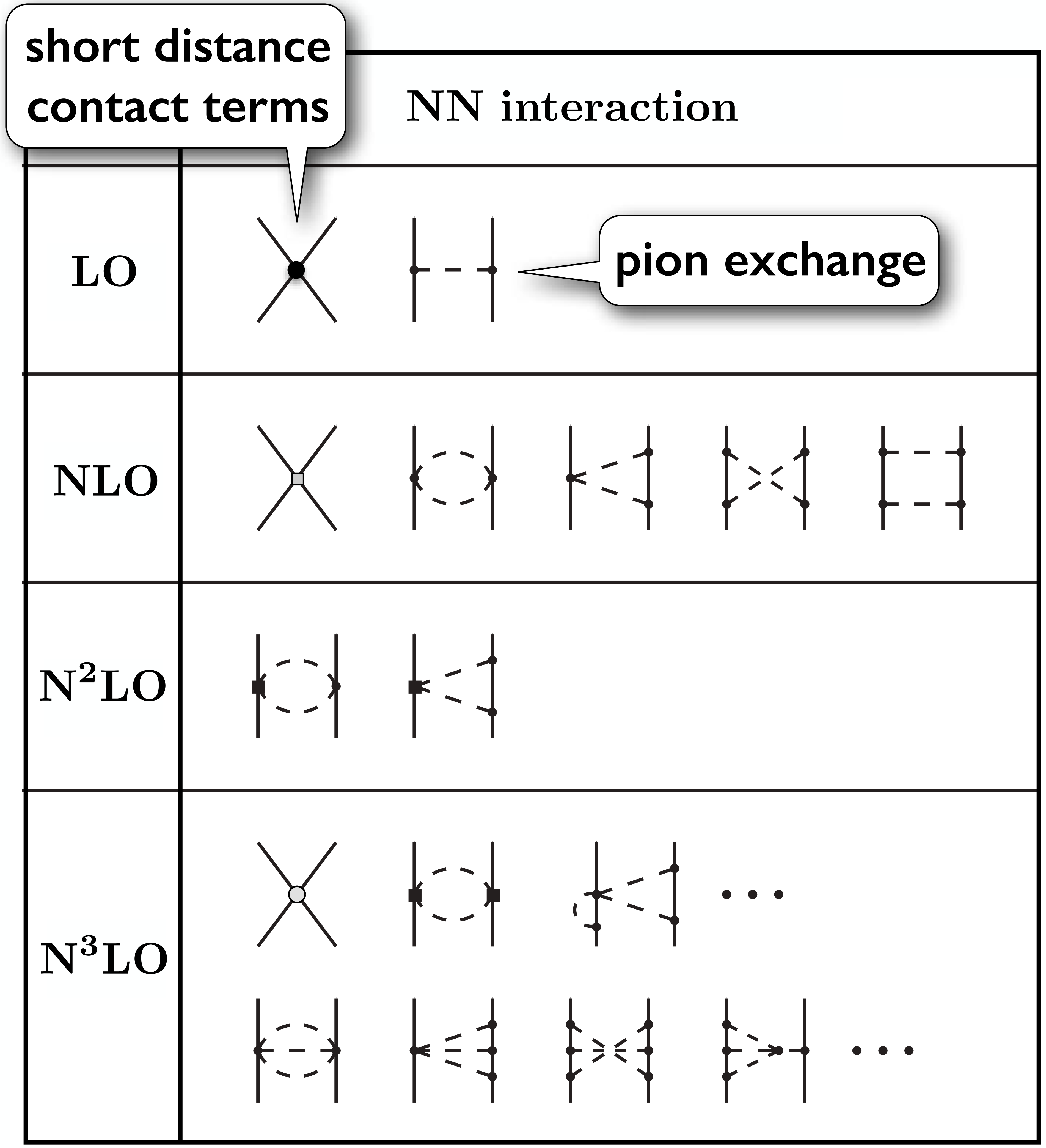
- **Pions** and **Nucleons** as active degrees of freedom in a **Nuclear Fermi Sea**

- Perturbative methods:
  Chiral Effective Field Theory
  and
  Nuclear Many-Body Problem
  (baryon densities $n_B \lesssim 1.5\, n_0$)

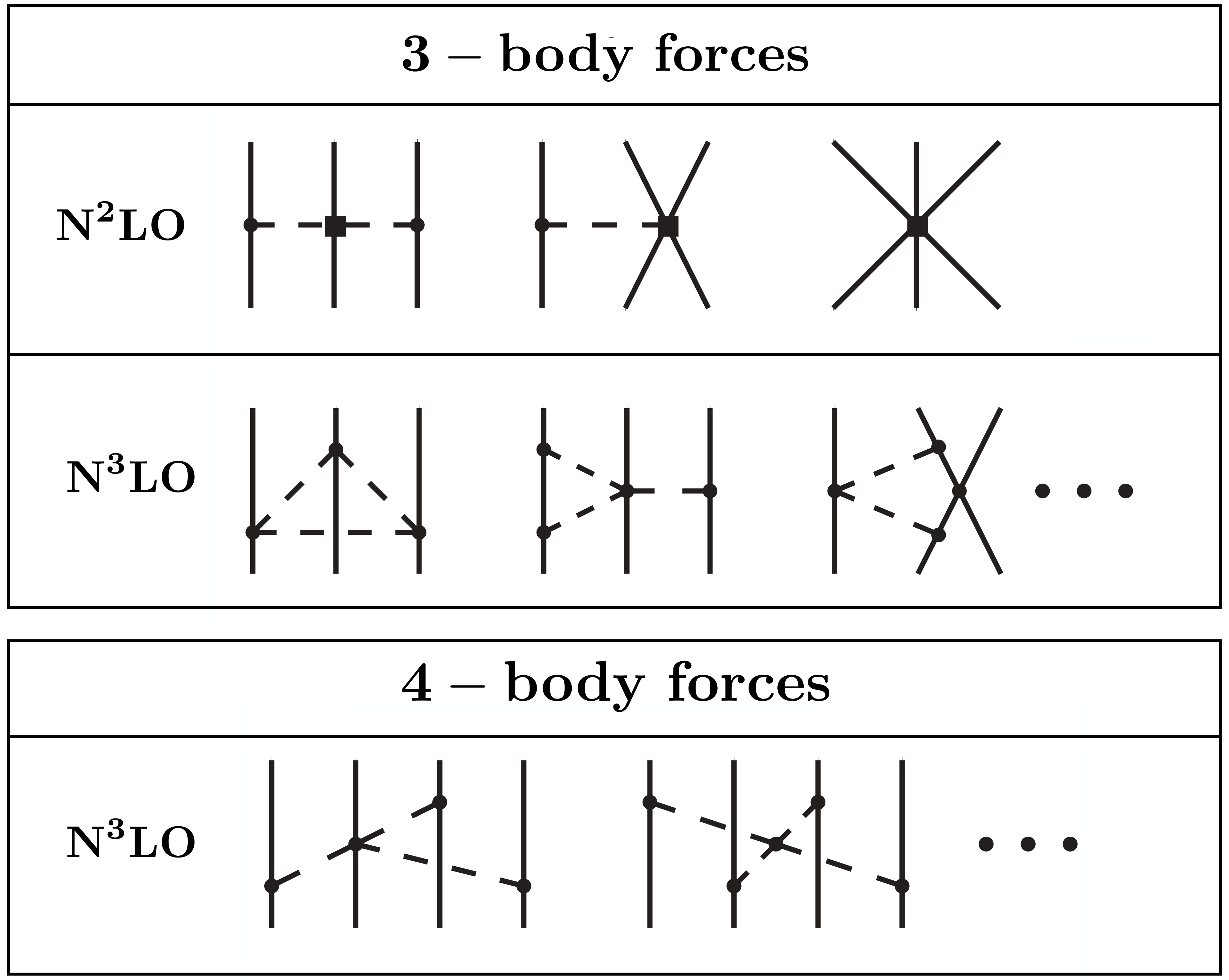
- Non-perturbative methods:
  Chiral Nucleon-Meson Field Theory
  and
  Functional Renormalisation Group
  (towards higher densities : $n_B \lesssim 5\, n_0$)

NUCLEON-NUCLEON INTERACTION from CHIRAL EFFECTIVE FIELD THEORY

Weinberg   Bedaque & van Kolck   Bernard, Epelbaum, Kaiser, Meißner



- **Systematically organized hierarchy in powers of $\frac{Q}{\Lambda}$**
  (Q: momentum, energy, pion mass ; $\Lambda < \Lambda_\chi = 4\pi\, f_\pi$)



- **NN interaction state-of-the-art: N^4LO**
  plus convergence tests at N^5LO

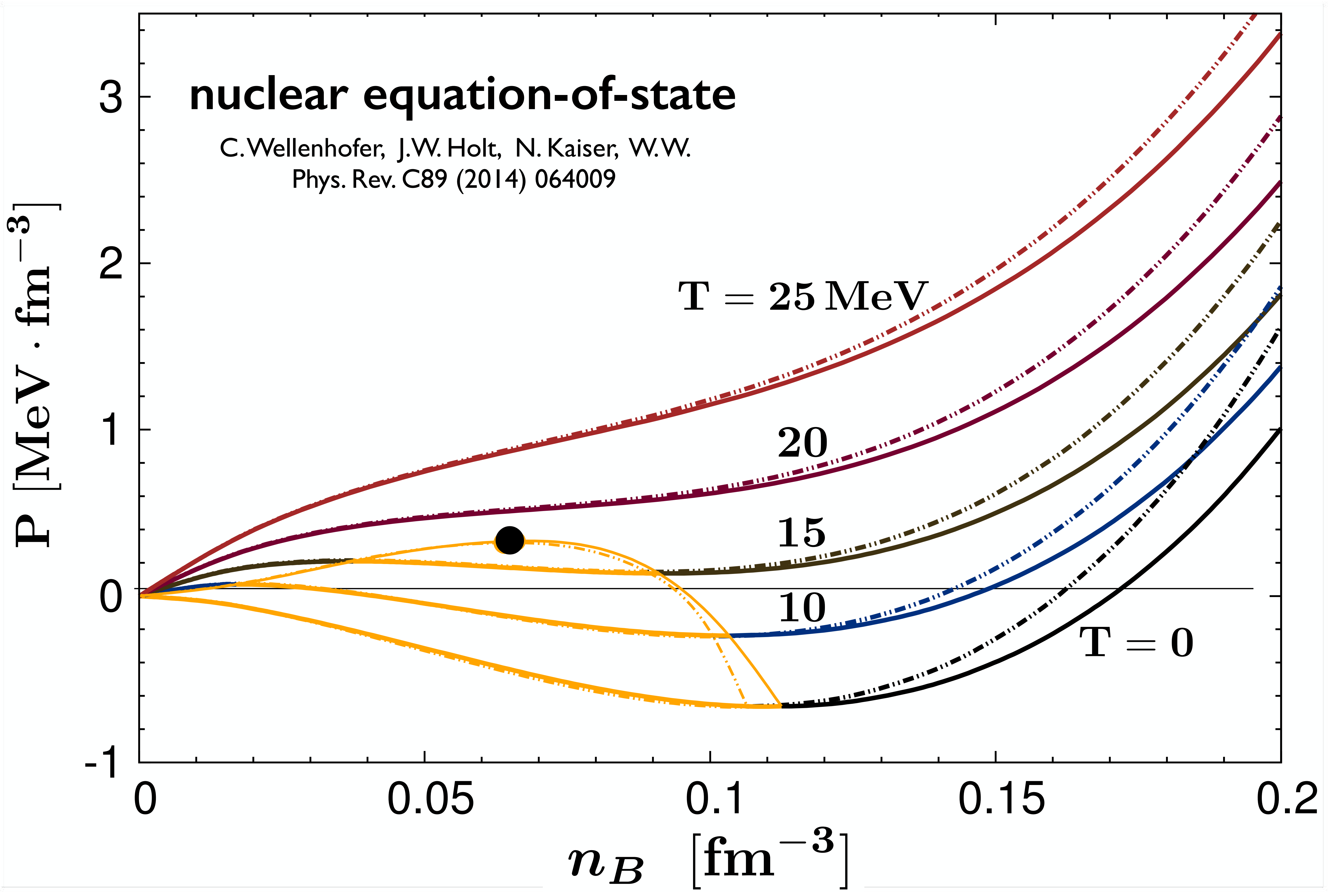
NUCLEAR THERMODYNAMICS from CHIRAL EFT

- Symmetric nuclear matter : first-order liquid-gas phase transition
- **N3LO chiral NN interactions** + **N2LO** 3-body forces
  Finite - T many-body perturbation theory calculations

Critical temperature of liquid-gas first-order transition :

$T_c = 17.4$ MeV

C. Wellenhofer, J.W. Holt, N. Kaiser, Phys. Rev. C92 (2015) 015801



Empirical position of liquid-gas critical point :

$T_c = 17.9 \pm 0.4$ MeV

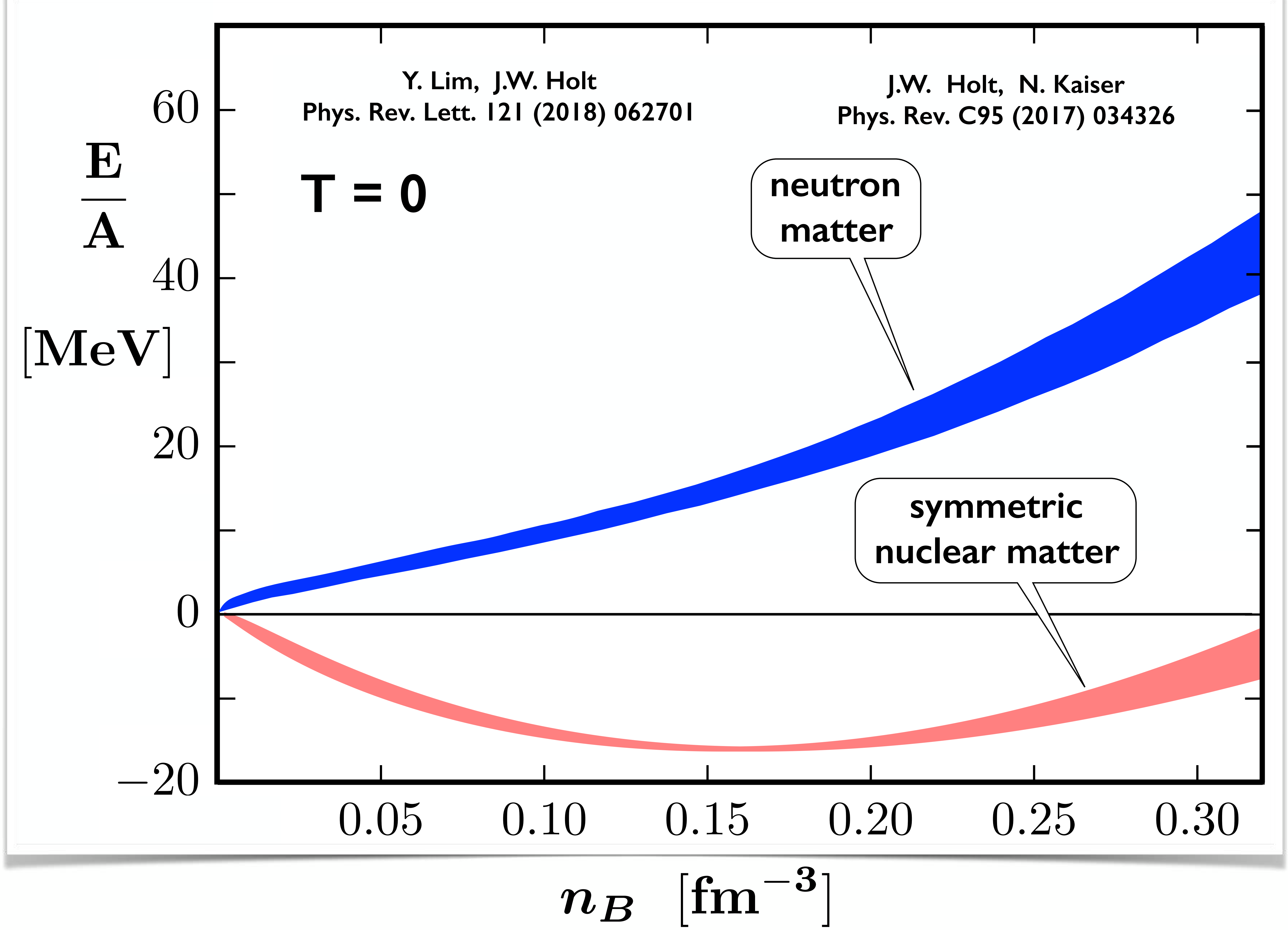
$n_c = 0.06 \pm 0.01\ \mathrm{fm}^{-3}$

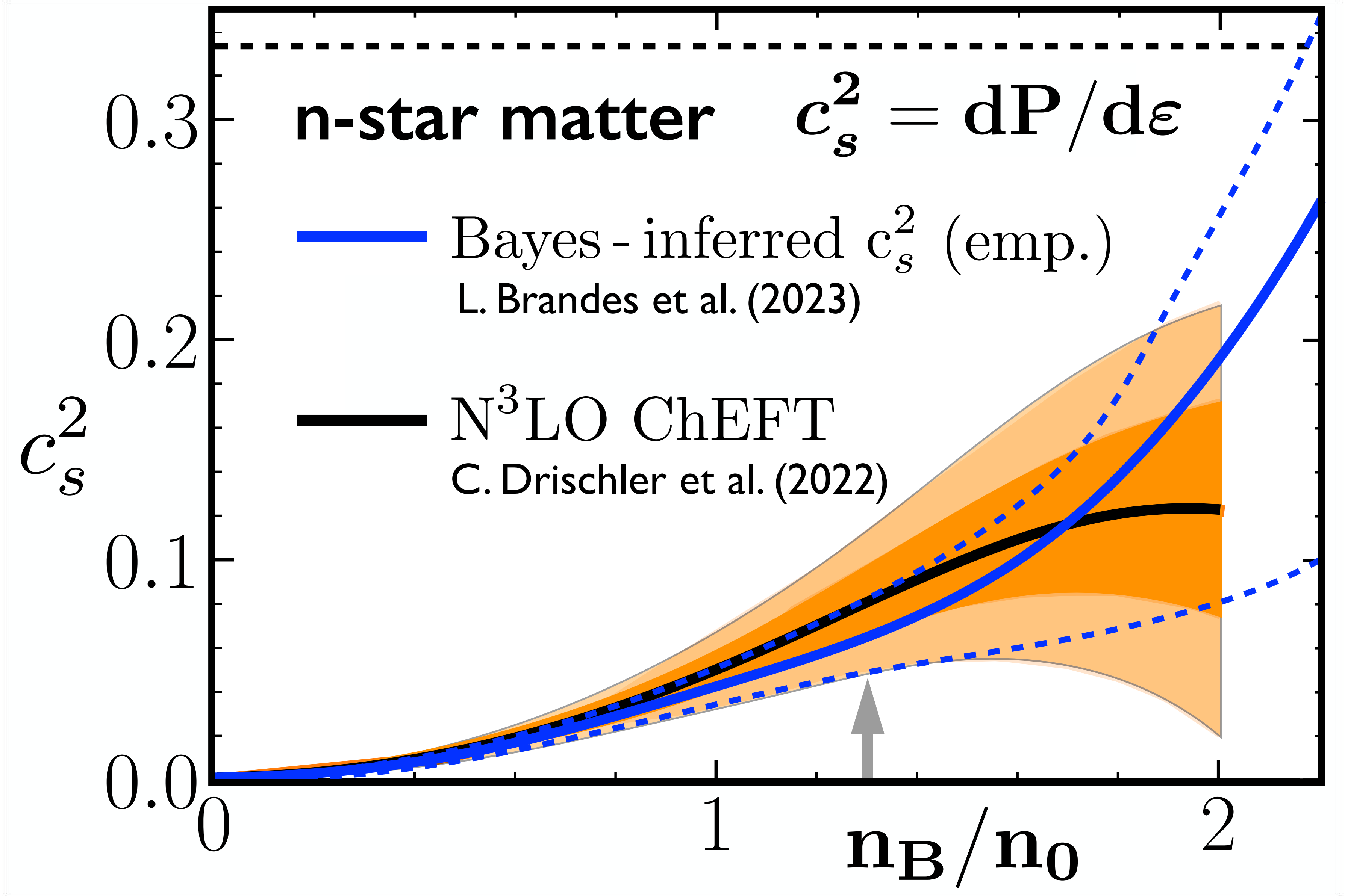
$P_c = 0.31 \pm 0.07\ \frac{\mathrm{MeV}}{\mathrm{fm}^3}$

J. B. Elliot et al. Phys. Rev. C87 (2013) 054622

NEUTRON and NUCLEAR MATTER from CHIRAL EFT

- **N3LO chiral NN interactions** + **N2LO** 3-body forces
- **Many-body perturbation theory** (3rd order)





- **Perturbative Chiral EFT :** applicable up to baryon densities $n_B \lesssim 1.5\, n_0$

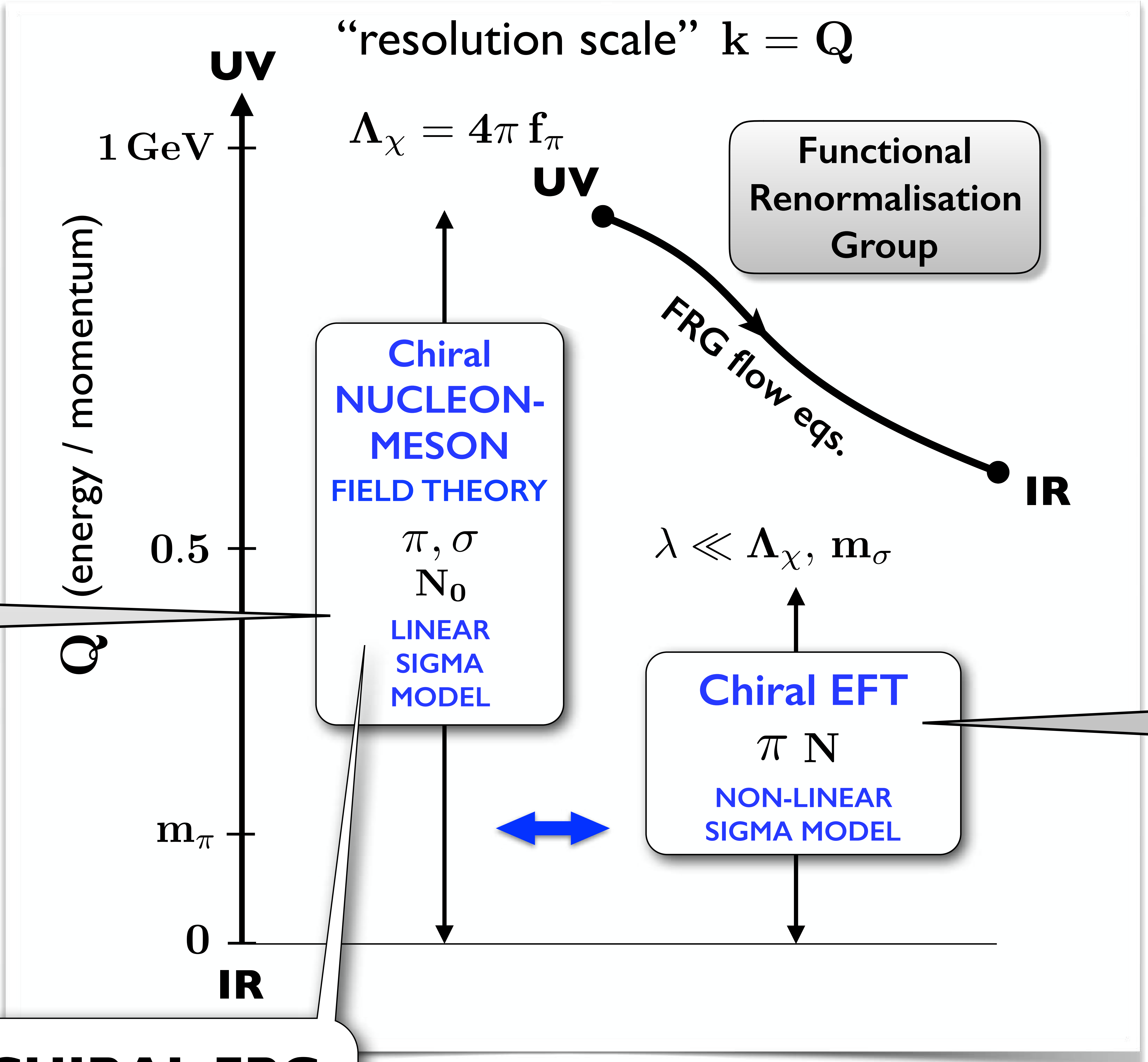
Theoretical FRAMEWORKS and METHODS

- **UV** initialization at :

$$\Lambda_{UV} \gtrsim$$
$$\Lambda_\chi = 4\pi f_\pi \sim 1\,\mathrm{GeV}$$

Non-perturbative methods:
Chiral Nucleon-Meson Field Theory
and
Functional Renormalisation Group
towards high densities

$$n_B \lesssim 5\, n_0$$



CHIRAL FRG

Perturbative methods:
Chiral Effective Field Theory
and
Nuclear Many-Body Problem
baryon densities

$$n_B \lesssim 1.5\, n_0$$

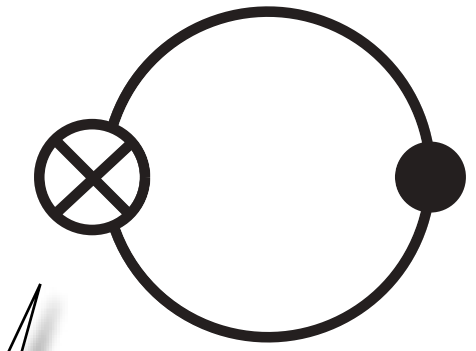
- **IR** limit : $Q \to 0$
full effective action

Renormalisation Group strategies

k-dependent action

Wetterich's FRG flow equations

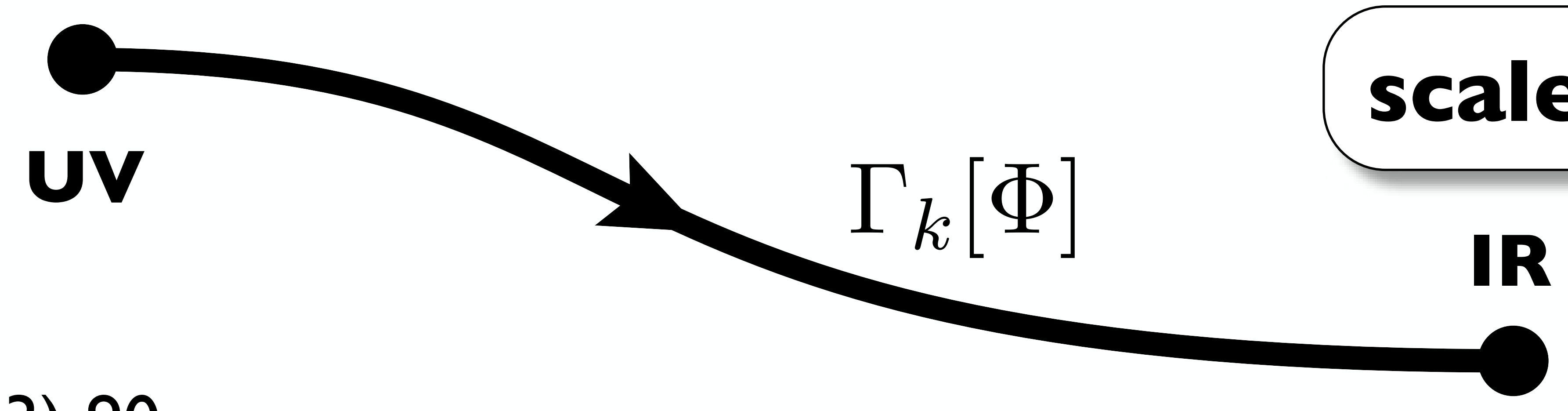
$$k\frac{\partial\Gamma_k[\Phi]}{\partial k} = \frac{1}{2}\mathrm{Tr}\left[k\frac{\partial R_k}{\partial k}\cdot\left(\Gamma_k^{(2)}[\Phi]+R_k\right)^{-1}\right] =$$



full propagator

scale regulator R_k

$\Gamma_{k=\Lambda}[\Phi] = S$

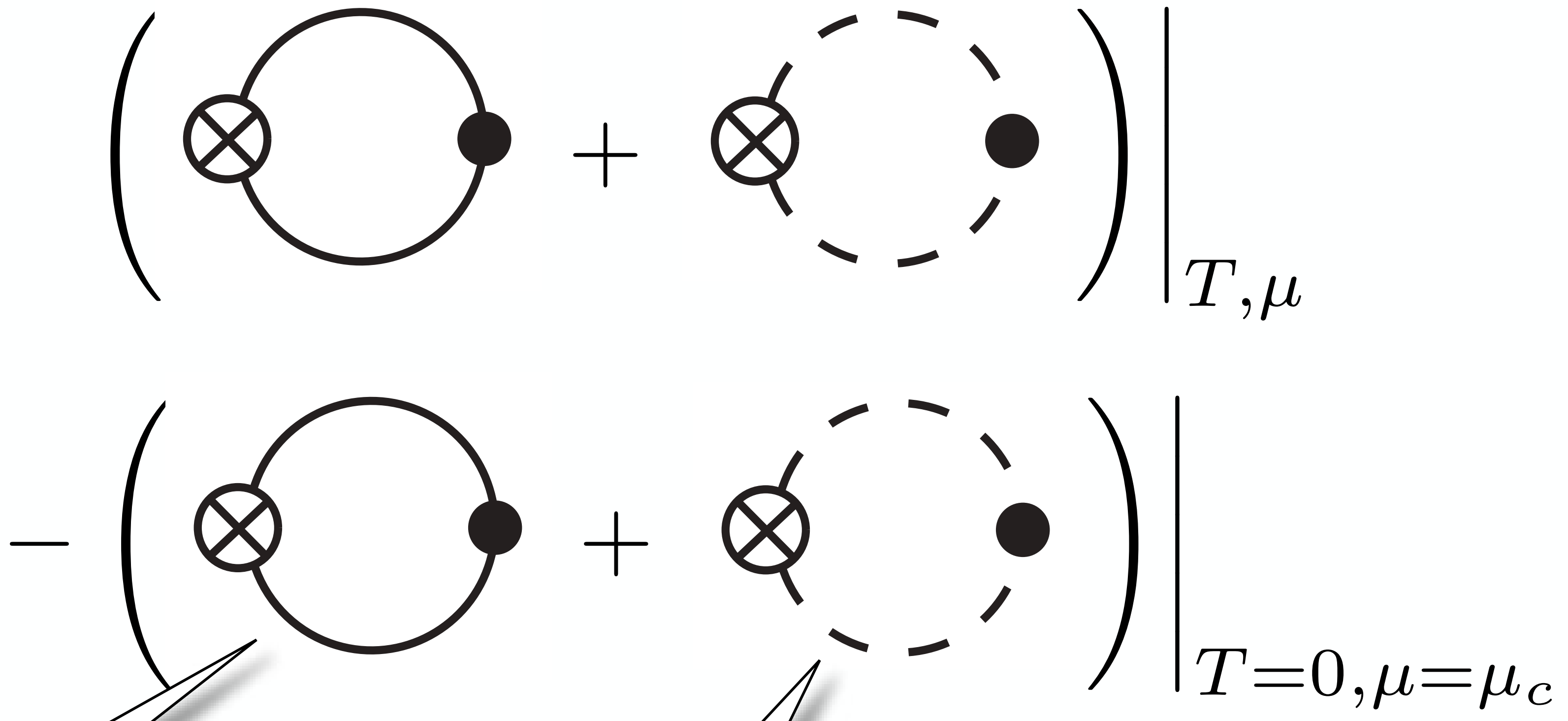


$\Gamma_{k=0}[\Phi] = \Gamma[\Phi]$

C. Wetterich
Phys. Lett. B 301 (1993) 90

- **Thermodynamics** :

$k\,\partial_k\bar{\Gamma}_k(T,\mu) =$



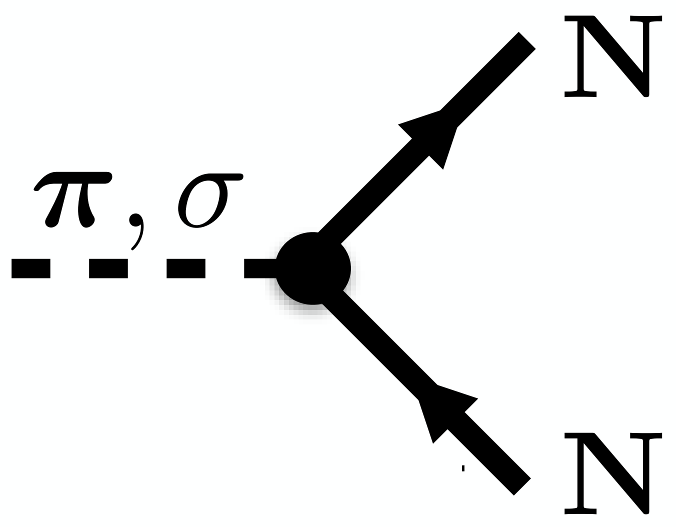
nucleons **pions**

- **multi-pion exchange** processes
- **nucleonic** excitations of Fermi sea
- **multi-nucleon** correlations

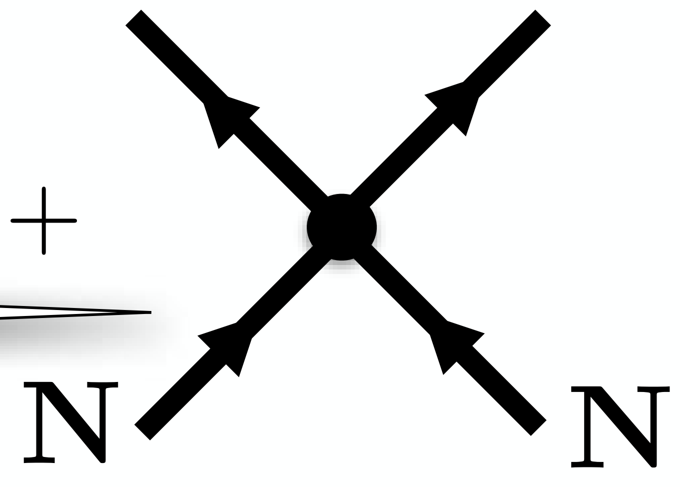
Mesons, Nucleons, Nuclear Matter and Functional Renormalization Group

- **Chiral Nucleon - Meson Lagrangian**

$$\mathcal{L} = \bar{\mathrm{N}}\, i\gamma_\mu \partial^\mu \mathrm{N} + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}\right) +$$

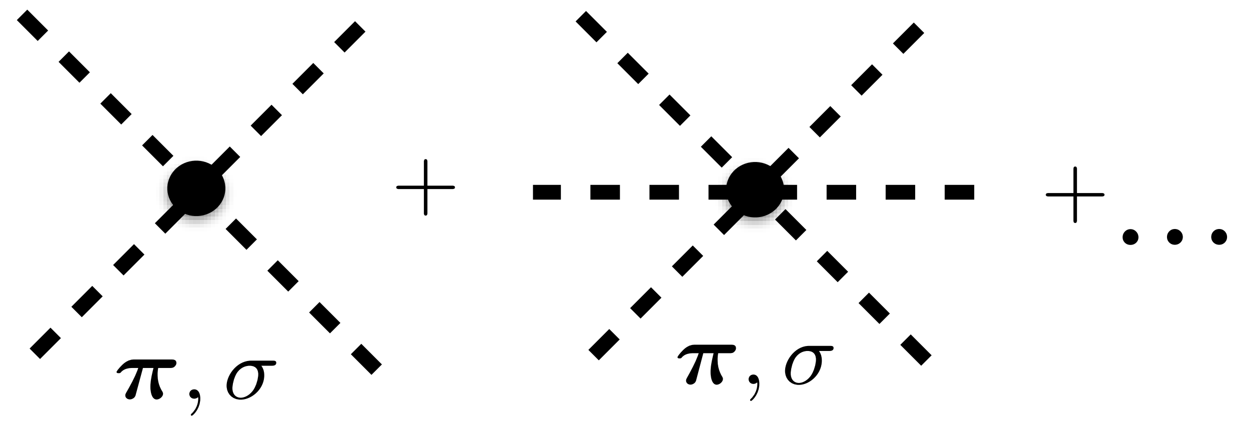


$+$



$- \mathcal{U}(\boldsymbol{\pi}, \sigma)$

isoscalar & isovector current-current interactions



- **Nambu-Goldstone bosons** $\boldsymbol{\pi}$ and "**heavy**" $\boldsymbol{\sigma}$
- **Potential** $\mathcal{U}(\sigma, \boldsymbol{\pi})$: **Polynomial** in $\chi = \boldsymbol{\pi}^2 + \sigma^2$

constructed to reproduce **vacuum physics** and **equilibrium nuclear matter**

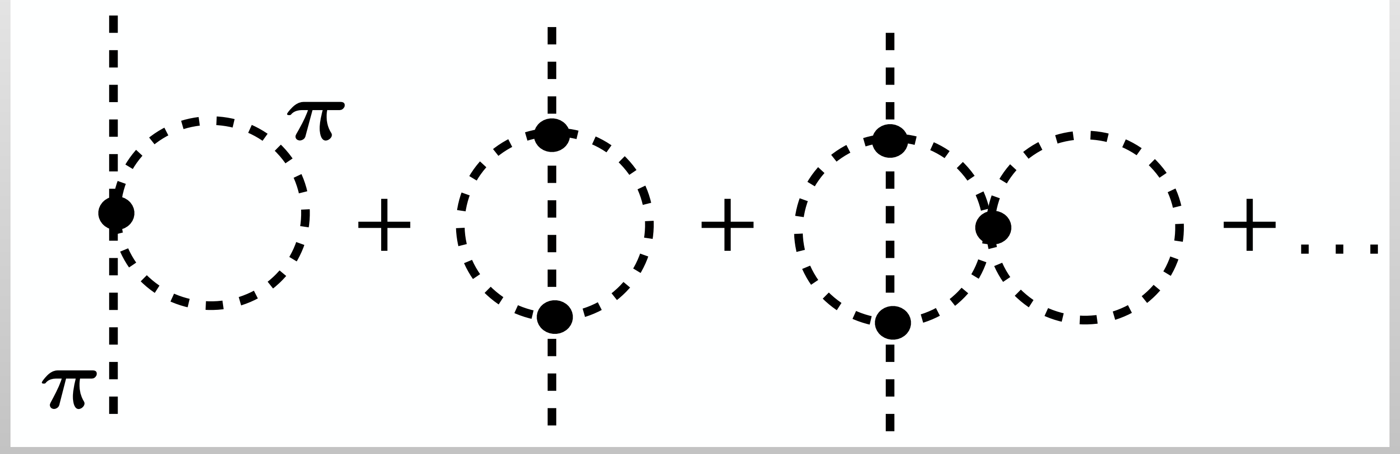
- **Pionic fluctuations** , **nucleonic particle-hole excitations** and **many-body correlations** treated **non-perturbatively** using **FRG**

Review: M. Drews, W.W. : Prog. Part. Nucl. Phys. 91 (2017) 347

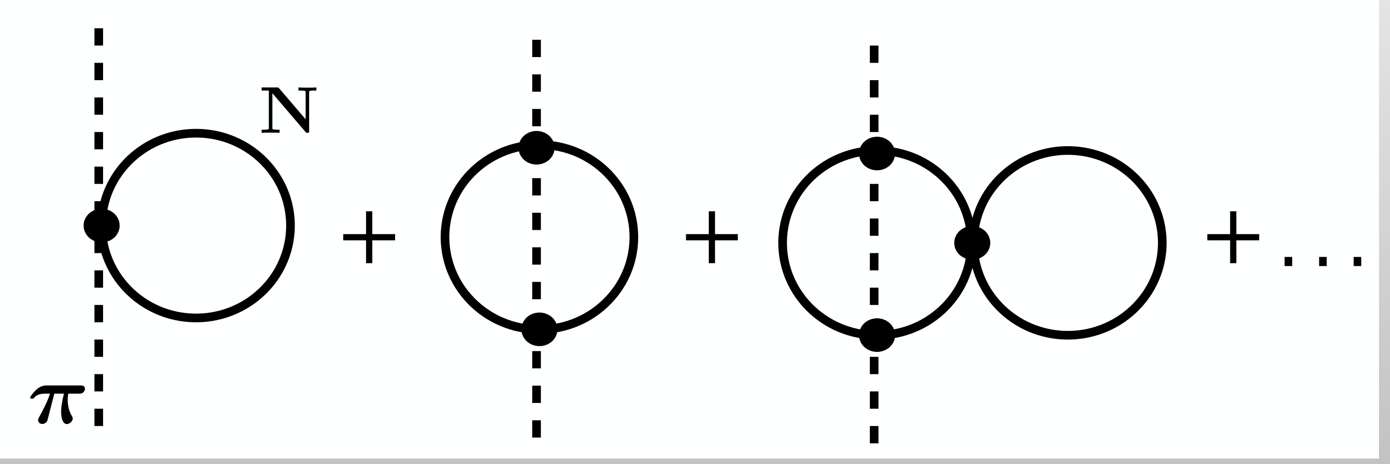
Categories of **mesonic** and **baryonic** fluctuations
treated non-perturbatively in the **FRG approach**

- **Pion propagator**

$$D_\pi(\mathbf{x_1},\mathbf{x_2}) = \left[\frac{\delta^2\Gamma}{\delta\pi(\mathbf{x_1})\,\delta\pi(\mathbf{x_2})}\right]^{-1}$$



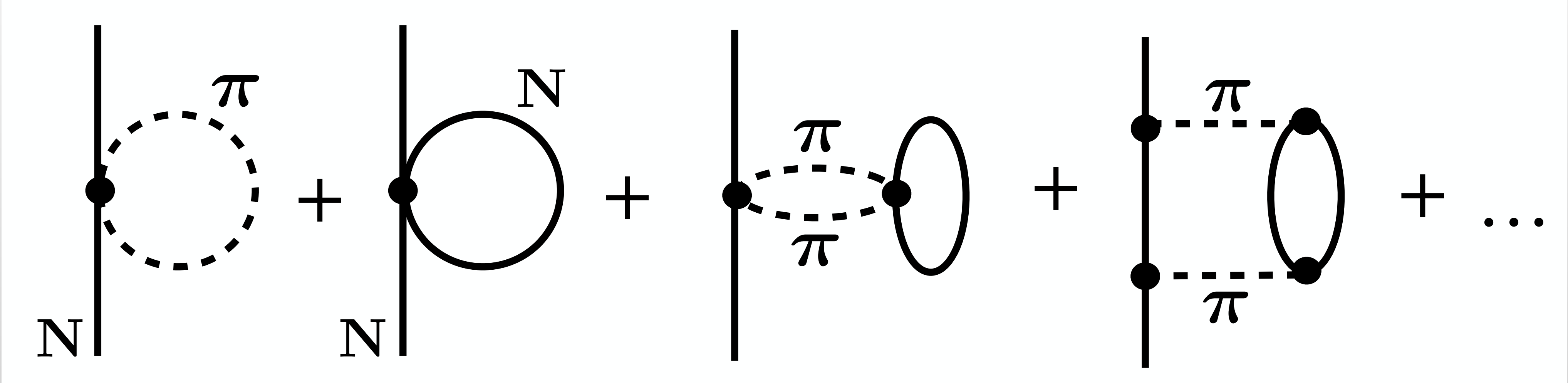
Pion propagation in **pionic heat bath**



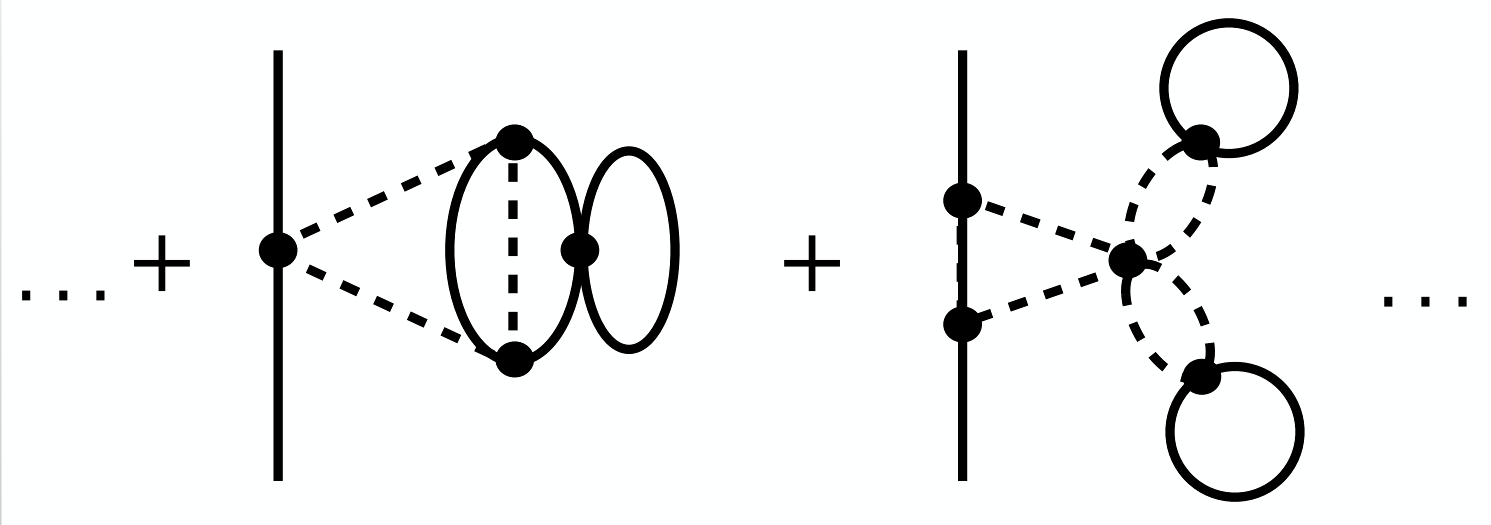
Pion propagation in **nuclear Fermi sea**

- **Nucleon propagator**

$$D_N(\mathbf{x_1},\mathbf{x_2}) = \left[\frac{\delta^2\Gamma}{\delta N(\mathbf{x_1})\,\delta N^\dagger(\mathbf{x_2})}\right]^{-1}$$



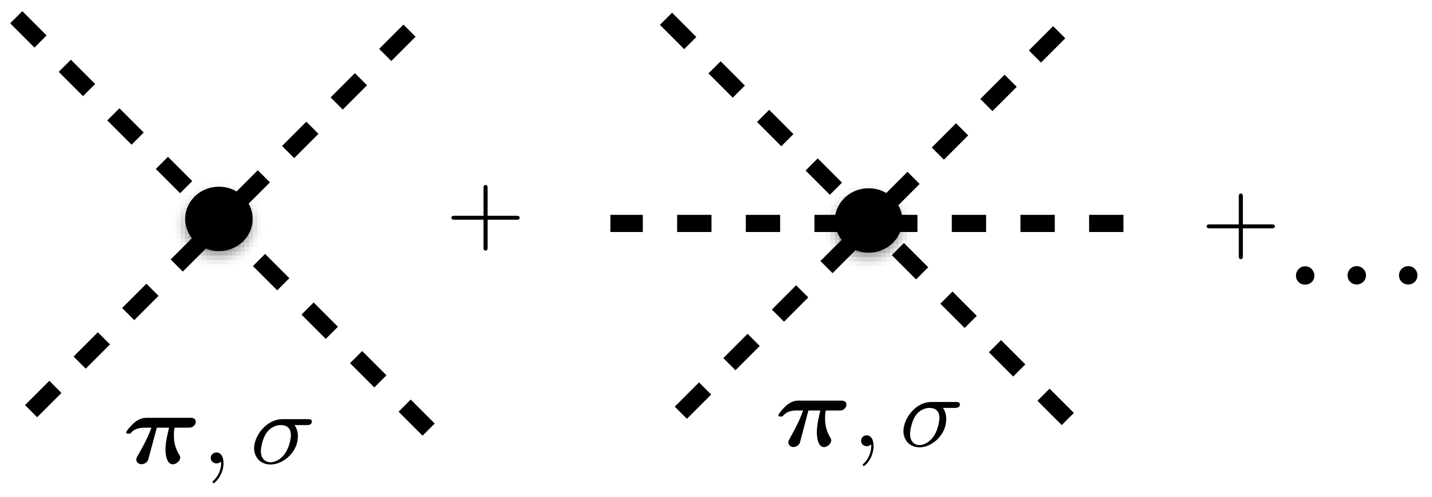
thermal pion cloud | Hartree (MF) | Two-pion exchange | 2nd order tensor force



multi-nucleon correlations

Loops & ladders involving **excitations** of the **nucleonic Fermi sea**

Fixing the input



- **Potential** $\mathcal{U}(\sigma, \boldsymbol{\pi}) = \mathcal{U}_0(\chi) - m_\pi^2 f_\pi(\sigma - f_\pi)$

explicit chiral symmetry breaking

Chiral invariant part :

$$\mathcal{U}_0(\chi) = \sum_n a_n(\chi - \chi_0)^n$$

expanded in powers of

$\chi = \frac{1}{2}(\sigma^2 + \boldsymbol{\pi}^2)$ $\chi_0 = f_\pi^2/2$ (vacuum)

$n_{max} = 4$

a_1, a_2
determined by vacuum constants (f_π, m_π, m_σ)

a_3, a_4
adjustable parameters

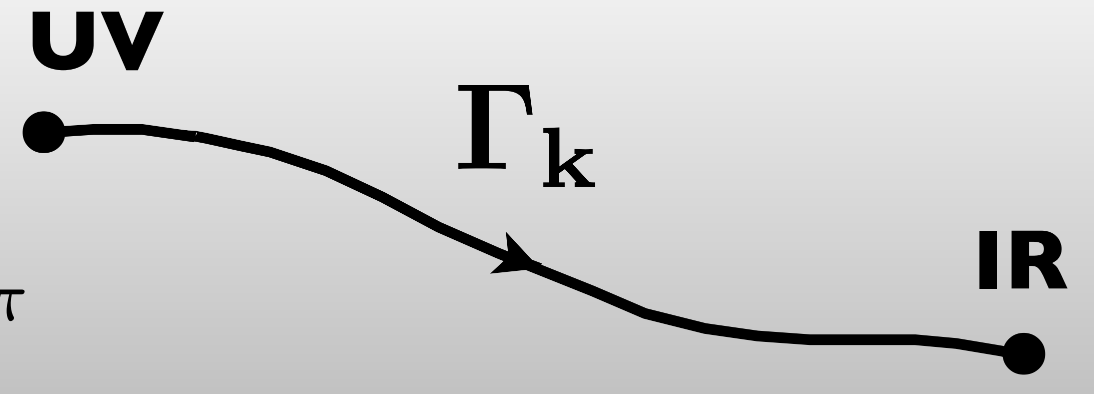
- **Scalar** ("sigma") **field** :
mean field (chiral **order parameter**) plus **fluctuating** pieces.

σ **mass**: **NOT** to be identified with "$\sigma(500)$" : $\mathbf{m}_\sigma \simeq \mathbf{0.9\ GeV}$

- **Nucleon mass** : $M_N = g\sqrt{2\chi}$ ... in **vacuum**: $M_N = g\, f_\pi$ (Goldberger - Treiman)

FRG Flow equations in practice & Fixing the input (contd.)

UV scale :
action $\Gamma_{\mathbf{k}=\Lambda}$
$\Lambda \simeq \Lambda_\chi = 4\pi\, f_\pi$



"**full**"
effective action
$\Gamma_{\mathbf{k}=0}$

- Simplifying assumptions and approximations :
  - **- derivative expansion**
  - **- no "k-running" of Yukawa coupling**

- **k-dependent effective action** :

$$\Gamma_k = \int d^4x \Big[\bar{\Psi} i\gamma_\mu \partial^\mu \Psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}$$
$$-\bar{\Psi}\left[g(\sigma + i\gamma_5\, \boldsymbol{\tau}\cdot\boldsymbol{\pi}) + \gamma_0(g_v\, v + g_w\, w\, \tau_3)\right]\Psi - \mathcal{U}_k(\boldsymbol{\pi},\sigma; v, w)\Big)$$

k-dependent potential

- **Vector fields** v (isoscalar) and w (isovector) encode **short-distance NN dynamics**:
  Self-consistently determined background **mean fields** (non-fluctuating)
  (**not** to be identified with physical ω and ρ mesons)

- **Effective chemical potentials** of neutrons and protons : $\mu_{n,p}^{\rm eff} = \mu_{n,p} - g_v\, v \pm g_w\, w$

Relevant quantities: $G_v = \dfrac{g_v^2}{m_V^2}$, $G_w = \dfrac{g_w^2}{m_V^2}$ $\longleftrightarrow$ **contact terms** in **ChEFT** $G_w \simeq G_v/4 \sim 1\,\mathrm{fm}^2$

FRG Flow equations in practice (contd.)

- **Flow** of **k-dependent effective potential**

$$k\frac{\partial \mathcal{U}_k}{\partial k}(T,\mu_p,\mu_n;\chi,v,w) = \frac{k^5}{12\pi^2}\left[\frac{1+2f_B(E_\sigma)}{E_\sigma} + \frac{3[1+2f_B(E_\pi)]}{E_\pi} - 4\sum_{i=p,n}\frac{1-f_F(E_N-\mu_i^{\text{eff}})}{E_N}\right]$$

$$f_{F,B}(E) = \left[\mathrm{e}^{E/T}\pm 1\right]^{-1} \quad \chi = \frac{1}{2}(\sigma^2+\boldsymbol{\pi}^2) \quad E_\pi^2 = k^2 + \mathcal{U}_k'(\chi)\,, \quad E_\sigma^2 = k^2 + \mathcal{U}_k'(\chi) + 2\chi\,\mathcal{U}_k''(\chi)\,,$$

$$E_N^2 = k^2 + 2g^2\chi\,, \qquad \mathcal{U}_k'(\chi) = \frac{\partial \mathcal{U}_k(\chi)}{\partial \chi}\,, \qquad \mu_{n,p}^{\text{eff}} = \mu_{n,p} - g_v\,v \pm g_w\,w$$

. . . plus vector field equations, then full system of coupled differential equations solved on a grid.

- **Thermodynamics** : Grand canonical potential $\Omega = -\frac{T}{V}\ln \mathcal{Z} = \Omega_F + \mathcal{U}_{k=0}$

pressure $P = -\Omega$

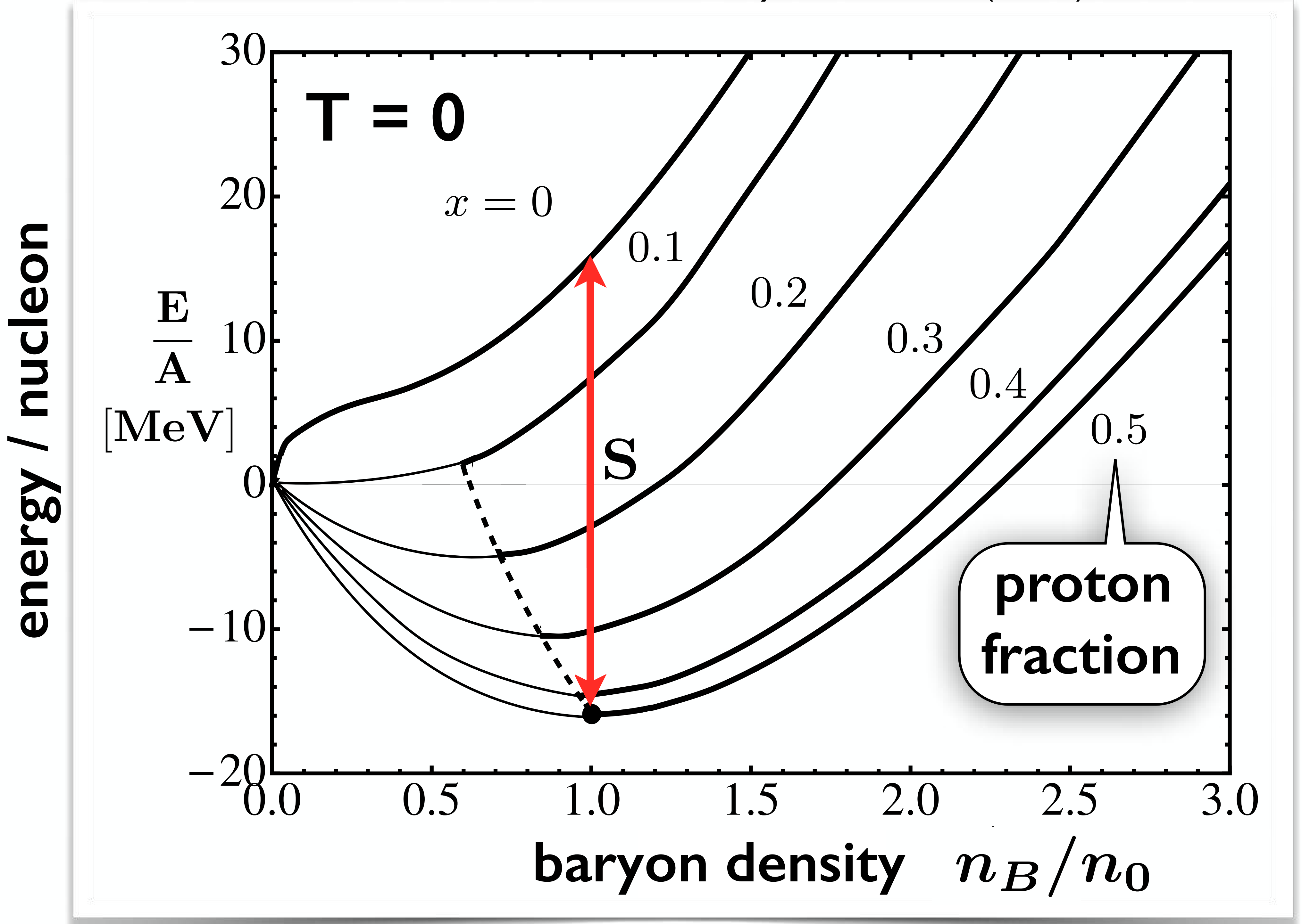
entropy density $s = -\frac{\partial \Omega}{\partial T}$

baryon densities $n_i = -\frac{\partial \Omega}{\partial \mu_i}$

energy density $\mathcal{E} = -P + \sum_{i=p,n}\mu_i n_i + Ts$

From symmetric to asymmetric nuclear matter
in the Chiral FRG approach

M. Drews, T. Hell, B. Klein, W.W. : Phys. Rev. D 88 (2013) 096011

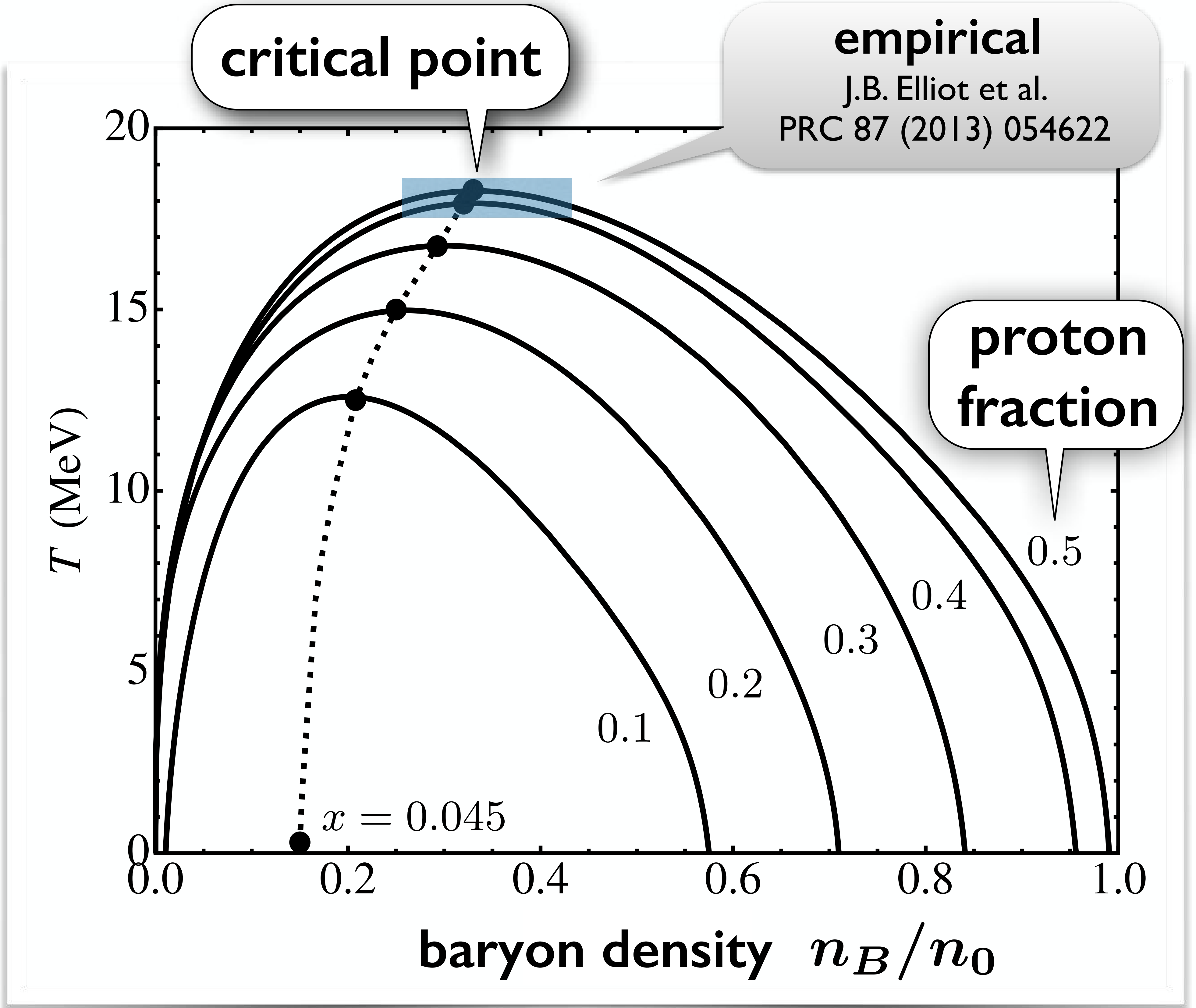


- **Symmetry energy** $S = 32\,\text{MeV}$
  (“empirical” : $S = 32 \pm 2$ MeV)
  M. Baldo, G.F. Burgo : Prog. Part. Nucl. Phys. 91 (2016) 203

- **FRG results** (**non-perturbative**) consistent with (perturbative) **Chiral EFT calculations**

M. Drews, W.W. : Phys. Rev. C91 (2015) 035802

- **Liquid-gas phase transition** :
  Evolution of coexistence regions from **symmetric** to **asymmetric** nuclear matter



FLUCTUATIONS and PHASES in BARYONIC MATTER
- a closer look -

L. Brandes, N.. Kaiser, W.W. : Eur. Phys. J. A57 (2021) 243

- **Vacuum Fluctuations** (nucleonic zero-point energy / one-loop fermionic effective potential)

  … treated as logarithmic correction to mean-field approximation : V. Skokov et al. : Phys. Rev. D82 (2010) 034029

  Vacuum part of fermionic grand potential :

$$\delta\Omega_{\rm vac} = -4\int \frac{d^3p}{(2\pi)^3} E_N = -\frac{2}{\pi^2}\int dp\, p^2 \sqrt{p^2 + M_N^2(\sigma)} \xrightarrow[\text{regularisation}]{\text{dimensional}} -\frac{M_N^4}{8\pi^2}\ln\frac{M_N^2}{\Lambda^2}$$

$$(\Lambda = g f_\pi)$$

extended mean-field appr. (EMF)

$$\Omega_{\rm EMF} = \Omega_{\rm MF} - \frac{(g\sigma)^4}{4\pi^2}\ln\frac{\sigma}{f_\pi}$$

vacuum fluctuations

mean-field

- Note :

  Functional Renormalisation Group automatically includes **vacuum fluctuations**

CHIRAL PHASE TRANSITION in DENSE BARYONIC MATTER ?

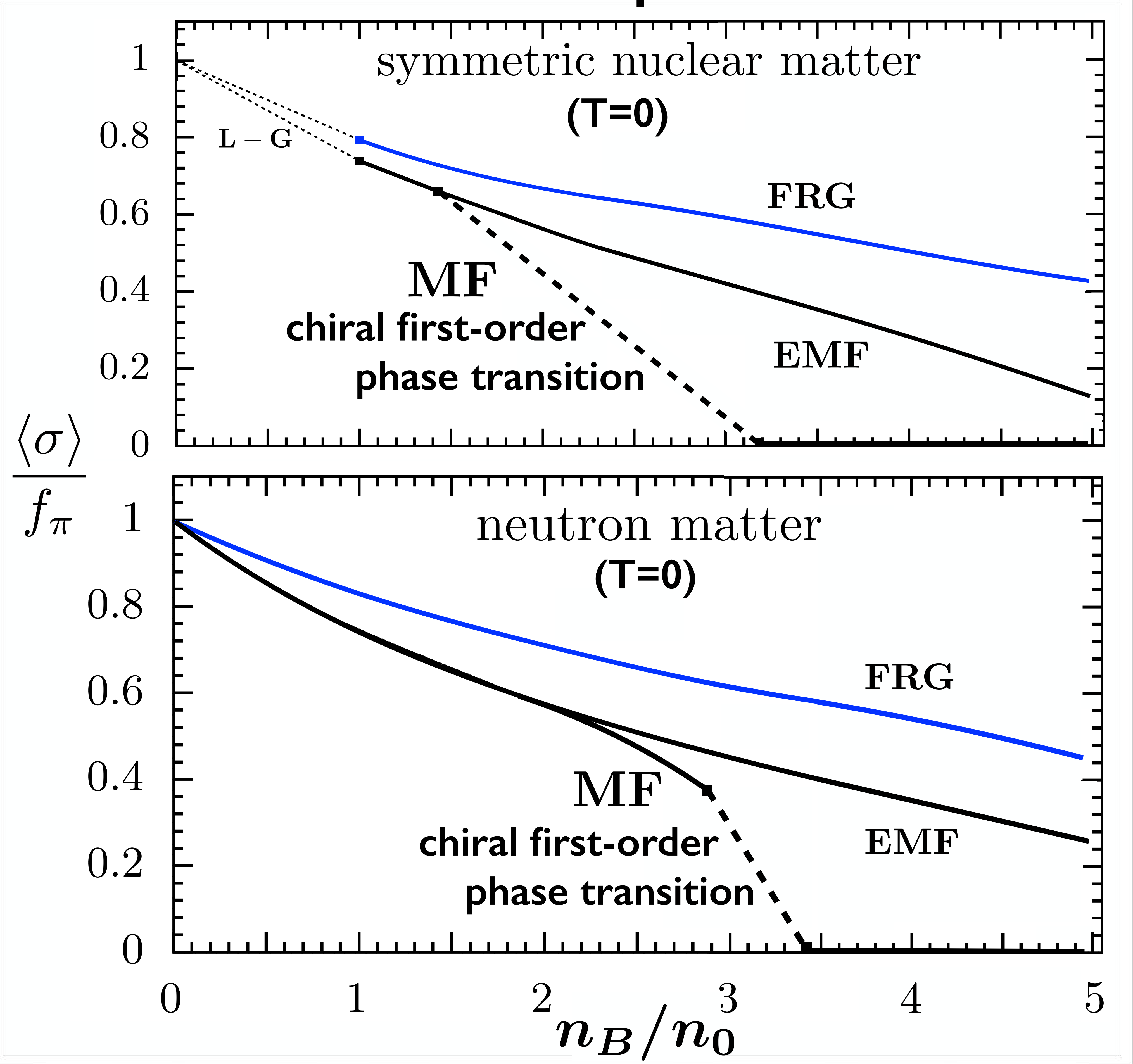
- **Studies in chiral nucleon-meson field theory**

M. Drews, W.W.: Prog. Part. Nucl. Phys. 93 (2017) 69 — L. Brandes, N. Kaiser, W.W.: Eur. Phys. J. A57 (2021) 243

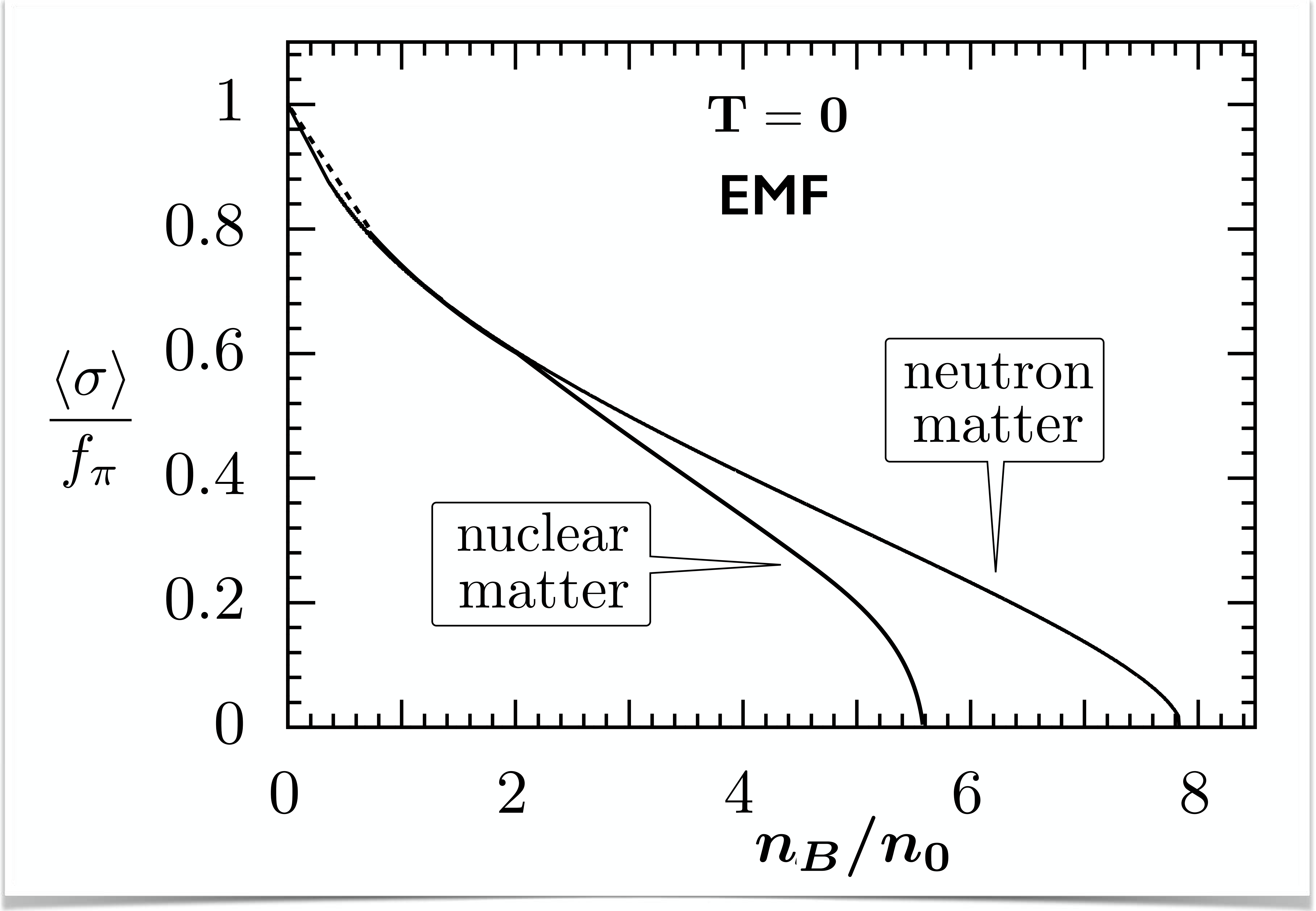
- **Mean-field** approximation **(MF)** :
  chiral first-order phase transition
  at baryon densities $n_B \sim 2 - 3\, n_0$
- **Vacuum fluctuations** (EMF) :
  shift chiral transition to **high density**
  → **smooth crossover**
- **Functional Renormalisation Group** **(FRG)** :
  non-perturbative loop corrections
  involving **pions** & **nucleon-hole** excitations
  → further reinforcement of **stabilising effects**

Chiral crossover transition at $n_B > 6\, n_0$
far beyond core densities in neutron stars

Chiral order parameter



CHIRAL LIMIT $(m_\pi \to 0)$
2nd order chiral phase transition in nuclear and neutron matter



L. Brandes, N.. Kaiser, W.W.: Eur. Phys. J. A57 (2021) 243

- **Chiral Nucleon-Meson Field Theory**
- **EMF calculations :** Extended Mean-Field including logarithmic vacuum fluctuations
- **Critical densities** (chiral limit)
  $$n_B^{cr} > 5\, n_0$$
- Alternative approach: **Parity-doublet model**
  $\{N(1/2^+) - N^*(1/2^-)\}$
  Critical densities $n_B^{cr} > 10\, n_0$

J. Eser, J.-P. Blaizot : Phys. Rev. C109 (2024) 045201
arXiv:2408.01302

7.

Neutron Star Matter as a Relativistic Fermi Liquid

Basics of (Relativistic) Fermi-Liquid Theory

G. Baym, S.A. Chin : Nucl. Phys. A262 (1976) 527

T. Matsui : Nucl. Phys. A370 (1981) 365

- **Variation of the energy** (T = 0)

$$\delta E = V\delta\mathcal{E} = \sum_p \varepsilon_p\, \delta n_p + \frac{1}{2V}\sum_{pp'} \mathcal{F}_{pp'}\delta n_p \delta n_{p'} + \ldots \qquad n_p = \Theta(\mu - \varepsilon_p)$$

quasiparticle energy $\varepsilon_p = \dfrac{\delta E}{\delta n_p}$

quasiparticle interaction

$$\mathcal{F}_{pp'} = V\frac{\delta^2 E}{\delta n_p \delta n_{p'}} = f_{pp'} + g_{pp'}\,\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'$$

- **Landau effective mass**

$$m^* = \sqrt{p_F^2 + M^2(\rho)}$$

- **Density of states** at the **Fermi surface** $N(0) = \dfrac{m^*\, p_F}{\pi^2}$

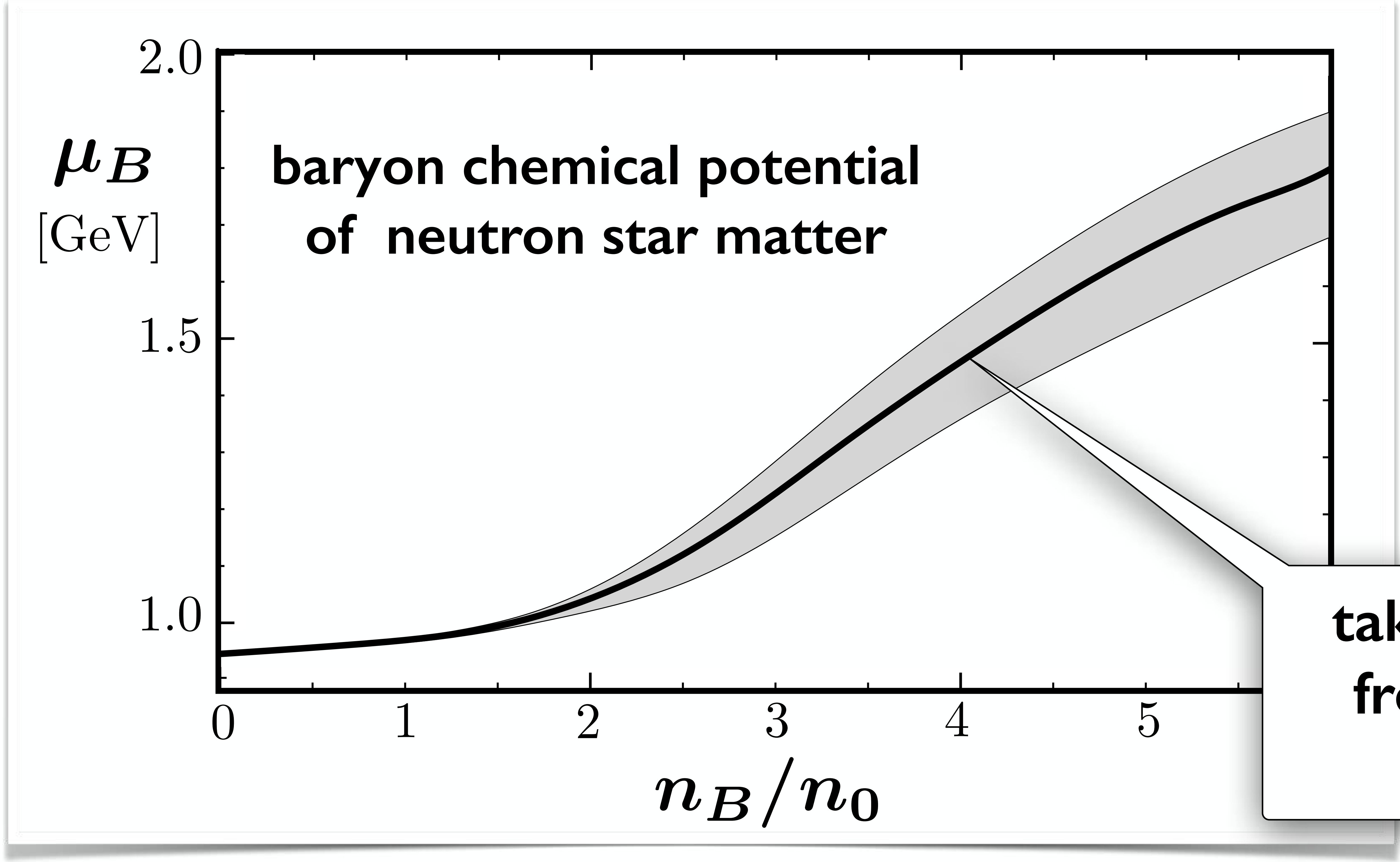
- **Landau parameters** **Quasiparticle interaction** expanded in **Legendre series**

$$f_{pp'} = \sum_{\ell=0}^{\infty} f_\ell\, P_\ell(\cos\theta_{pp'}) \qquad F_\ell = N(0)\, f_\ell$$

DENSE BARYONIC MATTER in NEUTRON STARS as a RELAVISTIC FERMI LIQUID

B. Friman, W.W. : Rhys. Rev. C100 (2019) 065807 L. Brandes, W.W. : Symmetry 16 (2024) 111

- **Neutron Star Matter : Fermi liquid** / dominantly neutrons + ca. 5 % protons
- **Baryonic Quasiparticles** :
  baryons "dressed" by their **strong interactions** and imbedded in **mesonic (multi-pion) field**



- **Landau effective mass**

$$m_L^*(n_B) = \sqrt{p_F^2 + M_N^2(n_B)}$$

- **Baryon chemical potential**

$$\mu_B = m_L^*(n_B) + \mathcal{U}(n_B)$$

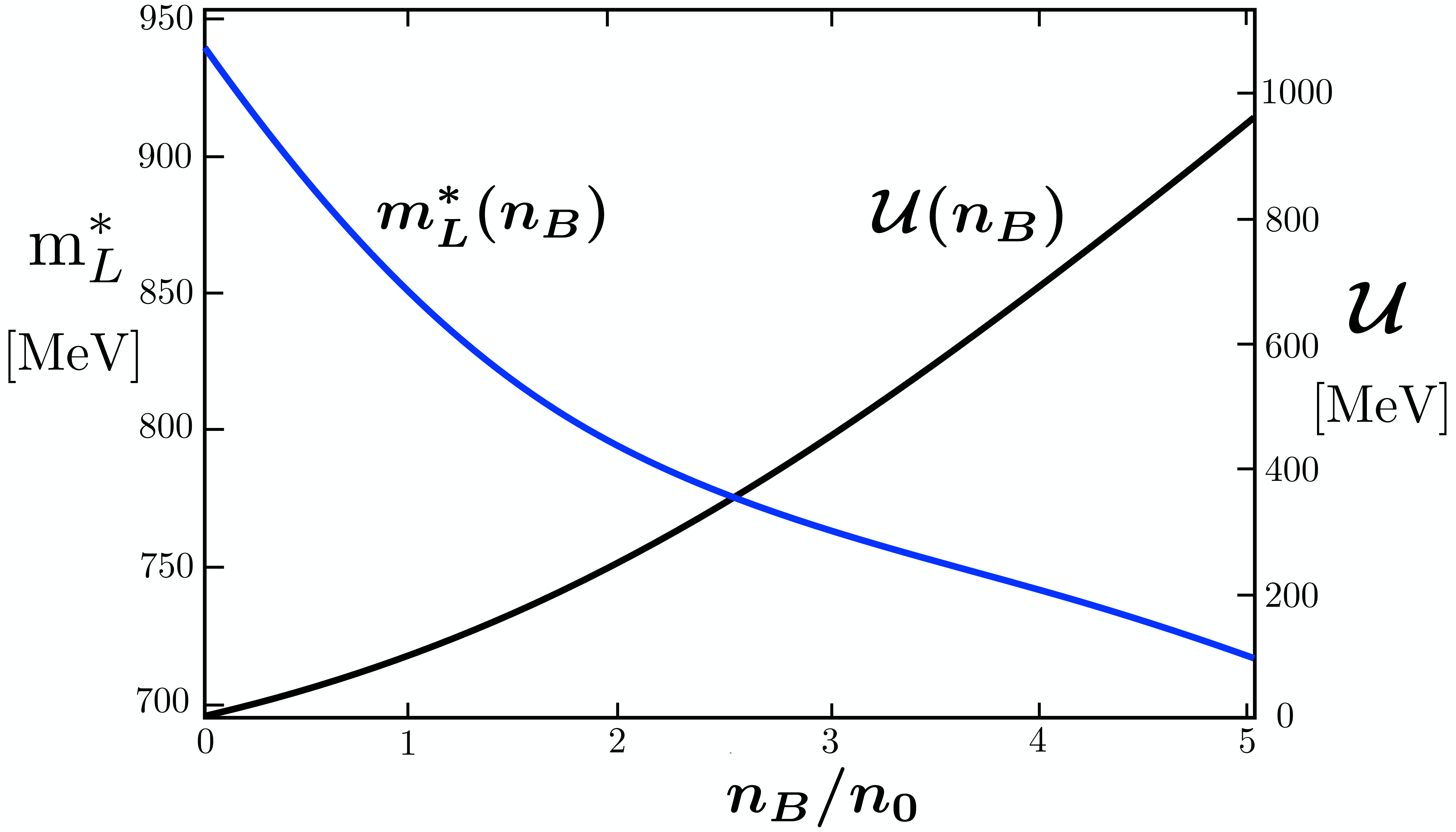
take median of $\mu_B(n_B)$ from Bayesian-inferred neutron star EoS

quasiparticle potential

QUASIPARTICLE POTENTIAL and FERMI-LIQUID PARAMETERS

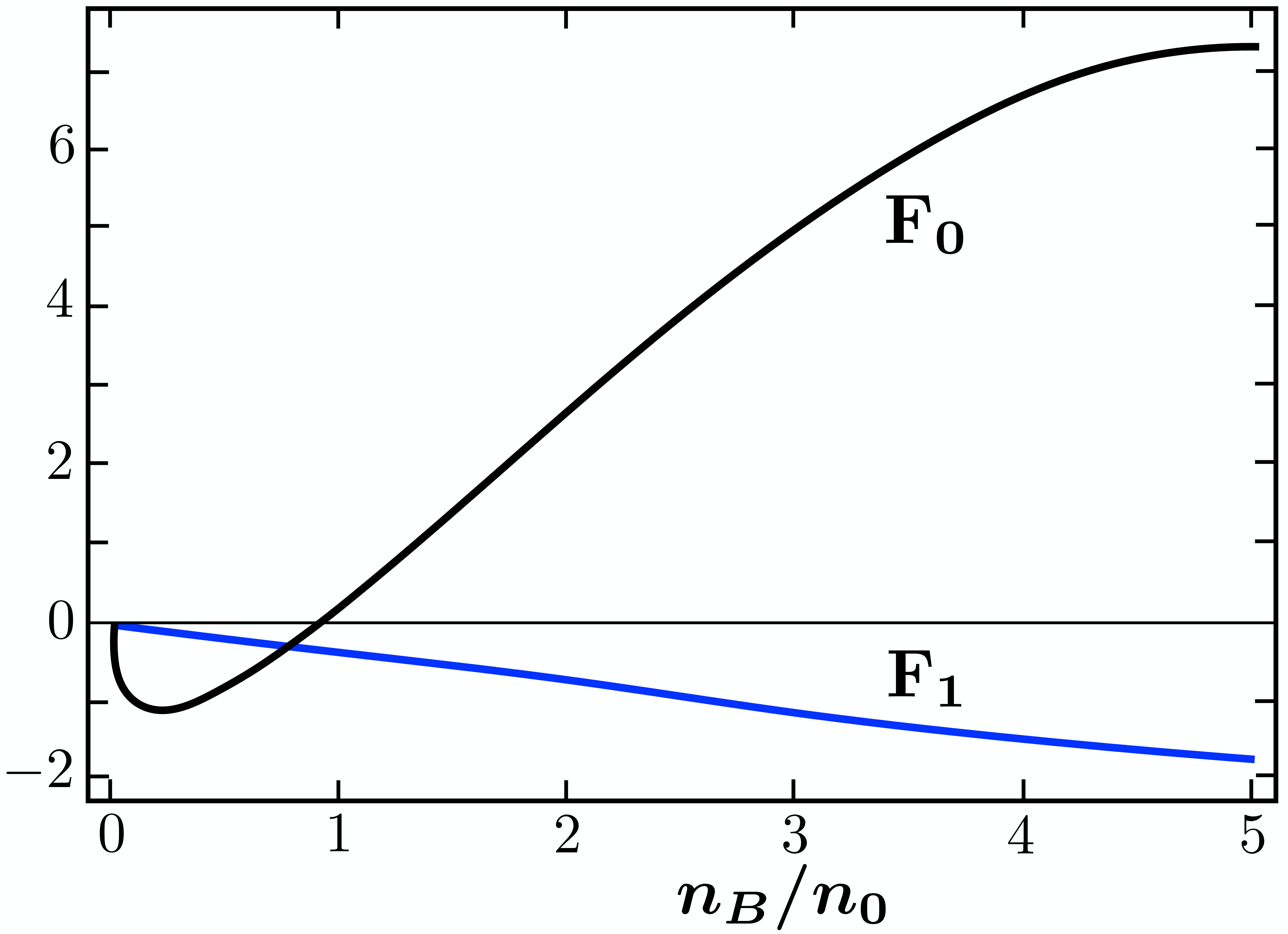
- $m_L^*(n_B)$ from **chiral nucleon-meson field theory** & Functional Renormalisation Group
- Quasiparticle effective potential

$$\mathcal{U}(n_B) = \sum_n u_n \left(\frac{n_B}{n_0}\right)^n$$



- Landau Fermi-Liquid parameters

$$F_0 = \frac{m_L^* \, p_F}{\pi^2} \frac{\partial \mu_B}{\partial n_B} - 1 \qquad F_1 = -\frac{3\mathcal{U}}{\mu_B}$$



→ **Strongly repulsive correlations** including **many-body forces** with $n \geq 2$

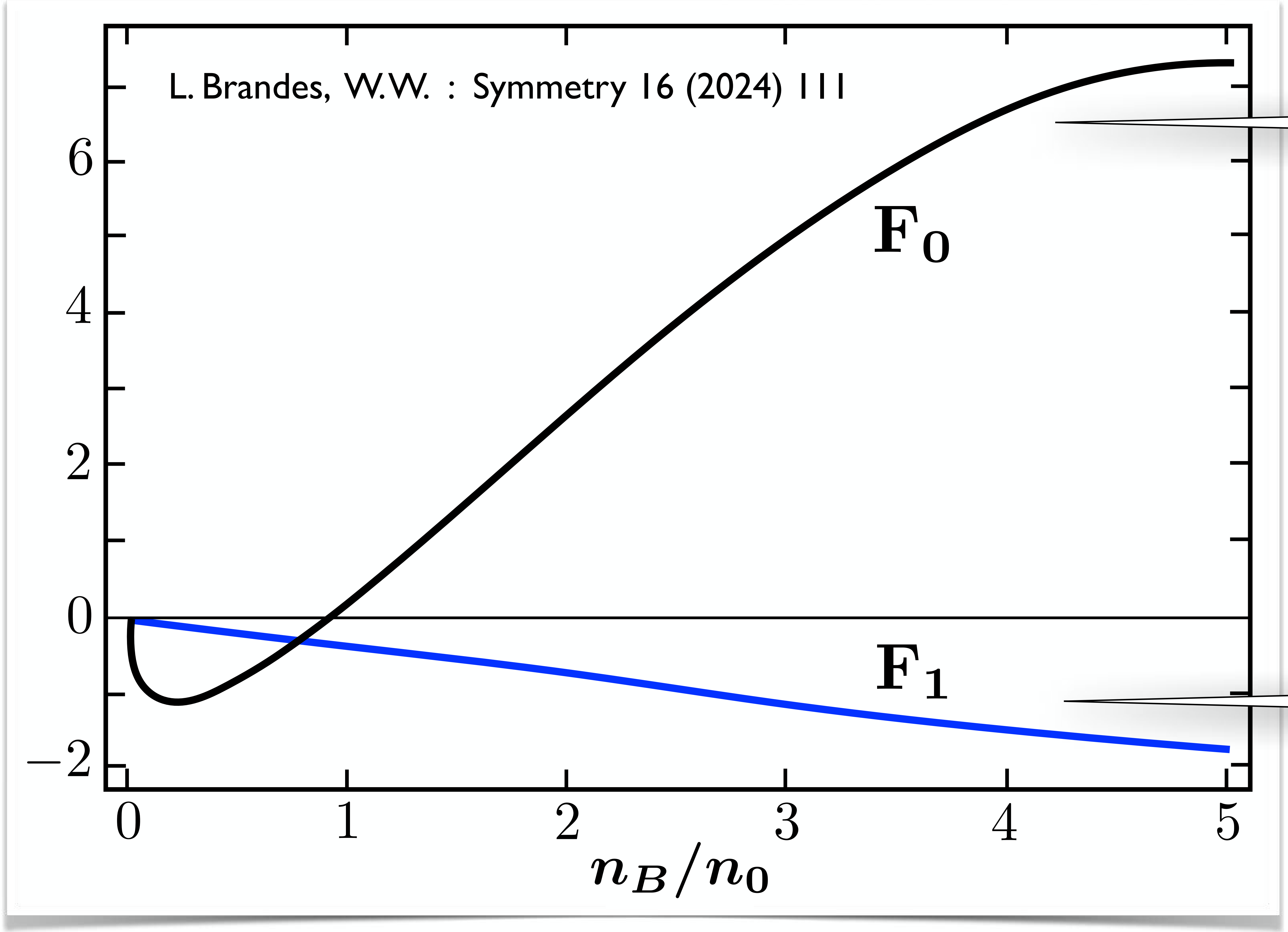
LANDAU FERMI-LIQUID PARAMETERS

- **Deduced from empirically inferred neutron star matter EoS**
  in combination with Chiral Nucleon-Meson Field Theory & Functional Renormalisation Group

neutron star matter :

a strongly correlated Fermi liquid
…

but not "extreme" !



strongly repulsive many-body correlations

decreasing effective Fermion mass

- **Low densities** $n_B \lesssim n_0$: good agreement with (perturbative) **ChEFT results**

J.W. Holt, N.Kaiser, W.W. : Phys. Rev. C87 (2013) 014338

LANDAU FERMI-LIQUID PARAMETERS (contd.)

- **Comparison with atomic liquid helium-3 in its normal phase at low temperature (3 K)**

G. Baym, Ch. Pethick : Landau Fermi-Liquid Theory (1991)

- **Interaction** between **He-3 atoms:**
attractive van der Waals potential plus strongly **repulsive short-range core**

- **Landau Fermi-Liquid parameters** of **liquid helium-3** at pressures P = (0 - 30) bar:

$$\mathbf{F_0(^3He) \sim 10 - 70} \qquad \mathbf{F_1(^3He) \sim 5 - 13}$$

D. S. Greywall, Phys. Rev. B33 (1986) 7520

… **much larger** by magnitude than Landau parameters of **neutron star matter** !

- **Neutron star matter** at central densities is a **strongly correlated Fermi system**
… but not as extreme as one might have thought !

SUMMARY

- **Neutron star matter :**
  - Equation-of-state constrained by steadily expanding observational data base (Multimessenger observations : **stiff EoS** required)
  - $\mathrm{M}_{max} \gtrsim 2\,\mathrm{M}_{\odot}\,,\ \mathrm{R} \simeq 11 - 13\,\mathrm{km}$
  - Strong first-order phase transition unlikely
  - Not ruled out: baryonic matter or smooth hadrons-to-quarks crossover

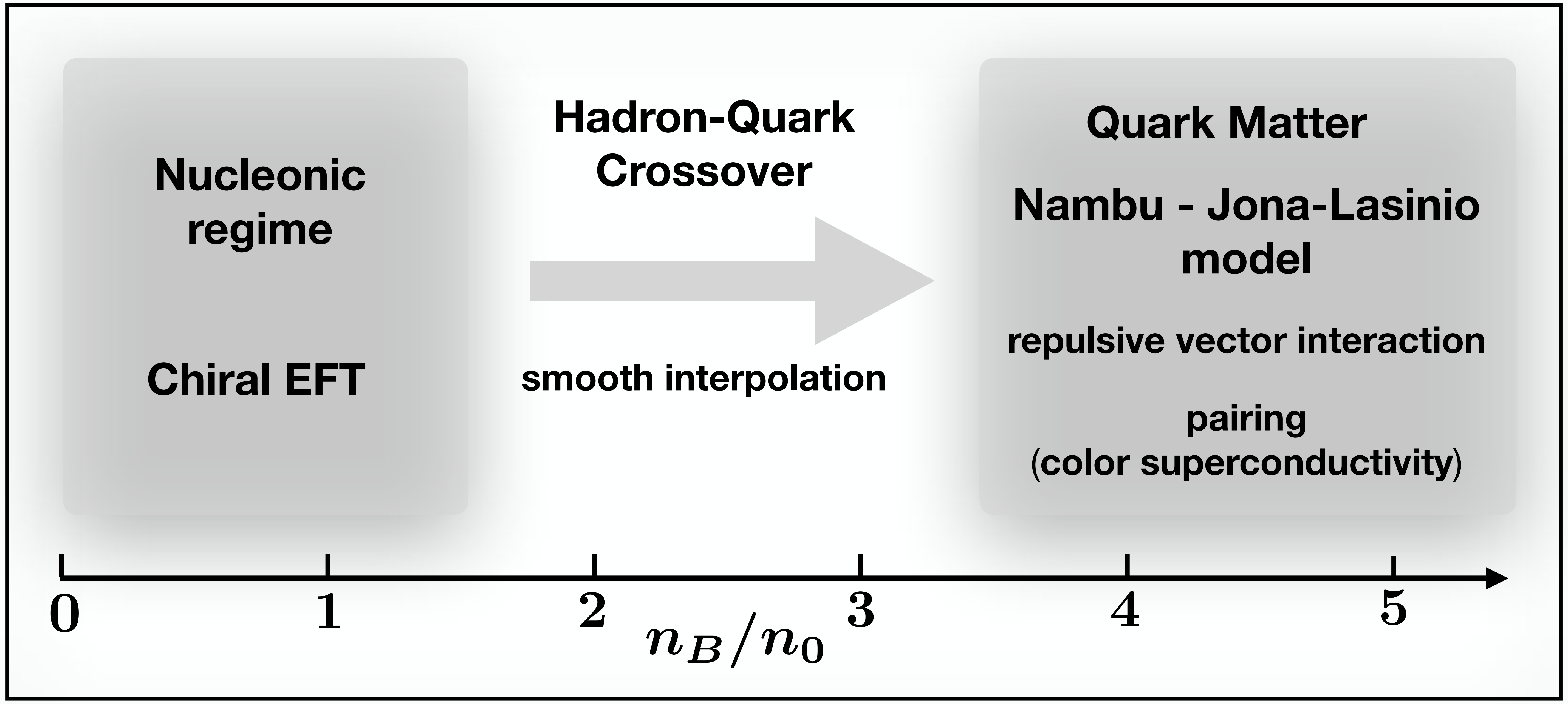
- **Theoretical concepts :**
  ChEFT (perturbative), FRG (non-perturbative), Fermi-Liquid Theory
  - **Conventional** (hadronic) **EoS** consistent with empirical "stiffness" constraints
  - **No chiral phase transition** in neutron-rich matter up to at least $n_B > 5\,n_0$

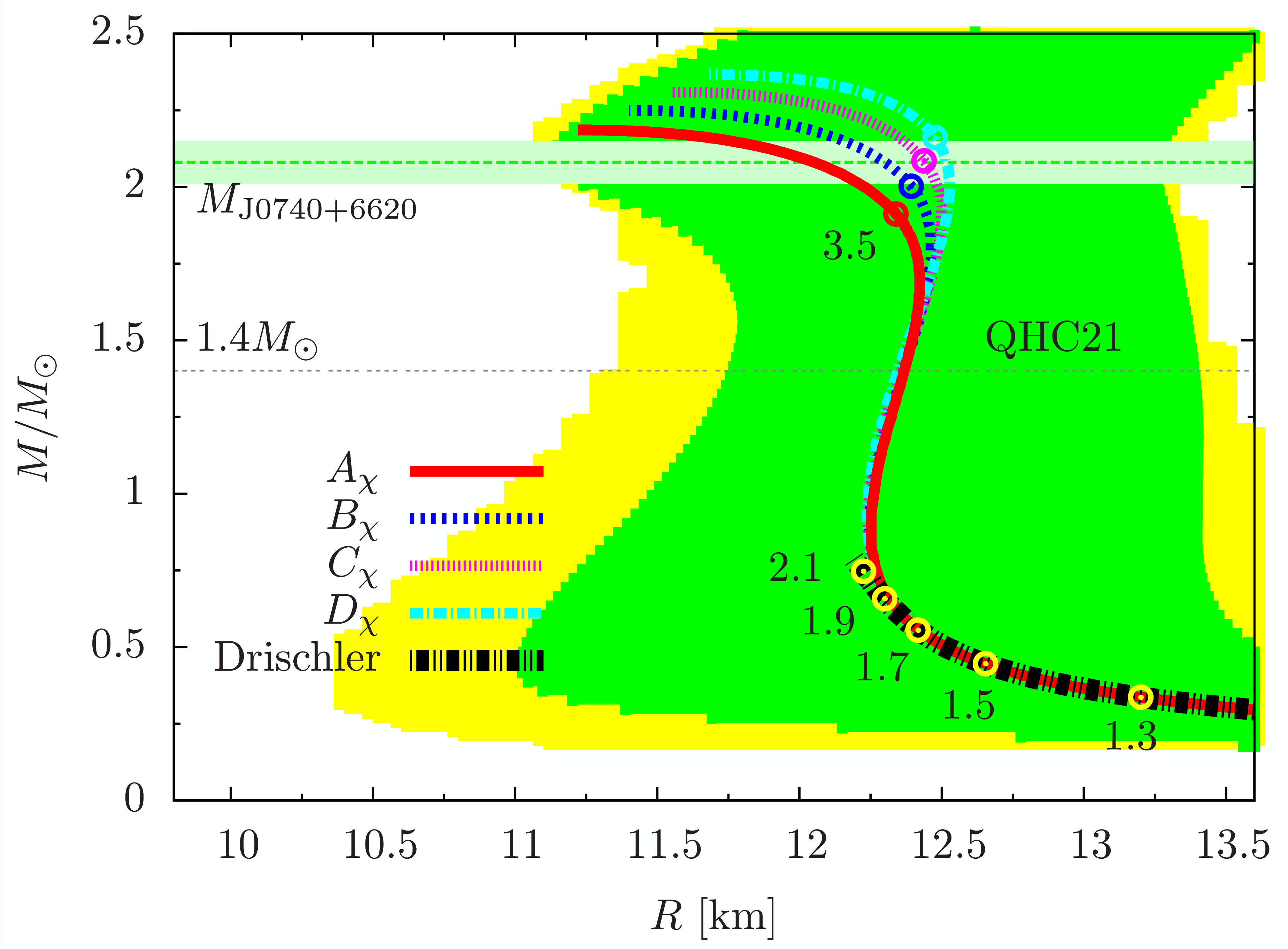
- **Notes added :**
  - **Hadron-Quark Continuity** (successful example: QHC21 equation-of-state)
  - **Superfluidity** (from neutron matter to 2-flavor quark matter)

COLD MATTER at EXTREME DENSITIES
Hadron - Quark Continuity

- **QHC21 Equation-of-State**

T. Kojo, G. Baym, T. Hatsuda : Astroph. J. 934 (2022) 46





- **NJL model features** : Chiral symmetry restoration   Scalar-pseudoscalar coupling G

  Vector coupling $g_V/G \simeq 1.0 - 1.3$ Pairing interaction $H/G \simeq 1.5 - 1.6$

- Intermediate crossover region may involve "quarkyonic" matter

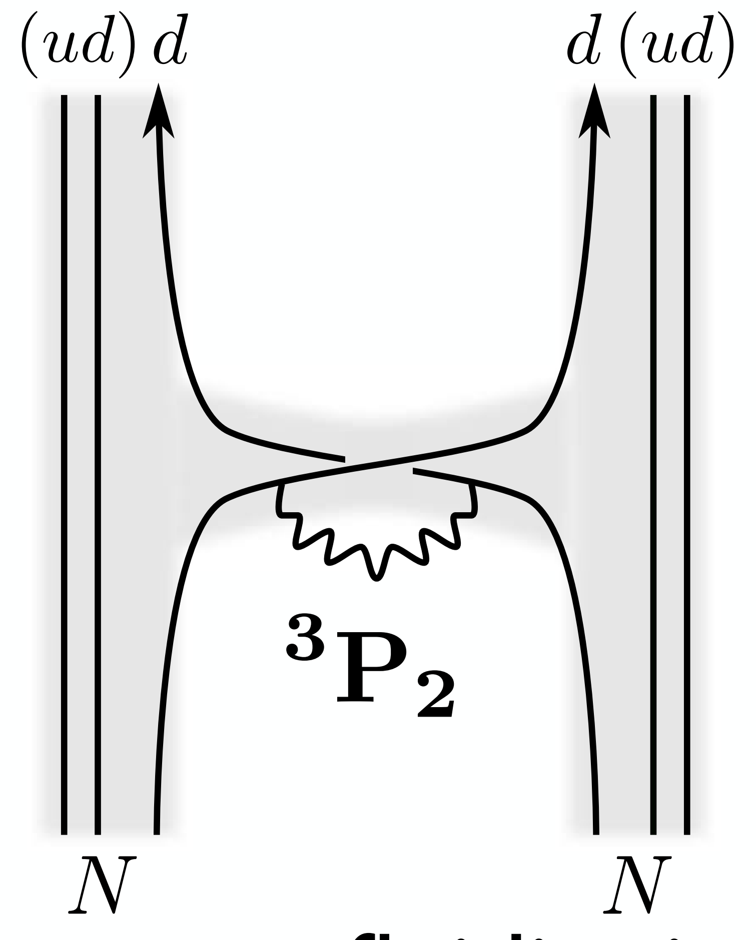
L. NcLerran, S. Reddy
Phys. Rev. Lett. 122 (2019) 122701

COLD MATTER at EXTREME DENSITIES
Hadron - Quark Continuity (contd.)

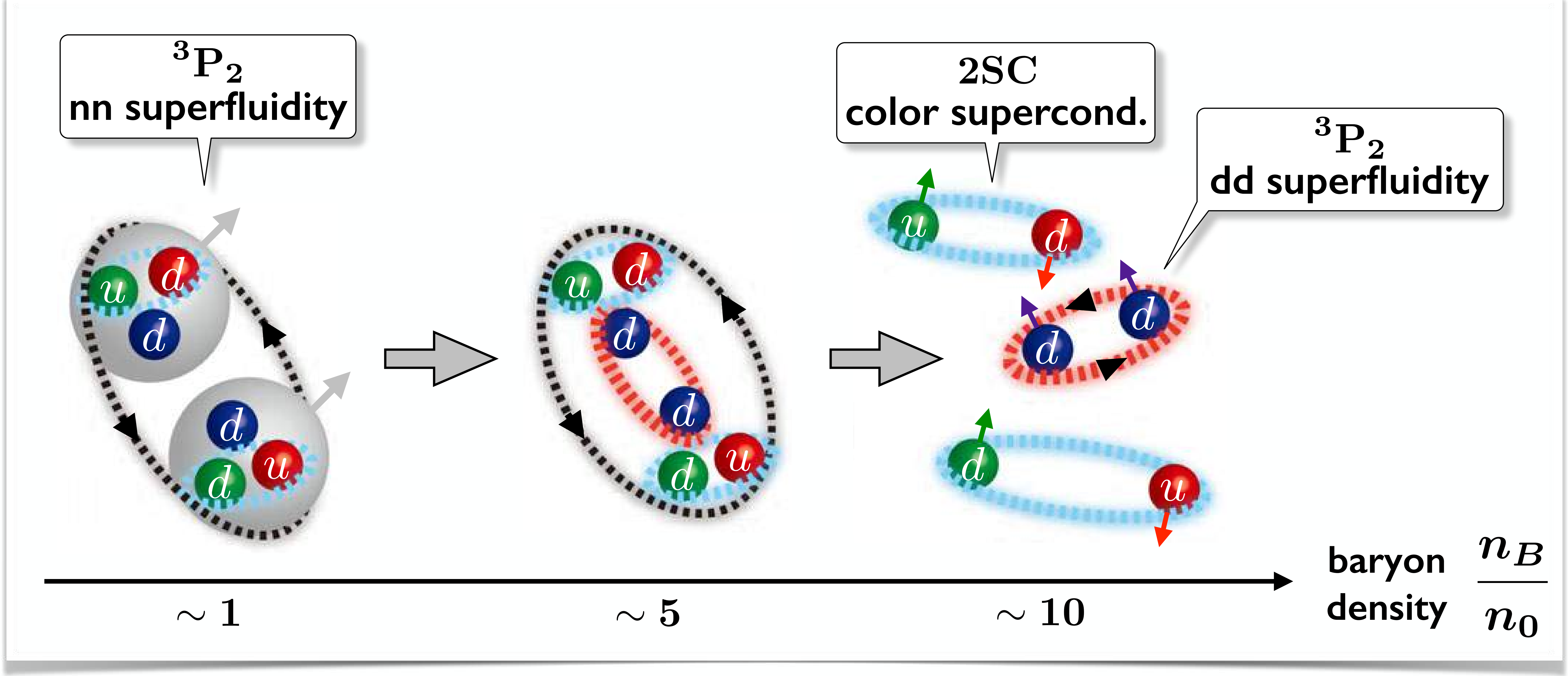
- **Continuity** test case : **superfluidity** from **neutron matter** to $N_f = 2$ **quark matter**

Y. Fujimoto, K. Fukushima, W.W. : Phys. Rev. D101 (2020) 094009

- Attractive **spin-orbit force** in $^3P_2$ **nn** and **dd** channels



induces superfluidity in both hadronic and quark phases



- **Continuous matching** of symmetry breaking patterns from **neutron matter** to $2SC + \langle dd \rangle$ **quark matter**

$$[SU(3)_C] \times SU(2)_L \times SU(2)_R \times U(1)_B \rightarrow [SU(2)_C] \times SU(2)_L \times SU(2)_R \times U(1)_{C+B}$$

global color — chiral symmetry — baryon no.

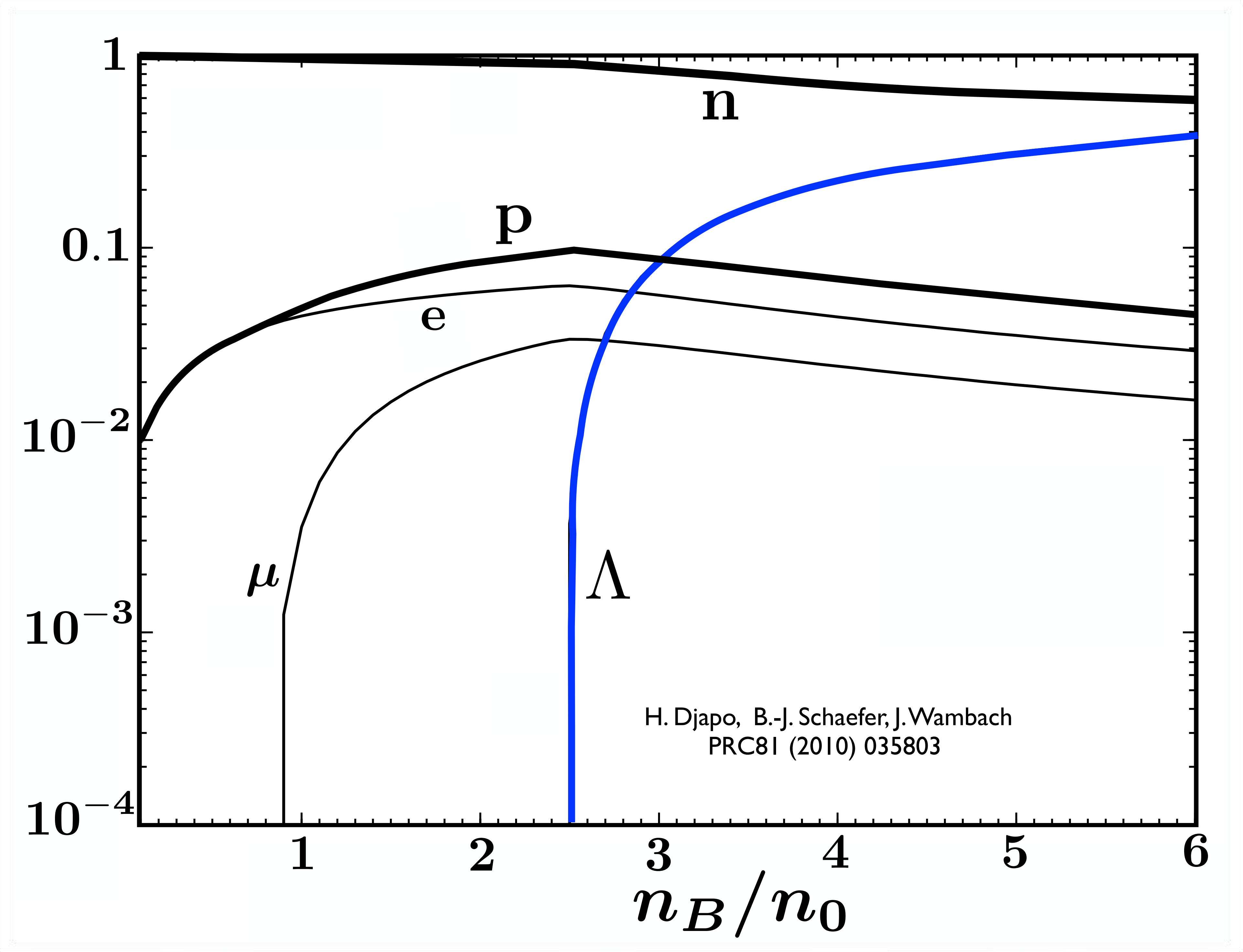
broken by $\langle dd \rangle$ condensate

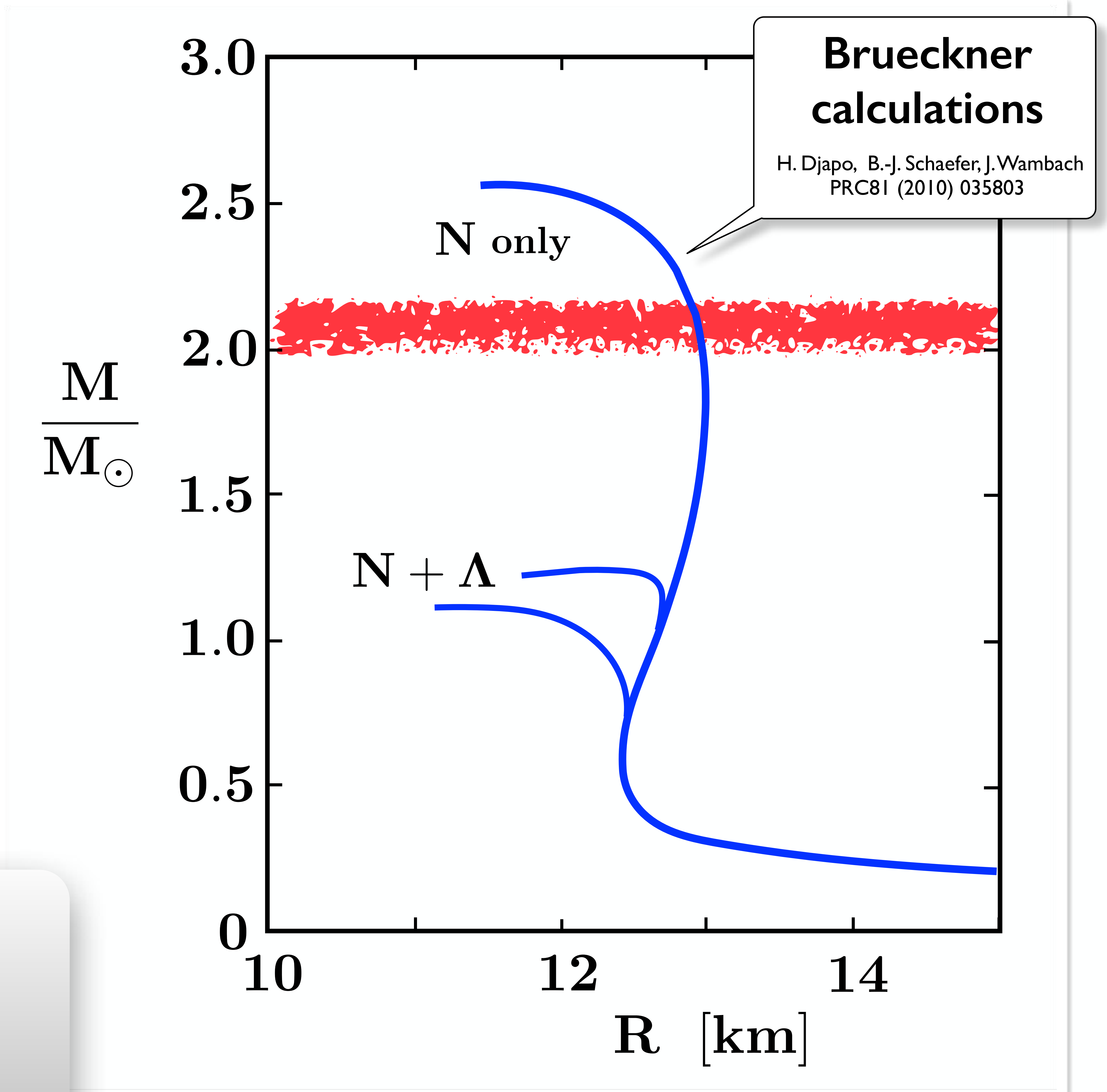
8.

Hyperon-Nuclear Interactions
and
Strangeness in Dense Matter

- **Chiral SU(3) Effective Field Theory of Hyperon-Nucleon Interactions**
- **Hyperon - NN Three-Body Forces**
- **"Hyperon Puzzle" in Neutron Stars ?**

NEUTRON STAR MATTER including HYPERONS





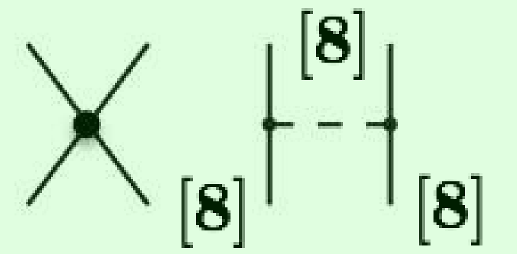
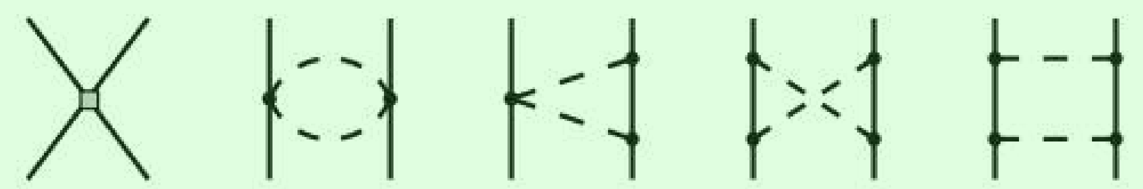
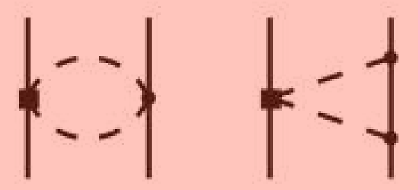
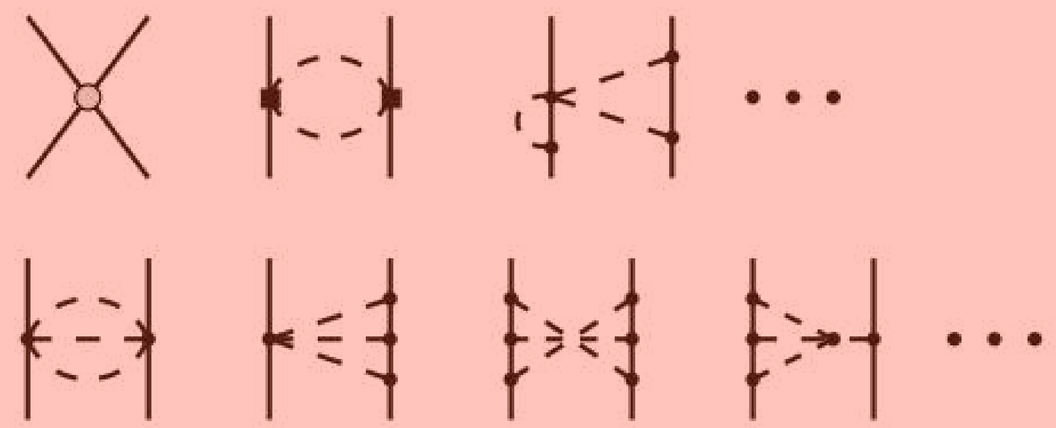
- Inclusion of **hyperons** :

EoS **too soft** to support 2-solar-mass n-stars, unless:
strong repulsion in **YN** and/or **YNN** interactions

$$\begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

BARYON-BARYON INTERACTIONS from CHIRAL $SU(3)_L \times SU(3)_R$ EFFECTIVE FIELD THEORY

$$\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

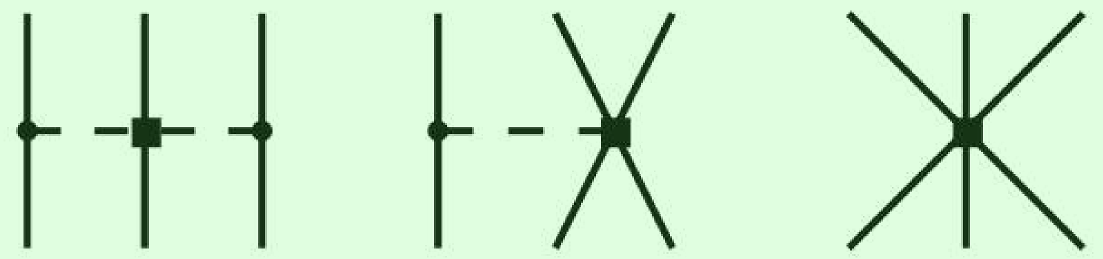
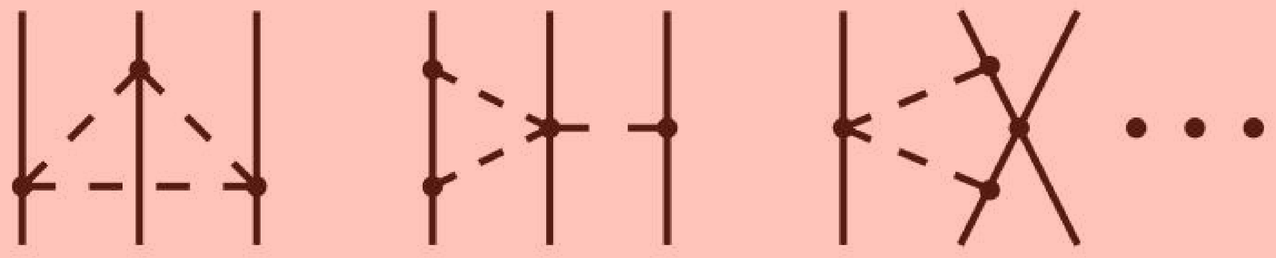
	BB interactions
LO	
NLO	
N^2LO	
N^3LO	

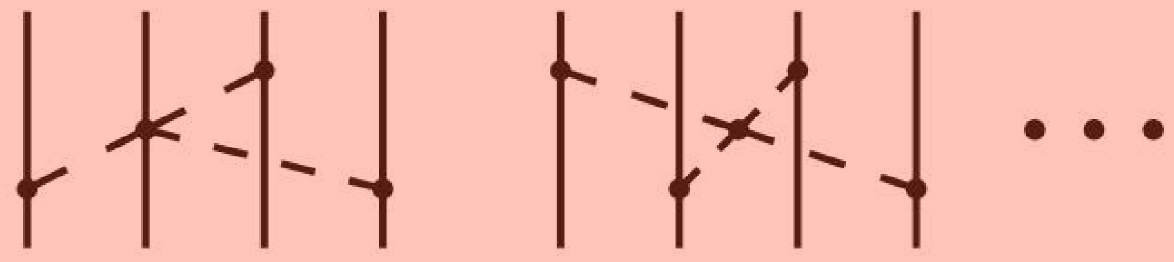
- Systematically organized hierarchy in powers of Q/Λ
  (Q: momentum, energy, pseudoscalar meson masses)

- Hyperon-Nucleon interaction :
  still very limited scattering data base

↓

- Restriction to
  NLO YN interactions plus three-body forces

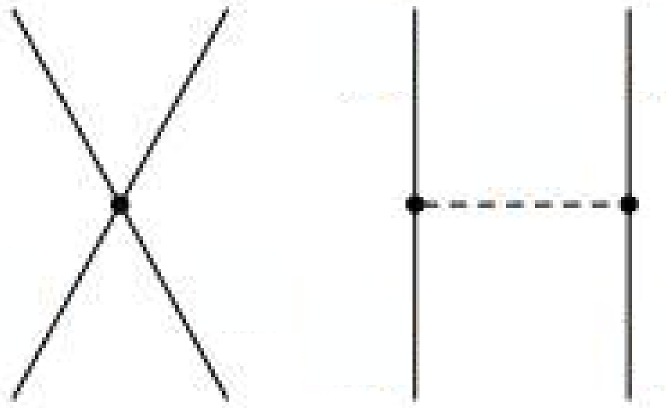
	$3 - \text{body forces}$
N^2LO	
N^3LO	

	$4 - \text{body forces}$
N^3LO	

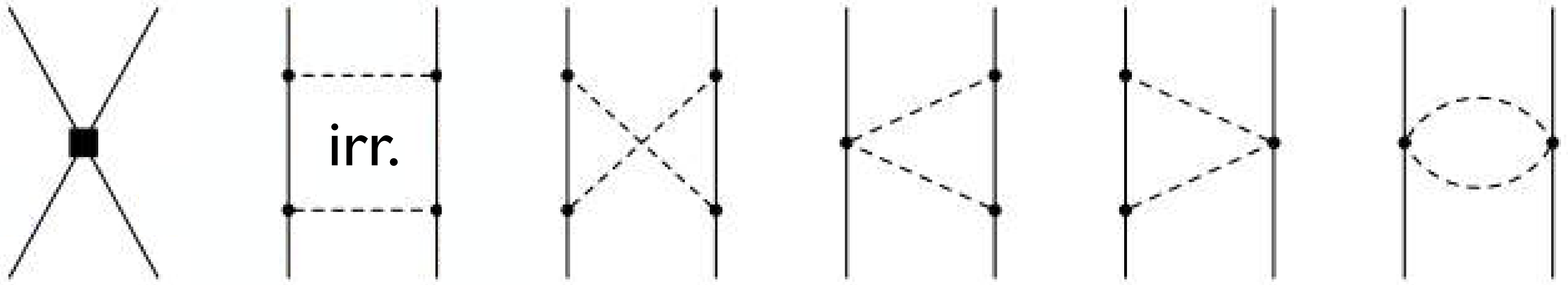
Technische Universität München

Hyperon - Nucleon Interaction
from **CHIRAL SU(3) Effective Field Theory**

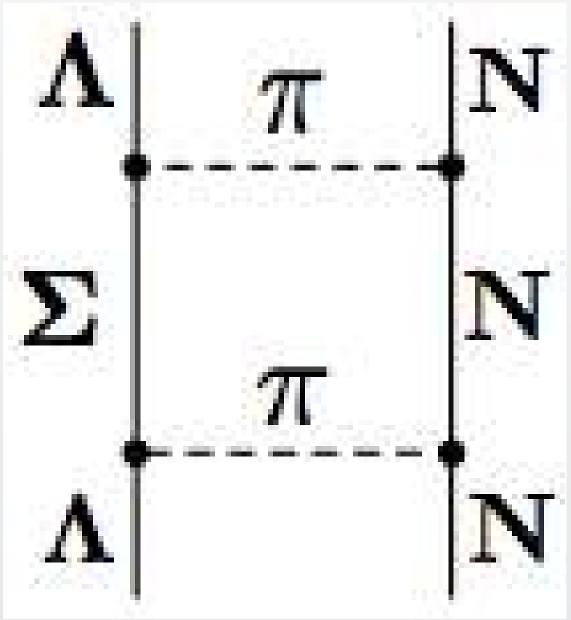
LO :



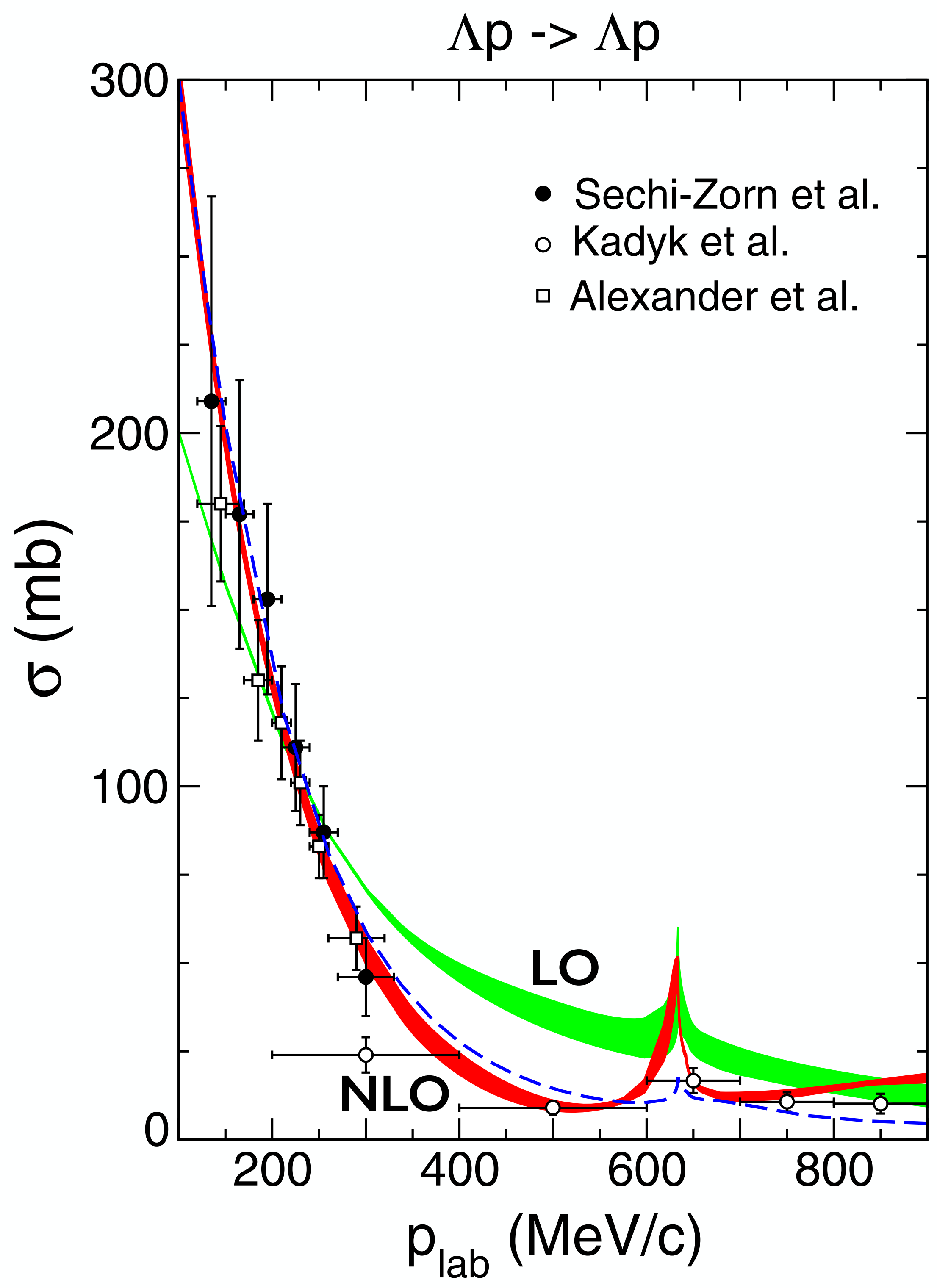
NLO :

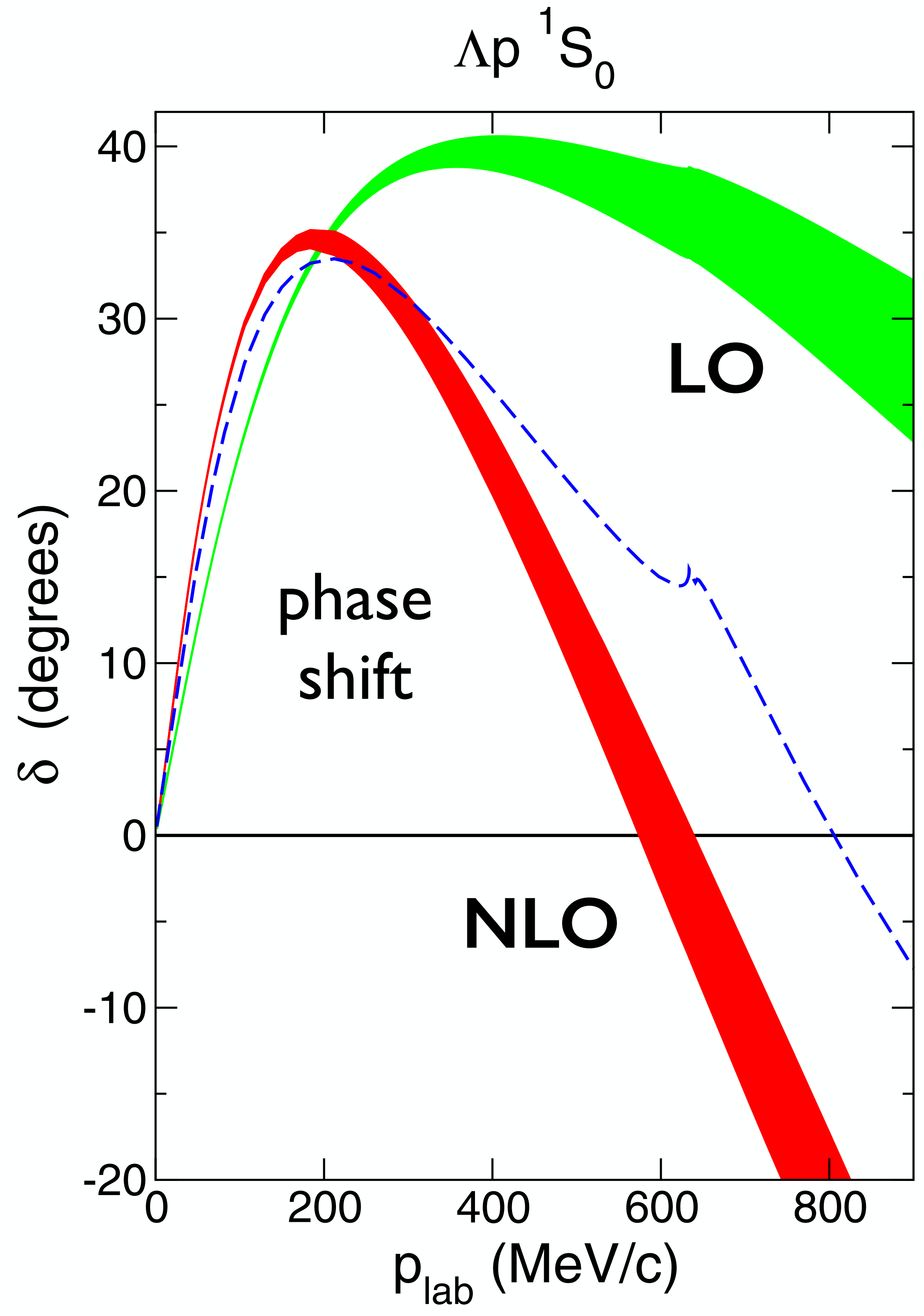


ΛN
scattering



Important role of $\Lambda N \leftrightarrow \Sigma N$ coupled channels





- **moderate attraction** at low momenta
  - → relevant for **hypernuclei**
- **increasing repulsion** at higher momenta
  - → relevant for **dense baryonic matter**

J. Haidenbauer, S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W.W.
Nucl. Phys. A 915 (2013) 24

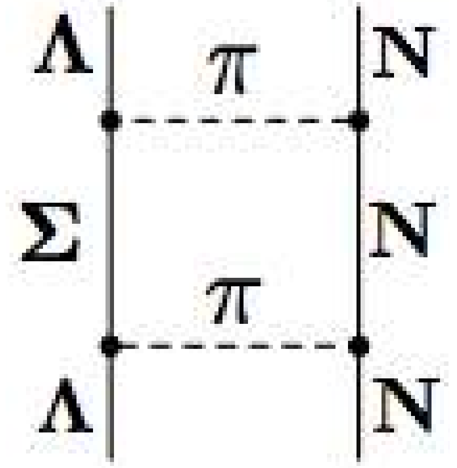
Λ Hyperon - Nucleon Interaction update

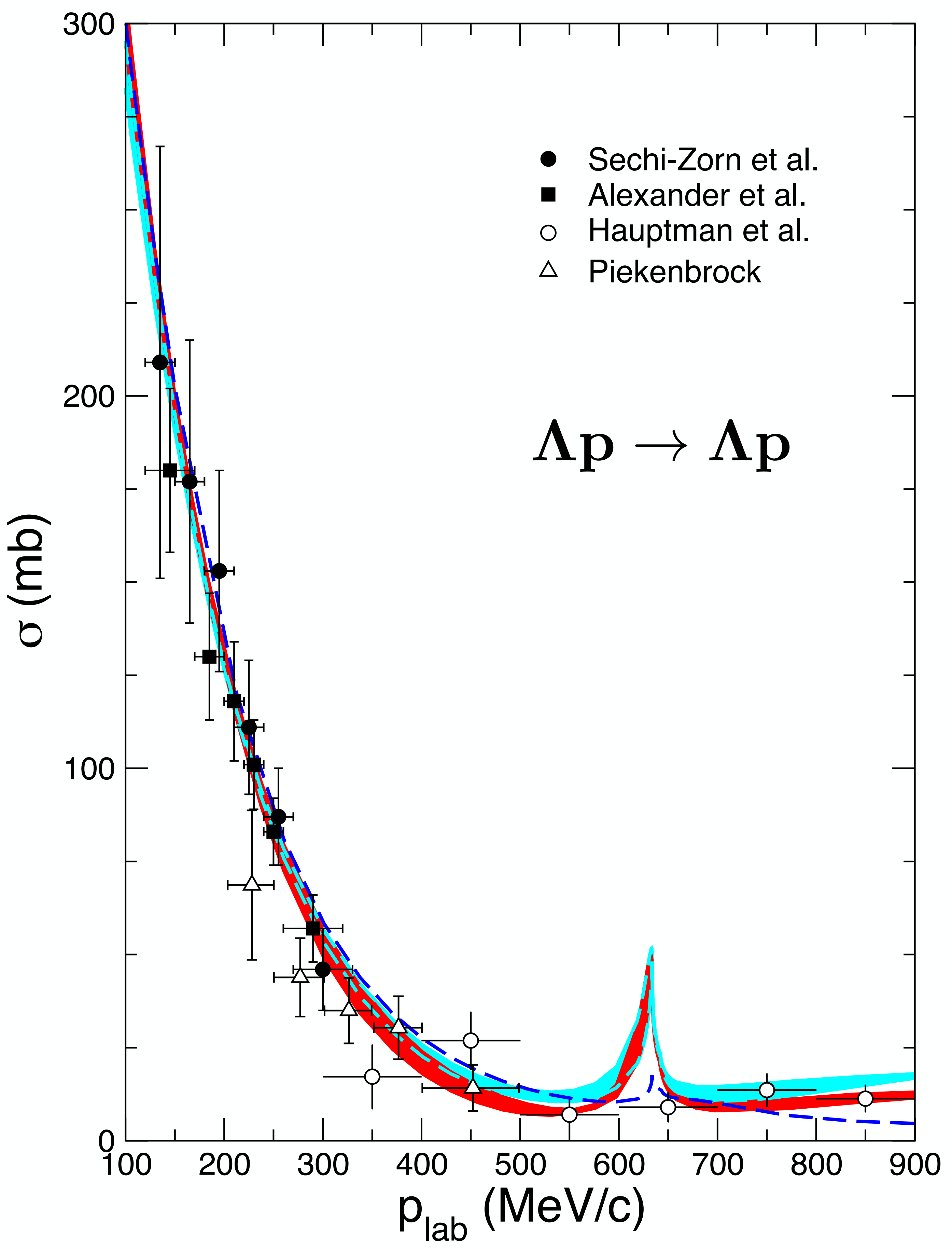
J. Haidenbauer, U.-G. Meißner, A. Nogga Eur. Phys. J. A56 (2020) 91

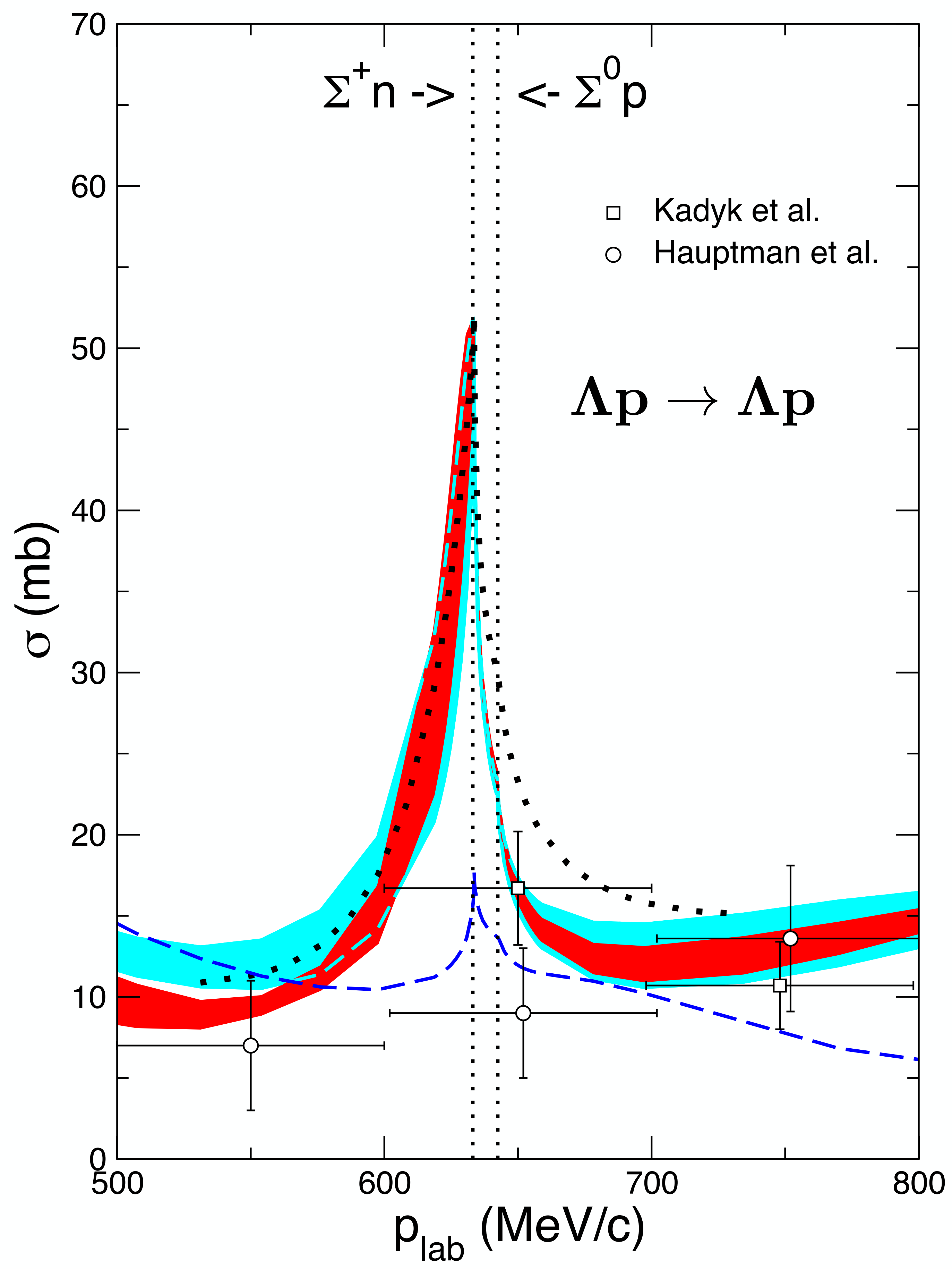
- Reduced no. of independent parameters (contact terms) at NLO by symmetries connecting NN and YN S-waves

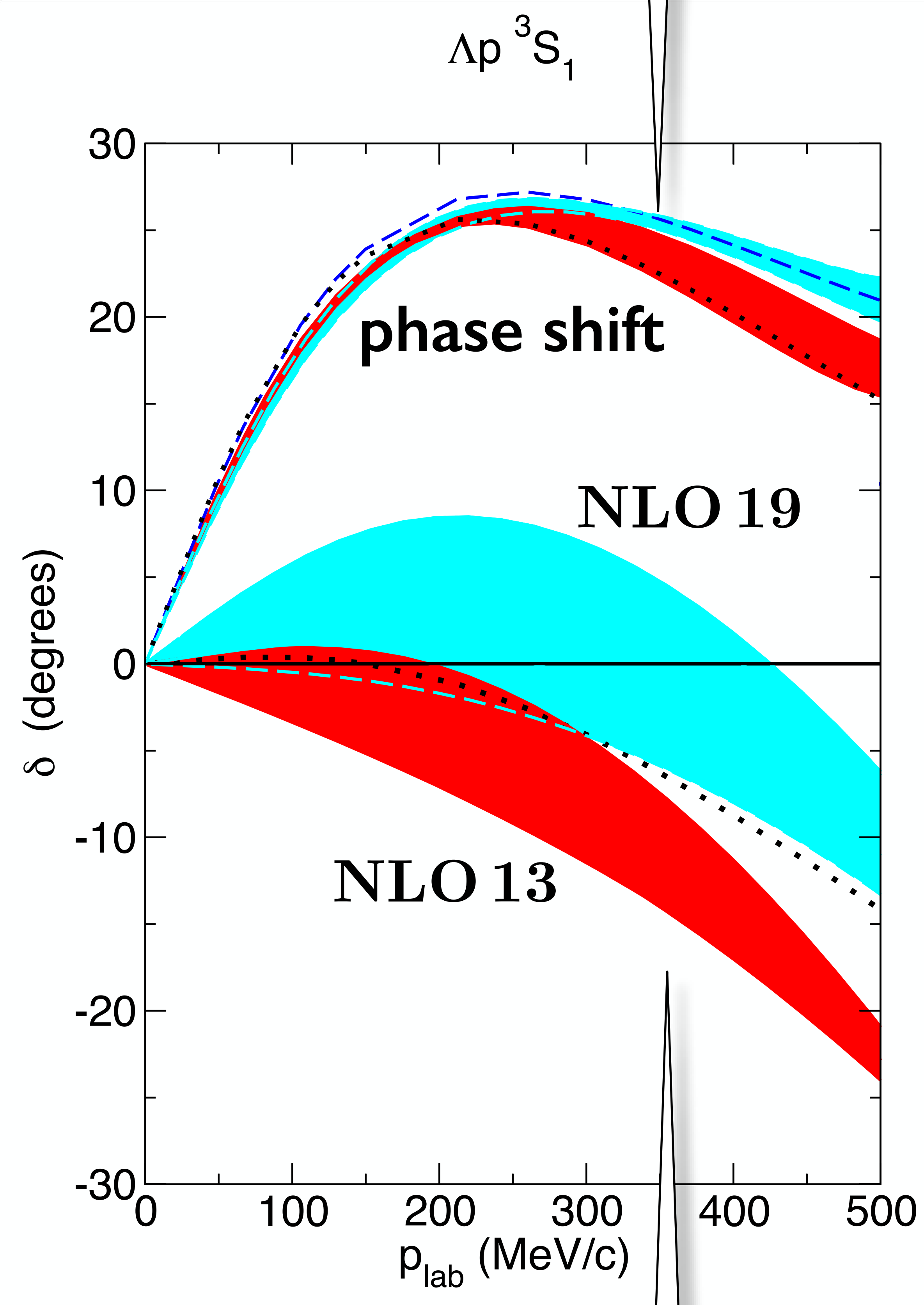
blue : NLO 19
red : NLO 13

including $\Lambda N \leftrightarrow \Sigma N$ coupling







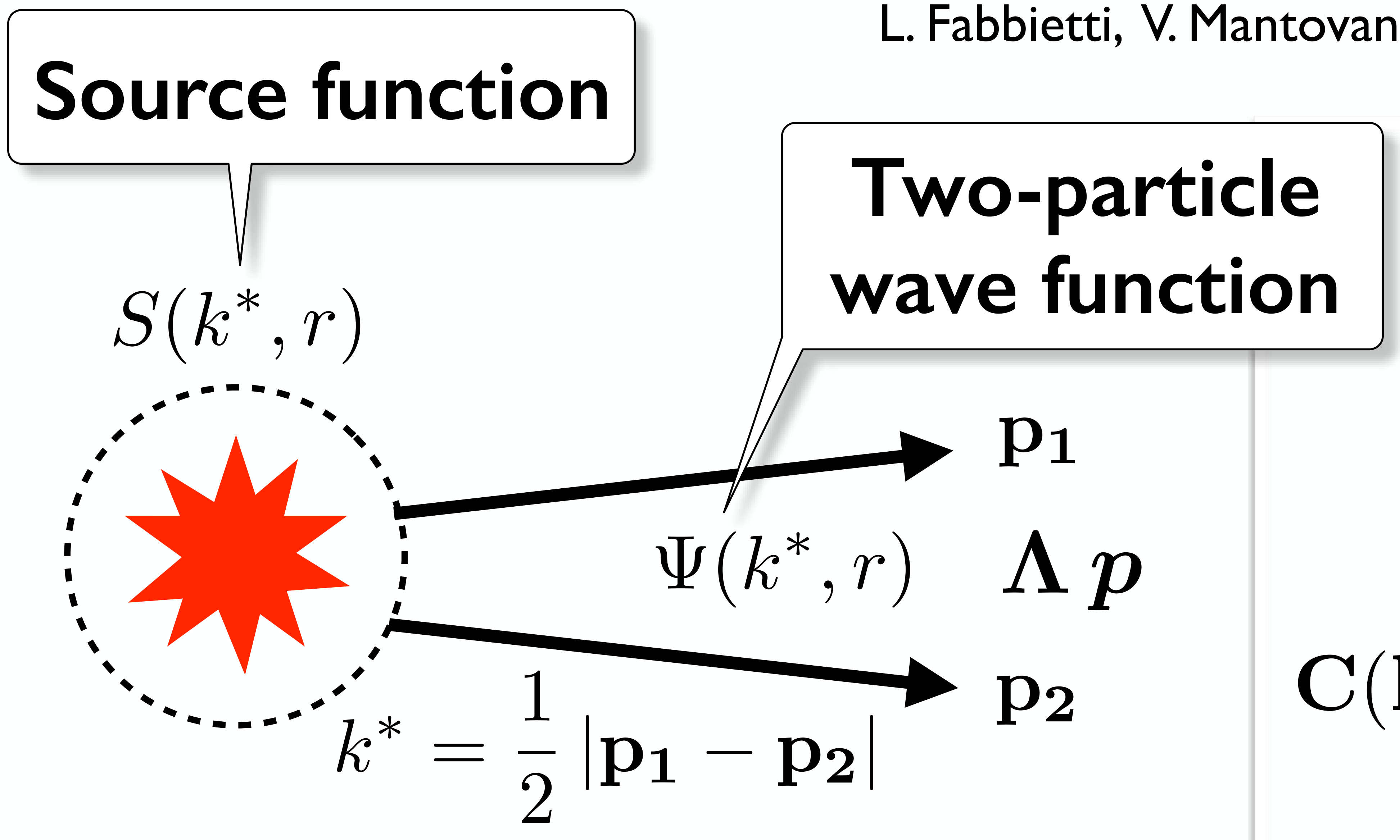


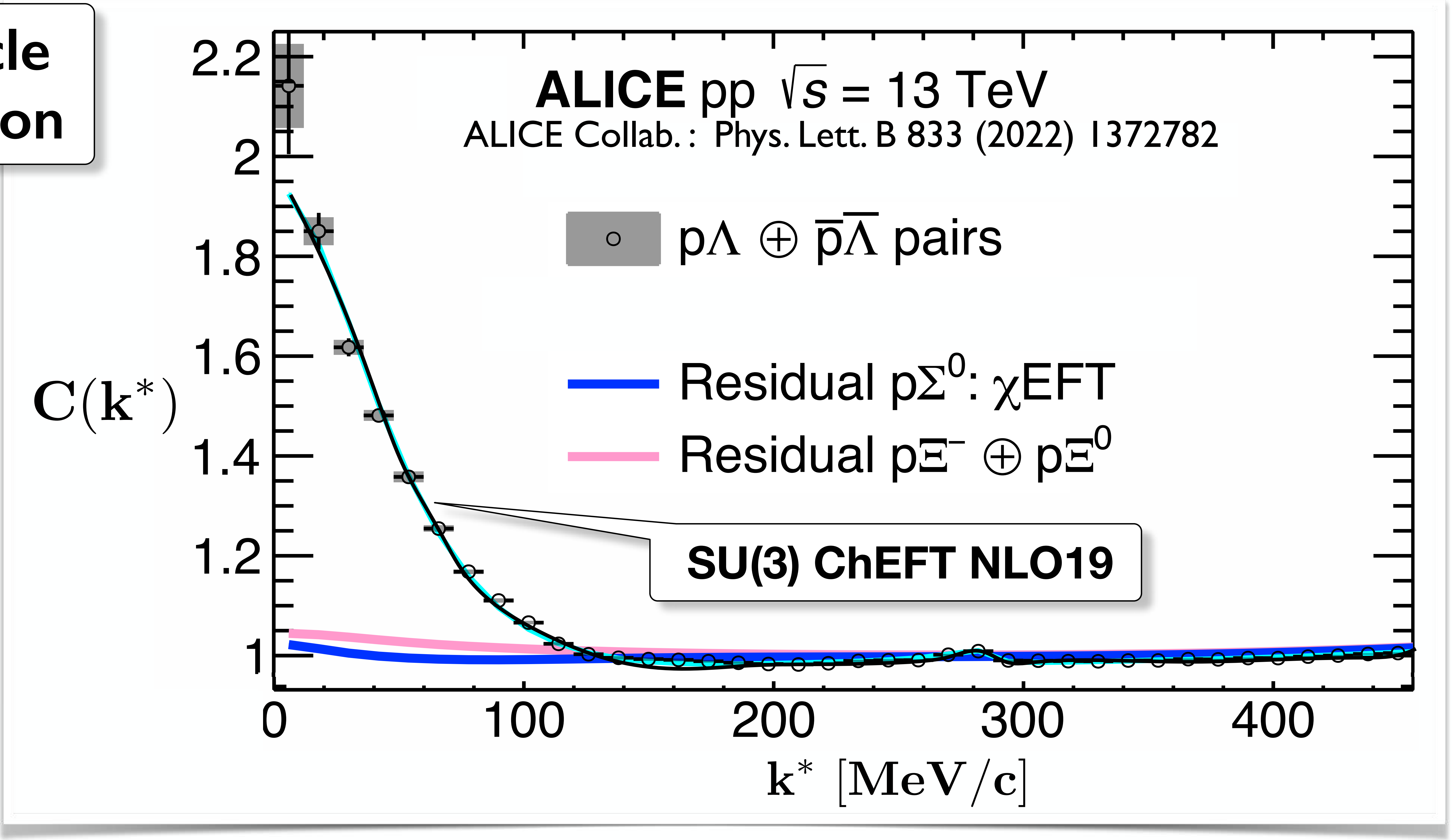
without $\Lambda N \leftrightarrow \Sigma N$ coupling

$\Lambda\, p$ CORRELATION FUNCTION

- Femtoscopy two-particle correlation studies from p p collisions with ALICE @ LHC

L. Fabbietti, V. Mantovani Sarti, O.Vazquez Doce : Ann. Rev. Nucl. Part. Sci. 71 (2021) 377





- Correlation function

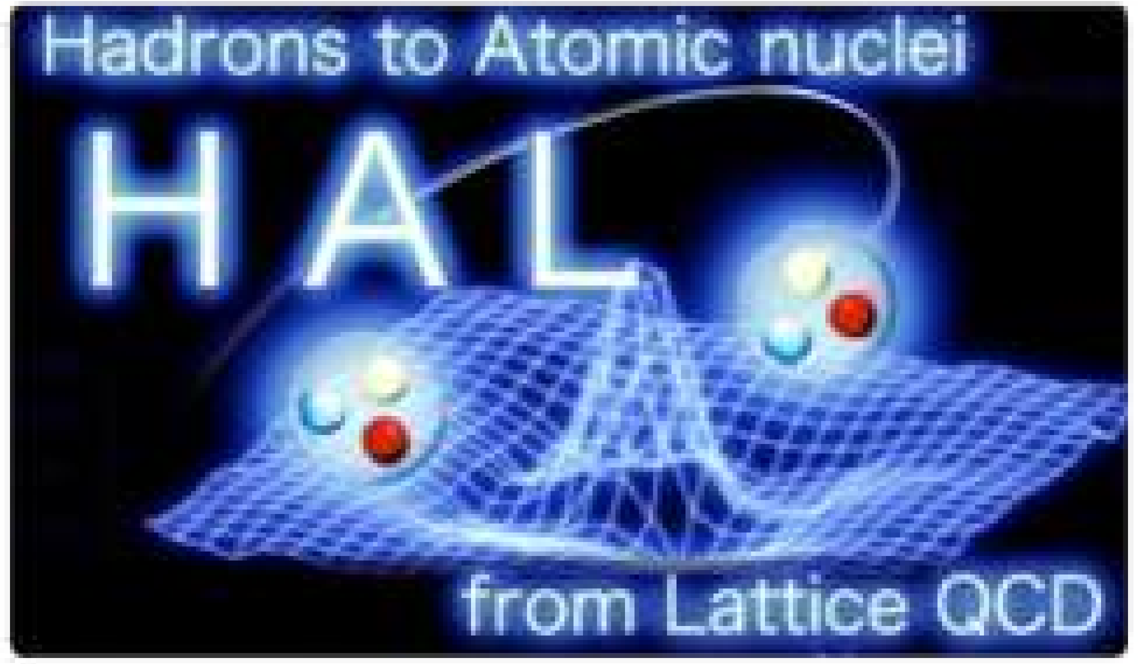
$$C(k^*) = \frac{\langle \mathcal{P}_1(k^*)\mathcal{P}_2(k^*)\rangle}{\langle \mathcal{P}_1(k^*)\rangle\langle \mathcal{P}_2(k^*)\rangle}$$
$$= \int d^3r\, S(k^* r)|\Psi(k^* r)|^2$$

- Accurate test of low-momentum $\Lambda\, p$ interaction

D.L. Mihaylov, J. Haidenbauer, V. Mantovani Sarti, Phys. Lett. B850 (2024) 138550

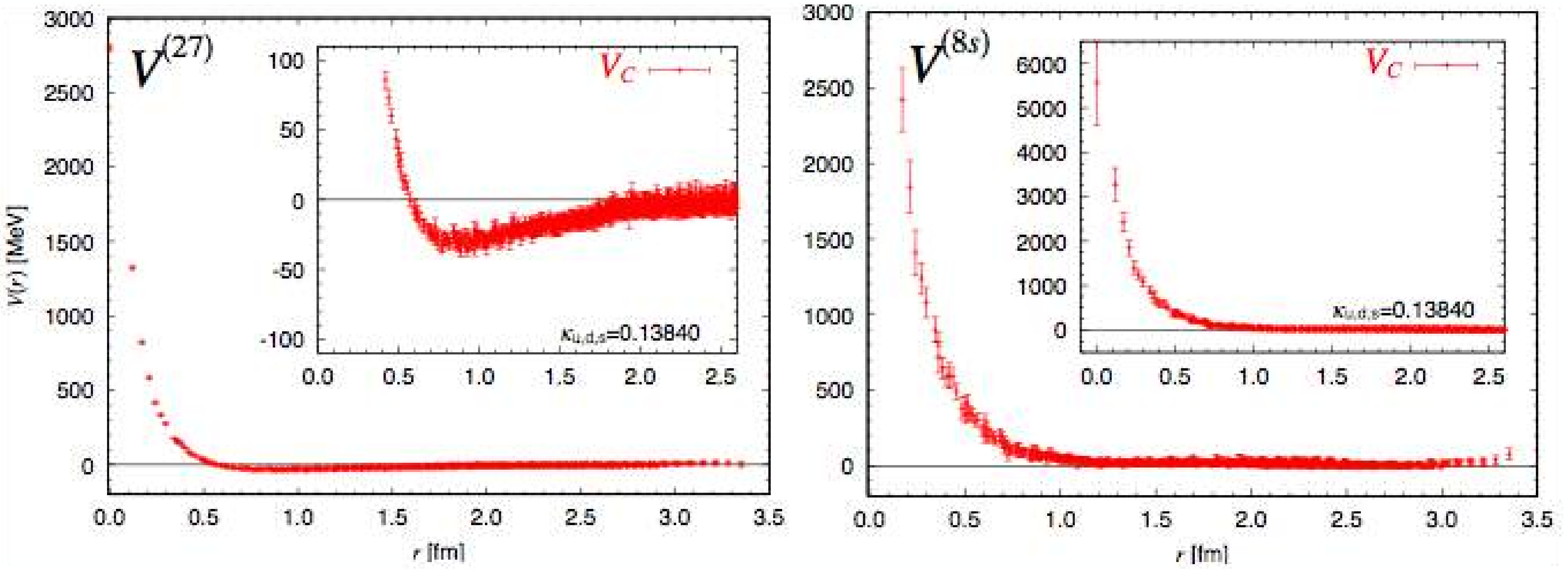
Hyperon - Nucleon Interactions from **Lattice QCD**

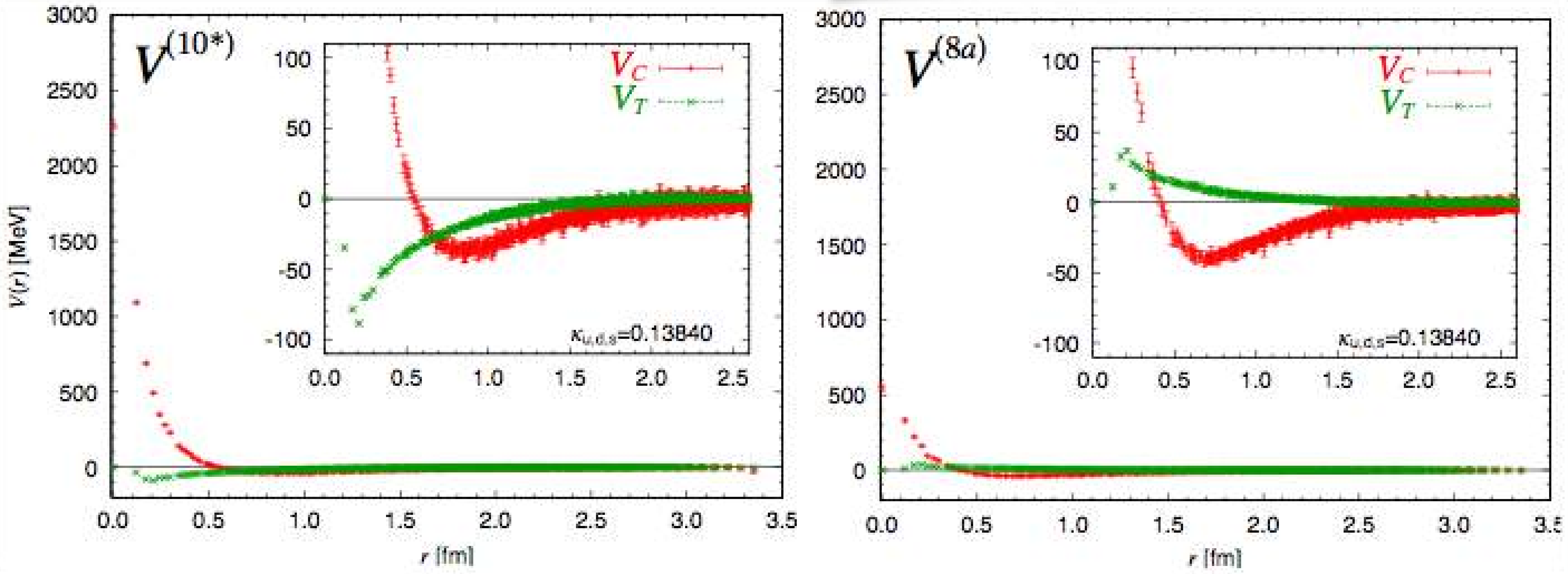
T. Inoue et al.   PTP 124 (2010) 591   Nucl. Phys. A881 (2012) 28



$\mathrm{m}_\pi = \mathrm{m}_K = 0.47\ \mathrm{GeV}$

towards physical quark masses





SU(3) decomposition

$$\Lambda N(^1S_0) = \frac{9}{10}[27] + \frac{1}{10}[8_s]$$

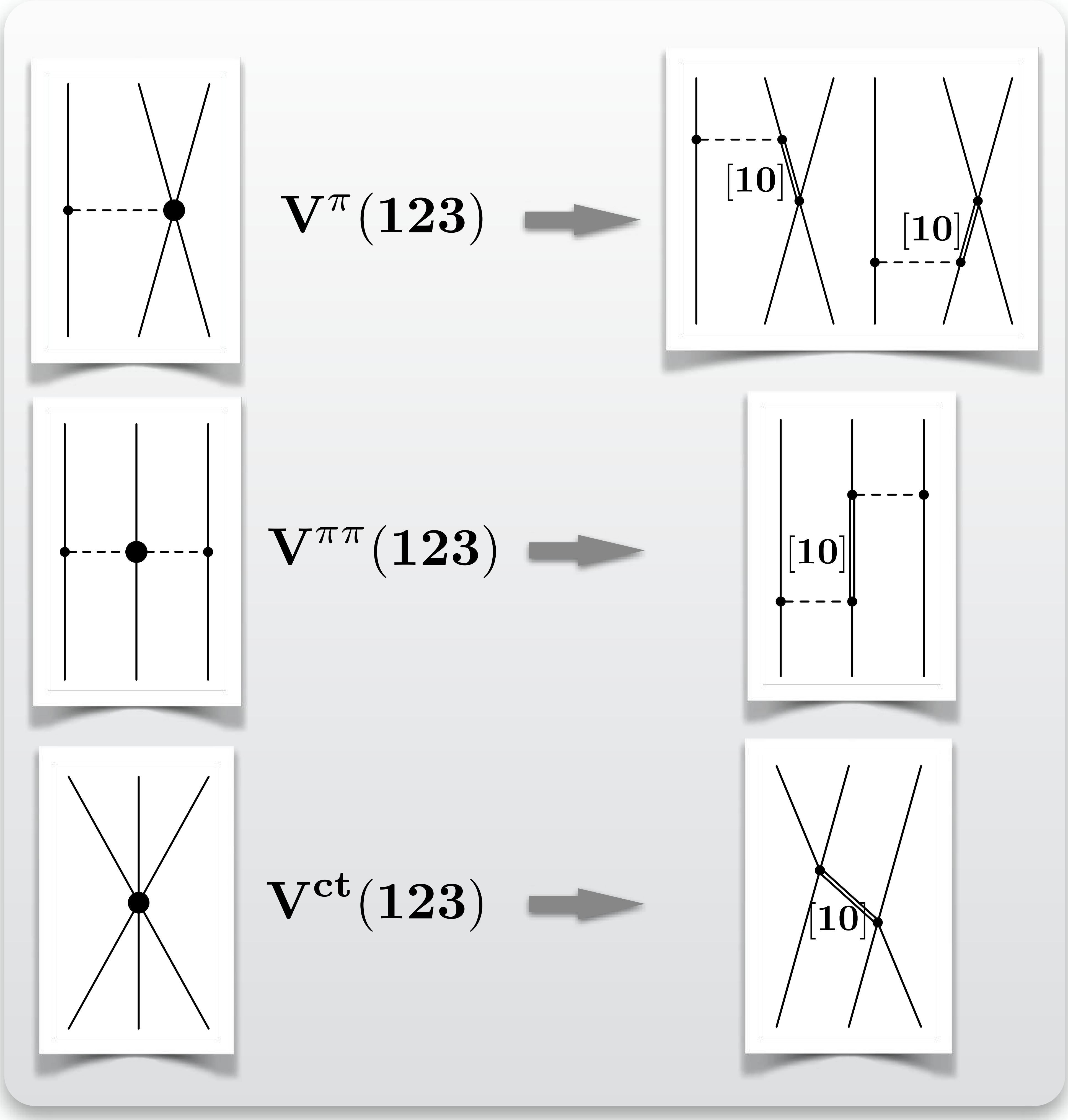
$$\Lambda N(^3S_1) = \frac{1}{2}[10^*] + \frac{1}{2}[8_a]$$

Short-range repulsive core in all channels

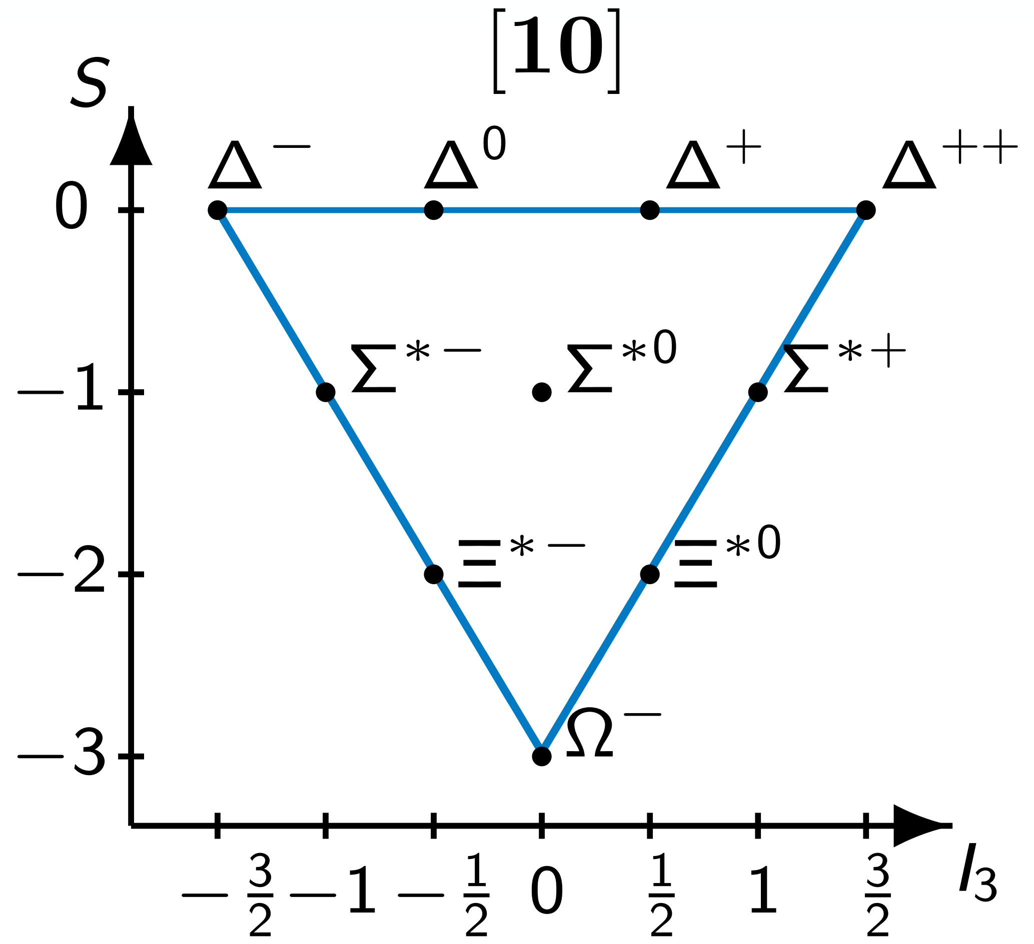
HYPERON-NUCLEON-NUCLEON THREE-BODY FORCES

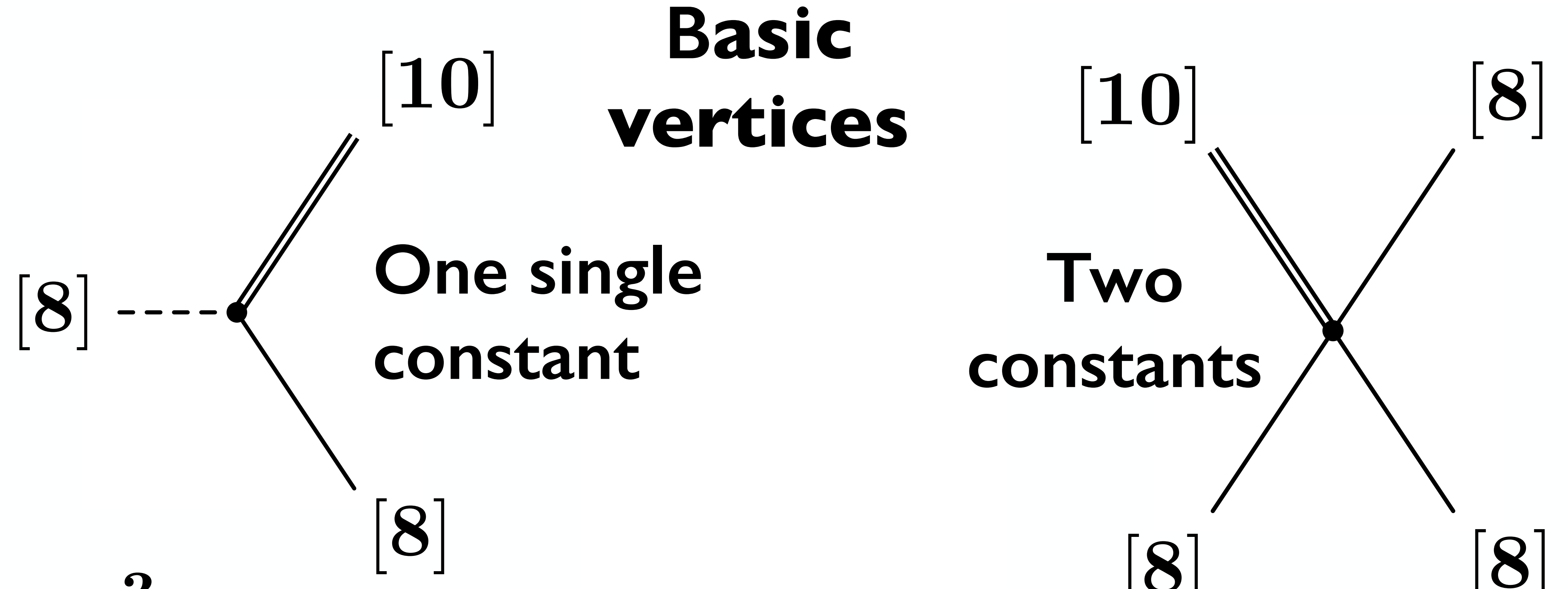
from Chiral $SU(3)_L \times SU(3)_R$ Effective Field Theory

S. Petschauer, N. Kaiser, J. Haidenbauer, U.-G. Meißner, W.W. : Phys. Rev. C93 (2016) 014001



- **Decuplet Dominance** in YNN three-body forces
- Estimates of **YNN** interactions assuming dominant (Σ^*, Δ) intermediate states



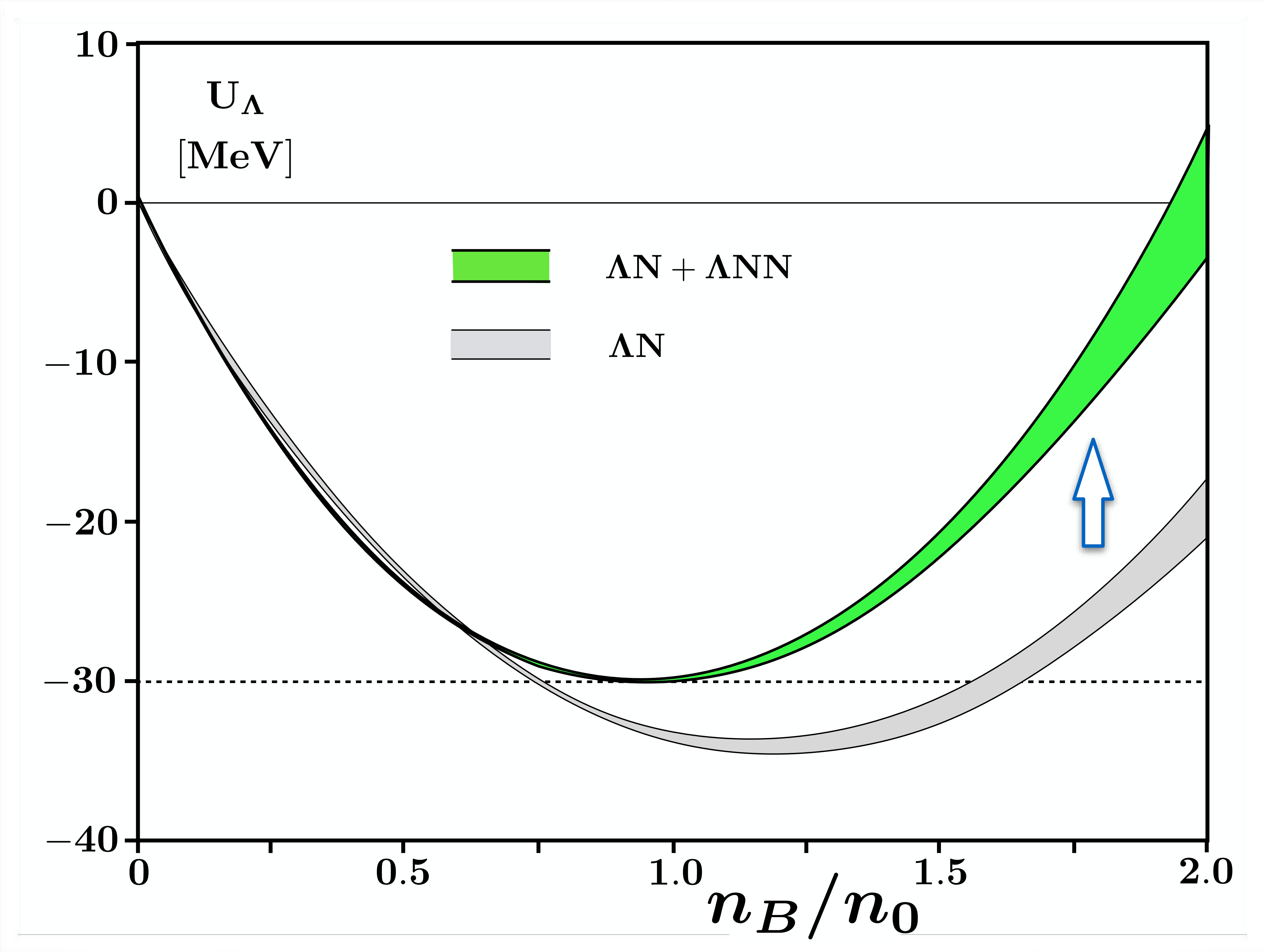


$$C = \frac{3}{4} g_A \simeq 1 \text{ from } \Delta \to \pi N$$

Density dependence of Λ single particle potential

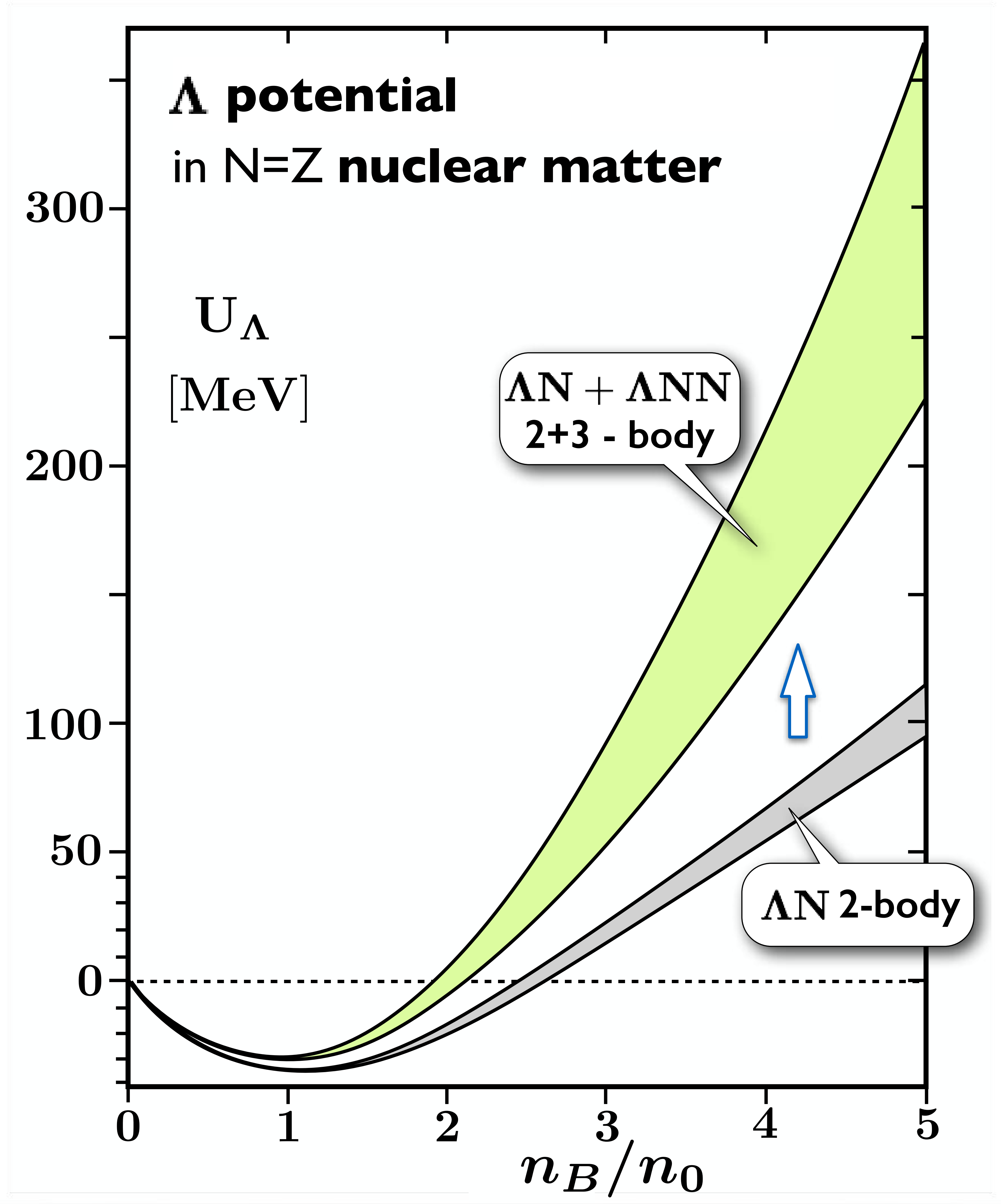
- Coupled-channels G-matrix including $\Lambda NN \leftrightarrow \Sigma NN$
- **Three-body interactions** treated as **density-dependent effective 2-body forces**

D. Gerstung, N. Kaiser, W.W. : Eur. Phys. J. A56 (2020) 175



Chiral
NN (N3LO)
+ YN (NLO)
+NNN+YNN
interactions

Strong additional repulsion from YNN three-body forces

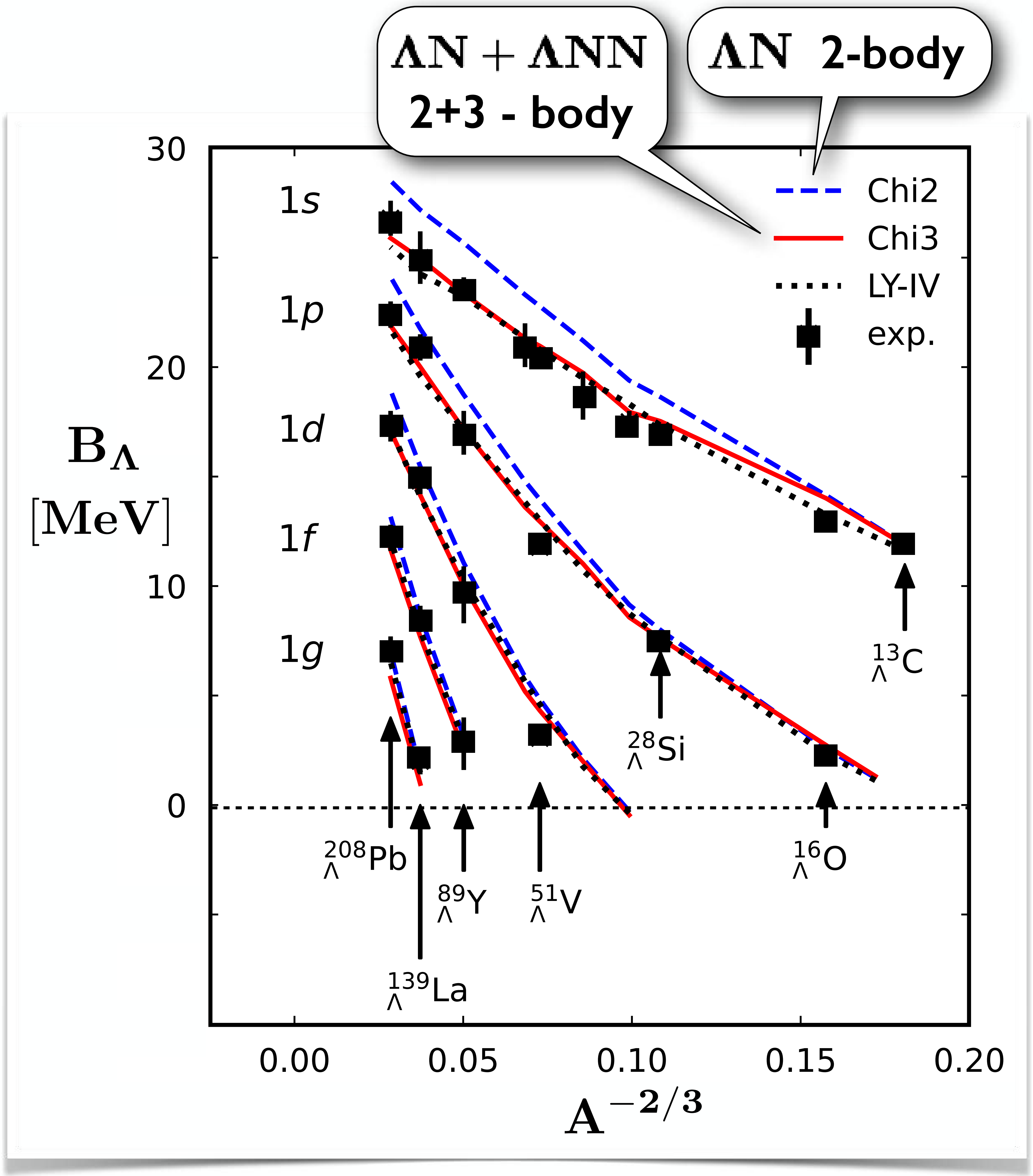


- Constrained by hypernuclear physics : $U_\Lambda(\rho = \rho_0) \simeq -30\,\mathrm{MeV}$

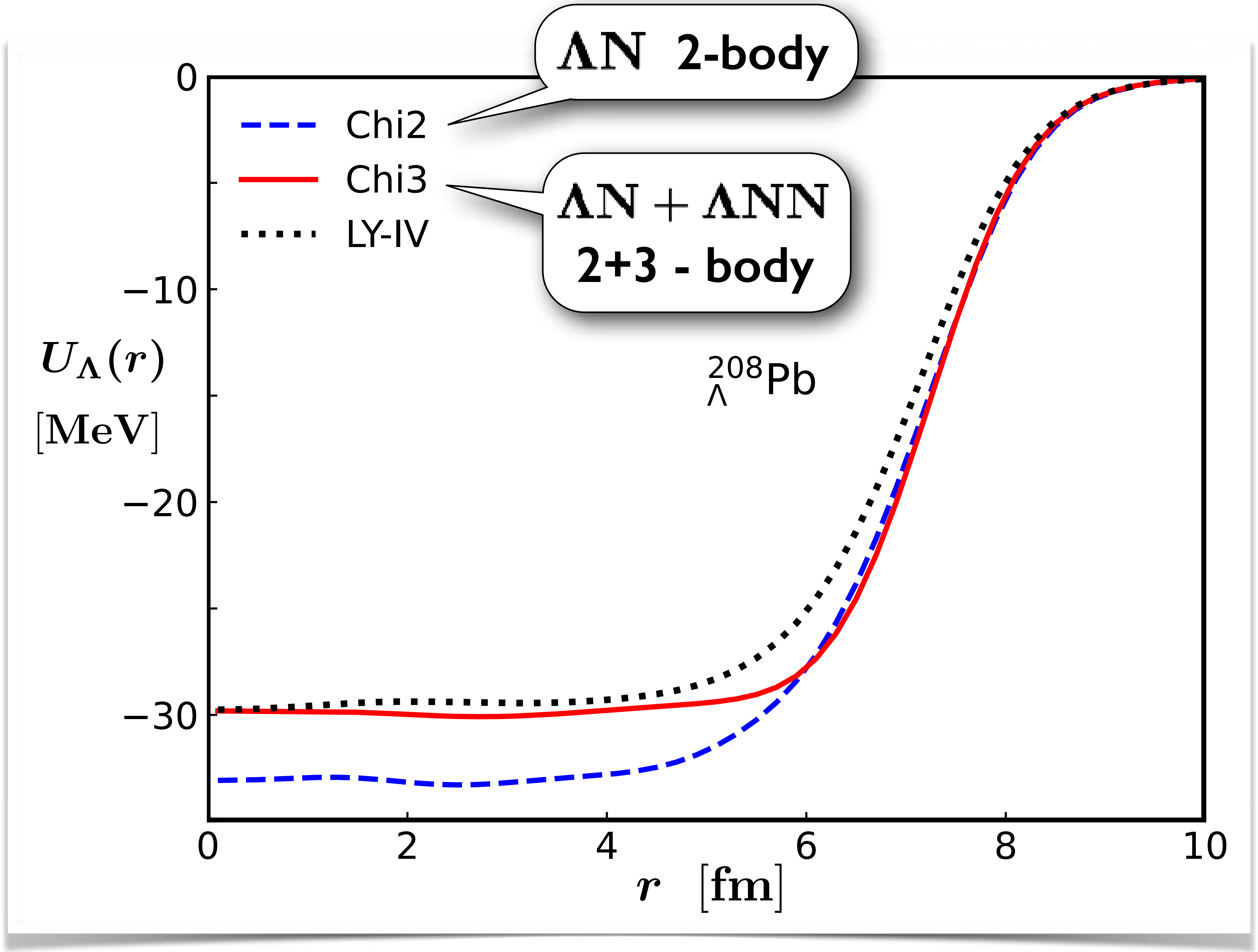
A. Gal, E. Hungerford, D. Millener
Rev. Mod. Phys. 88 (2016) 035004

Λ HYPERNUCLEI

- Translation of **Chiral NN (N3LO) + YN (NLO) + NNN + YNN interactions** into **Skyrme type energy density functional**



A. Jinno, K. Murase, Y. Nara, A. Ohnishi : Phys. Rev. C108 (2023) 065803



- **Overbinding of Λ with only two-body forces**

HYPERNUCLEAR PHENOMENOLOGY

- Recent update on Λ**- nuclear binding energies** including ΛNN **three-body forces**

- Two- and three-body **hyperon-nucleus potentials** using **realistic** (empirically constrained) **density distributions**

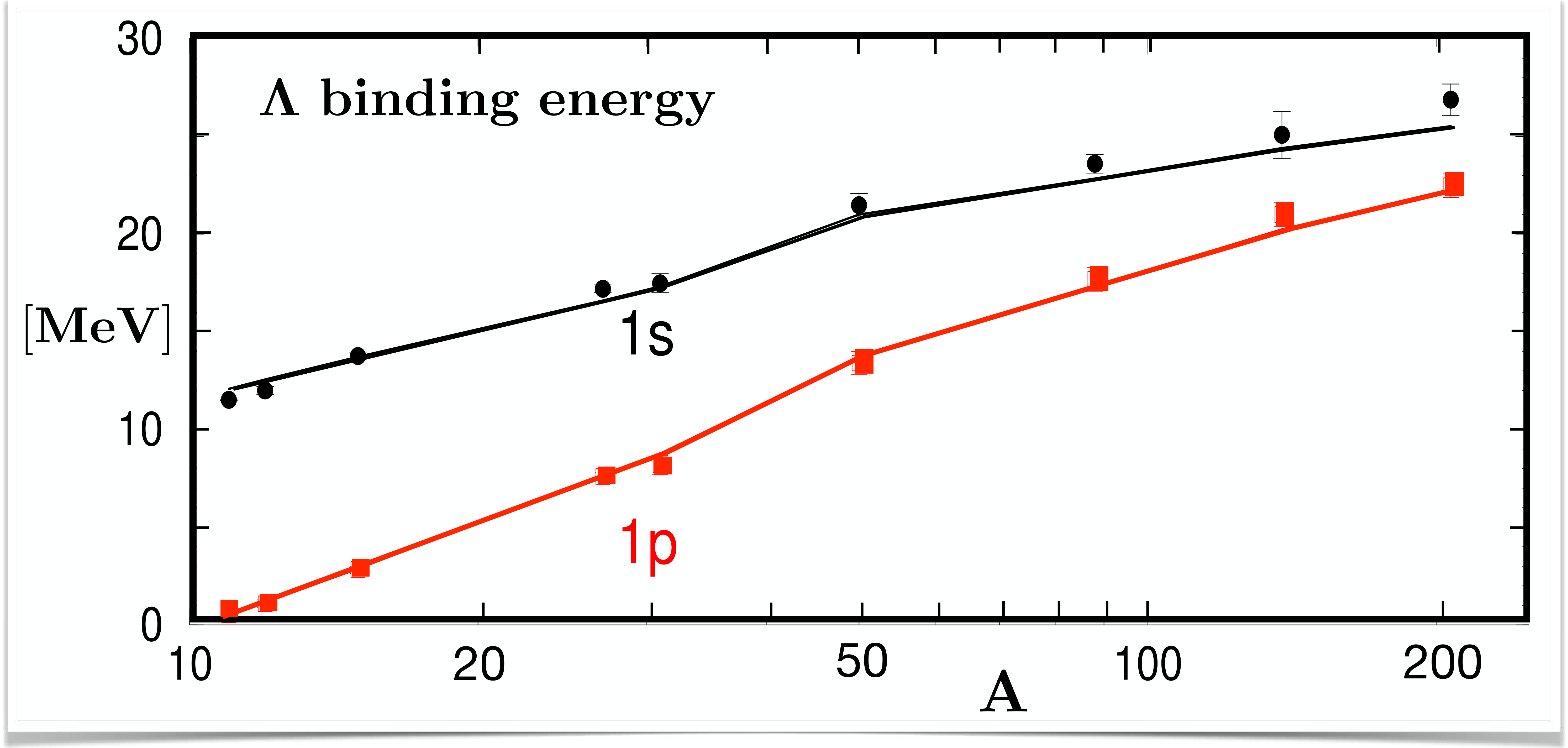
E. Friedman, A. Gal : Phys. Lett. B 837 (2023) 137669 ; Nucl. Phys. A1039 (2023) 122725

$$U_\Lambda(n_B) = U_0^{(2)}(p_F)\frac{n_B(r)}{n_0} + U_0^{(3)}\left(\frac{n_B(r)}{n_0}\right)^2$$

- **Pauli correlations important**

- **Best fit** :

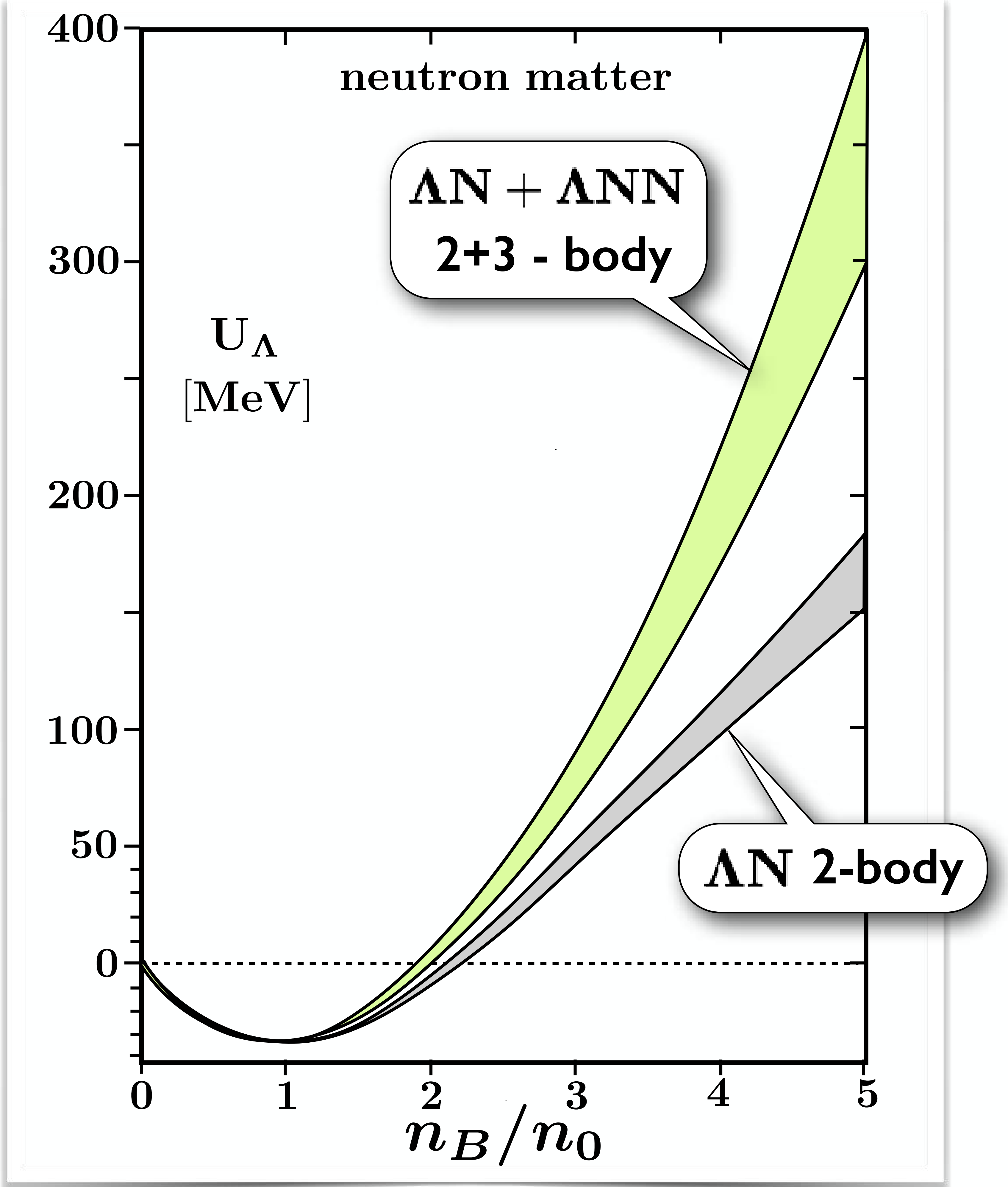
$$U_\Lambda(n_0) = -(27.3 \pm 0.6)\,\text{MeV}$$
$$U_0^{(2)} = -(38.6 \pm 0.8)\,\text{MeV}$$
$$U_0^{(3)} = (11.3 \pm 1.4)\,\text{MeV}$$



- Such **repulsive** ΛNN **three-body forces** can possibly solve the hyperon puzzle in n-stars

Λ HYPERONS in NEUTRON STARS ?

- Onset condition for appearance of Λ hyperons in neutron stars : Equality of chemical potentials



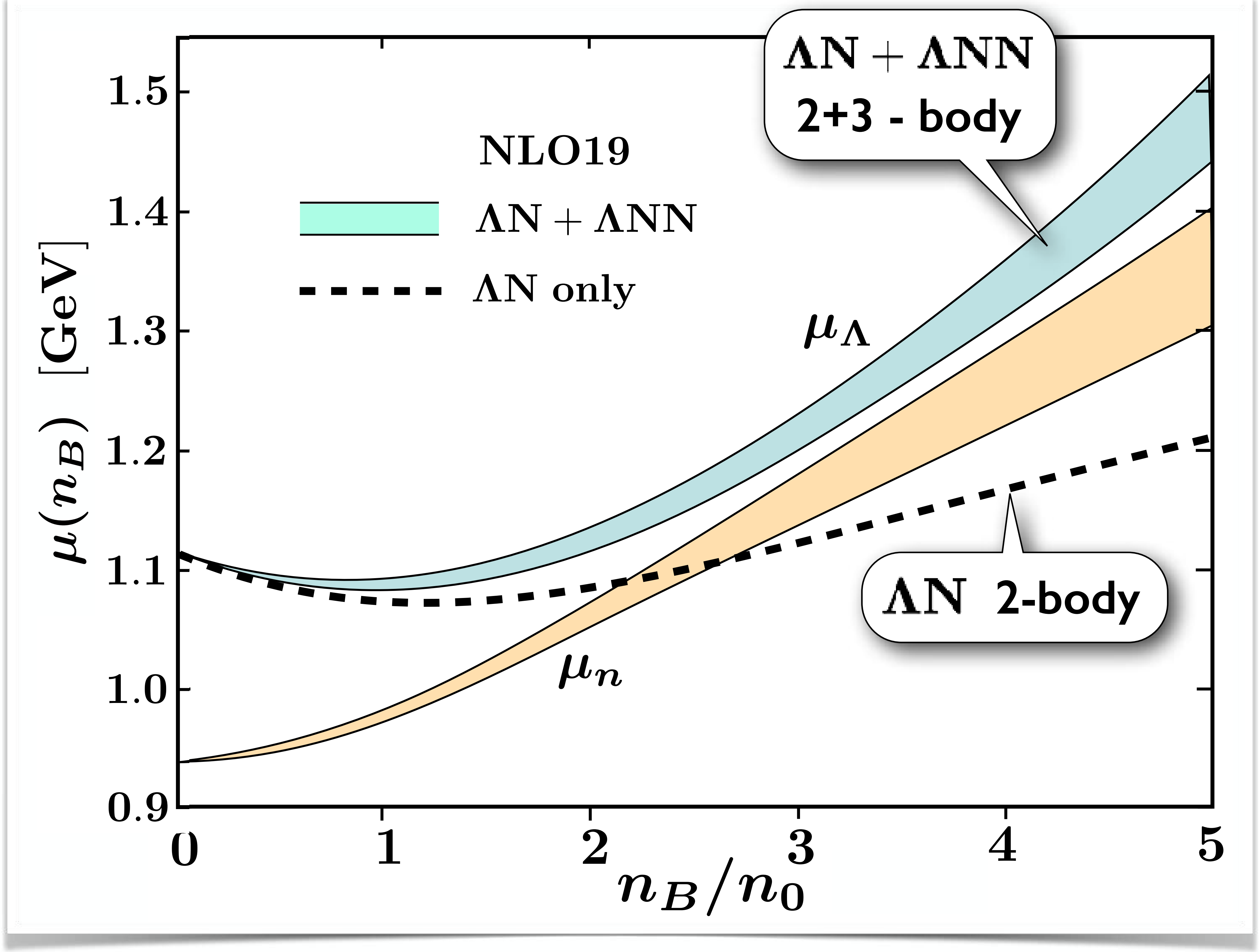
- **Hyperon chemical potential** in neutron star matter from **Chiral SU(3) EFT** interactions

D. Gerstung, N. Kaiser, W.W.
Eur. Phys. J. A56 (2020) 175

- **Neutron chemical potential** in neutron star matter from **Chiral EFT + FRG EoS**

M. Drews, W.W.
Prog. Part. Nucl. Phys. 93 (2017) 69

$$\mu_\Lambda = \mu_n \qquad \mu_i = \frac{\partial \varepsilon}{\partial n_i}$$



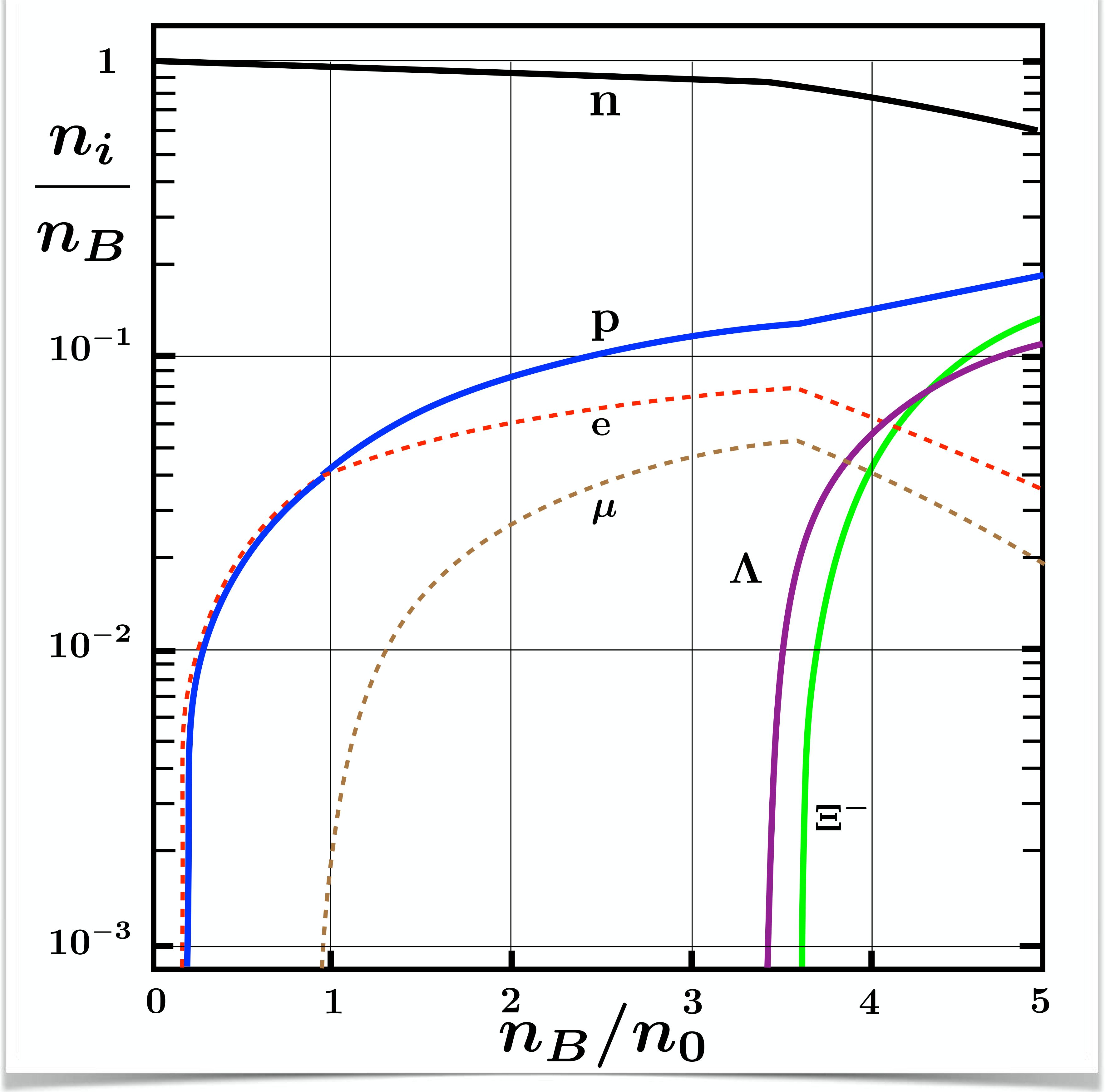
NEUTRON STAR MATTER including HYPERONS

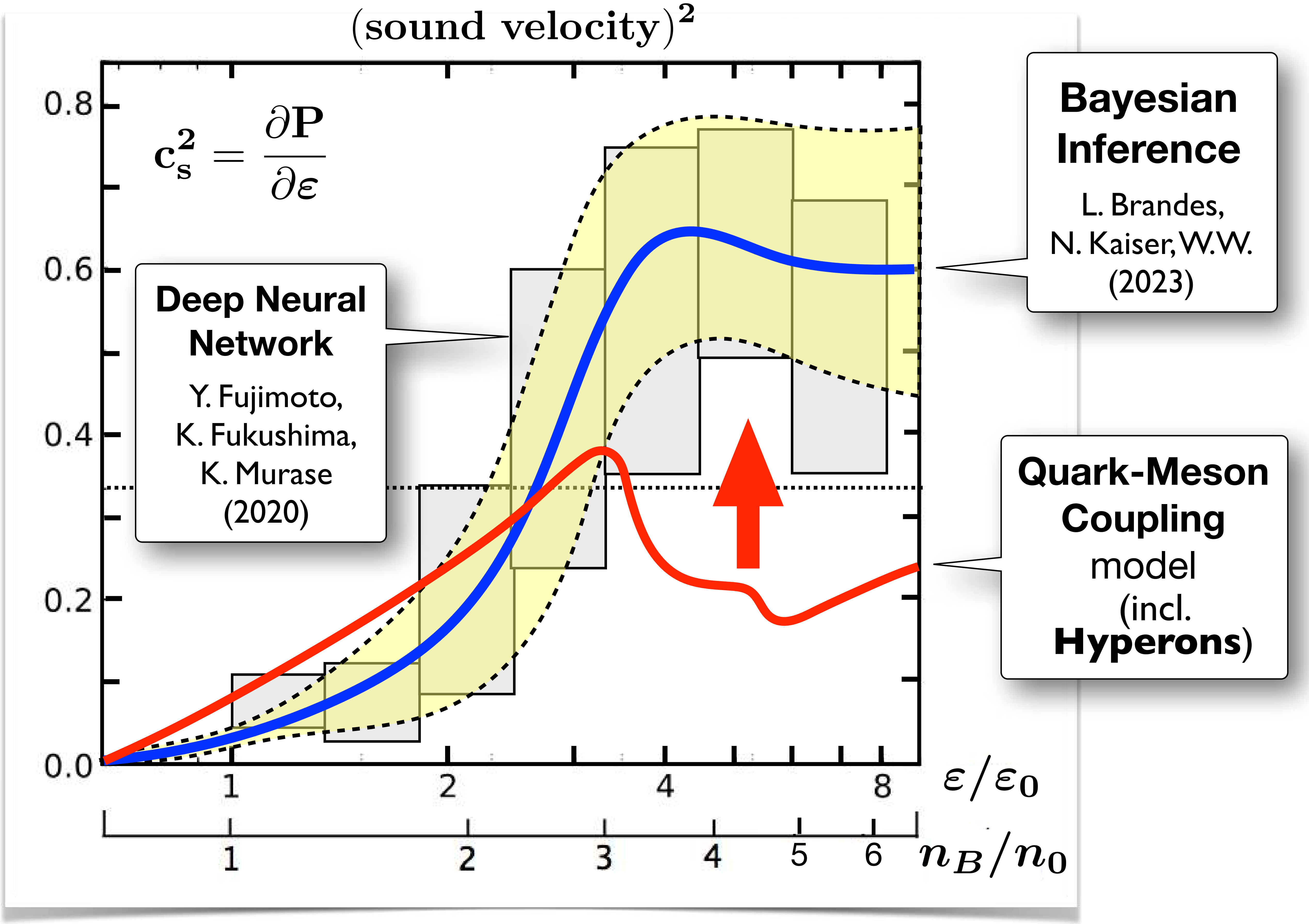
NO "Hyperon Puzzle"?

- **Quark - Meson Coupling model** $M_B^* = M_B^{(0)} - g_B\,\bar{\sigma} + \frac{d_B}{2}(g_B\,\bar{\sigma})^2$

T.F. Motta, P.A.M. Guichon, A.W. Thomas
Nucl. Phys. A1009 (2021) 122157

- **Effective in-medium baryon masses including non-linear dependence on σ field, with scalar polarizability d_B representing e.g. effects of three-body forces**





Strangeness **CONCLUSIONS** and **OUTLOOK**

- Key to **strangeness** in **neutron stars** :
  - → **balance** between **hyperon-nuclear 2-** and **3-body forces**
  - → **overbinding** in **hypernuclei** by two-body interactions compensated by **repulsive** hyperon-nuclear three-body forces
- **Equation-of-state** of **neutron star matter** :
  - → even **stiffer** than previously expected $(\mathrm{M_{max}} \simeq 2.3\,\mathrm{M_\odot})$
  - → increasingly repulsive hyperon-nuclear many-body forces can prevent appearance of hyperons in neutron star cores
- Further insights and constraints expected :
  - → expanded high-statistics **YN two-body data base**
  - → improved **high-resolution hypernuclear spectroscopy**
  - → growing quantity and quality of **astrophysical data** focus on EoS and **speed of sound** in **neutron stars**