Two-color QCD as a laboratory of cold and dense matter: Chiral effective model approach

Daiki Suenaga (KMI, Nagoya U.)

Suenaga-Murakami-Itou-Iida; Phys.Rev.D 107, 054001 (2023)
Kawaguchi-**Suenaga**; JHEP 08, 189 (2023)
Suenaga-Murakami-Itou-Iida; Phys.Rev.D 109, 074031 (2024)
Kawaguchi-**Suenaga**; Phys. Rev. D 109, 096034 (2024)
Fejos-**Suenaga**, in preparation,
etc.

my recent series of linear sigma model study on dense two-color QCD

Introduction

- **What is two-color QCD (QC_2D)?** = **Strong interaction with** $N_c = 2$

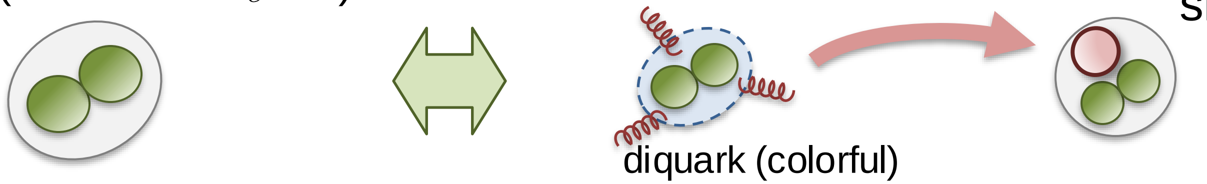
then

- Diquarks turn to be color-singlet baryons → well-defined!

diquark (hadron for $N_c = 2$)

for $N_c = 3$

singly heavy baryon (SHB) as a hadron



diquark (colorful)

- Diquark baryons and mesons are treated in a unified way

diquark baryon

meson

symmetric

$2 \simeq 2^*$:pseudoreality of color SU(2)

→ Chiral symmetry (flavor structure) is extended to $SU(2N_f)$ from $SU(N_f)_L \times SU(N_f)_R$

⋮

- **Why two-color QCD (QC_2D)?**

  - Useful to extract information of **singly heavy baryon (SHB) spectrum** from the viewpoint of chiral symmetry and $U(1)_A$ anomaly

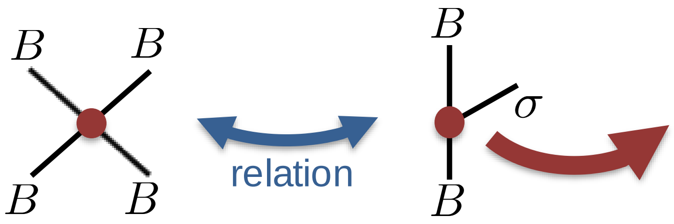
$0^{\pm}, 1^{\pm}, \cdots$ $\Lambda_c(1/2^+), \Sigma_c(1/2^+, 3/2^+), \Lambda_c^*(1/2^-, 3/2^-), \cdots$

$N_c = 2$ world $N_c = 3$ world

Dynamics of only  is important based on the heavy-quark effective theory (HQET) (“brown mock” picture)

  - The extended $SU(2N_f)$ symmetry doesn’t matter for the above motivation, since it just relates couplings among diquarks and mesons

eg

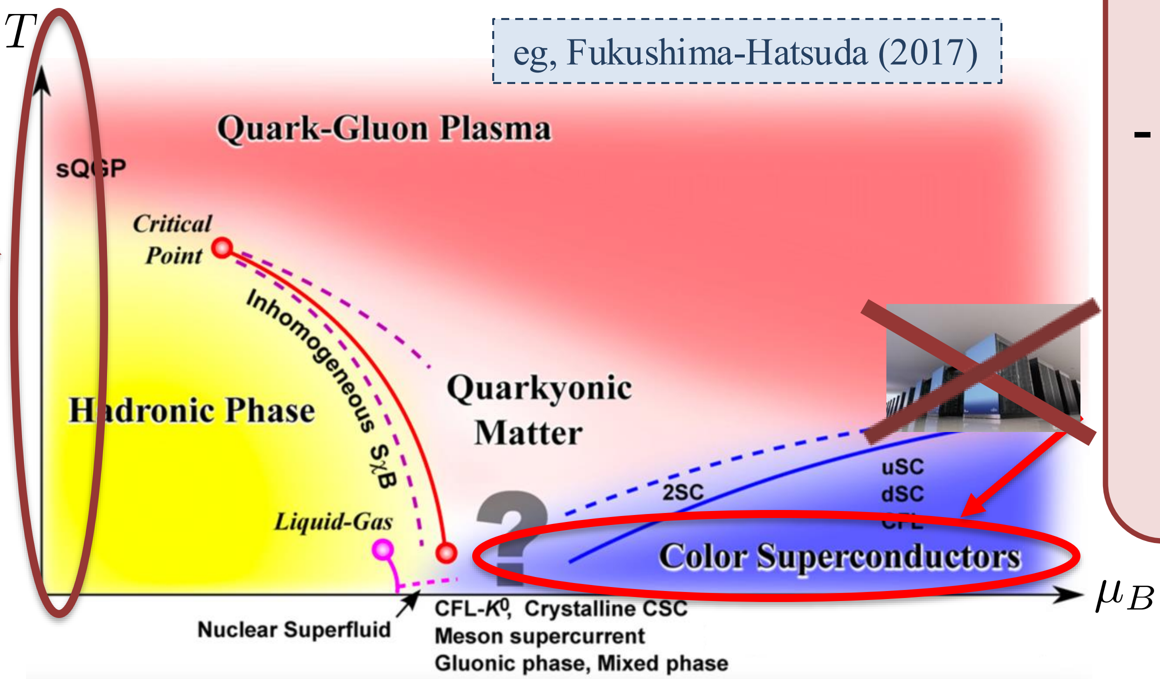


- From the viewpoint of mass generation, only this σBB coupling is important *regardless of the couplings relations*
- $U(1)_A$ anomaly *universally exists regardless of # of colors*

Introduction

- **Why two-color QCD (QC_2D)?**

Lattice QCD



For $N_c = 3$

- **Lattice QCD cannot apply due to the *sign problem***

⇨ **frontier of QCD** from lattice simulation point of view

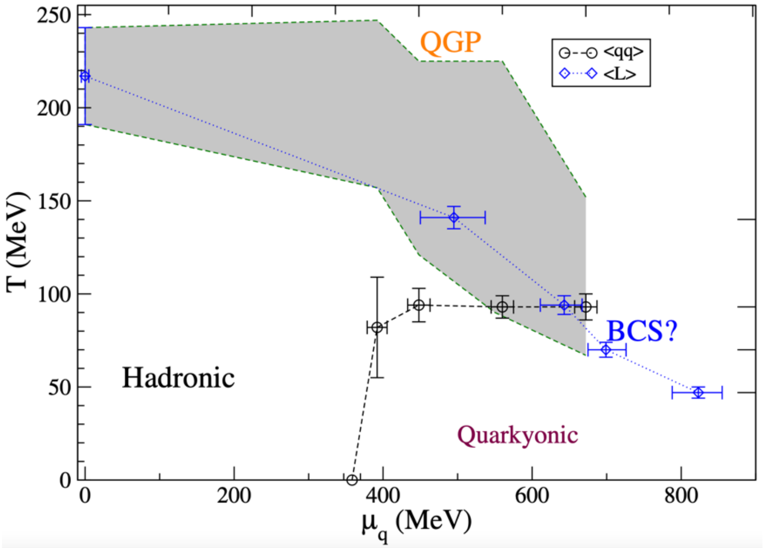
eg, G Aarts (2016), K. Nagata (2022).

⇔ In QC_2D world, the lattice simulation is possible thanks to the pseudoreality of $SU(2)_c$

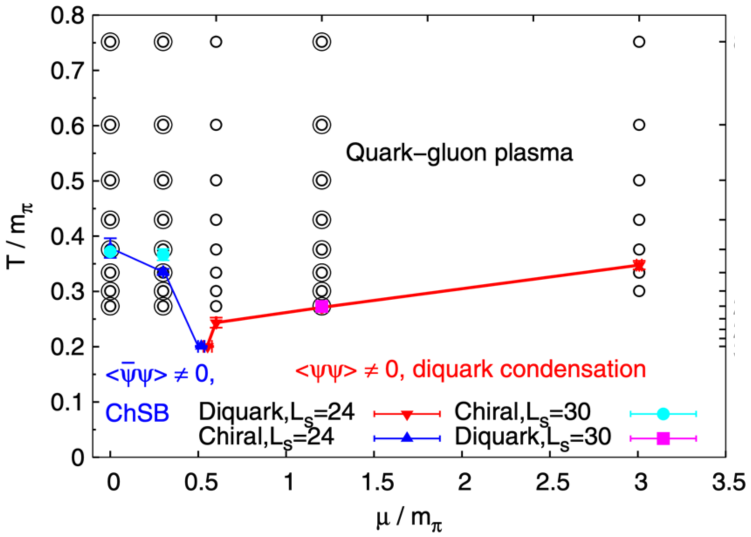
= **noteworthy advantage of QC_2D**

- **Phase diagram in QC_2D**

  - Examples of simulation results of phase diagram in QC_2D



Boz-Cotter-Fister-Mehta-Skullerud (2013)



Buividovich-Smith-Smekal (2020)

. . .

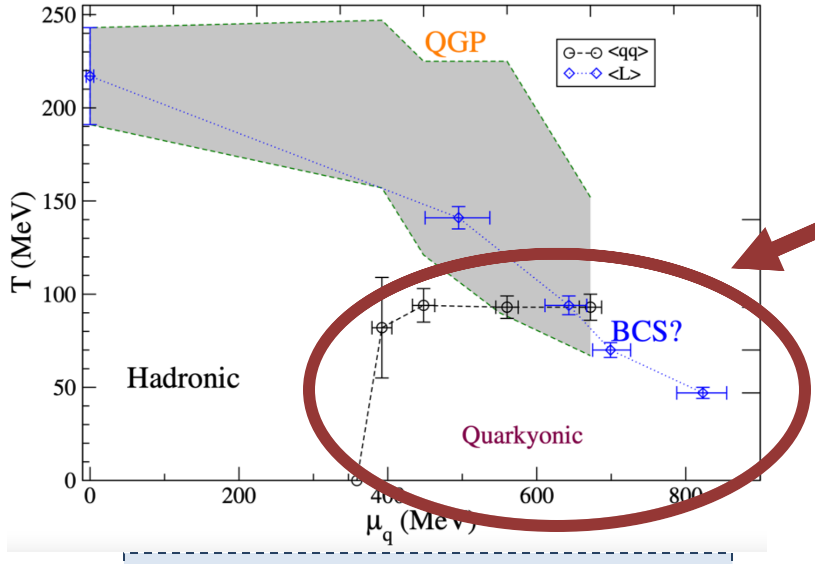
- Ireland/UK group (Hands, Skullerud, …)
- UK group (Buividovich, ...)
- Russian group (Bornyakov, ...)
- Japanese group (Iida-san, Itou-san, …), (+Nonaka-san)

etc.

- **Phase diagram in QC_2D**

  - Examples of simulation results of phase diagram in QC_2D

Baryon superfluid phase
(diquark condensed phase) $\langle\psi\psi\rangle \neq 0$



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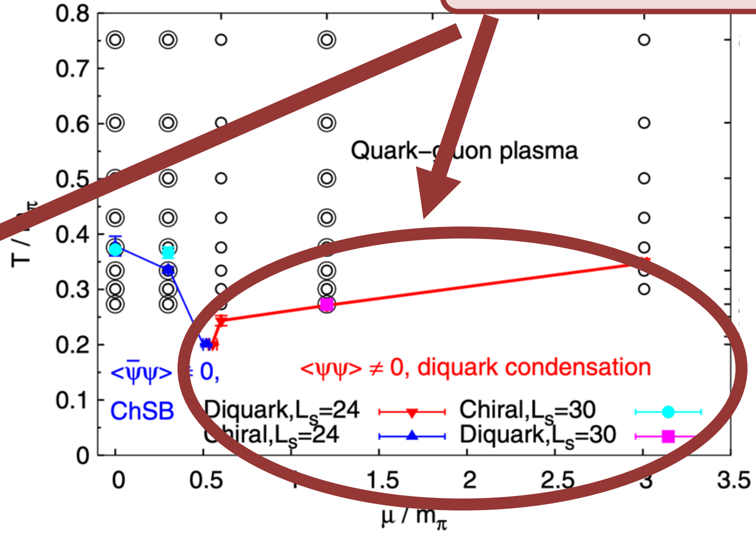

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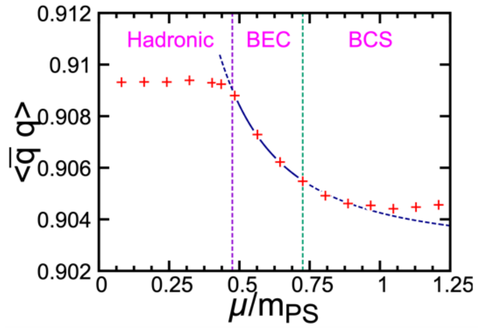
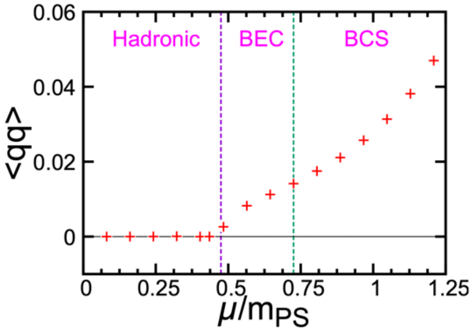
  (+Nonaka-san, …)

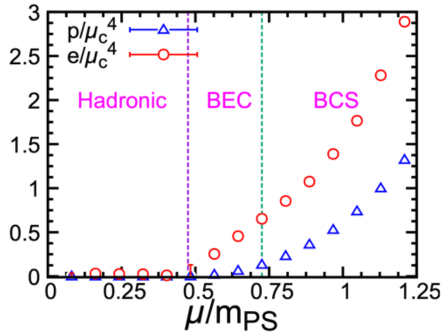
• Lattice results

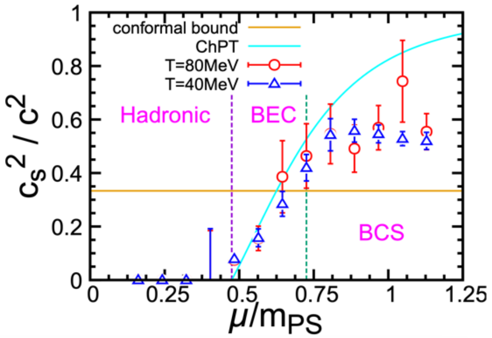
- In addition to phase diagram, hadron mass spectrum, gluon propagator, transport coefficient, EoS, sound velocity, $\langle\bar{\psi}\psi\rangle, \langle\psi\psi\rangle, \langle L\rangle$, etc. have been simulated

eg, Japanese group results









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<u>My approach</u>

– (i) Regard QC_2D lattice simulations as useful "numerical experiments" of cold and dense QCD, then (ii) give interpretation from symmetry viewpoints based on effective models

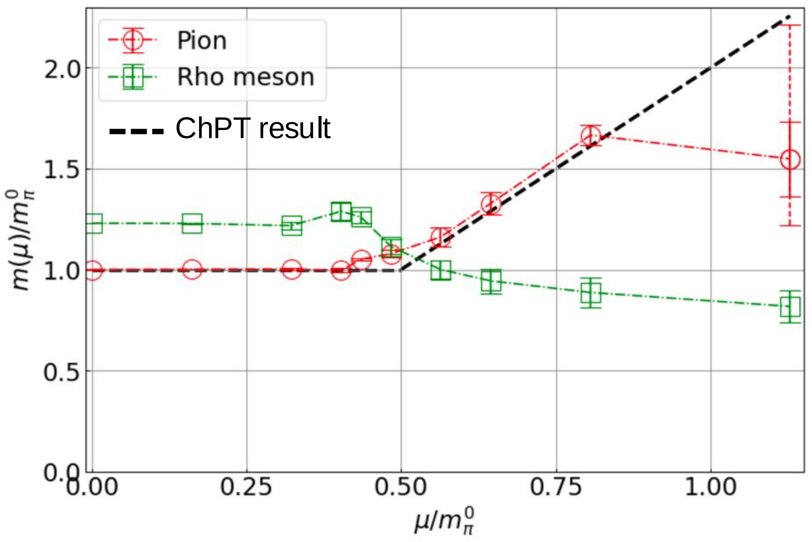
<u>My publications on QC_2D</u>

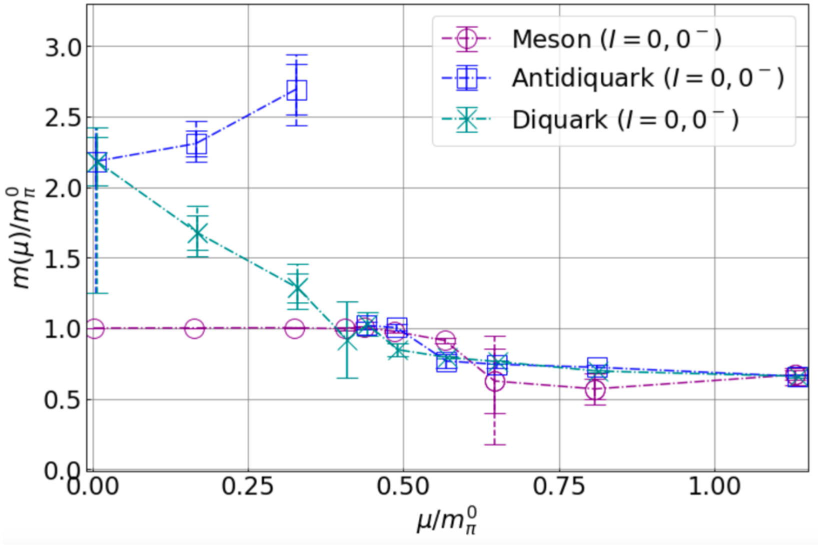
Gluon propagator: **Suenaga**-Kojo(2019), Kojo-**Suenaga**(2021), CSE effect: **Suenaga**-Kojo(2021), Sound velocity: Kojo-**Suenaga**(2022), Kawaguchi-**Suenaga**(2024), Topological susceptibility: Kawaguchi-**Suenaga**(2023), Hadron mass: **Suenaga**-Murakami-Itou-Iida (2023, 2024), and in-preparations.

Introduction

Lattice results on hadron masses

Murakami-Suenaga-Iida-Itou, PoS LATTICE2022 (2023) 154





- ChPT only describes pions (and 0^+ diquark baryons) as the low-energy EFT with systematic expansion

invented by eg Kogut-Stephanov-Toublan-Verbaarschot-Zhitnitsky(2000)

Introduction

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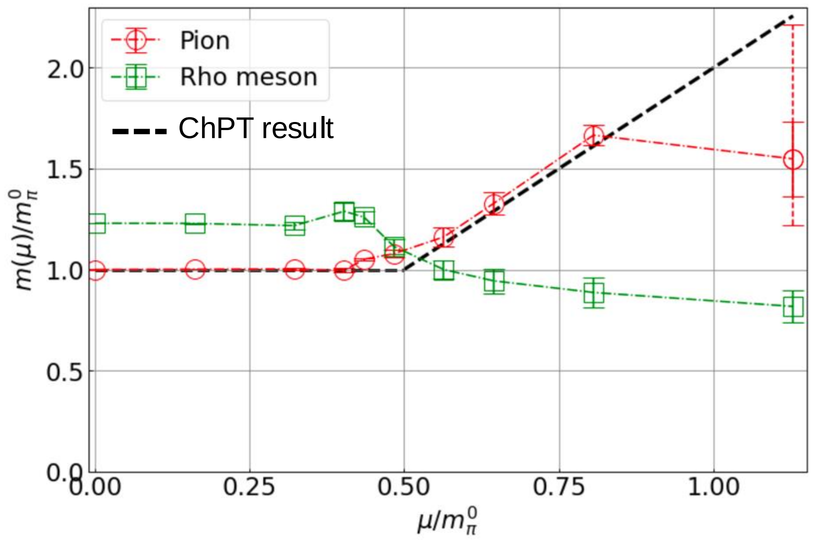
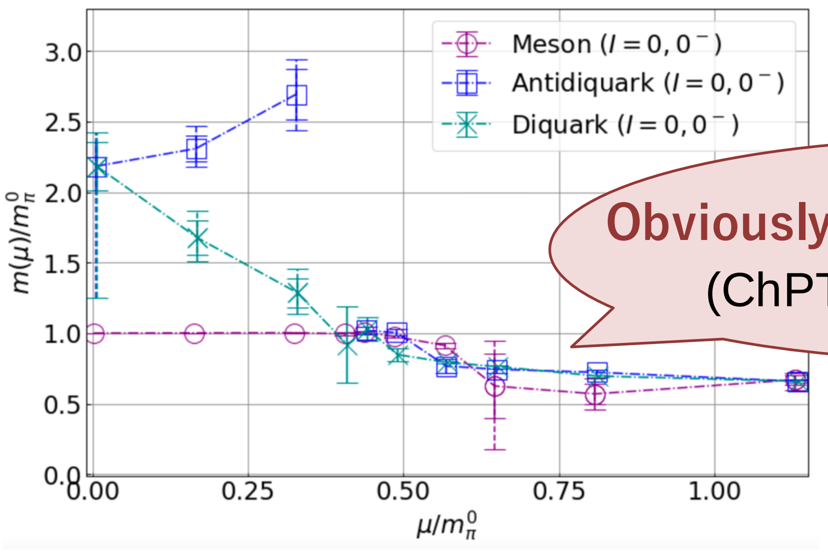
Murakami-Suenaga-Iida-Itou, PoS LATTICE2022 (2023) 154





Obviously lighter than pions!
(ChPT cannot treat them)

- ChPT only describes pions (and 0^+ diquark baryons) as the low-energy EFT with systematic expansion

invented by eg Kogut-Stephanov-Toublan-Verbaarschot-Zhitnitsky(2000)

HOWEVER…!

- Pion is no longer light in superfluid phase (for $m_\pi^0/2 \lesssim \mu$) ⇨ ChPT is no longer the correct low-energy EFT!

⇨ I constructed another model (linear sigma model) as a reasonable EFT in dense QC_2D (this talk)

Q: What is your ultimate goal?

A: To provide information on Neutron star physics

⋮

in a broad sense

A: To unveil $SU(N_c)$ Yang-Mills theory in many-body system of quarks/hadrons!

message of this talk:

→ There is no reason to ignore fruitful QC_2D numerical experiments!

Model

- **Pauli-Gursey SU(4) symmetry**

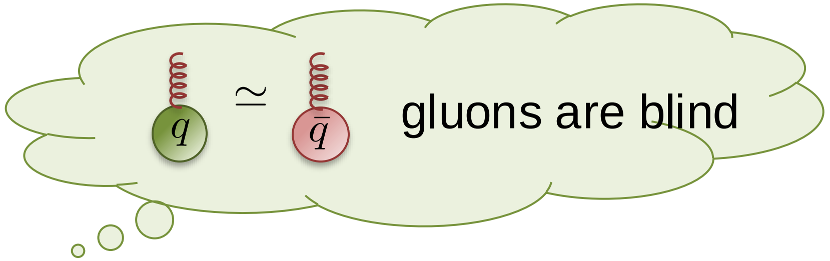
  - Pseudo reality of $SU(2)_c$ allows us to rewrite QC_2D Lagrangian with massless quarks as

$$\mathcal{L}_{\mathrm{QC_2D}} = \bar{\psi} i \partial\!\!\!/ \psi - g_s \bar{\psi} A\!\!\!/^a T_c^a \psi = \Psi^\dagger i \partial_\mu \sigma^\mu \Psi - g_s \Psi^\dagger A_\mu^a T_c^a \sigma^\mu \Psi$$

pseudoreality: $\sigma^2 \sigma^a \sigma^2 = -(\sigma^a)^*$

In two-flavor: $\Psi = (\psi_R, \tilde{\psi}_L)^T = (u_R, d_R, \tilde{u}_L, \tilde{d}_L)^T$ with $\tilde{\psi}_L = \sigma^2 \tau_c^2 \psi_L^*$

Four-dimensional Pauli matrix: $\sigma^\mu = (1, \sigma^i)$



  - $\mathcal{L}_{\mathrm{QC_2D}}$ is obviously invariant under $\Psi \to g\Psi$ $[g \in SU(4)]$

$SU(2)_L \times SU(2)_R$ chiral symmetry ⇨ (enlarged) Pauli-Gursey $SU(4)$ symmetry — Pauli (1957), Gursey (1958)

- All low-energy effective model of QC_2D is constructed to satisfy this symmetry

$\Gamma_{\mathrm{QC_2D}} = \Gamma_{\mathrm{EFT}}$ in the low-energy regime (matching condition)

• Linear sigma model (LSM)

- LSM is an effective model describing not only NG bosons (π etc.) but also their P-wave excitations

⟺ extended model including all order of ChPT

Black-Fariborz-Jora-Park-Schechter-Shahid (2009)

$\pi, \eta, B, \bar{B}$ parity(chiral) partner $\sigma, a_0, B', \bar{B}'$

(not in ChPT)

- Introduce a 4×4 Σ matrix

cf, $\Sigma = \sigma + i\pi^a\tau^a$ for $N_c = 3$

$$\Sigma_{ij} = \frac{1}{2}\begin{pmatrix} 0 & -\frac{B'-iB}{2\sqrt{2}} & \frac{\sigma-i\eta+a_0^0-i\pi^0}{4} & \frac{a_0^+-i\pi^+}{2\sqrt{2}} \\ \frac{B'-iB}{2\sqrt{2}} & 0 & \frac{a_0^--i\pi^-}{2\sqrt{2}} & \frac{\sigma-i\eta-a_0^0+i\pi^0}{4} \\ -\frac{\sigma-i\eta+a_0^0-i\pi^0}{4} & -\frac{a_0^--i\pi^-}{2\sqrt{2}} & 0 & -\frac{\bar{B}'-i\bar{B}}{2\sqrt{2}} \\ -\frac{a_0^+-i\pi^+}{2\sqrt{2}} & -\frac{\sigma-i\eta-a_0^0+i\pi^0}{4} & \frac{\bar{B}'-i\bar{B}}{2\sqrt{2}} & 0 \end{pmatrix}_{ij} \sim \Psi_j\sigma^2\tau_c^2\Psi_i$$

$\Sigma \to g\Sigma g^T$

$[g \in SU(4)]$

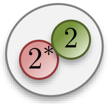
- Assignment of hadron fields

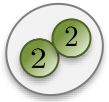
$B \sim -\frac{i}{\sqrt{2}}\psi^T C\gamma_5\tau_c^2\tau_f^2\psi$ $\quad B' \sim -\frac{1}{\sqrt{2}}\psi^T C\tau_c^2\tau_f^2\psi$ $\quad \sigma \sim \bar{\psi}\psi$

$a_0^a \sim \bar{\psi}\tau_f^a\psi$ $\quad \eta \sim \bar{\psi}i\gamma_5\psi$ $\quad \pi^a \sim \bar{\psi}i\gamma_5\tau_f^a\psi$

⟺

Hadron	J^P	Quark number	Isospin
σ	0^+	0	0
a_0	0^+	0	1
η	0^-	0	0
π	0^-	0	1
B ($\bar{B}$)	0^+	$+2(-2)$	0
B' ($\bar{B}'$)	0^-	$+2(-2)$	0

} mesons 

} baryons 

Model

- **Lagrangian of Linear sigma model (LSM)**

  - (approximately) $SU(4)$-invariant LSM Lagrangian is given by

$$\mathcal{L} = \mathrm{tr}[D_\mu \Sigma^\dagger D^\mu \Sigma] - m_0^2 \mathrm{tr}[\Sigma^\dagger \Sigma] - \lambda_1 \left(\mathrm{tr}[\Sigma^\dagger \Sigma]\right)^2 - \lambda_2 \mathrm{tr}[(\Sigma^\dagger \Sigma)^2] + \mathrm{tr}[H^\dagger \Sigma + \Sigma^\dagger H] + c(\det\Sigma + \det\Sigma^\dagger)$$

$U(1)_A$ anomaly

$$D_\mu \Sigma = \partial_\mu \Sigma - i\mu_q \delta_{\mu 0}\{J, \Sigma\} \quad \text{with } J = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}$$

<u>chemical potential effect</u>

$$H = h_q E \quad \text{with } E = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}$$

<u>current-quark mass effect</u>

- Advantage of LSM

→ iso-singlet 0^- hadrons are also treated (mandatory from lattice result!)

→ we can see mass relation between parity (chiral) partners

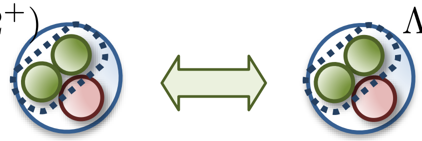
<u>parity (chiral) partner</u>

$$\eta, \pi \leftrightarrow \sigma, a_0$$

$$B(\bar{B}) \leftrightarrow B'(\bar{B}')$$

<u>in $N_c = 3$ world</u>

$\Lambda_c(1/2^+)$ HQS-singlet (observed)



$\Lambda_c(1/2^-)$ HQS-singlet (unobserved)

<u>My hope</u>

Hints from $B'(\bar{B}')$ analysis in QC_2D for the unobserved HQS-singlet $\Lambda_c(1/2^-)$?

Mean fields

- **Mean field**

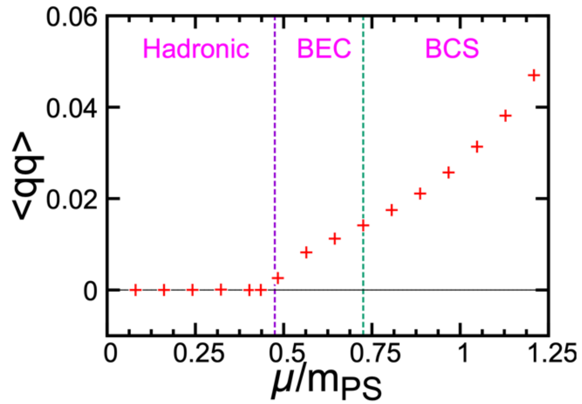
- The mean fields are $\sigma_0 \equiv \langle \sigma \rangle$ and $\Delta \equiv \left\langle \frac{B + \bar{B}}{\sqrt{2}} \right\rangle$

$\sigma_0 \sim \langle \bar{\psi}\psi \rangle$: chiral condensate

$\Delta \sim -\frac{i}{2}\langle \psi^T C \gamma_5 \tau_c^2 \tau_f^2 \psi \rangle + \mathrm{h.c.}$: diquark condensate

diquark condensate by lattice

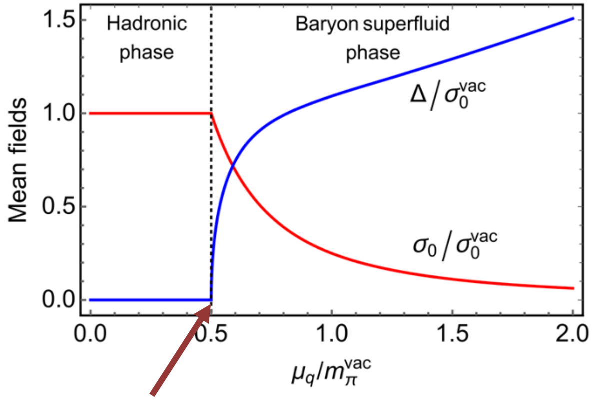
Iida et al, 2405.20566



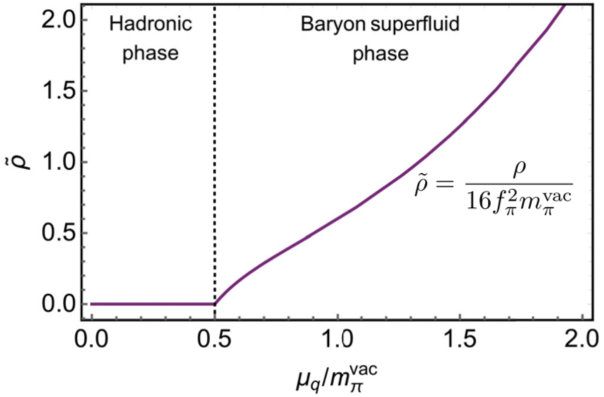
$\langle qq \rangle = 0$: hadronic phase

$\langle qq \rangle \neq 0$: baryon superfluid phase

σ_0 and Δ vs μ_q



density ρ vs μ_q



2[nd] order phase transition at $\mu_q = m_\pi^{\mathrm{vac}}/2$

<u>Input here</u>

$\sigma_0^{\mathrm{vac}} = 250\,\mathrm{MeV}$ (put by hand)

$\lambda_1 = c = 0$ (large Nc)

$m_\pi^{\mathrm{vac}} = 738\,\mathrm{MeV}$

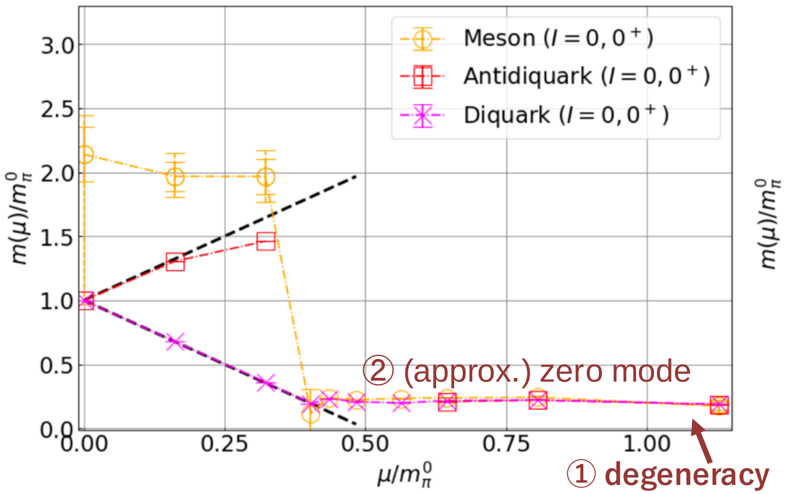
$m_{a_0}^{\mathrm{vac}}/m_\pi^{\mathrm{vac}} = 2.18$

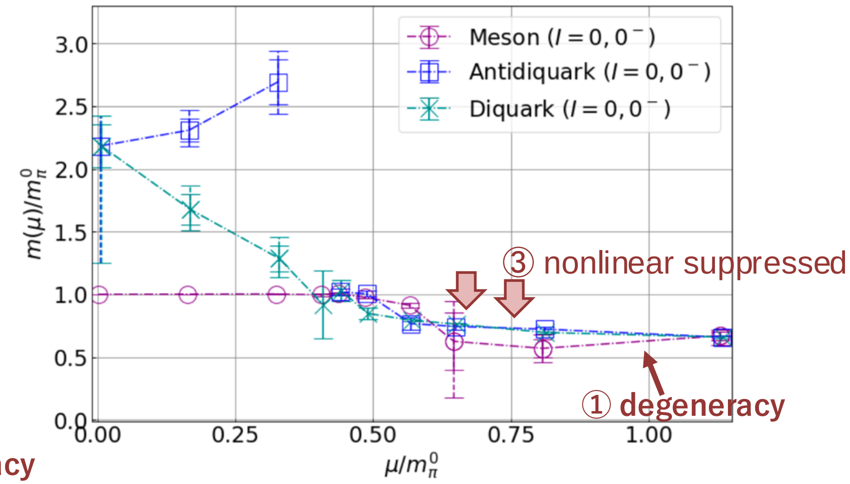
} lattice — Murakami et al

Hadron masses

- **Comparison with lattice** (normalized by $m_\pi^{\rm vac}$)

Lattice (Murakami et al)

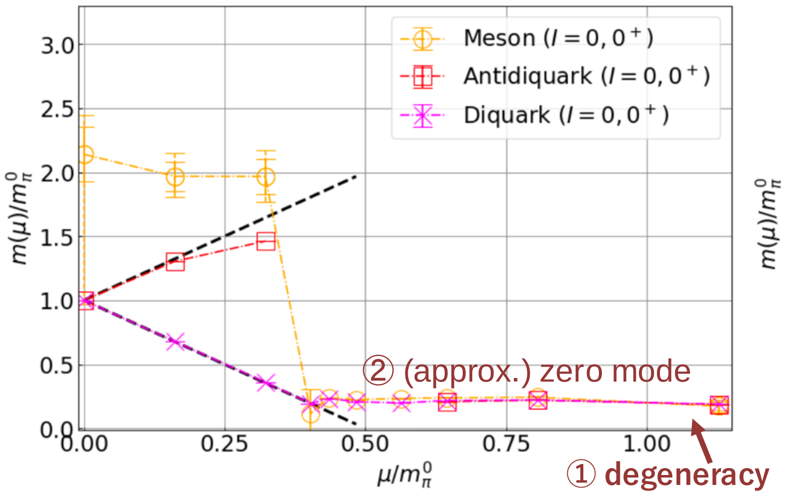


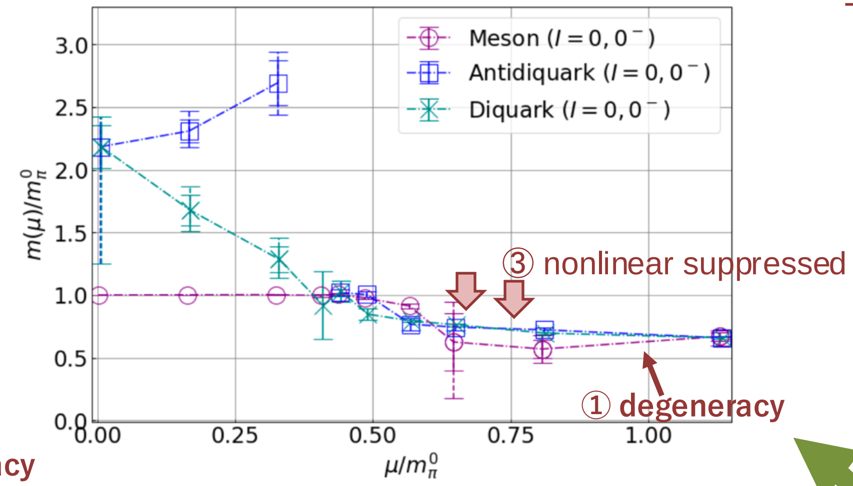


Hadron masses

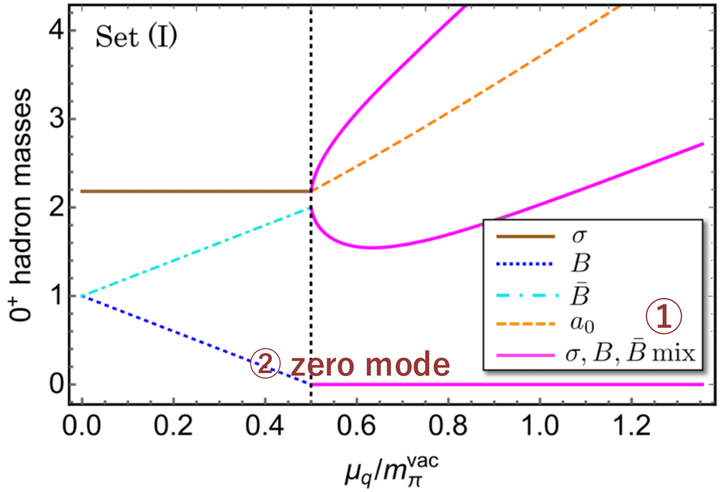
・Comparison with lattice (normalized by m_π^{vac})

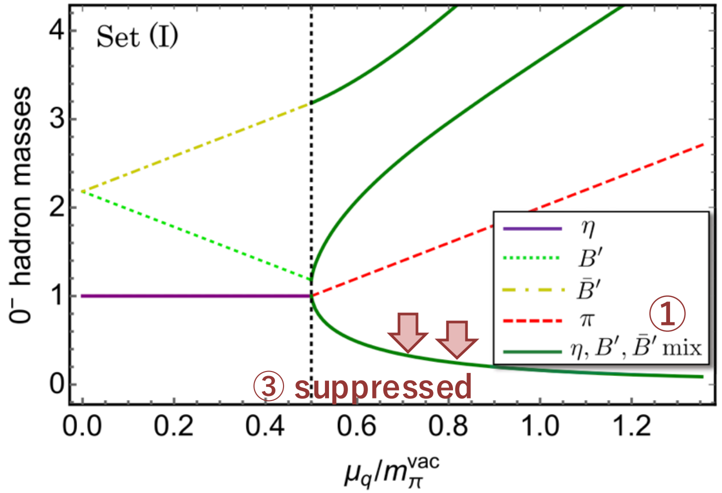
Lattice (Murakami et al)





My LSM





Baryon superfluidity

→ U(1) baryon number violation

① $\sigma, B, \bar{B}$ mixing (0^+)
$\eta, B', \bar{B}'$ mixing (0^-)

② Zero mode (NG boson) by $U(1)_B \to 1$

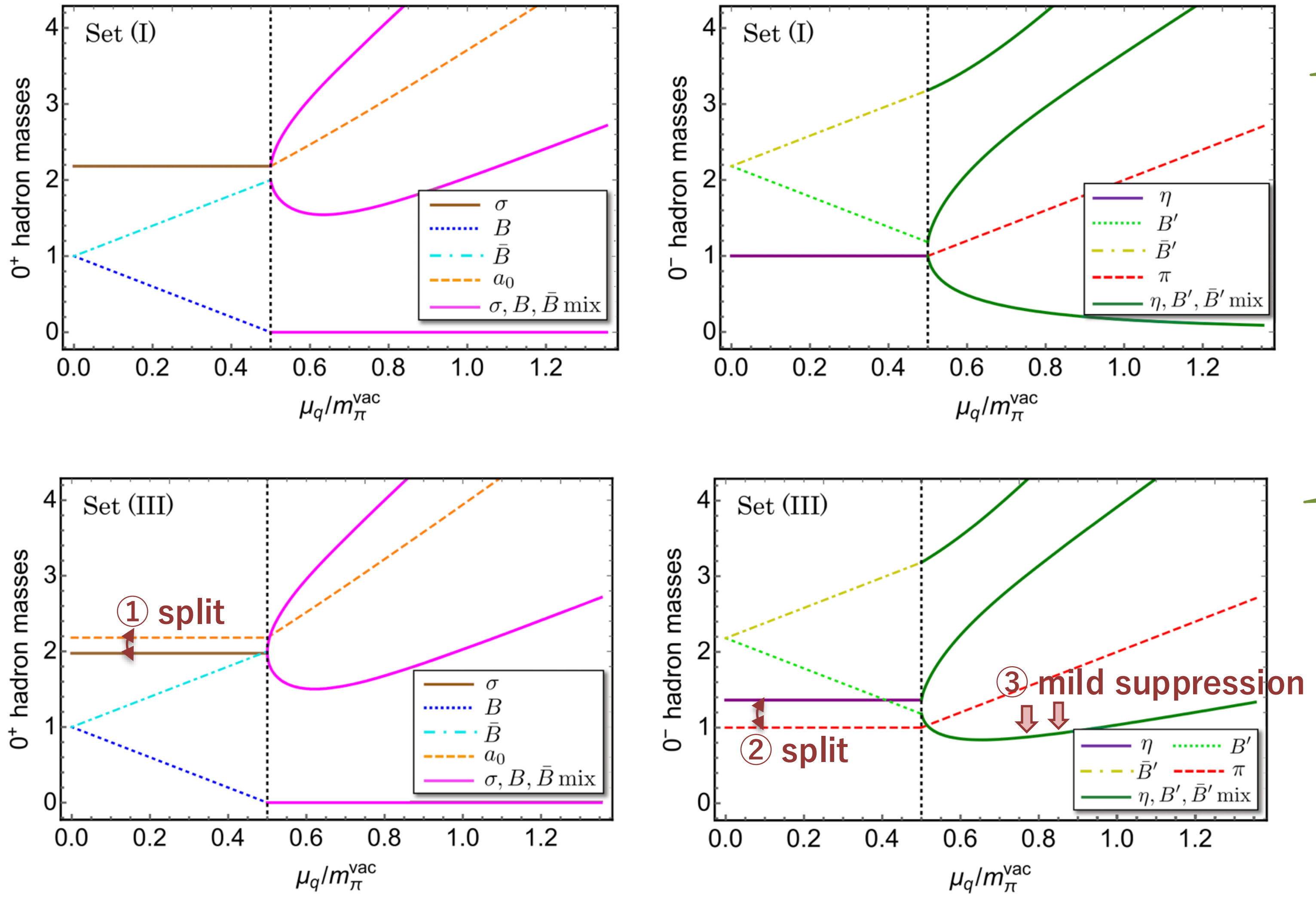
③ Nonlinear suppression of "η mode" mass

Qualitatively good agreement!

⇒ We are ready to apply my LSM to other quantities

Hadron masses

- **Parameter dependence**



$c = 0$ (no anomaly effect)

$c = 15$ (finite anomaly effect)

① σ and a_0 mass splitting

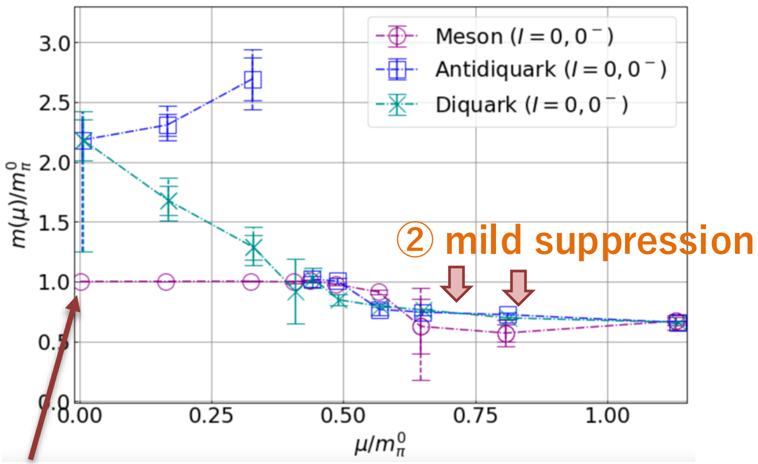
② π and η mass splitting

③ Milder suppression of “η mode”

Hadron masses

- **Comparison with lattice –focused on anomaly-**

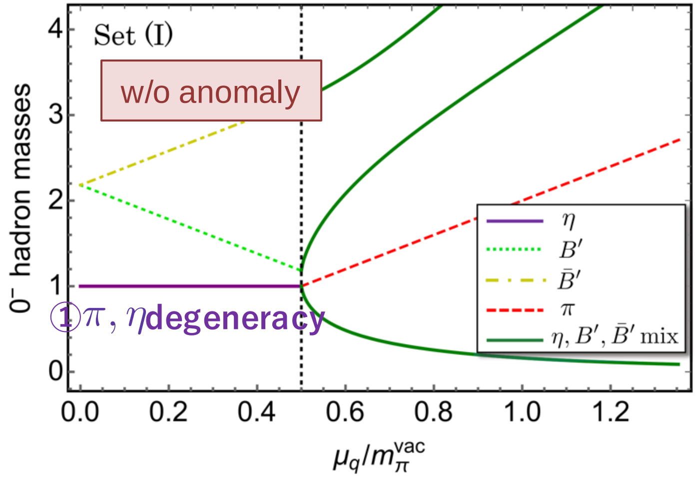
Lattice QCD (Murakami et al)



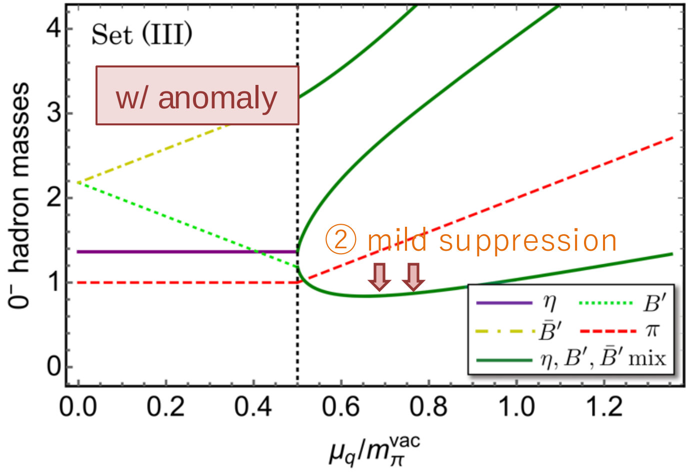
① degeneracy of π, η (← no disconnected diagrams)

⟺ dose not change significantly even when disc. diagrams are included

My model



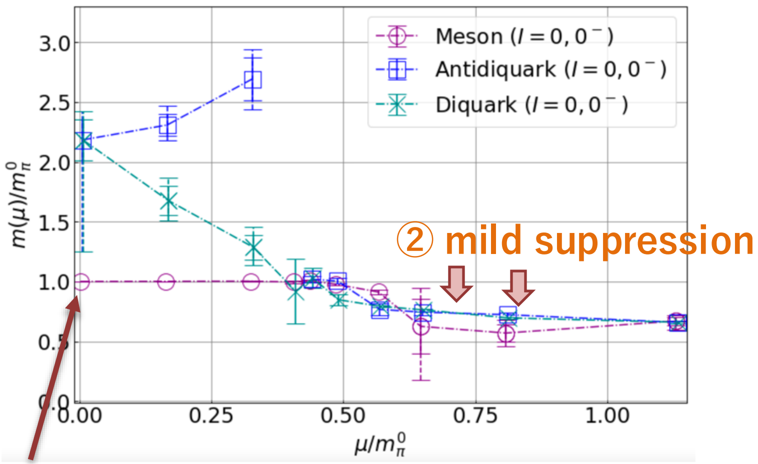
$c = 0$



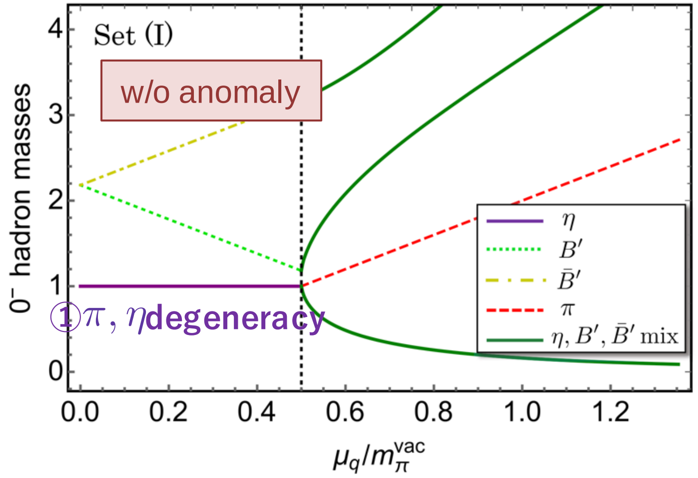
$c = 15$

- **Comparison with lattice –focused on anomaly-**

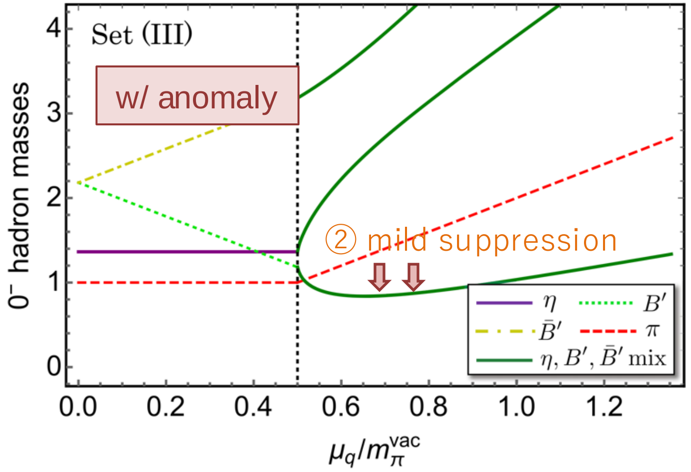
Lattice QCD (Murakami et al)



My model



$c = 0$



$c = 15$

① degeneracy of π, η (← no disconnected diagrams)

⇔ dose not change significantly even when disc. diagrams are included

- At zero density anomaly effect is suppressed,
  but at finite density anomaly would be enhanced

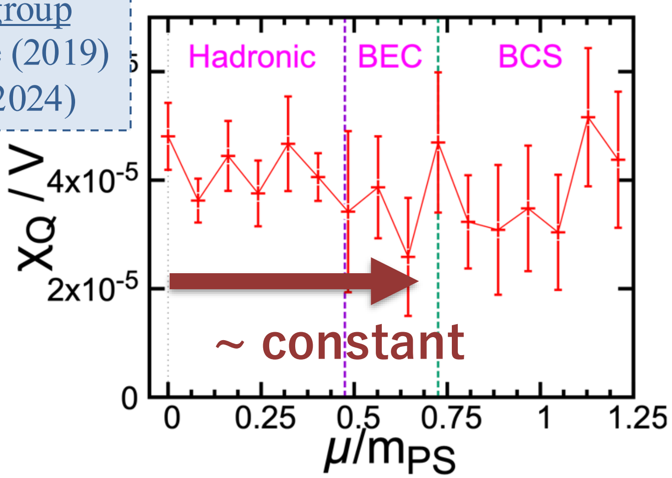
⇨ FRG analysis (work in progress)

Application 1: Top. susceptibility

・Topological susceptibility

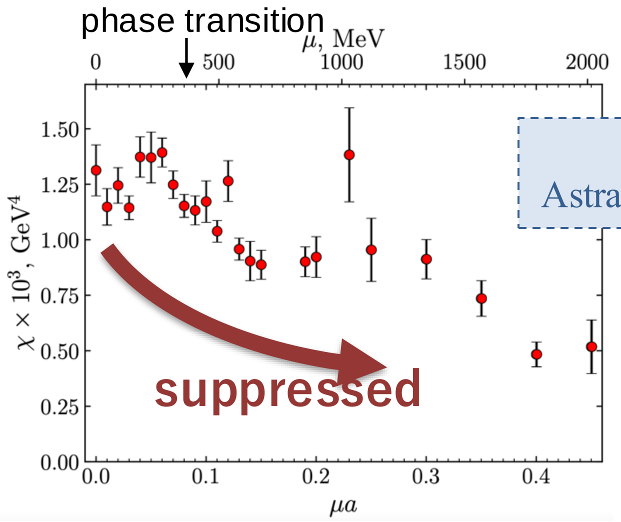
- Lattice results of topological susceptibility by two groups look inconsistent even at qualitative level

Japanese group
Iida-Itou-Lee (2019)
Iida et al (2024)



Why?

inconsistent



Russian group
Astrakhantsev et al (2020)

Definition of topological susceptibility

cf, Instanton, ‘tHooft (1986)

$$\chi_{\rm top} = -\int d^4x \frac{\delta^2 \Gamma_{\rm QC_2D}}{\delta\theta(x)\delta\theta(0)}\bigg|_{\theta=0} = -i\int d^4x \langle Q(x)Q(0)\rangle$$ with topological operator $Q = \frac{g^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}$

- We applied LSM to theoretically explore fate of $\chi_{\rm top}$ in dense QC_2D

Application 1: Top. susceptibility

- **Theoretical background of** $\chi_{\rm top}$

- QC$_2$D generating functional with a θ-term is

$$Z_{\rm QC_2D} = \int [d\bar{\psi}d\psi][dA]\exp\left[i\int d^4x\left(\bar{\psi}(i\not{D} - m_l)\psi - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu,a} + \theta\frac{g^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}\right)\right]$$

U(1)$_A$ axial transformation $\psi \to \exp\left(i\theta/4\gamma_5\right)\psi$

- θ dependence is absorbed into quark mass term via Fujikawa's method

$$Z_{\rm QC_2D} = \int [d\bar{\psi}d\psi][dA]\exp\left[i\int d^4x\left(\bar{\psi}i\not{D}\psi - m_l\bar{\psi}\exp\left(i\theta/2\gamma_5\right)\psi - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu,a}\right)\right]$$

Ward-Takahashi identity
$\langle\bar{\psi}\psi\rangle = -im_l\chi_\pi$

current quark mass

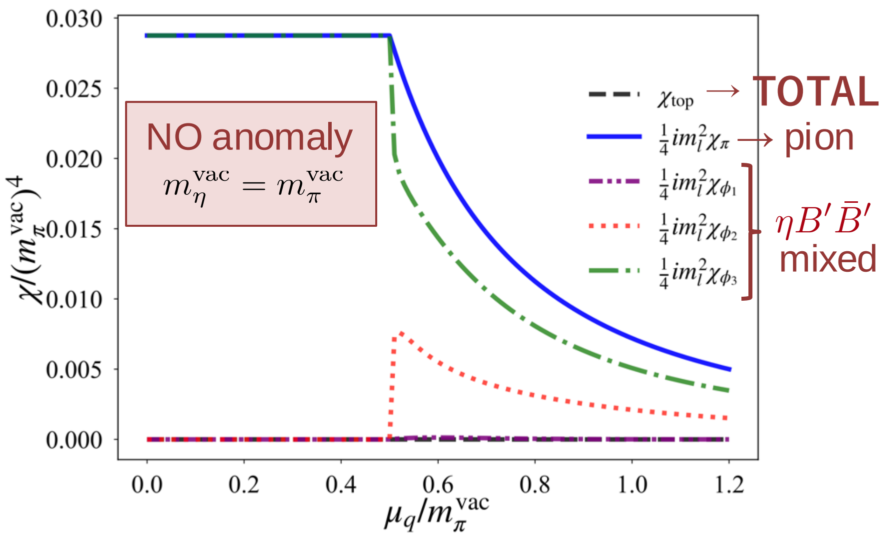
$$\chi_{\rm top} = -\int d^4x \frac{\delta^2\Gamma_{\rm QC_2D}}{\delta\theta(x)\delta\theta(0)}\bigg|_{\theta=0} = \frac{im_l^2}{4}(\chi_\pi - \chi_\eta) = \frac{f_\pi^2 m_\pi^2}{2}\left(1 - \frac{\chi_\eta}{\chi_\pi}\right)$$

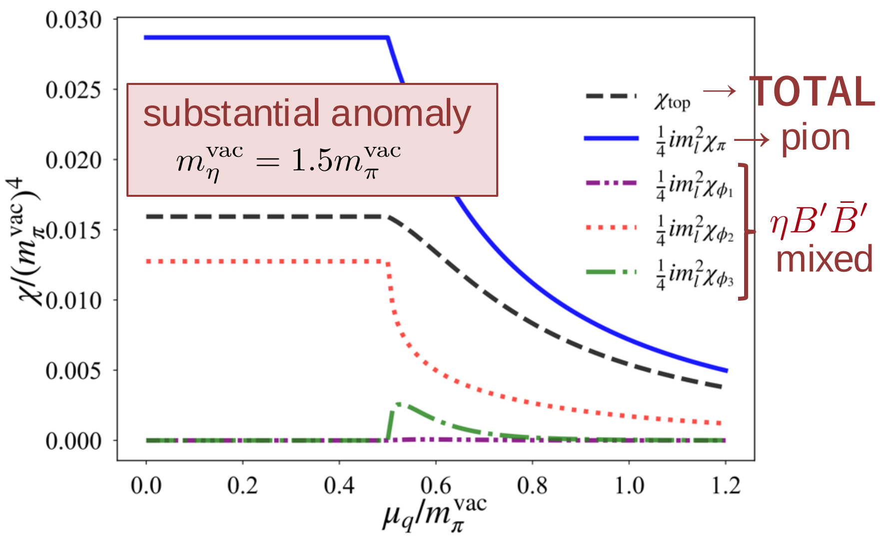
$$\begin{cases} \chi_\pi\delta^{ab} = \int d^4x\langle(\bar{\psi}i\gamma_5\tau_f^a\psi)(x)(\bar{\psi}i\gamma_5\tau_f^b\psi)(0)\rangle \\ \chi_\eta = \int d^4x\langle(\bar{\psi}i\gamma_5\psi)(x)(\bar{\psi}i\gamma_5\psi)(0)\rangle \end{cases}$$

- Matching $Z_{\rm QC_2D} = Z_{\rm LSM}$ enables us to evaluate χ_π and χ_η within LSM

Application 1: Top. susceptibility

- **Results**
  - $\chi_{\rm top}$ within LSM for $m_\eta^{\rm vac}/m_\pi^{\rm vac} = 1.0, 1.5$ reads



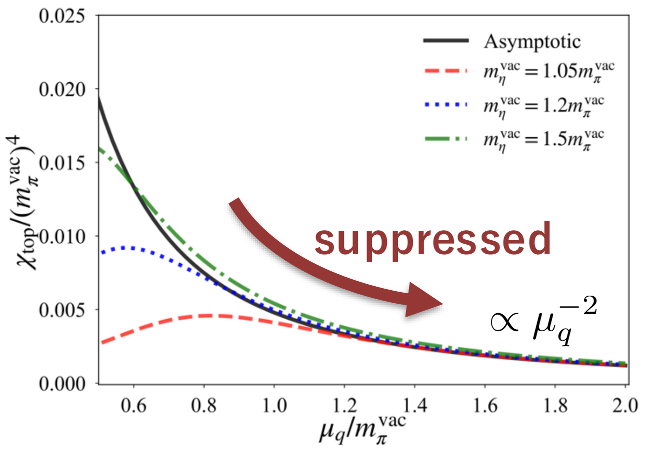


- Anomaly effect is absent $(m_\eta^{\rm vac} = m_\pi^{\rm vac})$ $\rightarrow$ $\chi_{\rm top}$ is always vanishing
- Anomaly effect is present $(m_\eta^{\rm vac} > m_\pi^{\rm vac})$ $\rightarrow$ $\chi_{\rm top}$ is positively induced
  - For $\mu_q \to \infty$, topological susceptibility asymptotically approaches zero

Application 1: Top. susceptibility

- **Asymptotic behavior**

- Asymptotic behavior of $\chi_{\rm top}$ for $m_\eta^{\rm vac}/m_\pi^{\rm vac} = 1.05,\ 1.2,\ 1.5$



- Black curve is analytic solution for large μ_q

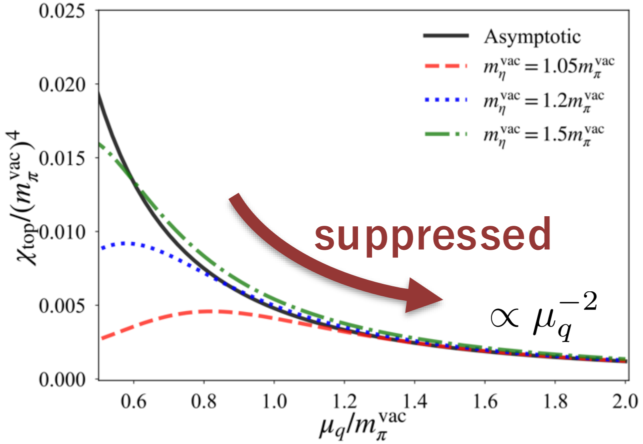
$$\chi_{\rm top} = -\frac{m_l\langle\bar{\psi}\psi\rangle}{4}\left(1-\frac{\chi_\eta}{\chi_\pi}\right) \to \frac{(f_\pi^{\rm vac})^2(m_\pi^{\rm vac})^4}{12}\mu_q^{-2}$$

essentially from the chiral restoration $\sigma_0 \propto \mu_q^{-2}$

Application 1: Top. susceptibility

・Asymptotic behavior

- Asymptotic behavior of $\chi_{\rm top}$ for $m_\eta^{\rm vac}/m_\pi^{\rm vac} = 1.05,\ 1.2,\ 1.5$



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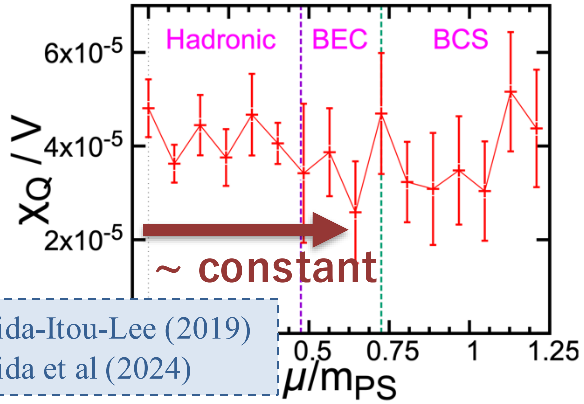
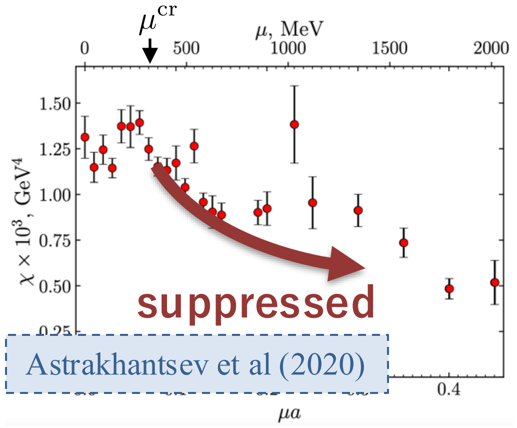
essentially from the chiral restoration $\sigma_0 \propto \mu_q^{-2}$

comparison with lattice



Iida-Itou-Lee (2019)
Iida et al (2024)



Astrakhantsev et al (2020)

- Russian group result shows μ_q^{-2} behavior?
- Japanese group result could suggest enhancement of U(1) anomaly at finite density?

- When $m_\eta^{\rm vac}/m_\pi^{\rm vac}$ is not so large, my LSM result does not yield the sizable suppression

[No conclusive statement yet]

Application 2: Sound velocity

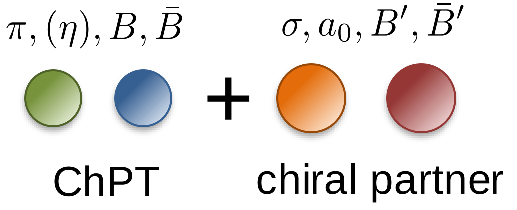
- **Sound velocity at mean-field level within the LSM**

$$\bar{\mu} = \mu/\mu_{\rm cr} = 2\mu/m_\pi$$
$$\delta\bar{m}^2_{\sigma-\pi} = (m_\sigma^2 - m_\pi^2)/\mu_{\rm cr}^2$$

pressure: $$p = \underbrace{f_\pi^2 m_\pi^2 \left(\bar{\mu}^2 + \frac{1}{\bar{\mu}^2}\right)}_{\text{ChPT result}} + f_\pi^2 m_\pi^2 \left[\frac{4}{\delta\bar{m}^2_{\sigma-\pi}}(\bar{\mu}^2 - 1)^2\right]$$

energy: $$\epsilon = \underbrace{f_\pi^2 m_\pi^2 \left[\frac{(\bar{\mu}^2+3)(\bar{\mu}^2-1)}{\bar{\mu}^2}\right]}_{\text{ChPT result}} + f_\pi^2 m_\pi^2 \left[\frac{4}{\delta\bar{m}^2_{\sigma-\pi}}(3\bar{\mu}^2+1)(\bar{\mu}^2-1)\right]$$

sound velocity: $$c_s^2 = \frac{(1 - 1/\bar{\mu}^4) + 8(\bar{\mu}^2 - 1)/\delta\bar{m}^2_{\sigma-\pi}}{(1 + 3/\bar{\mu}^4) + 8(3\bar{\mu}^2 - 1)/\delta\bar{m}^2_{\sigma-\pi}}$$



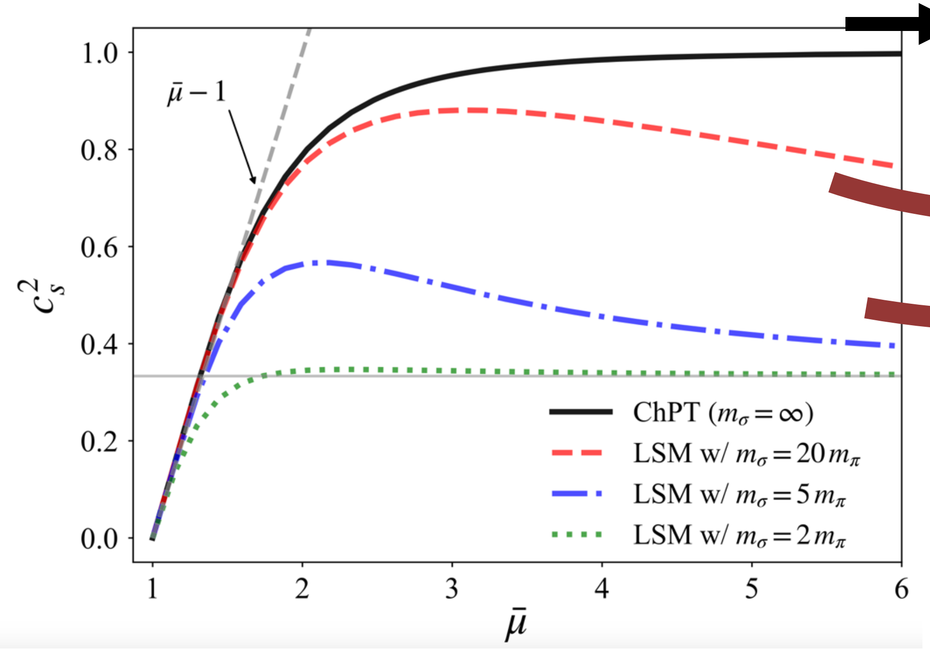
Universal structure: $(\text{LSM result}) = (\text{ChPT result}) + (1/\delta\bar{m}^2_{\sigma-\pi}\ \text{contribution})$

- Integrating out the chiral partners $(m_\sigma \to \infty)$ yields the ChPT results $(1/\delta\bar{m}^2_{\sigma-\pi} \to 0)$

Application 2: Sound velocity

- **Sound velocity peak**

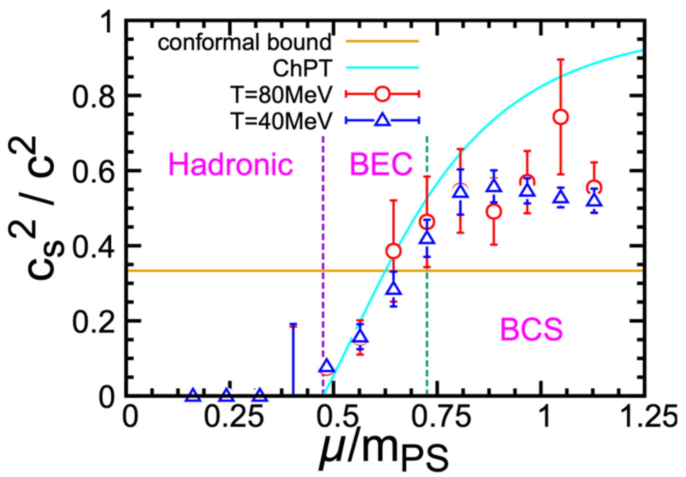
$$\bar{\mu} = \mu/\mu_{\rm cr} = 2\mu/m_\pi$$

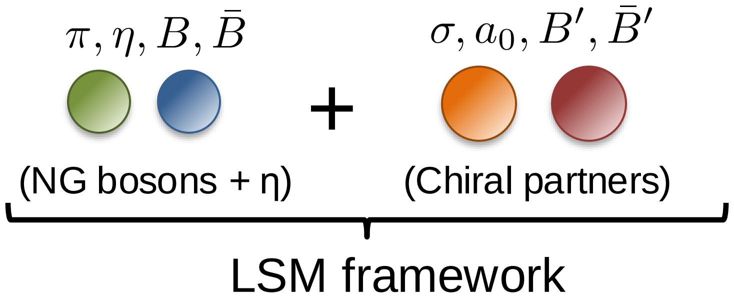


ChPT result $(c_s^2 \to 1)$: no peak

peak when $m_\sigma < \infty$ $(c_s^2 \to 1/3)$

cf, lattice: Iida-Itou-Murakami-Suenaga(2024)





- The peak structure is driven by contributions from chiral partners

- Any connection with crossover to quark matter ?

- Fluctuation and spin-1 hadron effect are needed for more quantitative comparison

Extension of my LSM

- **LSM with spin-1 hadrons**

  - Introduce the following 4×4 matrix representing spin-1 hadrons

$$\Psi = (\psi_R, \tilde{\psi}_L)^T = (u_R, d_R, \tilde{u}_L, \tilde{d}_L)^T$$
with $\tilde{\psi}_L = \sigma^2 \tau_c^2 \psi_L^*$

$$\Phi_{ij}^{\mu} \sim \Psi_j^{\dagger} \sigma^{\mu} \Psi_i = \frac{1}{2} \begin{pmatrix} \frac{\omega+\rho^0-(f_1+a_1^0)}{\sqrt{2}} & \rho^+ - a_1^+ & \sqrt{2} B_S^{I=+1} & B_S^{I=0} - B_{AS} \\ \rho^- - a_1^- & \frac{\omega-\rho^0-(f_1-a_1^0)}{\sqrt{2}} & B_S^{I=0} + B_{AS} & \sqrt{2} B_S^{I=-1} \\ \sqrt{2} \bar{B}_S^{I=-1} & \bar{B}_S^{I=0} + \bar{B}_{AS} & -\frac{\omega+\rho^0+f_1+a_1^0}{\sqrt{2}} & -(\rho^- + a_1^-) \\ \bar{B}_S^{I=0} - \bar{B}_{AS} & \sqrt{2} \bar{B}_S^{I=+1} & -(\rho^+ + a_1^+) & -\frac{\omega-\rho^0+f_1-a_1^0}{\sqrt{2}} \end{pmatrix}_{ij}^{\mu}$$

Hadron	J^P	Quark number	Isospin
ω	1^-	0	0
ρ	1^-	0	1
f_1	1^+	0	0
a_1	1^+	0	1
B_S ($\bar{B}_S$)	1^+	+2 (−2)	1
B_{AS} ($\bar{B}_{AS}$)	1^-	+2 (−2)	0

spin-1 mesons

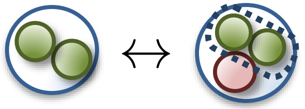
$$\omega^\mu \sim \bar{\psi}\gamma^\mu\psi , \quad f_1^\mu \sim \bar{\psi}\gamma_5\gamma^\mu\psi ,$$
$$\rho^{0,\mu} \sim \bar{\psi}\tau_f^3\gamma^\mu\psi , \quad \rho^{\pm,\mu} \sim \frac{1}{\sqrt{2}}\bar{\psi}\tau_f^{\mp}\gamma^\mu\psi ,$$
$$a_1^{0,\mu} \sim \bar{\psi}\tau_f^3\gamma_5\gamma^\mu\psi , \quad a_1^{\pm,\mu} \sim \frac{1}{\sqrt{2}}\bar{\psi}\tau_f^{\mp}\gamma_5\gamma^\mu\psi$$

spin-1 diquarks

$$B_S^{I=0,\mu} \sim -\frac{i}{\sqrt{2}}\psi^T C\gamma^\mu\tau_c^2\tau_f^1\psi$$
$$B_S^{I=\pm1,\mu} \sim -\frac{i}{2}\psi^T C\gamma^\mu\tau_c^2(\mathbf{1}_f \pm \tau_f^3)\psi ,$$
$$B_{AS}^\mu \sim -\frac{1}{\sqrt{2}}\psi^T C\gamma_5\gamma^\mu\tau_c^2\tau_f^2\psi$$
$$\bar{B}_S^{I=0,\mu} = (B_S^{I=0,\mu})^\dagger , \quad \bar{B}_S^{I=\pm1,\mu} = (B_S^{I=\mp1,\mu})^\dagger$$
$$\bar{B}_{AS}^\mu = (B_{AS}^\mu)^\dagger ,$$

correspondence to three-color QCD
$$B_S \leftrightarrow \Sigma_c(2455)\ [\Sigma_c(2520)]$$
$$B_{AS} \leftrightarrow \Lambda_c(2595)\ [\Lambda_c(2625)]$$

$N_c = 2$ $\qquad$ $N_c = 3$



Chiral model for SHB

Harada-Liu-Oka-Suzuki (2020)
Suenaga-Hosaka (2021,2022)
Suenaga-Oka, in preparation

Extension of my LSM

- **Extended linear sigma model (eLSM)**
  - Φ^μ transforms as $\Phi^\mu \to g\Phi^\mu g^\dagger \; [g \in SU(4)]$

cf, eLSM by Frankfurt group ↔ HLS by Harada-Nonaka-Yamaoka(2010)

$$\begin{aligned}\mathcal{L}_{\rm eLSM} &= {\rm tr}[D_\mu\Sigma^\dagger D^\mu\Sigma] - m_0^2{\rm tr}[\Sigma^\dagger\Sigma] - \lambda_1\left({\rm tr}[\Sigma^\dagger\Sigma]\right)^2 - \lambda_2{\rm tr}[(\Sigma^\dagger\Sigma)^2] + {\rm tr}[H^\dagger\Sigma + \Sigma^\dagger H] + c({\rm det}\Sigma + {\rm det}\Sigma^\dagger) \\ &- \frac{1}{2}{\rm tr}[\Phi_{\mu\nu}\Phi^{\mu\nu}] + m_1^2{\rm tr}[\Phi_\mu\Phi^\mu] + ig_3{\rm tr}\left[\Phi_{\mu\nu}[\Phi^\mu,\Phi^\nu]\right] + h_1{\rm tr}[\Sigma^\dagger\Sigma]{\rm tr}[\Phi_\mu\Phi^\mu] + h_2{\rm tr}[\Sigma\Sigma^\dagger\Phi_\mu\Phi^\mu] \\ &+ h_3{\rm tr}[\Phi_\mu^T\Sigma^\dagger\Phi^\mu\Sigma] + g_4{\rm tr}[\Phi_\mu\Phi_\nu\Phi^\mu\Phi^\nu] + g_5{\rm tr}[\Phi_\mu\Phi^\mu\Phi_\nu\Phi^\nu] + g_6{\rm tr}[\Phi_\mu\Phi^\mu]{\rm tr}[\Phi_\nu\Phi^\nu] + g_7{\rm tr}[\Phi_\mu\Phi_\nu]{\rm tr}[\Phi^\mu\Phi^\nu]\end{aligned}$$

$$\left\{\begin{aligned}\Phi_{\mu\nu} &\equiv D_\mu\Phi_\nu - D_\nu\Phi_\mu \\ D_\mu\Sigma &\equiv \partial_\mu\Sigma - iG_\mu\Sigma - i\Sigma G_\mu^T - ig_1\Phi_\mu\Sigma - ig_2\Sigma\Phi_\mu^T \\ D_\mu\Phi_\nu &\equiv \partial_\mu\Phi_\nu - i[G_\mu,\Phi_\nu]\end{aligned}\right.$$

and $G_\mu \to \mu_q\delta_{\mu 0}J$ with $J = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1}\end{pmatrix}$

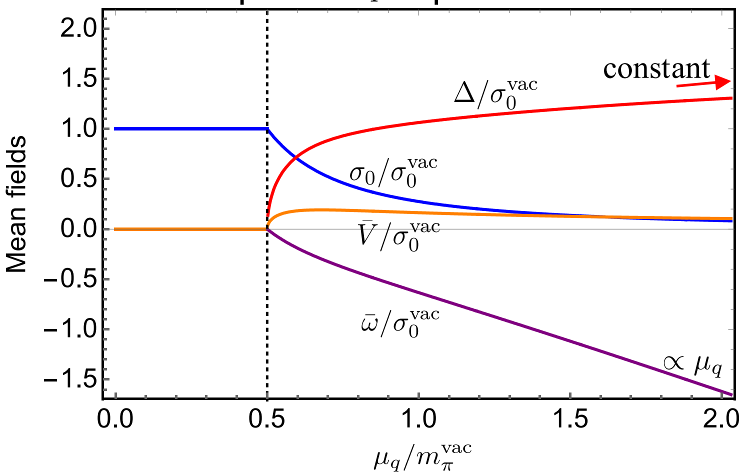
- There are four possible mean fields

$$\sigma_0 = \langle\sigma\rangle \qquad \bar\omega = \langle\omega^{\mu=0}\rangle$$

$$\Delta = \left\langle\frac{B+\bar B}{\sqrt{2}}\right\rangle \qquad \bar V = \left\langle\frac{\bar B_{AS}^{\mu=0} - B_{AS}^{\mu=0}}{\sqrt{2}i}\right\rangle$$

(↑ vector diquark)

Example of μ_q dep. of mean fields



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$$
\begin{aligned}
\mathcal{L}_{\rm eLSM} =\ & \mathrm{tr}[D_\mu\Sigma^\dagger D^\mu\Sigma] - m_0^2\mathrm{tr}[\Sigma^\dagger\Sigma] - \lambda_1\left(\mathrm{tr}[\Sigma^\dagger\Sigma]\right)^2 - \lambda_2\mathrm{tr}[(\Sigma^\dagger\Sigma)^2] + \mathrm{tr}[H^\dagger\Sigma + \Sigma^\dagger H] + c(\mathrm{det}\Sigma + \mathrm{det}\Sigma^\dagger) \\
& - \frac{1}{2}\mathrm{tr}[\Phi_{\mu\nu}\Phi^{\mu\nu}] + m_1^2\mathrm{tr}[\Phi_\mu\Phi^\mu] + ig_3\mathrm{tr}\left[\Phi_{\mu\nu}[\Phi^\mu,\Phi^\nu]\right] + h_1\mathrm{tr}[\Sigma^\dagger\Sigma]\mathrm{tr}[\Phi_\mu\Phi^\mu] + h_2\mathrm{tr}[\Sigma\Sigma^\dagger\Phi_\mu\Phi^\mu] \\
& + h_3\mathrm{tr}[\Phi_\mu^T\Sigma^\dagger\Phi^\mu\Sigma] + g_4\mathrm{tr}[\Phi_\mu\Phi_\nu\Phi^\mu\Phi^\nu] + g_5\mathrm{tr}[\Phi_\mu\Phi^\mu\Phi_\nu\Phi^\nu] + g_6\mathrm{tr}[\Phi_\mu\Phi^\mu]\mathrm{tr}[\Phi_\nu\Phi^\nu] + g_7\mathrm{tr}[\Phi_\mu\Phi_\nu]\mathrm{tr}[\Phi^\mu\Phi^\nu]
\end{aligned}
$$

$$
\left\{
\begin{aligned}
\Phi_{\mu\nu} &\equiv D_\mu\Phi_\nu - D_\nu\Phi_\mu \\
D_\mu\Sigma &\equiv \partial_\mu\Sigma - iG_\mu\Sigma - i\Sigma G_\mu^T - ig_1\Phi_\mu\Sigma - ig_2\Sigma\Phi_\mu^T \\
D_\mu\Phi_\nu &\equiv \partial_\mu\Phi_\nu - i[G_\mu,\Phi_\nu]
\end{aligned}
\right.
$$

and $G_\mu \to \mu_q\delta_{\mu 0}J$ with $J = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}$

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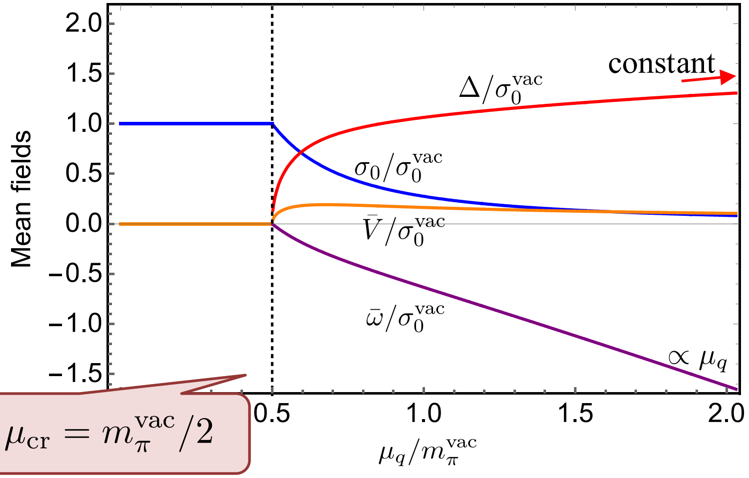
(↑ vector diquark)

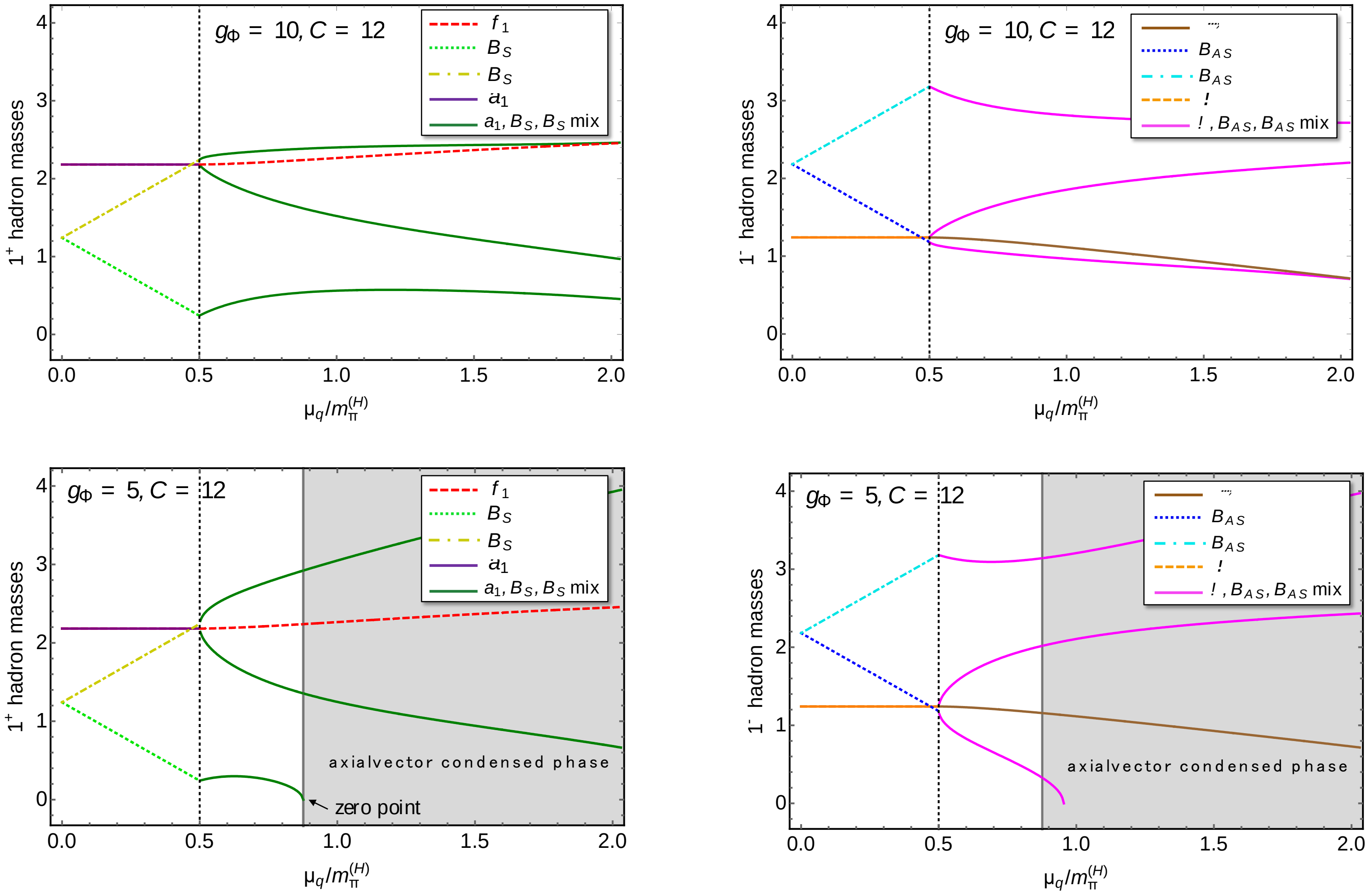
Example of μ_q dep. of mean fields



again $\mu_{\rm cr} = m_\pi^{\rm vac}/2$

Extension of my LSM

• Spin-1 mass spectrum



C ~ mixing strength between spin-0 and spin-1 hadrons
g_Φ ~ coupling strength among spin-1 hadrons

ρ mass reduction
→ consistent with lattice

Possibility of
(axial)vector condensate?

cf, Lenaghan-Sannino-Splittorff (2002)

I'm looking forward to further lattice data!

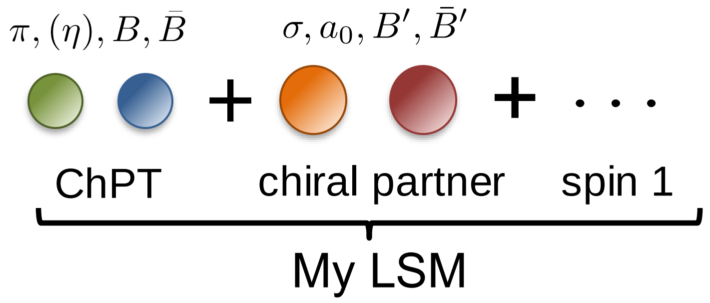
Conclusions

- I constructed the LSM as an effective model of cold and dense QC_2D

{ Not only NG bosons but also their chiral partners are described (= Extended model of ChPT)

- Qualitative understanding of $0^{\pm}$ hadron masses measured on the lattice

→ Good benchmark to explore dense QC_2D



- Suppression of the topological susceptibility for $\mu_q \to \infty$ caused by chiral restoration

- The sound velocity peak occurs from chiral-partner contributions

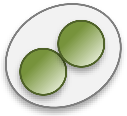
Q: Any connection with crossover to quark matter ?

- Extension to include spin-1 hadrons → possibility of (axial)vector condensation

Conclusions

- QC_2D is a good testing ground to explore diquark nature

$N_c = 2$



hint

$N_c = 3$

- Elucidation of SHB spectrum focusing on chiral-partner structure

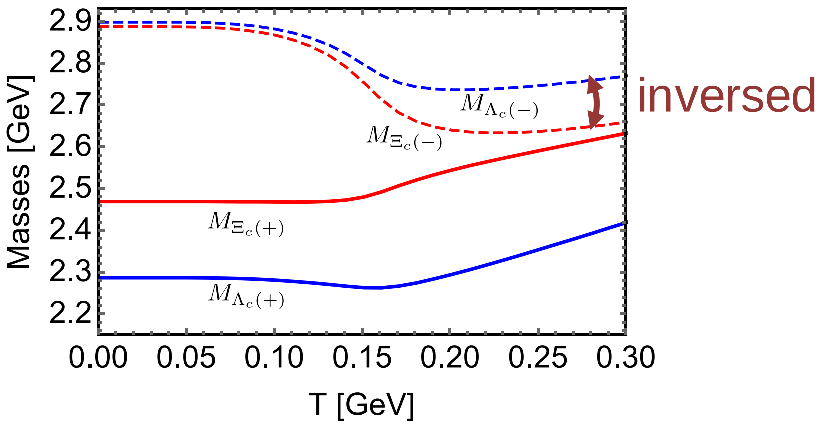
$\Lambda_c \eta$ channel

eg, examination of unobserved HQS-singlet $\Lambda_c(1/2^-)$ which is the chiral partner of $\Lambda_c(1/2^+)$

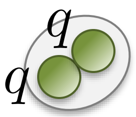
- EFT analysis suggests that $U(1)_A$ anomaly effect to generate "inverse mass hierarchy" of (unobserved) $1/2^-$ SHBs

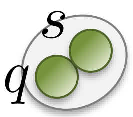
SHB mass at T

Suenaga-Oka (2023)



- 2+1 flavor QC_2D lattice simulation would be useful!
  (no sign problem as long as $\mu_q = 0$ even at finite T)





I hope!