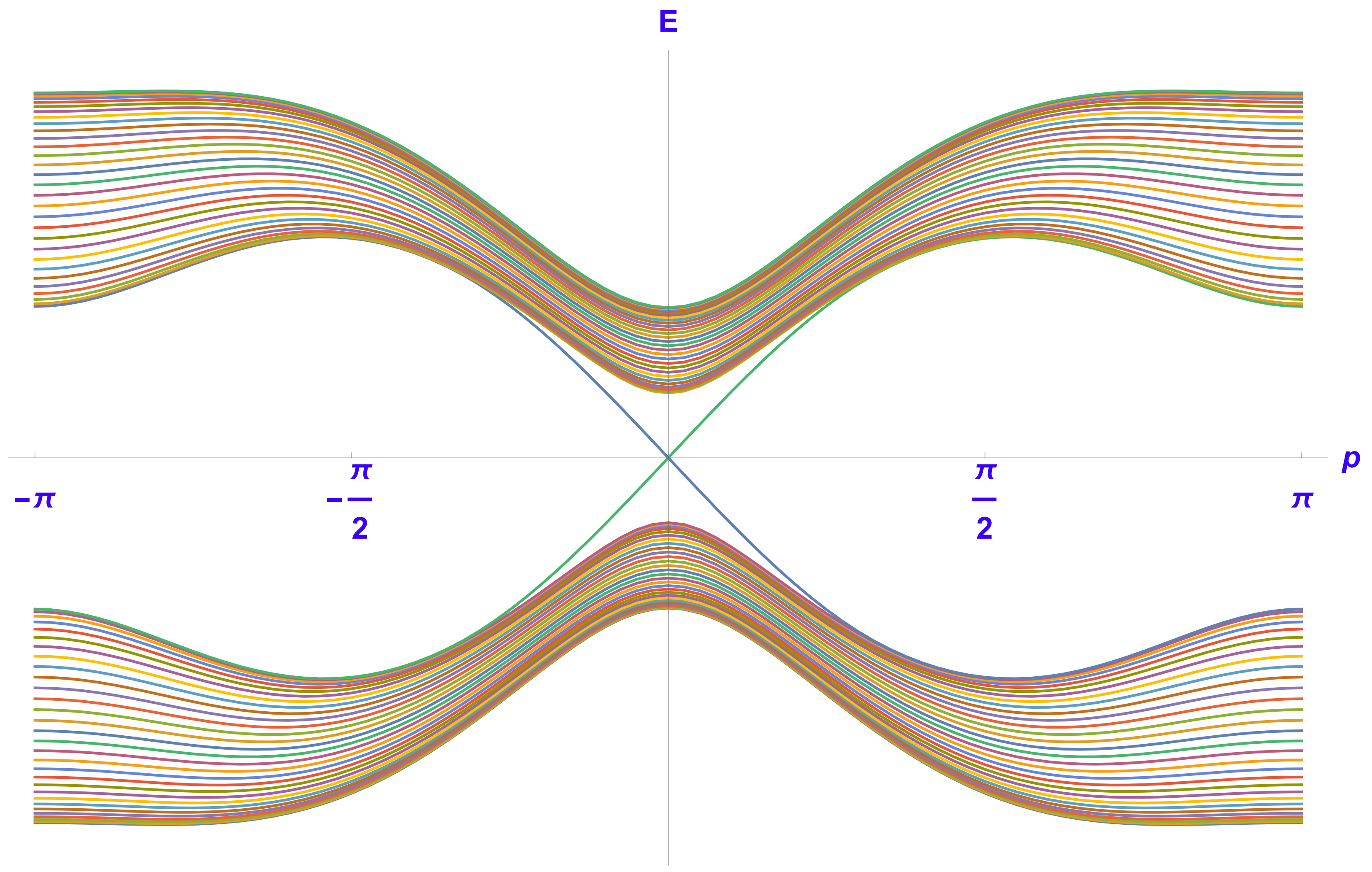
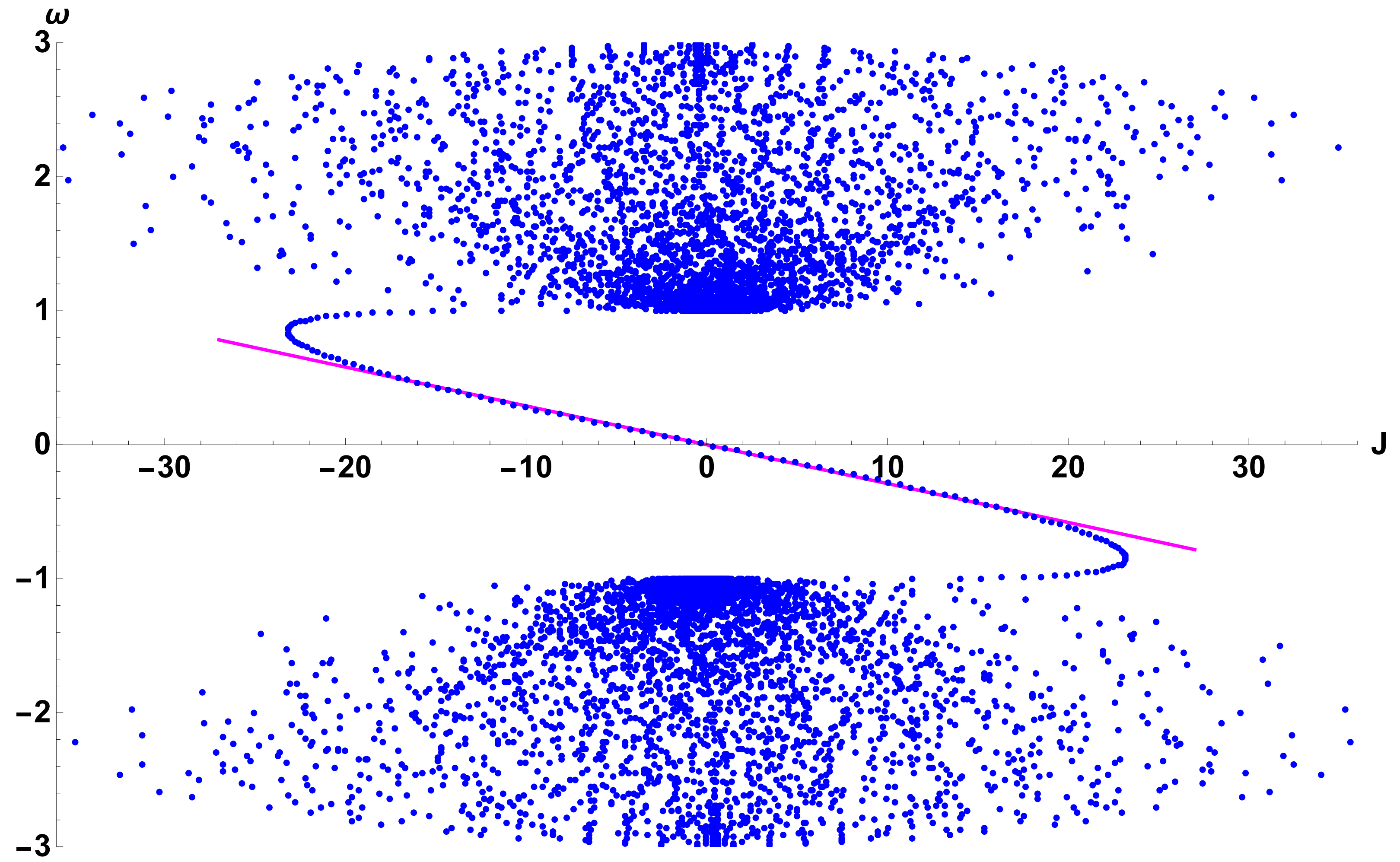
Chiral gauge theory on the lattice
(and questions for the continuum)





DB Kaplan: Phys. Rev. Lett. 132 (2024) 141603, arXiv:2312.01494

DB Kaplan, S. Sen: Phys. Rev. Lett. 132 (2024) 141604, arXiv:2312.04012

DB Kaplan, S. Sen: in progress

Outline of this talk:

- Chiral gauge theory and the Nielsen-Ninomiya theorem
- Edge states and topological phases
- A single connected phase boundary: a disc
- How to see free Weyl fermions on the lattice
- Gauging the theory
- A puzzle, and implications for the continuum?

Chiral gauge theory, Nielsen-Ninomiya theorem

A chiral gauge theory is one where a fermion mass term necessarily violates the gauge symmetry (i.e. the Standard Model)

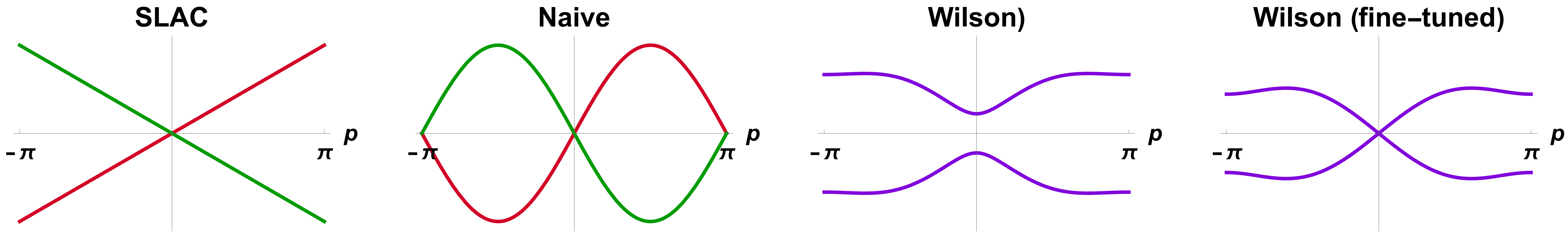
A nonperturbative regulator does not exist for such theories

- What does it mean to have a theory one cannot compute? Is it actually well-defined?
- How can we calculate nonperturbative physics without one? (E.g. EW baryon violation in the early universe)
- Might a definition on the computer imply the need for new physics we do not expect in our continuum definition?

Fundamental tension between the need for a UV mass scale to tame divergences, and a chiral gauge symmetry that forbids masses.

Nielsen-Ninomiya:

1. $\tilde{D}(\mathbf{p})$ is a periodic, analytic function of p_μ;
2. $D(\mathbf{p}) \propto \gamma_\mu p_\mu$ for $a|p_\mu| \ll 1$;
3. $\tilde{D}(\mathbf{p})$ invertible everywhere except $p_\mu = 0$;
4. $\{\Gamma, \tilde{D}(\mathbf{p})\} = 0$.



Examples (a=1):

"SLAC derivative" violates #1

$$\widetilde{D}(p) = \sum_\mu i\gamma_\mu p_\mu$$

Naive lattice fermions violate #3

$$\widetilde{D}(p) = \sum_\mu i\gamma_\mu \sin p_\mu$$

Wilson fermions violate #4

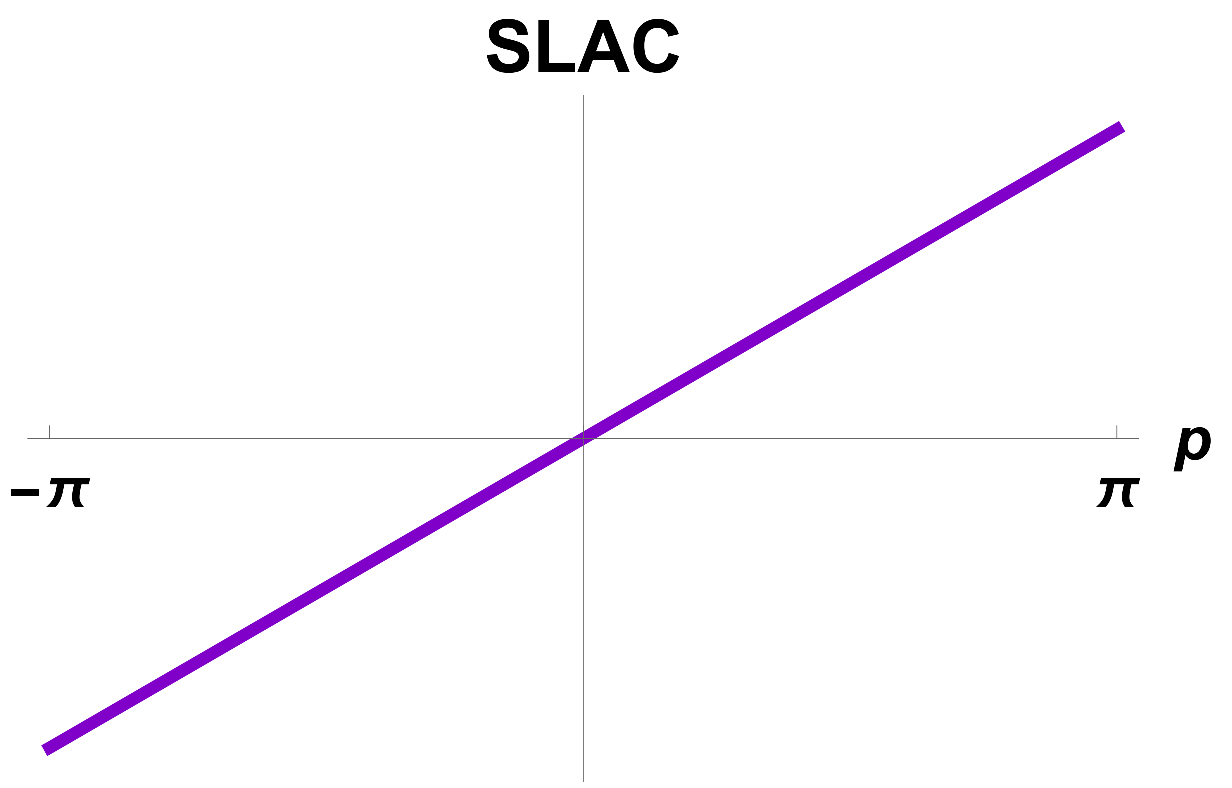
$$\widetilde{D}(p) = \sum_\mu i\gamma_\mu \sin p_\mu + M + \frac{r}{2}\sum_\mu (1 - \cos p_\mu)$$

Heuristic reasons behind NN theorem:

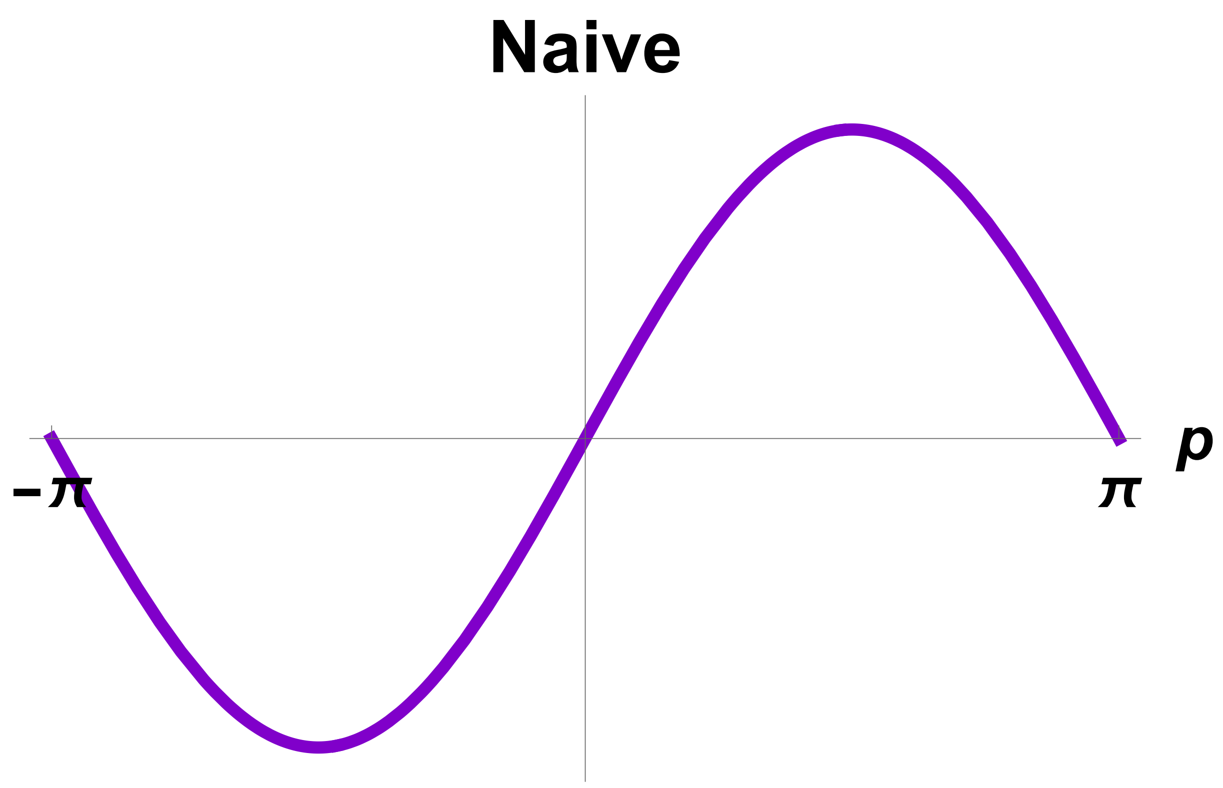
* If the lattice had exact chiral symmetry and its continuum limit gave a massless Dirac fermion, how could **anomalies** ever arise in the continuum?

* If a chiral Dirac fermion existed, one could consider a lattice Weyl fermion using $P_\pm = (1 \pm \Gamma)/2$ projectors...

...but how can a continuous periodic function $P_- D(p) P_+$ cross p=0 only once?



Nonlocal



Dirac, not Weyl

NN theorem tells us that there should be mirror fermions: incompatible with chiral gauge theory

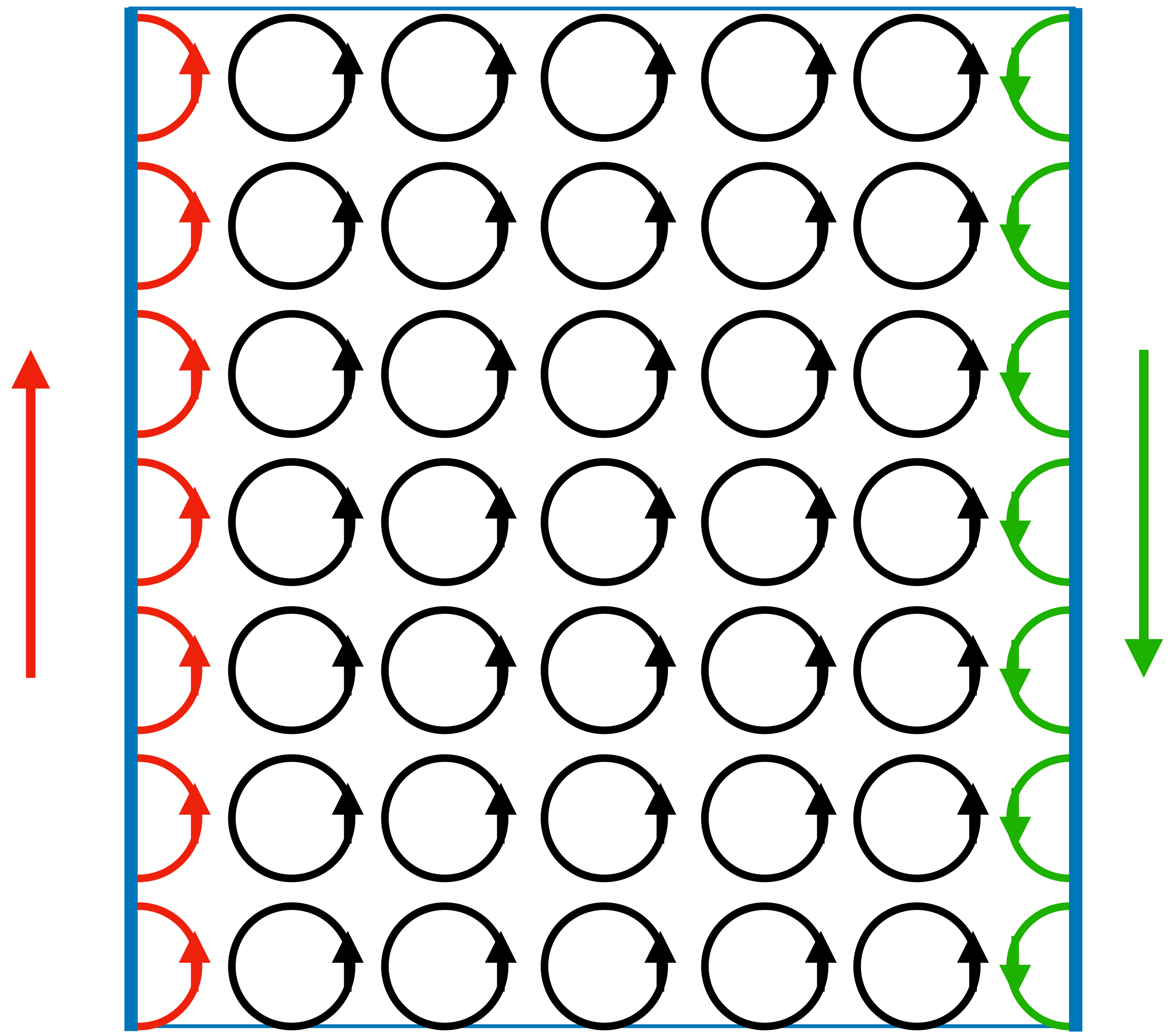
Attempts to get rid of mirror fermions on the lattice:

1. Decouple them by breaking gauge symmetry and giving them a mass; restore gauge symmetry in continuum limit
Golterman, Shamir

2. Gap the system and give masses to the mirrors without breaking gauge symmetry (many-body effects)
Eichten, Preskill
X.G. Wen

3. Eliminate mirror fermions by sacrificing locality (this work)

Chiral edge states appear naturally in the Integer Quantum Hall Effect:



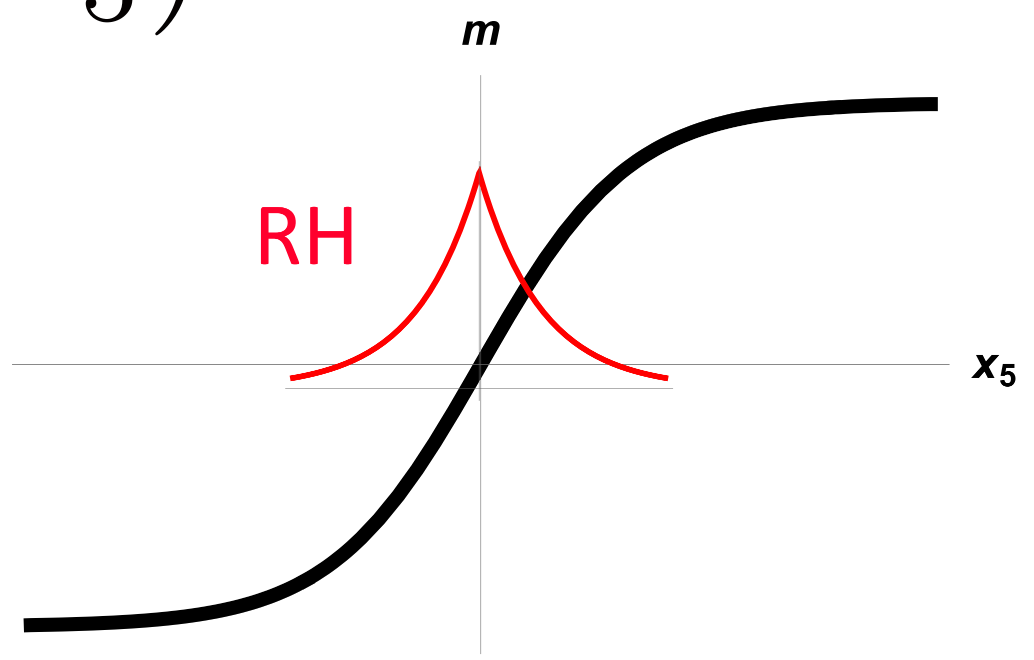
Analog for Dirac fermions with domain wall mass [Jackiw & Rebbi]:

$$\left[\not{\partial} + \gamma_5 \partial_5 + m(x_5)\right] \Psi = 0$$

Has solutions: $\Psi = \phi_\pm(x_5)\chi_\pm$

$$\gamma_5 \chi_\pm = \pm\chi_\pm$$

$$\phi_\pm(x_5) = e^{\mp \int^{x_5} m(s)\, ds}$$



With this domain wall mass profile, ϕ_+ is normalizable ➡ massless chiral edge state

Why does a Dirac equation have a massless chiral edge state?

Answer from condensed matter physics:

- A QFT with a free massive Dirac fermion in odd spacetime dimension can be in two different topological phases depending on the sign of the mass…
- ...so a domain wall is a boundary between two topological phases…
- …the only way to connect two topological phases is for the theory to go gapless at the interface

What is a topological phase?

Toy example: topological insulator in 0+1 dimensions — quantum mechanics with a gap

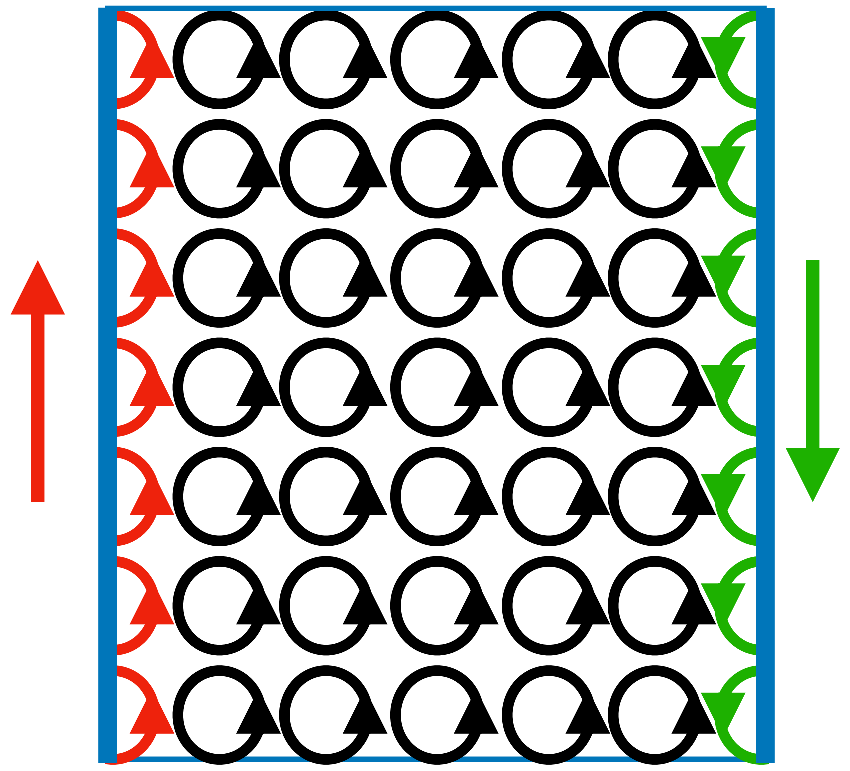
$$H(s)\psi = E(s)\psi \ , \quad |E(s)| > \Delta$$

Define topological quantum number: *ν = # of negative energy states.*

Theories with different parameter *s* are then topologically equivalent.

For the topology to change, e.g. # negative energy states, theory has to go gapless.

What is topologically quantized in a QFT of massive Dirac fermions?

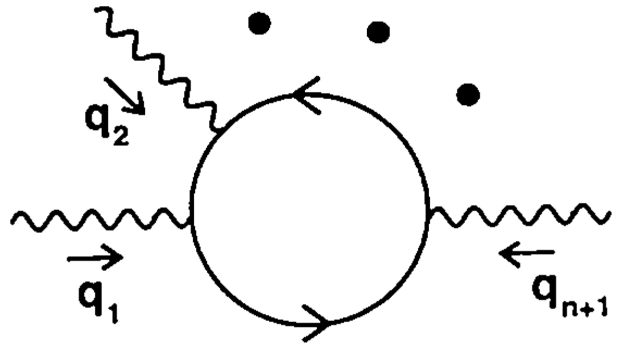


In the Integer Quantum Hall Effect it is the Hall conductivity

The QFT analog is the coefficient of the Chern-Simons term obtained by integrating out the massive fermion in a background gauge field.

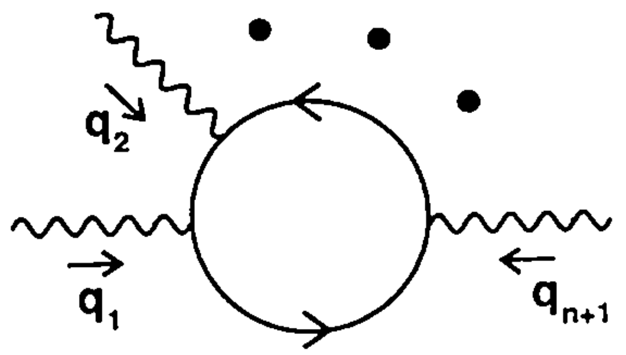
$$\kappa\epsilon_{abc\ldots}\mathrm{Tr}A_a\partial_b A_c\ldots$$

Using Ward identity, Chern-Simons coefficient in d= 2n+1 is proportional to



$$\epsilon_{\mu_1\ldots\mu_d}\int\frac{d^dp}{(2\pi)^d}\,\mathrm{Tr}\;S(p)\frac{\partial S^{-1}(p)}{\partial p_{\mu_1}}\cdots S(p)\frac{\partial S^{-1}(p)}{\partial p_{\mu_d}}$$

where S(p) is the fermion propagator. When the theory is regulated, this is a winding number for the map $S^{-1}(p)$ from S^d (momentum space) to $S^d = SO(d+1)/SO(d)$



$$\epsilon_{\mu_1 \ldots \mu_d} \int \frac{d^d p}{(2\pi)^d} \,\mathrm{Tr}\; S(p) \frac{\partial S^{-1}(p)}{\partial p_{\mu_1}} \cdots S(p) \frac{\partial S^{-1}(p)}{\partial p_{\mu_d}}$$

Remarkable fact:

Since the topology is in **momentum/spin space**, topological phases and massless edge states appear at domain wall boundaries on an infinite spacetime **lattice**

E.g. Wilson fermions (DBK 1992; K. Jansen, M. Schmaltz 1993; M. Golterman, K. Jansen, DBK, 1993):

$$\mathcal{D} = \gamma_\mu \partial + M + \frac{r}{2}\Delta$$

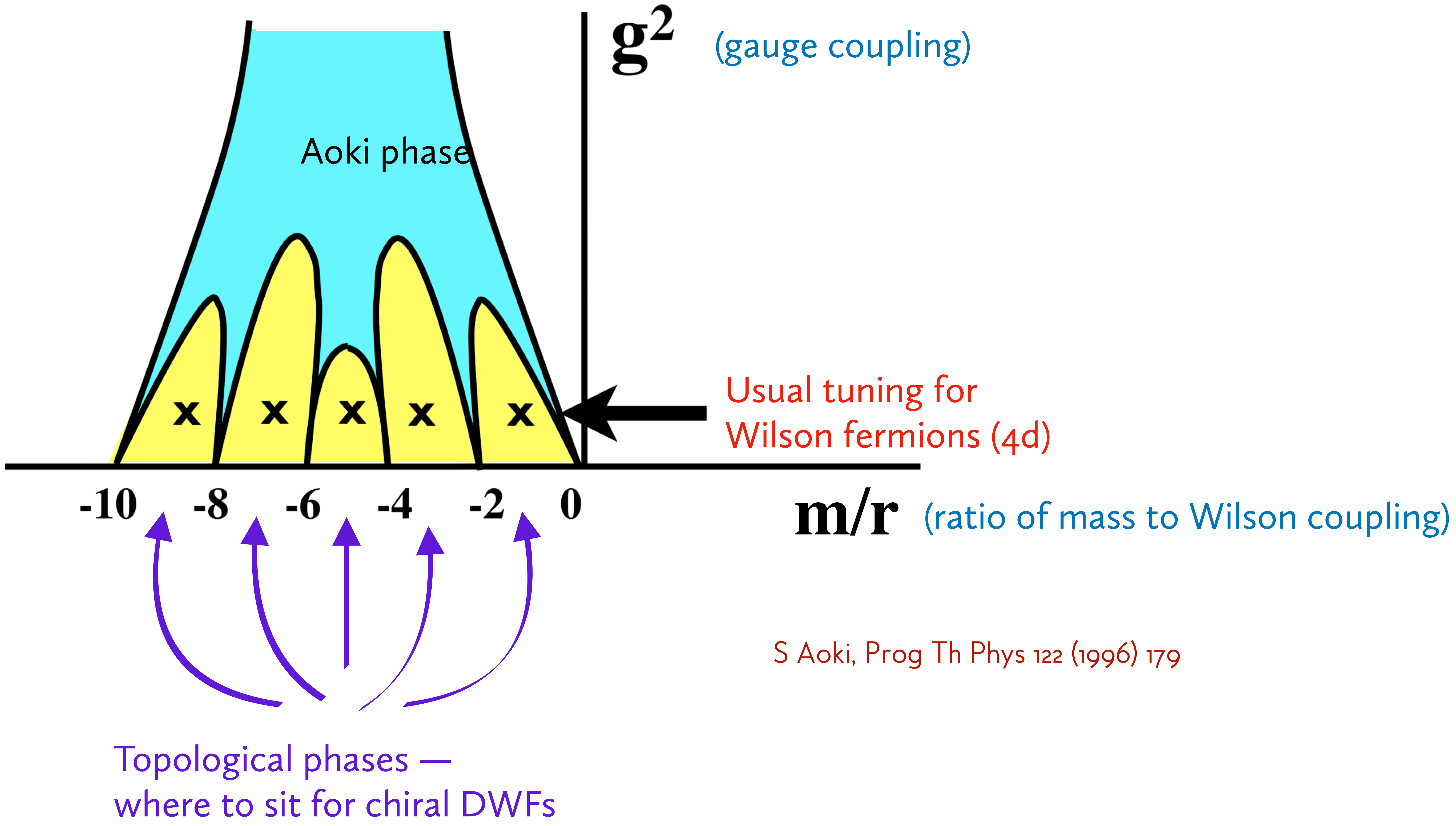
$$\tilde{\mathcal{D}}(p) = M + \sum_\mu \left[i \sin p_\mu \gamma_\mu + \frac{r}{2}(1 - \cos p_\mu) \right]$$

$$\partial_\mu \psi(x) = \frac{\psi(x + a\hat{\mu}) - \psi(x - a\hat{\mu})}{2a} \;,$$

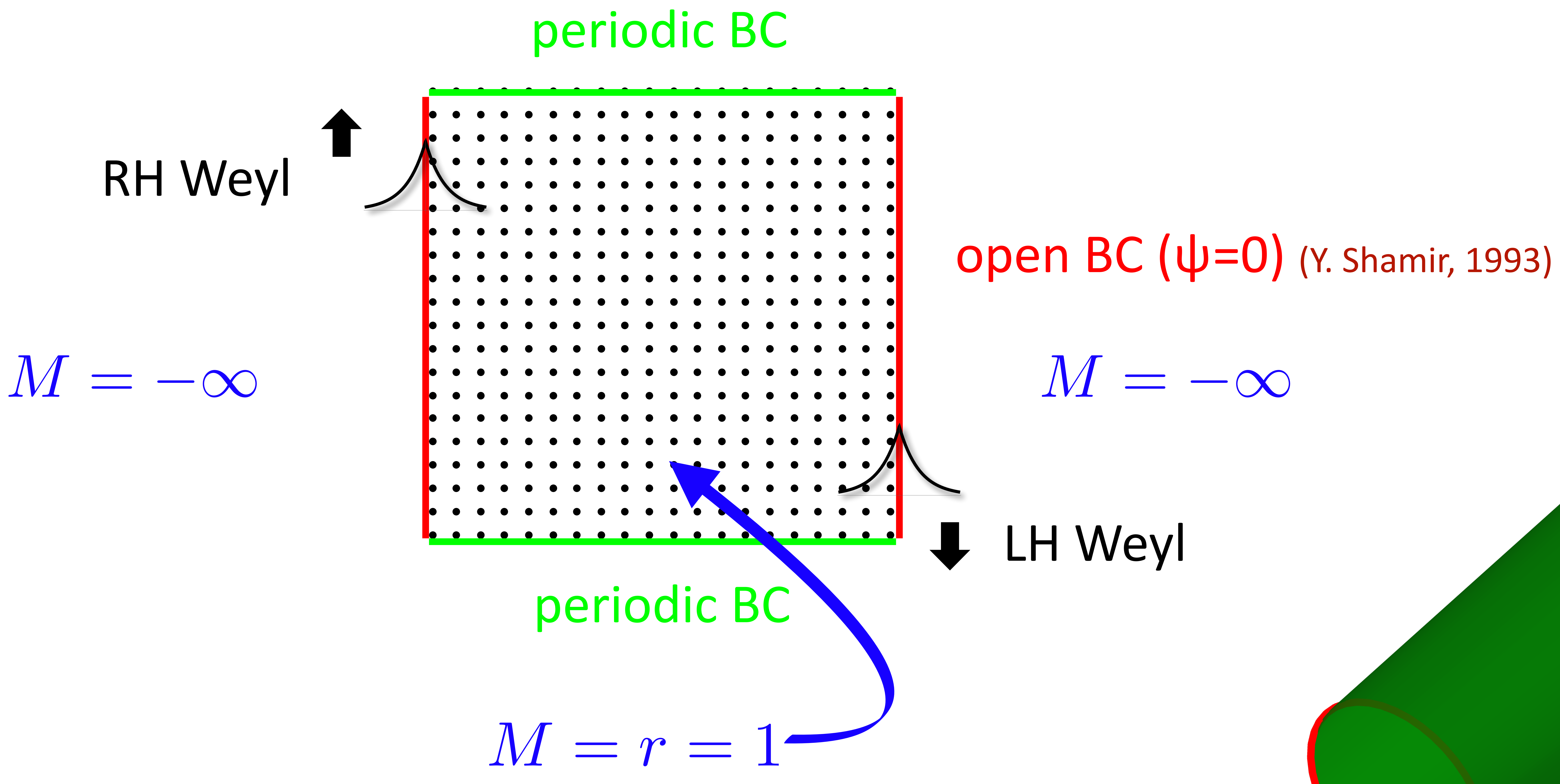
$$\Delta\psi(x) = \frac{\psi(x + a\hat{\mu}) - 2\psi(x) + \psi(x - a\hat{\mu})}{a^2}$$

Nontrivial topological phases for $0 < \dfrac{M}{r} < 2d$ with phase boundaries at $\dfrac{M}{r} = 0, 2, \ldots, 2d$

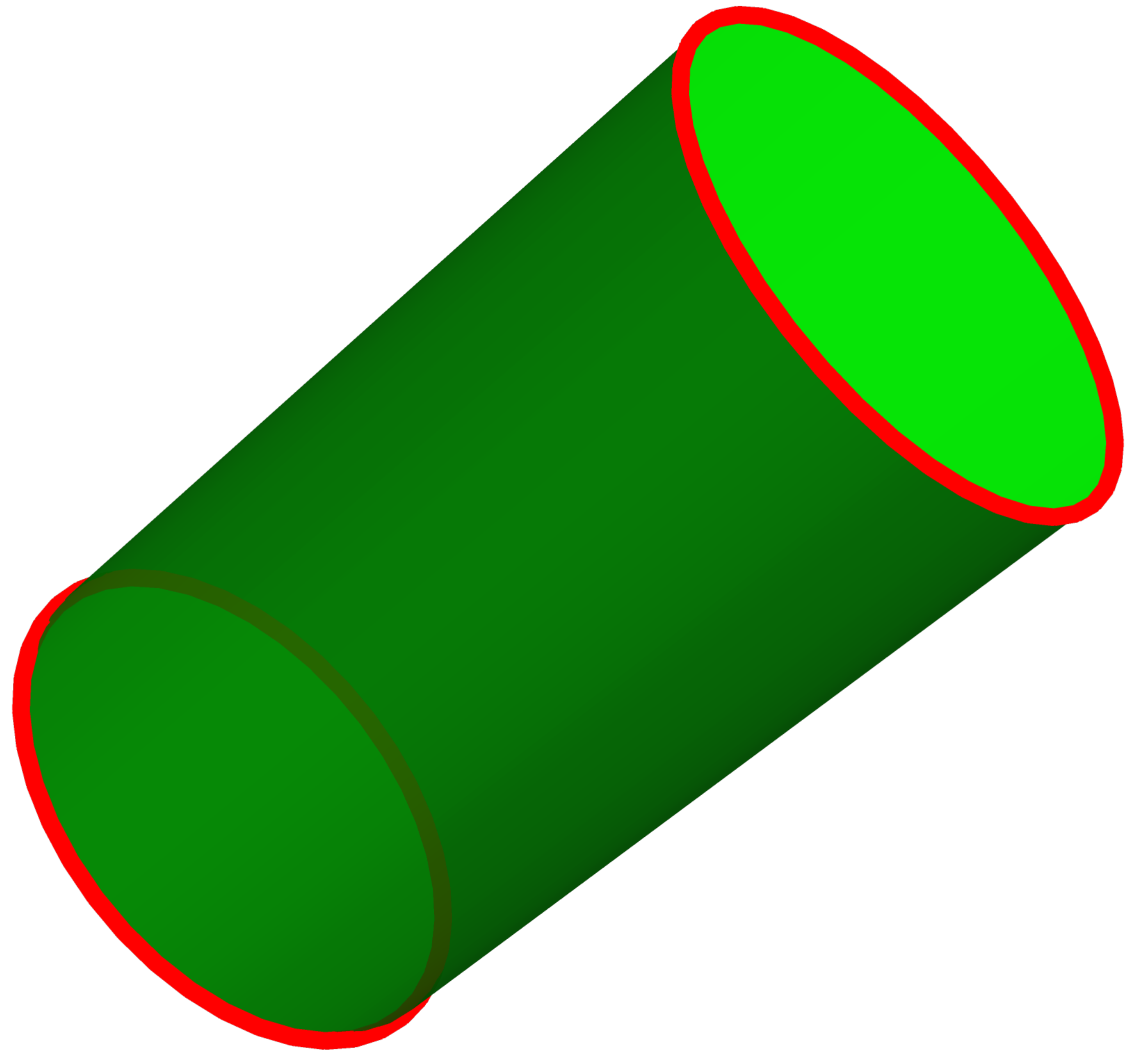
Phase diagram for QCD with Wilson fermions in 5d Euclidian spacetime



S Aoki, Prog Th Phys 122 (1996) 179

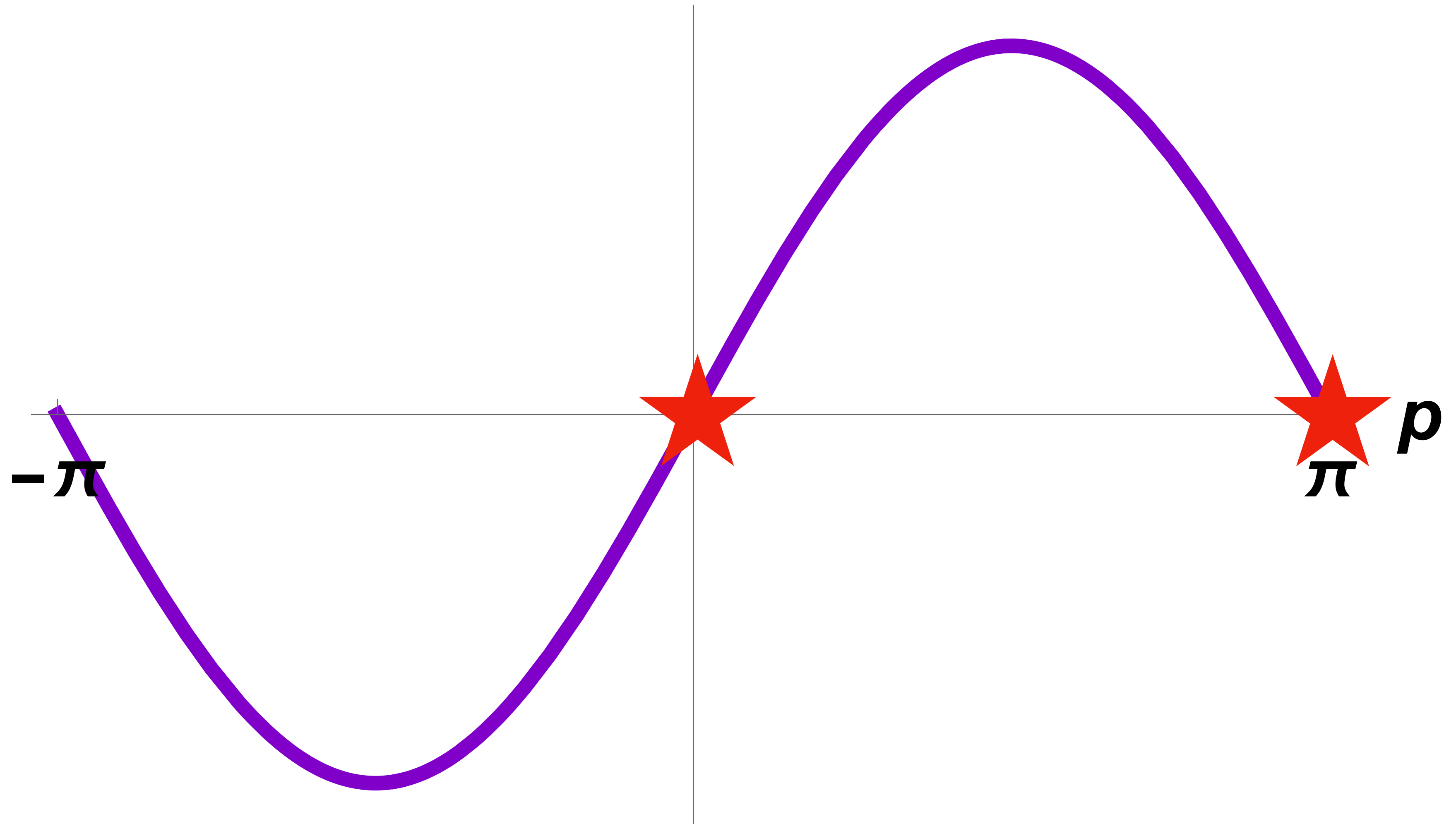


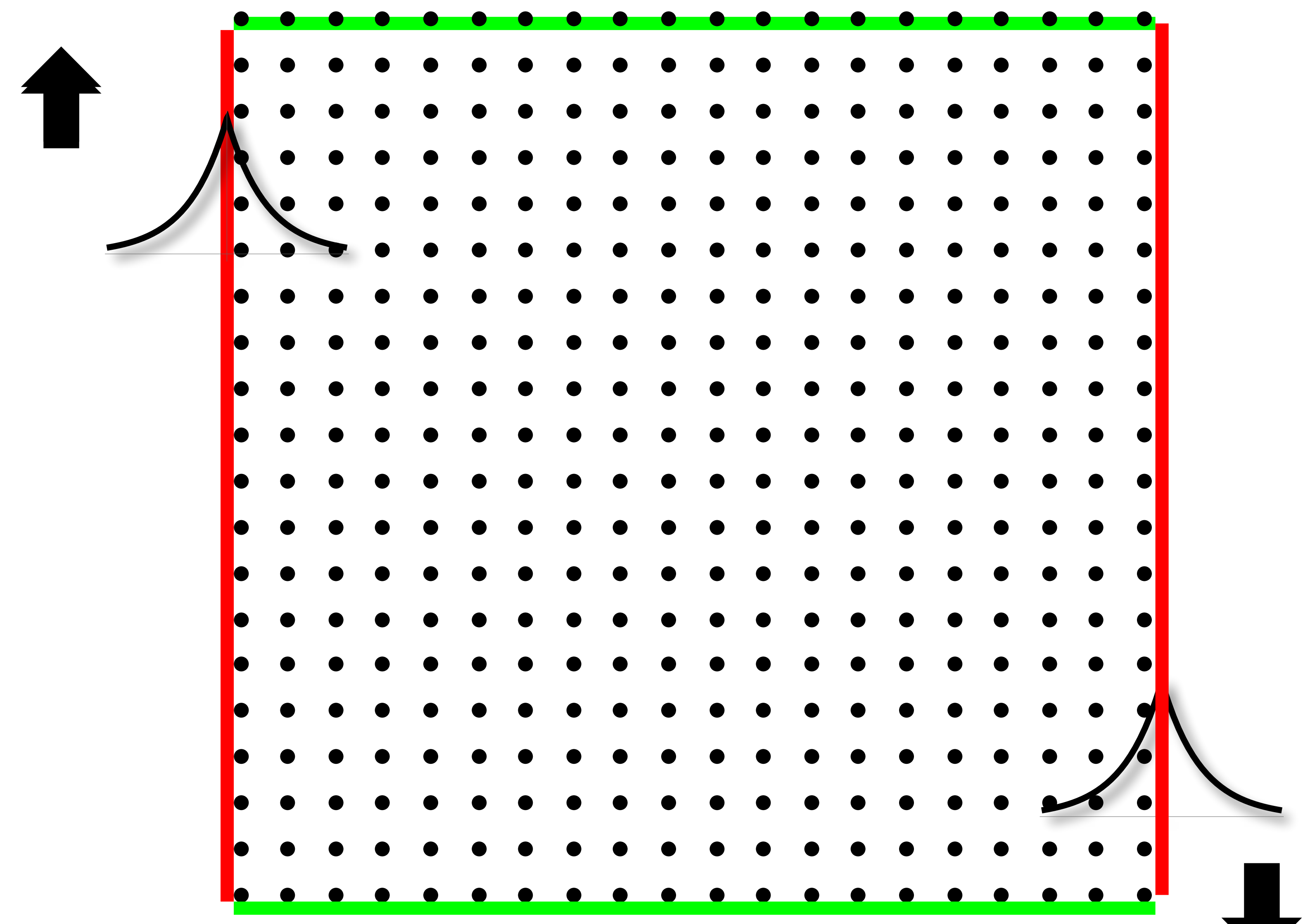
Obtain *almost* massless RH & LH Weyl fermions… mass $\propto e^{-2ML}$



Lattice has topology of an open cylinder with two boundaries

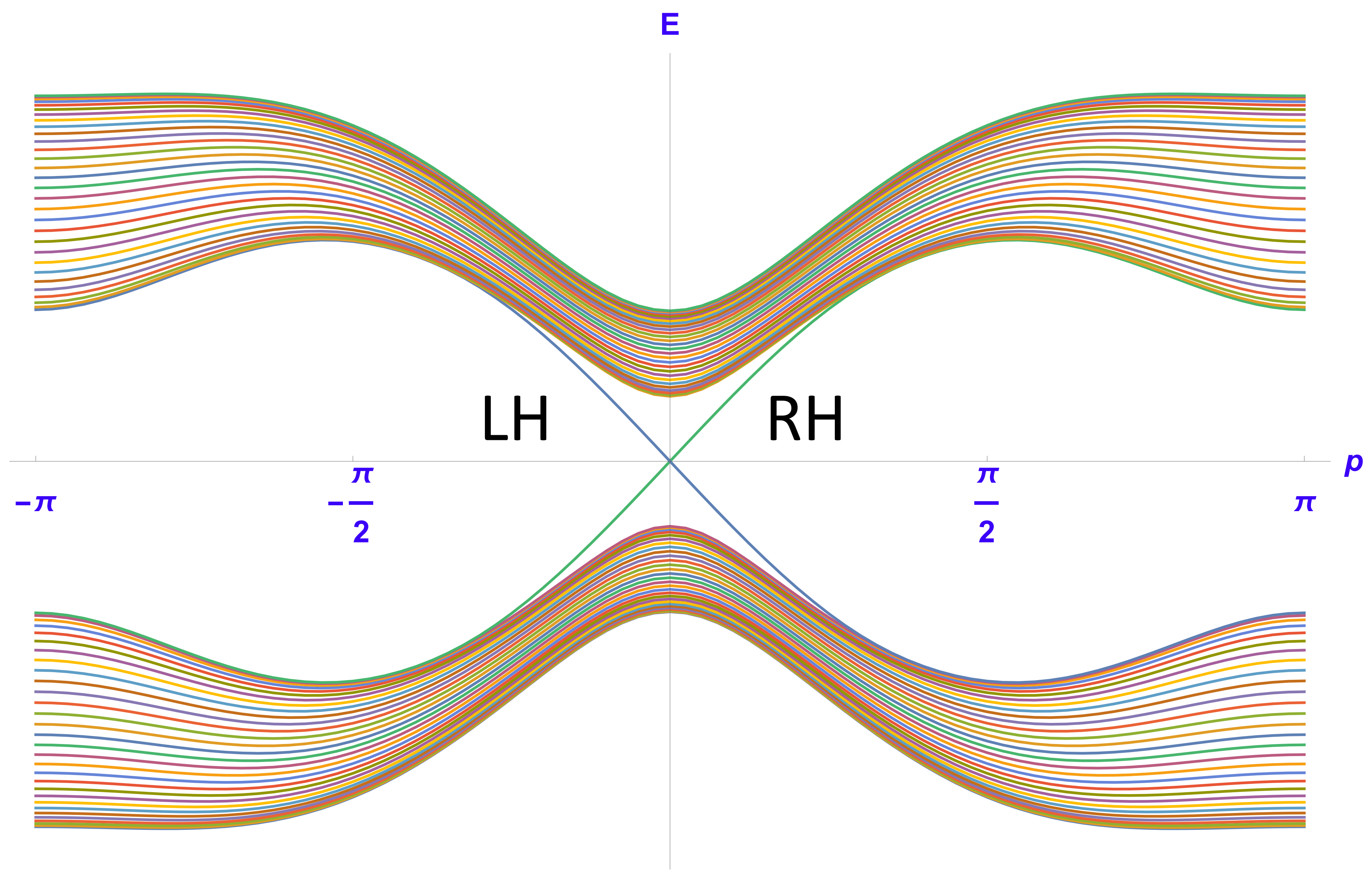
Won't there be doubled copies of fermions on each wall?





No! thanks to Wilson term, profile of zeromode $\propto e^{-M_{\rm eff} x_5}$

$$M_{\rm eff} \simeq M + r\sum_{i=1}^{d}(\cos p_i - 1)$$



At critical $|p_{crit}| < \pi$, $M_{\rm eff}$ changes sign, state **delocalizes**

What has been gained?? Wanted:

1. $\tilde{D}(\mathbf{p})$ is a periodic, analytic function of p_μ; — locality
2. $D(\mathbf{p}) \propto \gamma_\mu p_\mu$ for $a|p_\mu| \ll 1$; — correct continuum limit
3. $\tilde{D}(\mathbf{p})$ invertible everywhere except $p_\mu = 0$; — no doublers
4. $\{\Gamma, \tilde{D}(\mathbf{p})\} = 0$. — exact chiral symmetry ($\Gamma = \gamma_5$)

With exponentially light Dirac fermion, #4 is violated.

Any advantage of domain wall fermions over Wilson fermions?

Yes... $\left\{\tilde{D}, \Gamma\right\} = \tilde{D}\Gamma\tilde{D}$ Obeys "Ginsparg-Wilson" equation

- reproduces the correct chiral anomalies
- but still enforces multiplicative mass renormalization

Excellent for QCD...
BUT... apparently useless for constructing a chiral gauge theory?

Domain fermions have the attractive feature of being topological and "knowing" about anomalies

Proposals to use them for evading Nielsen-Ninomiya theorem and constructing a lattice chiral gauge theory:

- Ginsparg-Wilson approach (Lüscher): use GW fermions (Abelian chiral gauge theories constructed this way, but not non-Abelian). Sacrifices NN #4 (D anti-commuting with γ_5 … involves O(a) corrections)
- Symmetric mass generation (Eichten, Preskill, Wen, Cenke, You, Wang…): invoke many-body physics to gap unwanted mirror fermions when anomalies cancel
- Proposal here: use domain wall fermion with **single** connected boundary between topological phases to regulate chiral gauge theory.

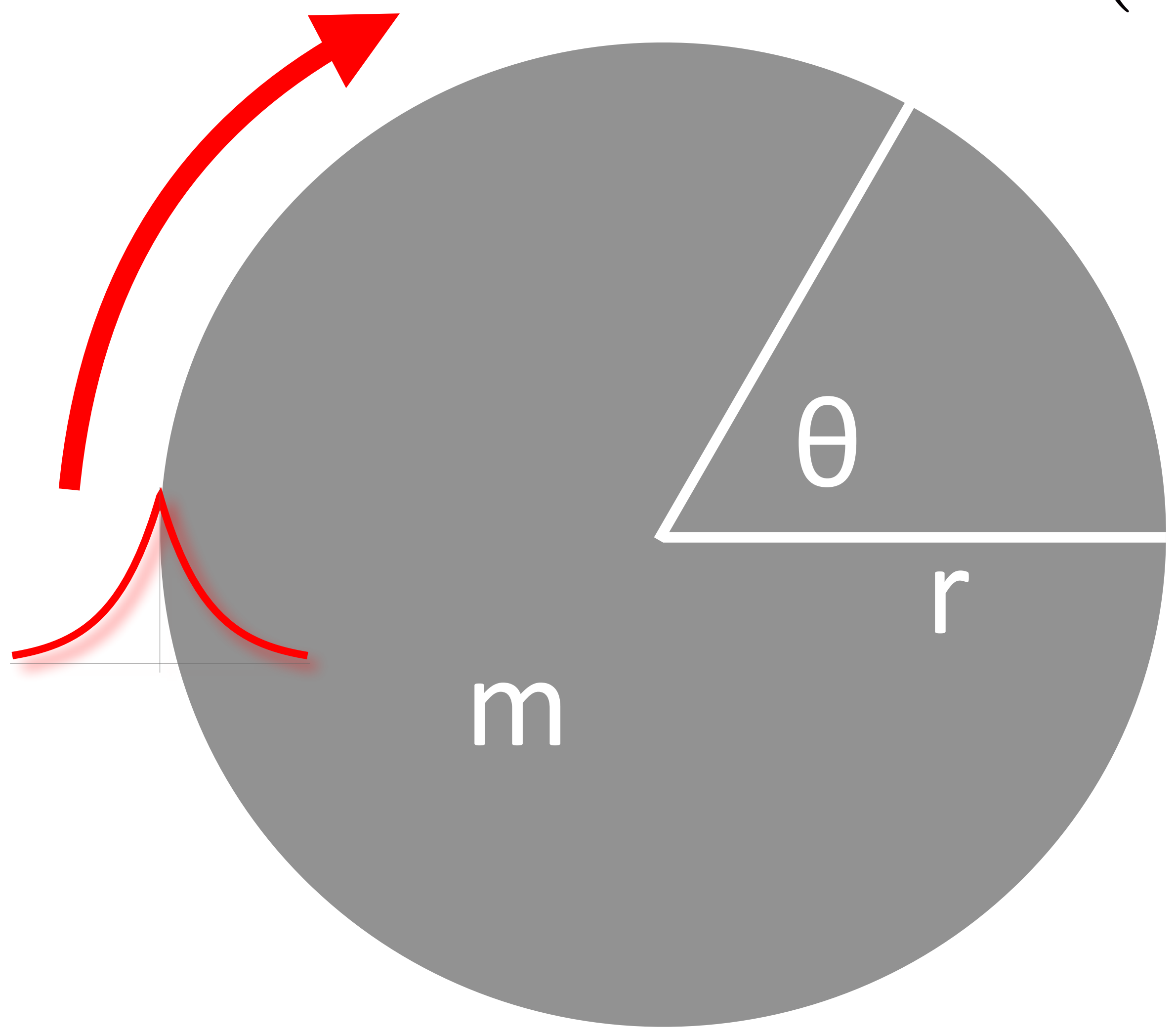
Edge states on manifold with a **single** boundary:

Consider Dirac fermion in d+1 *continuum* dimensions:

$M^{d-1} \times R^2$ with coordinates $\{\mathbf{x}_\perp, x, y\} = \{\mathbf{x}_\perp, r, \theta\}$

$$m(r) = \begin{cases} m & r < R \\ -M & r > R \end{cases}$$

$$-M \to -\infty$$

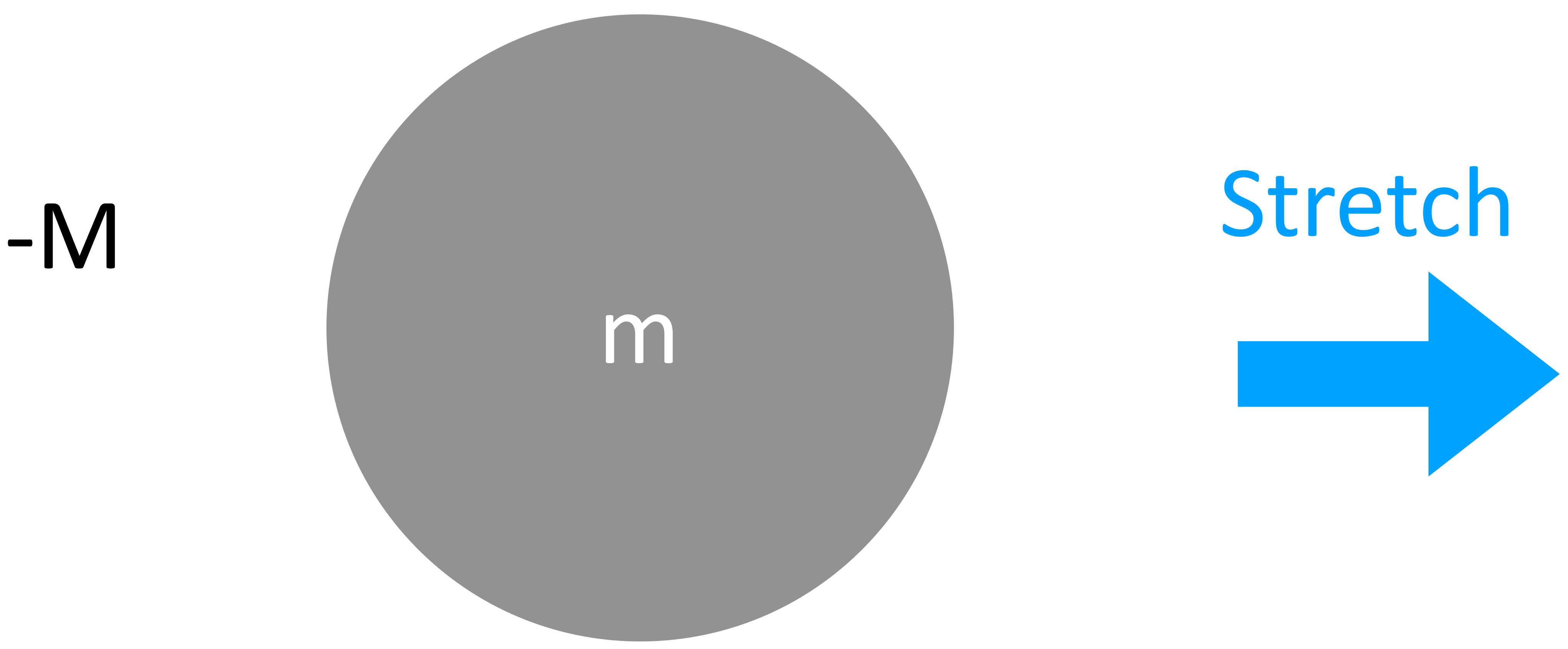


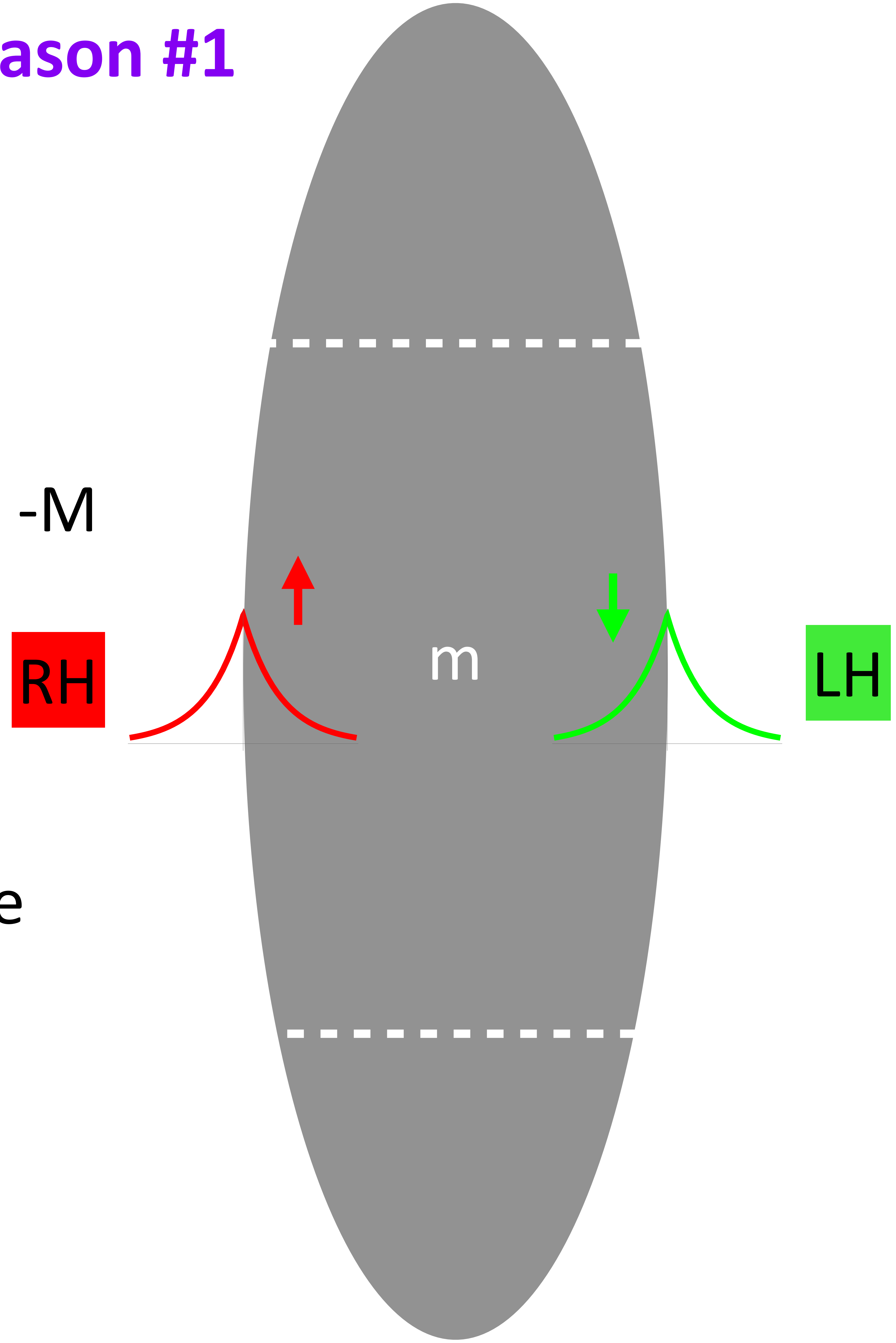
Shouldn't this have a single Weyl fermion edge state?

Which must be exactly massless?

Which can be realized with Wilson fermions on a lattice?

Why there can't be a chiral edge state: reason #1





...looks like wall/anti-wall system with finite size

...expect RH + LH modes with exponentially small chiral symmetry violating mass

Why there can’t be a chiral edge state: reason #2

- If there is an exact chiral edge state, then there must be a solution that is independent of angle (zero momentum) which is an exact zero-mode of the higher dimension Dirac operator on the disc

- Zeromode solutions are easy to solve for! 😀

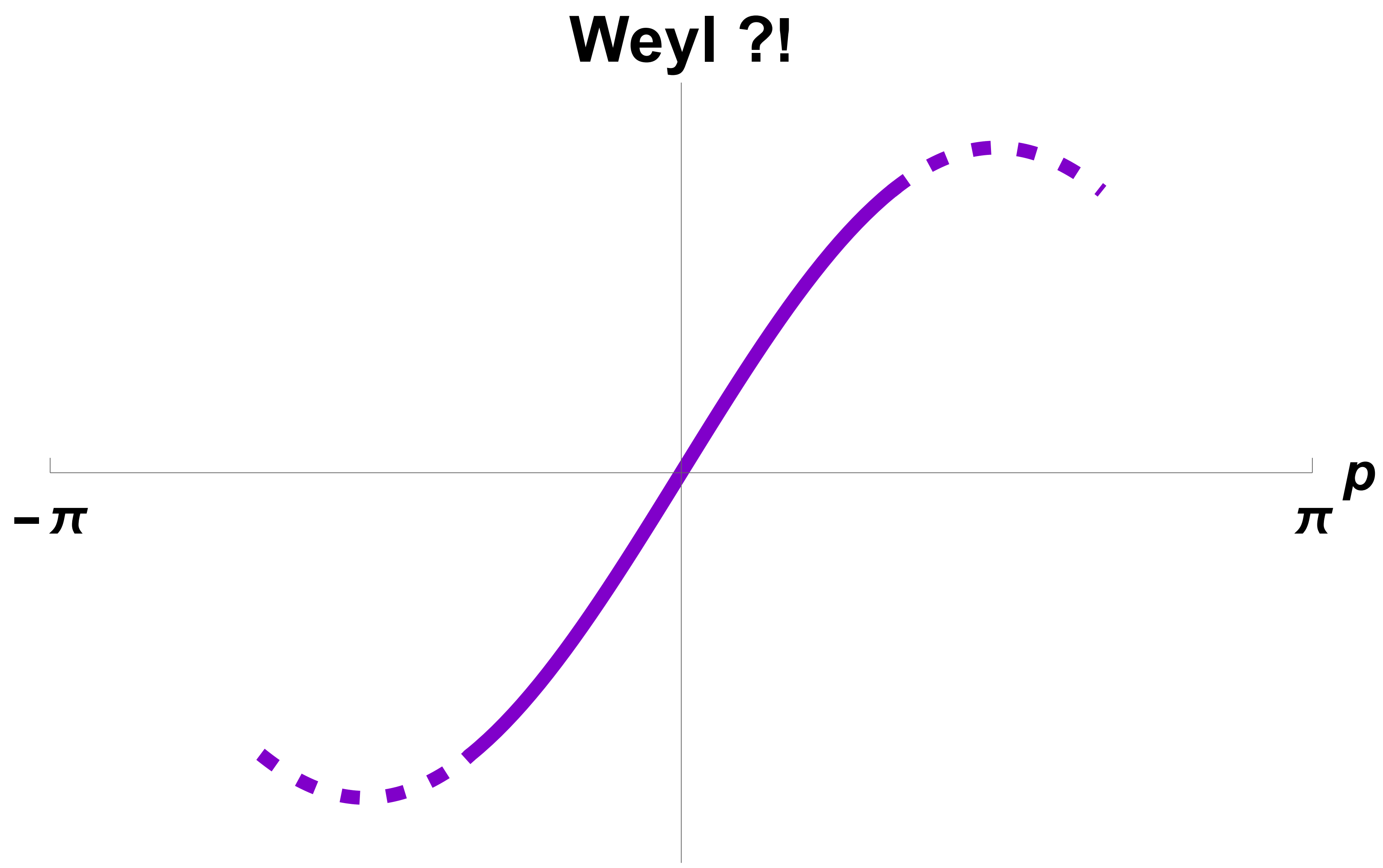
- And it is easy to show that there isn’t a zeromode for the Dirac operator on disc! 😩

$\Rightarrow$

NO ZERO
MOMENTUM EDGE
STATE

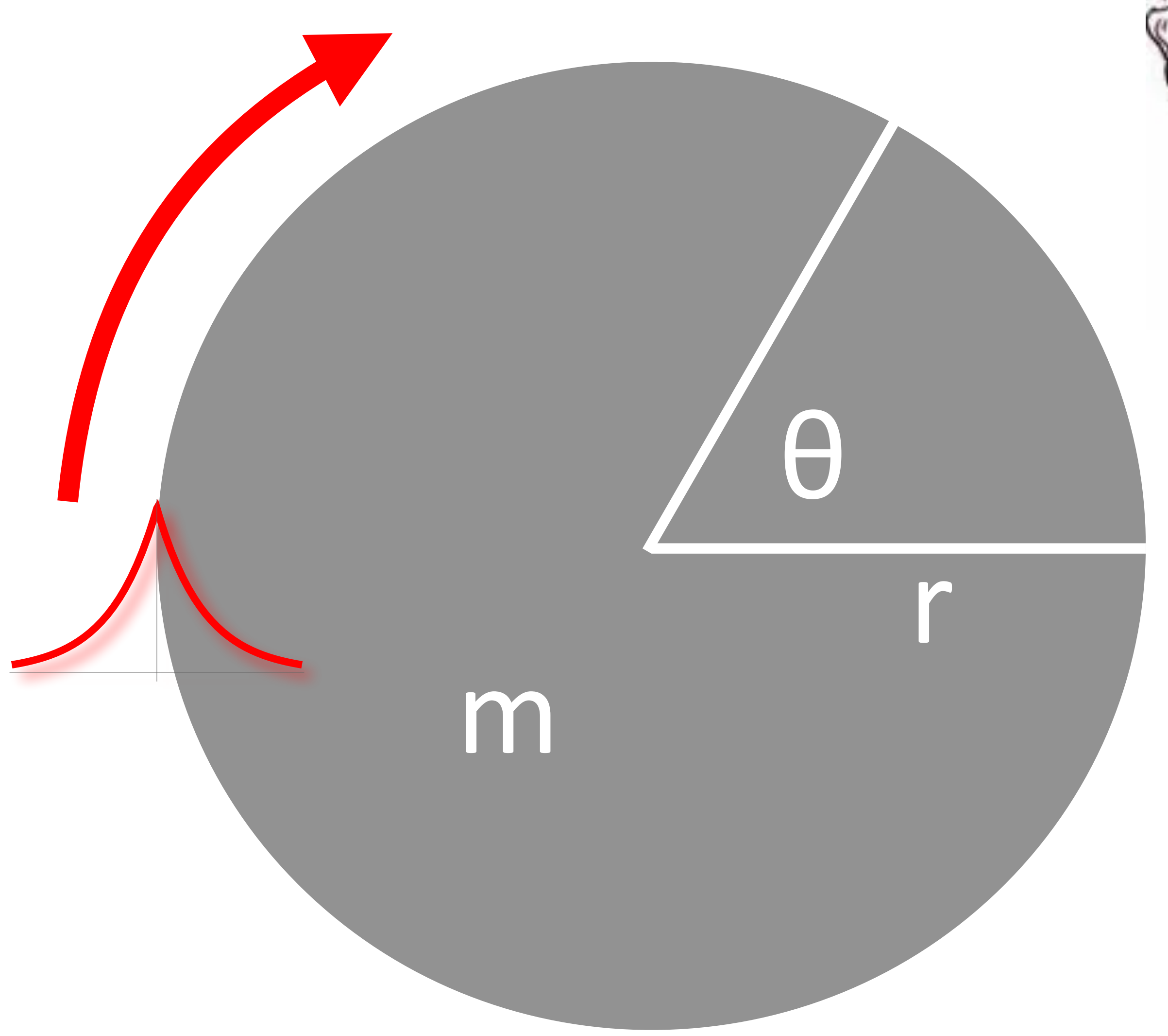
Why there can't be a chiral edge state: reason #3

- Since topological phases exist with Wilson fermions on a lattice, we should be able to easily construct disc edge states on a lattice if they exist in the continuum! 😀
- …but the Nielsen Ninomiya theorem says that we on a lattice we must have an analytic, periodic dispersion relation which cannot cross zero an odd number of times (eg, once) 😩



Think less, calculate more

$$-M \to -\infty$$



Solve the Dirac equation with this mass profile

(DB Kaplan: Phys. Rev. Lett. 132 (2024) 141603, arXiv:2312.01494)

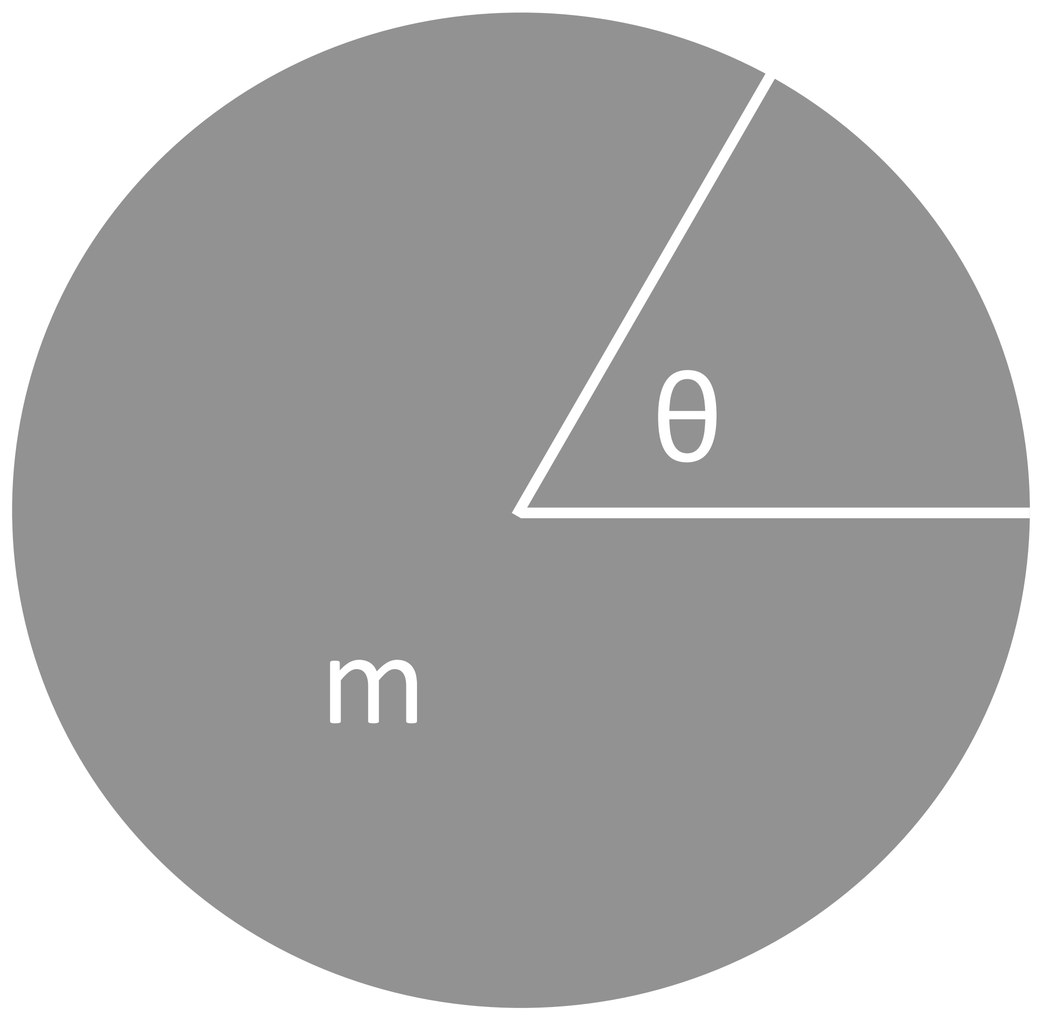
$$S = \int d\mathbf{x}_\perp \int r\, dr\, d\theta\, \overline{\psi} \left(\partial\!\!\!/_\perp + \mathcal{D} \right) \psi$$

dim=d-2 → $\partial\!\!\!/_\perp$; dim=2 → $\mathcal{D}$

$$\mathcal{D} = \gamma_x \partial_x + \gamma_y \partial_y + m(r)$$

$$= \gamma_r \left(\partial_r + \frac{1}{2r} \right) + \frac{i}{r} \gamma_\theta \mathcal{J} + m(r)$$

$-M \to -\infty$



$$\mathcal{J} = -i\partial_\theta + \tfrac{1}{2}\Sigma \,, \qquad \Sigma = -\frac{i}{2} [\gamma_x, \gamma_y]$$

A convenient basis:

$$\vec{\gamma}_\perp = \sigma_3 \otimes \vec{\Gamma} \,, \quad \gamma_x = \sigma_1 \otimes 1 \,, \quad \gamma_y = \sigma_2 \otimes 1 \,.$$

$$\gamma_r = \begin{pmatrix} 0 & e^{-i\theta} \\ e^{i\theta} & 0 \end{pmatrix} \,, \qquad \gamma_\theta = \begin{pmatrix} 0 & -ie^{-i\theta} \\ ie^{i\theta} & 0 \end{pmatrix}$$

- r plays the role of x_5
- γ_r plays the role of γ_5

Find:

- There is an exact Weyl edge mode circulating the disc in only one direction
- Its chiral symmetry is exact: part of the exact U(1) fermion number symmetry of the higher dimension theory
- The total angular momentum coordinate (*-j/R*) plays the role of linear momentum around the disc edge

Precisely: Euclidian action of edge mode is

$$S = \int d\mathbf{x}_\perp \sum_n \bar{\chi}_n \left(\vec{\Gamma} \cdot \vec{\partial}_\perp + \mu_n \right) \chi_n \ .$$

In d=1+1, $\vec{\Gamma} = 1$

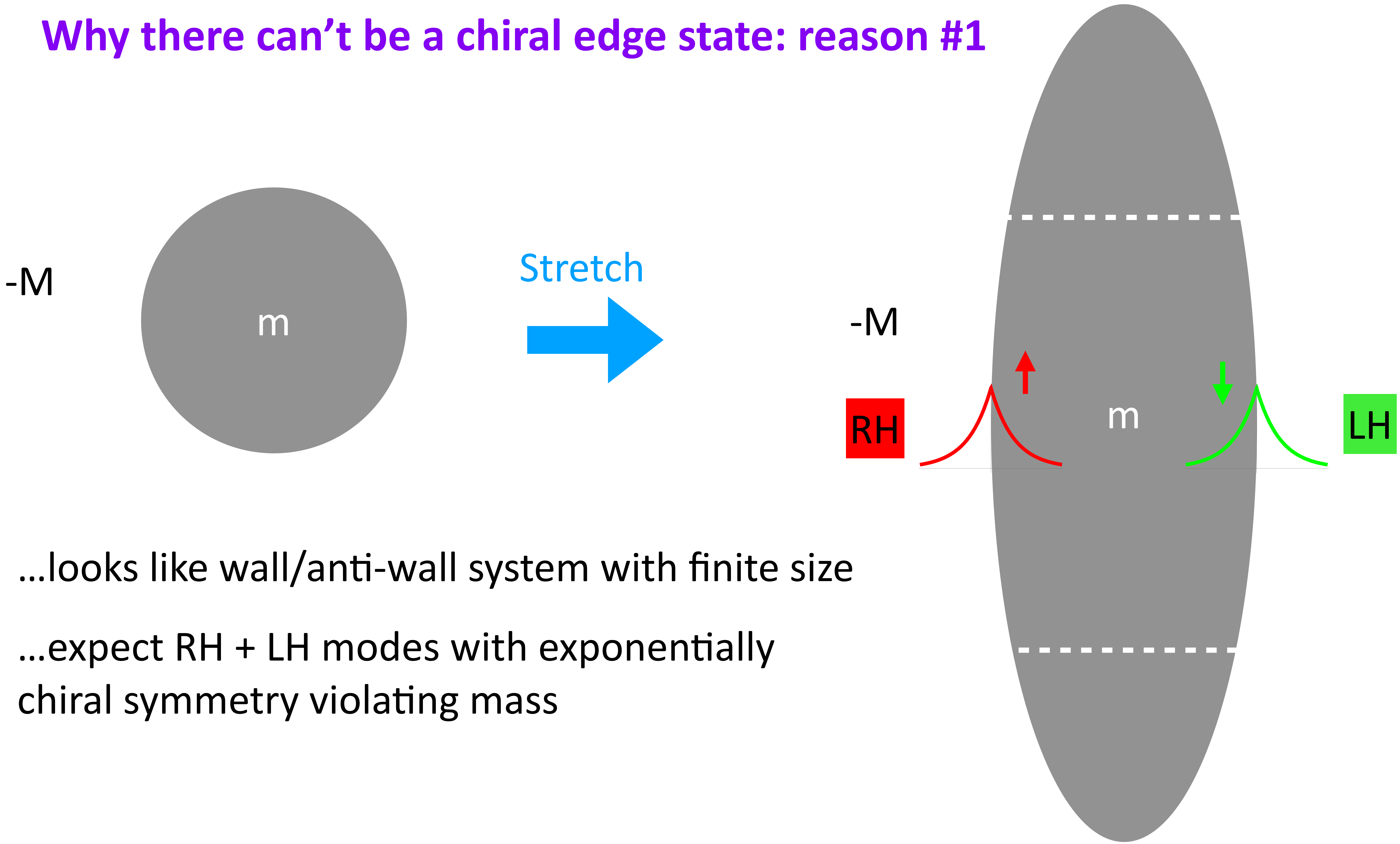
In d=3+1, $\vec{\Gamma} = \vec{\sigma}$

-j/R ~ momentum in boundary world

$$\mu_j = -\frac{j}{R}\left[1 + \frac{1}{2mR} + \frac{1}{2m^2R^2} + \frac{3}{4m^3R^3} + \frac{3}{2m^4R^4} + \frac{15}{4m^5R^5}\right] + \frac{j^3}{R}\left[\frac{1}{4m^4R^4} + \frac{3}{2m^5R^5}\right] + O\left((mR)^{-6}\right), \quad (21)$$

What happened to all those arguments that this shouldn't be possible?

Why there can't be a chiral edge state: reason #1



…looks like wall/anti-wall system with finite size

…expect RH + LH modes with exponentially chiral symmetry violating mass

…**but** the wall/anti-wall system had constant γ_5…

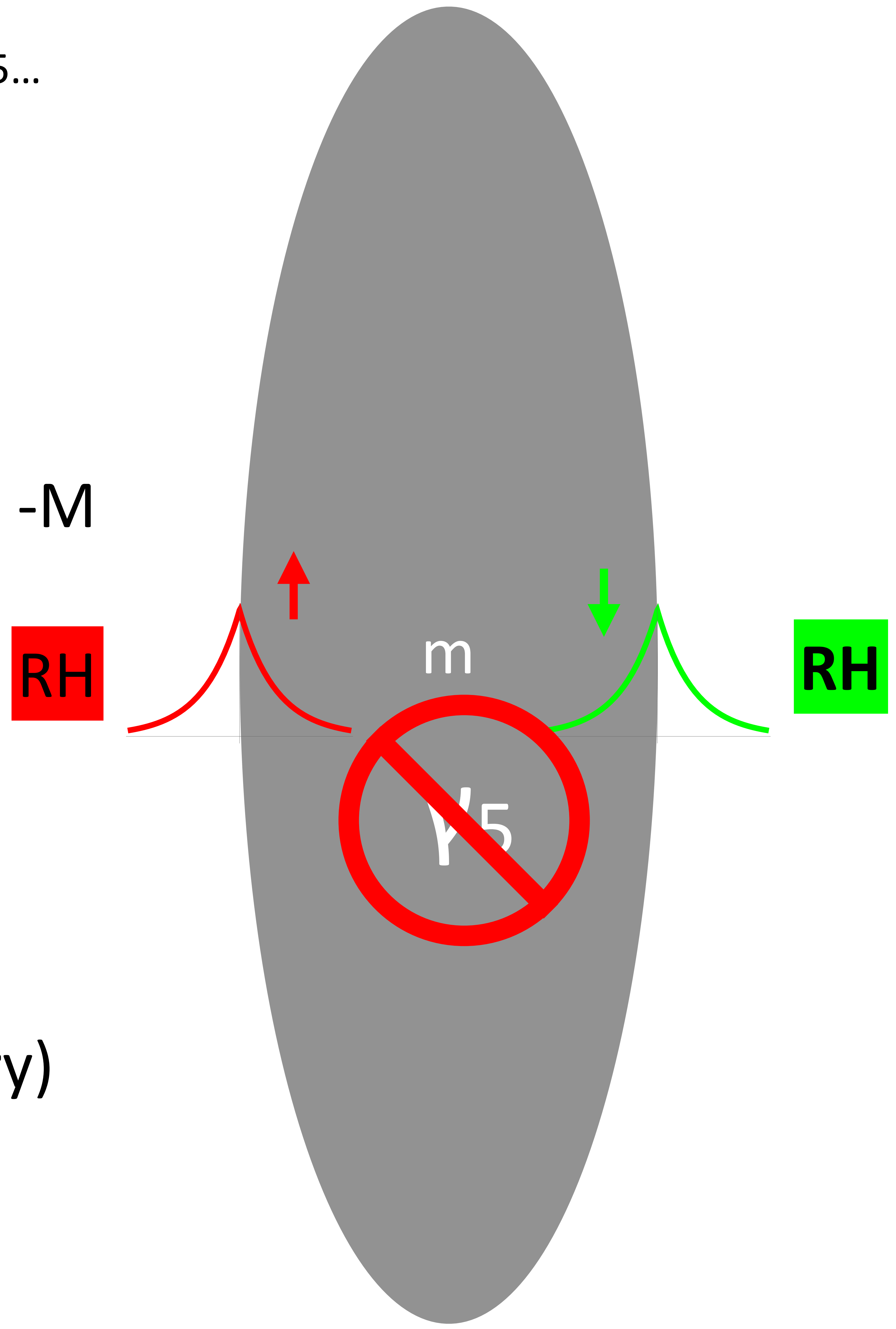
on disc, analog of γ_5 for edge states is

$$\gamma_r = \begin{pmatrix} 0 & e^{-i\theta} \\ e^{i\theta} & 0 \end{pmatrix}$$

which changes sign on opposite side of disc!

Exponentially small interaction is still there,
but preserves chirality 100%
(= fermion number in higher dimension theory)

…it violates locality though! 👻



Why there can't be a chiral edge state: reason #2

- If there is an exact chiral edge state, then there must be a field that is independent of angle (zero momentum) which is an exact zero-mode of the higher dimension Dirac operator on the disc
- Zeromode solutions are easy to solve! 😀
- And it is easy to show that there isn't one! 😩

…but we have seen that momentum about the edge is given by -j/R and j=±1/2, ±3/2…

There is no zero momentum edge state — it behaves as if anti-periodically quantized!
…and therefore no exact zeromode in Euclidian spacetime.

Why there can't be a chiral edge state: reason #3

- Since topological phases exist with Wilson fermions on a lattice, we should be able to easily construct disc edge states on a lattice if they exist in the continuum! 😀
- …but the Nielsen Ninomiya theorem says that on a lattice we must have an analytic, periodic dispersion relation which cannot cross zero an odd number of times (eg, once) 😩

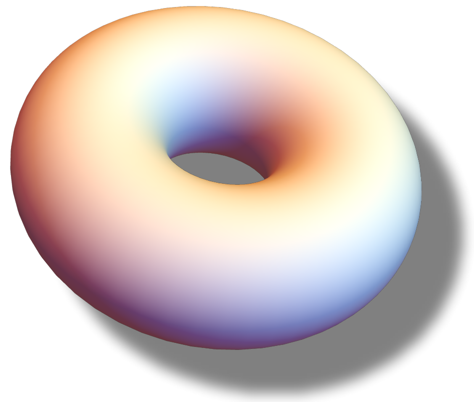
- …but we have already argued that there must be some nonlocality in the theory, so perhaps the dispersion is not analytic & periodic and that's OK?

Let's look at what happens on a lattice

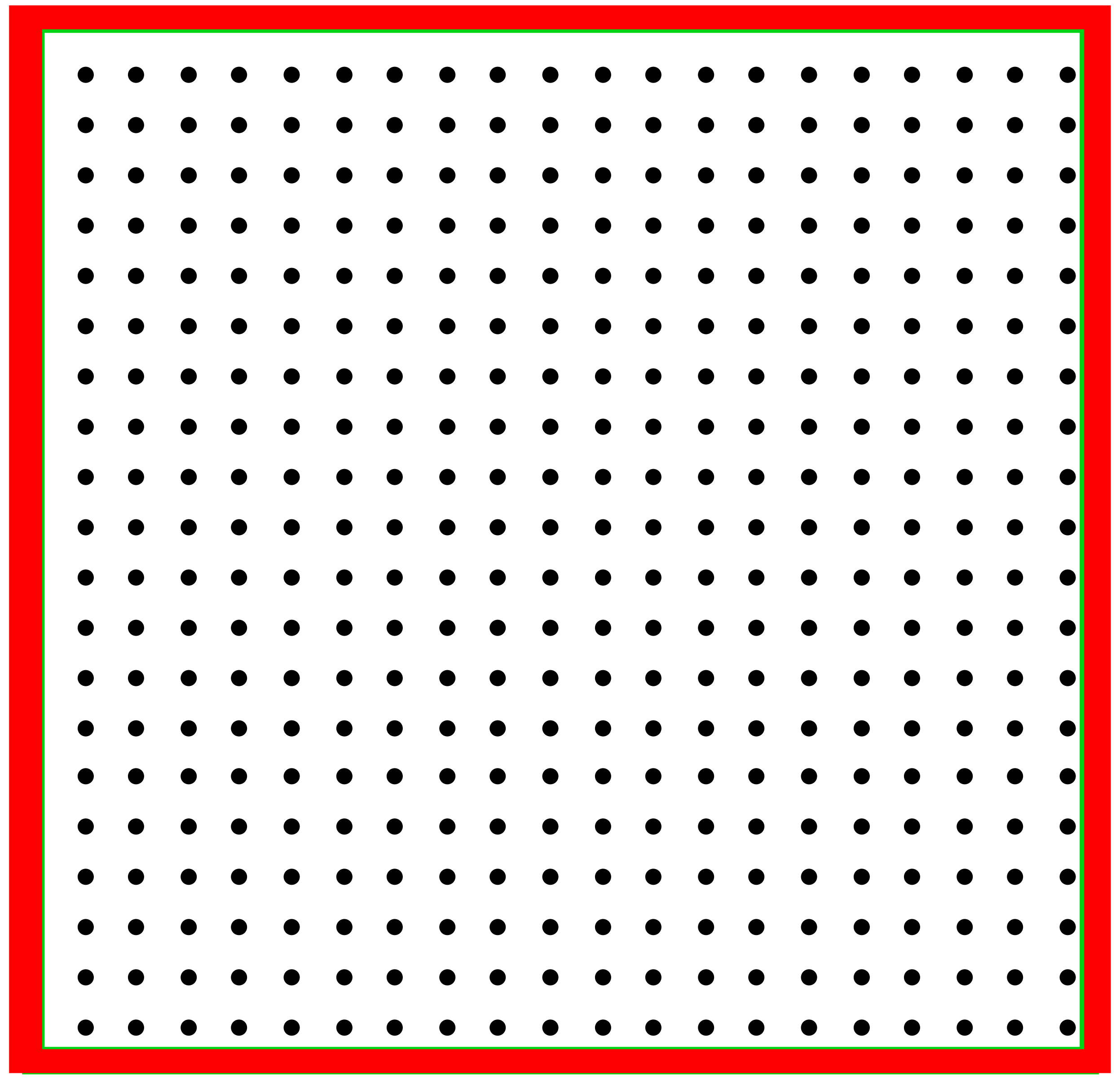
DB Kaplan, S. Sen: Phys. Rev. Lett. 132 (2024) 141604 arXiv:2312.04012

Free Wilson fermions on a 2d spatial lattice: consider 3 different boundary conditions

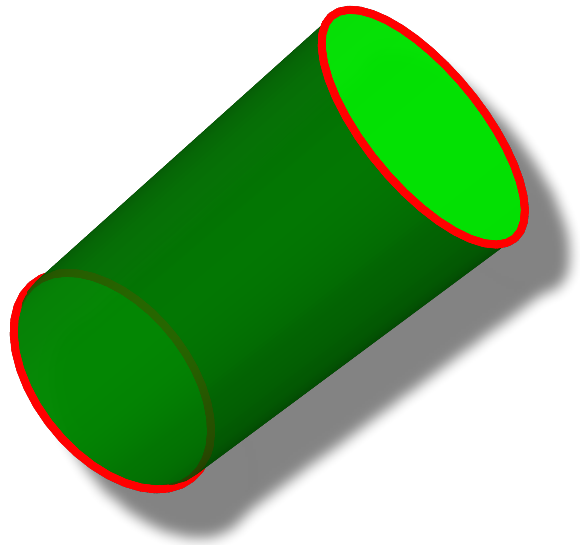
- **Periodic** boundary conditions in x & y



topology = torus, no boundaries

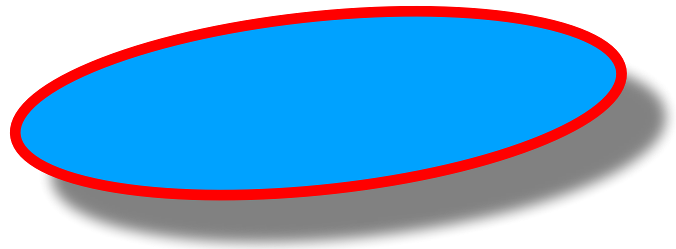


- Mixed: **periodic** in y + **open** BC in x



topology = open cylinder,
2 disconnected boundaries

- **Open** boundary conditions in x & y

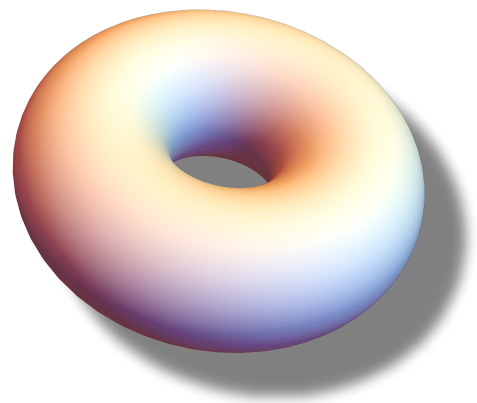


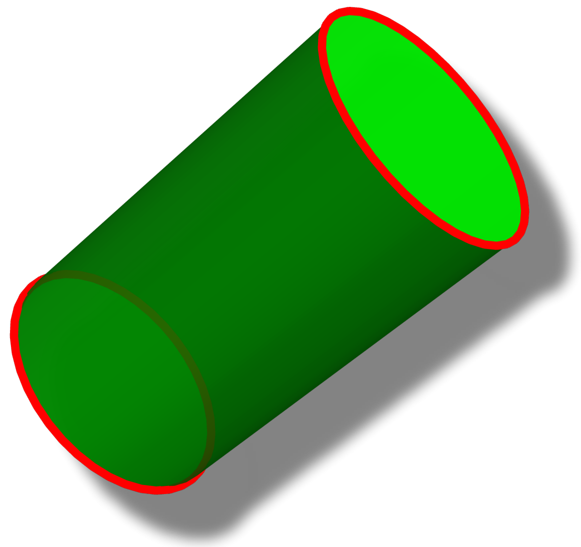
topology = disc,
1 connected boundary

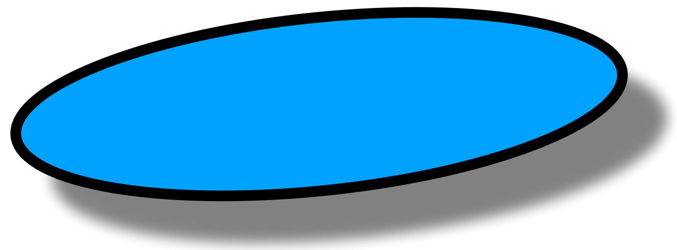
$$H = \gamma_0 \mathcal{D} ,$$
$$\mathcal{D} = \sum_{\mu=1}^{2} \gamma_\mu \partial_\mu + M + \frac{r}{2} \Delta$$

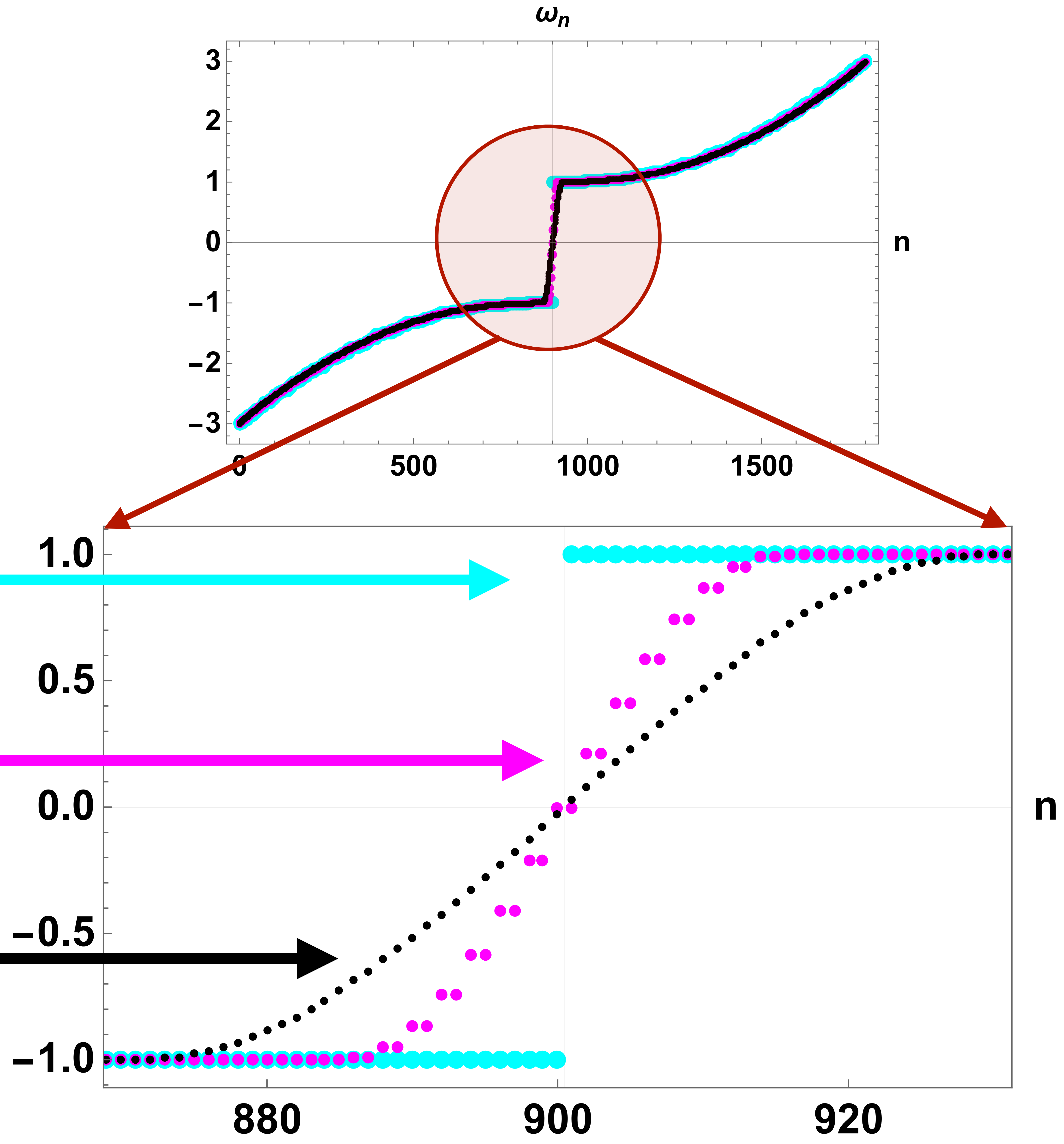
Spectrum of 2d Hamiltonian for massive Wilson fermions:

3 different lattice topologies (BC)

=  gapped

=  LH+RH, massless Dirac

=  one massless Weyl !?



Weyl edge state?
Look at 1+1 dispersion relation

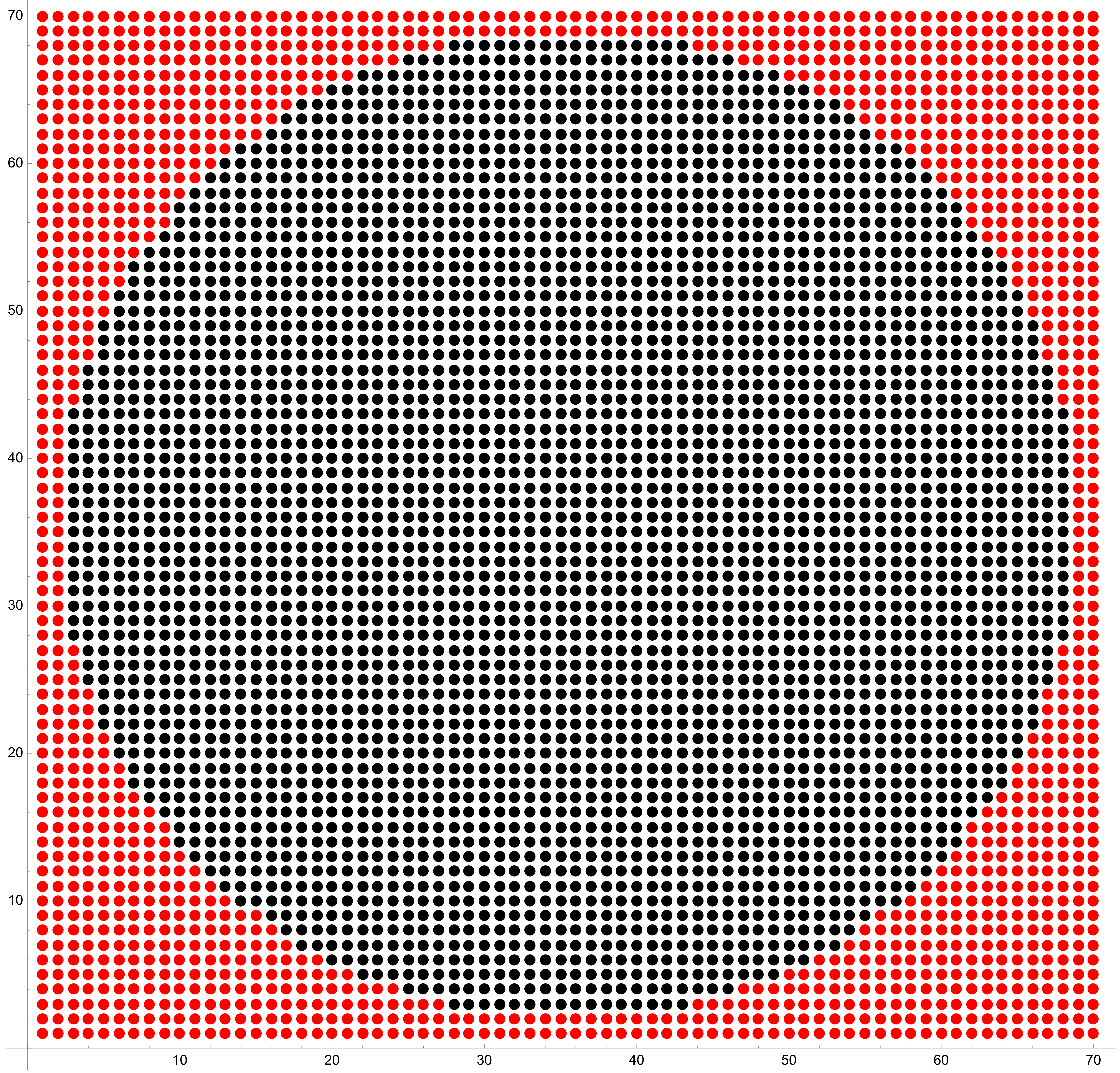
Work on a lattice disc with open BC

$$P_R = \begin{cases} 0 & x^2 + y^2 \geq R^2 \\ 1 & x^2 + y^2 < R^2 \end{cases}$$

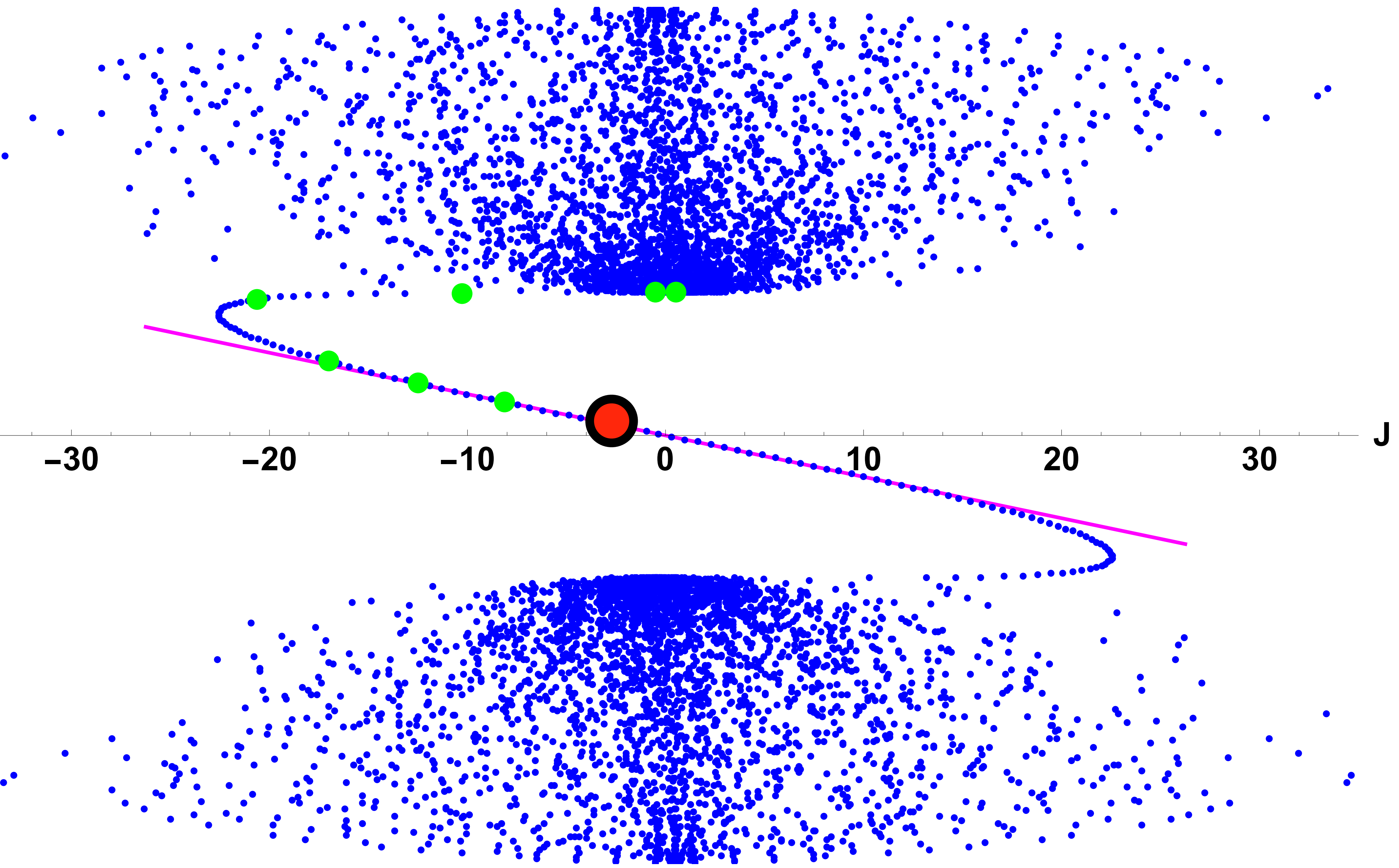
$$H_{\text{disc}} = P_R \, H_{L\times L} \, P_R$$

We took L=70, R = 34.

If you want E vs p for the edge state, plot E vs J

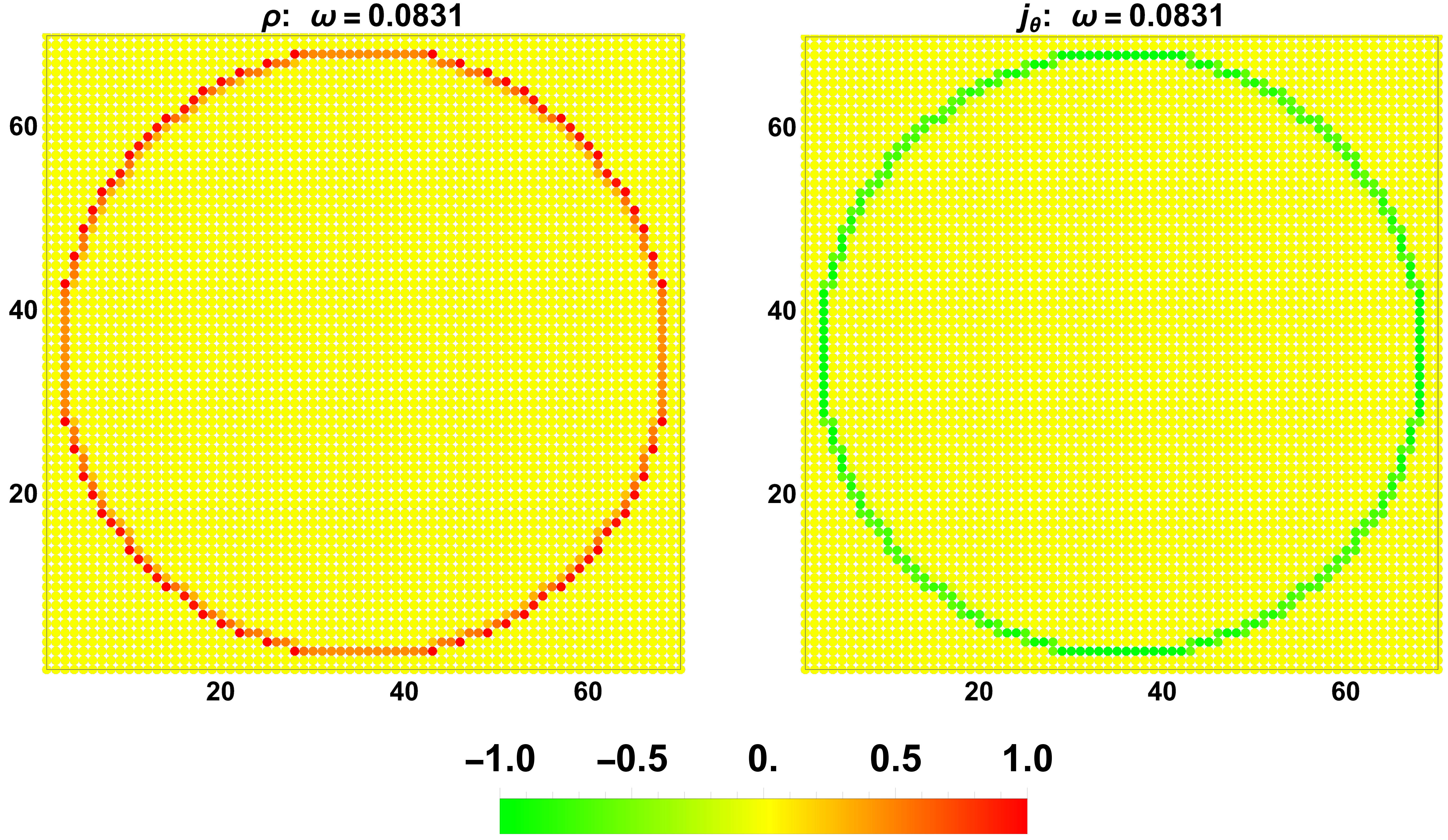


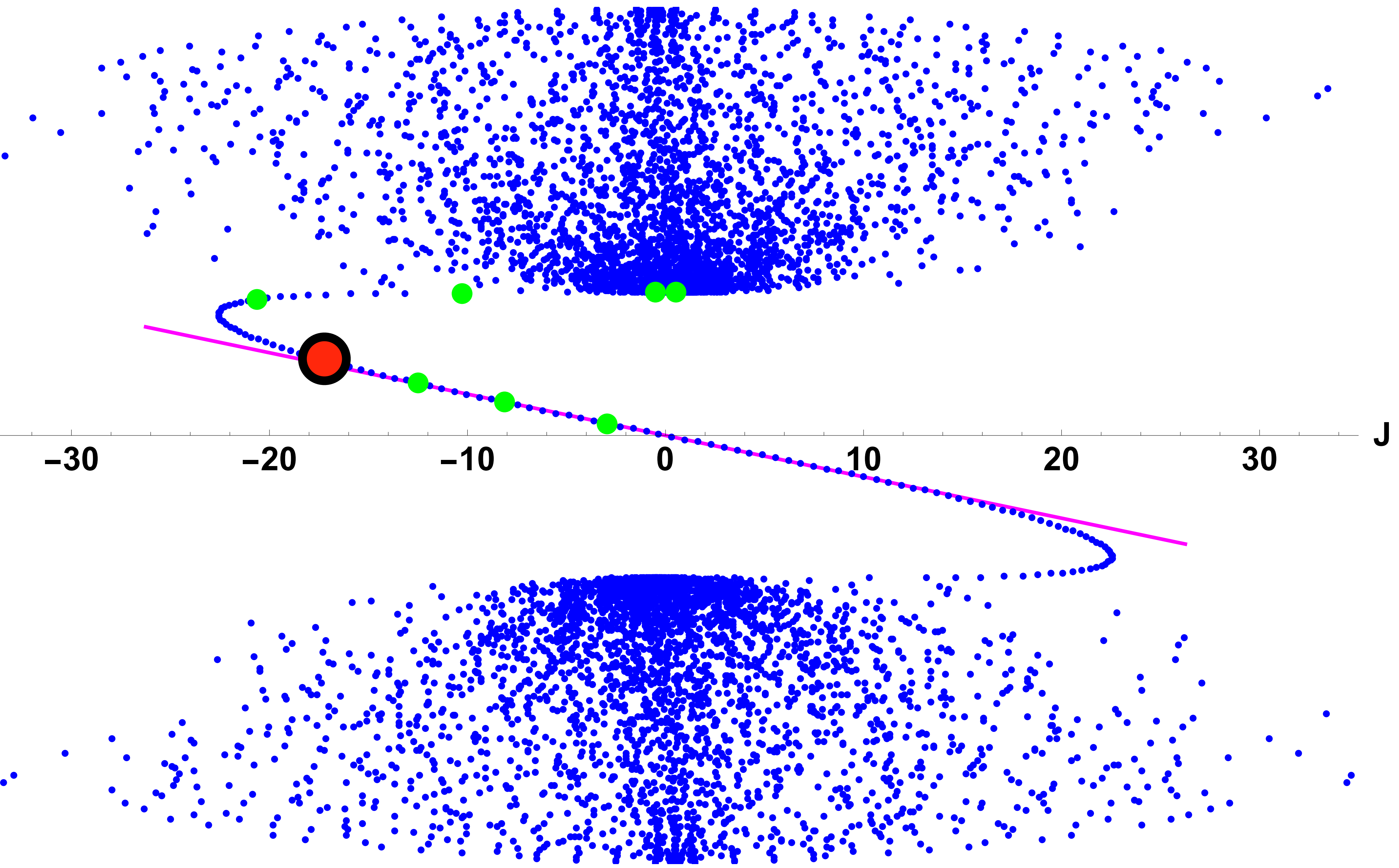
Nielsen-Ninomiya would have you believe this is not possible for sensible system



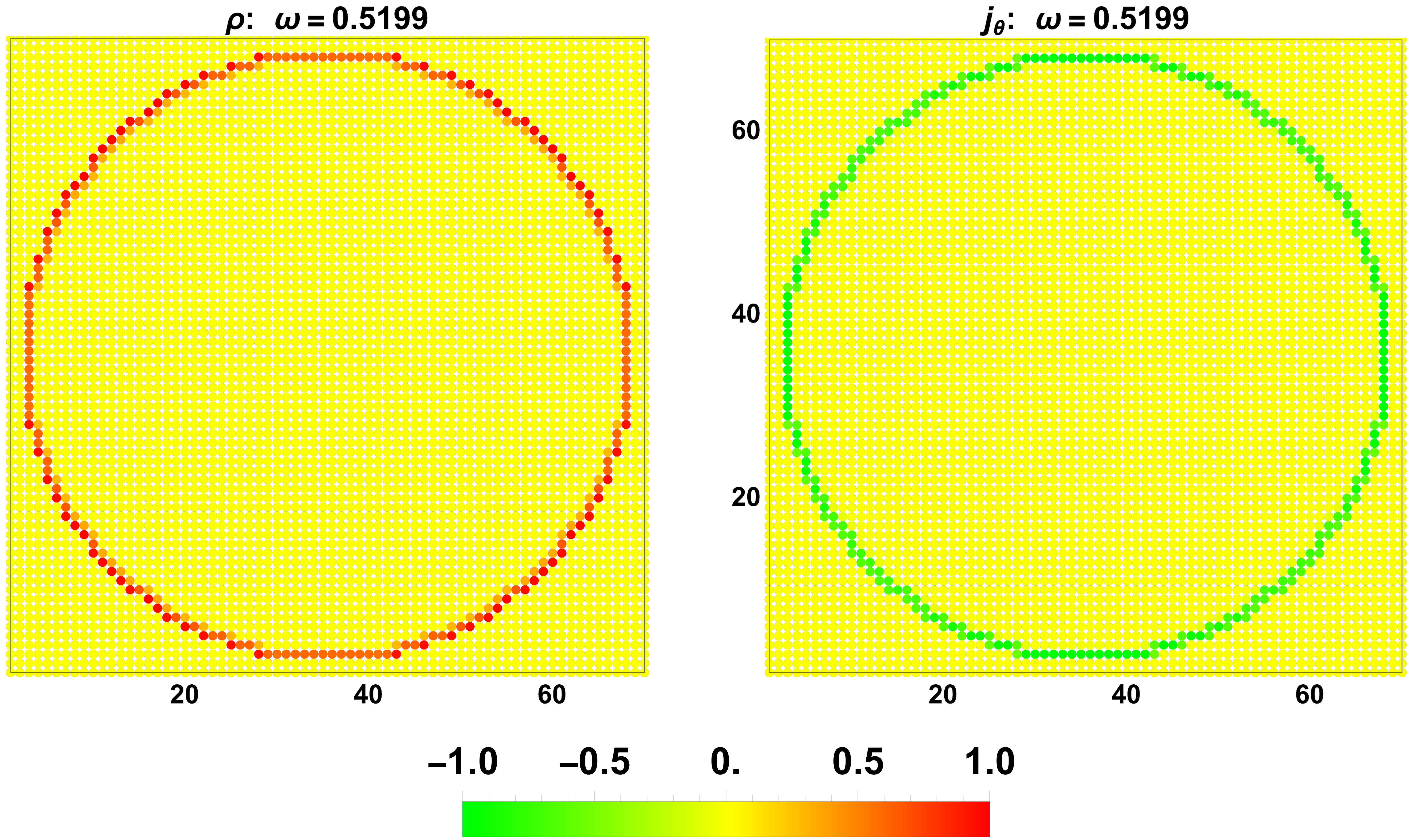
charge density ρ

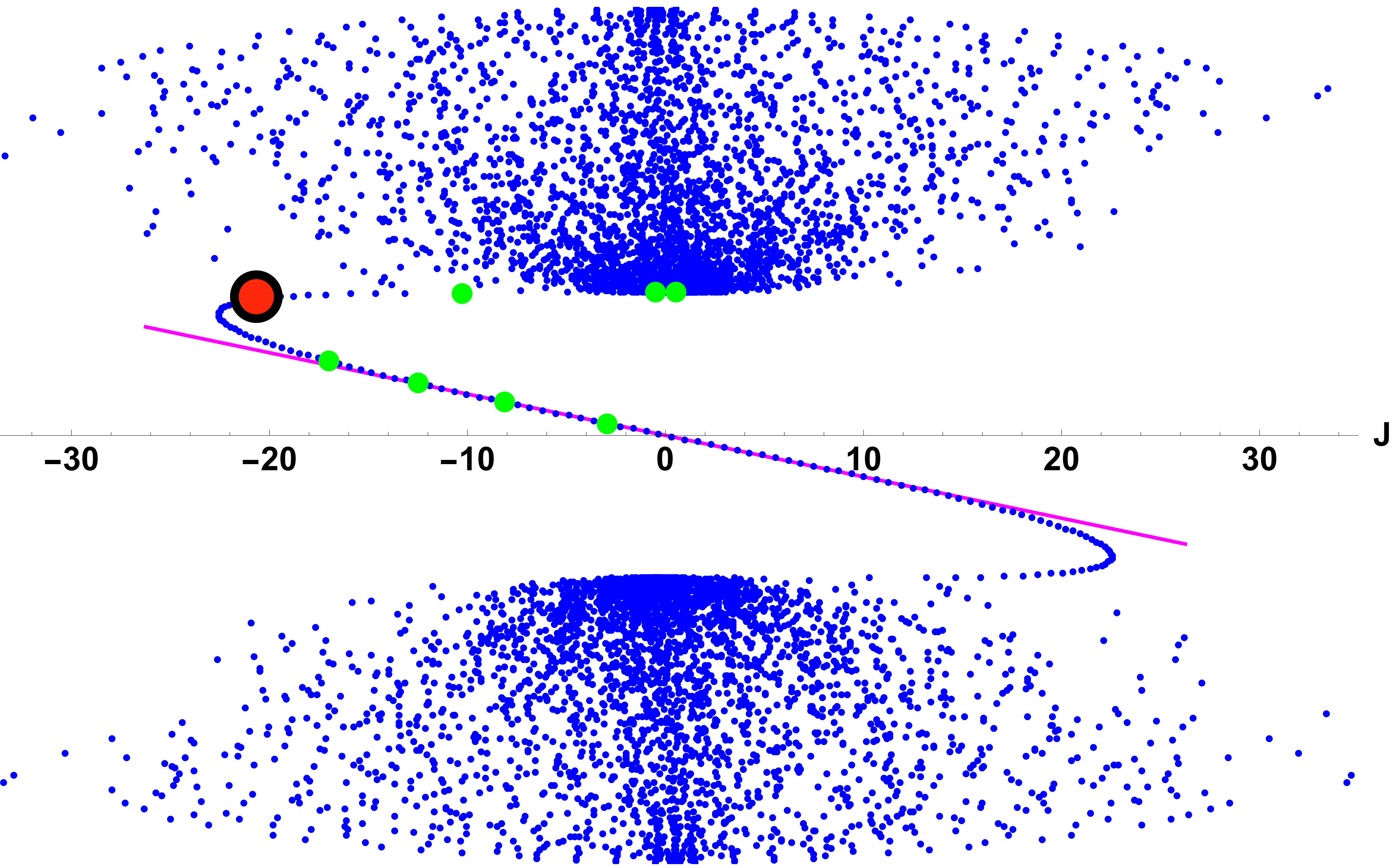
current density j_θ





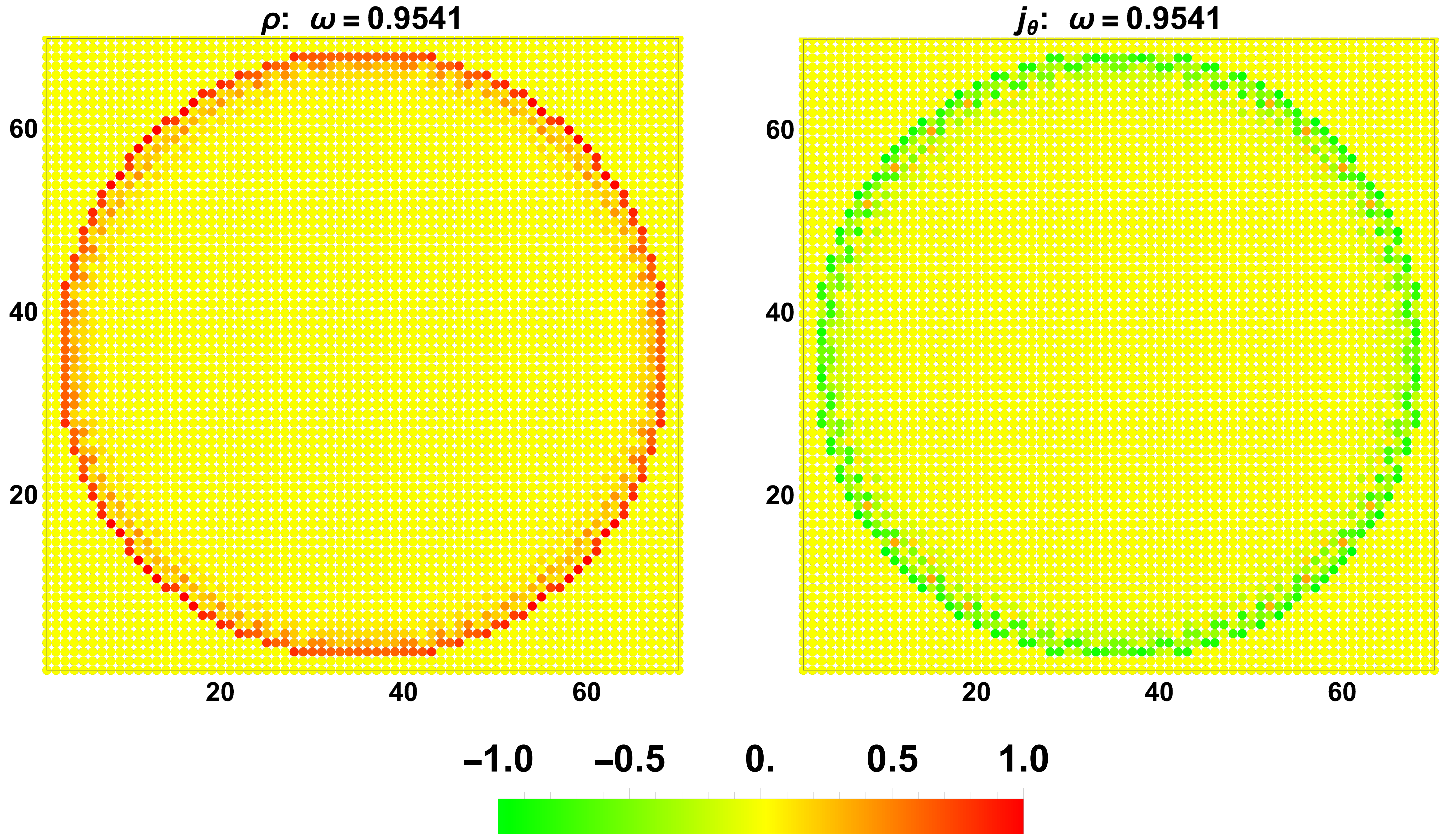
charge density ρ   current density j_θ

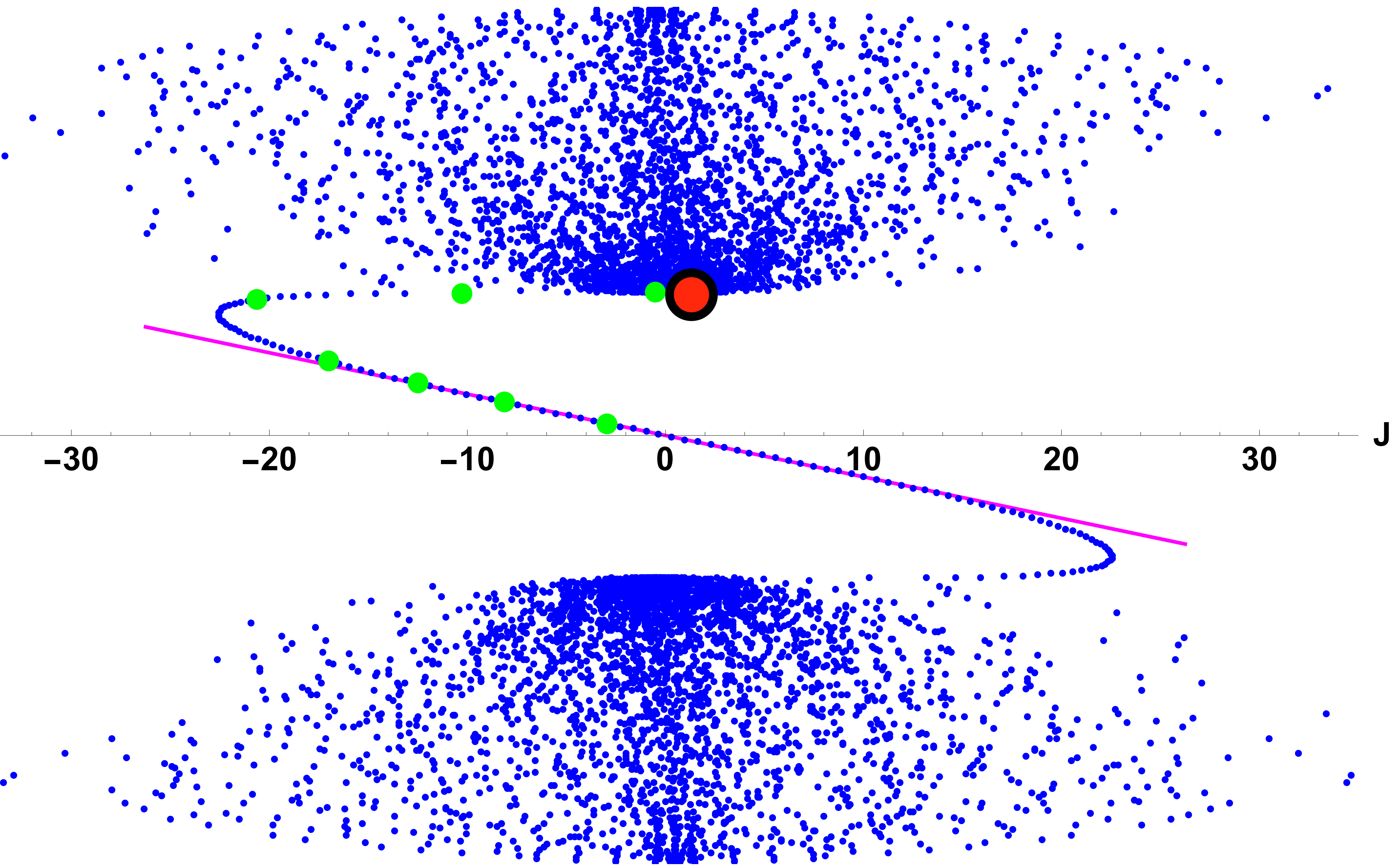




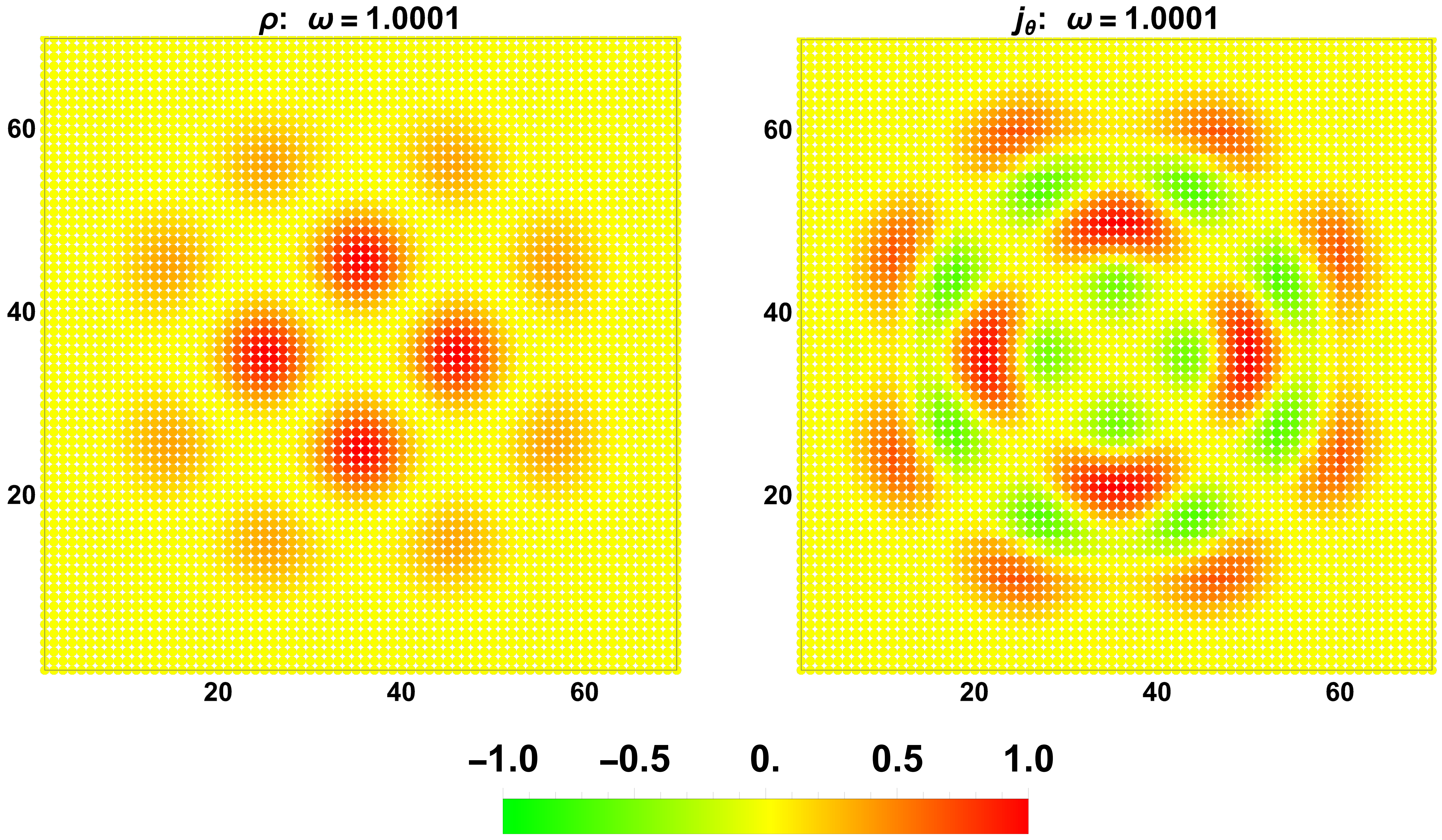
charge density ρ

current density j_θ





charge density ρ　　　current density j_θ



Note: chiral edge states on a 2-sphere boundary were previously discussed in the context of describing Weyl fermions in a gravitational background:

Shoto Aoki and Hidenori Fukaya, "Curved domain-wall fermions," Progress of Theoretical and Experimental Physics **2022**, 063B04 (2022).

Shoto Aoki and Hidenori Fukaya, "Curved domain-wall fermion and its anomaly inflow," Progress of Theoretical and Experimental Physics **2023**, 033B05 (2023).

Shoto Aoki, Hidenori Fukaya, and Naoto Kan, "A lattice formulation of weyl fermions on a single curved surface," Progress of Theoretical and Experimental Physics **2024**, 043B05 (2024).

Last (important!) piece of the puzzle: how to gauge?

d+1 theory with N_f flavors has exact $U(N_f)$ global symmetry…can easily gauge a subgroup in the continuum or the lattice. The gauge measure is well defined because its a regulated a theory of Dirac fermions

…but want a d-dimensional *gauge theory, not d+1…unlike CM systems*

Define bulk gauge fields B_μ to be functionals of the boundary values A_μ ; integrate only over the A_μ

$$B_\mu(\mathbf{x}_\perp, r, \theta)\Big|_{r=R} = A_\mu(\mathbf{x}_\perp, \theta)$$

For example, B_μ can be solution to Euclidian YM eq. subject to this BC.

$$B_\mu(\mathbf{x}_\perp, r, \theta)\Big|_{r=R} = A_\mu(\mathbf{x}_\perp, \theta)$$

For example, B_μ can be solution to Euclidian YM eq. subject to this BC.

Update boundary field A_μ

Compute bulk field B_μ subject to BC

Compute 5d fermion determinant Δ[B]

Multiply Δ[B] by exp(-S[A]) ☜ [4d YM action]

4d boundary fields are quantum; 5d bulk fields are classical subject to quantum BC

$$B_\mu(\mathbf{x}_\perp, r, \theta)\Big|_{r=R} = A_\mu(\mathbf{x}_\perp, \theta)$$

For example, B_μ can be solution to Euclidian YM eq. subject to this BC.

In general this will give a terribly nonlocal theory:

Generate Chern Simons operator in the bulk which is a function of B_μ and therefore a nonlocal functional of the d-dimensional gauge fields A_μ

*…but its coefficient vanishes if edge chiral gauge theory is anomaly-free**

Conjecture: this theory will be a local d-dimensional theory in the infrared *iff* the chiral gauge theory is anomaly-free

**More precisely: CS term -> $e^{i\pi\eta[A]}$, includes nonperturbative anomalies (see Witten, Yonekura)*

Criticisms by Aoki, Fukaya, Kan and by Golterman & Shamir:

Aoki, Fukaya, Kan:
When there are nontrivial gauge field configurations on the boundary, there fermion zeromodes on the surface are paired with zeromodes in the bulk interior

Shoto Aoki and Hidenori Fukaya, "Curved domain-wall fermion and its anomaly inflow," Progress of Theoretical and Experimental Physics **2023**, 033B05 (2023).
Shoto Aoki, Hidenori Fukaya, and Naoto Kan, "A lattice formulation of Weyl fermions on a single curved surface," (2024), arXiv:2402.09774 [hep-lat].
Shoto Aoki, "Study of curved domain-wall fermions on a lattice," (2023), arXiv:2404.01002 [hep-lat].

Golterman and Shamir:
There is an exactly conserved, gauge invariant current for every Weyl fermion on the boundary, unlike in target 4d theory

Maarten Golterman and Yigal Shamir, "Conserved currents in five-dimensional proposals for lattice chiral gauge theories," Phys. Rev. D **109**, 114519 (2024).

These criticisms are apparently related: bulk zeromodes appear because of conserved U(1)

Golterman and Shamir:

- For every boundary Weyl fermion, have one bulk massive Dirac fermion
- Exact global U(1) symmetry for each bulk fermion with 5d conserved current
- Can construct exactly conserved 4d currents by integrating 5d currents over r
- Leads to too much symmetry for boundary theory…eg, N_f=1 QCD on boundary has exact U(1) x U(1) symmetry

GS currents: $$j_\mu(x) = \int rdr\, J_\mu(x,r)\ , \quad \mu = 1,\ldots,4$$

5d conserved current

Can show: $$\partial_\mu j_\mu(x) = 0$$

Problem! E.g., 4d QCD with N_f=1 would have exact $U(1)_V$ x $U(1)_A$ symmetry

Bug or feature ?? Integrate out massive bulk modes, find for 5d conserved current:

$$J_\mu(x,r) = \underbrace{\delta(R-r)\bar\chi\sigma_\mu D_\mu\chi}_{\text{chiral edge state contribution}} + \underbrace{\kappa\,\theta(R-r)\epsilon_{\mu bcde}F_{bc}F_{de}}_{\text{bulk gauge field contribution}} \qquad \mu = 1,\ldots,4$$

$$J_5(x,r) = \underbrace{\kappa\,\theta(R-r)\epsilon_{5bcde}F_{bc}F_{de}}_{\text{bulk gauge field contribution}}$$

chiral edge state contribution

bulk gauge field contribution

$$J_\mu(x,r) = \delta(R-r)\bar\chi\sigma_\mu D_\mu\chi + \kappa\,\theta(R-r)\epsilon_{\mu bcde}F_{bc}F_{de} \qquad \mu = 1,\ldots,4$$

chiral edge state contribution

bulk gauge field contribution

$$J_5(x,r) = \kappa\,\theta(R-r)\epsilon_{5bcde}F_{bc}F_{de}$$

Golterman-Shamir equation $\partial_\mu j_\mu(x) = 0$

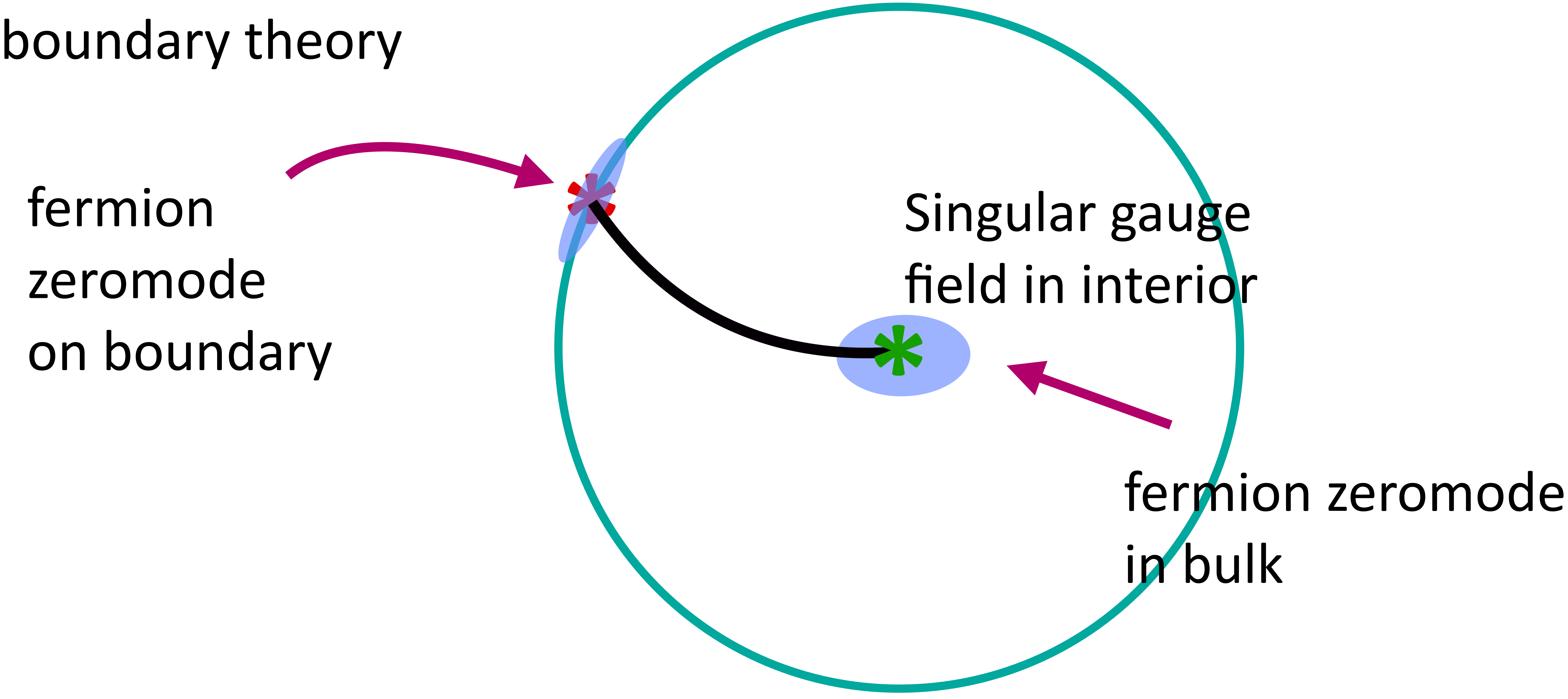
is found to be equivalent to the conventional anomalous Ward identity on the boundary

$$\partial_\mu\left(\bar\chi\sigma_\mu\chi(x)\right) = -\kappa\epsilon_{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}(x)$$

This is a **feature**, not a bug! Current conservation in the 5d theory = 4d “anomaly inflow”

However… integrating out bulk modes assumed no light states in interior…
What about Aoki-Fukaya-Kan-Golterman-Shamir zeromodes??

Instanton in boundary theory

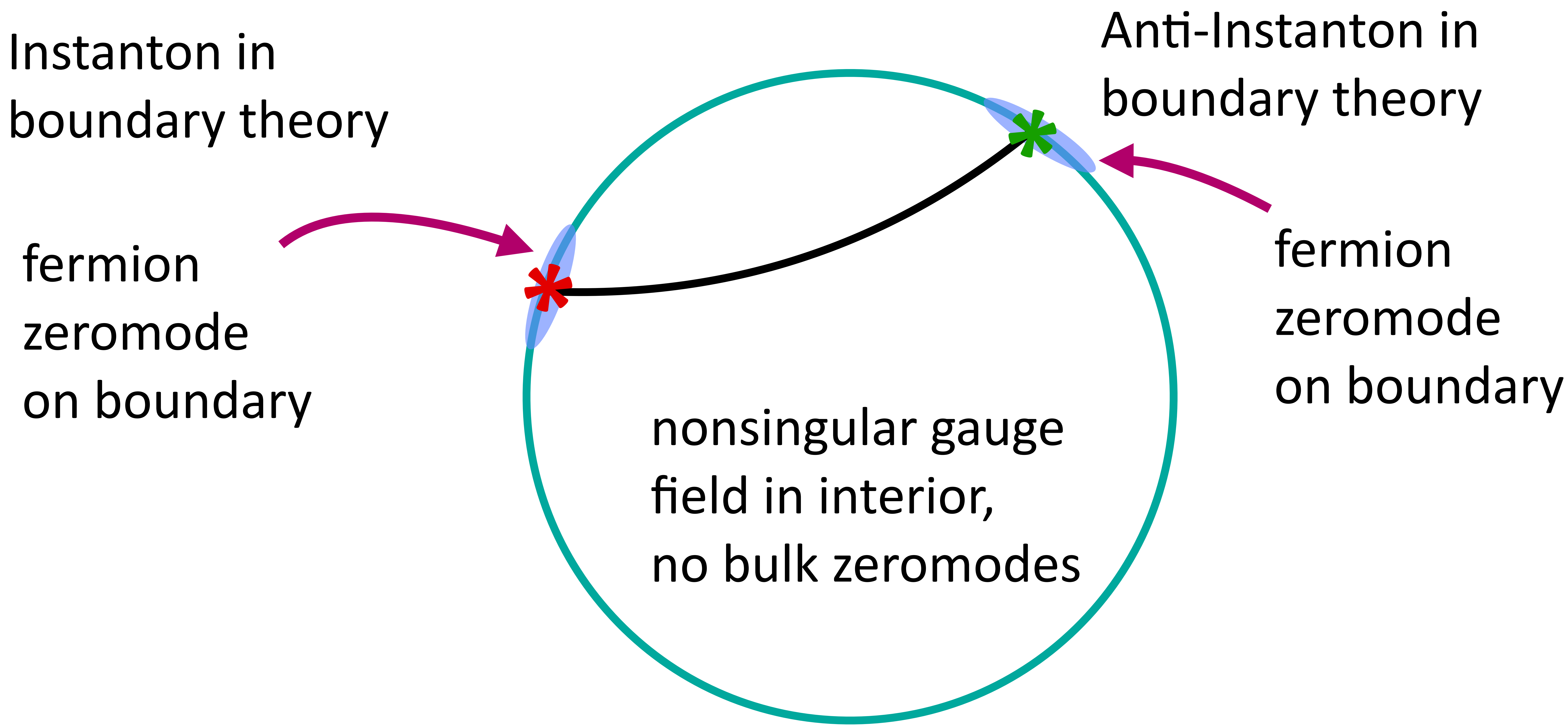


Integrating out bulk modes is not justified when boundary gauge field has nontrivial topology… Aoki-Fukaya-Kan-Golterman-Shamir criticism is a **bug** then, not a feature

It seems that expected theory cannot be achieved for nontrivial topology in boundary gauge field (e.g. instantons)

Very weird: whenever there are instantons, the 4d world becomes aware of mirror zeron lurking in the 5th dimensions?! …

For regulating the SM though, how about if we restrict to **trivial** topology?
(Eg, constrain number of instantons = number of anti-instantons)



Aoki-Fukaya-Kan-Golterman-Shamir problems seem to go away if the topology of boundary theory is trivial, Q=0.
(# instantons = # anti-instantons, imposed on boundary theory)

For regulating the SM though, how about if we restrict to trivial topology?
(Eg, constrain number of instantons = number of anti-instantons)

Has been shown that Q=0 QCD is equivalent to integrating over θ…
➙ yields θ=0 QCD + 1/volume corrections

Finite volume QCD at fixed topological charge
Sinya Aoki, Hidenori Fukaya, Shoji Hashimoto, Tetsuya Onogi,
PHYS. REV. D76, 054508 (2007)

Integrating over θ is equivalent to having an axion field…and then throwing away all of it except the p=0 mode

- Solves strong CP problem
- No axion particle
- Saturates Goldstone theorem for spontaneously broken exact U(1)
- Work in progress (DBK & S Sen)

Does this mean one can only regulate SM with θ=0??

Conclusions

An excitingly simple picture is emerging:
Chiral gauge theory as a boundary theory, without requiring new dynamics

Construction "understands" anomalies: local 4D theory emerges only if gauge anomalies cancel (discrete and perturbative)

Does it work? Too early to tell…
….but the Nielsen-Ninomiya theorem is no longer the obstacle.

- It appears that the theory is not purely 4d unless boundary gauge field topology is trivial (what does theory with nontrivial topology look like?? The η' portal 🤔)

- If gauge topology is trivial and anomalies cancel, it appears that chiral gauge theory can be regulated, giving θ=0 theory in large volume

- Could it be that 4d chiral gauge theory can only be regulated if anomalies cancel *and* θ=0? Is there a BSM scenario that realizes this physics?