YITP long-term and Nishinomiya-Yukawa memorial workshop

Hadrons and Hadron Interactions in QCD 2024 (HHIQCD 2024)

-- Experiments, Effective theories, and Lattice --

14th Oct. - 15th Nov., 2024
Yukawa Institute for Theoretical Physics, Kyoto University, Japan

Towards quantum simulating QCD: loop string hadron approach

BITS Pilani
Pilani | Dubai | Goa | Hyderabad | Mumbai
An Institution of Eminence

CRQIT
Center for Research in Quantum Information and Technology

Indrakshi Raychowdhury

Department of Physics,
BITS Pilani, K K Birla Goa Campus

November 13, 2024

innovate achieve lead

Classical Computation Era

Change of Paradigm

Quantum Computation Era

Classical Computation Era

Change of Paradigm

Quantum Computation Era

Our role

- Identifying the physics problem that would benefit from quantum computation
- Reformulating the problem suitable for quantum computation
- NISQ-era quantum simulation algorithms: analog and digital
- Think beyond NISQ era…

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Lattice gauge theory calculations without sign problem:
Real time dynamics

Classical Computation Era

Change of Paradigm

Quantum Computation Era

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Lattice gauge theory calculations without sign problem:
Real time dynamics

Classical Computation Era | **Lattice gauge theory calculations** | Quantum Computation Era

- Requires different theoretical framework.
- Addressed different objectives
- Computational Methods are entirely different.

Ultimate goal: performing LATTICE-QCD calculations using Quantum Computer

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Intermediate steps:

- Suitable development and choice of framework.
- Suitable choice of variables/basis.
- Algorithm development for various tasks- classical/quantum/hybrid.
- Quantum information theoretic understanding - connection to physics of QCD
- Quantum advantage - knowledge generation in fundamental laws of nature.

Ultimate goal: performing LATTICE-QCD calculations using Quantum Computer

Current Efforts: For simpler models such as Schwinger model, discrete gauge groups, low dimensional SU(2)/SU(3) gauge theory

Intermediate steps:

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Current Efforts: For arbitrary dimensional SU(2)/SU(3) gauge theories

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- Quantum advantage - knowledge generation in fundamental laws of nature.

Framework: Hamiltonian Formalism

PHYSICAL REVIEW D VOLUME 11, NUMBER 2 15 JANUARY 1975

Hamiltonian formulation of Wilson's lattice gauge theories

John Kogut*

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Leonard Susskind†

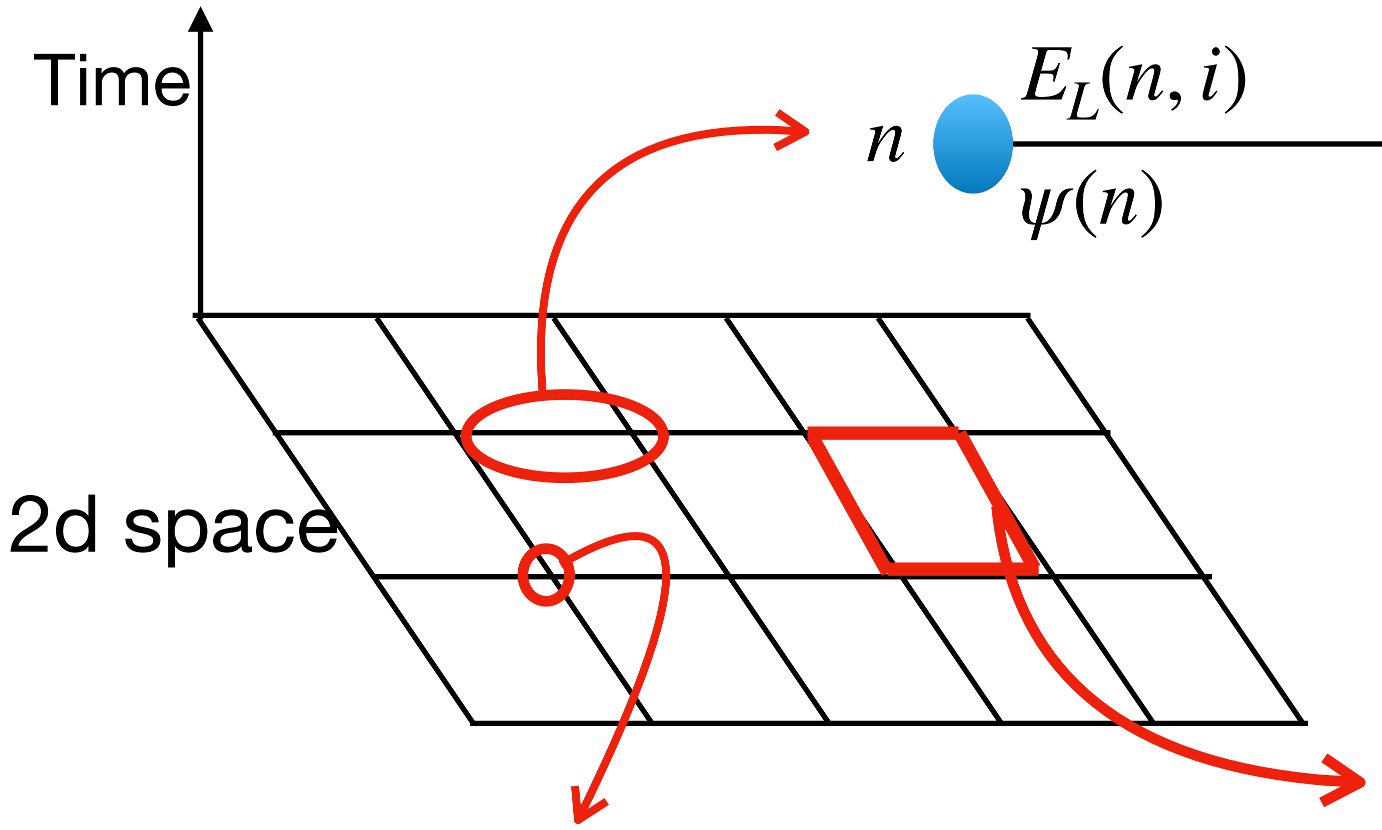
Belfer Graduate School of Science, Yeshiva University, New York, New York and Tel Aviv University, Ramat Aviv, Israel and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

(Received 9 July 1974)

Wilson's lattice gauge model is presented as a canonical Hamiltonian theory. The structure of the model is reduced to the interactions of an infinite collection of coupled rigid rotators. The gauge-invariant configuration space consists of a collection of strings with quarks at their ends. The strings are lines of non-Abelian electric flux. In the strong-coupling limit the dynamics is best described in terms of these strings. Quark confinement is a result of the inability to break a string without producing a pair.

Framework: Hamiltonian Formalism

Kogut-Susskind '74



n $E_L(n,i)$ $U(n,i)$ $E_R(n,i)$ $n+i$

$\psi(n)$ $\psi(n+i)$

$$[E_L, U] = U$$
$$[E_R, U] = -U$$
$$E_L^2 = E_R^2 = E^2$$
$$U_{plaquette} = U_1 U_2 U_3^\dagger U_4^\dagger$$

Gauss' law constraint:

$$G(n)|\Psi_{phys}\rangle = 0$$

$$[H, G(n)] = 0 \quad \forall n$$

$$G(n) = \sum_I \left[E_L(n,I) - E_R(n-I,I)\right] - \rho(n)$$

$$H = H_E + H_M + H_I + H_B$$

$$\frac{g^2 a}{2} \sum_{n,I} E^2(n,I)$$

$$m \sum (-1)^n \psi^\dagger(n)\psi(n)$$

Staggered fermion

$$\frac{1}{2a} \sum_{n,I} (-1)^n \psi^\dagger(n) U(n,I) \psi(n+I)$$

$$\frac{2a}{g^2} \sum_{plaquettes} \left[\mathrm{Tr} U_{plaquette} + h.c\right]$$

Framework: Hamiltonian Formalism

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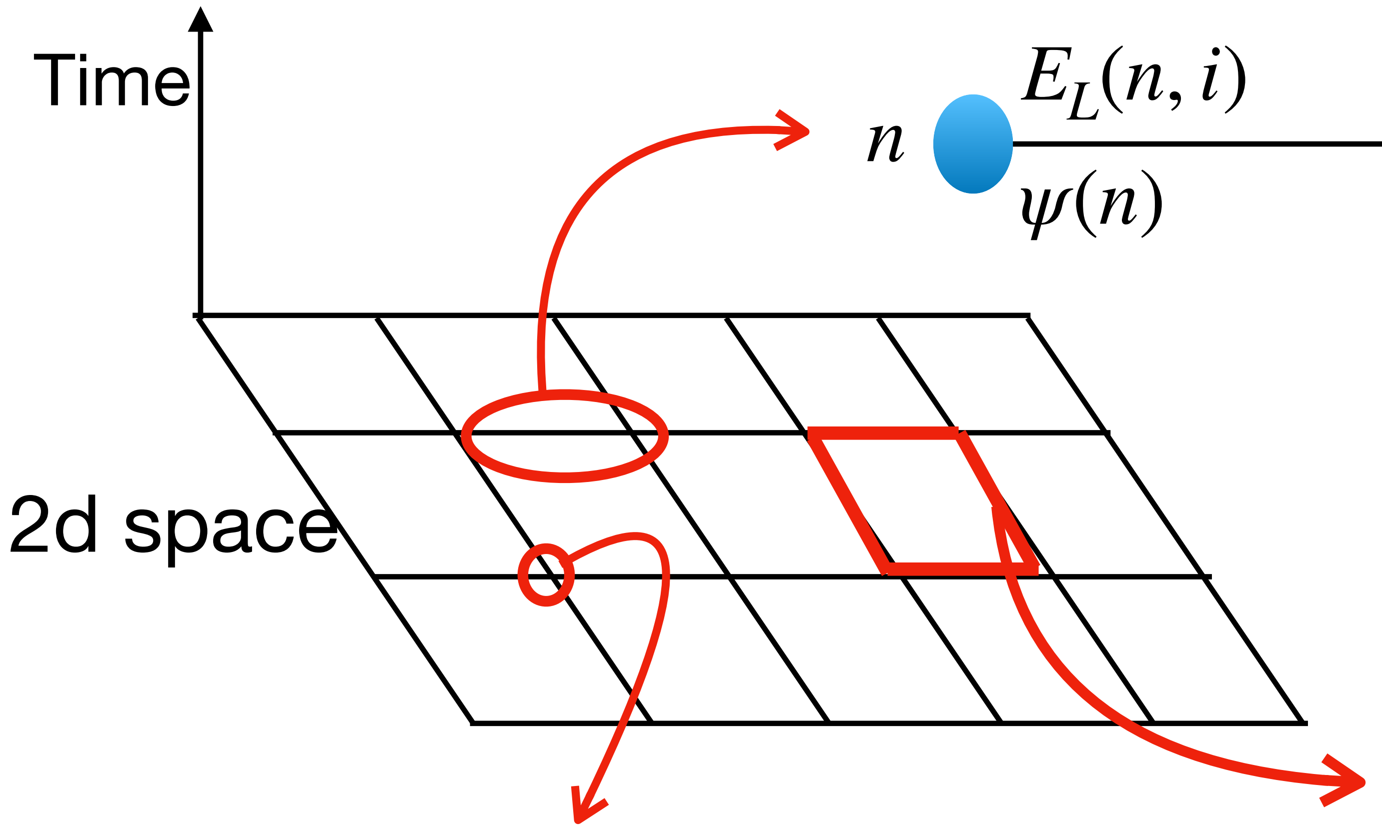

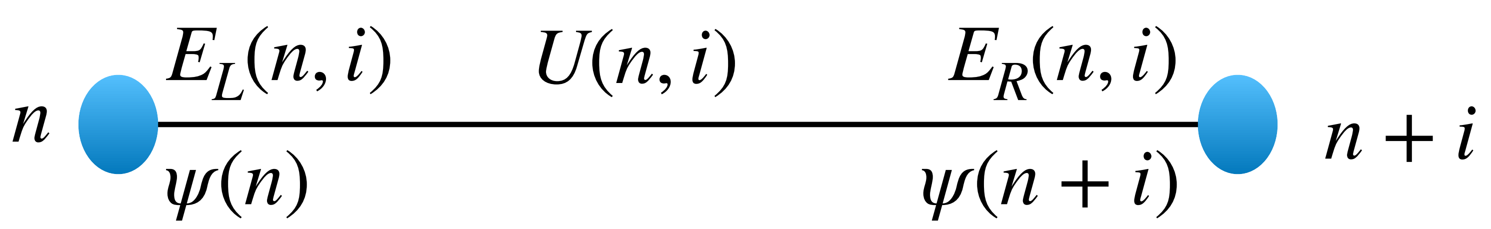

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$$H = H_E + H_M + H_I + H_B$$

H_E: $\frac{g^2 a}{2}\sum_{n,I} E^2(n,I)$

H_M: $m\sum (-1)^n\, \psi^\dagger(n)\psi(n)$

Staggered fermion

H_I: $\frac{1}{2a}\sum_{n,I} (-1)^n\, \psi^\dagger(n)U(n,I)\psi(n+I)$

H_B: $\frac{2a}{g^2}\sum_{plaquettes}\left[\mathrm{Tr}U_{plaquette} + h.c\right]$

U(1): $U(n,I) = e^{i\theta(n,I)}$

Schwinger Model : U(1) in 1+1d, H_B term absent

SU(2):

$E \to E^a, \quad a = 1,2,3$

$U \to U_{\alpha\beta}, \quad \alpha,\beta = 1,2$

$\psi \to \psi_\alpha, \quad \alpha = 1,2$

$$G(n) \to G^a(n) = \sum_I \left[E_L^a(n,I) + E_R^a(n-I,I)\right] + \psi(n)^\dagger \frac{\sigma^a}{2}\psi(n)$$

SU(3): $a = 1,2,3,\ldots,8.$

State-Of-The-Art

Limited progress with non-Abelian gauge theories, specifically SU(3)…

Limited progress with higher dimensional gauge theories..

The Path Towards Quantum simulating full QCD is still Unknown…

We choose a path:

Towards quantum simulating QCD: loop string hadron approach

Disclaimer: Other paths are also being explored towards the same goal…

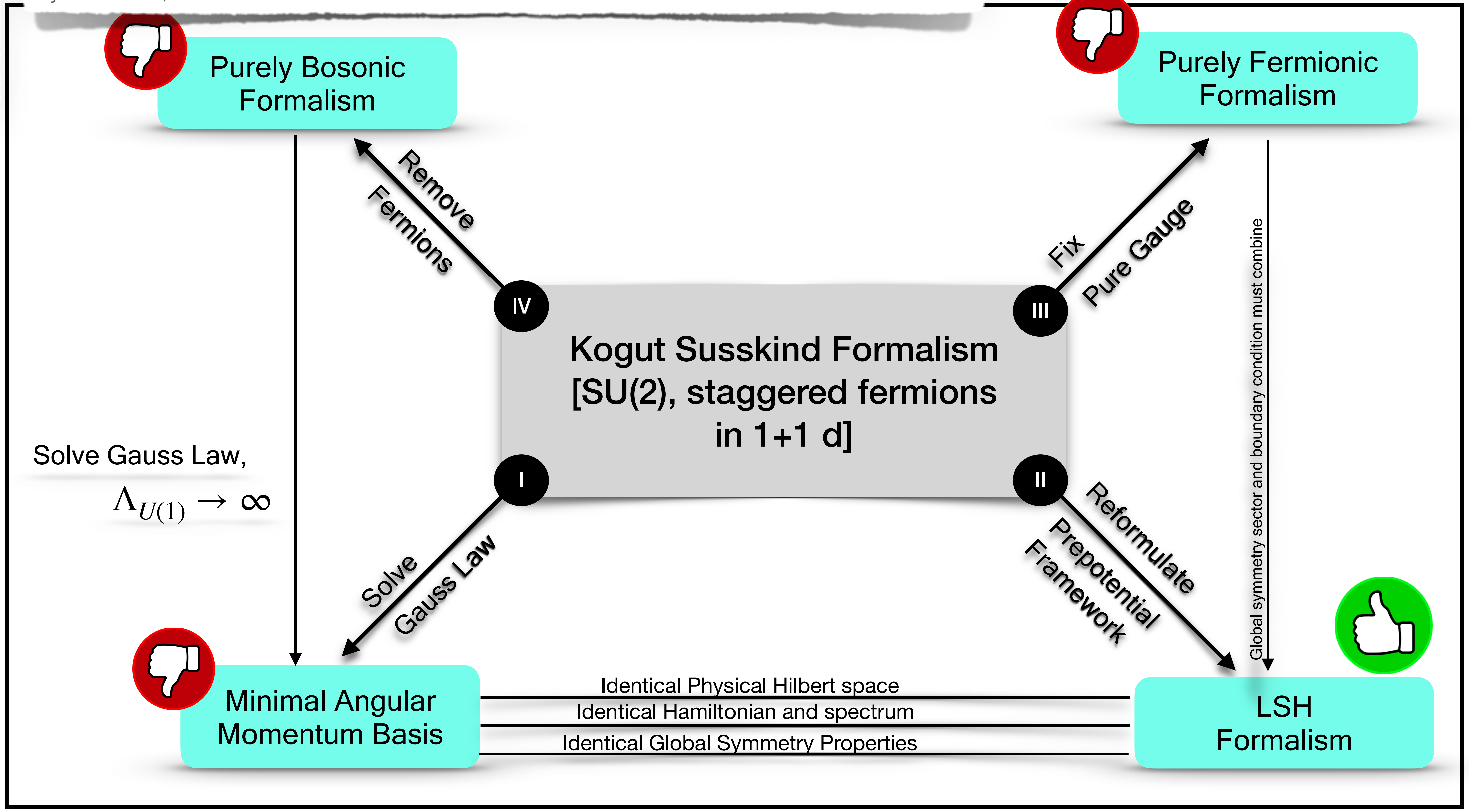
PHYSICAL REVIEW D **104,** 074505 (2021)

Search for efficient formulations for Hamiltonian simulation of non-Abelian lattice gauge theories

Zohreh Davoudi[1,2] Indrakshi Raychowdhury,[1] and Andrew Shaw[1]

Removing staggered fermionic matter in $U(N)$ and $SU(N)$ lattice gauge theories

Erez Zohar and J. Ignacio Cirac
Phys. Rev. D **99**, 114511 – Published 28 June 2019



Another (also most popular) candidate:
Quantum Link Model

QCD as a quantum link model

R. Brower, S. Chandrasekharan, and U.-J. Wiese
Phys. Rev. D **60**, 094502 – Published 27 September 1999

SU(2) rishon representation of gauge fields

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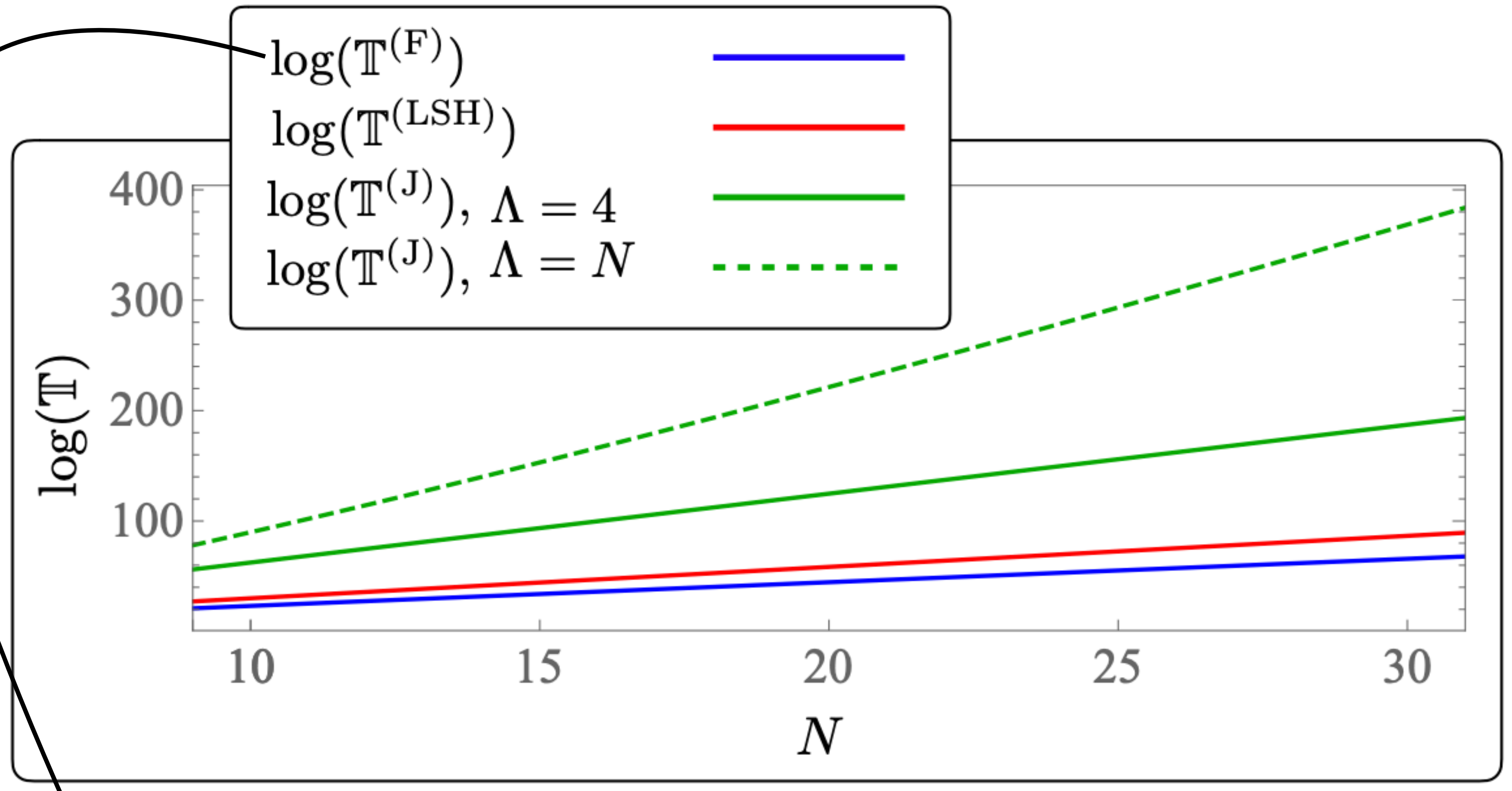

Not generalizable to higher dimension and general boundary condition

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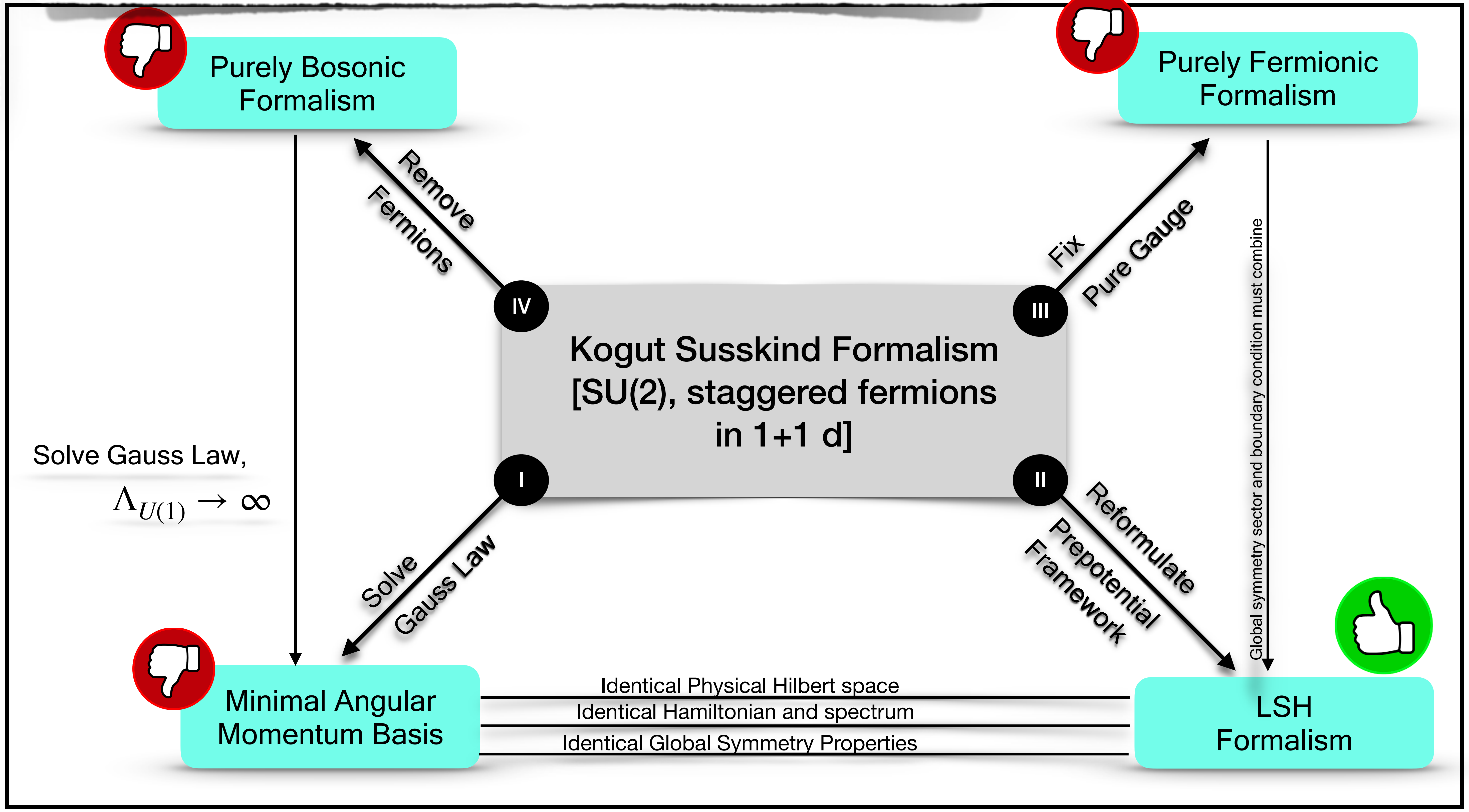

- LSH ⇒ cheapest and most promising framework.
- How? Ans. No need to impose Gauss law constraint: Significant reduction in the cost of Hilbert space generation, 1-sparse basis.

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SU(2) rishon representation of gauge fields

Prepotential Formulation of Gauge Theories

Reformulation of the original Kogut-Susskind Formalism in terms of Schwinger bosons

Formulated for SU(2), SU(3) and arbitrary SU(N)

Formulated for any dimension

GAUGE INVARIANCE + PROLIFERATION OF LOOP DEGREES OF FREEDOM

Describes dynamics of only physical degrees of freedom

Ref:
Manu Mathur, JPA 2005; NPB 2007;
*Ramesh Anishetty, Manu Mathur, **IR***
JPA 2009; JPA 2010; JMP 2009; JMP 2010; JMP 2011
***IR**, PhD Thesis, 2014;*
*Ramesh Anishetty, **IR**, PRD 2014;*
***IR**, arXiv: 1507.07305; EPJC 2019;*

+ **Staggered fermionic matter** → **Loop String Hadron (LSH) Formulation**

PHYSICAL REVIEW D **101,** 114502 (2020)

Loop, string, and hadron dynamics in SU(2) Hamiltonian lattice gauge theories

Indrakshi Raychowdhury[1,*] and Jesse R. Stryker[2,†]

PHYSICAL REVIEW D **107,** 094513 (2023)

Loop-string-hadron formulation of an SU(3) gauge theory with dynamical quarks

Saurabh V. Kadam,[1,*] Indrakshi Raychowdhury,[2,†] and Jesse R. Stryker[1,‡]
[1]*Maryland Center for Fundamental Physics, University of Maryland, College Park, Maryland 20742, USA*
[2]*Department of Physics, BITS-Pilani, K K Birla Goa Campus, Zuarinagar, Goa 403726, India*

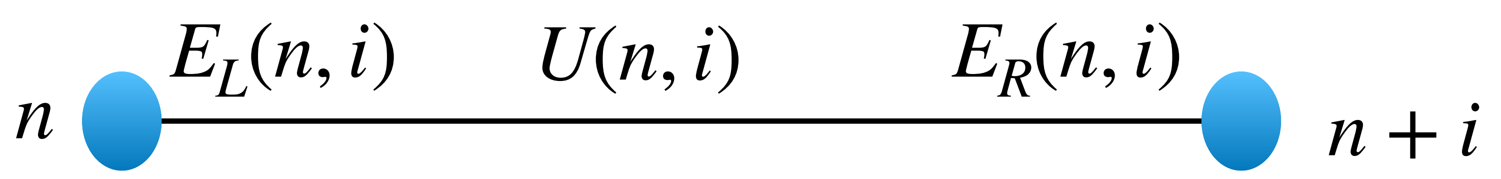
arXiv:2407.19181v1 [hep-lat] 27 Jul 2024

IQuS@UW-21-086

Loop-string-hadron approach to SU(3) lattice Yang-Mills theory: Gauge invariant Hilbert space of a trivalent vertex

Saurabh V. Kadam,[1,*] Aahiri Naskar,[2,†] Indrakshi Raychowdhury,[2,3,‡] and Jesse R. Stryker[4,5,§]

SU(2) Prepotential Formulation: 1d



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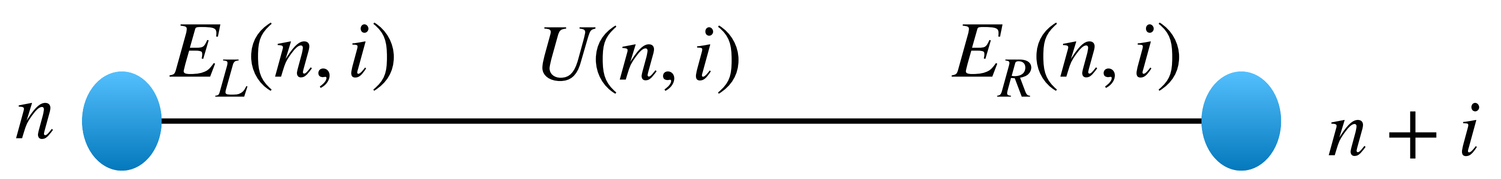

$$\hat{E}^{\mathrm{a}}_{L/R} \equiv \hat{a}^\dagger(L/R) T^{\mathrm{a}} \hat{a}(L/R)$$

$$[\hat{E}^{\mathrm{a}}_L, \hat{E}^{\mathrm{b}}_L] = i\epsilon^{\mathrm{abc}} \hat{E}^{\mathrm{c}}_L,$$

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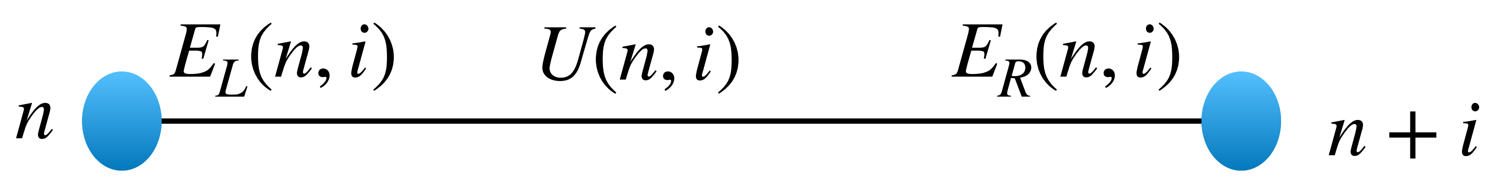

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Abelian Gauss' Law

$$N_L(x) = N_R(x)$$

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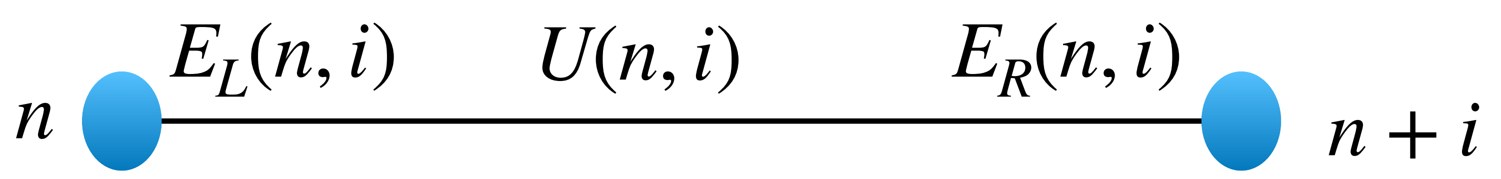

staggered site x staggered site $x+1$

$$\hat{N}_L(x) = \hat{N}_R(x)$$

$$\cdots \quad (\hat{\boldsymbol{E}}_R(x-1), \hat{U}_R(x-1)) \leftrightarrow \begin{pmatrix} \hat{a}^1_R(x-1) \\ \hat{a}^2_R(x-1) \end{pmatrix} \begin{pmatrix} \hat{a}^1_L(x) \\ \hat{a}^2_L(x) \end{pmatrix} \leftrightarrow (\hat{\boldsymbol{E}}_L(x), \hat{U}_L(x)) \; \vdots \; (\hat{\boldsymbol{E}}_R(x), \hat{U}_R(x)) \leftrightarrow \begin{pmatrix} \hat{a}^1_R(x) \\ \hat{a}^2_R(x) \end{pmatrix} \begin{pmatrix} \hat{a}^1_L(x+1) \\ \hat{a}^2_L(x+1) \end{pmatrix} \leftrightarrow (\hat{\boldsymbol{E}}_L(x+1), \hat{U}_L(x+1)) \quad \cdots$$

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⟷

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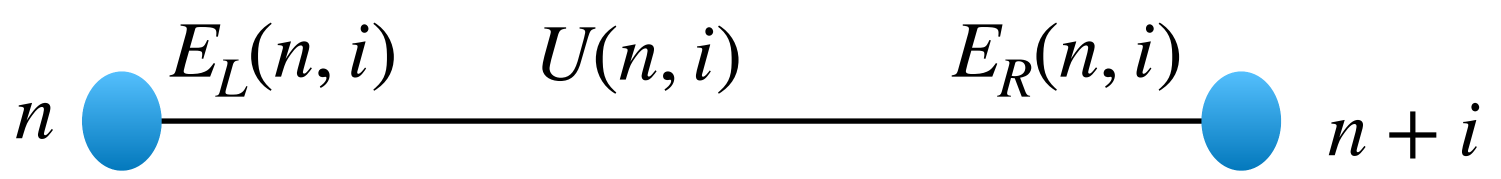
SU(2) LSH Formulation: 1d



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$\longleftrightarrow$

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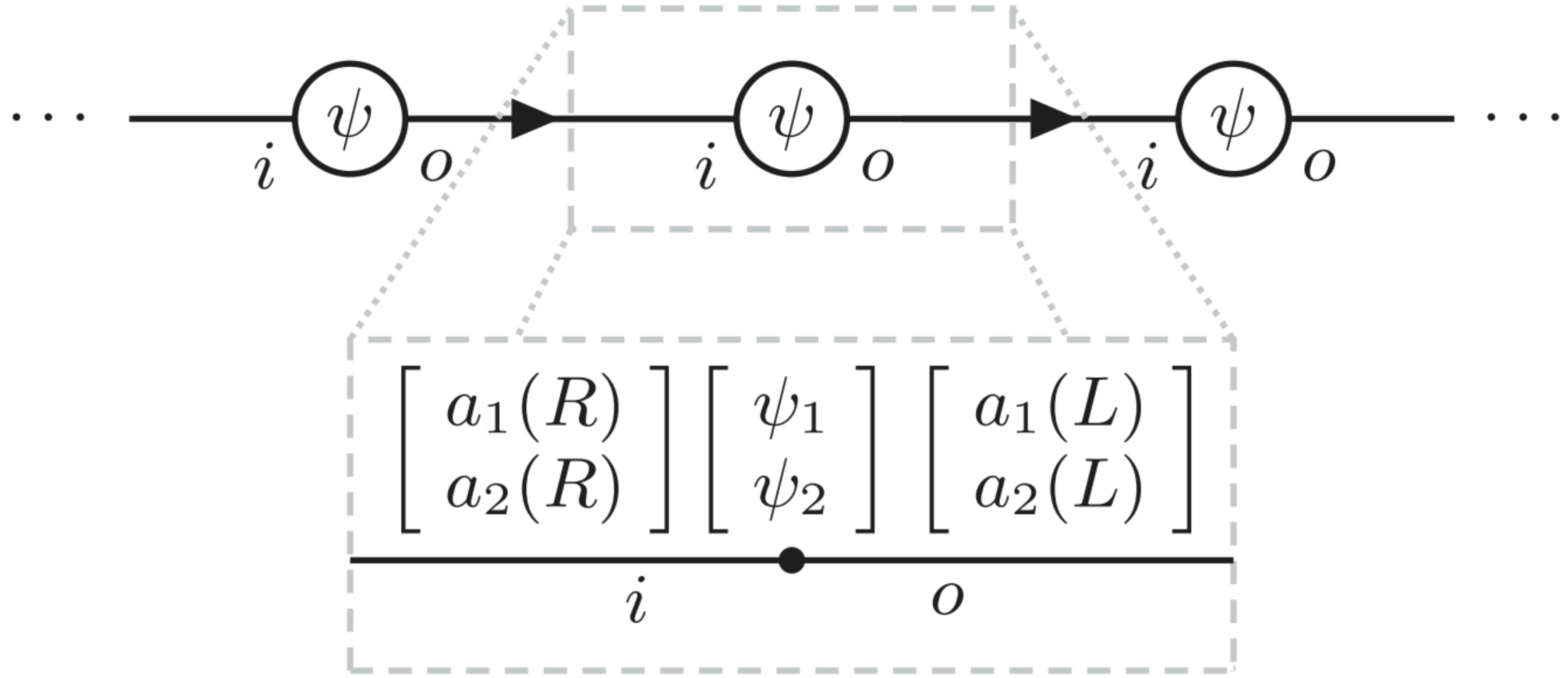
Local SU(2) Invariant Operators in 1d: loops-strings- hadrons

(i) *Pure gauge loop operators.*—$\mathcal{L}^{\sigma,\sigma'}$:

Gauge singlets constructed out of left and right bosons



(ii) *Incoming string operators.*—$\mathcal{S}_{\text{in}}^{\sigma,\sigma'}$:

Gauge singlets constructed out of left bosons and fermions

Outgoing string operators.—$\mathcal{S}_{\text{out}}^{\sigma,\sigma'}$:

Gauge singlets constructed out of Right boson and fermions

Hadron operators.—$\mathcal{H}^{\sigma,\sigma}$:

Gauge singlets constructed out of two fermions

$$(1/2)\mathcal{L}^{--}(\mathcal{S}_{\text{in}}^{++})^{n_i}(\mathcal{S}_{\text{out}}^{++})^{n_o}|0\rangle = \delta_{n_i,1}\delta_{n_o,1}\mathcal{H}^{++}|0\rangle$$

Local SU(2) Invariant Operators in 1d: loops-strings- hadrons

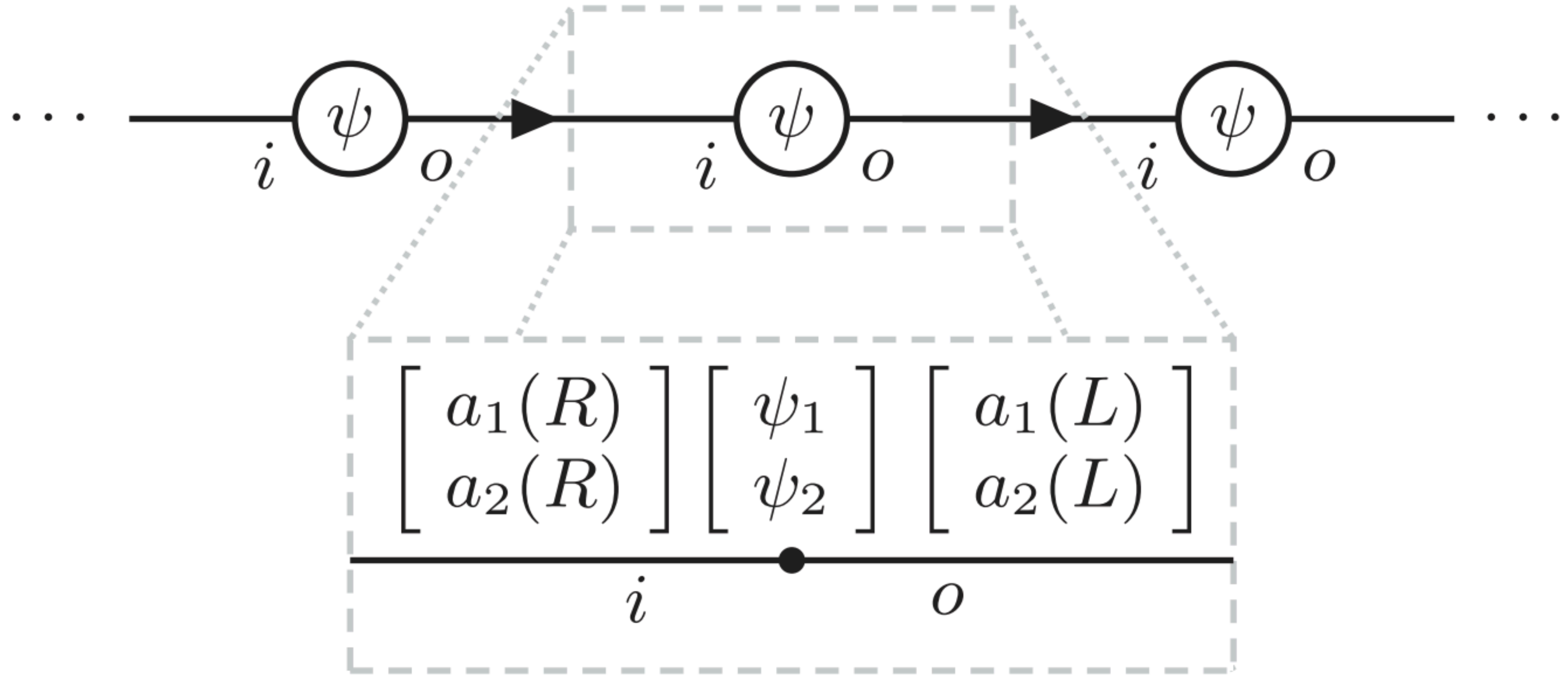
(i) *Pure gauge loop operators.*—$\mathcal{L}^{\sigma,\sigma'}$:

$$\mathcal{L}^{++} = a(R)^{\dagger}_{\alpha} a(L)^{\dagger}_{\beta} \epsilon_{\alpha\beta}$$

$$\mathcal{L}^{--} = a(R)_{\alpha} a(L)_{\beta} \epsilon_{\alpha\beta} = (\mathcal{L}^{++})^{\dagger}$$

$$\mathcal{L}^{+-} = a(R)^{\dagger}_{\alpha} a(L)_{\beta} \delta_{\alpha\beta}$$

$$\mathcal{L}^{-+} = a(R)_{\alpha} a(L)^{\dagger}_{\beta} \delta_{\alpha\beta} = (\mathcal{L}^{+-})^{\dagger}.$$



(ii) *Incoming string operators.*—$\mathcal{S}^{\sigma,\sigma'}_{\text{in}}$:

$$\mathcal{S}^{++}_{\text{in}} = a(R)^{\dagger}_{\alpha} \psi^{\dagger}_{\beta} \epsilon_{\alpha\beta}$$

$$\mathcal{S}^{--}_{\text{in}} = a(R)_{\alpha} \psi_{\beta} \epsilon_{\alpha\beta} = (\mathcal{S}^{++}_{\text{in}})^{\dagger}$$

$$\mathcal{S}^{+-}_{\text{in}} = a(R)^{\dagger}_{\alpha} \psi_{\beta} \delta_{\alpha\beta}$$

$$\mathcal{S}^{-+}_{\text{in}} = a(R)_{\alpha} \psi^{\dagger}_{\beta} \delta_{\alpha\beta} = (\mathcal{S}^{+-}_{\text{in}})^{\dagger}.$$

Outgoing string operators.—$\mathcal{S}^{\sigma,\sigma'}_{\text{out}}$:

$$\mathcal{S}^{++}_{\text{out}} = \psi^{\dagger}_{\alpha} a(L)^{\dagger}_{\beta} \epsilon_{\alpha\beta}$$

$$\mathcal{S}^{--}_{\text{out}} = \psi_{\alpha} a(L)_{\beta} \epsilon_{\alpha\beta} = (\mathcal{S}^{++}_{\text{out}})^{\dagger}$$

$$\mathcal{S}^{+-}_{\text{out}} = \psi^{\dagger}_{\alpha} a(L)_{\beta} \delta_{\alpha\beta}$$

$$\mathcal{S}^{-+}_{\text{out}} = \psi_{\alpha} a(L)^{\dagger}_{\beta} \delta_{\alpha\beta} = (\mathcal{S}^{+-}_{\text{out}})^{\dagger}.$$

Hadron operators.—$\mathcal{H}^{\sigma,\sigma}$:

$$\mathcal{H}^{++} = -\frac{1}{2!} \psi^{\dagger}_{\alpha} \psi^{\dagger}_{\beta} \epsilon_{\alpha\beta}$$

$$\mathcal{H}^{--} = \frac{1}{2!} \psi_{\alpha} \psi_{\beta} \epsilon_{\alpha\beta} = (\mathcal{H}^{++})^{\dagger}.$$

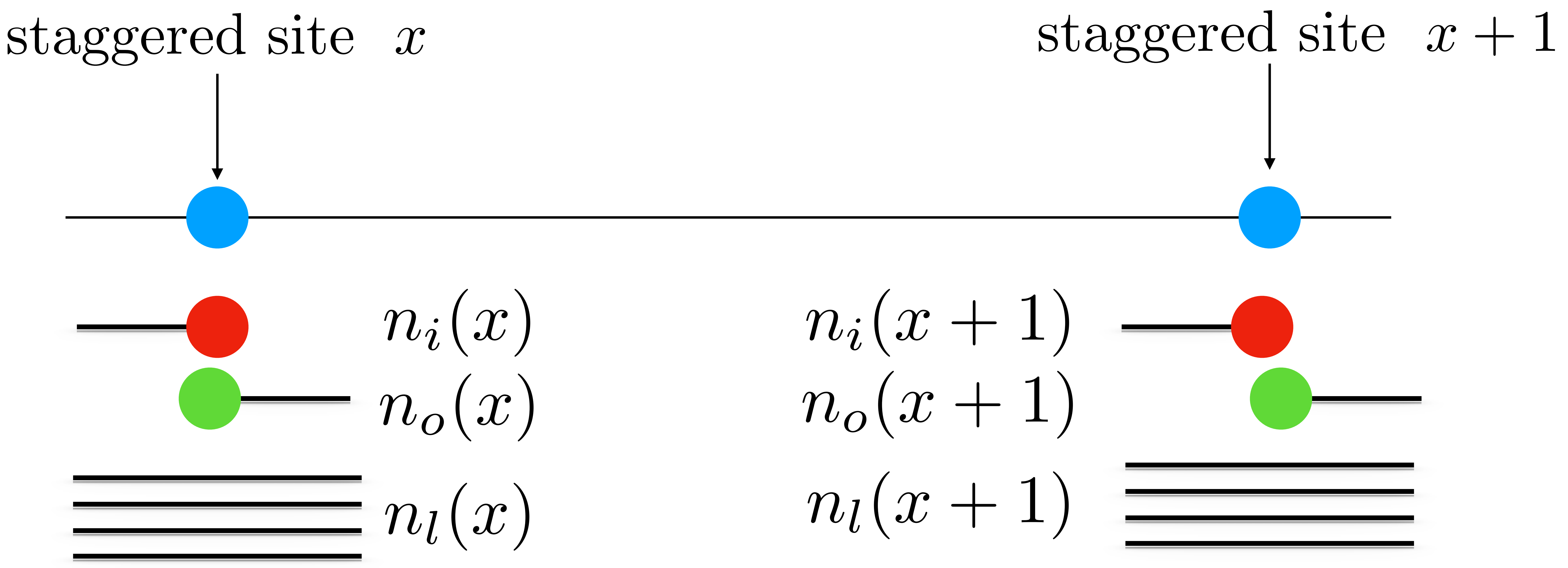
$$(1/2)\mathcal{L}^{--}(\mathcal{S}^{++}_{\text{in}})^{n_i}(\mathcal{S}^{++}_{\text{out}})^{n_o}|0\rangle = \delta_{n_i,1}\delta_{n_o,1}\mathcal{H}^{++}|0\rangle$$

LSH Formulation: local LSH basis for SU(2) in 1+1 dimension

At each site define: $n_l(x), n_i(x), n_o(x)$,

$$|n_l, n_i, n_o\rangle = \left(\mathcal{L}^{++}\right)^{n_l} \left(\mathcal{S}_i^{++}\right)^{n_i} \left(\mathcal{S}_o^{++}\right)^{n_o} |0\rangle$$

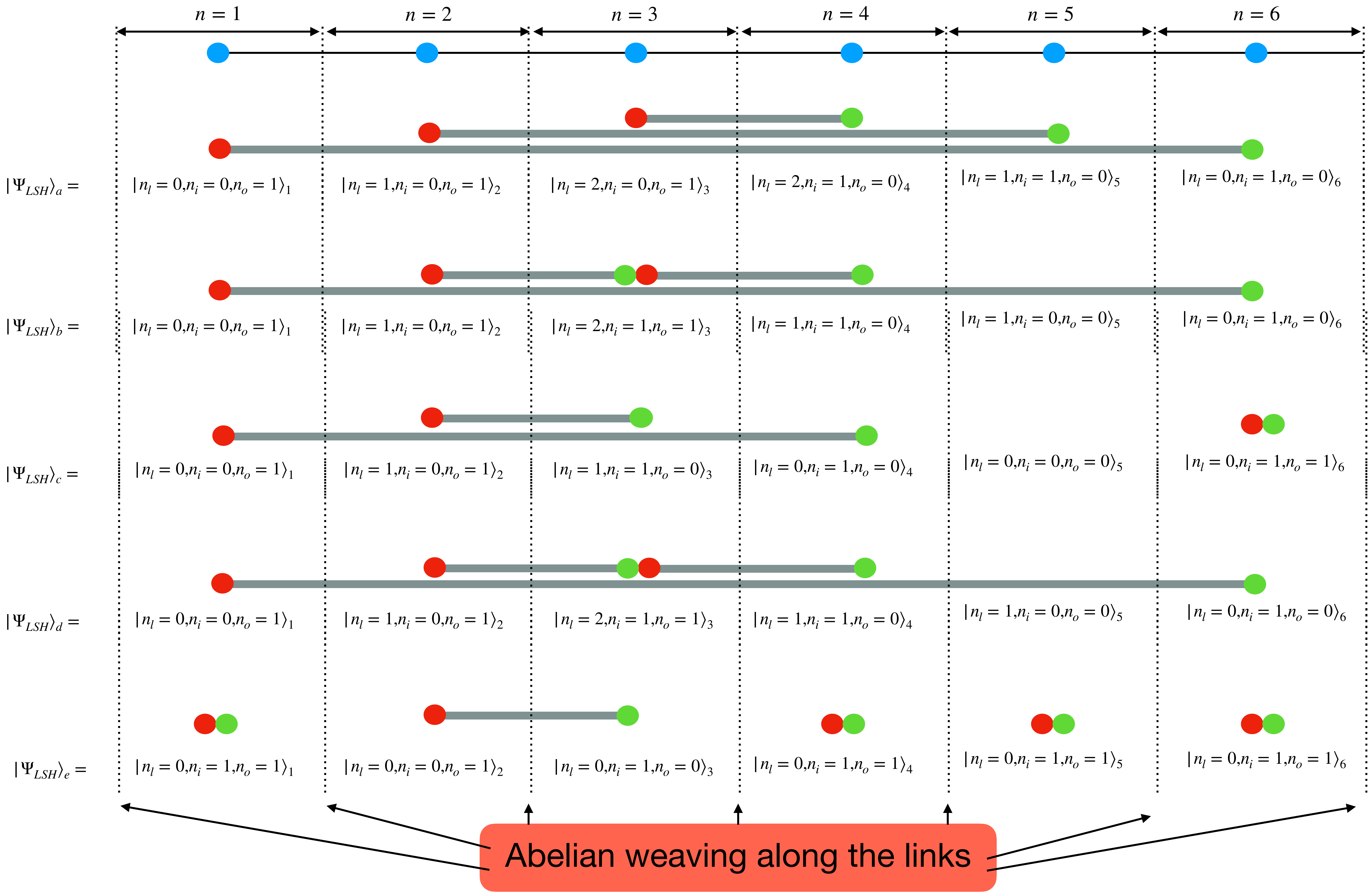
$0 \leq n_l(x) \leq \infty$,
$n_i(x) \in \{0, 1\}$,
$n_o(x) \in \{0, 1\}$.



Abelian weaving along the links

Pictorially global LSH states in 1d

$$|\Psi_{LSH}\rangle = |n_l, n_i, n_o\rangle_1 \otimes |n_l, n_i, n_o\rangle_2 \otimes |n_l, n_i, n_o\rangle_3 \otimes |n_l, n_i, n_o\rangle_4 \otimes |n_l, n_i, n_o\rangle_5 \otimes |n_l, n_i, n_o\rangle_6$$



LSH Formulation: key ingredients

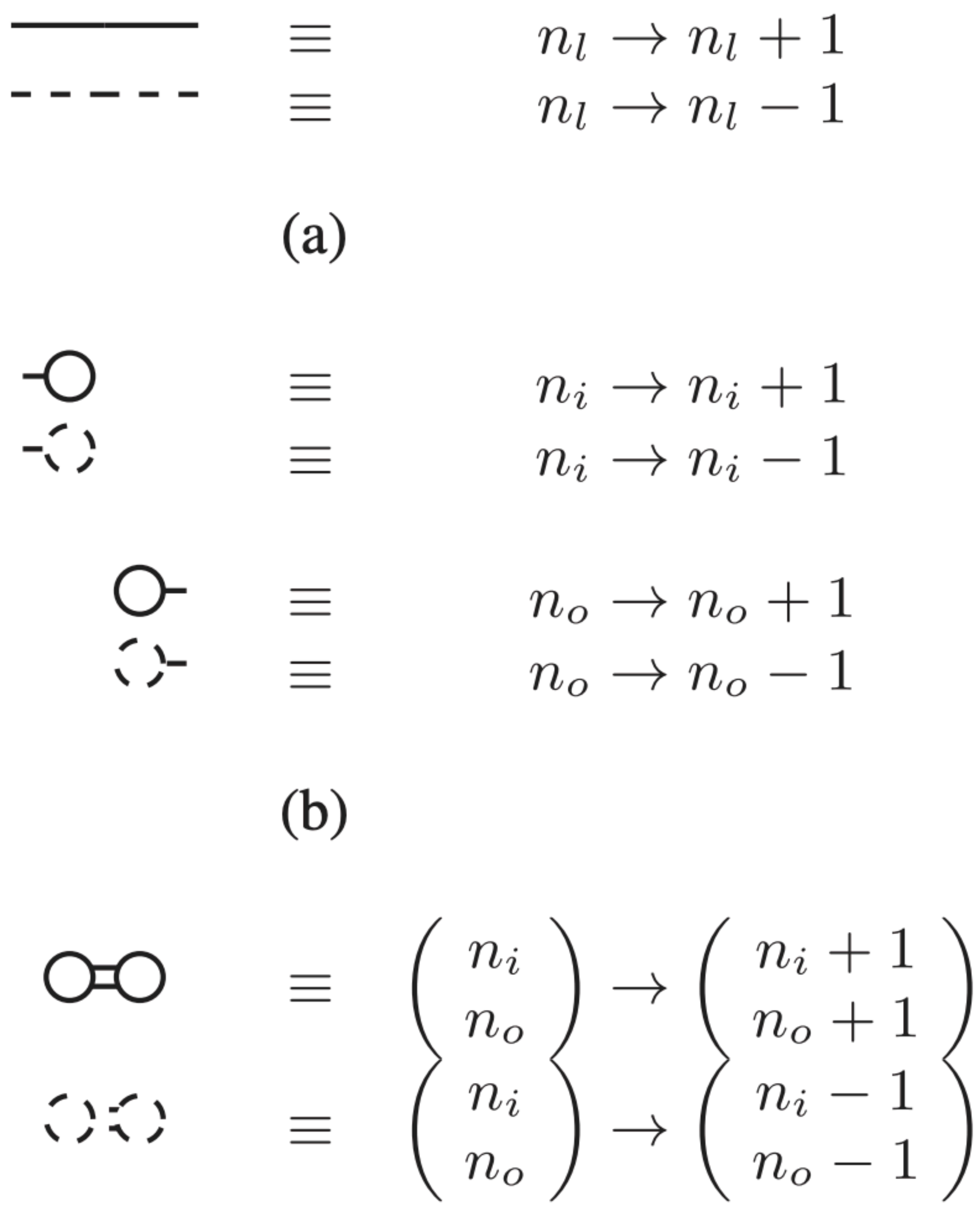
Local gauge invariant Hilbert space

Local constraint on each link: Abelian Gauss' law

LSH operators acting on the local basis

$$\hat{n}_l|n_l,n_i,n_o\rangle = n_l|n_l,n_i,n_o\rangle,$$
$$\hat{n}_i|n_l,n_i,n_o\rangle = n_i|n_l,n_i,n_o\rangle,$$
$$\hat{n}_o|n_l,n_i,n_o\rangle = n_o|n_l,n_i,n_o\rangle,$$

$$\hat{\lambda}^{\pm}|n_l,n_i,n_o\rangle = |n_l \pm 1,n_i,n_o\rangle,$$
$$\hat{\chi}_i^{+}|n_l,n_i,n_o\rangle = (1-\delta_{n_i,1})|n_l,n_i+1,n_o\rangle,$$
$$\hat{\chi}_i^{-}|n_l,n_i,n_o\rangle = (1-\delta_{n_i,0})|n_l,n_i-1,n_o\rangle,$$
$$\hat{\chi}_o^{+}|n_l,n_i,n_o\rangle = (1-\delta_{n_0,1})|n_l,n_i,n_o+1\rangle,$$
$$\hat{\chi}_o^{-}|n_l,n_i,n_o\rangle = (1-\delta_{n_0,0})|n_l,n_i,n_o-1\rangle.$$



Hamiltonian, describing dynamics of loops, strings and hadrons.

$$H^{(\mathrm{LSH})} = H_I^{(\mathrm{LSH})} + H_E^{(\mathrm{LSH})} + H_M^{(\mathrm{LSH})}$$

$$H_I^{(\mathrm{LSH})} = \frac{1}{2a}\sum_n \Bigg\{ \frac{1}{\sqrt{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x)) + 1}} \times \left[\hat{S}_o^{++}(x)\hat{S}_i^{+-}(x+1) + \hat{S}_o^{+-}(x)\hat{S}_i^{--}(x+1)\right] \times \frac{1}{\sqrt{\hat{n}_l(x+1) + \hat{n}_i(x+1)(1-\hat{n}_o(x+1)) + 1}} + \mathrm{h.c.}\Bigg\},$$

$$H_E^{(\mathrm{LSH})} = \frac{g^2 a}{2}\sum_n \left[\frac{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x))}{2} \times \left(\frac{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x))}{2} + 1\right)\right],$$

$$H_M^{(\mathrm{LSH})} = m\sum_n (-1)^x(\hat{n}_i(x) + \hat{n}_o(x)),$$

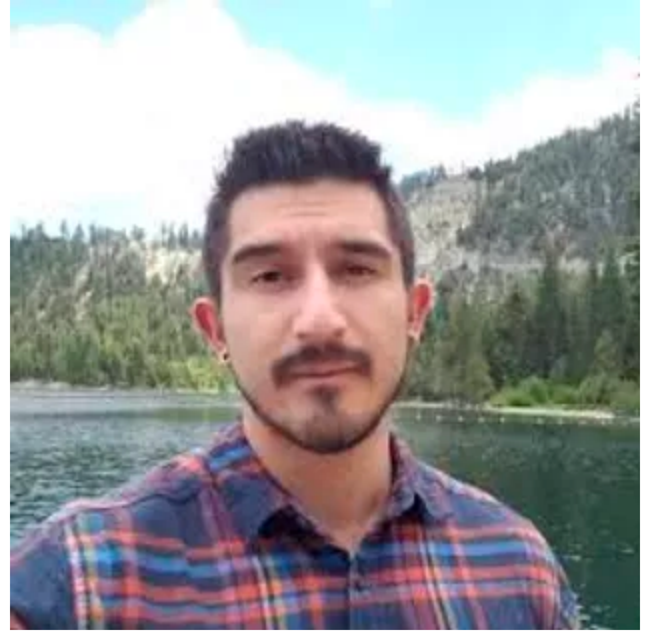
$$\begin{aligned}
\hat{S}_o^{++} &= \hat{\chi}_o^{+}(\lambda^+)^{\hat{n}_i}\sqrt{\hat{n}_l + 2 - \hat{n}_i},\\
\hat{S}_o^{--} &= \hat{\chi}_o^{-}(\lambda^-)^{\hat{n}_i}\sqrt{\hat{n}_l + 2(1 - \hat{n}_i)},\\
\hat{S}_o^{+-} &= \hat{\chi}_i^{+}(\lambda^-)^{1-\hat{n}_o}\sqrt{\hat{n}_l + 2\hat{n}_o},\\
\hat{S}_o^{-+} &= \hat{\chi}_i^{-}(\lambda^+)^{1-\hat{n}_o}\sqrt{\hat{n}_l + 1 + \hat{n}_o},
\end{aligned}$$

$$\begin{aligned}
\hat{S}_i^{+-} &= \hat{\chi}_o^{-}(\lambda^+)^{1-\hat{n}_i}\sqrt{\hat{n}_l + 1 + \hat{n}_i},\\
\hat{S}_i^{-+} &= \hat{\chi}_o^{+}(\lambda^-)^{1-\hat{n}_i}\sqrt{\hat{n}_l + 2\hat{n}_i},\\
\hat{S}_i^{--} &= \hat{\chi}_i^{-}(\lambda^-)^{\hat{n}_o}\sqrt{\hat{n}_l + 2(1 - \hat{n}_o)},\\
\hat{S}_i^{++} &= \hat{\chi}_i^{+}(\lambda^+)^{\hat{n}_o}\sqrt{\hat{n}_l + 2 - \hat{n}_o}.
\end{aligned}$$

The strong-coupling vacuum of the LSH Hamiltonian is given by

$$\begin{aligned}
n_l(x) &= 0, \text{ for all } x,\\
n_i(x) &= 0,\ n_o(x) = 0, \text{ for } x \text{ even},\\
n_i(x) &= 1,\ n_o(x) = 1, \text{ for } x \text{ odd}.
\end{aligned}$$

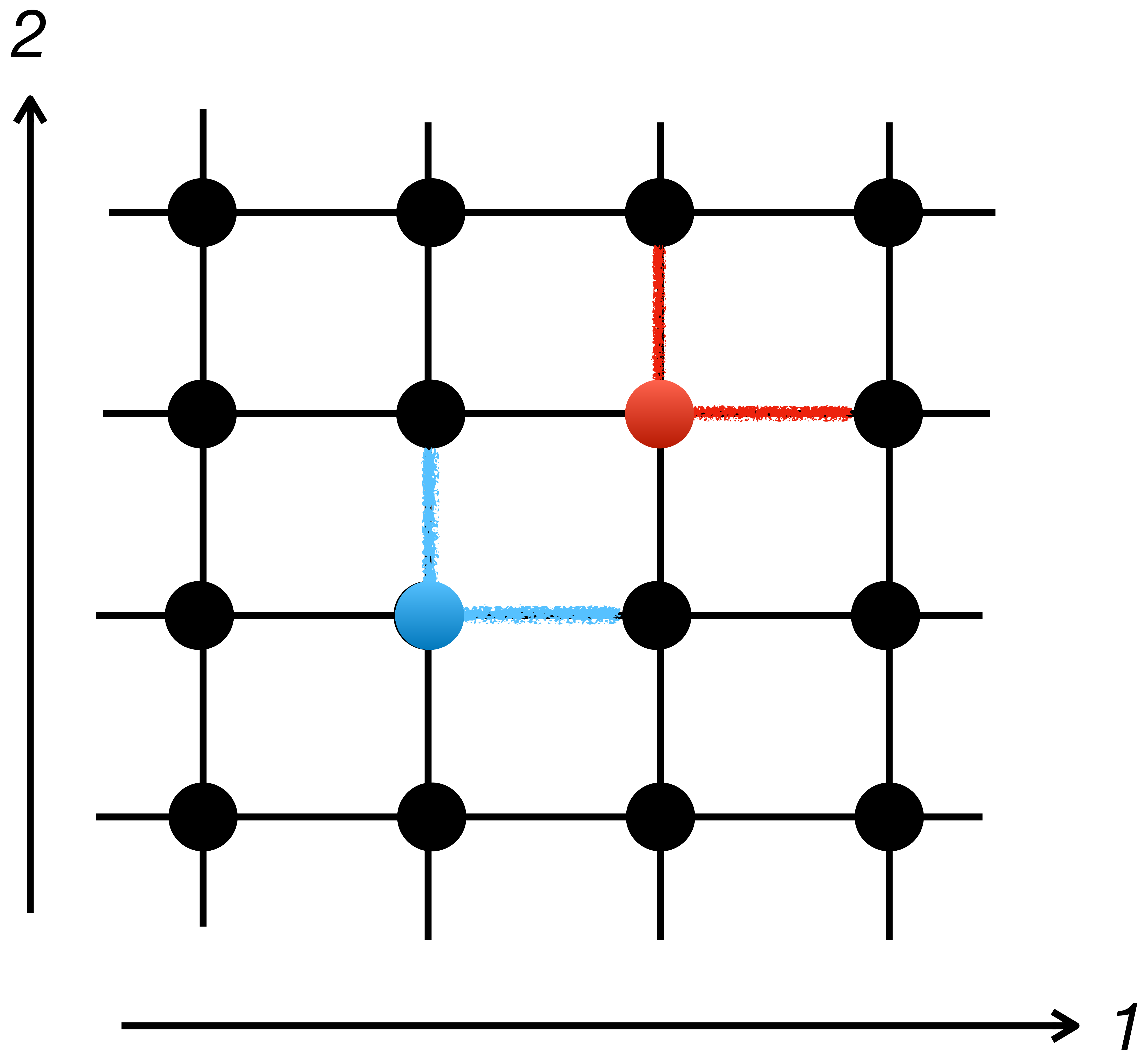
Collaborators:



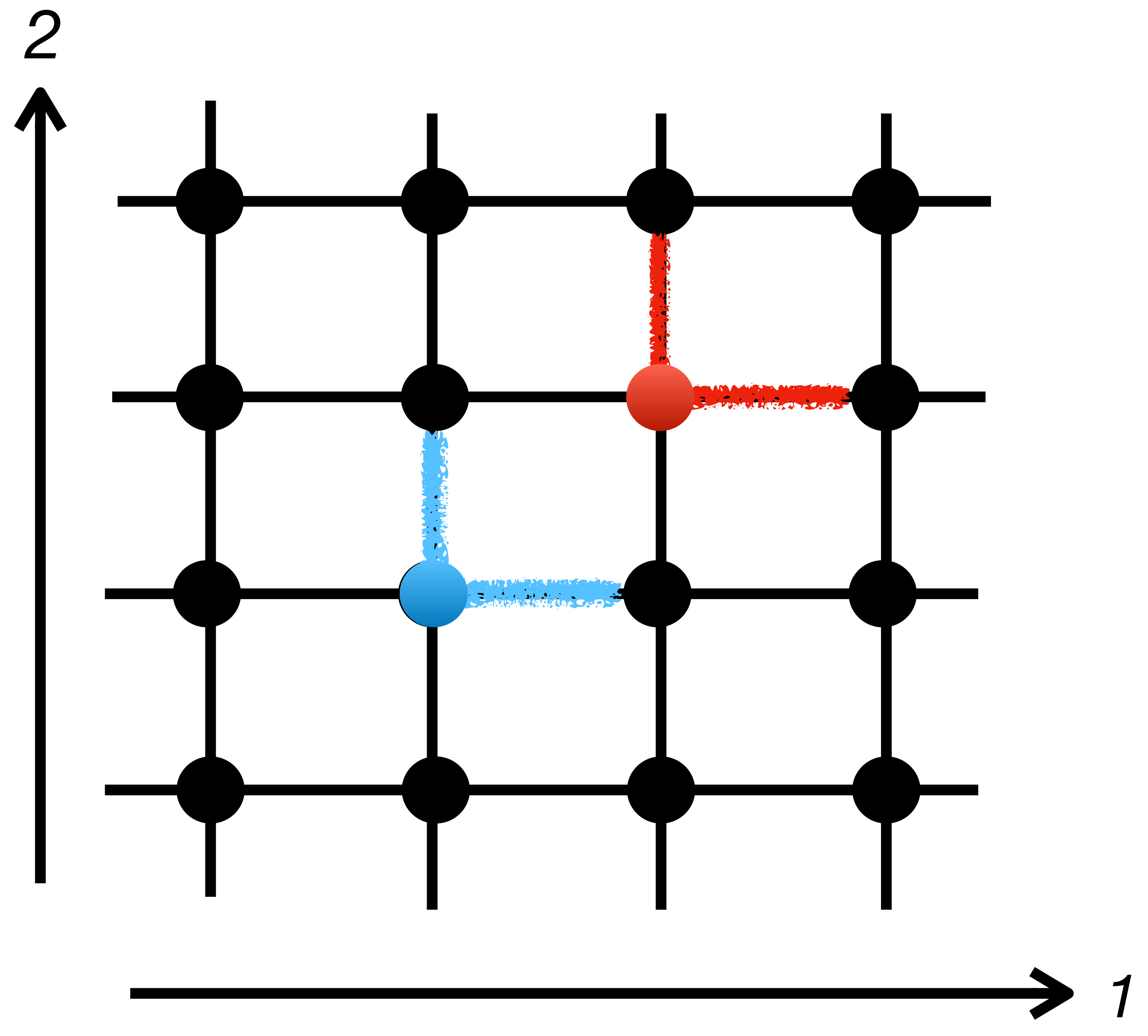
Jesse Stryker

Spectrum is identical to Kogut Susskind Hamiltonian

SU(2) LSH framework in $d > 1$



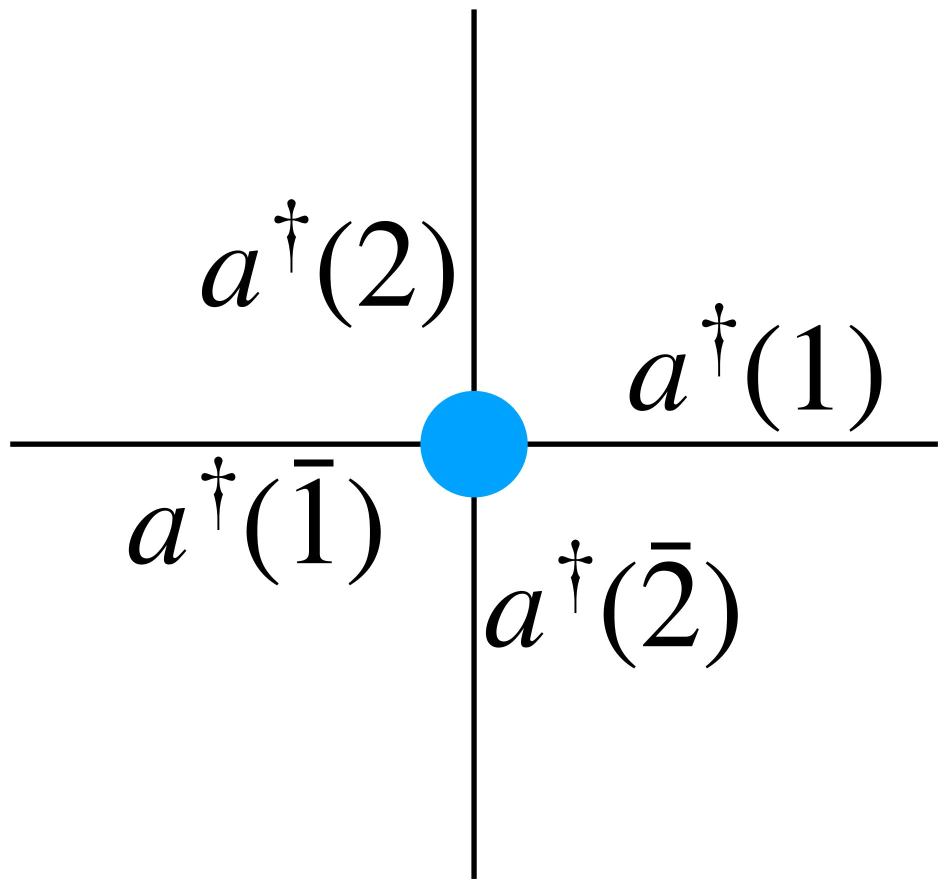
SU(2) LSH framework in $d > 1$



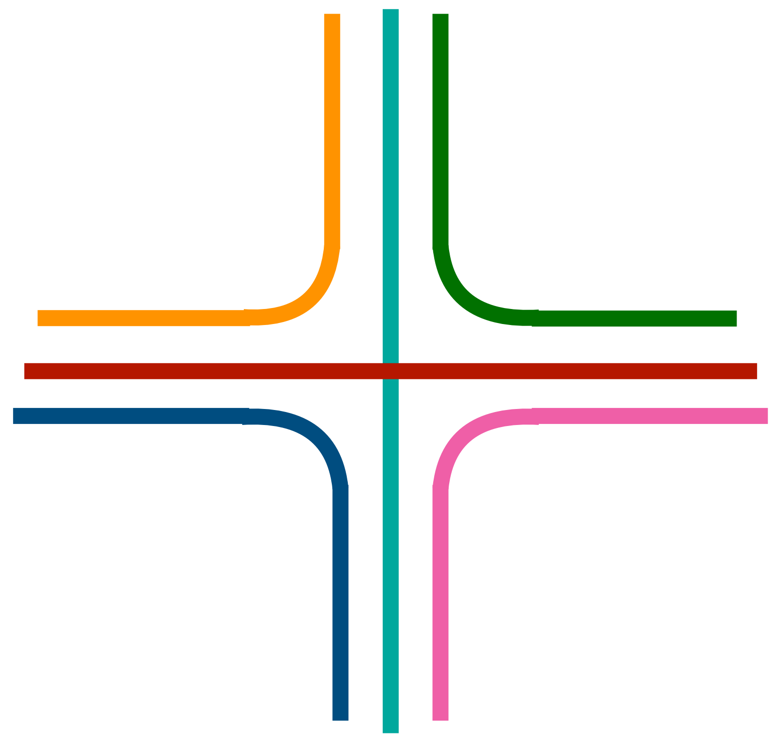
Local Loop Operator:

$$\mathscr{L}_{ij}^{++} = \epsilon^{\alpha\beta} a_\alpha^\dagger(i) a_\beta^\dagger(j)$$

Prepotential Formulation for 2+1 d:

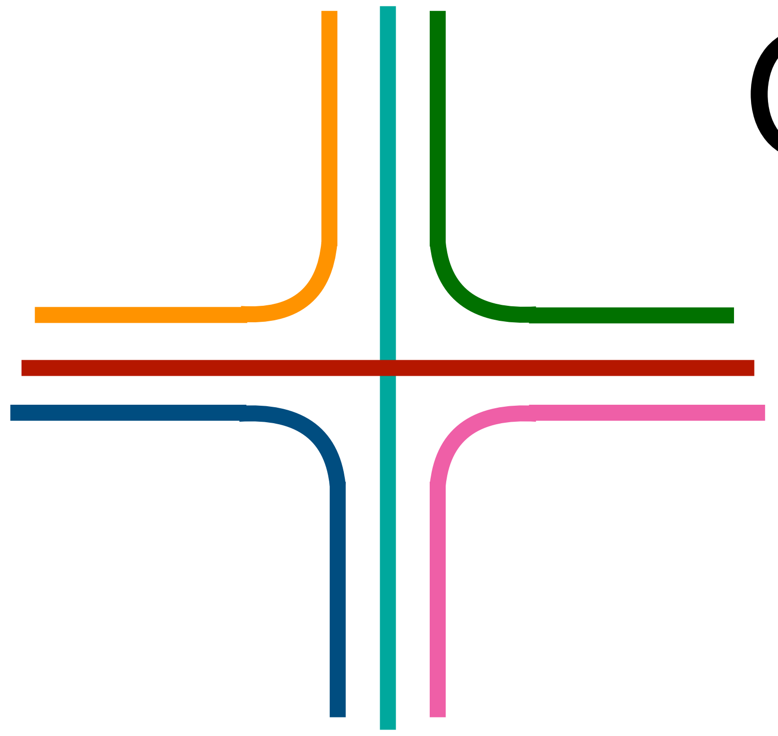


Pictorial representation:



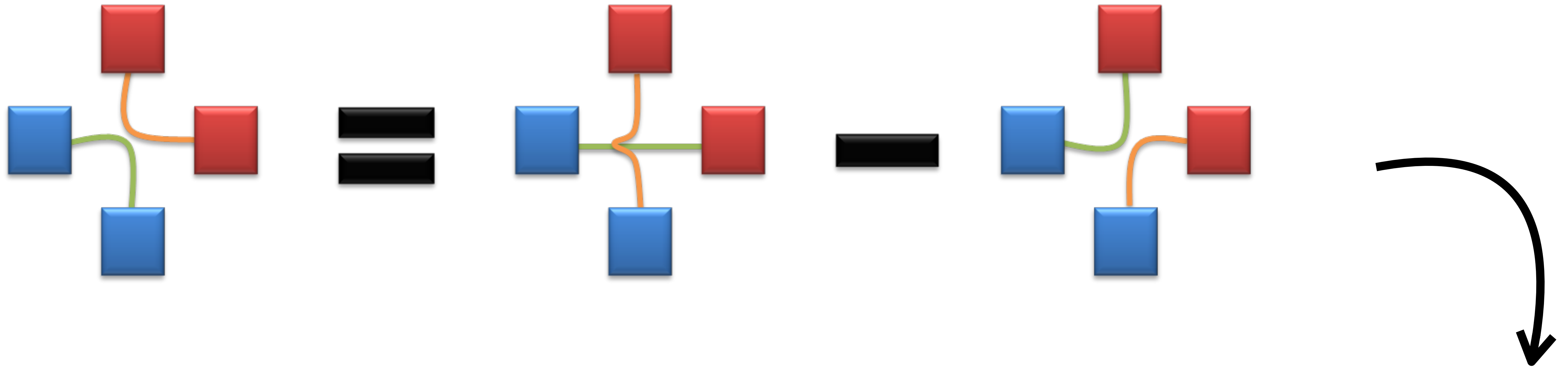
Overcomplete basis

SU(2) LSH framework in $d > 1$



Overcomplete basis

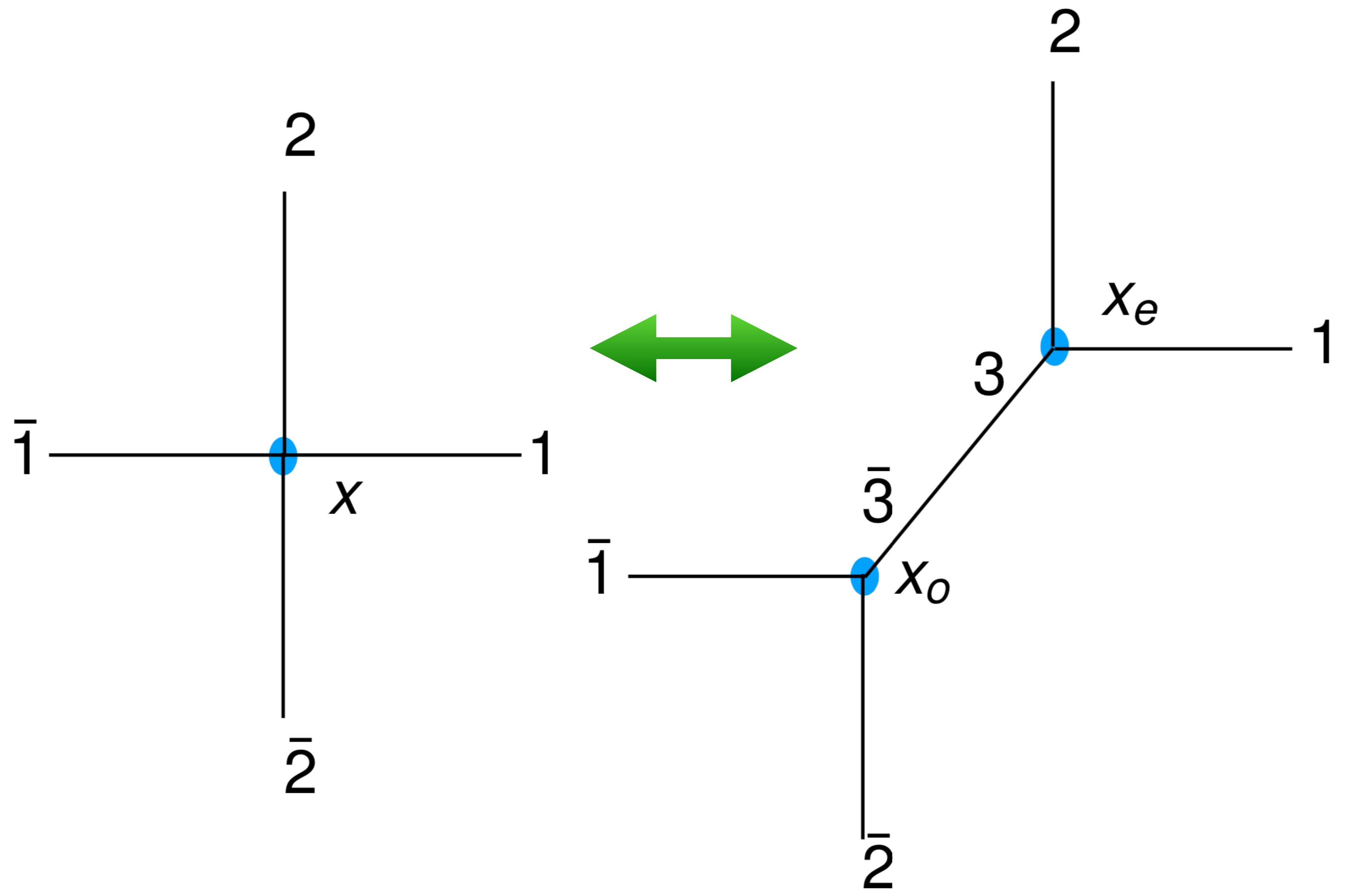
3 physical d.o.f = 6 (local loop quantum numbers in 2d)
- 2(Abelian Gauss' law constraint along 2 link directions)
-1 (**Mandelstam constraint**)



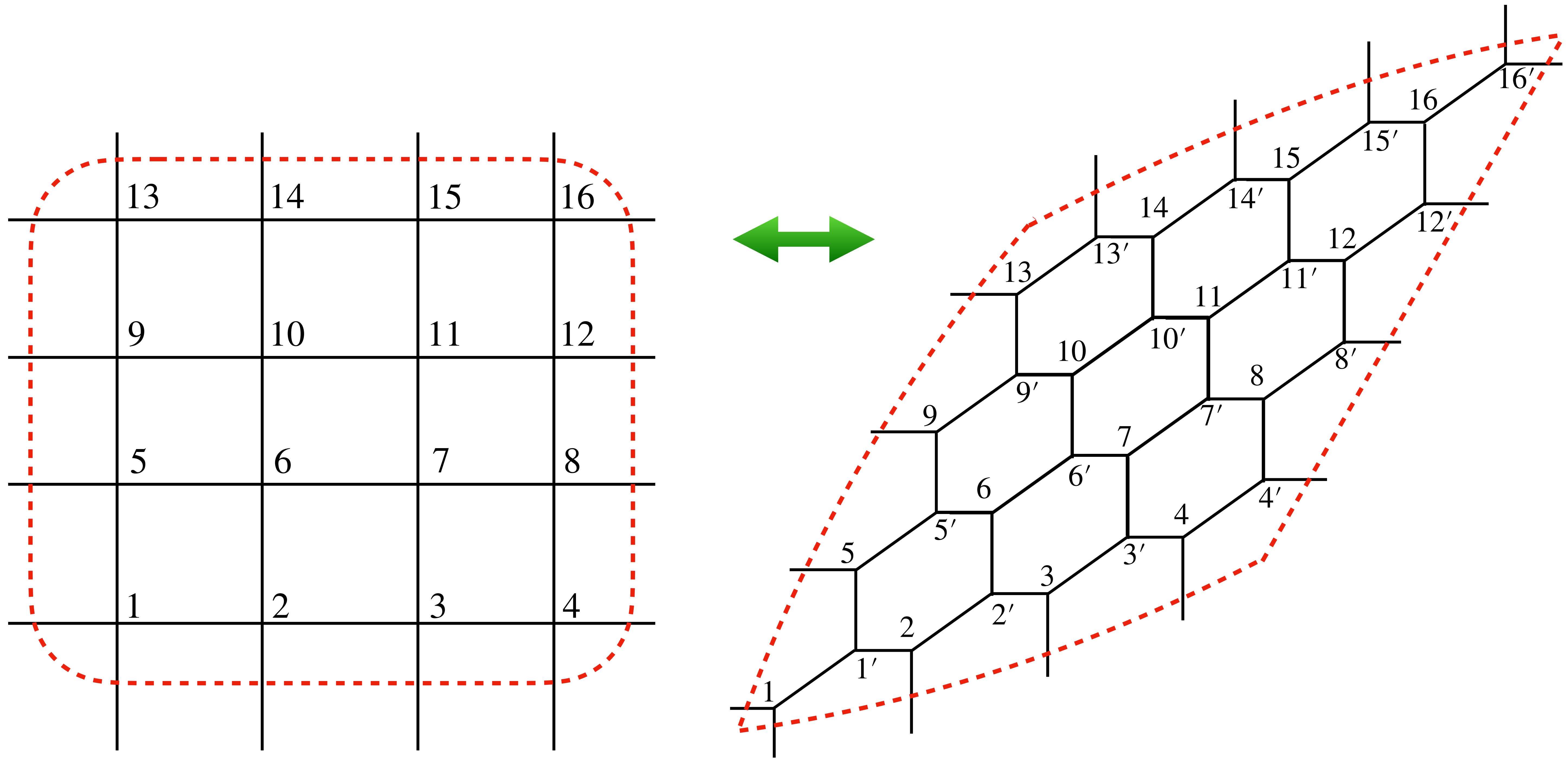
Non-linear constraints, become increasingly complicated with increasing dimension

SU(2) LSH framework in $d > 1$

Way out? Virtual point splitting scheme:

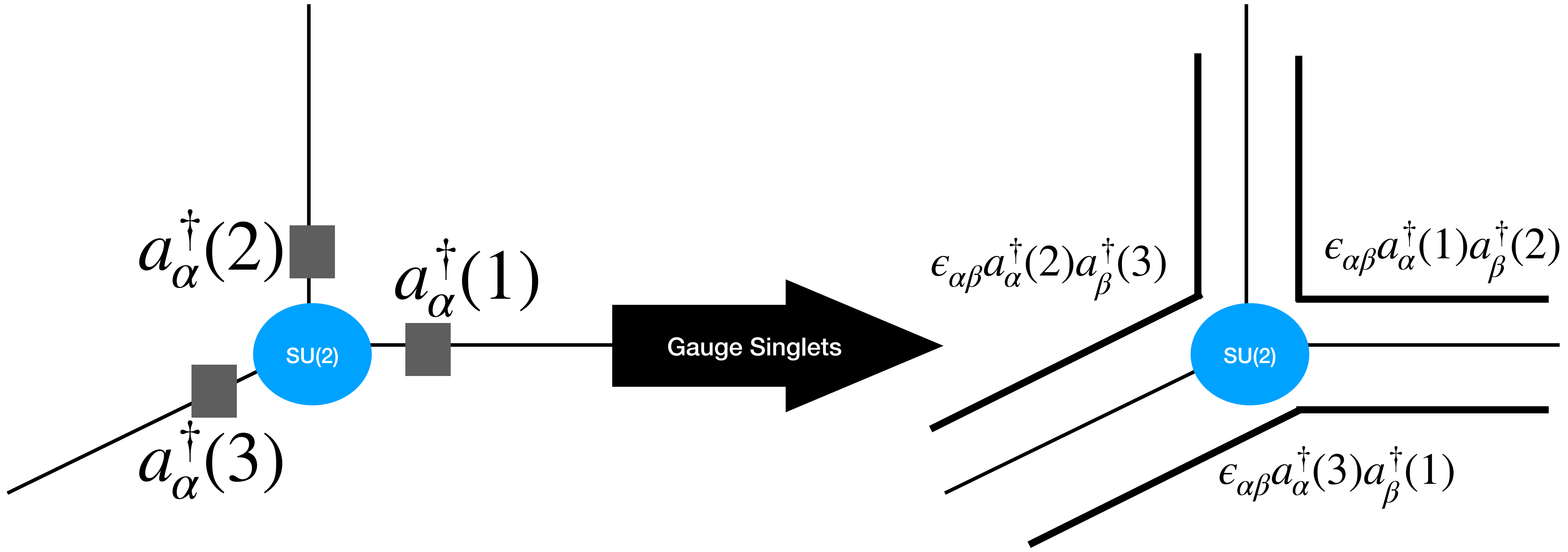


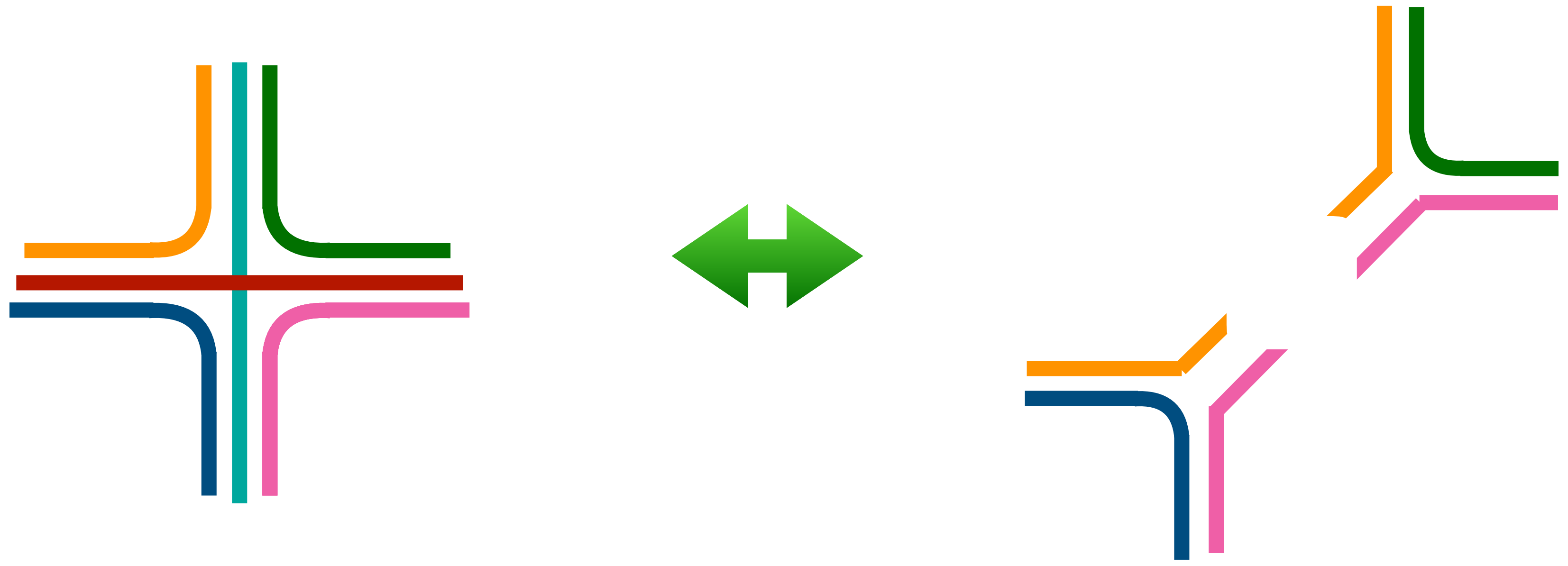
2-d LSH defined on the hexagonal lattice



SU(2) LSH framework in $d > 1$

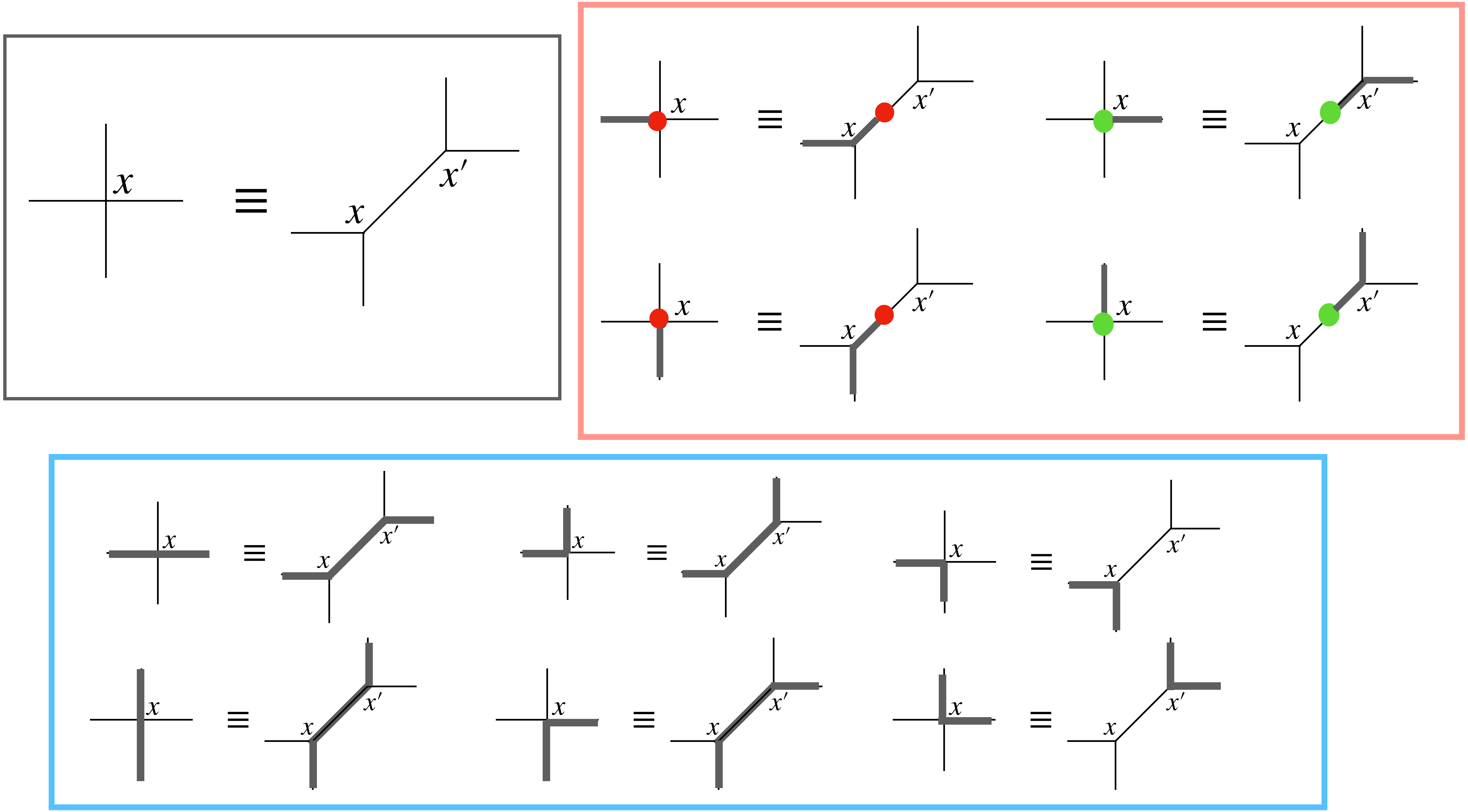
2d- LSH



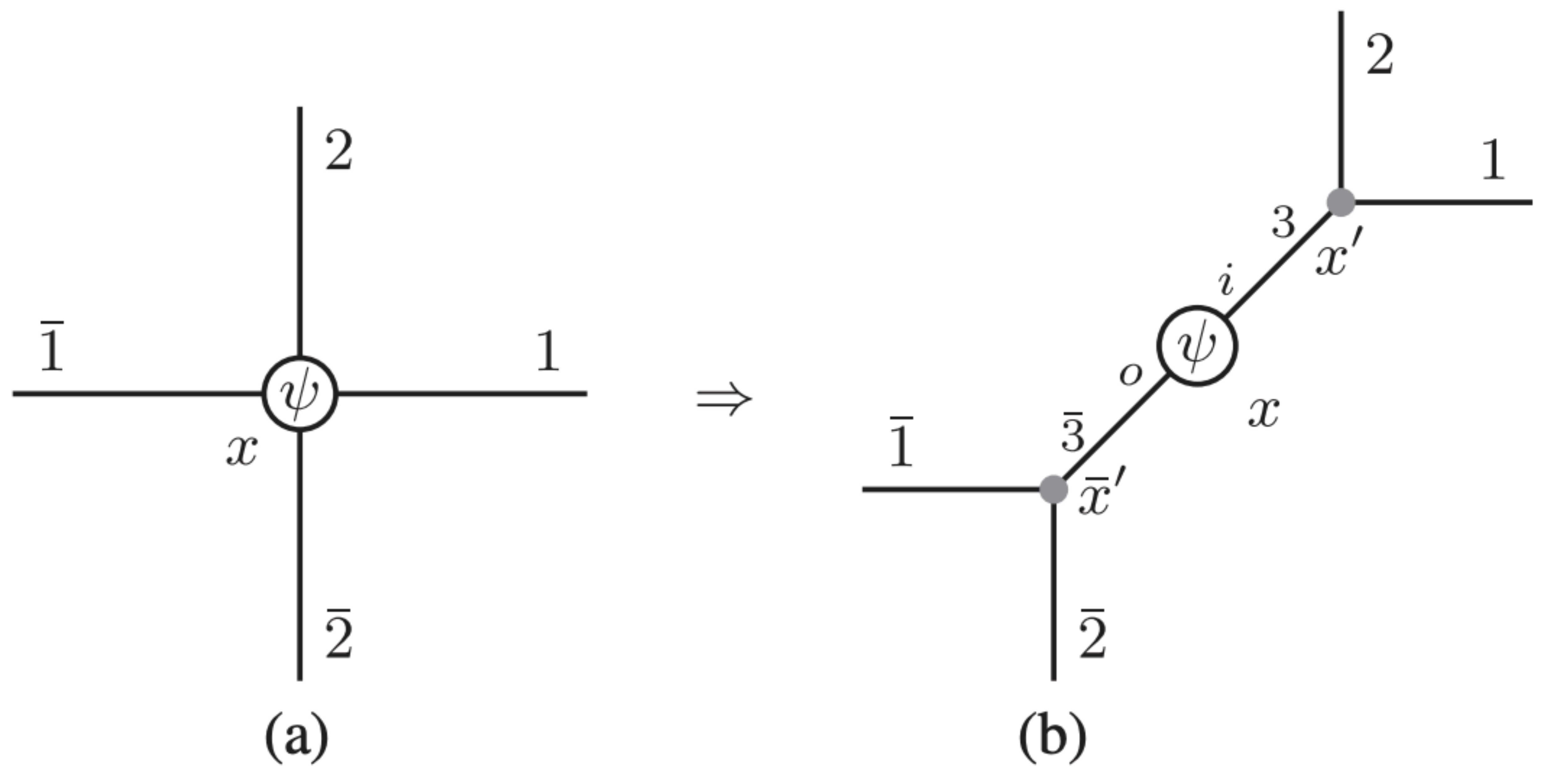


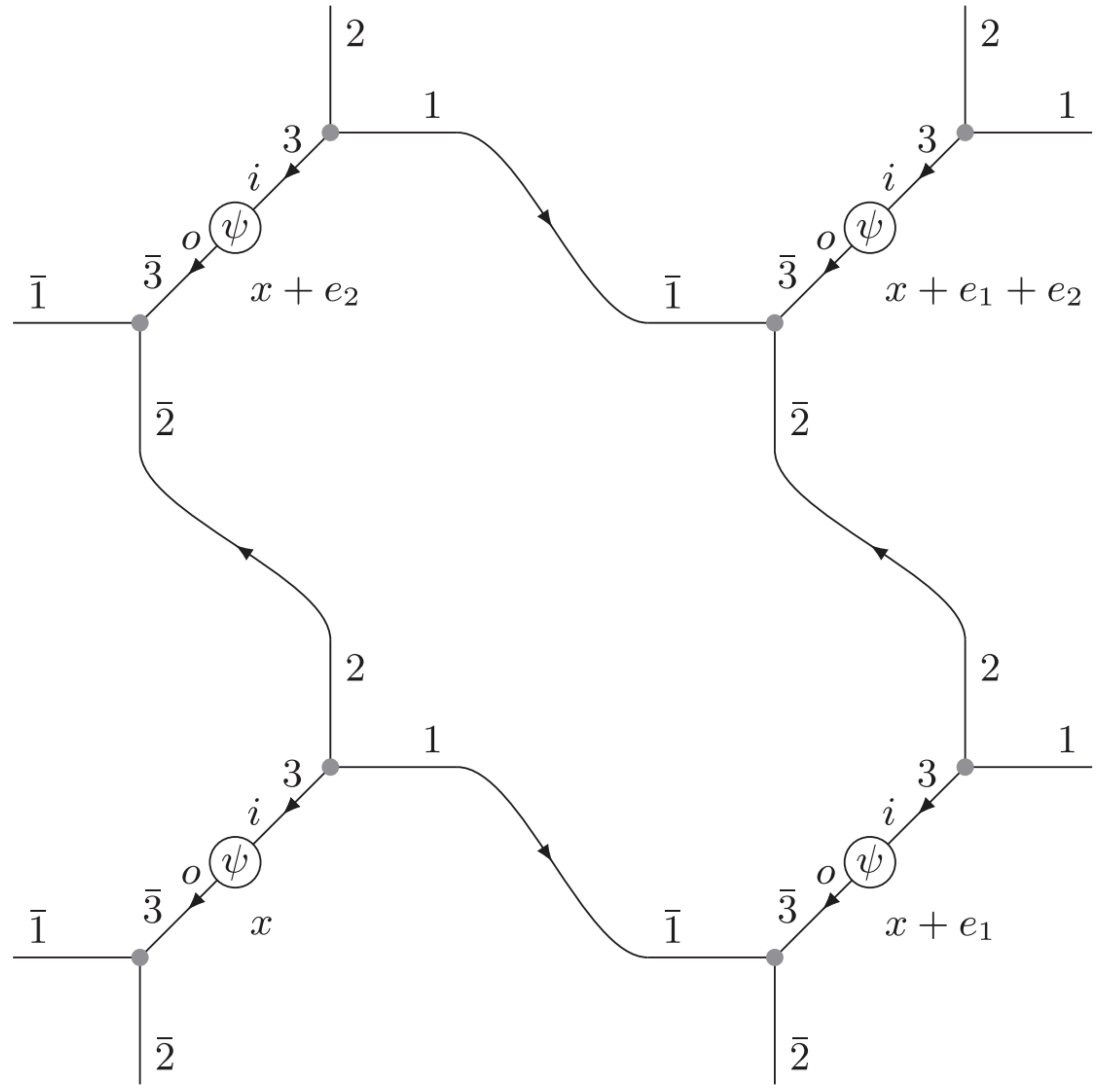
Generalized for arbitrary dimension!
Generalized to include matter!

3 physical d.o.f = 2 x 3 (local loop quantum numbers in 2d)
- 3(Abelian Gauss' law constraint)
+ **0** (**Mandelstam constraint**)


x
≡
x′

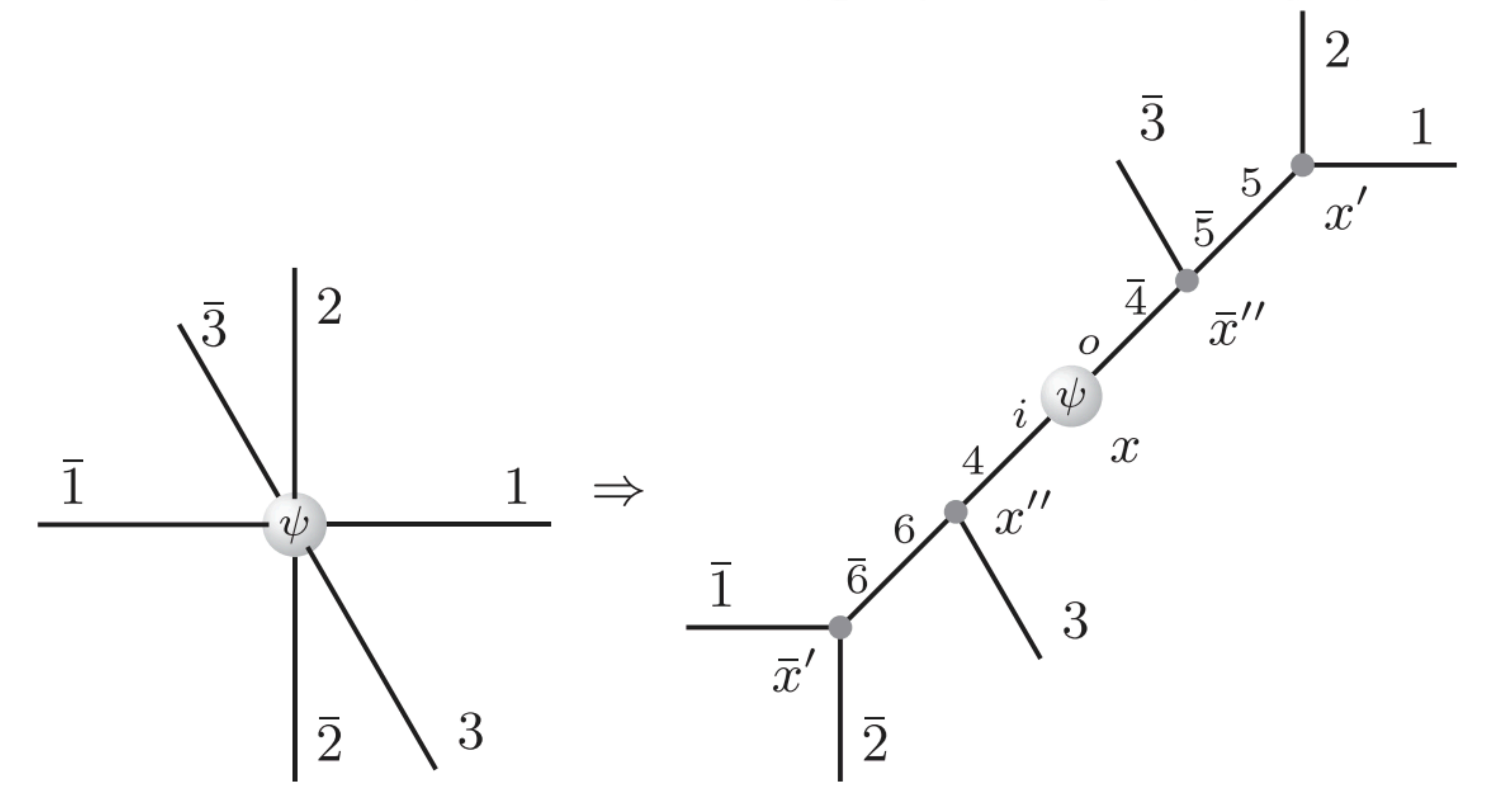
SU(2) LSH Formalism: 2+1 d





Matter-Gauge interactions
are same as in 1d

SU(2) LSH Formalism: 3+1 d



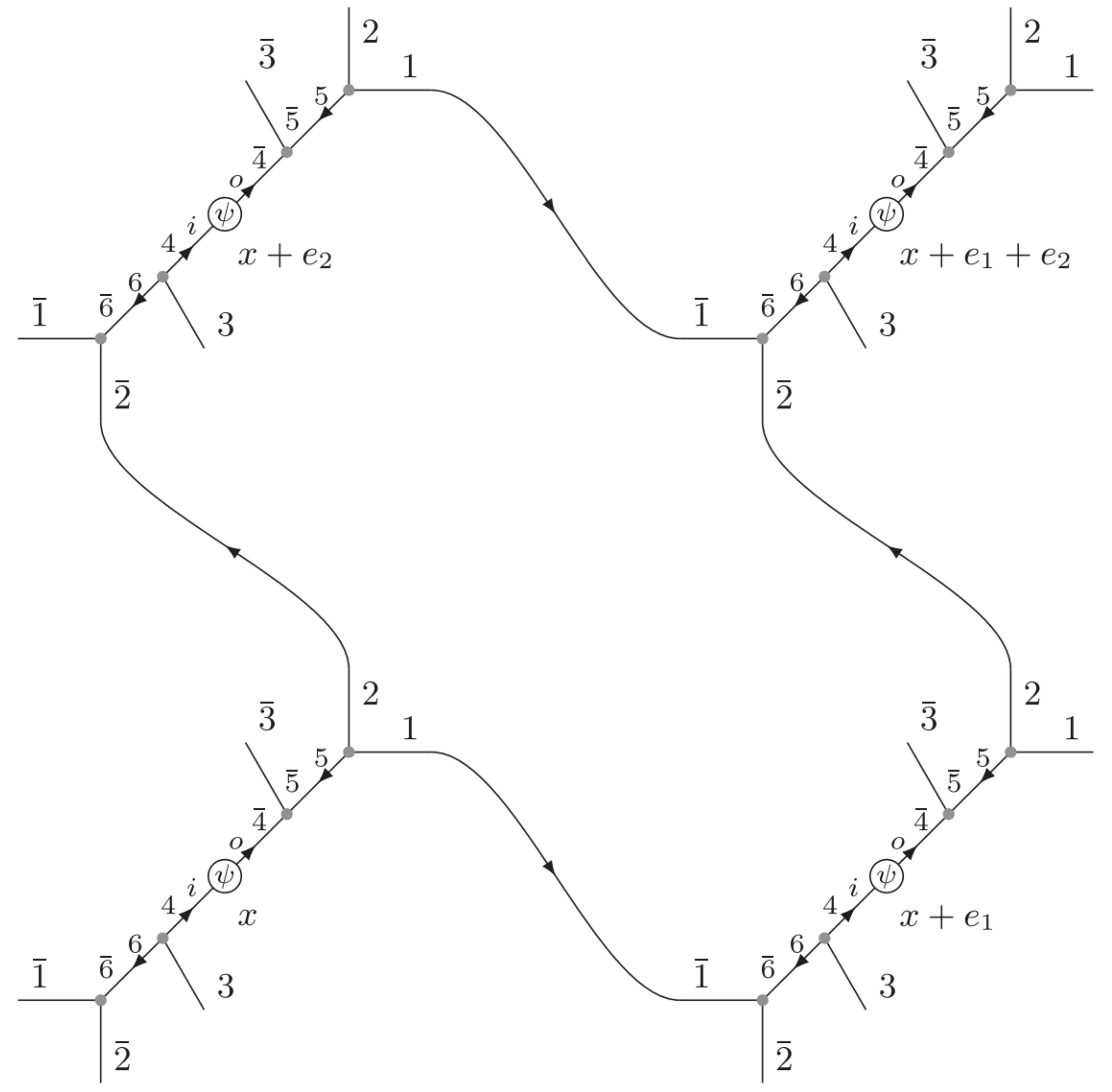


FIG. 7. Connectivity of a *xy*-plaquette in three dimensions.

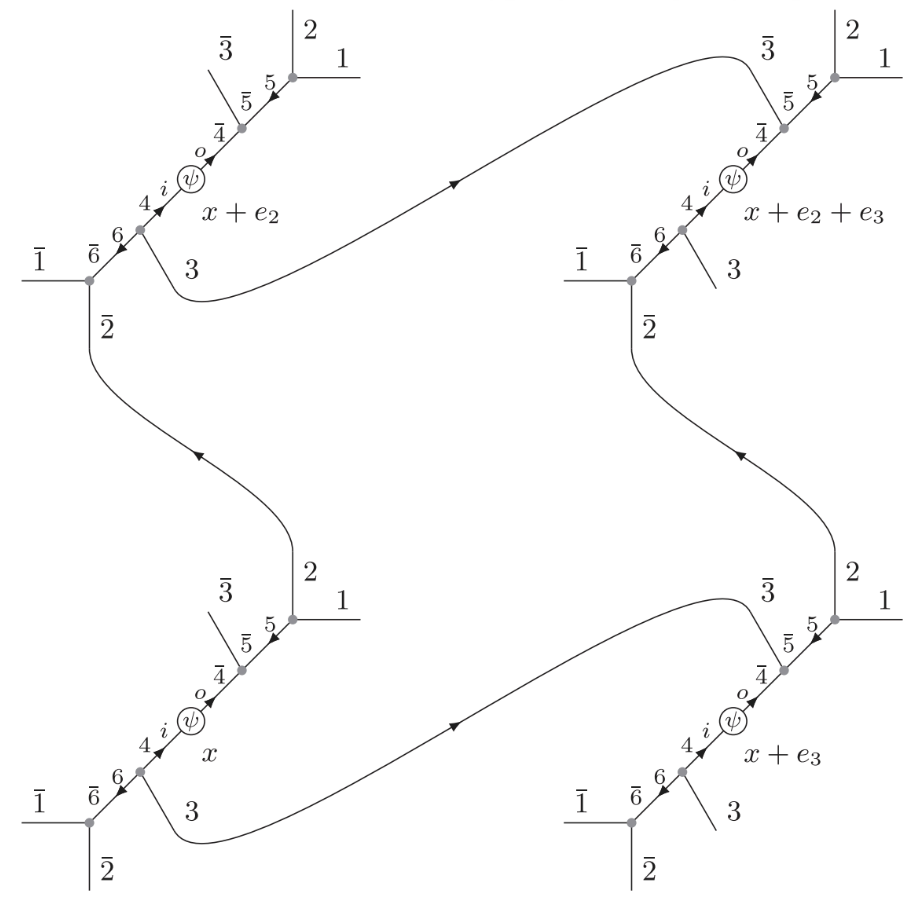


FIG. 8. Connectivity of a *yz*-plaquette in three dimensions.

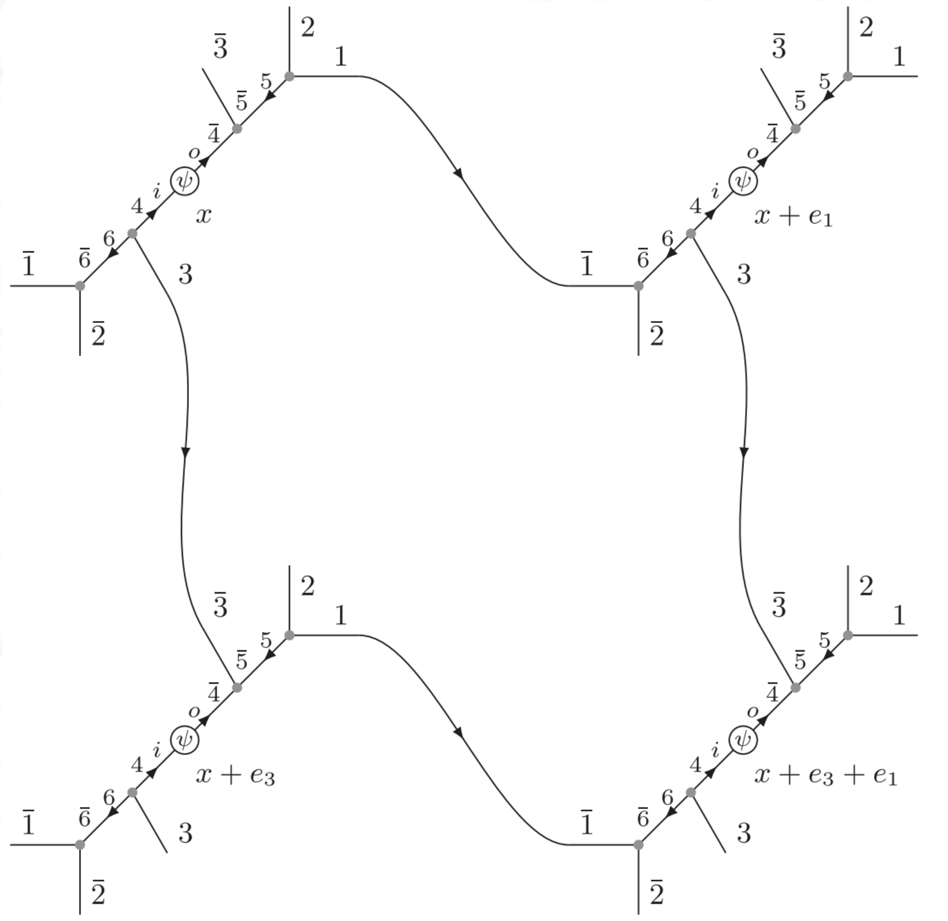


FIG. 9. Connectivity of a *zx*-plaquette in three dimensions.

- Matter-Gauge interactions are same as in 1+1d
- Pure gauge interactions are same as in 2+1d


Leads to exploring several new research directions:
LSH for full QCD
SU(3) gauge theory in 1+1d
Exploring interesting Physics
Global symmetries
Entanglement entropy
Thermalization
Quantum Algorithms
Tensor Network calculations
Analog quantum simulation using LSH
Developing algorithm for simulating SU(2) gauge theory on universal quantum computers

Loop-String-Hadron formulation of SU(3) gauge theory

PHYSICAL REVIEW D **107,** 094513 (2023)

Loop-string-hadron formulation of an SU(3) gauge theory with dynamical quarks

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Collaborators:

Jesse Stryker

Saurabh Kadam

The construction involves nontrivial complications over the SU(2) framework

Not yet the full-fledged QCD

In 1+1 dimension, to be generalised in d>2
Single quark flavour

However, the final construction retains all essential features of the SU(2) framework and can be used in straightforward manner like in SU(2)

The LSH Hamiltonian for (3+1)d SU(3) gauge theory → A concrete step towards quantum simulating QCD

LSH Formulation: key ingredients

Local gauge invariant Hilbert space

Local constraint on each link: Abelian Gauss' law

Manifestly gauge invariant dynamics within the LSH Hilbert space

Generalization to QCD

First attempt: SU(3) gauge theory in 1+1 dimension

Starting point: Prepotential formulation of SU(3) gauge theory

Prepotential formulation of SU(3) gauge theory

Ramesh Anishetty, Manu Mathur, IR, (2009), (2010)

Not a trivial generalisation of SU(2)

Kogut-Susskind Formulation → Schwinger boson representation of SU(3)

$\equiv$

Prepotential formulation for SU(3)

SU(3): rank 2

Fundamental and anti-fundamental representation:
Two Schwinger boson triplets $a^\dagger_\alpha, b^{\dagger\alpha}$

↓

Multiplicity problem caused by: $\sum_{\alpha=1}^{3} a^\dagger_\alpha b^{\dagger\alpha} \Rightarrow \equiv a^\dagger \cdot b^\dagger$

SU(3) singlets

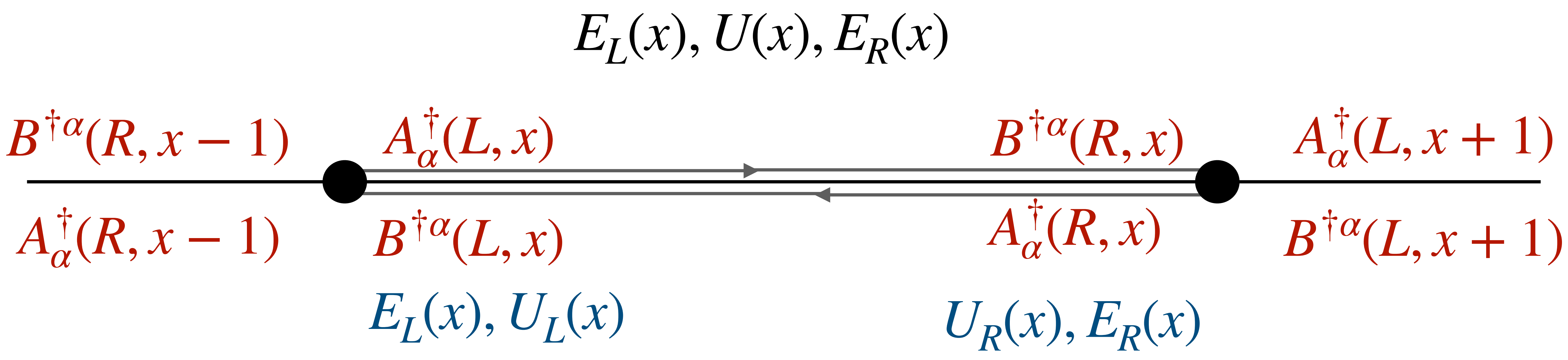
Solved ←

$A^\dagger_\alpha, B^{\dagger\alpha}$: Irreducible Schwinger boson representation of SU(3)
$A^\dagger_\alpha \cdot B^{\dagger\alpha} \approx 0$

↑ Prepotential formulation for SU(3)

Chaturvedi and Mukunda J. Math. Phys. 43, 5262 (2002)

Prepotential formulation of SU(3) gauge theory



Abelian Gauss' Law

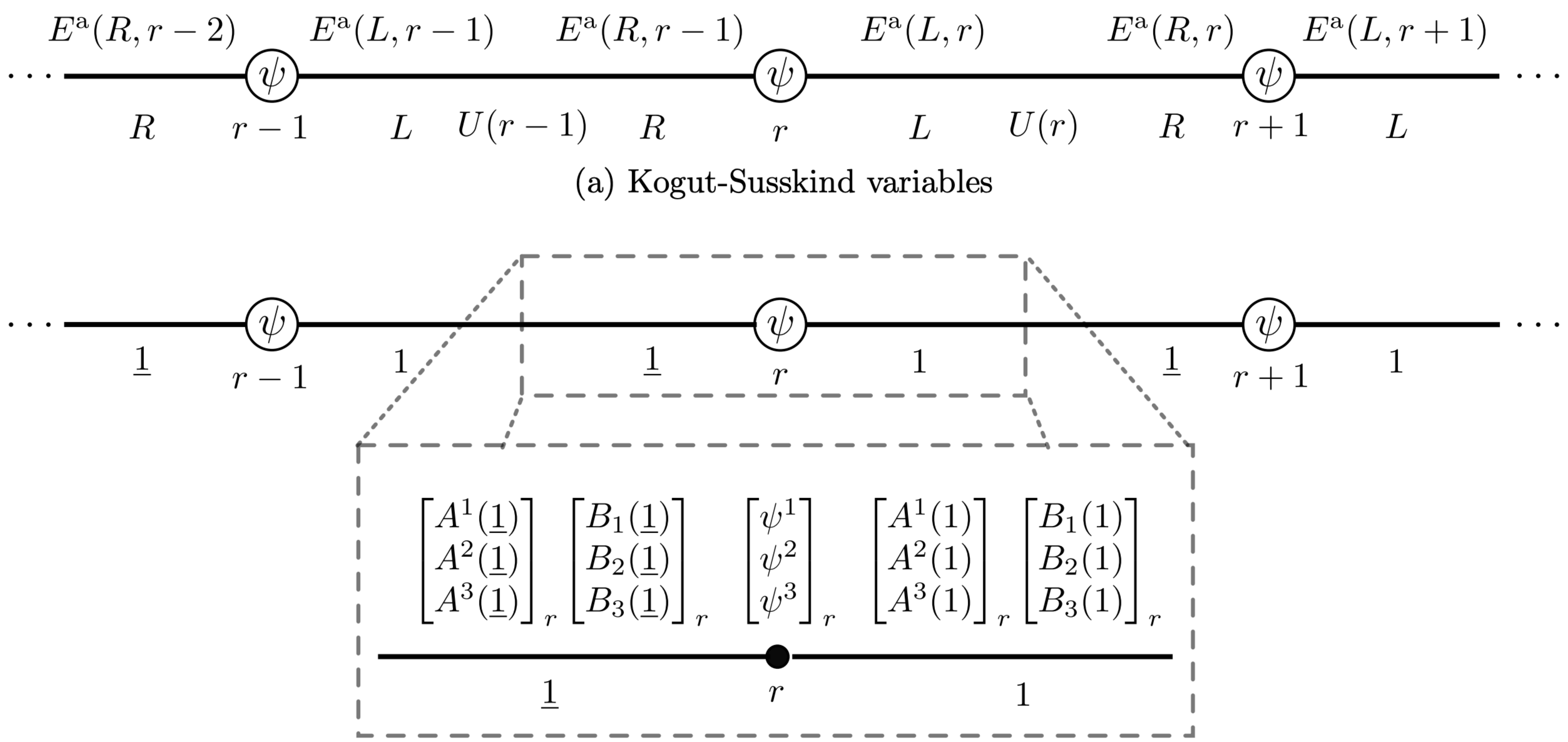
$$N_A(L,x) = N_B(R,x)$$

$$N_B(L,x) = N_A(R,x)$$

Imposes continuity of the flux lines

Directed flow of electric flux on a link: From triplet to anti-triplet

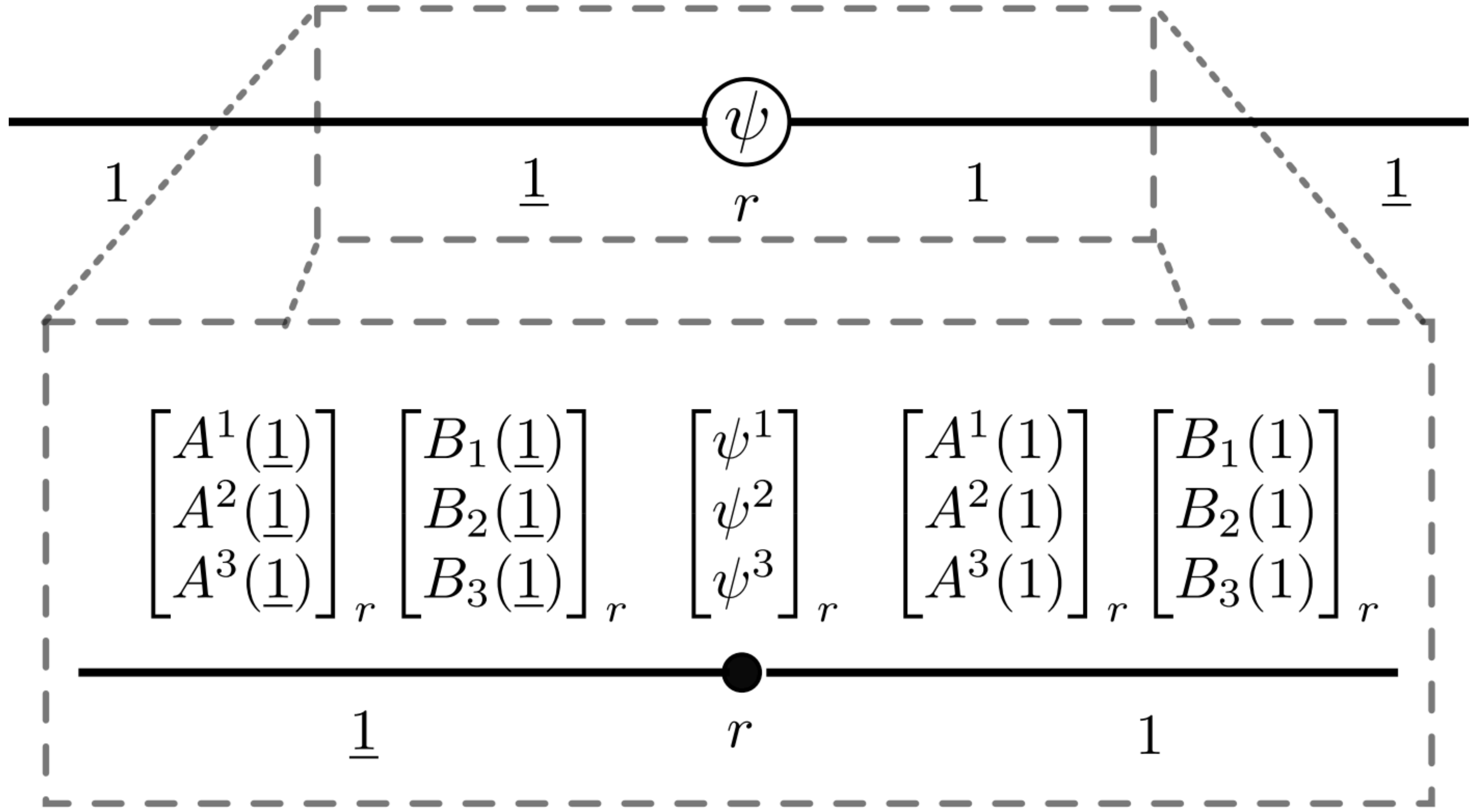
LSH formulation of SU(3) gauge theory



(a) Kogut-Susskind variables

(b) Prepotential variables.

Loop-String-Hadron formulation of SU(3) gauge theory



Local ingredients:

Fundamental $(1,0)$ or $\mathbf{3}$	Anti-fundamental $(0,1)$ or $\mathbf{3^*}$
$A^\dagger_\alpha(\underline{1}, r)$	$A^\alpha(\underline{1}, r)$
$A^\dagger_\alpha(1, r)$	$A^\alpha(1, r)$
$B_\alpha(\underline{1}, r)$	$B^{\dagger\alpha}(\underline{1}, r)$
$B_\alpha(1, r)$	$B^{\dagger\alpha}(1, r)$
$\psi^\dagger_\alpha(r)$	$\psi^\alpha(r)$

Singlets can be formed using:

$$\delta^\alpha{}_\beta \equiv \cdot$$

$\epsilon^{\alpha\beta\gamma}$ or $\epsilon_{\alpha\beta\gamma} \equiv \wedge$

Loop-String-Hadron basis: onsite SU(3) invariant basis

Local ingredients:

Fundamental $(1,0)$ or $\mathbf{3}$	Anti-fundamental $(0,1)$ or $\mathbf{3^*}$
$A^\dagger_\alpha(\underline{1},r)$	$A^\alpha(\underline{1},r)$
$A^\dagger_\alpha(1,r)$	$A^\alpha(1,r)$
$B_\alpha(\underline{1},r)$	$B^{\dagger\alpha}(\underline{1},r)$
$B_\alpha(1,r)$	$B^{\dagger\alpha}(1,r)$
$\psi^\dagger_\alpha(r)$	$\psi^\alpha(r)$

Singlets can be formed using:

$$\delta^\alpha{}_\beta \equiv \cdot$$

$\epsilon^{\alpha\beta\gamma}$ or $\epsilon_{\alpha\beta\gamma} \equiv \wedge$

$$\left[A^\dagger(\underline{1}) \cdot B^\dagger(1)\right]$$

$$\left[A^\dagger(1) \cdot B^\dagger(\underline{1})\right]$$

$$\psi^\dagger \cdot A^\dagger(\underline{1}) \wedge A^\dagger(1)$$

$$\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(1)$$

$$\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(\underline{1})$$

$$\psi^\dagger \cdot B^\dagger(\underline{1})$$

$$\psi^\dagger \cdot B^\dagger(1)$$

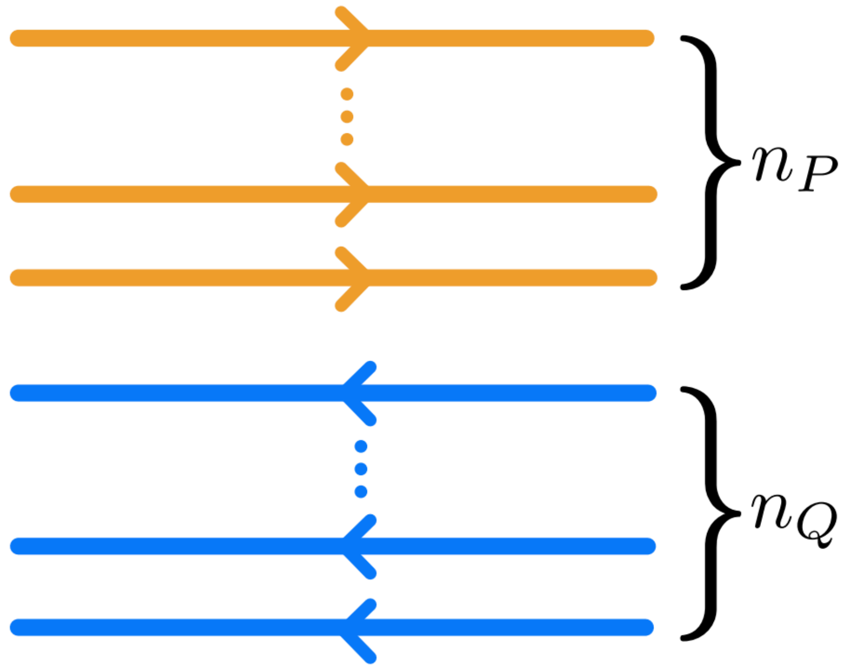
$$\psi^\dagger \cdot \psi^\dagger \wedge \psi^\dagger$$

LSH state:

$$\left|n_P, n_Q\,;\, \nu_{\underline{1}}, \nu_0, \nu_1\right\rangle_r, \qquad n_P, n_Q \in \{0,1,2,\cdots\}, \qquad \nu_{\underline{1}}, \nu_0, \nu_1 \in \{0,1\},$$

$$\left[A^\dagger(\underline{1})\cdot B^\dagger(1)\right]^{n_P} \longrightarrow$$

$$\left[A^\dagger(1)\cdot B^\dagger(\underline{1})\right]^{n_Q} \longrightarrow$$



$$|n_P, n_Q\rangle \propto |n_P, n_Q; 0,0,0\rangle \longrightarrow$$

$$\psi^\dagger\cdot B^\dagger(1)|n_P, n_Q\rangle \propto |n_P, n_Q; 0,0,1\rangle \longrightarrow$$

$$\psi^\dagger\cdot A^\dagger(\underline{1})\wedge A^\dagger(1)|n_P, n_Q\rangle \propto |n_P, n_Q; 0,1,0\rangle \longrightarrow$$

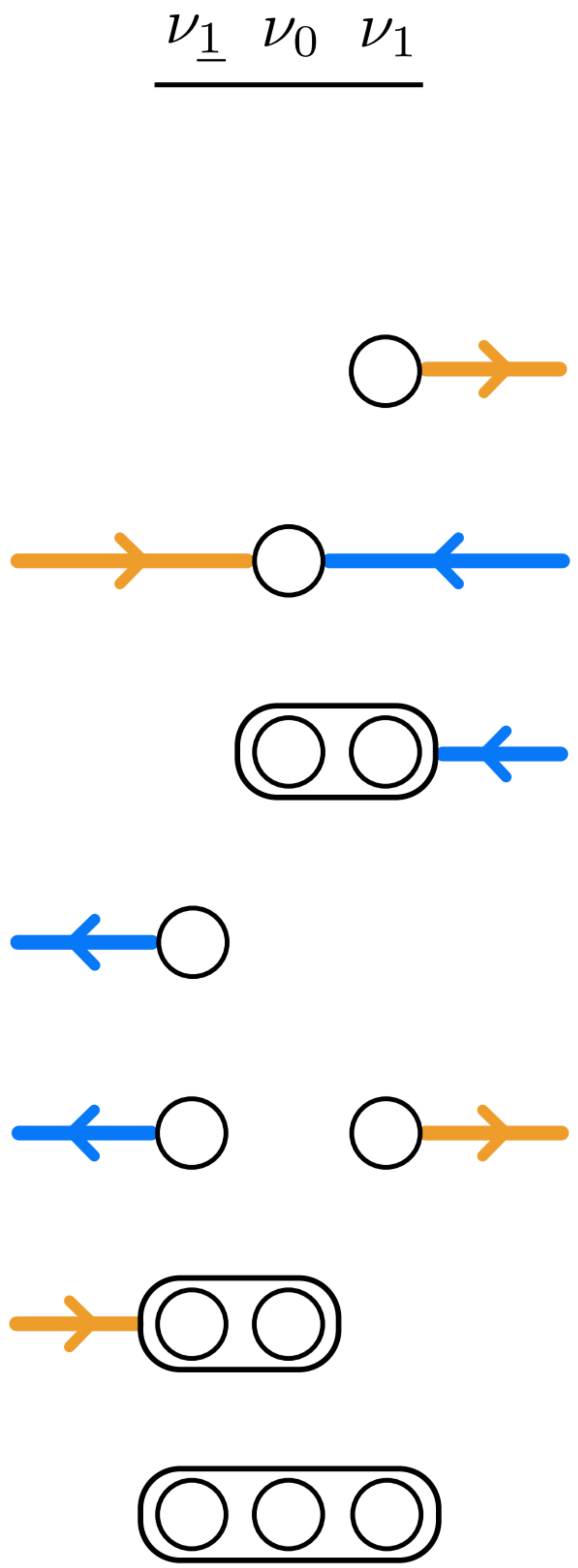
$$\psi^\dagger\cdot \psi^\dagger\wedge A^\dagger(1)|n_P, n_Q\rangle \propto |n_P, n_Q; 0,1,1\rangle \longrightarrow$$

$$\psi^\dagger\cdot B^\dagger(\underline{1})|n_P, n_Q\rangle \propto |n_P, n_Q; 1,0,0\rangle \longrightarrow$$

$$\psi^\dagger\cdot B^\dagger(\underline{1})\ \psi^\dagger\cdot B^\dagger(1)|n_P, n_Q\rangle \propto |n_P, n_Q; 1,0,1\rangle \longrightarrow$$

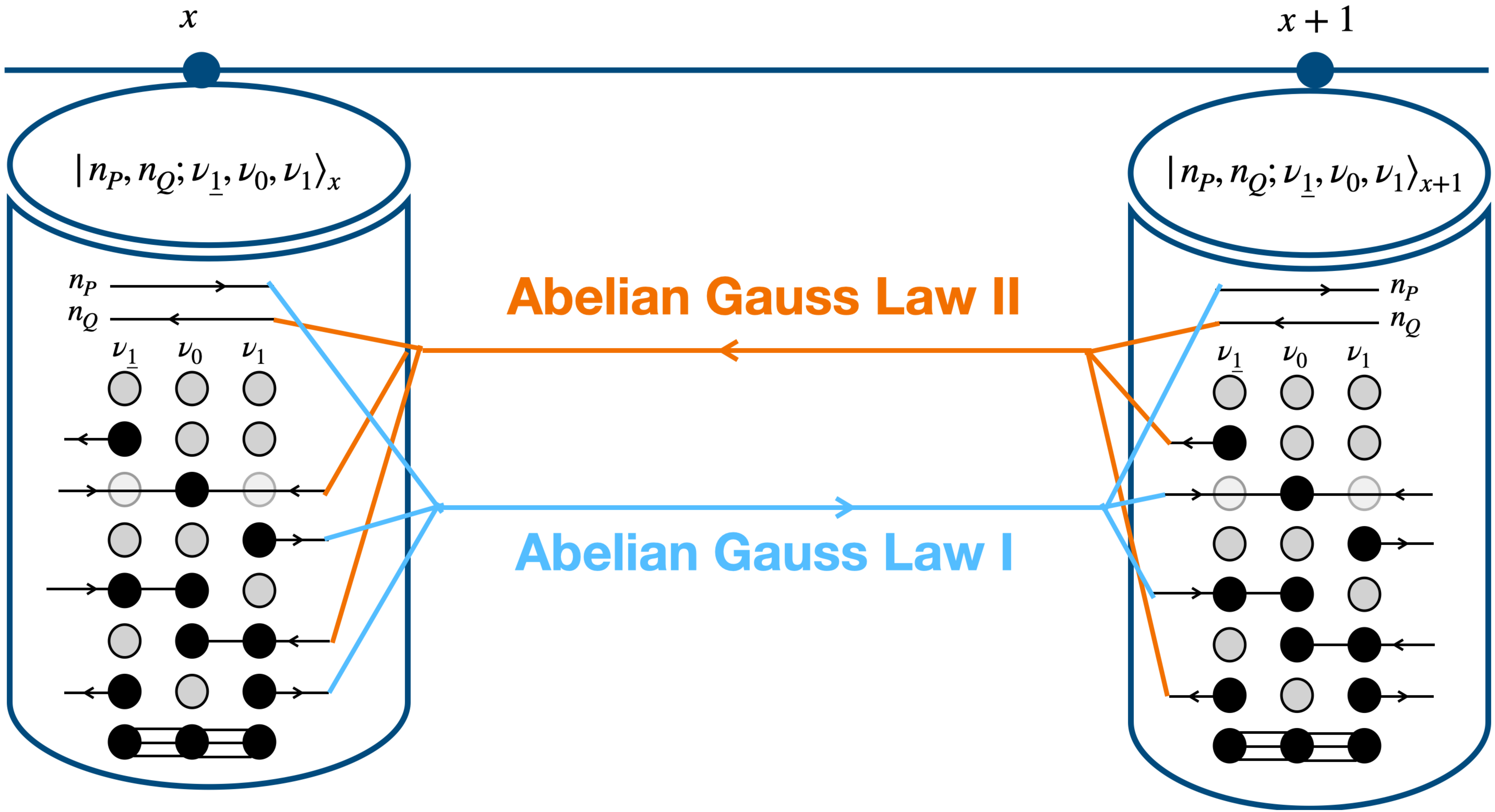
$$\psi^\dagger\cdot \psi^\dagger\wedge A^\dagger(\underline{1})|n_P, n_Q\rangle \propto |n_P, n_Q; 1,1,0\rangle \longrightarrow$$

$$\psi^\dagger\cdot \psi^\dagger\wedge \psi^\dagger|n_P, n_Q\rangle \propto |n_P, n_Q; 1,1,1\rangle \longrightarrow$$



Loop-String-Hadron basis: Pictorial Representation

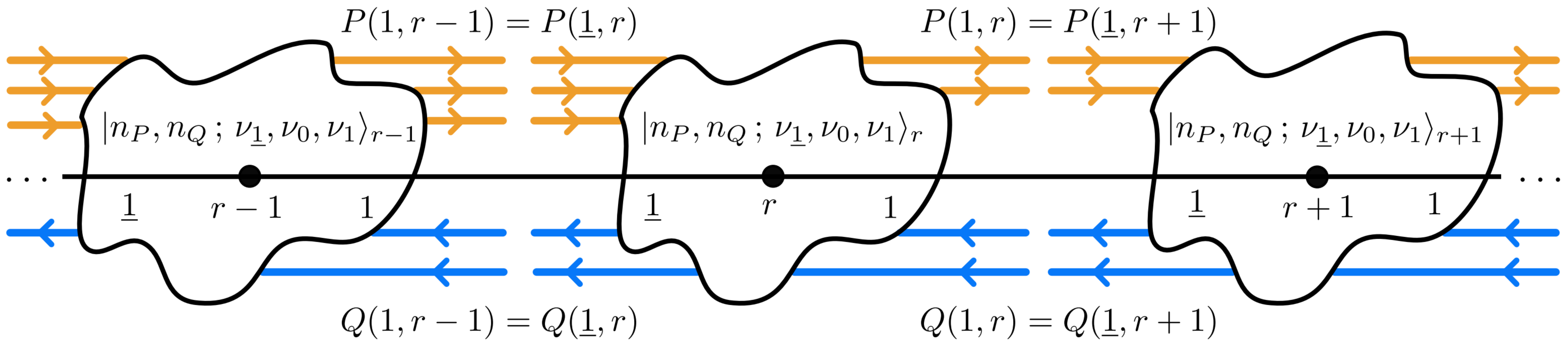
LSH Formulation: SU(3) in 1+1 dimension



LSH state:

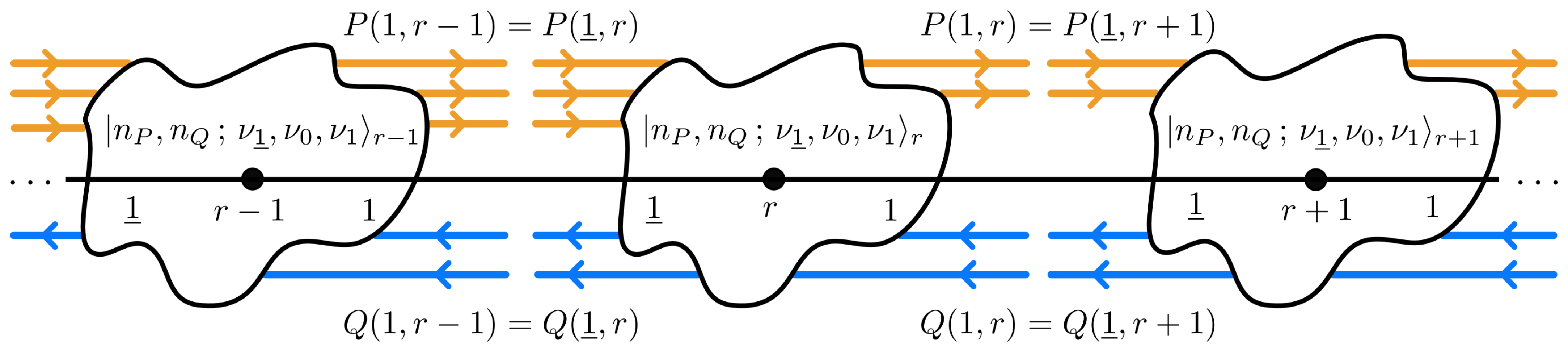
$$|n_P, n_Q\,;\, \nu_{\underline{1}}, \nu_0, \nu_1\rangle_r\,, \qquad n_P, n_Q \in \{0, 1, 2, \cdots\}, \qquad \nu_{\underline{1}}, \nu_0, \nu_1 \in \{0, 1\},$$

Snapshots of loops-strings-hadron configurations at each site → We further need to weave these along links **Abelian Gauss laws**



Global Loop-String-Hadron basis: Pictorial Representation

LSH Formulation: key ingredients for SU(3) in 1+1 dimension



Local LSH state:

$$\left|n_P, n_Q \,;\, \nu_{\underline{1}}, \nu_0, \nu_1\right\rangle_r , \qquad n_P, n_Q \in \{0, 1, 2, \cdots\}, \qquad \nu_{\underline{1}}, \nu_0, \nu_1 \in \{0, 1\},$$

Abelian Gauss laws

$$P(1, r) = P(\underline{1}, r+1) \quad \text{and} \quad Q(1, r) = Q(\underline{1}, r+1)$$

$$P(\underline{1}, r) = n_P(r) + \nu_0(r)\,(1 - \nu_1(r))$$

$$P(1, r) = n_P(r) + \nu_1(r)\,(1 - \nu_0(r))$$

$$Q(\underline{1}, r) = n_Q(r) + \nu_{\underline{1}}(r)\,(1 - \nu_0(r))$$

$$Q(1, r) = n_Q(r) + \nu_0(r)\,(1 - \nu_{\underline{1}}(r))$$

Towards building the LSH Hamiltonian

Kogut-Susskind Hamiltonian → Irreducible Schwinger boson representation of SU(3) coupled to on-site staggered fermions → Local LSH operators weaved together by the Abelian Gauss law

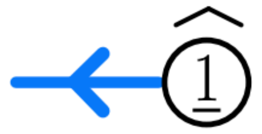
$$\psi^\dagger_\alpha(r)\,U^\alpha{}_\beta(r)\,\psi^\beta(r+1) = \Big[\psi^\dagger_\alpha\, B^{\dagger\alpha}(1)\;\eta(1)\Big]_r\Big[\eta(\underline{1})\;\psi^\beta\; A^\dagger_\beta(\underline{1})\Big]_{r+1} + \Big[\psi^\dagger_\alpha\, A^\alpha(1)\;\theta(1)\Big]_r\Big[\theta(\underline{1})\;\psi^\beta\; B_\beta(\underline{1})\Big]_{r+1}$$
$$+\Big[\psi^\dagger_\alpha\,(A^\dagger(1)\wedge B(1))^\alpha\;\delta(1)\Big]_r\Big[\delta(\underline{1})\;\psi^\beta\,(B^\dagger(\underline{1})\wedge A(\underline{1}))_\beta\Big]_{r+1}$$

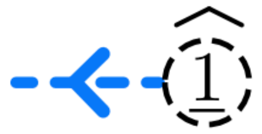
$$U^\alpha{}_\beta(r) = B^{\dagger\alpha}(L,r)\eta(r)A^\dagger_\beta(R,r+1) + A^\alpha(L,r)\theta(r)B_\beta(R,r+1) + (A^\dagger(L,r)\wedge B(L,r))^\alpha\delta(r)(B^\dagger(R,r+1)\wedge A(R,r+1))_\beta$$

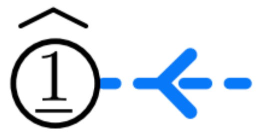
Local building blocks

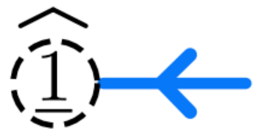
(a) $\psi^\dagger\cdot B^\dagger(1)\equiv$ 

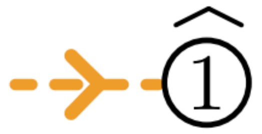
(b) $\psi\cdot B(1)\equiv$ 

(c) $\psi^\dagger\cdot B^\dagger(\underline{1})\equiv$ 

(d) $\psi\cdot B(\underline{1})\equiv$ 

(e) $\psi^\dagger\cdot A(1)\equiv$ 

(f) $\psi\cdot A^\dagger(1)\equiv$ 

(g) $\psi^\dagger\cdot A(\underline{1})\equiv$ 

(h) $\psi\cdot A^\dagger(\underline{1})\equiv$ 

(i) $\psi^\dagger\cdot A^\dagger(\underline{1})\wedge B(\underline{1})\equiv$ 

(j) $\psi\cdot B^\dagger(\underline{1})\wedge A(\underline{1})\equiv$ 

(k) $\psi^\dagger\cdot A^\dagger(1)\wedge B(1)\equiv$ 

(l) $\psi\cdot B^\dagger(1)\wedge A(1)\equiv$ 

$$\psi^\dagger_\alpha(r)\, U^\alpha{}_\beta(r)\, \psi^\beta(r+1) = \Big[\psi^\dagger_\alpha\, B^{\dagger\alpha}(1)\; \eta(1)\Big]_r \Big[\eta(\underline{1})\; \psi^\beta\; A^\dagger_\beta(\underline{1})\Big]_{r+1} + \Big[\psi^\dagger_\alpha\, A^\alpha(1)\; \theta(1)\Big]_r \Big[\theta(\underline{1})\; \psi^\beta\; B_\beta(\underline{1})\Big]_{r+1}$$
$$+ \Big[\psi^\dagger_\alpha\, (A^\dagger(1) \wedge B(1))^\alpha\; \delta(1)\Big]_r \Big[\delta(\underline{1})\; \psi^\beta\; (B^\dagger(\underline{1}) \wedge A(\underline{1}))_\beta\Big]_{r+1}$$

acting on $|n_P, n_Q\,;\, \nu_{\underline{1}}, \nu_0, \nu_1\rangle_r$

$$|n_P, n_Q\,;\, \nu_{\underline{1}}, \nu_0, \nu_1\rangle = (\hat\Gamma^\dagger_P)^{n_P} (\hat\Gamma^\dagger_Q)^{n_Q} (\hat\chi^\dagger_{\underline{1}})^{\nu_{\underline{1}}} (\hat\chi^\dagger_0)^{\nu_0} (\hat\chi^\dagger_1)^{\nu_1}\, |0,0\,;\,0,0,0\rangle$$

$$\hat\Gamma_P(r) = \sum_{n_P=1}^{\infty} \sum_{n_Q, \nu_{\underline{1}}, \nu_0, \nu_1} |n_P - 1, n_Q\,;\, \nu_{\underline{1}}, \nu_0, \nu_1\rangle\, \langle n_P, n_Q\,;\, \nu_{\underline{1}}, \nu_0, \nu_1|_r$$

$$\hat\Gamma_Q(r) = \sum_{n_Q=1}^{\infty} \sum_{n_P, \nu_{\underline{1}}, \nu_0, \nu_1} |n_P, n_Q - 1\,;\, \nu_{\underline{1}}, \nu_0, \nu_1\rangle\, \langle n_P, n_Q\,;\, \nu_{\underline{1}}, \nu_0, \nu_1|_r$$

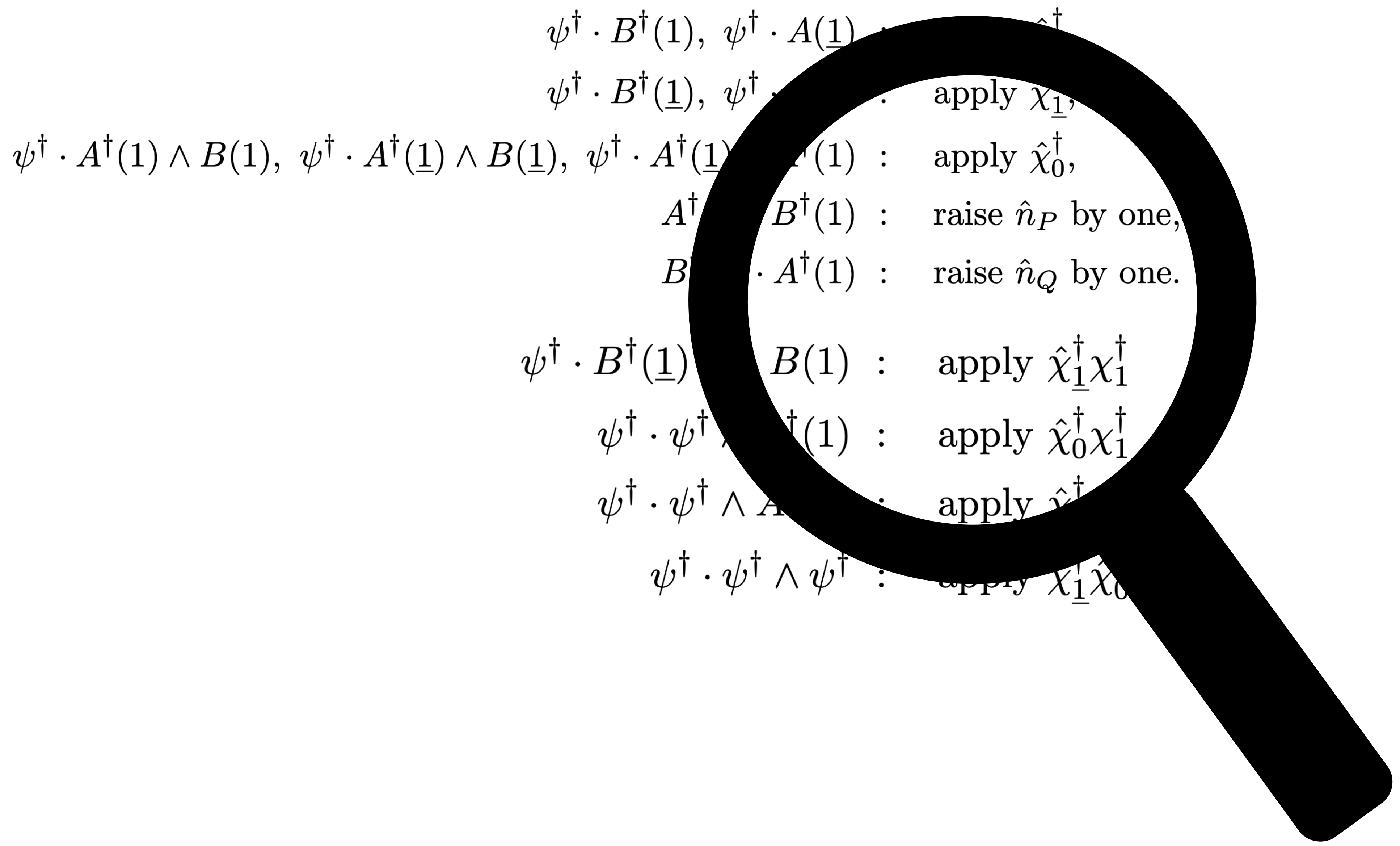
$$\hat\chi_{\underline{1}}(r) = \sum_{n_P, n_Q, \nu_0, \nu_1} |n_P, n_Q\,;\, 0, \nu_0, \nu_1\rangle\, \langle n_P, n_Q\,;\, 1, \nu_0, \nu_1|_r$$

$$\hat\chi_0(r) = \sum_{n_P, n_Q, \nu_{\underline{1}}, \nu_1} |n_P, n_Q\,;\, \nu_{\underline{1}}, 0, \nu_1\rangle\, \langle n_P, n_Q\,;\, \nu_{\underline{1}}, 1, \nu_1|_r\, (-1)^{\nu_{\underline{1}}}$$

$$\hat\chi_1(r) = \sum_{n_P, n_Q, \nu_{\underline{1}}, \nu_0} |n_P, n_Q\,;\, \nu_{\underline{1}}, \nu_0, 0\rangle\, \langle n_P, n_Q\,;\, \nu_{\underline{1}}, \nu_0, 1|_r\, (-1)^{\nu_{\underline{1}} + \nu_0}$$

$$\begin{aligned}
\psi^\dagger \cdot B^\dagger(1),\ \psi^\dagger \cdot A(\underline{1}) &: \quad \text{apply } \hat\chi^\dagger_1, \\
\psi^\dagger \cdot B^\dagger(\underline{1}),\ \psi^\dagger \cdot A(1) &: \quad \text{apply } \hat\chi^\dagger_{\underline{1}}, \\
\psi^\dagger \cdot A^\dagger(1) \wedge B(1),\ \psi^\dagger \cdot A^\dagger(\underline{1}) \wedge B(\underline{1}),\ \psi^\dagger \cdot A^\dagger(\underline{1}) \wedge A^\dagger(1) &: \quad \text{apply } \hat\chi^\dagger_0, \\
A^\dagger(\underline{1}) \cdot B^\dagger(1) &: \quad \text{raise } \hat n_P \text{ by one}, \\
B^\dagger(\underline{1}) \cdot A^\dagger(1) &: \quad \text{raise } \hat n_Q \text{ by one}. \\
\psi^\dagger \cdot B^\dagger(\underline{1})\, \psi^\dagger \cdot B(1) &: \quad \text{apply } \hat\chi^\dagger_{\underline{1}} \chi^\dagger_1 \\
\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(1) &: \quad \text{apply } \hat\chi^\dagger_0 \chi^\dagger_1 \\
\psi^\dagger \cdot \psi^\dagger \wedge A^\dagger(\underline{1}) &: \quad \text{apply } \hat\chi^\dagger_{\underline{1}} \chi^\dagger_0 \\
\psi^\dagger \cdot \psi^\dagger \wedge \psi^\dagger &: \quad \text{apply } \hat\chi^\dagger_{\underline{1}} \hat\chi^\dagger_0 \chi^\dagger_1
\end{aligned}$$

$$
\begin{aligned}
\psi^\dagger\cdot B^\dagger(1) &\mapsto \hat\chi_1^\dagger(\hat\Gamma_P^\dagger)^{\hat\nu_0}\sqrt{\hat n_P+2-\hat\nu_0}\sqrt{\frac{\hat n_P+\hat n_Q+3+\hat\nu_0}{\hat n_P+\hat n_Q+2+\hat\nu_{\underline{1}}+\hat\nu_0}}\\
\psi\cdot B(1) &\mapsto \hat\chi_1(\hat\Gamma_P)^{\hat\nu_0}\sqrt{\hat n_P+2(1-\hat\nu_0)}\sqrt{\frac{\hat n_P+\hat n_Q+3}{\hat n_P+\hat n_Q+2+\hat\nu_{\underline{1}}}}\\
\psi^\dagger\cdot B^\dagger(\underline{1}) &\mapsto \hat\chi_{\underline{1}}^\dagger(\hat\Gamma_Q^\dagger)^{\hat\nu_0}\sqrt{\hat n_Q+2-\hat\nu_0}\sqrt{\frac{\hat n_P+\hat n_Q+3+\hat\nu_0}{\hat n_P+\hat n_Q+2+\hat\nu_1+\hat\nu_0}}\\
\psi\cdot B(\underline{1}) &\mapsto \hat\chi_{\underline{1}}(\hat\Gamma_Q)^{\hat\nu_0}\sqrt{\hat n_Q+2(1-\hat\nu_0)}\sqrt{\frac{\hat n_P+\hat n_Q+3}{\hat n_P+\hat n_Q+2+\hat\nu_1}}\\
\psi^\dagger\cdot A(1) &\mapsto \hat\chi_{\underline{1}}^\dagger(\hat\Gamma_Q)^{1-\hat\nu_0}\sqrt{\hat n_Q+2\,\hat\nu_0}\sqrt{\frac{\hat n_P+\hat n_Q+2+\hat\nu_1}{\hat n_P+\hat n_Q+2}}\\
\psi\cdot A^\dagger(1) &\mapsto \hat\chi_{\underline{1}}(\hat\Gamma_Q^\dagger)^{1-\hat\nu_0}\sqrt{\hat n_Q+1+\hat\nu_0}\sqrt{\frac{\hat n_P+\hat n_Q+3-\hat\nu_0+\hat\nu_1}{\hat n_P+\hat n_Q+3-\hat\nu_0}}\\
\psi^\dagger\cdot A(\underline{1}) &\mapsto \hat\chi_1^\dagger(\hat\Gamma_P)^{1-\hat\nu_0}\sqrt{\hat n_P+2\,\hat\nu_0}\sqrt{\frac{\hat n_P+\hat n_Q+2+\hat\nu_{\underline{1}}}{\hat n_P+\hat n_Q+2}}\\
\psi\cdot A^\dagger(\underline{1}) &\mapsto \hat\chi_1(\hat\Gamma_P^\dagger)^{1-\hat\nu_0}\sqrt{\hat n_P+1+\hat\nu_0}\sqrt{\frac{\hat n_P+\hat n_Q+3-\hat\nu_0+\hat\nu_{\underline{1}}}{\hat n_P+\hat n_Q+3-\hat\nu_0}}\\
\psi^\dagger\cdot A^\dagger(1)\wedge B(1) &\mapsto -\hat\chi_0^\dagger(\hat\Gamma_P)^{1-\hat\nu_1}(\hat\Gamma_Q^\dagger)^{\hat\nu_{\underline{1}}}\sqrt{\hat n_P+2\,\hat\nu_1}\sqrt{\hat n_Q+2-\hat\nu_{\underline{1}}}\\
\psi\cdot B^\dagger(1)\wedge A(1) &\mapsto \hat\chi_0(\hat\Gamma_P^\dagger)^{1-\hat\nu_1}(\hat\Gamma_Q)^{\hat\nu_{\underline{1}}}\sqrt{\hat n_P+1+\hat\nu_1}\sqrt{\hat n_Q+2(1-\hat\nu_{\underline{1}})}\\
\psi^\dagger\cdot A^\dagger(\underline{1})\wedge B(\underline{1}) &\mapsto \hat\chi_0^\dagger(\hat\Gamma_P^\dagger)^{\hat\nu_1}(\hat\Gamma_Q)^{1-\hat\nu_{\underline{1}}}\sqrt{\hat n_P+2-\hat\nu_1}\sqrt{\hat n_Q+2\,\hat\nu_{\underline{1}}}\\
\psi\cdot B^\dagger(\underline{1})\wedge A(\underline{1}) &\mapsto -\hat\chi_0(\hat\Gamma_P)^{\hat\nu_1}(\hat\Gamma_Q^\dagger)^{1-\hat\nu_{\underline{1}}}\sqrt{\hat n_P+2(1-\hat\nu_1)}\sqrt{\hat n_Q+1+\hat\nu_{\underline{1}}}
\end{aligned}
$$



Diagonal functions are in terms of

$$
\begin{aligned}
\hat n_l\,|n_P,n_Q;\nu_{\underline{1}},\nu_0,\nu_1\rangle &= n_l\,|n_P,n_Q;\nu_{\underline{1}},\nu_0,\nu_1\rangle\,,\\
\hat\nu_f\,|n_P,n_Q;\nu_{\underline{1}},\nu_0,\nu_1\rangle &= \nu_f\,|n_P,n_Q;\nu_{\underline{1}},\nu_0,\nu_1\rangle\,.
\end{aligned}
$$

$$l\in\{P,Q\}$$

$$f\in\{\underline{1},0,1\}$$

Towards building the LSH Hamiltonian

KS Hamiltonian in LSH basis:

re-written in terms of LSH operators

$$H = H_M + H_E + H_I$$

$$H_M = \sum_{r=1}^{N} H_M(r) \equiv \mu \sum_{r=1}^{N} (-1)^r (\hat{\nu}_{\underline{1}}(r) + \hat{\nu}_0(r) + \hat{\nu}_1(r))$$

$$H_E = \sum_{r=1}^{N'} H_E(r) \equiv \sum_{r=1}^{N'} \frac{1}{3} \left(\hat{P}(1,r)^2 + \hat{Q}(1,r)^2 + \hat{P}(1,r)\hat{Q}(1,r) \right) + \hat{P}(1,r) + \hat{Q}(1,r)$$

$$H_I = \sum_{r=1}^{N'} H_I(r) \equiv \sum_r x \left[\hat{\chi}_1^\dagger (\hat{\Gamma}_P^\dagger)^{\hat{\nu}_0} \sqrt{1 - \hat{\nu}_0/(\hat{n}_P + 2)} \sqrt{1 - \hat{\nu}_{\underline{1}}/(\hat{n}_P + \hat{n}_Q + 3)} \right]_r \otimes \left[\sqrt{1 + \hat{\nu}_0/(\hat{n}_P + 1)} \sqrt{1 + \hat{\nu}_{\underline{1}}/(\hat{n}_P + \hat{n}_Q + 2)}\, \hat{\chi}_1 (\hat{\Gamma}_P^\dagger)^{1-\hat{\nu}_0} \right]_{r+1}$$

$$+ x \left[\hat{\chi}_{\underline{1}}^\dagger (\hat{\Gamma}_Q)^{1-\hat{\nu}_0} \sqrt{1 + \hat{\nu}_0/(\hat{n}_Q + 1)} \sqrt{1 + \hat{\nu}_1/(\hat{n}_P + \hat{n}_Q + 2)} \right]_r \otimes \left[\sqrt{1 - \hat{\nu}_0/(\hat{n}_Q + 2)} \sqrt{1 - \hat{\nu}_1/(\hat{n}_P + \hat{n}_Q + 3)}\, \hat{\chi}_{\underline{1}} (\hat{\Gamma}_Q)^{\hat{\nu}_0} \right]_{r+1}$$

$$+ x \left[\hat{\chi}_0^\dagger (\hat{\Gamma}_P)^{1-\hat{\nu}_1} (\hat{\Gamma}_Q^\dagger)^{\hat{\nu}_{\underline{1}}} \sqrt{1 + \hat{\nu}_1/(\hat{n}_P + 1)} \sqrt{1 - \hat{\nu}_{\underline{1}}/(\hat{n}_Q + 2)} \right]_r \otimes \left[\sqrt{1 - \hat{\nu}_1/(\hat{n}_P + 2)} \sqrt{1 + \hat{\nu}_{\underline{1}}/(\hat{n}_Q + 1)}\, \hat{\chi}_0 (\hat{\Gamma}_P)^{\hat{\nu}_1} (\hat{\Gamma}_Q^\dagger)^{1-\hat{\nu}_{\underline{1}}} \right]_{r+1}$$

Structurally identical to the SU(2) LSH construction

Towards building the LSH Hamiltonian

$$H = H_M + H_E + H_I$$

KS Hamiltonian in LSH basis:

re-written in terms of LSH operators

Numerically benchmarked with completely gauge fixed (pure gauge) Hamiltonian

$$H_M = \sum_{r=1}^{N} H_M(r) \equiv \mu \sum_{r=1}^{N} (-1)^r (\hat{\nu}_{\underline{1}}(r) + \hat{\nu}_0(r) + \hat{\nu}_1(r))$$

$$H_E = \sum_{r=1}^{N'} H_E(r) \equiv \sum_{r=1}^{N'} \frac{1}{3} \left(\hat{P}(1,r)^2 + \hat{Q}(1,r)^2 + \hat{P}(1,r)\hat{Q}(1,r) \right) + \hat{P}(1,r) + \hat{Q}(1,r)$$

$$\begin{aligned} H_I = \sum_{r=1}^{N'} H_I(r) \equiv \sum_r x & \left[\hat{\chi}_1^\dagger (\hat{\Gamma}_P^\dagger)^{\hat{\nu}_0} \sqrt{1 - \hat{\nu}_0/(\hat{n}_P + 2)} \sqrt{1 - \hat{\nu}_{\underline{1}}/(\hat{n}_P + \hat{n}_Q + 3)} \right]_r \otimes \left[\sqrt{1 + \hat{\nu}_0/(\hat{n}_P + 1)} \sqrt{1 + \hat{\nu}_{\underline{1}}/(\hat{n}_P + \hat{n}_Q + 2)} \, \hat{\chi}_1 (\hat{\Gamma}_P^\dagger)^{1-\hat{\nu}_0} \right]_{r+1} \\ + x & \left[\hat{\chi}_{\underline{1}}^\dagger (\hat{\Gamma}_Q)^{1-\hat{\nu}_0} \sqrt{1 + \hat{\nu}_0/(\hat{n}_Q + 1)} \sqrt{1 + \hat{\nu}_1/(\hat{n}_P + \hat{n}_Q + 2)} \right]_r \otimes \left[\sqrt{1 - \hat{\nu}_0/(\hat{n}_Q + 2)} \sqrt{1 - \hat{\nu}_1/(\hat{n}_P + \hat{n}_Q + 3)} \, \hat{\chi}_{\underline{1}} (\hat{\Gamma}_Q)^{\hat{\nu}_0} \right]_{r+1} \\ + x & \left[\hat{\chi}_0^\dagger (\hat{\Gamma}_P)^{1-\hat{\nu}_1} (\hat{\Gamma}_Q^\dagger)^{\hat{\nu}_{\underline{1}}} \sqrt{1 + \hat{\nu}_1/(\hat{n}_P + 1)} \sqrt{1 - \hat{\nu}_{\underline{1}}/(\hat{n}_Q + 2)} \right]_r \otimes \left[\sqrt{1 - \hat{\nu}_1/(\hat{n}_P + 2)} \sqrt{1 + \hat{\nu}_{\underline{1}}/(\hat{n}_Q + 1)} \, \hat{\chi}_0 (\hat{\Gamma}_P)^{\hat{\nu}_1} (\hat{\Gamma}_Q^\dagger)^{1-\hat{\nu}_{\underline{1}}} \right]_{r+1} \end{aligned}$$

Structurally identical to the SU(2) LSH construction

The LSH Hamiltonian | Numerically benchmarked with completely gauge fixed (pure gauge) Hamiltonian

Fermionic Hamiltonian with long range interaction

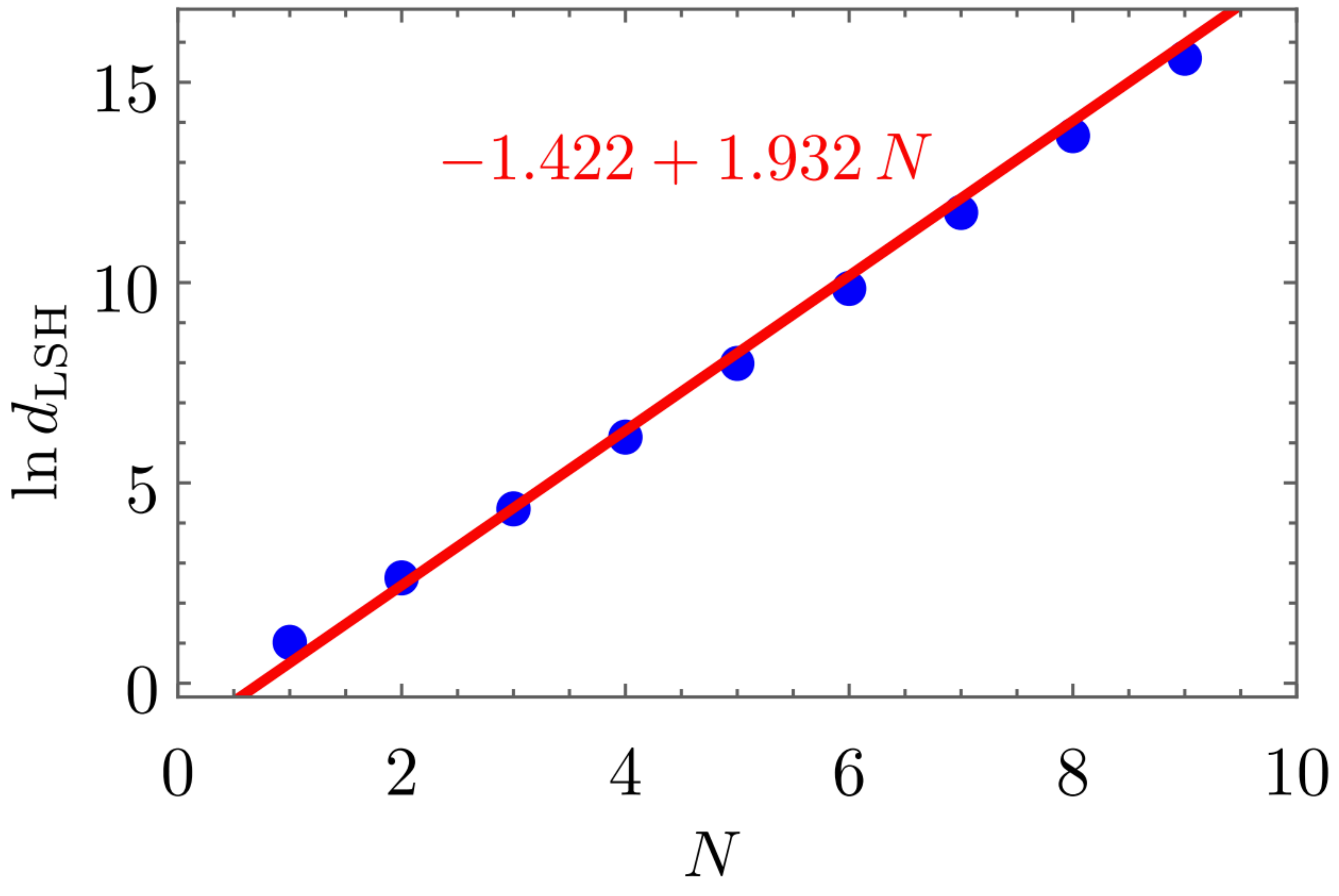
$$\psi(r) \to \psi'(r) = \left[\prod_{y<r} U(y)\right] \psi(r) \qquad \psi^\dagger(r) \to \psi^{\dagger\prime}(r) = \psi^\dagger(r) \left[\prod_{y<r} U(y)\right]^\dagger \qquad U(r) \to U'(r) = \left[\prod_{y<r} U(y)\right] U(r) \left[\prod_{z<r+1} U(z)\right]^\dagger$$

$$|\Psi\rangle^{(F)} = \prod_{x=0}^{N-1} |f_1, f_2, f_3\rangle_{(x)}$$

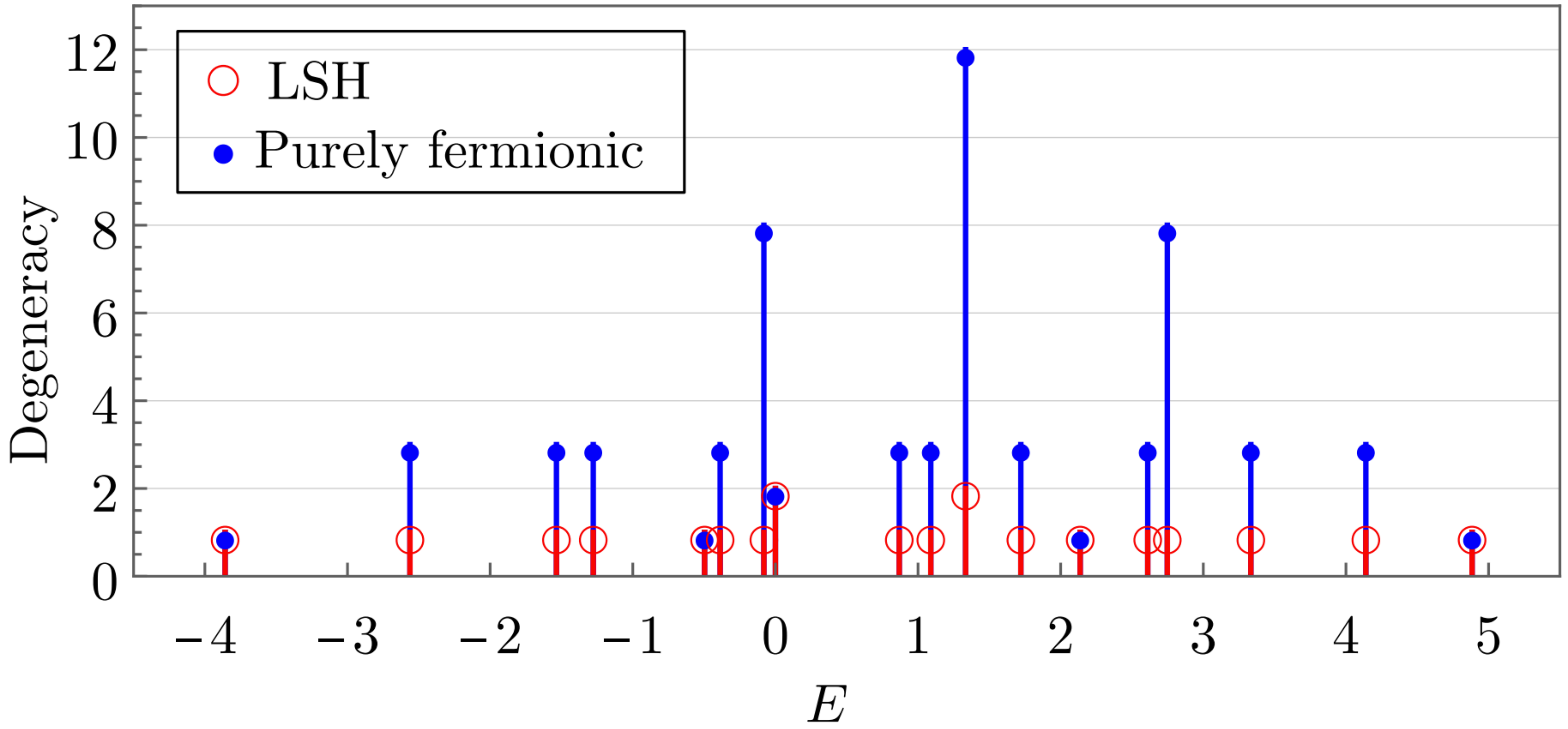
Contains degeneracy

Due to global symmetries

$$d(\mathcal{P}_f, \mathcal{Q}_f) = \frac{1}{2}(\mathcal{P}_f + 1)(\mathcal{Q}_f + 1)(\mathcal{P}_f + \mathcal{Q}_f + 2)$$



(a)



(b)

The LSH Hamiltonian

Global symmetries

$$H_I = \sum_{r=1}^{N'} H_I(r) \equiv \sum_r x \left[\hat{\chi}_1^\dagger (\hat{\Gamma}_P^\dagger)^{\hat{\nu}_0} \sqrt{1-\hat{\nu}_0/(\hat{n}_P+2)}\sqrt{1-\hat{\nu}_{\underline{1}}/(\hat{n}_P+\hat{n}_Q+3)}\,\right]_r \otimes \left[\sqrt{1+\hat{\nu}_0/(\hat{n}_P+1)}\sqrt{1+\hat{\nu}_{\underline{1}}/(\hat{n}_P+\hat{n}_Q+2)}\,\hat{\chi}_1 (\hat{\Gamma}_P^\dagger)^{1-\hat{\nu}_0}\right]_{r+1}$$

$$+\, x \left[\hat{\chi}_{\underline{1}}^\dagger (\hat{\Gamma}_Q)^{1-\hat{\nu}_0} \sqrt{1+\hat{\nu}_0/(\hat{n}_Q+1)}\sqrt{1+\hat{\nu}_1/(\hat{n}_P+\hat{n}_Q+2)}\,\right]_r \otimes \left[\sqrt{1-\hat{\nu}_0/(\hat{n}_Q+2)}\sqrt{1-\hat{\nu}_1/(\hat{n}_P+\hat{n}_Q+3)}\,\hat{\chi}_{\underline{1}} (\hat{\Gamma}_Q)^{\hat{\nu}_0}\right]_{r+1}$$

$$+\, x \left[\hat{\chi}_0^\dagger (\hat{\Gamma}_P)^{1-\hat{\nu}_1} (\hat{\Gamma}_Q^\dagger)^{\hat{\nu}_{\underline{1}}} \sqrt{1+\hat{\nu}_1/(\hat{n}_P+1)}\sqrt{1-\hat{\nu}_{\underline{1}}/(\hat{n}_Q+2)}\,\right]_r \otimes \left[\sqrt{1-\hat{\nu}_1/(\hat{n}_P+2)}\sqrt{1+\hat{\nu}_{\underline{1}}/(\hat{n}_Q+1)}\,\hat{\chi}_0 (\hat{\Gamma}_P)^{\hat{\nu}_1} (\hat{\Gamma}_Q^\dagger)^{1-\hat{\nu}_{\underline{1}}}\right]_{r+1}$$

Globally conserved charges

$$\sum_{r=1}^{N} \nu_{\underline{1}}(r)\,,\quad \sum_{r=1}^{N} \nu_0(r)\,,\quad \sum_{r=1}^{N} \nu_1(r)$$

Or,

$$\mathcal{F} = \sum_{r=1}^{N} \left(\nu_{\underline{1}}(r) + \nu_0(r) + \nu_1(r)\right)$$

$$\Delta\mathcal{P} = \sum_{r=1}^{N} \left(\nu_1(r) - \nu_0(r)\right),$$

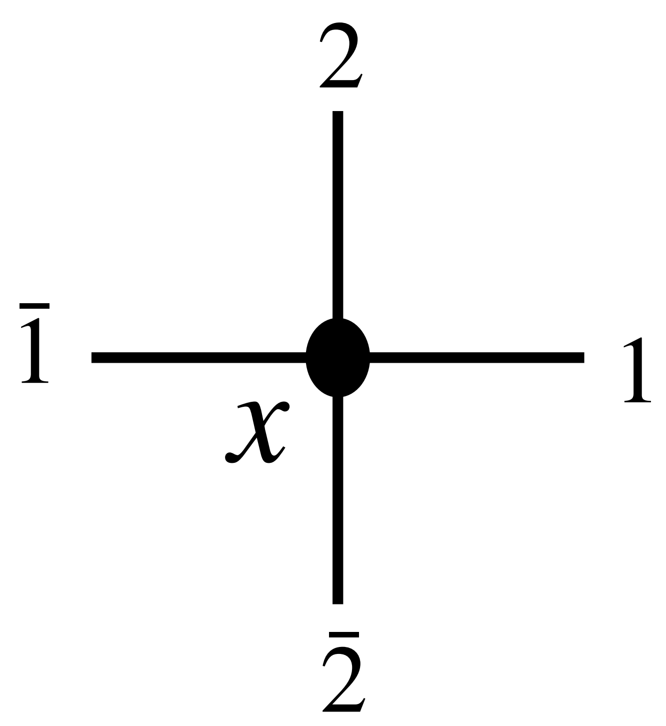
$$\Delta\mathcal{Q} = \sum_{r=1}^{N} \left(\nu_0(r) - \nu_{\underline{1}}(r)\right),$$

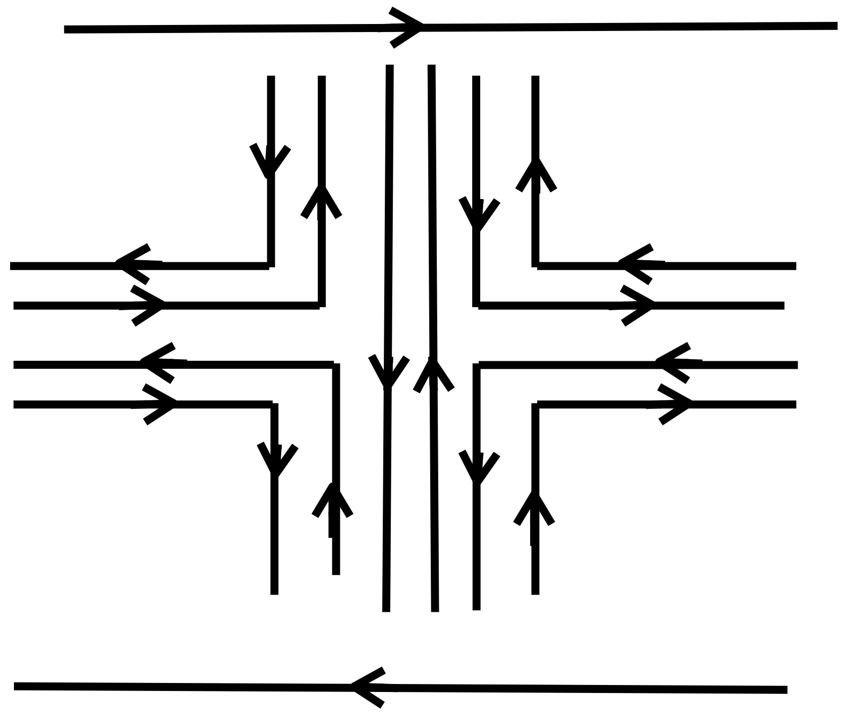
Degeneracy in fermionic formulation :

$$d(\mathcal{P}_f, \mathcal{Q}_f) = \frac{1}{2}(\mathcal{P}_f+1)(\mathcal{Q}_f+1)(\mathcal{P}_f+\mathcal{Q}_f+2)$$

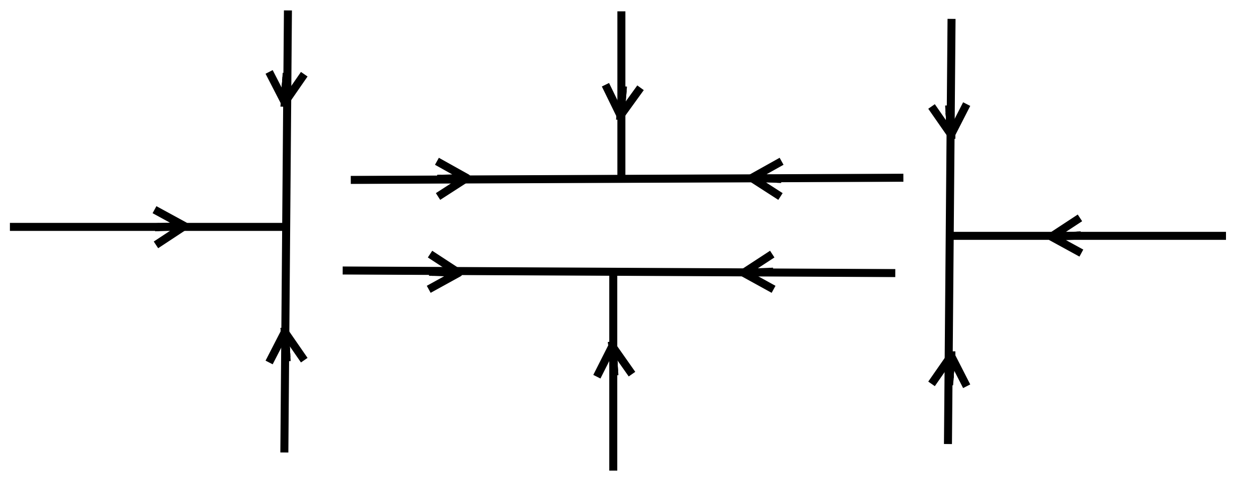
$$(\mathcal{P}_f, \mathcal{Q}_f) = (\mathcal{P}_0 + \Delta\mathcal{P}, \mathcal{Q}_0 + \Delta\mathcal{Q})$$

LSH Formulation: SU(3) in 2+1 dimension and beyond





(a)



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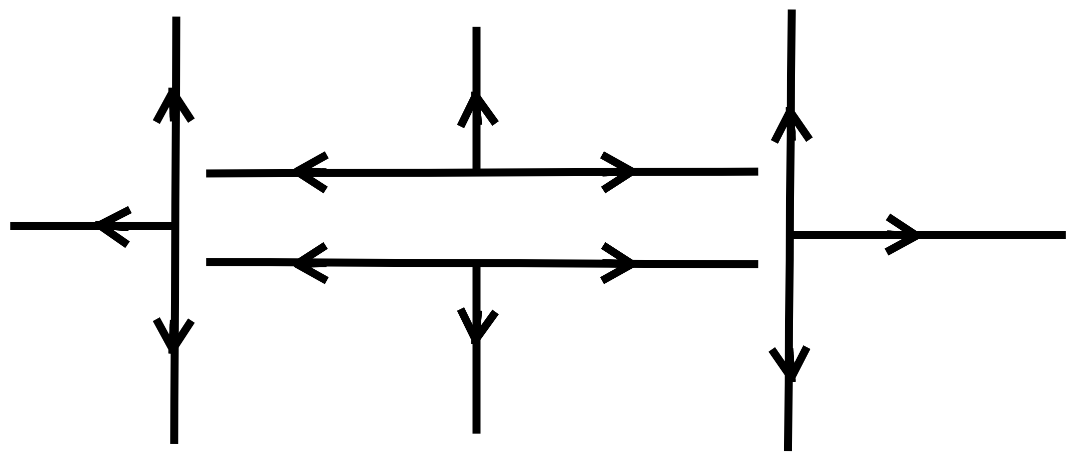
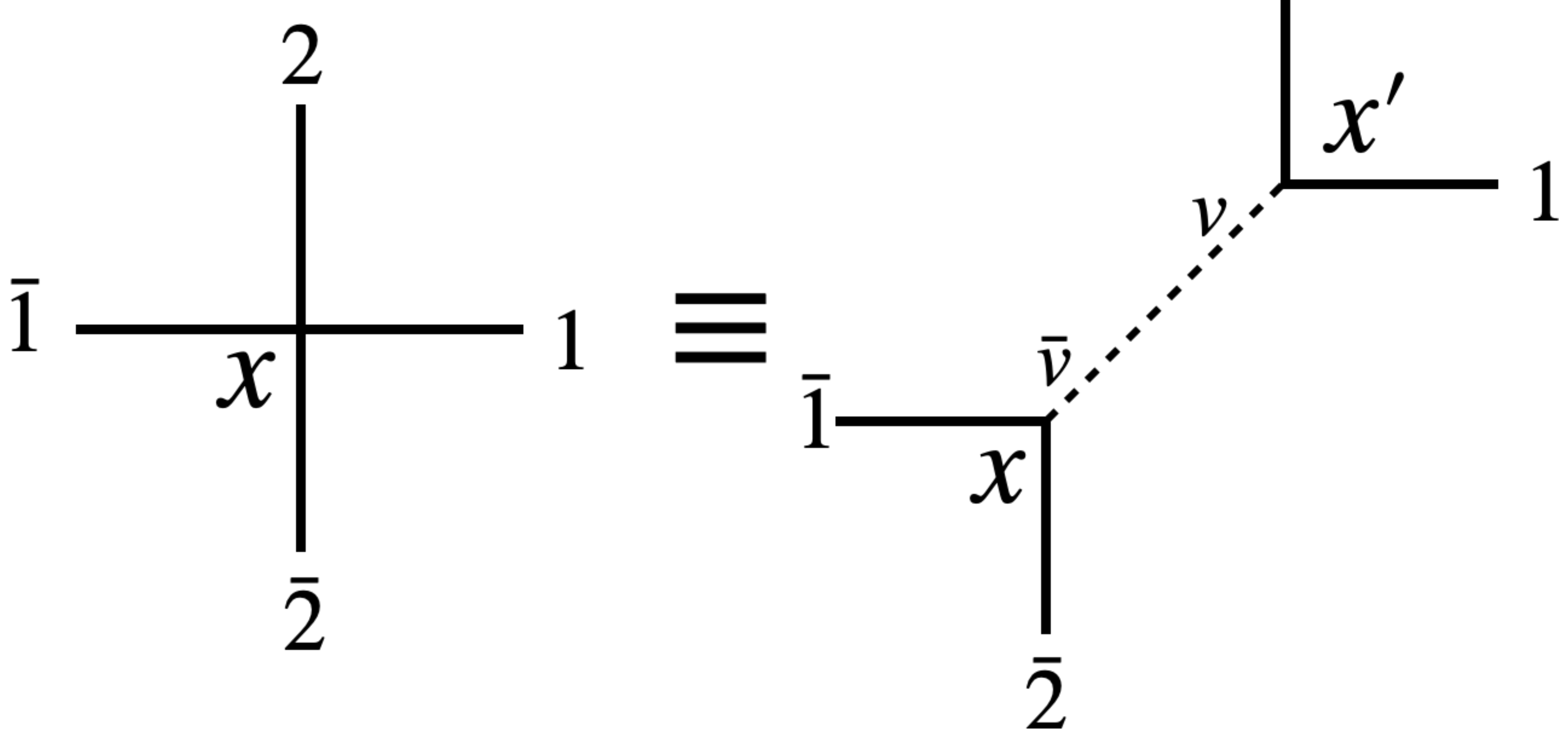


(c)

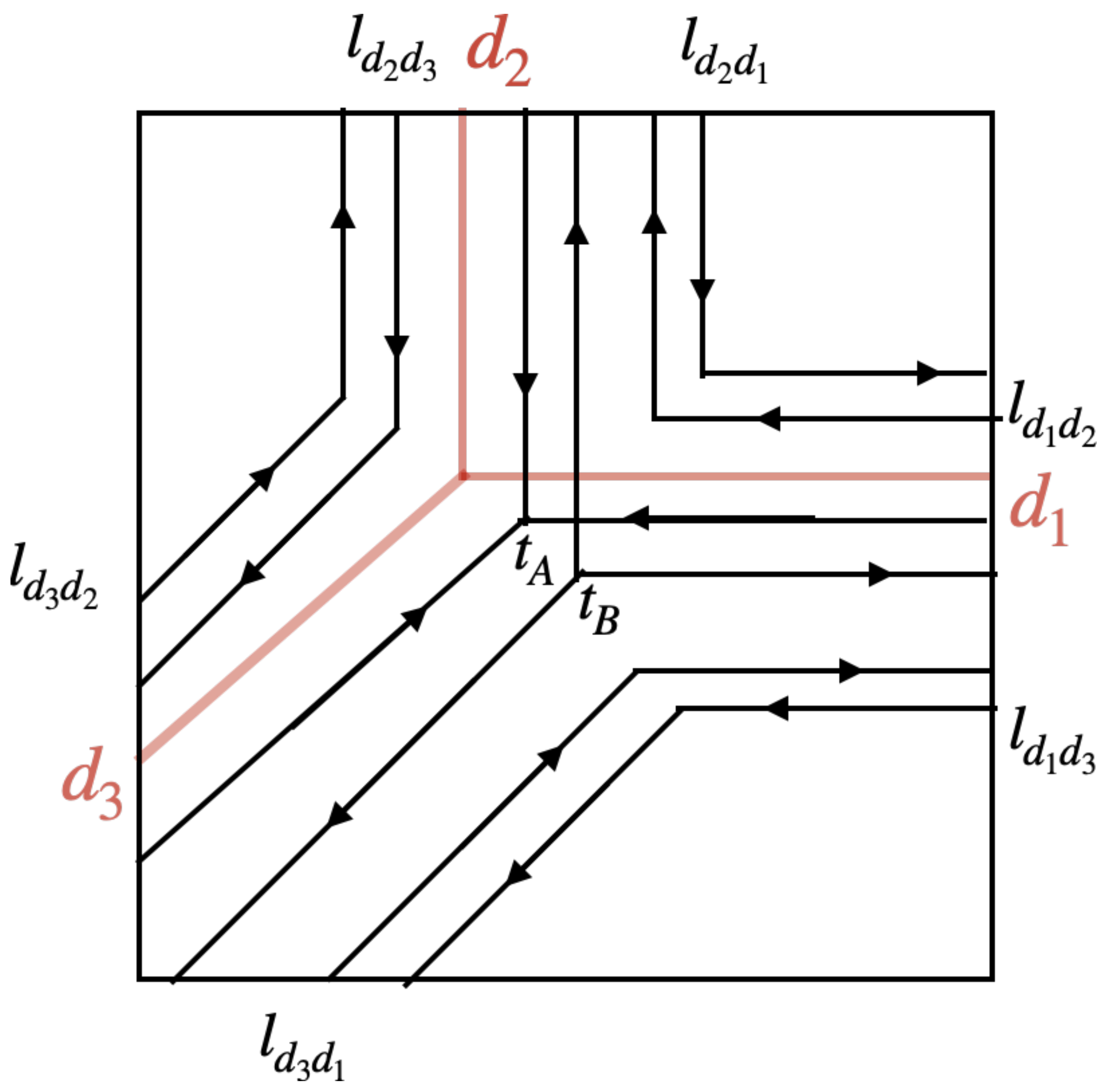
Loop configurations for 2d lattice

Too many loop degrees of freedom

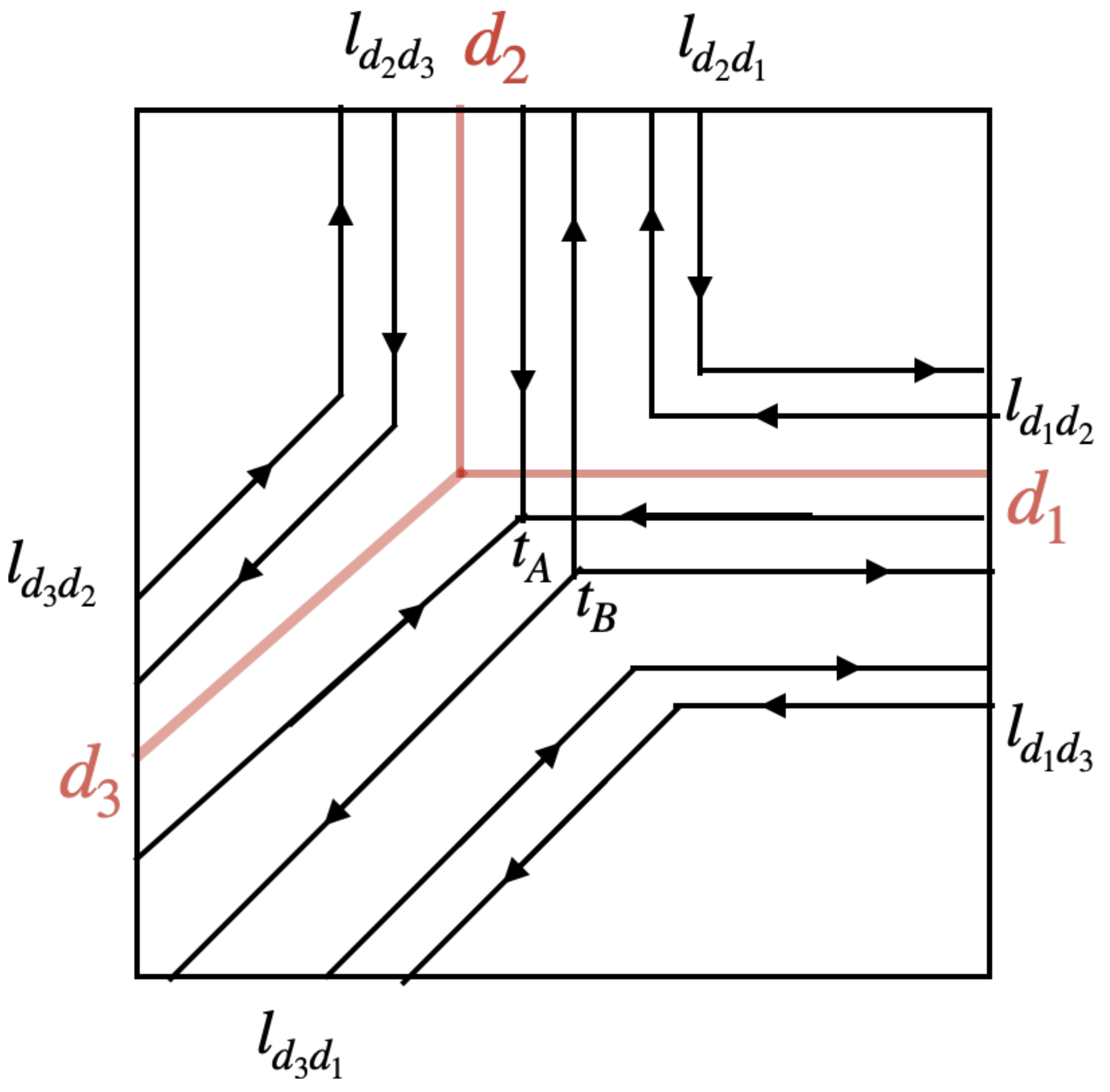
Perform point splitting



Loops on triangular lattice



LSH Formulation: SU(3) in 2+1 dimension and beyond



$$\hat{L}_{IJ} = \sum_{\alpha} A^{\dagger}_{\alpha}[I]B^{\dagger\alpha}[J] \text{ for} I \neq J$$

$$\hat{T}^{A}_{IJK} = \sum_{\alpha,\beta,\gamma} \epsilon^{\alpha\beta\gamma}A^{\dagger}_{\alpha}[I]A^{\dagger}_{\beta}[J]A^{\dagger}_{\gamma}[K]$$

$$\hat{T}^{B}_{IJK} = \sum_{\alpha,\beta,\gamma} \epsilon_{\alpha\beta\gamma}B^{\dagger\alpha}[I]B^{\dagger\beta}[J]B^{\dagger\gamma}[K]$$

Loops on triangular lattice : redundant loop d.o.f. still exist

Mandelstam constraint: $\hat{T}^A\hat{T}^B = \hat{L}_{12}\hat{L}_{2v}\hat{L}_{v1} + \hat{\hat{L}}_{12}\hat{\hat{L}}_{2v}\hat{\hat{L}}_v$

$$|l_{12}, l_{2v}, l_{v1}, l_{21}, l_{v2}, l_{1v}\, ;\, t_A, t_B\rangle$$
$$= \left(\hat{L}_{12}\right)^{l_{12}}\left(\hat{L}_{2v}\right)^{l_{2v}}\left(\hat{L}_{v1}\right)^{l_{v1}}$$
$$\left(\hat{\hat{L}}_{12}\right)^{l_{21}}\left(\hat{\hat{L}}_{2v}\right)^{l_{v2}}\left(\hat{\hat{L}}_{v1}\right)^{l_{1v}}$$
$$\left(\hat{T}^A\right)^{t_A}\left(\hat{T}^B\right)^{t_B}|0,0,0,0,0,0\, ;\, 0,0\rangle$$

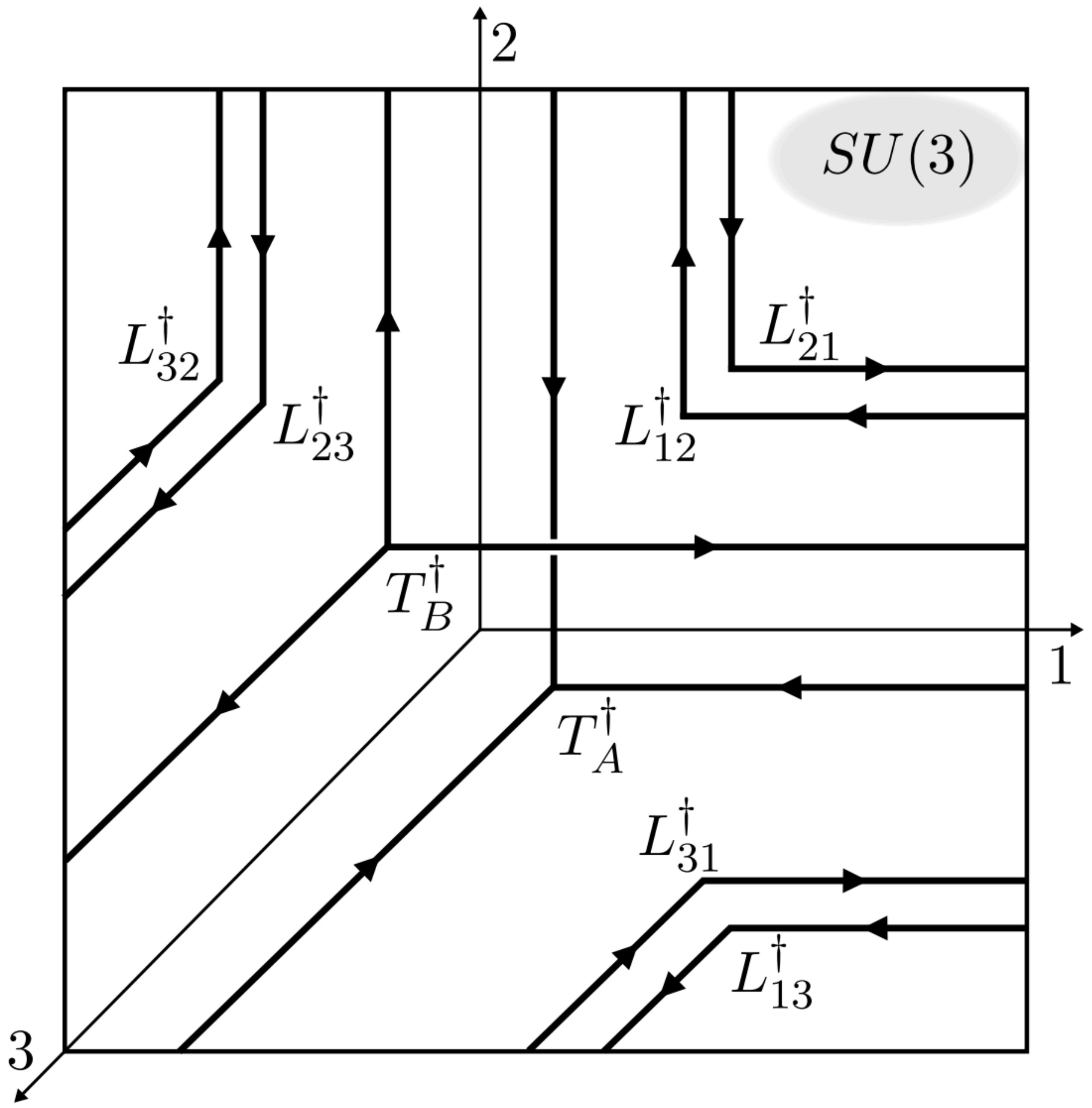
Occupation number basis: insufficient

$$p_1 = l_{12} + l_{1v} + t_A \quad , \quad q_1 = l_{21} + l_{v1} + t_B$$
$$p_2 = l_{21} + l_{2v} + t_A \quad , \quad q_2 = l_{12} + l_{v2} + t_B$$
$$p_v = l_{v1} + l_{v2} + t_A \quad , \quad q_v = l_{1v} + l_{2v} + t_B$$

$$\boxed{p_1 + p_2 + p_v = q_1 + q_2 + q_v + 3(t_A - t_B)}$$

LSH Formulation: SU(3) beyond 1+1 dimensions

First focus: trivalent vertex



$$L^\dagger_{12} = A^\dagger_\alpha(1)B^{\dagger\alpha}(2)$$
$$L^\dagger_{23} = A^\dagger_\alpha(2)B^{\dagger\alpha}(3)$$
$$L^\dagger_{31} = A^\dagger_\alpha(3)B^{\dagger\alpha}(1)$$
$$L^\dagger_{21} = A^\dagger_\alpha(2)B^{\dagger\alpha}(1)$$
$$L^\dagger_{32} = A^\dagger_\alpha(3)B^{\dagger\alpha}(2)$$
$$L^\dagger_{13} = A^\dagger_\alpha(1)B^{\dagger\alpha}(3)$$
$$T^\dagger_B = \epsilon_{\alpha\beta\gamma}B^{\dagger\alpha}(1)B^{\dagger\beta}(2)B^{\dagger\gamma}(3)$$
$$T^\dagger_A = \epsilon^{\alpha\beta\gamma}A^\dagger_\alpha(1)A^\dagger_\beta(2)A^\dagger_\gamma(3)$$

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IQuS@UW-21-086

Loop-string-hadron approach to SU(3) lattice Yang-Mills theory: Gauge invariant Hilbert space of a trivalent vertex

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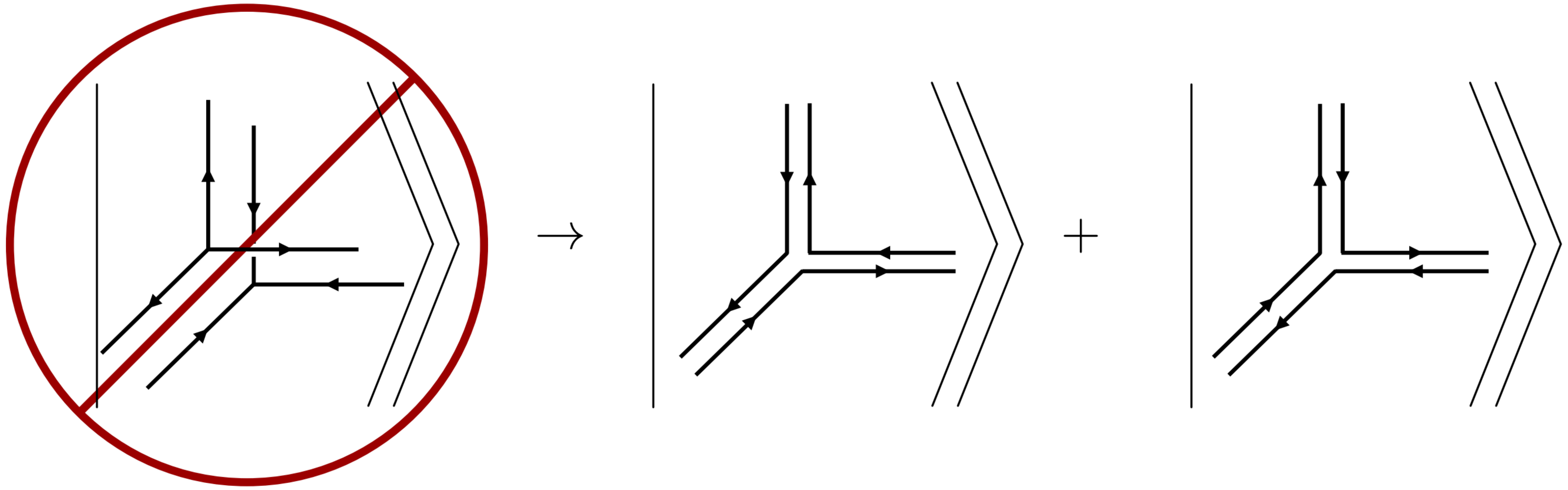
Remnant Mandelstam Constraint:

$$T^\dagger_A T^\dagger_B \simeq L^\dagger_{12}L^\dagger_{23}L^\dagger_{31} + L^\dagger_{21}L^\dagger_{32}L^\dagger_{13}$$

Choose a naive basis:



$$|\ell_{12}\,\ell_{23}\,\ell_{31};\ell_{21}\,\ell_{32}\,\ell_{13};t\rangle\rangle \equiv L^{\dagger\,\ell_{12}}_{12}L^{\dagger\,\ell_{23}}_{23}L^{\dagger\,\ell_{31}}_{31}L^{\dagger\,\ell_{21}}_{21}L^{\dagger\,\ell_{32}}_{32}L^{\dagger\,\ell_{13}}_{13} \times \begin{cases} T^{\dagger\,t}_A\,|0\rangle\,, & t \geq 0 \\ T^{\dagger\,-t}_B\,|0\rangle\,, & t < 0 \end{cases}$$

$$\ell_{IJ} \in \{0,\ 1,\ 2,\ 3,\ \ldots\},$$
$$t \in \{0,\ \pm1,\ \pm2,\ \pm3,\ \ldots\}$$

LSH Formulation: SU(3) beyond 1+1 dimensions

Is the naive basis a good choice?

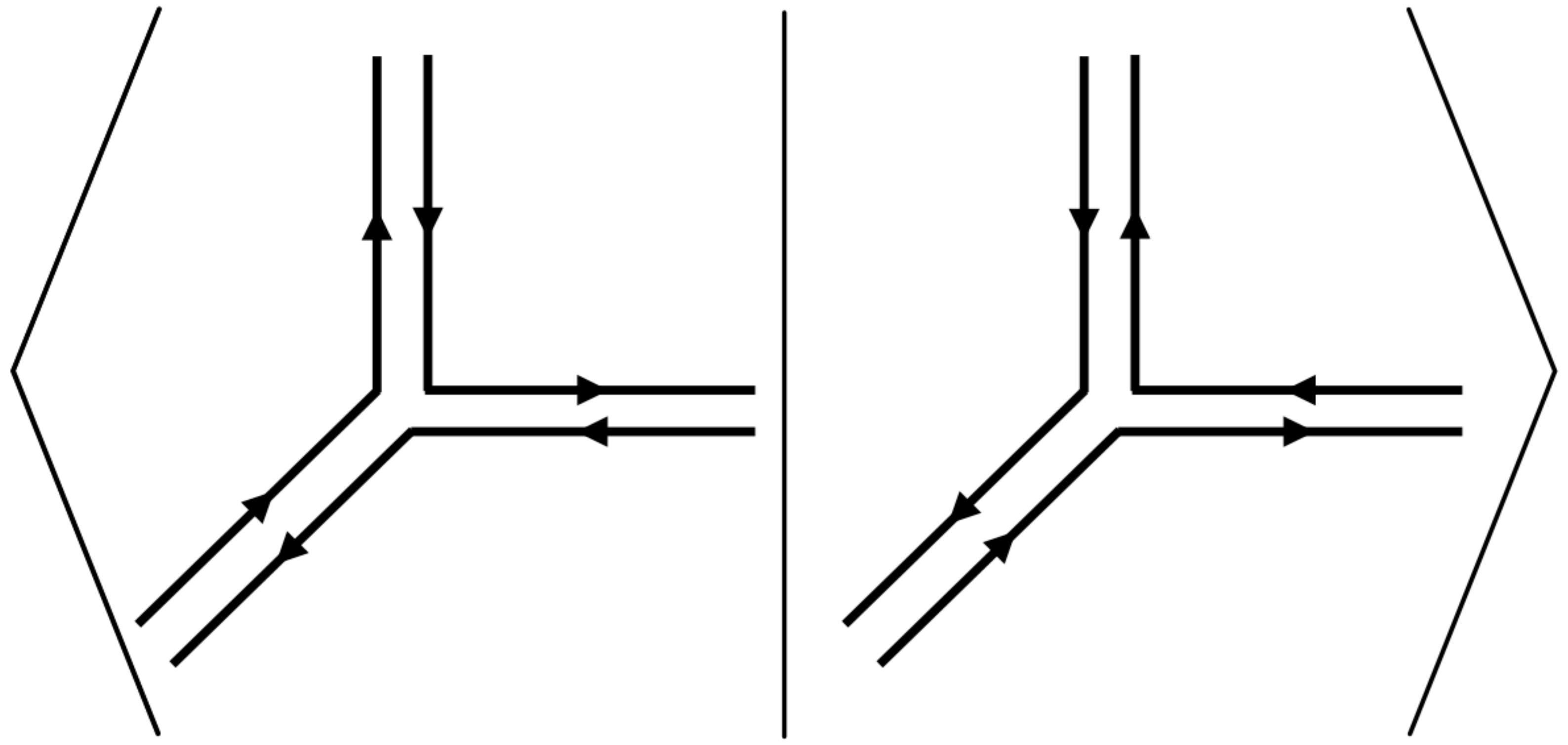
$$|\ell_{12}\,\ell_{23}\,\ell_{31};\ell_{21}\,\ell_{32}\,\ell_{13};t\rangle\rangle \equiv L_{12}^{\dagger\,\ell_{12}} L_{23}^{\dagger\,\ell_{23}} L_{31}^{\dagger\,\ell_{31}} L_{21}^{\dagger\,\ell_{21}} L_{32}^{\dagger\,\ell_{32}} L_{13}^{\dagger\,\ell_{13}} \times \begin{cases} T_A^{\dagger\,t}\,|0\rangle\,, & t \geq 0 \\ T_B^{\dagger\,-t}\,|0\rangle\,, & t < 0 \end{cases}$$

$$\ell_{IJ} \in \{0,\ 1,\ 2,\ 3,\ \ldots\},$$

$$t \in \{0,\ \pm 1,\ \pm 2,\ \pm 3,\ \ldots\}$$

Yes, it solves the Mandelstam constraint

But, not always orthogonal !!!



$$= -\frac{2}{7} \neq 0$$

$$|||$$

$$\langle 0\,0\,0;1\,1\,1;0|1\,1\,1;0\,0\,0;0\rangle$$

Way out: Brute force orthogonalisation - elegance of the framework is lost!

Alternatively: Find the hidden 7th quantum number

As the first attempt: Find orthogonal subspaces

LSH Formulation: SU(3) beyond 1+1 dimensions

- The problem is due to presence of the Littlewood Richardson Coefficients associated with direct product of irres:

$$\lambda \otimes \mu = \bigoplus_{\nu} d^{\nu}_{\lambda,\mu}\, \nu$$

- To find orthonormal states characterised by 7 quantum numbers at each site, in order to match with the physical degrees of freedom.

$$\boxed{|p_1, q_1, p_2, q_2, p_3, q_3, \rho\rangle}$$

- Seventh Casimir Candidate: $C_T \equiv (T_A T_B)^{\dagger} T_A T_B$

- The seventh Casimir is diagonalised in each $(p_1, q_1, p_2, q_2, p_3, q_3)$ sector.

- Examples: $\mathrm{Spec}_{111111}(C_T) = \left\{0, \frac{80}{3}\right\}$ $\qquad \mathrm{Spec}_{222222}(C_T) = \left\{0, \frac{1008}{5}, \frac{45684}{125}\right\}$

$$\mathrm{Spec}_{322322}(C_T) = \left\{0, \frac{7}{900}\left(59609 - \sqrt{383666161}\right), \frac{7}{900}\left(59609 + \sqrt{383666161}\right)\right\}$$

LSH Formulation: SU(3) beyond 1+1 dimensions

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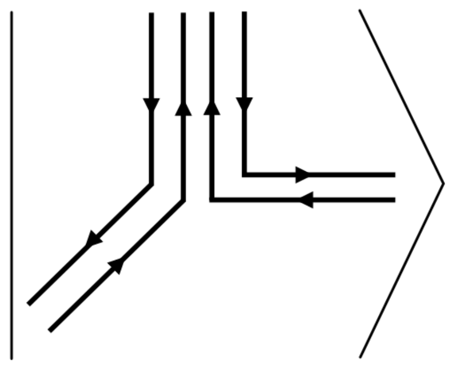
LSH Formulation: SU(3) beyond 1+1 dimensions

As the first attempt: Find orthogonal subspaces

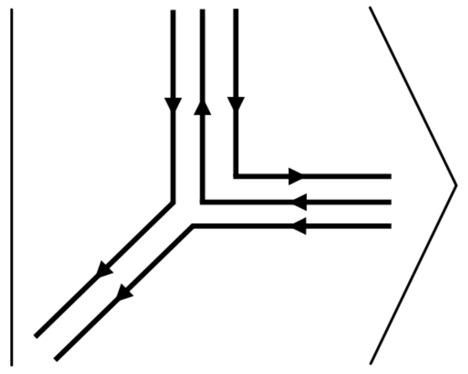
Has been normalised.



$= |\ell_{12} = \ell_{23} = \ell_{21} = \ell_{32} = 1\rangle$

Class I (0,0,0)

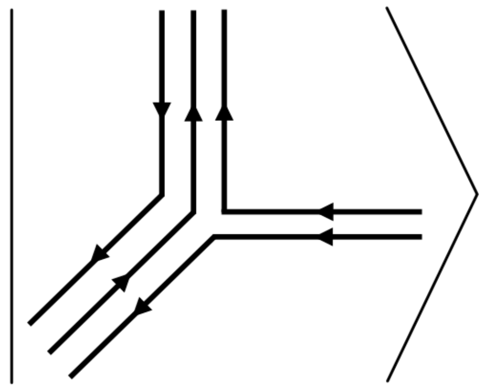
$$|\ell_{IJ}, \ell_{JK}, \ell_{JI}, \ell_{KJ}, t\rangle = \frac{(L^\dagger_{IJ})^{\ell_{IJ}}(L^\dagger_{JK})^{\ell_{JK}}(L^\dagger_{JI})^{\ell_{JI}}(L^\dagger_{KJ})^{\ell_{KJ}}|0\rangle}{\sqrt{\frac{1}{2}(\ell_{IJ}+\ell_{JI}+\ell_{JK}+\ell_{KJ}+|t|+2)\ell_{IJ}!\,\ell_{JK}!\,\ell_{JI}!\,\ell_{KJ}!\,|t|!\,(\ell_{IJ}+\ell_{KJ}+|t|+1)!\,(\ell_{JK}+\ell_{JI}+|t|+1)!}},$$



$= |\ell_{12} = \ell_{23} = \ell_{21} = \ell_{13} = 1\rangle$

Class IIa $(1,1,-2)$

$$|\ell_{IJ}, \ell_{JK}, \ell_{JI}, \ell_{IK}; t\rangle = \frac{(L^\dagger_{IJ})^{\ell_{IJ}}(L^\dagger_{JK})^{\ell_{JK}}(L^\dagger_{JI})^{\ell_{JI}}(L^\dagger_{IK})^{\ell_{IK}}|0\rangle}{\sqrt{\frac{1}{2}\frac{(\ell_{IJ}+\ell_{JK}+\ell_{JI}+\ell_{IK}+|t|+2)\,\ell_{IJ}!\,\ell_{JK}!\,\ell_{JI}!\,\ell_{IK}!\,|t|!\,(\ell_{IJ}+\ell_{IK}+|t|+1)!\,(\ell_{JK}+\ell_{JI}+|t|+1)!\,\binom{\ell_{IJ}+\ell_{JK}+\ell_{JI}+\ell_{IK}+|t|+1}{\ell_{IK}}}{\binom{\ell_{IJ}+\ell_{JI}+\ell_{IK}+|t|+1}{\ell_{IK}}}}},$$



$= |\ell_{12} = \ell_{23} = \ell_{32} = \ell_{13} = 1\rangle$

Class IIb $(2,-1,-1)$

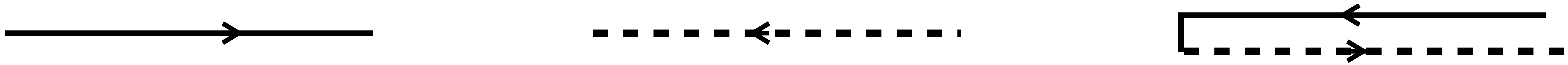
$$|\ell_{IJ}, \ell_{JK}, \ell_{KJ}, \ell_{IK}, t\rangle = \frac{(L^\dagger_{IJ})^{\ell_{IJ}}(L^\dagger_{JK})^{\ell_{JK}}(L^\dagger_{KJ})^{\ell_{KJ}}(L^\dagger_{IK})^{\ell_{IK}}|0\rangle}{\sqrt{\frac{1}{2}\frac{(\ell_{IJ}+\ell_{JK}+\ell_{KJ}+\ell_{IK}+|t|+2)\,\ell_{IJ}!\,\ell_{JK}!\,\ell_{KJ}!\,\ell_{IK}!\,|t|!\,(\ell_{JK}+\ell_{IK}+|t|+1)!\,(\ell_{IJ}+\ell_{KJ}+|t|+1)!\,\binom{\ell_{IJ}+\ell_{JK}+\ell_{KJ}+\ell_{IK}+|t|+1}{\ell_{IK}}}{\binom{\ell_{JK}+\ell_{KJ}+\ell_{IK}+|t|+1}{\ell_{IK}}}}}.$$

LSH Formulation: SU(3) beyond 1+1 dimensions

Dynamics of LSH operators: Plaquette / LSH at corners

- Link operator:

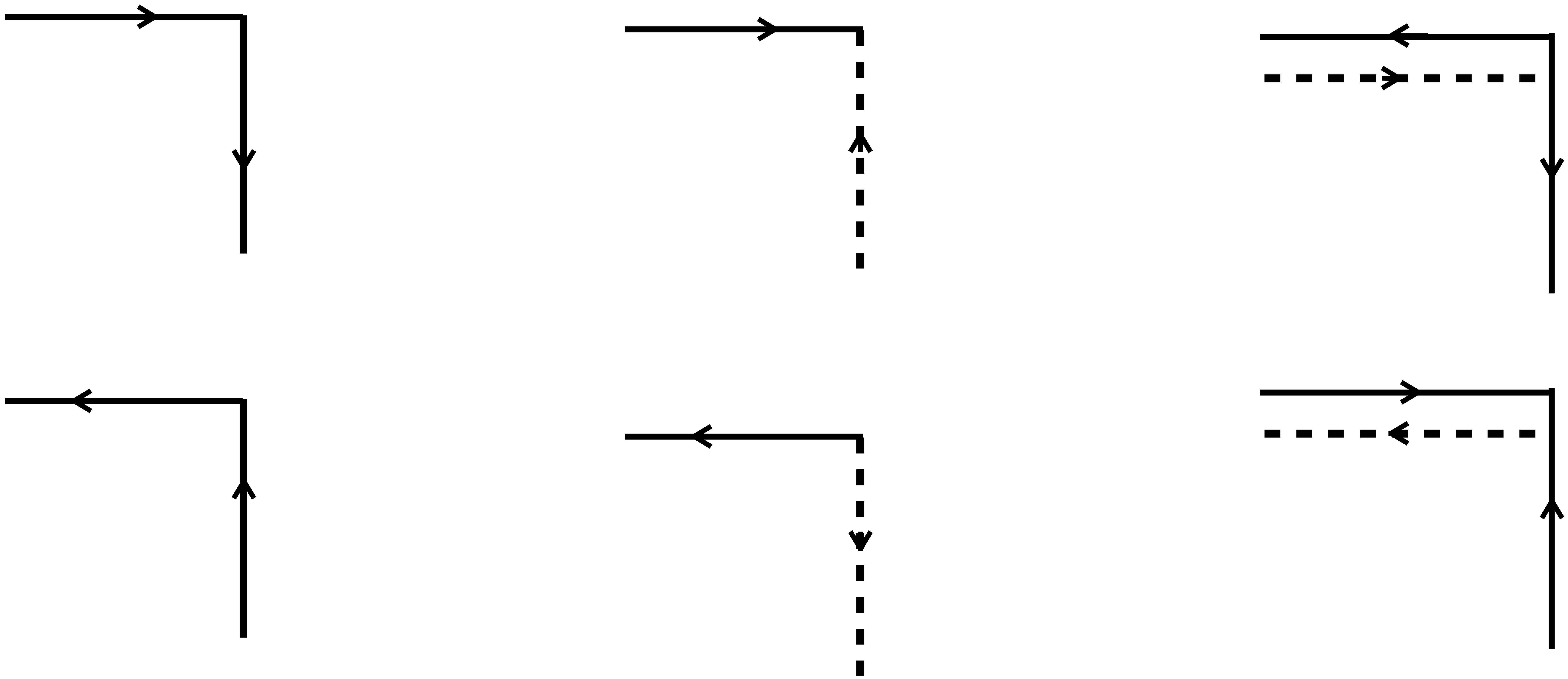
$$
\begin{aligned}
U^{\alpha}{}_{\beta}(r) = & \; B^{\dagger\alpha}(L,r)\eta(r)A^{\dagger}_{\beta}(R,r+1) \\
& + A^{\alpha}(L,r)\theta(r)B_{\beta}(R,r+1) \\
& + (A^{\dagger}(L,r) \wedge B(L,r))^{\alpha}\delta(r)(B^{\dagger}(R,r+1)\wedge A(R,r+1))_{\beta}
\end{aligned}
$$



LSH Formulation: SU(3) beyond 1+1 dimensions

Dynamics of LSH operators: Plaquette / LSH at corners

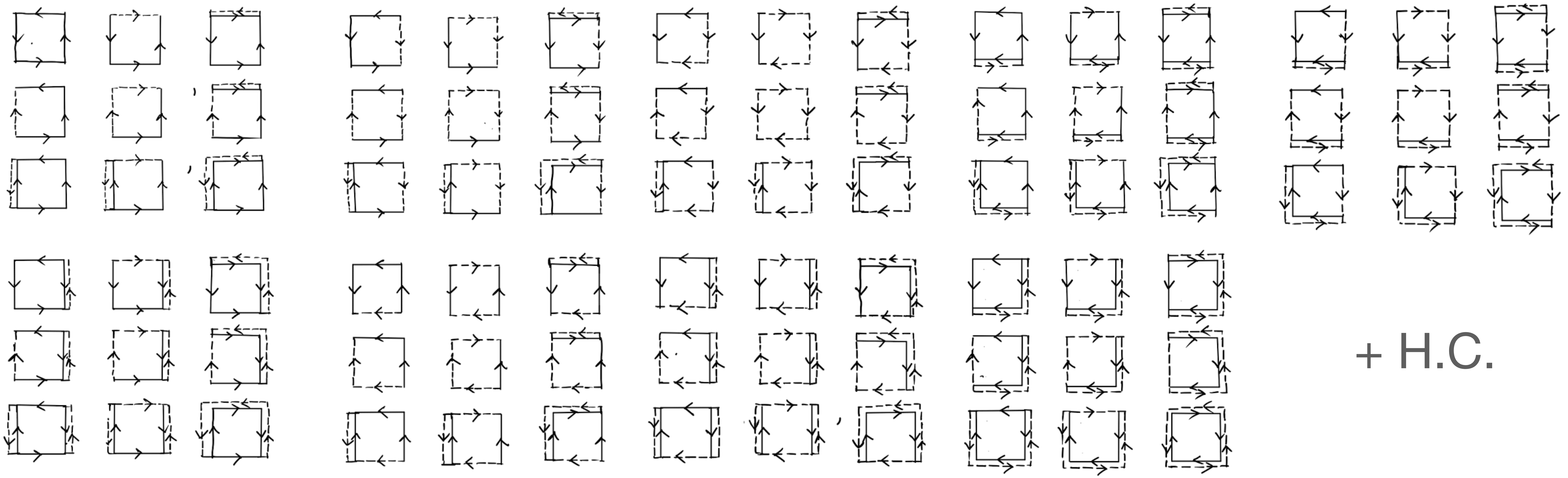
- Gauge Invariant LSH Corner operators at a trivalent vertex:



LSH Formulation: SU(3) beyond 1+1 dimensions

Dynamics of LSH operators: Plaquette / LSH at corners

- Magnetic Hamiltonian in LSH framework:



LSH Formulation: SU(3) beyond 1+1 dimensions

Dynamics of LSH operators: in the naive basis

- Fundamental LSH operators:

$$L^{\dagger}_{IJ} = A(I)^{\dagger}_{\alpha} B(J)^{\dagger\alpha},$$
$$T^{\dagger}_{A} = \epsilon^{\alpha\beta\gamma} A(1)^{\dagger}_{\alpha} A(2)^{\dagger}_{\beta} A(3)^{\dagger}_{\gamma}$$
$$T^{\dagger}_{B} = \epsilon_{\alpha\beta\gamma} B(1)^{\dagger\,\alpha} B(2)^{\dagger\,\beta} B(3)^{\dagger\,\gamma}$$

- On-site LSH operators in Hamiltonian:

$$\begin{aligned}
N_{IJ} &\equiv A(I)^{\dagger}_{\alpha} A(J)^{\alpha} \\
M_{IJ} &\equiv A(I)^{\dagger}_{\alpha} A(J)^{\alpha} \\
\epsilon A(I)^{\dagger} A(J)^{\dagger} B(J) &\equiv \epsilon^{\alpha\beta\gamma} A(I)^{\dagger}_{\alpha} A(J)^{\dagger}_{\beta} B(J)_{\gamma} \\
\epsilon B(I)^{\dagger} B(J)^{\dagger} A(J) &\equiv \epsilon^{\alpha\beta\gamma} B(I)^{\dagger}_{\alpha} B(J)^{\dagger}_{\beta} A(J)_{\gamma} \\
\epsilon A(I)^{\dagger} A(J)^{\dagger} B(K) &\equiv \epsilon^{\alpha\beta\gamma} A(I)^{\dagger}_{\alpha} A(J)^{\dagger}_{\beta} B(K)_{\gamma} \\
\epsilon B(I)^{\dagger} B(J)^{\dagger} A(K) &\equiv \epsilon^{\alpha\beta\gamma} B(I)^{\dagger}_{\alpha} B(J)^{\dagger}_{\beta} A(K)_{\gamma} \\
\epsilon A(I)^{\dagger} B(J) A(J)^{\dagger} &\equiv \epsilon^{\alpha\beta\gamma} A(I)^{\dagger}_{\alpha} B(J)_{\beta} A(J)^{\dagger}_{\gamma} \\
&= -\left(\frac{P_J + Q_J + 3}{P_J + Q_J + 2}\right) \epsilon A(I)^{\dagger} A(J)^{\dagger} B(J)
\end{aligned}$$

LSH Formulation: SU(3) beyond 1+1 dimensions

Dynamics of LSH operators: in the naive basis

- The algebra of all possible LSH operator closes: allows one to perform calculation of matrix elements in LSH basis

$$[L^-_{IJ}, L^+_{IJ}] = 3 + N_I + M_J - F_J N_{JI} N_{IJ} - F_I M_{IJ} M_{JI} - F_I F_J L^+_{JI} L^-_{JI}$$

$$[L^-_{IJ}, L^+_{JI}] = -F_I M_{IJ} N_{JI} - F_J N_{JI} M_{IJ} - F_I F_J L^+_{JI} L^-_{IJ}$$

$$[L^-_{IJ}, L^+_{KI}] = -F_I M_{IJ} N_{KI}$$

$$[L^-_{IJ}, L^+_{IK}] = M_{KJ} - F_I M_{IJ} M_{KI}$$

$$[L^-_{IJ}, L^+_{KJ}] = N_{KI} - F_J N_{JI} N_{KJ}$$

$$[L^-_{IJ}, L^+_{JK}] = -F_J N_{JI} M_{KJ}$$

$$[L^-_{KJ}, N_{IJ}] = 0$$

$$[L^-_{JK}, N_{IJ}] = 0$$

$$[L^-_{KI}, N_{IJ}] = -F_I N_{IK} L^-_{JI}$$

$$[L^-_{JI}, N_{IJ}] = -F_I N_{IJ} L^-_{JI}$$

$$[L^-_{IK}, N_{IJ}] = L^-_{JK} - F_I M_{IK} L^-_{JI}$$

$$[L^-_{IJ}, N_{IJ}] = -F_I M_{IJ} L^-_{JI}$$

$$[M_{IJ}, L^+_{IK}] = 0$$

$$[M_{IJ}, L^+_{KI}] = 0$$

$$[M_{IJ}, L^+_{JI}] = -F_J L^+_{JI} M_{IJ}$$

$$[M_{IJ}, L^+_{JK}] = -F_J L^+_{JI} M_{KJ}$$

$$[M_{IJ}, L^+_{KJ}] = L^+_{KI} - F_J L^+_{JI} N_{}$$

$$[M_{IJ}, L^+_{IJ}] = -F_J L^+_{JI} N_{IJ}$$

$$[N_{IJ}, L^+_{IK}] = 0$$

$$[N_{IJ}, L^+_{KI}] = 0$$

$$[N_{IJ}, L^+_{JK}] = L^+_{IK} - F_J L^+_{IJ} M$$

$$[N_{IJ}, L^+_{JI}] = -F_J L^+_{IJ} M_{IJ}$$

$$[N_{IJ}, L^+_{IJ}] = -F_J L^+_{IJ} N_{IJ}$$

$$[N_{IJ}, L^+_{KJ}] = -F_J L^+_{IJ} N_{KJ}$$

$$\left[T^{+\,+\,-}_{A_P A_Q B_Q}, T^{-\,+\,-}_{A_Q B_P A_P}\right] = -L^+_{PQ} N_P - N_Q M_{PQ} + M_{PQ} - 3M_{PQ}$$

$$\left[T^{+\,+\,-}_{A_P A_Q B_Q}, T^{-\,+\,-}_{A_P B_R A_R}\right] = F_P T^{+\,+\,-}_{B_P B_R A_R} T^{-\,+\,-}_{B_P A_Q B_Q} - L^+_{QR} L^-_{RQ} + N_{QR} M_{RQ}$$

$$\left[T^{+\,+\,-}_{A_P A_Q B_Q}, T^{-\,+\,-}_{A_R B_P A_P}\right] = L^+_{QP} L^-_{RQ} - N_{QR} M_{PQ}$$

$$\left[T^{+\,+\,-}_{A_P A_Q B_Q}, T^{-\,+\,-}_{A_Q B_R A_R}\right] = L^+_{PR} L^-_{RQ} - M_{RQ} N_{PR}$$

$$\left[T^{+\,+\,-}_{A_P A_Q B_Q}, T^{-\,+\,-}_{A_R B_Q A_Q}\right] = N_{QR} N_{PQ} - N_{PR} N_Q + L^+_{PQ} L^-_{RQ} - N_{PR} M_Q$$

$$\left[T^{-\,+\,-}_{B_P A_Q B_Q}, T^{-\,+\,-}_{A_P B_Q A_Q}\right] = N_{QP} L^-_{QP} - M_{QP} L^-_{PQ}$$

$$\left[T^{-\,+\,-}_{B_P A_Q B_Q}, T^{-\,+\,-}_{A_Q B_P A_P}\right] = (M_P + 3 - F_Q M_Q) L^-_{PQ} + F_Q N_P L^-_{PQ} - (N_Q + 3 - F_Q M_Q) L^-_{PQ} - F_Q M_Q L^-_{PQ}$$

$$\left[T^{-\,+\,-}_{B_P A_Q B_Q}, T^{+\,+\,-}_{B_P B_Q A_Q}\right] = (1 + F_Q) N_Q - F_Q N_Q^2 - N_Q (M_Q + 3 - F_Q N_Q) - F_Q (1 + F_Q) N_Q - F_Q^2 N_Q^2 - F_P T^{+\,+\,-}_{A_P A_Q B_Q} T^{-\,+\,-}_{A_P B_Q A_Q} + L^+_{QP} L^-_{QP} - M_P N_Q + M_P M_Q + M_Q - M_{QP} M_{PQ} - F_P L^+_{PQ} L^-_{PQ}$$

$$\left[T^{-\,+\,-}_{B_P A_Q B_Q}, T^{+\,+\,-}_{B_P B_P A_P}\right] = (M_P + 3 - F_P N_P) N_{QP} + F_P N_{QP} N_P - (1 + F_P) N_{QP} + M_Q N_{QP}$$

$$\left[T^{-\,+\,-}_{B_P A_Q B_Q}, T^{+\,+\,-}_{B_P B_R A_R}\right] = L^+_{QR} L^-_{PQ} - M_{RQ} N_{QR} - F_P L^+_{QR} L^-_{RQ} - F_P T^{+\,+\,-}_{A_P A_Q B_Q} T^{-\,+\,-}_{A_P B_R A_R}$$

$$\left[T^{-\,+\,-}_{B_P A_Q B_Q}, T^{+\,+\,-}_{B_R B_P A_P}\right] = N_{QP} M_{RQ} - L^+_{QR} L^-_{PQ}$$

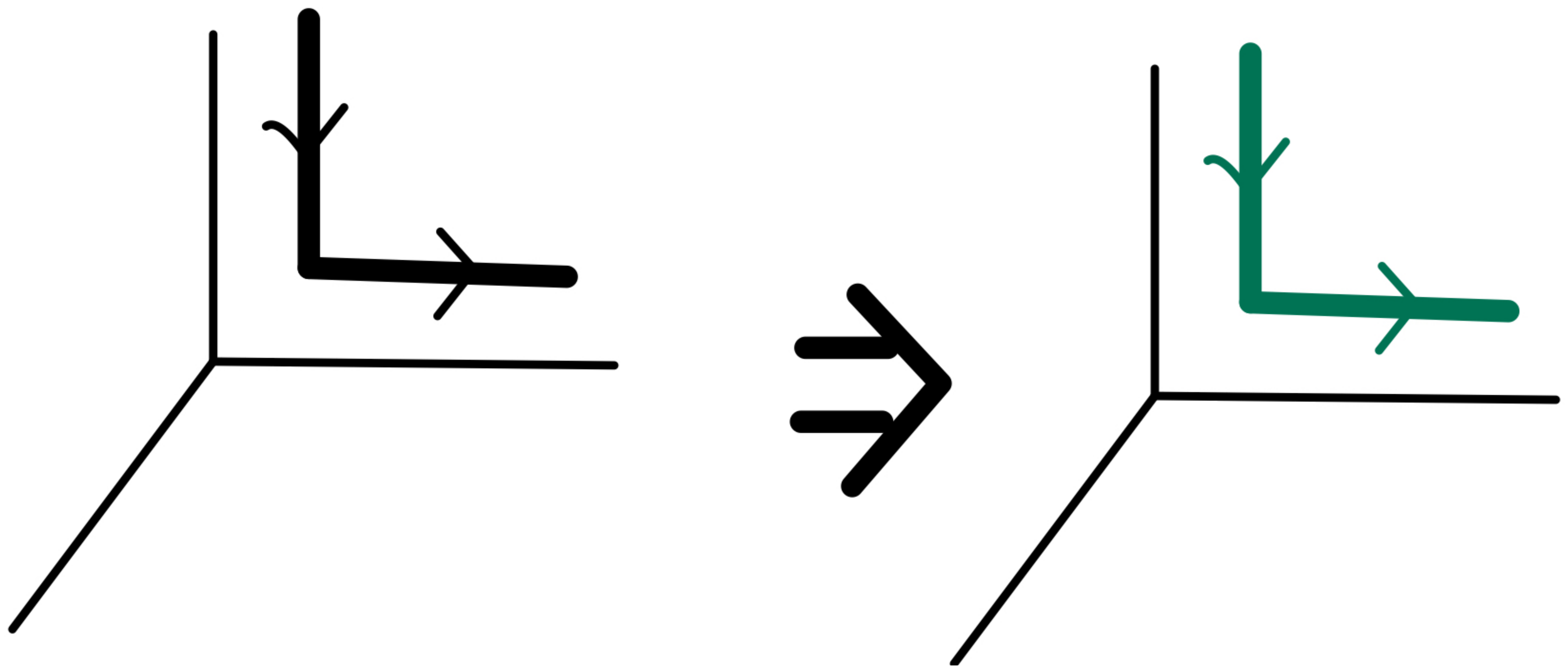
$$\left[T^{-\,+\,-}_{B_P A_Q B_Q}, T^{+\,+\,-}_{B_Q B_R A_P}\right] = M_{RP} N_{QR} - L^+_{RR} L^-_{RP}$$

$$\left[T^{-\,+\,-}_{B_P A_Q B_Q}, T^{+\,+\,-}_{B_R B_Q A_Q}\right] = L^+_{QR} L^-_{QP} - M_{RP} N_Q + M_{RP} M_Q - M_{RP} M_{RQ}$$

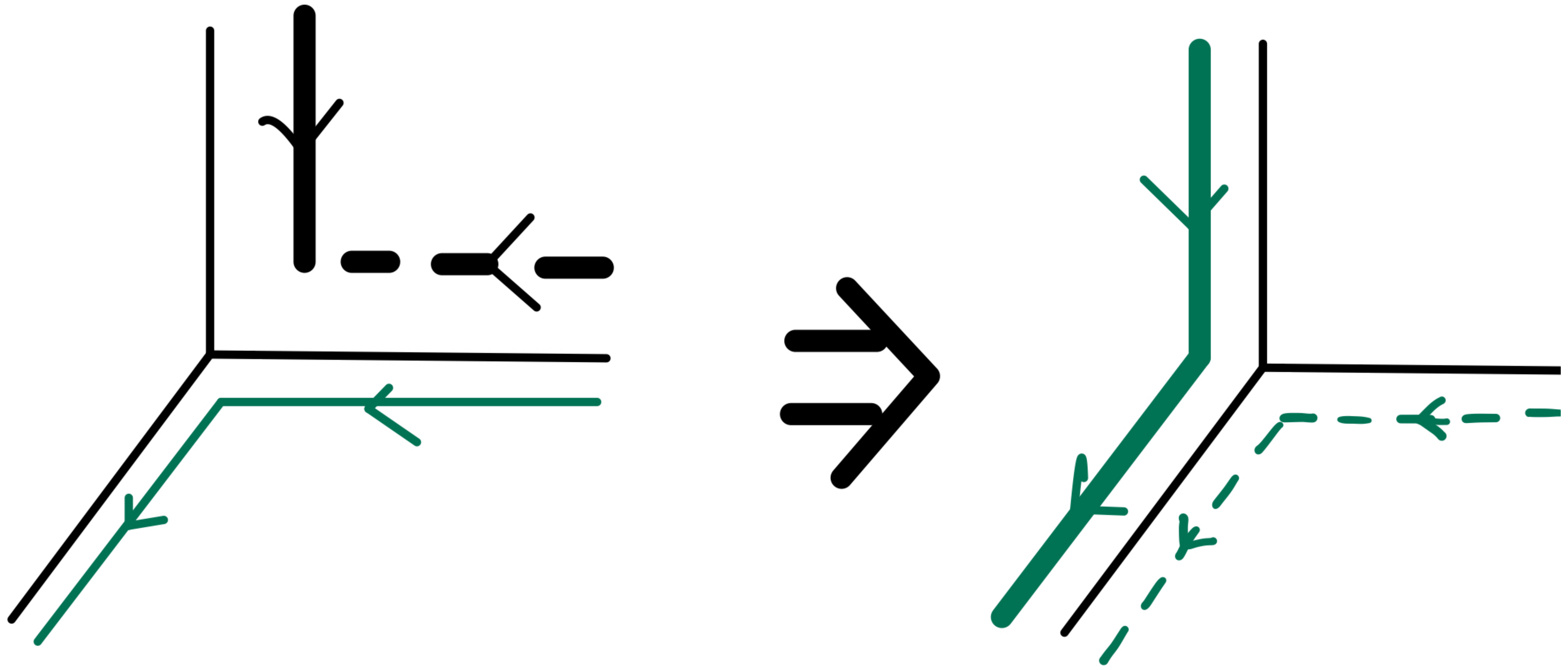
LSH Formulation: SU(3) beyond 1+1 dimensions

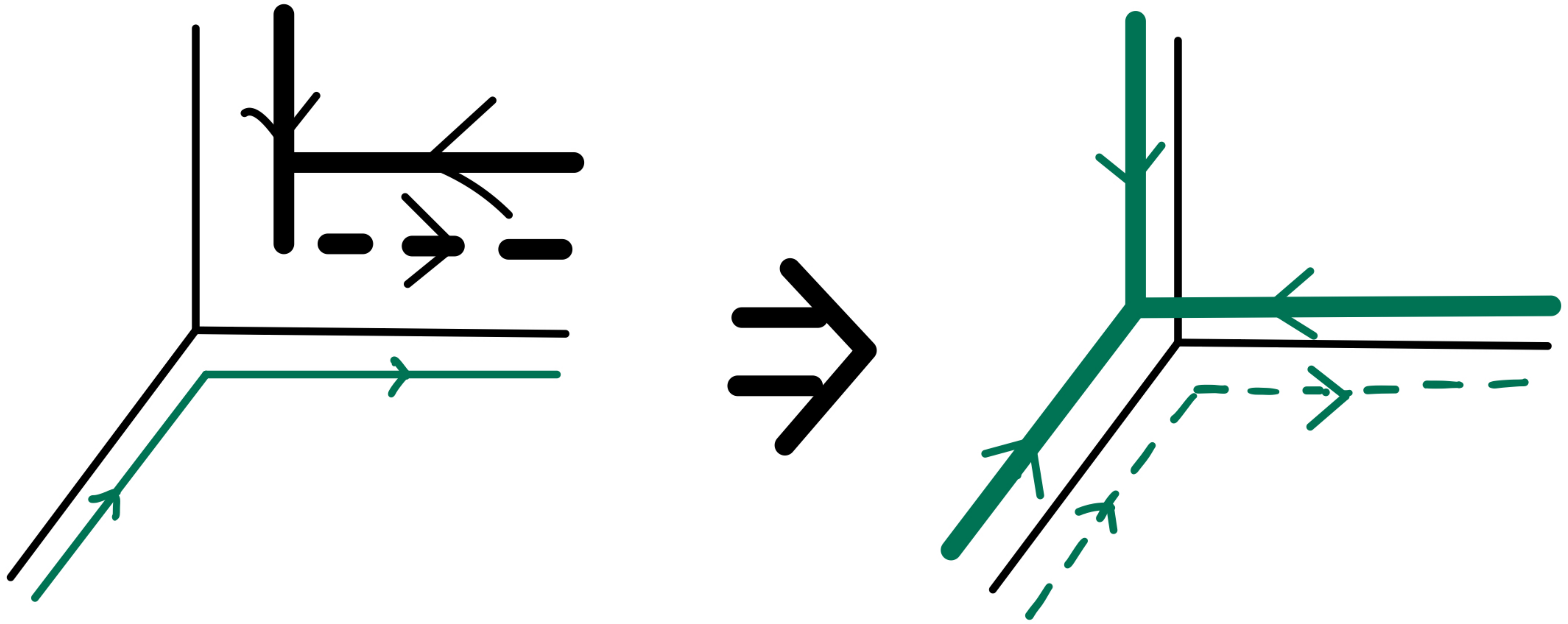
Dynamics of LSH operators: in the naive basis

- On-site LSH operators acting on LSH basis states at a trivalent vertex:



- Algebraic calculation of matrix element is tedious: This job has now been automatised.





LSH Formulation: SU(3) beyond 1+1 dimensions

Dynamics of LSH operators: in the naive basis

- The action of LSH operators on LSH basis: examples

$$\begin{aligned} N_{ij}\,|\{\ell\}\,;t\rangle\rangle = \left(\frac{1}{\ell_{ij}+\ell_{jk}+\ell_{ji}+\ell_{kj}+|t|+1}\right)\Big[&\ell_{jk}(\ell_{jk}+\ell_{ji}+\ell_{kj}+|t|+1)\,|\ell_{jk}-1,\ell_{ik}+1,\cdots;t\rangle\rangle - \\ &-\ell_{ji}\ell_{kj}\,|\ell_{ij}+1,\ell_{ki}+1,\ell_{ji}-1,\ell_{kj}-1,\cdots;t\rangle\rangle\Big] \end{aligned}$$

$$\begin{aligned} M_{ij}\,|\{\ell\}\,;t\rangle\rangle = \left(\frac{1}{\ell_{ij}+\ell_{jk}+\ell_{ji}+\ell_{kj}+|t|+1}\right)\Big[&\ell_{kj}(\ell_{ij}+\ell_{jk}+\ell_{kj}+|t|+1)\,|\ell_{ki}+1,\ell_{kj}-1,\cdots;t\rangle\rangle - \\ &-\ell_{ij}\ell_{jk}\,|\ell_{ij}-1,\ell_{jk}-1,\ell_{ji}+1,\ell_{ik}+1,\cdots;t\rangle\rangle\Big] \end{aligned}$$

$$\begin{aligned} &L_{ij}\,|\{\ell\}\,;t\rangle\rangle \\ &= \left(\frac{1}{\ell_{ij}+\ell_{jk}+\ell_{ji}+\ell_{kj}+|t|+1}\right)\times \\ &\quad\times\Big(|\ell_{jk}+1,\ell_{ki}+1,\ell_{ji}-1,\ell_{kj}-1,\ell_{ik}-1,\cdots;t\rangle\rangle\times \\ &\qquad\times\ell_{ji}\ell_{kj}\ell_{ik}\left(\frac{\ell_{ij}+\ell_{jk}+\ell_{kj}+|t|+2}{\ell_{ij}+\ell_{ji}+\ell_{ik}+|t|+1}\right)\left[\frac{\ell_{ki}}{\ell_{ij}+\ell_{ki}+\ell_{ji}+\ell_{ik}+|t|+1}-\left(\frac{\ell_{ij}+\ell_{ji}+\ell_{ik}+|t|+1}{\ell_{ij}+\ell_{jk}+\ell_{kj}+|t|+2}+1\right)\right]+ \\ &\quad+|\ell_{ij}-1,\cdots;t\rangle\rangle\times \\ &\qquad\times\ell_{ij}\Big\{\frac{(-\ell_{ki})}{\ell_{ij}+\ell_{ki}+\ell_{ji}+\ell_{ik}+|t|+1}\left[(\ell_{kj}+1)(\ell_{ij}+\ell_{jk}+\ell_{kj}+|t|+1)+\frac{\ell_{ji}\ell_{ik}(\ell_{jk}+1)}{\ell_{ij}+\ell_{ji}+\ell_{ik}+|t|+1}\right]+ \\ &\qquad\quad+\Big[\ell_{ik}(\ell_{ij}+\ell_{ji}+\ell_{kj}+|t|+1)+\left(\frac{\ell_{ik}\ell_{ji}(\ell_{jk}+1)}{\ell_{ij}+\ell_{ji}+\ell_{ik}+|t|+1}\right)+ \\ &\qquad\qquad+(\ell_{ij}+\ell_{kj}+|t|+1)(\ell_{ij}+\ell_{jk}+\ell_{ji}+\ell_{kj}+|t|+2)\Big]\Big\}+ \\ &\quad+|\ell_{ij}-2,\ell_{jk}-1,\ell_{ki}-1,\ell_{ji}+1,\ell_{kj}+1,\ell_{ik}+1;t\rangle\rangle\left(\frac{\ell_{ij}(\ell_{ij}-1)\ell_{jk}\ell_{ki}}{\ell_{ij}+\ell_{ki}+\ell_{ji}+\ell_{ik}+|t|+1}\right)\Big). \end{aligned}$$

$$T_B^\dagger\,|\{\ell\}\,;t\rangle\rangle = \begin{cases} t\le 0: & |\{\ell\}\,;t-1\rangle\rangle \\ t>0: & |\ell_{ij}+1,\ell_{jk}+1,\ell_{ki}+1,\cdots;t-1\rangle\rangle + |\ell_{ji}+1,\ell_{kj}+1,\ell_{ik}+1,\cdots;t-1\rangle\rangle \end{cases}$$

$$T_A^\dagger\,|\{\ell\}\,;t\rangle\rangle = \begin{cases} t\ge 0: & |\{\ell\}\,;t+1\rangle\rangle \\ t<0: & |\ell_{ij}+1,\ell_{jk}+1,\ell_{ki}+1,\cdots;t+1\rangle\rangle + |\ell_{ji}+1,\ell_{kj}+1,\ell_{ik}+1,\cdots;t+1\rangle\rangle \end{cases}$$

$$(\epsilon\,A_i^\dagger A_j^\dagger B_j)\,|\{\ell\}\,;t\rangle\rangle = \epsilon_{ijk}\begin{cases} t\ge 0: & \ell_{kj}\,|\ell_{kj}-1,\cdots;t+1\rangle\rangle \\ t<0: & (\ell_{kj}+|t|)\,|\ell_{ji}+1,\ell_{ik}+1,\cdots;t+1\rangle\rangle + \\ & +\ell_{kj}\,|\ell_{ij}+1,\ell_{jk}+1,\ell_{ki}+1,\ell_{kj}-1,\cdots;t+1\rangle\rangle \end{cases}$$

$$(\epsilon\,B_i^\dagger B_j^\dagger A_j)\,|\{\ell\}\,;t\rangle\rangle = \epsilon_{ijk}\begin{cases} t\le 0: & \ell_{jk}\,|\ell_{jk}-1,\cdots;t-1\rangle\rangle \\ t>0: & (\ell_{jk}+|t|)\,|\ell_{ij}+1,\ell_{ki}+1,\cdots;t-1\rangle\rangle + \\ & +\ell_{jk}\,|\ell_{ji}+1,\ell_{kj}+1,\ell_{ik}+1,\ell_{jk}-1,\cdots;t-1\rangle\rangle \end{cases}$$

LSH Formulation: SU(3) beyond 1+1 dimensions

Dynamics of LSH operators: in the naive basis

- The action of LSH operators on LSH basis: the resultant state can be found pictorially

- Algebraic calculation of matrix element is tedious: This job has now been automatised.

- Next task: to construct the Hamiltonian matrix.

LSH Formulation: SU(3) in 2+1 dimension and beyond

Challenges:

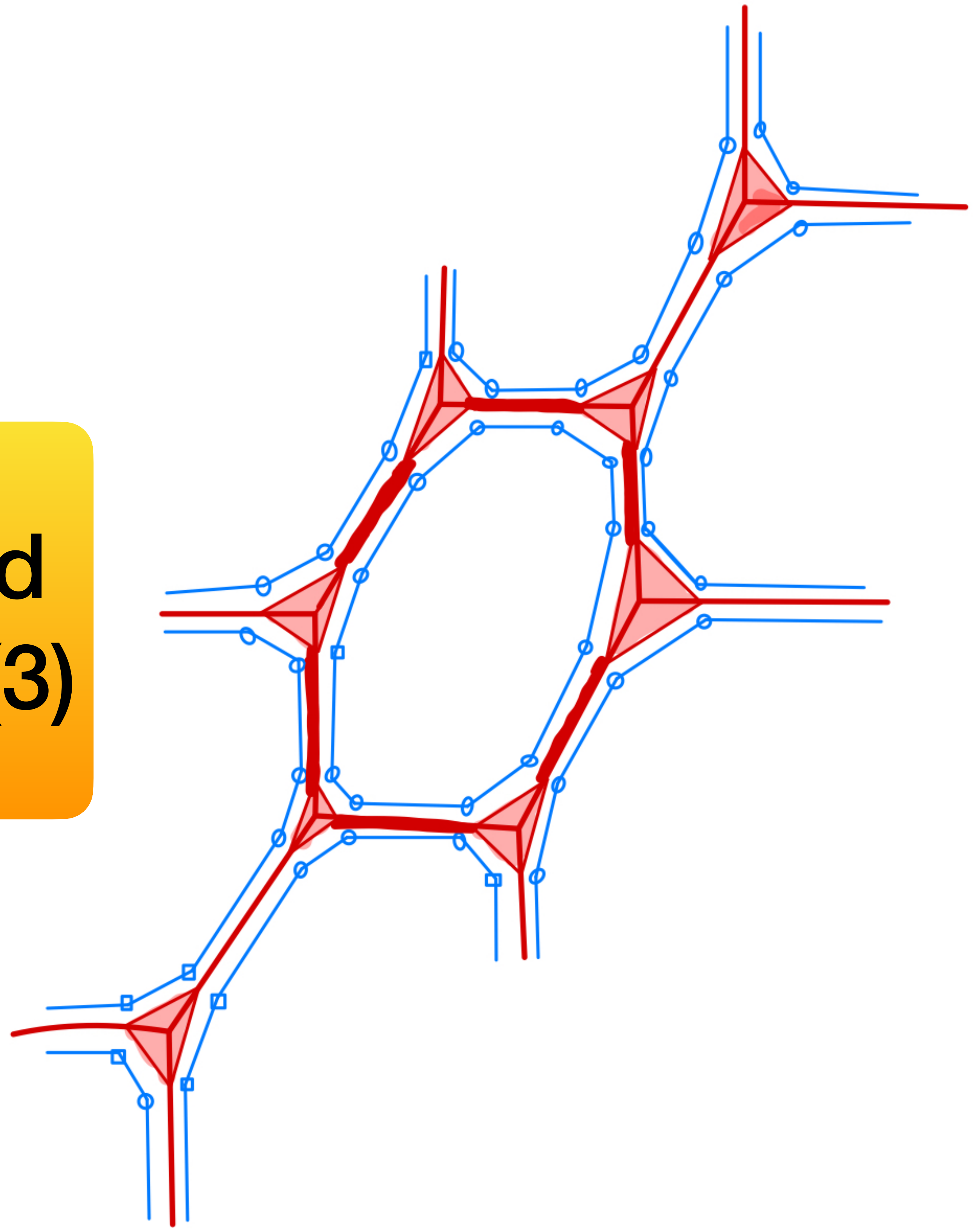
- Point splitting does not solve all the Mandelstam constraints.
- Still remain one unsolved constraint at each site.
- Naive basis is not orthonormal.

Aim:

- To come-up with an elegant Hamiltonian calculation for SU(3) in 3+1 dimension.

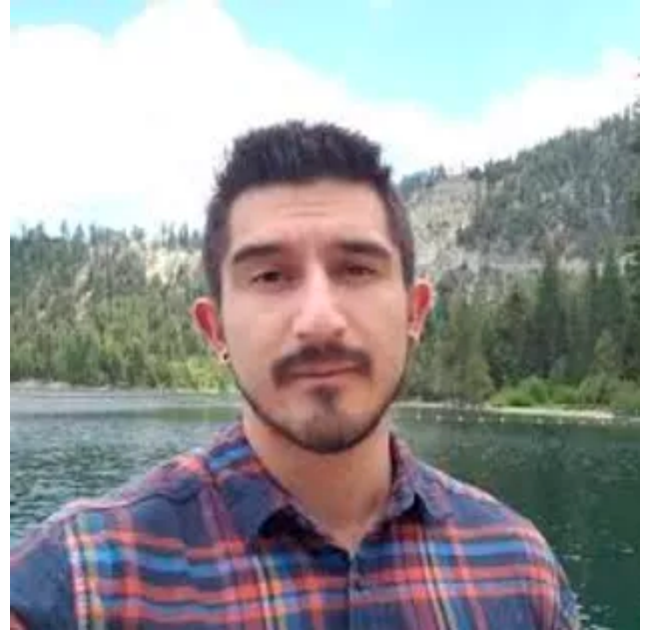
Work is in progress…

May require a second point splitting for SU(3)



Collaborators:

Aahiri Naskar, Grad student, BITS Goa



Jesse Stryker

Saurabh Kadam

Part II: Applications

Benefits of working in the LSH framework: Applications in quantum simulation

Symmetry protection protocol:

Already demonstrated for SU(2)

PHYSICAL REVIEW D **106**, 054510 (2022)

Protecting local and global symmetries in simulating (1+1)D non-Abelian gauge theories

Emil Mathew[*] and Indrakshi Raychowdhury[†]
Department of Physics, BITS-Pilani, K K Birla Goa Campus, Zuarinagar, Goa 403726, India

Local symmetries: AGL

$$n_{\text{out}}(x) = n_{\text{in}}(x+1)$$

LSH framework:
no local non-Abelian
symmetry

Global symmetries: global SU(2)

$$Q = \sum_{x=0}^{N-1} [n_i(x) + n_o(x)]$$

$$q = \sum_{x=0}^{N-1} [n_0(x) - n_i(x)]$$

For a particular Q value, q can take any value from $-Q$ to $+Q$ and defines different disconnected sectors of the larger gauge-invariant LSH Hilbert space.

Charge conjugation symmetry: The particle antiparticle symmetry of the theory identifies (Q, q) sector of the Hamiltonian to the $(Q, -q)$ sector.

Emil Mathew,
Grad. Student,
BITS-Pilani, Goa

Benefits of working in the LSH framework: Applications in quantum simulation

Symmetry protection protocol for SU(3):

arXiv:2404.12158v1

Protecting gauge symmetries in the the dynamics of SU(3) lattice gauge theories

Emil Mathew[1,2,*] and Indrakshi Raychowdhury[1,2,†]

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(Dated: April 19, 2024)

Global charges

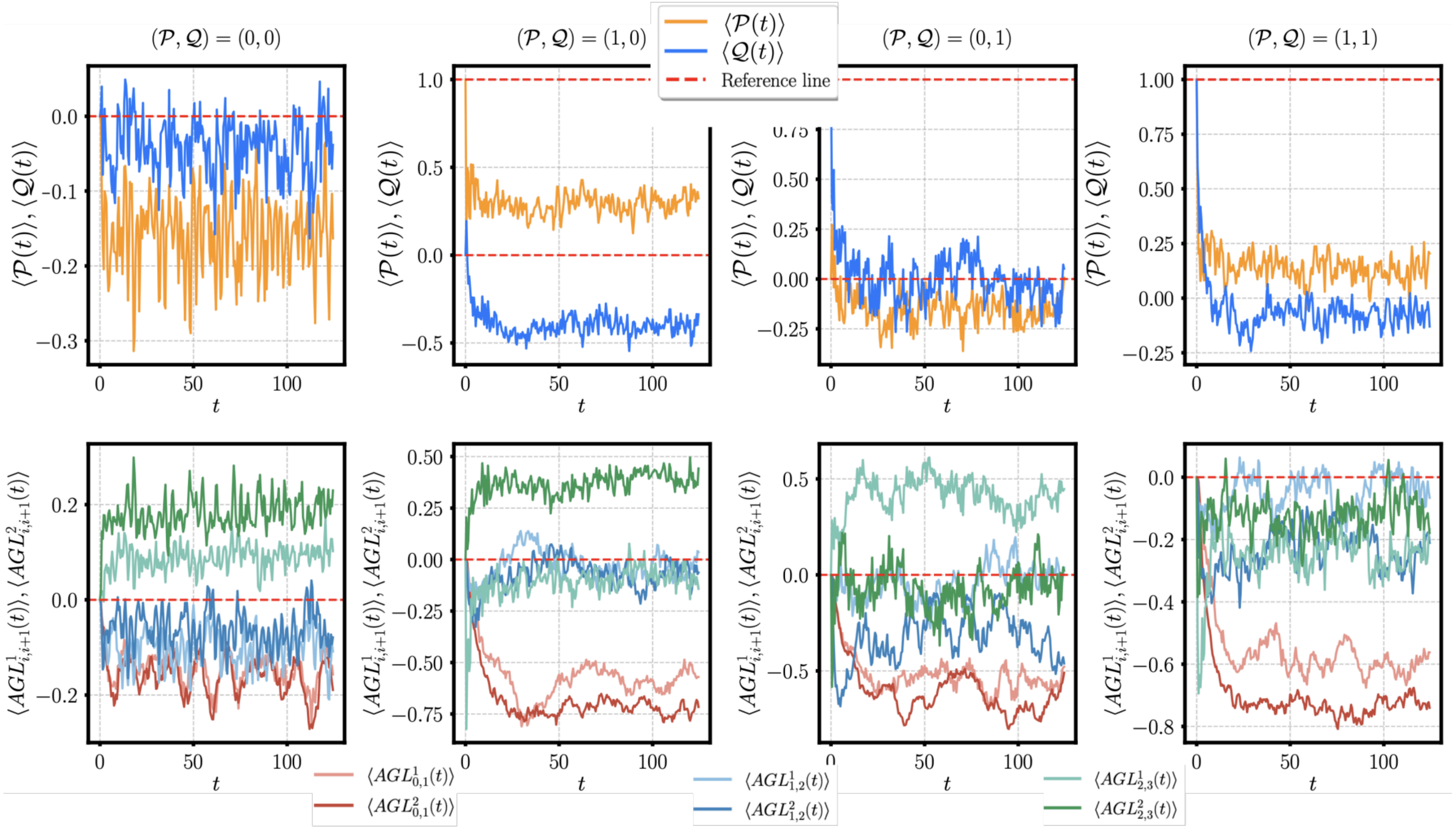
$$q_f = \sum_r \nu_f(r)$$

$$\begin{aligned} \mathcal{F} &= q_{\underline{1}} + q_0 + q_1 \\ \mathcal{P} &= q_1 - q_0 \\ \mathcal{Q} &= q_0 - q_{\underline{1}} \end{aligned}$$

Local charges

$$P_1(r) = P_{\underline{1}}(r+1)$$
$$Q_1(r) = Q_{\underline{1}}(r+1)$$

Symmetry protection protocol for SU(3):



Manifestly violating global symmetries leads to all local symmetries to be violated

arXiv:2404.12158v1

Applications in quantum simulation

Already demonstrated for SU(2)

Analog Quantum Computation

PHYSICAL REVIEW A **105**, 023322 (2022)

Raka Dasgupta

Cold-atom quantum simulator for string and hadron dynamics in non-Abelian lattice gauge theory

Raka Dasgupta[1,*] and Indrakshi Raychowdhury[2,3,†]
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[2]*Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, Maryland 20742, USA*
[3]*BITS-Pilani, K. K. Birla Goa Campus, Zuarinagar, Goa 403726, India*

(Received 11 October 2020; accepted 2 February 2022; published 22 February 2022)

We propose an analog quantum simulator for simulating real-time dynamics of $(1+1)$-dimensional non-Abelian gauge theory well within the existing capacity of ultracold-atom experiments. The scheme calls for the realization of a two-state ultracold fermionic system in a one-dimensional bipartite lattice, and the observation of subsequent tunneling dynamics. Being based on the loop string hadron formalism of $SU(2)$ lattice gauge theory, this simulation technique is completely $SU(2)$ invariant and simulates accurate dynamics of physical phenomena such as string breaking and/or pair production. The scheme is scalable and particularly effective in simulating the theory in the weak-coupling regime, and also a bulk limit of the theory in the strong-coupling regime up to certain approximations. This paper also presents a numerical benchmark comparison of the exact spectrum and real-time dynamics of lattice gauge theory to that of the atomic Hamiltonian with an experimentally realizable range of parameters.

DOI: 10.1103/PhysRevA.105.023322

Key advantage:

1+1d dynamics: dynamics of strings

In LSH: string ends are purely fermionic object

Continuity of strings are guaranteed by AGL: protected by global symmetries

Application: Analog Quantum Simulation

Experimental Demonstration: minor modification/combination of

PRL 115, 115303 (2015) PHYSICAL REVIEW LETTERS week ending 11 SEPTEMBER 2015

Exploring Competing Density Order in the Ionic Hubbard Model with Ultracold Fermions

Michael Messer,[1] Rémi Desbuquois,[1] Thomas Uehlinger,[1] Gregor Jotzu,[1] Sebastian Huber,[2] Daniel Greif,[1] and Tilman Esslinger[1]
[1]*Institute for Quantum Electronics, ETH Zurich, 8093 Zurich, Switzerland*
[2]*Institute for Theoretical Physics, ETH Zurich, 8093 Zurich, Switzerland*

(Received 18 March 2015; revised manuscript received 1 July 2015; published 9 September 2015)

We realize and study the ionic Hubbard model using an interacting two-component gas of fermionic atoms loaded into an optical lattice. The bipartite lattice has a honeycomb geometry with a staggered energy offset that explicitly breaks the inversion symmetry. Distinct density-ordered phases are identified using noise correlation measurements of the atomic momentum distribution. For weak interactions the geometry induces a charge density wave. For strong repulsive interactions we detect a strong suppression of doubly occupied sites, as expected for a Mott insulating state, and the externally broken inversion symmetry is not visible anymore in the density distribution. The local density distributions in different configurations are characterized by measuring the number of doubly occupied lattice sites as a function of interaction and energy offset. We further probe the excitations of the system using direction dependent modulation spectroscopy and discover a complex spectrum, which we compare with a theoretical model.

DOI: 10.1103/PhysRevLett.115.115303 PACS numbers: 67.85.Lm, 71.10.Fd, 71.30.+h, 73.22.Pr

Pair creation dynamics in lattice gauge theory

PHYSICAL REVIEW LETTERS 121, 130402 (2018)

Nonequilibrium Mass Transport in the 1D Fermi-Hubbard Model

S. Scherg,[1,2] T. Kohlert,[1,2] J. Herbrych,[3,4] J. Stolpp,[1,5,6] P. Bordia,[1,2] U. Schneider,[1,2,7] F. Heidrich-Meisner,[5] I. Bloch,[1,2] and M. Aidelsburger[1,2,*]
[1]*Fakultät für Physik, Ludwig-Maximilians-Universität München, Munich, Germany*
[2]*Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany*
[3]*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*
aterials Science and Technology Division Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA
[5]*Institute for Theoretical Physics, Georg-August-Universität Göttingen, 37077 Göttingen, Germany*
ld Sommerfeld Center for Theoretical Physics, Ludwig-Maximilians-Universität München, 80333 Munich, Germany
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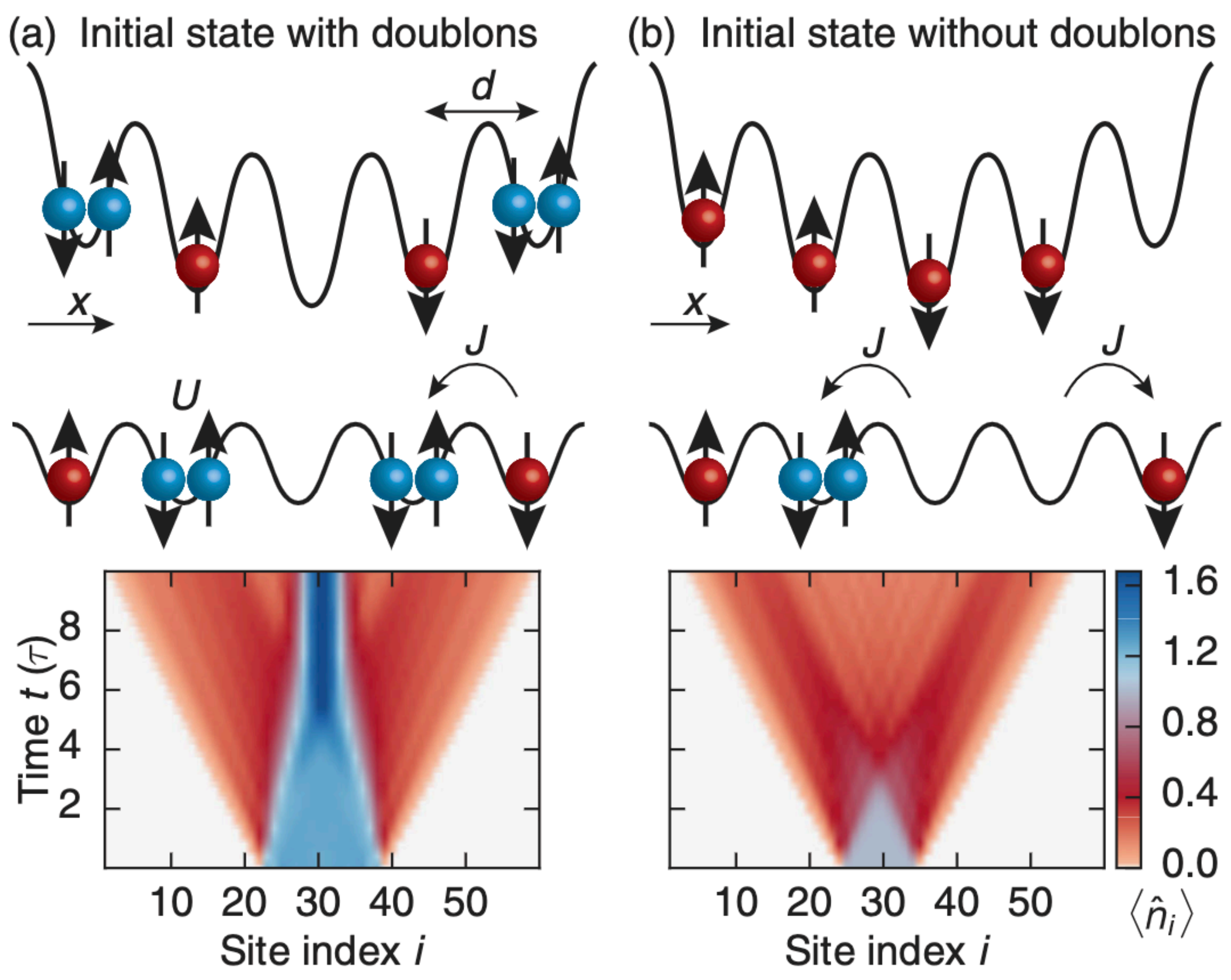
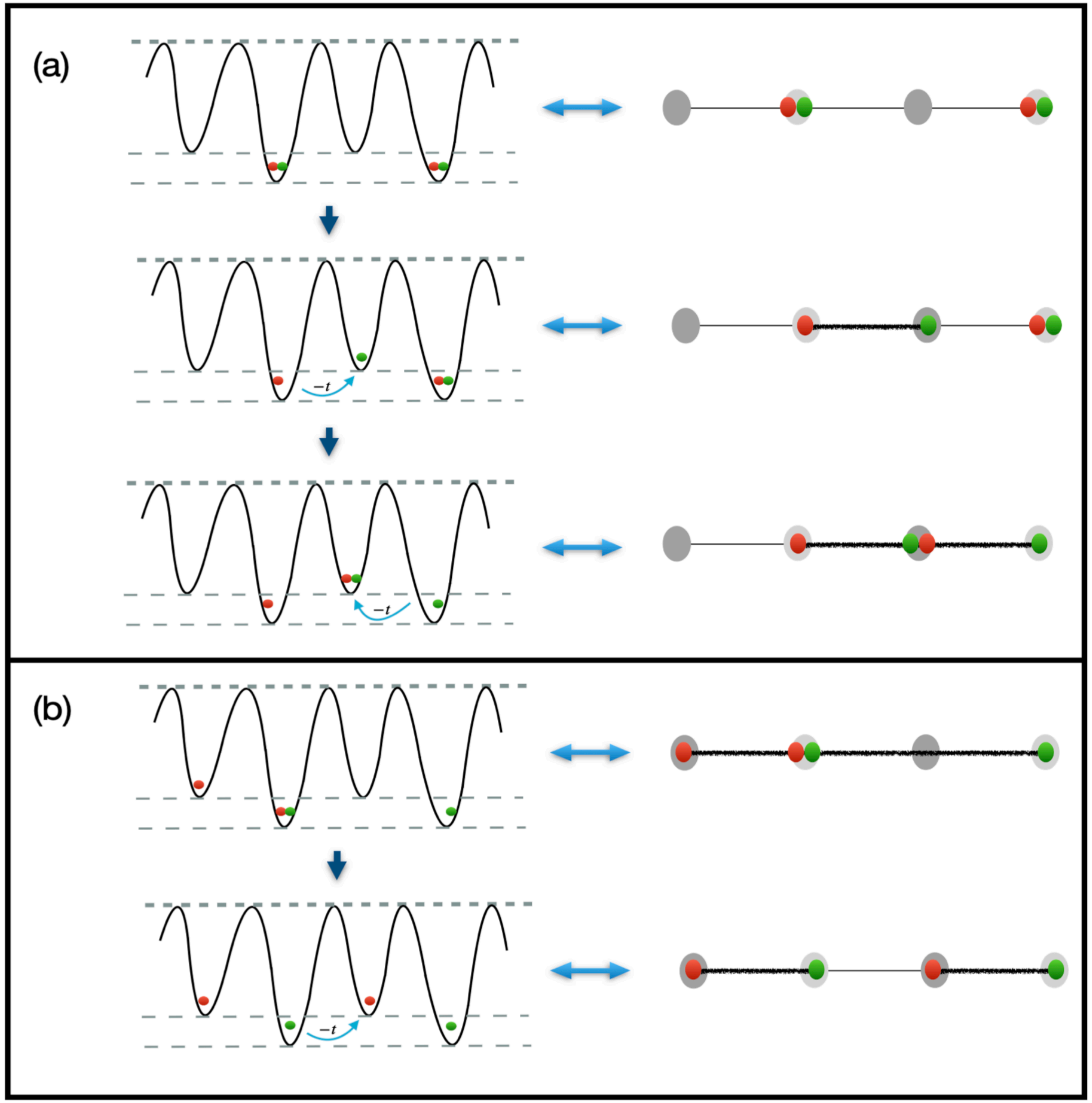
(Received 20 June 2018; revised manuscript received 21 August 2018; published 25 September 2018)

We experimentally and numerically investigate the sudden expansion of fermions in a homogeneous one-dimensional optical lattice. For initial states with an appreciable amount of doublons, we observe a dynamical phase separation between rapidly expanding singlons and slow doublons remaining in the trap center, realizing the key aspect of fermionic quantum distillation in the strongly interacting limit. For initial states without doublons, we find a reduced interaction dependence of the asymptotic expansion speed compared to bosons, which is explained by the interaction energy produced in the quench.



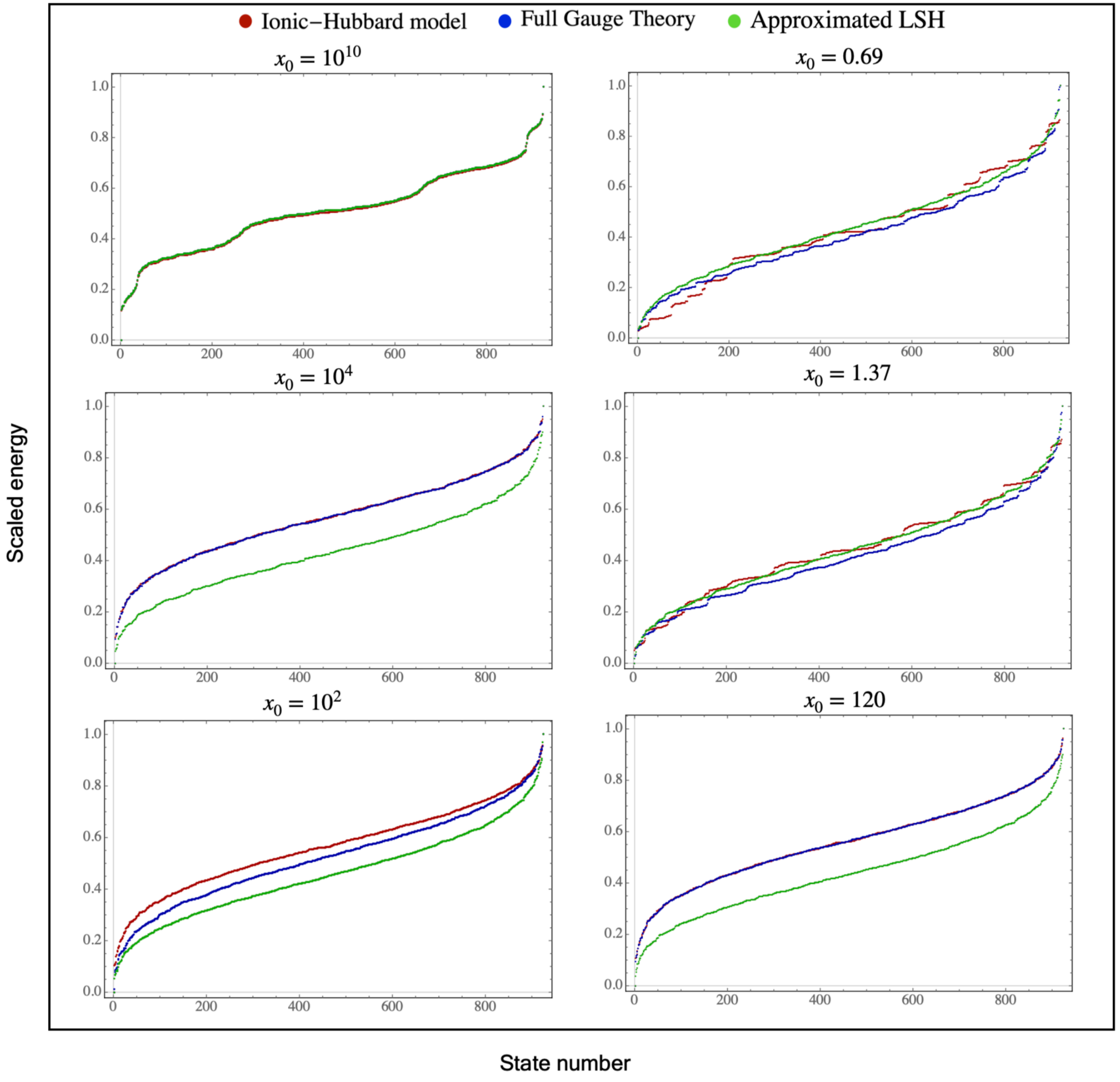
Application: Analog Quantum Simulation

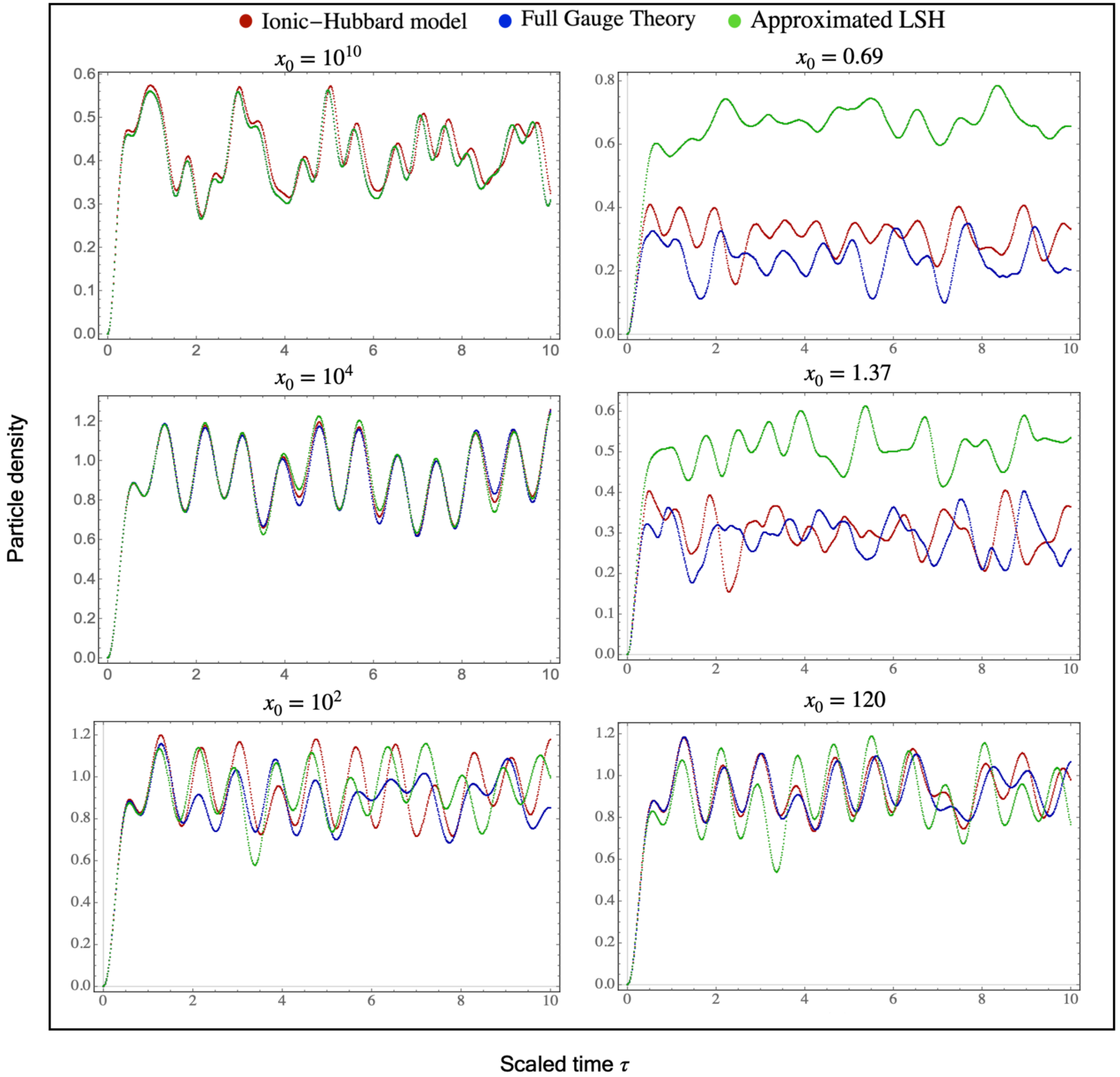
Simulated Dynamics: cartoon



Application: Analog Quantum computation

Numerical Comparison: Exact diagonalization on 6 site lattice





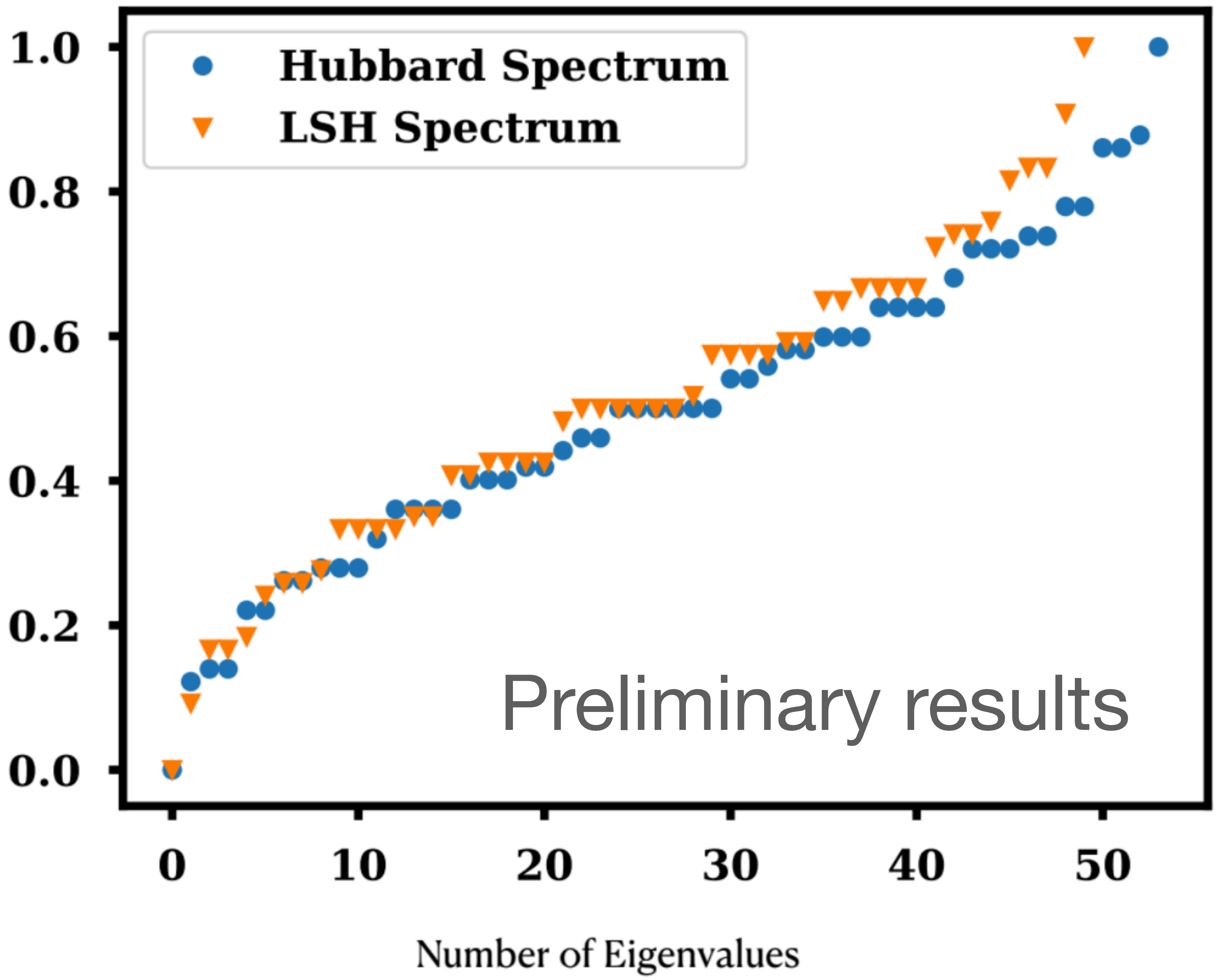
$$\tau_{\text{exp}} = \frac{\hbar \tau_{\text{atomic}}}{t} \equiv \frac{\tau_{\text{atomic}}}{1.5716} \text{ ms}$$
$$\Rightarrow \equiv \frac{2a\tau_{\text{gauge}}}{1.5716} \text{ms.}$$

Generalization of this scheme for SU(3): under investigation

1+1d dynamics: dynamics of SU(3) strings and hadrons in LSH framework

Continuity of strings are guaranteed by AGL: protected by global symmetries

Simulated by SU(3) Fermi-Hubbard model



Collaborators: Madhumita Kabiraj, Emil Mathew, Raka Dasgupta

Benefits of working in the LSH framework: Digital quantum simulation

Already demonstrated for SU(2)

PHYSICAL REVIEW RESEARCH **2**, 033039 (2020)

Solving Gauss's law on digital quantum computers with loop-string-hadron digitization

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Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195, USA

(Received 22 April 2020; accepted 4 June 2020; published 9 July 2020)

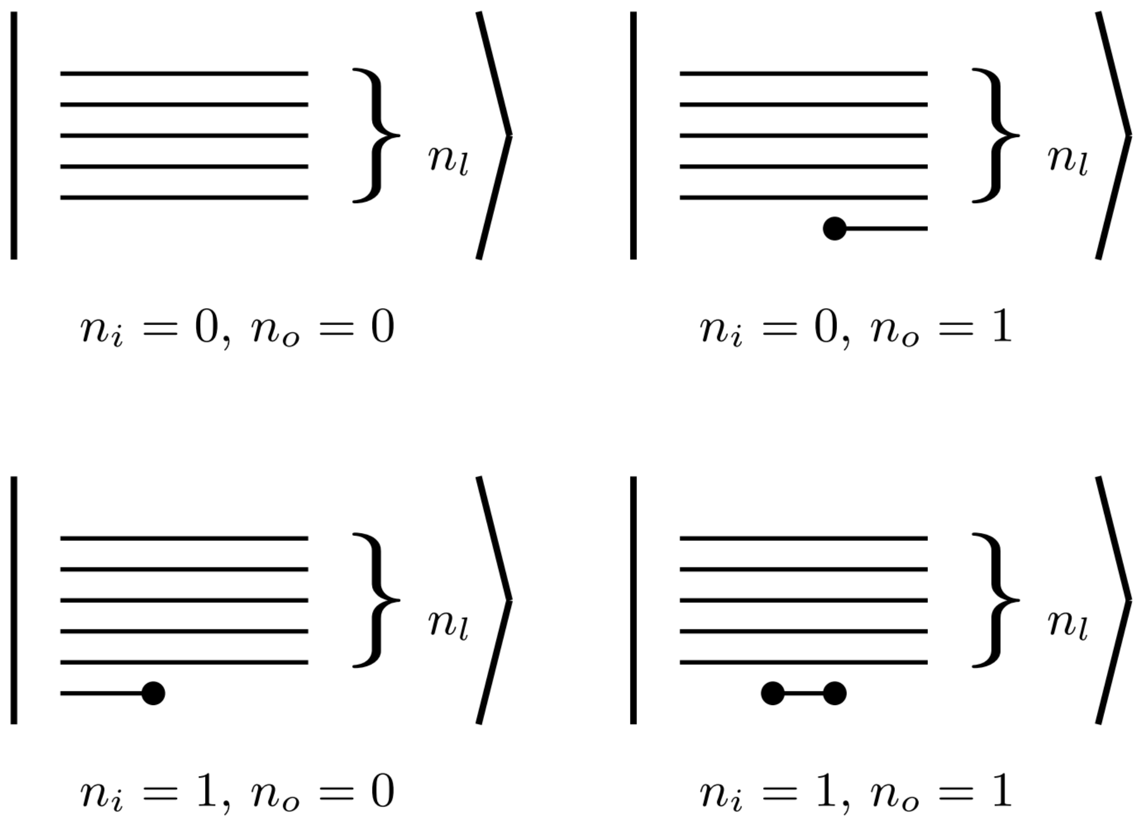
We show that using the loop-string-hadron (LSH) formulation of SU(2) lattice gauge theory (I. Raychowdhury and J. R. Stryker, Phys. Rev. D **101**, 114502 (2020)) as a basis for digital quantum computation easily solves an important problem of fundamental interest: implementing gauge invariance (or Gauss's law) exactly. We first discuss the structure of the LSH Hilbert space in d spatial dimensions, its truncation, and its digitization with qubits. Error detection and mitigation in gauge theory simulations would benefit from physicality "oracles," so we decompose circuits that flag gauge-invariant wave functions. We then analyze the logical qubit costs and entangling gate counts involved with the protocols. The LSH basis could save or cost more qubits than a Kogut-Susskind-type representation basis, depending on how the bases are digitized as well as the spatial dimension. The numerous other clear benefits encourage future studies into applying this framework.

DOI: 10.1103/PhysRevResearch.2.033039

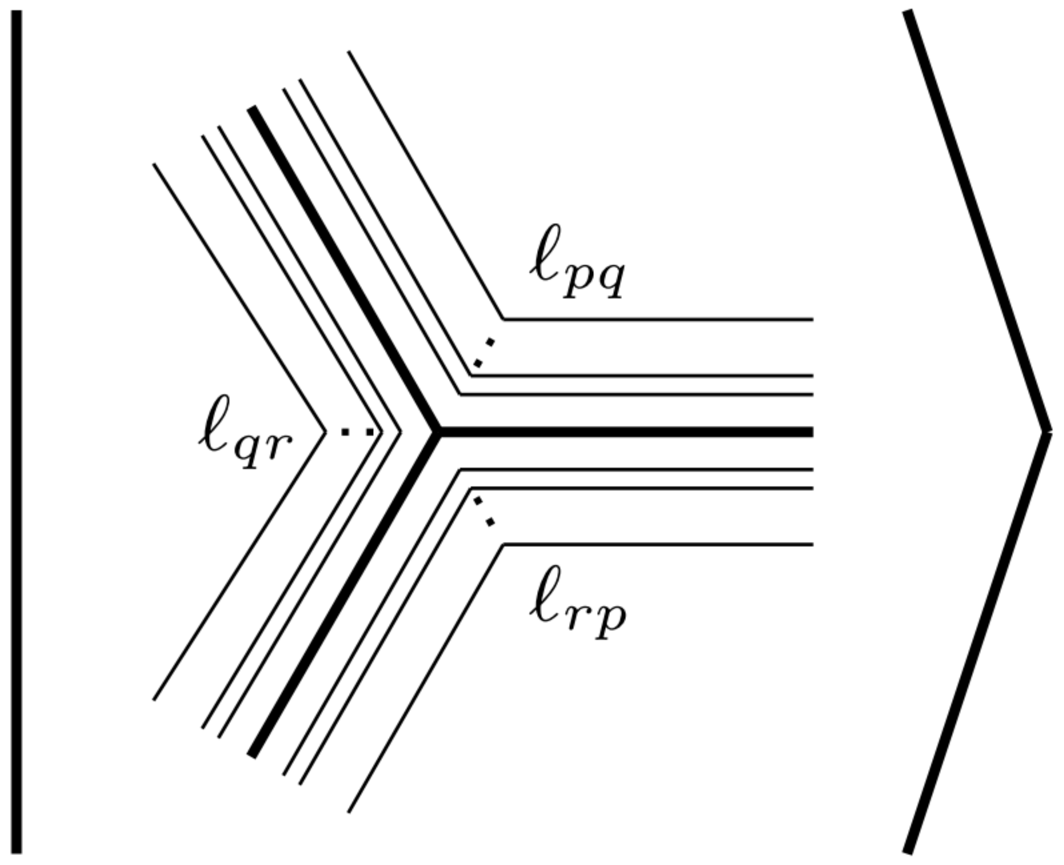
Digitization of LSH Hilbert space

Binary Representation of Loop Quantum Numbers

Matter site



Gluonic site



$$\sim |\ell_{pq}, \ell_{qr}, \ell_{rp}\rangle$$

(i) $N+1$ qubits per quark site loop number n_ℓ,
(ii) N qubits per gluonic site loop number ℓ_{ij},

where

$$N = \lceil \log_2(\bar{j}+1) \rceil .$$

The quark occupancy numbers require no truncation.

$$n_\ell = \sum_{m=0}^{N} 2^m n_{\ell,m} \quad (n_{\ell,m} = 0,\, 1),$$

$$\ell_{ij} = \sum_{m=0}^{N-1} 2^m \ell_{ij,m} \quad (\ell_{ij,m} = 0,\, 1)$$

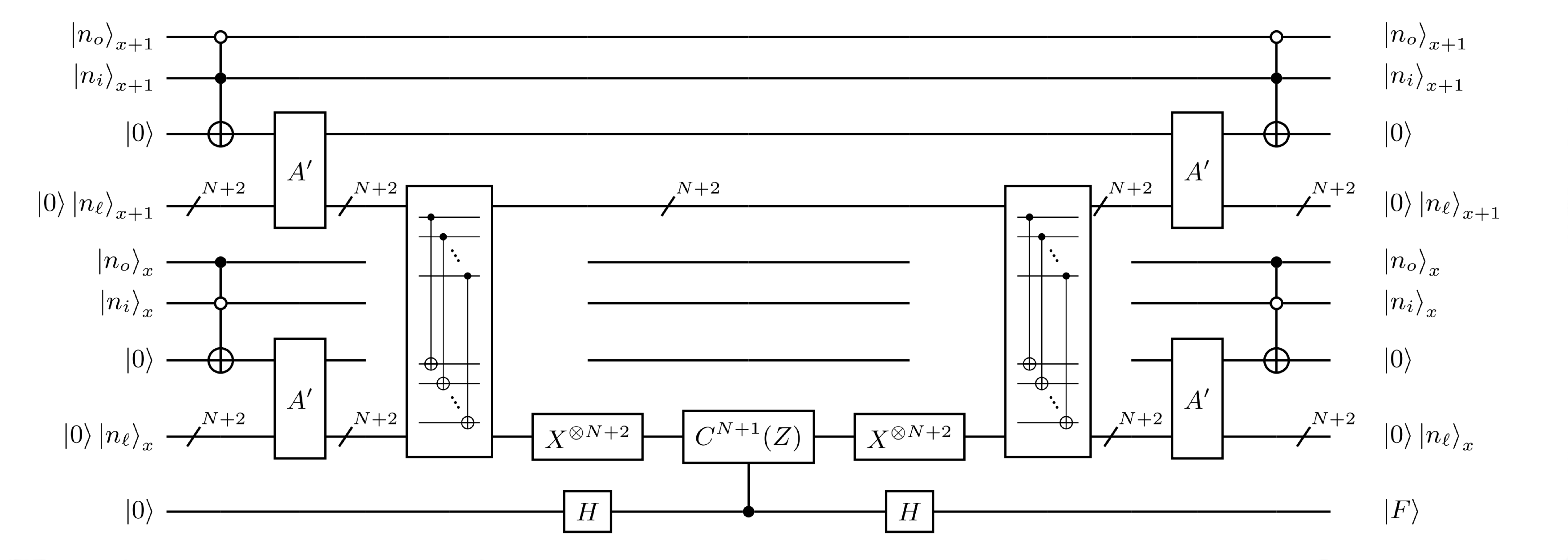
$$|n_\ell\rangle = \bigotimes_{m=0}^{N} |n_{\ell,m}\rangle \,,$$

$$|\ell_{ij}\rangle = \bigotimes_{m=0}^{N-1} |\ell_{ij,m}\rangle \,.$$

Oracle to check the Abelian Gauss law

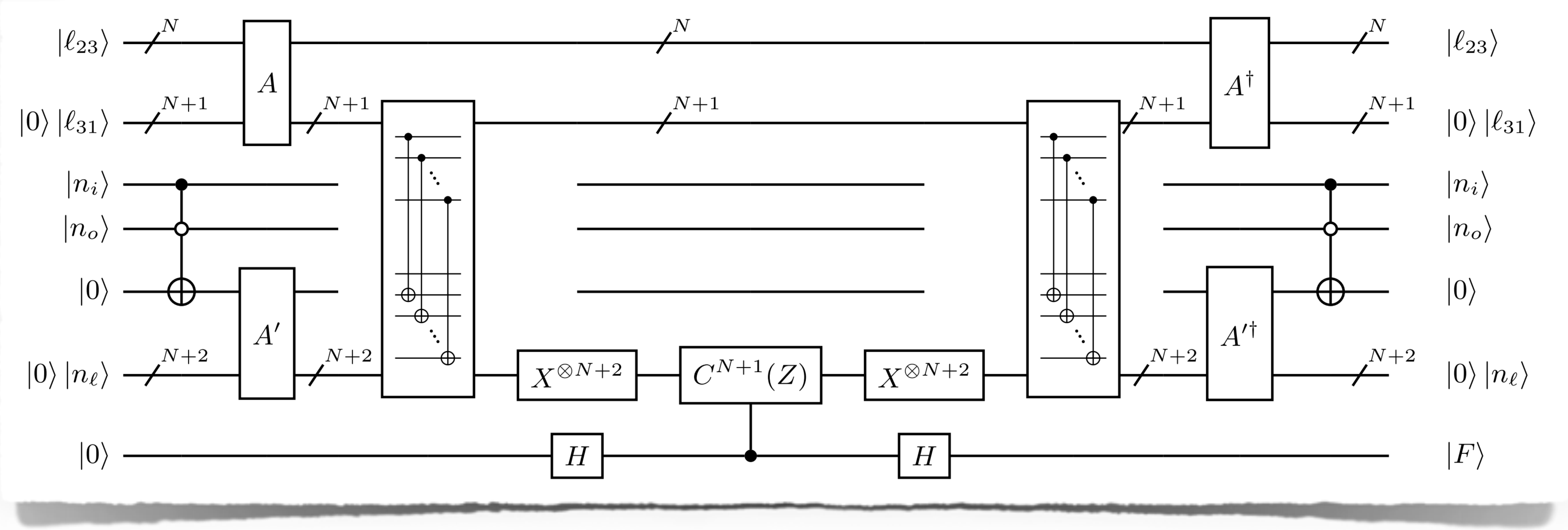
1d

qq

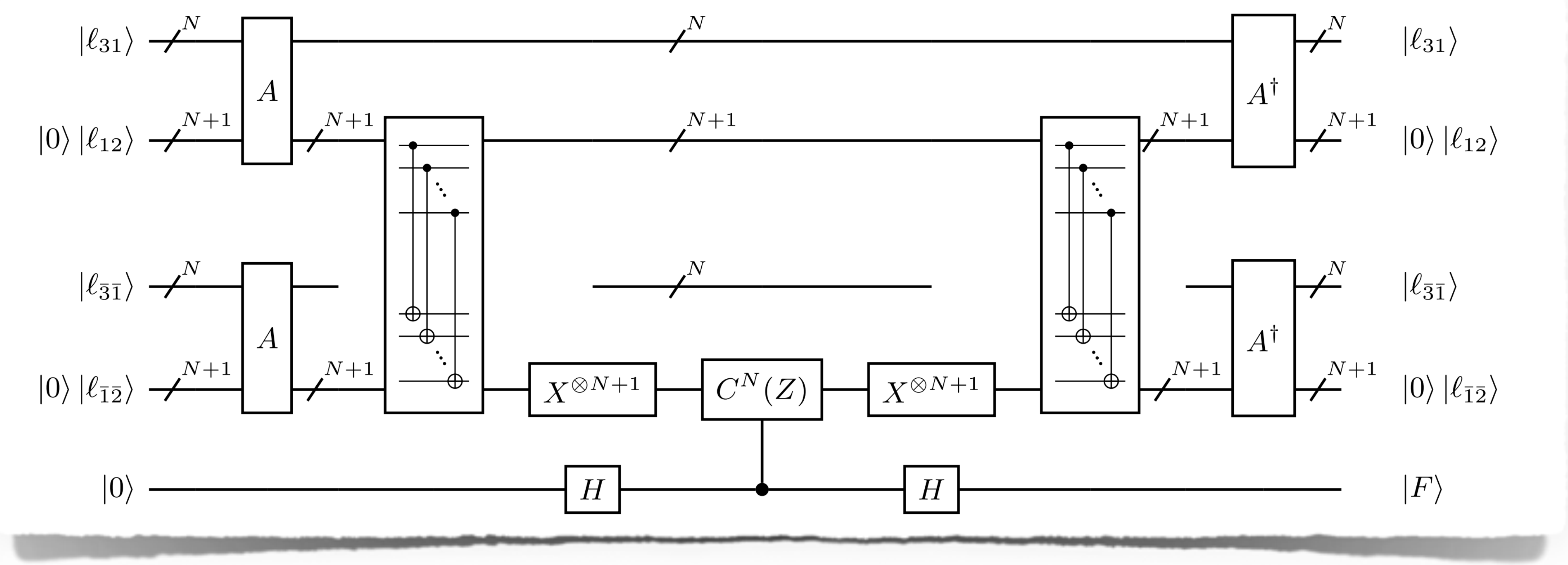


d >1

qg



gg

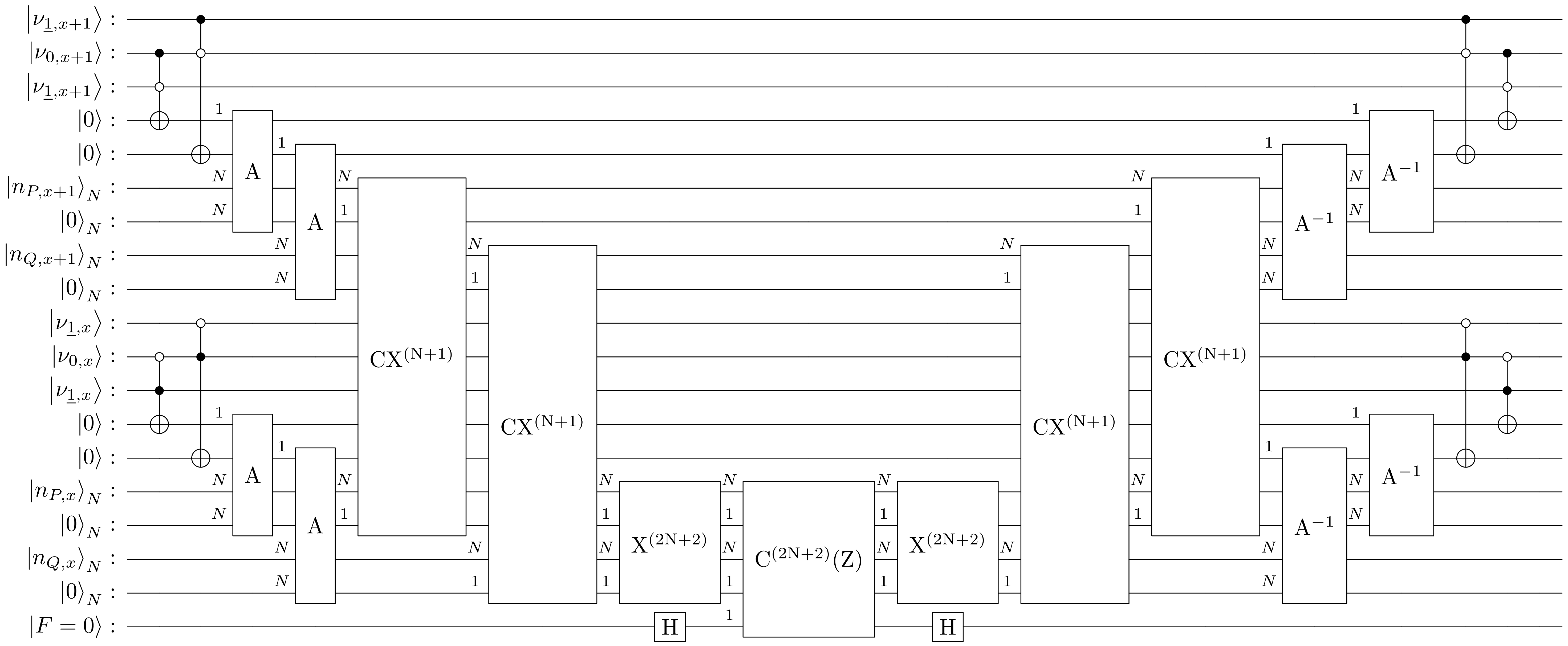


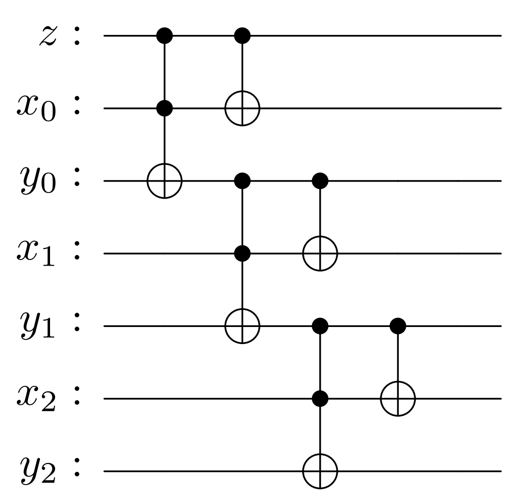
Benefits of working in the LSH framework: Applications in quantum simulation

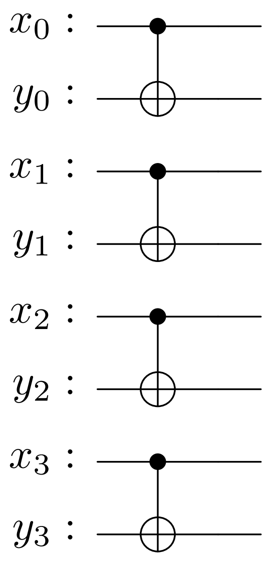
Work in progress for SU(3)

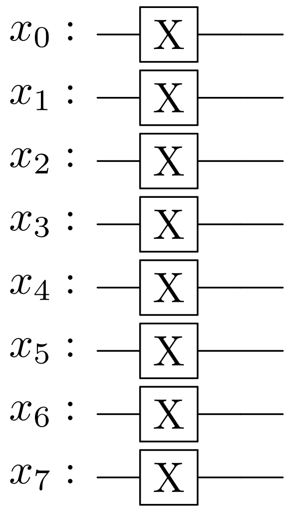
Fran Ilčić,
Grad. student,
BITS Goa

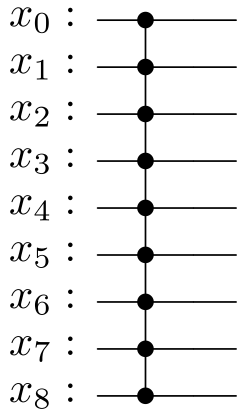
Physicality Oracle: Preliminary construction











Useful component for state preparation algorithms such as QAOA and error detection in a simulation

Benefits of working in the LSH framework: Applications in quantum simulation

Already demonstrated for SU(2)

Digital Quantum Computation

Quantum
the open journal for quantum science

UMD-PP-022-13, IQuS_WKSHP@UW-22-001

General quantum algorithms for Hamiltonian simulation with applications to a non-Abelian lattice gauge theory

Zohreh Davoudi,[1,2,*] Alexander F. Shaw,[3,†] and Jesse R. Stryker[1,‡]

A detailed analysis establishes benefits of using LSH framework on universal quantum computers both in

near-term

m/g	$\Delta_{\text{Trot.}}$	x	L	η	t/a_s	Schwinger bosons Qubits	Schwinger bosons Min. s	Schwinger bosons Min. CNOTs	LSH Qubits	LSH Min. s	LSH Min. CNOTs
1	10%	0.1	10	2	1	92	186	4.8613×10^6	40	63	2.63088×10^5
1	10%	0.1	10	2	5	92	2072	5.41538×10^7	40	702	2.93155×10^6
1	10%	0.1	10	4	1	164	433	5.21403×10^8	60	136	1.64261×10^6
1	10%	0.1	10	4	5	164	4841	5.82936×10^9	60	1519	1.83465×10^7
1	10%	0.1	20	2	1	192	262	1.44561×10^7	80	89	7.84624×10^5
1	10%	0.1	20	2	5	192	2929	1.61611×10^8	80	993	8.75429×10^6
1	10%	0.1	20	4	1	344	613	1.55832×10^9	120	193	4.92111×10^6
1	10%	0.1	20	4	5	344	6846	1.74034×10^{10}	120	2149	5.47952×10^7

far-term

m/g	x	η	L	t/a_s	Δ	$\alpha_{\text{Trot.}}$	$\alpha_{\text{Newt.}}$	Schwinger bosons Qubits	Schwinger bosons T gates	LSH Qubits	LSH T gates
1	1	4	100	1	0.01	90%	9%	2626	8.19713×10^{11}	1319	3.91817×10^{10}
1	1	4	100	1	0.001	90%	9%	2704	3.09951×10^{12}	1397	1.5172×10^{11}
1	1	4	100	10	0.01	90%	9%	2704	3.0993×10^{13}	1397	1.51643×10^{12}
1	1	4	100	10	0.001	90%	9%	2808	1.2146×10^{14}	1475	5.76229×10^{12}
1	1	4	1000	1	0.01	90%	9%	18904	3.12769×10^{13}	6797	1.53099×10^{12}
1	1	4	1000	1	0.001	90%	9%	19008	1.22564×10^{14}	6875	5.81562×10^{12}
1	1	4	1000	10	0.01	90%	9%	19008	1.22564×10^{15}	6875	5.81468×10^{13}
1	1	4	1000	10	0.001	90%	9%	19086	4.48657×10^{15}	6979	2.29217×10^{14}

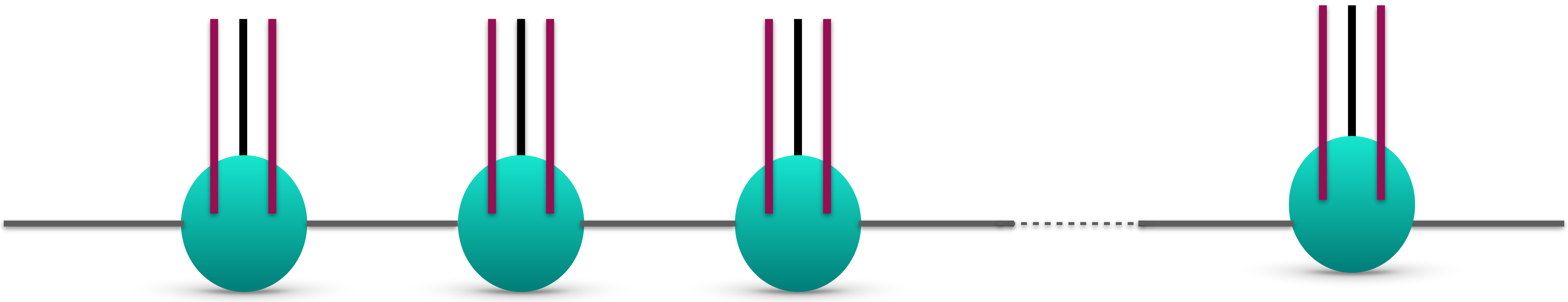
"The loop-string-hadron formulation further retains the non-Abelian gauge symmetry despite the inexactness of the digitized simulation, without the need for costly controlled operations. Such theoretical and algorithmic considerations are likely to be essential in quantumly simulating other complex theories of relevance to nature."

Benefits of working in the LSH framework: Applications in quantum simulation

Other ongoing works:

Tensor network calculations for non-Abelian gauge theories

Matrix Product State Ansatz for LSH in one spatial dimension



On-site tensor with three physical indices:
1 bosonic and 2 fermionic

Other ongoing works:

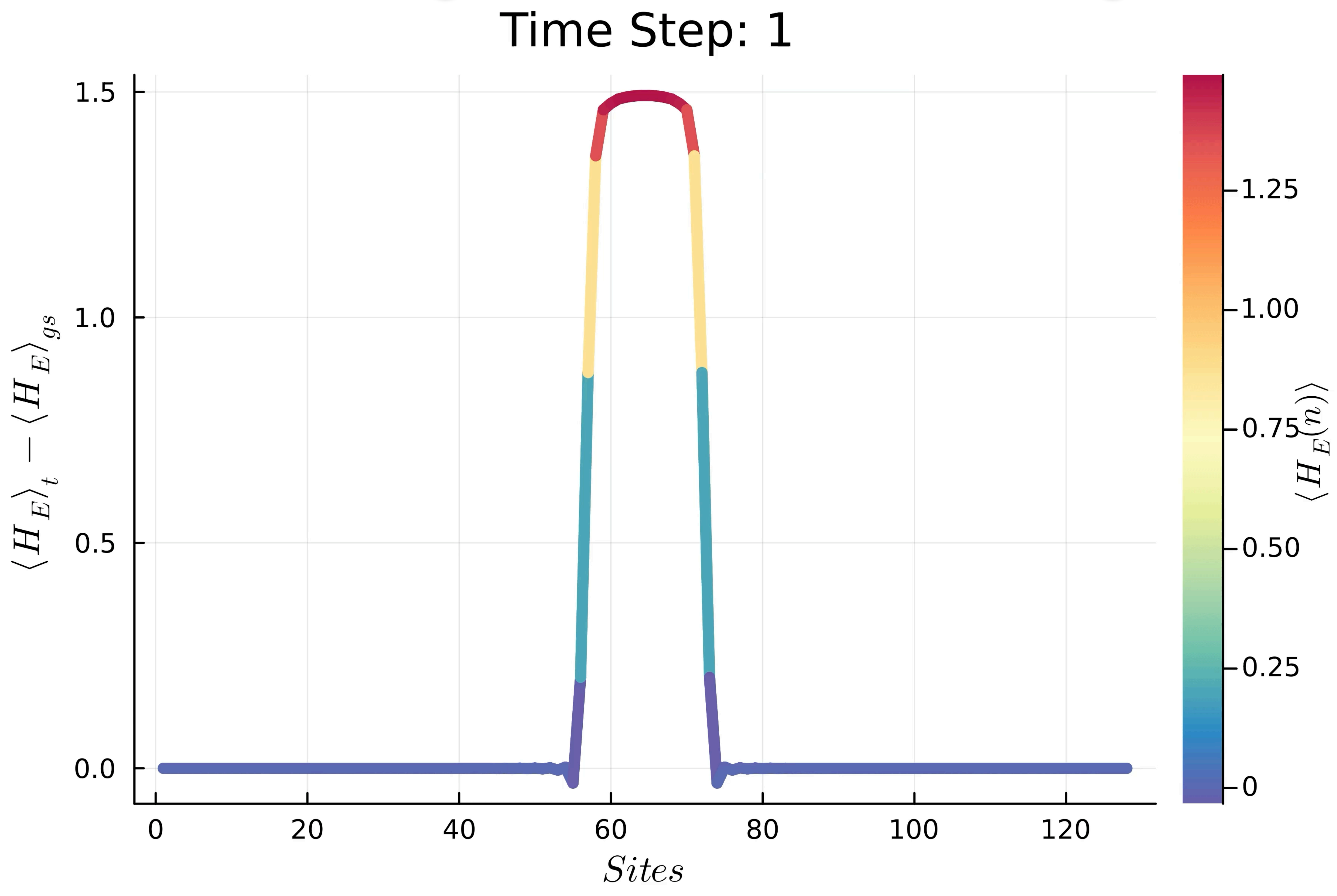
Tensor network calculations for non-Abelian gauge theories

Code is developed and benchmarked with exact-diagonalization for small systems

Produces static and dynamic results

MPS Calculations using LSH framework

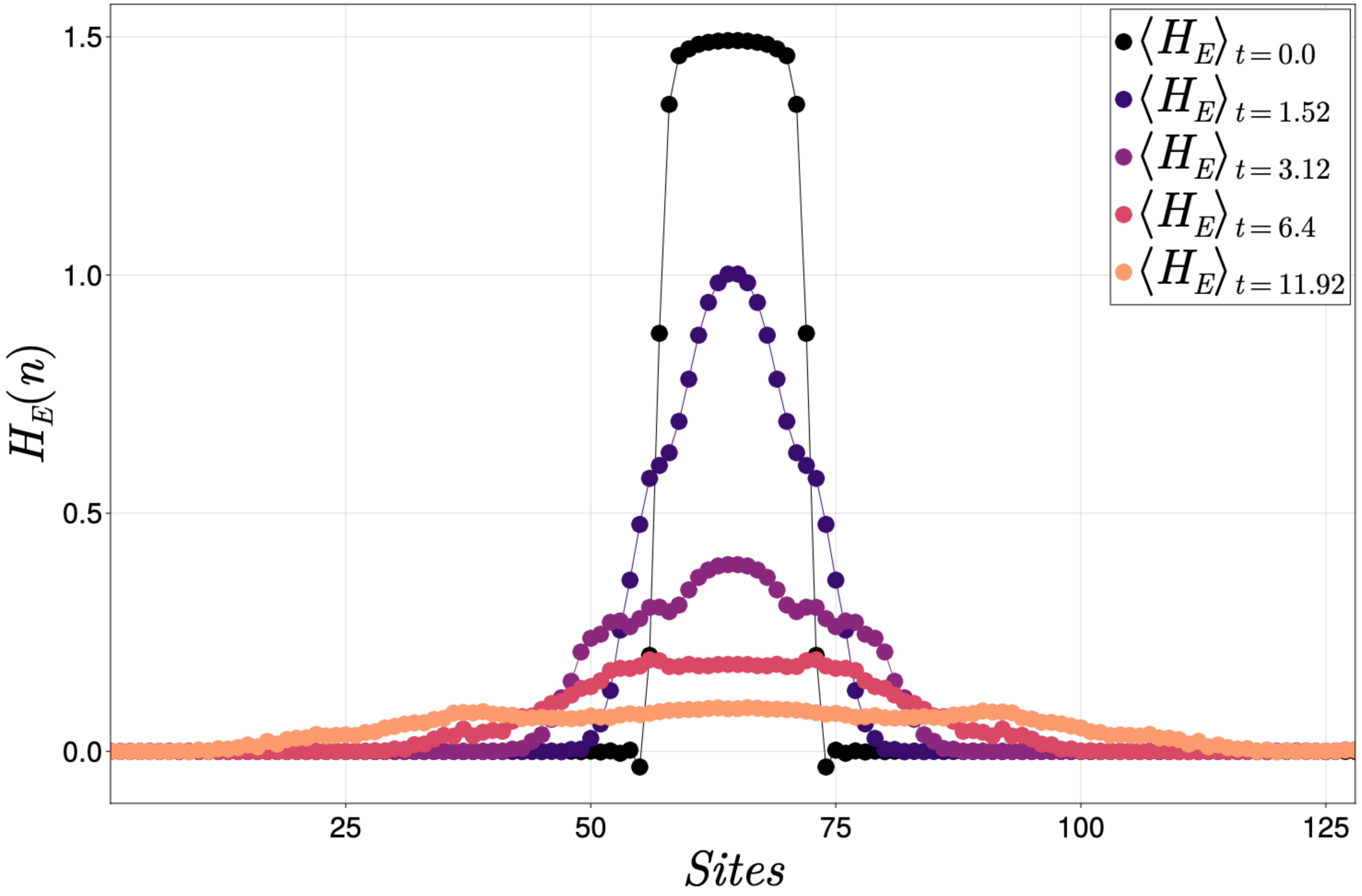
Time-evolution of a string state on the interacting vacuum

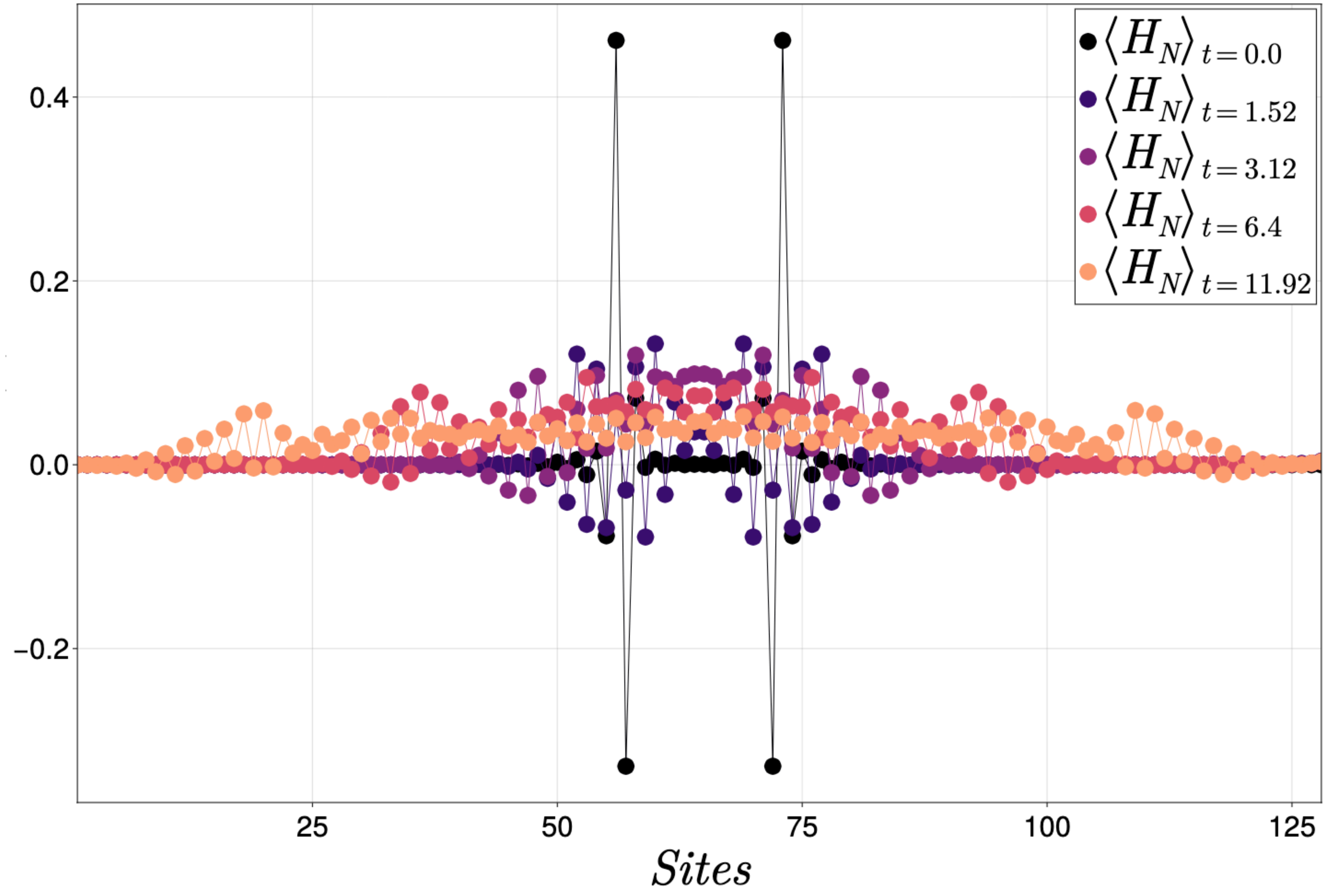


Benefits of working in the LSH framework: Applications in quantum simulation

Other ongoing works:

Time-evolution of a dynamical string state

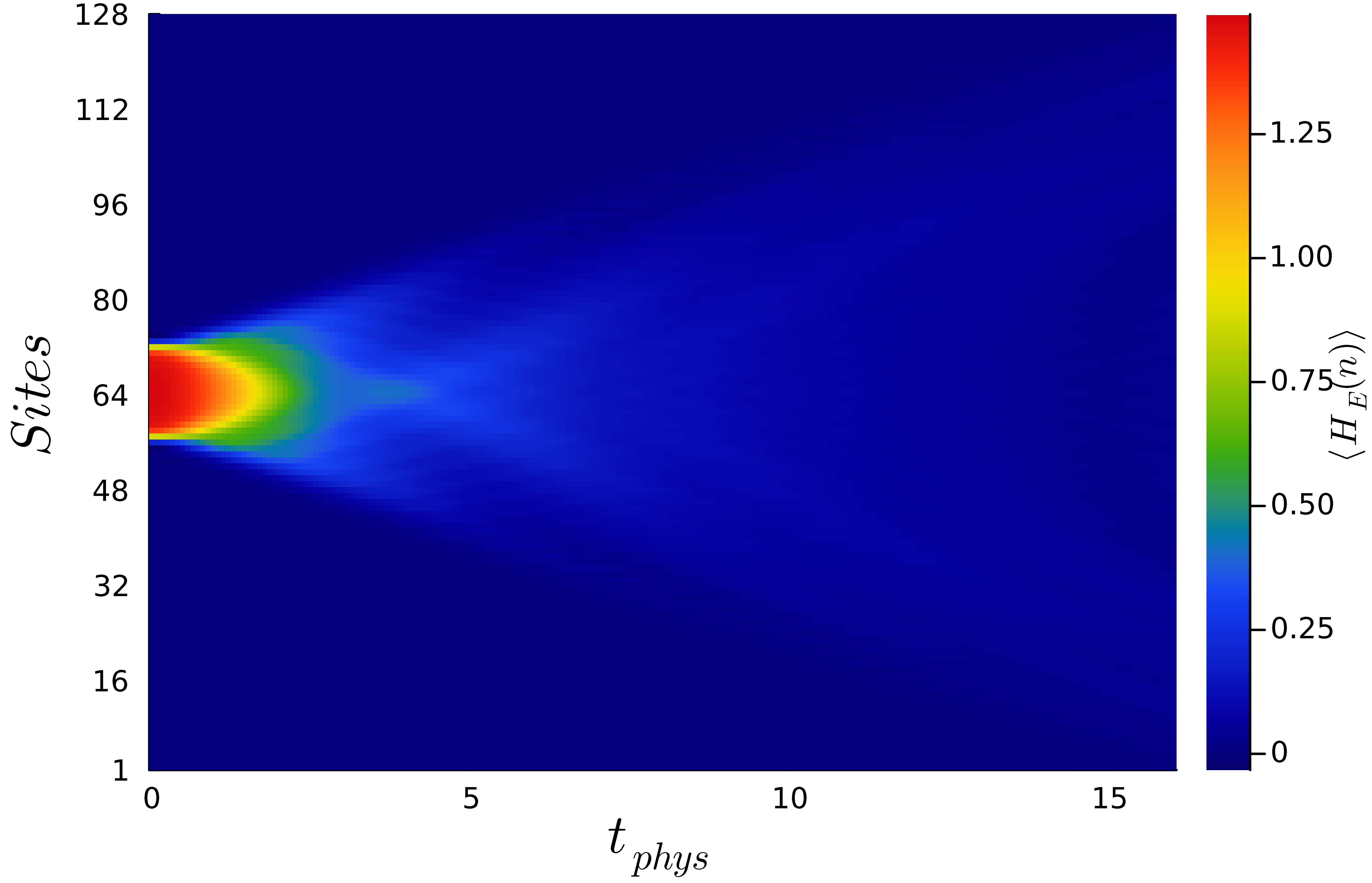


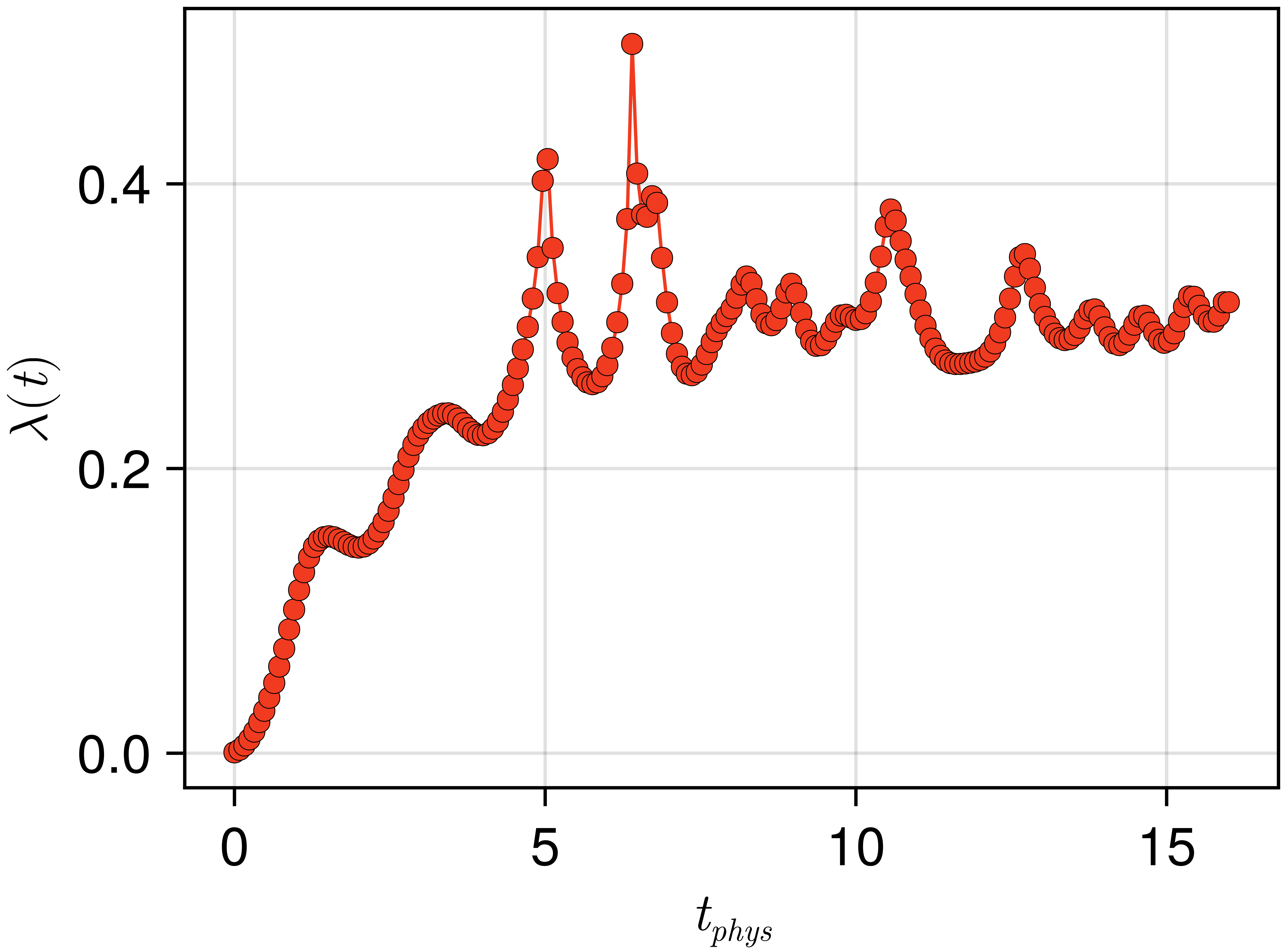


MPS Calculations using LSH framework

MPS preparation of interacting vacuum

Time-evolution of a string state on the interacting vacuum

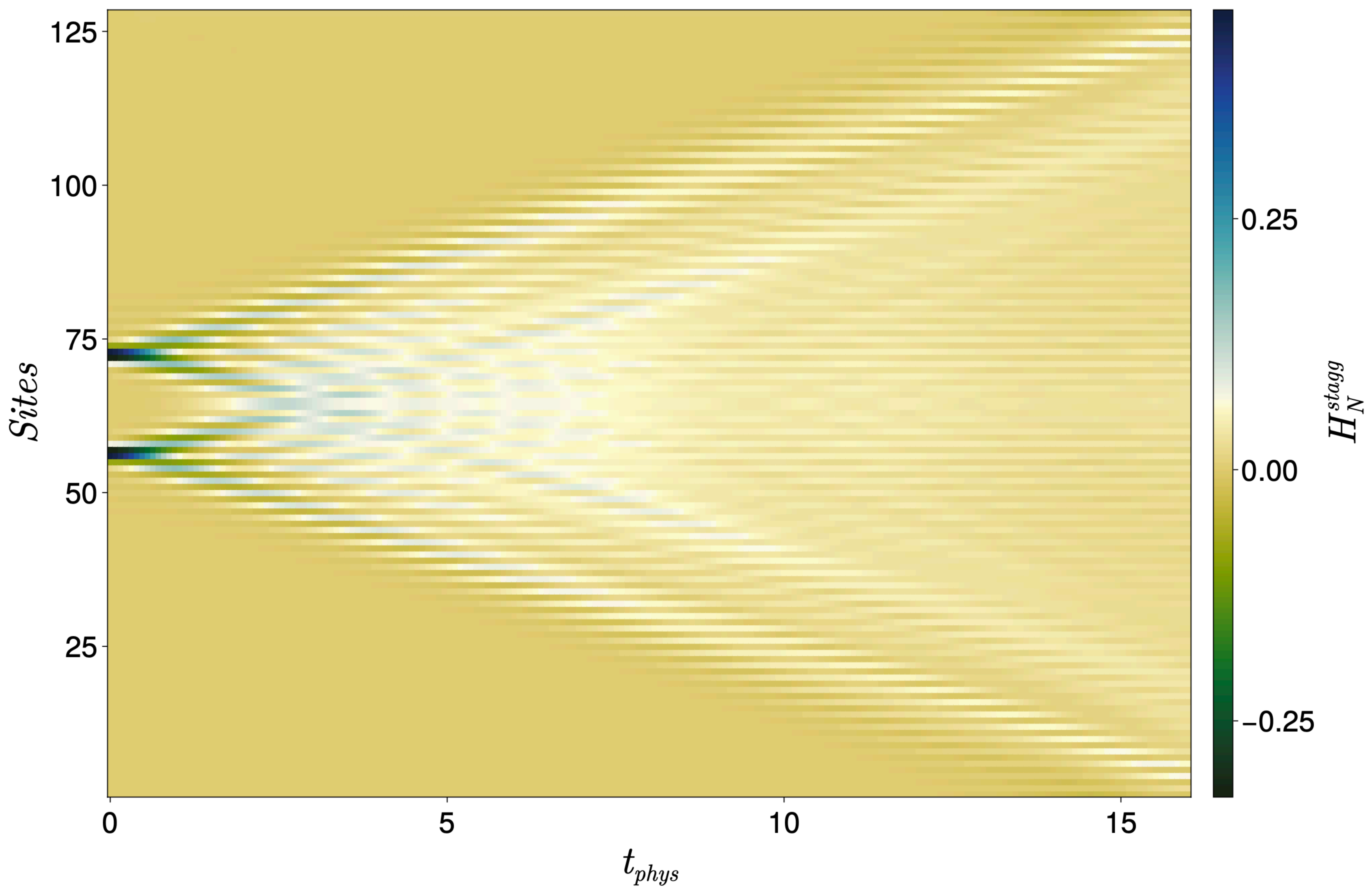


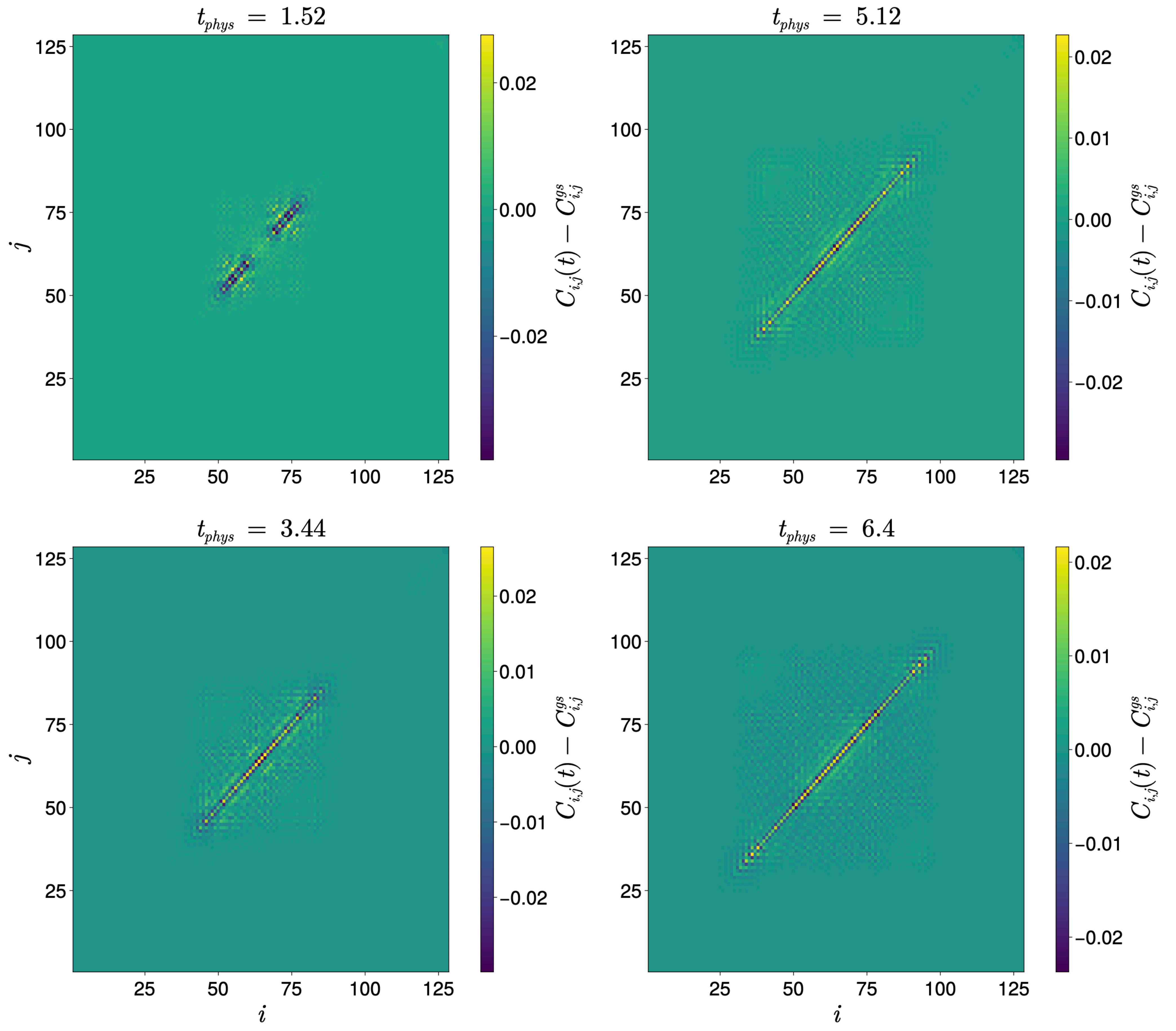


Benefits of working in the LSH framework: Applications in quantum simulation

Other ongoing works:

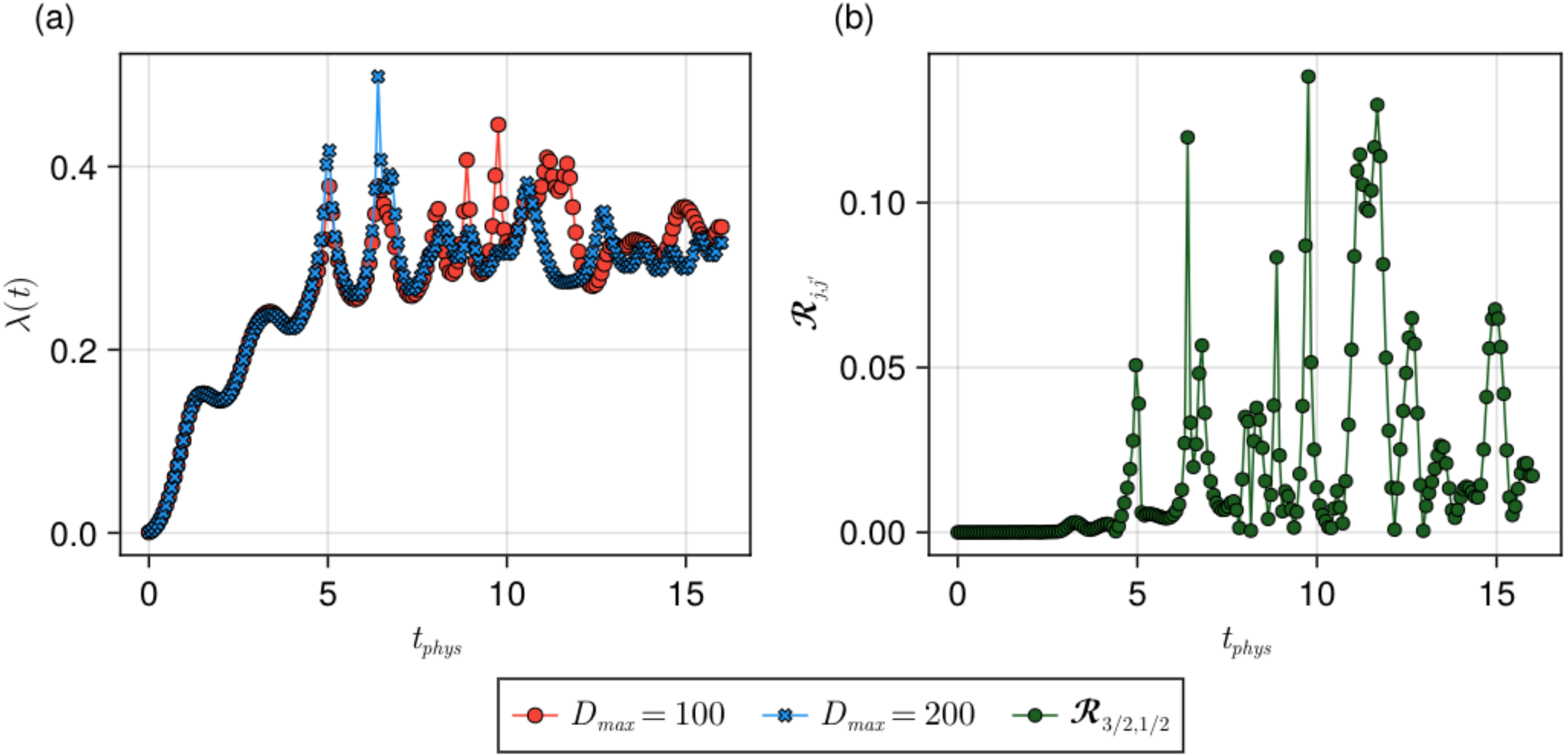
Time-evolution of a dynamical string state





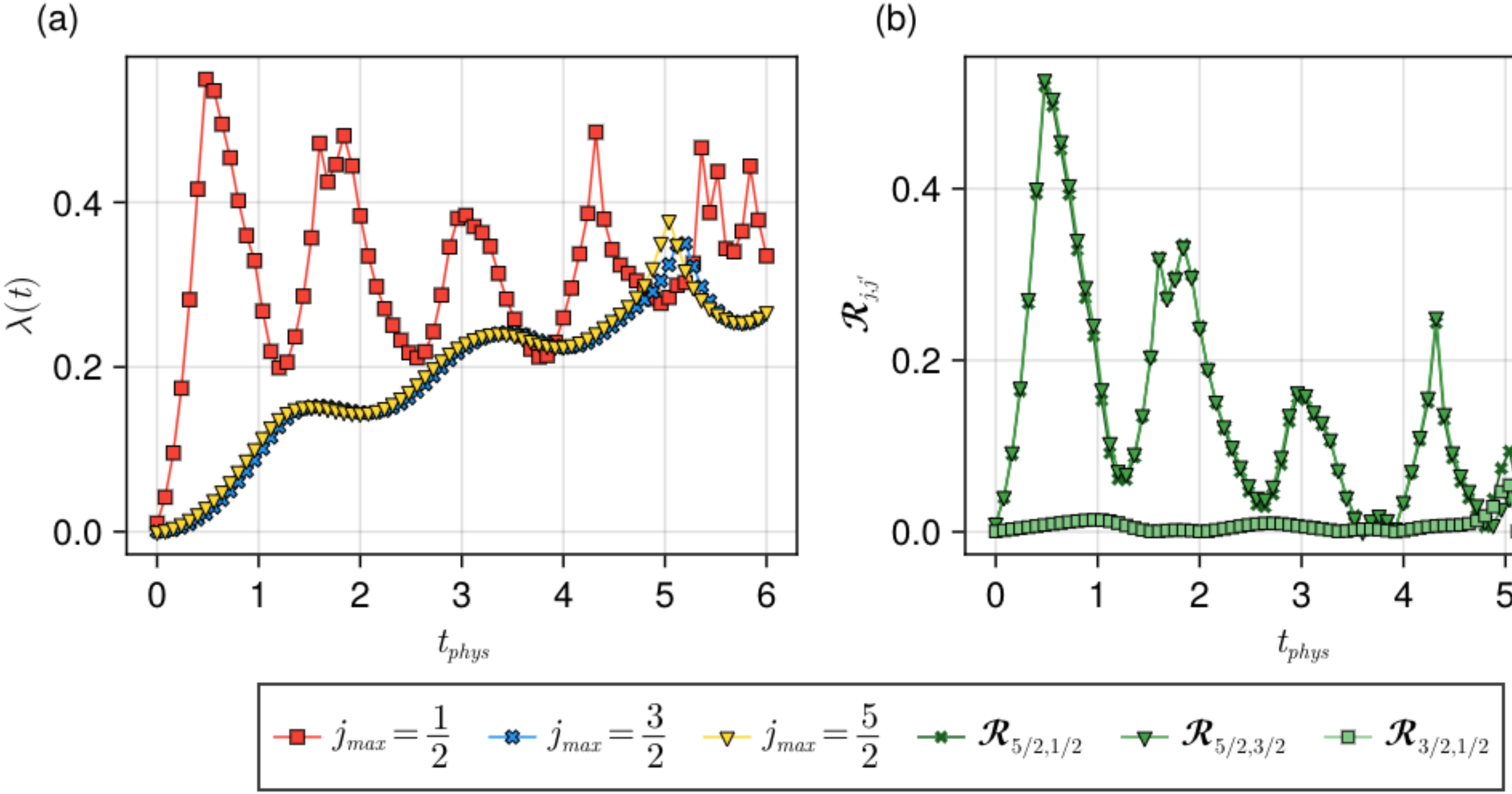
MPS Calculations using LSH framework

Probing effect of finite bond dimension: N=128



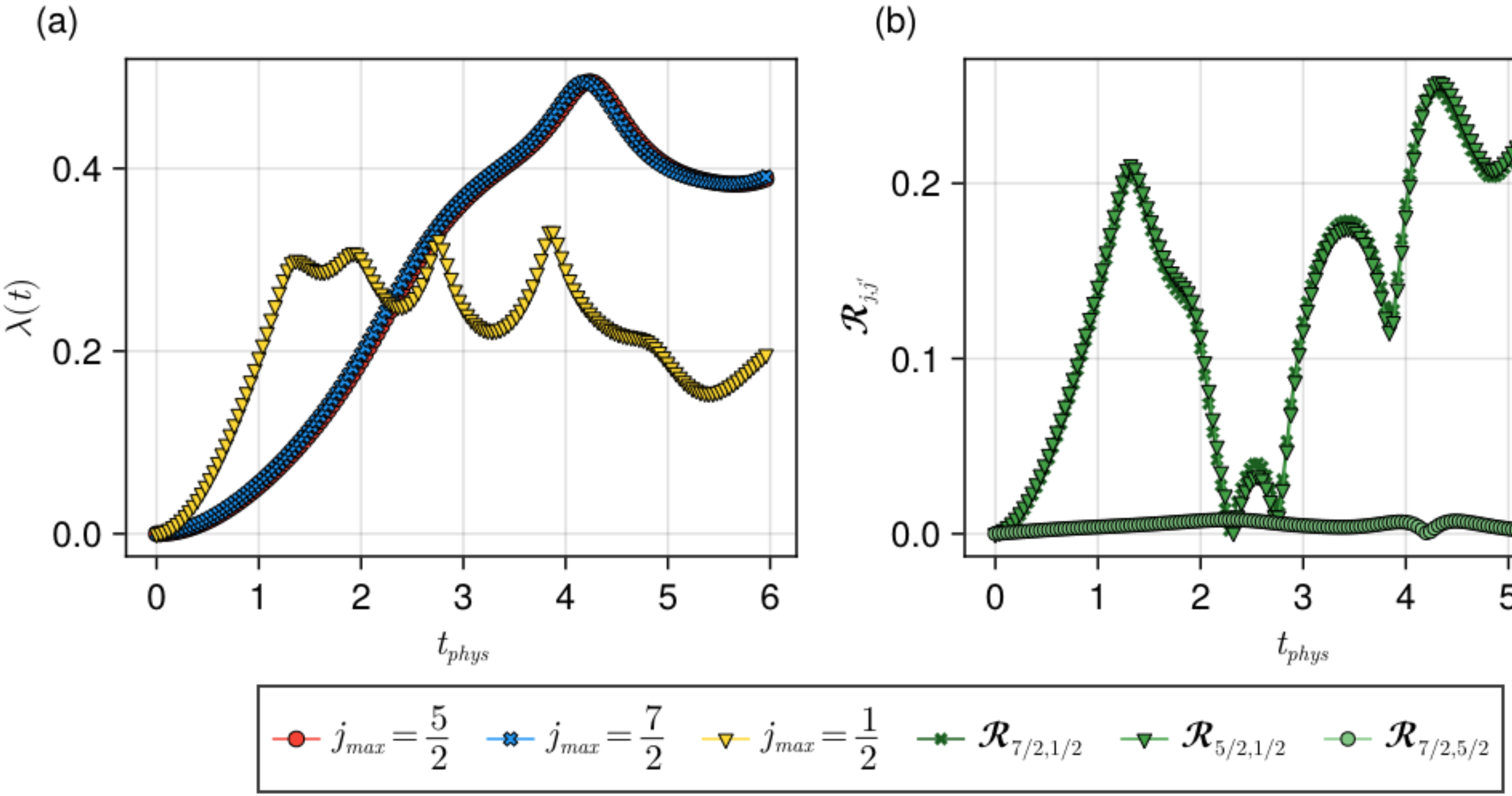
MPS Calculations using LSH framework

Probing cut-off dependence in dynamics: N=128



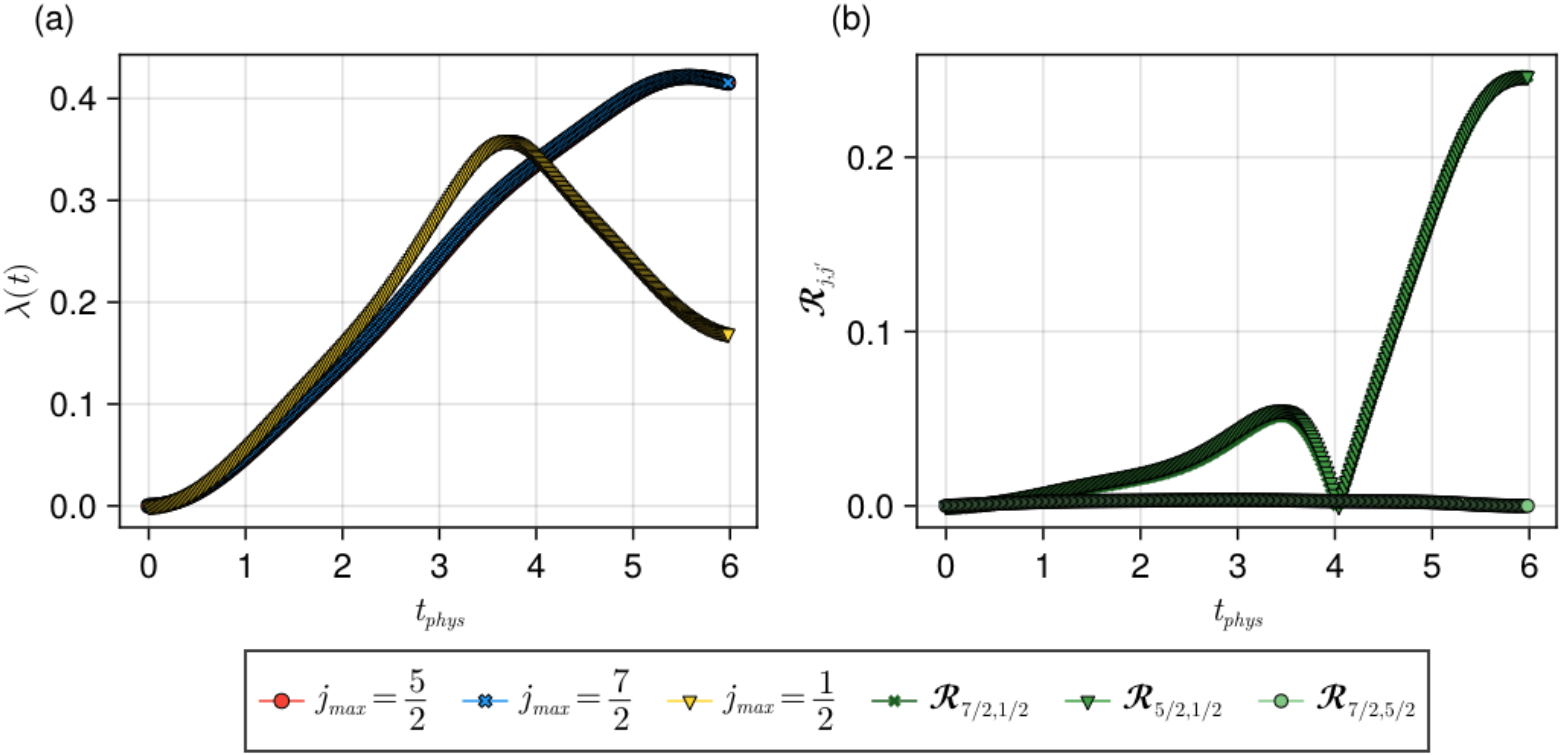
MPS Calculations using LSH framework

Probing cut-off dependence in dynamics: N=64



MPS Calculations using LSH framework

Probing cut-off dependence in dynamics: N=32



Benefits of working in the LSH framework: Applications in quantum simulation

Other ongoing works:

MPS calculations for non-Abelian gauge theories

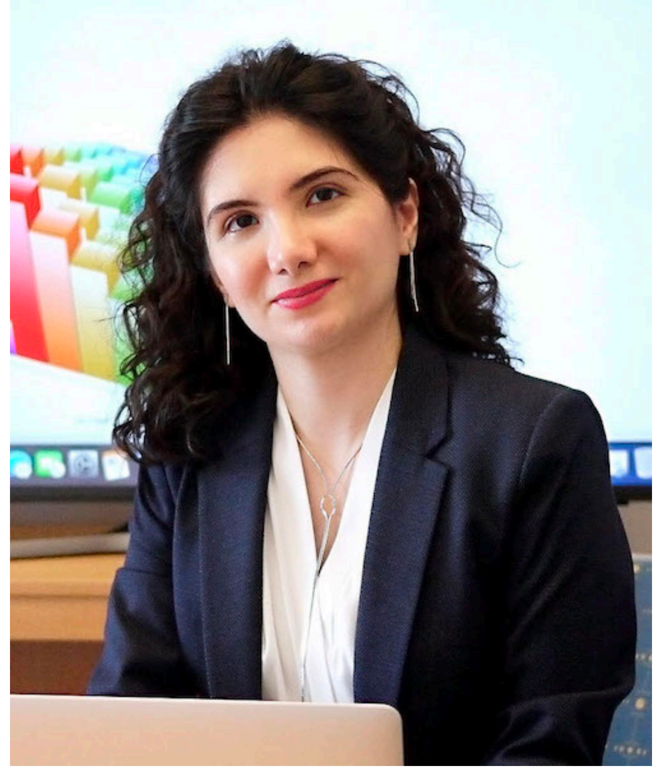
Collaborators:


Emil Mathew


Navya Gupta


Aniruddha Bapat


Saurabh Kadam

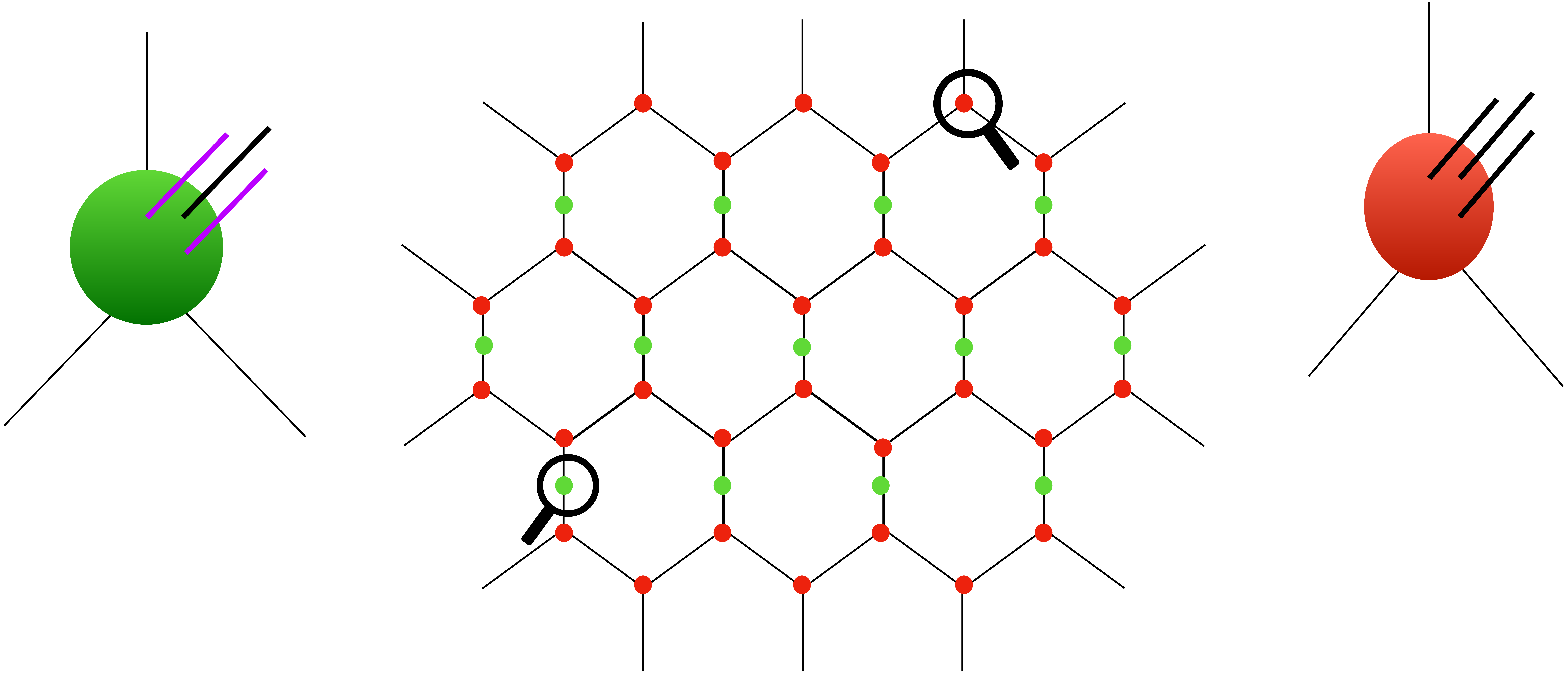

Zohreh Davoudi


Jesse Stryker

Benefits of working in the LSH framework: Applications in quantum simulation

Under construction:

PEPS Ansatz for LSH

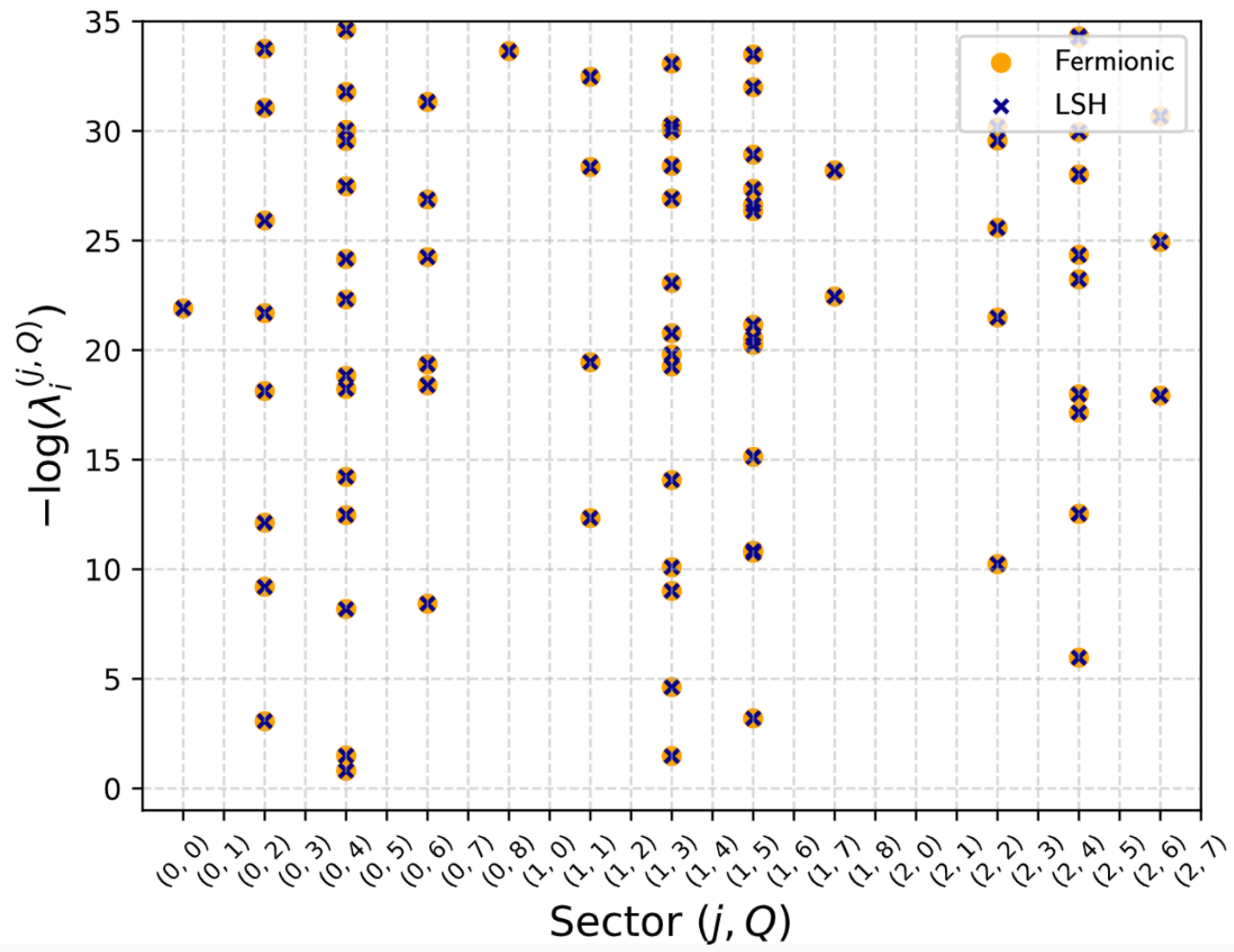


Benefits of working in the LSH framework: Applications in quantum simulation

Other ongoing works:

Different distillation procedure for different choice of framework: symmetry resolved entanglement entropy for non-Abelian gauge theories

Understanding entanglement structure for non-Abelian gauge theories



LSH framework: being abelianized, involve much simpler distillation procedure

Ongoing work:

Thermalization of gauge theories

Q. Does non-Abelian gauge theories exhibit quantum chaos?

Attempt to find if the eigenstate thermalisation hypothesis (ETH) hold for non-Abelian gauge theories

Check for ETH Markers: Level-spacing statistics, diagonal ETH, off-diagonal ETH

Computational tool: Exact diagonalization/ block diagonalization Using LSH framework

Using LSH framework allows to push the boundary in terms of lattice size, cut-off and going beyond SU(2)

Conclusions

Hamiltonian simulation of SU(2) gauge theory is a tough job

$\Rightarrow$ Considerably less progress in quantum simulating the same using angular momentum basis within KS framework

Conclusions

Hamiltonian simulation of SU(2) gauge theory is a tough job

$\Longrightarrow$ Considerably less progress in quantum simulating the same using angular momentum basis within KS framework

LSH framework of SU(2) LGT shows considerable advantage

$\Longrightarrow$ Significant progress in the last couple of years in digital and analog quantum simulating the same

Conclusions

Hamiltonian simulation of SU(2) gauge theory is a tough job

$\Rightarrow$ Considerably less progress in quantum simulating the same using angular momentum basis within KS framework

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Hamiltonian simulation of SU(3) gauge theory is almost an impossible job

$\Rightarrow$ Anaogous SU(3) angular momentum basis is not well understood, No progress so far beyond fully gauge removed 1d lattice

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$\Rightarrow$ Following the path of applications of SU(2) LSH, one can make the first concrete step towards quantum simulating QCD

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$\Rightarrow$ Following the path of applications of SU(2) LSH, one can make the first concrete step towards quantum simulating QCD

LSH framework in 3+1 dimension including multiple quark flavours: QCD

Thank You

BITS Pilani
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