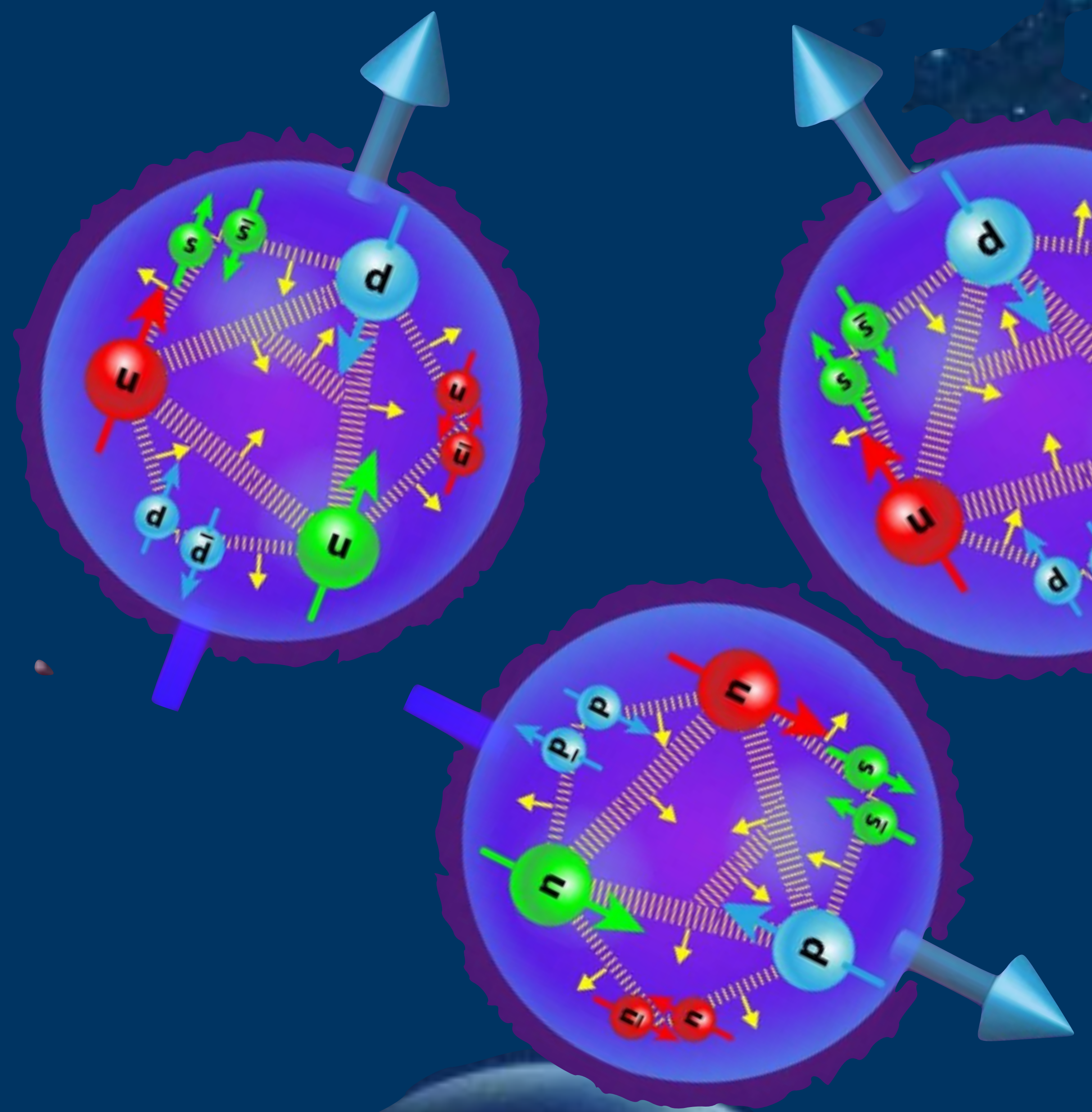
Connecting Quarks to the Cosmos: The Role of Quark Mass

Sanjay Reddy, Institute for Nuclear Theory, University of Washington, Seattle.

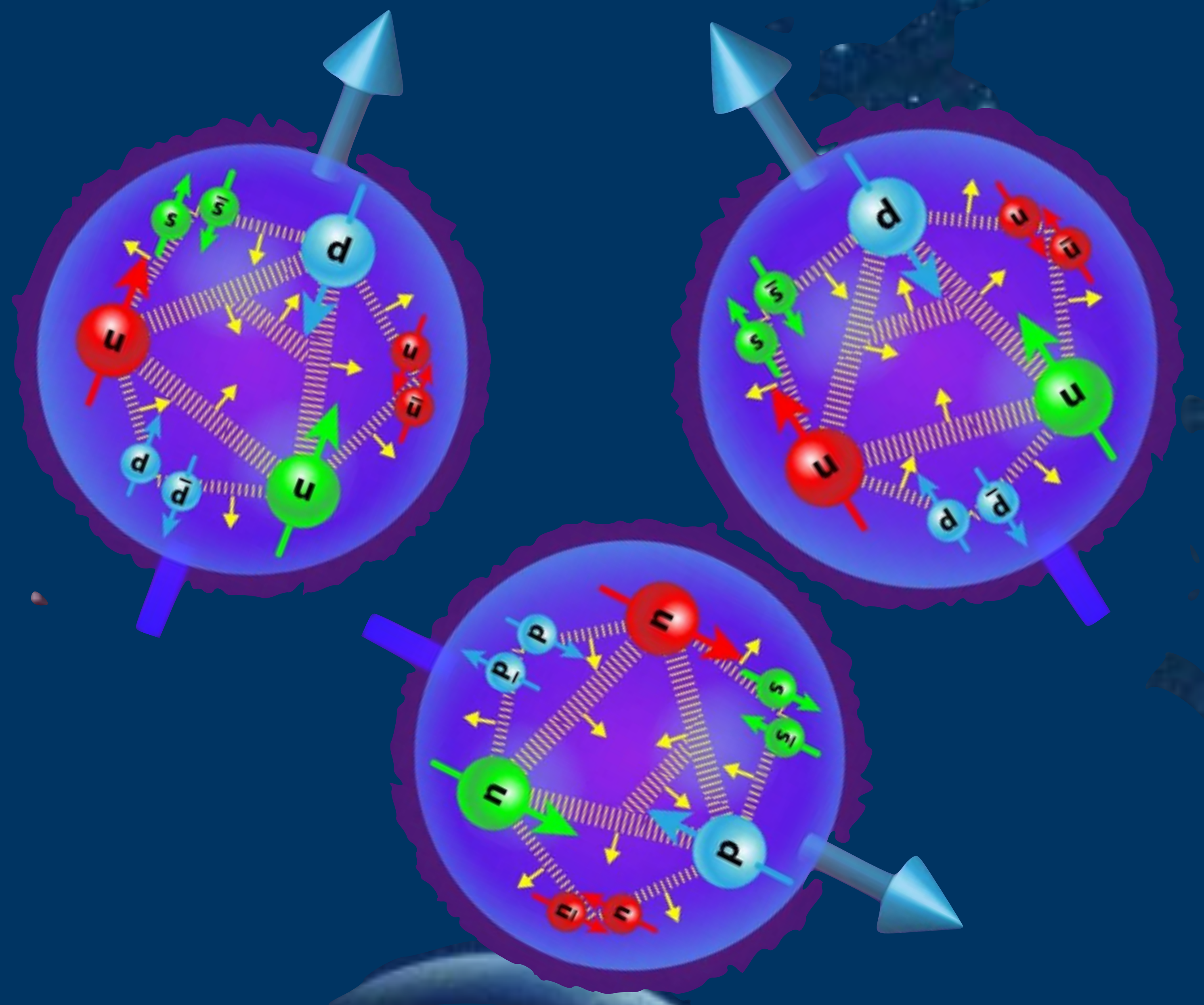


YITP Nishinomiya-Yukawa Symposium
Kyoto, October 31, 2024

INSTITUTE for NUCLEAR THEORY
U.S. DEPARTMENT OF ENERGY Office of Science
N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries PHYSICS FRONTIER CENTER
NSF

Connecting Quarks to the Cosmos: The Role of Quark Mass

Sanjay Reddy, Institute for Nuclear Theory, University of Washington, Seattle.



- Introduction.
- Quark mass in EFT and pion coupling to two nucleons.
- A new class of three-nucleon forces.
- Axion condensation in neutron stars.
- Conclusions.

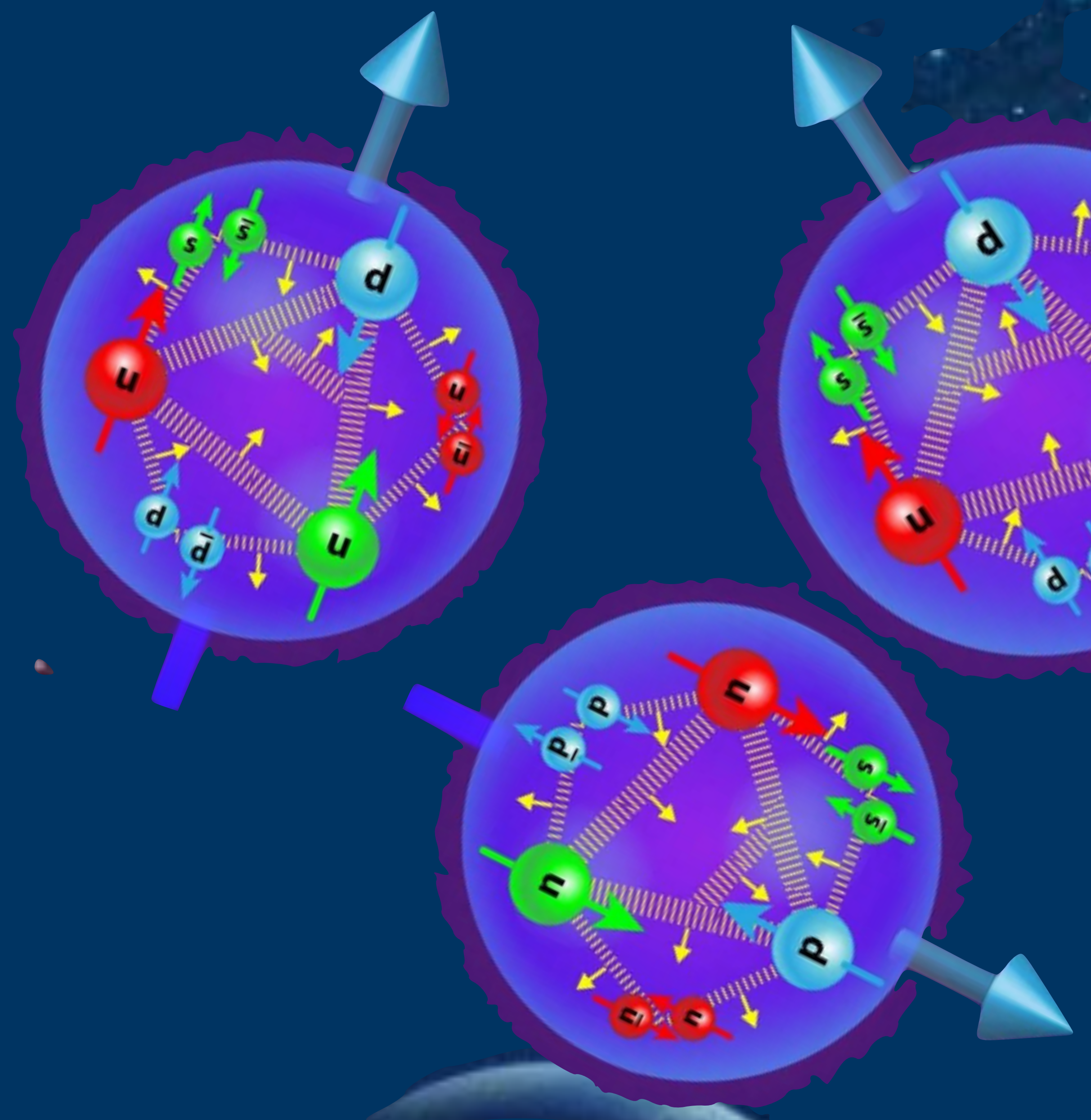
INSTITUTE for NUCLEAR THEORY
U.S. DEPARTMENT OF ENERGY Office of Science

YITP Nishinomiya-Yukawa Symposium
Kyoto, October 31, 2024

N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries
PHYSICS FRONTIER CENTER
NSF

Connecting Quarks to the Cosmos: The Role of Quark Mass

Sanjay Reddy, Institute for Nuclear Theory, University of Washington, Seattle.



- Introduction.
- Quark mass in EFT and pion coupling to two nucleons.
- A new class of three-nucleon forces.
  arXiv: 2410.2xxxx
- Axion condensation in neutron stars.
- Conclusions.

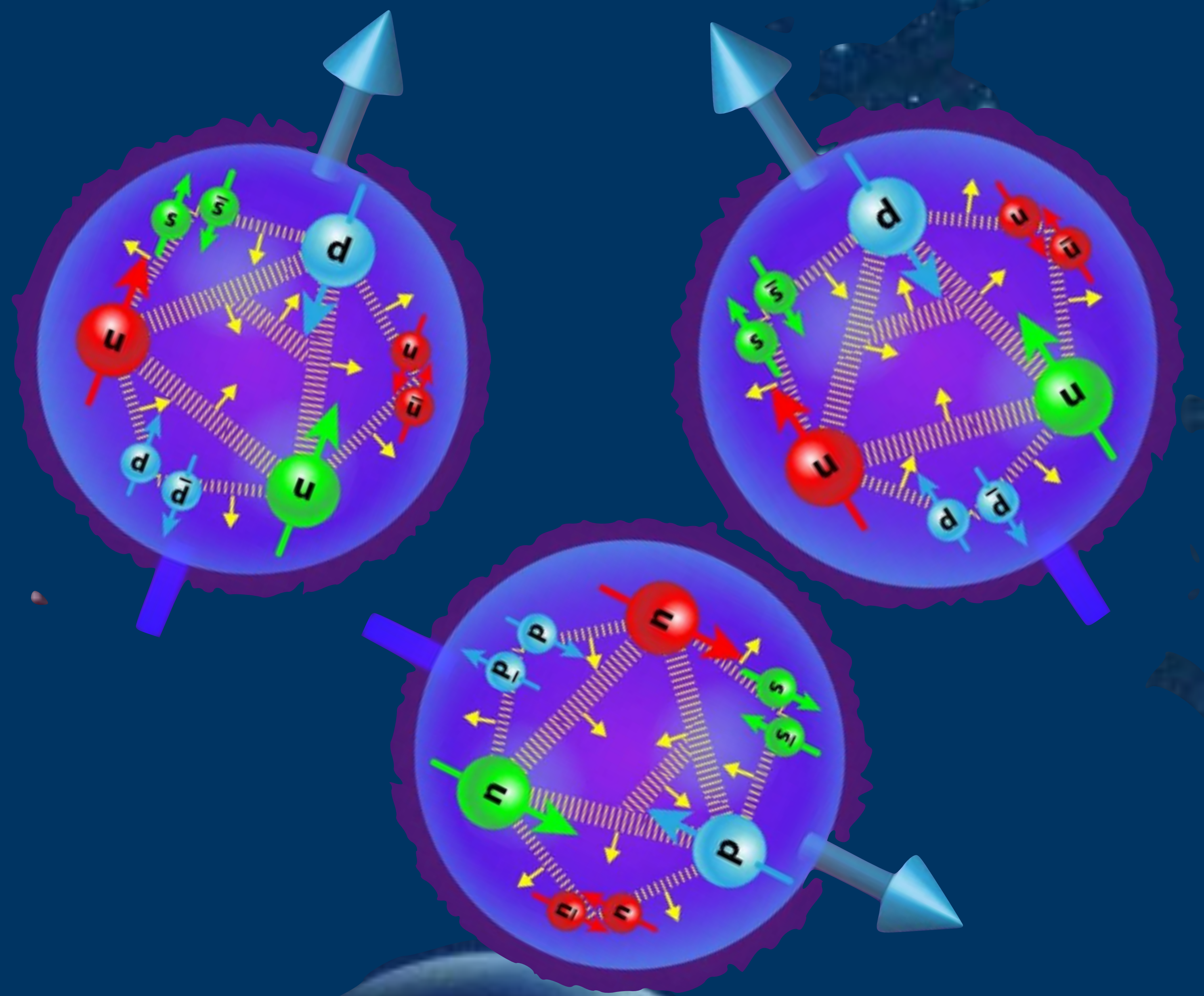
INSTITUTE for NUCLEAR THEORY
U.S. DEPARTMENT OF ENERGY Office of Science

YITP Nishinomiya-Yukawa Symposium
Kyoto, October 31, 2024

N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries PHYSICS FRONTIER CENTER
NSF

Connecting Quarks to the Cosmos: The Role of Quark Mass

Sanjay Reddy, Institute for Nuclear Theory, University of Washington, Seattle.



- Introduction.
- Quark mass in EFT and pion coupling to two nucleons.
- A new class of three-nucleon forces.
  arXiv: 2410.2xxxx
- Axion condensation in neutron stars.
  arXiv: 2410.21590
- Conclusions.

INSTITUTE for NUCLEAR THEORY

U.S. DEPARTMENT OF ENERGY
Office of Science

YITP Nishinomiya-Yukawa Symposium
Kyoto, October 31, 2024

N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries
PHYSICS FRONTIER CENTER

NSF

QCD

A Simple Lagrangian with Marvelous Emergent Complexity at Low-Energy:

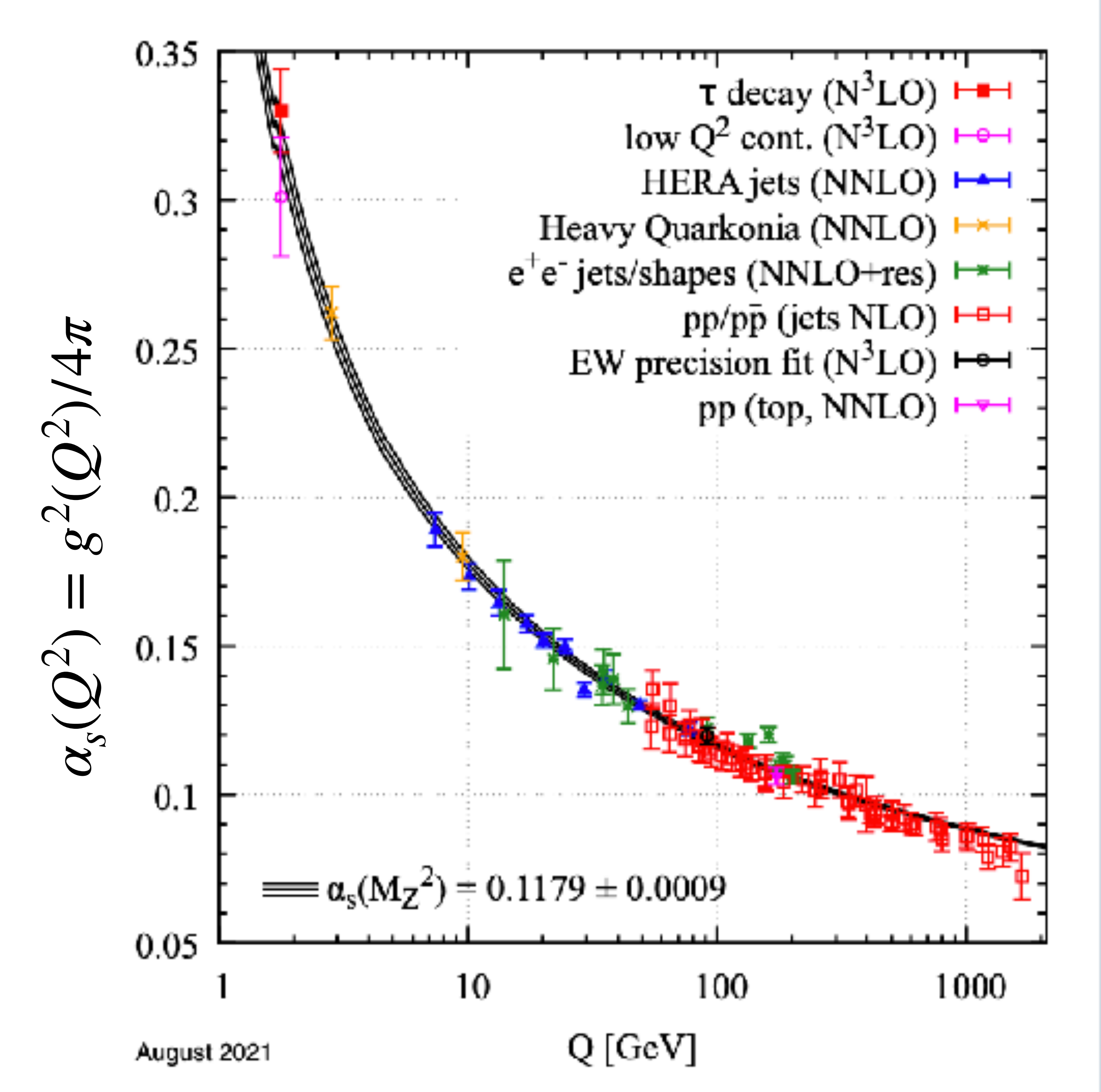
$$\mathscr{L} = \sum_f \bar{\psi}_{\alpha f} \left(\gamma^\mu (\delta_{\alpha\beta} \partial_\mu - g \, (T_a G^a_\mu)_{\alpha\beta} + m_f \right) \psi_{\beta f} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

QCD

A Simple Lagrangian with Marvelous Emergent Complexity at Low-Energy:

$$\mathscr{L} = \sum_f \bar{\psi}_{\alpha f} \left(\gamma^\mu (\delta_{\alpha\beta} \partial_\mu - g \, (T_a G^a_\mu)_{\alpha\beta} + m_f \right) \psi_{\beta f} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Running Coupling



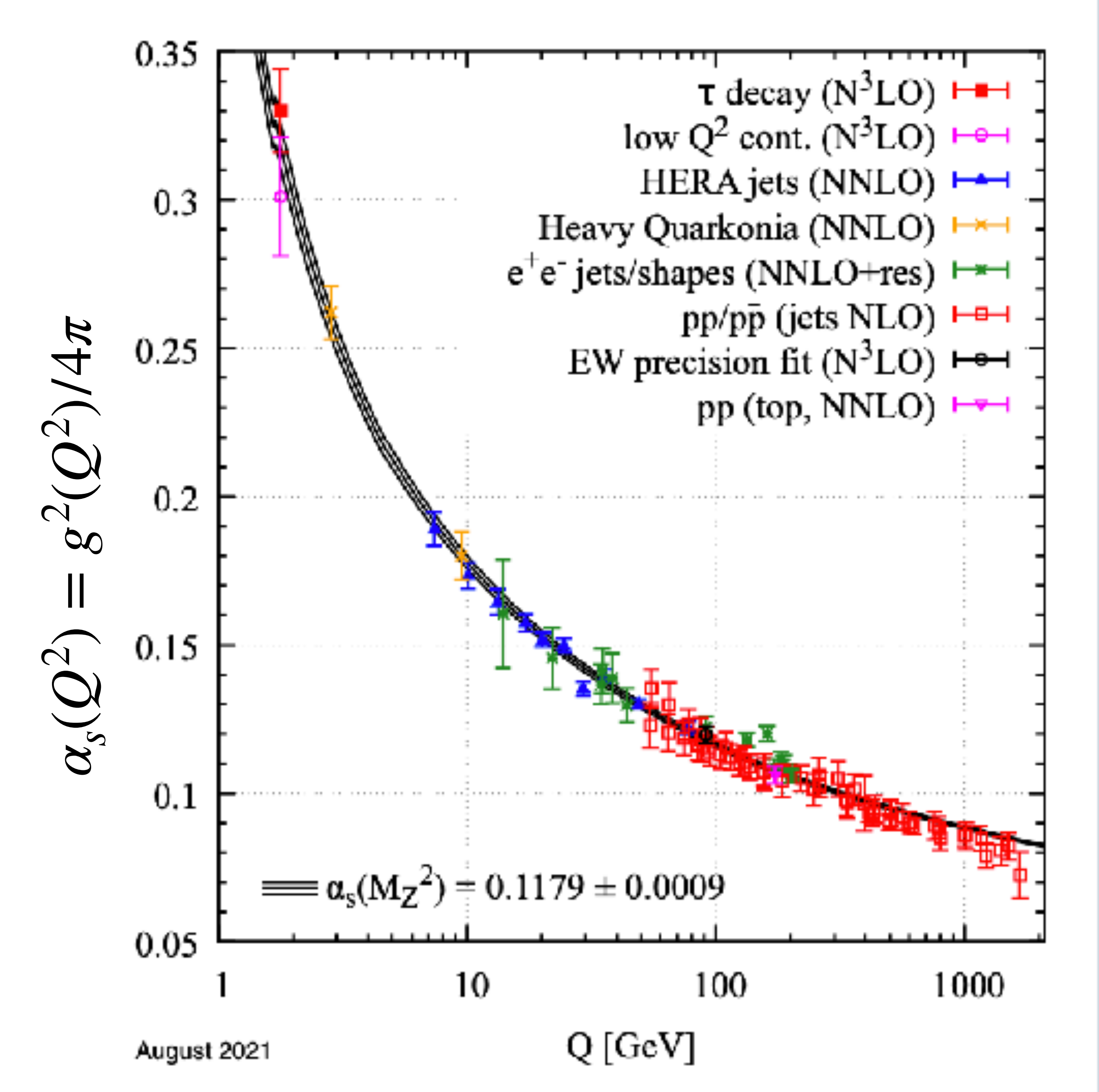
https://pdg.lbl.gov/2021/reviews/

QCD

A Simple Lagrangian with Marvelous Emergent Complexity at Low-Energy:

$$\mathscr{L} = \sum_f \bar{\psi}_{\alpha f}\left(\gamma^\mu(\delta_{\alpha\beta}\partial_\mu - g\ (T_a G^a_\mu)_{\alpha\beta} + m_f\right)\psi_{\beta f} - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$

Running Coupling



https://pdg.lbl.gov/2021/reviews/

Quark Mass Matrix

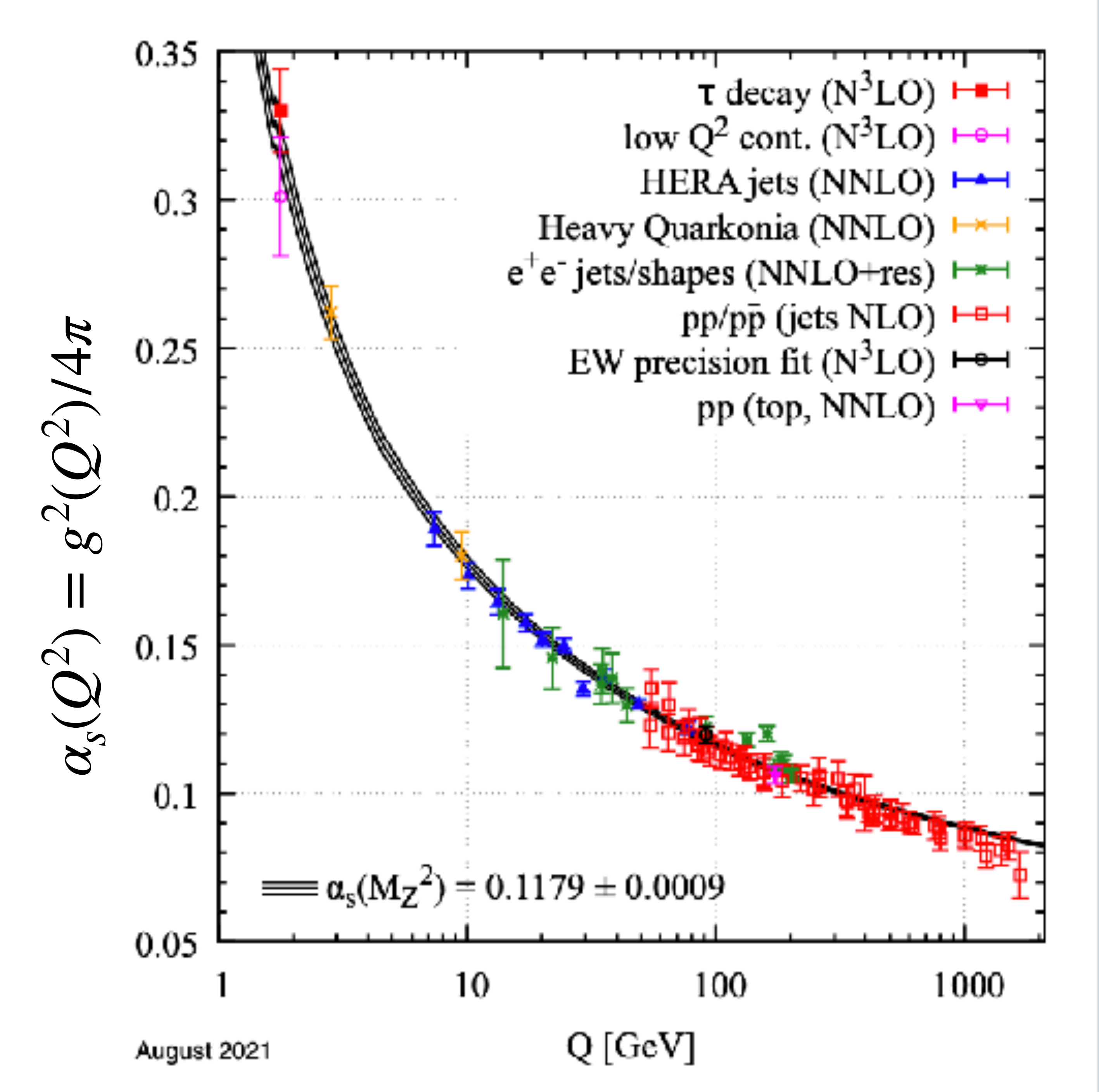
$$\begin{pmatrix} m_u \approx 2.5\ \text{MeV} & 0 & 0 \\ 0 & m_d \approx 5\ \text{MeV} & 0 \\ 0 & 0 & m_s \approx 100\ \text{MeV} \end{pmatrix}$$

QCD

A Simple Lagrangian with Marvelous Emergent Complexity at Low-Energy:

$$\mathscr{L} = \sum_f \bar{\psi}_{\alpha f} \left(\gamma^\mu (\delta_{\alpha\beta} \partial_\mu - g\ (T_a G^a_\mu)_{\alpha\beta} + m_f \right) \psi_{\beta f} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

<u>Running Coupling</u>



https://pdg.lbl.gov/2021/reviews/

<u>Quark Mass Matrix</u>

$$\begin{pmatrix} m_u \approx 2.5\ \text{MeV} & 0 & 0 \\ 0 & m_d \approx 5\ \text{MeV} & 0 \\ 0 & 0 & m_s \approx 100\ \text{MeV} \end{pmatrix}$$

Heavy Quarks (unimportant at low-energy):

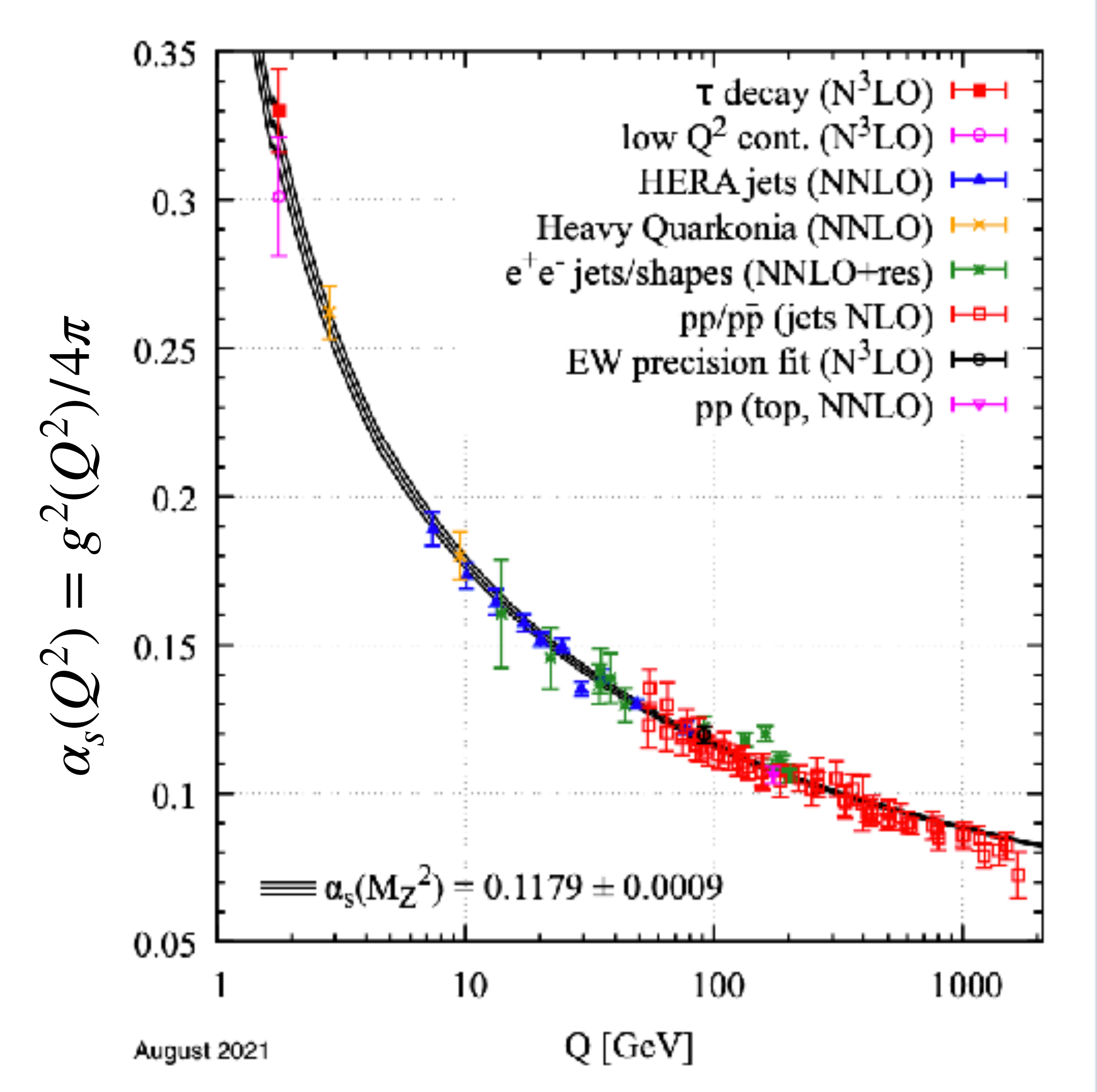
$m_c \approx 1.3$ GeV $\quad m_b \approx 4$ GeV $\quad m_t \approx 170$ GeV

QCD

A Simple Lagrangian with Marvelous Emergent Complexity at Low-Energy:

$$\mathscr{L} = \sum_f \bar{\psi}_{\alpha f} \left(\gamma^\mu (\delta_{\alpha\beta}\partial_\mu - g\ (T_a G^a_\mu)_{\alpha\beta} + m_f \right) \psi_{\beta f} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \theta\ \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$$

Running Coupling



https://pdg.lbl.gov/2021/reviews/

Quark Mass Matrix

$$\begin{pmatrix} m_u \approx 2.5 \text{ MeV} & 0 & 0 \\ 0 & m_d \approx 5 \text{ MeV} & 0 \\ 0 & 0 & m_s \approx 100 \text{ MeV} \end{pmatrix}$$

Heavy Quarks (unimportant at low-energy):

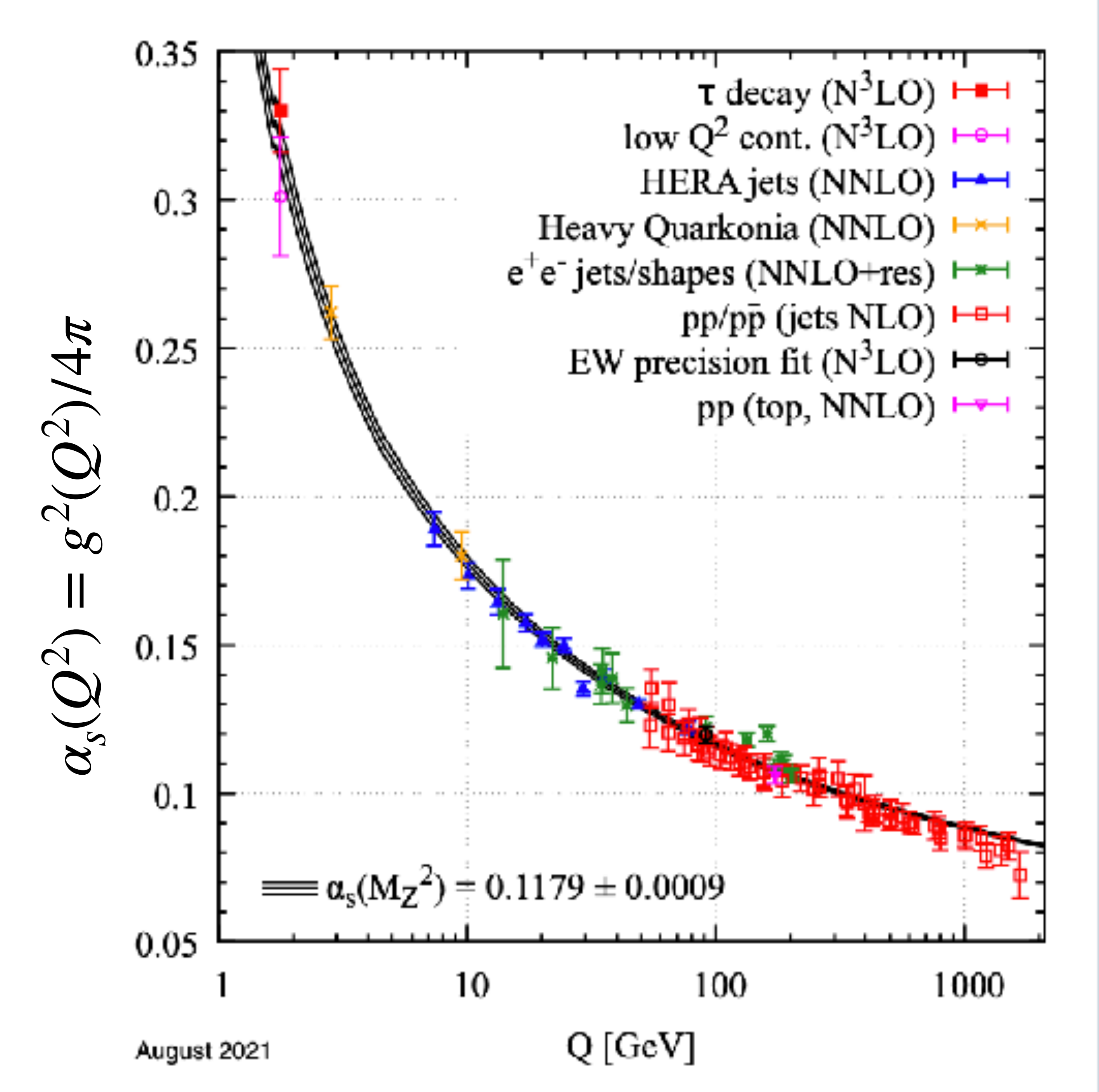
$m_c \approx 1.3$ GeV $\quad m_b \approx 4$ GeV $\quad m_t \approx 170$ GeV

QCD

A Simple Lagrangian with Marvelous Emergent Complexity at Low-Energy:

$$\mathscr{L} = \sum_f \bar{\psi}_{\alpha f}\left(\gamma^\mu(\delta_{\alpha\beta}\partial_\mu - g\,(T_a G^a_\mu)_{\alpha\beta} + m_f\right)\psi_{\beta f} - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a + \theta\,\frac{g^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$

Running Coupling



https://pdg.lbl.gov/2021/reviews/

Quark Mass Matrix

$$\begin{pmatrix} m_u \approx 2.5\text{ MeV} & 0 & 0 \\ 0 & m_d \approx 5\text{ MeV} & 0 \\ 0 & 0 & m_s \approx 100\text{ MeV} \end{pmatrix}$$

Heavy Quarks (unimportant at low-energy):

$m_c \approx 1.3$ GeV $\quad m_b \approx 4$ GeV $\quad m_t \approx 170$ GeV

θ-QCD

- Source of CP violation
- Induces neutron EDM:

  $d_n \approx 3 \times 10^{-16}$ e cm
- Experimental bound:

  $d_n \lesssim 10^{-26}$ e cm

  or $\theta \lesssim 10^{-10}$

How do (light) quark masses affect low-energy nuclear physics?

$$m_\pi^2 = -\frac{\langle \bar{q}q \rangle}{3f_\pi^2}\left(m_u + m_d\right) + \cdots \quad \Rightarrow \quad K_{m_\pi} = \frac{m_q}{m_\pi}\frac{\delta m_\pi}{\delta m_q} \simeq 0.5$$

$$m_n = m_0 + \underbrace{\sigma_{\pi n}}_{\simeq 50 \pm 10\ \mathrm{MeV}}\frac{m_\pi^2}{(m_\pi^2)_{\mathrm{phys}}} + \cdots \quad \Rightarrow \quad K_{m_n} = \frac{m_q}{m_n}\frac{\delta m_n}{\delta m_q} \approx \frac{m_\pi^2}{m_n}\frac{\delta m_n}{\delta m_\pi^2} \simeq 0.05$$

J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U.-G. Meißner, J. Nebreda, and J. R. Pela´ez (2013), J. Donoghue (2006) , Beane and Savage (2003),

How do (light) quark masses affect low-energy nuclear physics?

$$m_\pi^2 = -\frac{\langle \bar{q}q \rangle}{3f_\pi^2}\left(m_u + m_d\right) + \cdots \quad \Rightarrow \quad K_{m_\pi} = \frac{m_q}{m_\pi}\frac{\delta m_\pi}{\delta m_q} \simeq 0.5$$

$$m_n = m_0 + \underbrace{\sigma_{\pi n}}_{\simeq 50 \pm 10 \text{ MeV}} \frac{m_\pi^2}{(m_\pi^2)_{\text{phys}}} + \cdots \quad \Rightarrow \quad K_{m_n} = \frac{m_q}{m_n}\frac{\delta m_n}{\delta m_q} \approx \frac{m_\pi^2}{m_n}\frac{\delta m_n}{\delta m_\pi^2} \simeq 0.05$$

Masses of heavier vector mesons are relatively insensitive to the quark mass.

$$K_{m_\rho} = \frac{m_q}{m_\rho}\frac{\delta m_\rho}{\delta m_q} \simeq 0.05$$

J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U.-G. Meißner, J. Nebreda, and J. R. Pela´ez (2013), J. Donoghue (2006) , Beane and Savage (2003),

How do (light) quark masses affect low-energy nuclear physics?

$$m_\pi^2 = -\frac{\langle \bar{q}q \rangle}{3f_\pi^2}\left(m_u + m_d\right) + \cdots \quad \Rightarrow \quad K_{m_\pi} = \frac{m_q}{m_\pi}\frac{\delta m_\pi}{\delta m_q} \simeq 0.5$$

$$m_n = m_0 + \underset{\uparrow \atop \simeq 50 \pm 10\ \mathrm{MeV}}{\sigma_{\pi n}}\frac{m_\pi^2}{(m_\pi^2)_{\mathrm{phys}}} + \cdots \quad \Rightarrow \quad K_{m_n} = \frac{m_q}{m_n}\frac{\delta m_n}{\delta m_q} \approx \frac{m_\pi^2}{m_n}\frac{\delta m_n}{\delta m_\pi^2} \simeq 0.05$$

Masses of heavier vector mesons are relatively insensitive to the quark mass.

$$K_{m_\rho} = \frac{m_q}{m_\rho}\frac{\delta m_\rho}{\delta m_q} \simeq 0.05$$

Mass of the scalar sigma meson

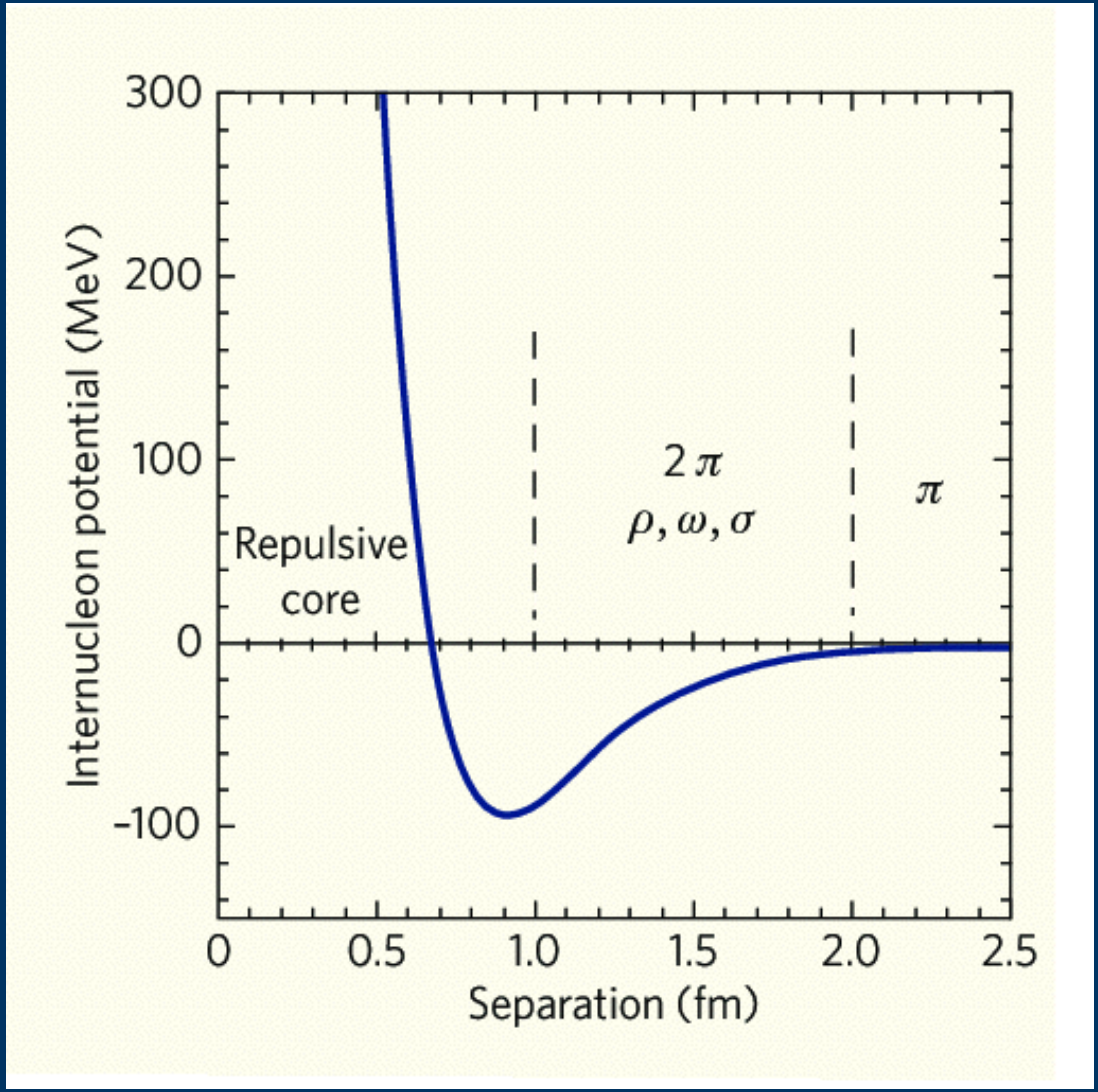
$$K_{m_\sigma} = \frac{m_q}{m_\sigma}\frac{\delta m_\sigma}{\delta m_q} \simeq 0.1$$

J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U.-G. Meißner, J. Nebreda, and J. R. Pela´ez (2013), J. Donoghue (2006) , Beane and Savage (2003),

How do quark masses affect nuclear interactions at low-energy?

Short answer: We do not really know.

- The effect on pion-exchange is easy to implement, but effects at short distances are not.

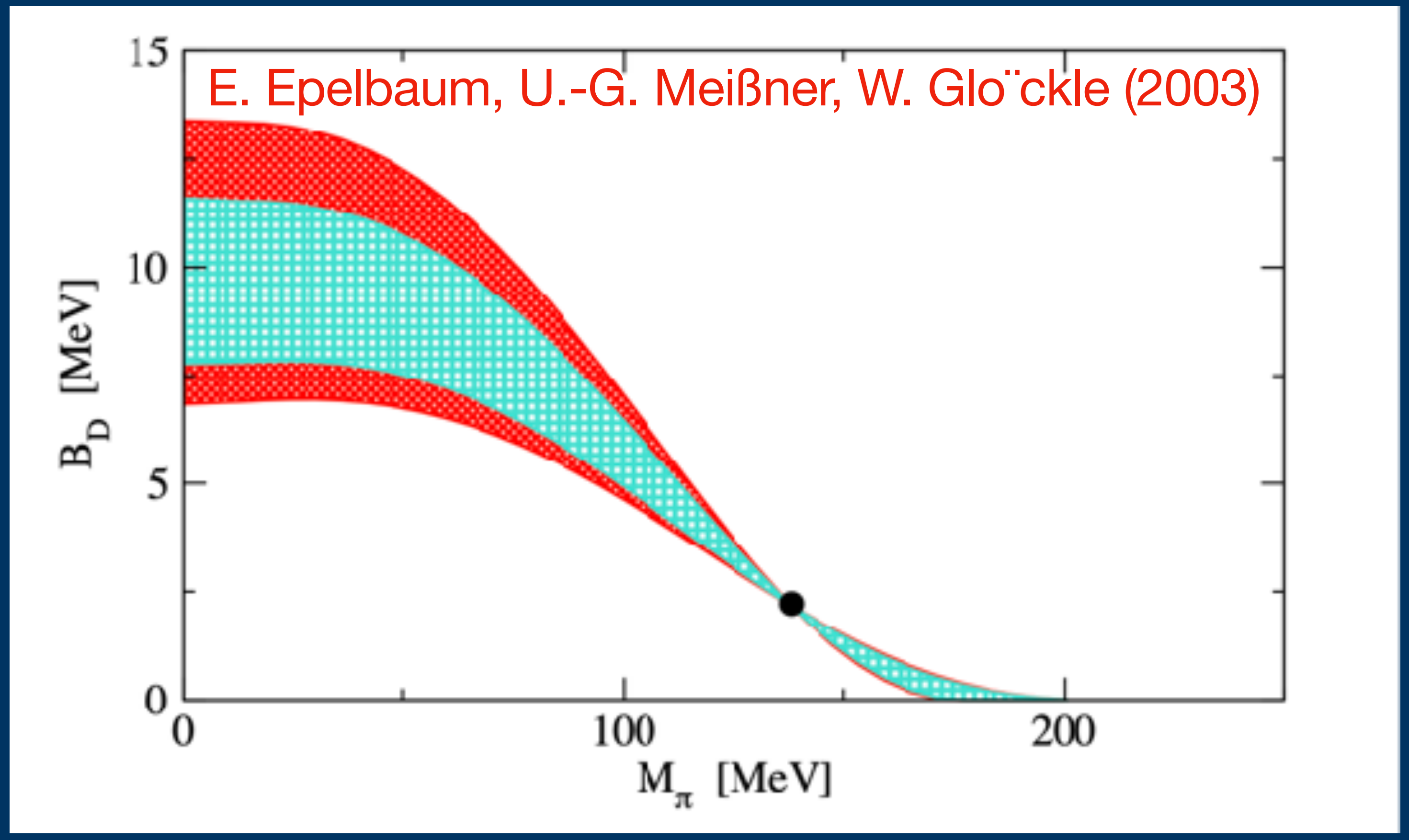


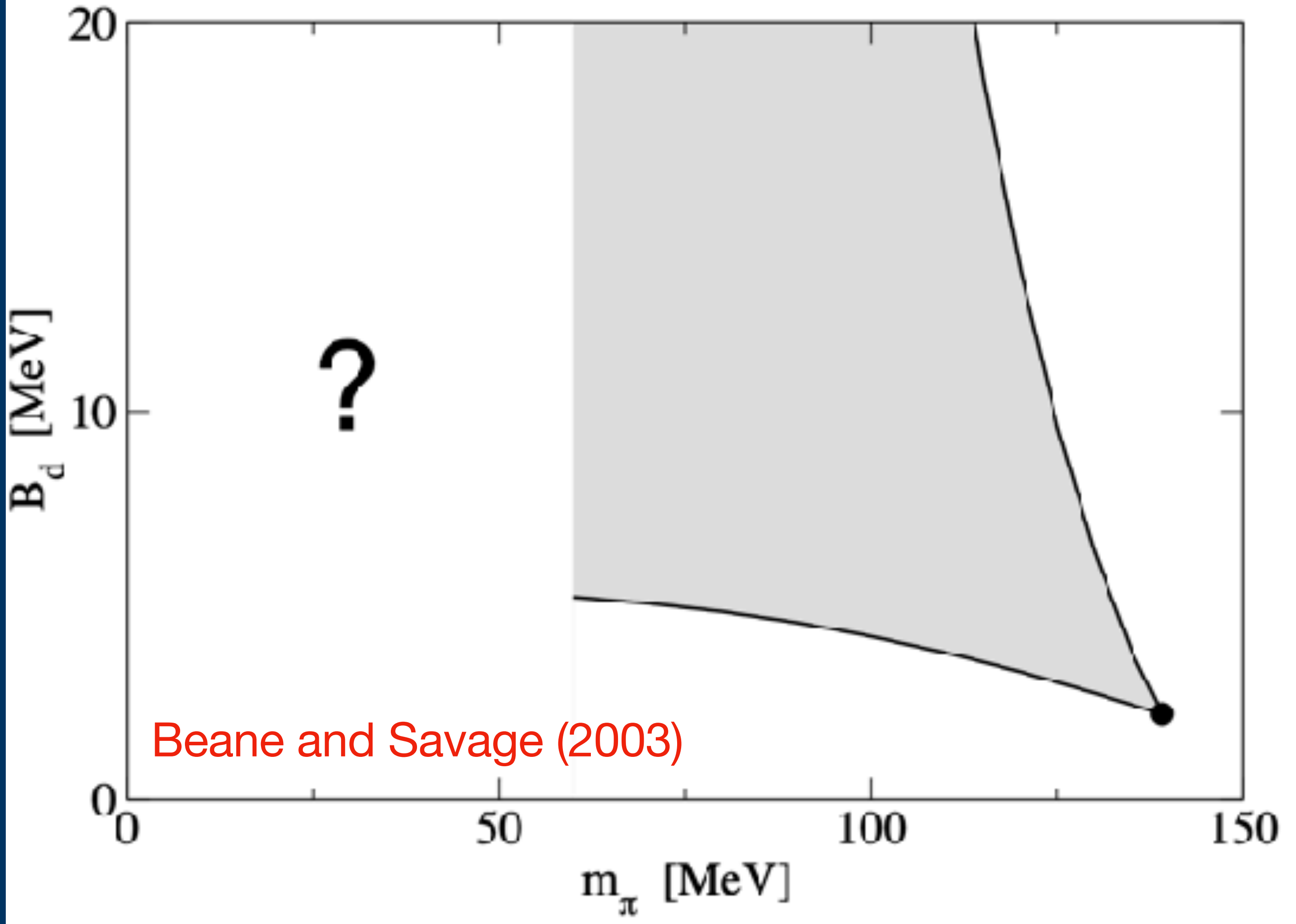
J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U.-G. Meißner, J. Nebreda, and J. R. Pela´ez (2013), E. Epelbaum and J. Gegelia (2013), J. Donoghue (2006) , E. Epelbaum, U.-G. Meißner, W. Glo¨ckle (2003), Beane and Savage (2003), Bulgac, Miller, Strikman (1997).

How do quark masses affect nuclear interactions at low-energy?

Short answer: We do not really know.

- The effect on pion-exchange is easy to implement, but effects at short distances are not.





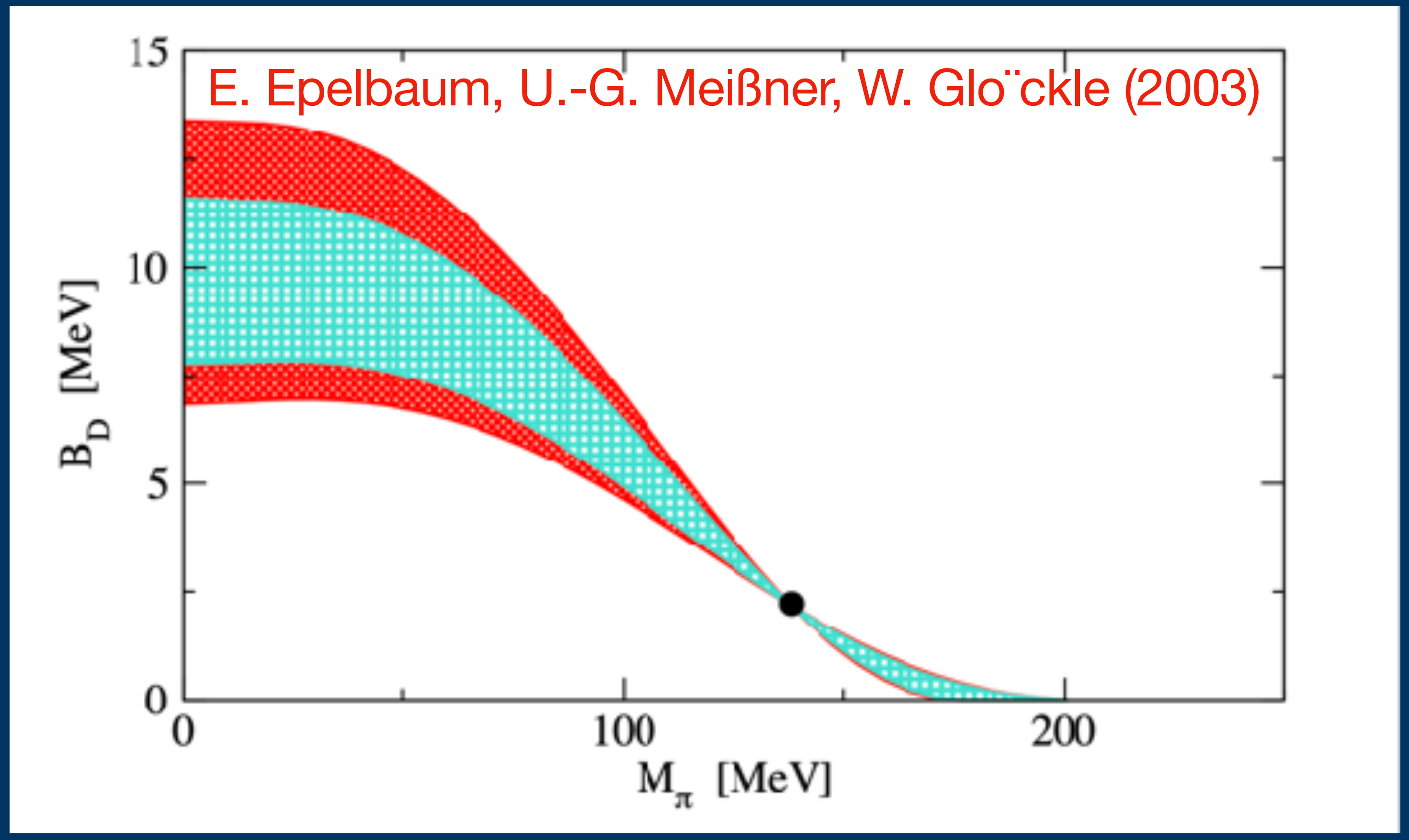
J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U.-G. Meißner, J. Nebreda, and J. R. Pela´ez (2013), E. Epelbaum and J. Gegelia (2013), J. Donoghue (2006) , E. Epelbaum, U.-G. Meißner, W. Glo¨ckle (2003), Beane and Savage (2003), Bulgac, Miller, Strikman (1997).

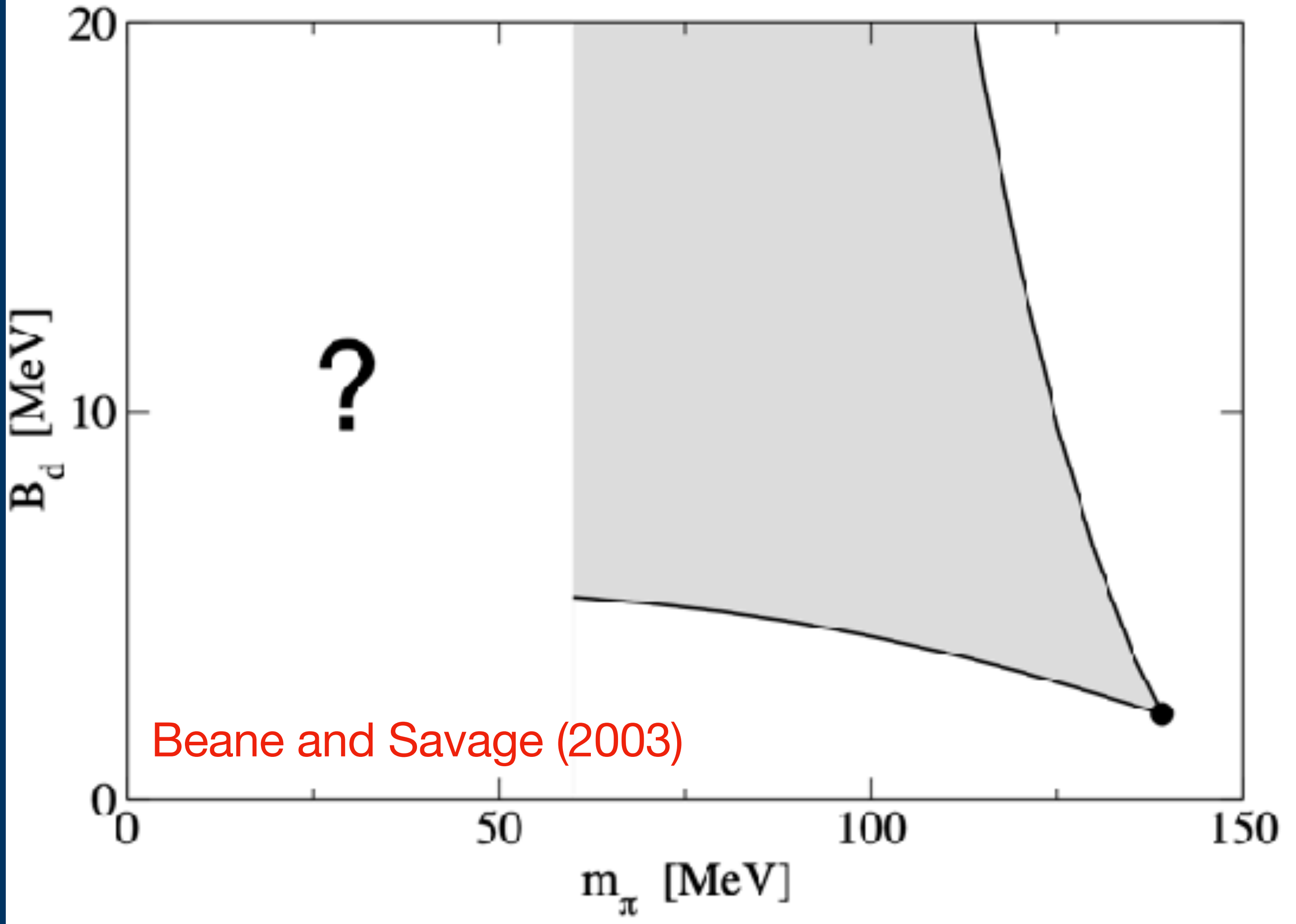
How do quark masses affect nuclear interactions at low-energy?

Short answer: We do not really know.

- The effect on pion-exchange is easy to implement, but effects at short distances are not.

- Models with reasonable assumptions suggest that the deuteron binding energy increases with decreasing pion mass.





J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U.-G. Meißner, J. Nebreda, and J. R. Pela´ez (2013), E. Epelbaum and J. Gegelia (2013), J. Donoghue (2006) , E. Epelbaum, U.-G. Meißner, W. Glo¨ckle (2003), Beane and Savage (2003), Bulgac, Miller, Strikman (1997).

How do quark masses affect nuclear interactions at low-energy?

Short answer: We do not really know.

- The effect on pion-exchange is easy to implement, but effects at short distances are not.

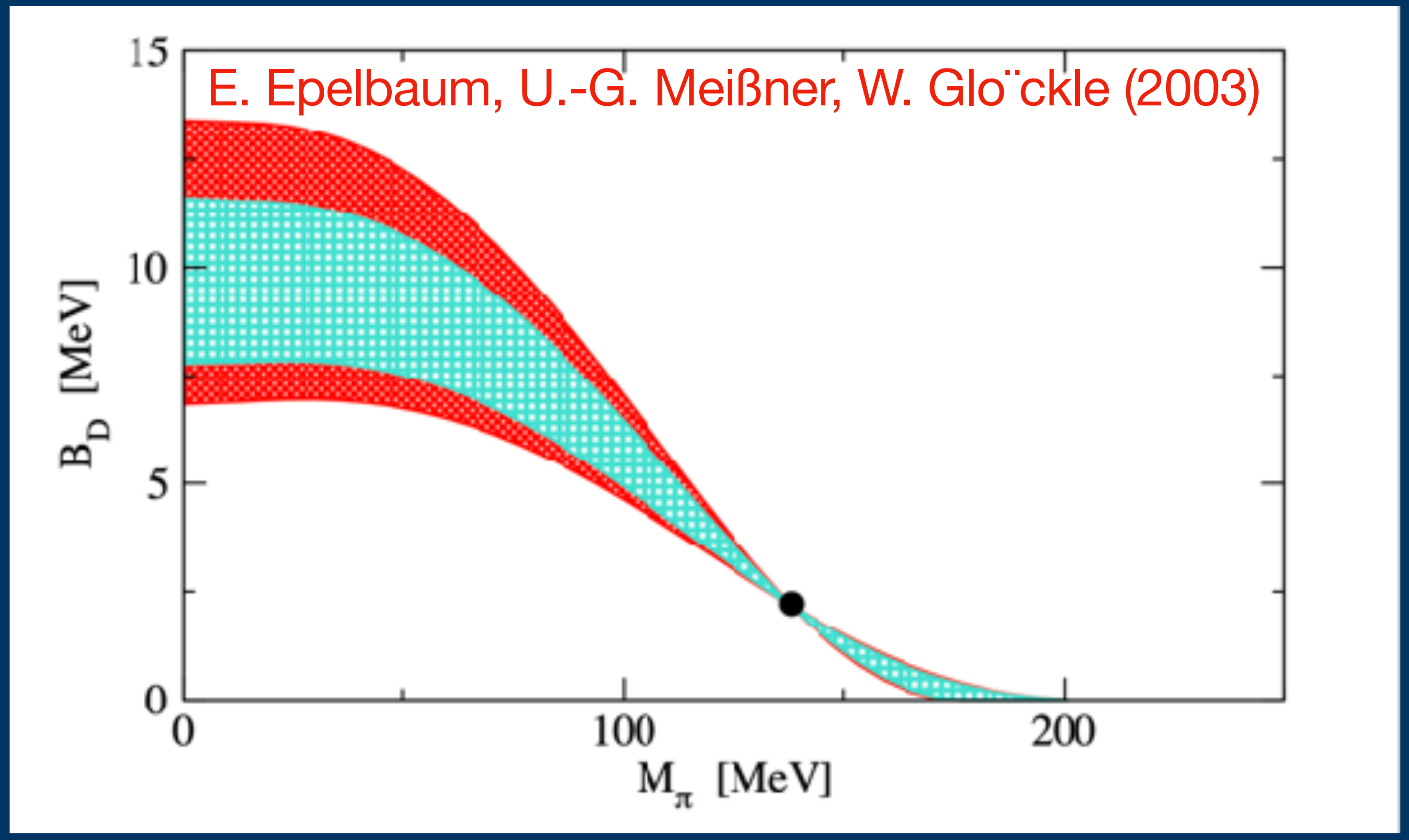
- Models with reasonable assumptions suggest that the deuteron binding energy increases with decreasing pion mass.

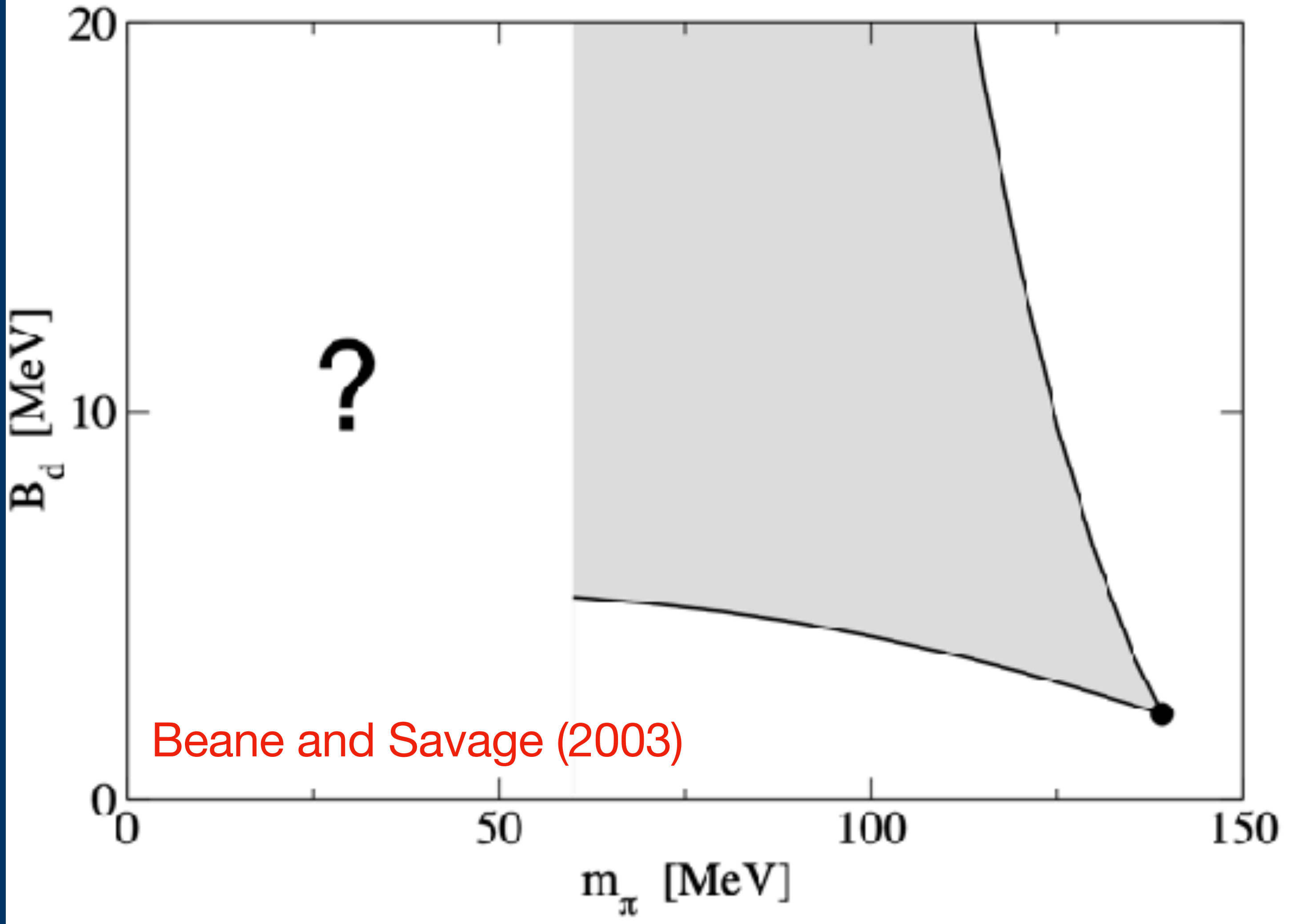
Effect of quark mass (pion mass) on the scattering length:

$$K_{a_s} = \frac{m_q}{a_s}\frac{\delta a_s}{\delta m_q} \simeq 2.4 \pm 3$$ J. C. Berengut, E. Epelbaum, et al. (2013)

$$\simeq 5 \pm 5$$ Beane and Savage (2003)

$$\simeq 2.3 \pm 1.9$$ E. Epelbaum, U.-G. Meißner, W. Glöckle (2003)





J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U.-G. Meißner, J. Nebreda, and J. R. Peláez (2013), E. Epelbaum and J. Gegelia (2013), J. Donoghue (2006) , E. Epelbaum, U.-G. Meißner, W. Glöckle (2003), Beane and Savage (2003), Bulgac, Miller, Strikman (1997).

Variations of Quark Mass in the Early Universe?

Earlier studies*: Measured abundances of BBN and estimates of the quark mass dependence of 1 and 2 nucleon systems to constrain variation for quark mass

$$-2\,\% < \frac{\delta m_q}{m_q} < 6\,\%$$

J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U.-G. Meißner, J. Nebreda, and J. R. Pela´ez (2013)

*J. C. Berengut, E. Epelbaum, V. V. Flambaum, C. Hanhart, U.-G. Meißner, J. Nebreda, and J. R. Pela´ez (2013), P.F. Bedaque, T. Luu, and L. Platter (2011), V. V. Flambaum and R. B. Wiringa (2007), J. C. Berengut, V. V. Flambaum, and V. F. Dmitriev (2010), O. Civitarese, M. A. Moline´ , and M. E. Mosquera (2010), A. Coc, P. Descouvemont, K.A. Olive, J.-P. Uzan, and E. Vangioni (2012), V. V. Flambaum and E. V. Shuryak (2003), V. V. Flambaum, A. Holl, P. Jaikumar, C. D. Roberts, and S. V. Wright (2006), A. Holl, P. Maris, C. D. Roberts, and S. V. Wright (2006),V. V. Flambaum (2005),V. V. Flambaum, D. B. Leinweber, A. W. Thomas, and R. D. Young (2004).

Variations of Quark Mass & Triple-apha Reaction

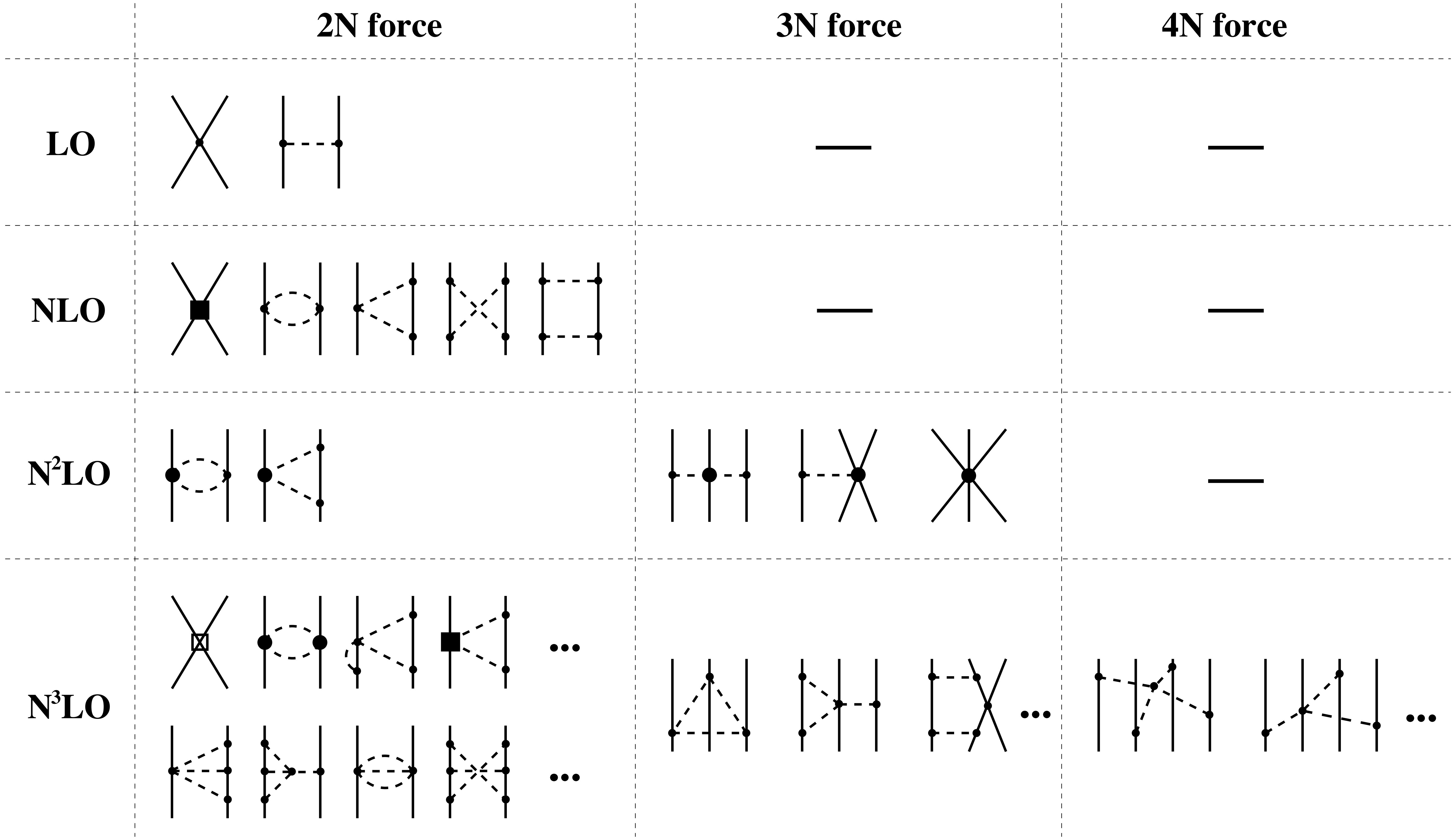
Earlier studies** showed that a few percent variation in the quark masses would do damage to the triple-alpa reaction in stars. If the Hoyle state was not at the finely tuned energy there would be either too little or too much Carbon.

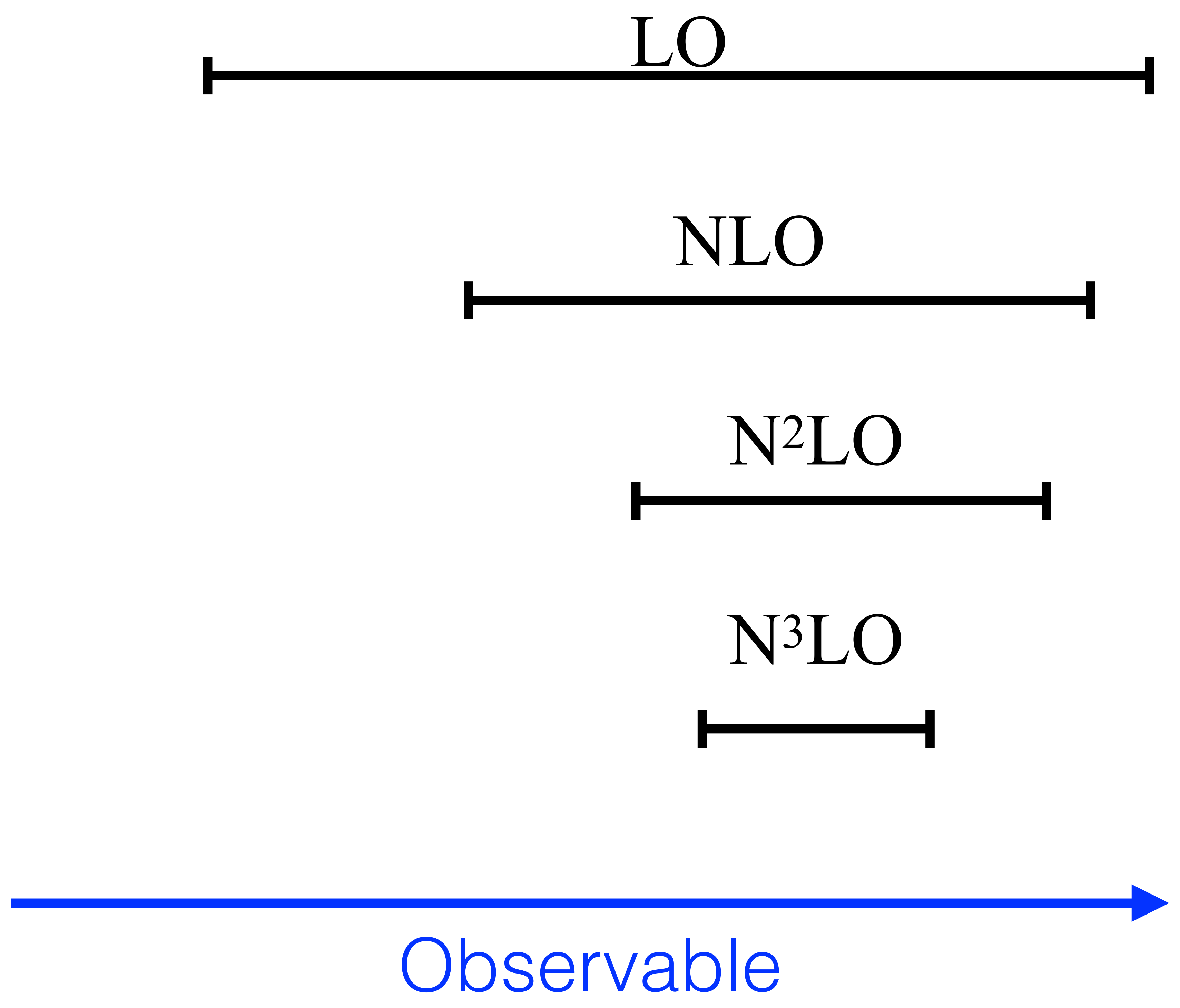
**D. Lee, E. Epelbaum, H. Krebs,T. A. Laehde, Ulf-G. Meissner (2013)

Part 1 : Quark Mass Dependence of Short-Range Nuclear Forces and its implications.

Nuclear Forces from Effective Field Theory (EFT)

EFT Hamiltonians organizes operators in powers of the momentum: $\dfrac{Q}{\Lambda_B}$



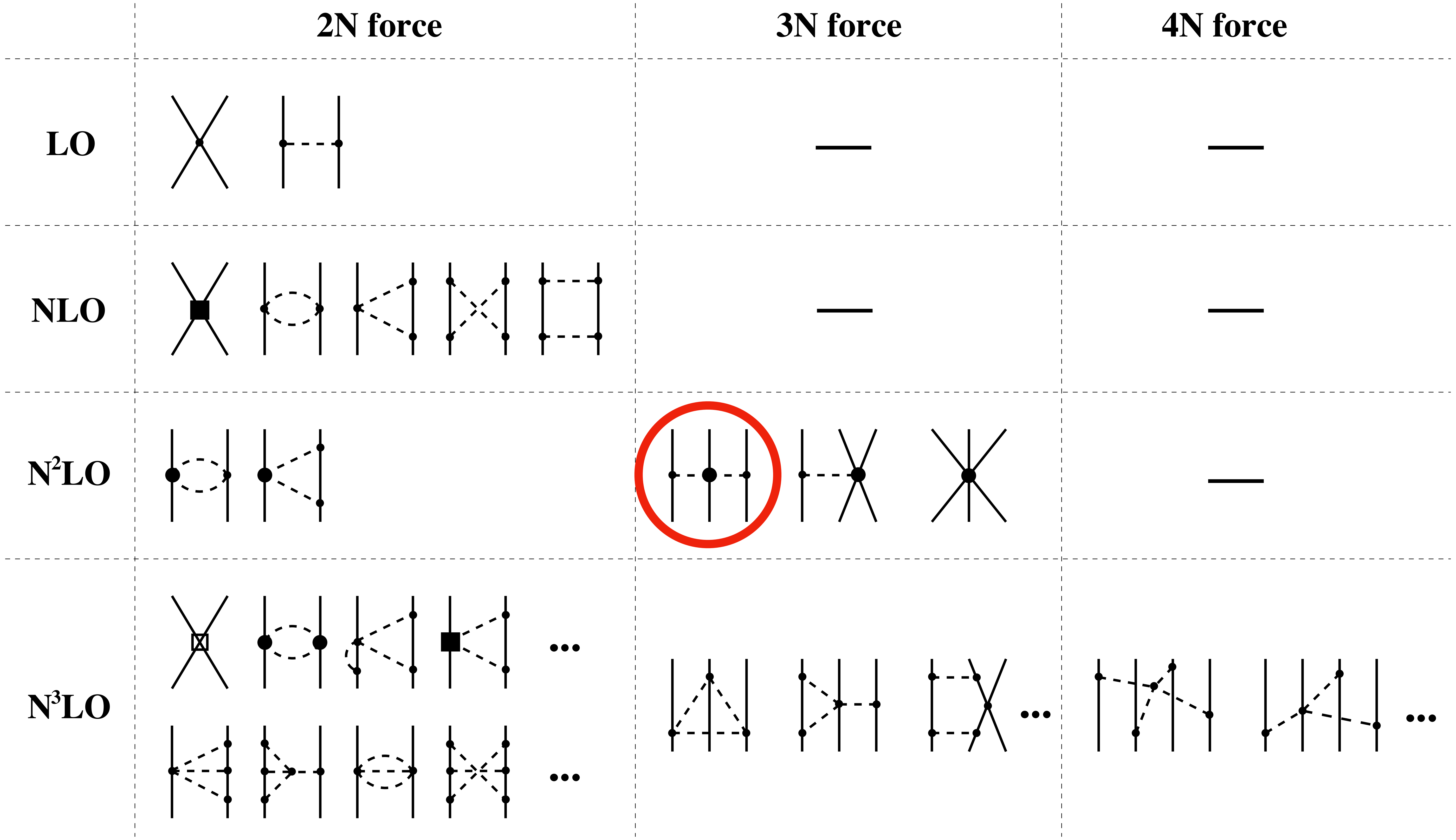


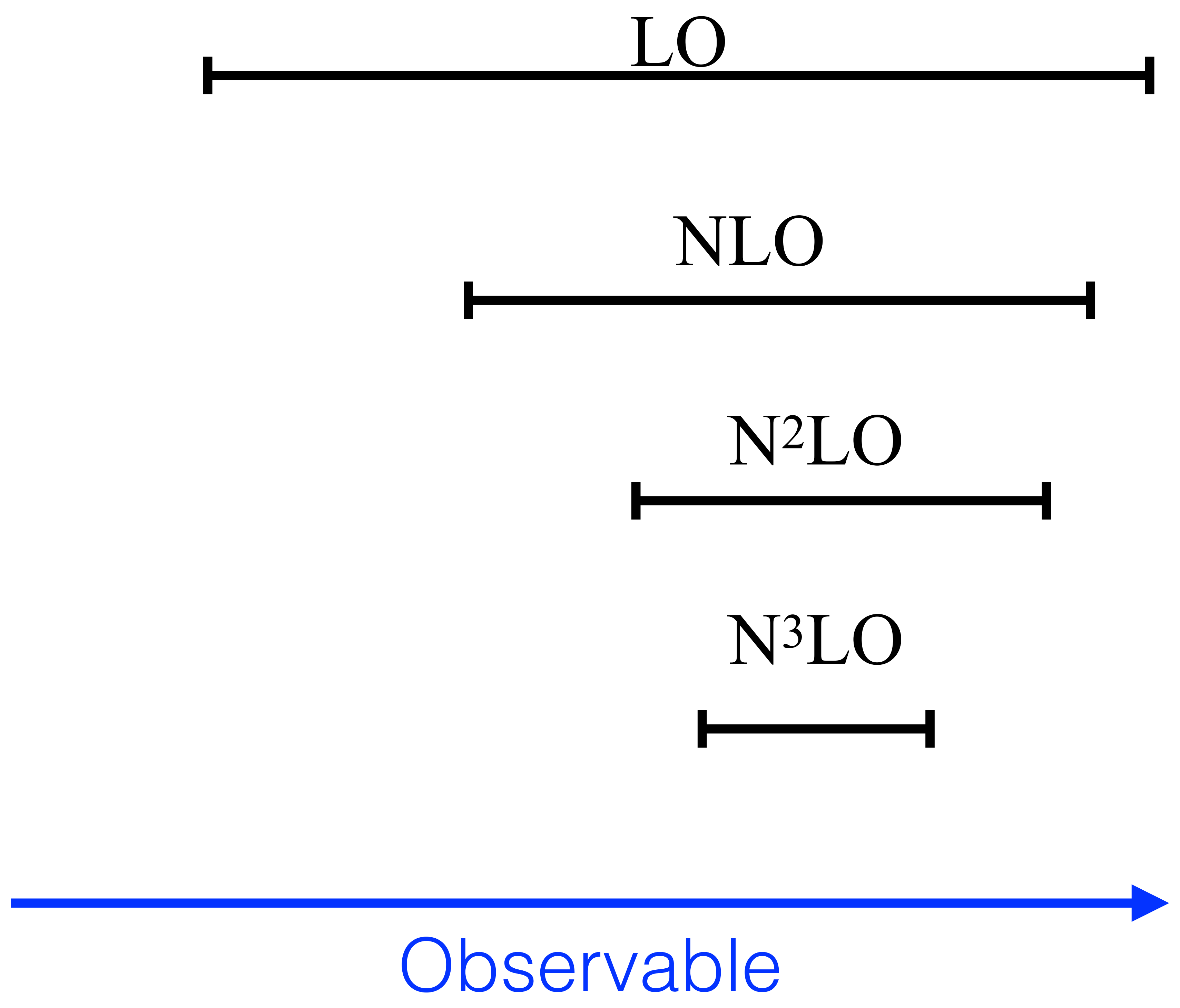
Allows for error estimation*. Provides guidance for the structure of three and many-body forces.

Beane, Bedaque, Epelbaum, Kaplan, Machliedt, Meisner, Phillips, Savage, **van Klock**, **Weinberg,** Wise ..

Nuclear Forces from Effective Field Theory (EFT)

EFT Hamiltonians organizes operators in powers of the momentum: $\dfrac{Q}{\Lambda_B}$

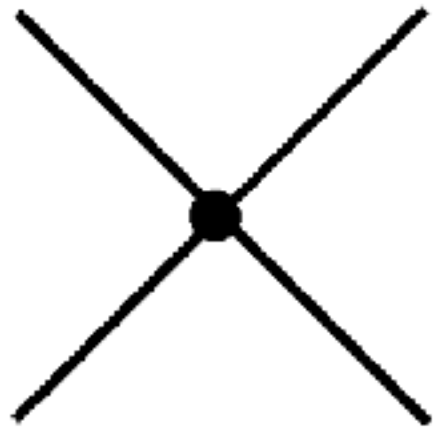




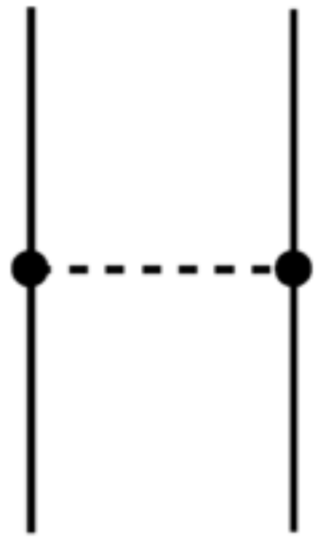
Allows for error estimation*. Provides guidance for the structure of three and many-body forces.

Beane, Bedaque, Epelbaum, Kaplan, Machliedt, Meisner, Phillips, Savage, **van Klock**, **Weinberg**, Wise ..

Quark (pion) mass-dependence of NN interaction in EFT



$+$

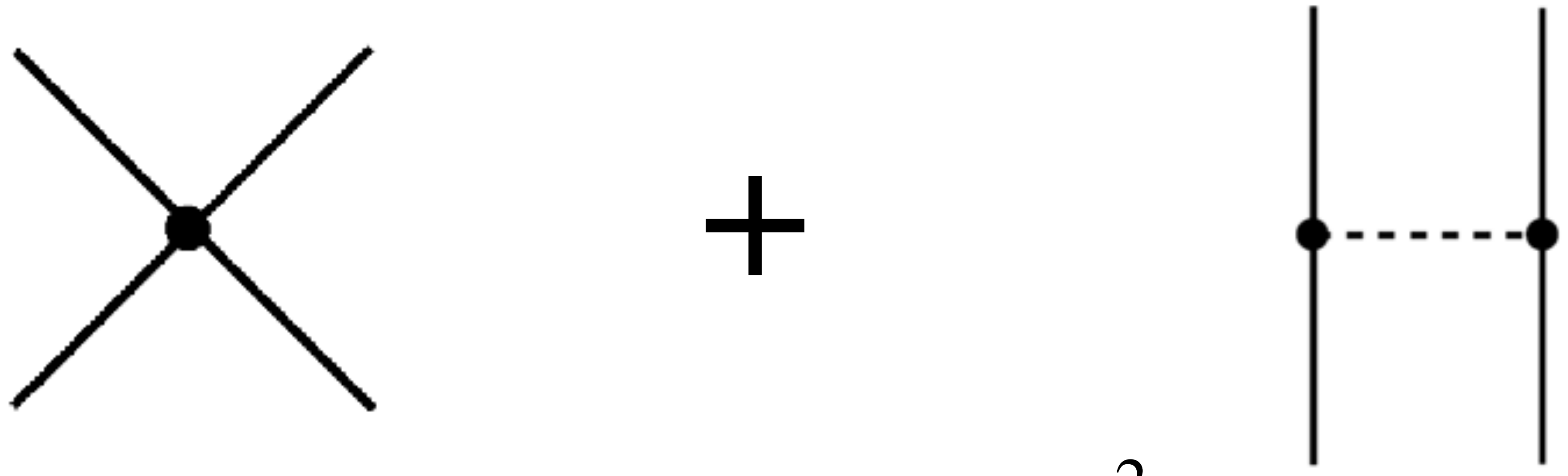


$$V_{\mathrm{LO}}(q) = \quad C_0 + D_2\, m_\pi^2 \quad + \quad \frac{g_A^2}{4f_\pi^2}\,\frac{\sigma_1\cdot\mathbf{q}\;\sigma_2\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\,\tau_1\cdot\tau_2$$

Analysis of 2-nucleon scattering in Lattice QCD for different values m_π could, in principle, determine D_2 but systematics are too large at this time.

Beane, Bedaque, Detmold, Savage (NPLQCD), Walker-Loud (Cal-Lat), Aoki, Hatsuda, Ishii (HAL QCD Collaboration), ….

Quark (pion) mass-dependence of NN interaction in EFT

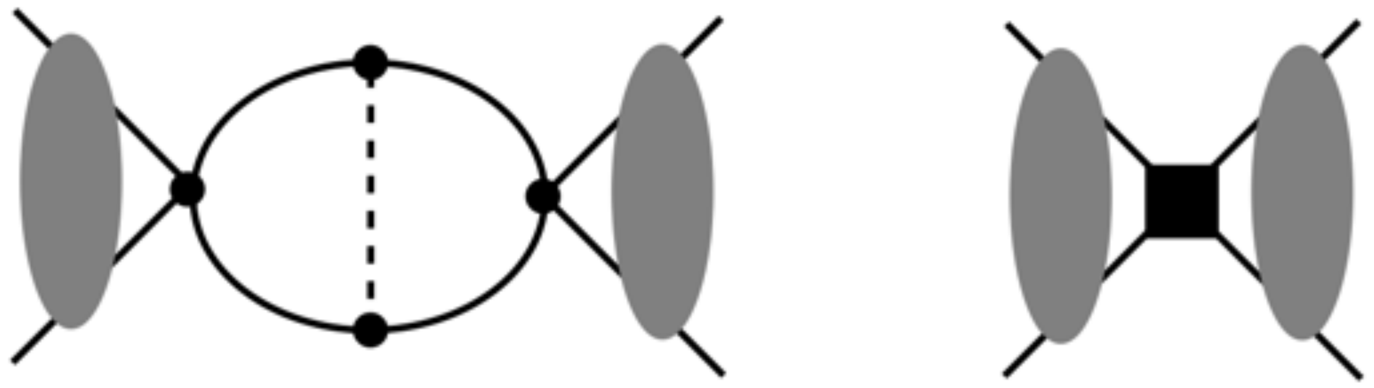


$$V_{\rm LO}(q) = \quad C_0 + D_2\, m_\pi^2 \quad + \quad \frac{g_A^2}{4f_\pi^2}\,\frac{\boldsymbol{\sigma_1}\cdot\mathbf{q}\;\boldsymbol{\sigma_2}\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\;\tau_1\cdot\tau_2$$

Renormalization requires D_2 :

Kaplan, Savage, Wise (1998)

To obtain a scattering amplitude that is independent of regularization or cut-off Λ requires:

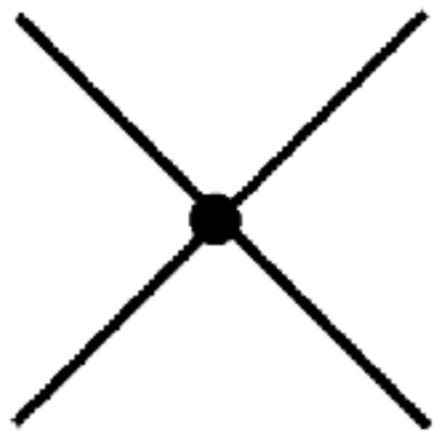


$$\Lambda\frac{d}{d\Lambda}\left(\frac{D_2}{C_0^2}\right)_{\rm KSW} = \frac{g_A^2 m_N^2}{64\pi^2 f_\pi^2} \quad \Longrightarrow \quad \frac{|D_2|}{C_0^2} \approx \frac{g_A^2 m_N^2}{64\pi^2 f_\pi^2} \simeq \frac{1}{4}$$

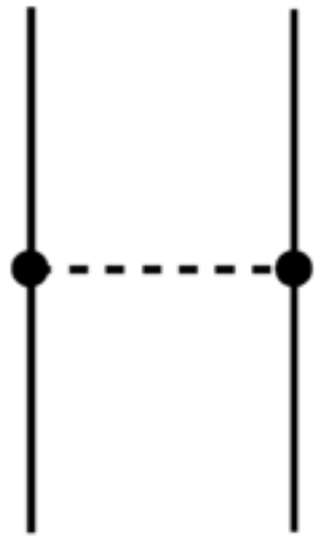
Analysis of 2-nucleon scattering in Lattice QCD for different values m_π could, in principle, determine D_2 but systematics are too large at this time.

Beane, Bedaque, Detmold, Savage (NPLQCD), Walker-Loud (Cal-Lat), Aoki, Hatsuda, Ishii (HAL QCD Collaboration), ….

Quark (pion) mass-dependence of NN interaction in EFT



+

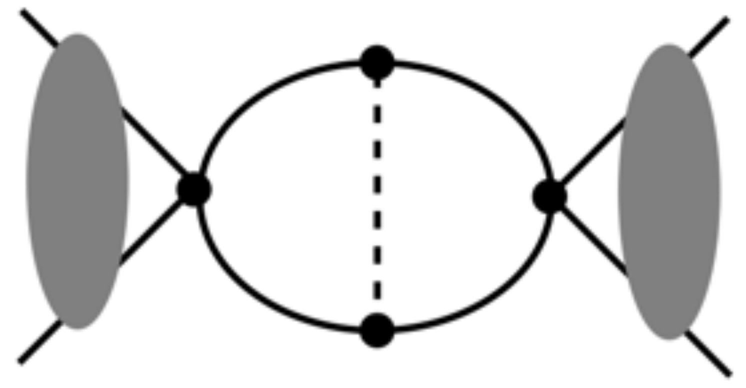


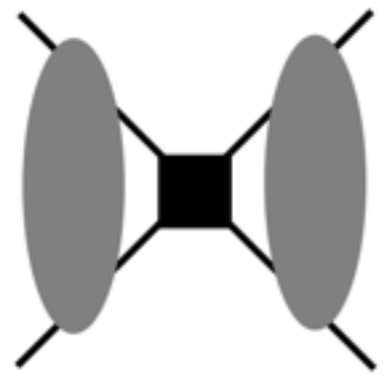
$$V_{\mathrm{LO}}(q) = \quad C_0 + D_2\, m_\pi^2 \quad + \quad \frac{g_A^2}{4f_\pi^2}\,\frac{\boldsymbol{\sigma_1}\cdot\mathbf{q}\;\boldsymbol{\sigma_2}\cdot\mathbf{q}}{\mathbf{q}^2 + m_\pi^2}\;\tau_1\cdot\tau_2$$

Renormalization requires D_2 :

Kaplan, Savage, Wise (1998)

To obtain a scattering amplitude that is independent of regularization or cut-off Λ requires:





$$\Lambda\frac{d}{d\Lambda}\left(\frac{D_2}{C_0^2}\right)_{\mathrm{KSW}} = \frac{g_A^2 m_N^2}{64\pi^2 f_\pi^2} \quad \Longrightarrow \quad \frac{|D_2|}{C_0^2} \approx \frac{g_A^2 m_N^2}{64\pi^2 f_\pi^2} \simeq \frac{1}{4} \qquad |D_2| \approx \frac{1}{5f_\pi^4}$$

Analysis of 2-nucleon scattering in Lattice QCD for different values m_π could, in principle, determine D_2 but systematics are too large at this time.

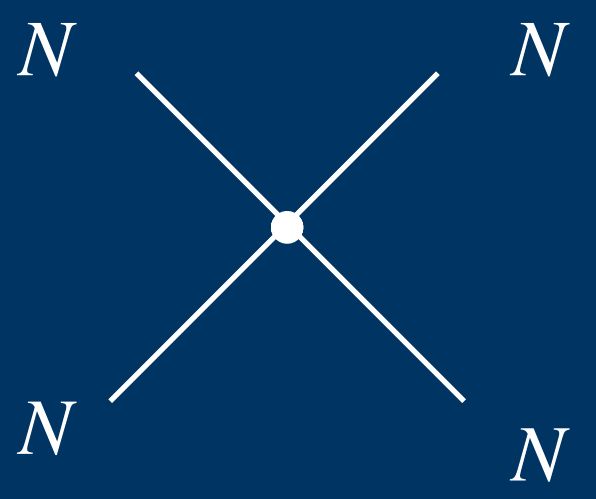
Beane, Bedaque, Detmold, Savage (NPLQCD), Walker-Loud (Cal-Lat), Aoki, Hatsuda, Ishii (HAL QCD Collaboration), ….

D_2 and Coupling to Pions

Chiral symmetry requires that pion mass terms only appear in a specified form:

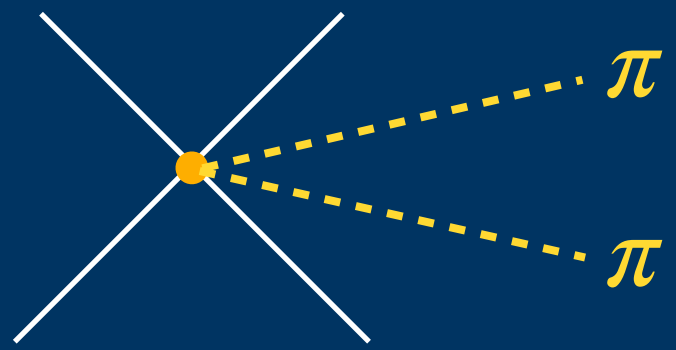
$$m_\pi^2 \Rightarrow m_\pi^2 \left(1 + \frac{\pi_a \pi_b}{2 f_\pi^2} \delta_{ab} + \cdots \right)$$

This induces a coupling of pions to two-nucleons:



$D_2\, m_\pi^2$

$+$

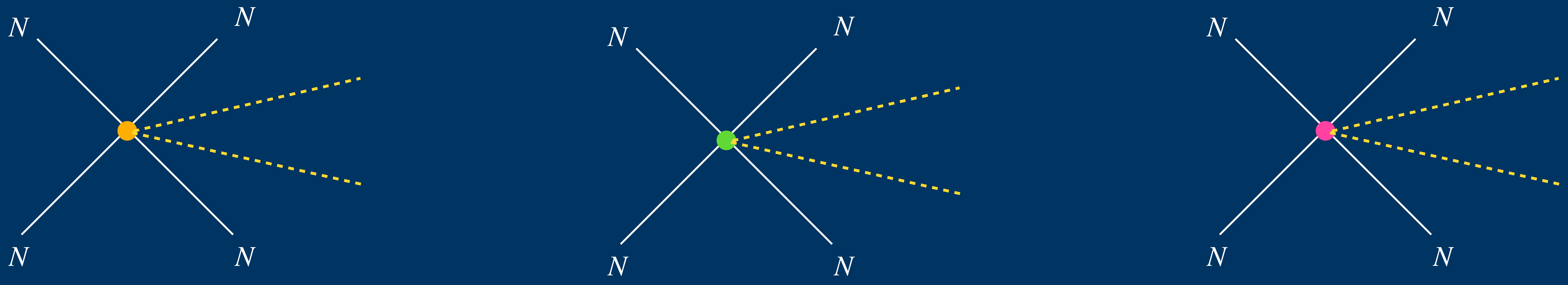


$D_2 \frac{m_\pi^2}{2 f_\pi^2}$

The LEC D_2 could be determined by pion-nucleus scattering but it will likely be challenging.

Two more enhanced pion-two-nucleon couplings: E_2 and F_2

B. Borasoy and H. W. Griesshammer (2001), (2003)



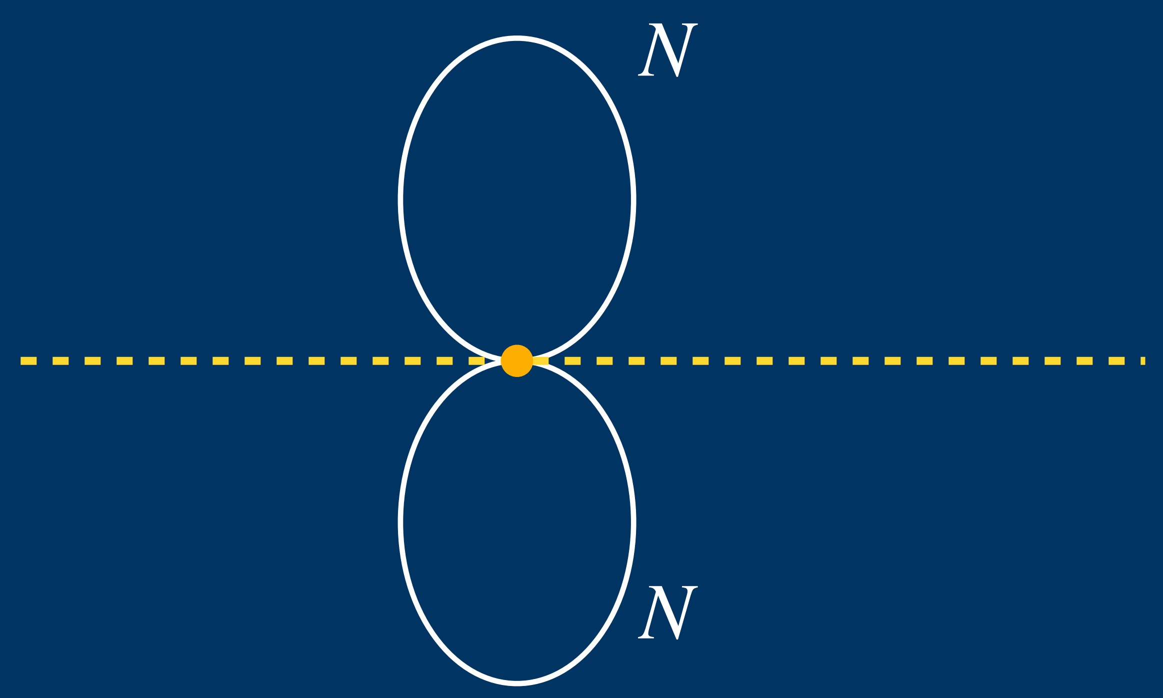
$D_2\, m_\pi^2$ $\quad$ $E_2\, \omega^2$ $\quad$ $F_2\, q^2$

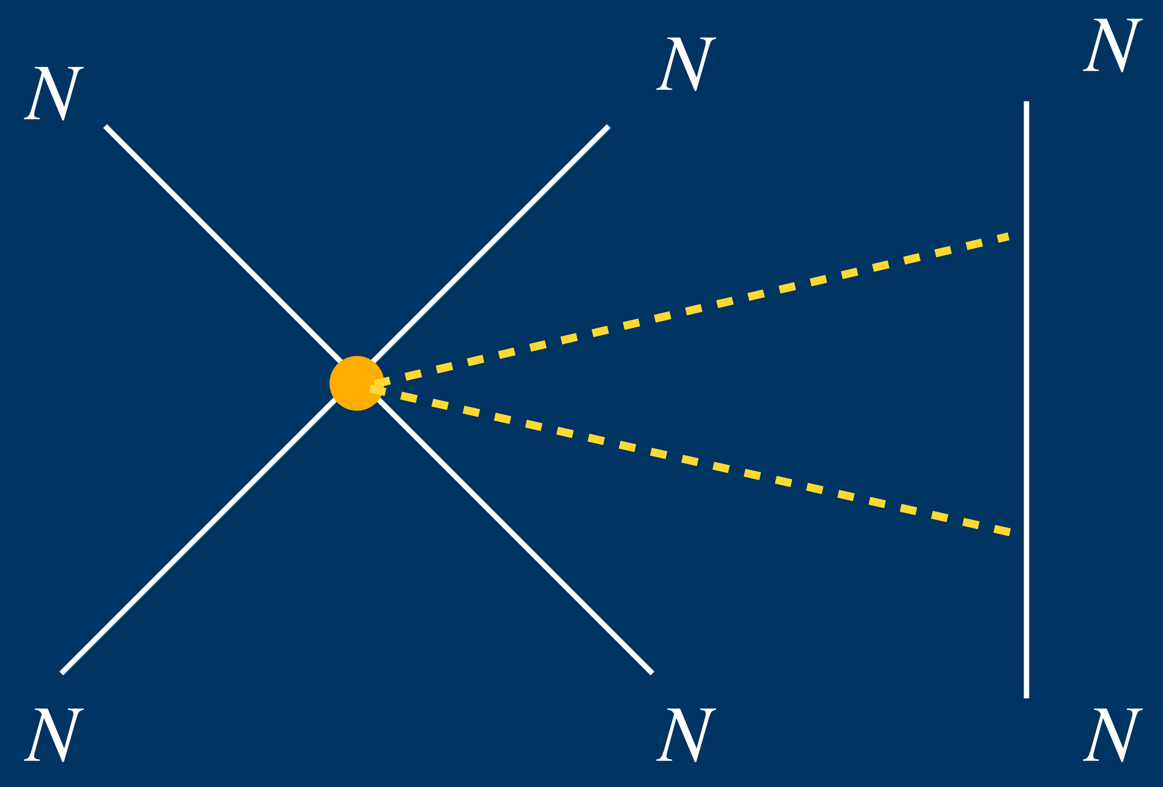
$D_2, E_2, \&\ F_2$ are enhanced for the same reason and apriori expected to be of similar size.

Typical size of these LECs: $\quad D_2 \approx E_2 \approx F_2 \approx \dfrac{1}{5 f_\pi^4}$

In Weinberg counting (which discounts RG invariance): $\quad D_2 \approx E_2 \approx F_2 \approx \dfrac{1}{\Lambda_N^2 f_\pi^2} \approx \dfrac{1}{50\ f_\pi^4}$

Implications of a Stronger Pion Coupling to Two Nucleons.





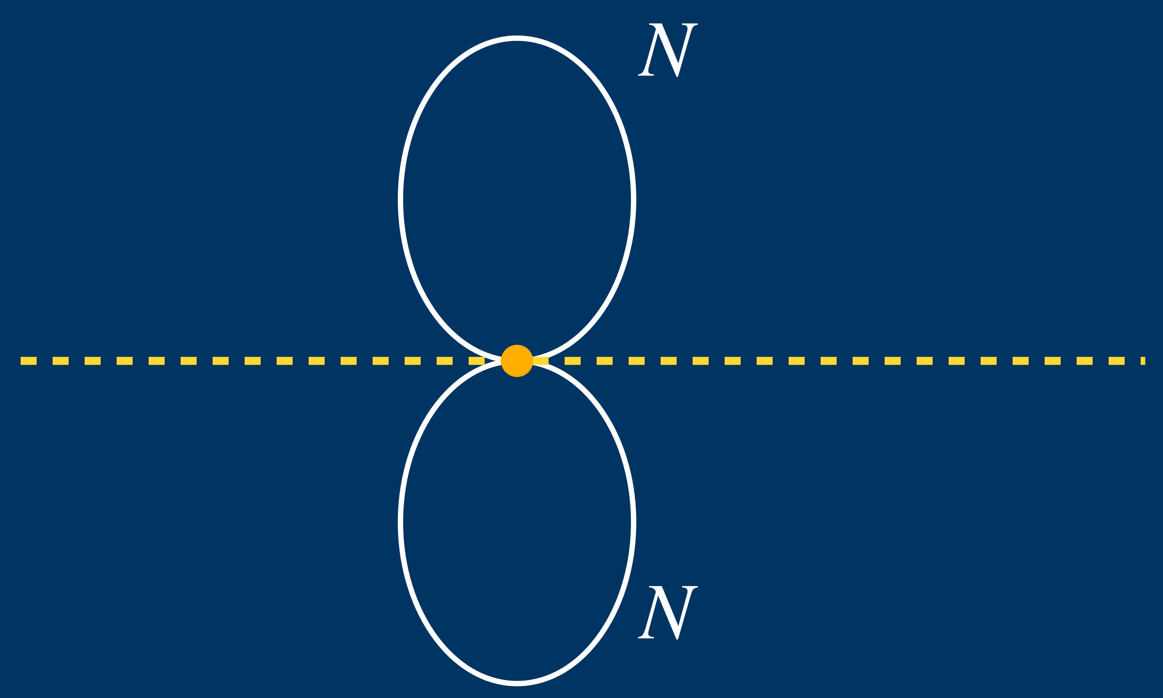
Pion-mass shift at finite density:

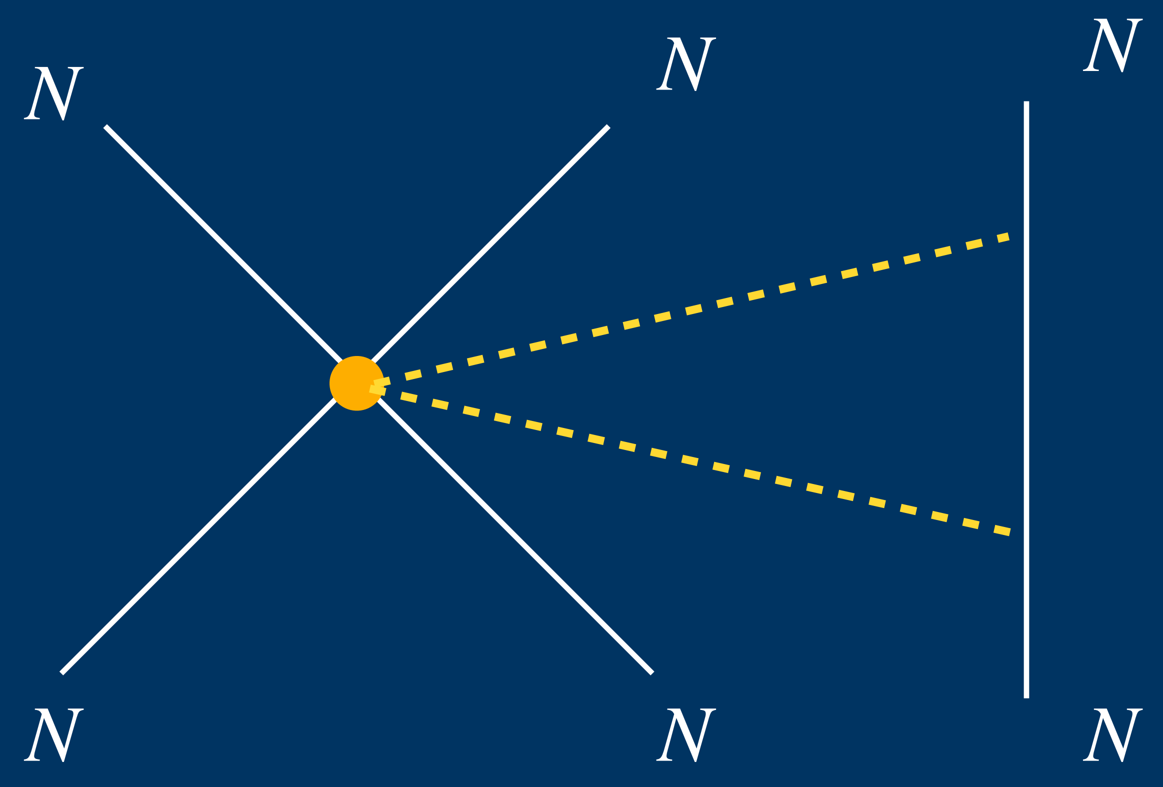
$$\delta m_\pi^2 \approx \frac{(D_2 m_\pi^2 + E_2 \omega^2)}{2 f_\pi^2}\, n_B^2$$

Enhanced contribution to the 3NF:

$$V_{3N} \approx \frac{9 D_2 g_A^2 m_\pi^3}{128 \pi f_\pi^4}\, \mathcal{J}(q^2/2m_\pi^2) + \frac{15 F_2 g_A^2 m_\pi^3}{16 \pi f_\pi^4}\, \mathcal{J}(q^2/2m_\pi^2)$$

Implications of a Stronger Pion Coupling to Two Nucleons.





Pion-mass shift at finite density:

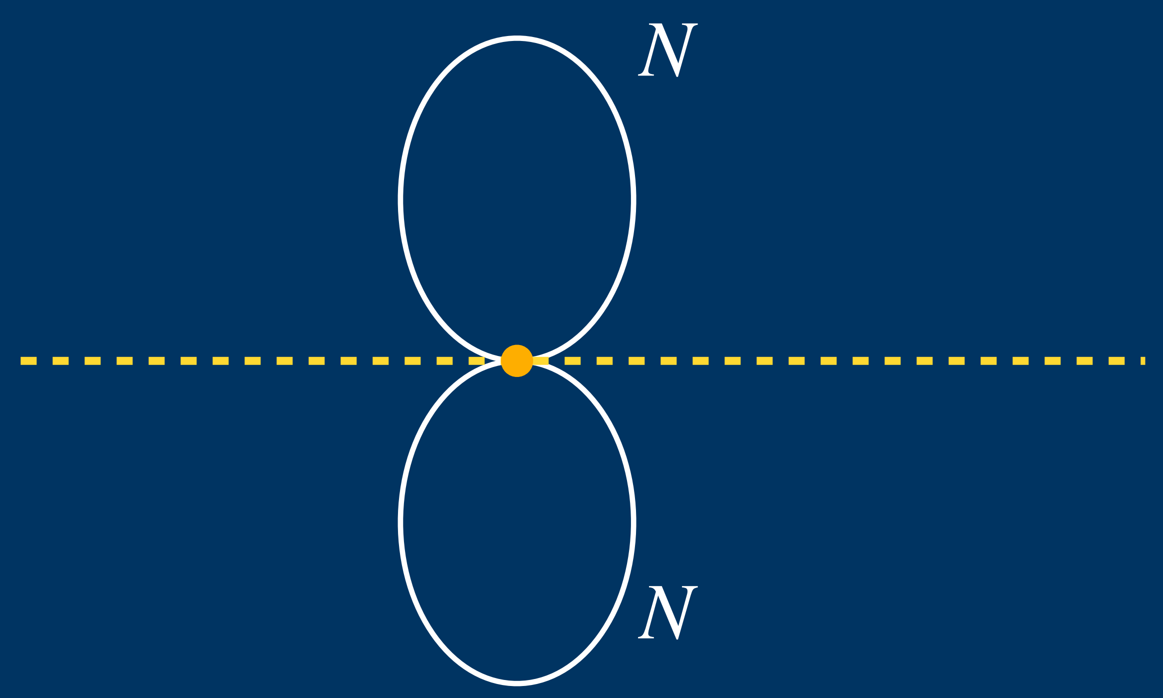
$$\delta m_\pi^2 \approx \frac{(D_2 m_\pi^2 + E_2 \omega^2)}{2 f_\pi^2}\, n_B^2$$

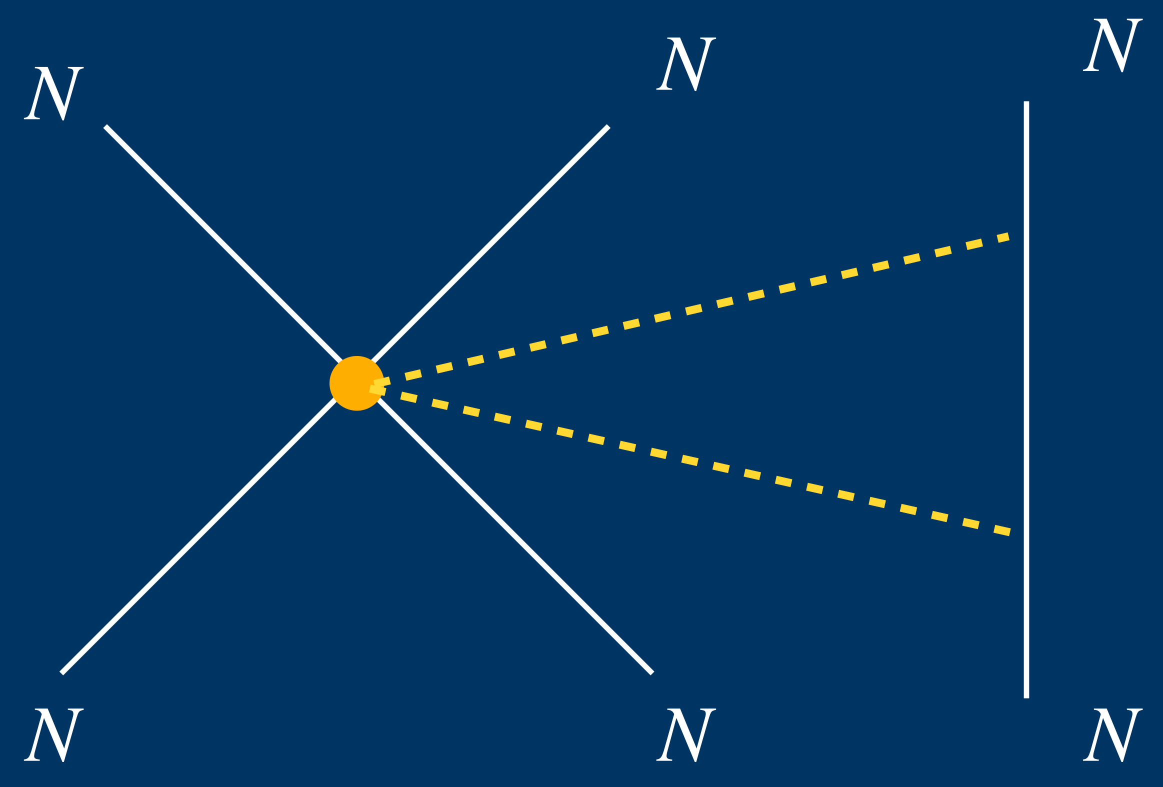
Enhanced contribution to the 3NF:

$$V_{3N} \approx \frac{9 D_2 g_A^2 m_\pi^3}{128 \pi f_\pi^4}\, \mathcal{J}(q^2/2m_\pi^2) + \frac{15 F_2 g_A^2 m_\pi^3}{16 \pi f_\pi^4}\, \mathcal{J}(q^2/2m_\pi^2)$$

Fore, Kaiser, Reddy, Warrington (2024).

Implications of a Stronger Pion Coupling to Two Nucleons.





Pion-mass shift at finite density:

$$\delta m_\pi^2 \approx \frac{(D_2 m_\pi^2 + E_2 \omega^2)}{2 f_\pi^2} \; n_B^2$$

Fore, Kaiser, Reddy, Warrington (2024).

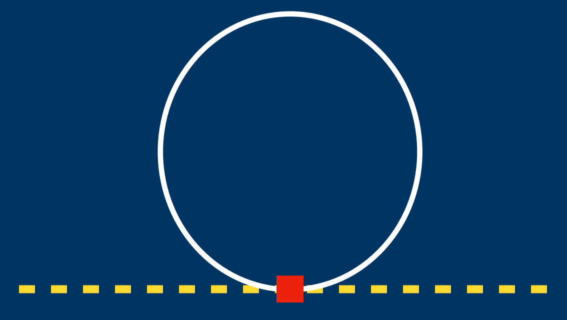
Enhanced contribution to the 3NF:

$$V_{3N} \approx \frac{9 D_2 g_A^2 m_\pi^3}{128 \pi f_\pi^4} \; \mathcal{J}(q^2/2m_\pi^2) + \frac{15 F_2 g_A^2 m_\pi^3}{16 \pi f_\pi^4} \; \mathcal{J}(q^2/2m_\pi^2)$$

Cirigliano, Dawid, Dekens, Reddy (2024 in prep.)

Pion Mass in Symmetric Nuclear Matter

The pion dispersion relation in nuclear matter is given by $\omega^2 - m_\pi^2 - \Pi_{\rm sym}(\omega, k_F) = 0$



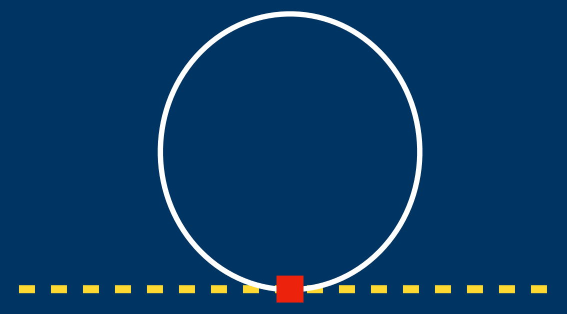
linear density

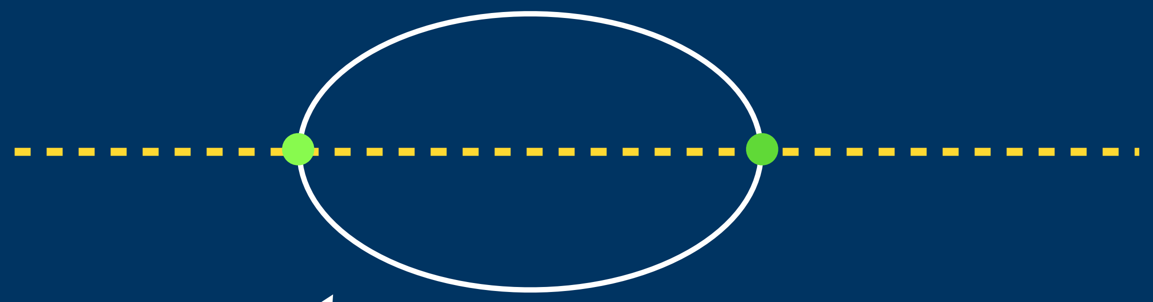
Pion self-energy: $\Pi_{\rm sym}(\omega, k_F) \approx -\,T^+(\omega)\; n_B$

N. Kaiser, W. Weise (2001), E. E. Kolomeitsev, N. Kaiser, W. Weise (2003)

Pion Mass in Symmetric Nuclear Matter

The pion dispersion relation in nuclear matter is given by $\omega^2 - m_\pi^2 - \Pi_{\mathrm{sym}}(\omega, k_F) = 0$





linear density

double-scattering

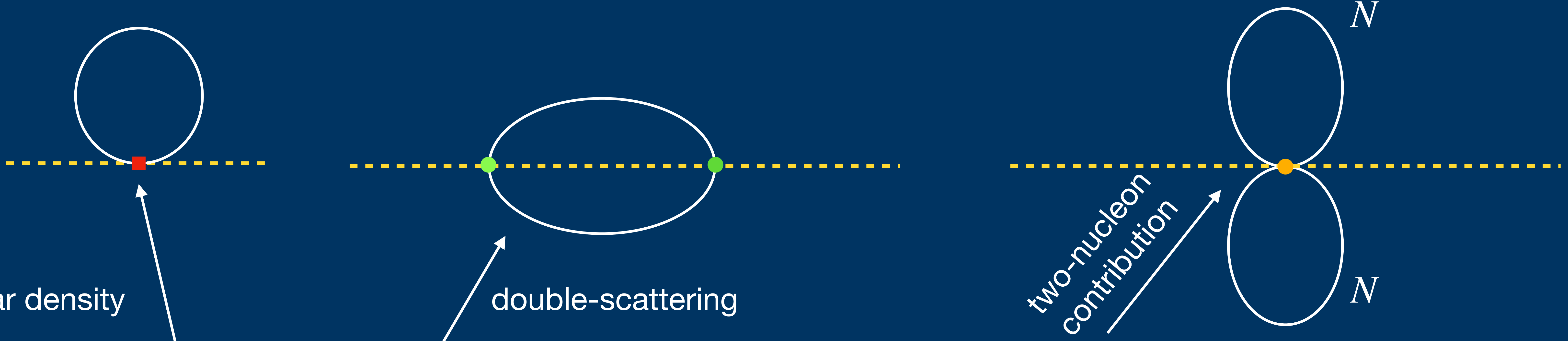
Pion self-energy: $$\Pi_{\mathrm{sym}}(\omega, k_F) \approx -T^+(\omega)\; n_B + \frac{2\omega^2 k_F^4}{(2\pi f_\pi)^4}\left[1 + \frac{Q^2}{2k_F^2}\left(2\ln\frac{|Q|}{2k_F} - 1\right)\right]$$

N. Kaiser, W. Weise (2001), E. E. Kolomeitsev, N. Kaiser, W. Weise (2003)

Pion Mass in Symmetric Nuclear Matter

The pion dispersion relation in nuclear matter is given by $\omega^2 - m_\pi^2 - \Pi_{\mathrm{sym}}(\omega, k_F) = 0$

Neill Warrington



Pion self-energy: $$\Pi_{\mathrm{sym}}(\omega, k_F) \approx -T^+(\omega)\; n_B + \frac{2\omega^2 k_F^4}{(2\pi f_\pi)^4}\left[1 + \frac{Q^2}{2k_F^2}\left(2\ln\frac{|Q|}{2k_F} - 1\right)\right] - \frac{3n_B^2}{16 f_\pi^2}(D_2 m_\pi^2 + E_2\omega^2)\,.$$

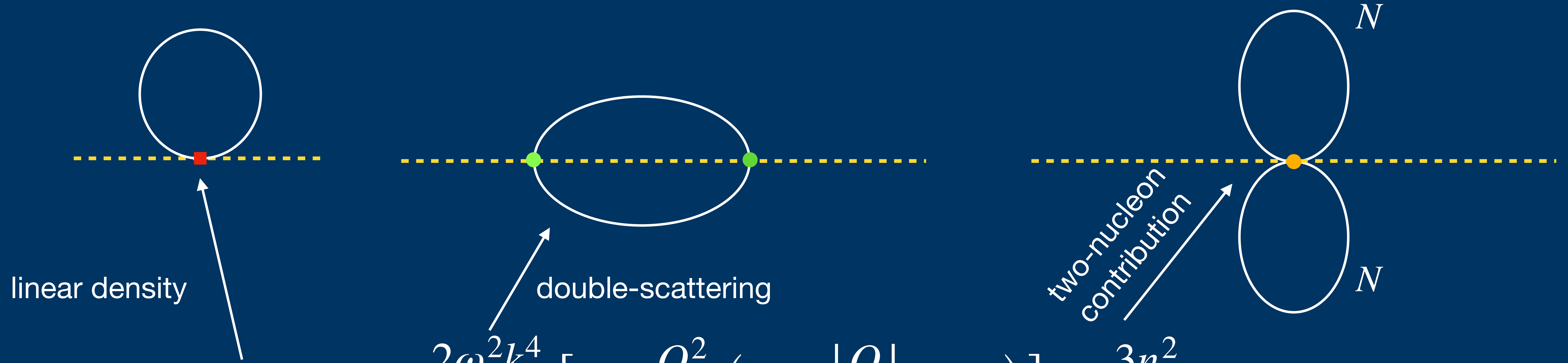
N. Kaiser, W. Weise (2001), E. E. Kolomeitsev, N. Kaiser, W. Weise (2003)

B. Fore, N. Kaiser, S. Reddy, N. Warrington (2024).

Pion Mass in Symmetric Nuclear Matter

The pion dispersion relation in nuclear matter is given by $\omega^2 - m_\pi^2 - \Pi_{\rm sym}(\omega, k_F) = 0$

Neill Warrington



Pion self-energy: $$\Pi_{\rm sym}(\omega, k_F) \approx -\, T^{+}(\omega)\; n_B + \frac{2\omega^2 k_F^4}{(2\pi f_\pi)^4}\left[1 + \frac{Q^2}{2k_F^2}\left(2\ln\frac{|Q|}{2k_F} - 1\right)\right] - \frac{3n_B^2}{16 f_\pi^2}(D_2 m_\pi^2 + E_2\omega^2)\,.$$

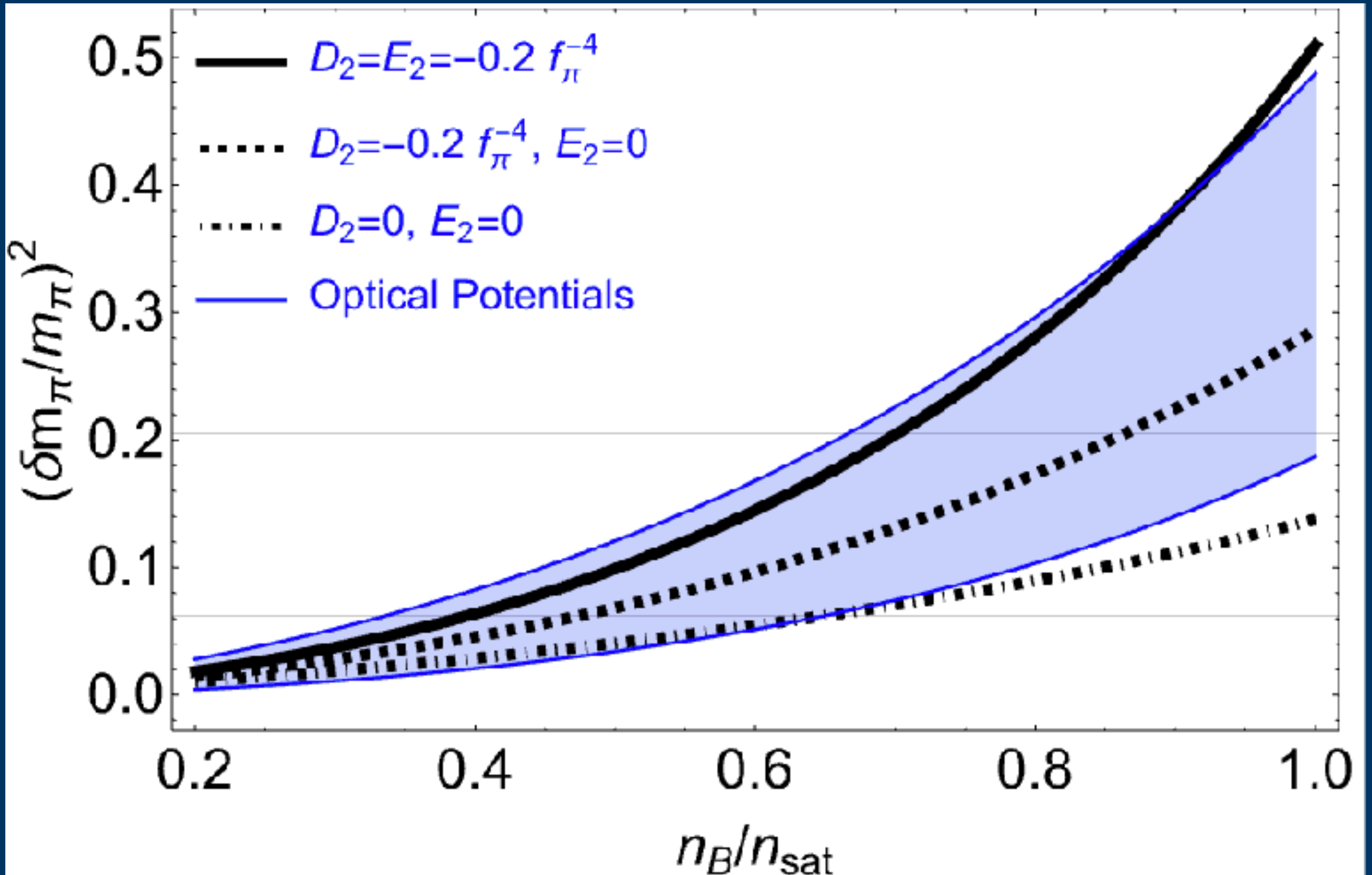
N. Kaiser, W. Weise (2001), E. E. Kolomeitsev, N. Kaiser, W. Weise (2003)

B. Fore, N. Kaiser, S. Reddy, N. Warrington (2024).

Mass shift due to interactions: $\delta m_\pi^2 = \omega^2 - m_\pi^2$

Optical potential fits to pion atoms provide constraints on the pion mass shift.

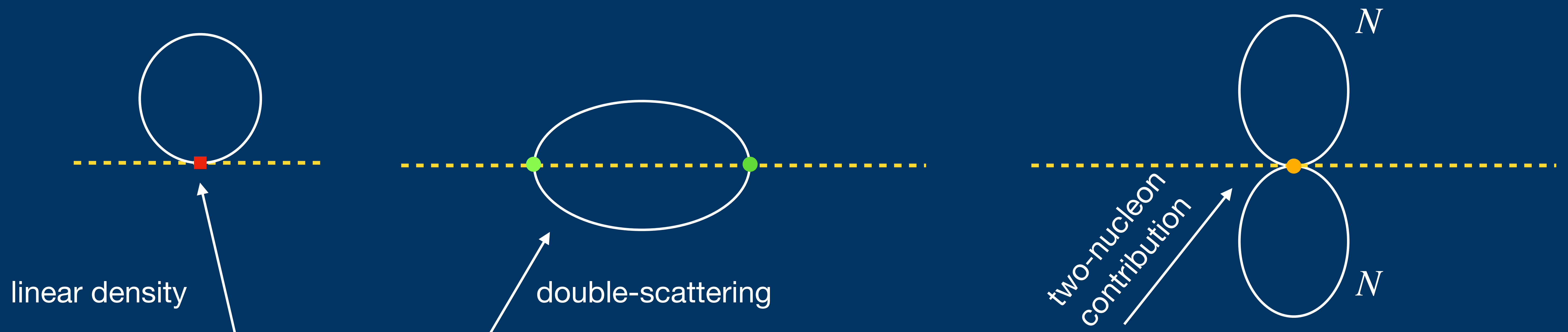
E. Friedman (2001,2002), E. Friedman and A. Gal (2007), Nishi et al. (2023)



Pion Mass in Symmetric Nuclear Matter

The pion dispersion relation in nuclear matter is given by $\omega^2 - m_\pi^2 - \Pi_{\rm sym}(\omega, k_F) = 0$

Neill Warrington



Pion self-energy: $$\Pi_{\rm sym}(\omega, k_F) \approx -T^+(\omega)\, n_B + \frac{2\omega^2 k_F^4}{(2\pi f_\pi)^4}\left[1 + \frac{Q^2}{2k_F^2}\left(2\ln\frac{|Q|}{2k_F} - 1\right)\right] - \frac{3n_B^2}{16 f_\pi^2}(D_2 m_\pi^2 + E_2\omega^2)\,.$$

N. Kaiser, W. Weise (2001), E. E. Kolomeitsev, N. Kaiser, W. Weise (2003)

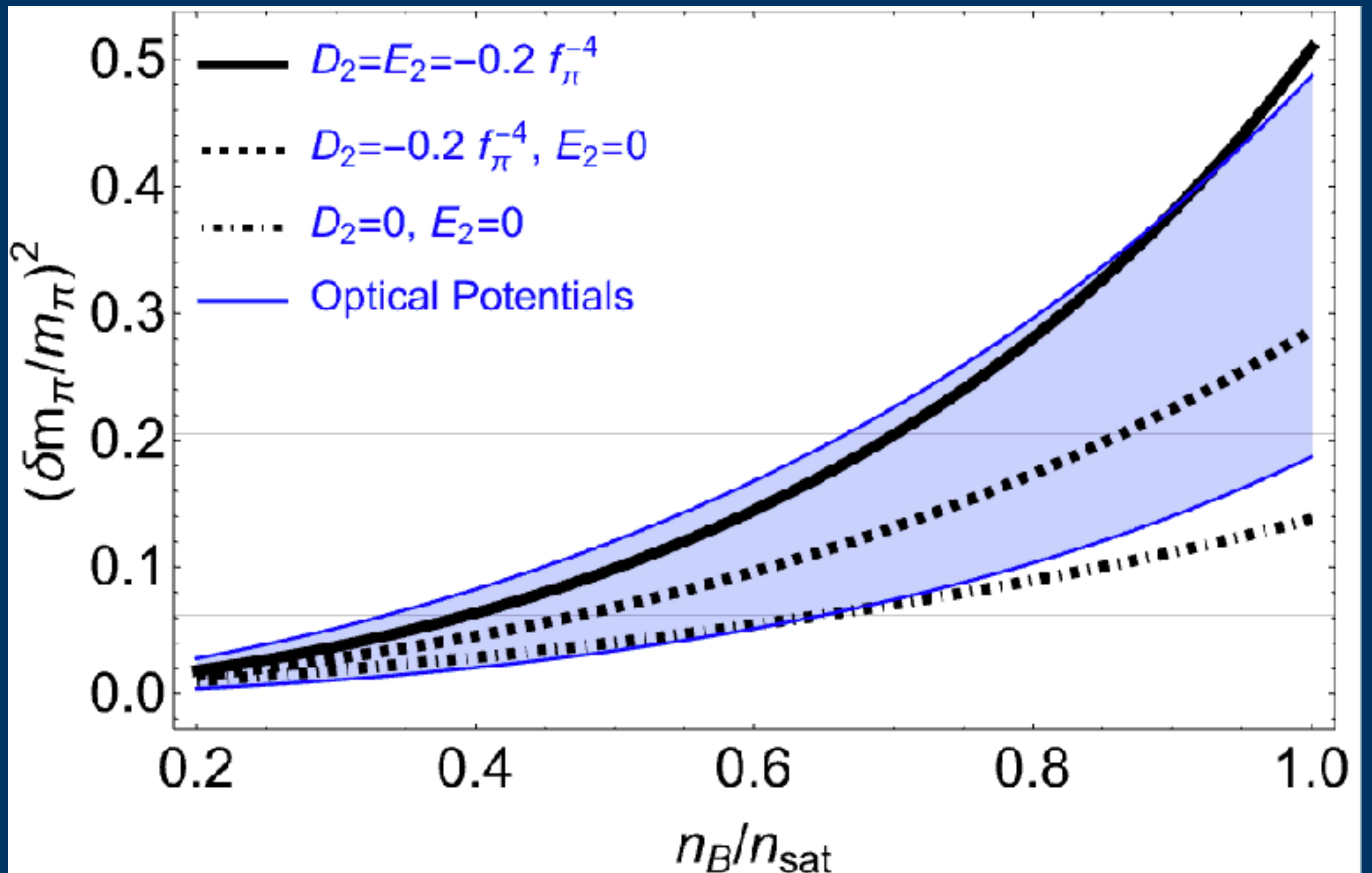
B. Fore, N. Kaiser, S. Reddy, N. Warrington (2024).

Mass shift due to interactions: $\delta m_\pi^2 = \omega^2 - m_\pi^2$

Optical potential fits to pion atoms provide constraints on the pion mass shift.

E. Friedman (2001,2002), E. Friedman and A. Gal (2007), Nishi et al. (2023)

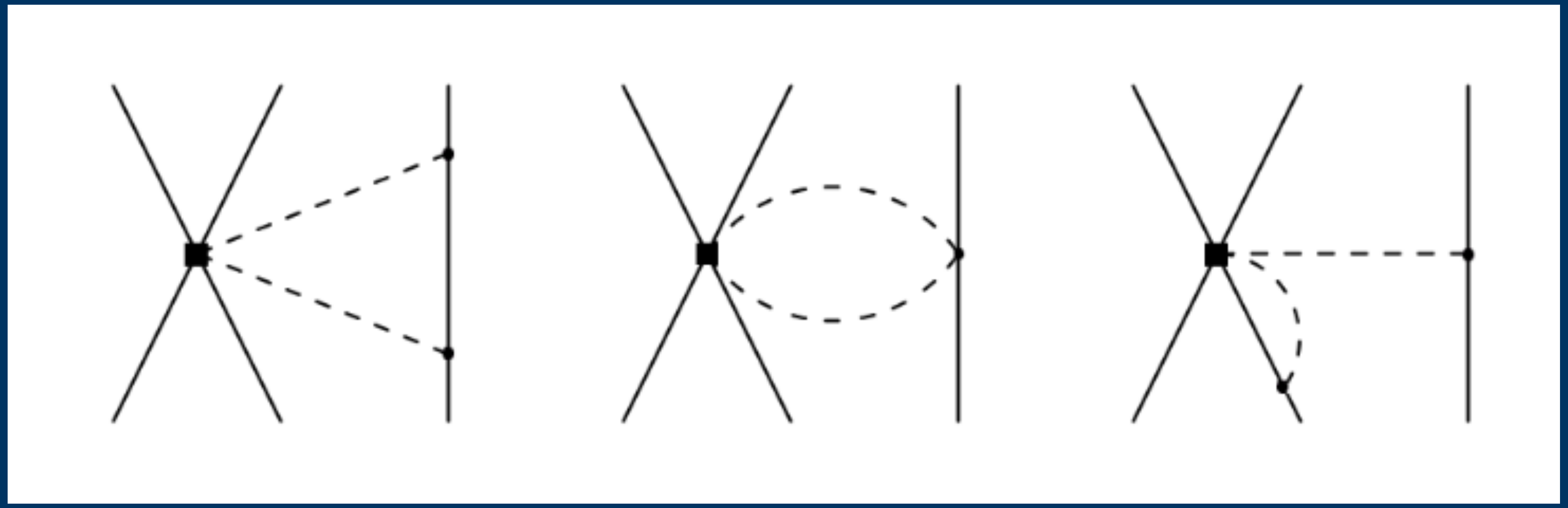
Negative values of D_2 & E_2 can account for observed repulsion in pionic atoms.



A New Class of Three Nucleon Forces


Maria Dawid


Wouter Dekens


3NF due to pion coupling to two nucleons

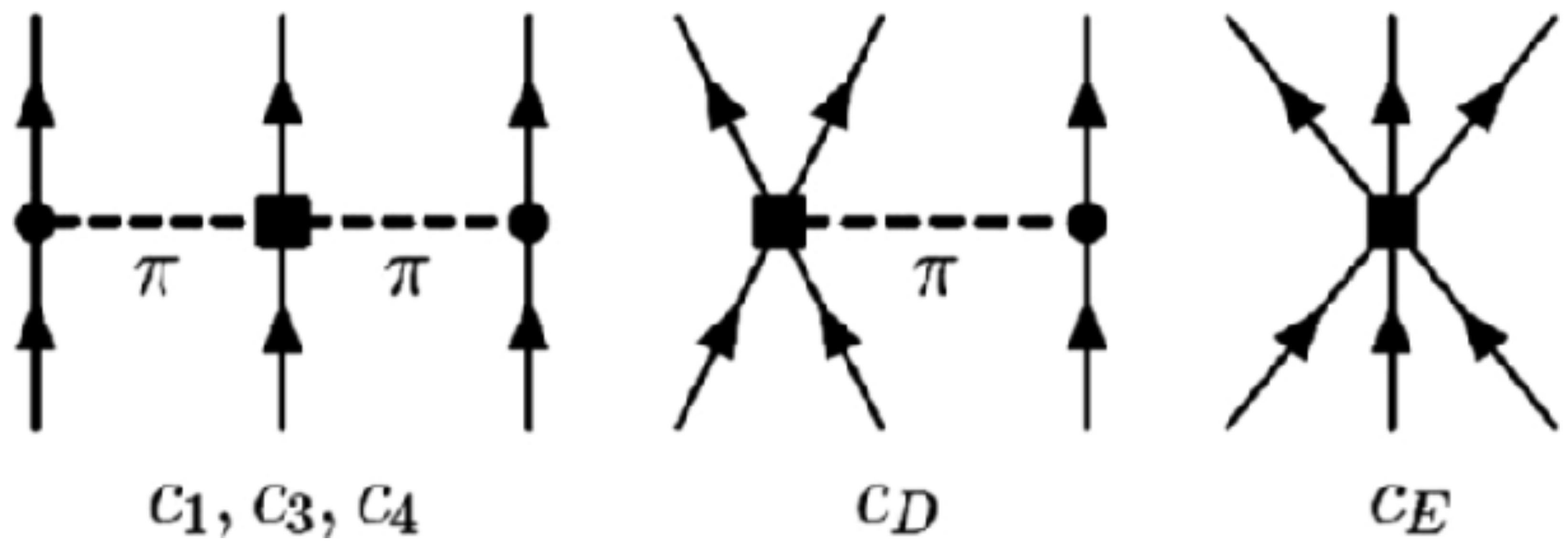
$$V_{ijk}^{i'j'k'}(\vec{q}_1,\vec{q}_2,\vec{q}_3) = -\frac{9g_A^2 D_2 m_\pi^3}{128\pi f_\pi^4}\kappa_{ij}^{i'j'}\delta_{kk'}\,\mathcal{I}\left(\frac{\vec{q}_3^{\,2}}{4m_\pi^2}\right) \quad \text{where} \quad \mathcal{I}(b) = \frac{2}{3}\left(1+\left(\frac{1}{2\sqrt{b}}+\sqrt{b}\right)\cot^{-1}(1/\sqrt{b})\right)$$

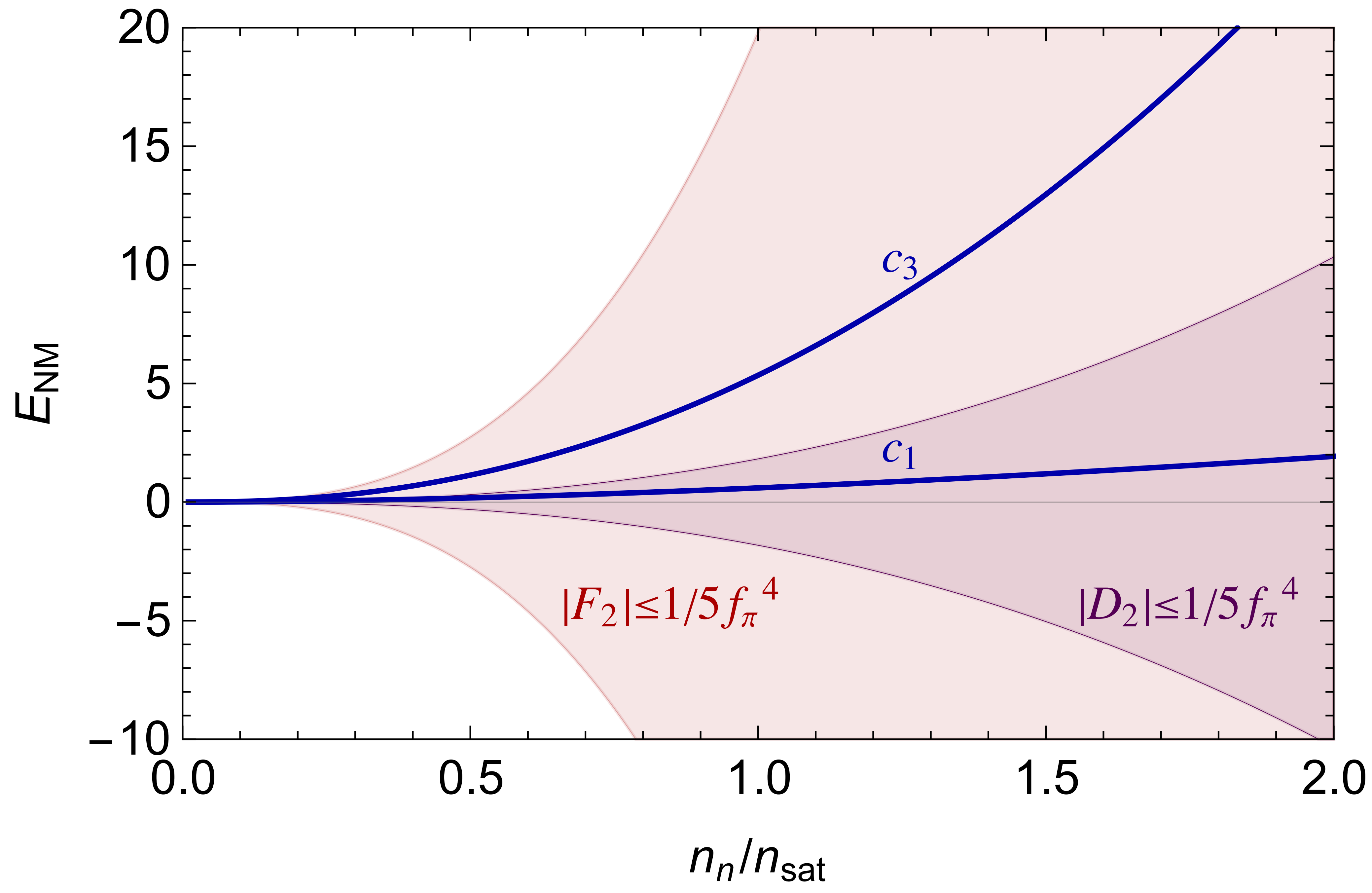
$$V_{ijk}^{i'j'k'}(\vec{q}_1,\vec{q}_2,\vec{q}_3) = -\frac{15F_2 g_A^2 m_\pi^3}{16\pi f_\pi^4}\delta_{kk'}\left(\bar{f}_2^S\delta_{ii'}\delta_{jj'}+\bar{f}_2^T\vec{\sigma}_{i'i}\cdot\vec{\sigma}_{j'j}\right)\mathcal{J}\left(\frac{\vec{q}_3^{\,2}}{4m_\pi^2}\right) \quad \text{where} \quad \mathcal{J}(b) = \frac{3}{5}\left((1+2b)\mathcal{I}(b)+\frac{2}{3}\right)$$

V. Cirigliano, M. Dawid, W. Dekens, S. Reddy (2024)

D_2 and F_2 Contributions to the Energy are Large

In neutron and nuclear matter, the leading 3NF plays a critical role.





The new 3NF can be large enough to compete with the NNLO forces currently employed in Chiral EFT.

The uncertainty is large because D_2 & F_2 are not yet known.

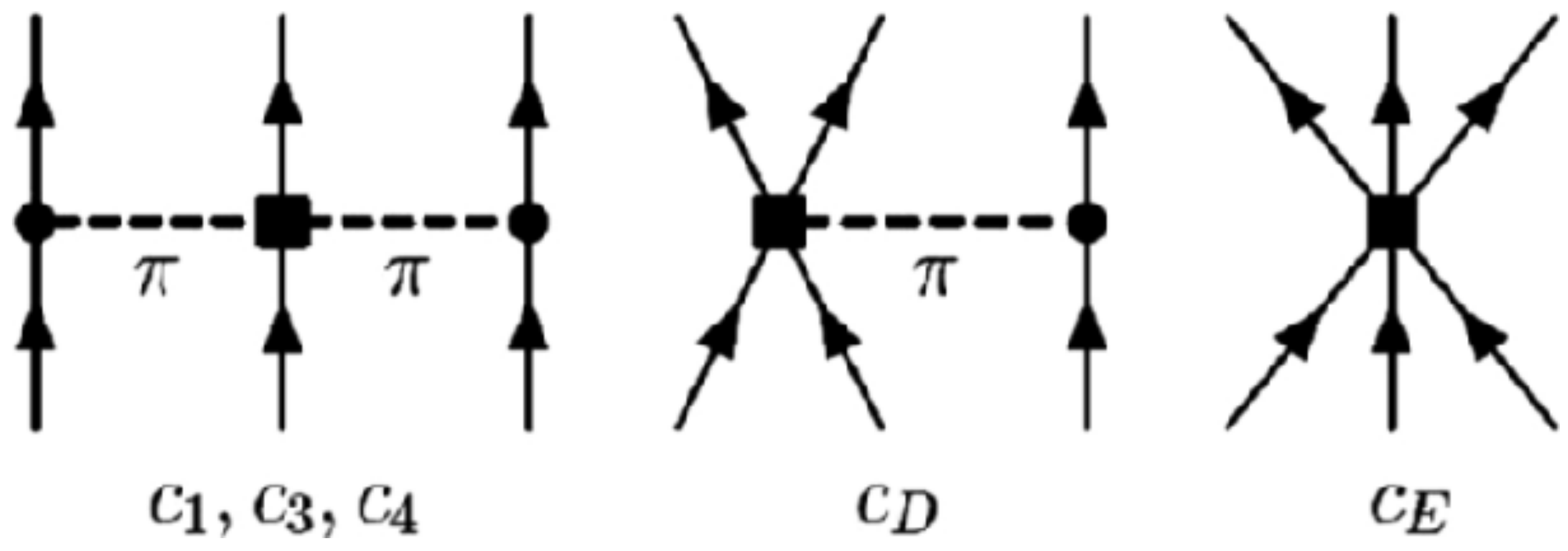
Nuclear structure and pion-nucleus scattering data can independently constrain D_2 & F_2.

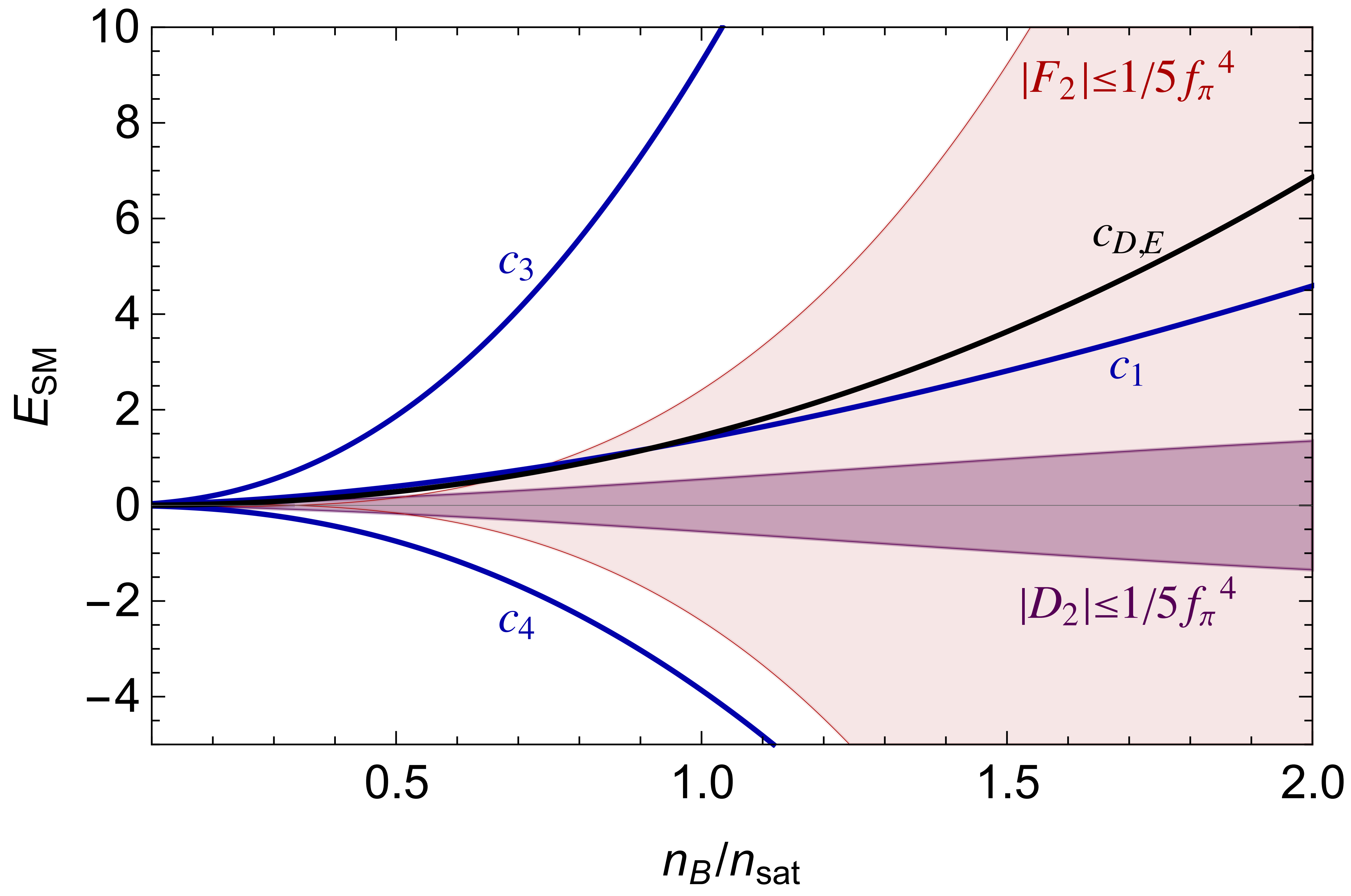
Independent determinations would test the convergence of EFT and estimates of truncation errors.

V. Cirigliano, M. Dawid, W. Dekens, S. Reddy (2024)

D_2 and F_2 Contributions to the Energy are Large

In neutron and nuclear matter, the leading 3NF plays a critical role.





The new 3NF can be large enough to compete with the NNLO forces currently employed in Chiral EFT.

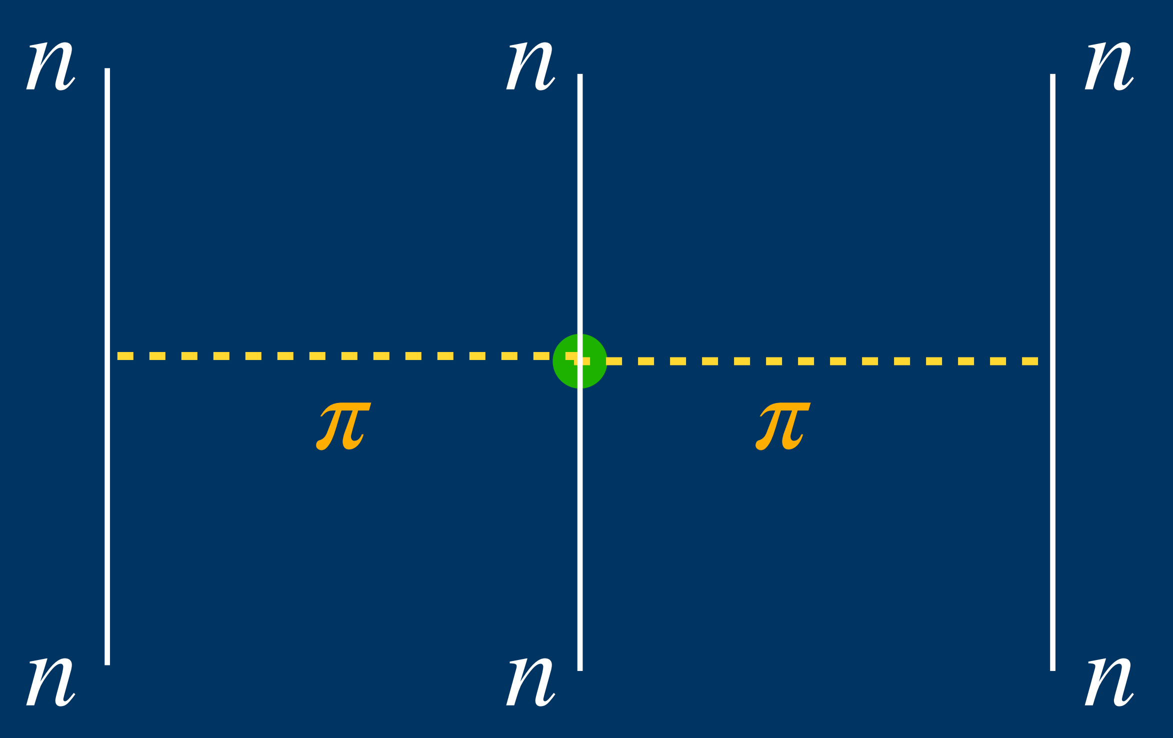
The uncertainty is large because D_2 & F_2 are not yet known.

Nuclear structure and pion-nucleus scattering data can independently constrain D_2 & F_2.

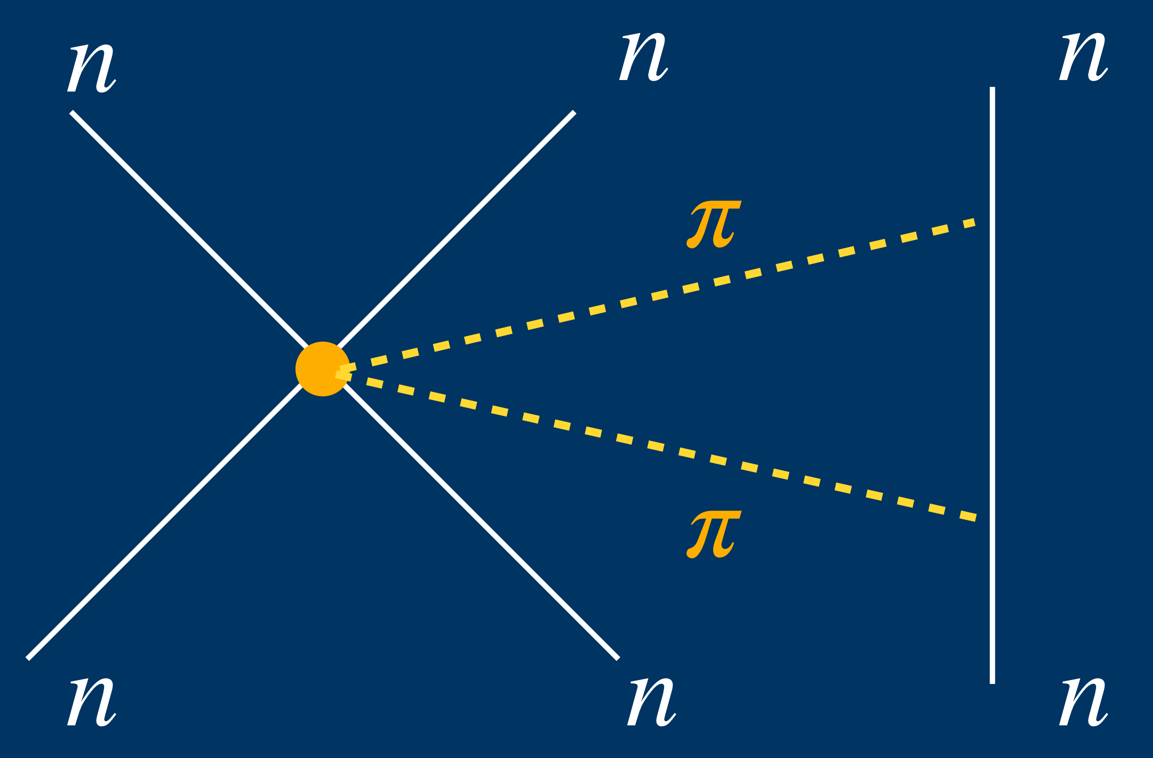
Independent determinations would test the convergence of EFT and estimates of truncation errors.

V. Cirigliano, M. Dawid, W. Dekens, S. Reddy (2024)

Neutron Matter: Underestimating Errors?



c_1 & c_3



D_2 & F_2

Current Paradigm:

Leading 3NF is determined by pion-nucleon scattering data. Independent of multi-nucleon information . Errors are small becasue there are no 3NF short-distance contributions.

Our calculation:

Pion coupling to two-nucleons can play a role. Information about two-nucleon dynamics influences 3NF to ensure proper renormalization. Error estimates will likely need revsion.

Chiral EFT at N²L0 predicts

$$P(n_{\rm sat}) = 3.1 \pm 0.5 \ {\rm MeV/fm^3}$$

C. Drischler, R. J. Furnstahl, J. A. Meleldez, D. R. Phillips (2021)

$$P(n_{\rm sat}) = 2.2 \pm 0.4 \ ({\rm MeV/fm^3})$$

I. Tews,R. Somasundaram,D. Lonardoni,H. Göttling, R. Seutin, J. Carlson S. Gandolfi,K. Hebeler, A. Schwenk (2024)

We estimate the contribution to the pressure from our new 3NFs to be:

$$\delta P_{\rm 3NF} = \left[0.7\left(\frac{D_2}{D_2^{\rm ref}}\right) + 8.8\left(\frac{F_2}{F_2^{\rm ref}}\right)\right] \frac{\rm MeV}{\rm fm^3}$$

where $D_2^{\rm ref} = F_2^{\rm ref} = \dfrac{1}{5f_\pi^4}$

V. Cirigliano, M. Dawid, W. Dekens, S. Reddy (2024)

Nuclear Symmetry Energy

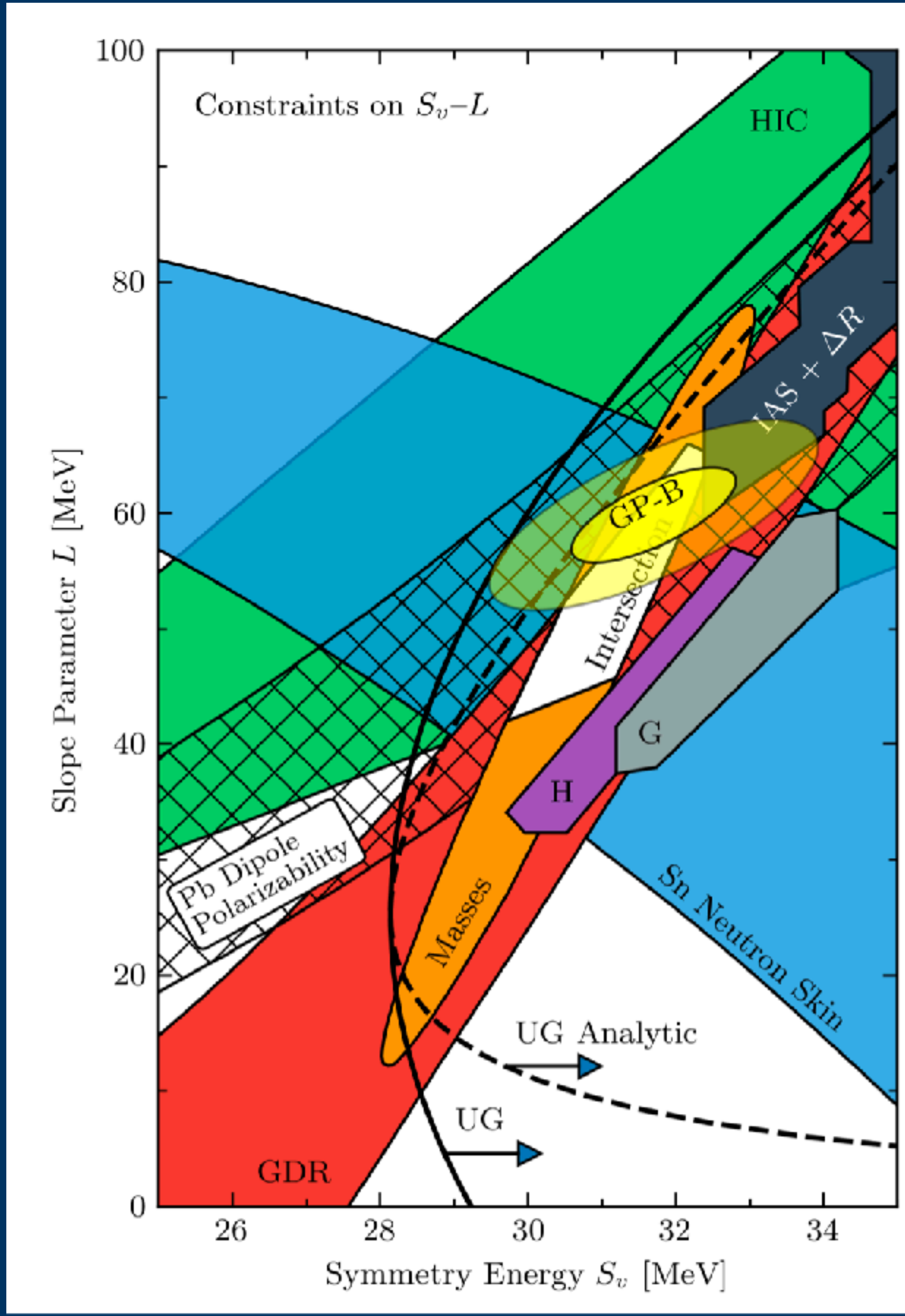
The new 3NFs can significantly alter the symmetry energy and it slope at saturation density:

$$\frac{\delta S_0}{\mathrm{MeV}} \simeq (9\ D_2 + 96\ F_2)\, f_\pi^4$$

$$\frac{\delta L}{\mathrm{MeV}} \simeq (13\ D_2 + 166\ F_2)\, f_\pi^4$$

Approximate agreement between current ChiEFT calcualtions of neutron matter and experiment suggests either a large cancellation or anamolosly small values for the D_2 and F_2

Constraints on D_2 and F_2 from light-nuclei would provide useful guidance.



C. Drischler, R. J. Furnstahl, J. A. Meleldez, D. R. Phillips (2021)

Part 2 : Axion Condensation in Neutron Stars

θ and Axions

To explain $\theta < 10^{-10}$ a mechanism was proposed: Make θ a dynamical quantity.
Introduced a new field that relaxes to zero to minimize the free energy:

$$\theta = \frac{a}{f_a}$$

Axion field

A new high energy scale

The axion is a pseudo-scalar particle that arises as a Goldstone boson from the breaking of a new U(1) symmetry introduced by Pecci and Quinn.

R. Peccei and H. R. Quinn (1977), S. Weinberg (1978), F. Wilczek (1978)

θ and Axions

$$\mathscr{L}_\theta = \theta \, \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$$

To explain $\theta < 10^{-10}$ a mechanism was proposed: Make θ a dynamical quantity.
Introduced a new field that relaxes to zero to minimize the free energy:

$$\theta = \frac{a}{f_a}$$

Axion field

A new high energy scale

The axion is a pseudo-scalar particle that arises as a Goldstone boson from the breaking of a new U(1) symmetry introduced by Pecci and Quinn.

R. Peccei and H. R. Quinn (1977), S. Weinberg (1978), F. Wilczek (1978)

Axion Mass and Energy

The axion coupling to gluons can be elimanted by a transformation of the quark mas matrix M_q

$$M_q \to M_q \ \exp\left(2i \frac{a}{f_a} \ Q_a\right) \qquad \text{where} \qquad Q_a = \frac{M_q^{-1}}{\mathrm{Tr}\ M_q^{-1}} = \frac{1}{m_u + m_d}\begin{pmatrix} m_d & & \\ & m_u & \\ & & 0 \end{pmatrix} + \mathcal{O}[m_u/m_s, m_d/m_s]$$

This leads to an axion mass which can be calculated from the Chiral Lagrangian

$$m_a^2 = \frac{f_\pi^2}{f_a^2}\left(\frac{m_u m_d}{(m_u + m_d)^2}\right) m_\pi^2$$

And a corresponding contribution to the energy density or an axion potential

$$V\left(\theta = \frac{a}{f_a}\right) = f_\pi^2 m_\pi^2 \left[1 - \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left[\frac{\theta}{2}\right]}\right] = \frac{1}{2} f_a^2 m_a^2 \theta^2 + \cdots$$

Which is minimized at $\theta = 0$.

Hadrons at $\theta \neq 0$

Because $M_q \to M_q \, \exp\left(2i\theta \, Q_a\right)$

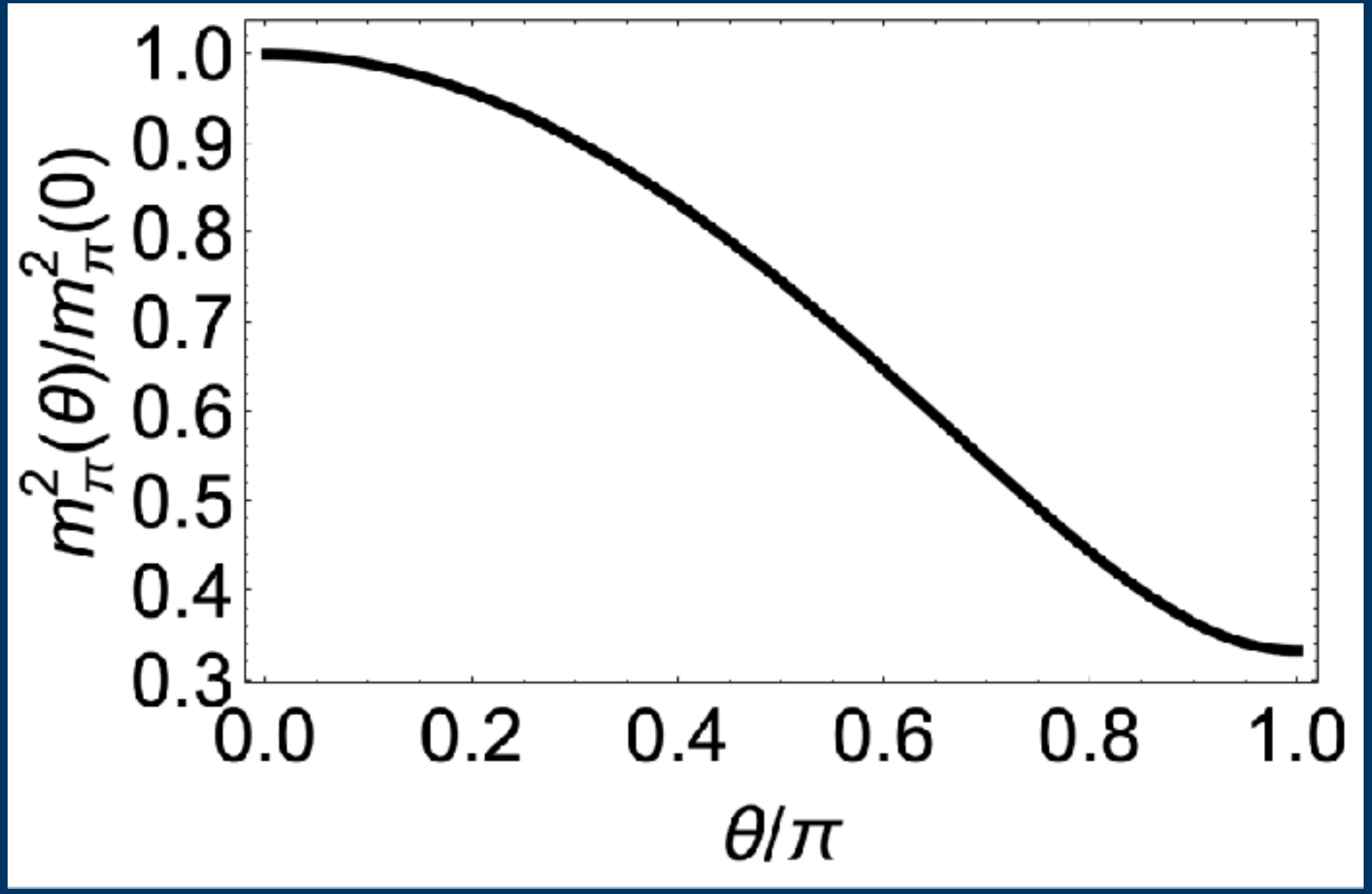
the pion mass decreases with θ:

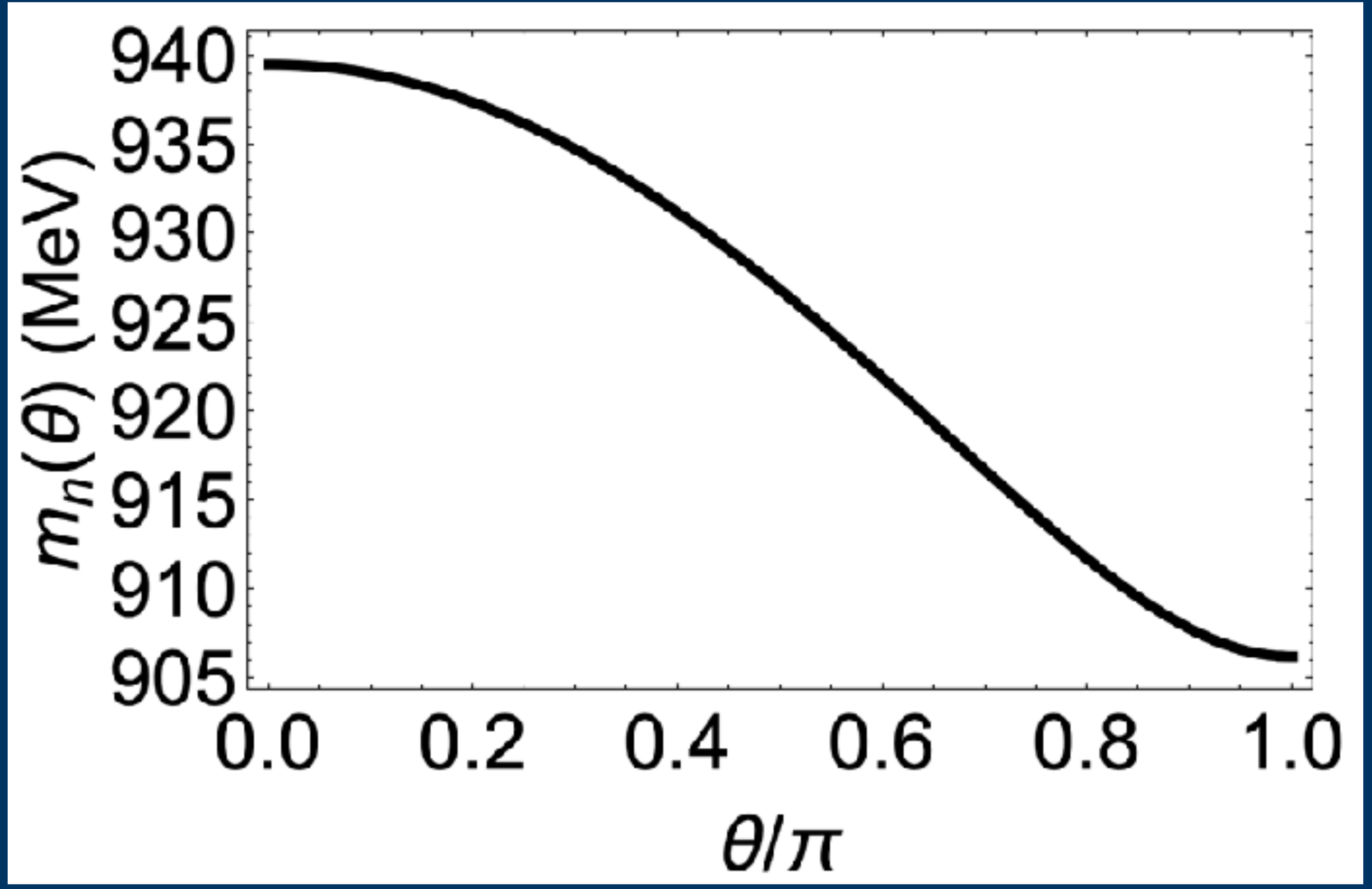
$$m_\pi^2(\theta) = m_\pi^2(\theta = 0) \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left[\frac{\theta}{2}\right]}$$

$$\frac{m_\pi^2(\theta = \pi)}{m_\pi^2(\theta = 0)} = \frac{m_d - m_u}{m_d + m_u} \approx \frac{1}{3}$$

The resulting decrease in the nucleon mass

$$m_n(\theta) = m_0 + \sigma_{\pi n} \frac{m_\pi^2(\theta)}{m_\pi^2(\theta = 0)} + \cdots$$





Axion Condensation

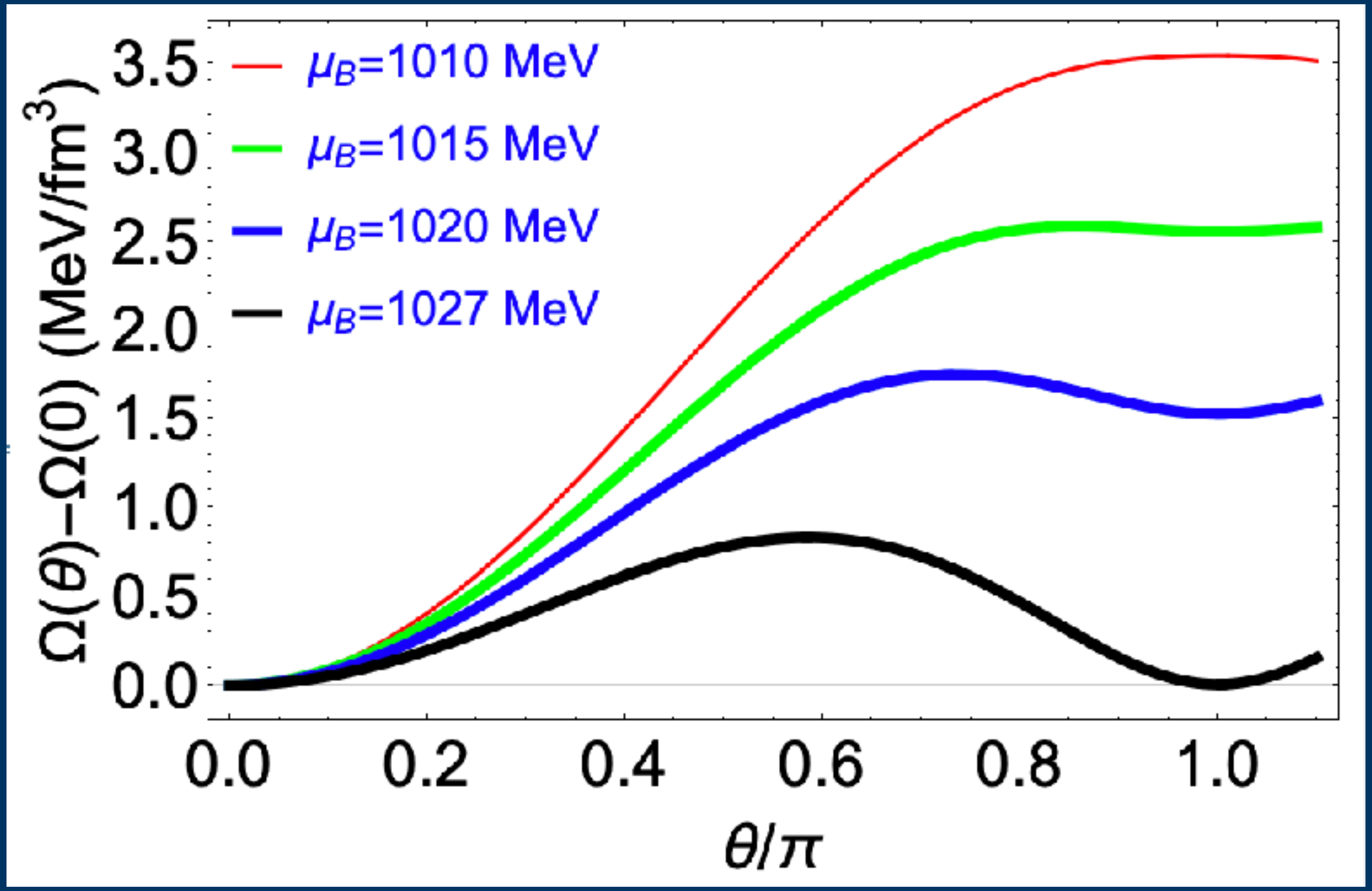
The decrease in the nucleon mass

$$m_n(\theta) = m_0 + \sigma_{\pi n} \frac{m_\pi^2(\theta)}{m_\pi^2(\theta = 0)} + \cdots$$

favors a first-order transition to a ground state with $\theta = \pi$

If interactions are neglected the transiton occurs at

$$n_B \simeq 2.6 \ n_{\text{sat}}$$



To address if axions can condense in neutron stars we need to know how nuclear interactions are modified when the pion mass is reduced to about 80 MeV.

If the nuclear interaction energy decreases with decreasing pion mass, axions would condense at

$$n_B < 2.6 \ n_{\text{sat}}$$

Can interactions favor axion condensation?

How does the interaction energy at nuclear density change with m_π?

$$\Delta E_{\rm int} = E_{\rm int}(m_\pi) - E_{\rm int}(m_\pi^{\rm phys})$$

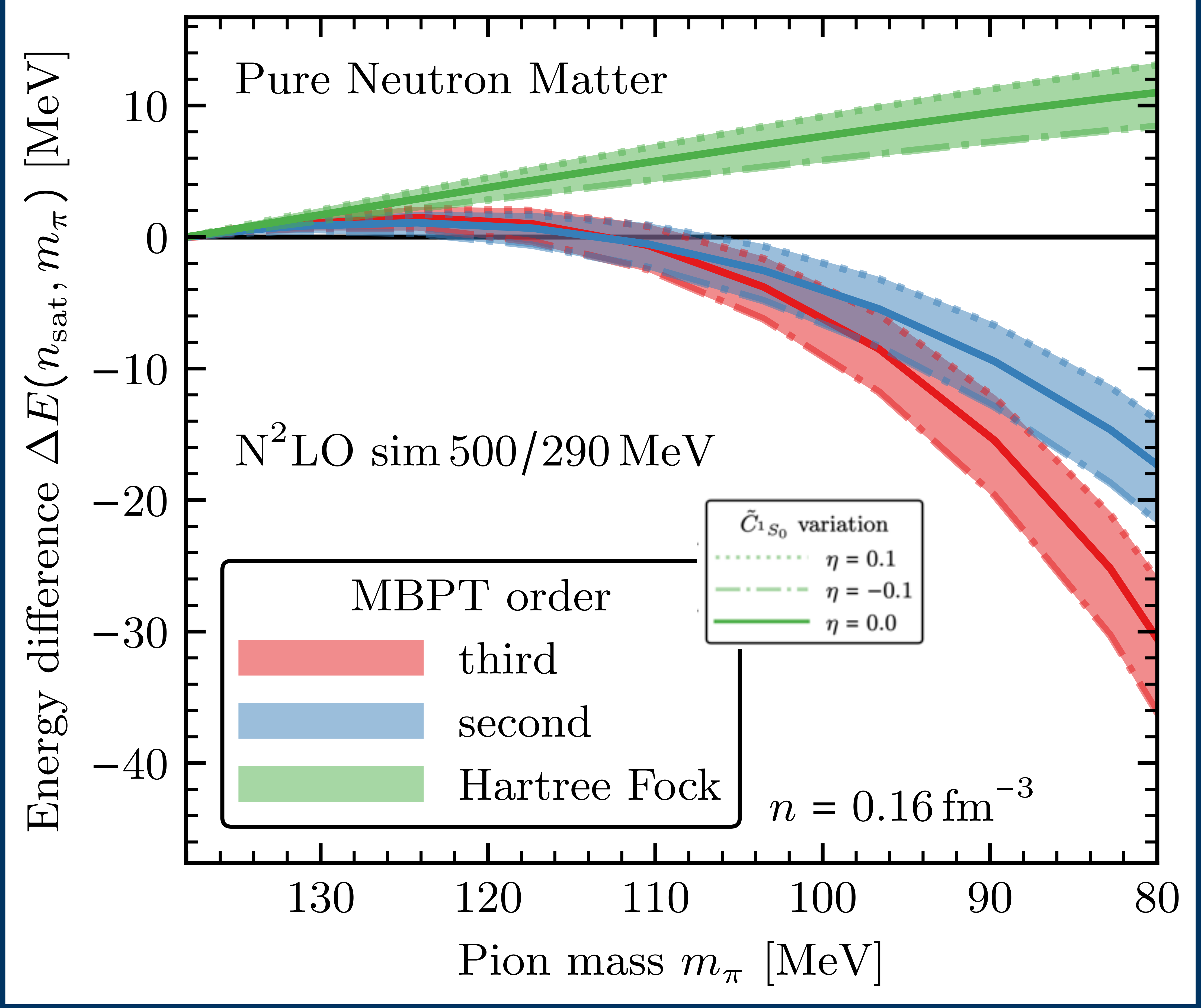
Christian Drischler

- In ChiEFT, interaction increases at first, and then decreases with decreasing m_π

- Uncertainty due D_2 does not alter this conclusion.

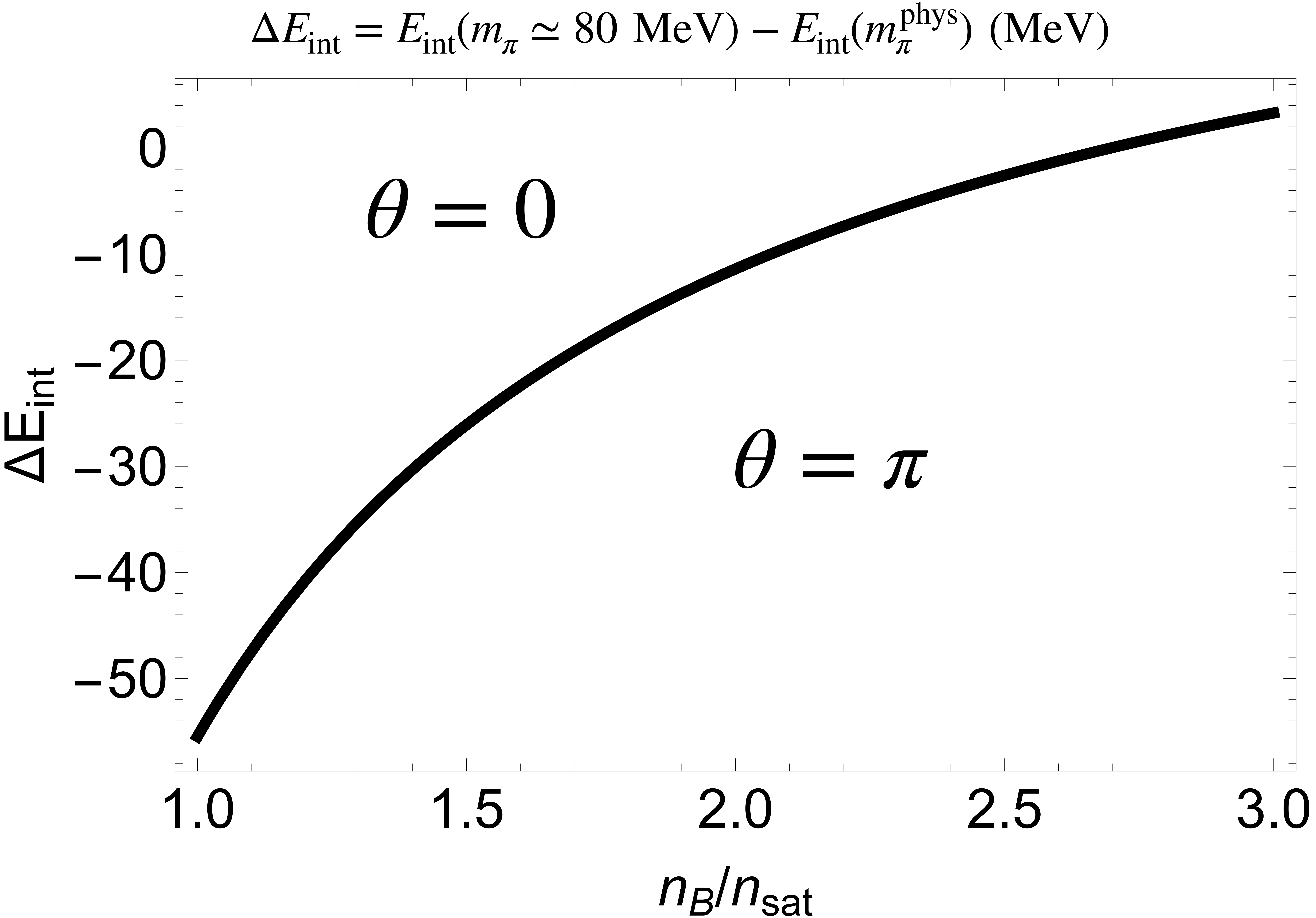
$$-0.1 < \eta = \frac{D_2 m_\pi^2}{\tilde{C}_{^1S_0}} < 0.1$$

- A large cutoff dependence suggets that a systematic study of the pion-mass dependence of short-range components (LECs) is warranted.

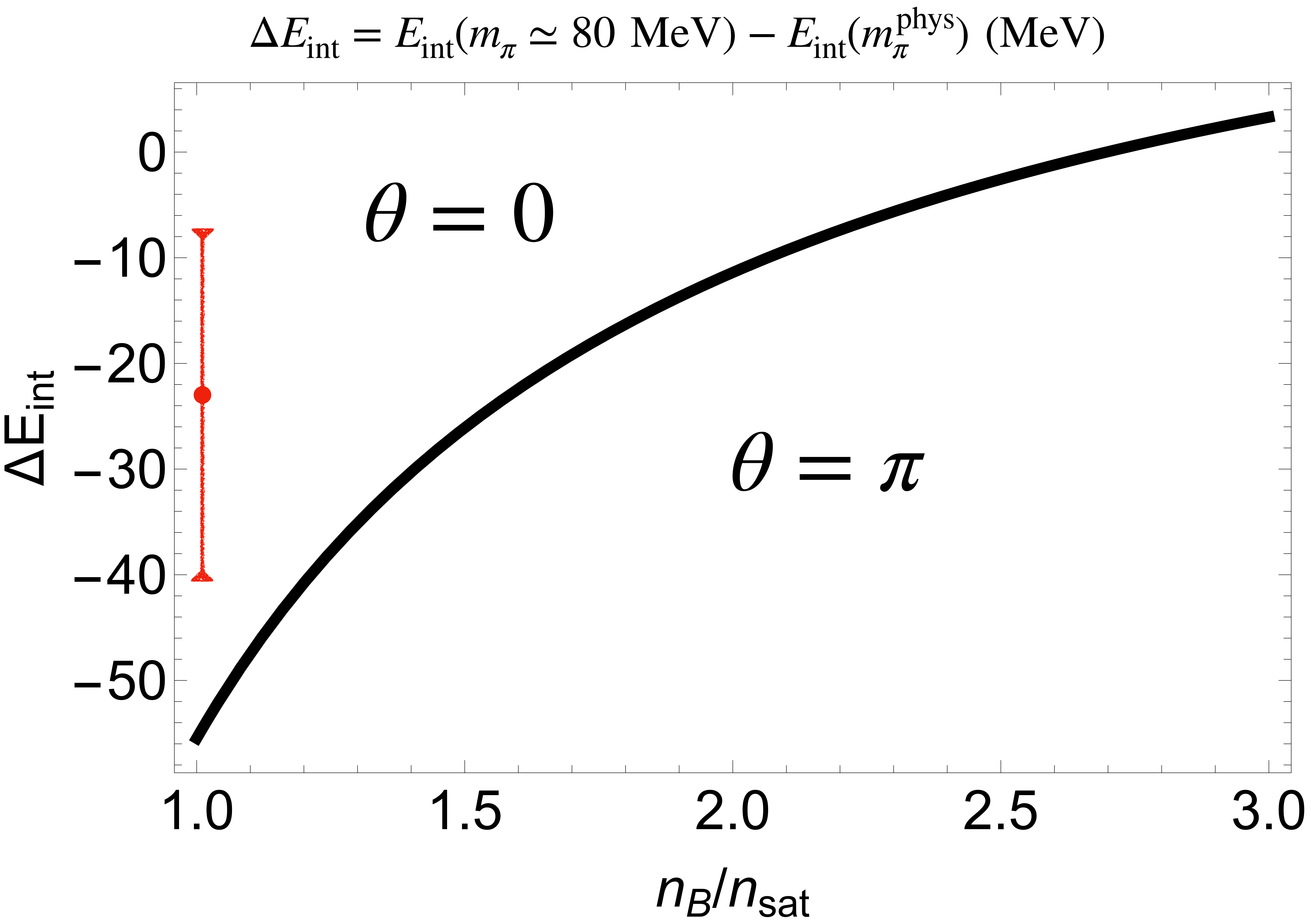


M. Kumamoto, J. Huang, C. Drischler, M. Baryakhtar, S. Reddy (2024)

Consistent inclusion of $M_q \rightarrow M_q\ \exp\left(2i\theta\ Q_a\right)$ in Chiral EFT warrants more work.



$\Delta E_{\rm int} = E_{\rm int}(m_\pi \simeq 80 \text{ MeV}) - E_{\rm int}(m_\pi^{\rm phys})$ (MeV)
$\theta = 0$
$\theta = \pi$
$\Delta E_{\rm int}$
0
−10
−20
−30
−40
−50
1.0
1.5
2.0
2.5
3.0
$n_B/n_{\rm sat}$



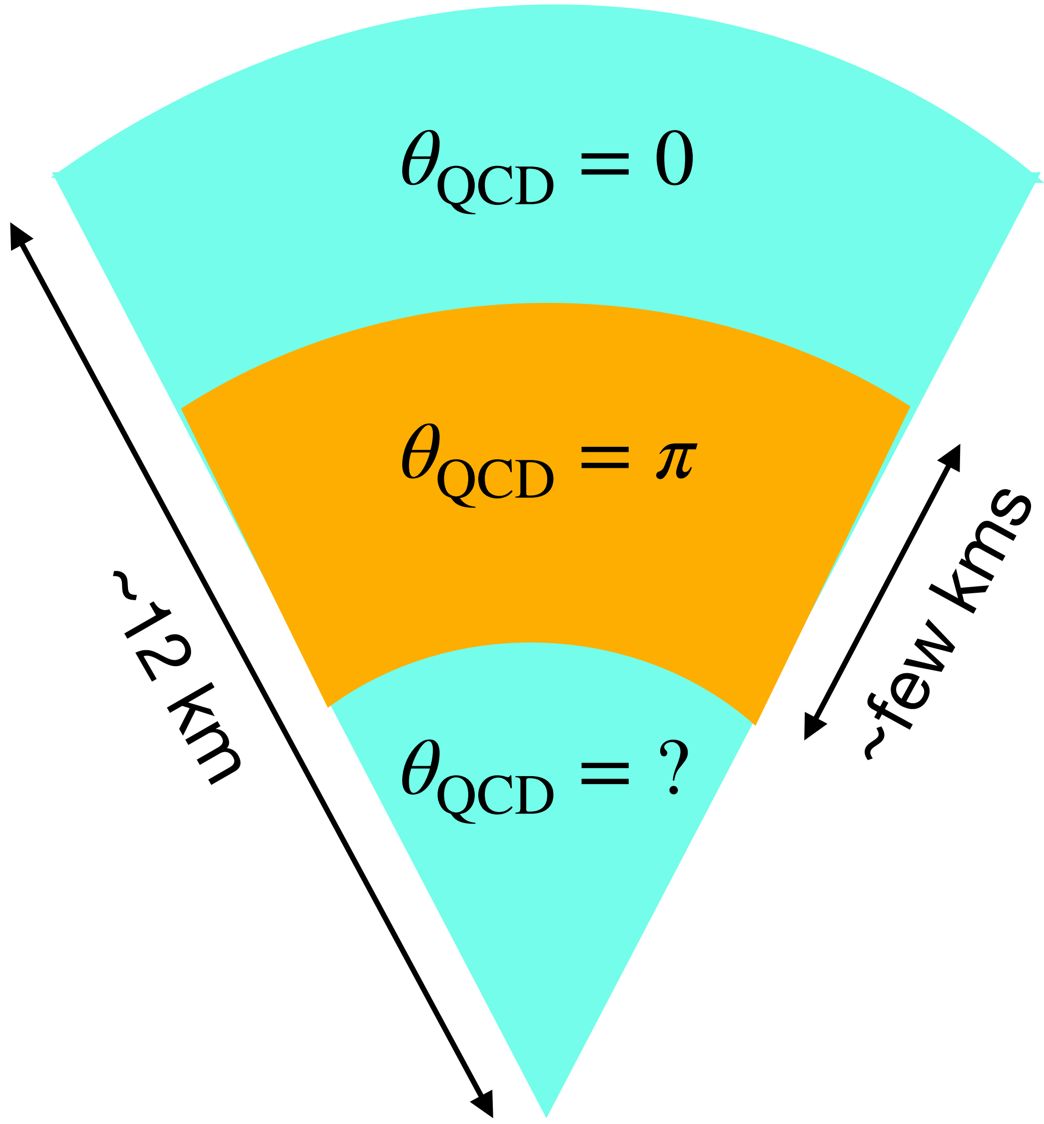
$\Delta E_{\rm int} = E_{\rm int}(m_\pi \simeq 80 \text{ MeV}) - E_{\rm int}(m_\pi^{\rm phys})$ (MeV)
$\theta = 0$
$\theta = \pi$
$\Delta E_{\rm int}$
0
−10
−20
−30
−40
−50
1.0
1.5
2.0
2.5
3.0
$n_B/n_{\rm sat}$

Axion Condensation in Mean Field Models

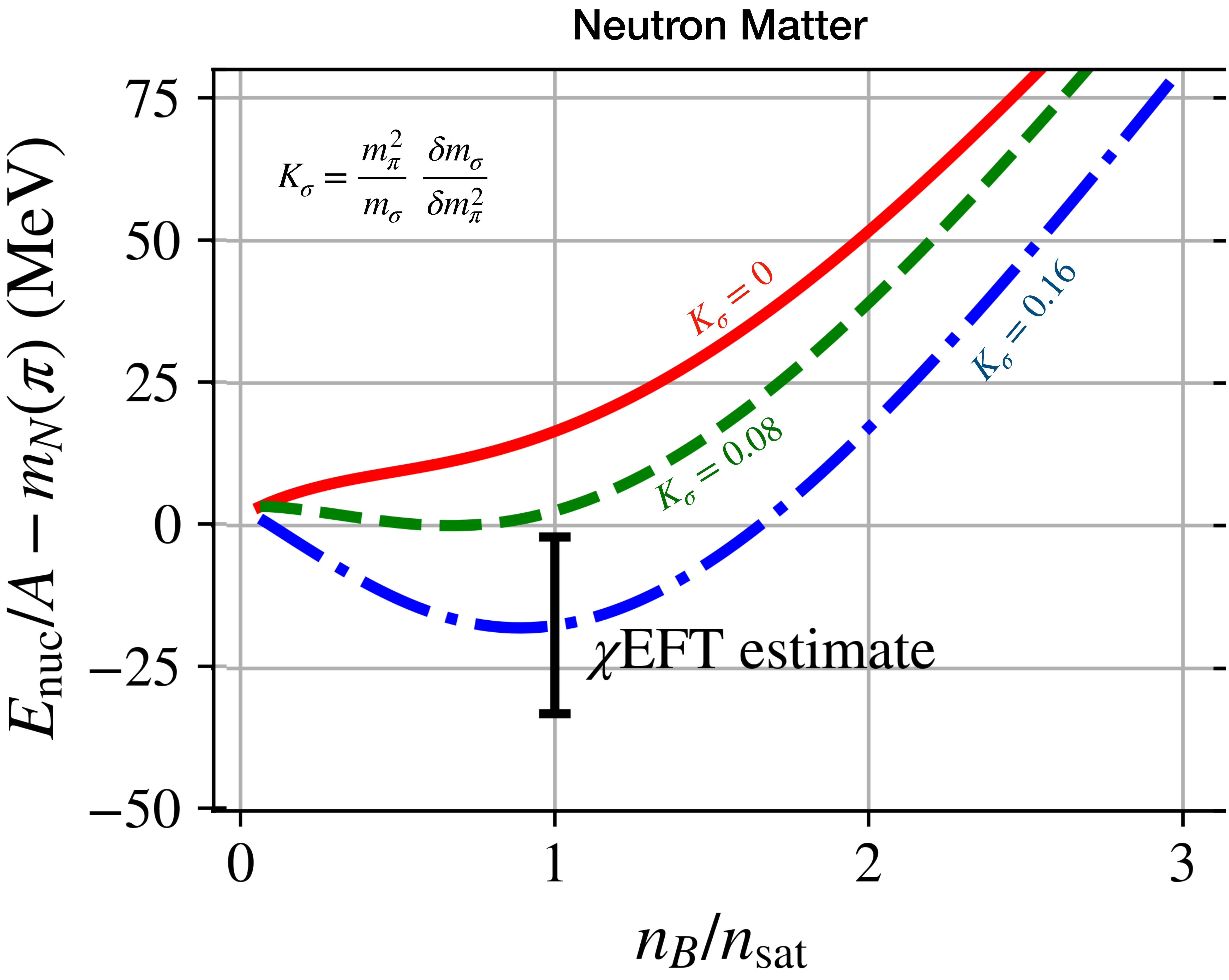
Condensation is realized in Relativistic Mean Field models of neutron-rich matter when the σ meson mass is lowered.

A 6% reduction of m_σ at $\theta = \pi$ would favor axion condensation at

$$n_B^c \lesssim 2\ n_{\rm sat}$$



Mia Kumamoto



M. Kumamoto, J. Huang, C. Drischler, M. Baryakhtar, S. Reddy (2024)

Conclusions

- The enhanced coupling of pions to two nucleons can be important for pion-nucleus scattering, pionic atoms with implications for pion in dense nuclear matter.

- Three nucleon forces that originate from D_2 and F_2 are significant. They are likley to influence all aspects of nuclear structure and dynamics. Their size will be scheme and regulator dependent.

- Independent determination of D_2, E_2, and F_2 from nuclear structure and pion-nucleus scattering in needed to quantify uncertatinties in EFT calculations.

- Understanding the quark or pion mass dependence of nuclear forces is important to address the possibility of axion condnesation in neutron stars.

Conclusions

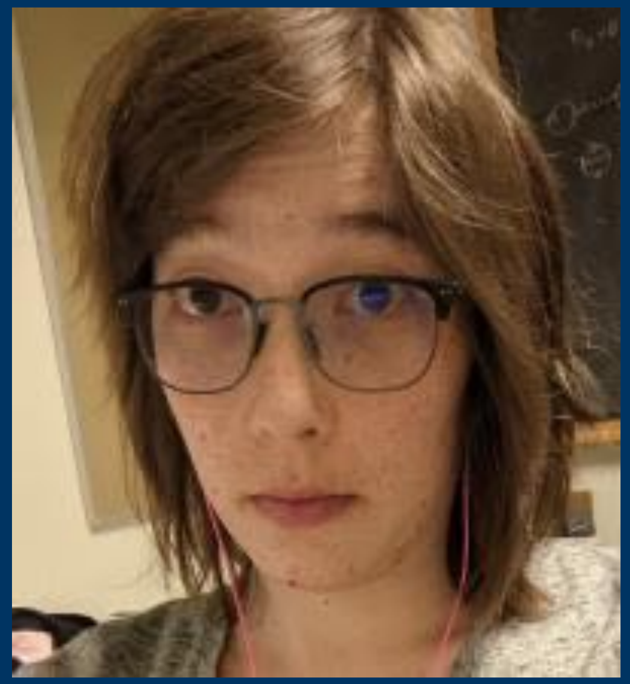
- The enhanced coupling of pions to two nucleons can be important for pion-nucleus scattering, pionic atoms with implications for pion in dense nuclear matter.

- Three nucleon forces that originate from D_2 and F_2 are significant. They are likley to influence all aspects of nuclear structure and dynamics. Their size will be scheme and regulator dependent.

- Independent determination of D_2, E_2, and F_2 from nuclear structure and pion-nucleus scattering in needed to quantify uncertatinties in EFT calculations.

- Understanding the quark or pion mass dependence of nuclear forces is important to address the possibility of axion condnesation in neutron stars.

Maria Dawid



Mia Kumamoto

Wouter Dekens

Neill Warrington