

The phi-N and phi-nucleus interaction from theory and experiment

Philipp Gubler (JAEA)



P. Gubler, M. Ichikawa, T. Song and E. Bratkovskaya, arXiv:2408.15364 [hep-ph].

L.M. Abreu, P. Gubler, K.P. Khemchandani, A. Martínez Torres and A. Hosaka, arXiv:2409.05170 [hep-ph].

R. Ejima, C. Sasaki, P. Gubler and K. Shigaki, in preparation.

Talk at the YITP long-term workshop
"Hadrons and Hadron Interactions in QCD 2024 (HHIQCD2024)",
YITP, Kyoto University, Japan
October 31, 2024

Work done in
collaboration
with:

M. Ichikawa (JAEA)

T. Song (GSI)

E. Bratkovskaya (Goethe U. Frankfurt)

L.M. Abreu (U. Federal da Bahia)

K.P. Khemchandani (U. F. de Sao Paulo)

A. Martínez Torres (U. de Sao Paulo)

A. Hosaka (Osaka U./RCNP)

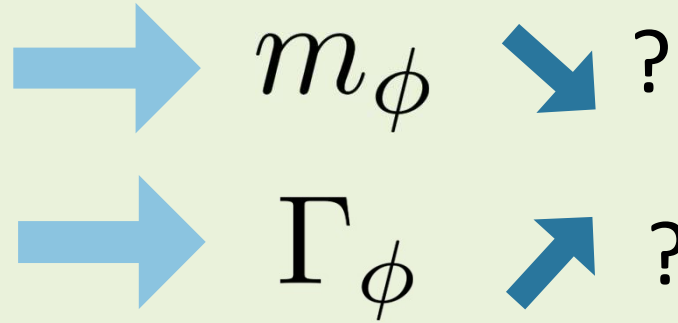
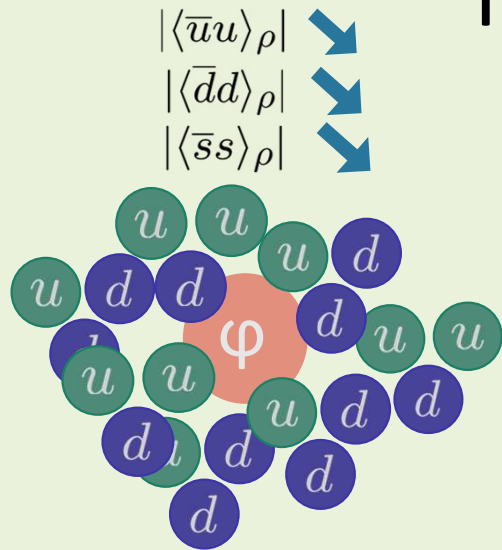
R. Ejima (Hiroshima U.)

C. Sasaki (U. of Wroclav/Hiroshima U.)

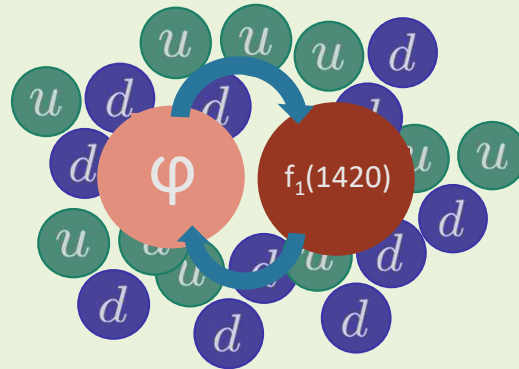
K. Shigaki (Hiroshima U.)

Topics of this talk

1.

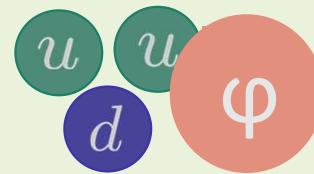


2.



Measurable at J-PARC?

3.

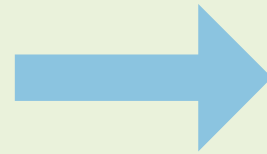
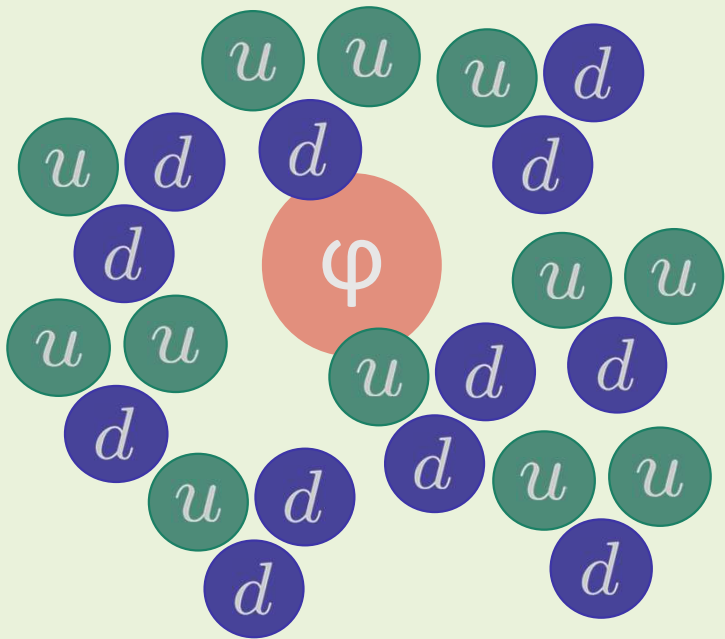


How attractive is this interaction?

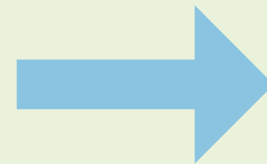
$$|\langle \bar{u}u \rangle_\rho| \searrow$$

$$|\langle \bar{d}d \rangle_\rho| \searrow$$

$$|\langle \bar{s}s \rangle_\rho| \searrow$$



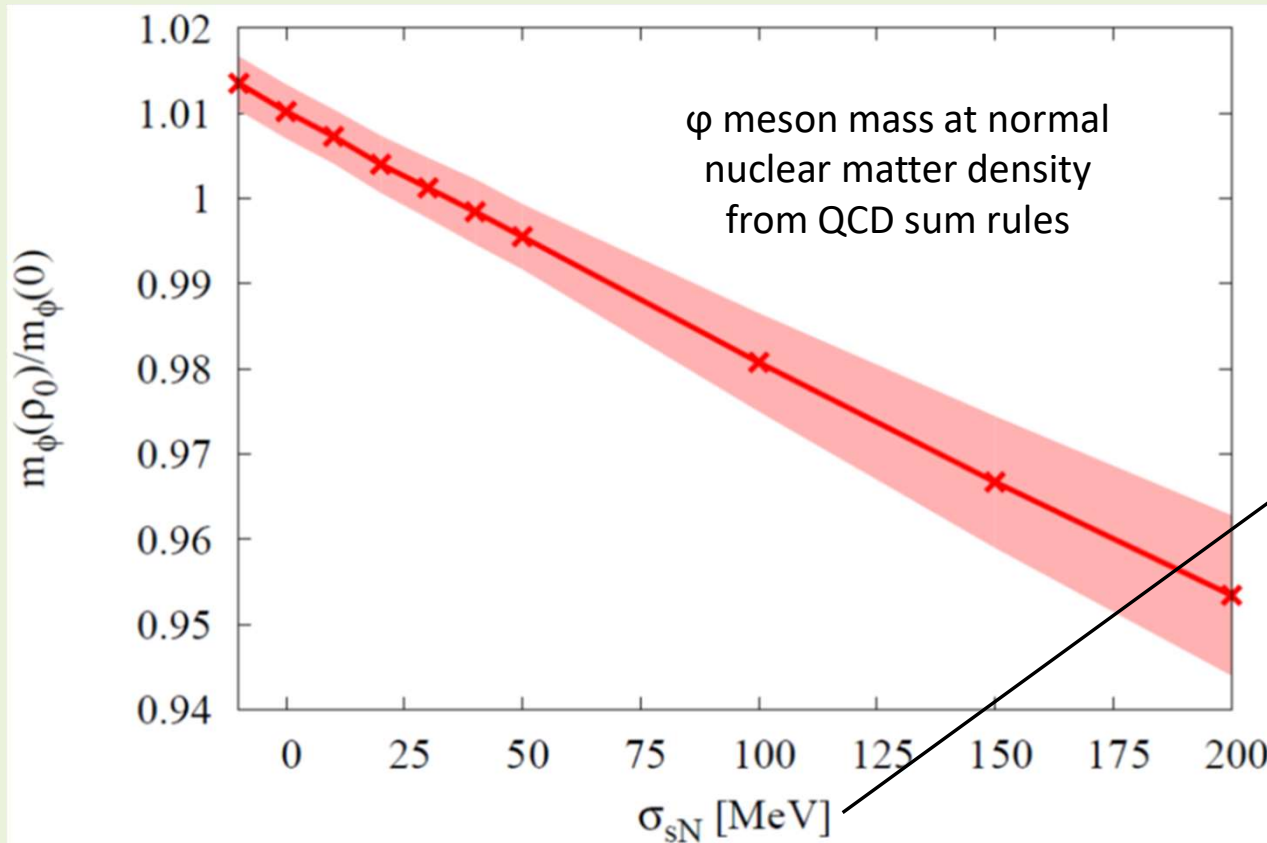
$$m_\phi \searrow ?$$



$$\Gamma_\phi \nearrow ?$$

Why should we be interested?

The ϕ meson mass in nuclear matter probes the strange quark condensate at finite density!



P. Gubler and K. Ohtani, Phys. Rev. D **90**, 094002 (2014).

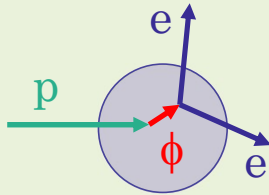
T. Hatsuda and S.H. Lee,
Phys. Rev. C **46**, R34 (1992).

$$\sigma_{sN} = m_s \langle N | \bar{s}s | N \rangle$$

$$|\langle \bar{s}s \rangle_\rho| = |\langle \bar{s}s \rangle_0| - \frac{\rho}{m_s} \sigma_{sN} + \dots$$

Previous experimental results

KEK
E325



12 GeV
pA-reaction

slow φs

Pole mass:

$$\frac{m_\phi(\rho)}{m_\phi(0)} = 1 - k_1 \frac{\rho}{\rho_0}$$

0.034 ± 0.007

intermediate
φs

Pole width:

$$\frac{\Gamma_\phi(\rho)}{\Gamma_\phi(0)} = 1 + k_2 \frac{\rho}{\rho_0}$$

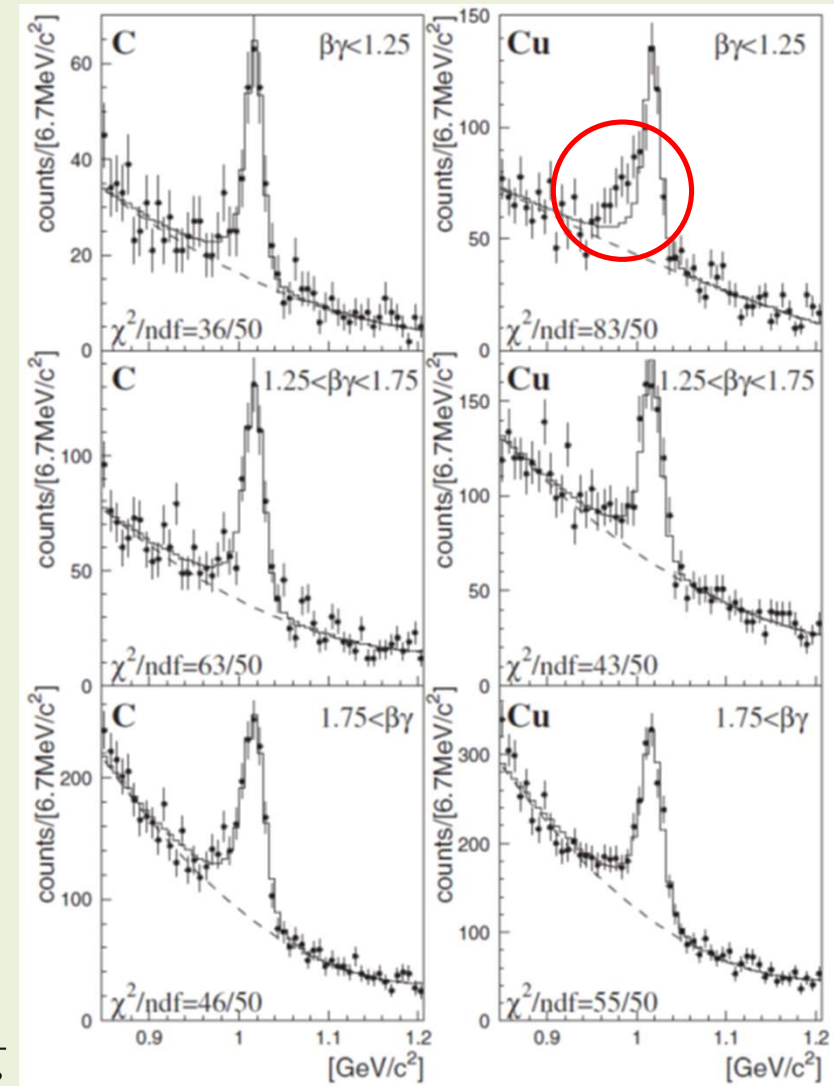
2.6 ± 1.5



Measurement is being repeated with
~100x increased statistics at the
J-PARC E16 experiment!

fast φs

$$\beta\gamma = \frac{|\vec{p}|}{m_\phi}$$

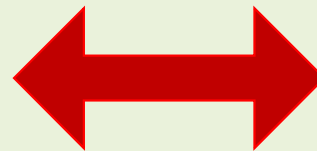


R. Muto et al. (E325 Collaboration), Phys. Rev. Lett. **98**, 042501 (2007).

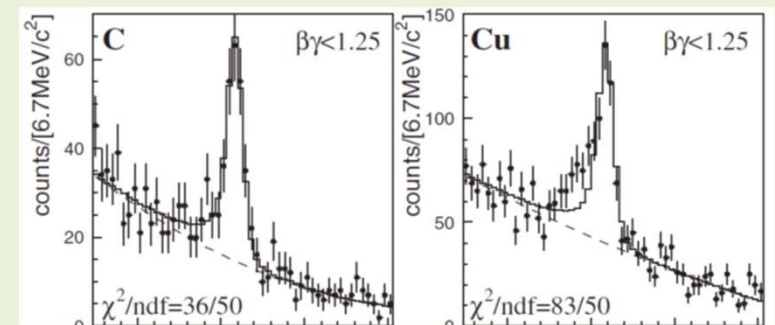
Comparison of theory and experiment

Information useful for theory

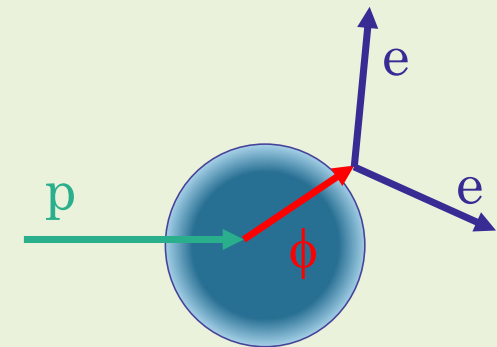
- ★ Spectral function as a function of density
- ★ Mass at normal nuclear matter density
- ★ Decay width at normal nuclear matter density



Experimental data



Realistic simulation of pA reaction is needed!



Our tool: transport simulation PHSD (Parton Hadron String Dynamics)

E.L. Bratkovskaya and W. Cassing, Nucl. Phys. A **807**, 214 (2008).
W. Cassing and E.L. Bratkovskaya, Phys. Rev. C **78**, 034919 (2008).

Off-shell dynamics of vector mesons and kaons
(dynamical modification of the mesonic spectral function during the simulated reaction)

Used spectral function:

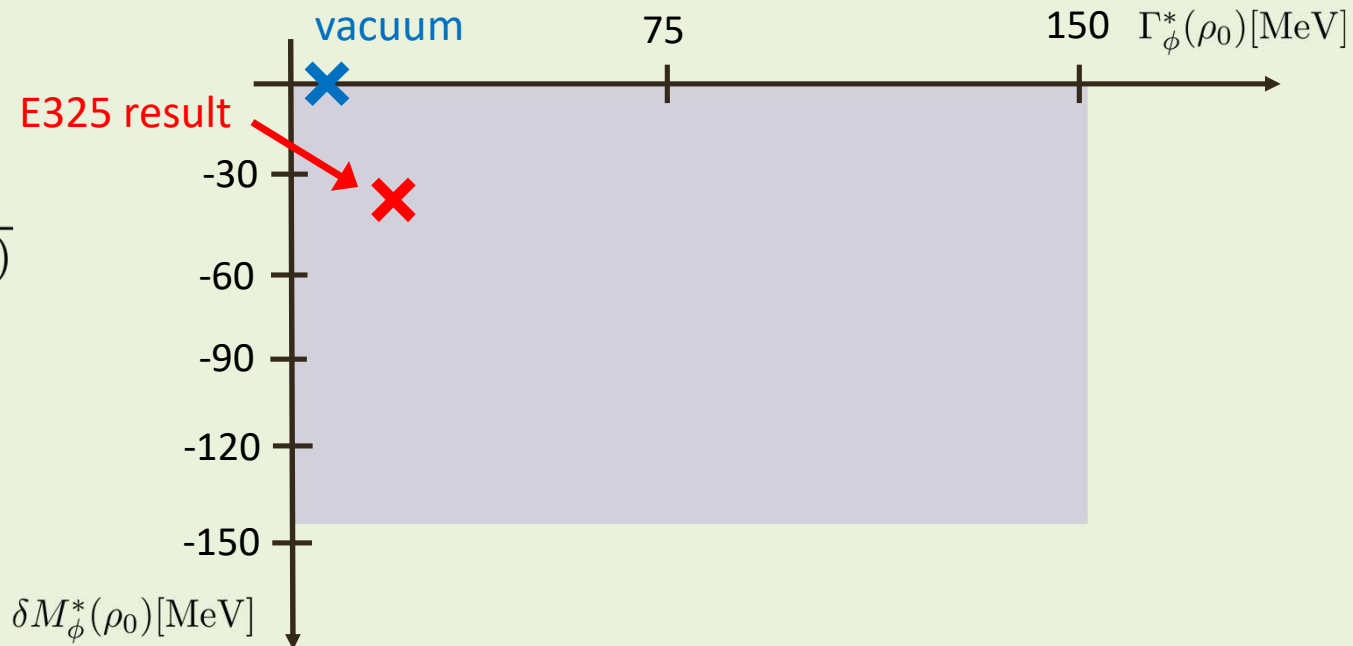
Relativistic Breit-Wigner with density dependent mass and width

$$C \frac{2}{\pi} \frac{M^2 \Gamma_\phi^*(M, \rho)}{[M^2 - M_\phi^{*2}(\rho)]^2 + M^2 \Gamma_\phi^{*2}(M, \rho)}$$

with

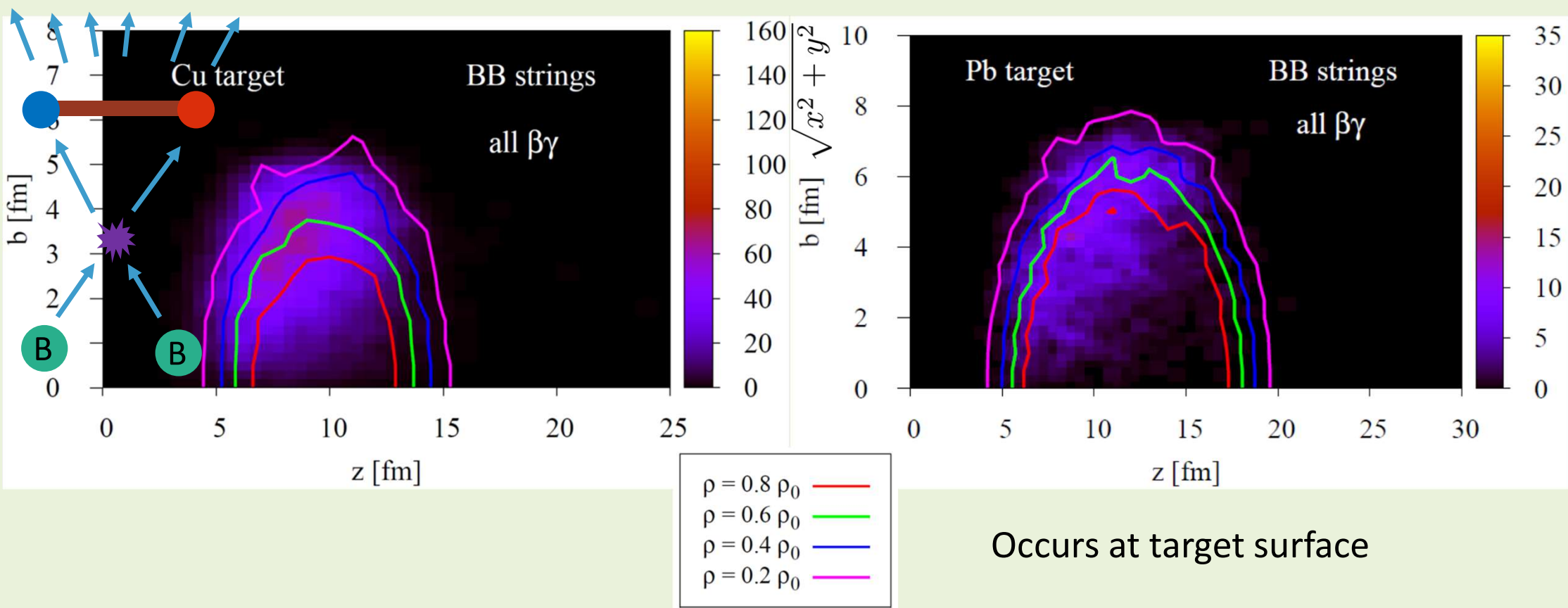
$$\begin{cases} M_\phi^*(\rho) = M_\phi^{\text{vac}} \left(1 - \alpha^\phi \frac{\rho}{\rho_0}\right), \\ \Gamma_\phi^*(M, \rho) = \Gamma_\phi^{\text{vac}} + \alpha_{\text{coll}}^\phi \frac{\rho}{\rho_0} \end{cases}$$

Simulated scenarios:



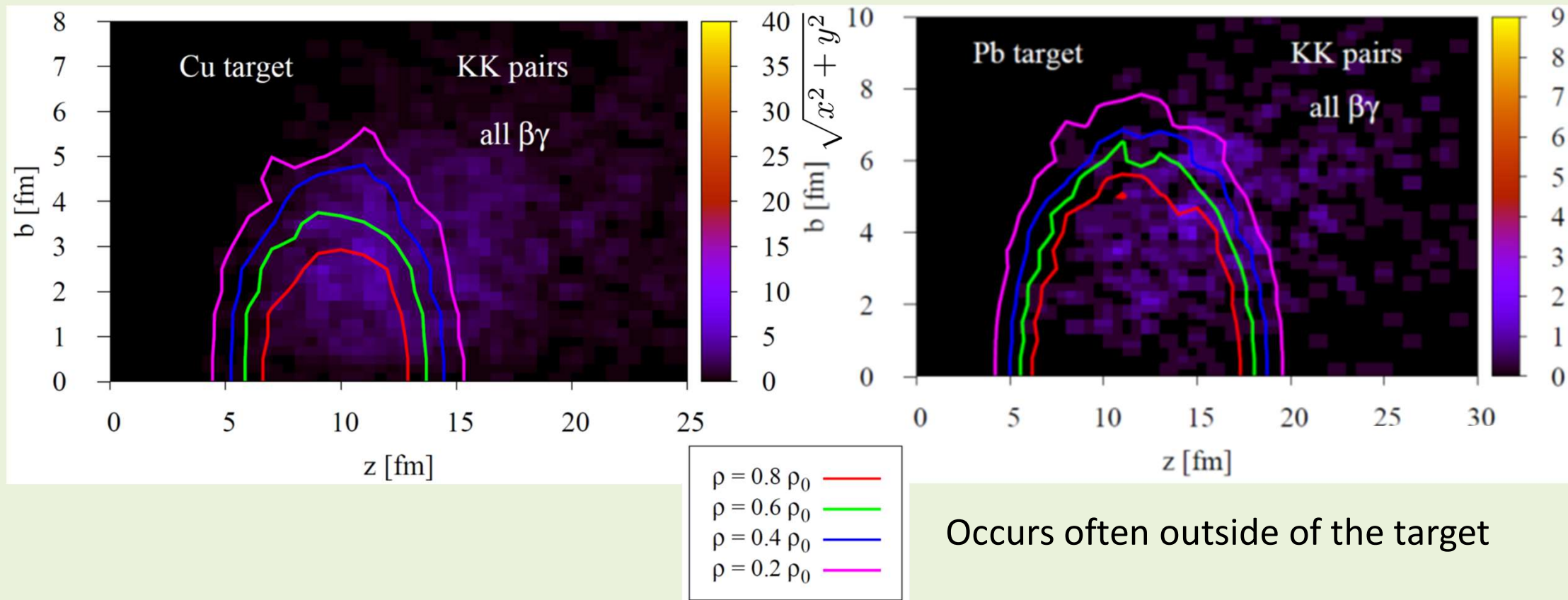
How are φ mesons produced in 12 GeV pA collisions?

Production through initial high-energy collisions (via strings)



How are ϕ mesons produced?

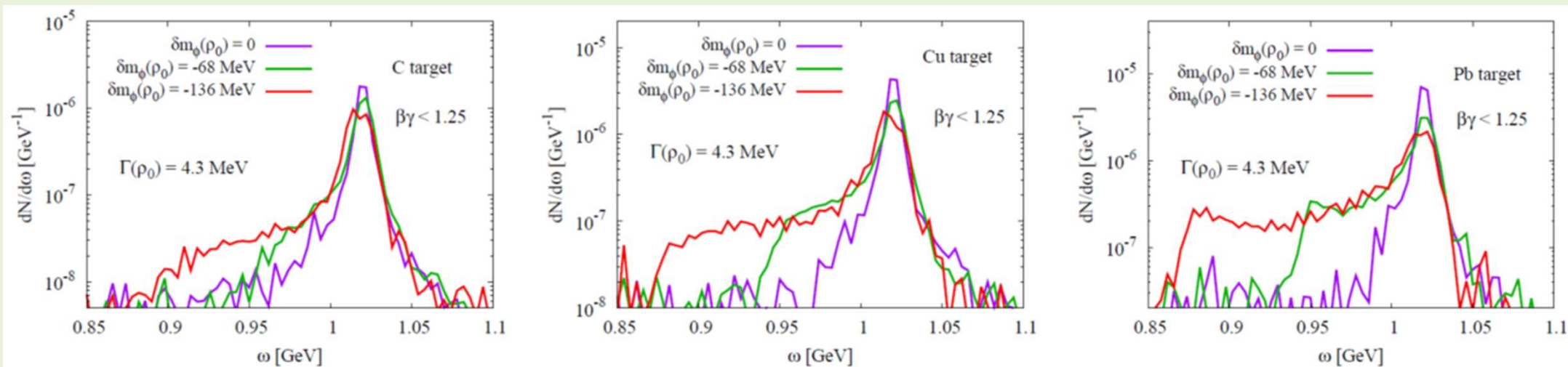
Production through secondary low-energy hadron collisions



Occurs often outside of the target

The obtained dilepton spectrum (without experimental effects)

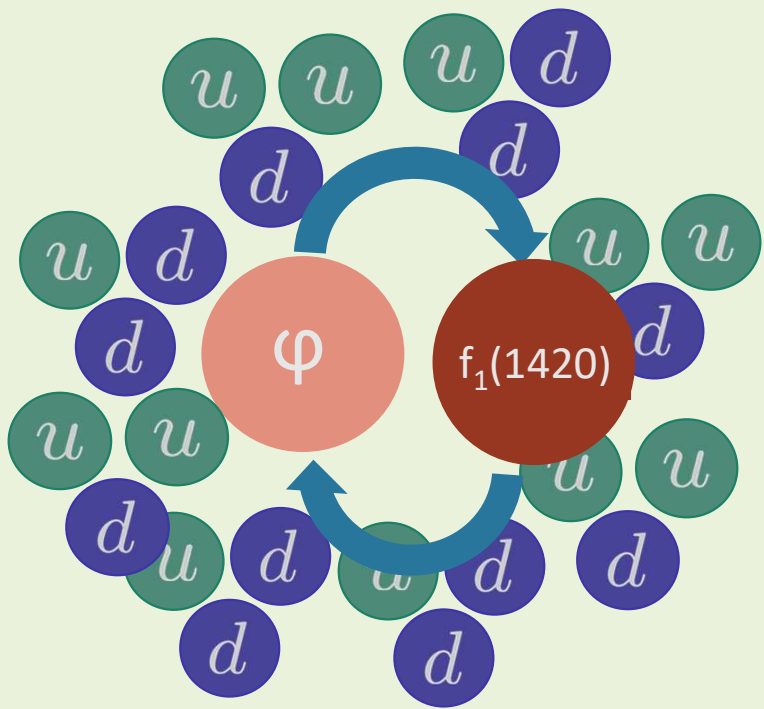
Pure mass shift scenarios (no broadening)



No second peak, but only shoulder structure for mass shift scenarios (even before considering experimental resolution effects)

Secondary peak can be generated for sufficient large mass shift scenario if the target is large enough (Pb here)

Direct comparison with experimental data is coming soon!



New peak in dilepton spectrum??

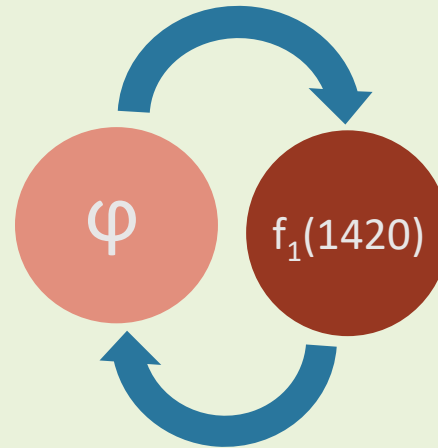
Another possible effect: chiral mixing

Simple idea:

Charge conjugation (C) symmetry is broken in nuclear matter

➔ Non-trivial mixing between different modes can occur

Here we consider the
Vector – Axial-vector
mixing in the strange
quark sector



R. Ejima, C. Sasaki, P. Gubler and K. Shigaki, in preparation.

Simple hadronic model including C-symmetry breaking

$$\mathcal{L} = 2c\epsilon^{0\mu\nu\lambda}\text{tr}\left[\partial_\mu V_\nu \cdot A_\lambda + \partial_\mu A_\nu \cdot V_\lambda\right]$$

Can be understood from an anomalous ω - ϕ -f1 coupling with a coherent ω -field: $\langle\omega_0\rangle \sim \rho$



tree-level V-A mixing!

C. Sasaki, Phys. Rev. D **106**, 054034 (2022).

However, the coupling c is model dependent:

$$c = 1.0 \frac{\rho}{\rho_0} \text{ [GeV]}$$

from holographic QCD

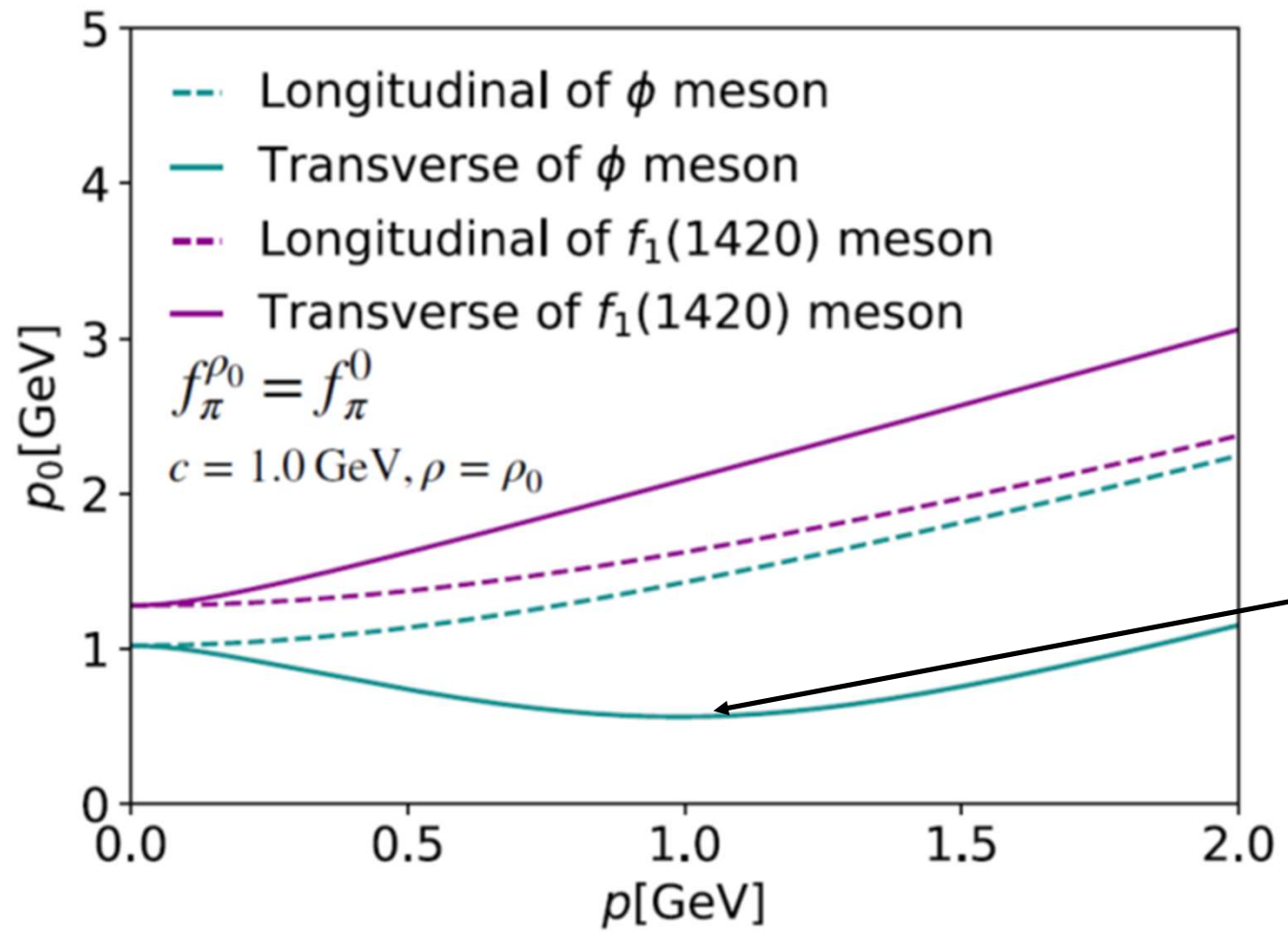
S. K. Domokos and J. A. Harvey,
Phys. Rev. Lett. **99**, 141602 (2007).

$$c = 0.1 \frac{\rho}{\rho_0} \text{ [GeV]}$$

from gauged WZW action

M. Harada and C. Sasaki,
Phys. Rev. C **80**, 054912 (2009).

Resulting dispersion relations (at $\rho = \rho_0$)



Moat!



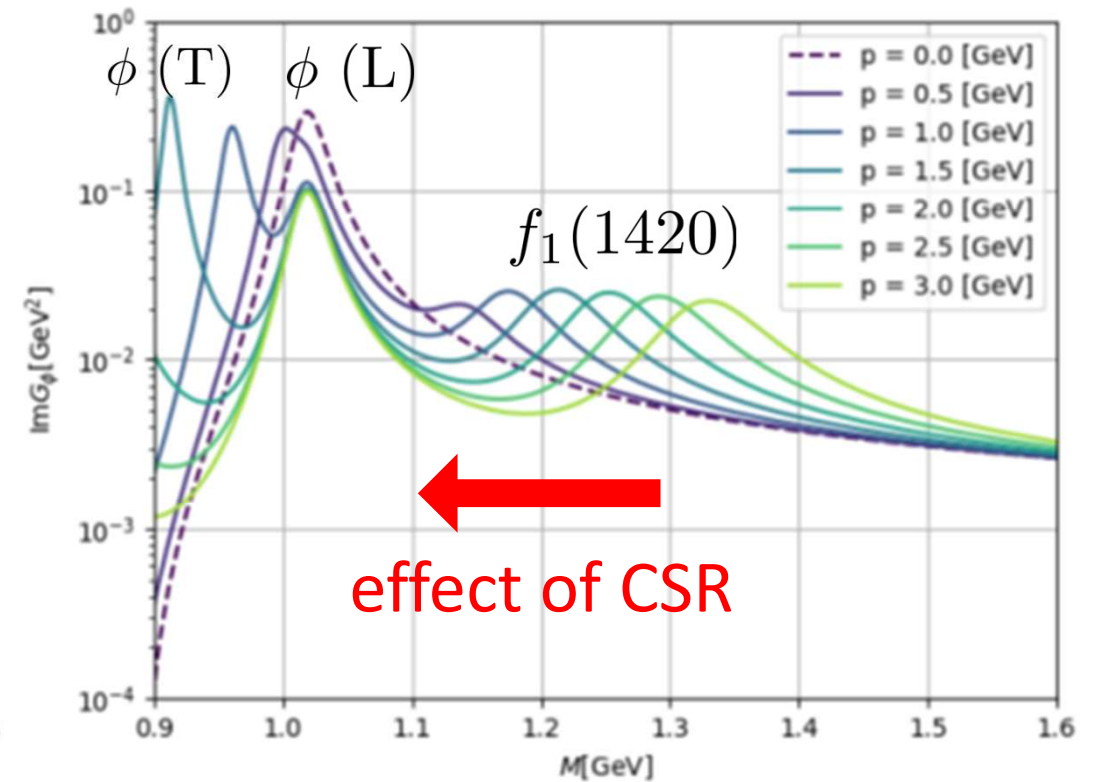
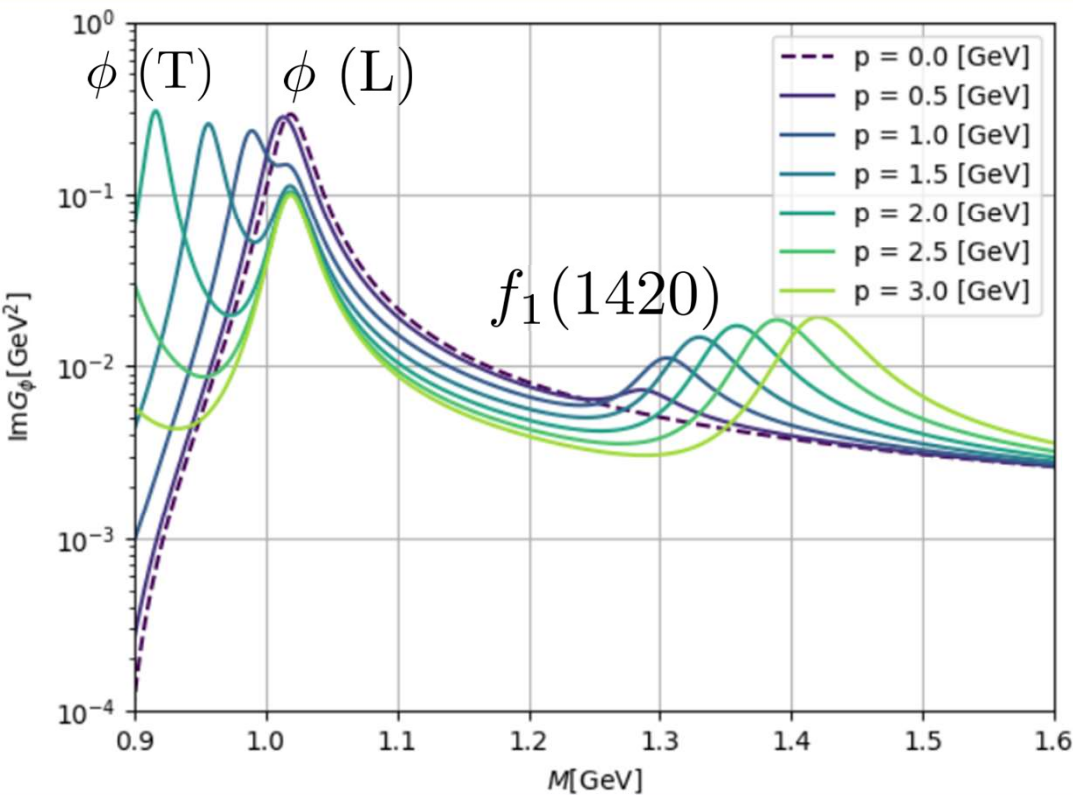
See Rob's talk

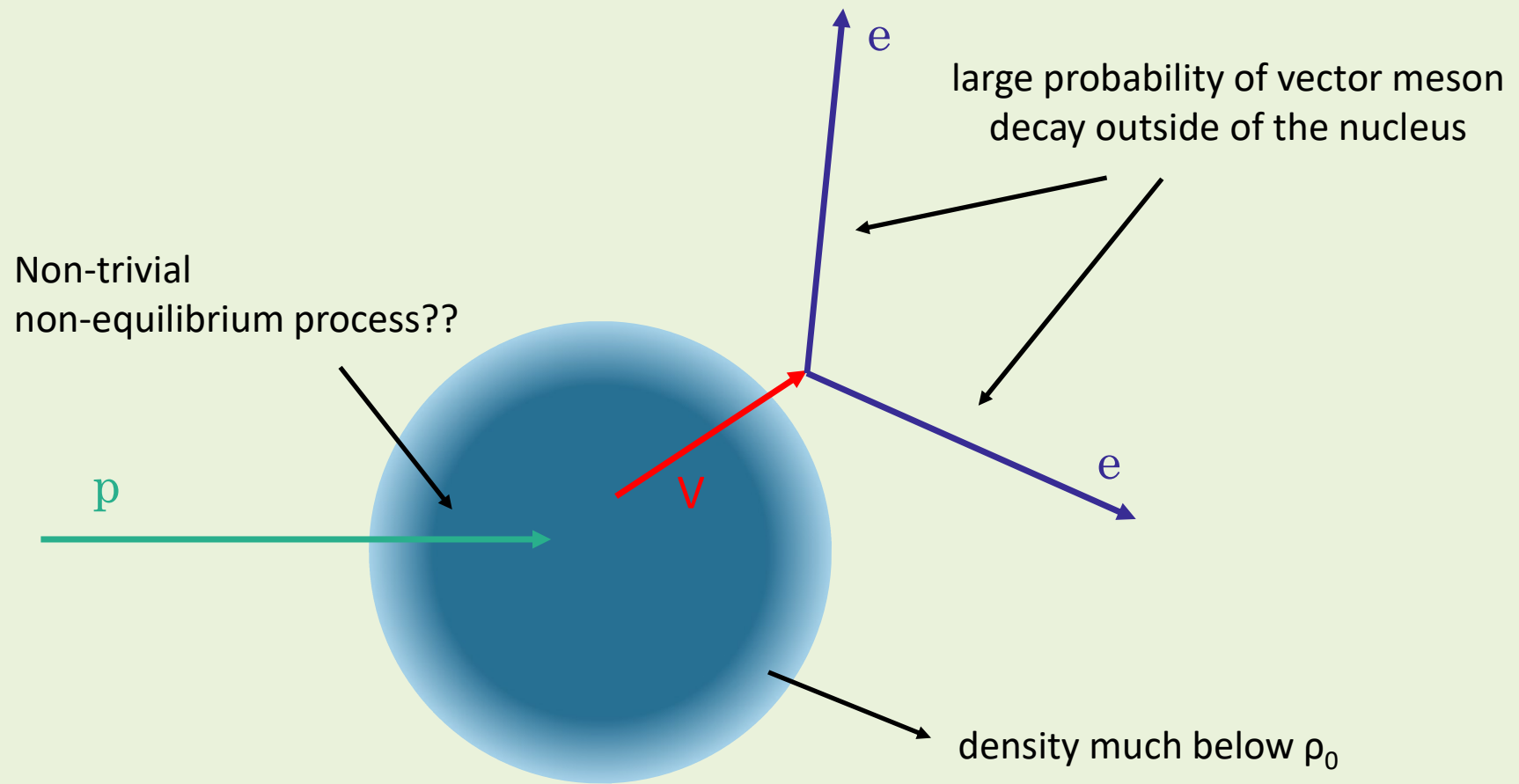
Resulting spectral functions (at $\rho = \rho_0$)

$$c = 0.1 \frac{\rho}{\rho_0} \text{ [GeV]}$$

without CSR

with CSR





Experimentally measurable invariant mass distribution

Computed using a state-of-the-art transport simulation (PHSD)

Convolved by Gaussian to take into account experimental resolution

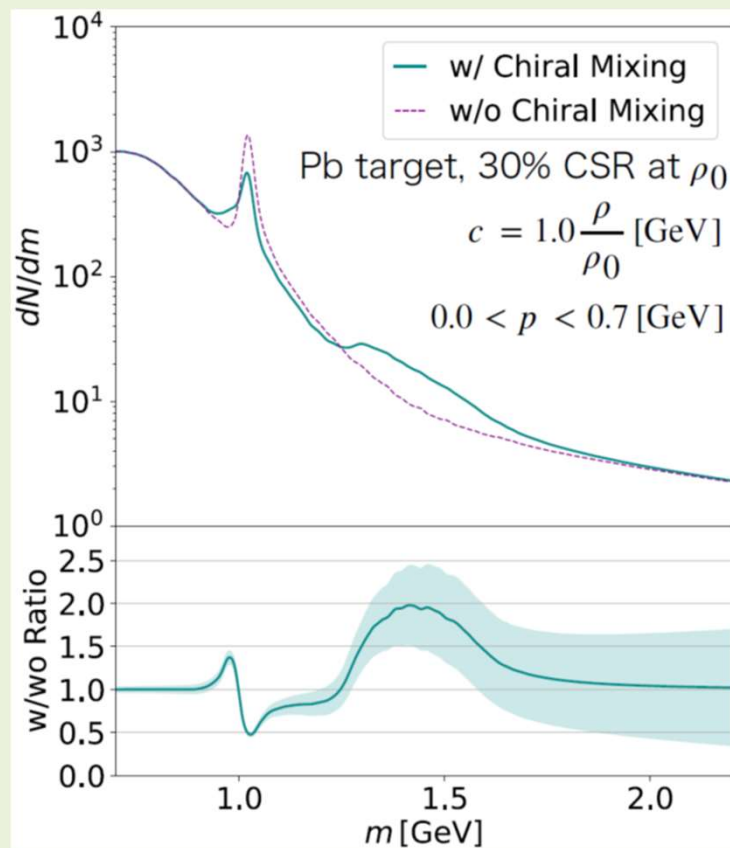
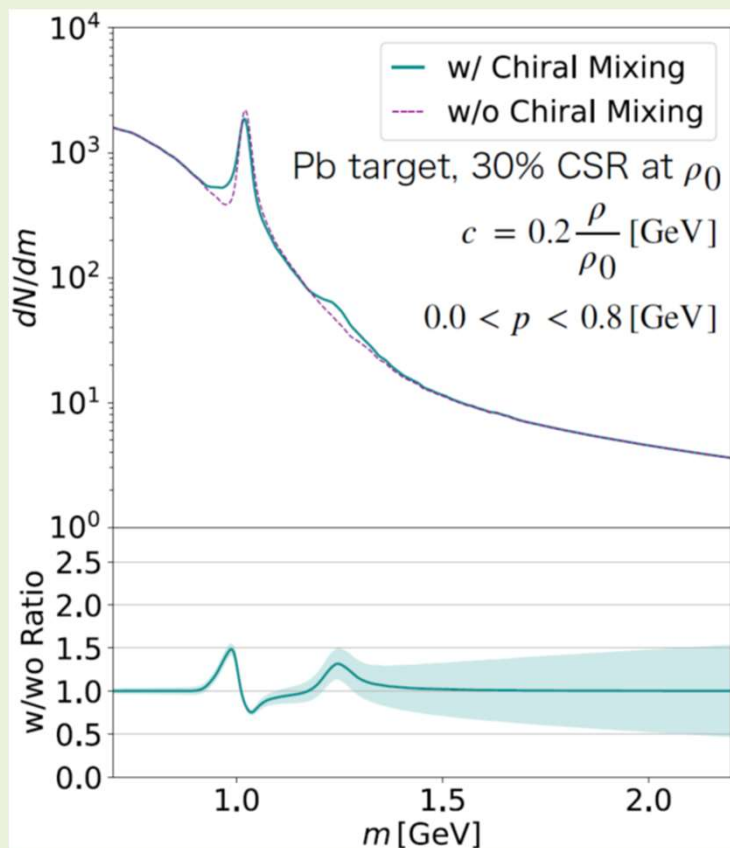
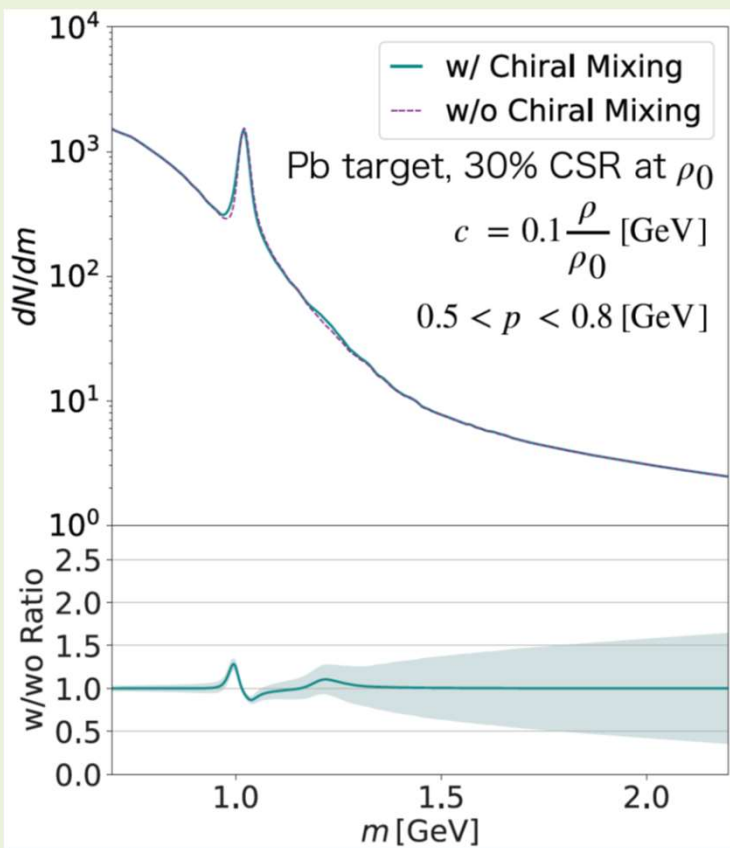
$$I(M) = \int \left[\int \text{Im}G_V(s, \vec{p}, \rho) \frac{dN}{d\vec{p}d\rho dt} \frac{d\vec{p}}{2p_0} d\rho dt + \int \text{Bkg}(s, \vec{p}) d\vec{p} \right] g(M - s) ds$$

Taken from the Sasaki-model (previous slide)

Background, obtained simulation of experimental conditions:
JAM \rightarrow Geant4

Invariant mass distributions for different mixing strengths

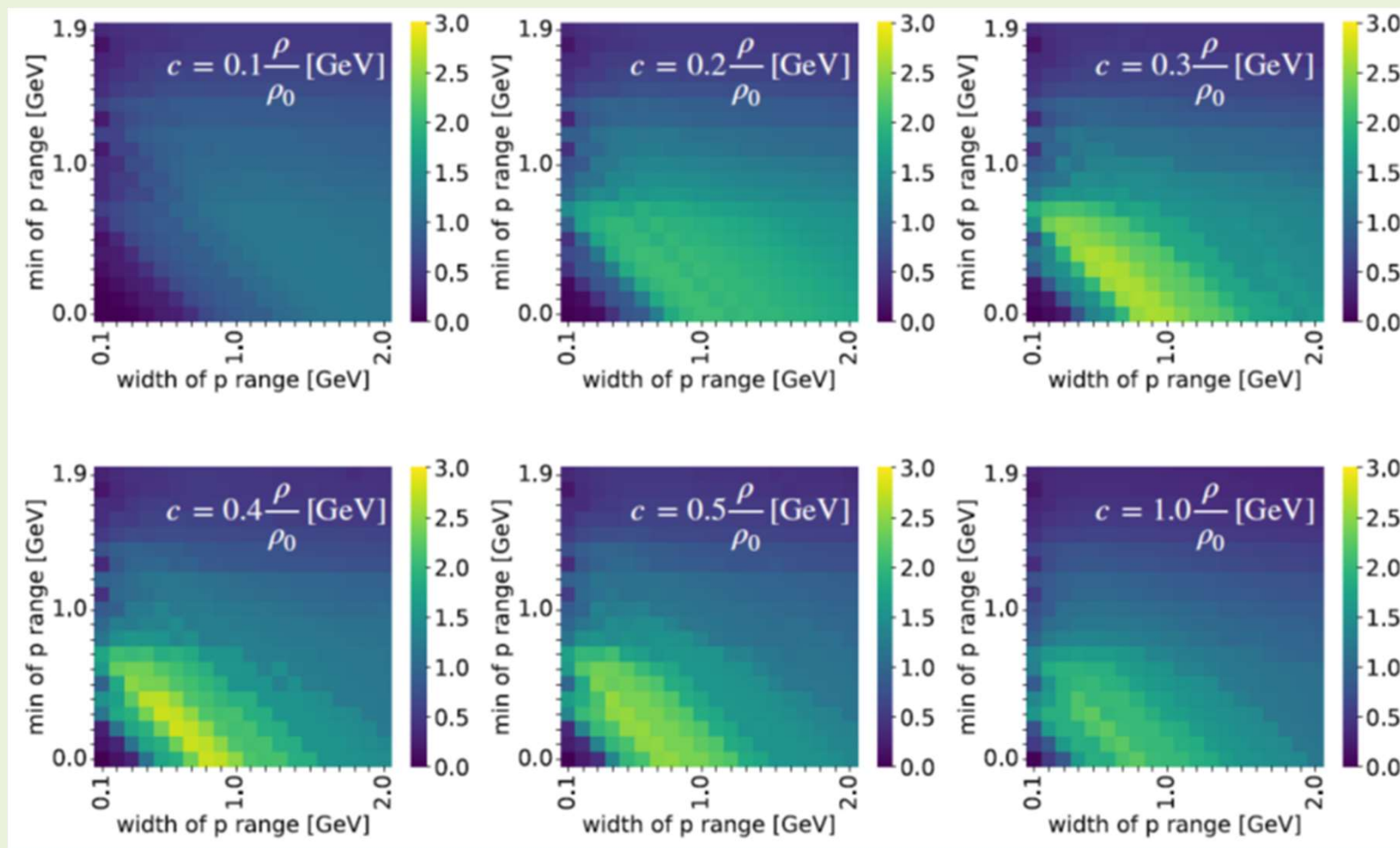
30 GeV pA collisions, Pb target, E16 Run2 statistics



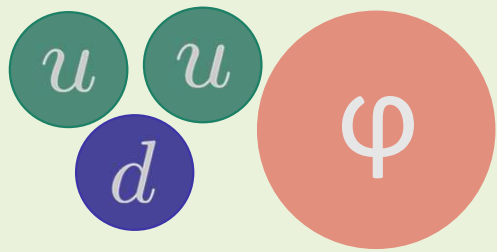
R. Ejima, C. Sasaki, P. Gubler and K. Shigaki, in preparation.

Potential signal strengths for different scenarios

30 GeV pA collisions, Pb target, E16 Run2 statistics



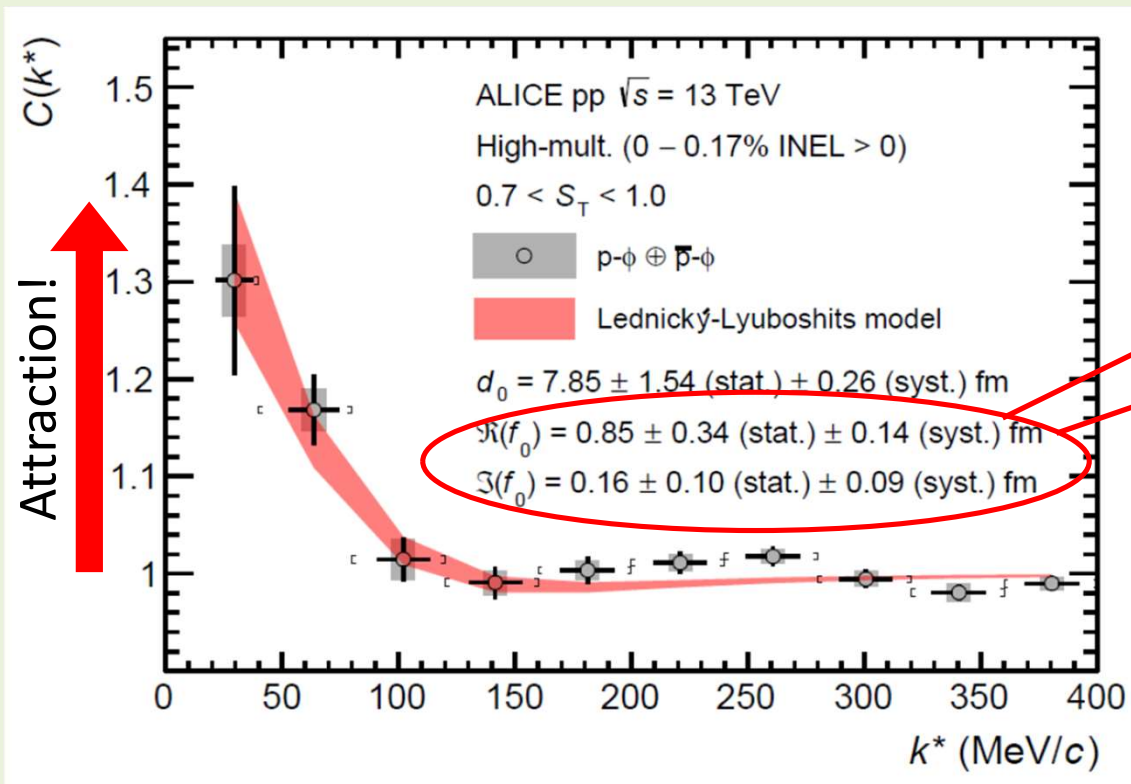
R. Ejima, C. Sasaki, P. Gubler and K. Shigaki, in preparation.



How strong is this interaction?

ALICE: pp

ϕ N correlation function



★ Strongly attractive

★ Small absorption

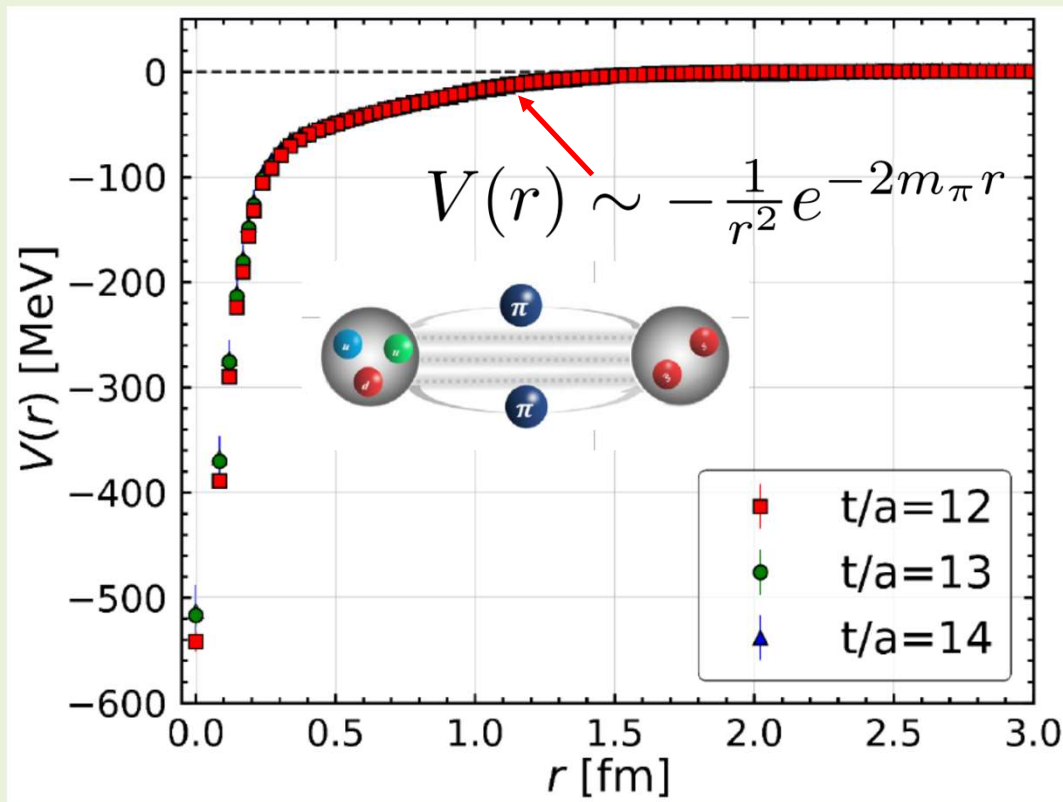
Caution:

Spin averaged value with
non-trivial weights
(thanks to Kamiya-san!)

S. Acharya et al. (ALICE Coll.), Phys. Rev. Lett. **127**, 172301 (2021).

φN potential from HAL QCD

Spin 3/2 channel

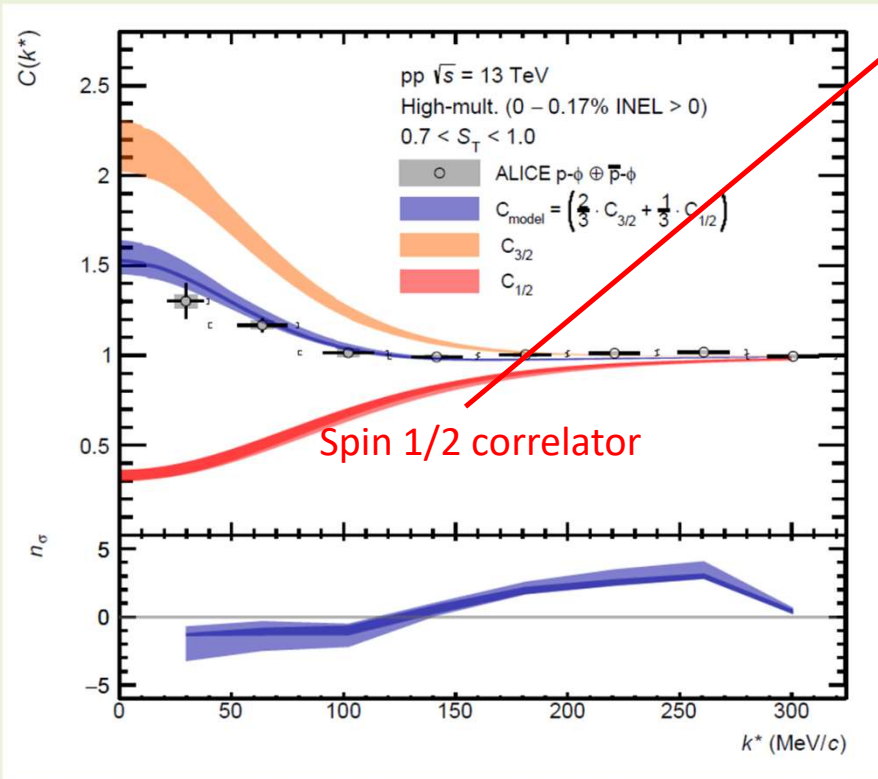


→ $a_0^{3/2} = 1.43(23)_{\text{stat.}} \left(\begin{smallmatrix} +36 \\ -06 \end{smallmatrix} \right)_{\text{syst.}} \text{ fm}$

Indication for a quite strong and attractive interaction!

Even more recent results

Combination of ALICE pp-data and HAL QCD (spin 3/2) calculation

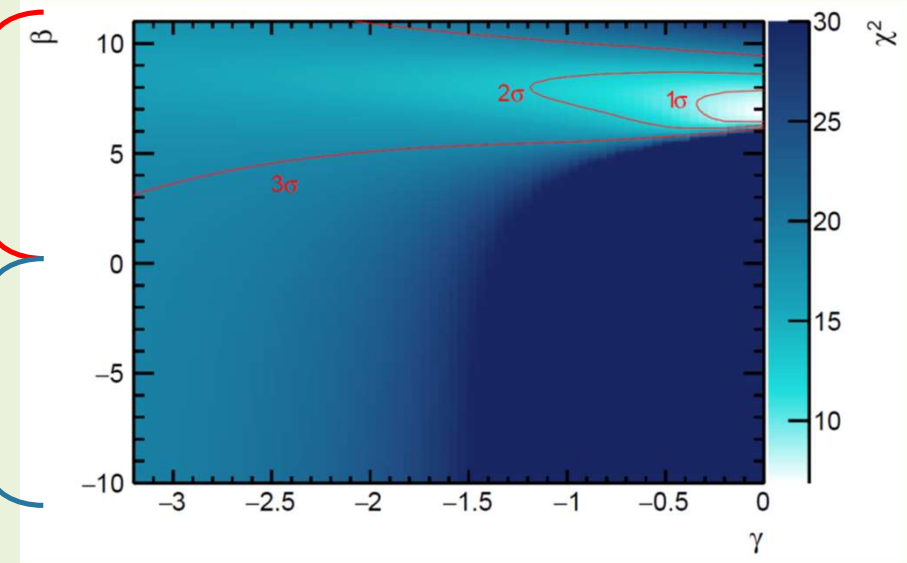


E. Chizzali et al., Phys. Lett. B **848**, 138358 (2024).

Bound state?
Repulsive?

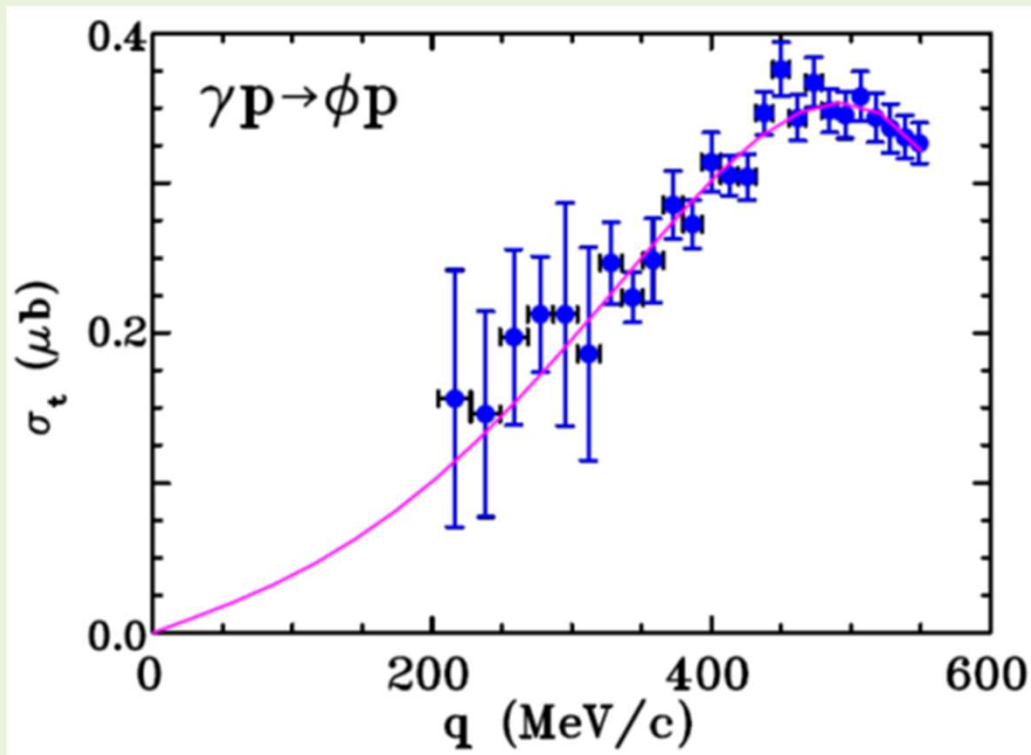
Attractive

Repulsive



➔ Evidence for ϕ -N bound state!

Photoproduction measurement (CLAS)



Vector meson
dominance is assumed

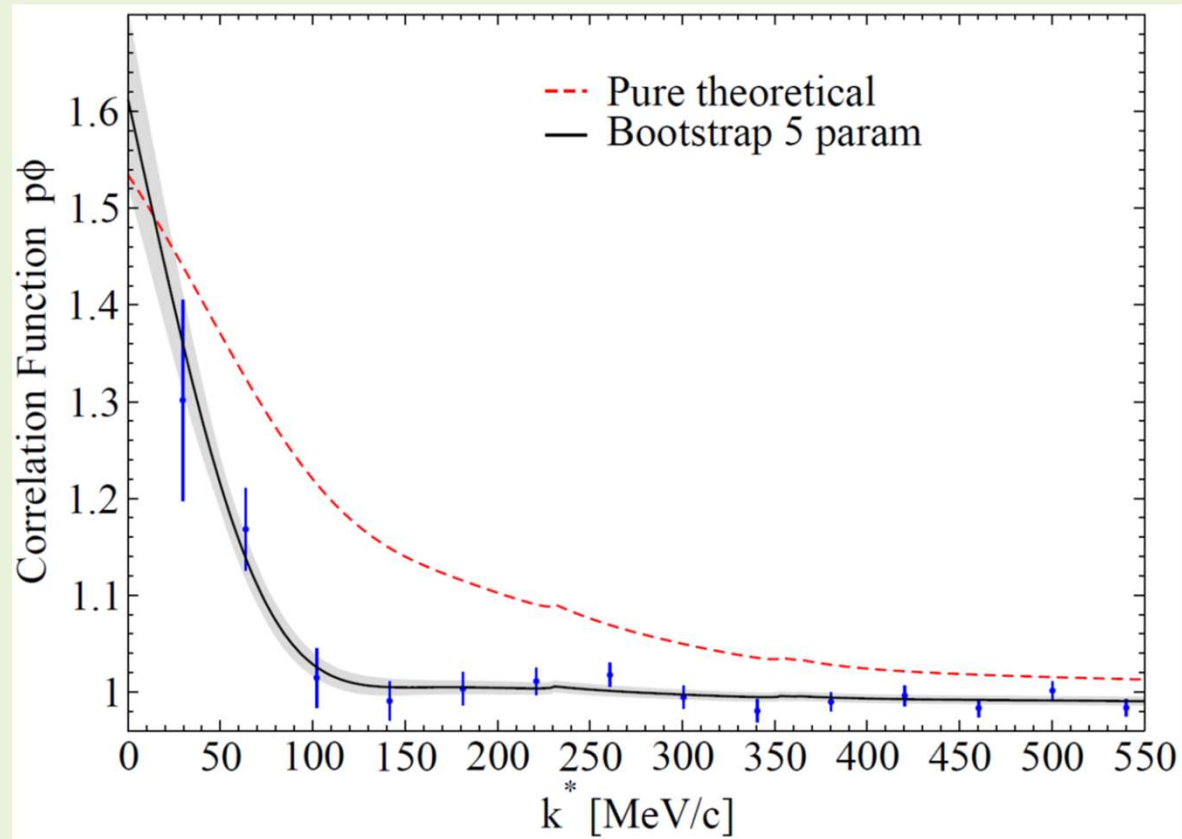
$$\begin{aligned} \frac{d\sigma^{\gamma p \rightarrow V p}}{d\Omega} \Big|_{\text{thr}} &= \frac{q}{k} \frac{1}{64\pi} |T^{\gamma p \rightarrow V p}|^2 \\ &= \frac{q}{k} \frac{\pi \alpha}{g_V^2} \frac{d\sigma^{V p \rightarrow V p}}{d\Omega} \Big|_{\text{thr}} = \frac{q}{k} \frac{\pi \alpha}{g_V^2} |\alpha_{V p}|^2 \end{aligned}$$

$$|a_0| = 0.063 \pm 0.010 \text{ fm}$$

Consistent with weak ϕN interaction

New analysis of the ALICE data

A. Feijoo, M. Korwieser and L. Fabbietti, arXiv:2407.01128 [hep-ph].



Coupled channel approach, with subtraction constants as fittable parameters:

	Pure theoretical	Bootstrap
$a_{\rho N}$	-2 (fixed)	-2 (fixed)
$a_{\omega N}$	-2 (fixed)	-3.04 ± 0.73
$a_{\phi N}$	-2 (fixed)	-3.15 ± 0.37
$a_{K^* \Lambda}$	-2 (fixed)	-1.98 ± 0.08
$a_{K^* \Sigma}$	-2 (fixed)	-1.95 ± 0.08
N_D	1 (fixed)	0.988 ± 0.004

New analysis of the ALICE data

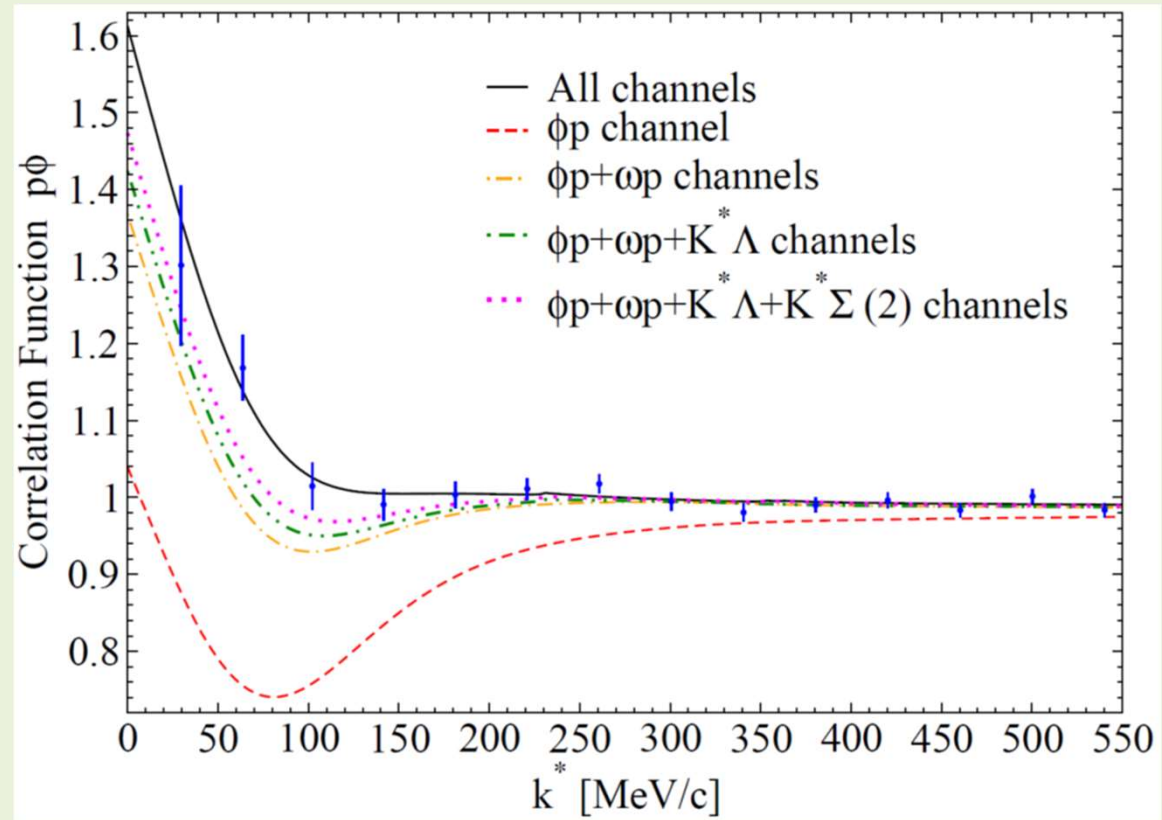
A. Feijoo, M. Korwieser and L. Fabbietti, arXiv:2407.01128 [hep-ph].

Obtained scattering length and effective range

Table 5: Effective range, r_{eff} (fm), and scattering length, a_0 (fm), for the ϕp and $\rho^0 p$ channels.

	Pure theoretical	Bootstrap
$a_0^{\phi p}$	$0.272 + i0.189$	$(-0.034 \pm 0.035) + i(0.57 \pm 0.09)$
$r_{eff}^{\phi p}$	$-7.20 - i0.09$	$(-8.06 \pm 2.57) + i(0.05 \pm 0.53)$
$a_0^{\rho^0 p}$	$0.090 + i0.568$	$(0.09 \pm 0.03) + i(0.56 \pm 0.05)$
$r_{eff}^{\rho^0 p}$	$-3.01 + i98.39$	$(-3.05 \pm 0.28) + i(98.40 \pm 0.12)$

Decomposition into different hadronic channels



An even newer analysis of the ALICE data

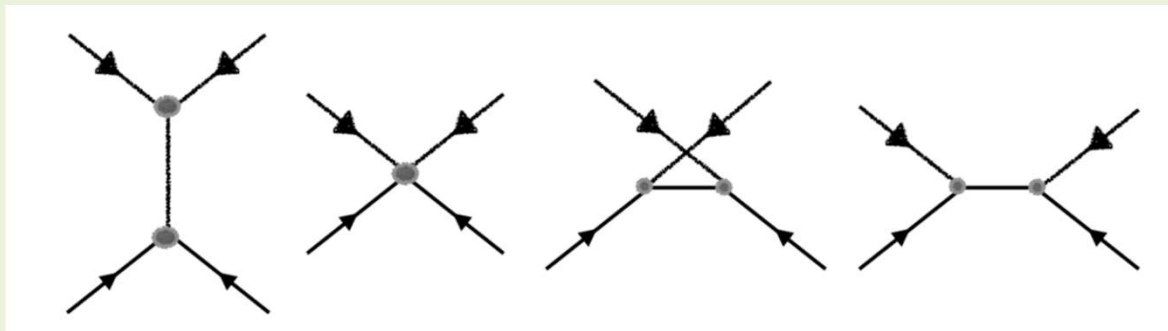
L.M. Abreu, P. Gubler, K.P. Khemchandani, A. Martínez Torres and A. Hosaka, arXiv:2409.05170 [hep-ph].

Starting point:

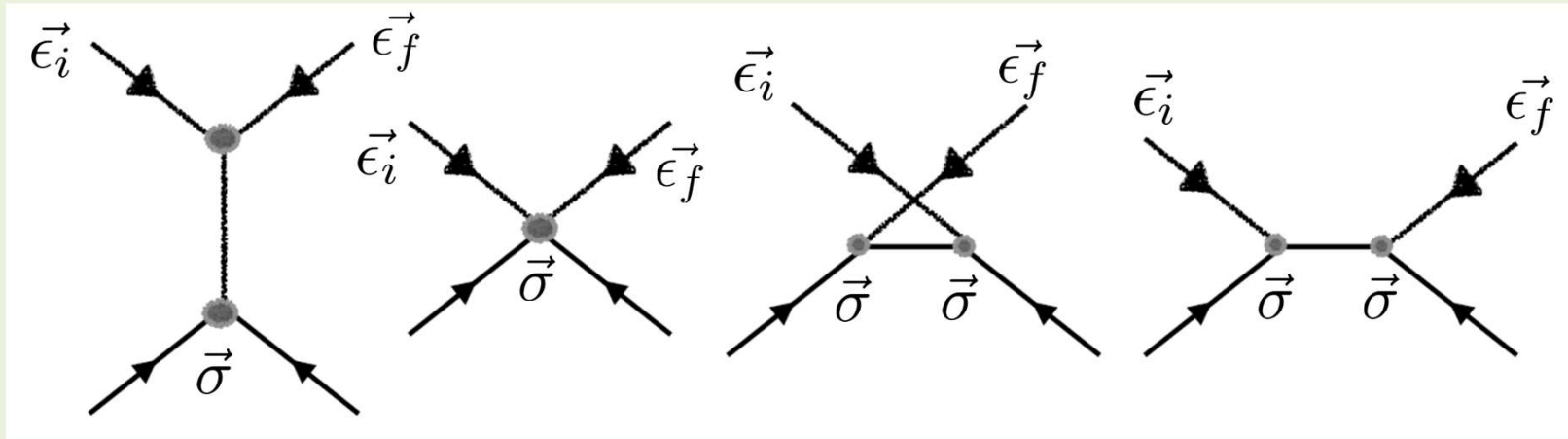
Hadronic Meson-Baryon interaction Lagrangian

1) Vector Meson-Baryon interaction: Based on Hidden Local Symmetry

$$\mathcal{L}_{VB} = -g \left\{ \langle \bar{B} \gamma_\mu [V_8^\mu, B] \rangle + \langle \bar{B} \gamma_\mu B \rangle \langle V_8^\mu \rangle + \frac{1}{4M} \left(F \langle \bar{B} \sigma_{\mu\nu} [V_8^{\mu\nu}, B] \rangle \right. \right. \\ \left. \left. + D \langle \bar{B} \sigma_{\mu\nu} \{V_8^{\mu\nu}, B\} \rangle \right) + \langle \bar{B} \gamma_\mu B \rangle \langle V_0^\mu \rangle + \frac{C_0}{4M} \langle \bar{B} \sigma_{\mu\nu} V_0^{\mu\nu} B \rangle \right\},$$



Crucial ingredient: spin dependent vector meson-baryon interactions



$$\vec{\epsilon}_i \cdot \vec{\epsilon}_f$$

Considered so far
in Feijoo et al.

$$\vec{\sigma} \cdot (\vec{\epsilon}_i \times \vec{\epsilon}_f) \quad (\vec{\sigma} \cdot \vec{\epsilon}_i)(\vec{\sigma} \cdot \vec{\epsilon}_f)$$

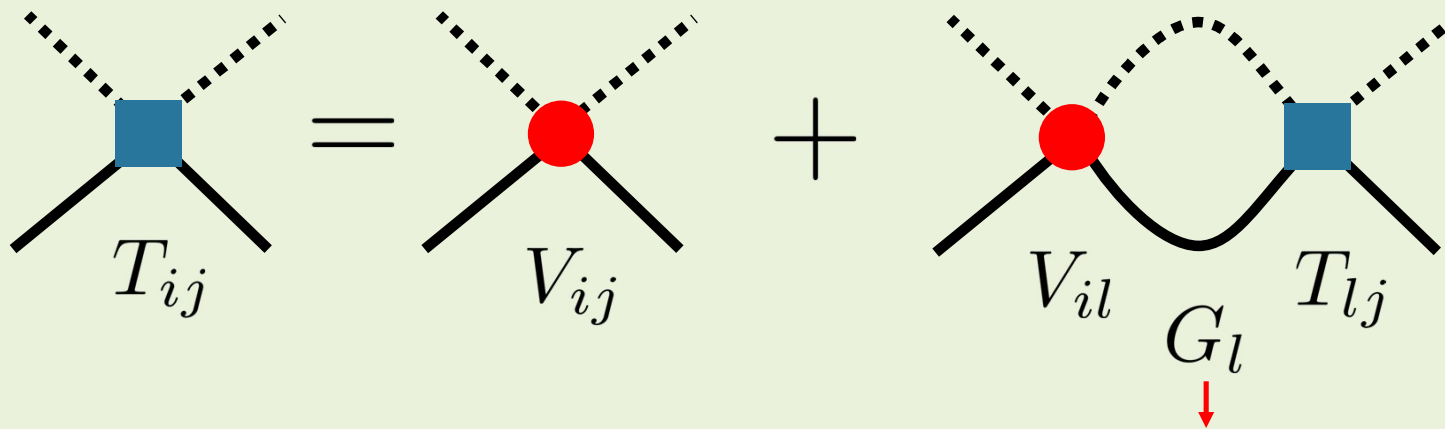
New!

Depend on spin configuration of the ϕN system (spin 1/2 or 3/2)

Next step:

Solve the Bethe-Salpeter equation in the Vector Meson-Baryon channel of interest to obtain the full scattering amplitude T

$$T_{ij} = V_{ij} + V_{il}G_l T_{lj}$$



Emergence of subtraction constants, which are parameters of the model



Loop contains divergence that needs to be regulated. We use dimensional regularization

For the spin 3/2 channel, we use two data sets to evaluate the corresponding uncertainty:

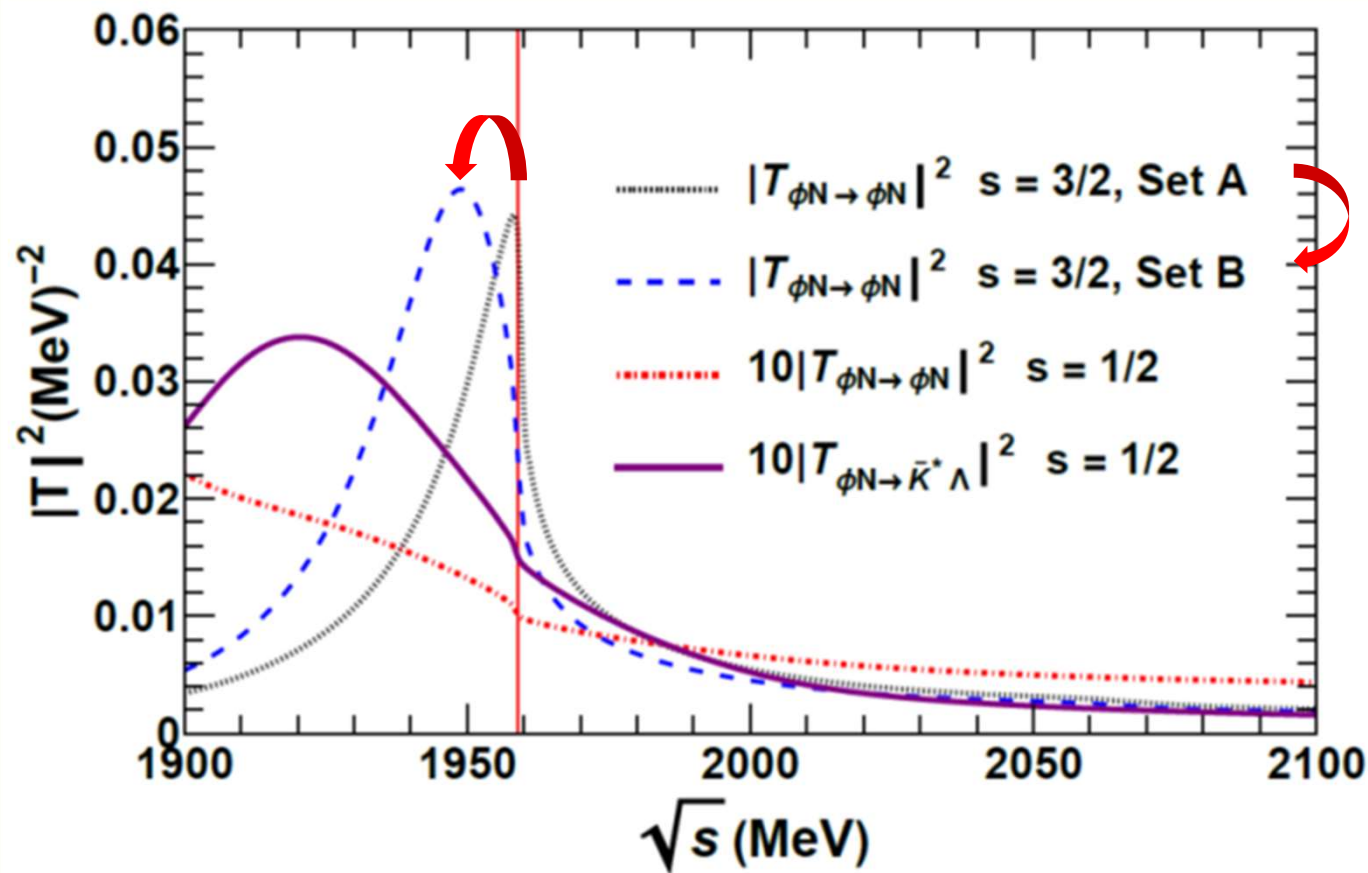
Channel (i)	a_i (Set A)	a_i (Set B)
ρN	-2.0	-2.0
ωN	-2.0	-2.0
ϕN	-1.7	-2.0
$K^* \Lambda$	-2.1	-2.1
$K^* \Sigma$	-2.0	-2.0



All are close to the “natural” value of $a_i = -2$

Only the value in the ϕN channel is modified, to study the change in the respective interaction strength

The obtained scattering amplitudes

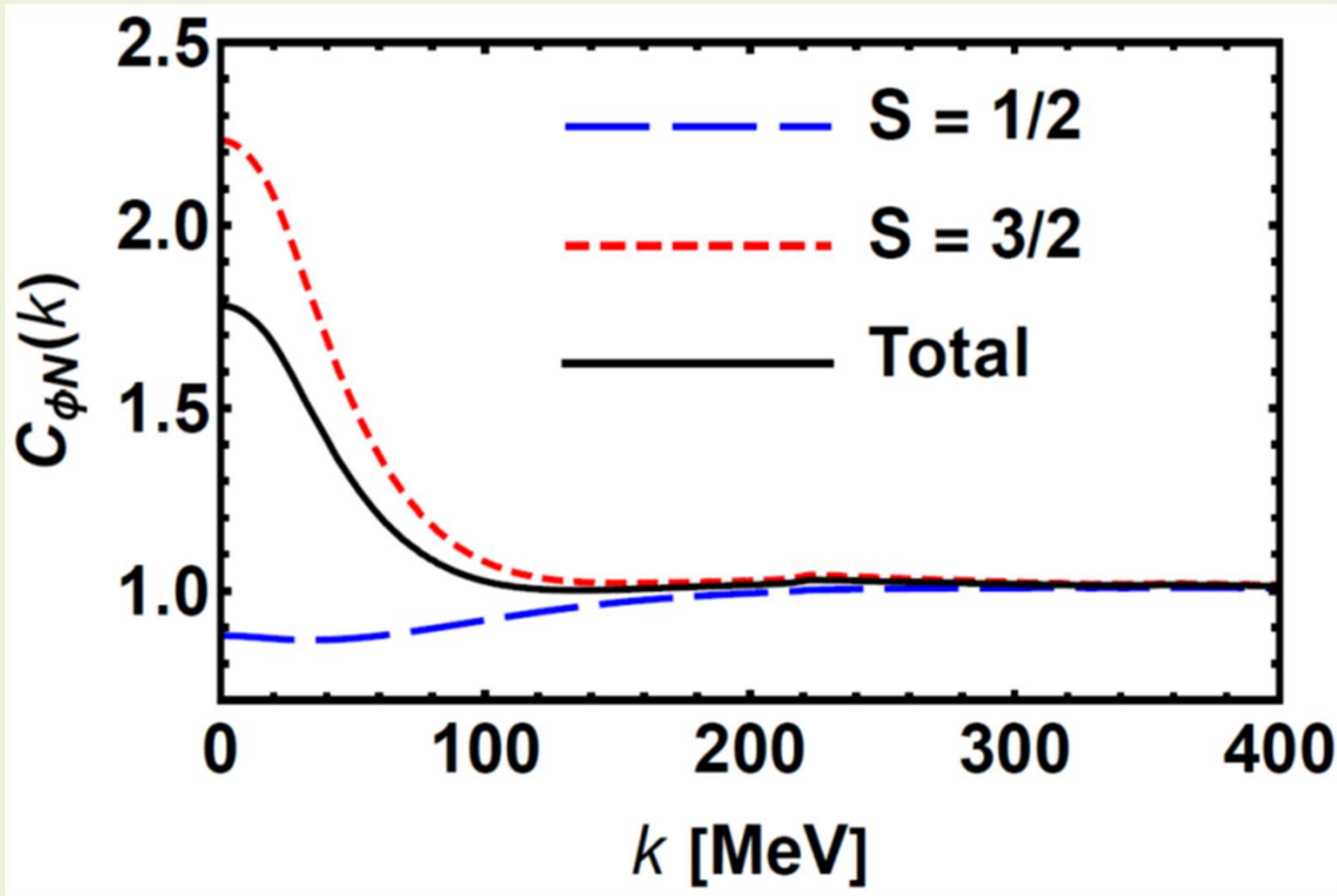


★ For spin 3/2, a state below the ϕN threshold can be found, depending on the values of the subtraction constants.

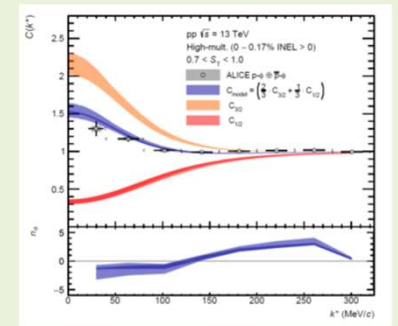
However, above the threshold, the differences between the two cases are small.

★ The spin 1/2 scattering amplitudes are much weaker than their spin 3/2 counterparts

The obtained correlation function (spin decomposed)

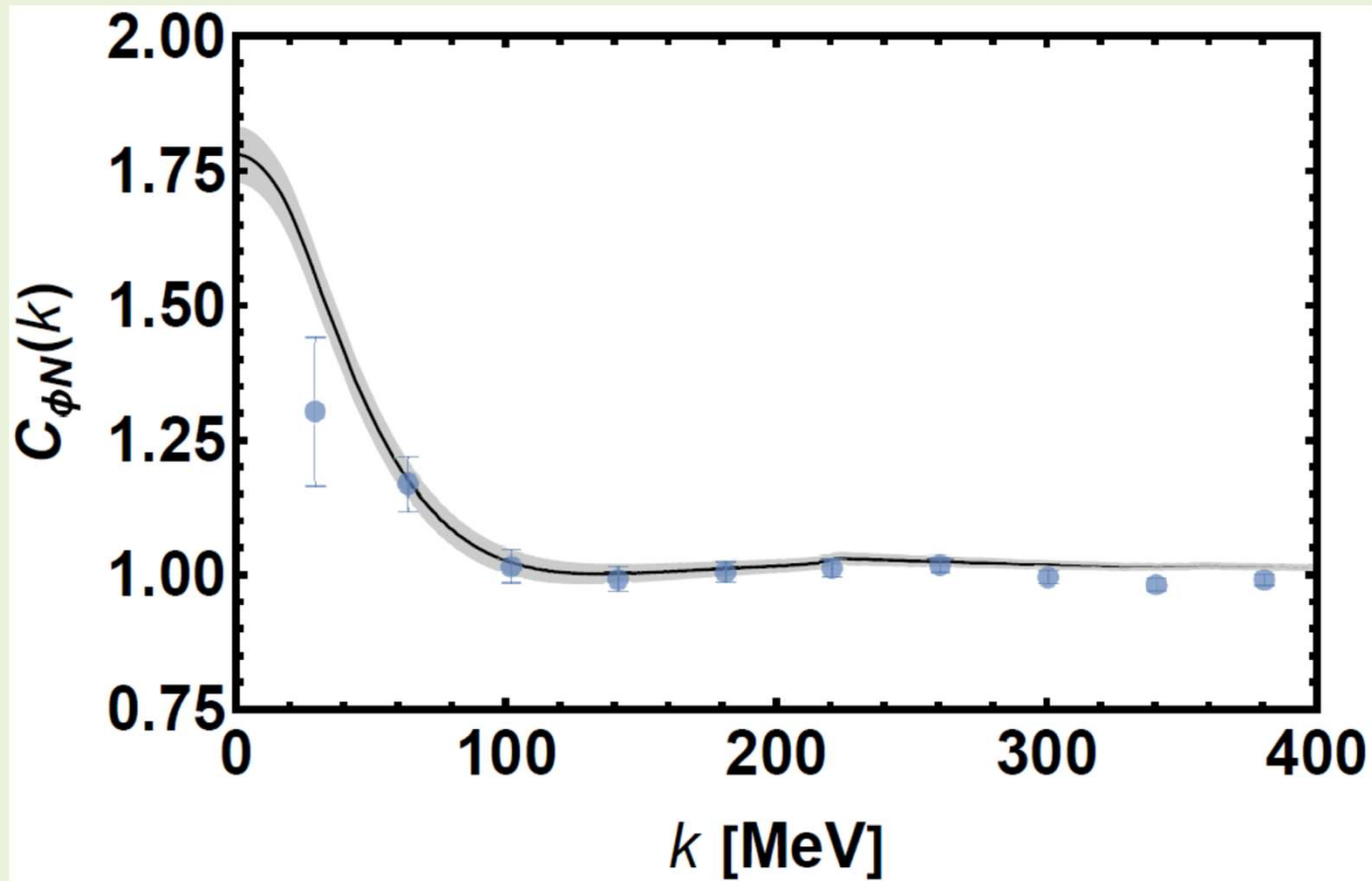


Qualitatively similar to
E. Chizzali et al.,
Phys. Lett. B **848**, 138358 (2024).



L.M. Abreu, P. Gubler, K.P. Khemchandani, A. Martínez Torres and A. Hosaka, arXiv:2409.05170 [hep-ph].

The obtained correlation function (compared with ALICE data)



Reasonably good agreement
without any parameter fitting!

The obtained scattering lengths

$$a_{\phi N}^{s=1/2} = -0.22 + i0.00 \text{ fm},$$

$$a_{\phi N}^{s=3/2, \text{set A}} = -0.30 + i1.50 \text{ fm},$$

$$a_{\phi N}^{s=3/2, \text{set B}} = -0.79 + i0.83 \text{ fm}.$$

Large model parameter dependence!



Correlation function is not very sensitive to the scattering length

Simple relation between ϕ N scattering length and ϕ meson mass shift in nuclear matter

$$\begin{aligned} V_\phi(\rho) &= -\frac{2\pi}{m_\phi} \rho \left(1 + \frac{m_\phi}{m_N}\right) a_0 \\ &\simeq -85 \frac{\rho}{\rho_0} \left(\frac{a_0}{\text{fm}}\right) \text{MeV} \end{aligned} \quad \left. \vphantom{\begin{aligned} V_\phi(\rho) &= -\frac{2\pi}{m_\phi} \rho \left(1 + \frac{m_\phi}{m_N}\right) a_0 \\ &\simeq -85 \frac{\rho}{\rho_0} \left(\frac{a_0}{\text{fm}}\right) \text{MeV} \end{aligned}} \right\} \text{Valid within the linear density approximation}$$

Larger than 100 MeV IF HAL QCD result is true for all spin configurations!

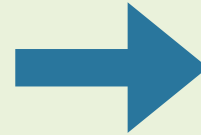
However, the above prescription seems problematic if a ϕ N bound state (or resonance) is formed.



Need better theoretical understanding!

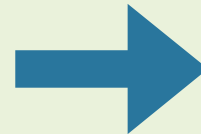
Summary and conclusions

★ A lot of new theoretical and experimental information about the ϕ N interaction is becoming available (LHC, HAL QCD)



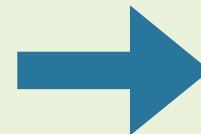
Strong hadronic medium effect?

★ Several works have by now studied the ALICE Correlation Function data, but the results largely disagree



**Need better data?
More reliable theory?**

★ With the state-of-the-art PHSD transport approach, we can now study pA reactions more reliably



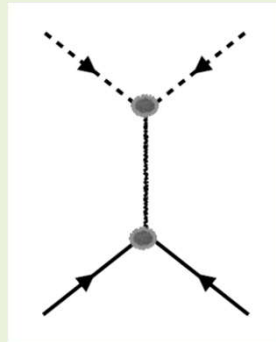
Many opportunities for new studies and projects!

Backup slides

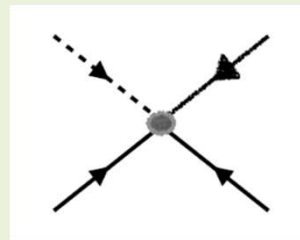
A lot of recent theoretical activity

- ★ Testing the φ -nuclear potential in pion-induced φ meson production on nuclei near threshold, E. Y. Paryev, Nucl. Phys. A **1032**, 122624 (2023).
- ★ The φp bound state in the unitary coupled-channel approximation, B.-X. Sun, Y.-Y. Fan, Q.-Q. Cao, Commun. Theor. Phys. **75**, 055301 (2023).
- ★ Possible ${}^3_{\varphi}H$ hypernucleus with the HAL QCD interaction, I. Filikhin, R. Y. Kezerashvili, B. Vlahovic, Phys. Rev. D **110**, L031502 (2024).
- ★ φ -p bound state and completeness of quantum states, A. Kuros, R. Maj, S. Mrowczynski, arXiv:2408.11941 [hep-ph].
- ★ Bound states of ${}^9_{\varphi}Be$ and ${}^6_{\varphi\varphi}He$ with $\varphi+\alpha+\alpha$ and $\varphi+\varphi+\alpha$ cluster models, I. Filikhin, R. Y. Kezerashvili, B. Vlahovic, arXiv:2408.13415 [nucl-th].
- ★ Relevance of the coupled channels in the φp and $\rho^0 p$ Correlation Functions , A. Feijoo, M. Korwieser, L. Fabbietti, arXiv:2407.01128 [hep-ph].

2) Pseudoscalar Meson-Baryon interaction: Based on Chiral Symmetry



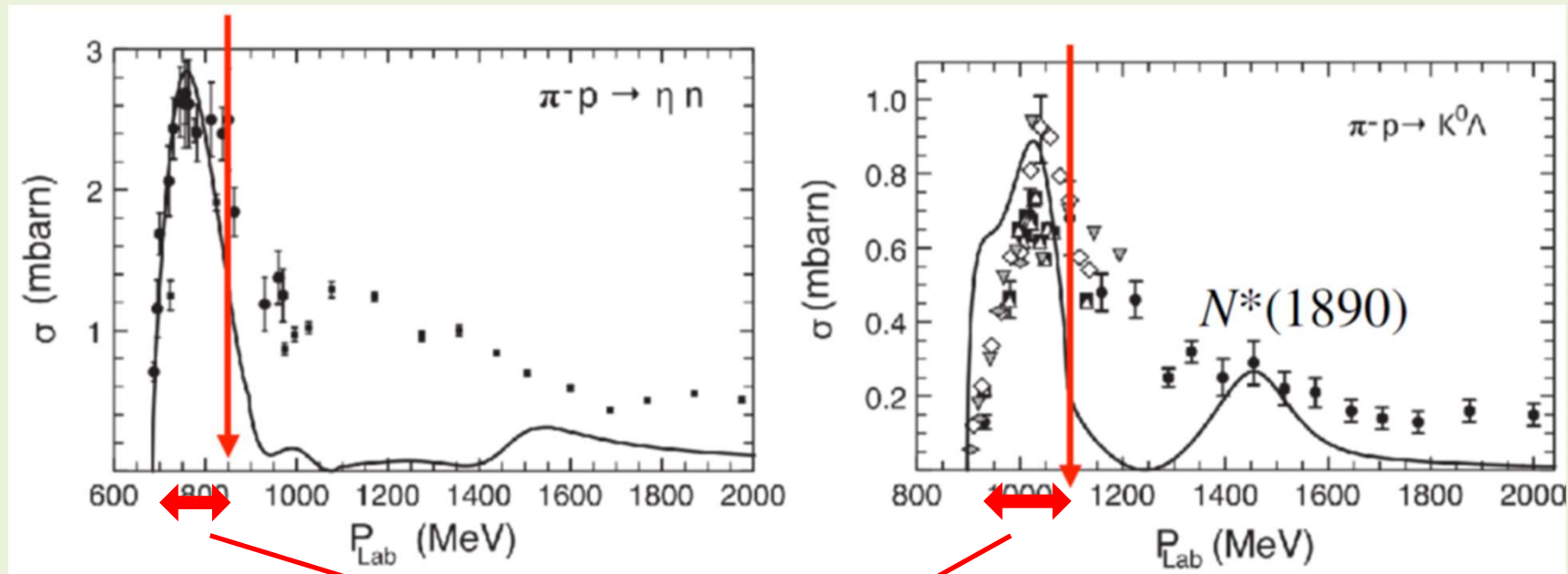
3) Transition between Pseudoscalar and Vector Mesons: Treating Vector meson as gauge boson in nonlinear sigma model (pion photoproduction + vector meson dominance)



Next step:

If possible, determine the parameters (subtraction constants) of the model from experimental data

For the spin 1/2 channel, scattering data are available for constraining the model parameters:



Regions used for the fit

K.P. Khemchandani, A. Martínez Torres, H. Nagahiro and A. Hosaka, Phys. Rev. D **88**, 114016 (2013).

L.M. Abreu, P. Gubler, K.P. Khemchandani, A. Martínez Torres and A. Hosaka, arXiv:2409.???? [hep-ph].

Next step:

Calculate the correlation function and compare with the ALICE data

We use a generalized Koonin-Pratt formula:

$$C_i(k_i) = 1 + 4\pi\theta(q_{max} - k_i) \int_0^\infty dr r^2 S_{12}(\vec{r}) \left(\sum_j w_j |j_0(k_i r) \delta_{ji} + T_{ji}(\sqrt{s}) \tilde{G}_j(r; s)|^2 - j_0^2(k_i r) \right)$$

Scattering amplitudes:
See previous slide

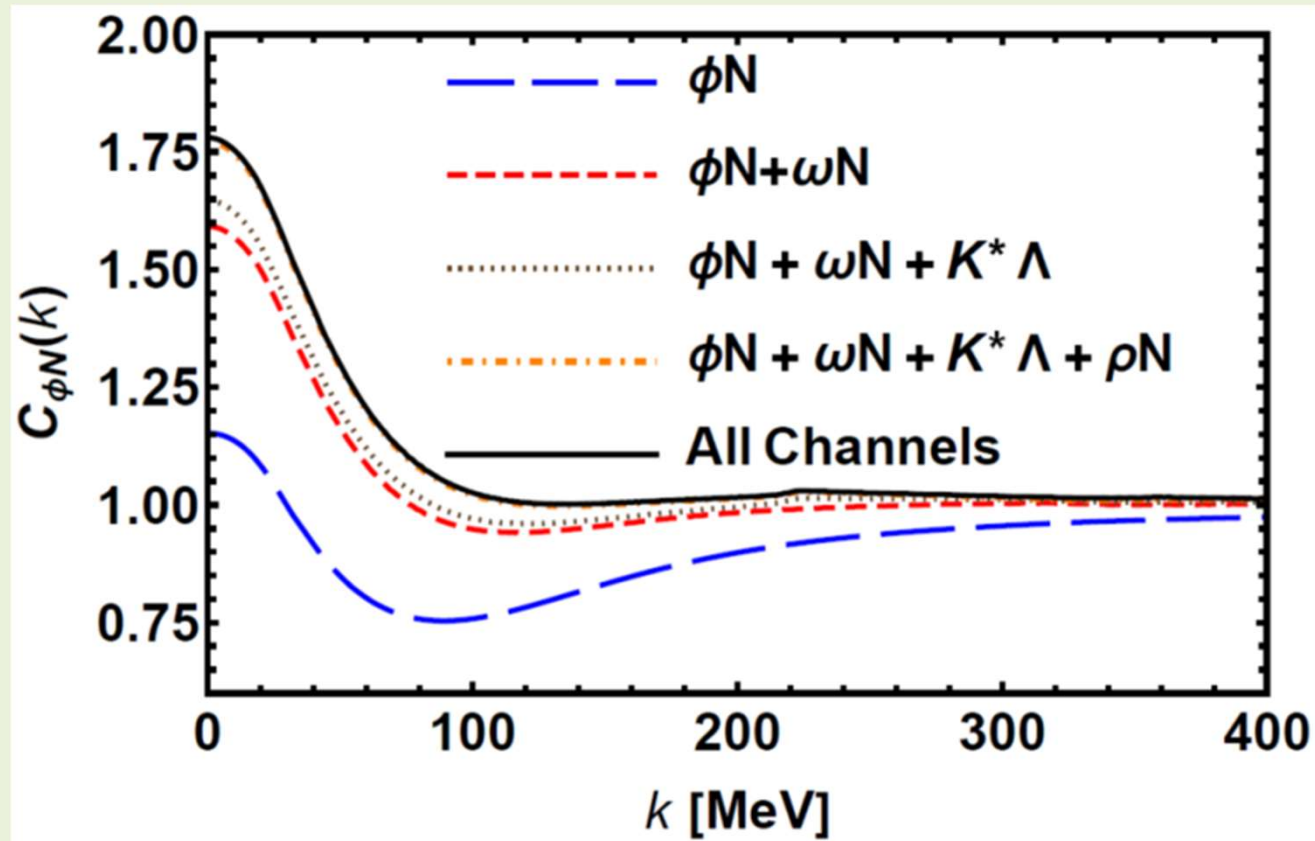
Source function:
Gaussian with radius of approx. 1 fm

Weights related to the multiplicity of pairs of primary particles created at the initial stage of the collision

<i>j</i> -th channel	$w_j^{(\frac{1}{2})}$	$w_j^{(\frac{3}{2})}$
πN	71	—
ηN	1	—
$K\Lambda$	5	—
$K\Sigma$	5	—
ρN	6.24	6.24
ωN	5.77	5.77
ϕN	1	1
$K^*\Lambda$	0.65	0.65
$K^*\Sigma$	0.42	0.42

Obtained from Thermal-Fist package

The obtained correlation function (channel decomposed)

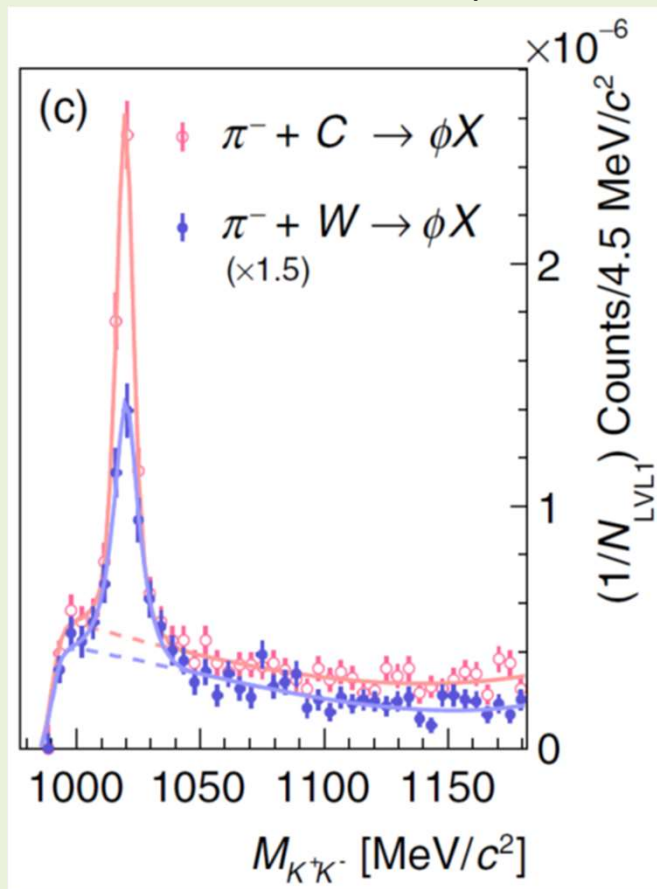


Similar to
A. Feijoo et al.,
arXiv:2407.01128 [hep-ph].

More recent results

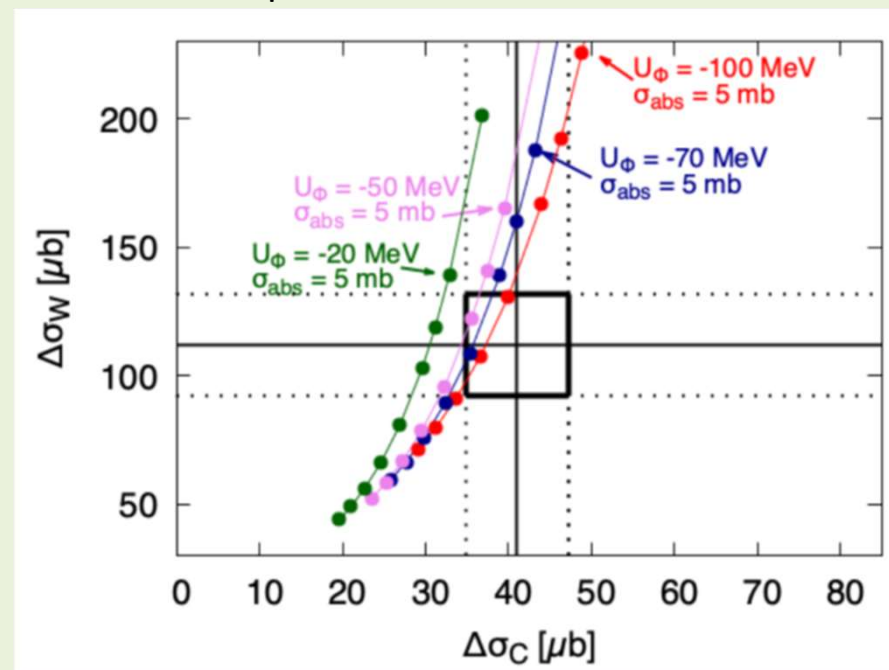
HADES: 1.7 GeV π^- -A-reaction

K^+K^- - invariant mass spectrum



J. Adamczewski-Musch et al. (HADES Coll.),
 Phys. Rev. Lett. **123**, 022002 (2019).

Theoretical analysis of the of the total ϕ meson production cross section:

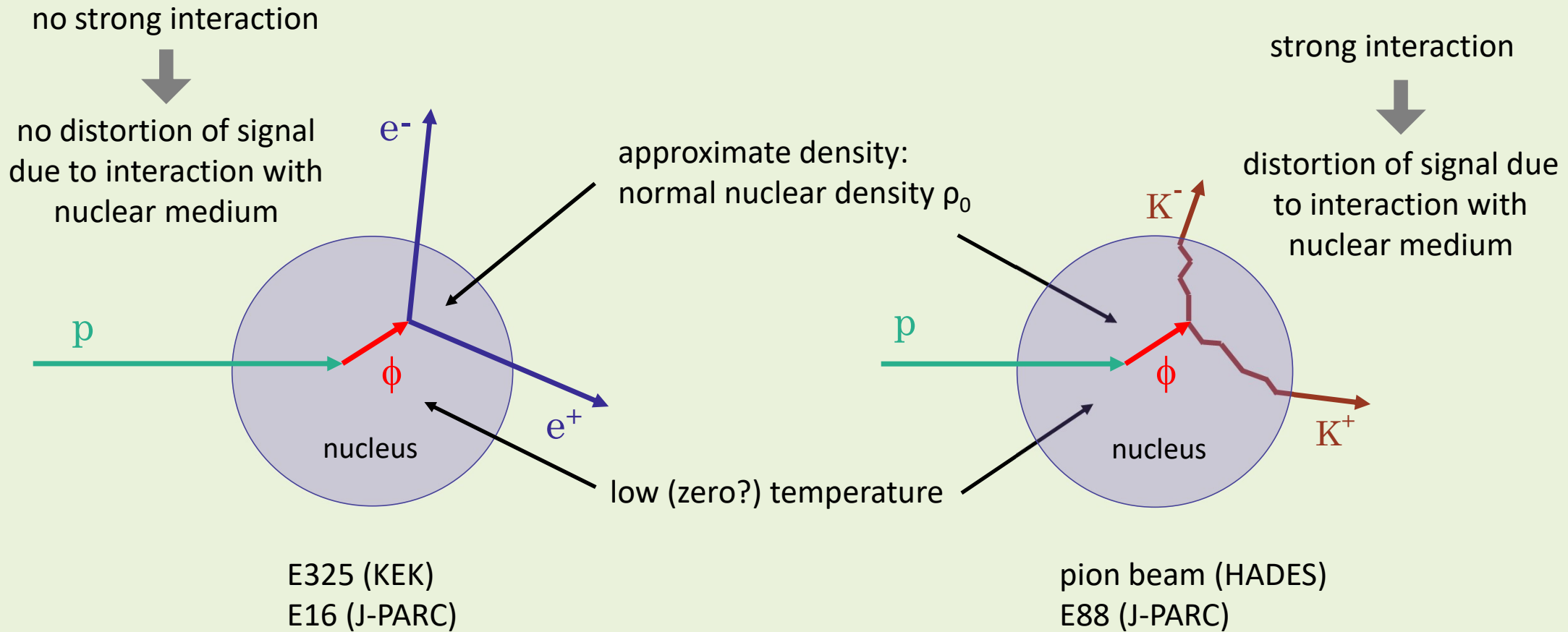


E. Ya. Paryev, Nucl. Phys. A **1032**, 122624 (2023).

- ➔ **Attractive ϕ -nucleus potential:**
 -(50 - 100) MeV
- ➔ **Relatively small imaginary part:**
 20 – 25 MeV

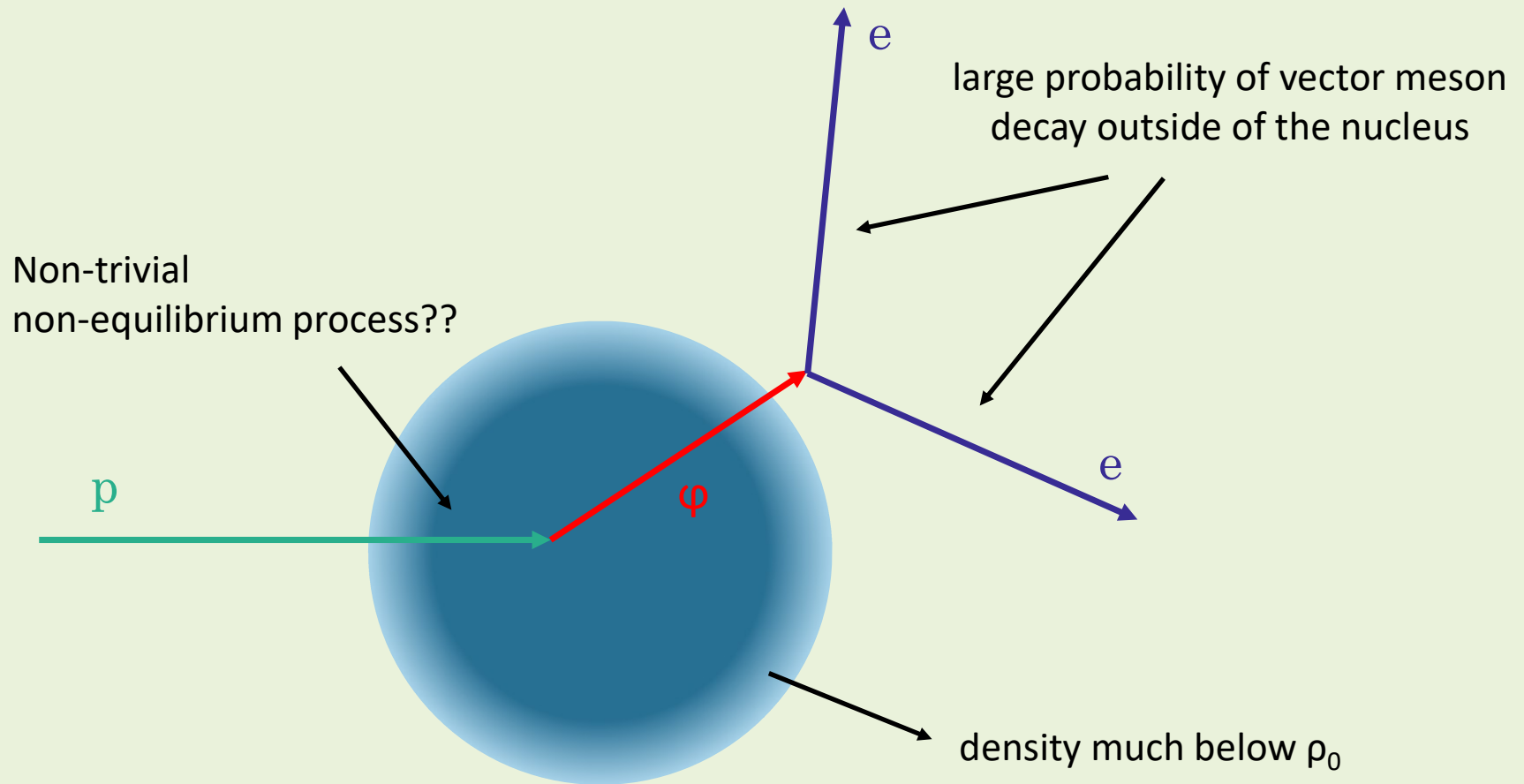
The ϕ meson in pA collisions

Experiments to be discussed in this talk



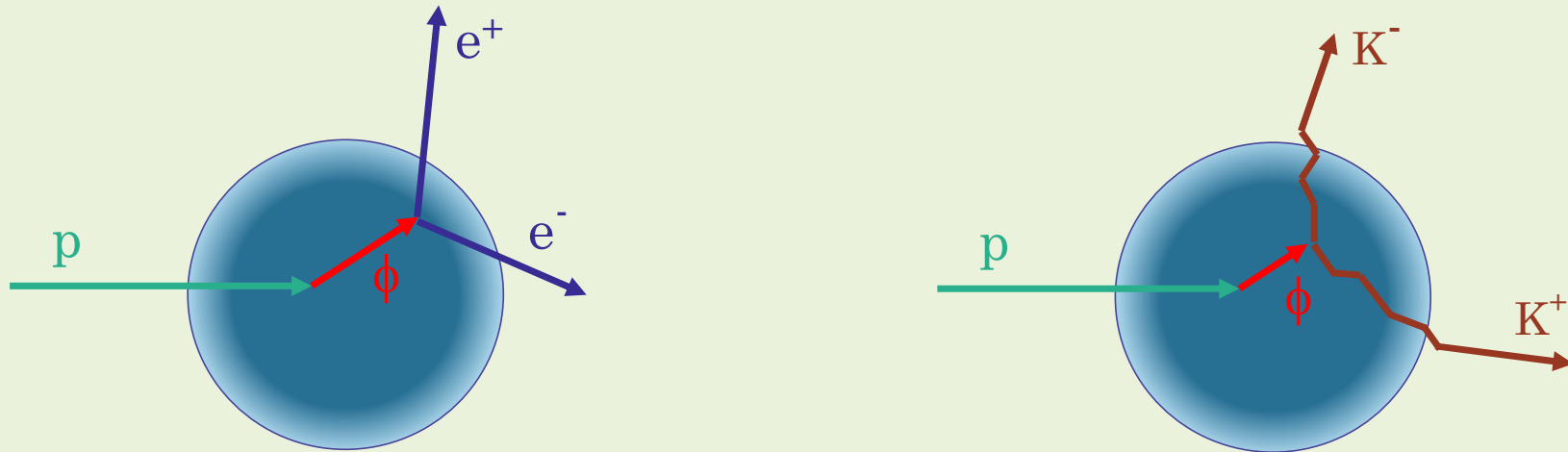
In reality, things are more complicated...

Proton induced generation of vector mesons in nuclei



Further tasks for theory

Have a good understanding of the production mechanisms of the ϕ mesons in nuclei from pA reactions.

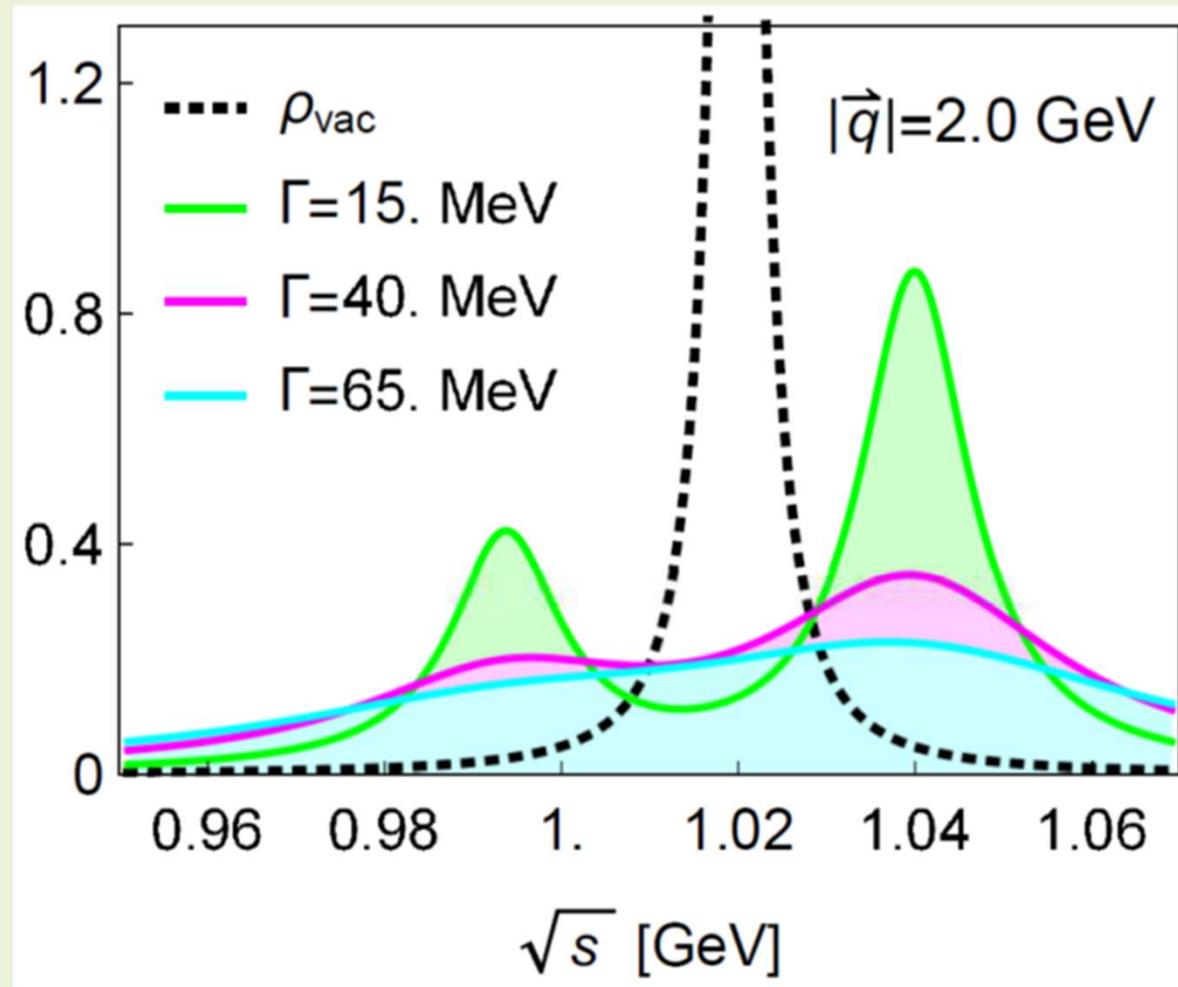


- ★ Where (and at what densities) is the ϕ meson produced and where does it decay?
- ★ How do the final state interactions of the decay particles influence the decay spectrum (especially for K^+K^-)?

➡ Realistic transport simulations using a transport approach (calculations using the PHSD code are ongoing)

➡ See talk by L. Oliva

The angle-averaged di-lepton spectrum



A double peak?

Computed at
normal nuclear
matter density

First application of our formalism

Thermal model with single freeze-out to describe soft hadron production for Au + Au collisions at top RHIC energies:

$$f_V(q, X) = e^{-q^\mu \beta_\mu(x) - \xi(x)} \quad (\text{Jüttner distribution})$$

$$\beta^\mu = \frac{u^\mu}{T}, \quad \xi = \frac{\mu}{T}$$

elliptical
asymmetry: $\left\{ \begin{array}{l} x = r_{\max} \sqrt{1 - \epsilon} \cos \phi, \\ y = r_{\max} \sqrt{1 - \epsilon} \sin \phi \end{array} \right.$

elliptical flow: $u^\mu = \frac{1}{N} (t, x\sqrt{1 + \delta}, y\sqrt{1 - \delta}, z)$

c %	ϵ	δ	τ_f [fm]	r_{\max} [fm]
0 – 15	0.055	0.12	7.666	6.540
15 – 30	0.097	0.26	6.258	5.417
30 – 60	0.137	0.37	4.266	3.779

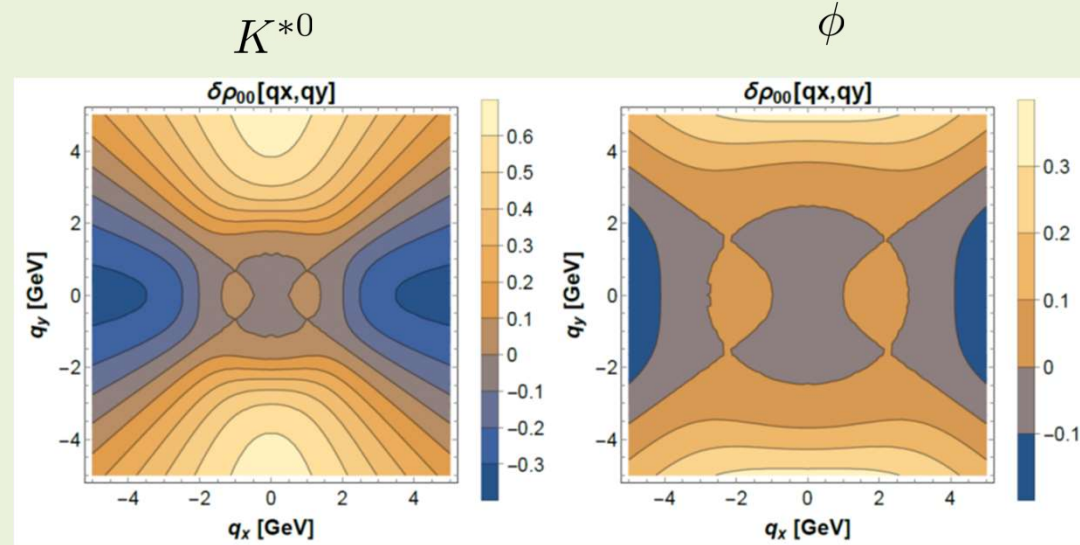
PHOENIX data at
 $\sqrt{s_{NN}} = 130$ GeV

A. Kumar, D.-L. Yang and P. Gubler, 2312.16900 [nucl-th], to be published in PRD.

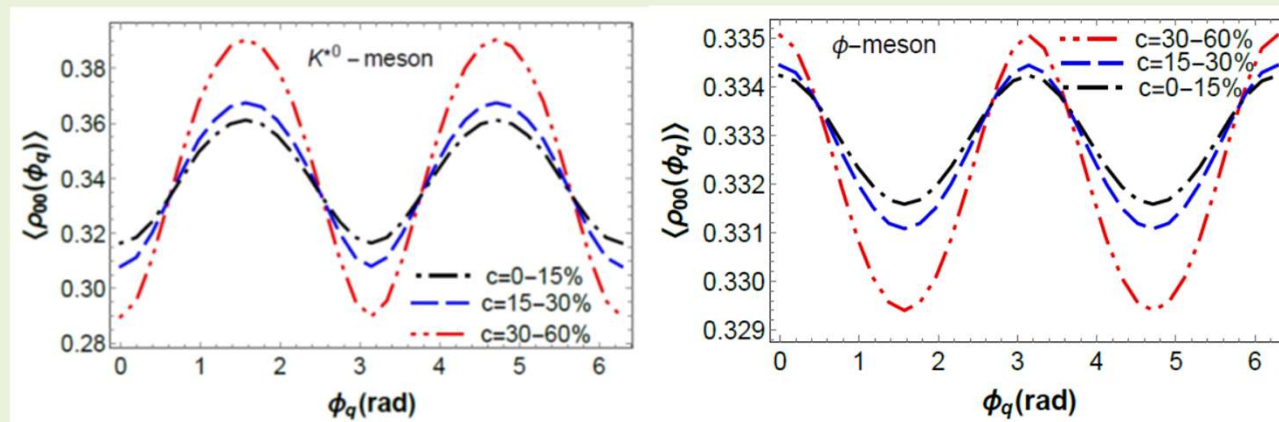
First application of our formalism

$$\rho_{00} = \frac{1}{3}$$

$c = 30 - 60\%$



ρ_{00}
Azimuthal angle
dependence



A. Kumar, D.-L. Yang and P. Gubler, 2312.16900 [nucl-th], to be published in PRD.

ϕ meson at rest in nuclear matter

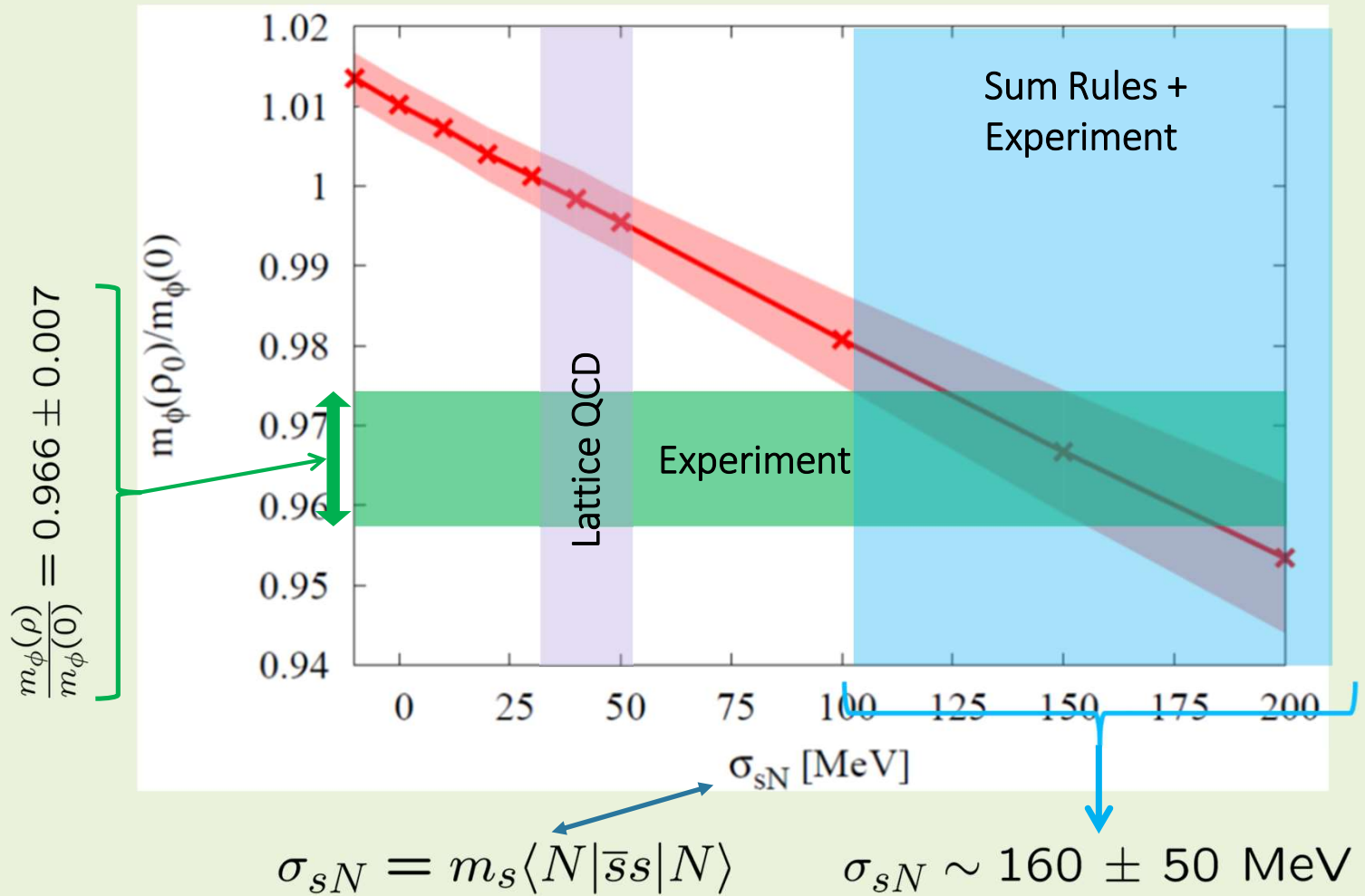
The ϕ meson mass in nuclear matter probes the strange quark condensate at finite density!

Not consistent?

R. Muto et al.
(KEK, E325 Collaboration),
Phys. Rev. Lett. **98**,
042501 (2007).



Measurement will be repeated at the J-PARC E16 experiment (with 100 times increased statistics!)



Condensates that appear in the vector channel

Quark condensates

$$\langle \bar{q}jq \rangle \equiv \langle g \bar{q} \gamma_\mu (D_\nu G_{\mu\nu}) q \rangle,$$

$$\langle j_5 j_5 \rangle \equiv \langle g^2 \bar{q} t^a \gamma_5 \gamma_\mu q \bar{q} t^a \gamma_5 \gamma_\mu q \rangle,$$

$$A_{\alpha\beta} \equiv \langle g \bar{q} (D_\mu G_{\alpha\mu}) \gamma_\beta q |_{ST} \rangle,$$

$$B_{\alpha\beta} \equiv \langle g \bar{q} \{iD_\alpha, \tilde{G}_{\beta\mu}\} \gamma_5 \gamma_\mu q |_{ST} \rangle,$$

$$C_{\alpha\beta} \equiv \langle m \bar{q} D_\alpha D_\beta q |_{ST} \rangle,$$

$$F_{\alpha\beta} \equiv \langle \bar{q} \gamma_\alpha i D_\beta q |_{ST} \rangle,$$

$$H_{\alpha\beta} \equiv \langle g^2 \bar{q} t^a \gamma_5 \gamma_\alpha q \bar{q} t^a \gamma_5 \gamma_\beta q \rangle,$$

$$K_{\alpha\beta\gamma\delta} \equiv \langle \bar{q} \gamma_\alpha D_\beta D_\gamma D_\delta q |_{ST} \rangle$$

scalar

non-scalar

Gluon condensates

$$\langle G^2 \rangle \equiv \langle g^2 G_{\mu\nu}^a G_{\mu\nu}^a \rangle,$$

$$\langle G^3 \rangle \equiv \langle g^3 f^{abc} G_{\mu\nu}^a G_{\nu\lambda}^b G_{\lambda\mu}^c \rangle,$$

$$\langle j^2 \rangle \equiv \langle g^2 (D_\mu G_{\alpha\mu}^a) (D_\nu G_{\alpha\nu}^a) \rangle,$$

$$G_{2\alpha\beta} \equiv \langle g^2 G_{\alpha\mu}^a G_{\beta\mu}^a |_{ST} \rangle,$$

$$X_{\alpha\beta} \equiv \langle g^2 G_{\mu\nu}^a D_\beta D_\alpha G_{\mu\nu}^a |_{ST} \rangle,$$

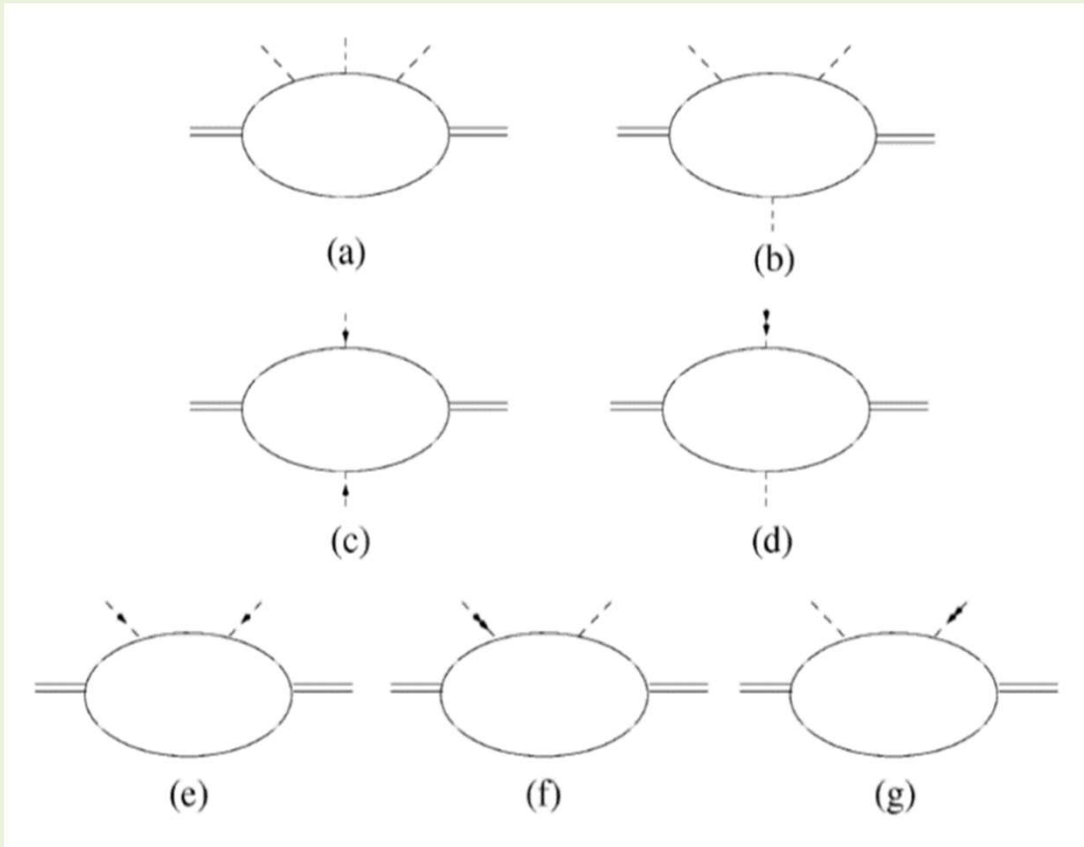
$$Y_{\alpha\beta} \equiv \langle g^2 G_{\alpha\mu}^a D_\mu D_\nu G_{\beta\nu}^a |_{ST} \rangle,$$

$$Z_{\alpha\beta} \equiv \langle g^2 G_{\alpha\mu}^a D_\beta D_\nu G_{\mu\nu}^a |_{ST} \rangle,$$

$$G_{4\alpha\beta\gamma\delta} \equiv \langle g^2 G_{\alpha\mu}^a D_\delta D_\gamma G_{\beta\mu}^a |_{ST} \rangle$$

Wilson coefficients were not yet available until recently

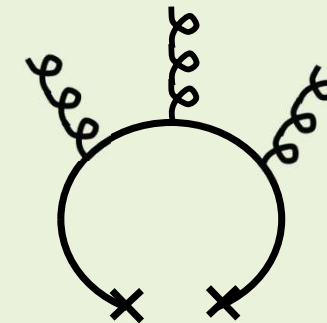
OPE calculation



Mass singularities in
chiral limit!

$$\frac{1}{m^2}, \log\left(\frac{\mu^2}{m^2}\right), \dots$$

Subtract corresponding quark
condensate contribution

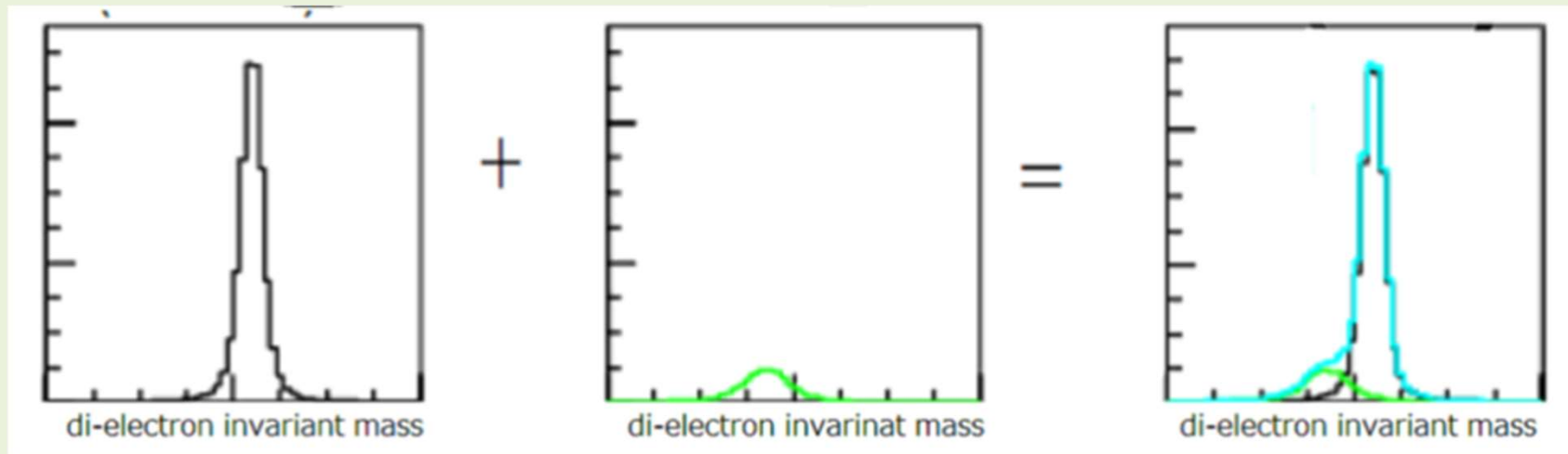
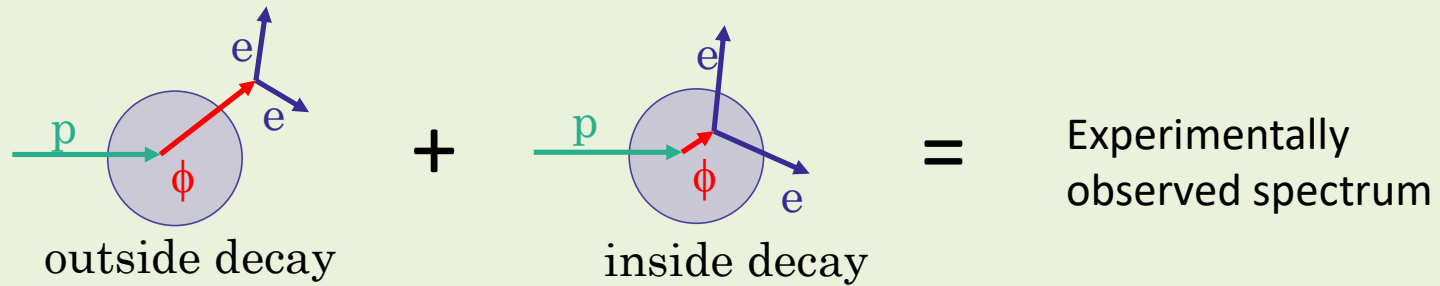


$$\langle \bar{q} \Gamma(D, G) q \rangle$$

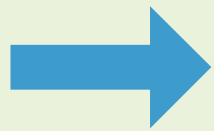
S. Kim and S.H. Lee, Nucl. Phys. **679**, 517 (2001).

H.J. Kim, P. Gubler and S.H. Lee, Phys. Lett. B **772**, 194 (2017).

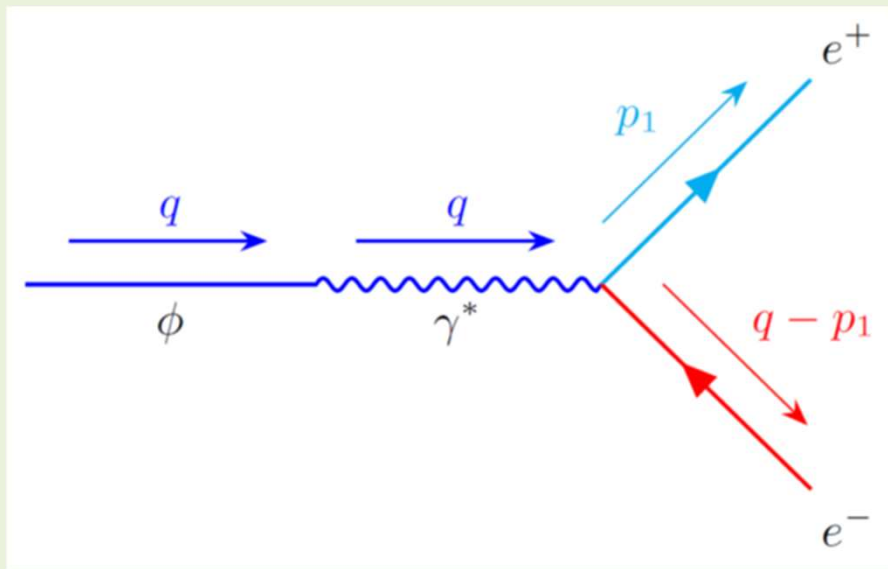
Experimental di-lepton spectrum



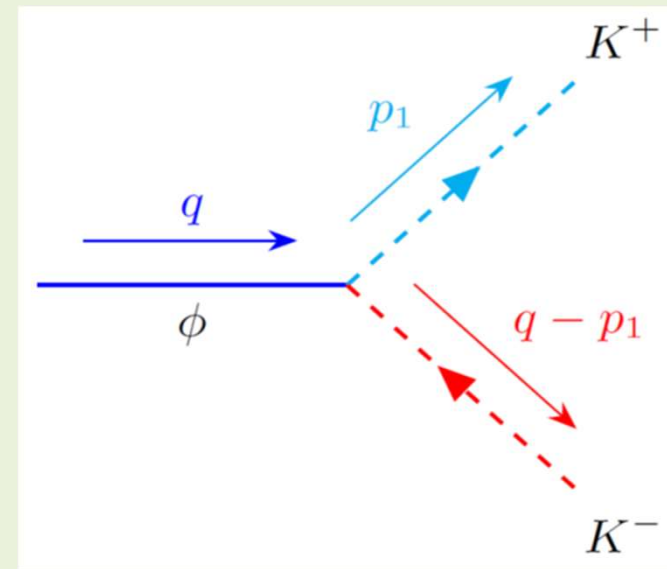
Can the two polarizations be disentangled?



Look at the angular distributions of various decay channels

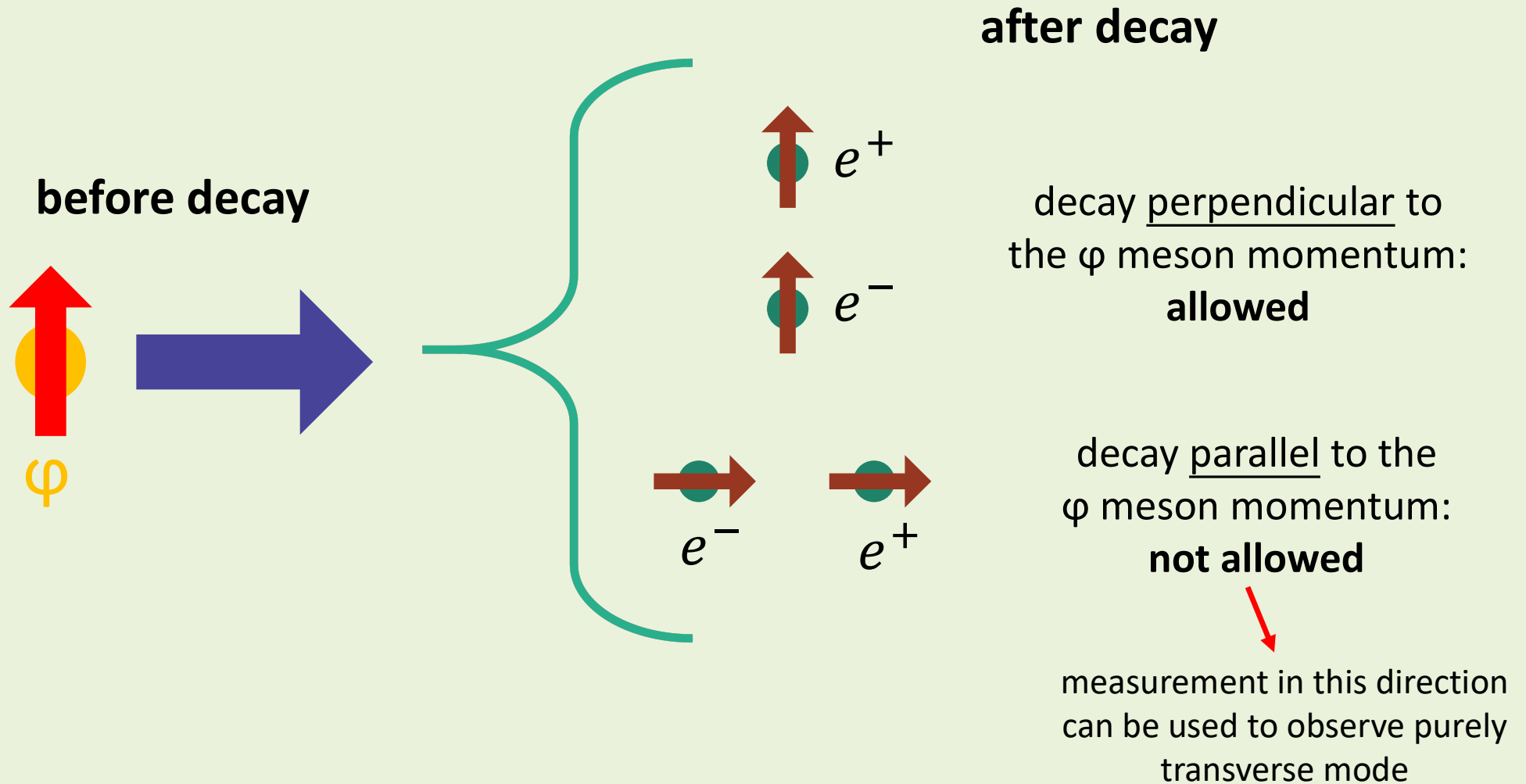


To be measured soon at the J-PARC E16 experiment

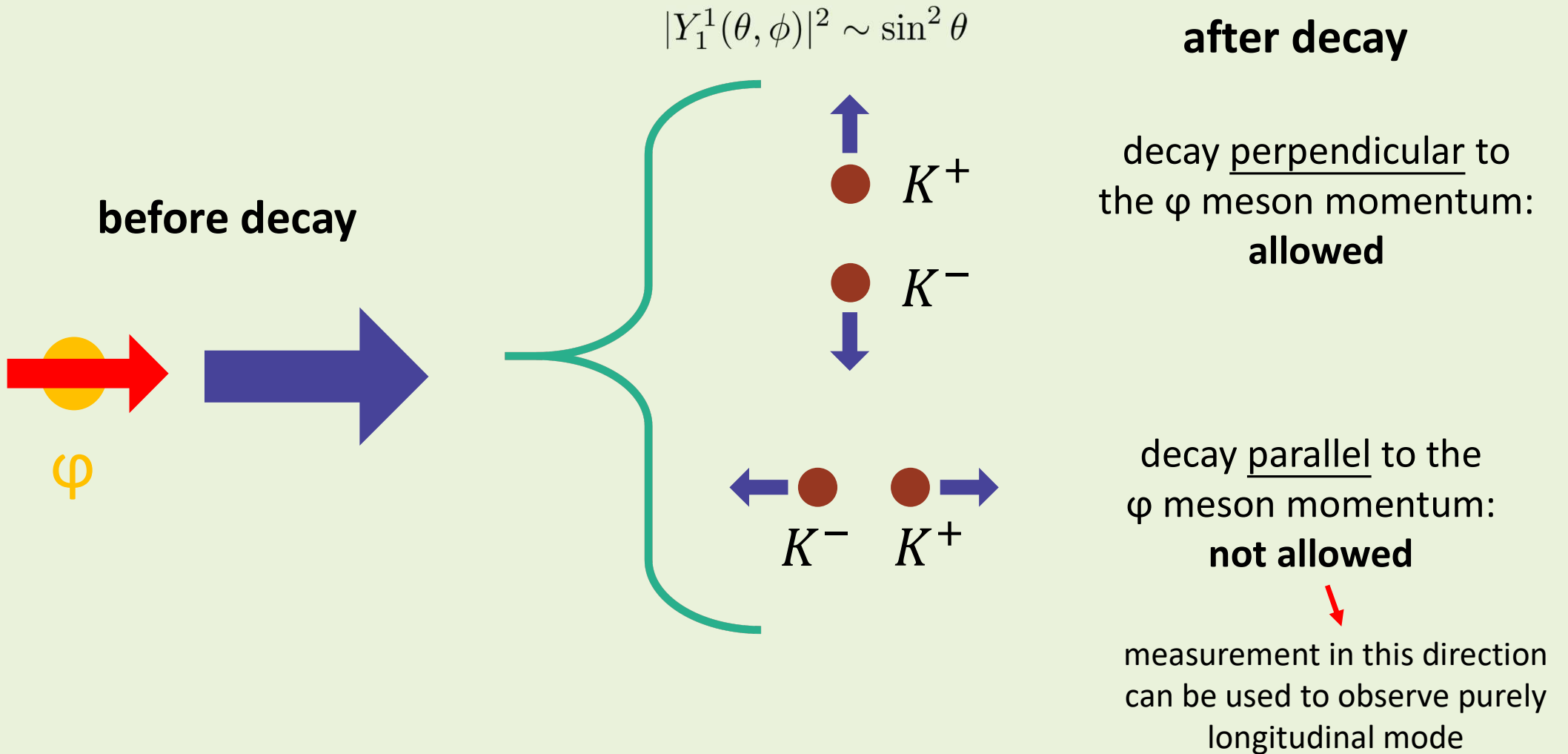


New E88 experiment at J-PARC (in a few years)

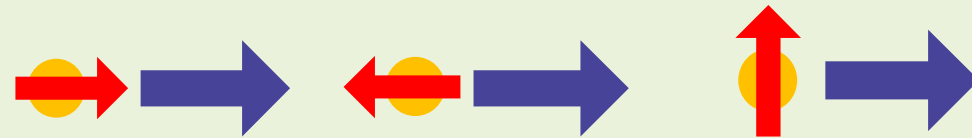
A simple example of dilepton decay of a longitudinally polarized ϕ



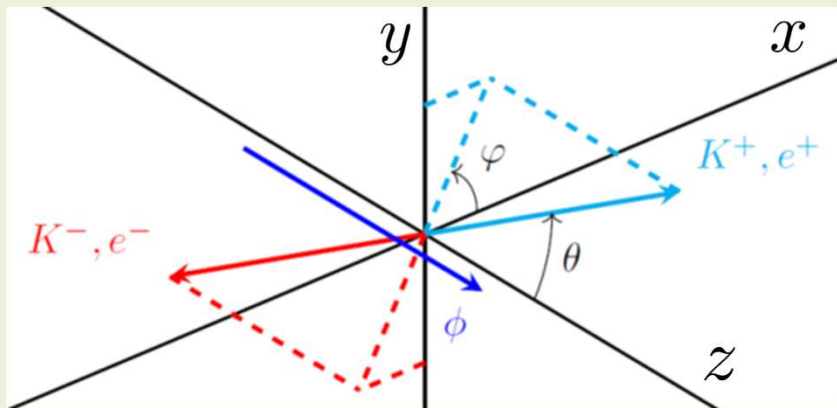
A simple example of K^+K^- decay of a transversely polarized φ



Full angular distribution of dilepton decay



Initial polarization: $|V\rangle = a_{+1}|+1\rangle + a_{-1}|-1\rangle + a_0|0\rangle$



Transverse polarization

Longitudinal polarization

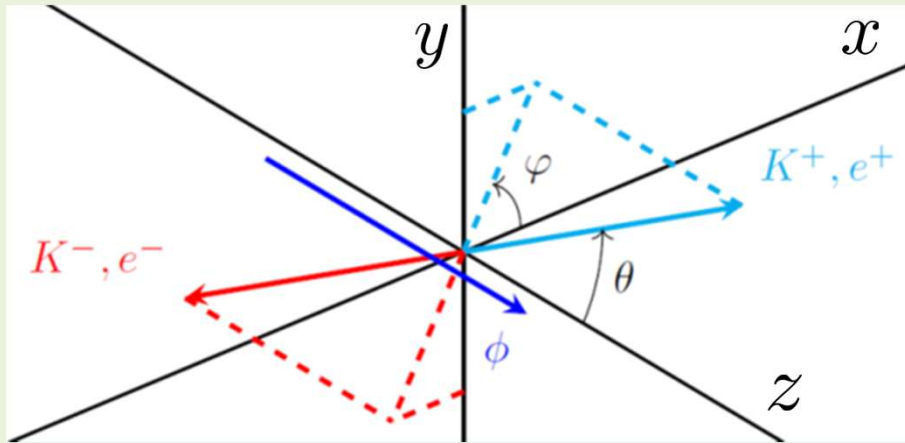
θ : polar angle

ϕ : azimuthal angle

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\Omega} = \frac{3}{16\pi} \left[(|a_{+1}|^2 + |a_{-1}|^2)(1 + \cos^2 \theta) + 2|a_0|^2(1 - \cos^2 \theta) + 2\text{Re}(a_{+1}a_{-1}^*) \sin^2 \theta \cos 2\phi + \dots \right]$$

other ϕ -dependent terms

Full angular distribution of dilepton decay



θ : polar angle
 ϕ : azimuthal angle

With

$$|a_{+1}|^2 + |a_{-1}|^2 + |a_0|^2 = 1, \quad |a_0|^2 = \rho_{00}$$

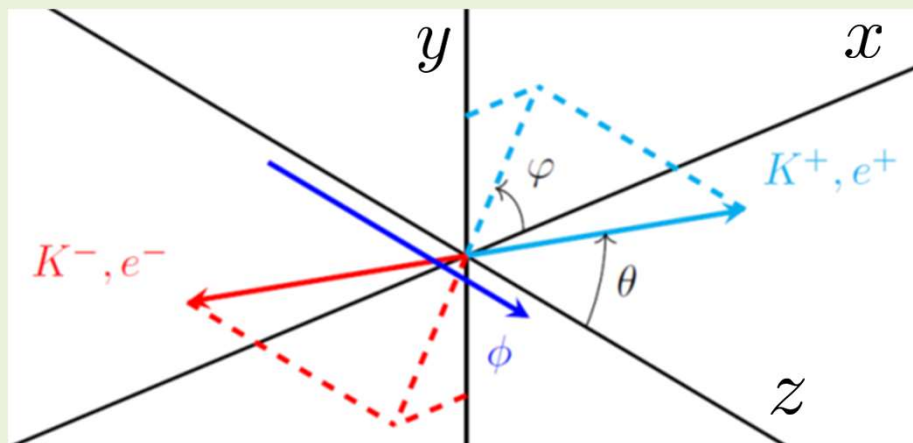
00-component of spin-density matrix

$$\rightarrow \frac{1}{\Gamma} \frac{d\Gamma}{d\Omega} = \frac{3}{16\pi} \left[1 + \cos^2 \theta + \rho_{00} (1 - 3 \cos^2 \theta) + \dots \right]$$

$$\rightarrow \rho_{00} = \frac{1}{3} \quad \text{Unpolarized case: vanishing } \theta\text{-dependence}$$

ϕ -dependent terms

Full angular distribution of K^+K^- decay



θ : polar angle
 ϕ : azimuthal angle

Transverse modes

Longitudinal mode

$$\begin{aligned} \frac{1}{\Gamma} \frac{d\Gamma}{d\Omega} &= \frac{3}{16\pi} \left[\underbrace{(|a_{+1}|^2 + |a_{-1}|^2)}_{\text{Transverse modes}} \sin^2 \theta + \overset{\text{Longitudinal mode}}{2|a_0|^2} \cos^2 \theta \right. \\ &\quad \left. - 2\text{Re}(a_{+1}a_{-1}^*) \sin^2 \theta \cos 2\phi + \dots \right] \\ &= \frac{3}{16\pi} \left[1 - \cos^2 \theta - \rho_{00}(1 - 3\cos^2 \theta) + \dots \right] \end{aligned}$$

ϕ -dependent terms