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HHIQCD@YITP

Equation of state in neutron stars from a bottom-up holographic QCD model

Akihiro Iwanaka (RCNP, Osaka U.)

Collaborators：
Daisuke Fujii (JAEA), Atsushi Hosaka (RCNP, JAEA),
Tadakatsu Sakai (KMI, Nagoya U.), Motoi Tachibana (Saga U.)

An ultimate purpose of QCD studies

To obtain the QCD phase diagram

▼

To challenge it

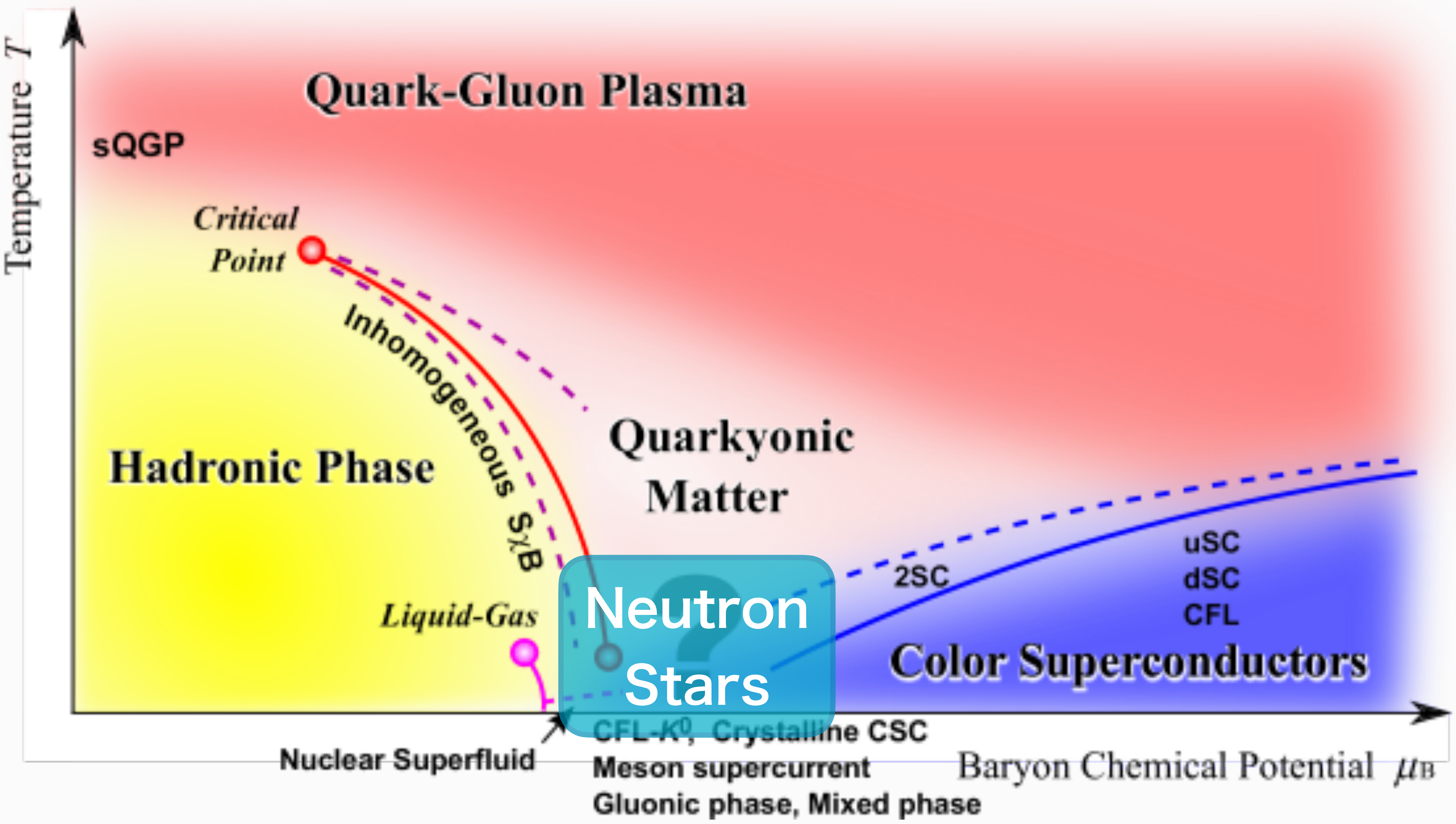
Focus on neutron stars

QCD phase diagram

K. Fukushima and T. Hatsuda,
Rep. Prog. Phys. **74** 014001 (2011)

Temperature

T

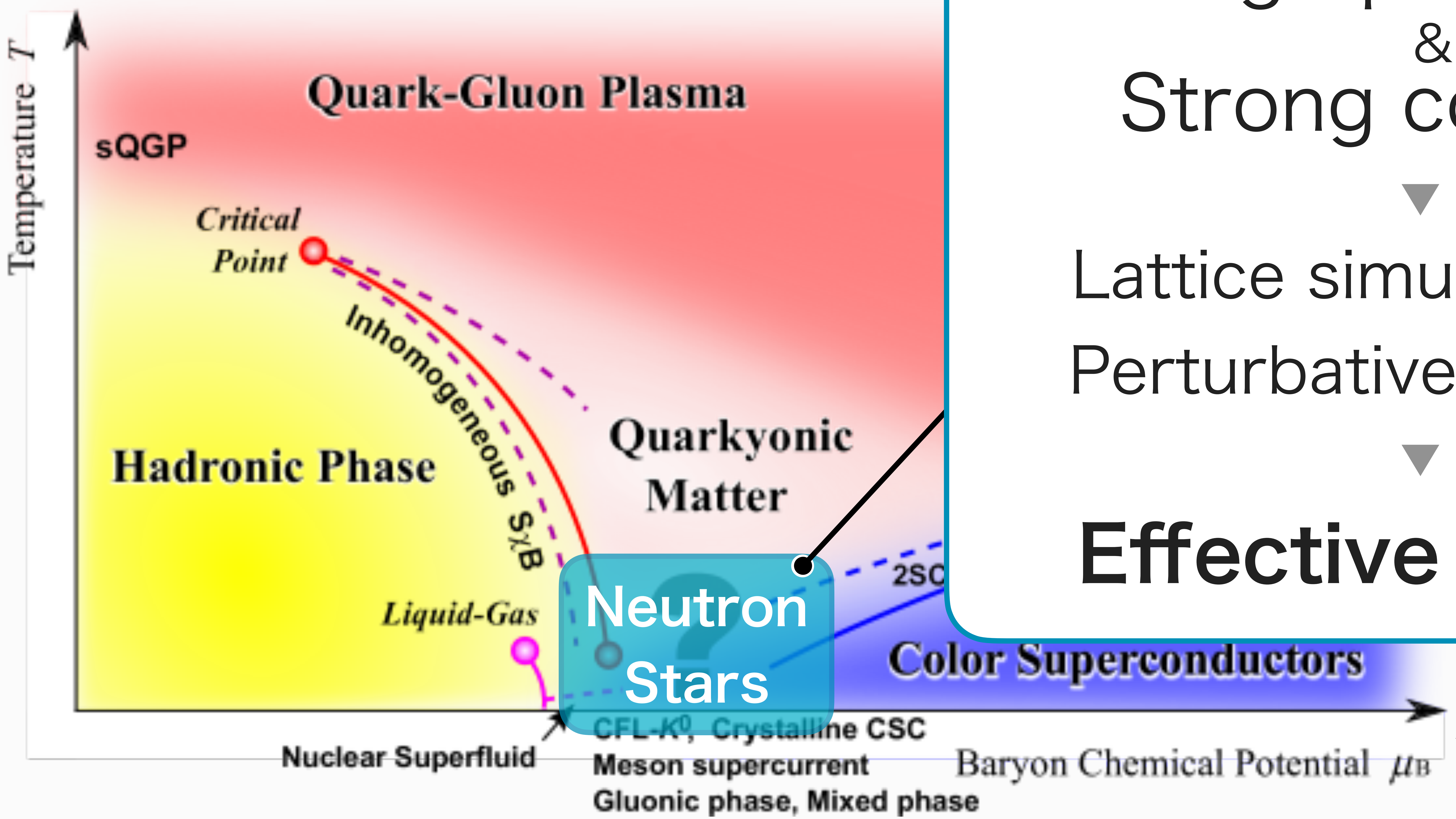


μ_B Baryon chemical potential

QCD phase diagram

K. Fukushima and T. Hatsuda,
Rep. Prog. Phys. **74** 014001 (2011)

Temperature
T



μ_B Baryon chemical potential

Sign problem
&
Strong coupling

▼

Lattice simulation → ×

Perturbative QCD → ×

▼

Effective models

Neutron Star for QCD

P. Demorest, et al., Nature **467,** 1081 (2010)

Effective model

gives EoS ↓ ↑ gives constraints

01. Equation of state

02. TOV equation

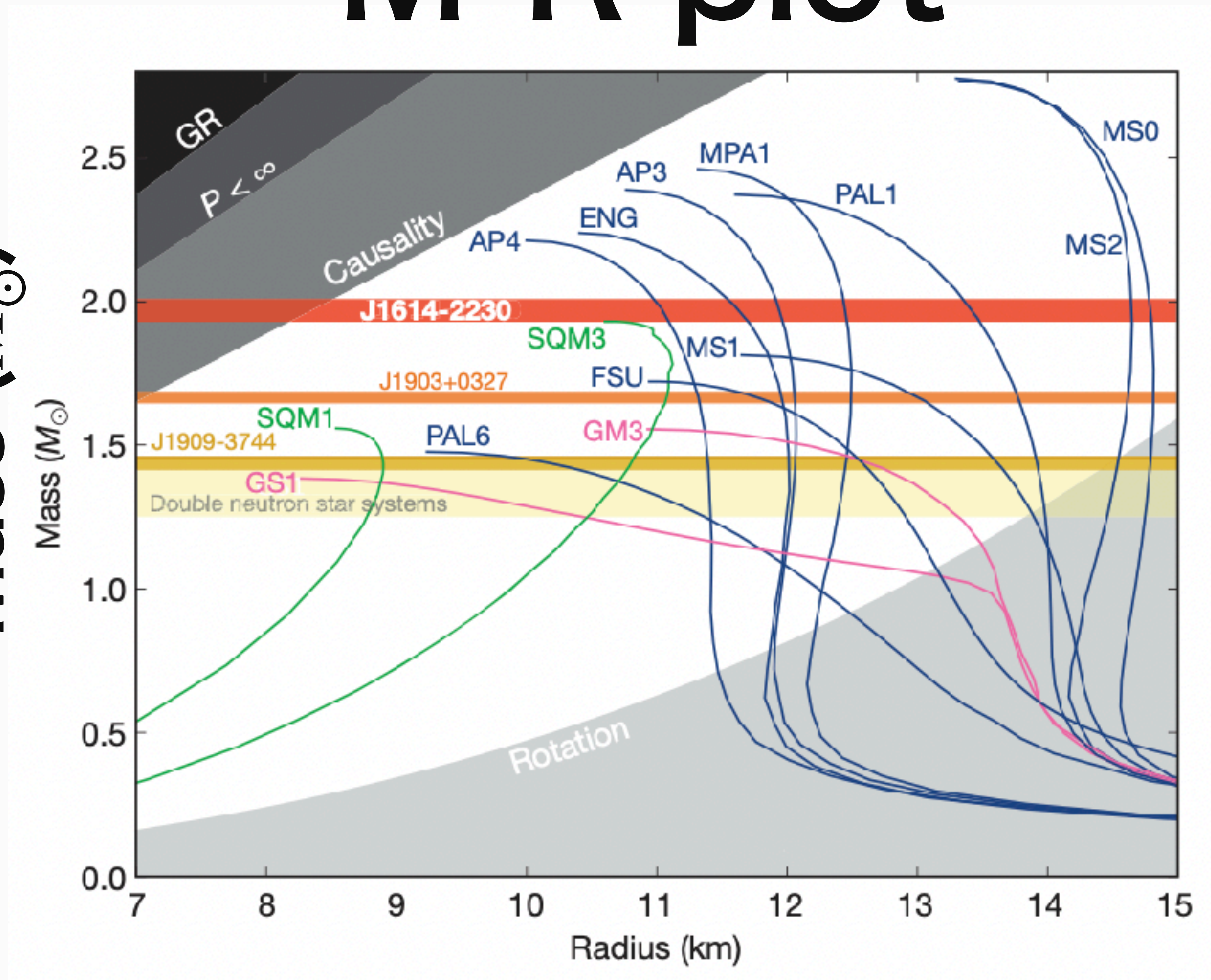
03. Energy conservation

observations ←

→ Calculations

M-R plot

Mass ($M_\odot$)



Radius (km)

Which model is better?

Holographic QCD

01 Finite density

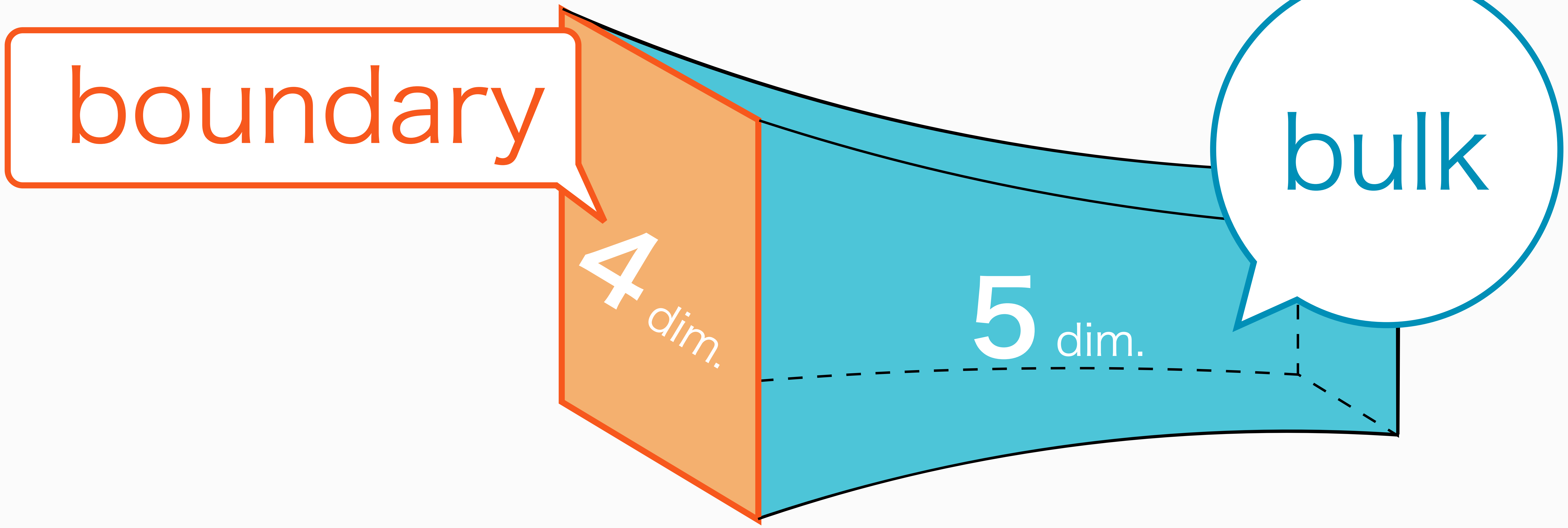
02 Strong coupling

03 Chiral transition

Holographic QCD

Note: Large N_c limit, $\lambda = g^2 N_c$ is fixed

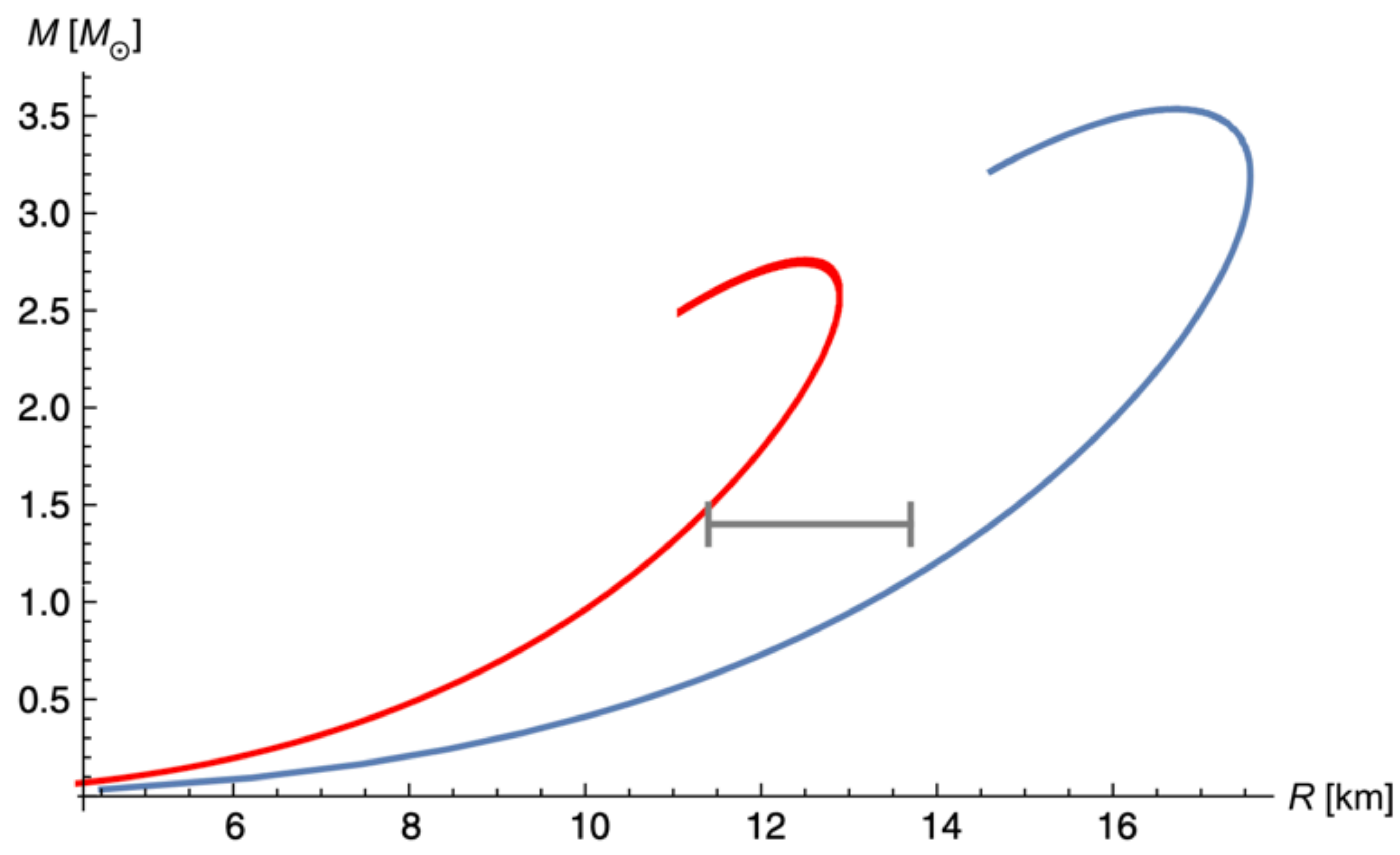
4 dim. QCD Strong	⟷	5 dim Gravity Weak



Previous study

Hard-wall model

Lorenzo Bartolini, et al., Phys. Rev. D **105**, 126014 (2022)



Massless quarks

▼

Introducing the quark mass

An advantage of bottom-up models,
it is hard for the D4-D8 model

Method

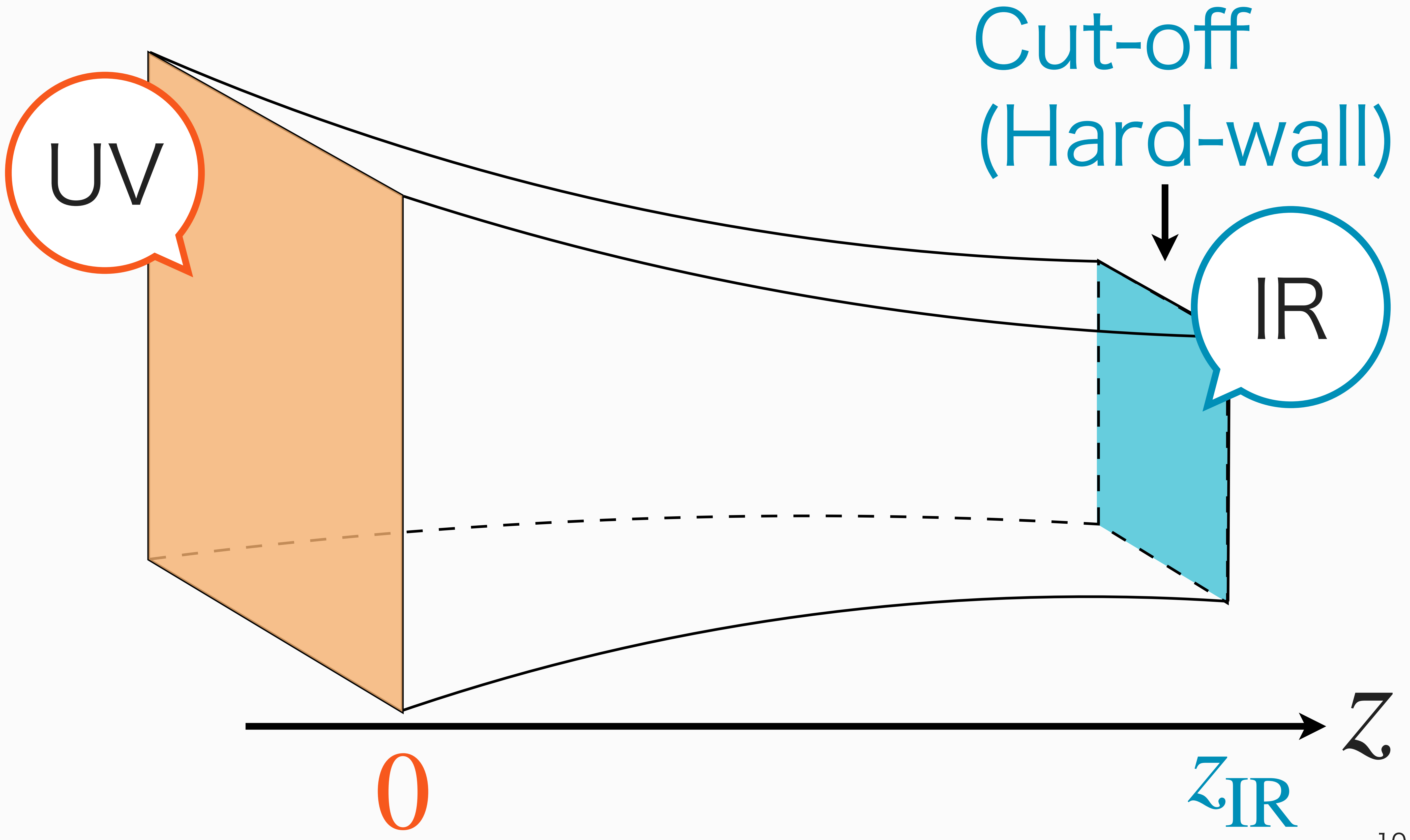
Hard-wall model

J. Erlich, et al., PRL **95**, 261602 (2005)

Cut-off AdS

(Confined phase)

$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right), \quad 0 \le z \le z_{\rm IR}$$



Procedure

J. Erlich, et al., PRL **95**, 261602 (2005)

Cut-off AdS
(Confined phase)

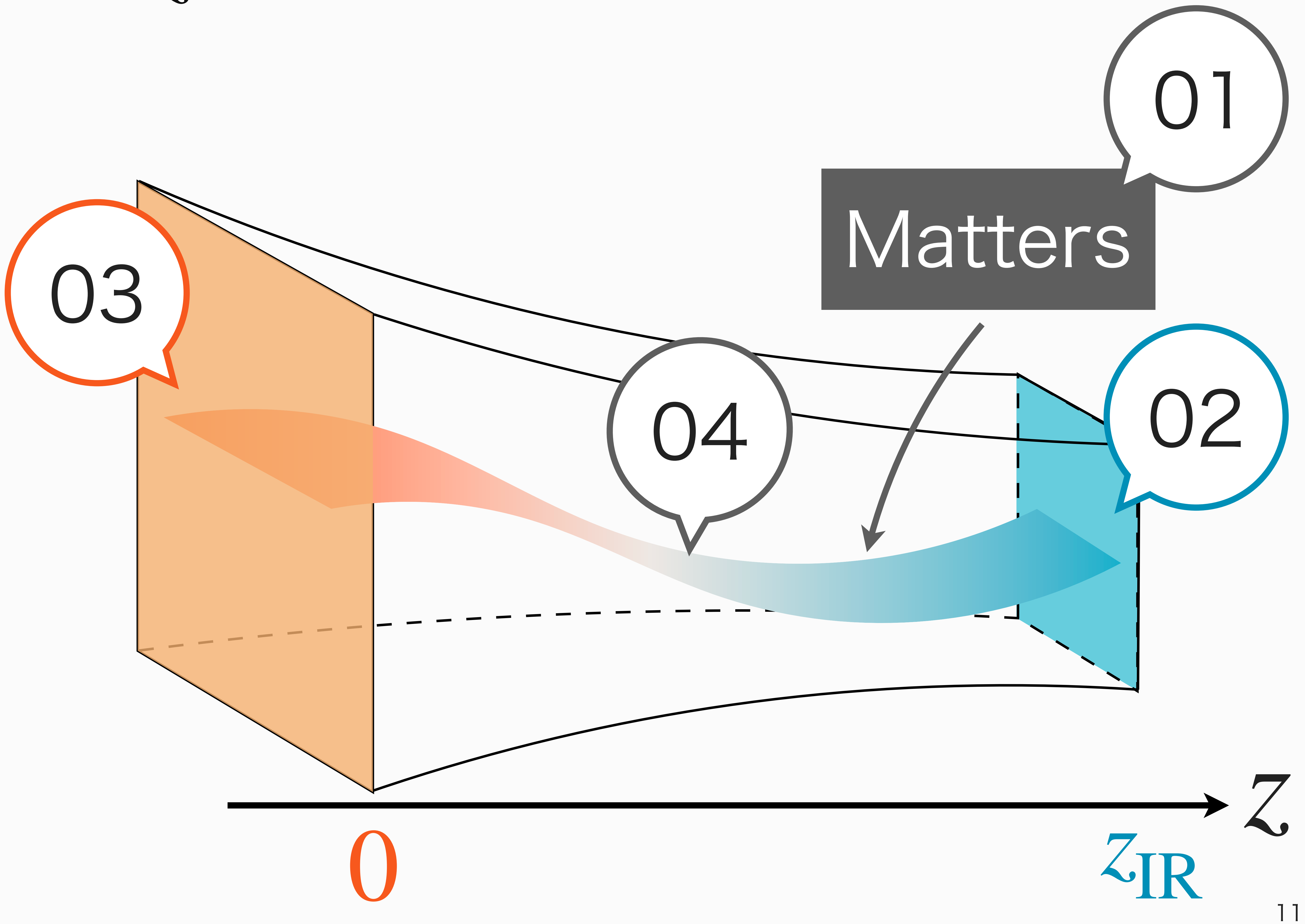
$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right), \quad 0 \le z \le z_{\rm IR}$$

01 Add matters

02 IR b.c.

03 UV b.c.

04 Solving EoM



01. Action of matters

$U(2)$ Flavor gauge fields (Left&Right)

Bi-fundamental Scalar fields

$$S = S_{\rm g} + S_{\rm CS} + S_{\Phi}$$

Chern-Simons term

$$S_{\rm g} = -\frac{N_c}{12\pi^2}\int d^4xdz\sqrt{-g}\left[\frac{1}{2}{\rm Tr}(L_{MN}L^{MN}) + \frac{1}{4}\hat{L}_{MN}\hat{L}^{MN} + \{R \leftrightarrow L\}\right],$$

$$S_{\rm CS} = \frac{N_c}{16\pi^2}\int d^4xdz\,\epsilon_{MNPQR}\left[\frac{1}{4}\hat{L}_M\left({\rm Tr}[L_{NP}L_{QR}] + \frac{1}{6}\hat{L}_{NP}\hat{L}_{QR}\right) - \{R \leftrightarrow L\}\right],$$

$$S_{\Phi} = \frac{N_c}{12\pi^2}\int d^4xdz\sqrt{-g}\left\{{\rm Tr}\left[(D_M\Phi)^\dagger D^M\Phi\right] + 3{\rm Tr}[\Phi^\dagger\Phi]\right\}.$$

L_M, R_M :$SU(2)$ gauge field

$\hat{L}_M$, $\hat{R}_M$:$U(1)$ gauge field

Φ :scalar field

$M, N, \cdots = 0,1,2,3,z$

$$L_{MN} = \partial_M L_N - \partial_N L_M - i[L_M, L_N],$$

$$L^a_{MN} = \partial_M L^a_N - \partial_N L^a_M + f^{abc}L^b_M L^c_N,$$

$$D_M\Phi = \partial_M\Phi - i\mathscr{L}_M\Phi + i\Phi\mathscr{R}_M,$$

$$\mathscr{L}_M = L^a_M\frac{\tau^a}{2} + \hat{L}_M\frac{I_2}{2}$$

(τ^a :Pauli matrix, $a = 1,2,3$),

$N_c = 3$, $L = 1$.

$$\mathscr{L}_z = \mathscr{R}_z = 0 \quad \text{(gauge fixing)}$$

Ansatz

Homogeneous Ansatz

"Mean-field approximation"

$\Phi = \omega_0(z)\frac{I_2}{2}$	$\longleftrightarrow$	Current quark mass Chiral condensate
$\mathscr{L}_0 = -\mathscr{R}_0 = \hat{a}_0(z)\frac{I_2}{2}$	$\longleftrightarrow$	Baryon chemical potential Baryon number density
$\mathscr{L}_i = -\mathscr{R}_i = -H(z)\frac{\tau^i}{2}$	$\longleftrightarrow$	Axial vector potential Axial vector meson condensate

Action with Ansatz

$U(2)$ Flavor gauge fields (Left&Right)

Bi-fundamental Scalar fields

$$S = S_{\rm g} + S_{\rm CS} + S_{\Phi}$$

Chern-Simons term

$$S_{\rm g} = -\frac{N_c}{12\pi^2}\int d^4x dz\, \frac{1}{z}\left\{3H^4(z) + 3[\,\partial_z H(z)\,]^2 - [\,\partial_z \hat{a}_0(z)\,]^2\right\},$$

$$S_{\rm CS} = \frac{3N_c}{8\pi^2}\int d^4x dz\, \hat{a}_0(z)\, H^2(z)\, \partial_z H(z),$$

$$S_{\Phi} = -\frac{N_c}{12\pi^2}\int d^4x dz\, \frac{1}{z^3}\left\{\frac{3}{2}H^2(z)\omega_0^2(z) + \frac{1}{2}[\,\partial_z \omega_0\,]^2 - \frac{3}{2}\frac{1}{z^2}\omega_0^2(z)\right\}$$

More two terms

Potential on the hard-wall

Counterterm

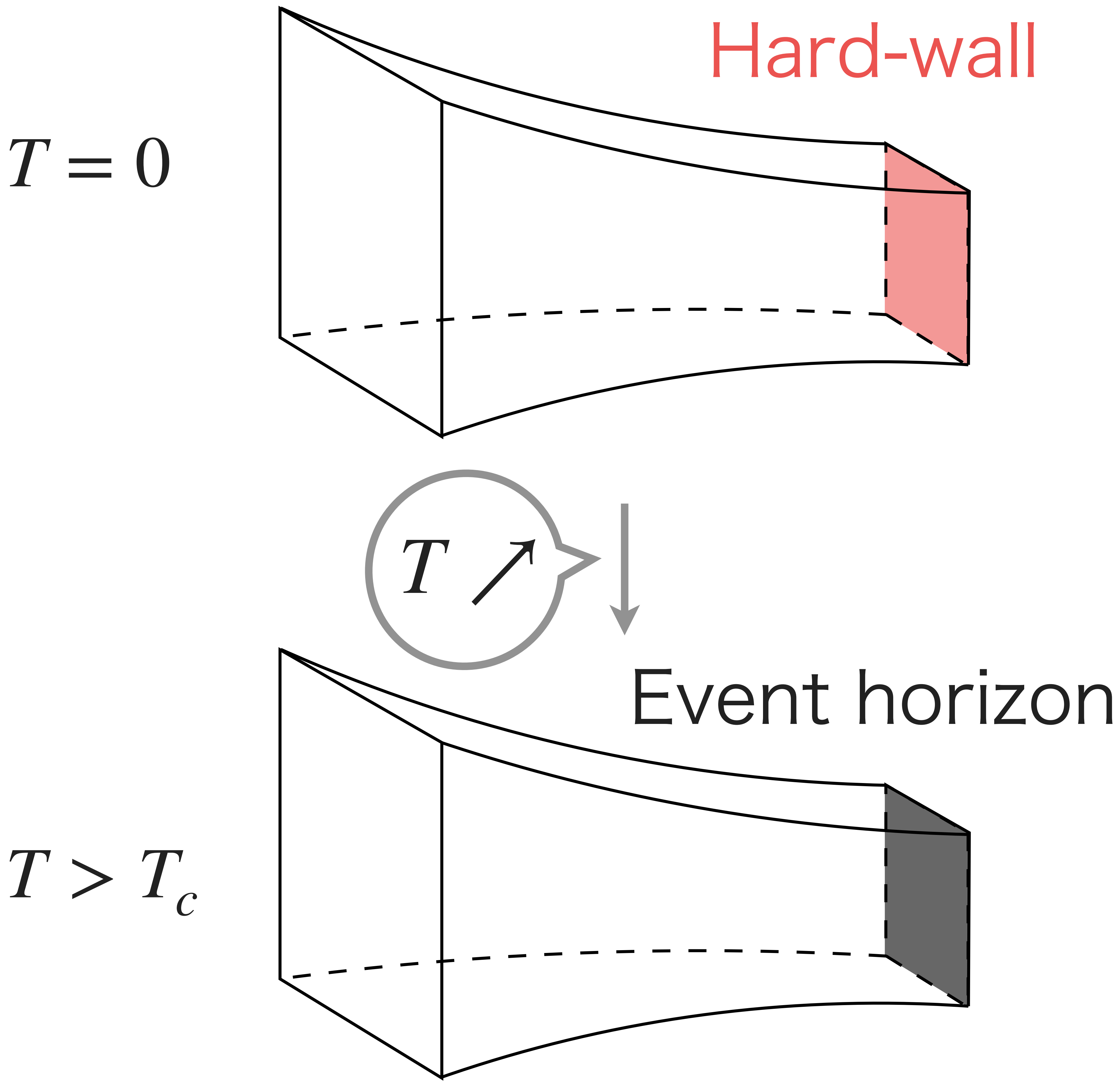
$$S = S_{\mathrm{g}} + S_{\mathrm{CS}} + S_{\Phi} + S_{\mathrm{IR}} + S_{\mathrm{c}}$$

$$S_{\mathrm{IR}} = -\int_{z=z_{\mathrm{IR}}} d^4x \left[\frac{k_2}{2} H^4(z) + \frac{m_b^2}{2z^4}\, \omega_0^2(z) \right],$$

$$S_c = -\int_{z=\epsilon} d^4x \, \frac{N_c}{12\pi^2} \left(\frac{1}{2\epsilon^4} \omega_0^2(\epsilon) + \frac{3\log\epsilon}{2\epsilon^2} \omega_0^2(\epsilon)\, H^2(\epsilon) + 3\log\epsilon\, H^4(\epsilon) \right)$$

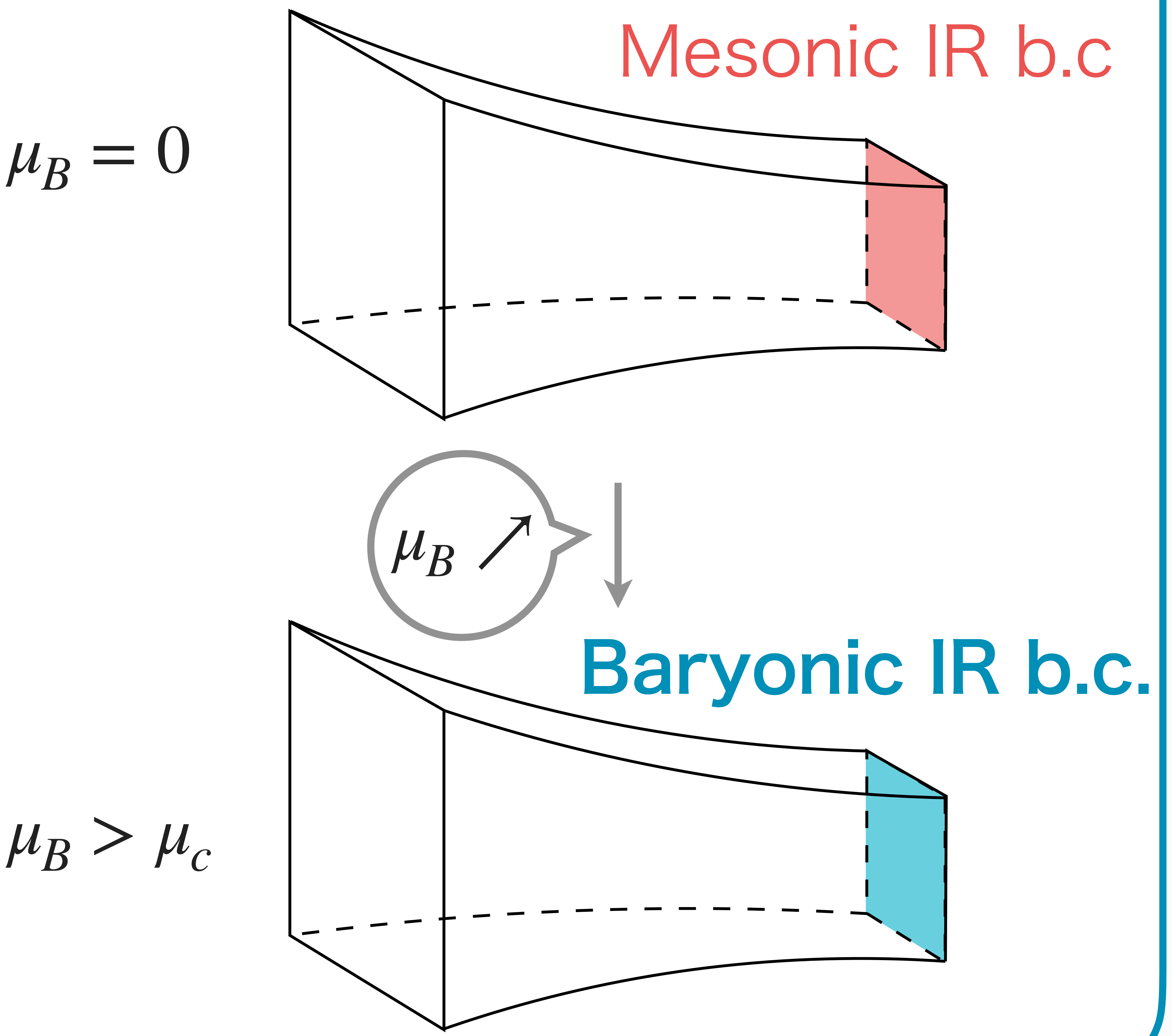
Our proposal

finite **Temperature**



New!

finite **Density**

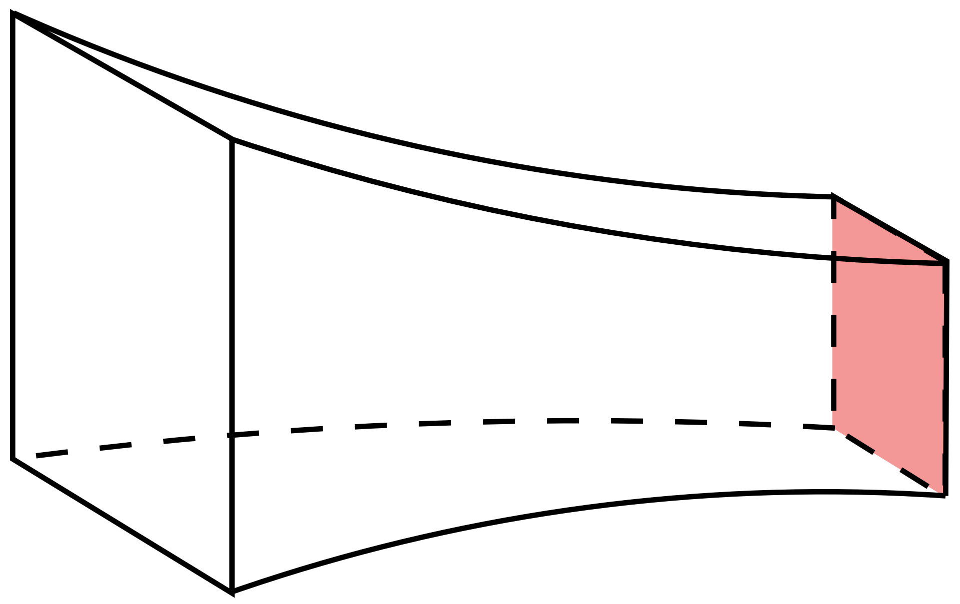


02. IR b.c.

Note: $z_{\mathrm{IR}} = 1$

Mesonic IR b.c. $(z = z_{\mathrm{IR}})$

$\mu_B < \mu_c$

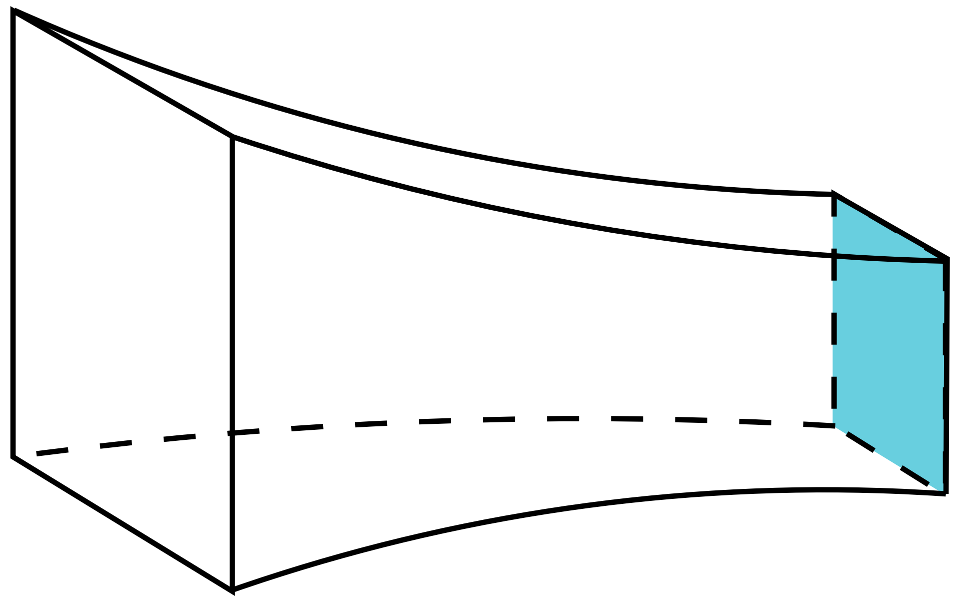


b.c. Neumann

$$\partial_z \omega_0(z_{\mathrm{IR}}) = -\frac{12\pi^2}{N_c}\left(3kH^2\omega_0 + m_b^2\omega_0 + \frac{\lambda}{4}\omega_0^3\right),$$
$$\partial_z \hat{a}_0(z_{\mathrm{IR}}) = 0,$$
$$\partial_z H(z_{\mathrm{IR}}) = 0.$$

Baryonic IR b.c. $(z = z_{\mathrm{IR}})$

$\mu_B \geq \mu_c$



b.c. Neumann + Dirichlet

$$\partial_z \omega_0(z_{\mathrm{IR}}) = -\frac{12\pi^2}{N_c}\left(3kH^2\omega_0 + m_b^2\omega_0 + \frac{\lambda}{4}\omega_0^3\right),$$
$$\hat{a}_0(z_{\mathrm{IR}}) = A = 4,$$
$$H(z_{\mathrm{IR}}) = B\,.$$

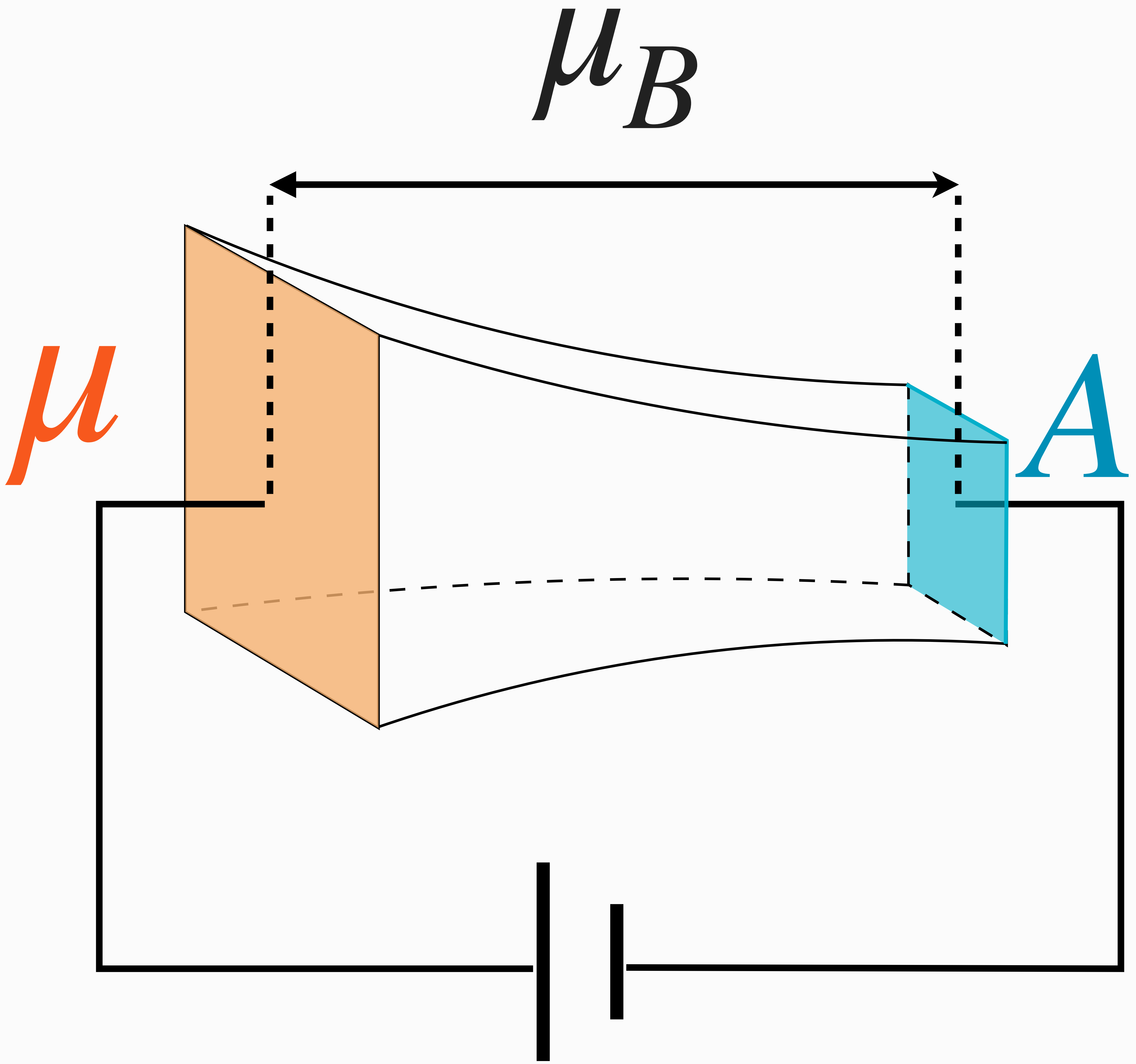
03. UV b.c.

Values of bulk fields on the UV boundary $\longleftrightarrow$ External fields in dual QFT

UV b.c. $(z = 0)$

$\partial_z \omega_0(0) = m = 3\ \mathrm{MeV}$	$\longleftrightarrow$	Current quark mass
$\hat{a}_0(0) = \mu$	$\longleftrightarrow$	Baryon chemical potential
$H(0) = \phi = B$	$\longleftrightarrow$	Axial vector potential

Gauge independent external fields



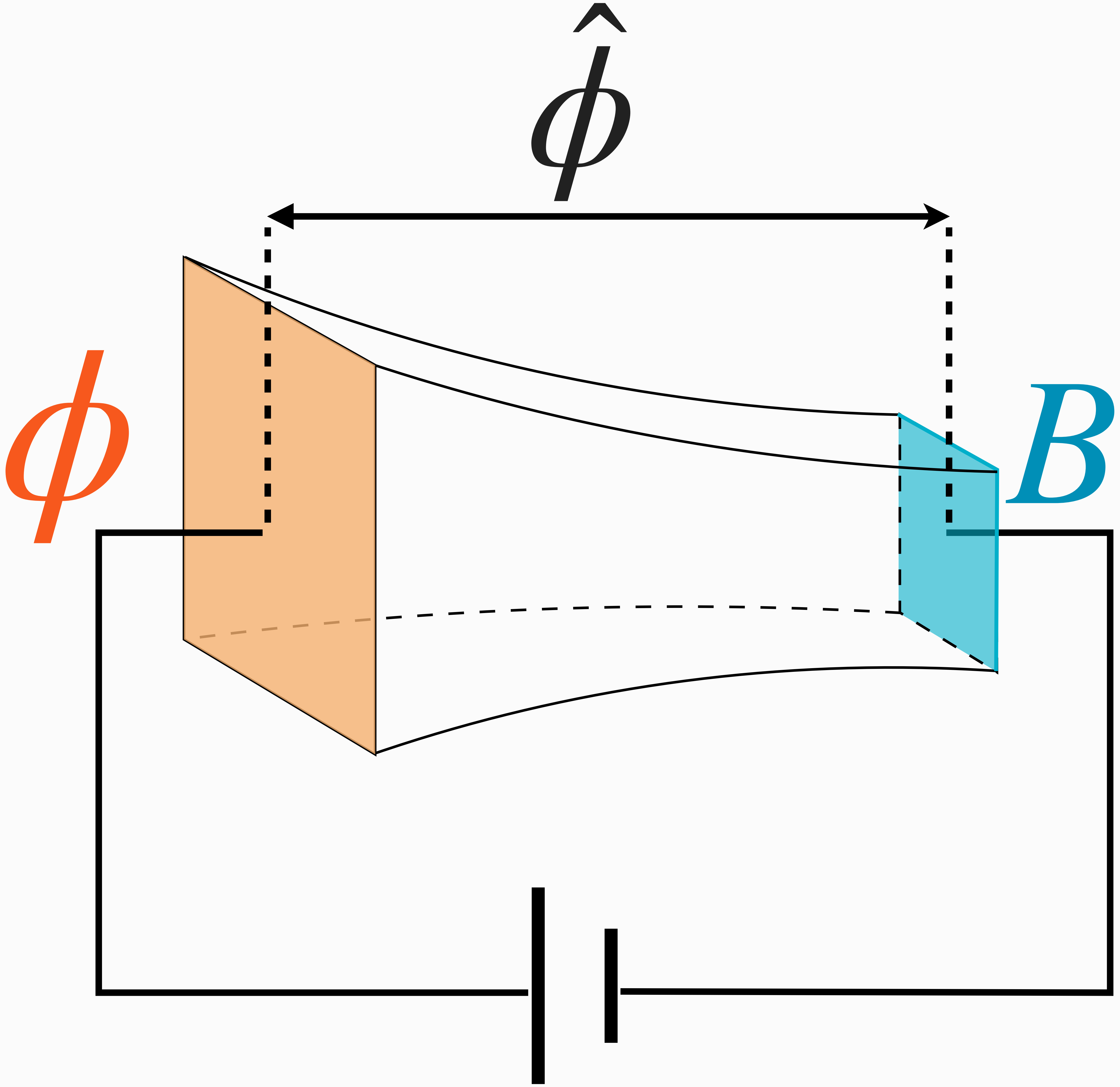
The physical chemical potential

$$\mu_B$$

Gauge independence gives

$$\mu_B \propto \mu - A = \int_0^{z_{\mathrm{IR}}} dz\, \hat{F}_{0z}$$

Gauge independent external fields



The physical axial vector potential

$$\hat{\phi}$$

Hedgehog structure gives

$$\hat{\phi}\, T^i \propto (\phi - B) T^i = \int_0^{z_{\mathrm{IR}}} dz\, F_{iz}$$

To exclude the axial external field

$$\phi = B$$

GKP-W method

S. S. Gubser, et al., Phys. Lett. B 428 (1998) 105–114
E. Witten, Adv. Theor. Math. Phys. 2 (1998)

Gradients of bulk fields near the UV boundary ⟷ **Expectation values of observables**

$a_2,\ h_2,\ w_2$

Gubser-Klebanov-Polyakov-Witten method

$$\xi = \frac{N_c}{4\pi^2} w_2 + \frac{N_c}{8\pi^2} m\phi^2$$ ⟷ Chiral condensate

$$d_B = -\frac{2}{3\pi^2} a_2$$ ⟷ Baryon number density

$$J = \frac{N_c}{\pi^2} h_2 + \frac{N_c}{2\pi^2}\phi\left(\frac{m^2}{4} + \phi^2\right) - \frac{3N_c}{8\pi^2}\mu\phi^2$$ ⟷ Axial vector meson condensate

Parameters

01 Meson mass spectrum

▼

AdS radius
&
place of the hard-wall

$$L^{-1} = z_{\rm IR}^{-1} = 323\ {\rm MeV}$$

02 Lattice result
of chiral condensate

H. Fukaya, et al., PRL **98**, 172001 (2007)

▼

Chiral condensate
in the mesonic phase

$$\xi_0 = (251\ {\rm MeV})^3$$

03 Critical chemical potential

▼

Parameter k_2 in $S_{\rm IR}$ & IR b.c. $H(z_{\rm IR}) = B$

$$(k_2,\ B) = (15,\ 0.4/L)$$

Results

Grand potential density Ω/V

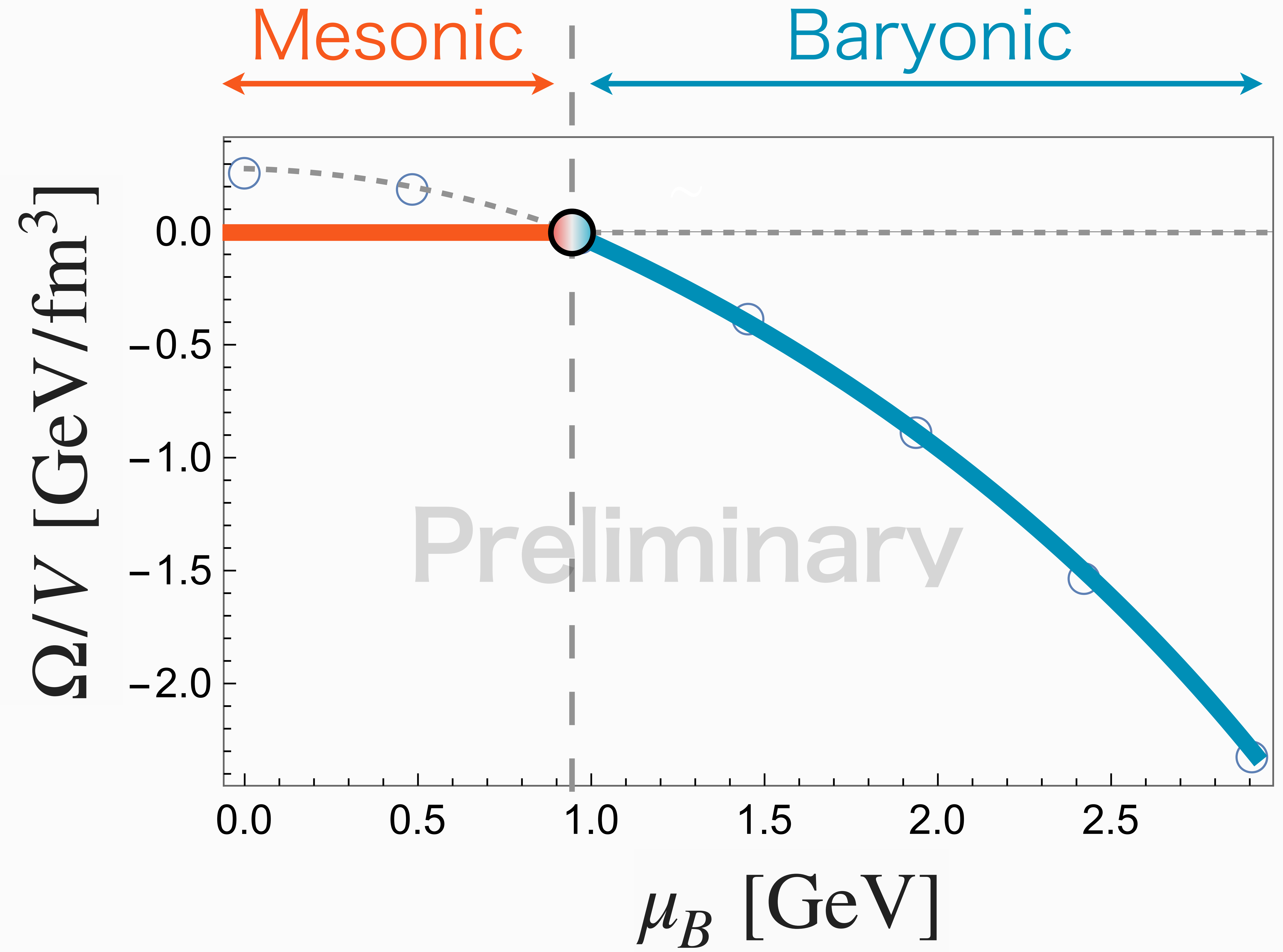
Two transitions

- Chirality
- Baryon number density

All transitions are

1 st

phase transitions



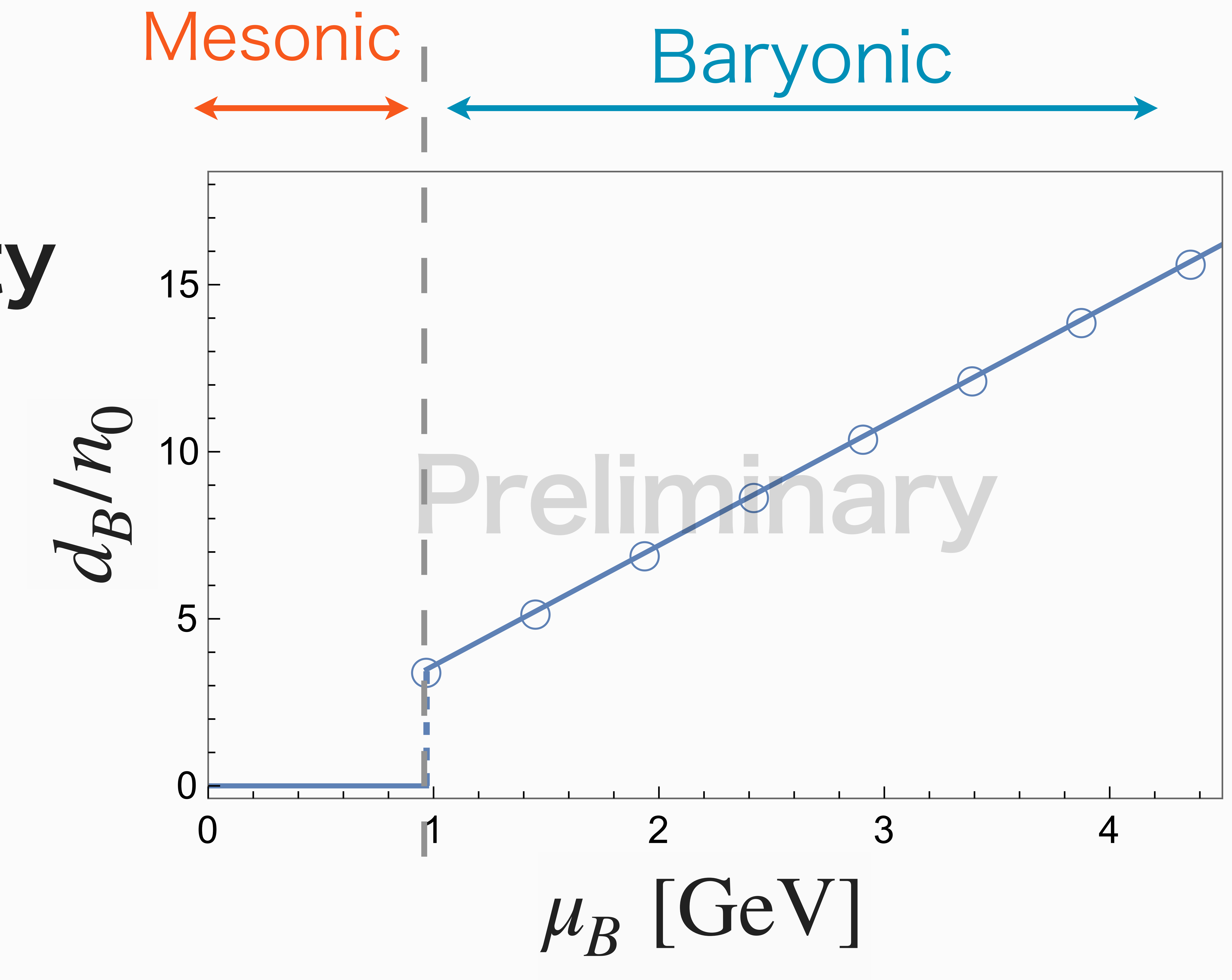
Baryon number denstiy n_B

Critical baryon density

$$n_c \sim 3.5 n_0$$

$$(n_0 = 0.17\ \mathrm{fm}^{-3})$$

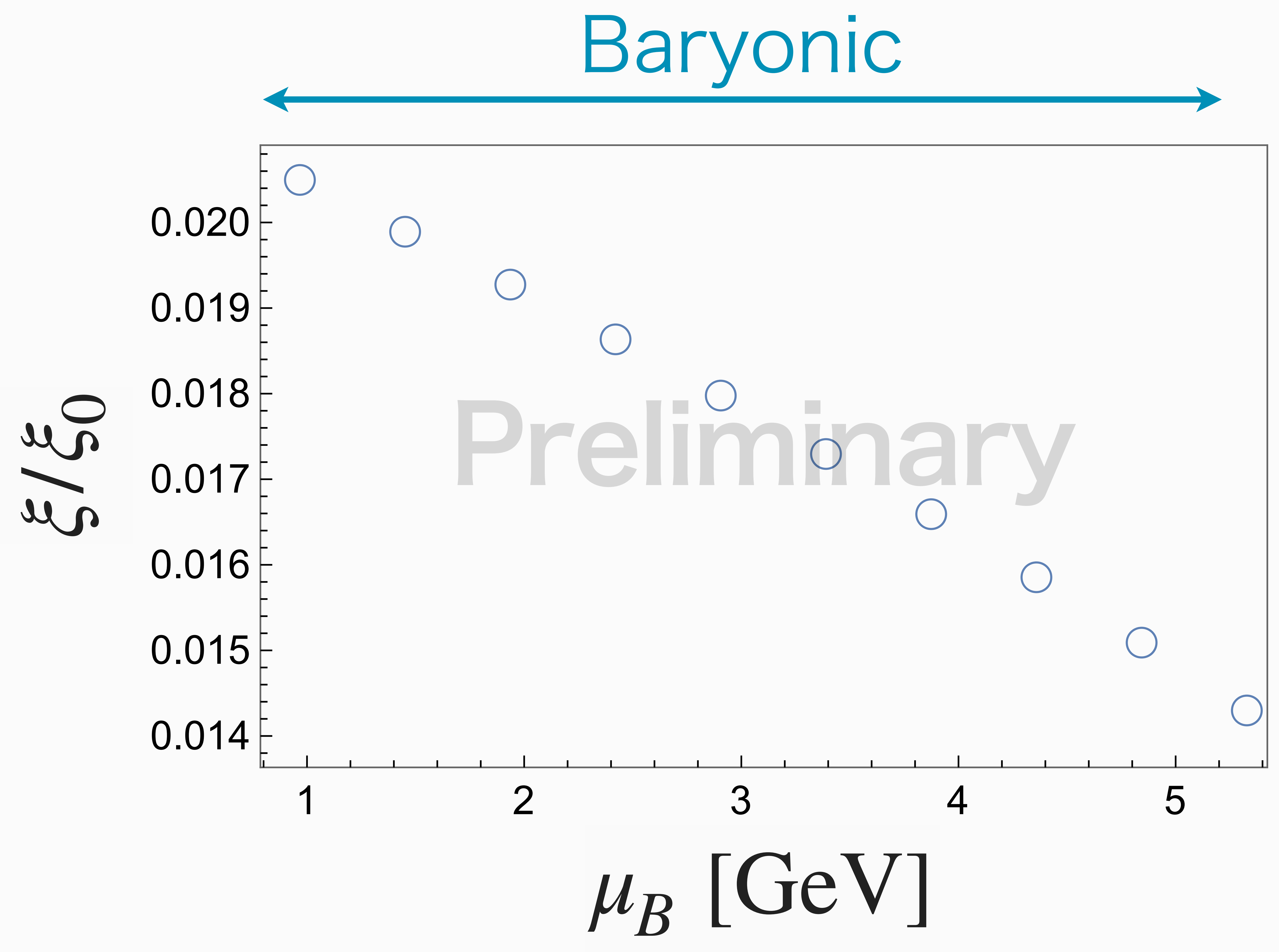
Increasing linearly



Chiral condensate ξ

Chiral symmetry
is almost restored

Decreasing function

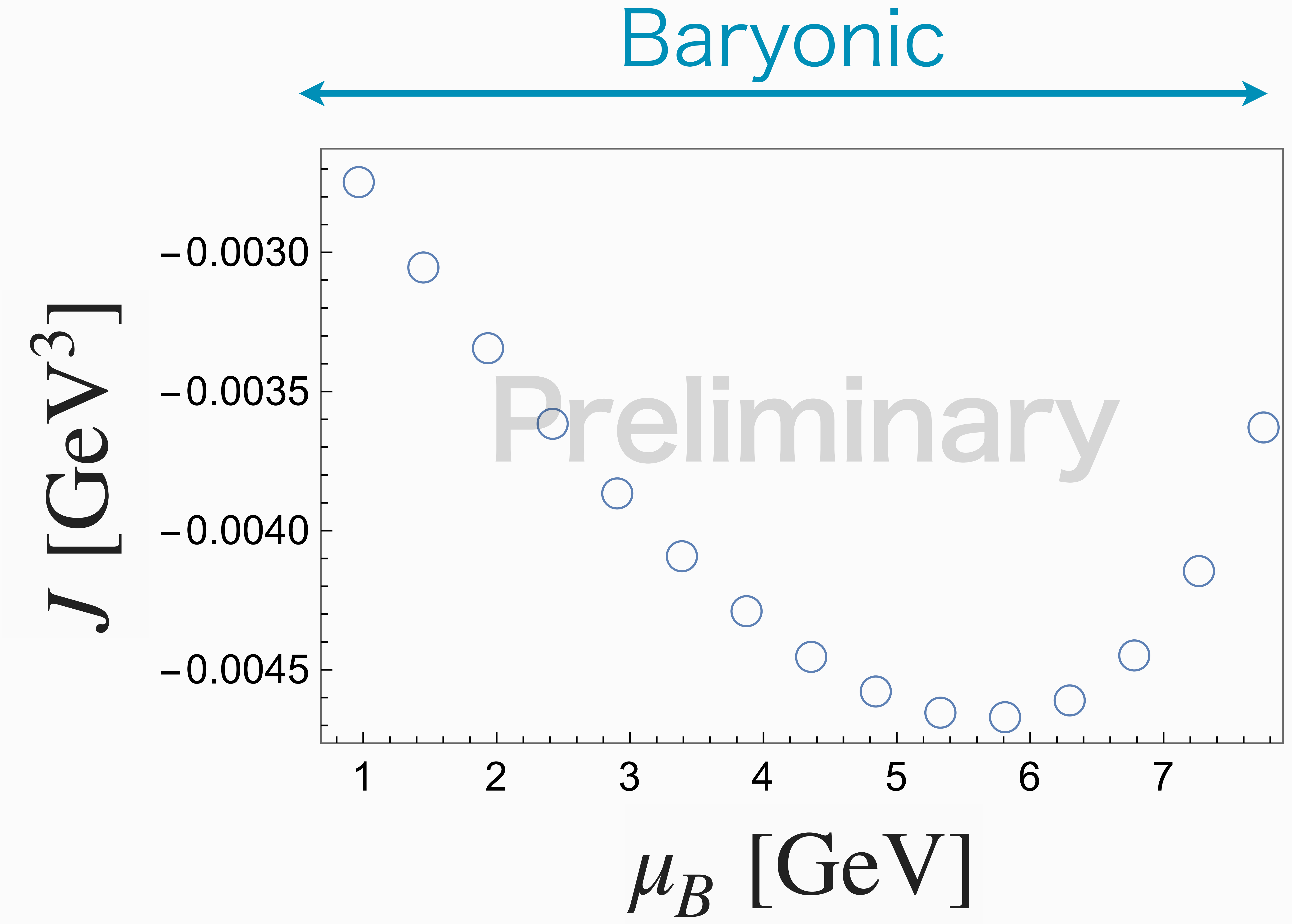


Axial vector condensate J

Axial vector mesons condense

It has a lower bound

at $\mu_B \sim 6$ GeV



Equation of state

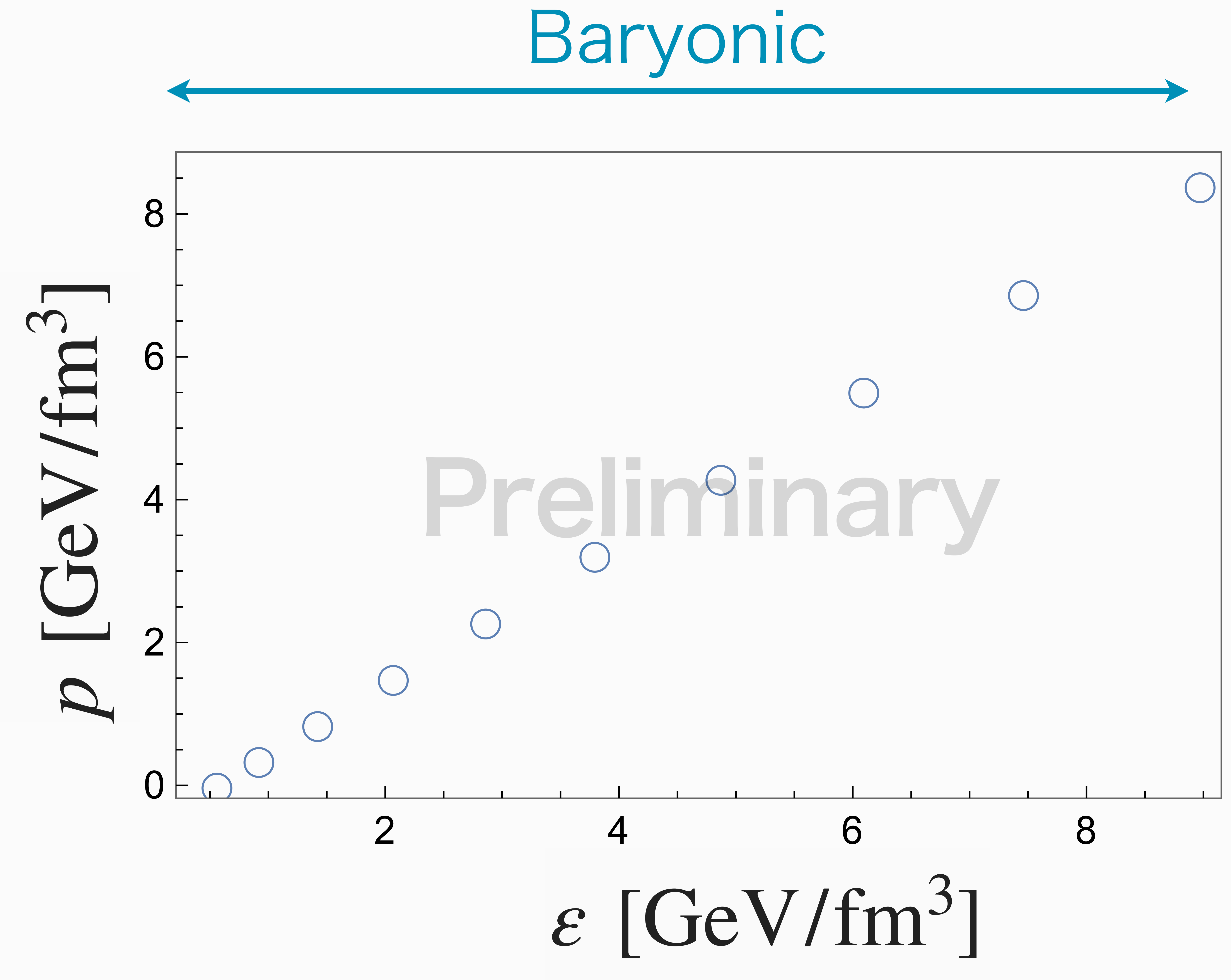
Almost linear

with gradient $\simeq 1$

▼

Speed of sound

$$c_s^2 = \frac{dp(\varepsilon)}{d\varepsilon} \simeq 1$$



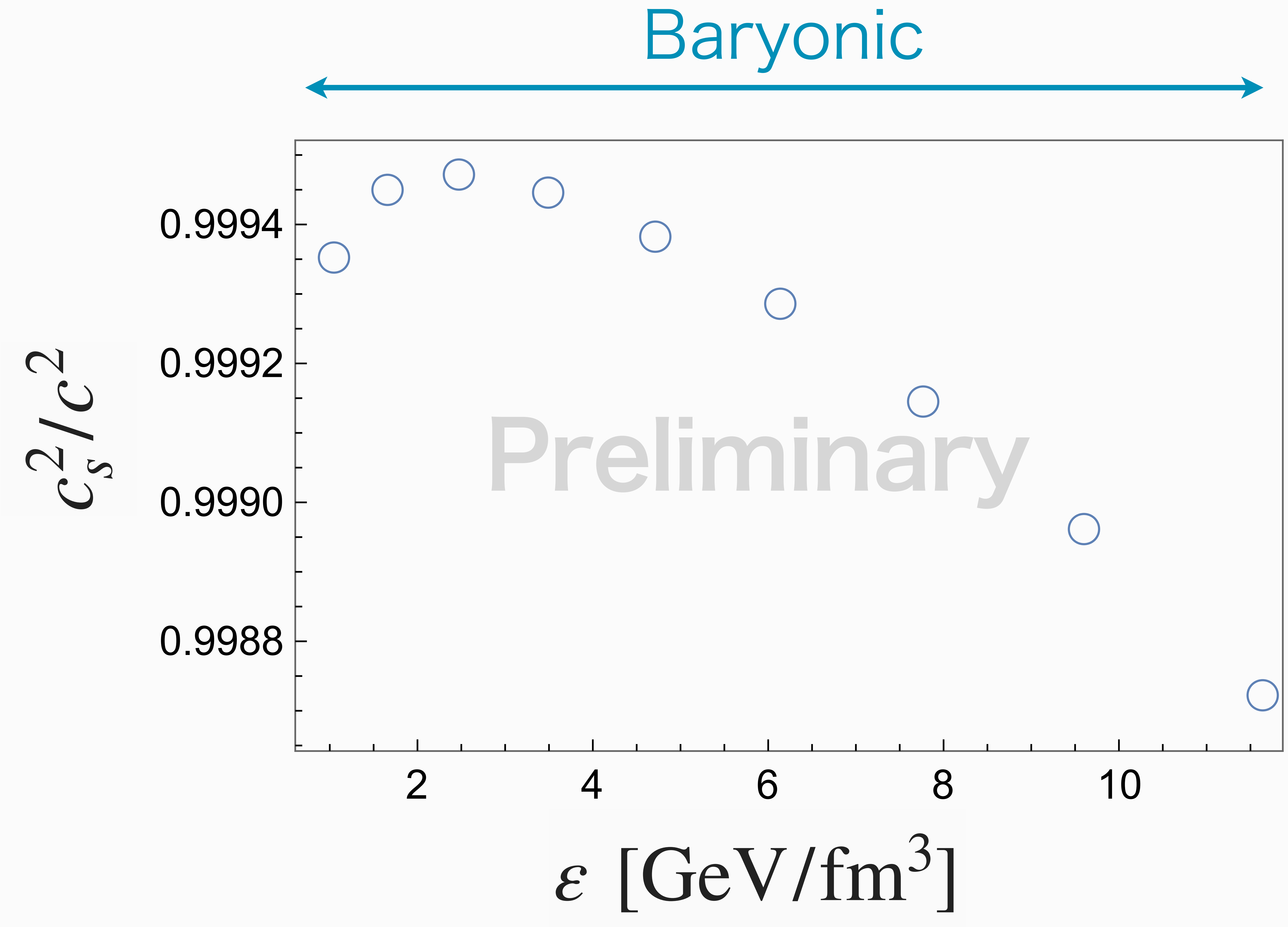
Speed of sound

Speed of sound

$$c_s^2 = \frac{dp(\varepsilon)}{d\varepsilon} \simeq 1$$

It has a peak

at $\mu_B \sim 2 \text{ GeV}$

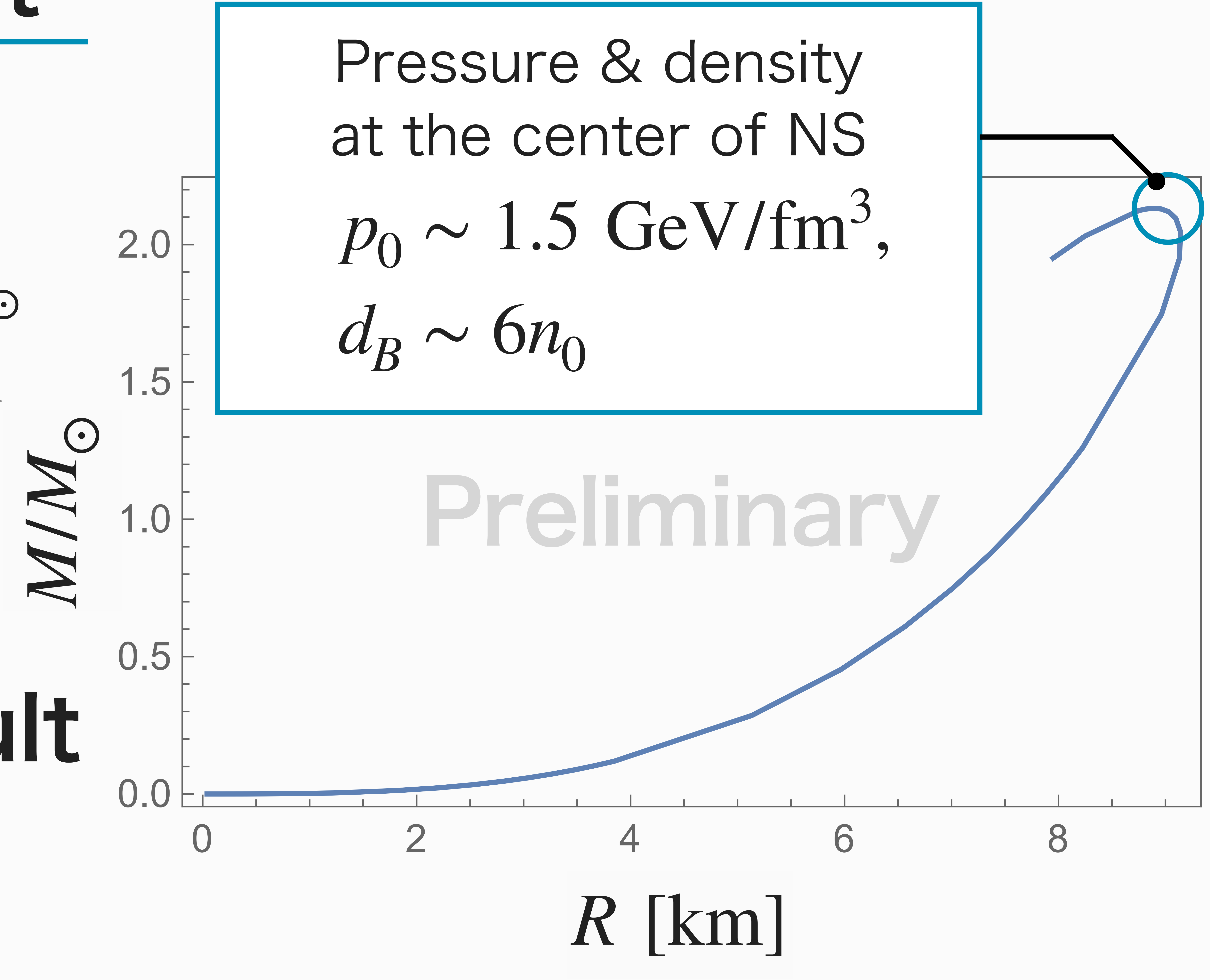


Mass-Radius plot

Maximal mass $\sim 2.2\,M_\odot$

Maximal radius $\sim 9\,\mathrm{km}$

Acceptable result



Summary

Pourpose

- Studying the QCD EoS from holographic QCD

Method

- Hard-wall model + Switching IR b.c.

Result

- Baryonic matter appears with first transition
- Acceptable M-R curve is obtained

Outlook

- Introducing strange quark

Fin.