Study on hadron-hadron interaction with femtoscopic technique ~Interaction of hyperons and Kaon~

東北大学

Yuki Kamiya

Tohoku Univ.

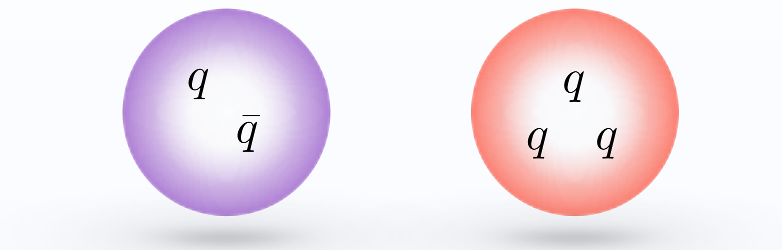
Hadrons and Hadron Interactions in QCD 2024 (HHIQCD 2024)
@ Kyoto 21, Nov., 2024

Contents

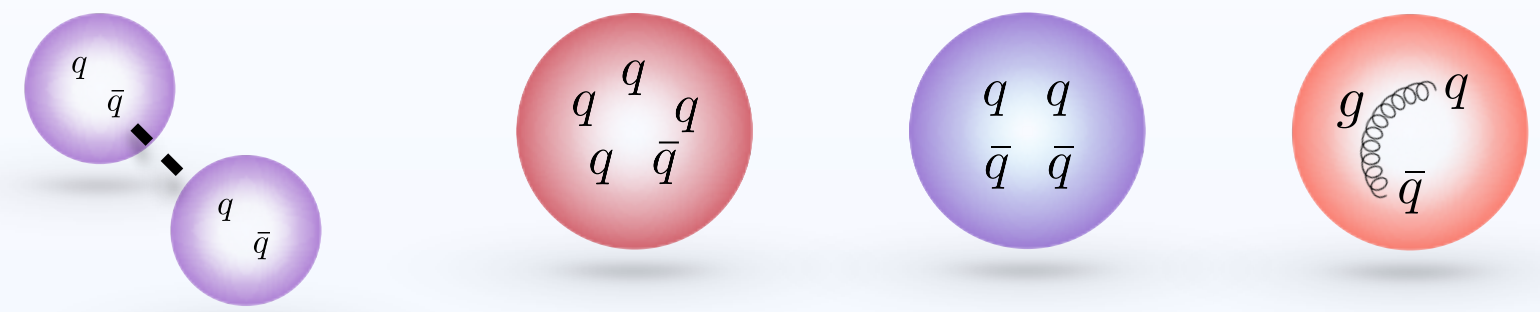
- Introduction on Femtoscopy : hadron-hadron momentum correlation function in high-energy nuclear collisions
- $\bar{K}N$ correlation function and coupled-channel effect
- $\Lambda\alpha$ and $\Xi\alpha$ correlation function
- Summary

Study of hadron resonances and interaction

- Stable hadrons
  - Described by simple quark configurations ($q\bar{q}$, qqq)



- Hadron resonances
  - Quark excitations
  - Hadron molecules
  - Exotic hadrons



Hadron molecules

Pentaquark state

Tetraquark state

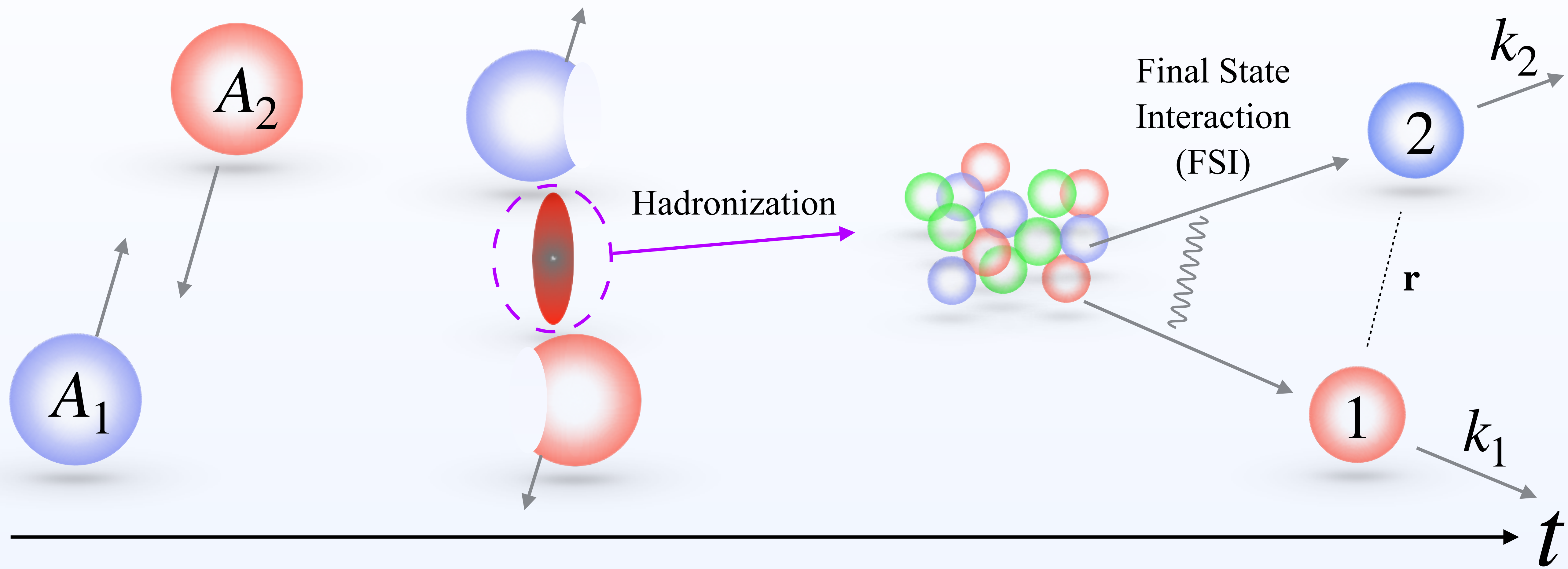
Hybrids

- Hadron resonances and hadron interaction
  - Resonances : Eigenstates
  - In scattering problem : Described as poles of scattering amplitude ($\mathscr{F}$)
  - Detailed nature : Related with the detailed structure of amplitude

<u>Understanding of resonance nature</u> ⟷ <u>Understanding of hadron interaction</u>

Femtoscopy

- High energy nuclear collision and FSI



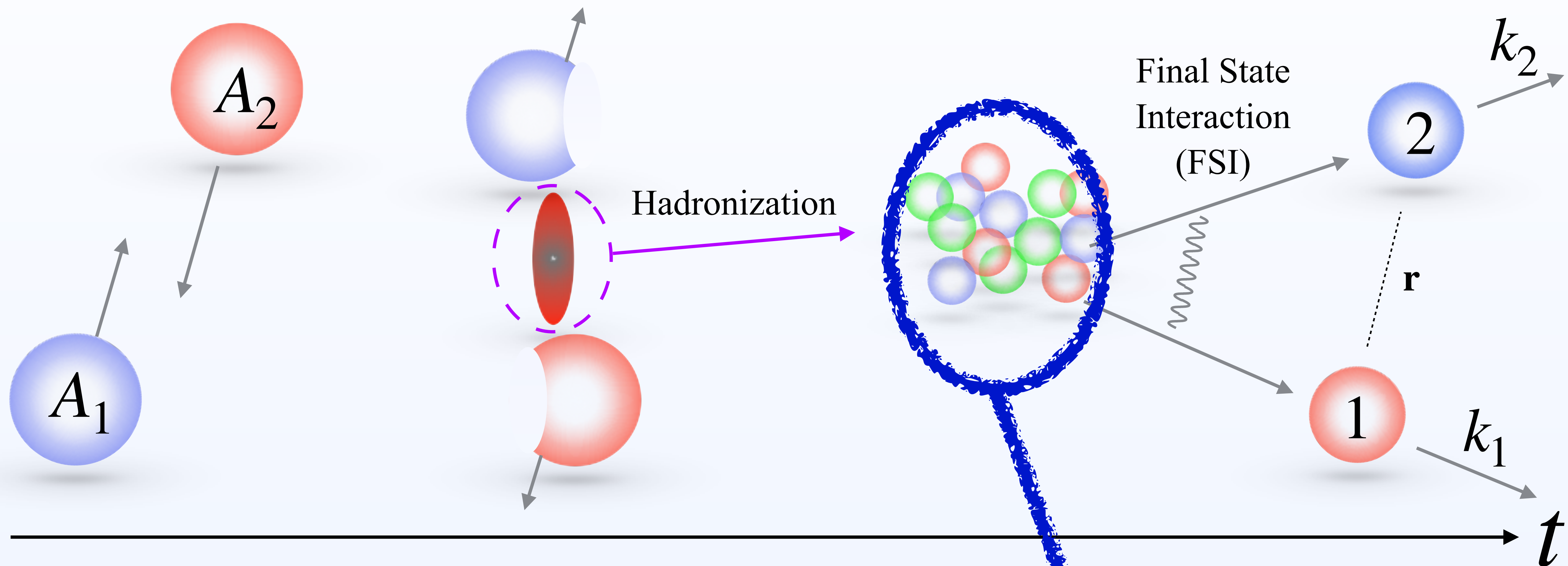
- Hadron-hadron correlation

$$C_{12}(k_1, k_2) = \frac{N_{12}(k_1, k_2)}{N_1(k_1)N_2(k_2)}$$

$$= \begin{cases} 1 & \text{(w/o correlation)} \\ \text{Others (w/ correlation)} & \end{cases}$$

Femtoscopy

- High energy nuclear collision and FSI



- Hadron-hadron correlation
  - Koonin-Pratt formula : S.E. Koonin, PLB 70 (1977) S. Pratt et. al. PRC 42 (1990)

$$C(\mathbf{q}) \simeq \int d^3\mathbf{r}\ S(\mathbf{r})\,|\,\varphi^{(-)}(\mathbf{q},\mathbf{r})\,|^2$$

$$\mathbf{q} = (m_2\mathbf{k}_1 - m_1\mathbf{k}_2)/(m_1+m_2)$$

$S(\mathbf{r})$: Source function

$\varphi^{(-)}(\mathbf{q},\mathbf{r})$: Relative wave function

- Depends on …

  Collision detail (A_i, energy, centrality)

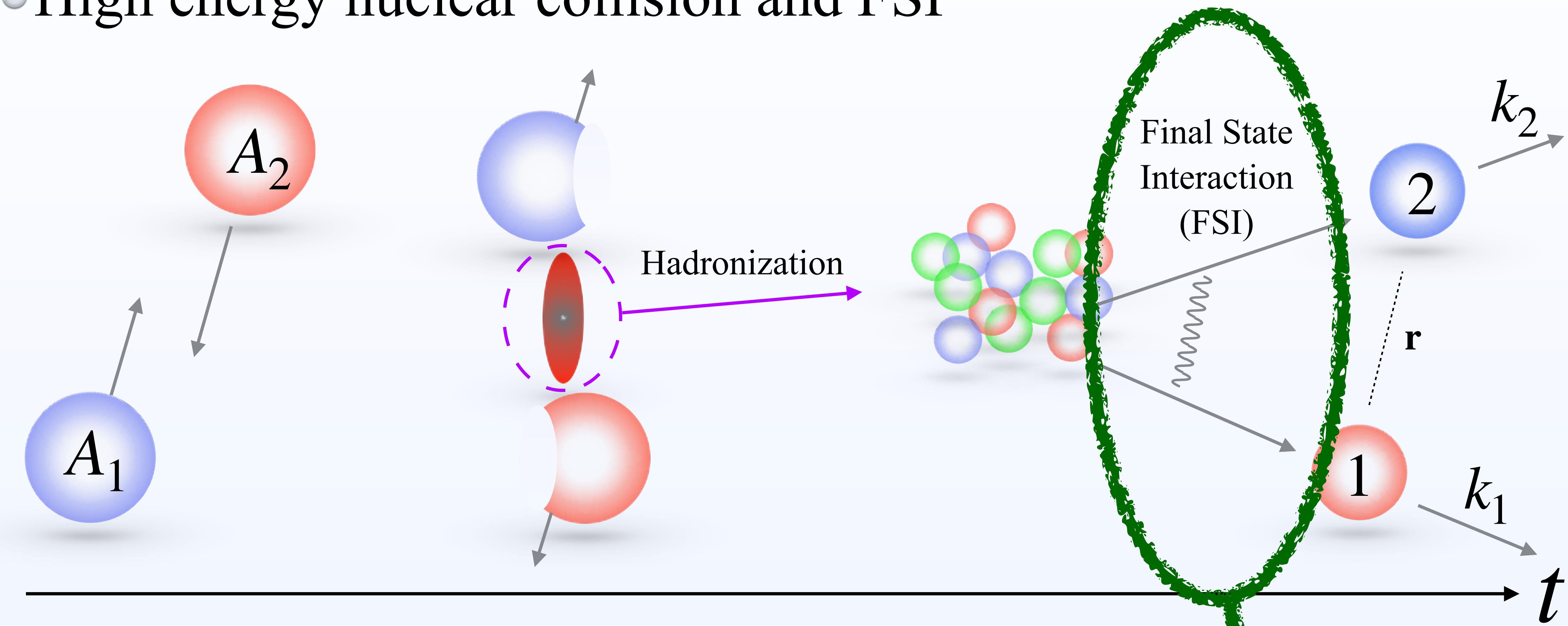
- Including information of…

  size of hadron source,

  momentum dependence, weight…

Femtoscopy

- High energy nuclear collision and FSI



- Hadron-hadron correlation
  - Koonin-Pratt formula : S.E. Koonin, PLB 70 (1977) S. Pratt et. al. PRC 42 (1990)

$$C(\mathbf{q}) \simeq \int d^3\mathbf{r}\ S(\mathbf{r})\,|\varphi^{(-)}(\mathbf{q},\mathbf{r})|^2$$

$$\mathbf{q} = (m_2\mathbf{k}_1 - m_1\mathbf{k}_2)/(m_1+m_2)$$

$S(\mathbf{r})$: Source function

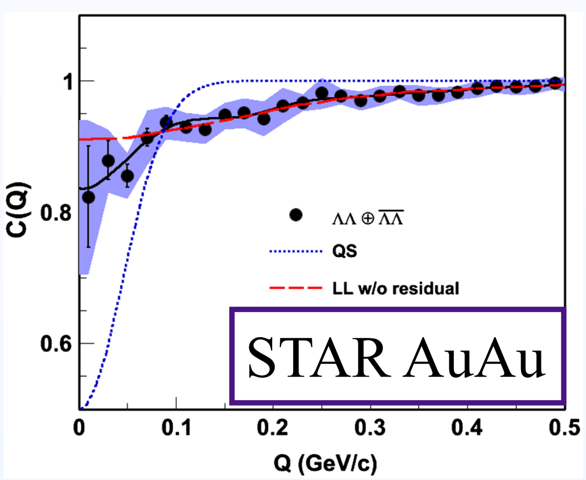
$\varphi^{(-)}(\mathbf{q},\mathbf{r})$: Relative wave function

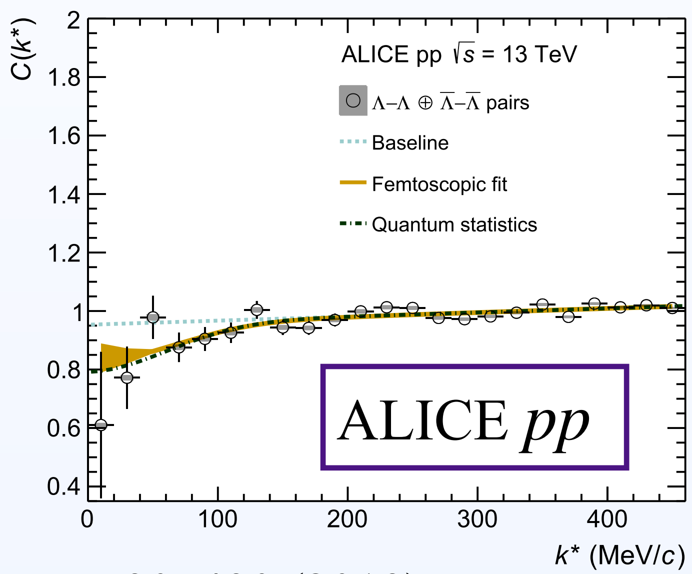
- Depends on …
  Interaction (strong and Coulomb)
  quantum statistics (Fermion, boson)

Femtoscopic data

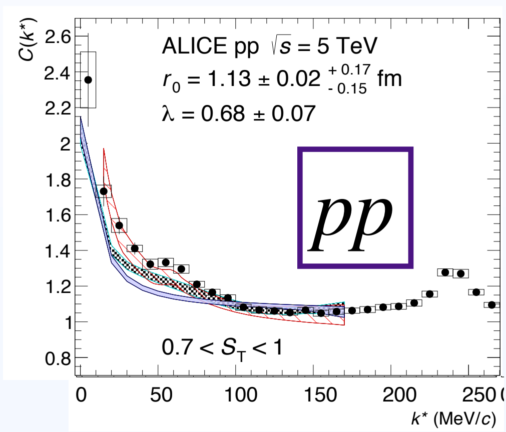
- Experimental data in various sectors

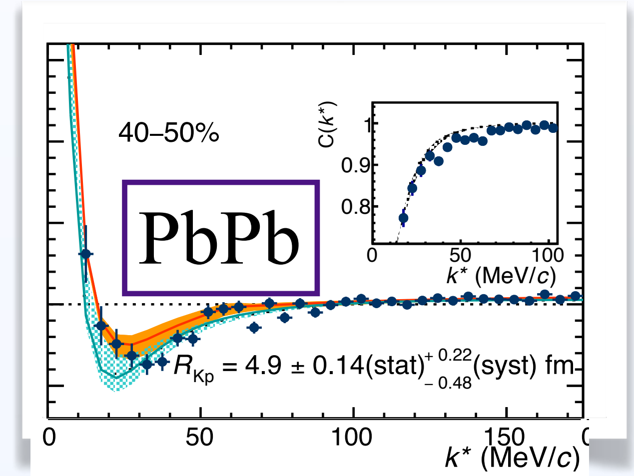
- $\Lambda\Lambda$ STAR AuAu: PRL 114,022301(2015)
  ALICE *pp*: PLB 797 (2019) 134822
  PbPb: PRC99, 024001 (2019)



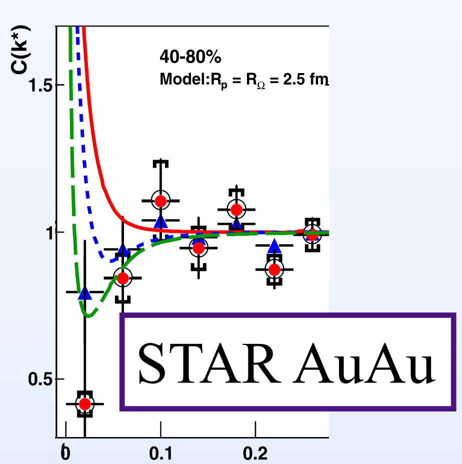


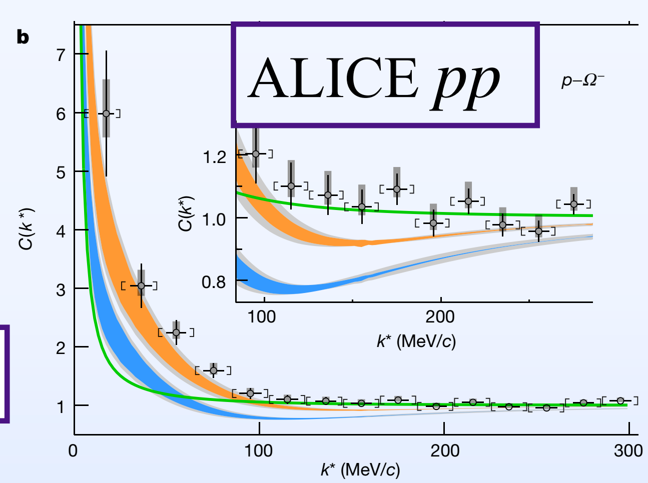
- $K^{\pm}p$ ALICE *pp*: PRL 124 (2020) 9, 092301
  PbPb: PLB 822 (2021) 136708
  STAR AuAu: NPA 982 (2019) 359



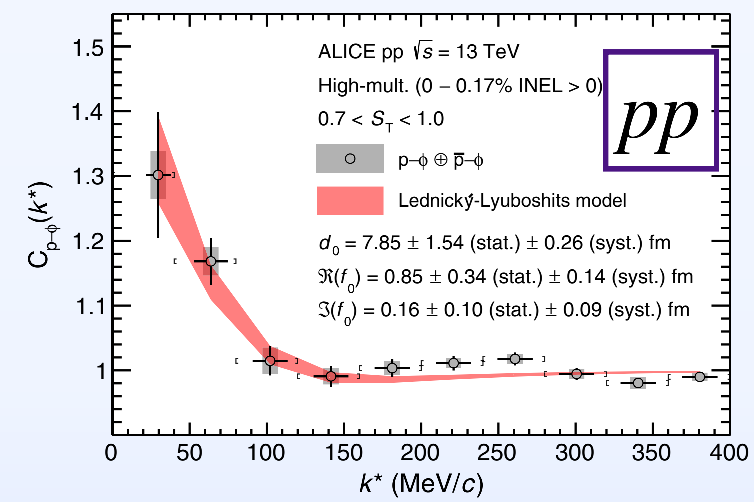


- $p\Omega$ STAR AuAu: PLB 790, 490 (2019)
  ALICE *pp*: Nature 588 (2020) 232





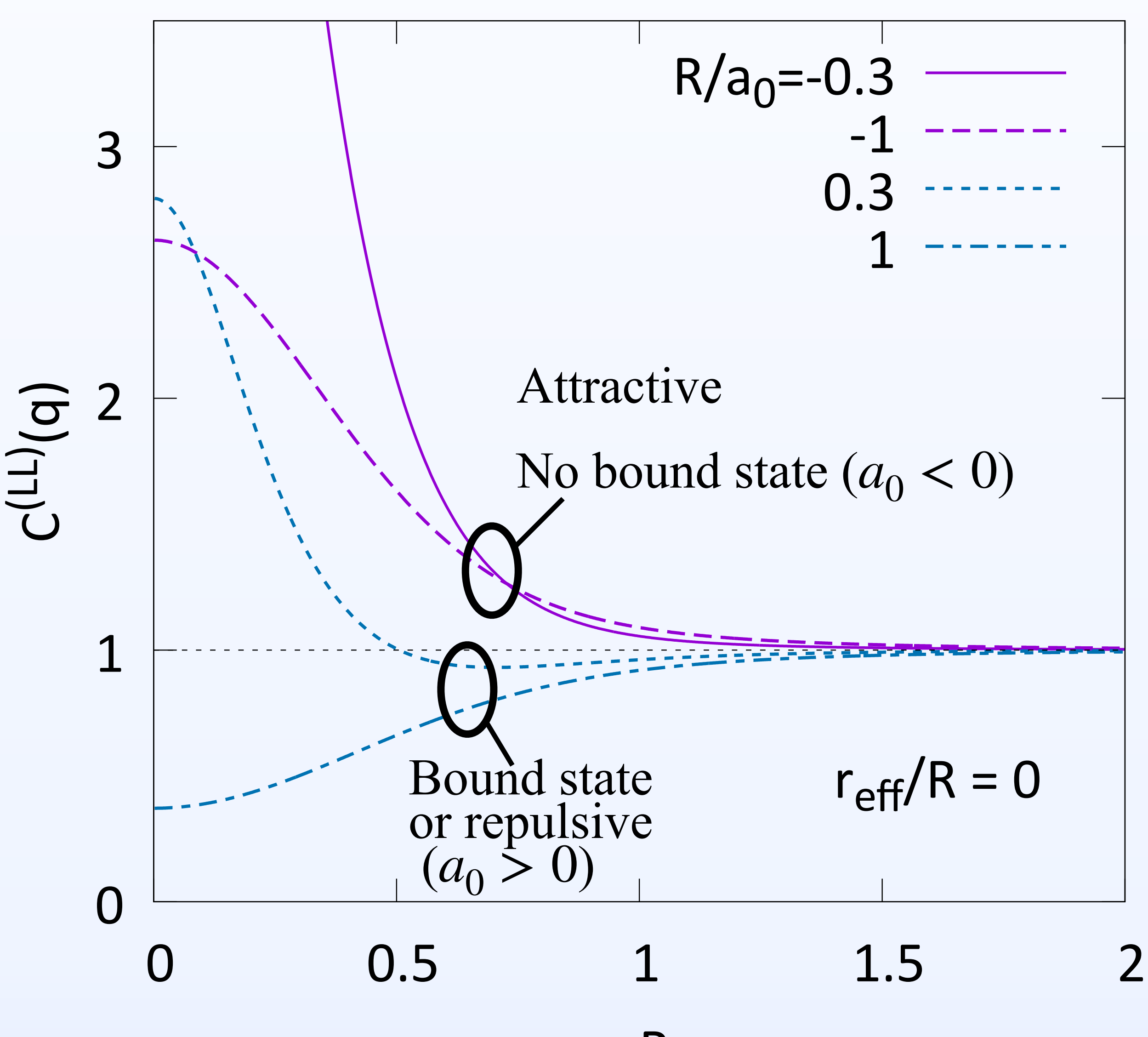
- $p\phi$ ALICE *pp*: PRL 127 (2021) 17, 172301



Source size dependence

- Line shapes of $C(q)$: relation to interaction

$$C(\mathbf{q}) \simeq \int d^3\mathbf{r}\; S(\mathbf{r})\,|\varphi^{(-)}(\mathbf{q},\mathbf{r})|^2$$



Morita, et al., PRC101 (2020)

- Scattering length a_0 and source size R determines the suppression/enhancement of line shape * $a_0 = -\mathscr{F}(q=0)$

- Repulsive int. ($a_0 > 0$, small $|a_0|$)
  → Suppressed $C(q)$

- Attractive int. w/ bound state ($a_0 > 0$, large $|a_0|$)
  → Suppressed $C(q)$ for Large R
  Enhanced $C(q)$ for small R

- Attractive int. w/o bound state ($a_0 < 0$)
  → Enhanced $C(q)$

Source size dependence

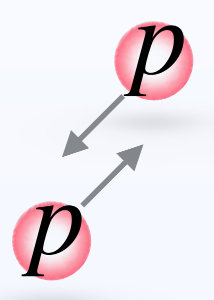
- Line shapes of $C(q)$: relation to interaction

• Attractive interaction

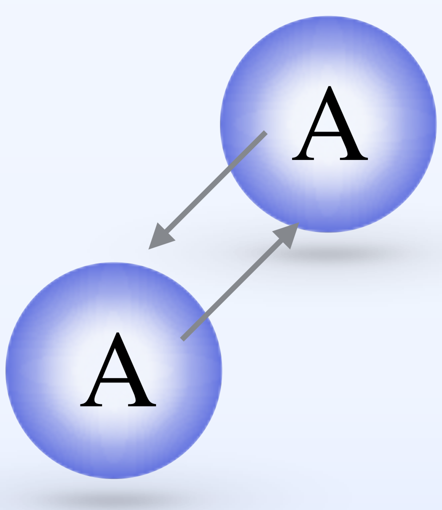
w/ bound state | w/o bound state

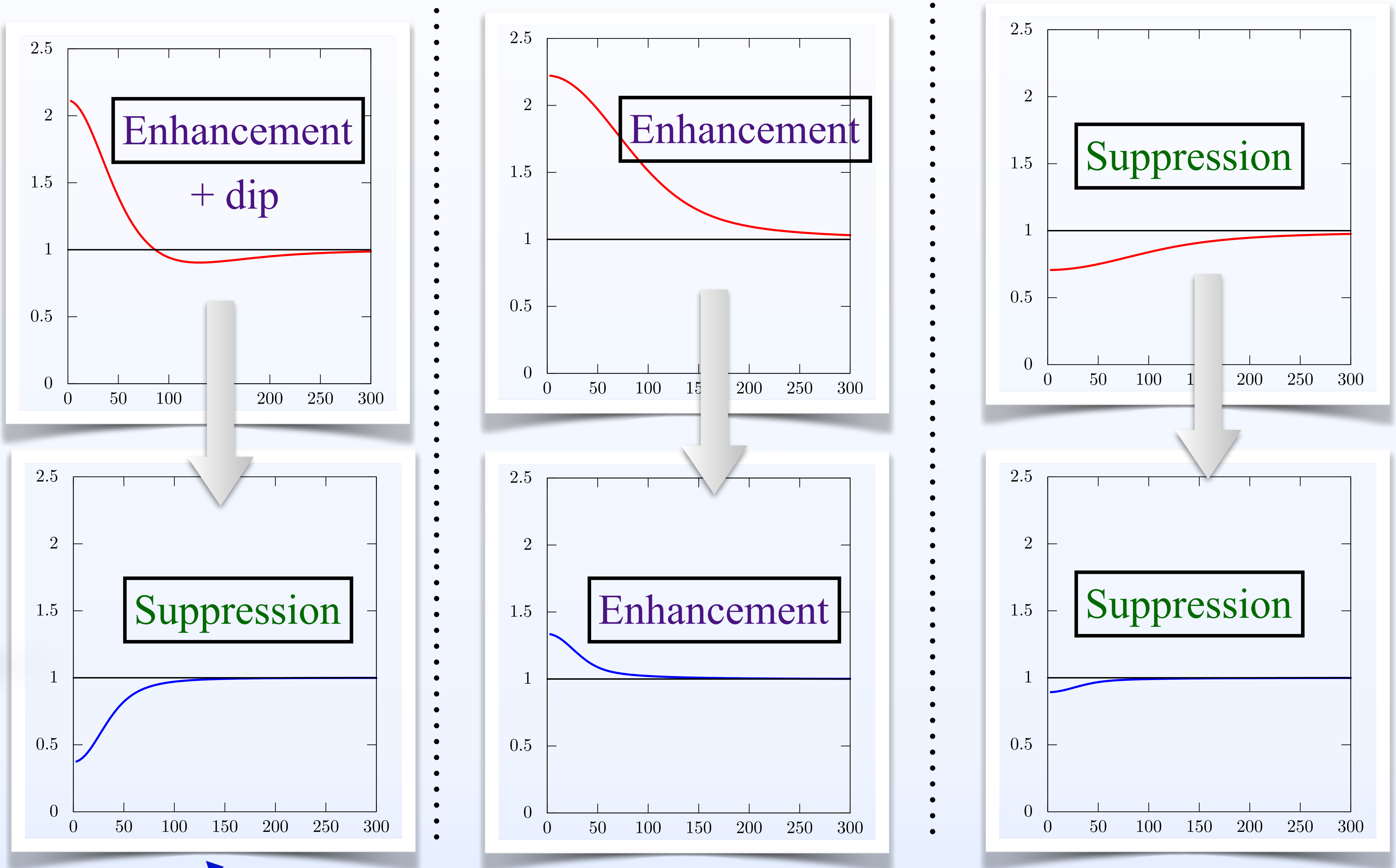
• Repulsive interaction

• Small source



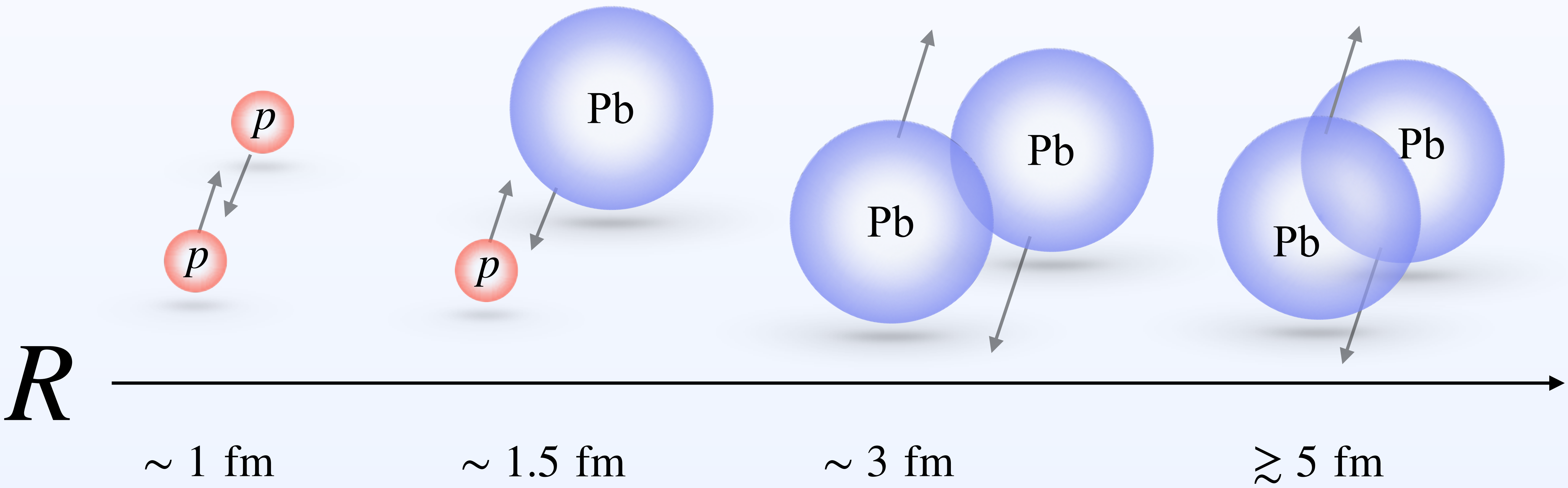
• Large source





Source size dependence for typical for bound state cases!

- How to control source size R
  - pp collisions
  - pPb collisions
  - PbPb collisions
    - peripheral
    - central



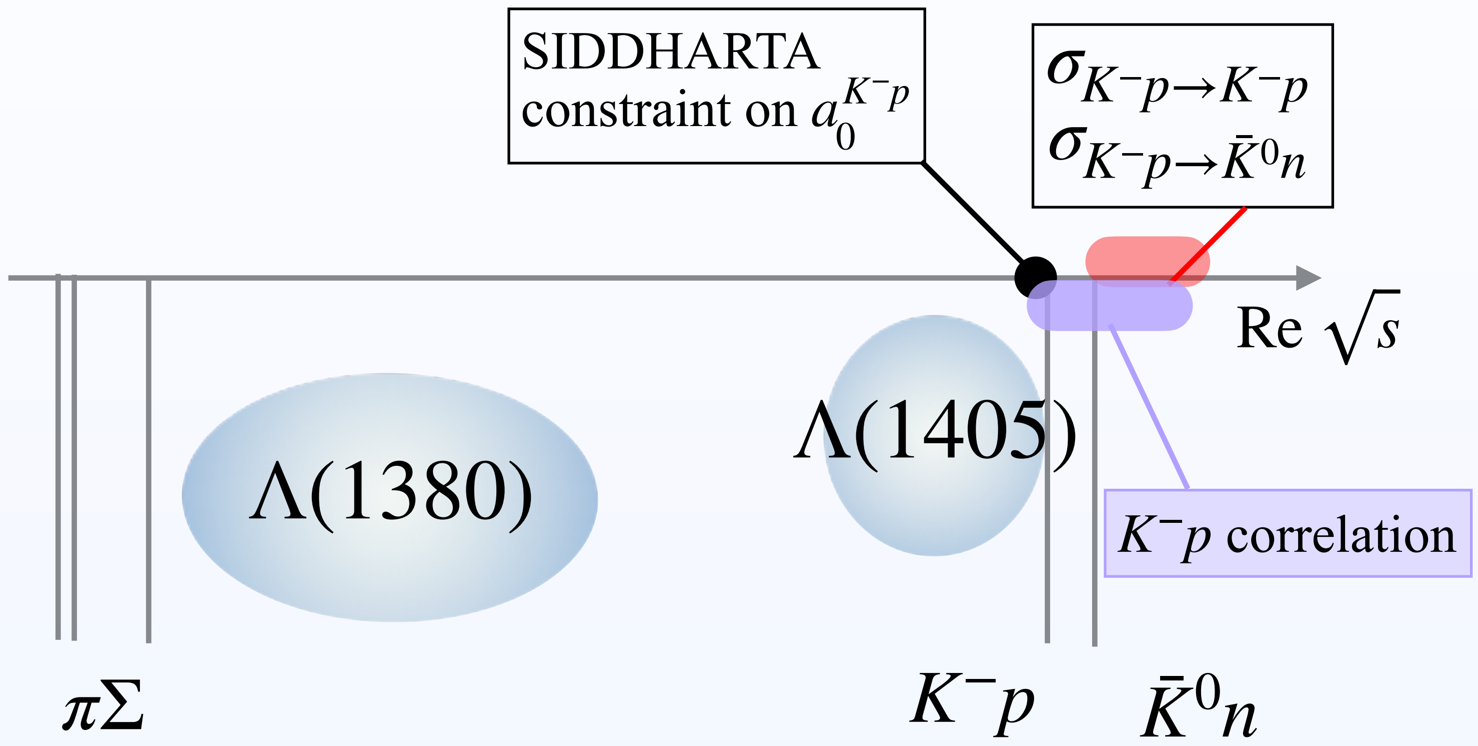
Contents

- Introduction on Femtoscopy : hadron-hadron momentum correlation function in high-energy nuclear collisions
- $\bar{K}N$ correlation function and coupled-channel effect
- $\Xi\alpha$ correlation function
- Summary

$\bar{K}N$ interaction and K^-p correlation

$\bar{K}N$ interaction and $\Lambda(1405)$

- Coupled-channel system of $\pi\Sigma$-$\pi\Lambda$-$\bar{K}N$
- Strong attraction reproducing quasi-bound state $\Lambda(1405)$
- Strong constraint on $a_0^{K^-p}$ by SIDDHARTA experiment of Kaonic hydrogen
  M. Bazzi, et al.. PLB 704 (2011)
- Structure of $\Lambda(1405)$ and $\Lambda(1380)$
  - two pole structure
    J. A. Oller and U. G. Meißner, PLB500, 263 (2001)
  - $\bar{K}N$ molecular picture
    R.H. Dalitz, S.F. Tuan, PRL 425 (1959).



Chiral SU(3) based $\bar{K}N$-$\pi\Sigma$-$\pi\Lambda$ potential

Miyahara, Hyodo, Weise, PRC 98 (2018)

- Constructed based on the amplitude with NLO chiral SU(3) dynamics
  Ikeda, Hyodo, Weise, NPA881 (2012)
- Coupled-channel, energy dependent as

$$V_{ij}^{\mathrm{strong}}(r,E) = e^{-(b_i/2+b_j/2)r^2} \sum_{\alpha=0}^{\alpha_{\max}} K_{\alpha,ij}\,(E/100\ \mathrm{MeV})^{\alpha}$$

- Constructed to reproduce the chiral SU(3) amplitude around the $\bar{K}N$ sub-threshold region

$\bar{K}N$ interaction and K^-p correlation

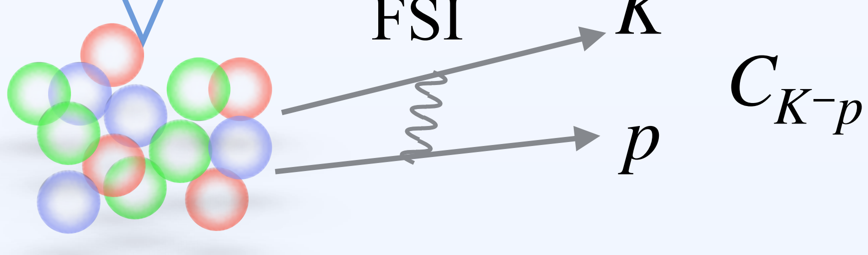
- Koonin-Pratt-Lednicky-Lyuboshits-Lyuboshits (KPLLL) formula

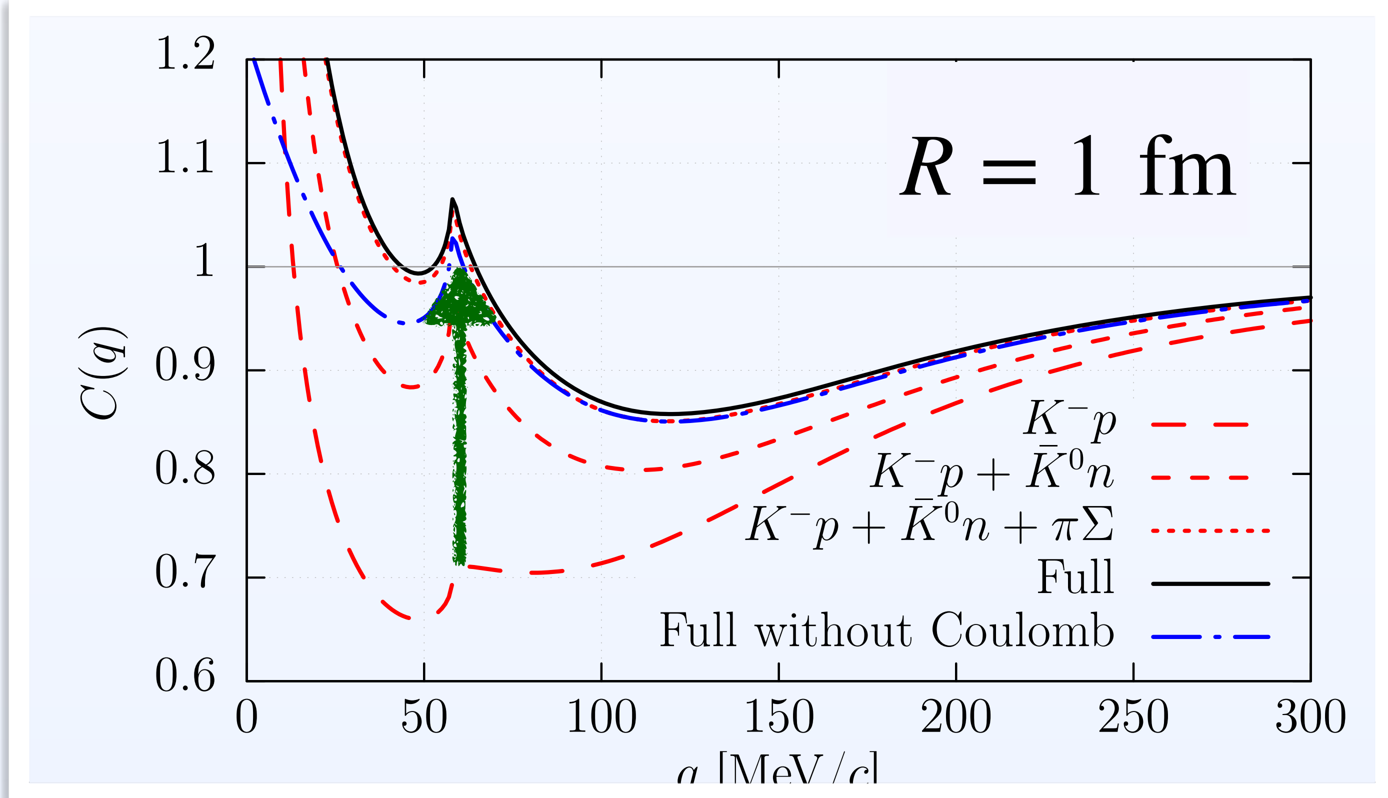
$$C(\mathbf{q}) = \int d^3\mathbf{r}\ S(\mathbf{r})\,|\psi^{(-)}(q;r)|^2 + \sum_{j\neq K^-p} \omega_j \int d^3\mathbf{r}\ S_j(\mathbf{r})\,|\psi_j^{(-)}(q;r)|^2$$

S.E. Koonin, PLB 70 (1977)
S. Pratt et. al. PRC 42 (1990)
R. Lednicky, et.al. Phys. At. Nucl. 61(1998)

- Contribution from coupled-channel source

$K^-p,\ \bar{K}^0n,\ \pi^0\Sigma^0,\ \pi^+\Sigma^-,\ \pi^-\Sigma^+,\ \pi^0\Lambda$

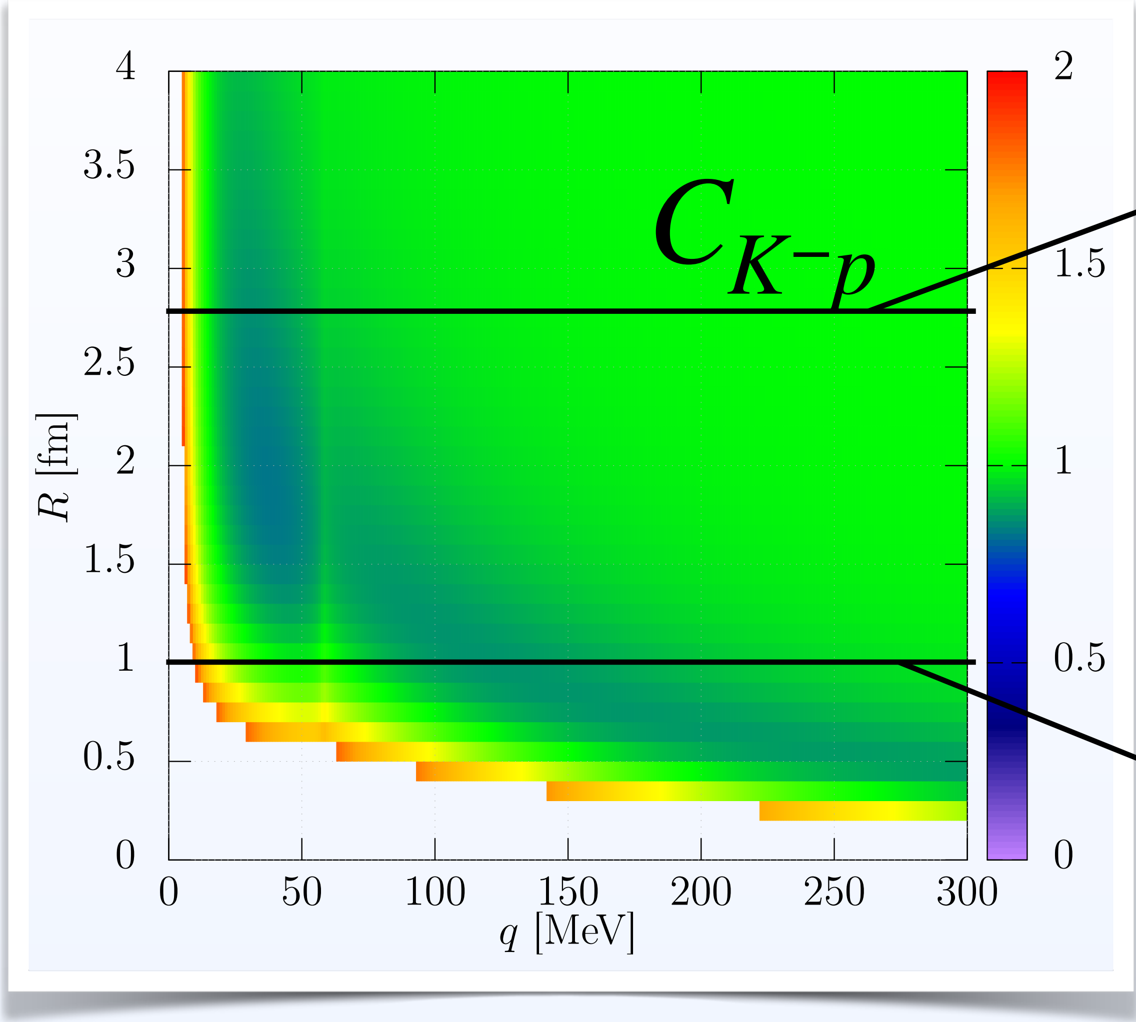


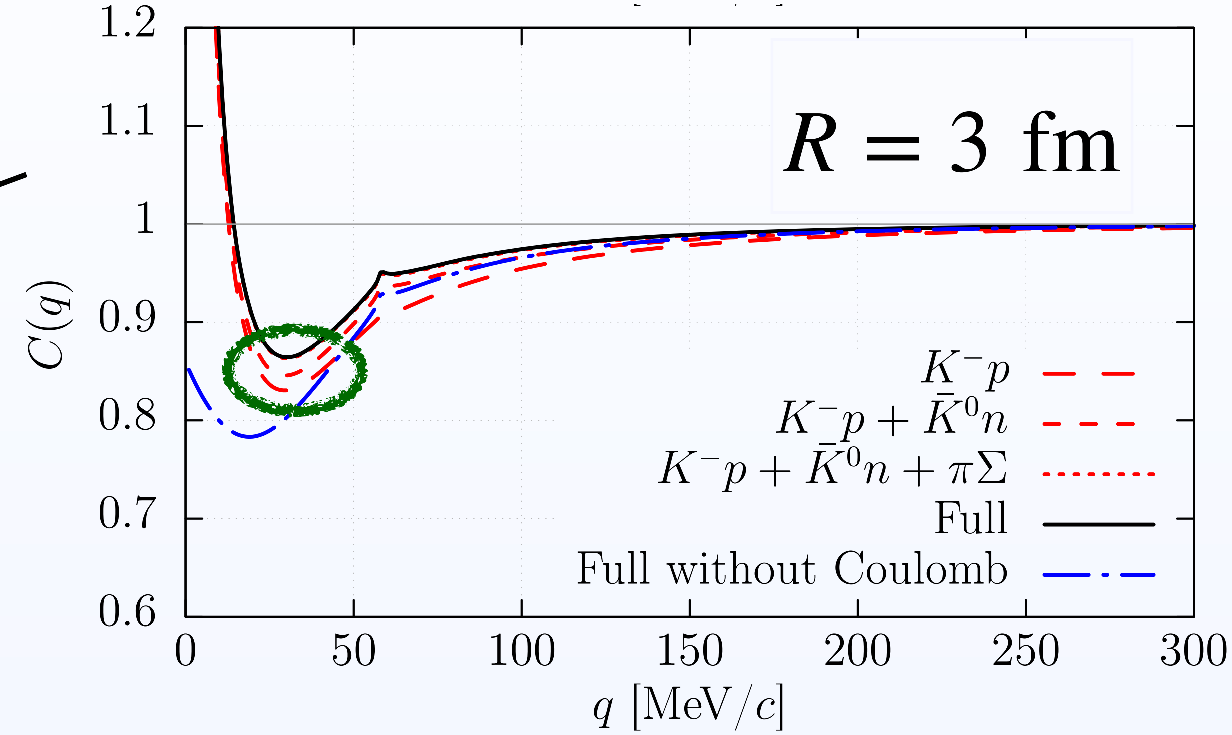


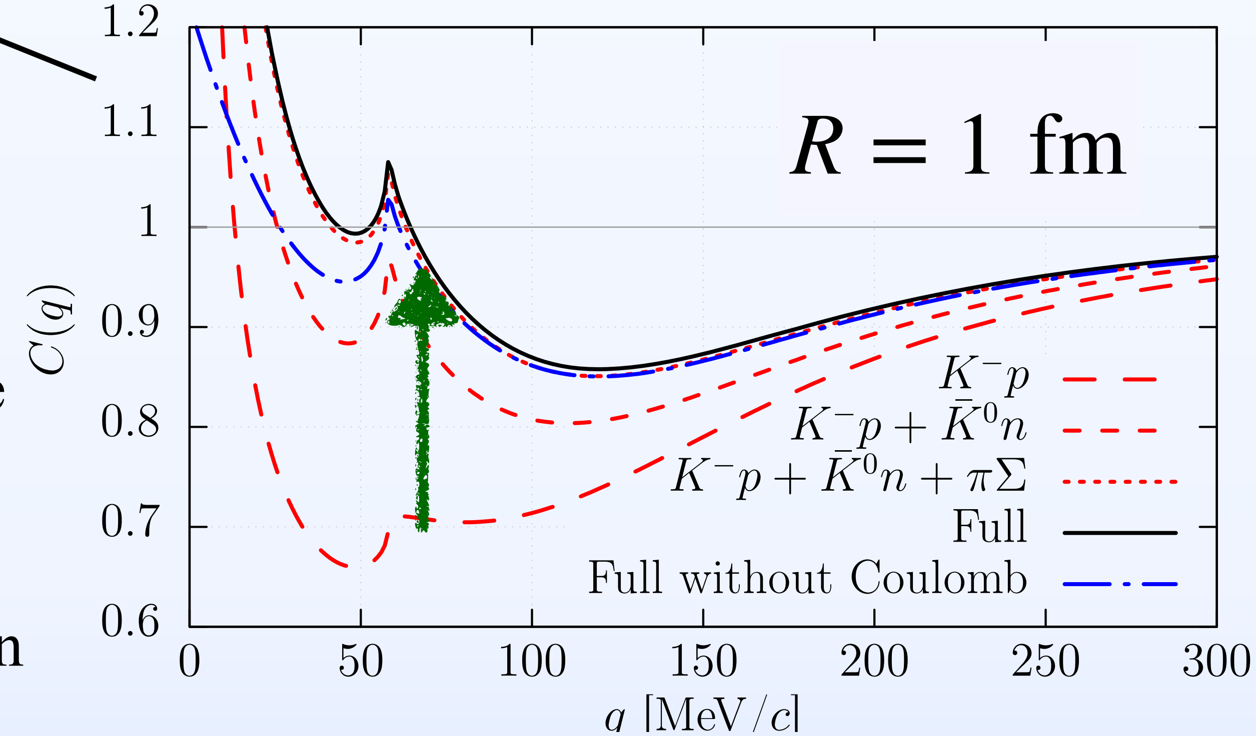
- Enhance $C(q)$
- Enhance cusp structure
- ω_i : production rate (compared to measured channel)

$\bar{K}N$ interaction and K^-p correlation

- Source size dependence of coupled-channel effect







- Strong source size dependence
  <== Due to the near-threshold Λ(1405) pole

- $C(q)$ with large source
  - Less prominent cusp structure
  - Weaker coupled-channel source contribution

•Large source：K^-p scattering

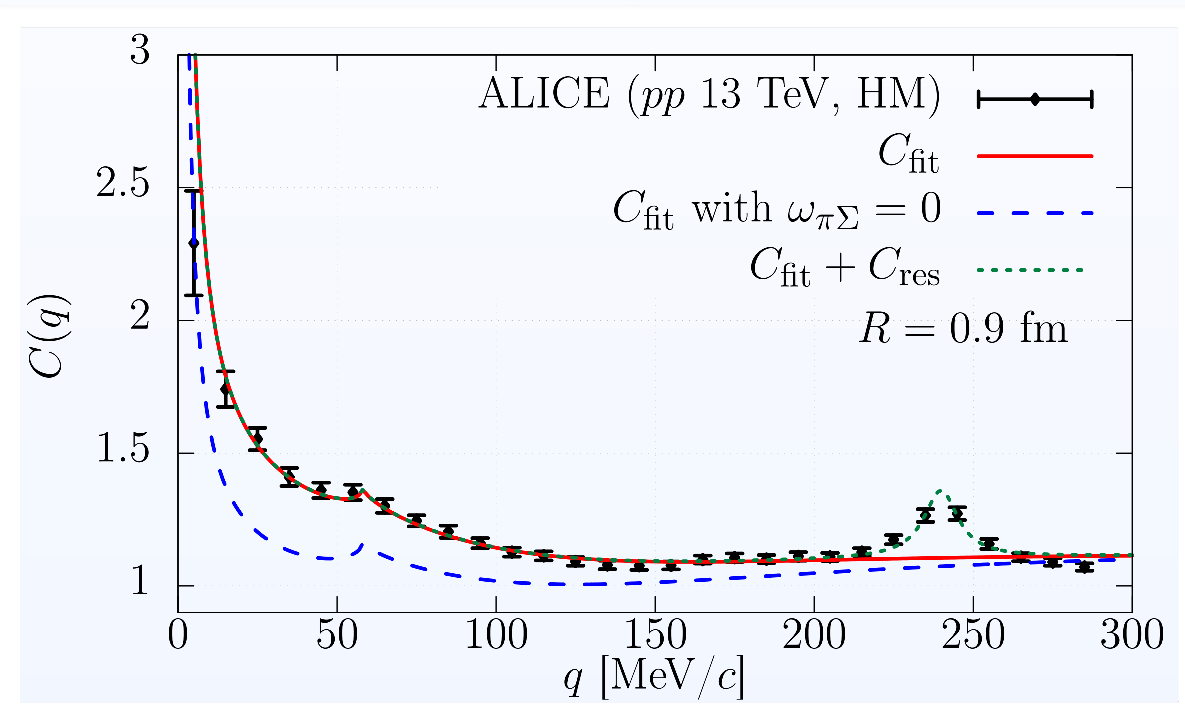
•Small source：detailed coupled-channel effect

$\bar{K}N$ interaction and K^-p correlation

- Source size dependence with K^-p data

- ALICE *pp* collision data

ALICE PRL 124, 092301 (2020)

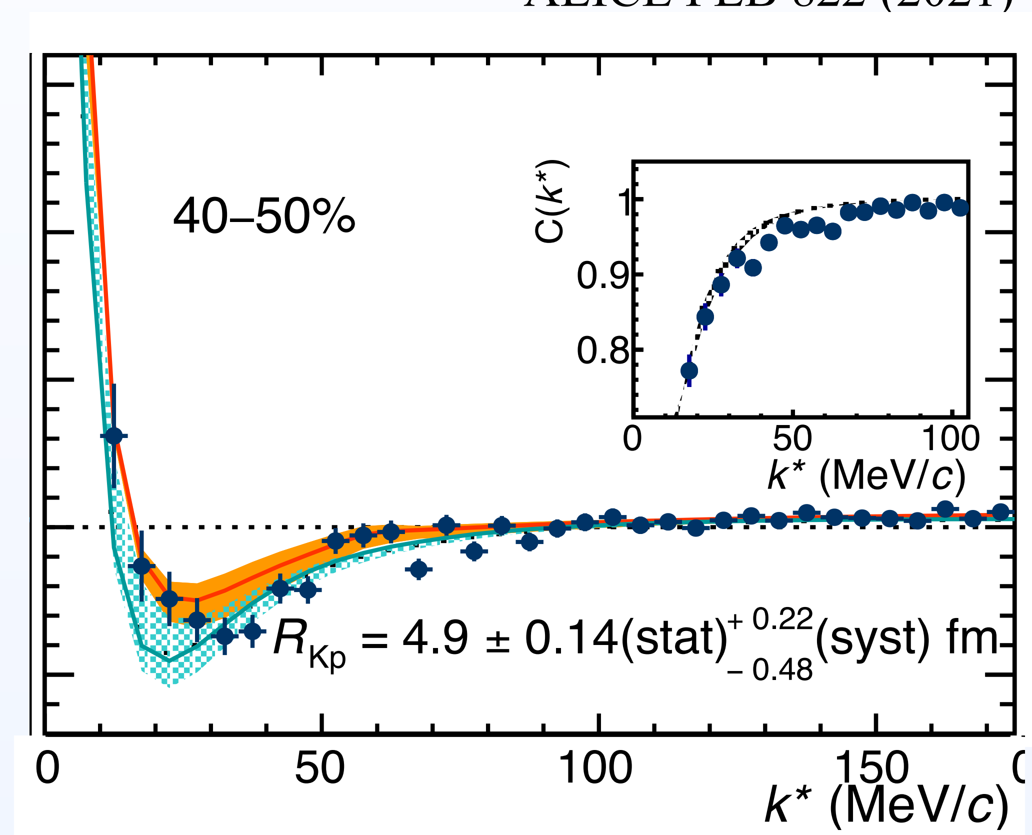


Kamiya, Hyodo, Morita, Ohnishi, Weise, PRL 124 (2020) 13, 132501

- Small source
  - Clear $\bar{K}^0 n$ cusp structure
  - Sizable contribution from coupled-channel source required to reproduce data

- ALICE PbPb collision data

ALICE PLB 822 (2021) 136708



- Large source
  - Weaker cusp
  - Consistent with analysis only with K^-p source

- Chiral SU(3) dynamics describes the both correlation data well.

$\bar{K}N$ interaction and $K^{-}p$ correlation

- Latest $K^{-}p$ correlation results

  ALICE, EPJC 83 (2023)

  - pPb : 0-20%, 20-40% 40-100%
  - PbPb : 60-70%, 70-80% 80-90%

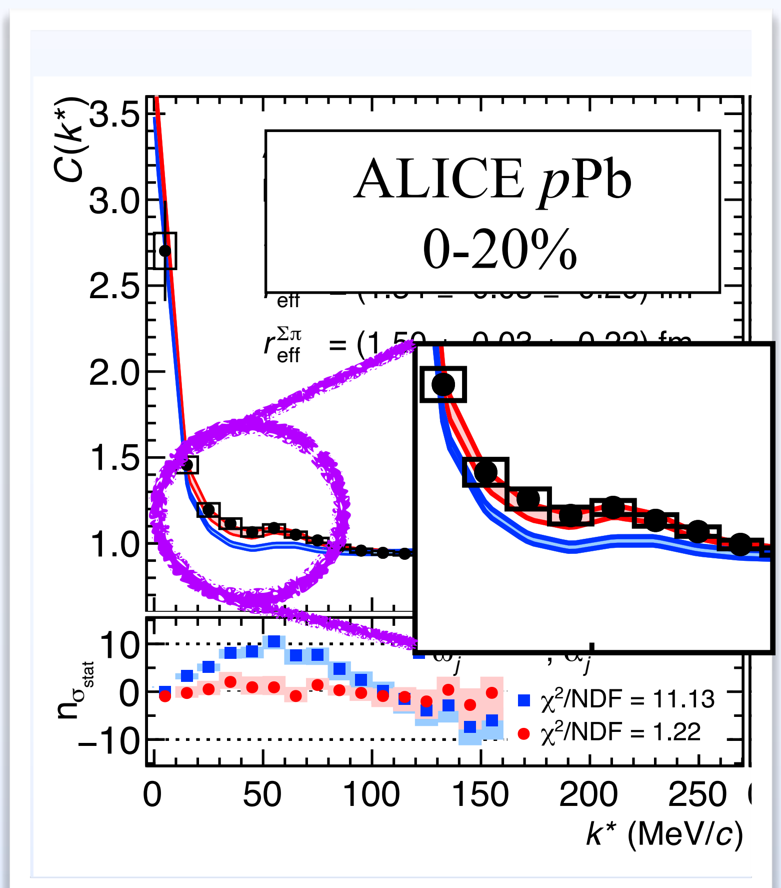
- Discrepancy around $\bar{K}^0 n$ threshold between chiral SU(3) model and exp. data for small source data

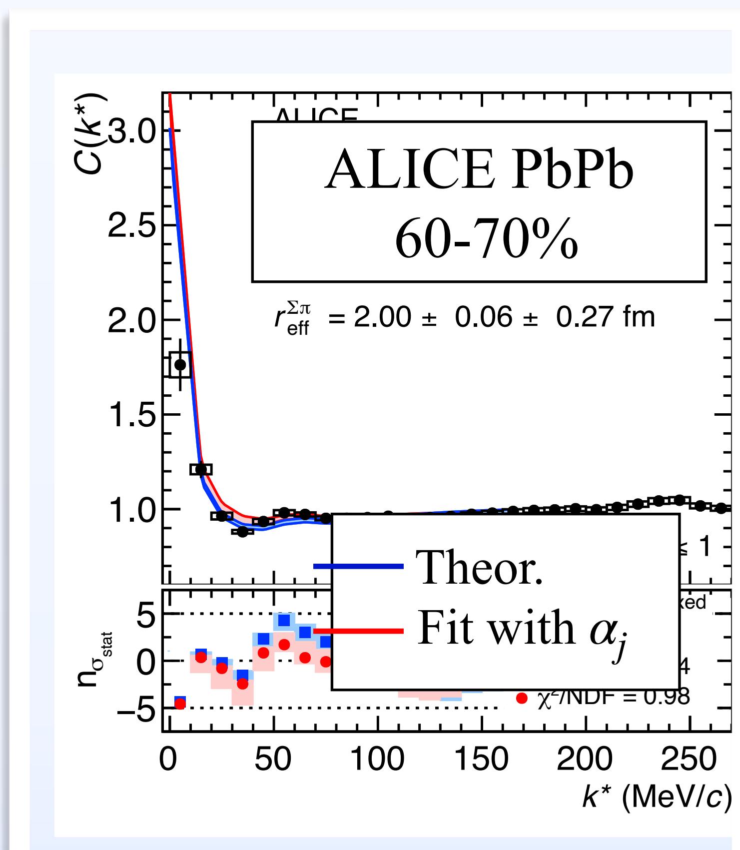
- Analysis with scale factor α_j
  - Scale the coupled-channel source contribution by scaling factor

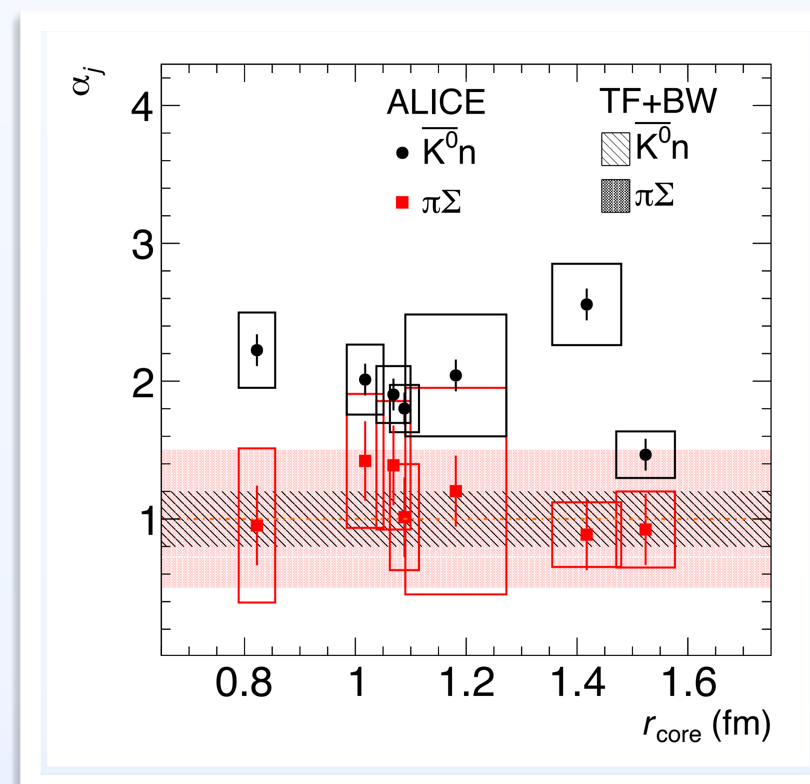
$$C_{K^-p} = C^{\mathrm{el}}_{K^-p} + \sum_j \alpha_j C^{\mathrm{inel}}_j$$

  - $\alpha_{\bar{K}^0 n} \sim 2$ gives better agreement

    implying the stronger coupling







Contents

- Introduction on Femtoscopy : hadron-hadron momentum correlation function in high-energy nuclear collisions

- $\bar{K}N$ correlation function and coupled-channel effect

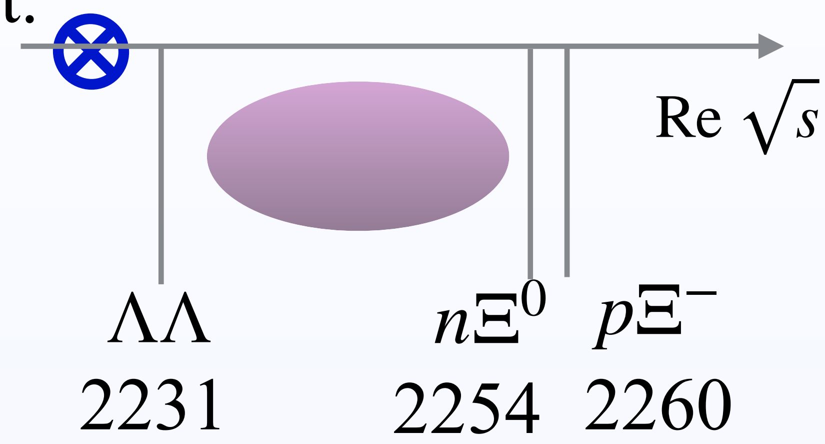
- $\Xi\alpha$ correlation function

- Summary

- $\Lambda\Lambda$-$N\Xi$ interaction ($S = -2$) and H-dibaryon
  - $J = 0$: Unique sector in flavor Octet-Octet baryon int.

    $8 \otimes 8 = \underline{1} \oplus 8_A \oplus 8_S \oplus 10 \oplus \overline{10} \oplus 27$

    - Pauli arrowed
    - Attractive color-magnetic int.



  - Flavor-singlet dihyperon "H" R. L. Jaffe, PRL 38 (1977), 195.

    Predicted as "single hadron" below $\Lambda\Lambda$
    - Binding energy of double Λ hypernucleus

      Takahashi et al., PRL87 (2001) 212502

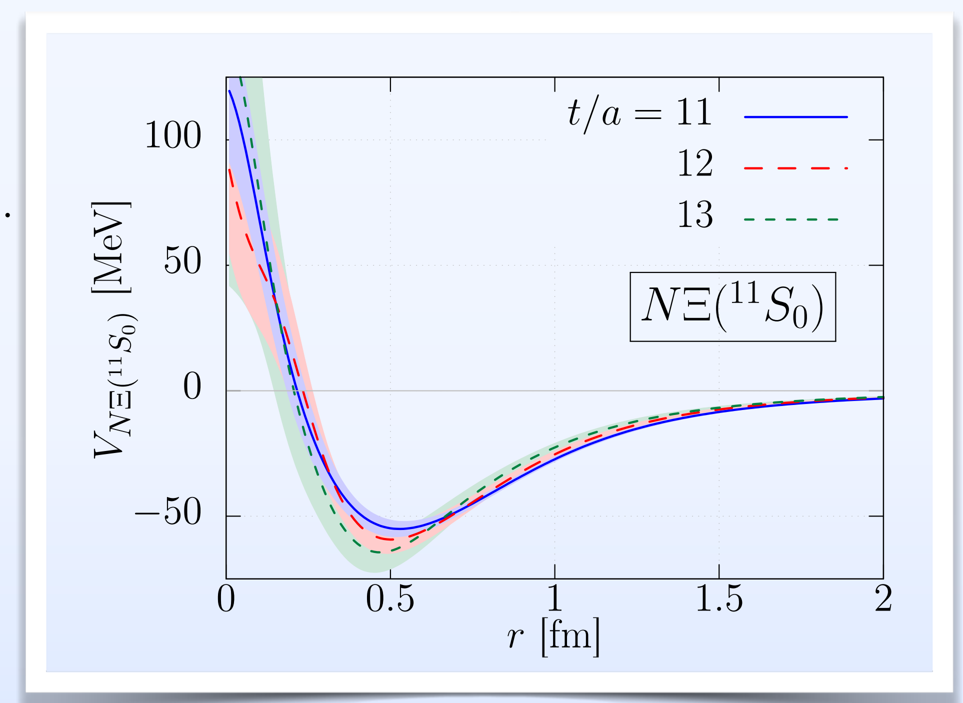
    ⟶ $\Lambda\Lambda$ does NOT form (deep) bound state

  - HAL QCD $\Lambda\Lambda$-$N\Xi$ coupled-channel potential

    K. Sasaki et al. [HAL QCD], NPA 998 (2020), 121737.
    - Strong attraction in $J = 0$, $I = 0$ $N\Xi$ channel

      $a_0^{p\Xi^-(J=0)} = -1.21 - i1.52$

    H dibaryon state is just barely unbound.



<u>Fate of H-dibaryon?</u>

$p\Xi^-$ correlation function

- $p\Xi^-$ correlation function

$$C_{p\Xi^-} = \frac{1}{4} C_{p\Xi^-,\mathrm{singlet}} + \frac{3}{4} C_{p\Xi^-,\mathrm{triplet}}$$

Couples to $\Lambda\Lambda$
(H-dibaryon channel)

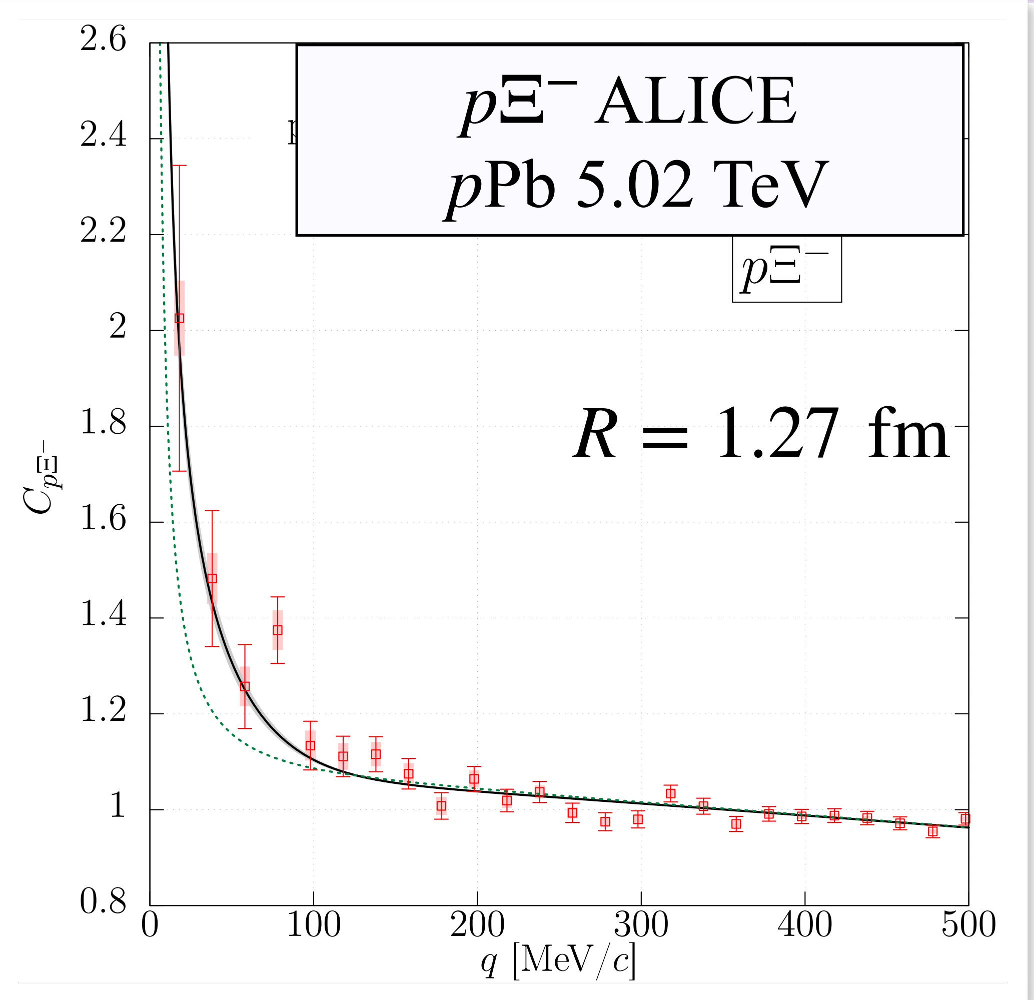
- Enhancement from pure Coulomb case
- Comparison with ALICE data

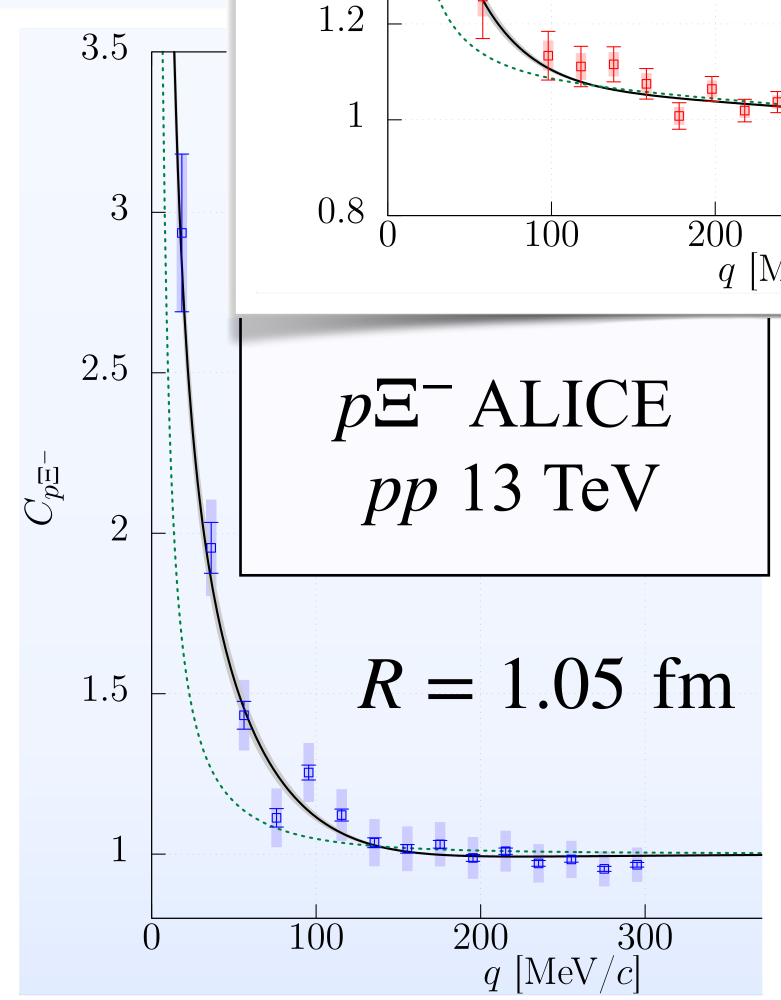
  pPb 5.02 TeV, *pp* 13 TeV collisions :
  S. Acharya et al. [ALICE], PLB 797 (2019).

- Spin channel reduction

  Singlet : Stronger enhancement

  Triplet : Weaker enhancement



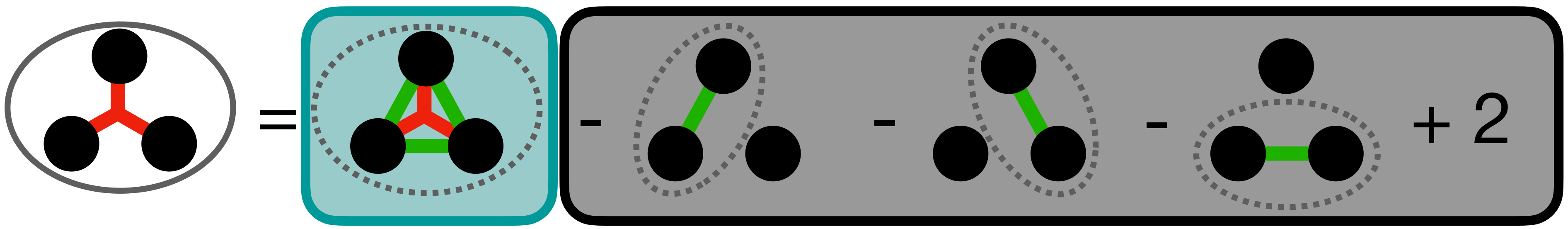


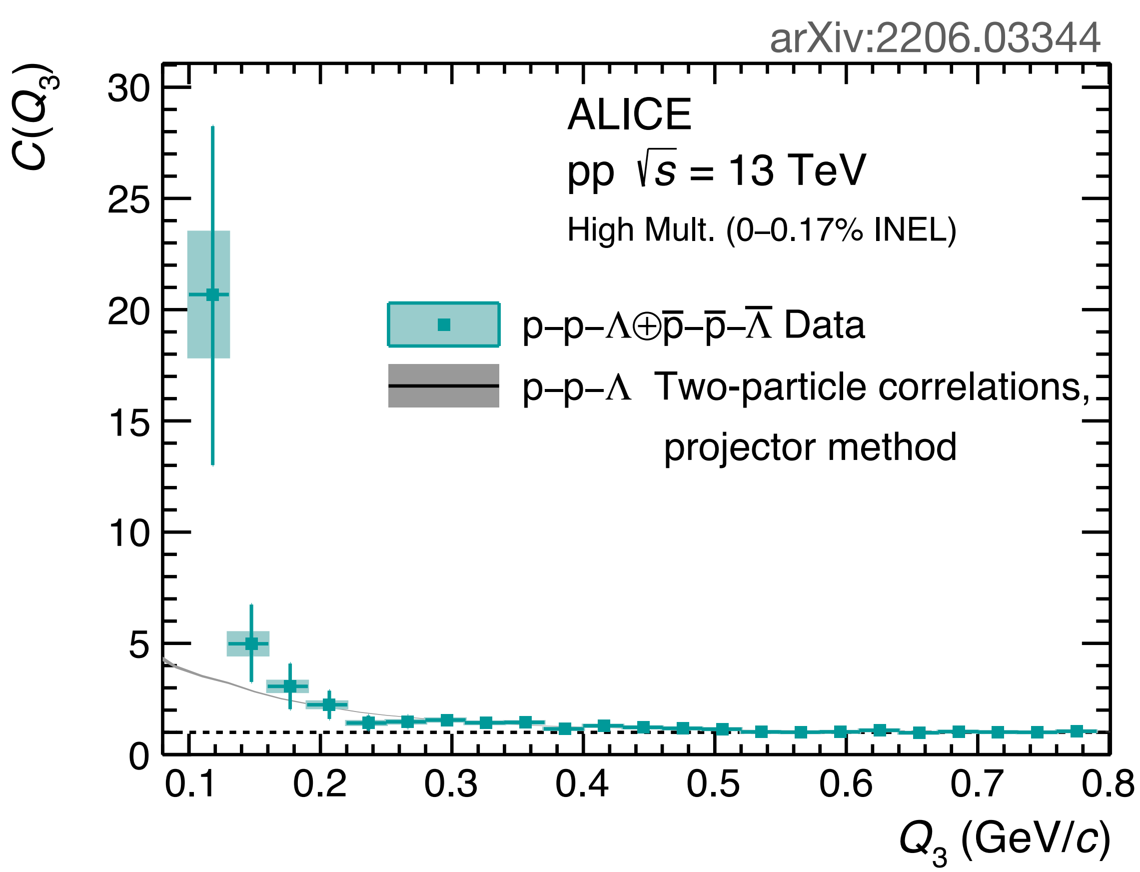
Y. Kamiya, et al. PRC 105, 014915 (20

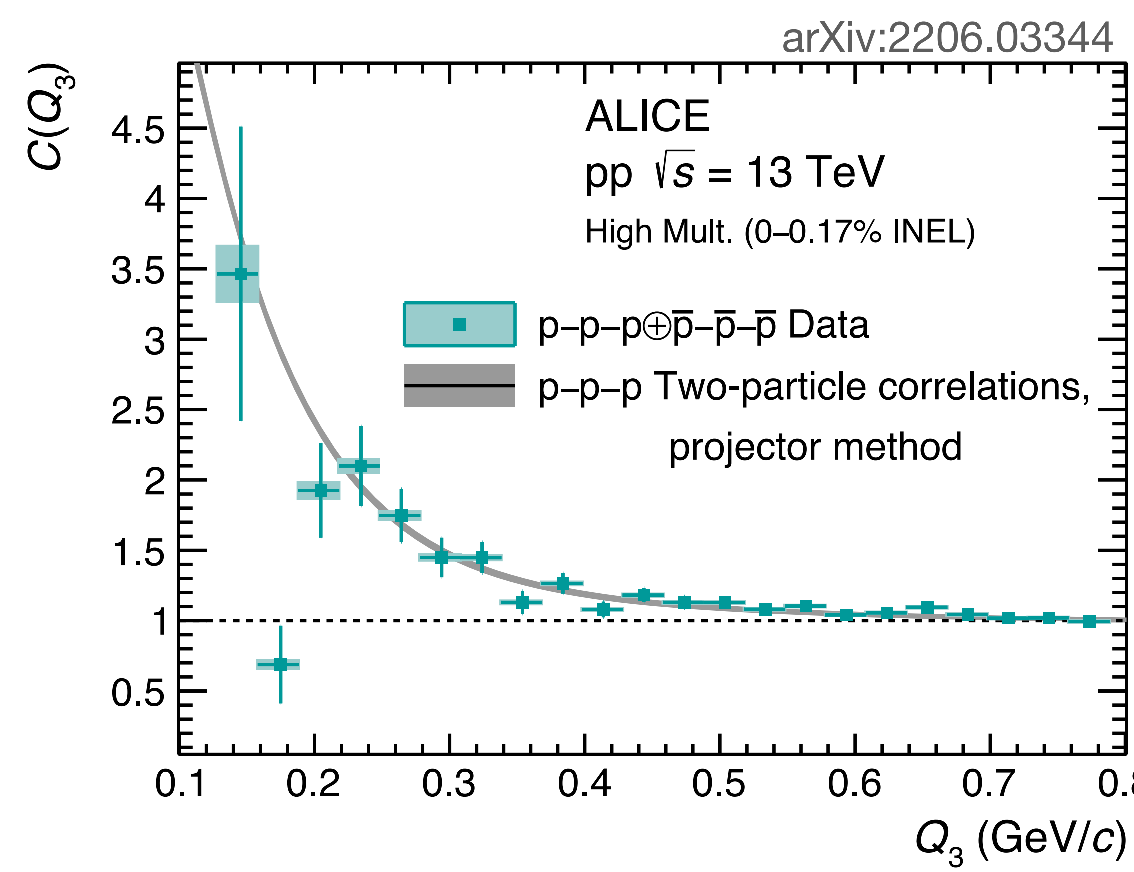
Correlation with few body systems

Three body correlation : $C\left(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\right) \equiv \frac{P\left(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\right)}{P\left(\mathbf{p}_1\right) P\left(\mathbf{p}_2\right) P\left(\mathbf{p}_3\right)}$

p-p-Λ and p-p-p correlations



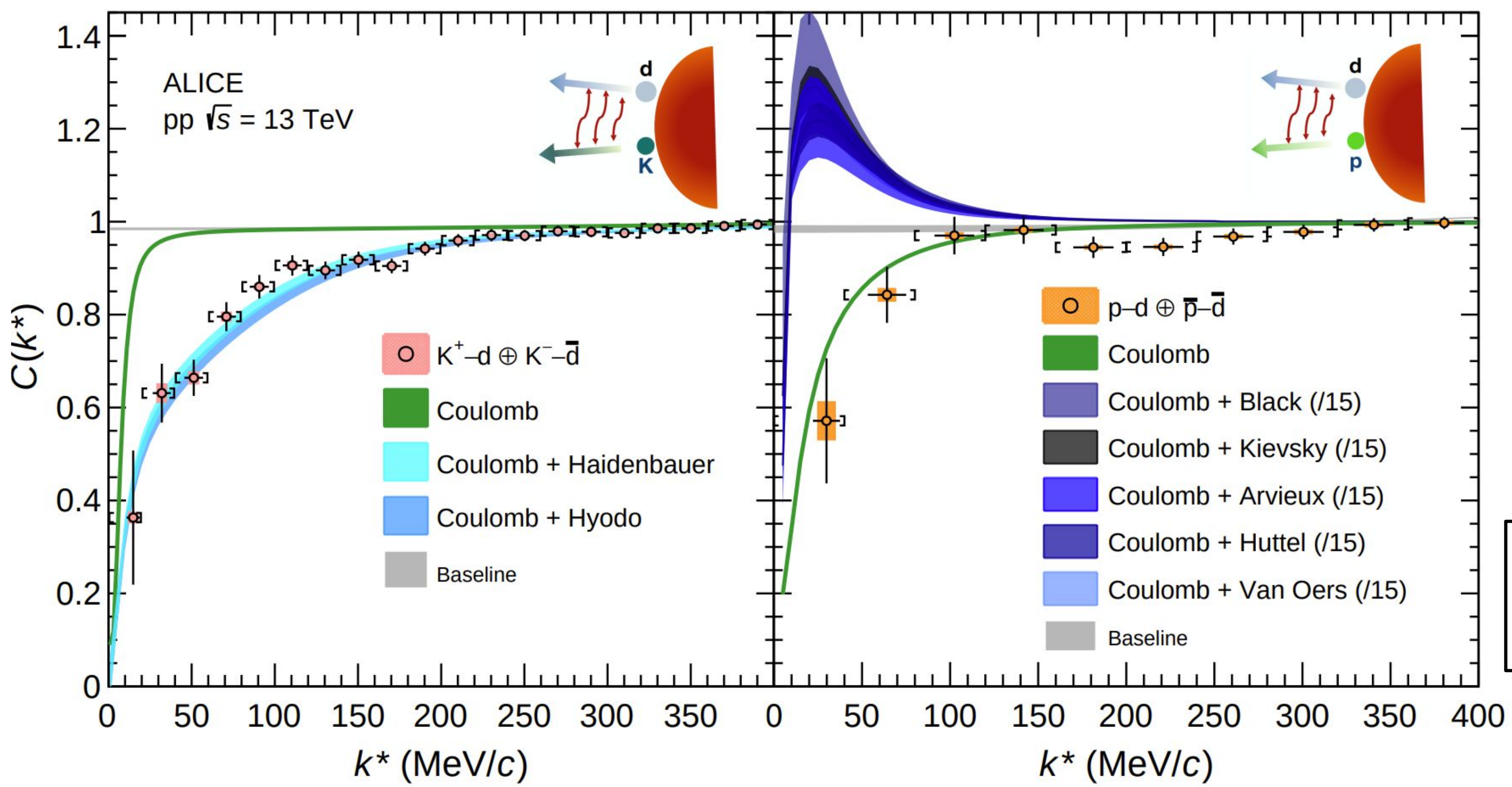




Talk slide from Laura Šerkšnytė and Raffaele Del Grande in FemTUM2022

Correlation with few body systems

Lednicky model vs ALICE data



r_{K+d}=1.41±0.04 fm

r_{pd}=1.08±0.06 fm

pd model calculations scaled by 1/15

Kd data well reproduced

⇒ fully formed deuterons present assuming small source

pd data not described

⇒ pd can't be treated as effective two-body system

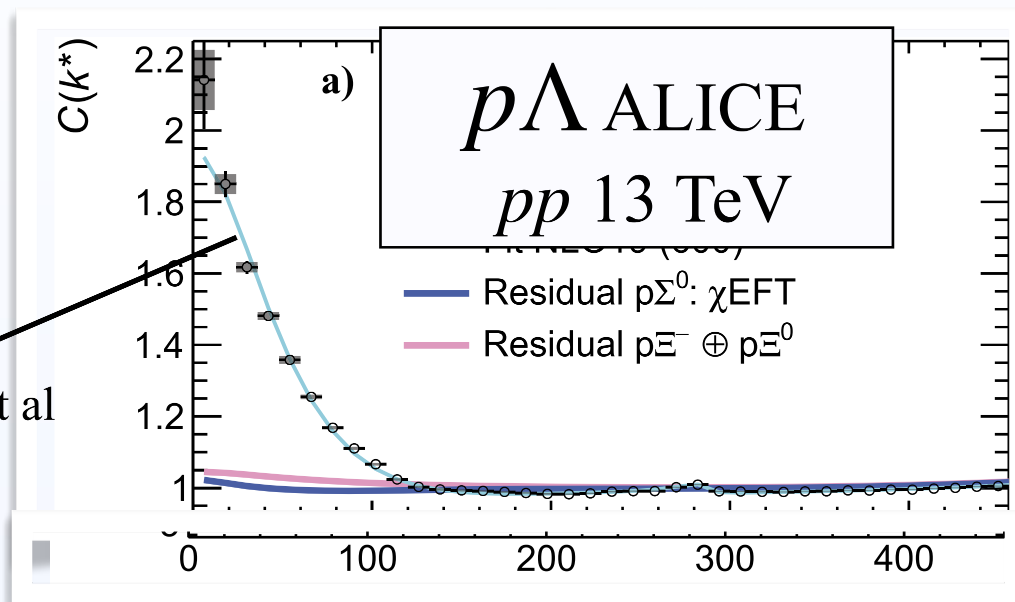
Considering protons, deuterons as distinguishable point-like particles leads to huge discrepancy

arXiv:2206.03344 (ALICE)

Talk slide from Oton Vazquez Doce's in FemTUM2022

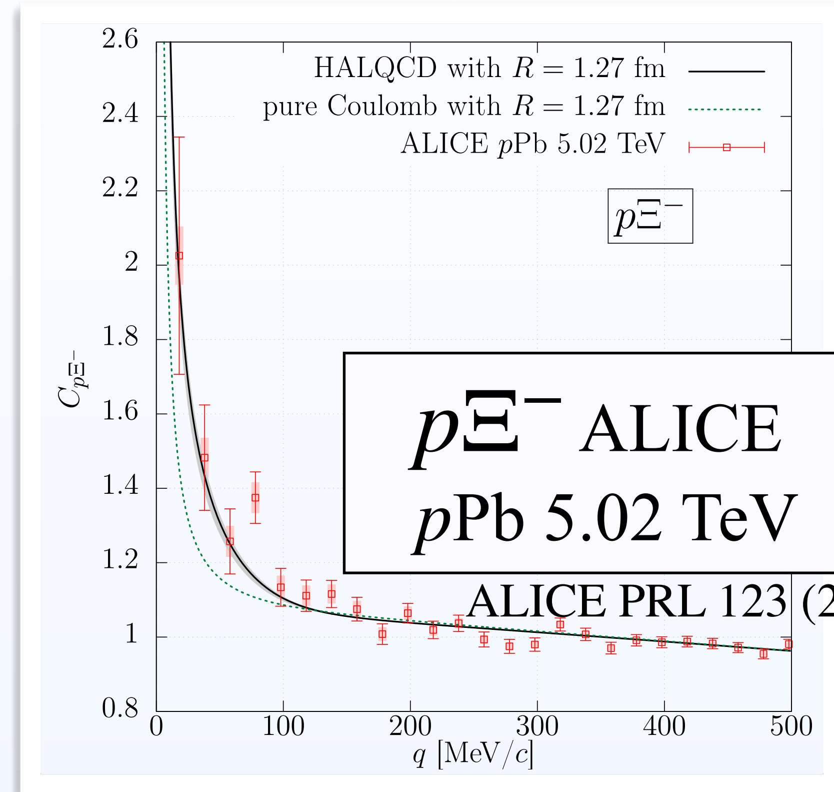
Y-α correlation

- Good agreement of Y-N correlation function



NLO 19
J. Haidenbauer, et al
EPJA 56(2020)

ALICE PLB 805 (2020) 135419

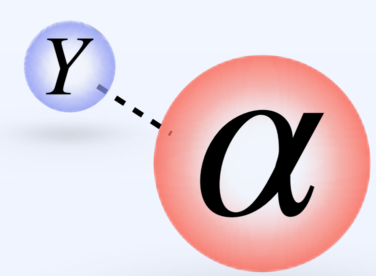


ALICE PRL 123 (2019).

Y. Kamiya, et al. PRC 105, 014915 (2022)

Y-α(^{4}He) correlation

- Large binding energy of α
  —> • Good description by two body treatment



- Y-α potential: smeared potential range
  —> • Detailed potential shape may be investigated

Further constraint on the YN(YY) int?

$\Lambda\alpha$ correlation

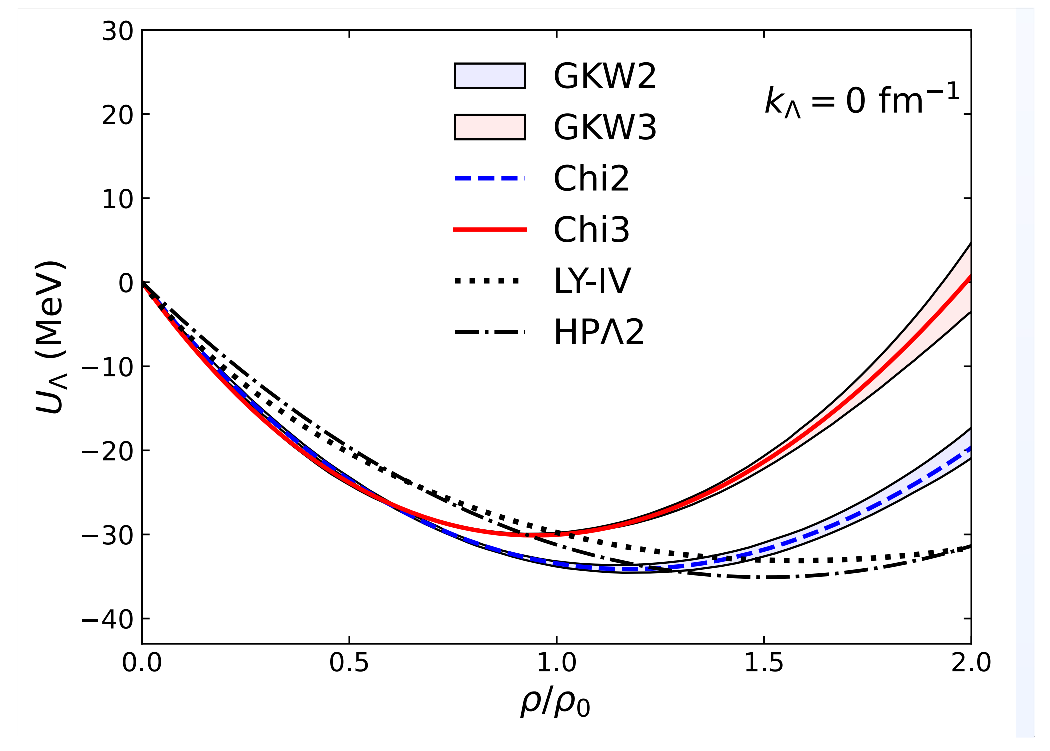
$N\Lambda$ interaction at finite density

- Chiral EFT with NLO D. Gerstung, N. Kaiser, W. Weise, EPJA 55 (2020)
  —> ΛNN three body interaction gives the additional repulsion
  —> stiffer EOS
- Chi3: Skyrme type Λ potential based on Chiral EFT with three body
  A. Jinno, K. Murase, Y. Nara, and A. Ohnishi, PRC 108 (2023) 6, 065803

$$U_\Lambda^{\rm local} = a_1^\Lambda \rho_N + a_2^\Lambda \tau_N - a_3^\Lambda \triangle \rho_N + a_4^\Lambda \rho_N^{4/3} + a_5^\Lambda \rho_N^{5/3}$$

  - Well reproduces the binding energy of Λ in hypernuclei

A. Jinno, Y. Kamiya, T. Hyodo, A. Ohnishi, PRC 110 (2024), 014001



- $N\Lambda$ potential model with different density dependence
  D. E. Lanskoy and Y. Yamamoto, PRC 55, 2330 (1997)
  N. Guleria, S. K. Dhiman, and R. Shyam, Nucl. Phys. A 886, 71 (2012)
  - HPΛ2
  - LY-IV

  Weaker desnity dependence

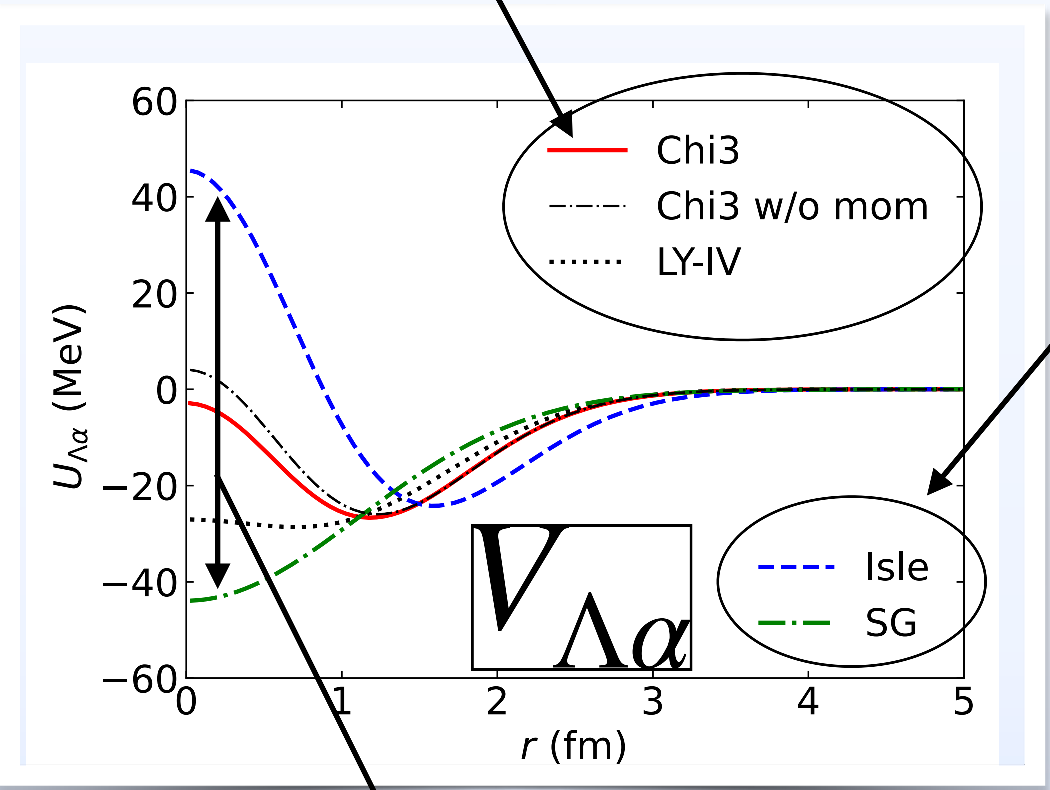
$\Lambda\alpha$ correlation

- $\Lambda\alpha$ potential w/ Skyrme type pot.
  - Nucleon density with Gaussian form:
    $$\rho(r) = A(2\nu_c/\pi)^{3/2} e^{-2\nu_c r^2}$$
    - high central density $\sim 2\rho_0$
  - Unknown a_3^Λ: fit to reproduce the $^5_\Lambda$He experimental $E_B = 3.12$ MeV

- Simple potential models

Kumagai-Fuse, S. Okabe, Y. Akaishi, PLB 345 (1995)

  - Isle potential
    $$V(r) = V_1 e^{-r^2/b_1^2} + V_2 e^{-r^2/b_2^2}$$
    repulsive core (short range) — attractive part (long range)
  - Single Gaussian (Isle)
    $$V(r) = V e^{-r^2/b^2}$$
    - parameters are chosen to reproduce E_B



Large difference in strength of repulsive core

- Difference of ρ dependence of Skyrme pot. appear in the strength of repulsive core.

- Strength of repulsive core

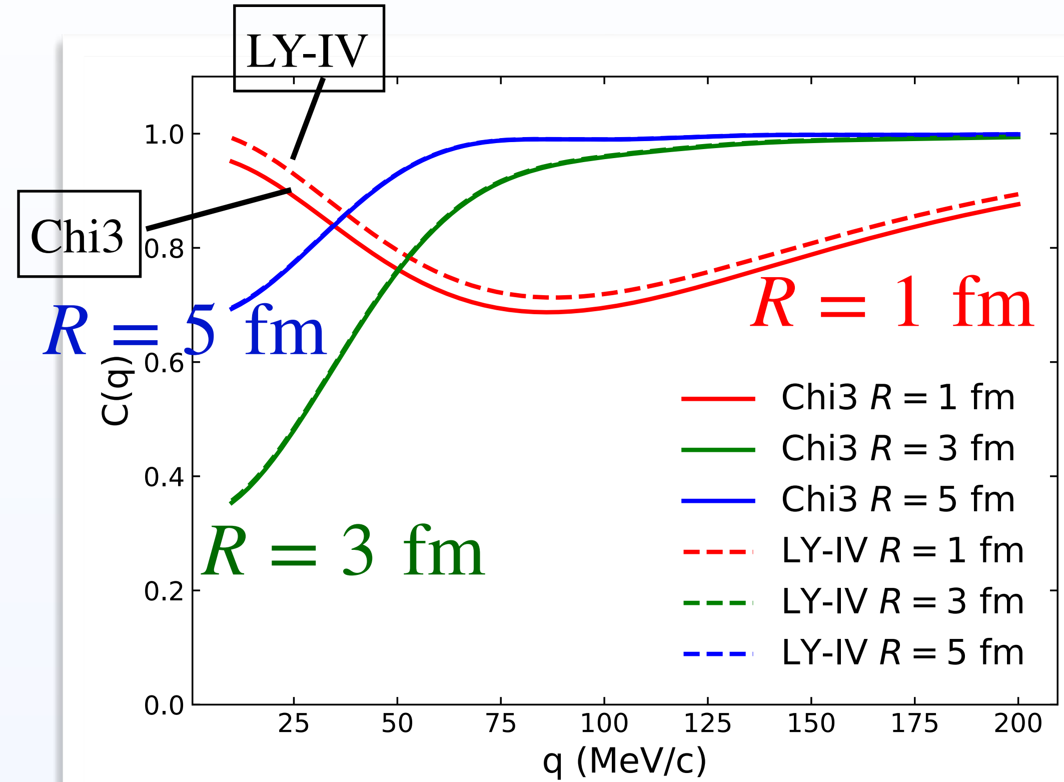
Isle > Chi3 > LY-IV> SG (No core)

- Potential shape dependence of $C_{\Lambda\alpha}$?

$\Lambda\alpha$ correlation

- Source size dependence of $C_{\Lambda\alpha}$
  - Characteristic lineshapes for weak binding system ($^{5}_{\Lambda}$He)
    - Dip for small source
    - Suppression for large source
  - Potential difference appear only in small source results

→ <u>Large source results are useful to check E_B of $^{5}_{\Lambda}$He</u>



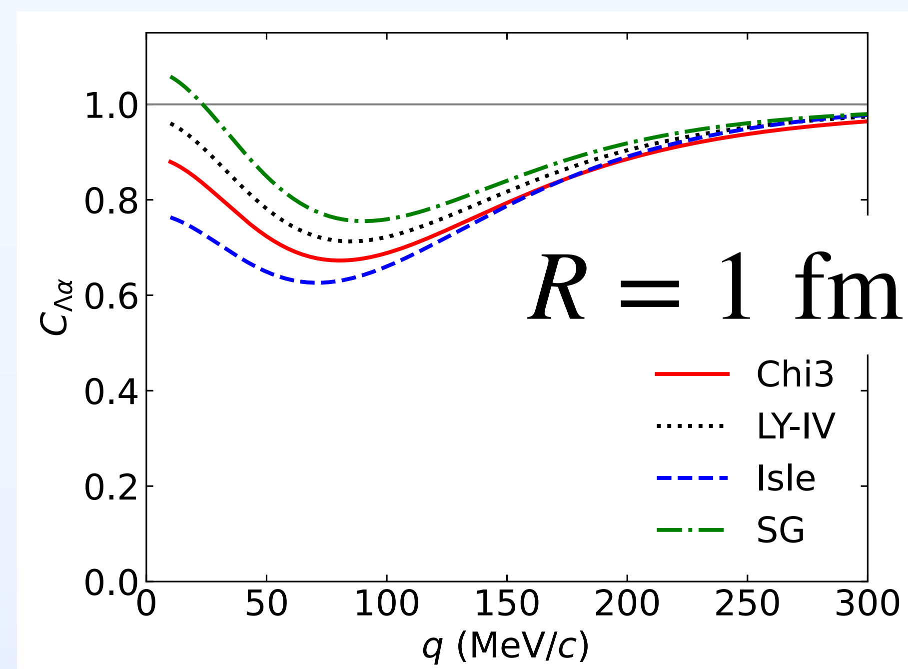
- Effect of repulsive core
  - $C(q)$ are ordered from bottom to top as

    Isle -> Chi3 -> LY-IV -> SG (No core)

    Same ordering with the strength of repulsive core

    —> Stronger core causes Stronger suppression

→ <u>Strength of the repulsive core can be tested with $C_{\Lambda\alpha}(q)$ from small source.</u>

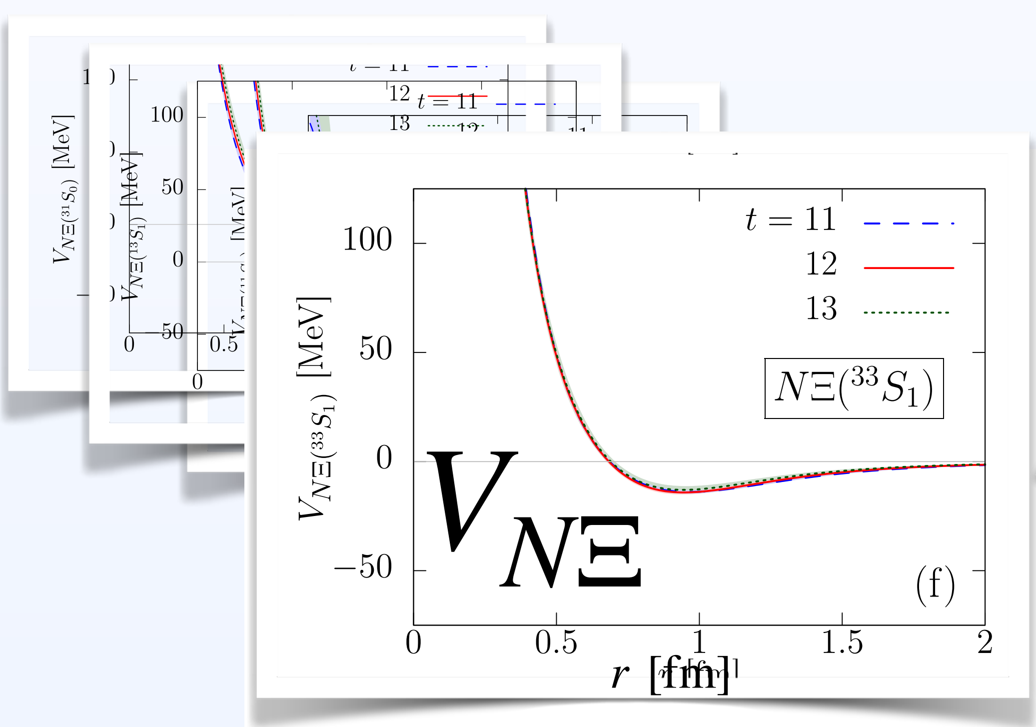


A. Jinno, Y. Kamiya, T. Hyodo, A. Ohnishi, PRC 110 (2024), 014001

$\Xi\alpha$ correlation

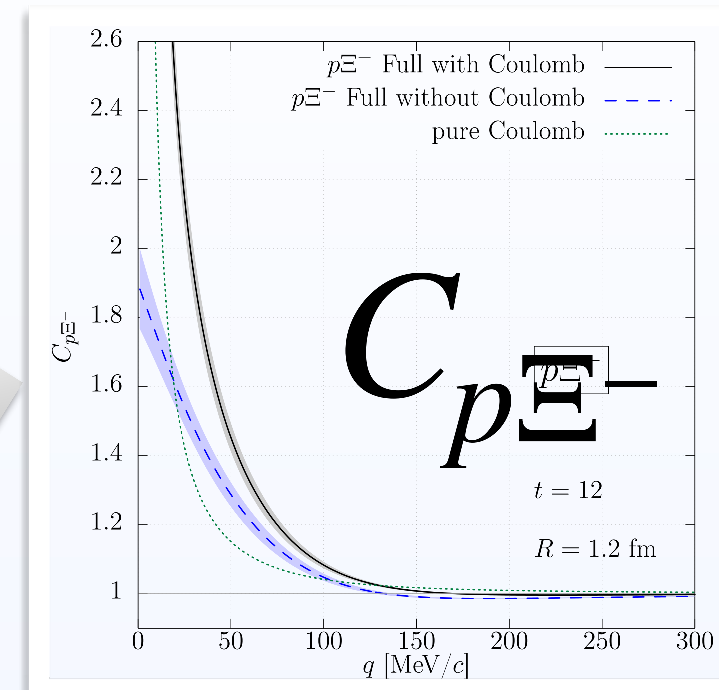
- $N\Xi$ potential and $\Xi\alpha$ potential

• HAL QCD $N\Xi$ potential
K. Sasaki et al., NPA, 121737 (2019).
4 components for s-wave : ${}^{11}S_0$, ${}^{13}S_1$, ${}^{31}S_0$, ${}^{33}S_1$



• Repulsive core
• Long tail attraction by π exchange

- $p\Xi^-$ correlation
$\sim [C(^{11}S_0) + 3C(^{13}S_1)$
$C(^{31}S_0) + 3C(^{33}S_1)]/8$

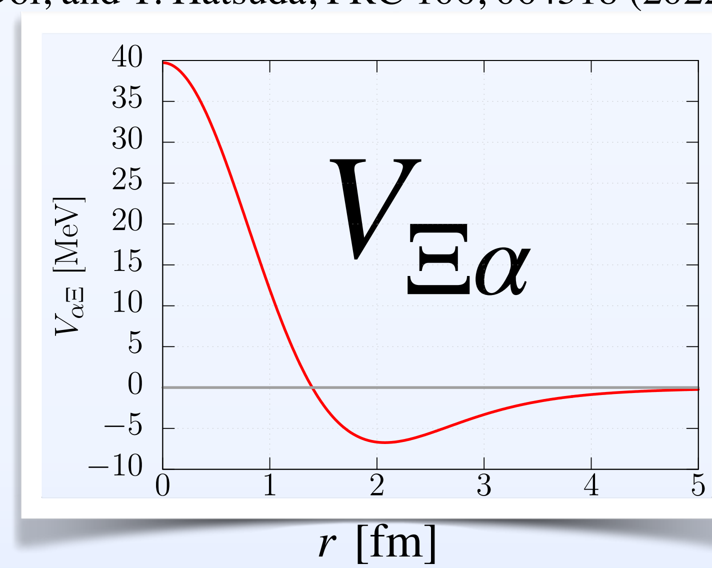


• Large enhancement from ${}^{11}S_0$

- Folding $\Xi\alpha$ potential
E. Hiyama, M. Isaka, T. Doi, and T. Hatsuda, PRC 106, 064318 (2022).

$[V(^{11}S_0) + 3V(^{13}S_1)$
$3V(^{31}S_0) + 9V(^{33}S_1)]/16$

• Large weight of ${}^{11}S_0$



• <u>Different channel weight</u>

• <u>Effect of smeared repulsive core/attraction?</u>

$\Xi\alpha$ correlation

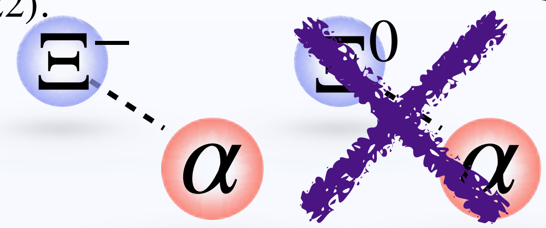
Predictions for $\Xi\alpha$ bound state: ${}^{5}_{\Xi}\mathrm{H}$

- Coulomb assisted bound state <— HAL QCD pot.
  - Bound state found only for Coulomb attractive pair

    E. Hiyama, et al PRC 106, 064318 (2022).

    K. Sasaki et al., NPA, 121737 (2019).

    $E_B = 0.47$ MeV



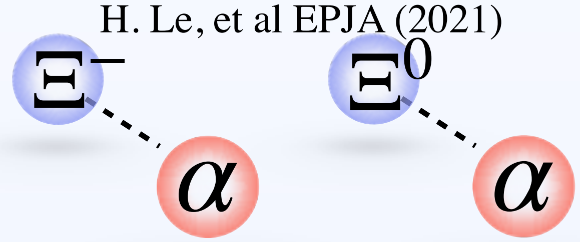
Large difference comes from ${}^{33}S_1$

H. Le, et al EPJA (2021)

- Deeper bound state <— chiral effective SU(3) pot.

  H. Le, et al EPJA (2021)

  $E_B = 2.16$ MeV



- Behavior for Coulomb assisted bound state?
- Can we distinguish ${}^{5}_{\Xi}\mathrm{H}$ with C_{Ξ^-}

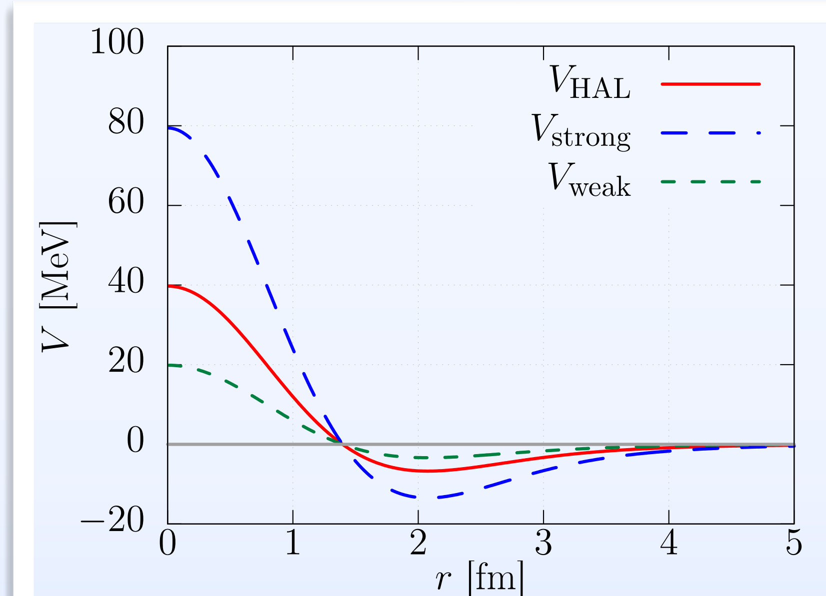
Folding potential and variations

- V_{HAL}: Folding potential based on $S = -2$ HAL QCD potential

  E. Hiyama, M. Isaka, T. Doi, and T. Hatsuda, PRC 106, 064318 (2022).

  K. Sasaki et al., NPA, 121737 (2019).

potential	E_B ($\Xi^0\alpha$) [MeV]	E_B ($\Xi^-\alpha$) [MeV]
VHAL	(Unbound)	0.47
Vstrong = 2 *VαΞ	1.15	2.08
Vweak = VαΞ /2	(Unbound)	0.18



- Check V/E_B dependence of $C_{\Xi\alpha}$

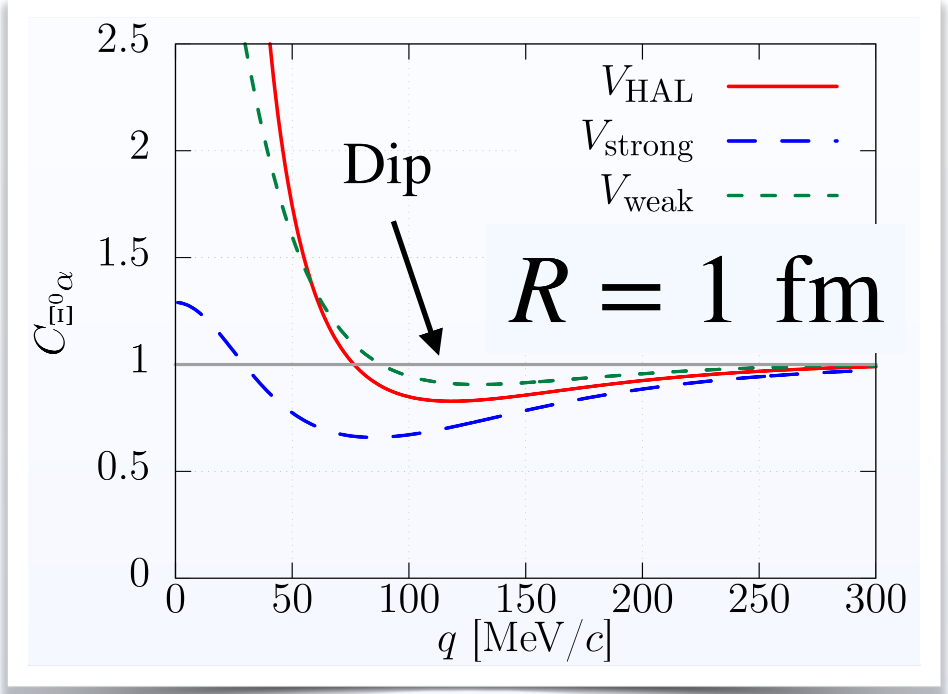
$\Xi\alpha$ correlation

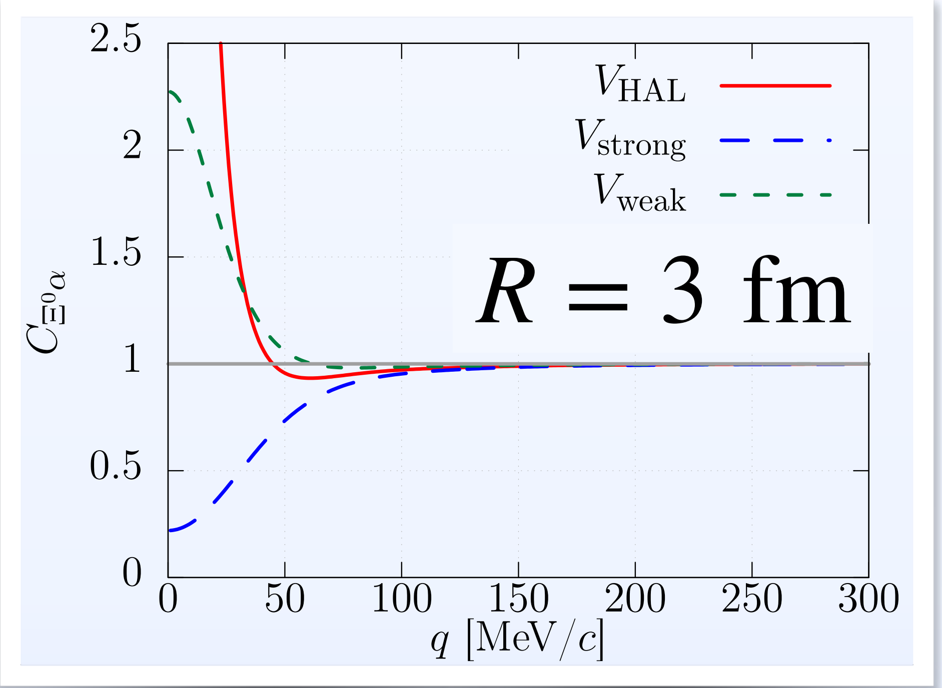
$\Xi^0\alpha$ correlation

potential	E_B [MeV]
VHAL	(Unbound)
Vstrong	1.15
Vweak	(Unbound)

- V_{strong}: Typical source size dependence with bound state
  - Suppression for large R
  - Enhancement and dip for for small R
- V_{HAL}, V_{weak}: strong enhancement
  - consistent with No $^5_\Xi$H
- Dip in $q \sim 100$ MeV/c for V_{HAL} and V_{weak}
  - Suppression by repulsive core?

→
- Source size dependence can
- Effect of detailed potential shape?





Y. Kamiya, A. Jinno, T. Hyodo, A. Ohnishi, 2409.13207

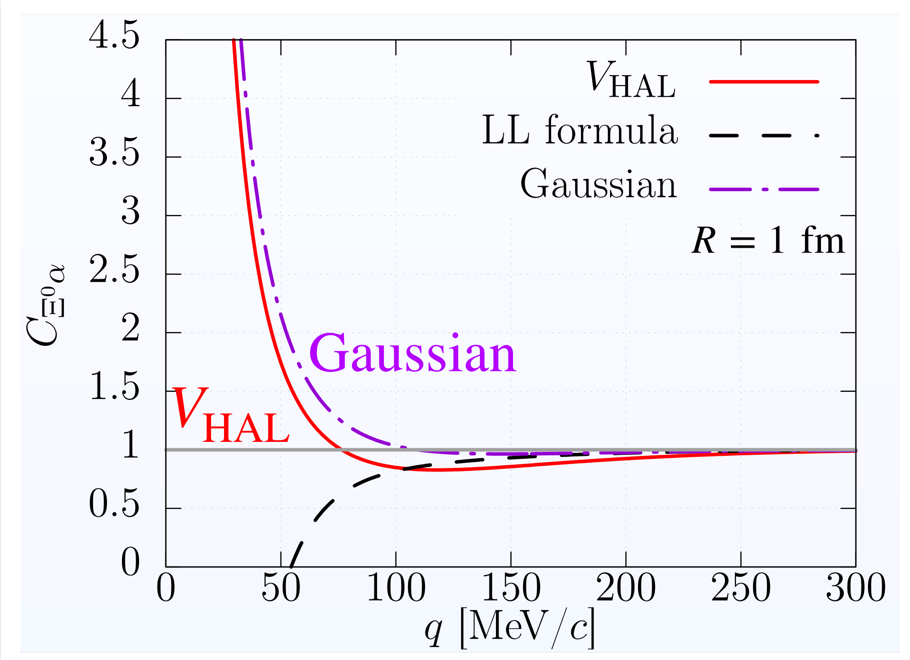
Ξα correlation

- Detailed potential dependence
  - Compare the folding potential results with simpler models
  - Purely attractive Gaussian potential

$$V_{\mathrm{Gaussian}}(r) = V_0 \exp(-r^2/b^2),$$

    - Larger $C(q)$ than the folding potentials
    - No dip structure at $q \sim 100$ MeV/c

⇒ Repulsive core causes dip in $C_{\Xi\alpha}$!



  - Lednicky-Lyuboshitz (LL) formula

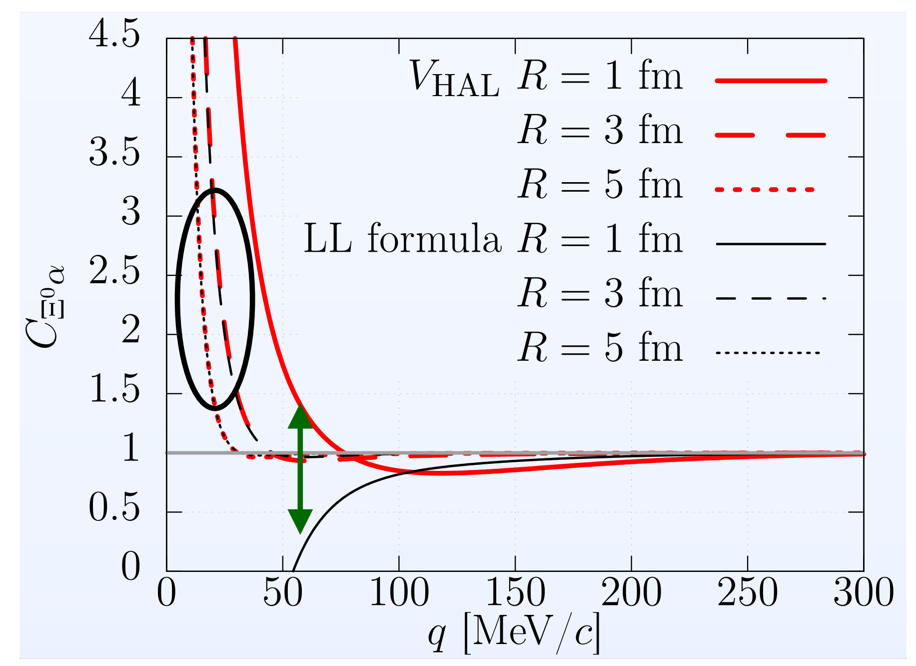
R. Lednicky, et al. Sov. J. Nucl. Phys. 35(1982).

    - approximation by asymptotic wave function

—> Good description for short range potential

    - Large deviation due to the large effective range for small source

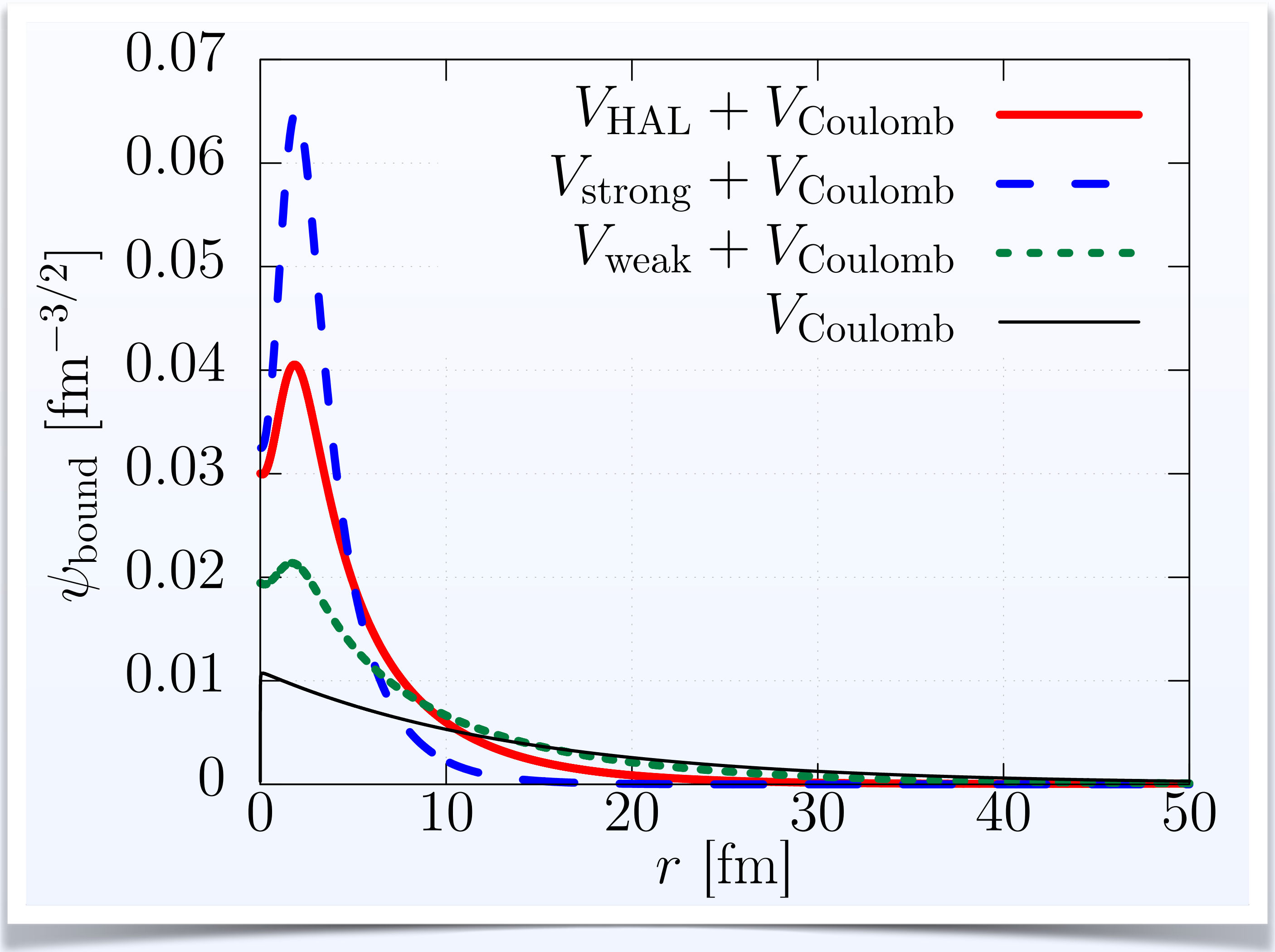
$r_e = 4.5$ fm (V_{HAL})



⇒ LL formula does not work for $C(q)$ from small source.

$\Xi\alpha$ correlation

- $\Xi^{-}\alpha$ bound state and Coulomb effect



potential	<Vshort>	<VCoulomb>	*B*
VHAL	**-0.93**	-0.63	0.47
Vstrong	**-4.36**	-0.94	2.08
Vweak	-0.14	**-0.36**	0.18

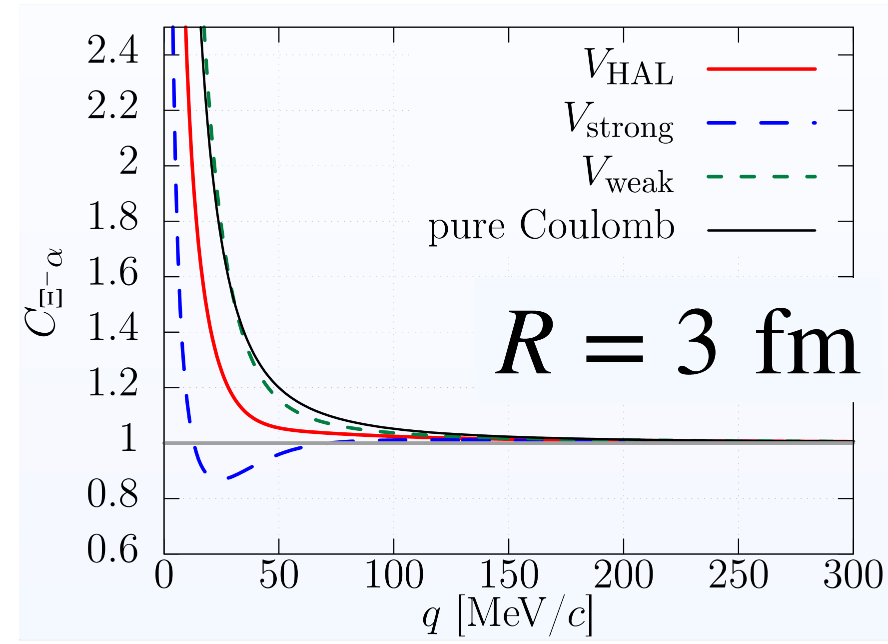
[MeV]

- V_{HAL} and V_{strong} : W.f. strongly localized in strong int. range.
  → Short range int. is dominant.
- V_{weak} : long range tail similar to pure Coulomb case
  → Coulomb int. is dominant.

$\Xi\alpha$ correlation

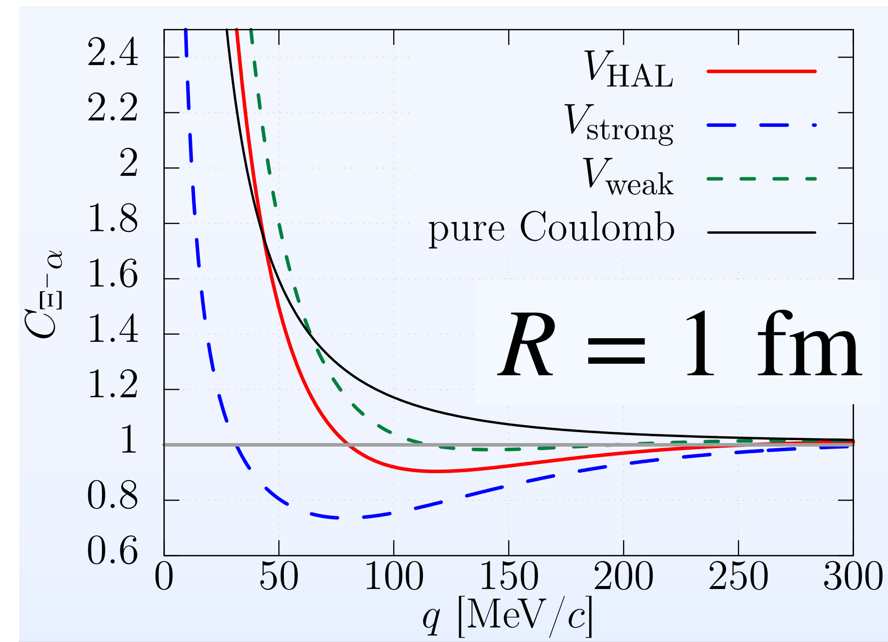
- $\Xi^{-}\alpha$ correlation

potential	E_B [MeV]
VHAL	0.47
Vstrong	2.16
Vweak	0.18



- Coulomb int. added:
  —> Strong int. effect appear as deviation from pure Coulomb
- $V_{\rm strong}$ and $V_{\rm weak}$: Coulomb enhancement added to $C_{\Xi^{0}\alpha}$

- $V_{\rm HAL}$: $C(q)$ with $R = 3$ fm turns to be suppressed
  —> Typical source size dependence with bound state



➡ <u>${}^{5}_{\Xi}$H can be distinguished by the source size dependence</u>

- Dip structure at $q \sim 100$ MeV/c for $R = 1$ fm

➡ <u>Repulsion core effect can be investigated with small source</u>

Summary

- Femtoscopic study on the hadron interaction
  - Direct approach to the low-energy interaction
  - Sensitive to the near-threshold resonance

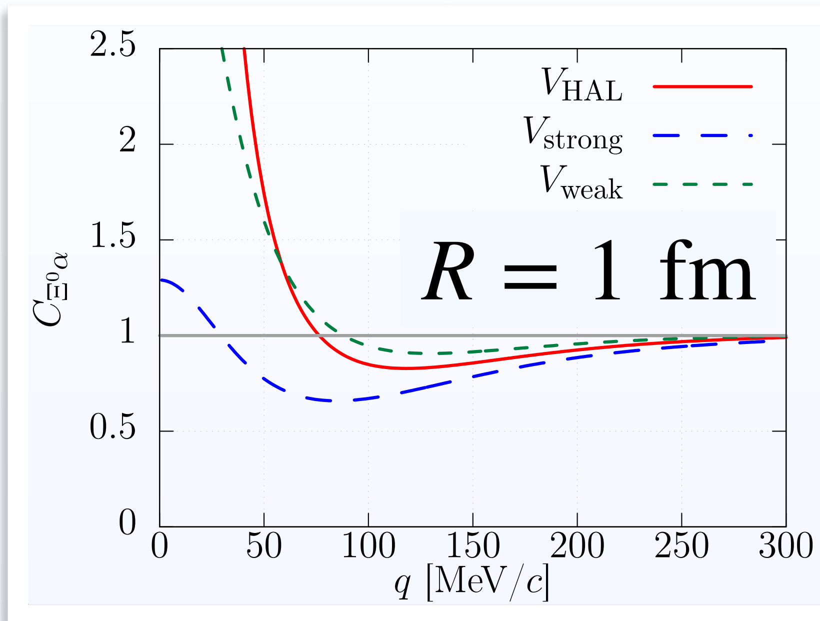
- $K^{-}p$ correlation
  - Chiral SU(3) model give the good agreement with the various $K^{-}p$ data
  - Finite deviation in small source indicates the stronger coupling

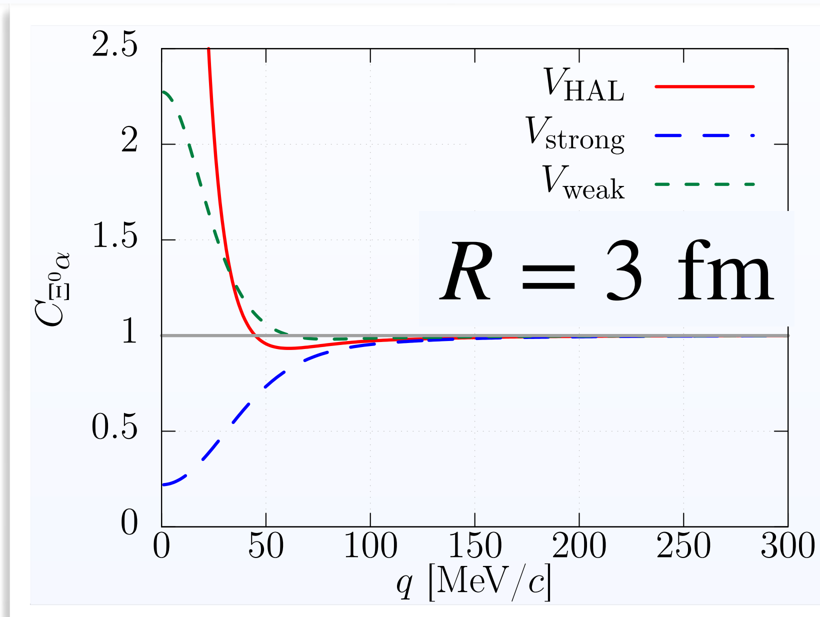
- $\Xi\alpha$ correlation function
  - Existence of ${}^{5}_{\Xi}\mathrm{H}$ can be tested with the source size dependence
  - Dip structure at intermediate momentum by the repulsive core

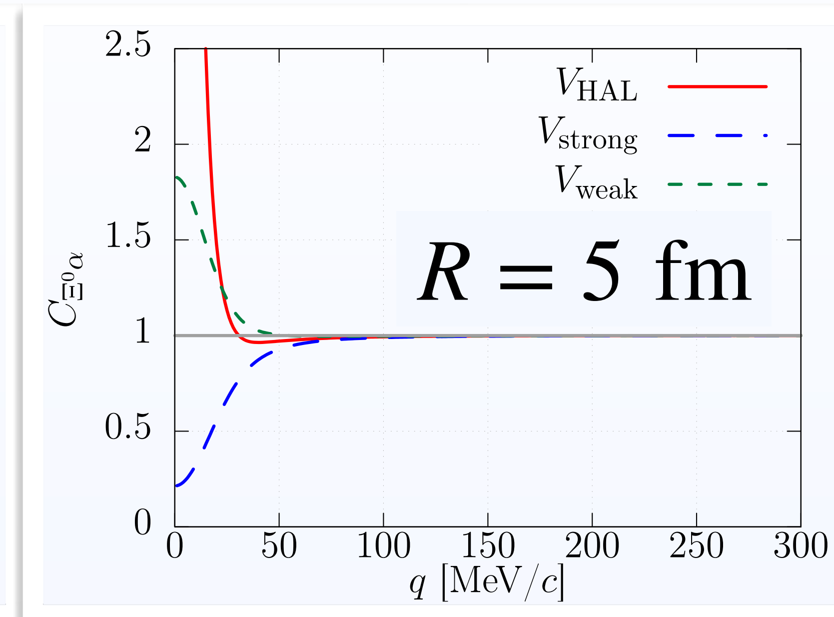
Thank you for your attention!

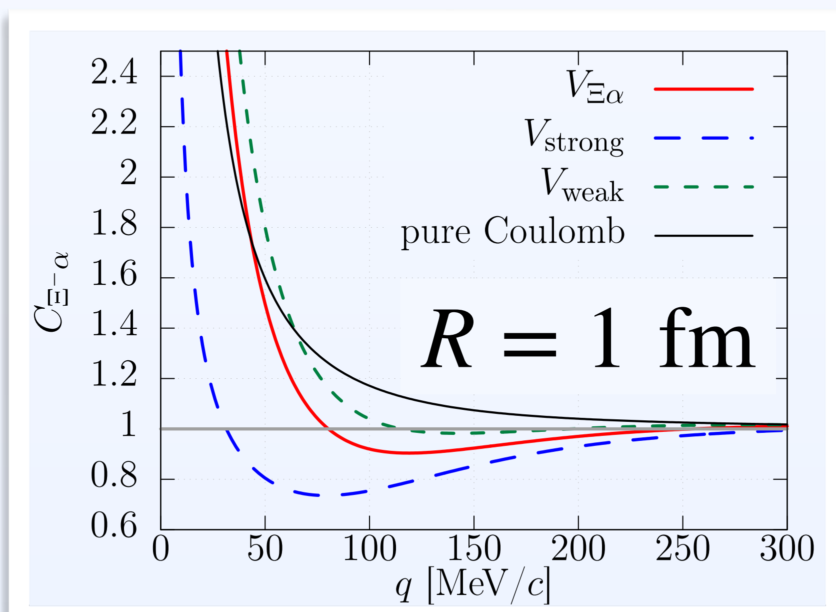
Thank you!

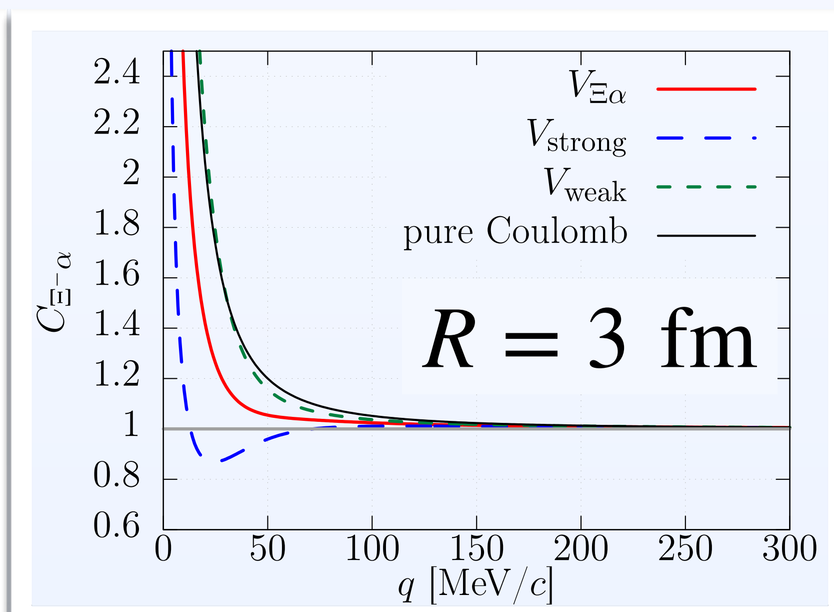
$\Xi\alpha$ correlations

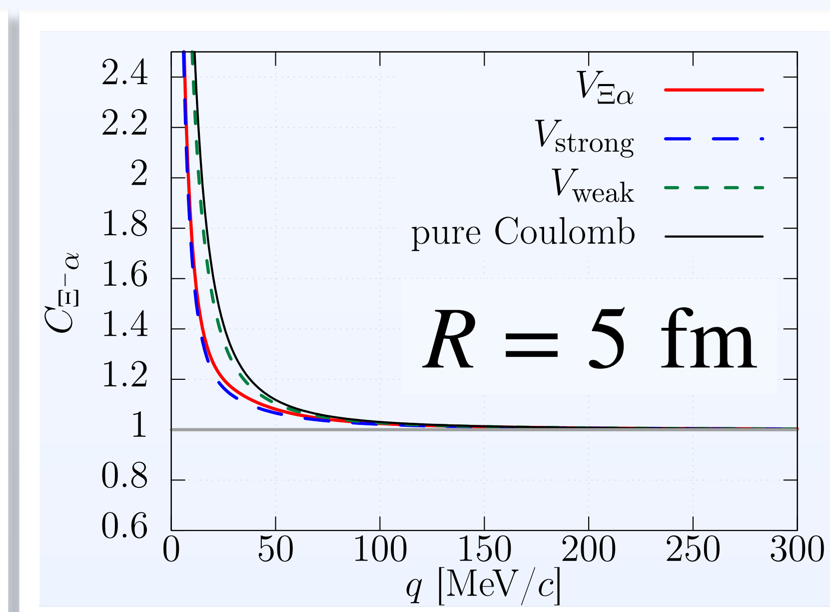


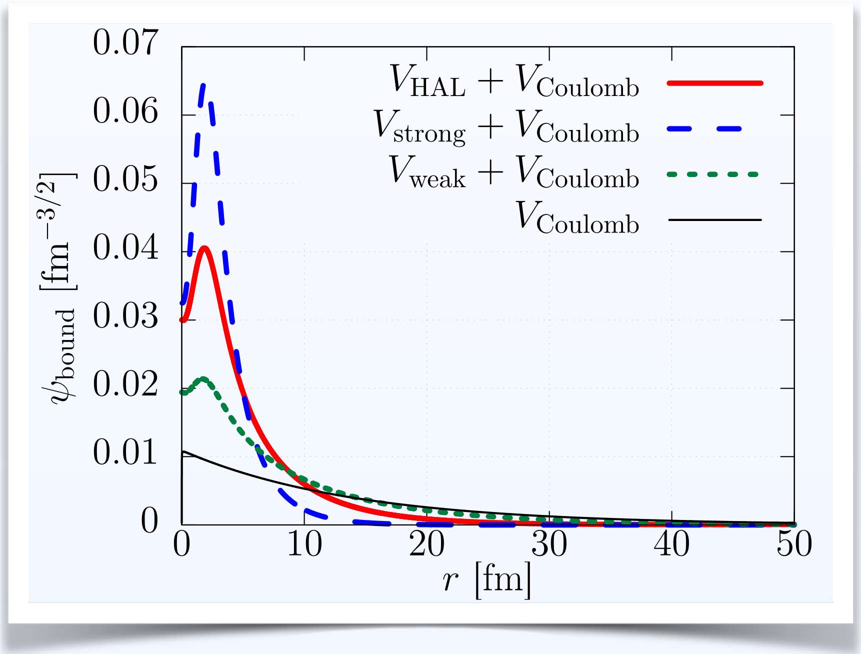


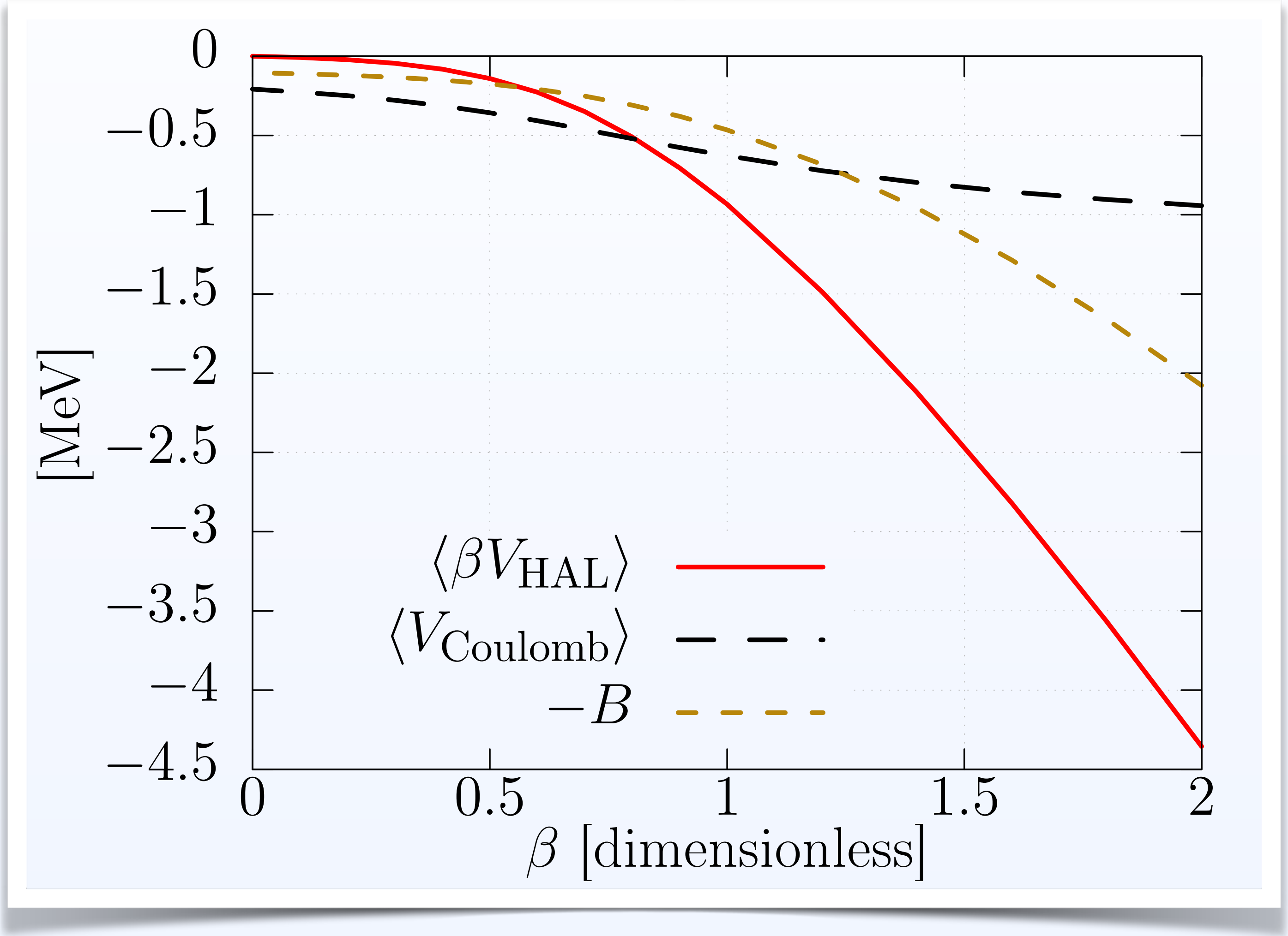




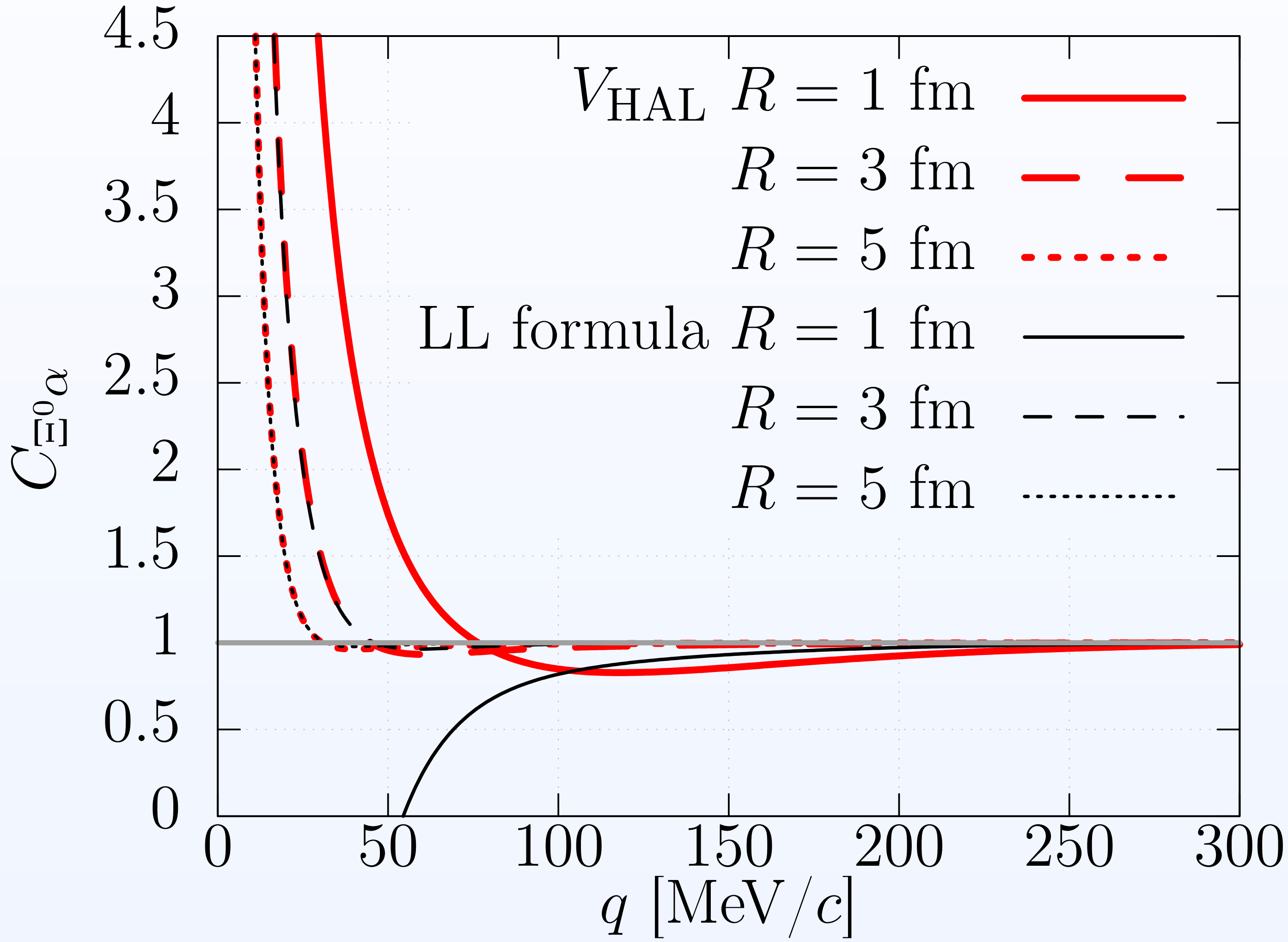





0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
10
20
30
40
50
r [fm]
ψ_{bound} [fm$^{-3/2}$]
$V_{\mathrm{HAL}} + V_{\mathrm{Coulomb}}$
$V_{\mathrm{strong}} + V_{\mathrm{Coulomb}}$
$V_{\mathrm{weak}} + V_{\mathrm{Coulomb}}$
V_{Coulomb}


0
−0.5
−1
−1.5
−2
−2.5
−3
−3.5
−4
−4.5
[MeV]
0
0.5
1
1.5
2
β [dimensionless]
$\langle \beta V_{\mathrm{HAL}} \rangle$
$\langle V_{\mathrm{Coulomb}} \rangle$
$-B$

LL formula for R = 1,3,5 fm



Gaussian potenial for V_{HAL}