Moats, moats, everywhere

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= NOPPRSW arXiv: 2410.xxxxx

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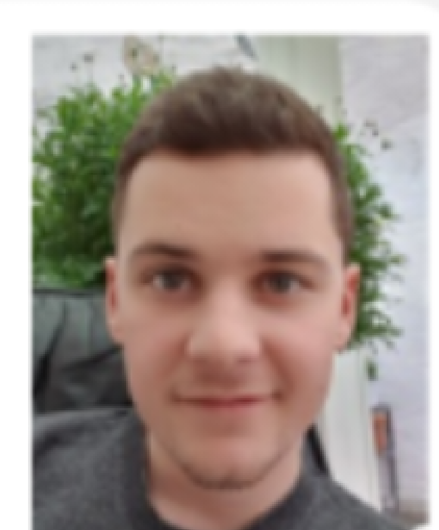


BROOKHAVEN NATIONAL LABORATORY

Rob Pisarski

MIT

Los Alamos NATIONAL LABORATORY EST. 1943

Stella Schindler & Frodo

Critical End Point (CEP):

Asakawa & Yazaki '89

Rajagopal, Shuryak & Stephanov '98, '99 + ...

$\vec{\phi} = (\sigma, \vec{\pi}) \qquad m_\pi = 0$

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial_\mu \vec{\phi})^2 + m^2 \vec{\phi}^2 + \lambda(\vec{\phi}^2)^2 + \kappa(\vec{\phi}^2)^3$$

Usually: $\lambda > 0$. $m^2 < 0$ - broken

$>$ - symmetric

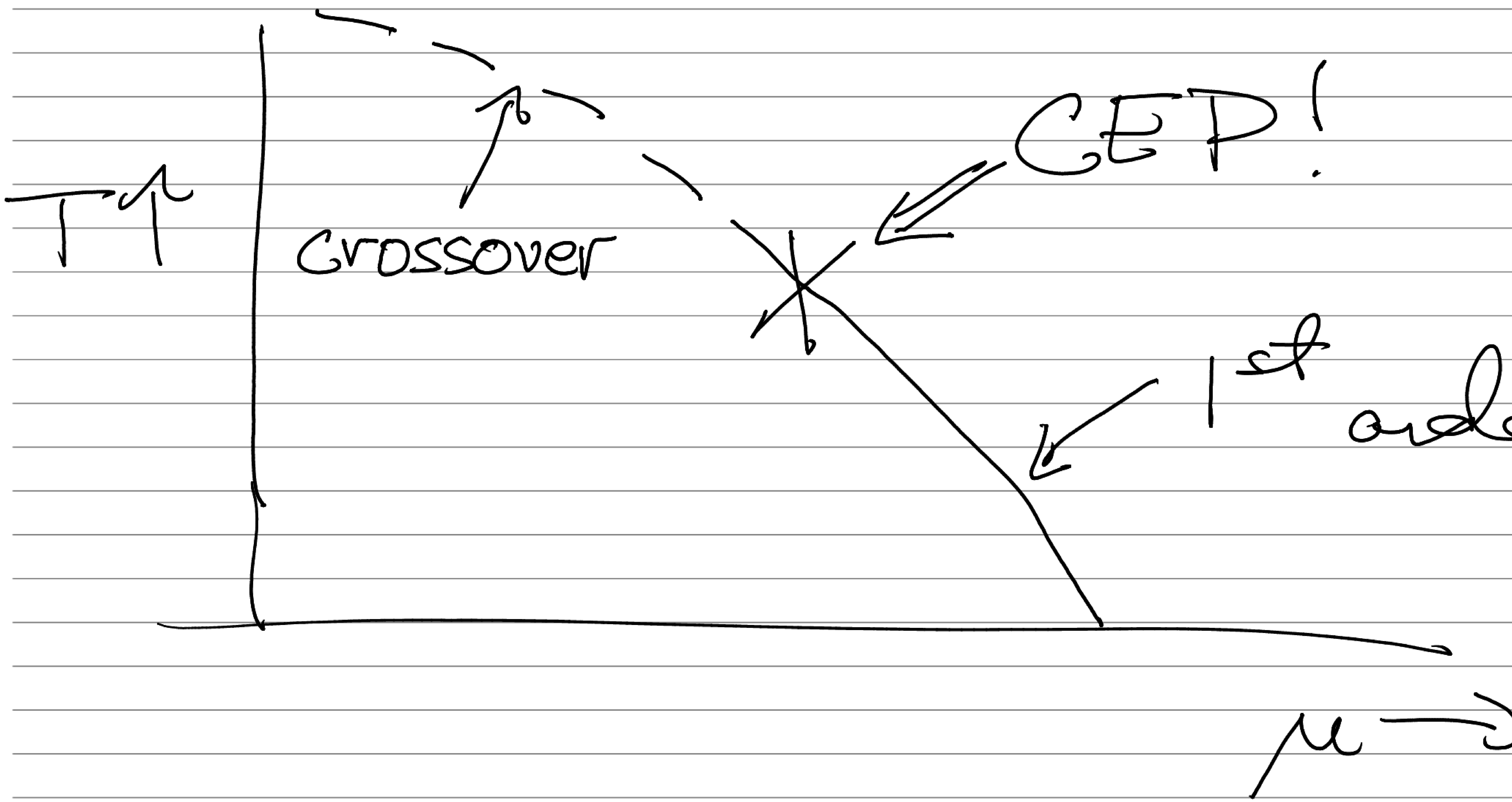
$=$ - 2nd order

BUT: $\lambda < 0$ $(\kappa > 0)$ ok $\Rightarrow$ 1st order

$\lambda = 0$ = tri-critical point

But $m_\pi \neq 0$! Still, can have $\lambda < 0$

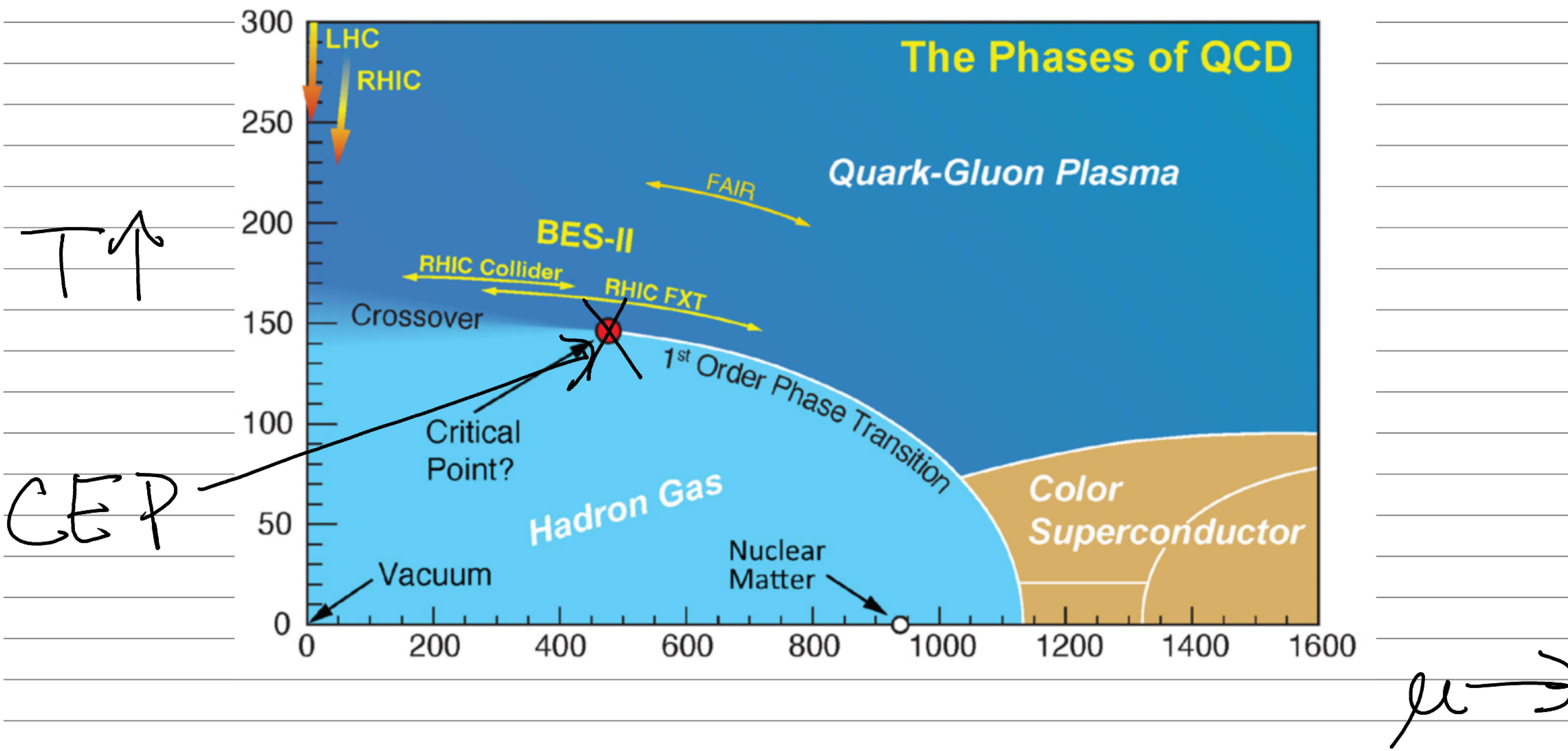
$\lambda = 0$ = CEP, True 2nd order PT



CEP & QCD phase diagram.

Can diagnose CEP from fluctuations

Stephanov '08 + ...



CEP : Functional Renormalization Group = FRG

Wei-jie Fu, Pawlowski, Rennecke 1909.02991

$T_{CEP} \sim 110$ $\mu^{Baryon}_{CEP} \sim 630$ $\pm 5\%$

All later values consistent

e.g. Shah + ... 2410.16206

Basar 2312.06952 :

Lee-Yang edge sing. + Pade

No direct evidence from lattice:

compute moments $\left.\frac{\partial^{2n} p(T,\mu)}{\partial \mu^{2n}}\right|_{\mu=0}$ & extrapolate

CEP – width?

But: in vacuum, σ meson is heavy & broad

CEP: $m_\sigma = 0$, $m_\pi \neq 0$

Going from vacuum → CEP ⇒

$m_\sigma = m_\pi$ only very close to (T_{CEP}, μ_{CEP})

Suggests narrow $\Delta T, \Delta\mu$. FRG: $\Delta T, \Delta\mu \sim \pm 2$ MeV!

Affects fluc's in broader range 1906.00936

Probed by BES @ RHIC, CBM @ FAIR

CEP: "Is that all there is, let's keep ..."

Old story: chiral spirals (spatially inhomogeneous crystals)

Here, new story: moats!

Moats!

Bodiam Castle in East Sussex

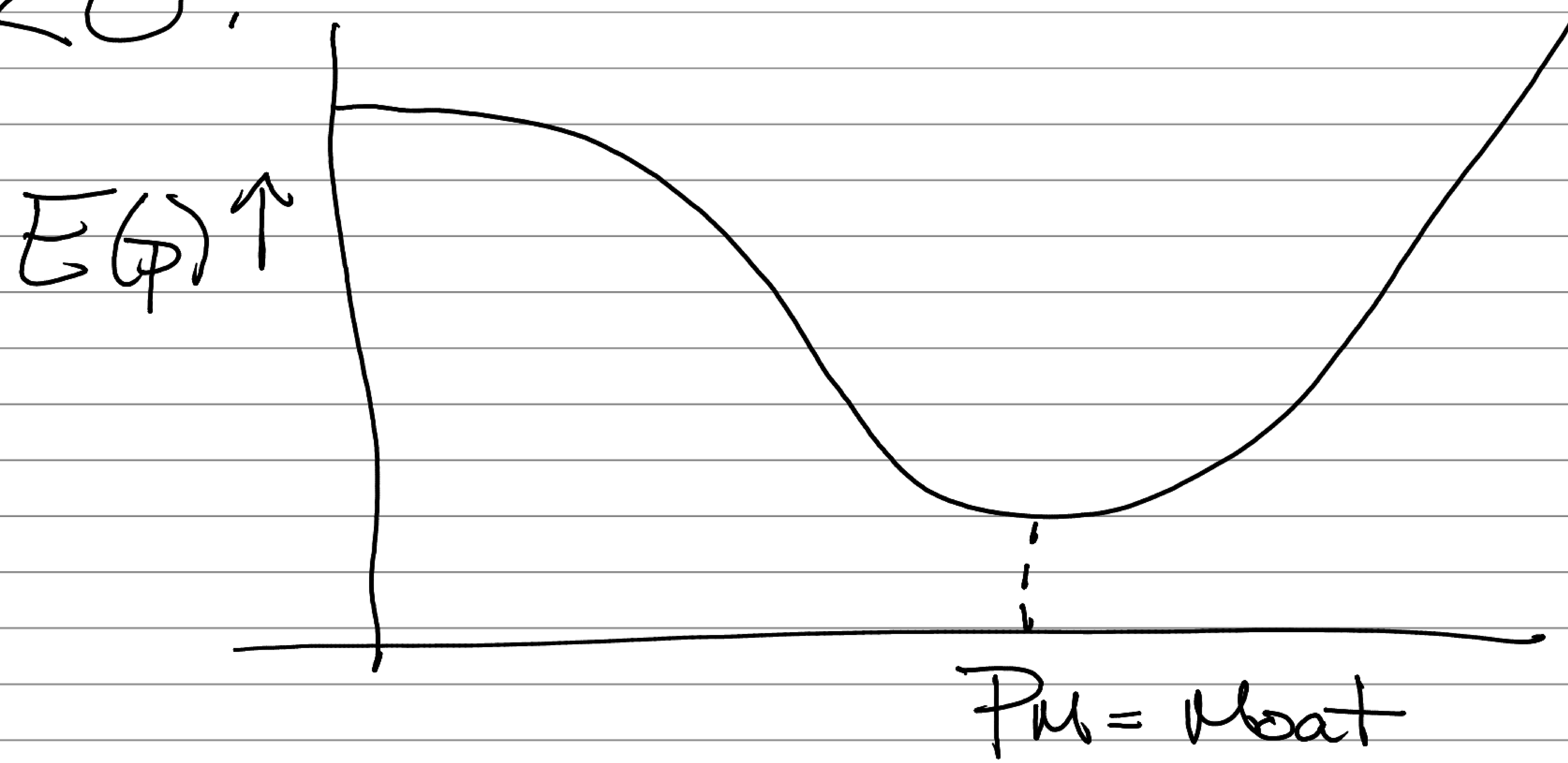
Moats in physics

Term from CM : MANY systems.

NEED T &/or $\mu \neq 0$ - lose Lorentz inv.

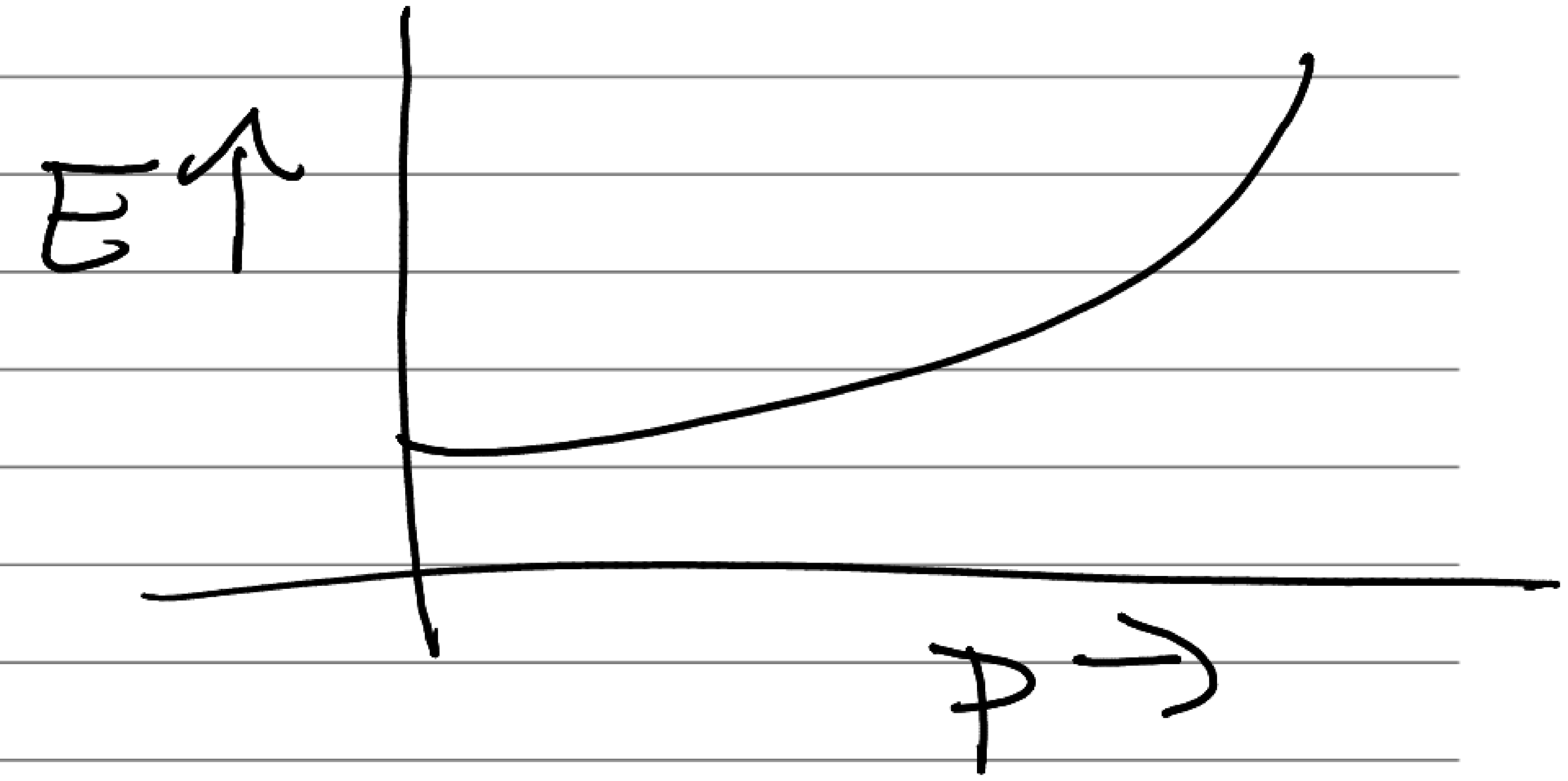
$$\mathcal{L}_{eff} = (\partial_0 \Phi)^2 + Z(\partial_i \Phi)^2 + \frac{1}{M^2}(\partial_i^2 \Phi)^2 + V(\phi)$$

If $Z < 0$:

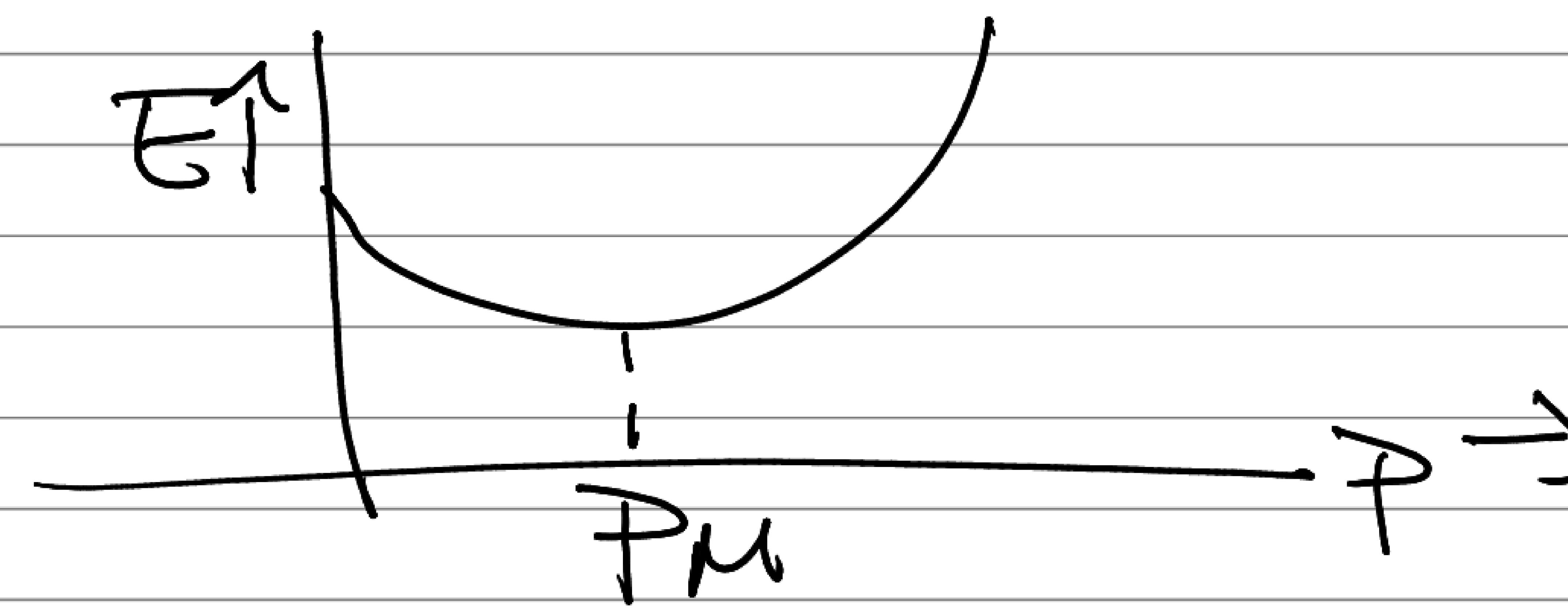


☆ Basic physics ☆

"Normal" phase:
Lightest particle
@ $p = 0$

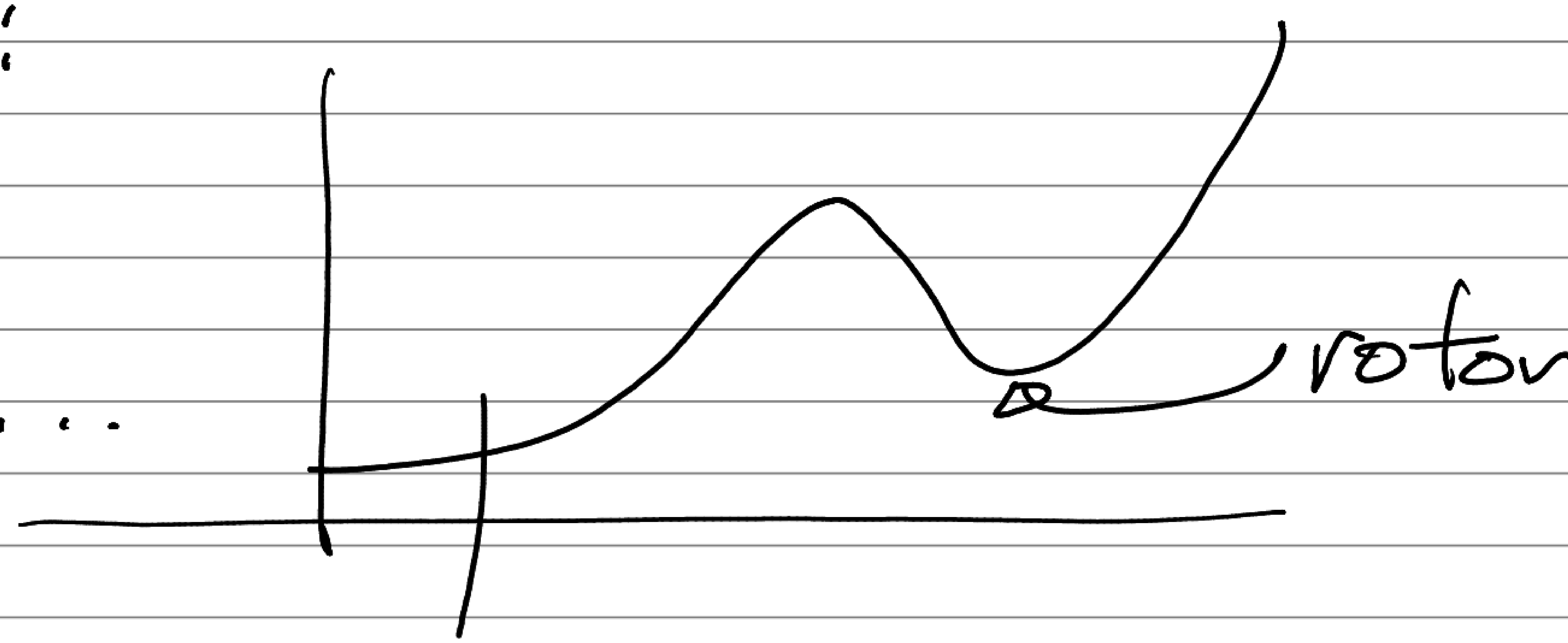


"Moat": ~~Lightest~~
@ $\underline{p_M \neq 0}$



Not roton:

Many possibilities...



Moats in Gross-Neveu

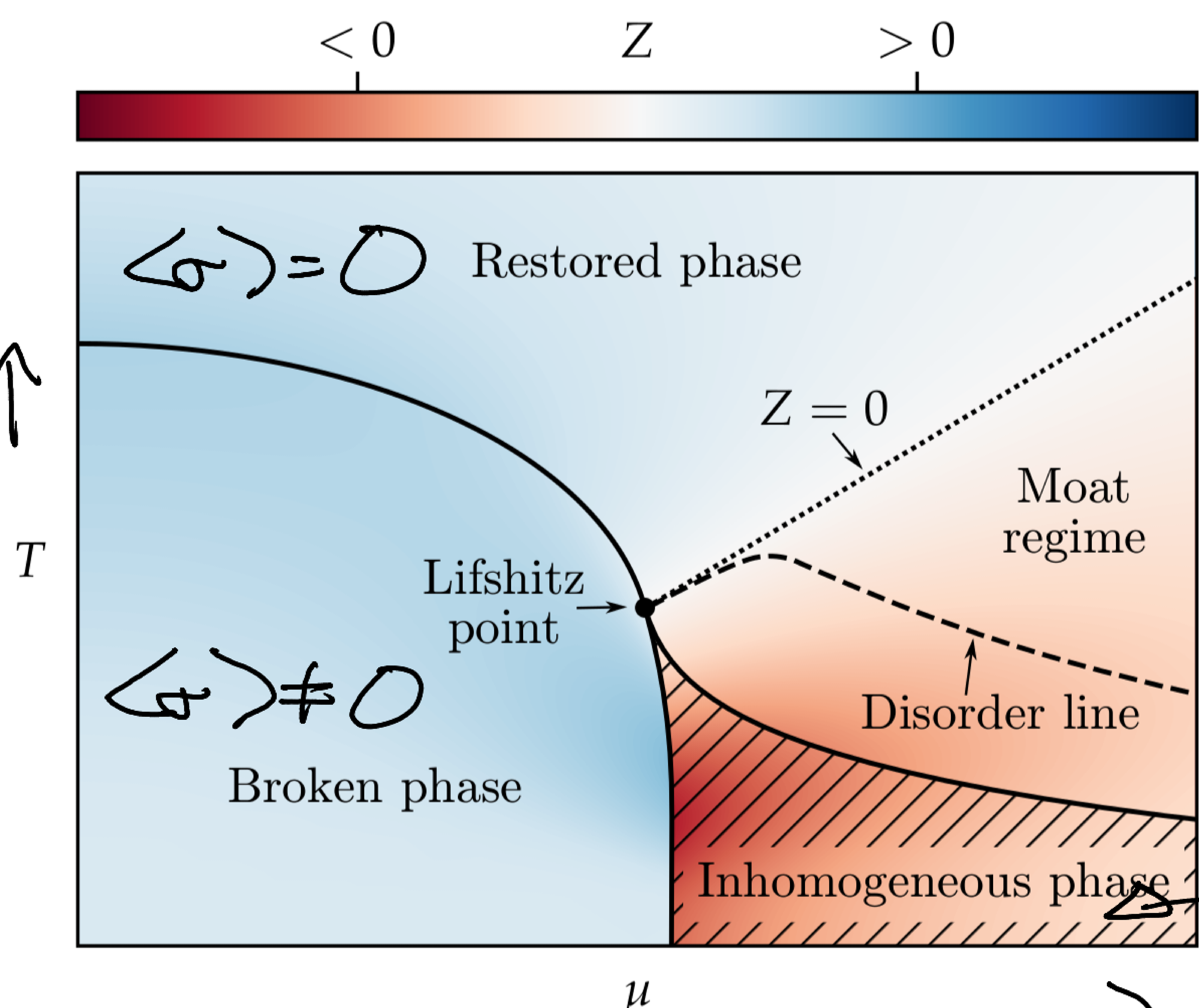
GN: 1+1 dim, $\mathcal{L}_{GN} = \bar{\psi}\, i \partial\!\!\!/\, \psi + g^2 (\bar{\psi}\psi)^2$

Asymp. free, soluble as $N = \#$ fermions $\to \infty$

Dynamical mass gen. $\Rightarrow$ χ sym. broken

KPRSW 2112.07024

$T \uparrow$



$\mu \to$

Moat regime ☆ HUGE ☆

1+1 dim's $\Rightarrow$ no CEP, Lifschitz point

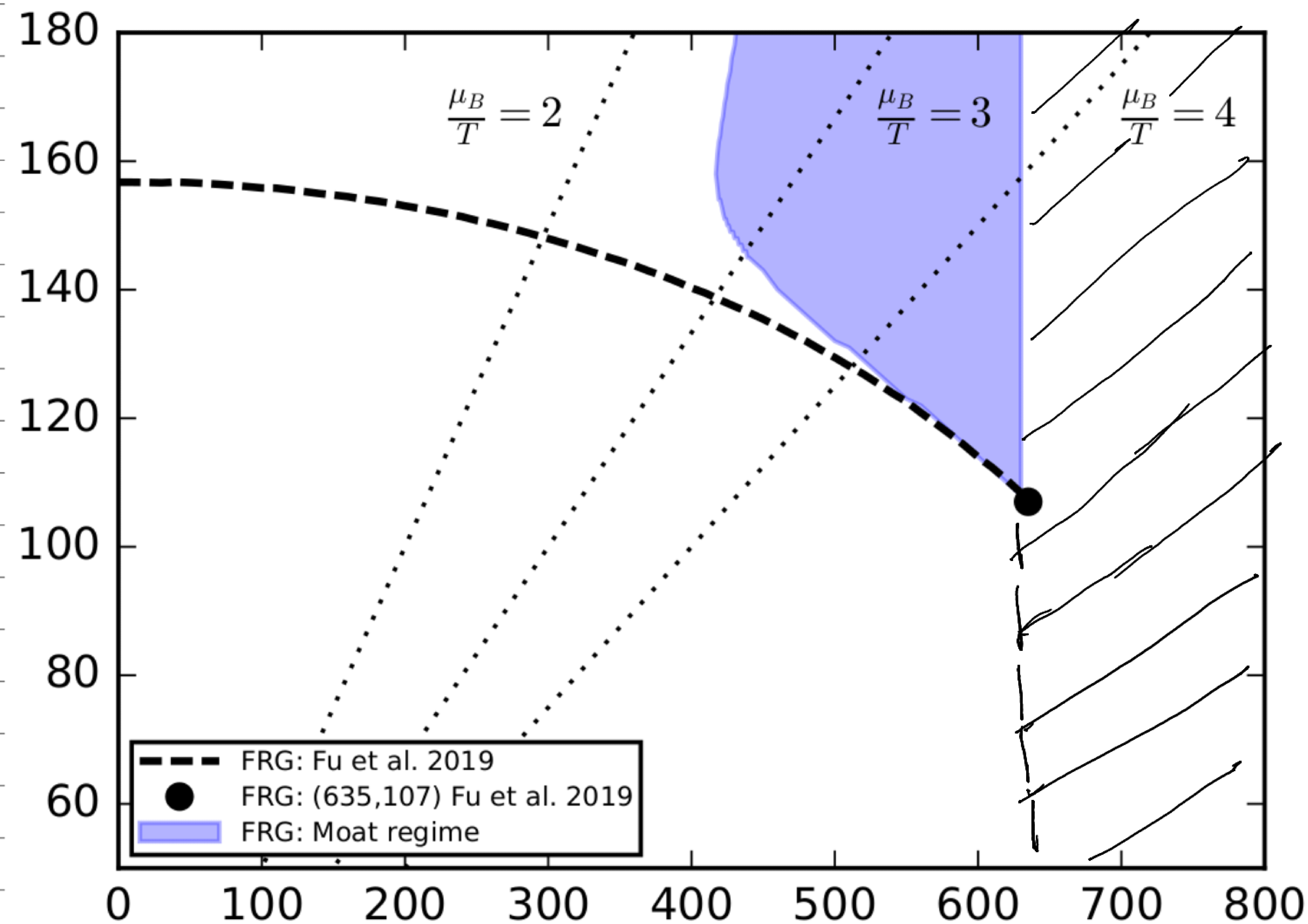
kink crystal

Moats in QCD

Wei-jie Fu, Pawlowski, RDP, Rennecke, Rui Wen, Shi Yin

2411.xxxxx ?

Using FRG: Cannot calculate (yet) for $\mu > \mu_{CEP}$



But find ☆ HUGE ☆ moat regime for $\mu < \mu_{CEP}$, $T > T_{CEP}$!

Surely extends to $\mu > \mu_{CEP}$, $T \gtrless T_{CEP}$

Dileptons in a moat

NOPPRSW: original $\mathcal{L} =$

$$\mathcal{L}_{eff} = (\partial_0 \phi)^2 + Z(\partial_i \phi)^2 + \frac{1}{M^2}(\partial_i^2 \phi)^2 + V(\phi)$$

Gauge invariance $\Rightarrow$ $\partial_i \to D_i$

$$\mathcal{L}^0_{eff} = (D_0 \phi)^2 + Z(D_i \phi)^2 + \frac{1}{M^2}(D_i^2 \phi)^2 + V$$

Added one new operator = dimension 6

Eight other operators = dim. 6

8 new operators

No Lorentz inv. ($T, \mu \neq 0$). $D_0 \neq D_i$

Six boring operators dim. 6:

$$(D_0\vec{\phi})^2 |\vec{\phi}|^2 , \ (D_i\vec{\phi})^2 |\vec{\phi}|^2$$

$$(\vec{\phi} \cdot D_0\vec{\phi})^2 , \ (\vec{\phi} \cdot D_i\vec{\phi})^2 ,$$

$$F_{0i}^2 |\phi|^2 , \ F_{ij}^2 |\phi|^2$$

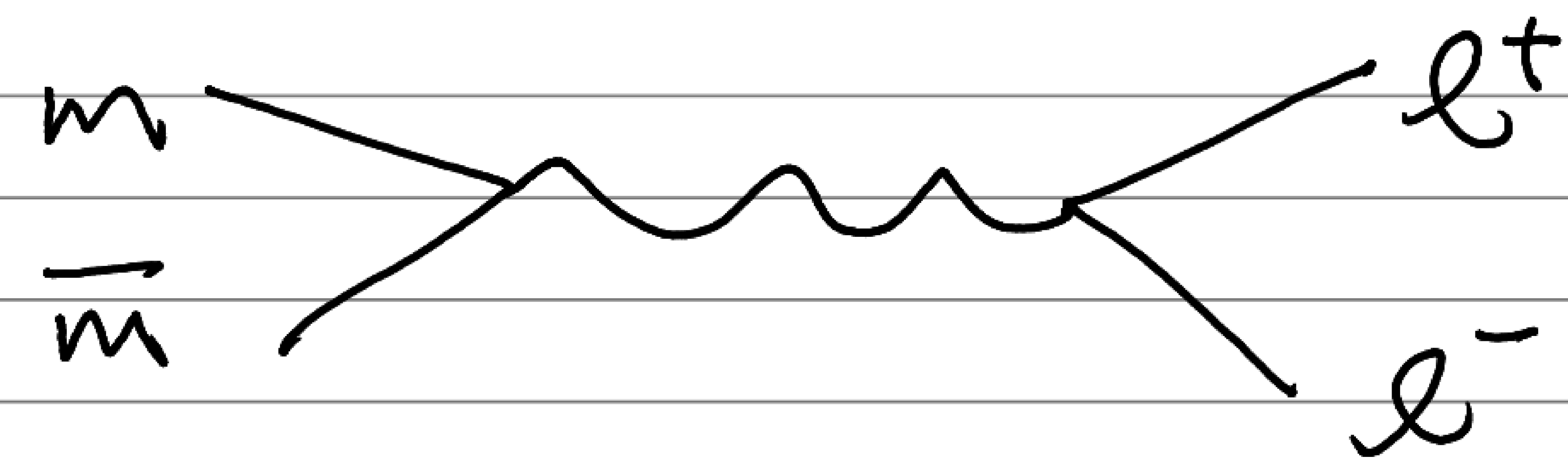
Two novel:

$$ie F_{0i} \left((D_0\phi)^* D_i\phi - \text{c.c.} \right) , \ ie F_{ij} (D_i\phi)^* D_j\phi$$

C = charge conjugation: $F_{\mu\nu} \to -F_{\mu\nu}$, $\phi \leftrightarrow \phi^*$

$\Rightarrow$ novel terms C-even.

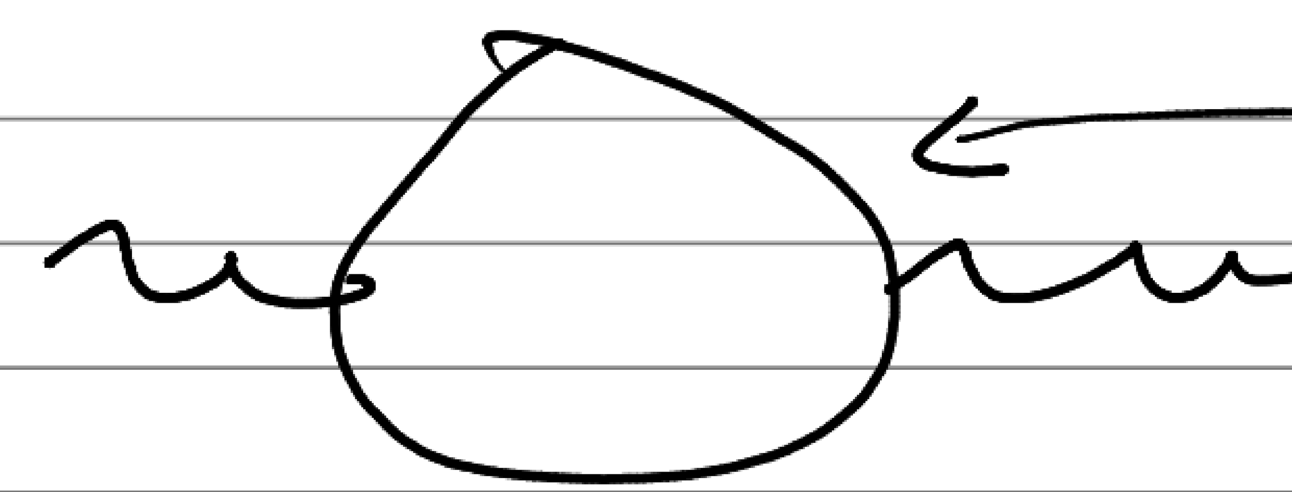
Dilepton rate

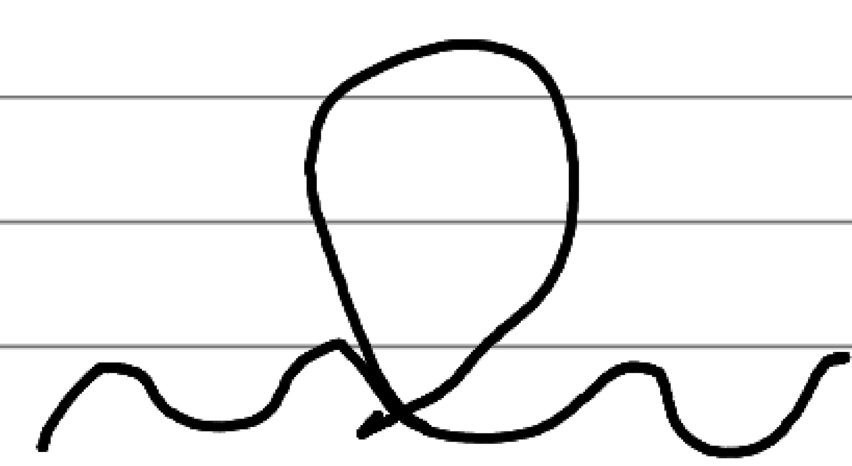
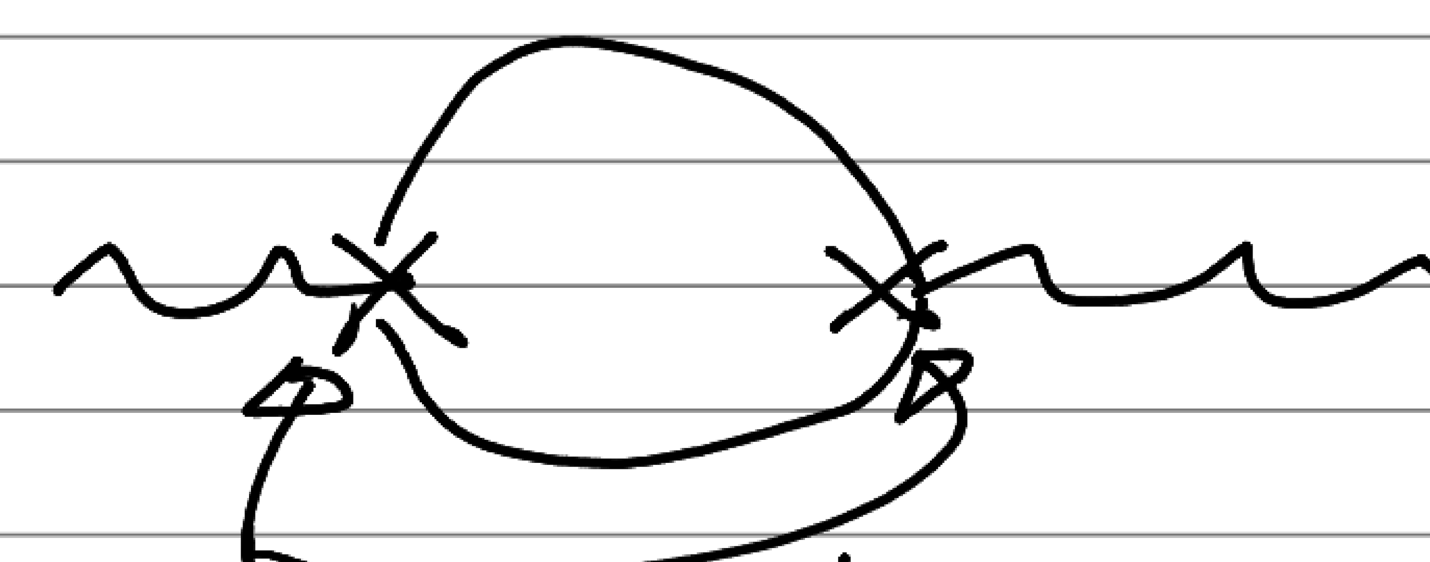
Rate $\sim \Bigg|$ 

$\Bigg|^2$

$\sim \propto \mathrm{Im}\, \Pi^{\mu\mu}(\omega, \bar{p})$ $\quad p^\mu$ of $\ell^+ \ell^-$

Take $\bar{p} = 0 \Rightarrow$ back to back $\ell^+ \ell^-$ in rest frame

$\mathrm{Im}\,\Pi \sim \mathrm{Im}$  $\mathcal{L}^0_{eff}$

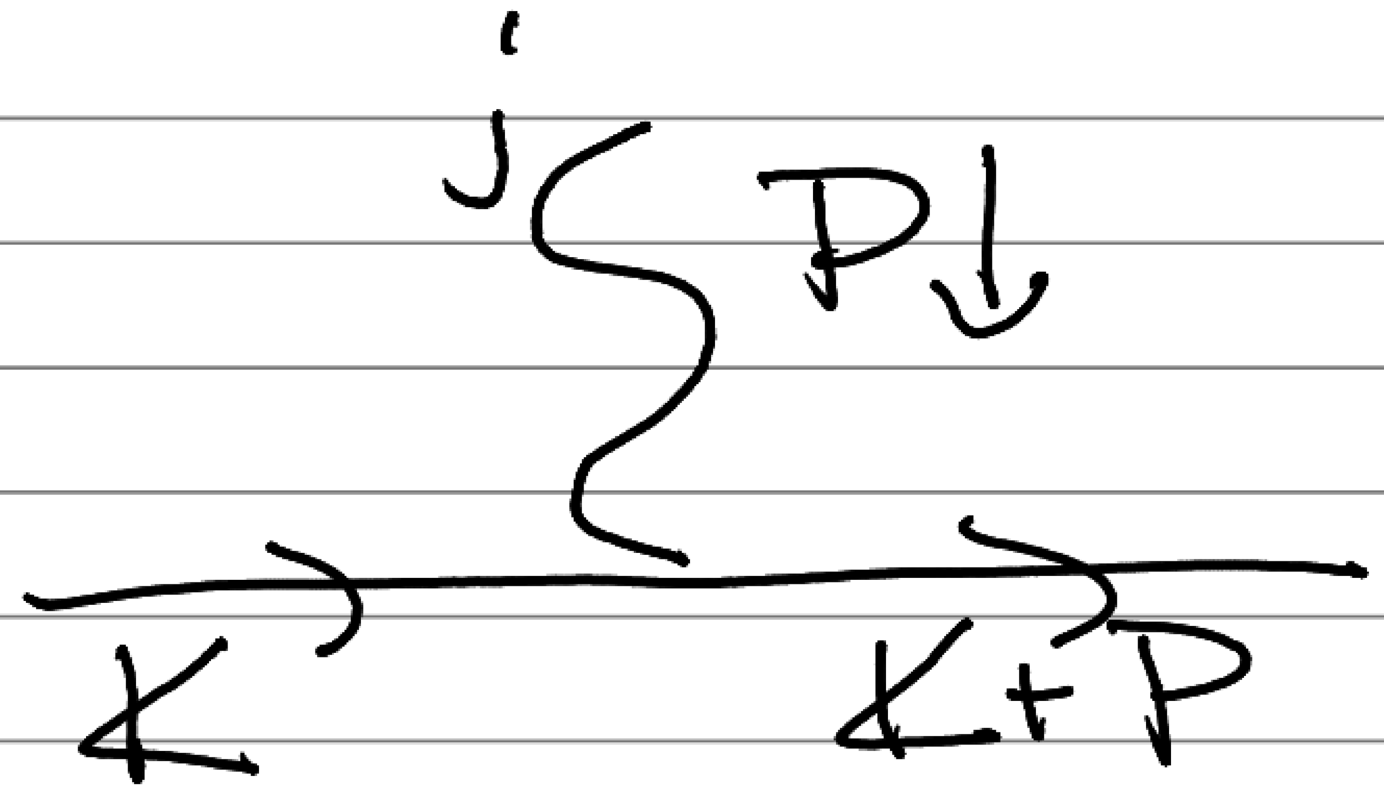
$+ \,\mathrm{Im}$  $+ \,\mathrm{Im}$ 

6 boring op's = tadpoles $= 0$

2 novel op's $= 0$ @ $\bar{p} = 0$ (not obvious)

Back to back $\ell^+\ell^-$

$\mathcal{L}^0_{\text{eff}}$: $\quad \Delta^{-1} = p_0^2 + \dfrac{(\bar{p}^2)^2}{M^2} + Z\bar{p}^2 + m^2$



$$= \Gamma^j(P,K)$$

$$= ie\,(2k+p)^j \Big(\underbrace{Z}_{|D\phi|^2} + \underbrace{\frac{1}{M^2}\big(\bar{k}^2 + (\bar{k}+\bar{p})^2\big)}_{|D^2\phi|^2} \Big)$$

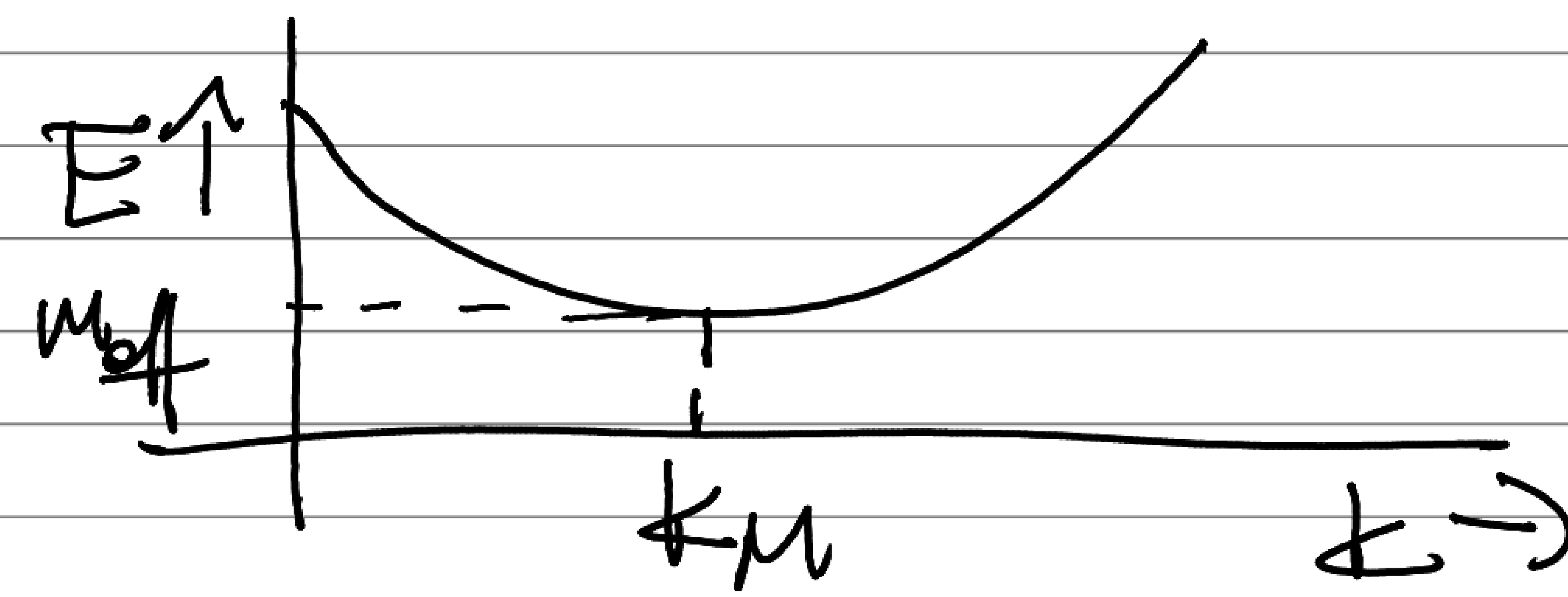
Satisfies Ward id.:

$$P^\mu \Gamma^\mu = ie\left(\Delta^{-1}(K+P) - \Delta^{-1}(K)\right)$$

$\Rightarrow P^\mu \Pi^{\mu\nu} = 0$ which is good.

Bottom of the Moat

If $Z<0$



$$E^2 = \frac{(k^2)^2}{M^2} + Z k^2 + m^2 = \frac{1}{M^2}\left(k^2 - k_M^2\right)^2 + m_{\rm eff}^2$$

About the bottom of the moat,

$$\vec{k} \simeq k_M \hat{k} + \delta\vec{k}$$

$$E \sim m_{\rm eff} + \frac{2k_M^2}{m_{\rm eff} M}\left(\hat{k}\cdot\delta\vec{k}\right)^2 + \ldots$$

$=$ quadratic in $(\delta k)^2$. Of course...

van Hove singularity?

Near threshold, $\omega = 2m_{\text{eff}} + \delta\omega$

$$\delta(\omega - 2m_{\text{eff}}) \sim \delta(\delta\omega + \dots (\delta k)^2)$$

$$\frac{1}{\sqrt{\omega - 2m_{\text{eff}}}} \quad \delta(\delta k)$$

= (van Hove) sing.! **BUT:**

$$\Gamma^{ij}(P, k) \sim 2iek^{j}\left(\frac{4k_M}{M^2}\, k \cdot \delta k\right)$$

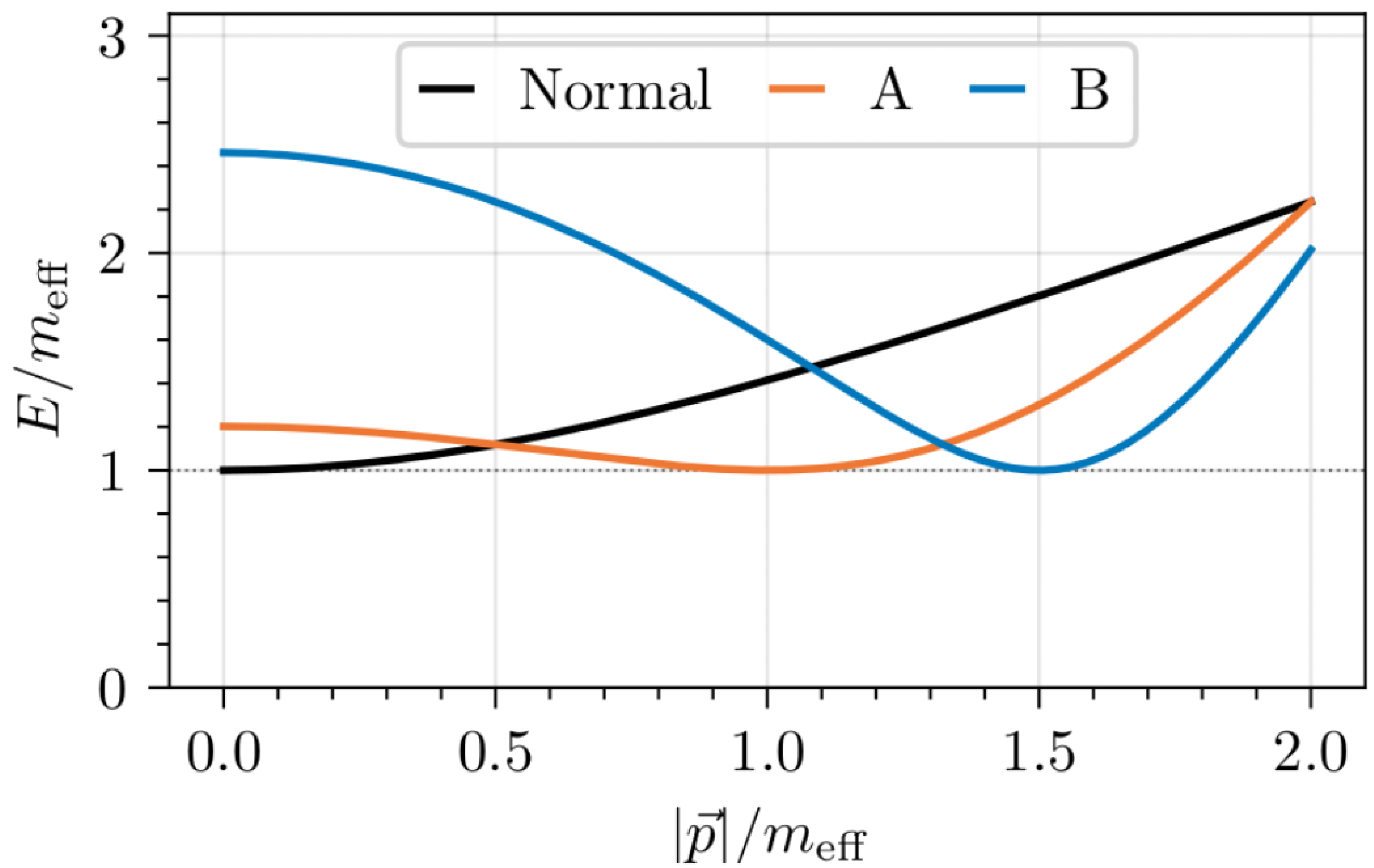
2 vertices $\Rightarrow$ NO sing.

Moaton-Moaton production is enhanced relative to normal π's!

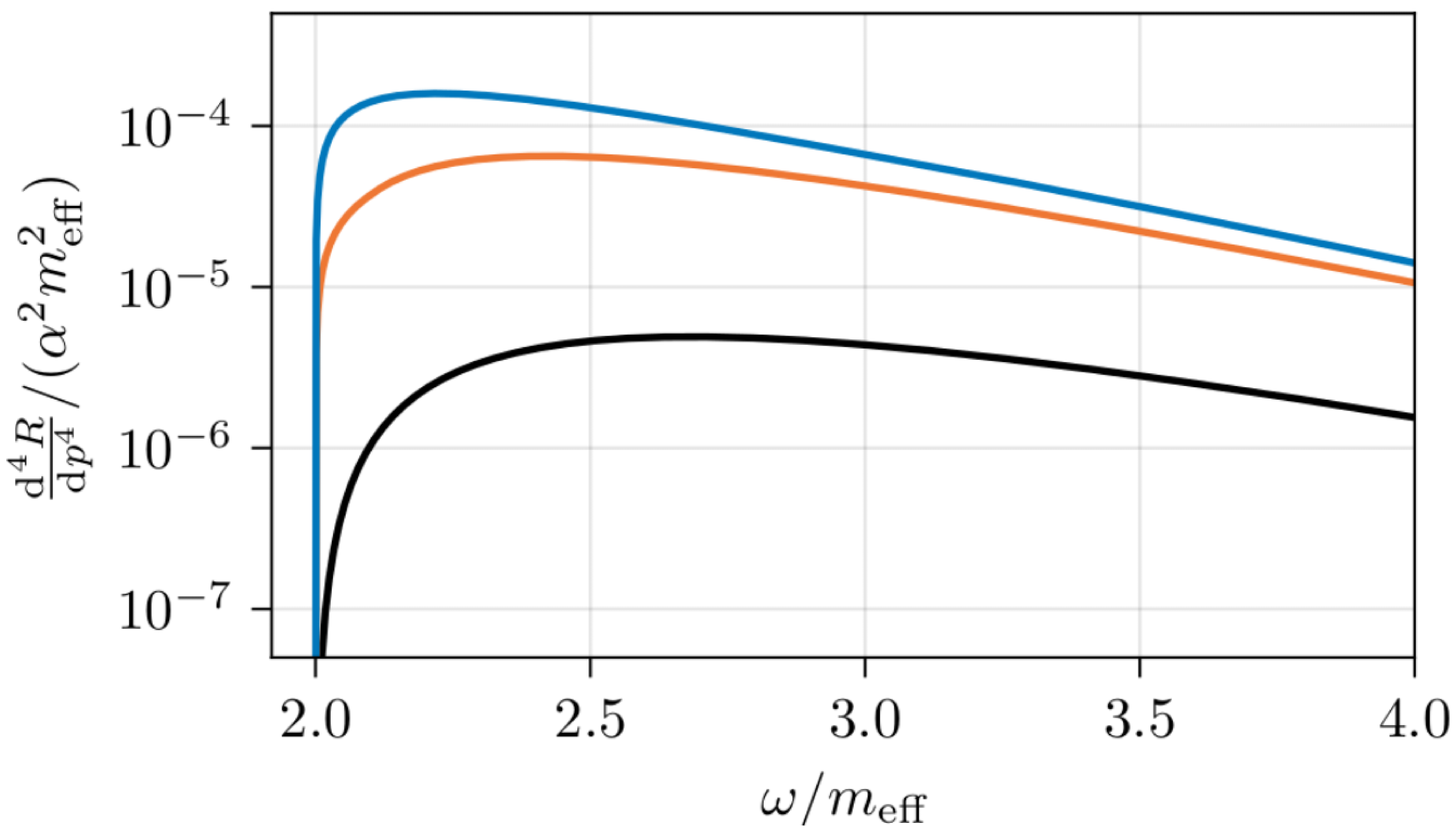
But no van Hove sing.

Moatons enhance $\ell^+\ell^-$

Disp. relation



$\ell^+\ell^-$ rate



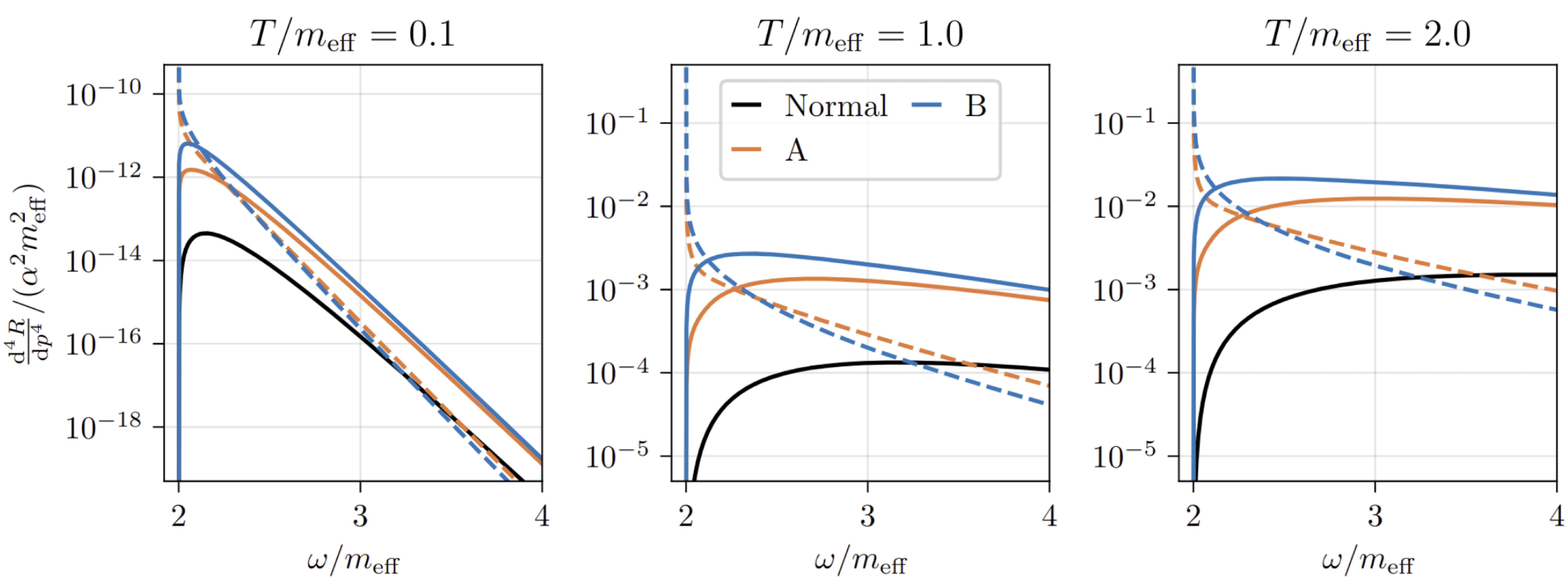
N.B.: m_{eff} is NOT constant, function of T & μ!

Without correct Γ^μ, van Hove sing.'s

Hayashi & Tsue 2407.08523

$\ell^+\ell^-$ from chiral spiral; do not gauge $(\partial^2\phi)^2$

van Hove sing.'s appear. But $P_\mu \Pi^{\mu\nu} \neq 0$.

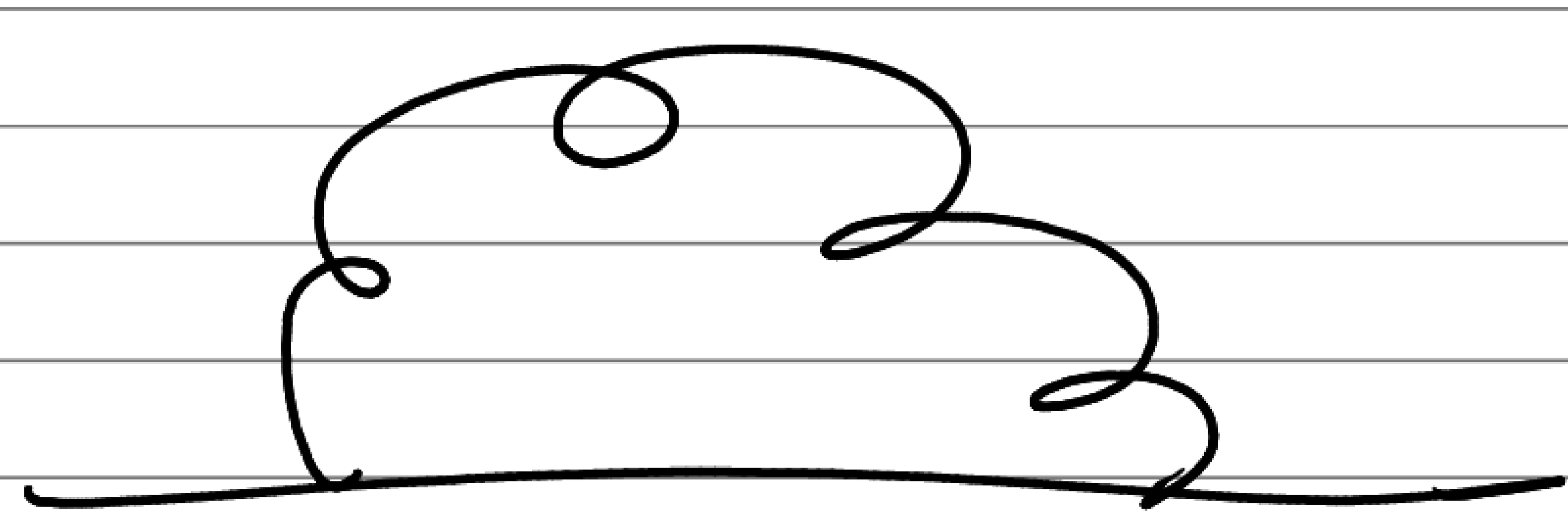


van Hove sing.'s ?

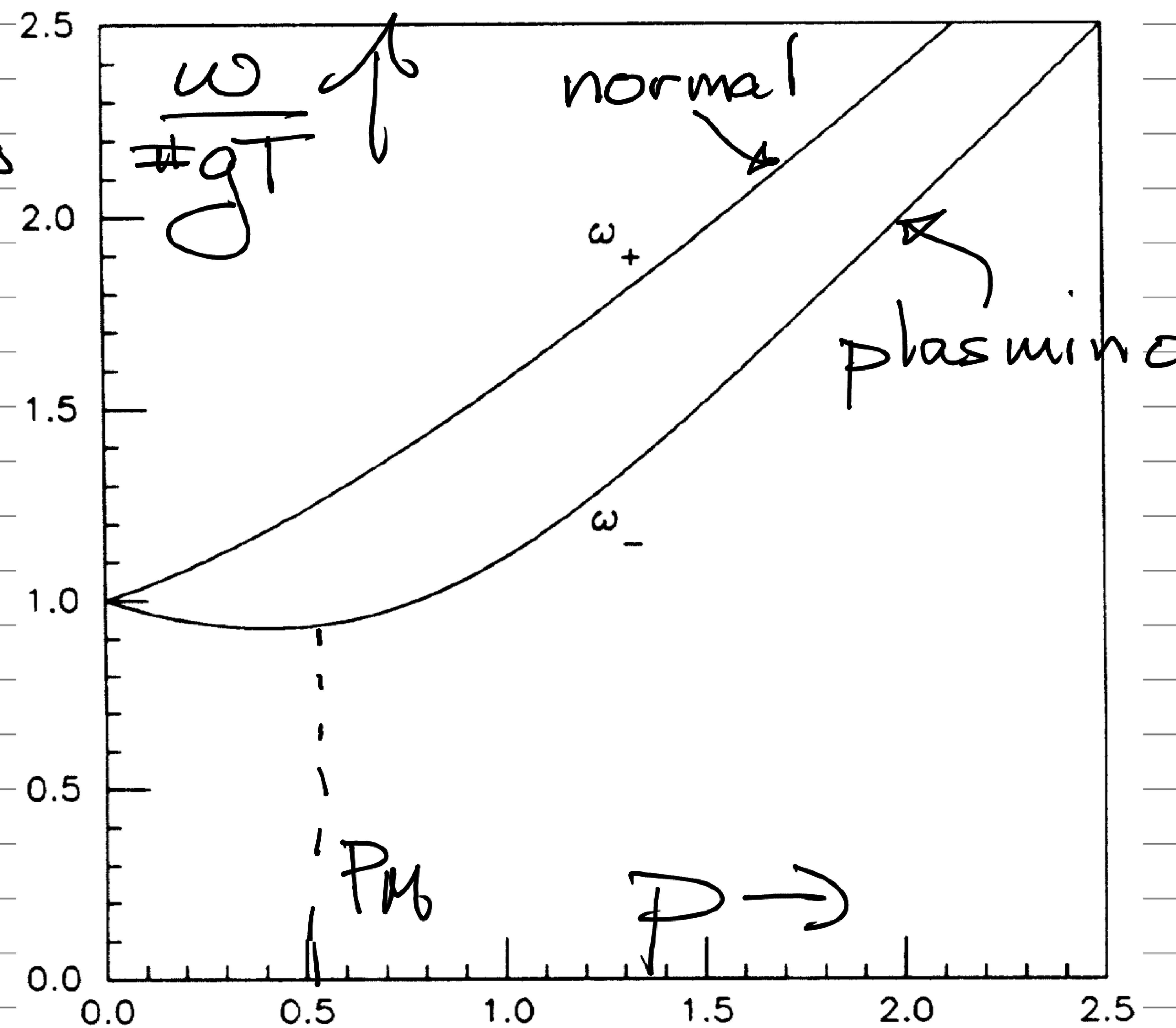
Do van H sing.'s always cancel? No!

"Plasmino" in Hard Thermal Loops

To $\sim gT$, massless quarks get thermal "mass" $\sim gT$



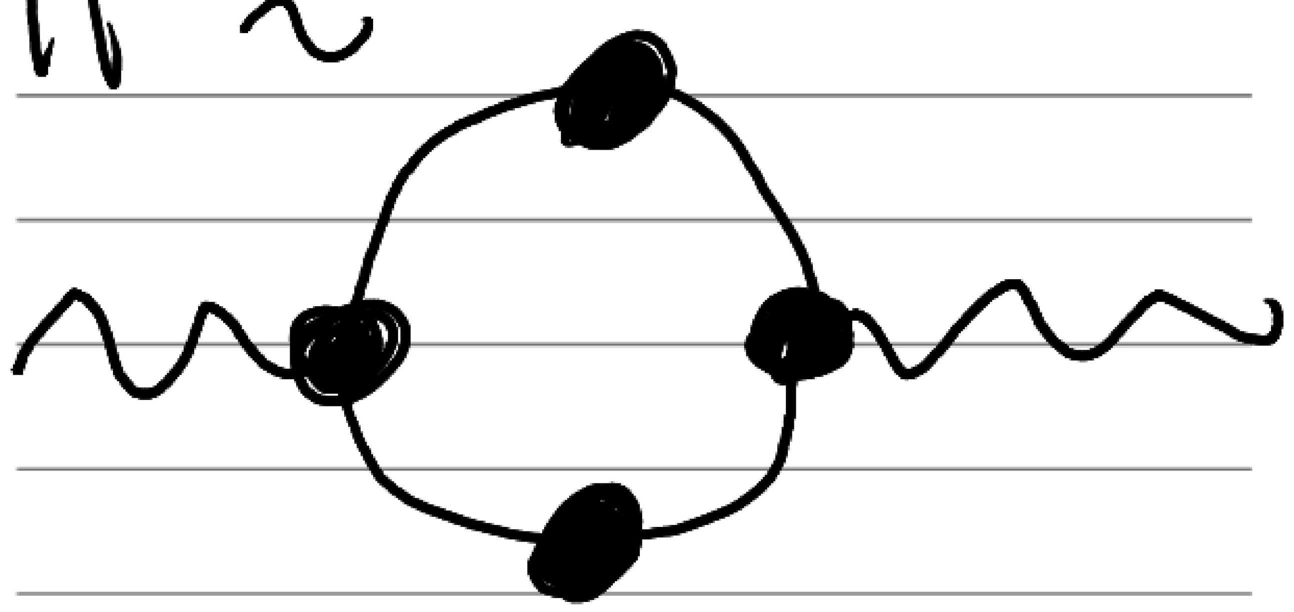
Klimov & Weldon
'81 '82

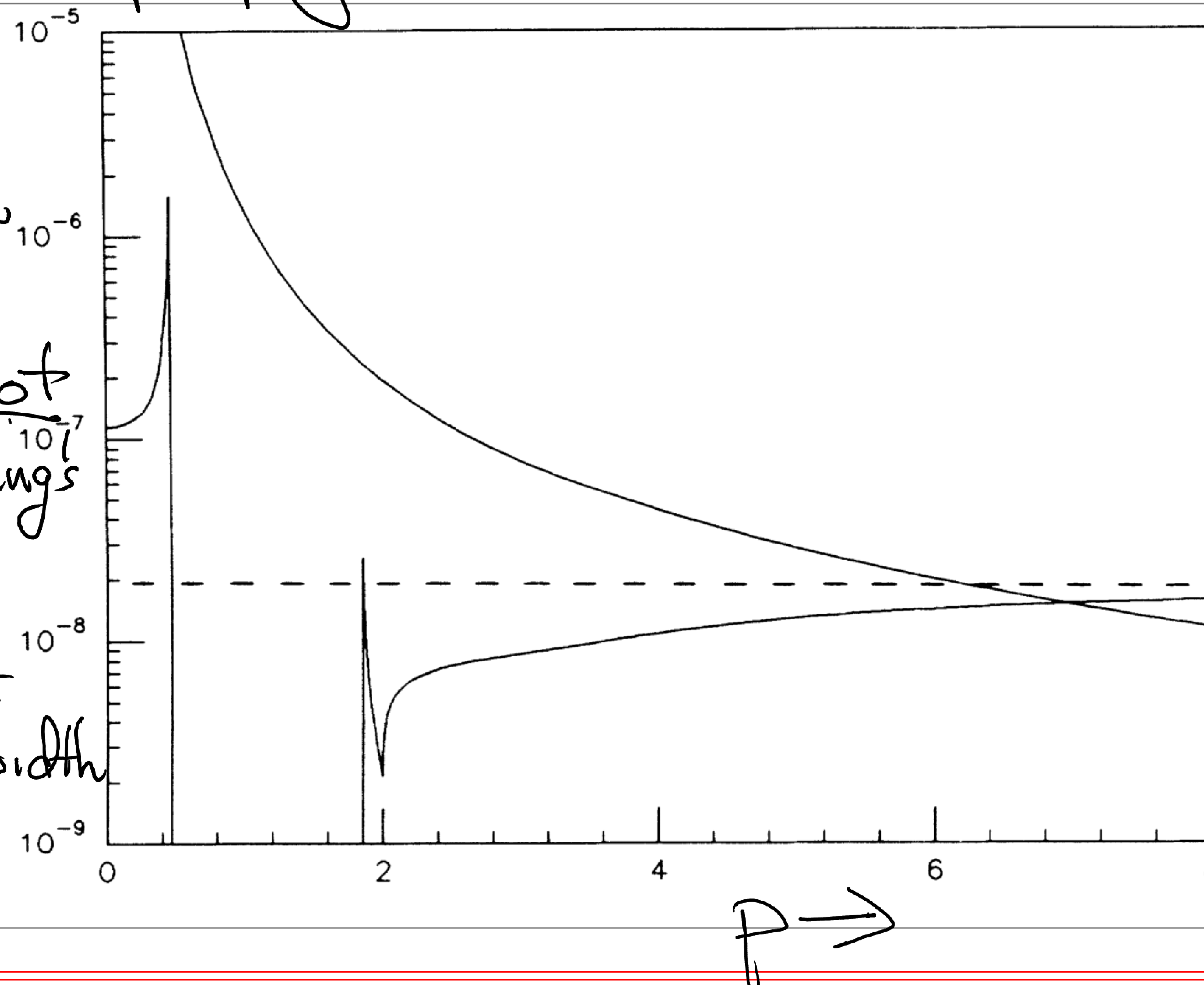


Plasminos do give van H sing.'s !

Braaten, RDP & Yuan '90
HTL resummed propagators and vertices

$\Pi \sim$



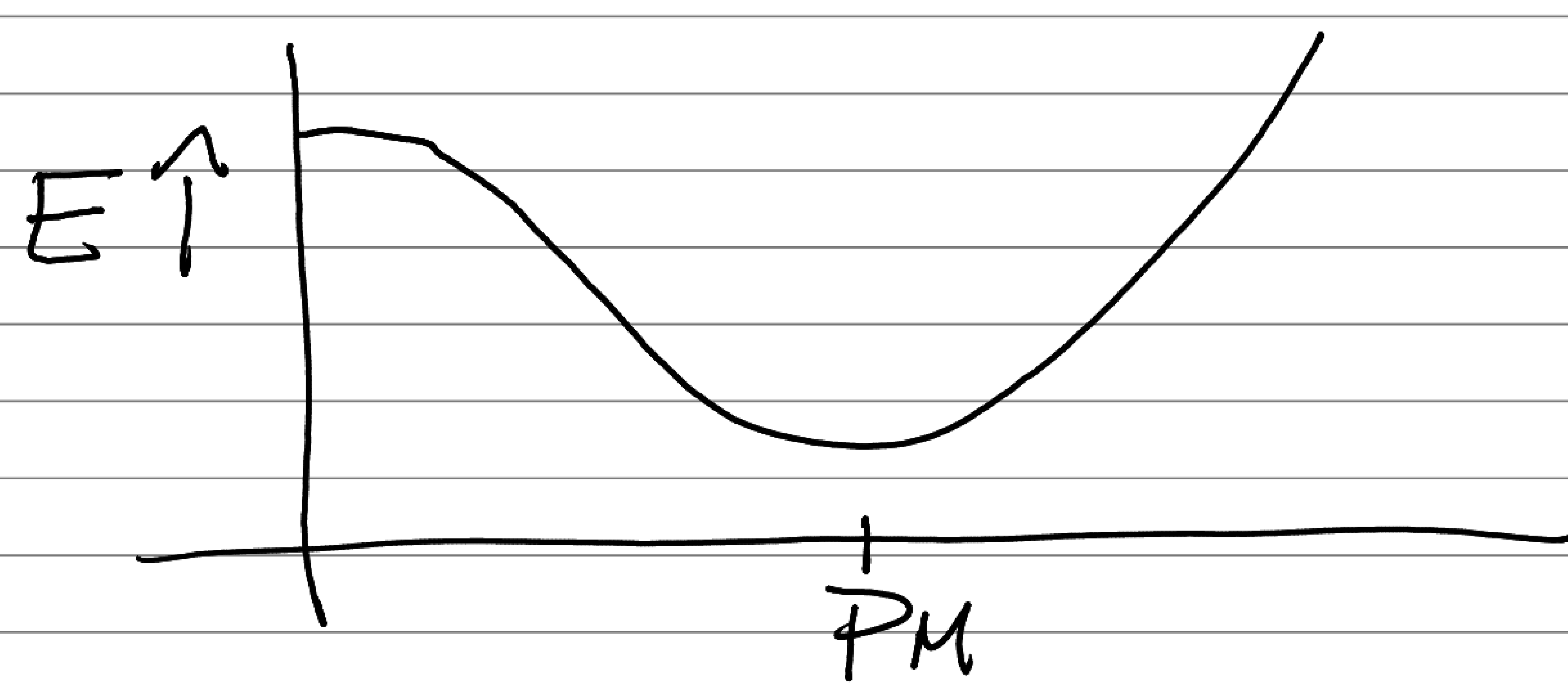


Vertices do not cancel vanH sings

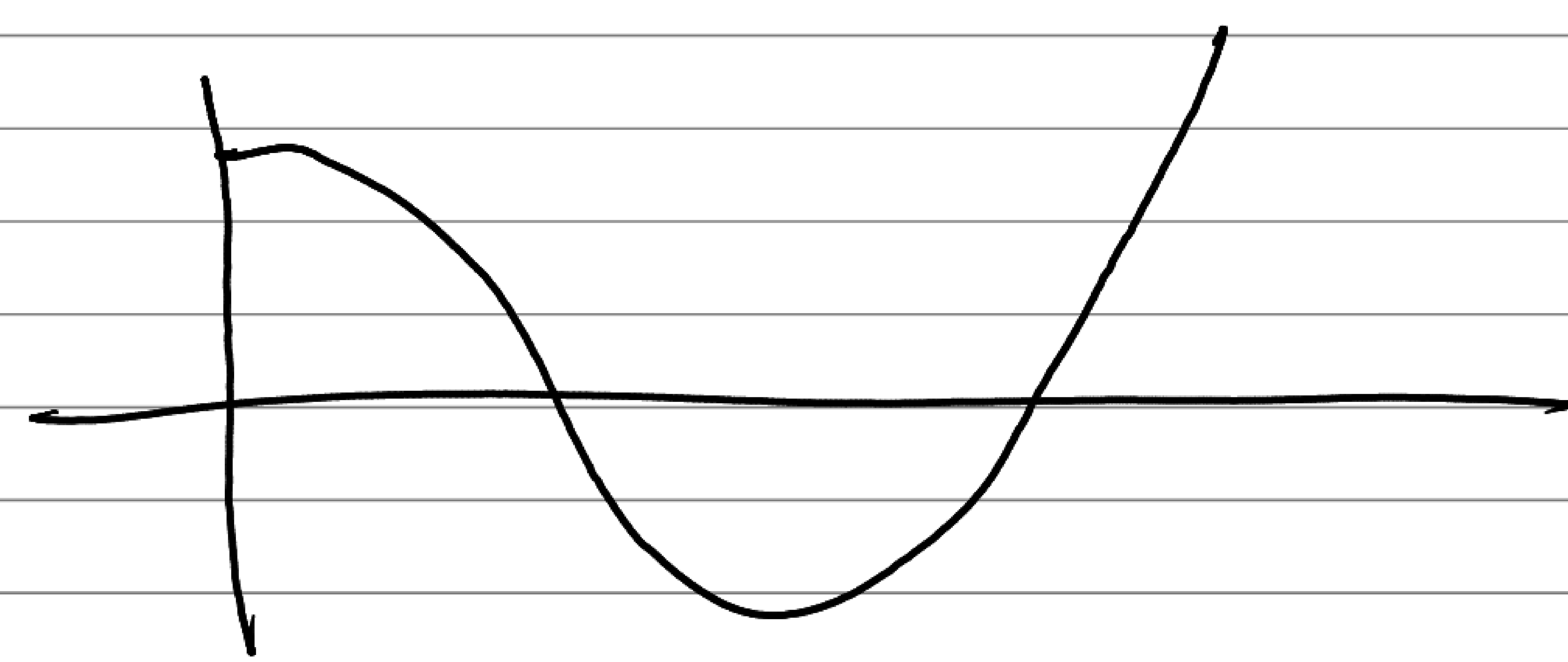
van H sing. smoothed out by thermal width anyway

Moats & χ spirals

Moat:



What if $E<0$?



$\Rightarrow$ spatially inhomogeneous condensate
usually 1-dim. (spon.'y breaks rot. sym.)
$O(1)$: kink crystal
$O(N\geq 2)$: χ spiral

seen in Gross-Neveu

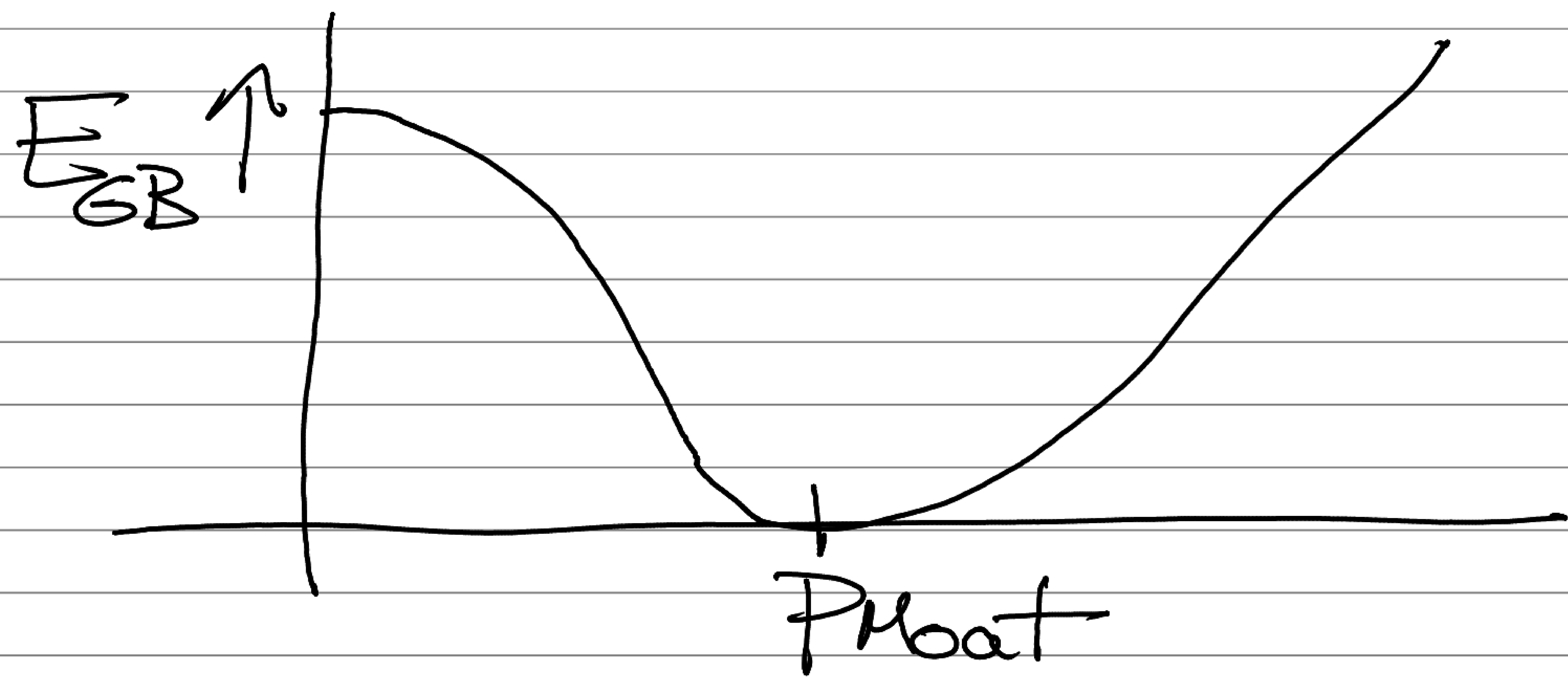
GB's in a Moat

For $O(N>2)$, χ spiral has Goldstone Bosons (GB)

At what p_{GB} is $E_{GB} = 0$?

RDP, S. Valgushev & A. Tsvelik 2005.10259

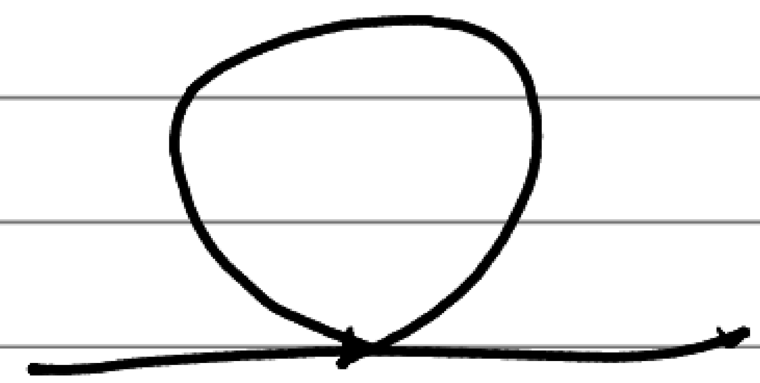
With specific ansatz for χ spiral,



GB's are massless @ p_M!

GB's $\Rightarrow$ Quantum π Liquid

E_{GB} quadratic about p_m; $\vec{p} \approx \vec{p}_m + \delta\vec{p}$

 $\sim T\int \frac{d^3p}{(\delta p_\perp)^2} \sim T p_m^2 \int \frac{d\delta p_\perp}{(\delta p_\perp)^2}$.

GB's disorder the system; linear IR divergence

Like Quantum Spin Liquid $\Rightarrow$ "Quantum π Liquid"

$$\langle \phi(x)\phi(0)\rangle \underset{x\to\infty}{\sim} e^{-m_{real}x}\cos(m_{imag}\,x)$$

$m_{imag}\neq 0$: defines "disorder line"

Distinct from disorder of X-spiral by trans. fluc's

Tong-Gyu Lee +, ,, 1504.03185

HBT in a Moat

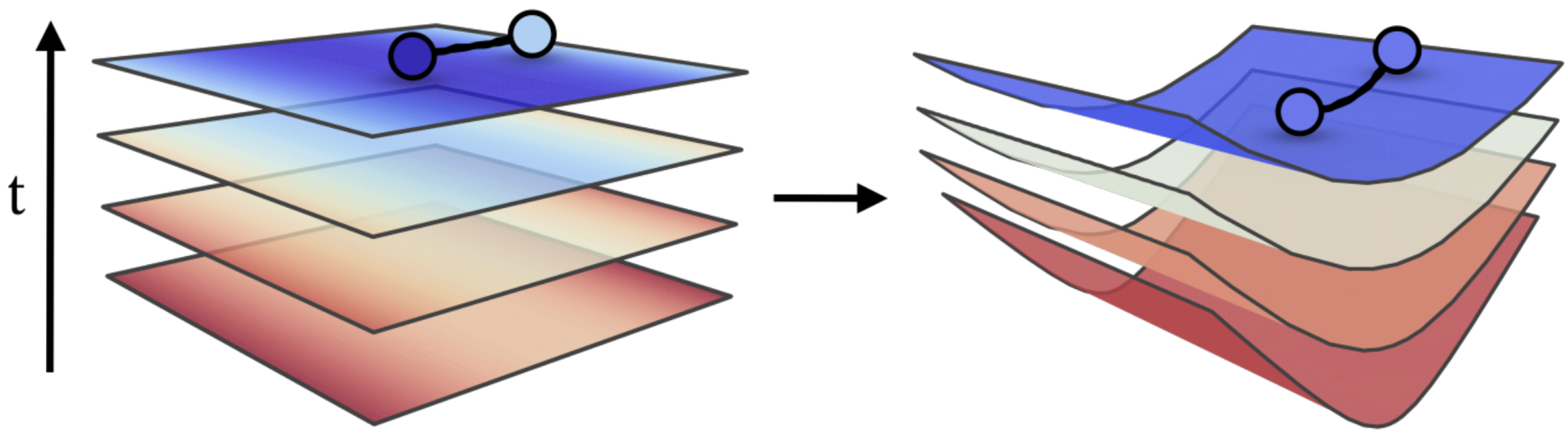
F. Rennecke, RDP, D. Rischke 2301.11484

Hanbury-Brown-Twiss (HBT) interferometry

Consider emission not surface @ constant time

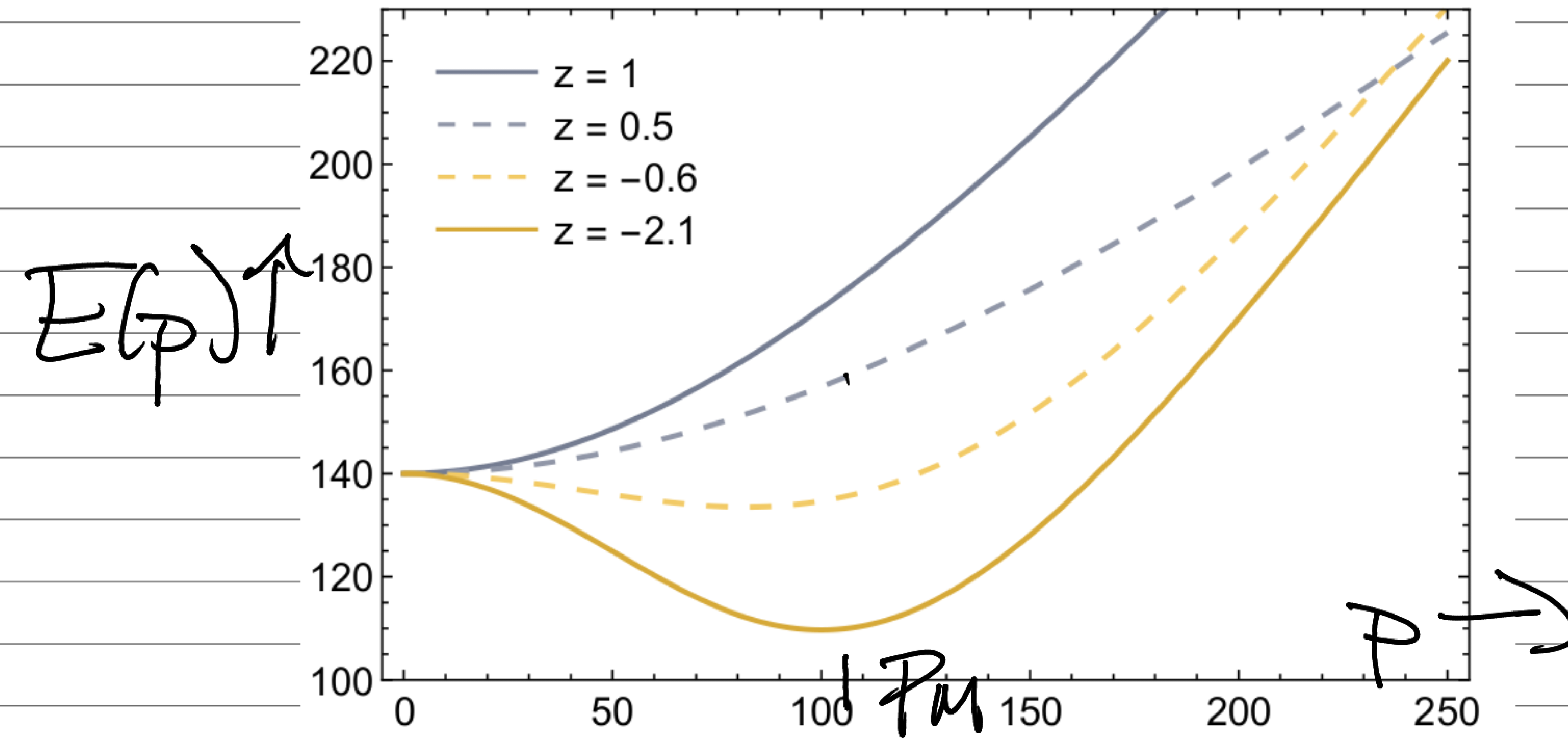
But on curved hypersurface
defined by same T & μ:

Cooper-Frye

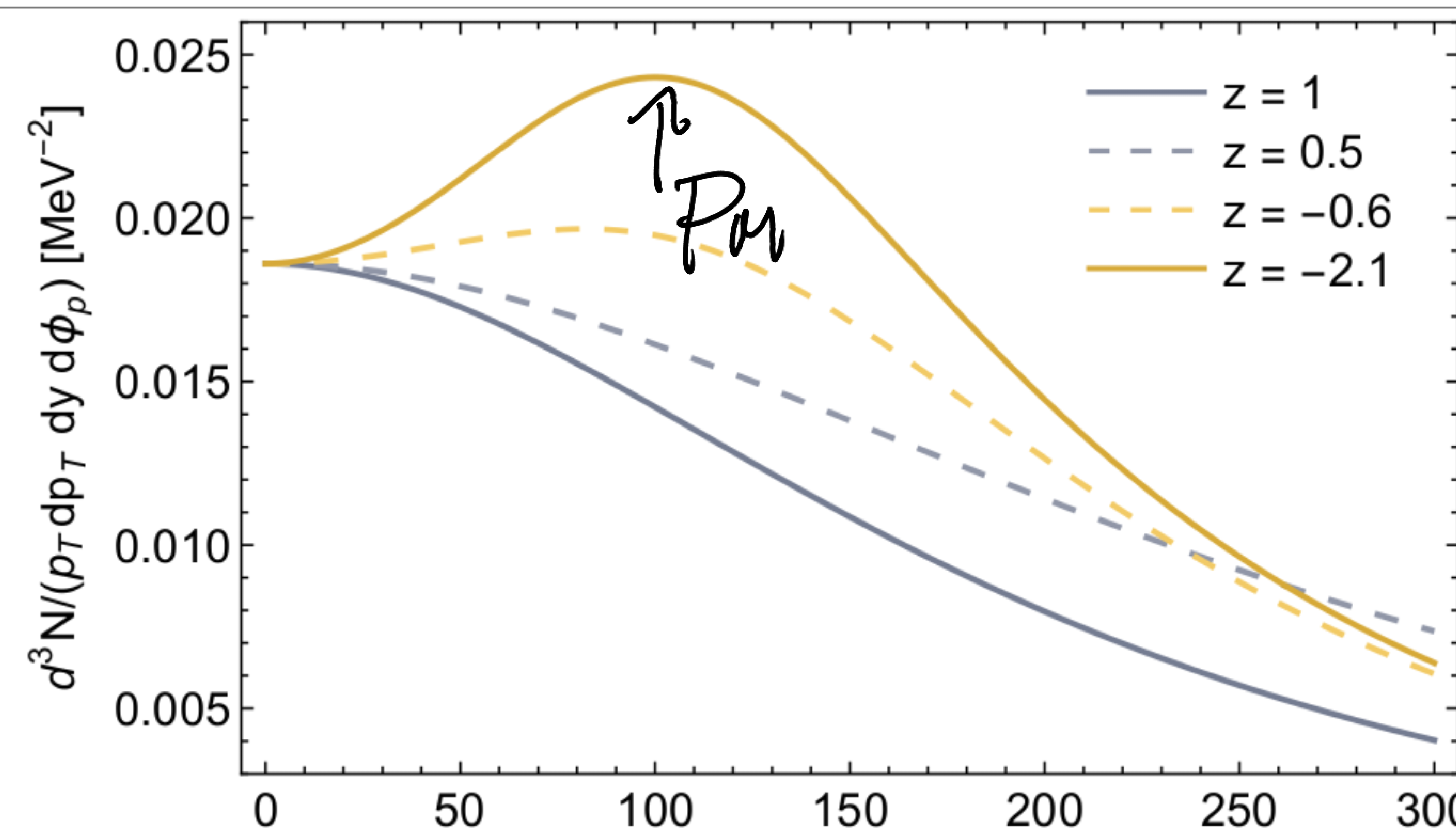


Pions in a Moat

Assume π's emitted in a moat,



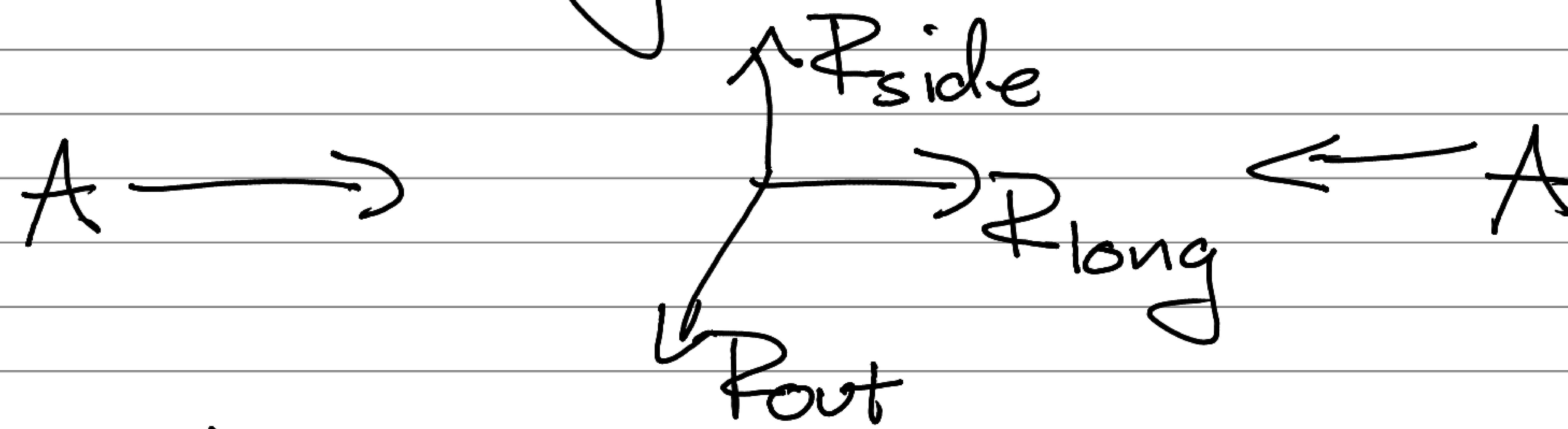
Then single particle fnc. has a maximum @ p_M



Exp'y, could be obscured by emission of π's not in a moat

Moaty HBT

HBT: interferometry btwn identical particles



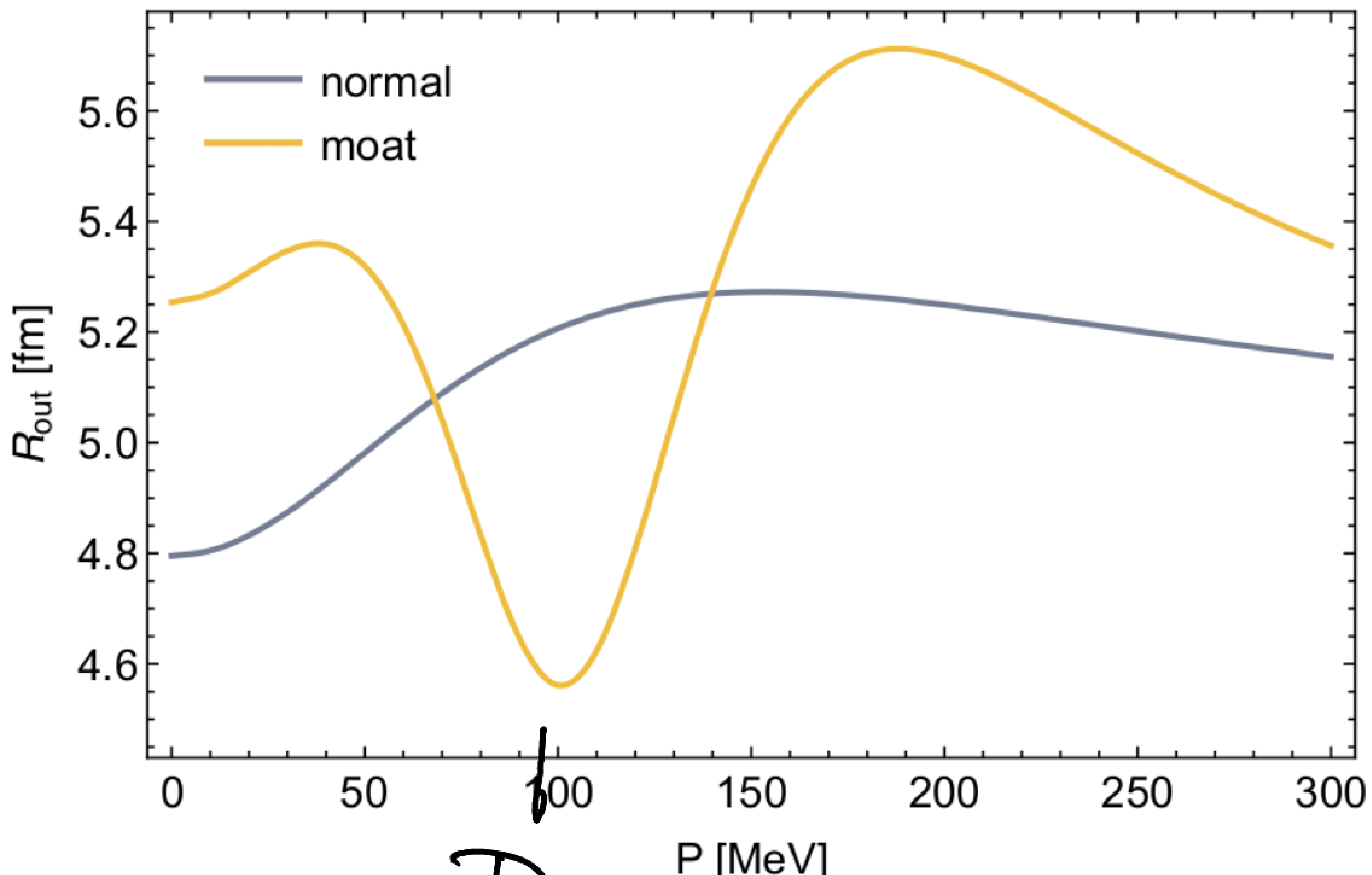
$R = \pi\pi$ pair,

If single particle dist. fnc. peaked @ P_{Moat}, must show up in two particle correlation fnc's

Rout in a Moat

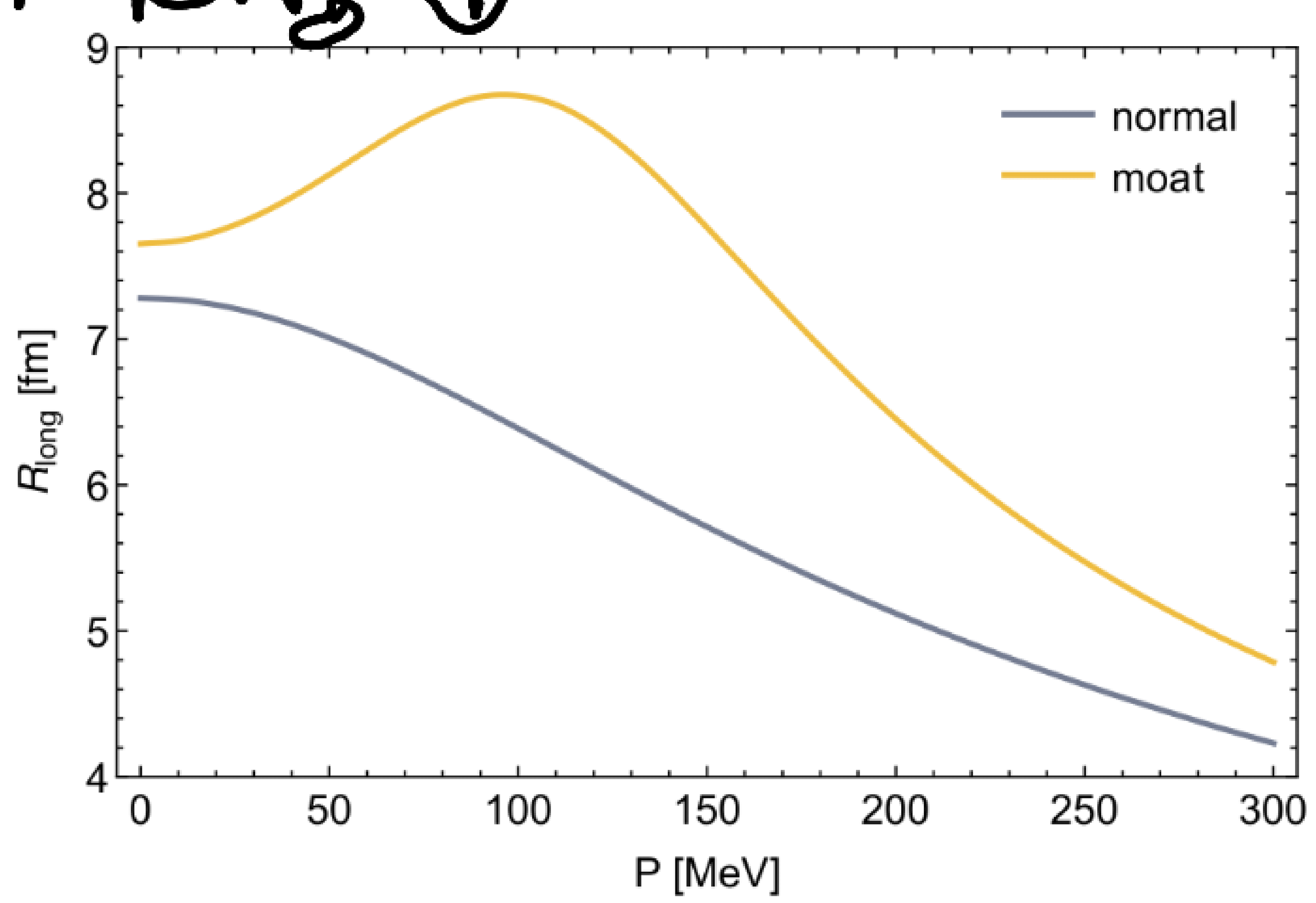
HBT radius where 2-particle corr. fnc. falls to 1/2 maximal size

Rout ↑



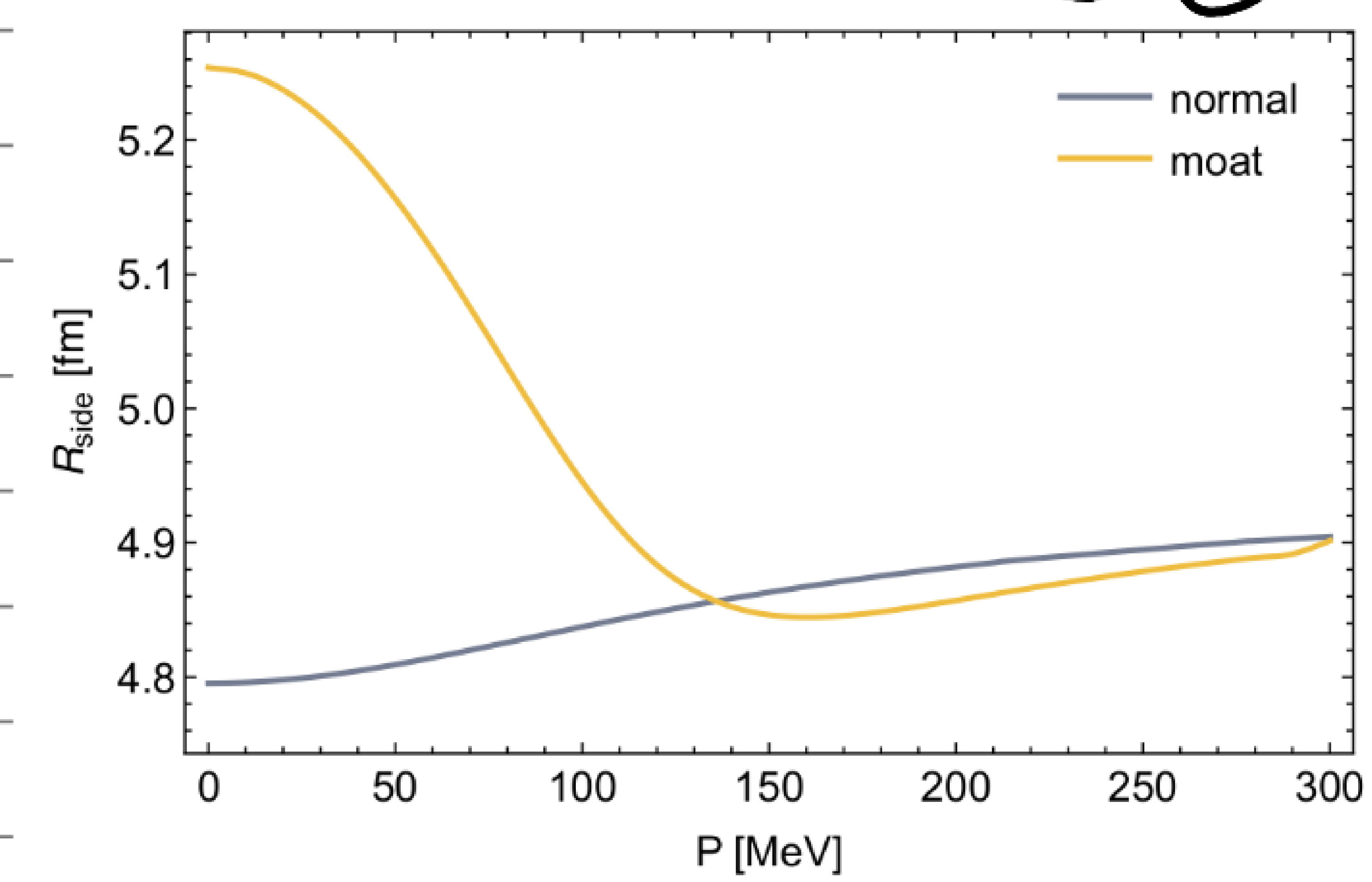
Rout signal most dramatic

Rlong ↓



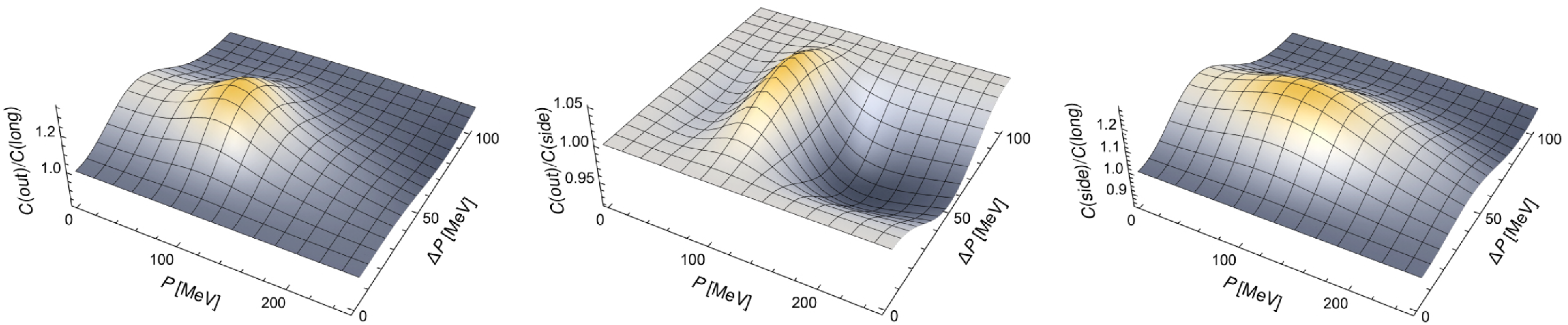
Pmoat

Rside ↓

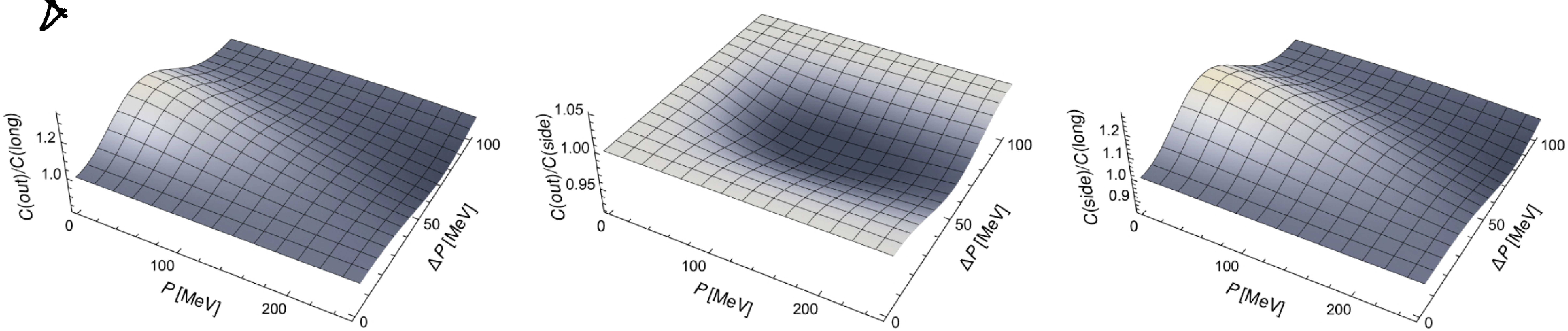


Ratio of C's in a Moat

Moaty π's



C_{out} / C_{long} $\qquad$ C_{out} / C_{side} $\qquad$ C_{side} / C_{long}



Normal π's