Interplay between the weak-coupling results and the lattice data in dense QCD

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References:

[1] Y. Fujimoto, PRD 109 (2024), arXiv:2312.11443; arXiv:2408.12514.
[2] Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, PRL 129 (2022), arXiv:2207.06753.
[3] Y. Fujimoto, S. Reddy, PRD 109 (2024) (selected as Editors' Suggestion), arXiv:2310.09427.

October 31, 2024 - HHIQCD 2024 @ YITP, Kyoto U

Neutron stars: why do we study now?

Holy grail of neutron stars: equation of state (EoS)

Now is the most exciting period because of…

- Recent advances in astrophysics

- Recent advances in QCD

Recent advances in QCD

- Higher-order computations of perturbative QCD (pQCD) EoS

  Freedman,McLerran(1977); Baluni(1978); Kurkela,Romatschke,Vuorinen (2009); Gorda,Säppi,Paatelainen,Seppänen,Österman,Schicho,Navarrete (2018-)

- Nuclear EoS from chiral effective field theory (χEFT)

  Tews,Krüger,Hebeler,Schwenk(2013);Drischler,Furnstahl,Melendez,Philips(2020); Keller,Hebeler,Schwenk(2022); … many others

- Lattice simulations of QCD at finite isospin density

  Kogut,Sinclair (2002); NPLQCD collaboration (2007-); Brandt,Chelnokov,Cuteri,Endrodi,… (2014-);

- Lattice simulations of two-color QCD at finite baryon density

  e.g. Iida,Itou,Murakami,Suenaga (2024)

- Hadron-hadron interaction from the lattice QCD

  HAL QCD collaboration (2006-)

- Hamiltonian lattice simulations of QCD in (1+1)-dimensions

  Hayata,Hidaka,Nishimura (2023)

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QCD at finite isospin density

Alford,Kapustin,Wilczek (1999); Kogut,Sinclair (2002-);
Beane,Detmold,Savage et al. (2007-);
Endrodi et al. (2014-)…

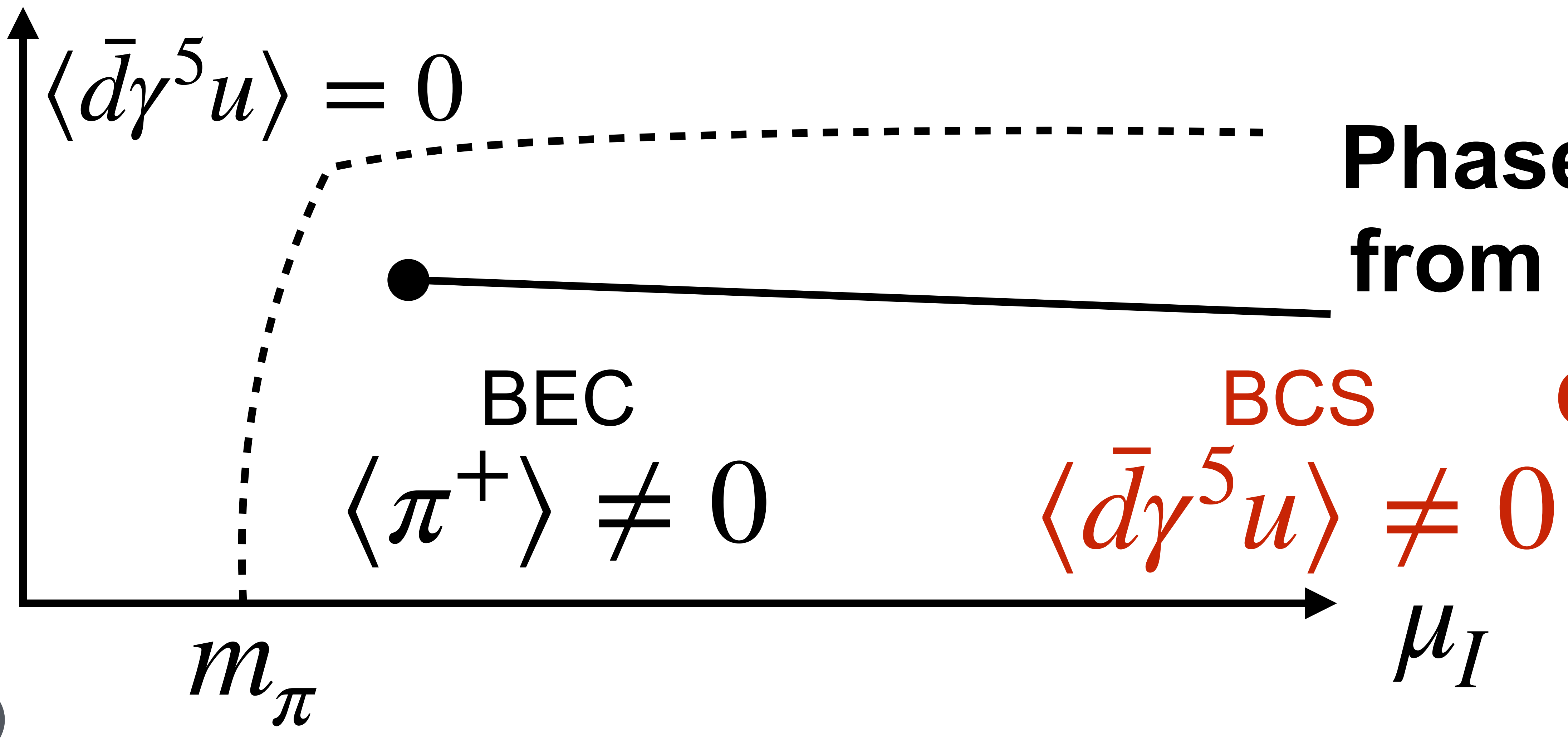
- **No sign problem** → EoS can be measured on the lattice!

- Isospin chemical potential (conjugate to isospin density I_3):

$$\mu_u = \frac{\mu_I}{2}, \quad \mu_d = -\frac{\mu_I}{2}$$ … Fermi surface of u & $\bar{d}$

- Phase structure:

Son,Stephanov (2000)



Phase structure is totally different from QCD at finite baryon density

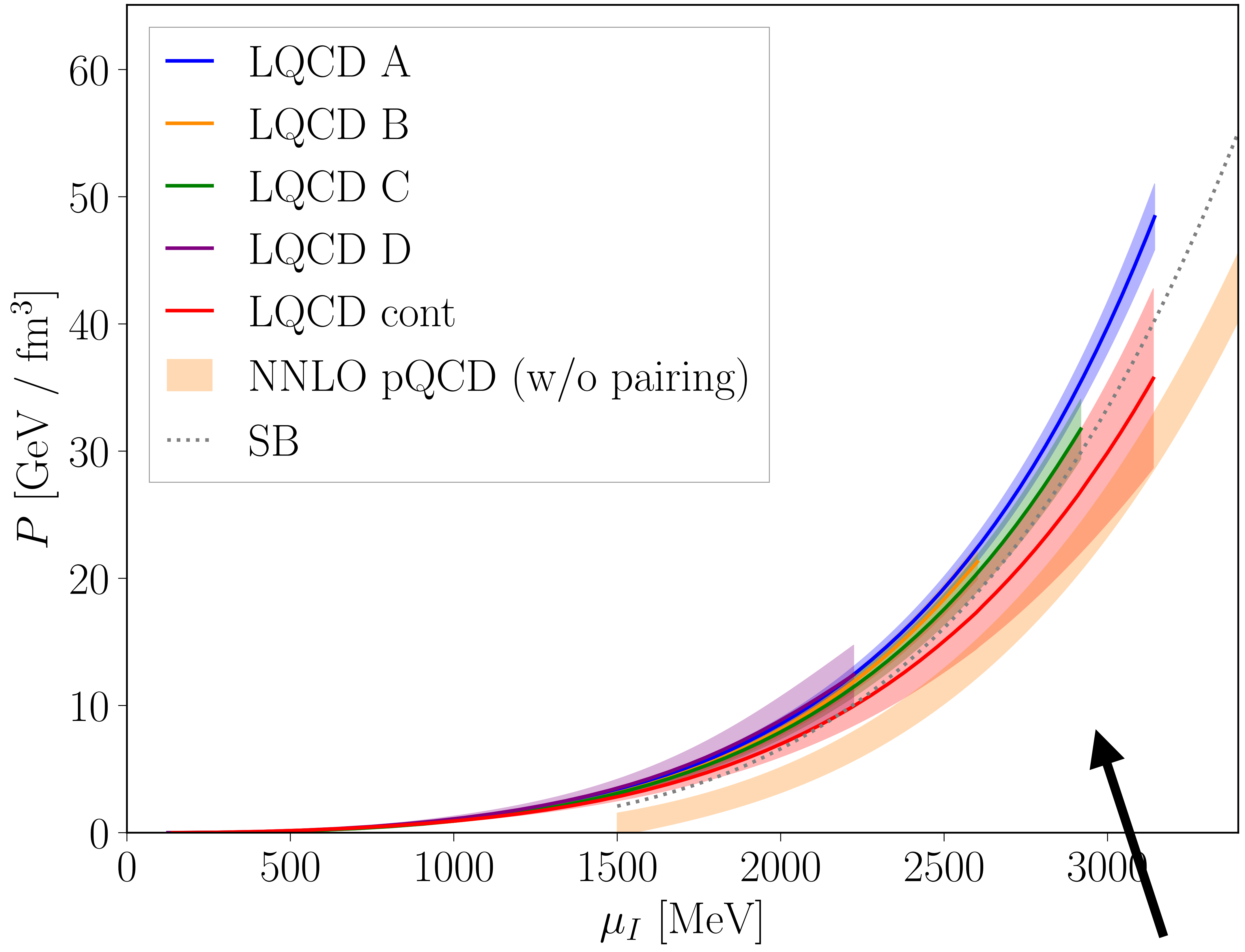
Cooper pairing

QCD at finite isospin density

Recent impact:

Abbott et al. (NPLQCD) (2023, 24)

EoS is calculated up to $\mu_I \sim 3$ GeV by lattice QCD in the continuum limit



weak-coupling regime

What can we learn about NSs from the lattice data?

- Ground states of finite-μ_B QCD and finite-μ_I QCD are totally different
  → Naive comparison of EoS is meaningless

- There are (at least) two ways to utilize the finite-μ_I lattice data:

1. **QCD inequality**

   robust way of comparing the pressure of finite-μ_B QCD and finite-μ_I QCD

2. **Comparison in the perturbative regime**

   finite-μ_B QCD and finite-μ_I QCD have the common weak-coupling expansion

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QCD inequality and bounds on the EoS

Abbott et al. (NPLQCD) (2023, 24)

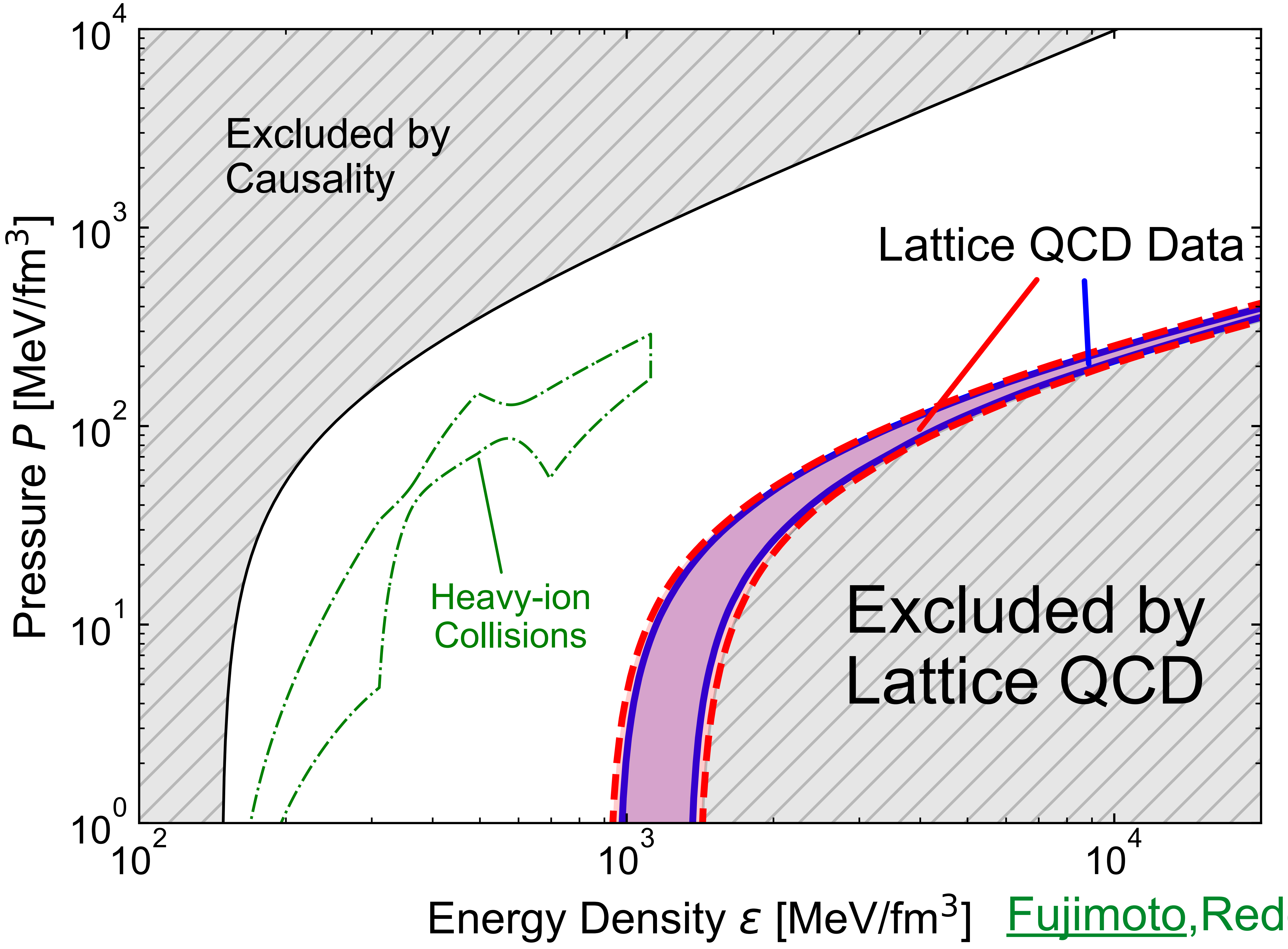
Bounds on the symmetric nuclear matter EoS:

QCD inequality:

$$P(\mu_B) \leq P_{\text{lattice}}\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Pro: Robust approach,
no systematic errors
apart from lattice uncertainties

Con: Not as constraining as
heavy-ion phenomenology



Cohen (2003);
Fujimoto,Reddy, PRD 109 (2023)

cf. Moore,Gorda (2023)

Heavy-ion:
Oliinychenko et al.(2022)

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Main topic for the rest of the talk

Notation

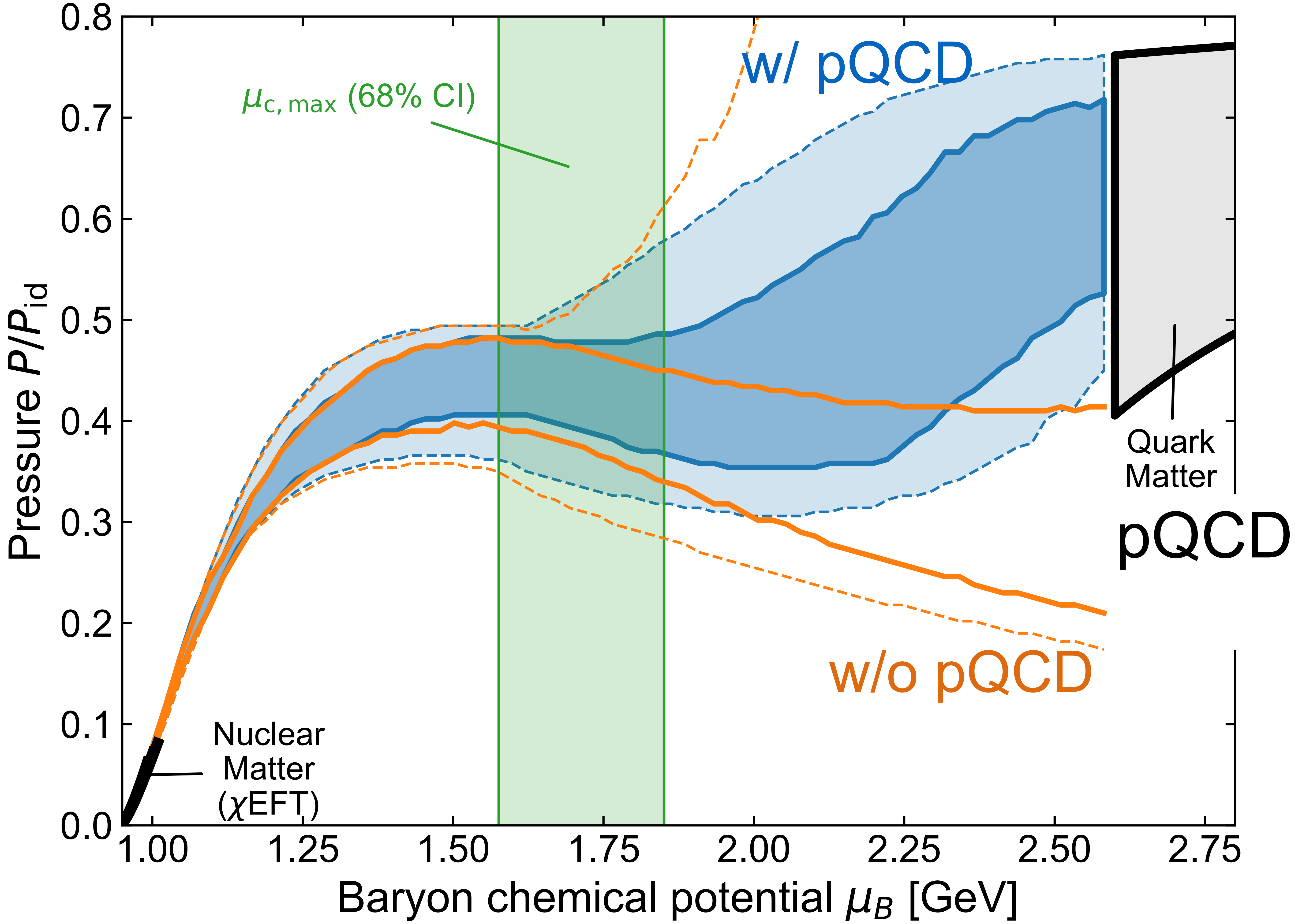
- QCD_I: QCD at finite μ_I and zero μ_B
- QCD_B: QCD at finite μ_B and zero μ_I
- μ: quark chemical potential

$$(\mu_B = N_c\mu,\ \mu_I = 2\mu)$$

Role of pQCD in constraining the NS EoS

- pQCD input is useful in NS EoS
- Consider pressure P normalized by the ideal quark gas value: $P_{\rm id} = \dfrac{N_c N_f \mu^4}{12\pi^2}$
- Without pQCD constraint, $P/P_{\rm id}$ is too small at high μ_B
- The pQCD constraint requires $P/P_{\rm id}$ to be large at high μ_B
  → favors soft EoS in the NS core

Fujimoto, Fukushima, McLerran, Praszalowicz, PRL129 (2022)



See also: Annala,Gorda,Hirvonen,Komoltsev,Kurkela,Nättilä,Vuorinen (2023); Komoltsev,Somasundaram,Gorda,Kurkela,Margueron,Tews (2023)

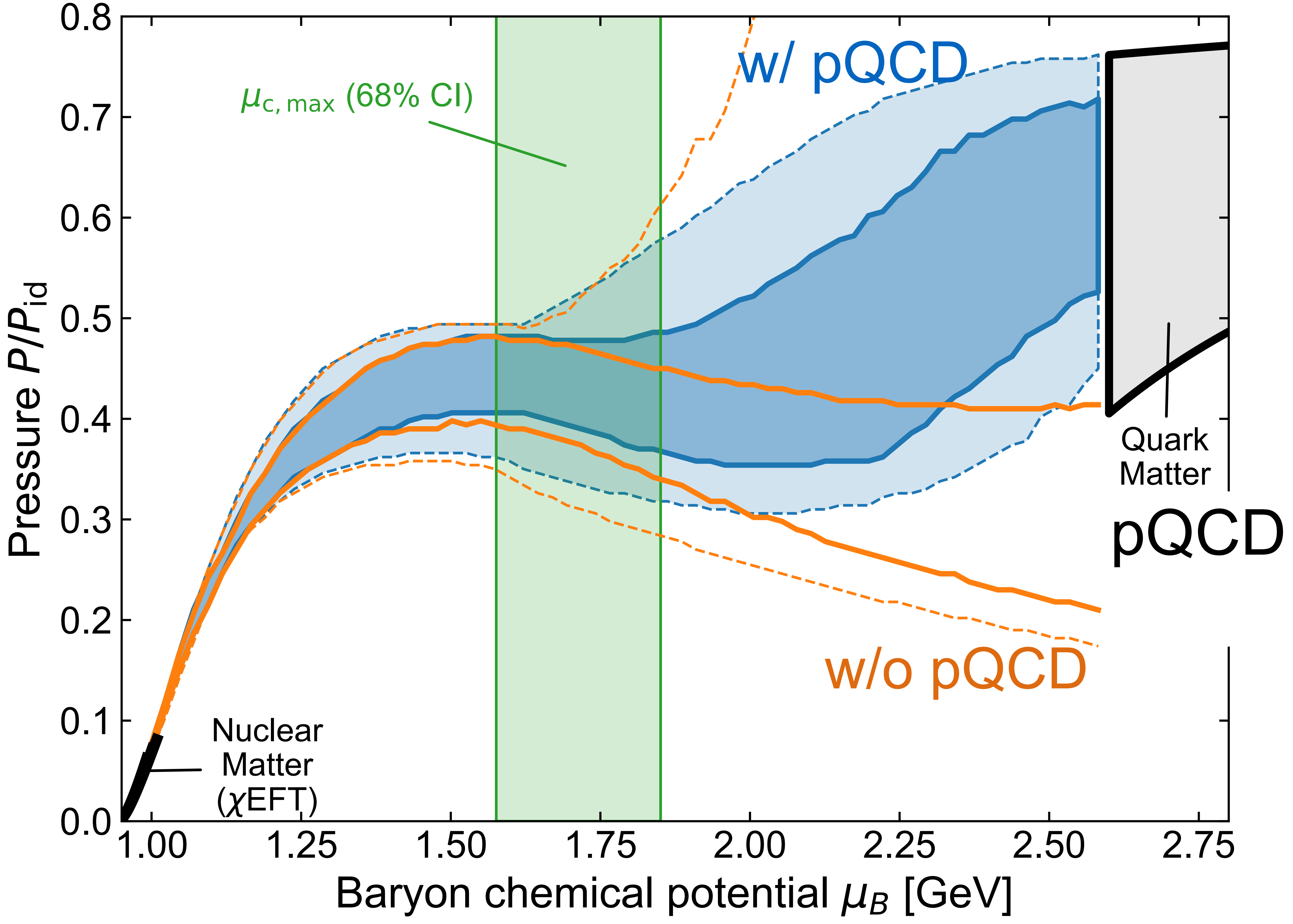
Role of pQCD in constraining the NS EoS

- **Trace anomaly:**
  related to the changes in $P/P_{\rm id}$

$$\varepsilon - 3P \propto \frac{d(P/P_{\rm id})}{d\ln\mu}$$

- $P/P_{\rm id}(\mu_B)$ monotonically increases as a function of μ_B by pQCD effect
  → **Positive $\varepsilon - 3P$ favored**

Fujimoto, Fukushima, McLerran, Praszalowicz, PRL129 (2022)



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Weak-coupling results in high-density QCD

Freedman,McLerran (1977); Baluni (1978); Kurkela et al. (2009-)

QCD EoS in weak-coupling α_s expansion:

$$P_{\rm QCD}(\mu) = \frac{3\mu^4}{4\pi^2}\left[1 - \mathcal{O}(\alpha_s)\right] + \frac{3\mu^2\Delta^2}{2\pi^2}\left[1 + \mathcal{O}(\alpha_s^{1/2})\right], \quad \ln\left(\frac{\Delta_{\rm gap}}{\mu}\right) = -b_{-1}\left(\frac{\alpha_s}{\pi}\right)^{-1/2} - b_0$$

Son (1998), Pisarski,Rischke (1998)
Brown,Liu,Ren (1999); Wang,Rischke (2001)
Review: Alford,Rajagopal,Schafer,Schmitt (2008);
Fujimoto (2023)

Applicability at low μ?

- Usually, it is used down to $\mu \sim 0.9$ GeV for the input of neutron stars

Kurkela,Fraga,Vuorinen (2014)

Weak-coupling formula is **universal** for QCD_B and QCD_I up to $\mathcal{O}(\alpha_s^2)$

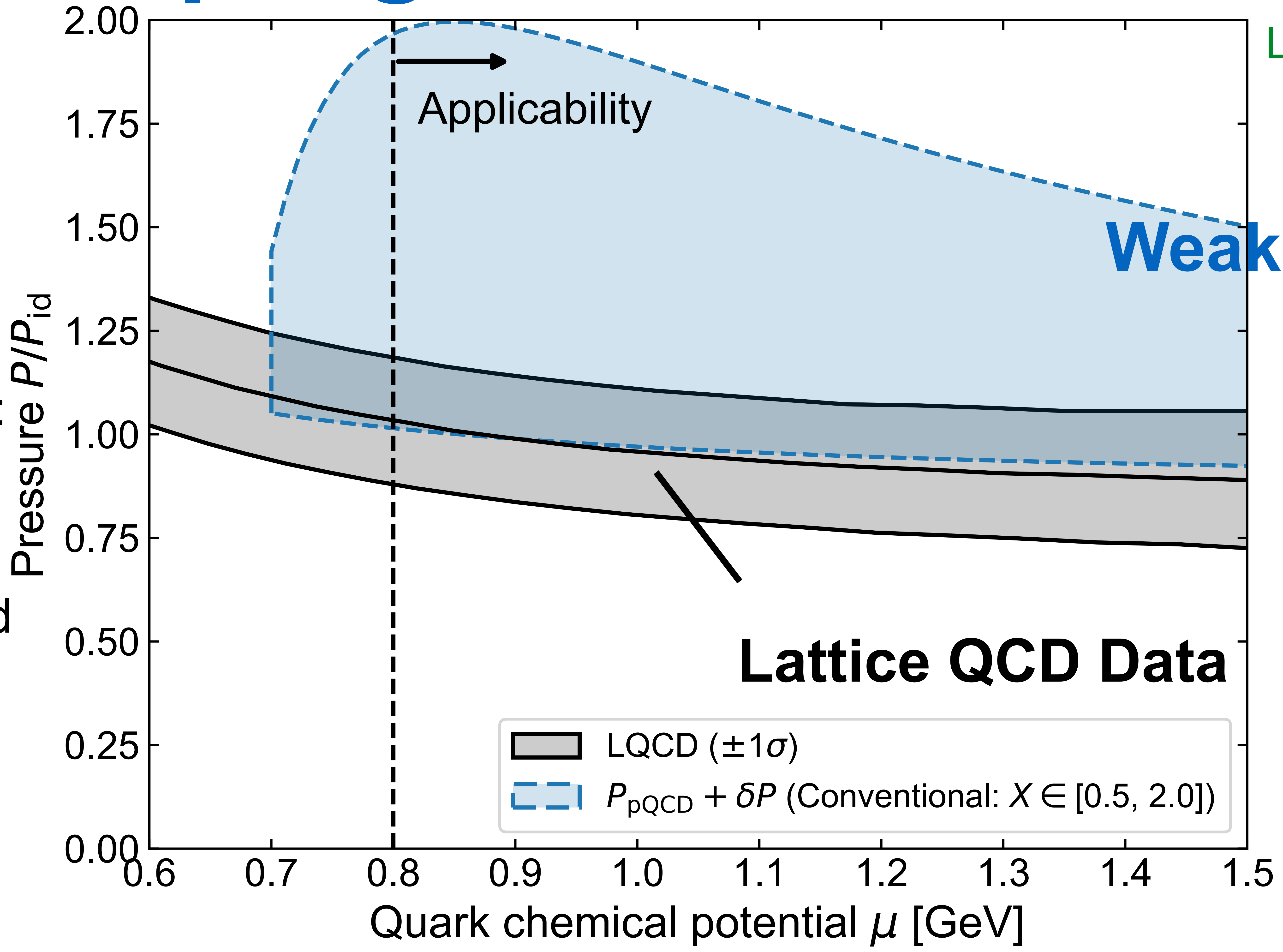
→ Lattice QCD_I can be used as a benchmark

Weak-coupling results vs lattice QCD_I data

Lattice data: Abbott et al. (2023, 24); Fujimoto (2023, 24)

Uncertainty in weak-coupling results: varying the renormalization scale $\bar{\Lambda}$ by a factor 2 around its typical scale

$\bar{\Lambda} = 2\mu$



Empirical evidence for the dense-QCD weak-coupling results to be applicable down to $\mu \sim 0.8$ GeV

At least the magnitude is correct

"Uncertainty" in pQCD

Fraga,Pisarski,Schaffner-Bielich(2001);
Kurkela,Romatschke,Vuorinen (2009)

$$\alpha_s \simeq \frac{1}{\beta_0 \ln\left(\frac{\bar{\Lambda}}{\Lambda_{\overline{\rm MS}}}\right)}$$

- **$\bar{\Lambda}$: renormalization scale**

  … only ambiguity in pQCD from perturbative series truncation

- Canonical choice: $\bar{\Lambda} = 2\mu$ (typical hard interaction scale)

- "Uncertainty" quantified by varying by factor 2

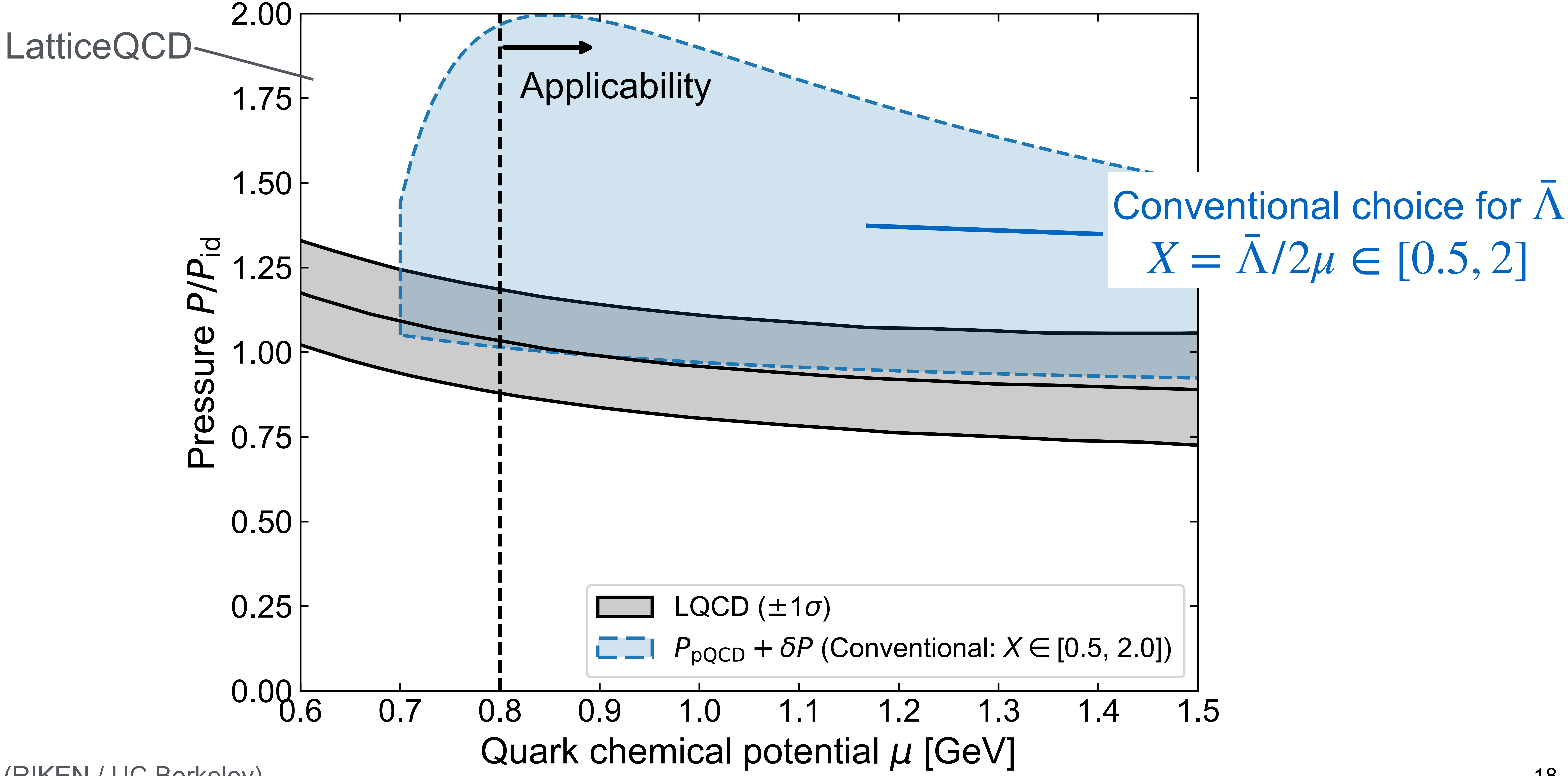
  i.e. $X \in [0.5,\ 2]$ with $X \equiv \bar{\Lambda}/(2\mu)$

  … ad hoc procedure, purely based on historical practice

cf. Gorda,Komoltsev,Kurkela,Mazeliauskas (2022)

Prescription for $\bar{\Lambda}$ determination

Fujimoto, 2408.12514



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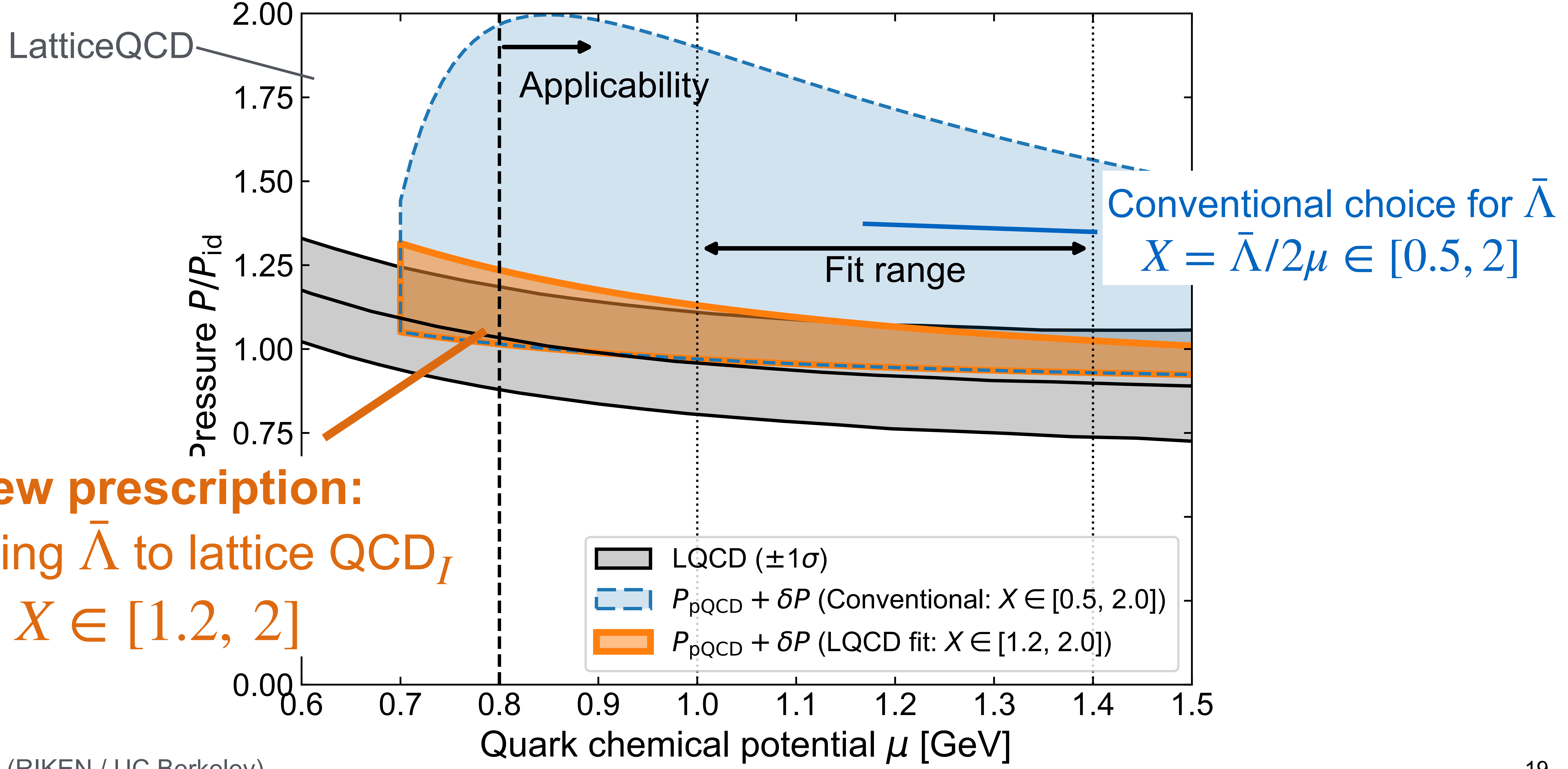


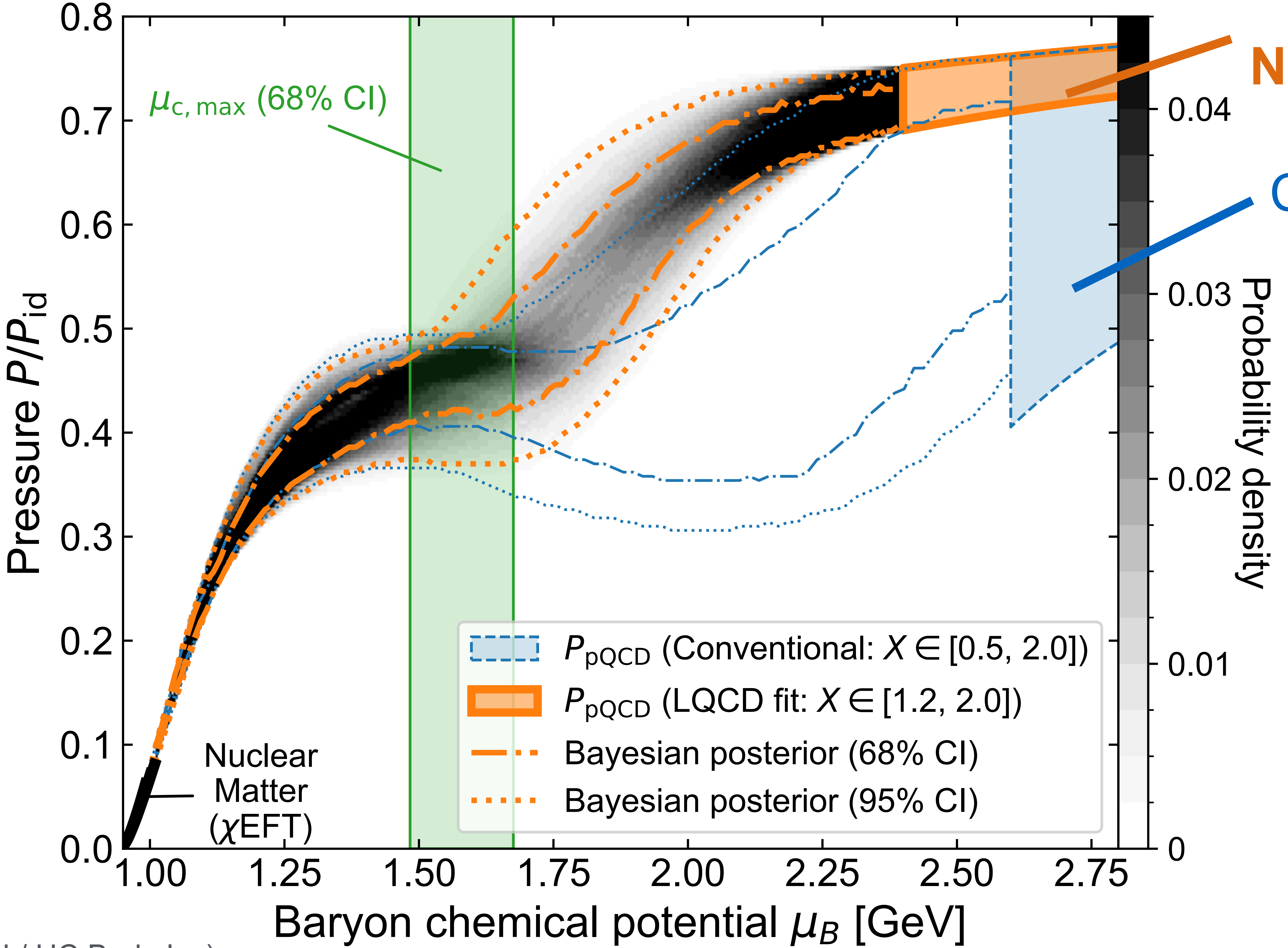
New prescription:

Matching $\bar{\Lambda}$ to lattice QCD_I

$X \in [1.2, 2]$

Effect on QCD_B: NS EoS

Fujimoto, 2408.12514

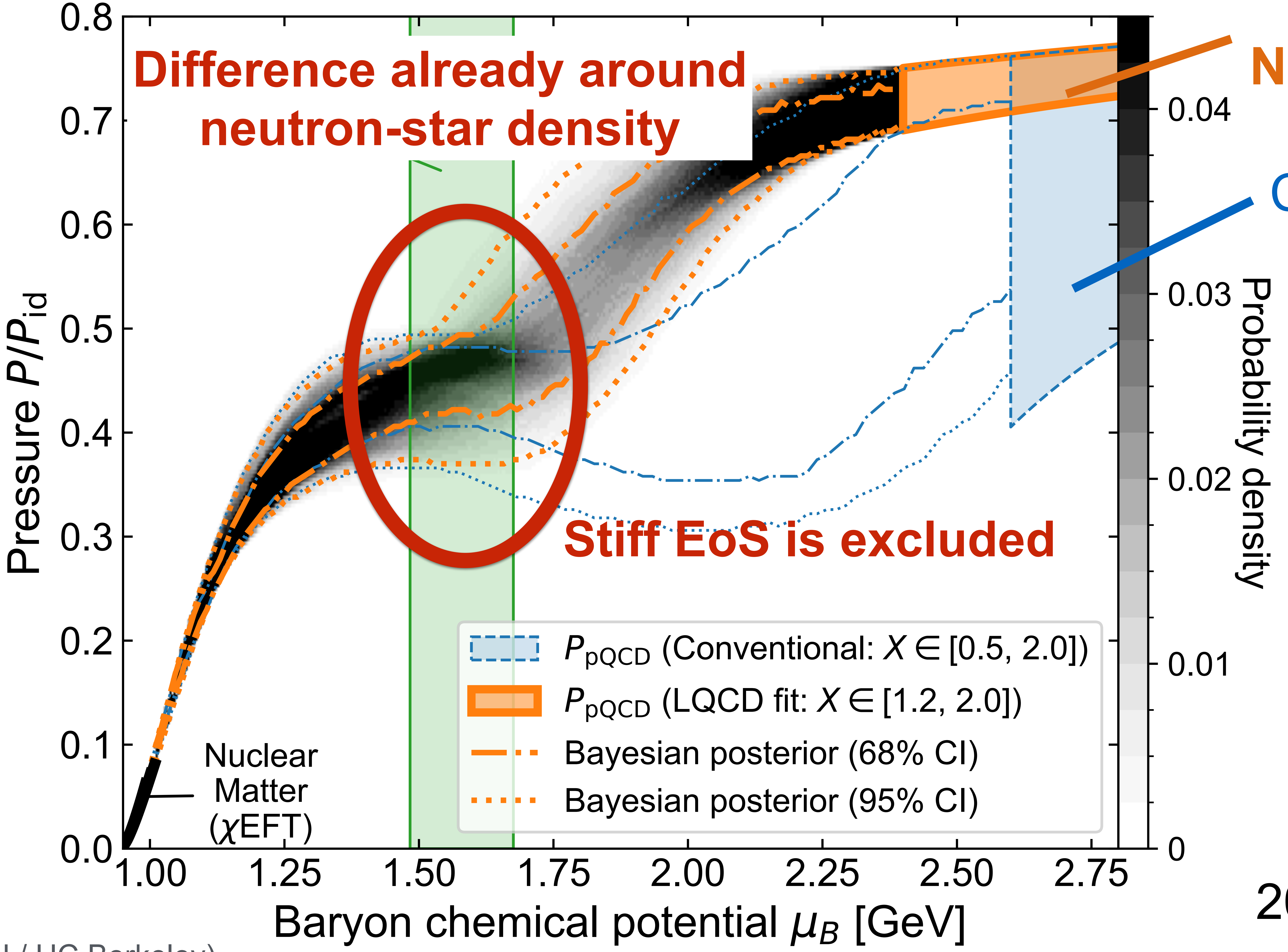


New choice

Conventional choice

Effect on QCD_B: NS EoS

Fujimoto, 2408.12514



With new choice, Bayes factor for $\varepsilon - 3P \geq 0$ is 20.3 (strong evidence)

Color superconductivity in weak coupling

Fujimoto, 2408.12514

$\Delta_{\mathrm{CFL}} \sim 1\ \mathrm{MeV}$ at $\mu = 0.8\ \mathrm{GeV}$

cf. $\Delta \lesssim 200$ MeV from astrophysical bound
Kurkela,Rajagopal,Steinhorst (2024)

- A negligibly **small** contribution to bulk thermodynamics in weak coupling:

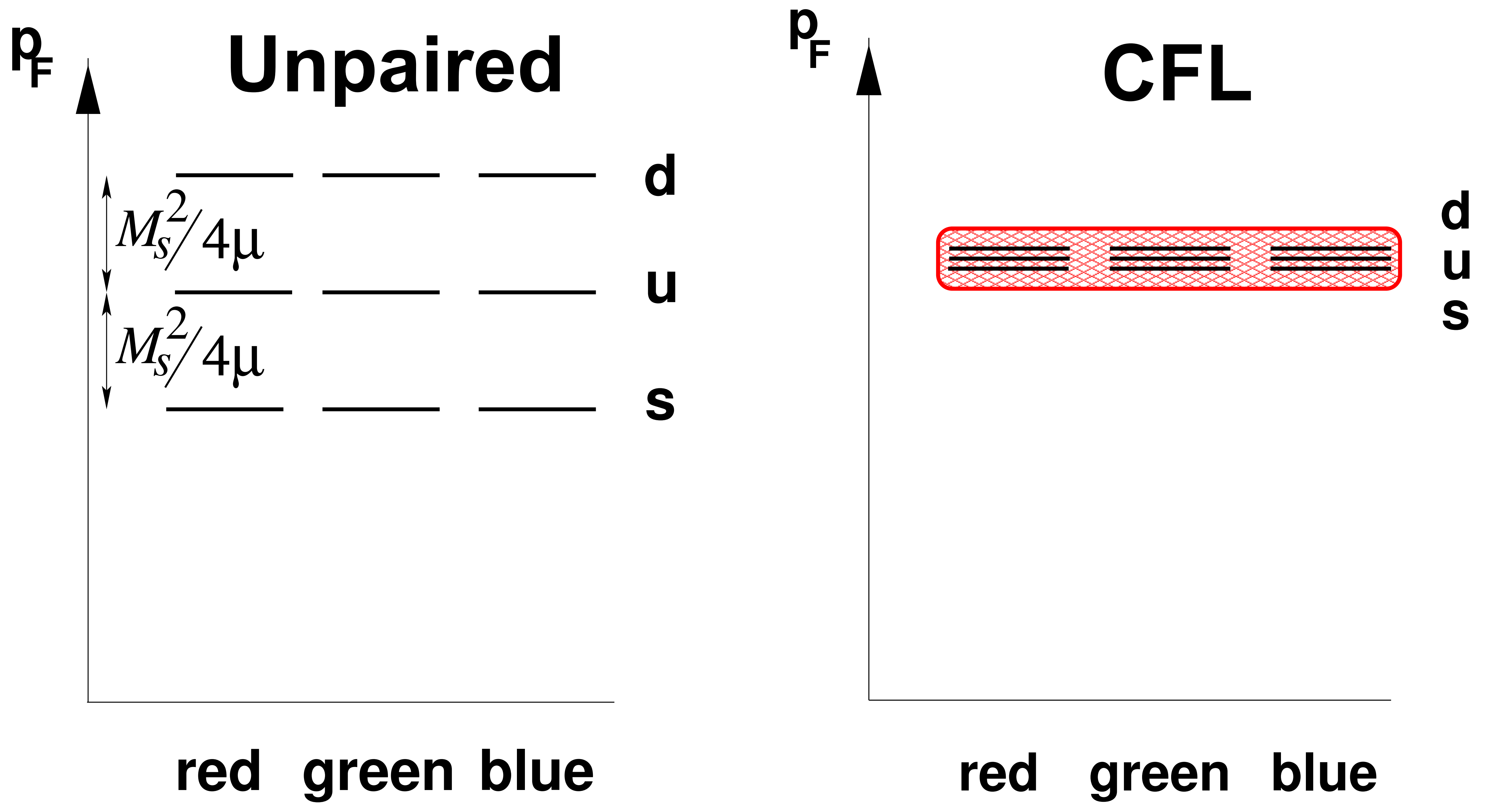
$$P_{\mathrm{QCD}}(\mu) = \frac{3\mu^4}{4\pi^2}\left[1 - \mathcal{O}(\alpha_s)\right] + \frac{3\mu^2\Delta^2}{2\pi^2}\left[1 + \mathcal{O}(\alpha_s)\right]$$

Color superconductivity in weak coupling

Fujimoto, 2408.12514

$$\Delta_{\mathrm{CFL}} \sim 1\ \mathrm{MeV} \text{ at } \mu = 0.8\ \mathrm{GeV}$$

- Δ_{CFL} is comparable to the stress induced by strange quark mass $\sim m_s^2/4\mu$
  → **CFL may not be the ground state even at $\mu_B = 2.4\ \mathrm{GeV}$**

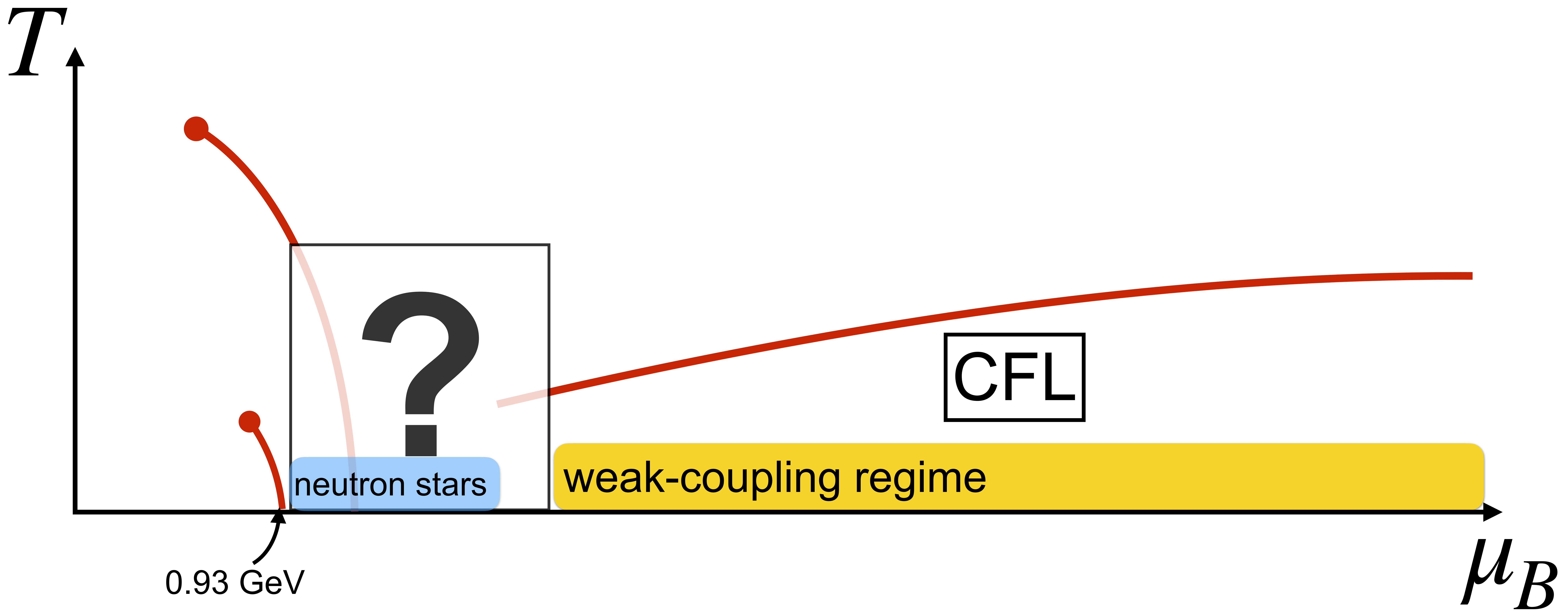


- NB: CFL and color superconductor may still be realized in NSs due to the nonperturbative enhancement from instantons

Alford,Rajagopal,Wilczek (1997);
Rapp,Schafer,Shuryak,Velkovsky (1997)

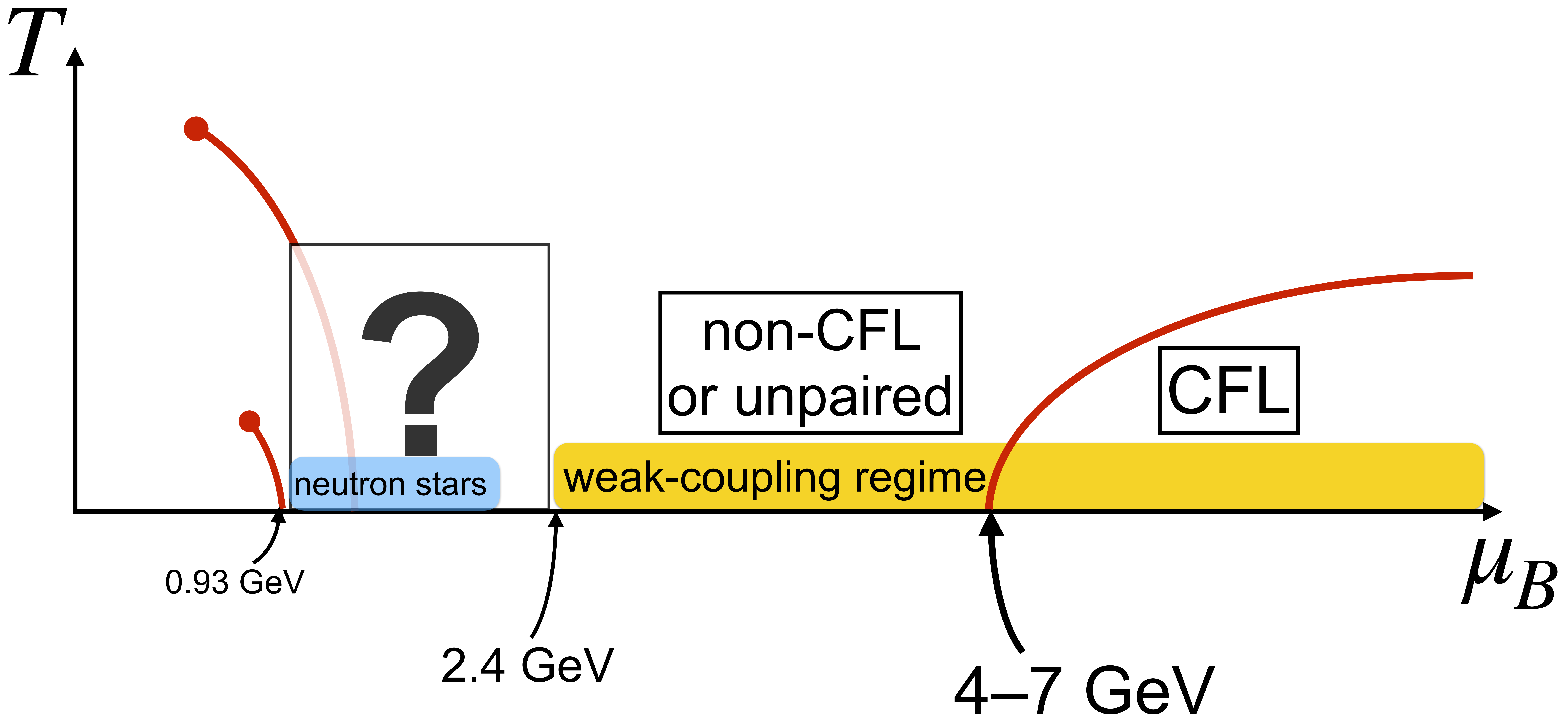
Effect on the QCD phase diagram

Common understanding:



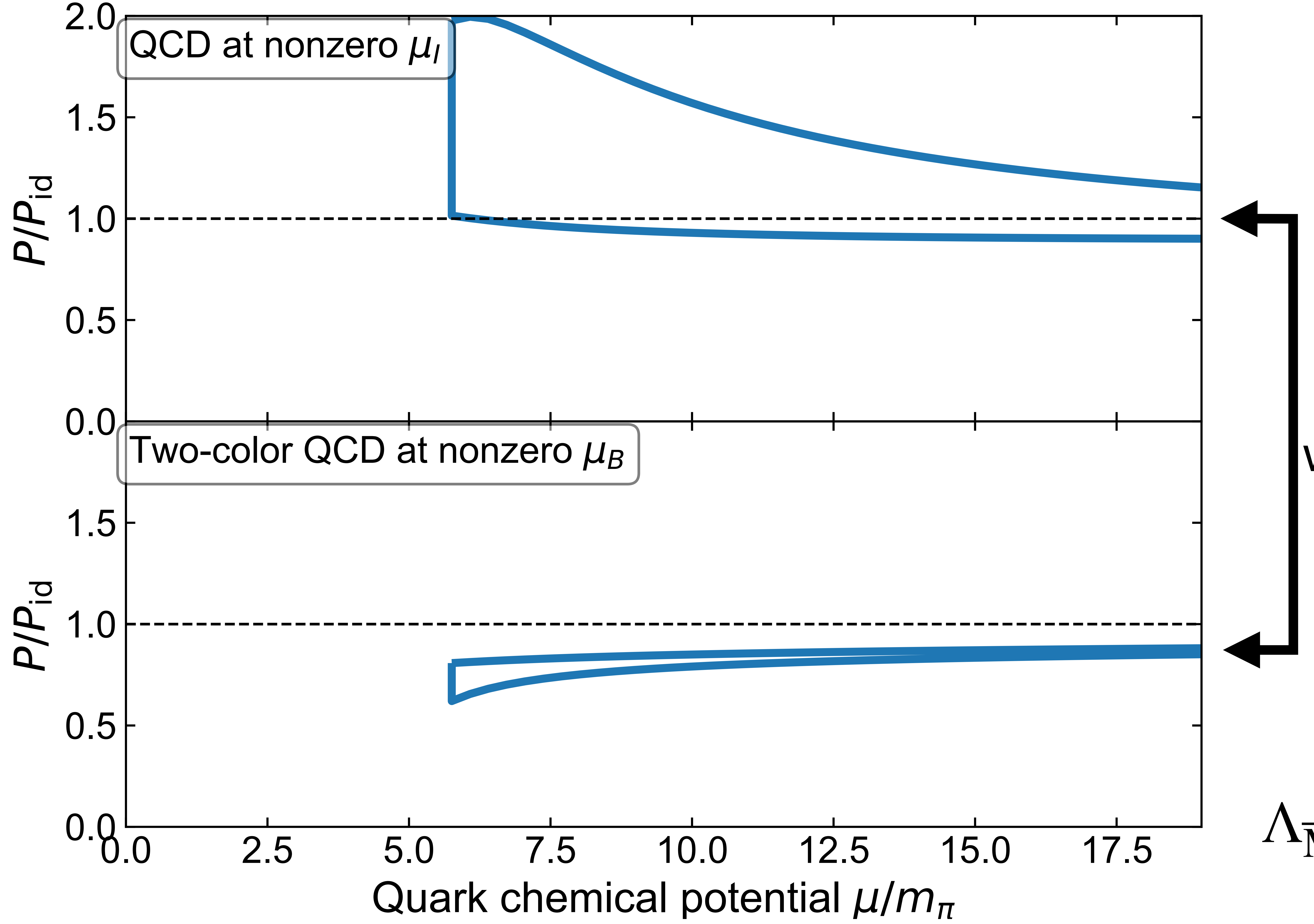
Effect on the QCD phase diagram

Based on weak-coupling calculation:



Weak-coupling EoS in two-color QCD

Fujimoto, 2408.12514



Completely different behavior in weak-coupling regime

Wanted:
$\Lambda_{\overline{\rm MS}}$ in two-color QCD

Summary

- **QCD at finite isospin density:** useful nonperturbative piece of information

- **QCD inequality:** robust bounds on the symmetric nuclear matter EoS

- **Weak-coupling results:** Matches with lattice data at finite isospin density
  - Empirical evidence for the validity down to $\mu \sim 0.8\ \mathrm{GeV}$.
  - Color-superconducting gap is negligible in the weak coupling limit.
  - CFL phase may be unstable against unpairing induced by the stress from strange quark mass
  - Two-color QCD shows qualitatively different behavior in weak coupling

Bonus materials

QCD at finite *isospin* density (QCD$_I$)

- Isospin chemical potential (conjugate to I_3):

$$\mu_u = \mu_I/2, \quad \mu_d = -\mu_I/2$$

- Partition function at finite isospin and **zero** baryon density:

$$Z_I(\mu_I) = \int [dA] \det \mathscr{D}(\frac{\mu_I}{2}) \det \mathscr{D}(-\frac{\mu_I}{2}) e^{-S_\mathrm{G}}$$

$$= \int [dA] \left| \det \mathscr{D}(\frac{\mu_I}{2}) \right|^2 e^{-S_\mathrm{G}}$$

$\det \mathscr{D}(-\mu) = [\det \mathscr{D}(\mu)]^*$

Positive real value
→ **NO sign problem**

Alford,Kapustin,Wilczek (1999);
Kogut,Sinclair (2002), Beane,Detmold,Savage (2008-); Brandt,Endrodi… (2014-)…

QCD at finite isospin density can be simulated on lattice

QCD inequality

Inequality among observables from path integrals Weingarten (1983); Witten (1983)

Inequality considered here:

QCD inequality for pressure $P \propto \log Z$:

$$P_B(\mu_B) \leq P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Pressure of dense QCD$_B$ matter
(what we want to know)

Pressure of dense QCD$_I$ matter
(what we already know from lattice QCD)

Cohen (2003); Fujimoto,Reddy (2023);
see also: Moore,Gorda (2023)

QCD inequality: derivation

Cohen (2003); Fujimoto,Reddy (2023);
see also: Moore,Gorda (2023)

- **Dirac operator**: $\mathscr{D}(\mu) \equiv \gamma^\mu D_\mu + m - \mu\gamma^0$, **property**: $\det \mathscr{D}(-\mu) = [\det \mathscr{D}(\mu)]^*$

- **QCD$_I$**: $Z_I(\mu_I) = \int [dA]\ \det \mathscr{D}(\frac{\mu_I}{2}) \det \mathscr{D}(-\frac{\mu_I}{2}) e^{-S_G} = \int [dA]\ \left|\det \mathscr{D}(\frac{\mu_I}{2})\right|^2 e^{-S_G}$

u quark   d quark   charge conjugation symmetry $\mu_B \to -\mu_B$

- **QCD$_B$**: $Z_B(\mu_B) = \int [dA]\ \det \mathscr{D}(\frac{\mu_B}{N_c}) \det \mathscr{D}(\frac{\mu_B}{N_c})\ e^{-S_G} = \int [dA]\, \mathrm{Re} \left[\det \mathscr{D}(\frac{\mu_B}{N_c})\right]^2 e^{-S_G}$

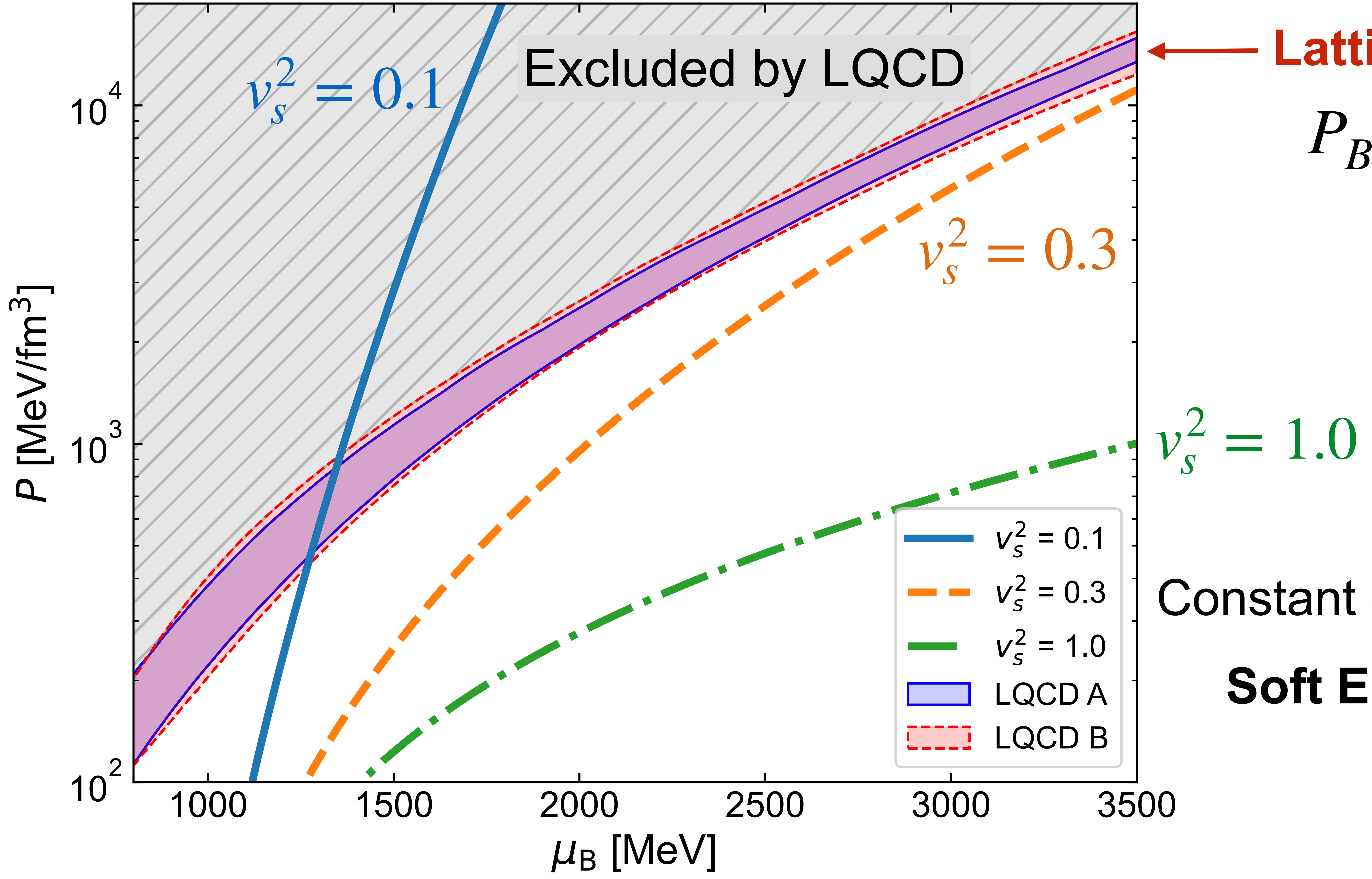
Note: this is **isospin symmetric** because there is no isospin imbalance

- From the relation $\mathrm{Re}\, z^2 \le |z^2| = |z|^2$:

$$Z_B(\mu_B) \le \int [dA]\ \left|\det \mathscr{D}(\frac{\mu_B}{N_c})\right|^2 e^{-S_G} = Z_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Direct use of QCD inequality

Lattice data: Abbott et al. (2023); Fujimoto,Reddy (2023)



Lattice data: upper bound

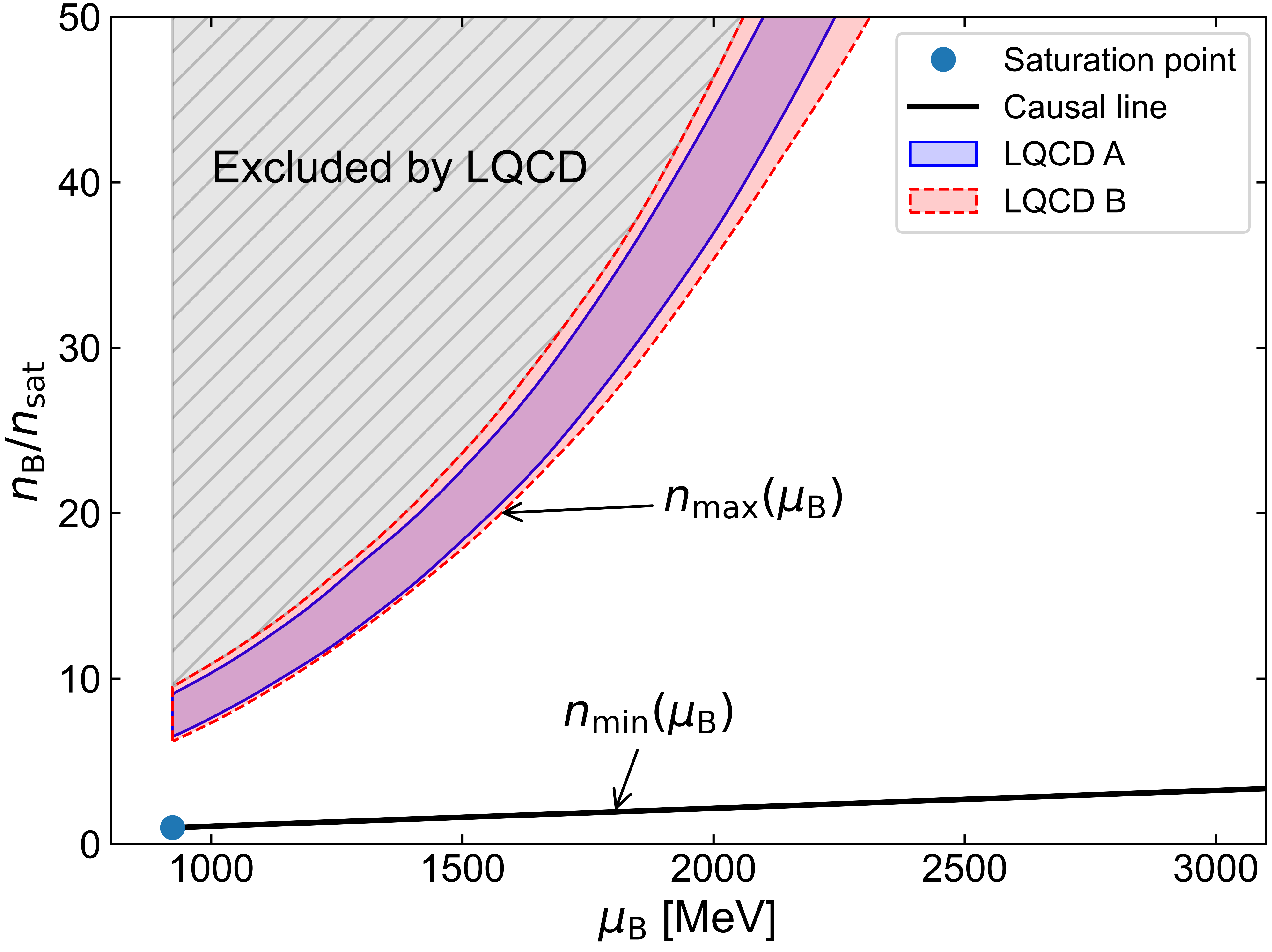
$$P_B(\mu_B) \le P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Constant sound speed EoS: $P(\varepsilon) \propto v_s^2 \varepsilon$

Soft EoS (smaller P at a given ε) is excluded

Bounds on $n_B(\mu_B)$

Komoltsev,Kurkela (2021); Fujimoto,Reddy (2023)



Properties $n_B(\mu_B)$ must satisfy:

① Stability:

$$\frac{d^2P}{d\mu_B^2} \geq 0 \quad \Rightarrow \quad \frac{dn_B}{d\mu_B} \geq 0$$

② Causality $v_s^2 \leq 1$:

$$v_s^2 = \frac{n_B}{\mu_B}\frac{d\mu_B}{dn_B} \leq 1 \quad \Rightarrow \quad \frac{dn_B}{d\mu_B} \geq \frac{n_B}{\mu_B}$$

③ QCD inequality on the integral:

$$\int_{\mu_{sat}}^{\mu_B} d\mu' \, n_B(\mu') \; \leq P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

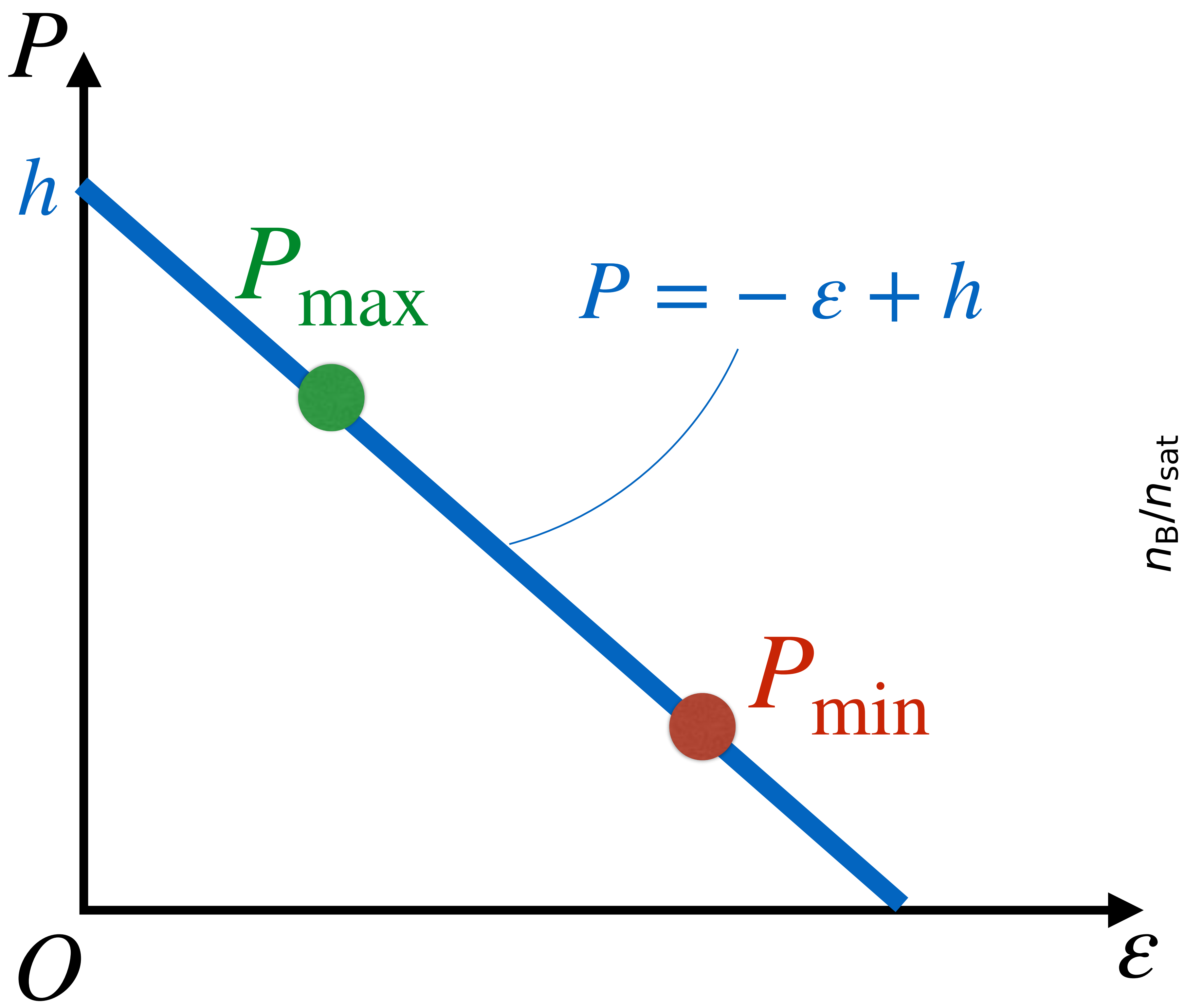
Lower bound of the integral must be specified

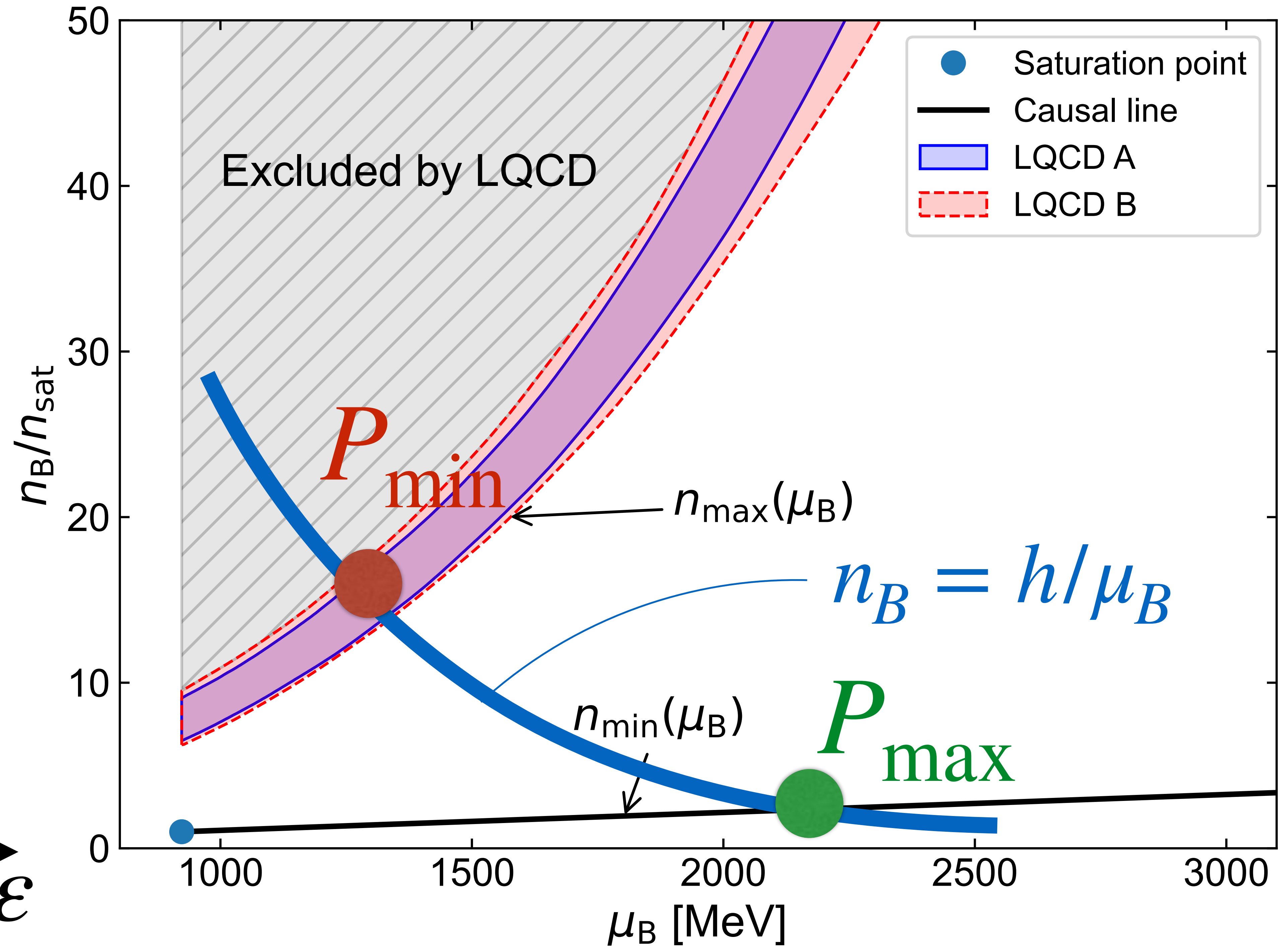
fix it to the **empirical saturation property**

Bounds on $P(\varepsilon)$

Isenthalpic line: $h = \mu_B n_B = \varepsilon + P = \text{const}$

Komoltsev,Kurkela (2021); Fujimoto,Reddy (2023)



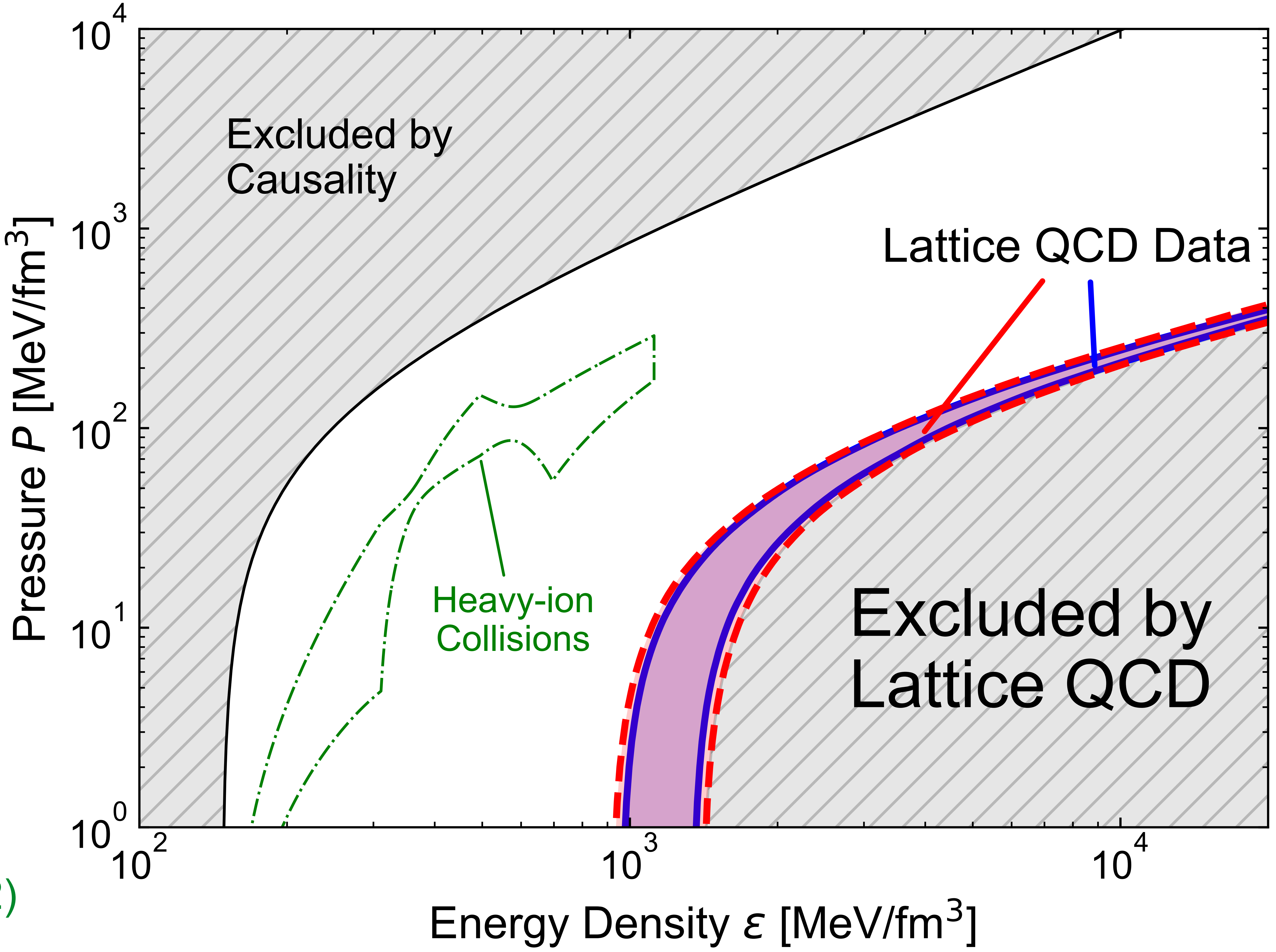


by changing value of h, the trajectories of $P_{\min}$ ($P_{\max}$) gives the lower (upper) bound for $P(\varepsilon)$

Robust bounds on $P(\varepsilon)$

Fujimoto,Reddy (2023)

From the relation $\varepsilon = -P + \mu_B n_B$:



Soft EoS at large ε is excluded

Heavy-ion:
Oliinychenko et al.(2022)