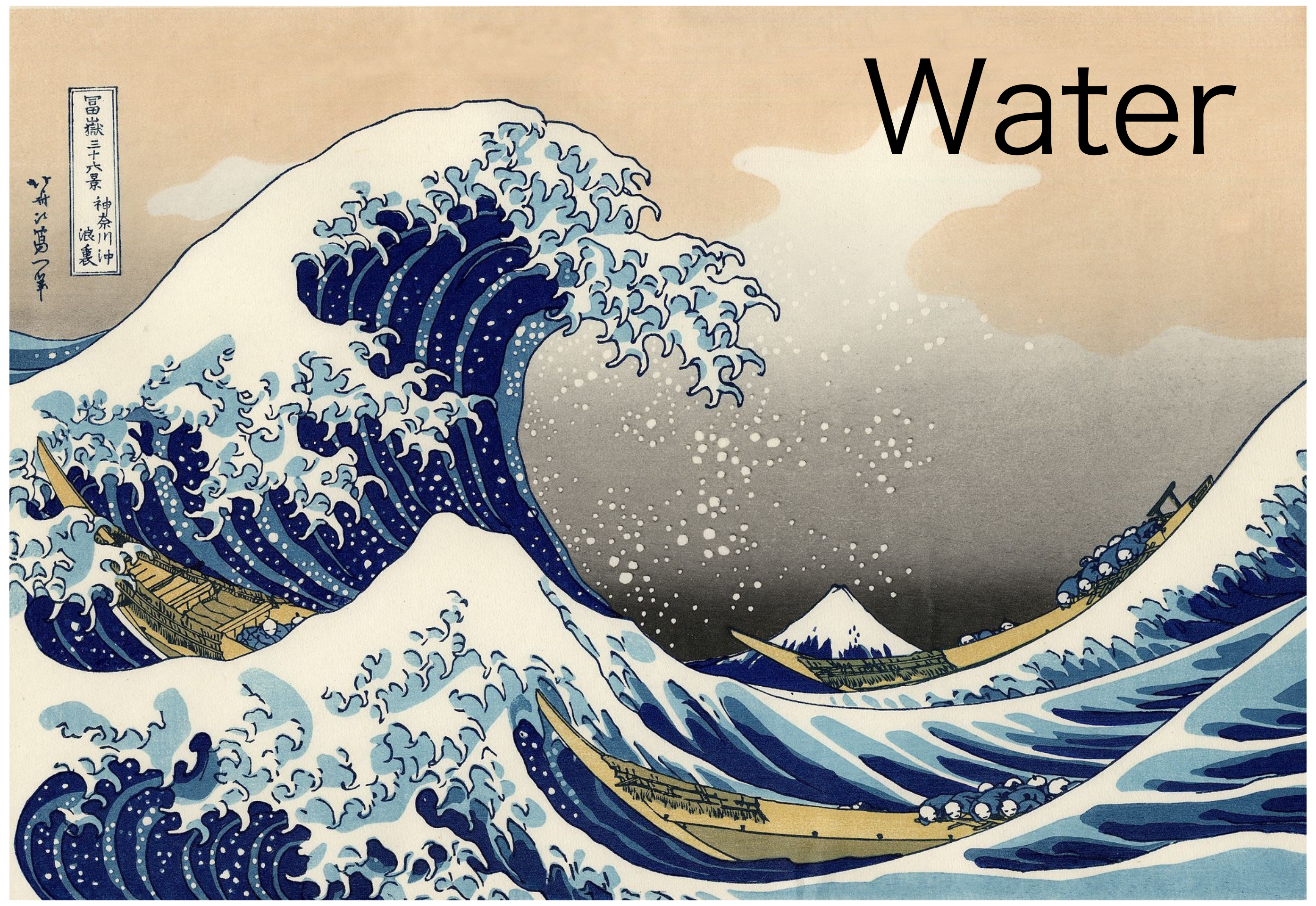
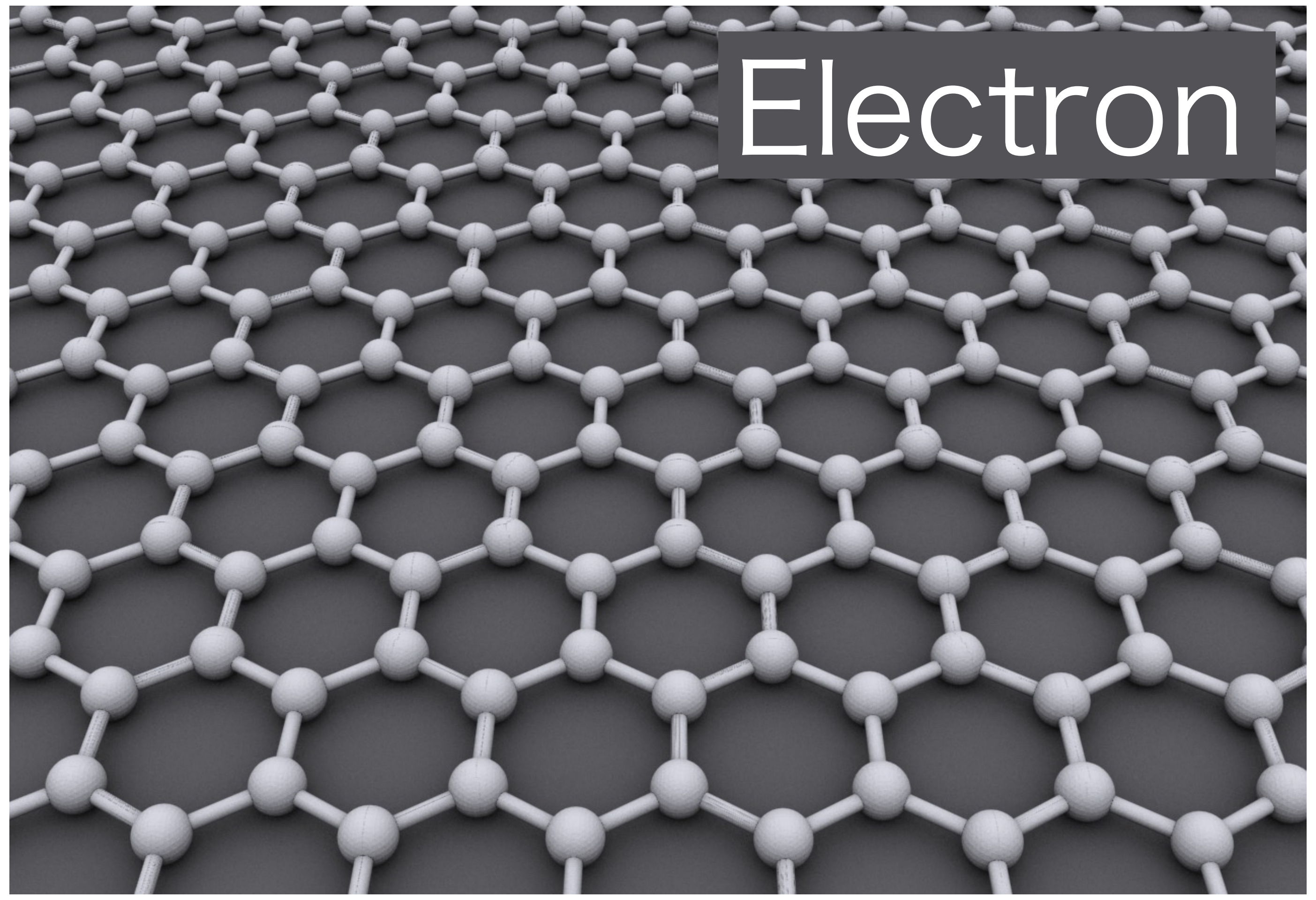
Hydrodynamics of superfluid phases in neutron star inner crusts

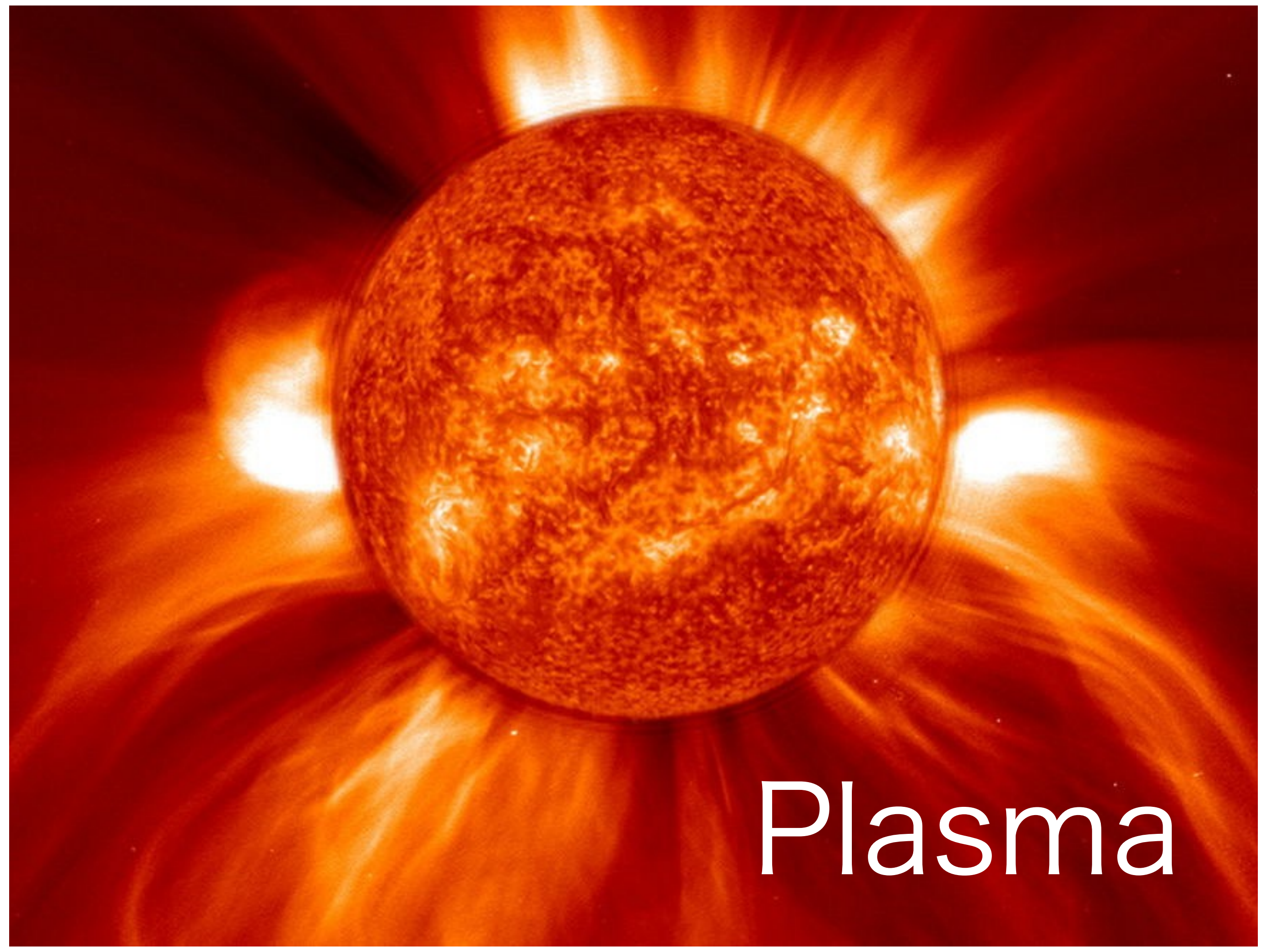
Masaru Hongo (Niigata University/RIKEN iTHEMS)

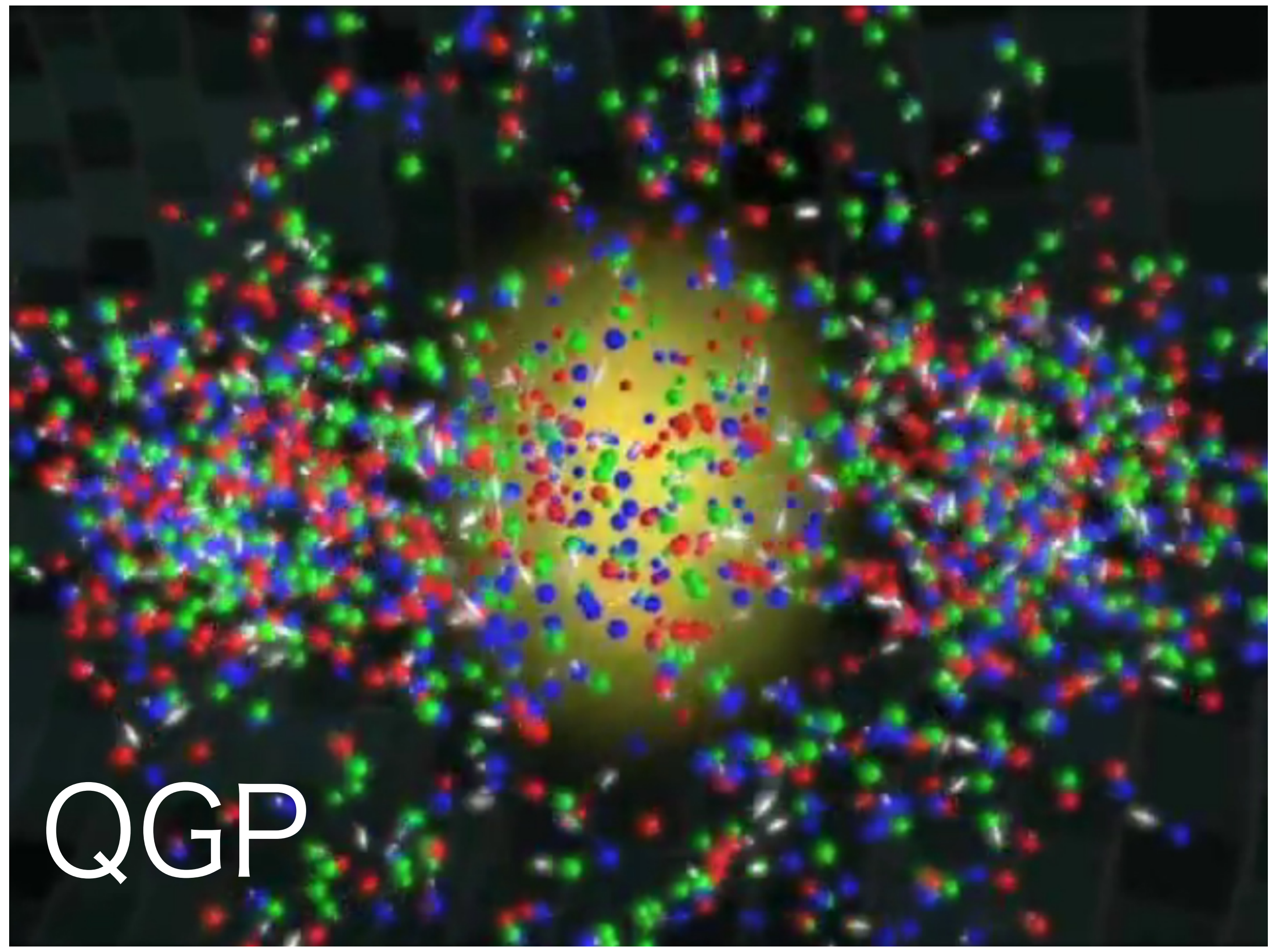
2024/10/31, Hadrons and Hadron Interactions in QCD 2024

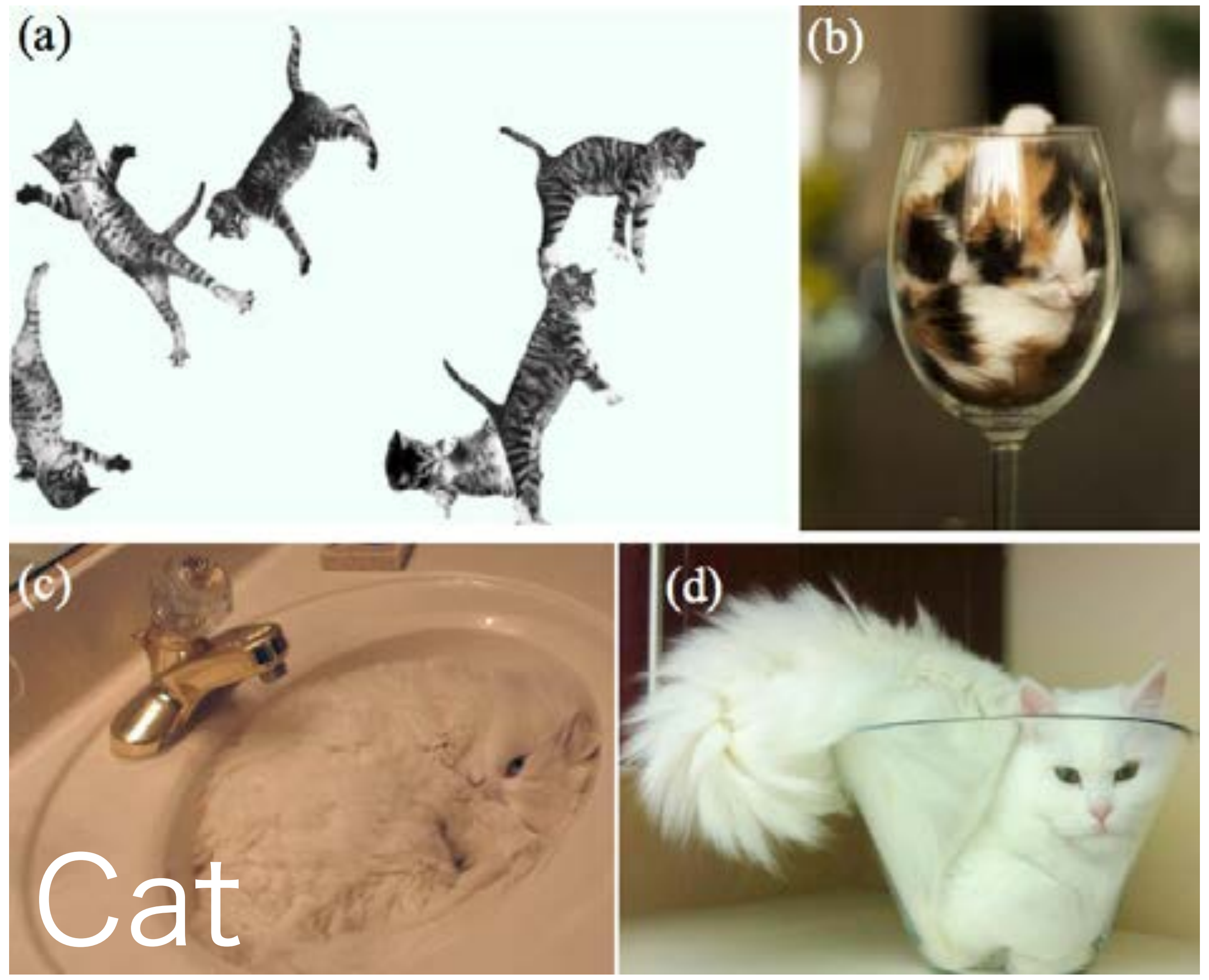
Hydrodynamics at work











Ig Nobel Prize 2017 [M.A. Fardin (2014)]

Let us apply hydrodynamics to nuclear matter in neutron stars!

WHAT and WHY hydro?

The oldest but state-of-the-art phenomenological field theory

Pascal's law

Hydro*dynamics*

Euler equations
(Perfect fluid)

Navier-Stokes equations
(Viscous fluid)

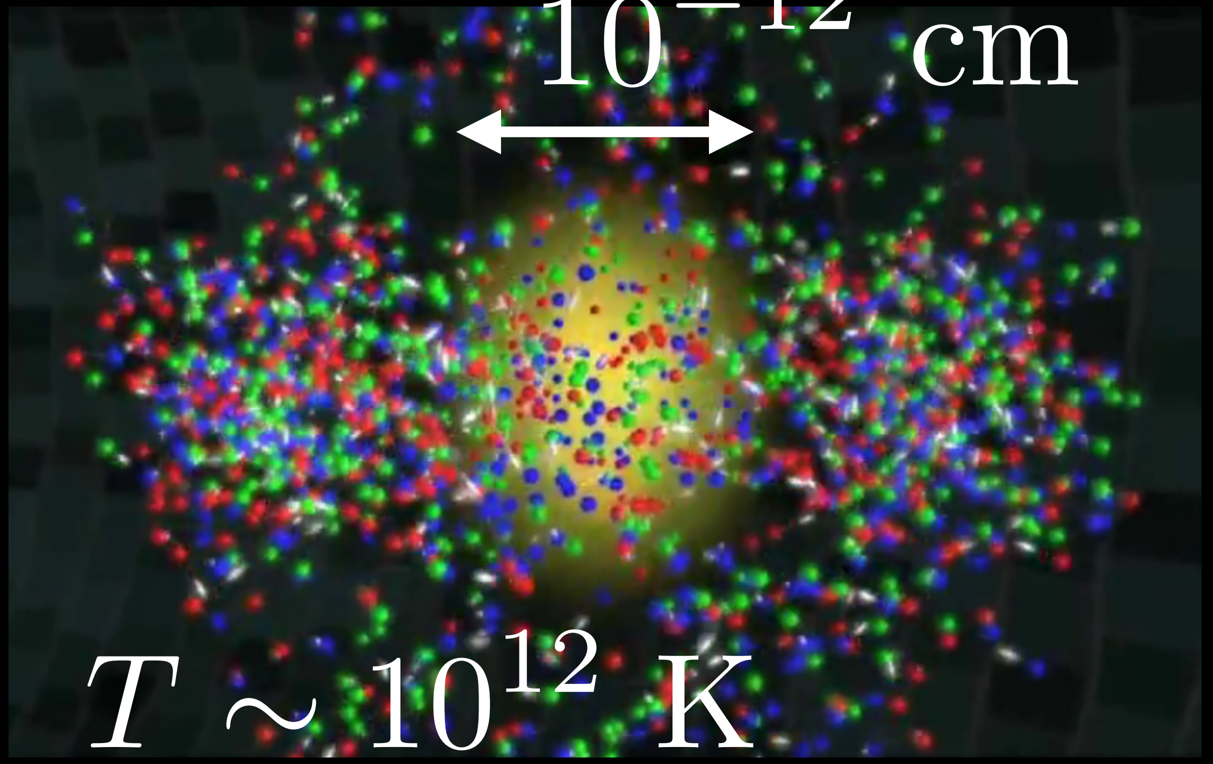
1600 — 1700 — 1800 — 1900

WHAT and WHY hydro?

Hydrodynamics = Low-energy EFT for real-time dynamics

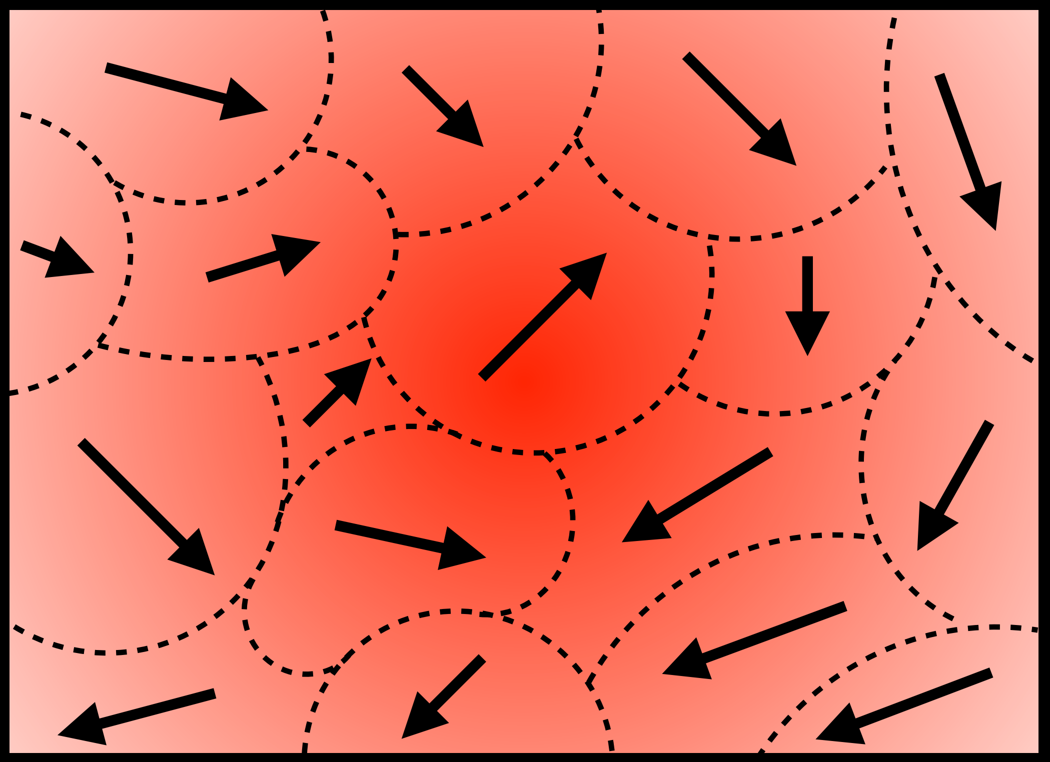
- Effective theory for macroscopic dynamics
- Universal description, not depending on details
- Only conserved quantity ~ ~~symmetry~~ of system

Quark-Gluon Plasma | **Hydro: $\{\beta(x),\ \vec{v}(x)\}$** | **Neutron Star**

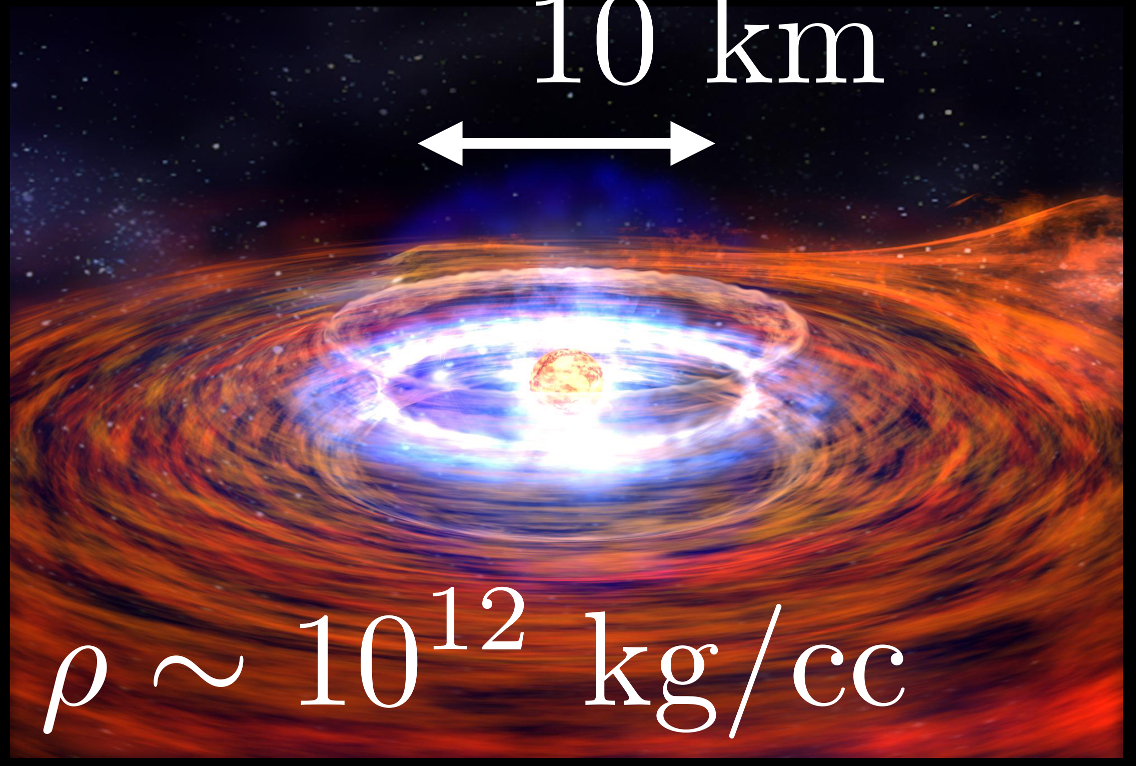


http://www.bnl.gov/rhic/news2/news.asp?a=1403&t=pr

$\simeq$



$\simeq$



http://newsoffice.mjitugenn.edu/2012/model-bursting-star-0302

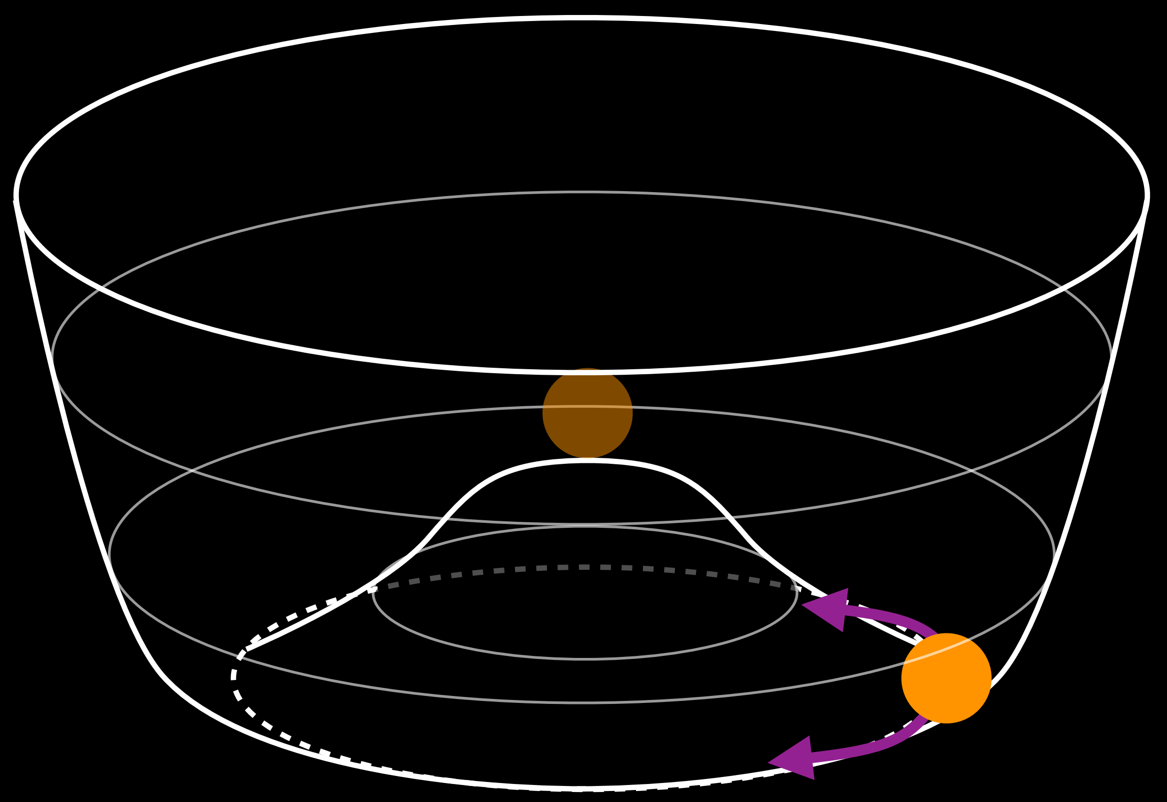
Hydrodynamics with SSB

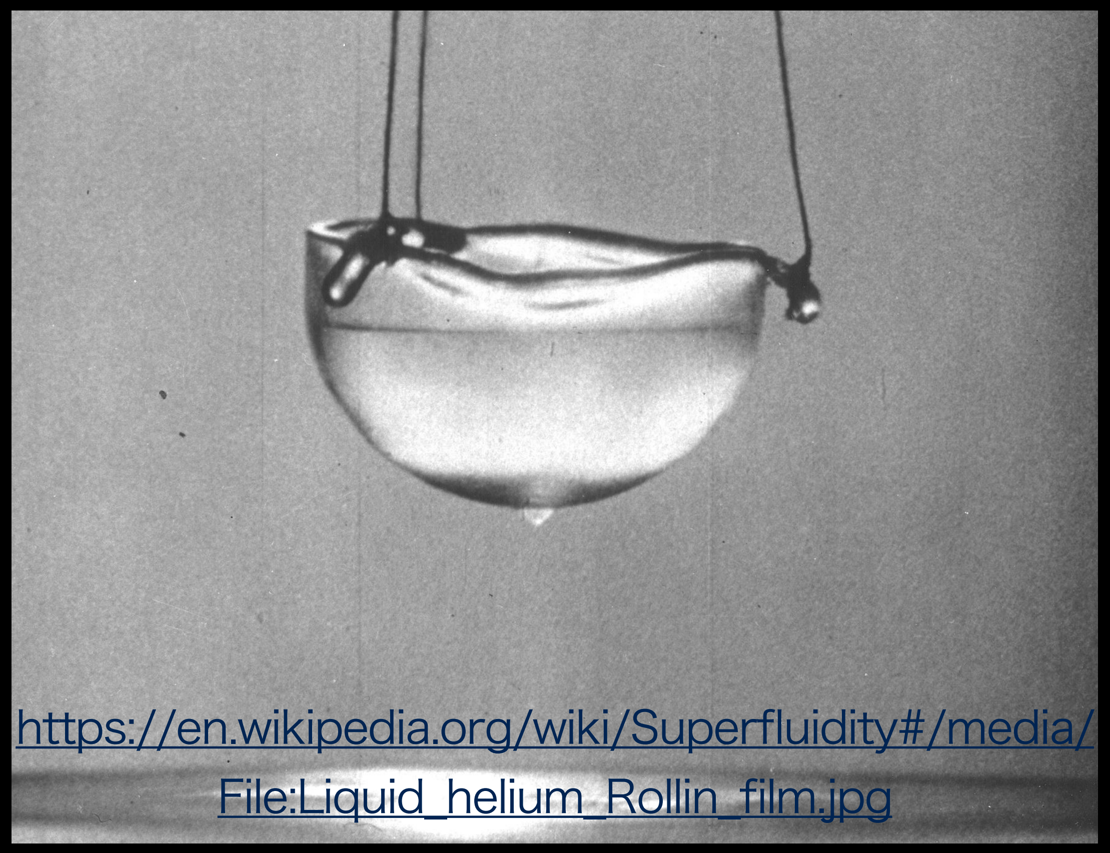
When continuous global symmetry is spontaneously broken,
hydrodynamic equation is strongly modified!

Ex. Helium II = U(1) symmetry breaking in ^{4}He

Superfluid Hydrodynamics (Two-fluid model) by Tisza, Landau

General consequence resulting from the Nambu-Goldstone theorem



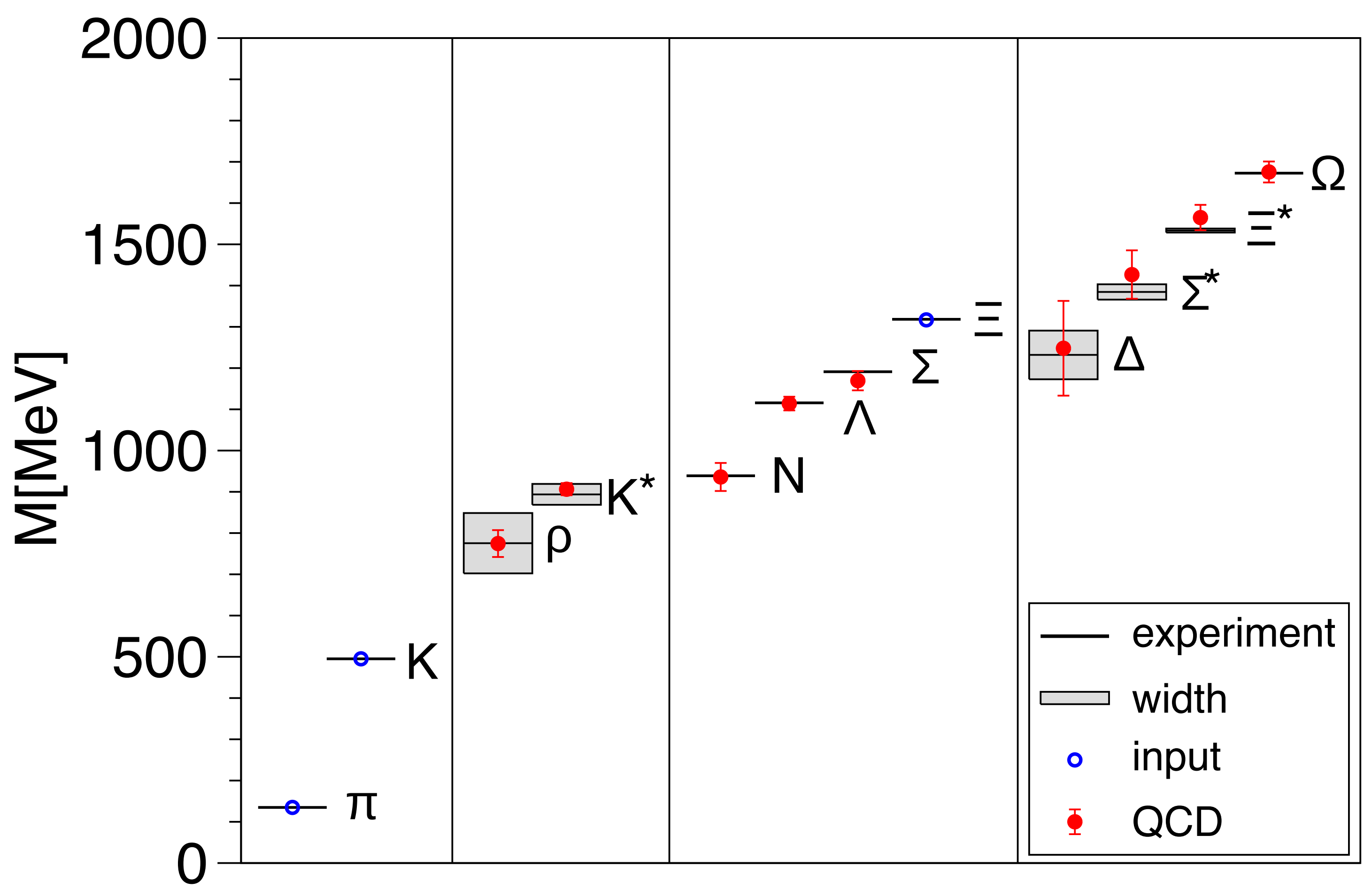


SSB patterns of QCD matter

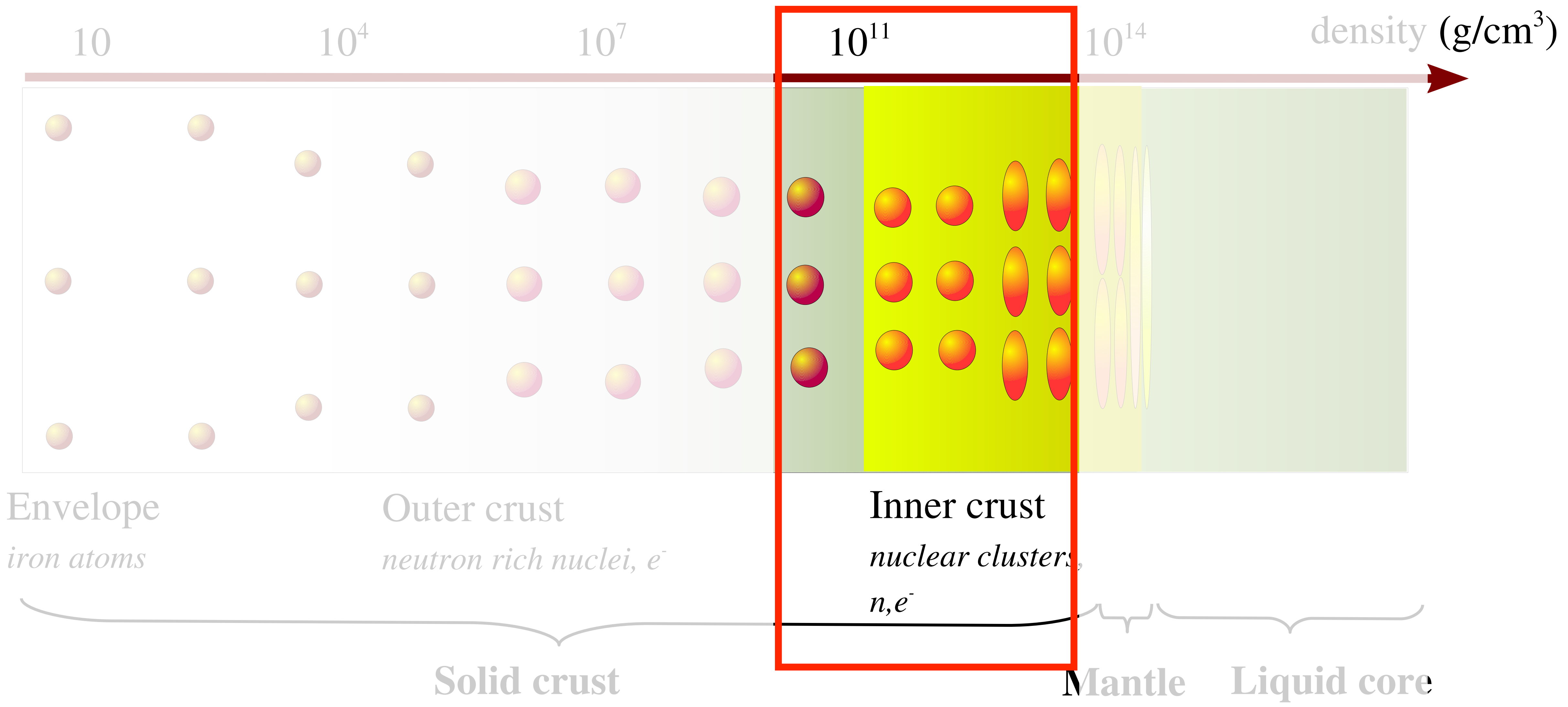
QCD enjoys Poincare $\times U(1)_B \times SU(2)_R \times SU(2)_L$ symmetry

- Low-temperature QCD breaks (approximate) chiral symmetry → Pions
- Neutron star inner crust → Superfluid and lattice phonons + Pions
- Liquid core the neutron star → Superfluid phonon + angulons (+ Pions)

Q. What are hydrodynamic equations in symmetry-broken phases?



From Budapest-Marseille-Wuppertal Collaboration (2008)

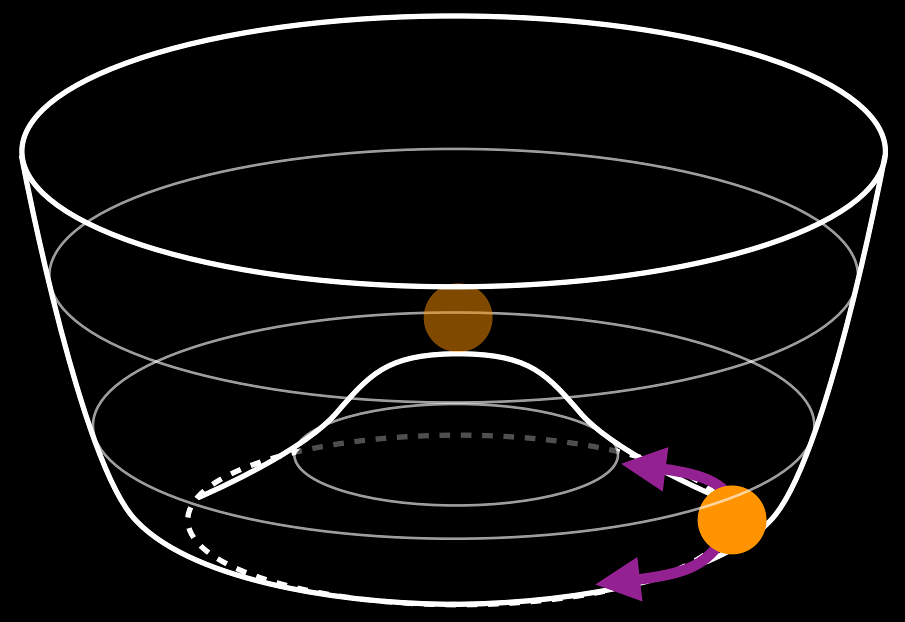


From Chamel-Haensel (2008)

Outline

Motivation:

Hydrodynamics for
symmetry-broken phases?



Approach:

Semi-phenomenology based on local thermodynamics

Result & Outlook:

HOW to derive hydrodynamics

- **Kinetic-theory derivation based on the Boltzman equation**

  [Tsumura et al, PLB (2007), Denicol et al, PRD (2012), …]

- **Nonequilibrium statistical operator approach**

  [Becattini et al, EPJC (2008), Hayata et al, PRD (2015), …]

- **Holographic-derivation based on fluid/gravity correspondence**

  [Baier et al, JHEP (2008), Bhattacharyya et al, JHEP (2008), …]

- **Projection operator/Poisson bracket approach**

  [Son PRL (2000), Hayata-Hidaka PRD (2015), …]

- **Phenomenological derivation based on local thermodynamics**

  [Son-Stephanov PRD (2002), Grossi et al., PRD (2021), …]

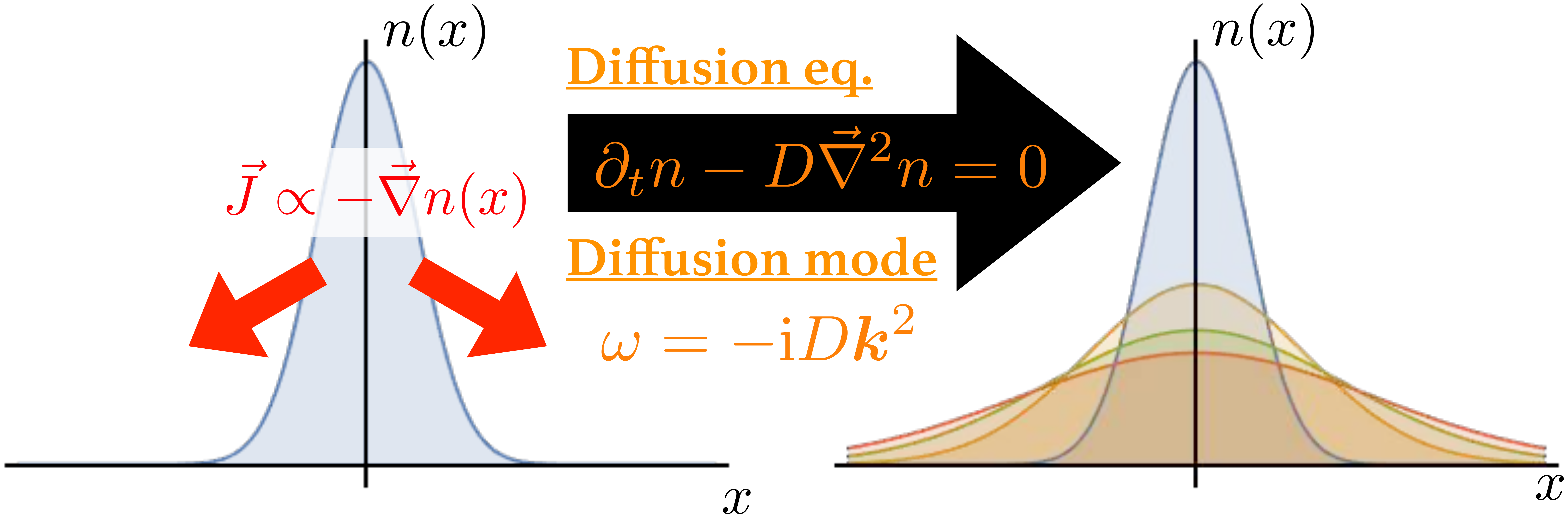
Prototype: Charge diffusion

◆ **Bulding blocks of hydrodynamic equation**

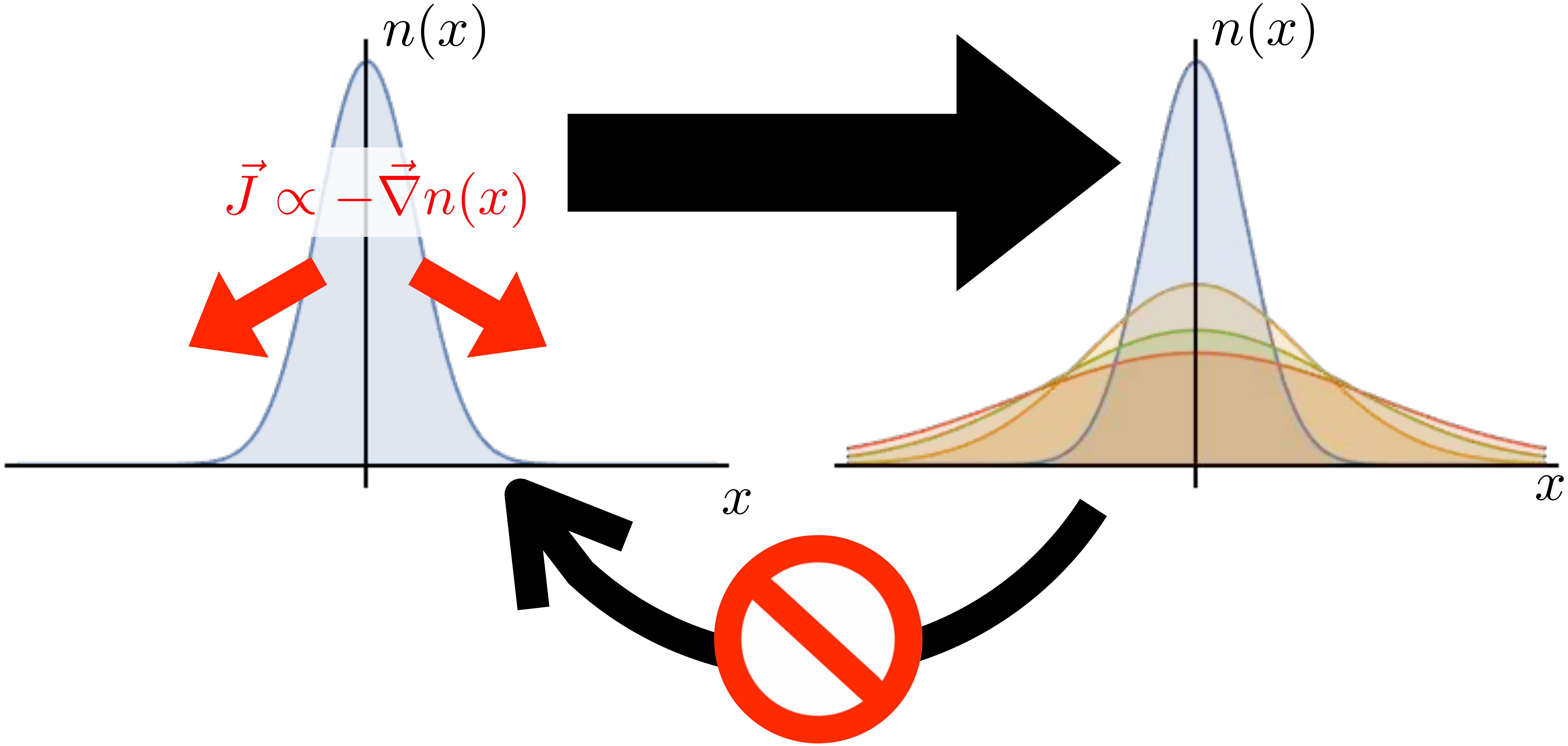
(1) Conservation law: $\partial_t n + \vec{\nabla} \cdot \vec{J} = 0$

(2) Constitutive relation: $\vec{J} = -T\kappa_n \vec{\nabla}(\beta\mu) \simeq -D\vec{\nabla} n$

(3) Physical properties: Values of $\kappa_n, \ \chi_n \quad (D = \kappa_n/\chi_n)$



Irreversibility of diffusion



No-go for time-reversal process!

Thermodynamic concepts, especially, The 2nd law, should be there!

Phenomenological derivation

Step 1. Determine dynamical d.o.m (& its equation of motion)

Charge density: $n(x)$ EoM: $\partial_t n + \vec{\nabla} \cdot \vec{J} = 0$

Step 2. Introduce entropy & conjugate variable with 1st law

Entropy density: $s(n)$ ⇒ ($T\mathrm{d}s = -\mu \mathrm{d}n$) Chemical pot.: $\beta\mu \equiv -\dfrac{\partial s}{\partial n}$

Step 3. Find $\vec{J}$ up to finite derivatives compatible with 2nd law

$\exists\, s^\mu$ such that $\partial_t s + \vec{\nabla} \cdot \vec{s} \geq 0$ ⇒ $\vec{J} \simeq -T\kappa_n \vec{\nabla}(\beta\mu)$

Step 4. Identify how parameters (e.g., κ_n) can be matched

Green-Kubo formula: $\kappa_n = \lim_{\omega\to 0} \dfrac{1}{\omega} \mathrm{Im}\, G_R^{J^x J^x}(\omega, \boldsymbol{k} = 0)$

QFT interpretation

$\simeq$ Ward-Takahashi identity

$\simeq$ Effective Lagrangian (Hamiltonian)

$\simeq$ A kind of symmetry constraints

$\simeq$ Matching condition for low-energy coeff.

Outline

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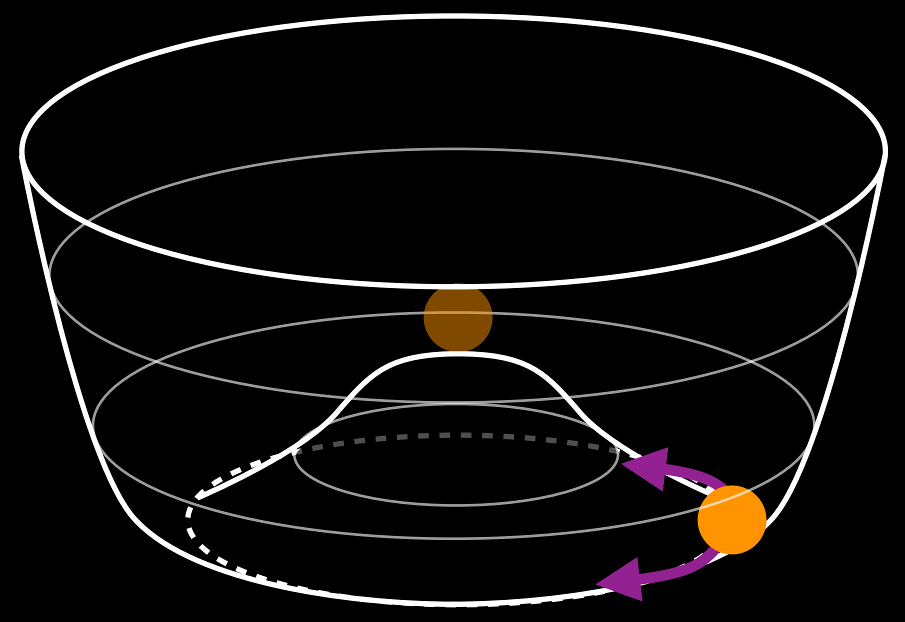
Hydrodynamics for
symmetry-broken phases?



Approach:

Semi-phenomenology based on local thermodynamics

Result & Outlook:

Derivation of hydrodynamics for symmetry-broken phases

Matching condition (Kubo formula) for all Onsager coeff.

Application to NS physics (e.g., neutrino reaction, ...)

Application to U(1)-symmetry breaking

⇓

In addition to the conserved charge density

superfluid phonon φ appears!

Phenomenological derivation

QFT interpretation

Step 1. Determine dynamical d.o.m (& its equation of motion)

Charge density: $n(x)$ EoM: $\partial_t n + \vec{\nabla} \cdot \vec{J} = 0$

≃ Ward-Takahashi identity

Step 2. Introduce entropy & conjugate variable with 1st law

Entropy density: $s(n)$ $T\mathrm{d}s = -\mu \mathrm{d}n$ ⇒ Chemical pot.: $\beta\mu \equiv -\dfrac{\partial s}{\partial n}$

≃ Effective Lagrangian (Hamiltonian)

Step 3. Find $\vec{J}$ up to finite derivatives compatible with 2nd law

$\exists\, s^\mu$ such that $\partial_t s + \vec{\nabla} \cdot \vec{s} \geq 0$ ⇒ $\vec{J} \simeq -T\kappa_n \vec{\nabla}(\beta\mu)$

≃ A kind of symmetry constraints

Step 4. Identify how parameters (e.g., κ_n) can be matched

Green-Kubo formula: $\kappa_n = \lim_{\omega\to 0} \dfrac{1}{\omega} \mathrm{Im}\, G_R^{J^x J^x}(\omega, \boldsymbol{k} = 0)$

≃ Matching condition for low-energy coeff.

Phenomenological derivation

Step 1. Determine dynamical d.o.m (& its equation of motion)

Charge density: n & Superfluid phonons: φ EoM: $\partial_t n + \partial_i J^i = 0$ & $\partial_t \varphi = \Pi$

Step 2. Introduce entropy & conjugate variable with 1st law

Entropy density: $s = s(n, v)$ with $v = \frac{1}{2}(\partial_i \varphi)^2, \quad \beta\mu = -\frac{\partial s}{\partial n}, \quad \beta f^2 = -\frac{\partial s}{\partial v}$

Step 3. Find $\{J^i, \Pi\}$ up to finite derivatives compatible with 2nd law

$\exists\, s^\mu$ such that $\partial_t s + \vec{\nabla} \cdot \vec{s} \geq 0$ ⇒ $J^i = f^2 \partial^i \varphi - \kappa_n \partial^i \mu, \ \Pi = -\mu + \zeta_s \partial_i u (f^2 \partial^i \varphi)$

$$\partial_t s = \frac{\partial s}{\partial n}\partial_t n + \frac{\partial s}{\partial v}\partial_t v = \beta\mu\partial_i J^i - \beta f^2 \partial_i \varphi \partial^i \Pi = \partial_i(\beta\mu J^i) + \beta[-J^i \partial_i \mu - f^2 \partial_i \varphi \partial^i \Pi]$$

Choosing $s^i := -\beta\mu J^i + \beta f^2 \Pi \partial^i \varphi, \ J^i = f^2 \partial^i \varphi - \kappa_n \partial^i \mu, \ \Pi = -\mu + \zeta_s \partial_i u (f^2 \partial^i \varphi)$ works!

Result for the simple superfluid

◆**Equation of motion**

$$\partial_t n + \partial_i J^i = 0, \quad \partial_t \varphi = \Pi$$

◆**Constitutive relation**

$$J^i = f^2 \partial^i \varphi - \kappa_n \partial^i \mu, \quad \Pi = -\mu + \zeta_s \partial_i u (f^2 \partial^i \varphi)$$

Supercurrent/Diffusion   Josephson eq./Damping effect

◆ **Onsager coefficient**

Charge conductivity: κ_n, Damping coefficient: ζ_s

➡ **Gapless mode:** $\omega \simeq \pm c_s |\boldsymbol{k}| - \frac{\mathrm{i}}{2}(D + f^2 \zeta_s)\boldsymbol{k}^2$ **appears!** $\left[c_s := \frac{f}{\sqrt{\chi}},\ D := \frac{\sigma}{\chi} \right]$

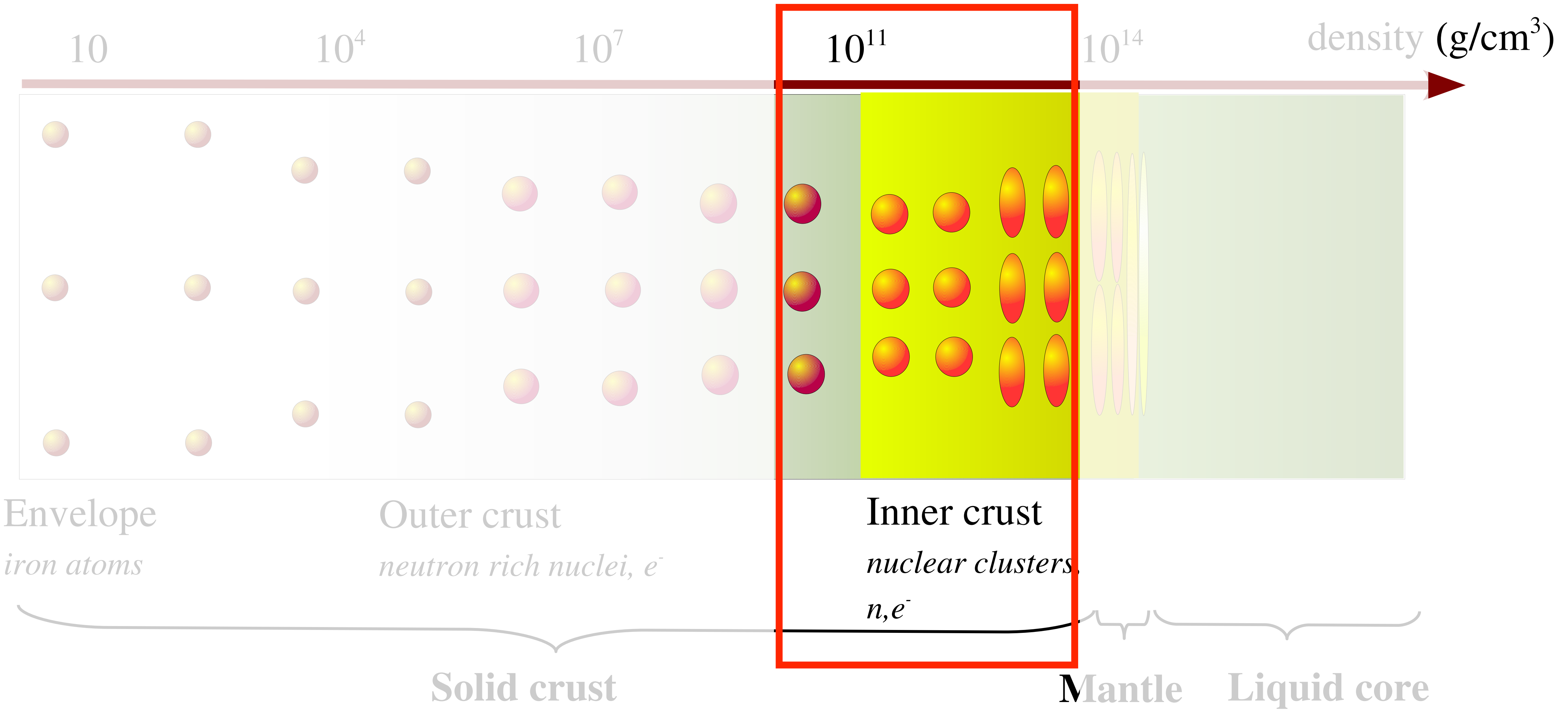
Application to Hydrodynamics in the neutron star inner crust

SSB pattern in the NS inner crust

Key properties: [See Cirigliano et al. et al, PRC (2011) for low-energy EFT]

- Nuclei form a Coulomb lattice:
  Translational symmetry is spontaneously broken → Lattice phonons ξ^i appears!
- Cooper pairs of dripped neutrons realize an s-wave condensate:
  $U(1)_n$ symmetry is spontaneously broken → Superfluid phonon φ appears!

➡ What is the corresponding hydrodynamics at $T \neq 0$ for the inner crust?



From Chamel-Haensel (2008)

Phenomenological derivation

Step 1. Determine dynamical d.o.m (& its equation of motion) — $(T^{\mu}_{\ \nu}u^{\nu} = -eu^{\mu})$

Charge densities: $c_a = \{T^0_{\ \mu}, \rho_n\}$ & Phonons: $\{\varphi, \xi^i\}$ EoM: $\partial_t c_a + \partial_i J^i_{\ a} = 0$ & $u^\mu\partial_\mu\varphi = \Pi,\ u^\mu\partial_\mu\xi^i = h^i$

Step 2. Introduce entropy & conjugate variable with 1st law — [Cirigliano et al. et al, PRC (2011)]

Entropy density
$$s \simeq s_0(e, \rho_n - g\partial_i\xi^i) - \frac{\beta f^2}{2}(\partial_i\varphi)^2 - \frac{\beta}{2}\mu^{ijkl}\partial_i\xi_j\partial_k\xi_l \quad \text{with} \quad \beta = \frac{\partial s}{\partial e},\ \beta\mu_n = -\frac{\partial s}{\partial \rho_n}$$

Step 3. Find $\{J^i_{\ a}, \Pi, h^i\}$ up to finite derivatives compatible with 2nd law

$$\exists\, s^\mu \text{ such that } \partial_t s + \vec{\nabla}\cdot\vec{s} \geq 0 \quad \Longrightarrow \quad J^i_{\ a} = \cdots,\ \Pi = \cdots,\ h^i = \cdots$$

The procedure looks complicated in this case, but we can do it!

Hydrodynamics for inner crust (preliminary)

◆ Equation of motion

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu J_n^\mu = 0, \quad u^\mu \partial_\mu \varphi = \Pi, \quad u^\mu \partial_\mu \xi^i = h^i$$

◆ Constitutive relation

$$T^{\mu\nu} = (e+p)u^\mu u^\nu + p\eta^{\mu\nu} + f^2 \partial^\mu \varphi \partial^\nu \varphi + T\frac{\partial s}{\partial v_{\mu\nu}} + T\frac{\partial s}{\partial v_{\mu\lambda}}\partial^\nu \xi_\lambda$$
$$-T\eta^{\mu\nu\rho\sigma}\partial_\rho(\beta u_\sigma) - T\zeta_\times h^{\mu\nu}\beta\partial_\mu(f^2\partial^\mu\varphi)$$

$$J^\mu = nu^\mu + f^2\partial^\mu\varphi - T\kappa_n \partial_{\perp\mu}\nu$$

$$\left[v_{\mu\nu} = \frac{1}{2}(\partial_\mu \xi_\nu + \partial_\nu \xi_\mu)\right]$$

$$\Pi = -\mu + T\zeta_s \partial_\mu(f^2\partial^\mu\varphi) + T\zeta_\times h^{\mu\nu}\partial_\mu(\beta u_\nu)$$

$$h^i = u^i - T\gamma_{ij}\frac{\partial s}{\partial \xi_j}$$

◆ Onsager coefficient

$$\eta,\ \zeta,\ \kappa_n,\ \zeta_s,\ \zeta_\times,\ \gamma_{ij}$$

Potential application to NS inner crusts?

◆ Hydrodynamic modes

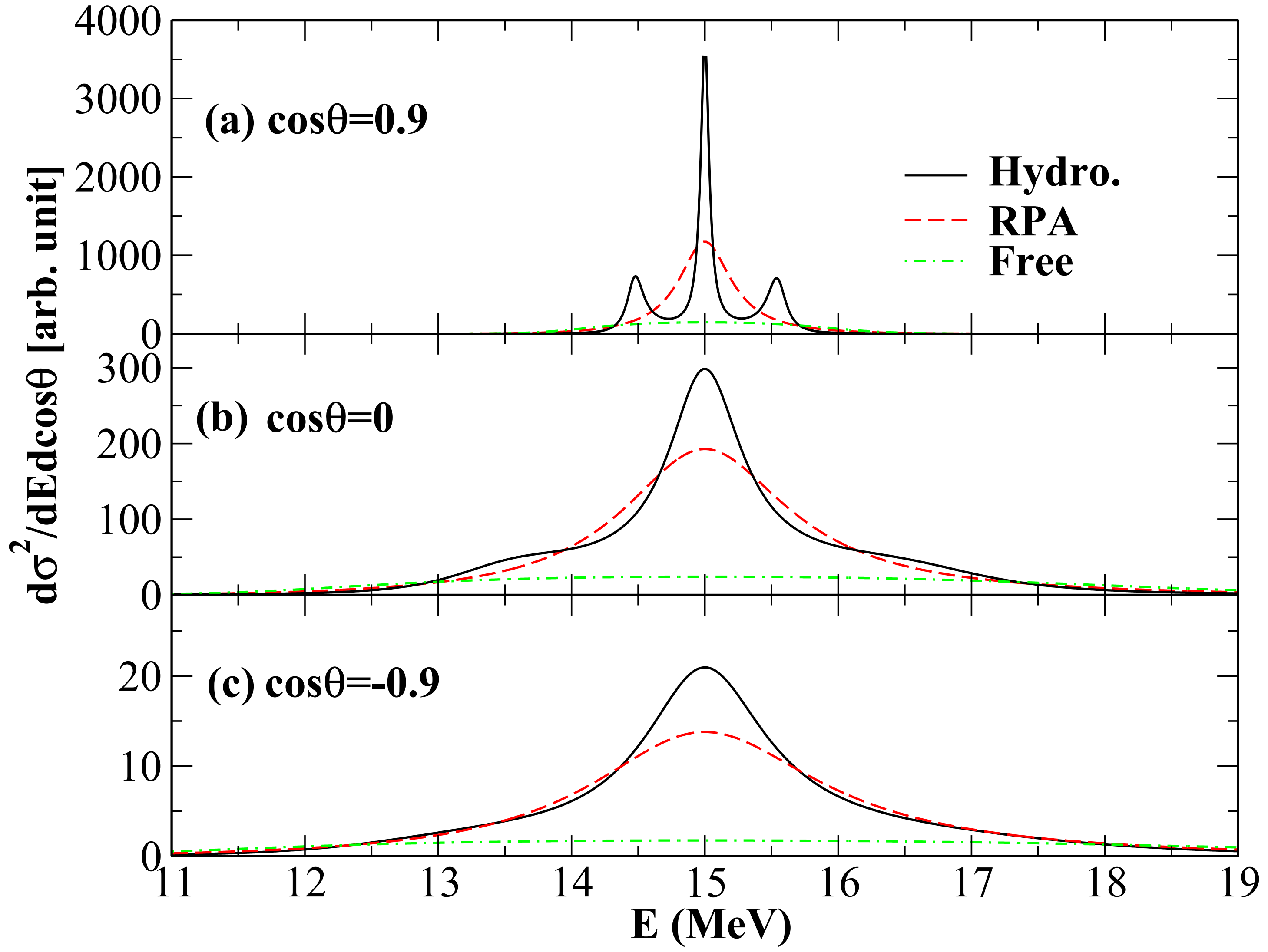
1 Diffusion + 4 Propagating modes (Entrainment at $T \neq 0$)

Carter-Chamel-Haensel NPA (2005), Cirigliano-Reddy-Sharma PRC (2011)

◆ Electromagnetic and neutrino coupling

Electron and photon dynamics

Low-energy neutrino reaction



Shen-Reddy PRC (2014)

Summary

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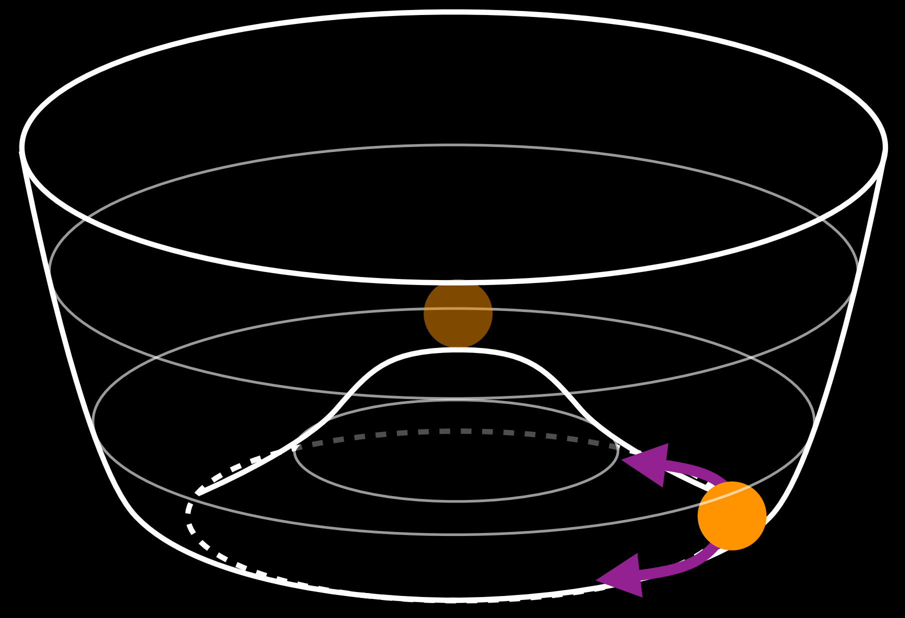

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