Where are we in understanding parton distributions from the first principles?

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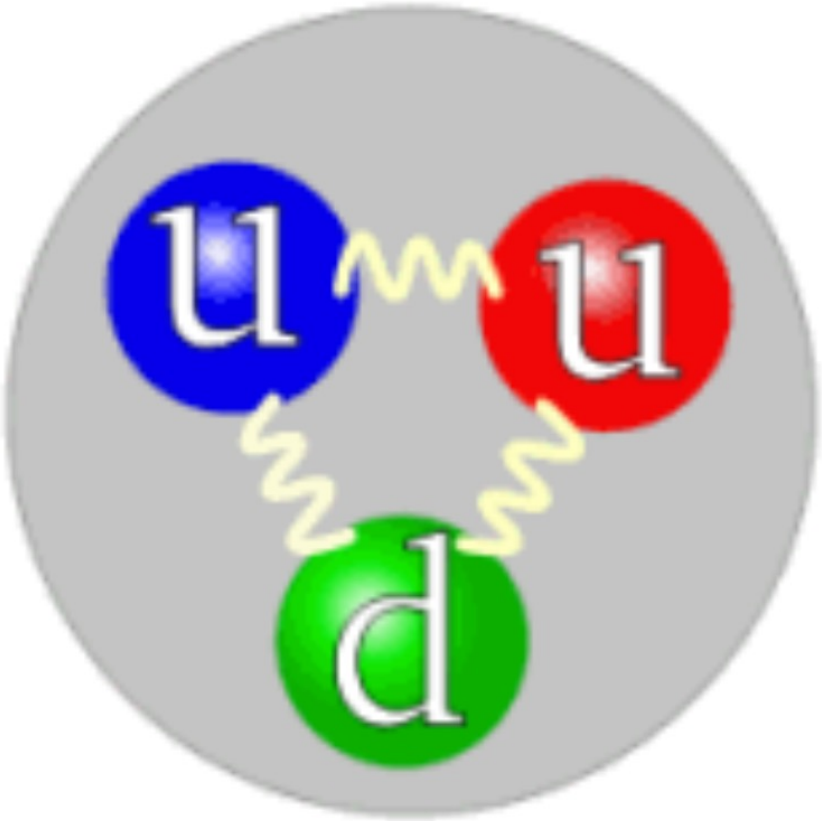
Yi-Bo Yang, Jianhui Zhang, Rui Zhang, Yong Zhao, Wei-Yang Liu, Chien-Yu Chou, Yi-Xian Chen

Gell-Mann's Triumph



Proton

What is a proton made of?

…and Feynman's puzzle:

How come there are only three quarks?

- QM: summing all the paths
- SR: no inertia frame is more special than the others

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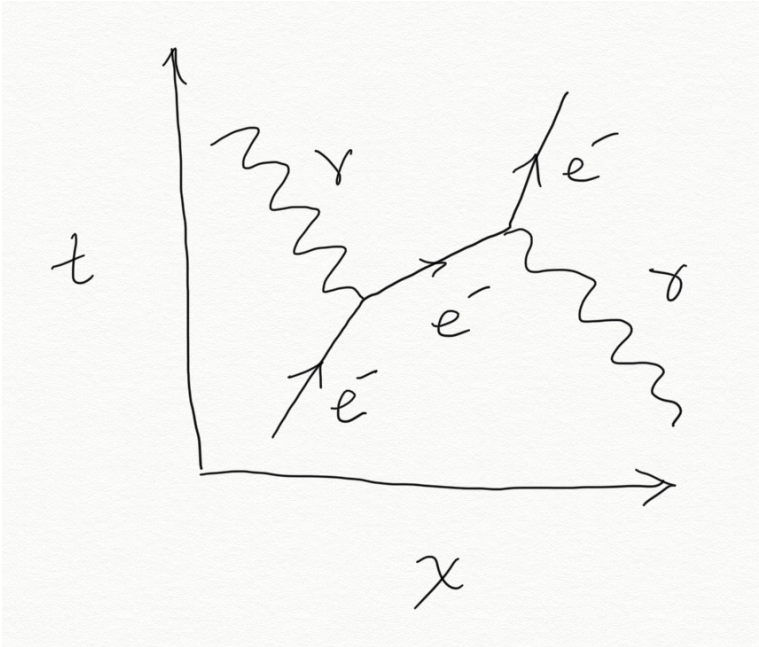
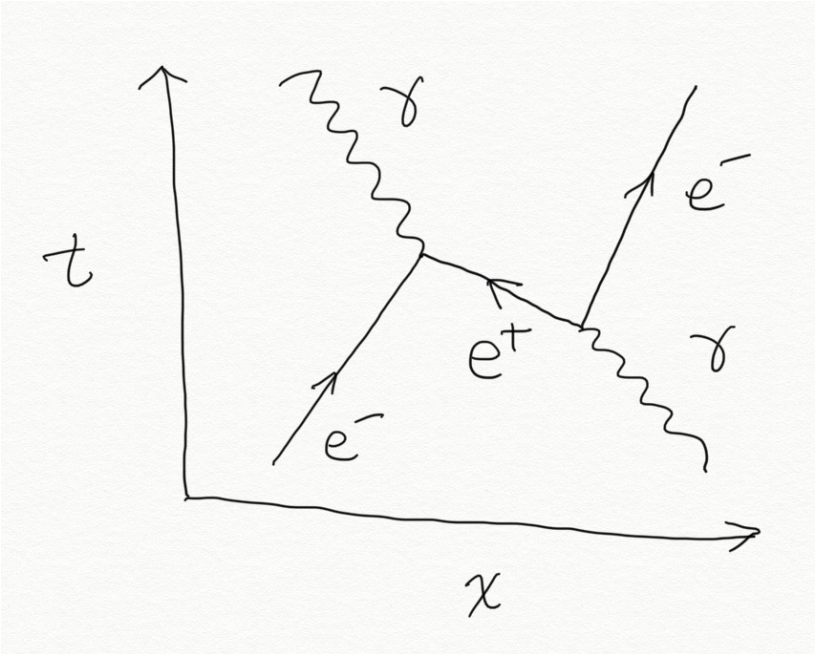
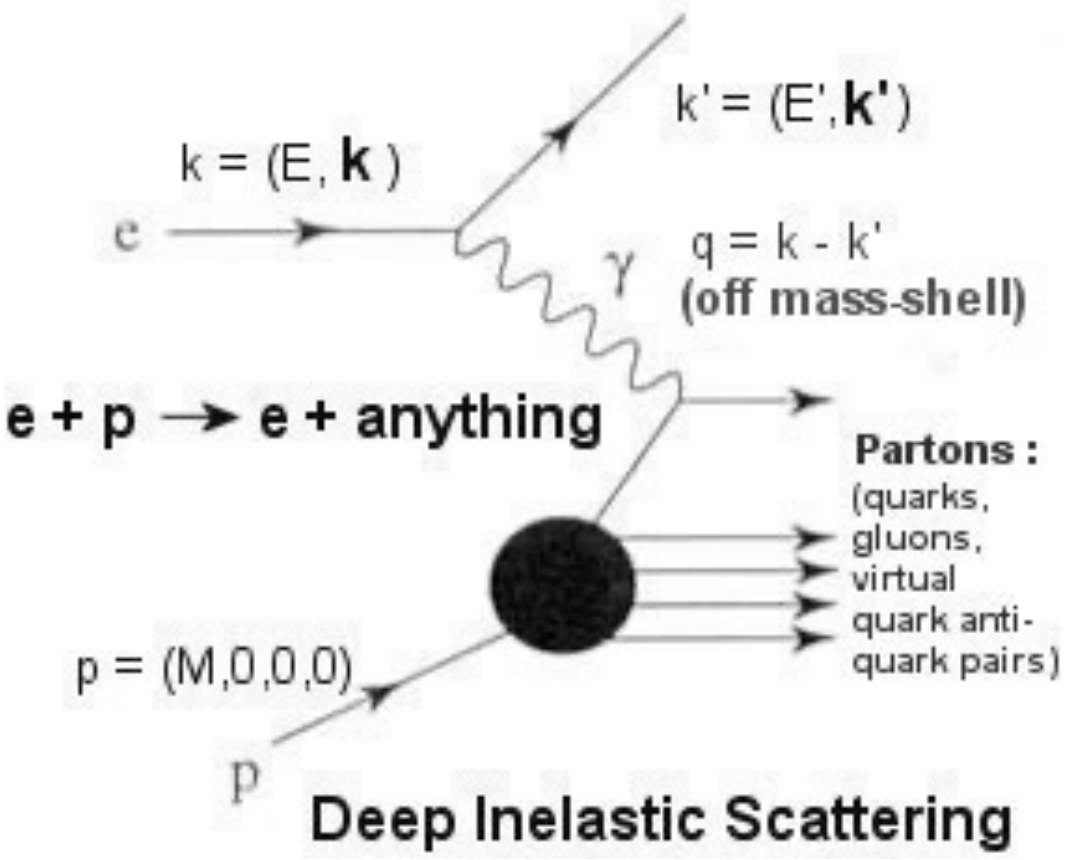




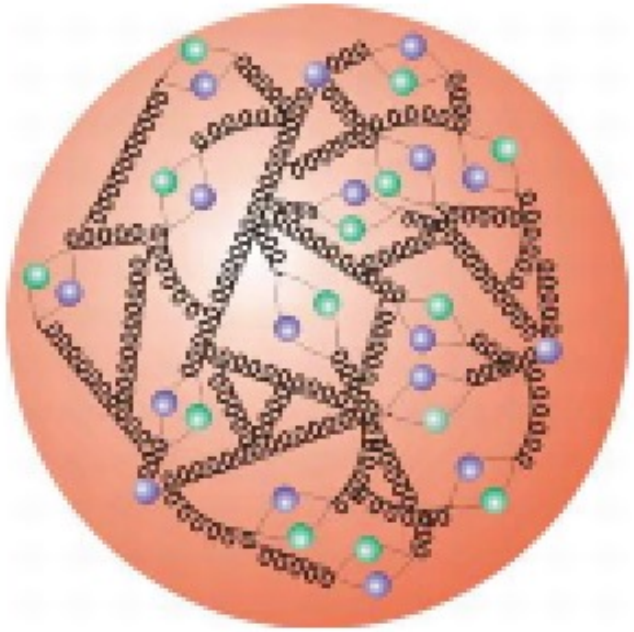
Particle anti-particle pair production should be included!

Smaller particles inside a nucleon?

Really?



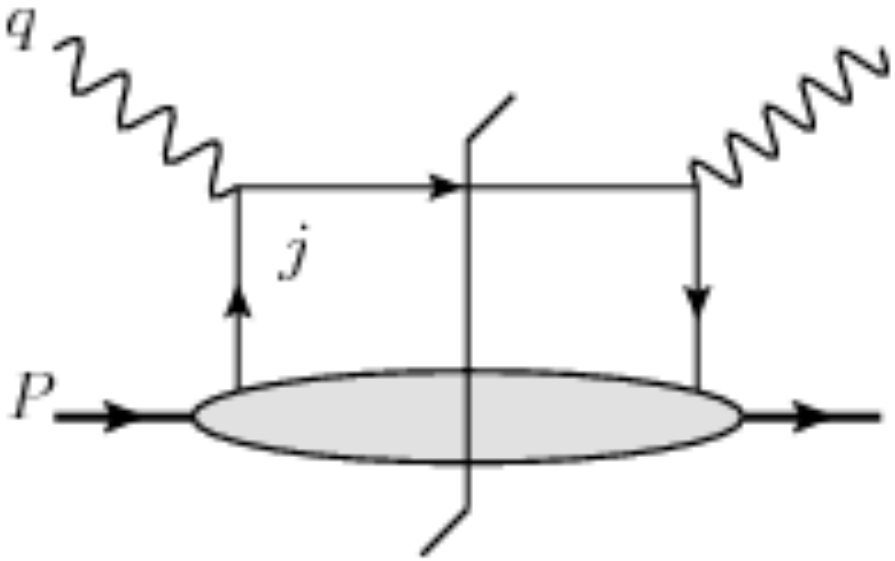
Proton is made by partons (quarks and gluons)



Parton structures: 1d mom+spin PDF to 3d GPD & TMD to Wigner (and beyond?) [BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …] to applications (Higgs, new physics…)

The timing is just right to tackle this problem.

Parton Distribution Function (PDF) in QCD



$$x = \frac{-q^2}{2P\cdot q}$$

The struck parton moves on a light cone at the leading order in the twist-expansion.

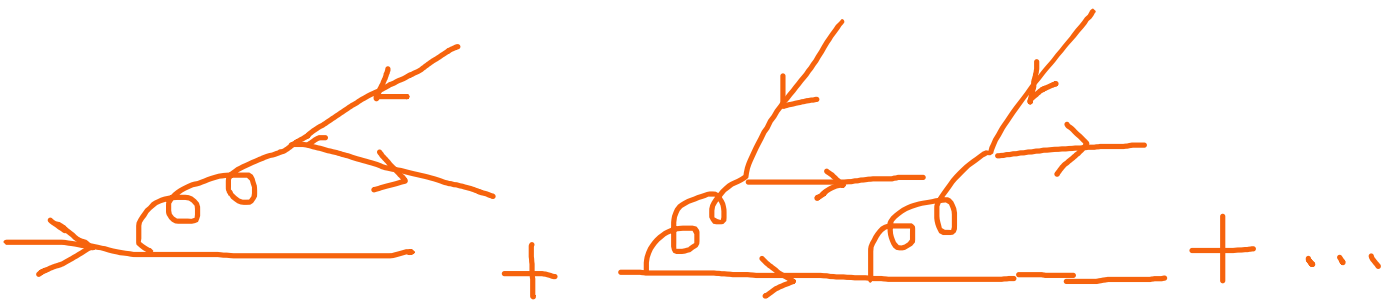
Wilson line

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \left\langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda) \right| P \right\rangle$$

PDFs from QCD---Why is it so hard?

- The number of quark anti-quark pairs diverges (manifestation of non-perturbative nature of the problem): an infinite body problem!
- Lattice QCD
- Euclidean lattice: light cone operators cannot be distinguished from local operators

$$t^2 - \mathbf{r}^2 = 0$$
$$-t_E^2 - \mathbf{r}^2 = 0$$



PDFs from QCD

- Moments of PDF given by local twist-2 operators (twist = dim - spin); limited to first few moments but carried out successfully

$$\langle x^n \rangle$$

Beyond the first few moments

- Smeared sources: Davoudi & Savage
- Gradient flow: Monahan & Orginos
- Current-current correlators: K.-F. Liu & S.-J. Dong; Braun & Müller; Detmold & Lin; QCDSF; Qiu & Ma
- Xiangdong Ji (Phys. Rev. Lett. 110 (2013) 262002): quasi-PDF: computing the x-dependence directly. (variation: pseudo-PDF, Radyushkin; w/ Karpie, Orginos, Zafeiropoulos)

Ji's idea

- Quark PDF in a proton: $(\lambda^2 = 0)$

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \left\langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda) \right| P \right\rangle$$

- Boost invariant in the z-direction, rest frame OK
- Quark bilinear op. always on the light cone
- What if the quark bilinear is slightly away from the light cone (space-like) in the proton rest frame?

- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.

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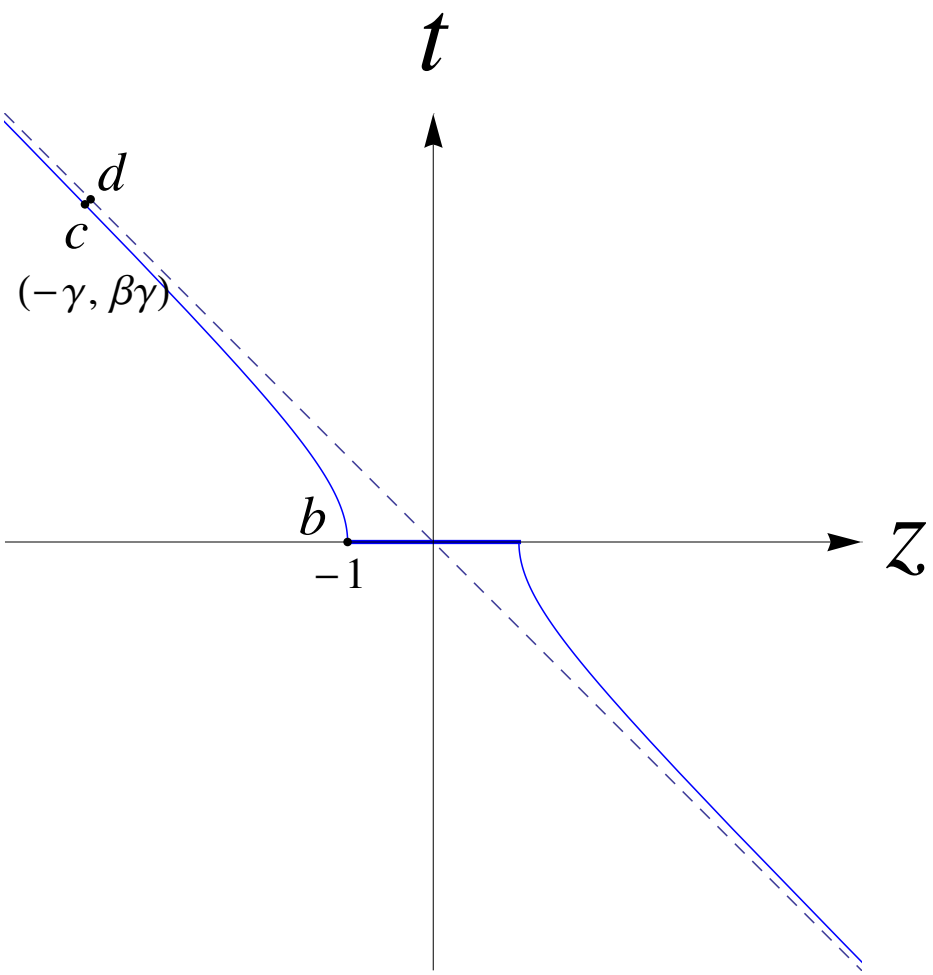

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- Analogous to HQET: need power corrections & matching---LaMET (Large Momentum Effective Theory)

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UV, PQCD

$$\tilde{q}(x,\Lambda,P_z)=\int\frac{dy}{|y|}Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right)q(y,\mu)+\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2},\frac{M^2}{P_z^2}\right)+\ldots.$$

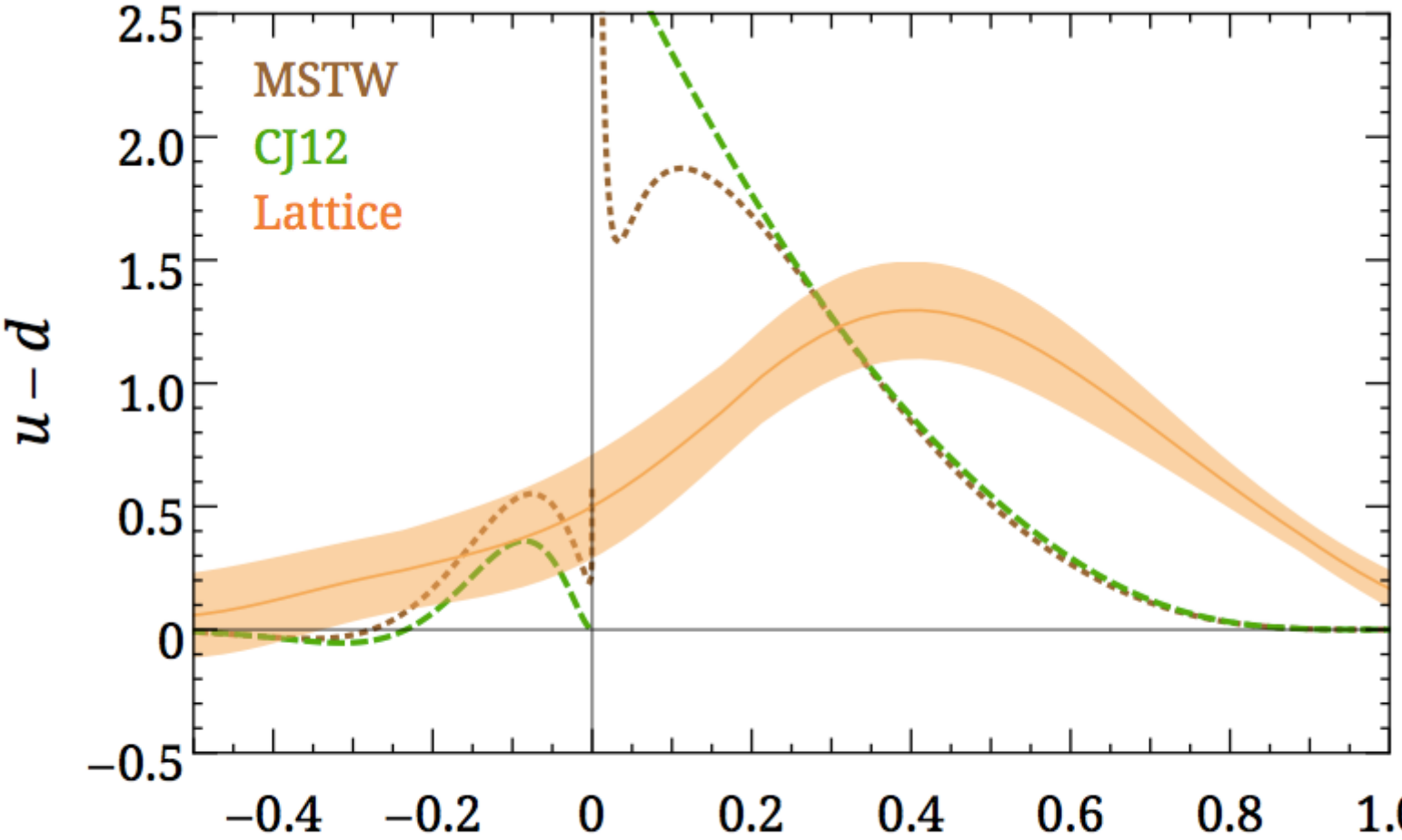
$$\widetilde{q}(x,\mu^2,P^z) \quad = \quad \int\frac{dz}{4\pi}e^{-ixzP^z}\left\langle P\left|\overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(z\lambda)\right|P\right\rangle$$

$$\lambda^\mu=(0,0,0,1)$$

LaMET 1.0

Proton Unpolarized PDF

LP3 (1402.1462)



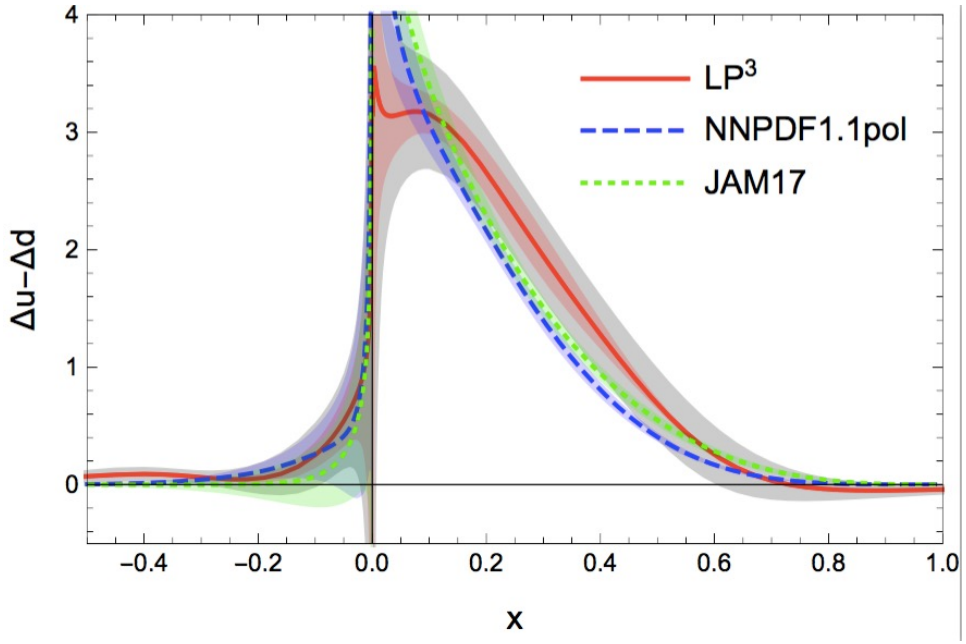
$$q(-x) = -\bar{q}(x)$$

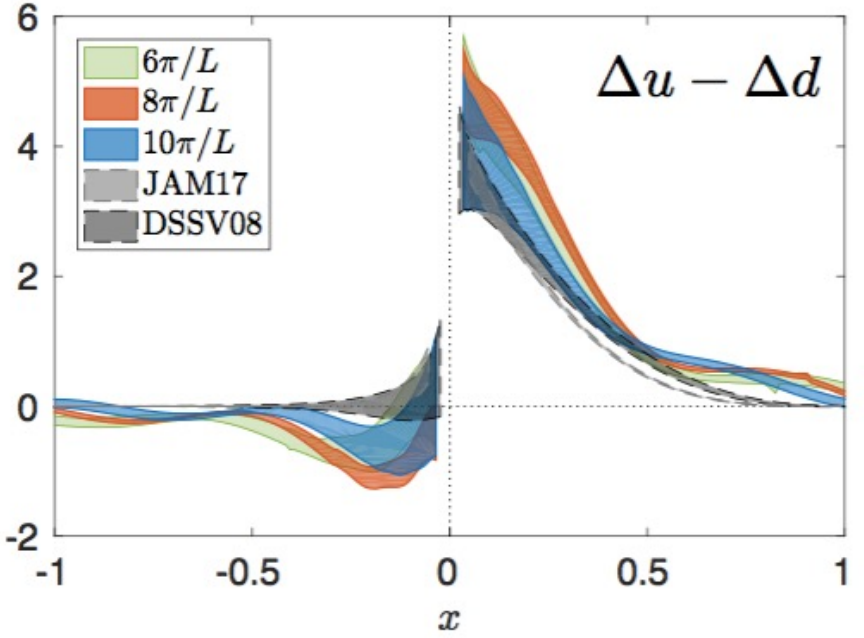
power corrections: JWC, Cohen, Ji, Lin, Zhang

matching: Xiong, Ji, Zhang, Zhao

LaMET 2.0

Proton Helicity PDF

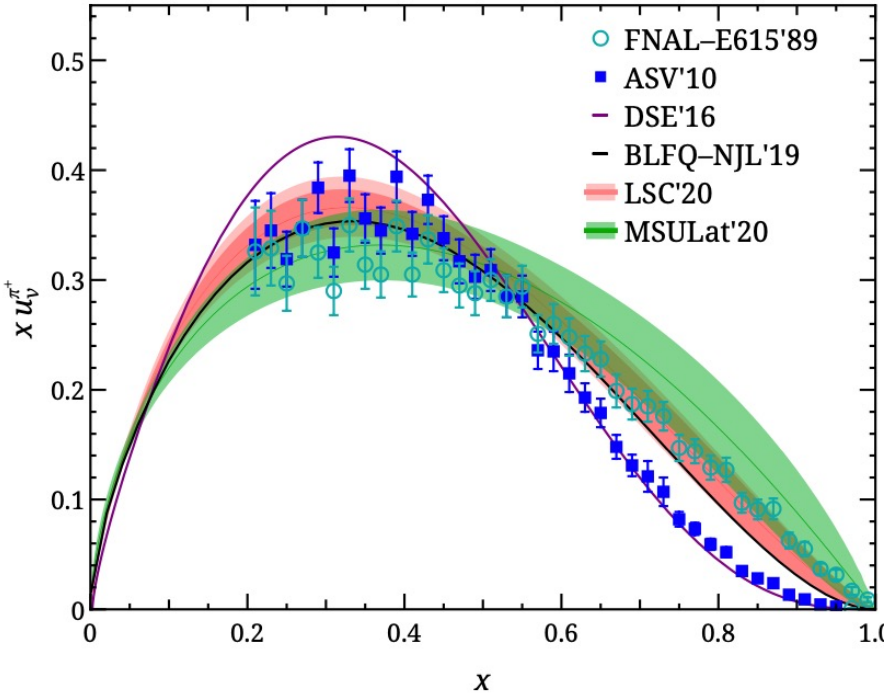


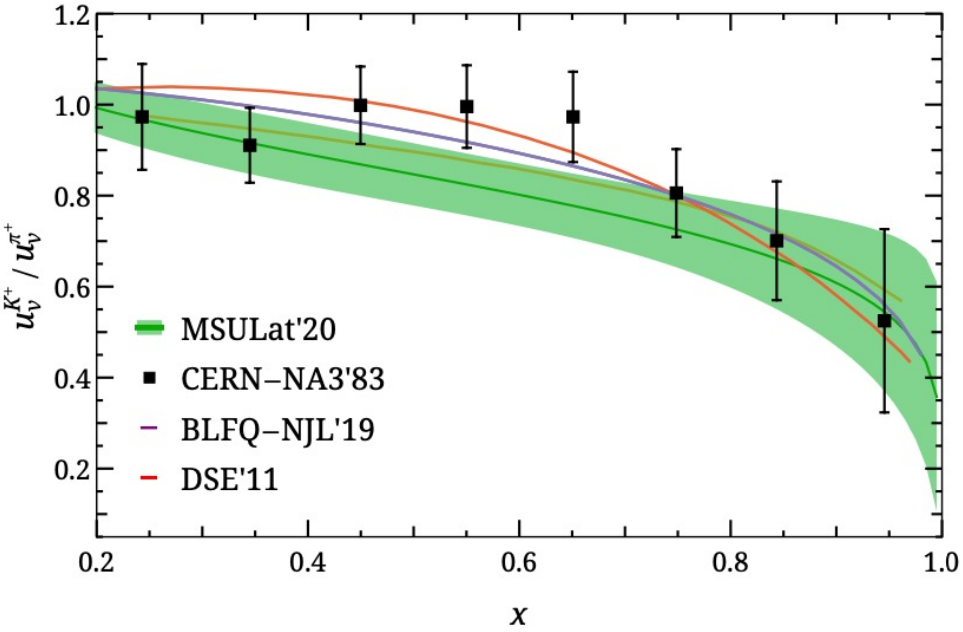


LP3(1807.07431,PRL) ETMC(1803.02685,PRL)

Momentum smearing: Bali, Lang, Musch, Schafer
Factorization: Ma, Qiu; Izubuchi, Ji, Jin, Stewart, Zhao
Wilson line mass subtraction: JWC, Ji, Zhang
LPT: Ishikawa, Ma, Qiu, Yoshida; Xiong, Luu, Meissner; Constantinou et al.
Multiplicative renormalizability: Ji, Zhang, Zhao
NPR: RI/MOM: Yong, Stewart; Constantinou et al.; Ratio: Radyushkin

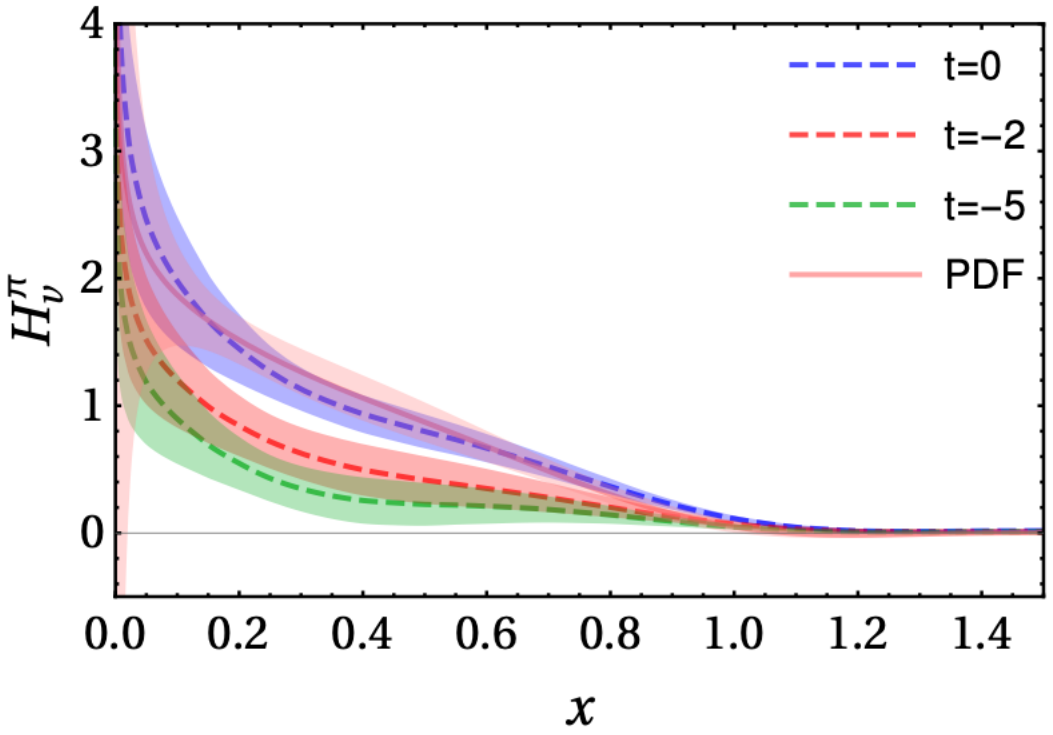
Meson Valence Quark Distributions





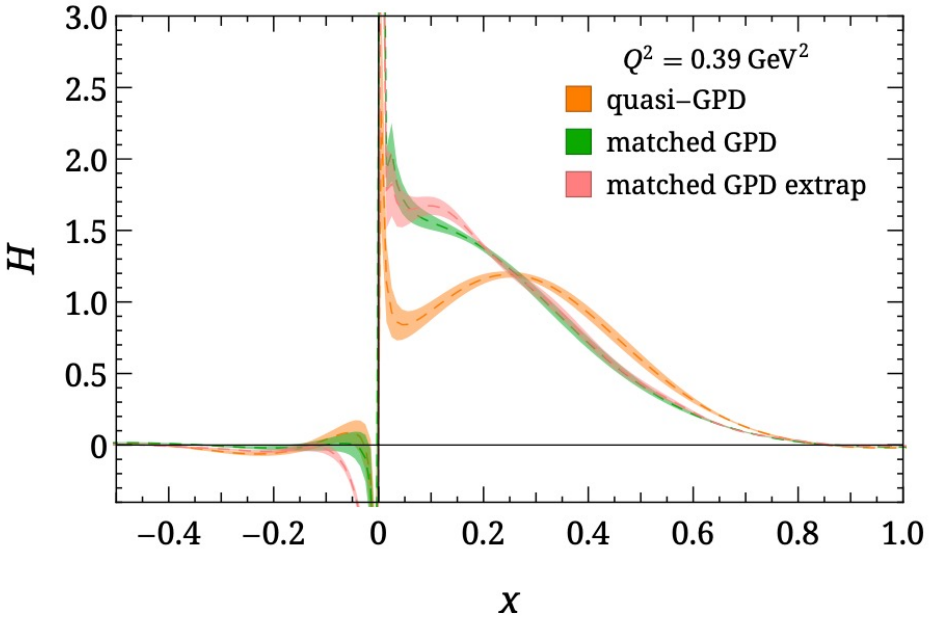
HW Lin, JWC, Z Fan, JH Zhang, R Zhang (2003.14128)

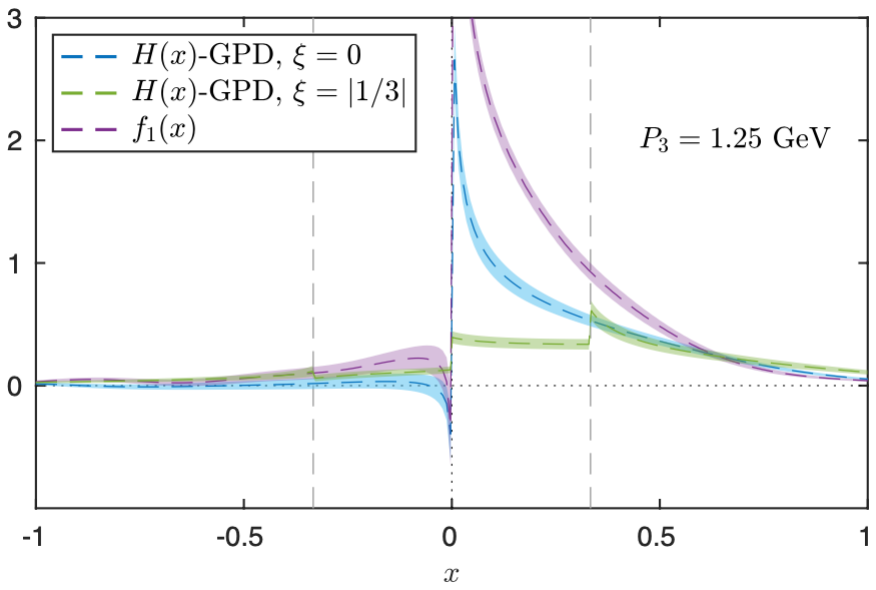
Pion Skewless Valance GPD



JWC, HW Lin, JH Zhang (1904.12376)

Nucleon GPD at Physical Pion Mass



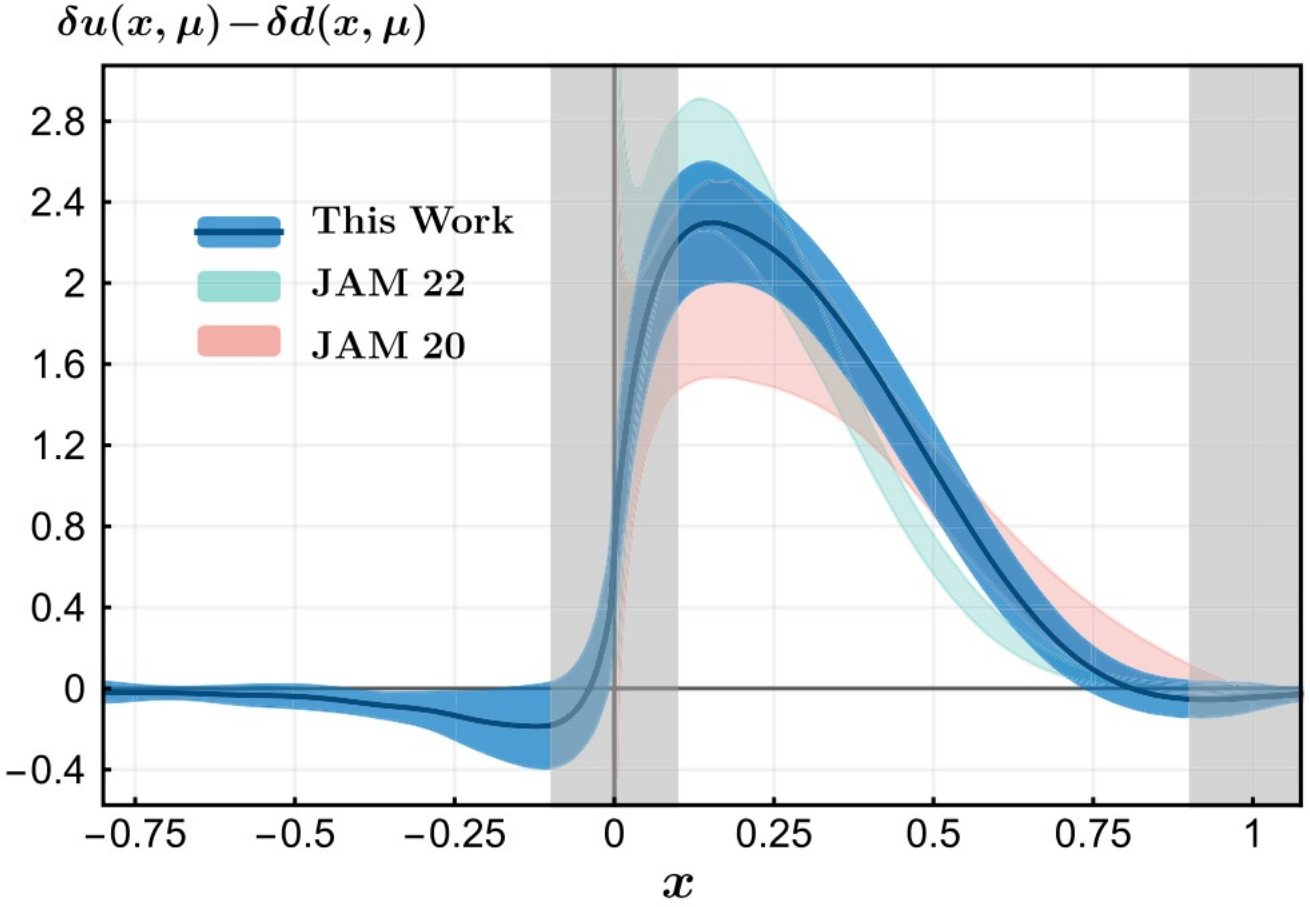


HW Lin (2008.12474, PRL)
M_pi =140 MeV

ETMC (2008.10573, PRL)
M_pi = 260 MeV

LaMET 3.0

Proton Transversity PDF in the Continuum and Physical Mass Limit

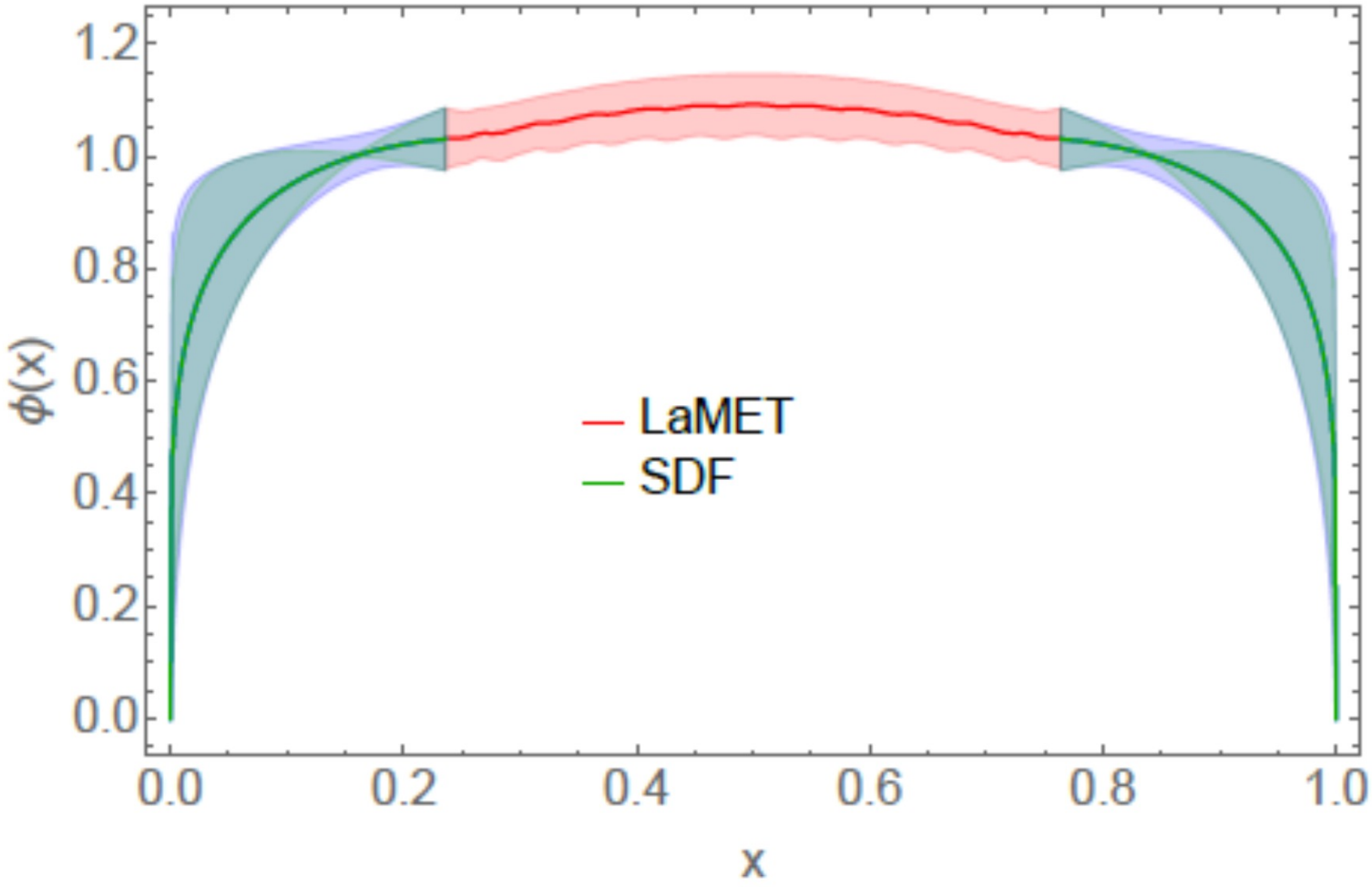


Yao, Walter, JWC, et al. (LPC), PRL 2023

Renormalon: Braun, Vladimirov, Zhang

Hybrid renormalization: Ji, Liu, Schäfer, Wang, Yang, Zhang, Zhao

Pion Distribution Amplitude

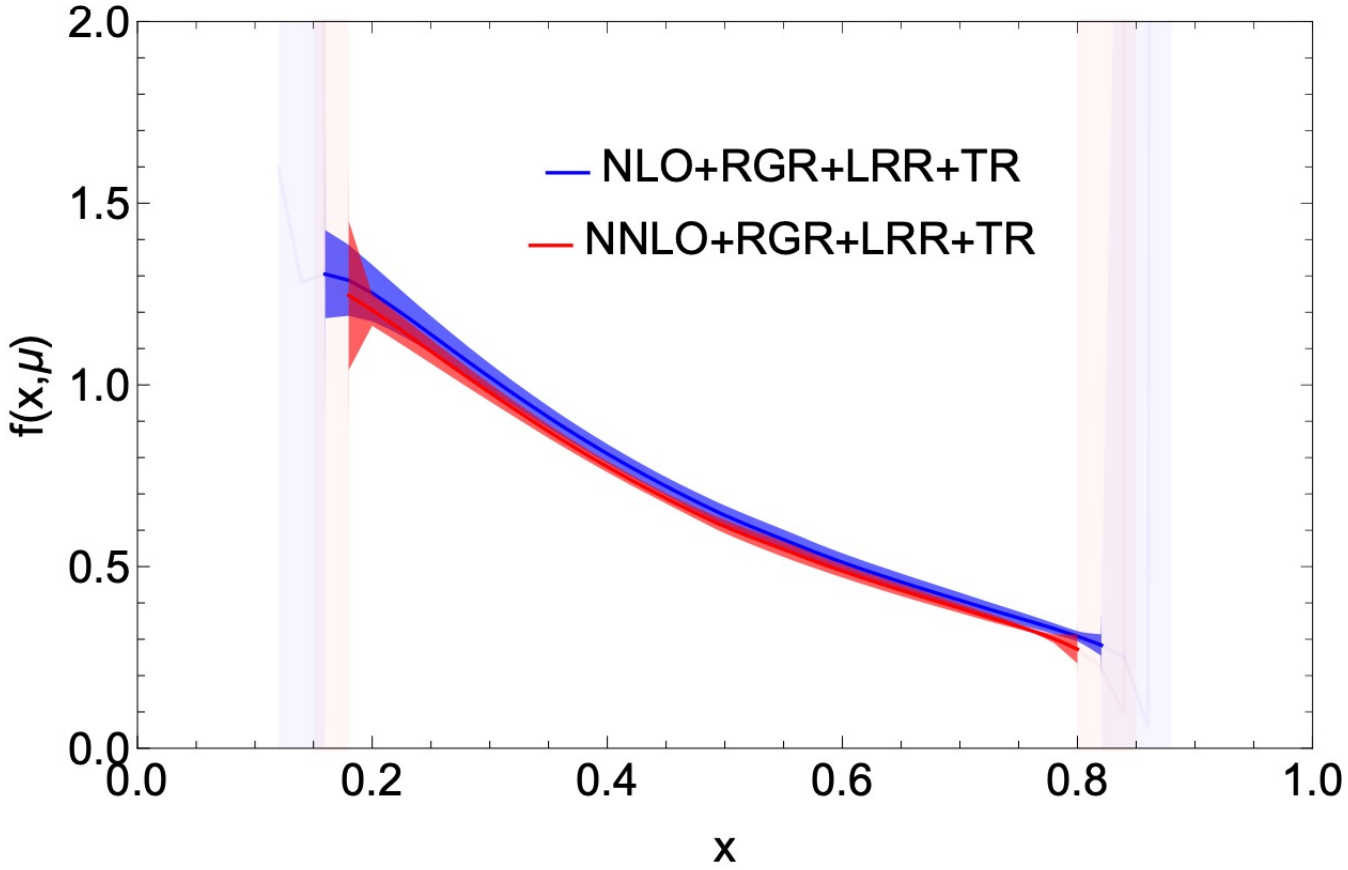


J. Holligan et al 2301.10372

Most recent developments

- Precision frontier: NNLO matching, Resummations, no Wilson line, Lanczos (D. Hackett, M. Wagman), high precision gauge fixing (YB Yang)
- Higher dimensional frontier: TMD, GPD
- Flavor frontier? gluon, u+d, s, c, b(?)

Resummations



Xiangdong Ji, Yizhuang Liu, Yushan Su, Rui Zhang (2410.12910)

NNLO kernel: ZY Li, YQ Ma, JW.Qiu; LB Chen, W Wang, R Zhu

Resummations: RGR: ln(x) powers; TR: ln(1-x) powers; LRR: leading renormalon

No Wilson Line

(Xiang Gao, Wei-Yang Liu, Yong Zhao)

Boosting brings the equal time correlator

$$\tilde{h}(z, P^z, \mu) = \frac{1}{2P^t}\langle P|\bar{\psi}(z)\gamma^t\psi(0)\Big|_{\vec{\nabla}\cdot\vec{A}=0}|P\rangle$$

in Coulomb gauge w/o Wilson line towards the light cone correlator

$$h(\lambda, \mu) = \frac{1}{2P^+}\langle P|\bar{\psi}(\xi^-)W(\xi^-, 0)\gamma^+\psi(0)|P\rangle$$

w/ Wilson line

Outlook

- Rapid progress made since 2013
- Systematic error study carried out (non-singlet)

  Know whether it works within two year (~20%)?
- Singlet PDF's: s, c, b and gluons

  Additional 5 yrs?
- If it works, complimentary to exp.: PDF (non-valence partons), DA, GPD, TMD, Wigner distributions …