Pions in nuclear and neutron star matter and χEFT

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Agenda:

- Symmetry energy and *pionization* of neutron star matter
- Pion-nucleon interaction and the chiral symmetry
- Pion mass in medium at order $O(p_F^3)$ and at order $O(p_F^{5/3})$
- P-wave and new s-wave pion condensation

Symmetry energy. Correlation among parameters

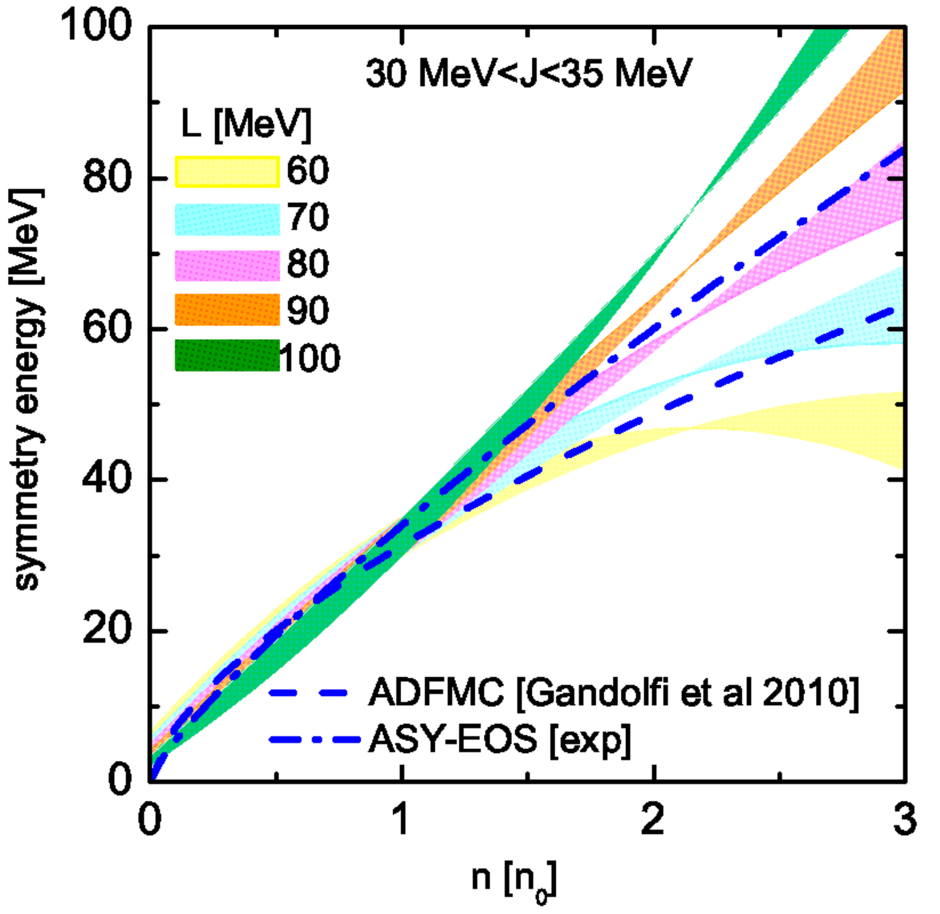
$$\varepsilon_S[n] = J + \frac{L}{3}\frac{n-n_0}{n_0} + \frac{K_{\rm sym}}{18}\frac{(n-n_0)^2}{n_0^2} + \ldots$$

If we assume some model for the density dependence of the symmetry energy

$$E_S(n) = C_k\,(n/n_0)^{2/3} + C_1\,n/n_0 + C_2\,(n/n_0)^{\gamma}$$

$$J = C_1 + C_2 + C_k \qquad 3L = C_1 + 2C_k + 3\gamma C_2$$

$$K = -2C_k + 9C_2(\gamma-1)\gamma$$



Eliminate C_1 and C_2

$$L = \left(\frac{2}{3\gamma} - 1\right) C_k + 3\,J + \frac{K_{\rm sym}}{3\,\gamma}$$

Taking L, J, K_{sym} from models fitted to empirical data we see that this relation works and obtain parameters C_k, γ

RMF: $$L = -11.76\ \mathrm{MeV} + 3\,J + \frac{K_{\rm sym}}{4.55} \qquad \gamma = 1.5$$

Skyrme $$L = -19.5\ \mathrm{MeV} + 3\,J + \frac{K_{\rm sym}}{5.50} \qquad \gamma = 1.8$$

$$\frac{4}{3} < \gamma$$

Constraint on the symmetry energy. NS Cooling

n+p+e+μ NS matter

Proton concentration/electron chemical potential

β-equilibrium: $$\mu_e = \mu_n - \mu_p = 4\,\varepsilon_S(n)\,(1-2\,x)$$

electroneutrality: $$n_e(\mu_e) + n_\mu(\mu_e) = n\,x$$

Neutrino emissivities $\quad T_9 = T/10^9 K \ll 1$

Direct URCA $\quad n \leftrightarrow p + e + \bar{\nu}_e$

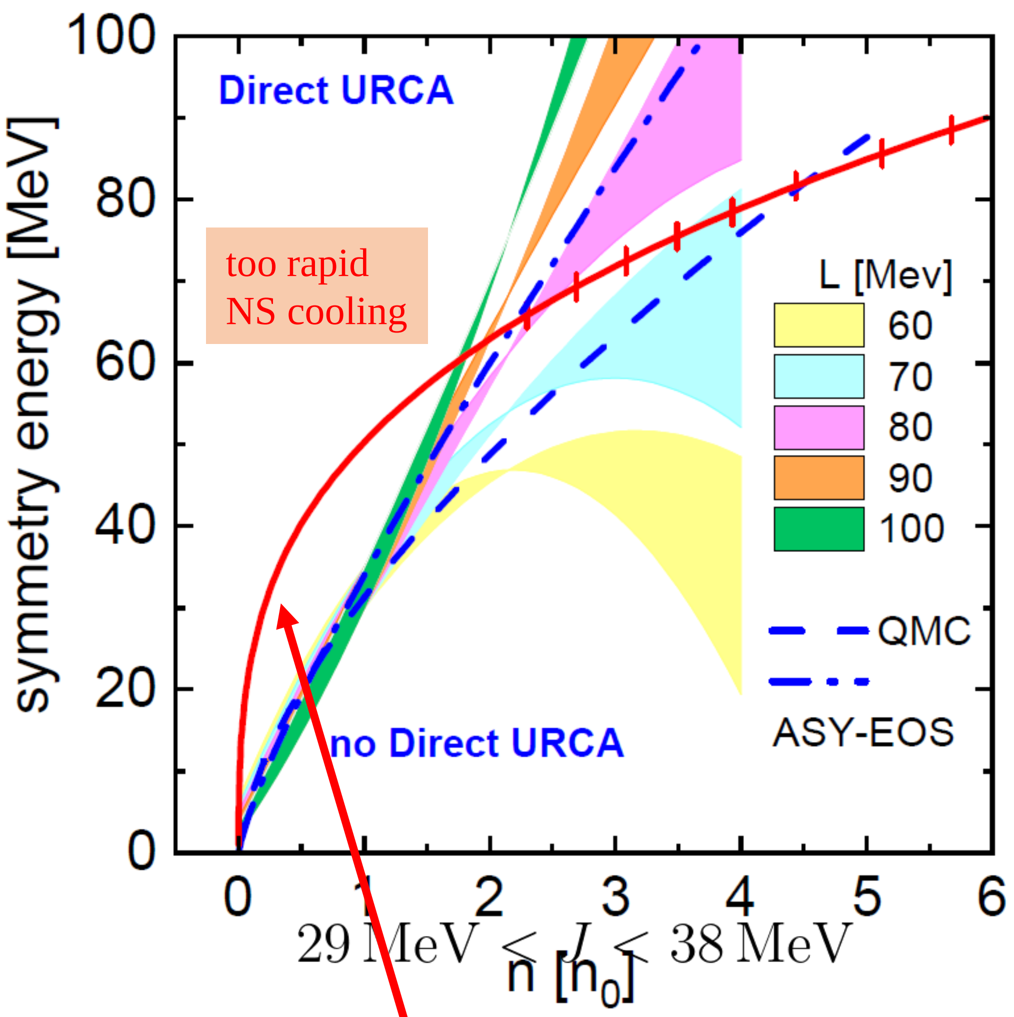
$$\varepsilon = 10^{27} \times \left(\frac{m_N^*}{m_N}\right)^2 T_9^6 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \quad \frac{\mathrm{erg}}{\mathrm{cm}^3\,\mathrm{s}}$$

Modified URCA $\quad n + N \leftrightarrow p + N + e + \bar{\nu}_e$

$$\varepsilon = 10^{22} \times \left(\frac{m_N^*}{m_N}\right)^4 T_9^8 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \quad \frac{\mathrm{erg}}{\mathrm{cm}^3\,\mathrm{s}}$$

Direct URCA threshold $\quad p_{\mathrm{F},n} \le p_{\mathrm{F},p} + p_{\mathrm{F},e}$

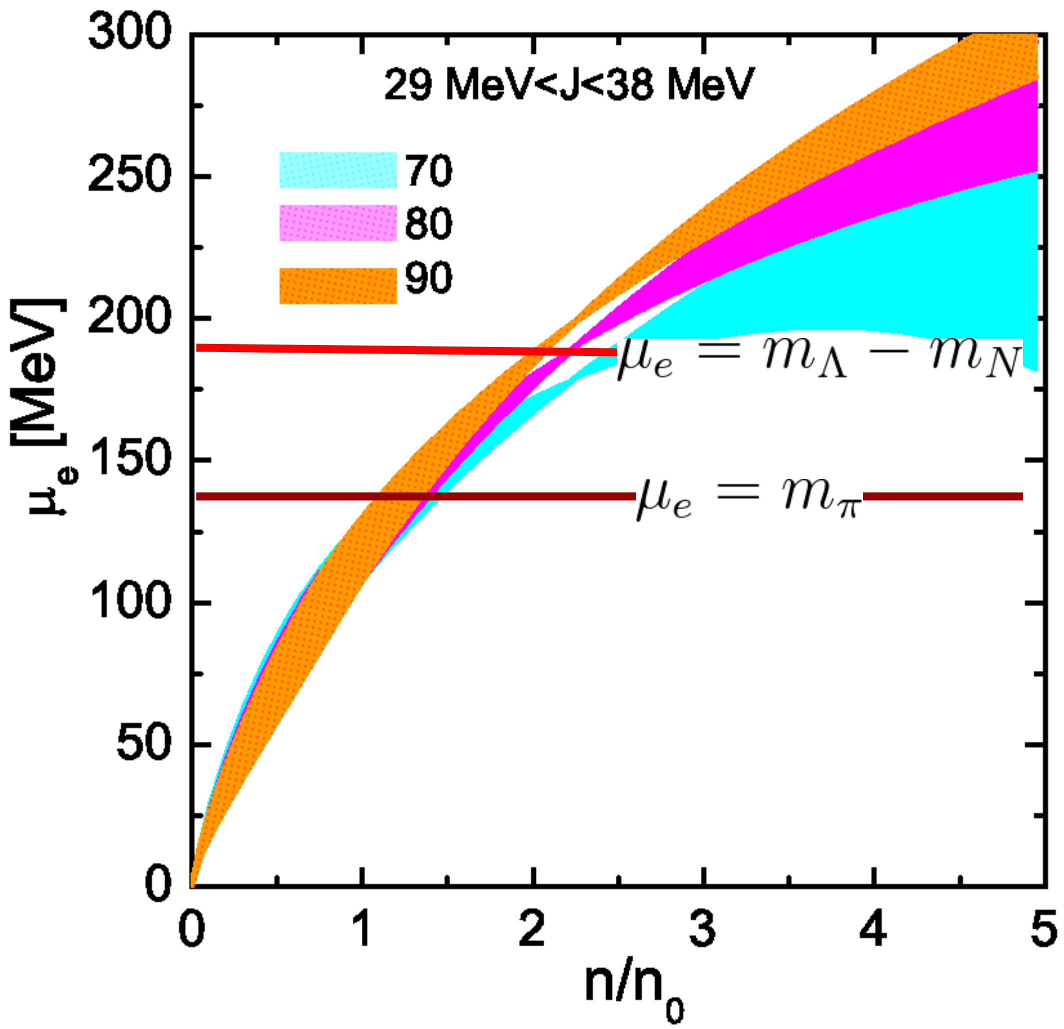
$$x = \frac{1}{1 + \left(1 + x_e^{1/3}\right)^3}\,, \quad x_e = \frac{n_e}{x\,n}$$



$29\,\mathrm{MeV} \le J \le 38\,\mathrm{MeV}$

From these 3 equations we express $\varepsilon_{\mathrm{sym}}^{(\mathrm{max})}[n]$

- *Pionization of the NS matter* $n+p+e+\mu$ matter



$$e^- + p \longrightarrow \Lambda + \nu_e$$ **Hyperonization** of the NS matter

$$e^- \longrightarrow \pi^- + \nu_e$$ **Pionization** of the NS matter

Softening of the EoS
Lowering of the maximum NS mass

Pion-nucleon interaction

isospin even and odd amplitudes $T^{(\pm)} = \frac{1}{2}\left[T^{(\pi^- p)} \pm T^{(\pi^- n)}\right]$

At the threshold: scattering amplitudes $T^{(\pm)}(\sqrt{s} = m_\pi + m_N) = 4\pi\left(1 + \frac{m_\pi}{m_N}\right)a_S^{\pm}$

$T^{(+)}[m_\pi]$ –0.122[KA86]; 0 [SP98]; 0.06 [EM98]; –0.003 [PSI]

$T^{(-)}[m_\pi]$ 1.32 [KA86]; 1.27 [SP98]; 1.11 [EM98]; 1.26 [PSI]

Current algebra prediction
Soft-pion theorem

$$\boxed{T^{(-)}(\omega) \approx \frac{\omega}{2 f_\pi^2}}$$

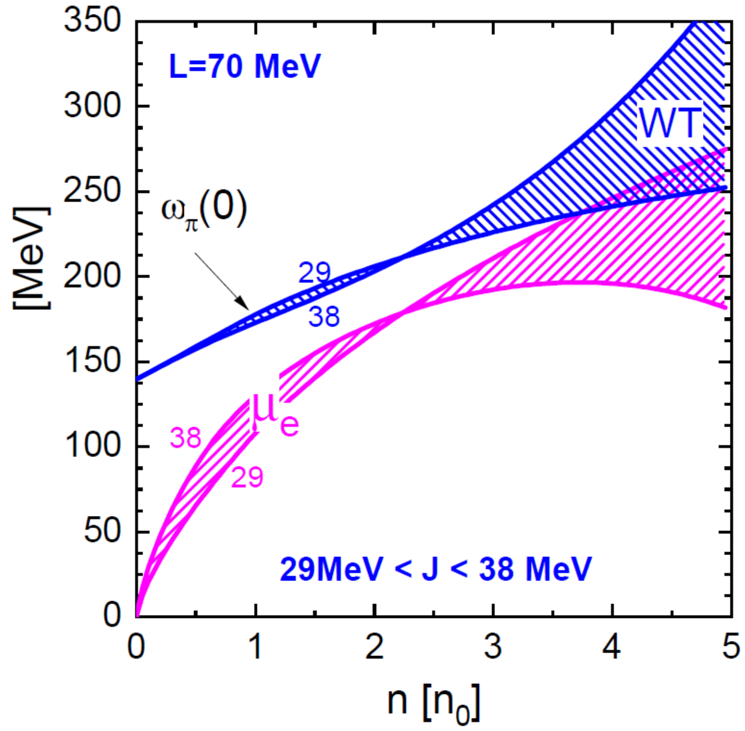
$$T^{(+)} \approx 0$$

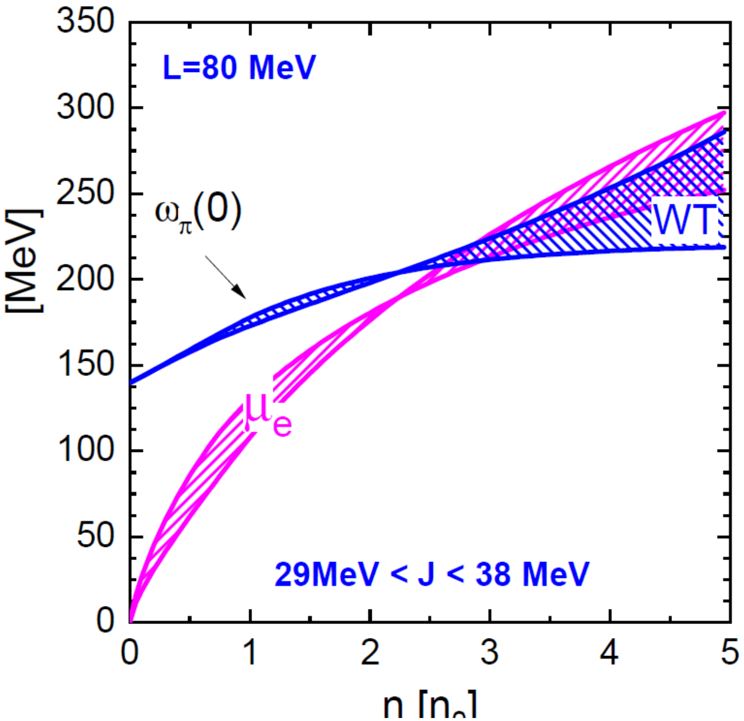
- polarization operator

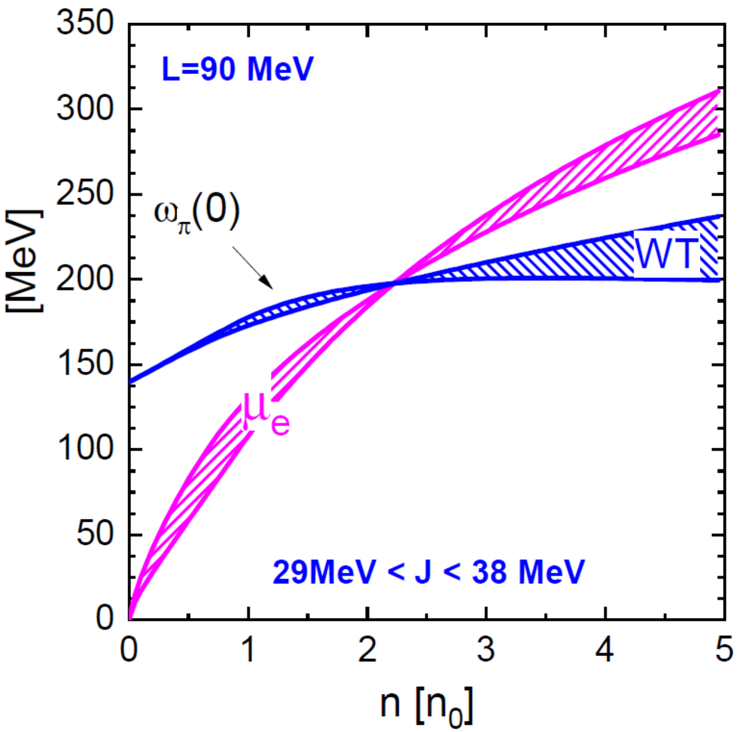
$$\Pi_S(\omega) = -T^{(-)}(\omega)\,(n_p - n_n)$$

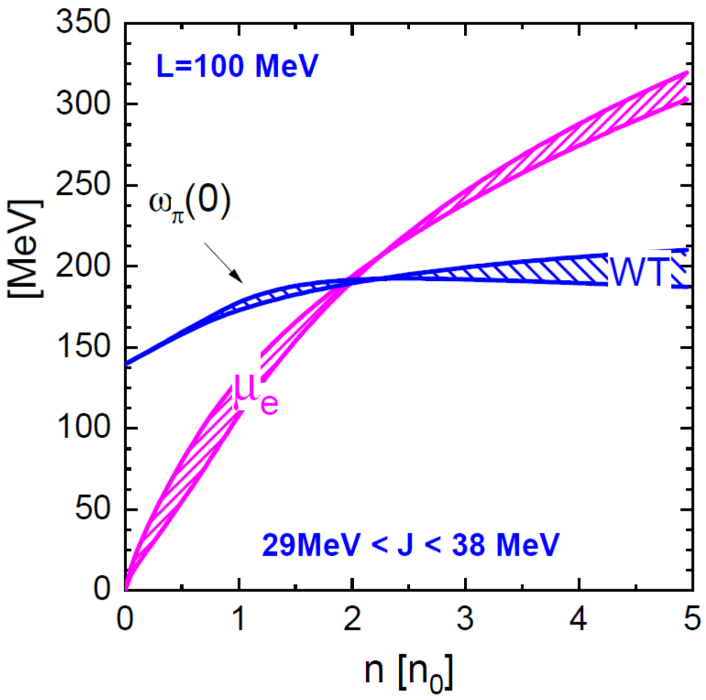
repulsive in neutron reach matter

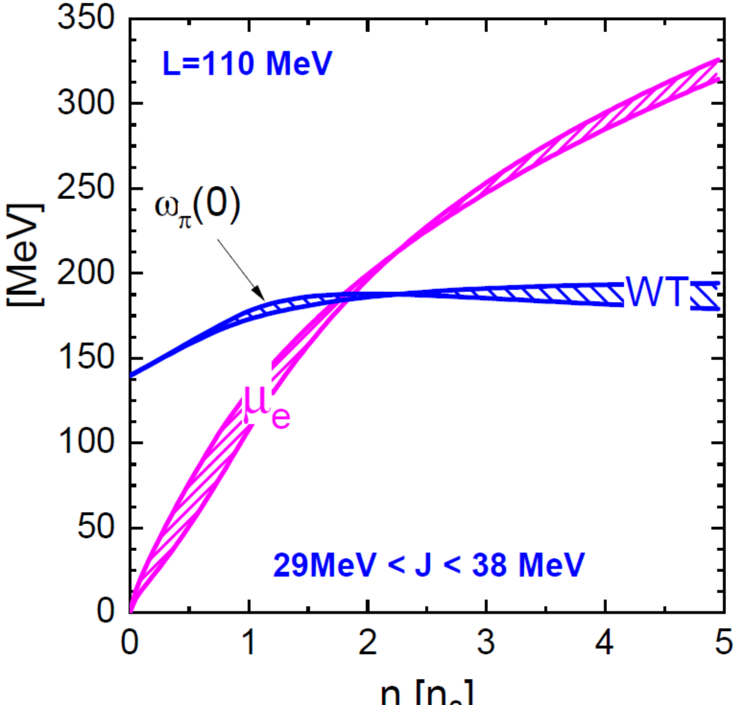
- spectrum $D^{-1}(\omega, k = 0) = \omega^2 - m_\pi^2 - \Pi_S(\omega) = 0$











Weinberg-Tomazawa term does not protect against pionization for too stiff symmetry energy, L>90 MeV

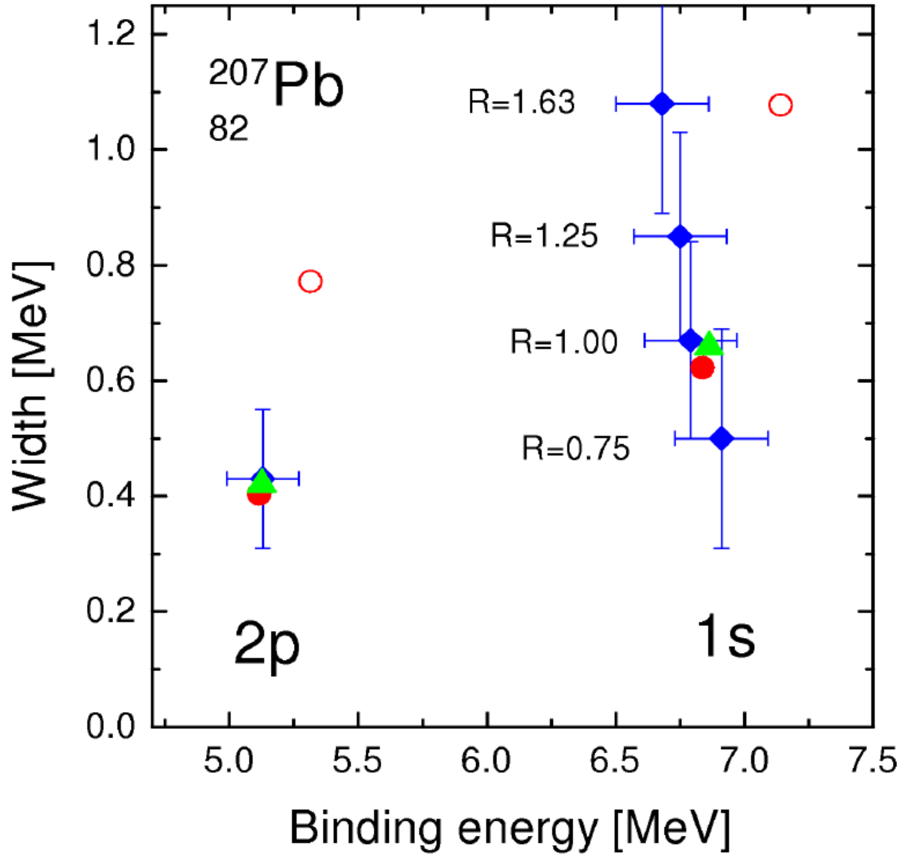
● Deeply bound pionic atoms

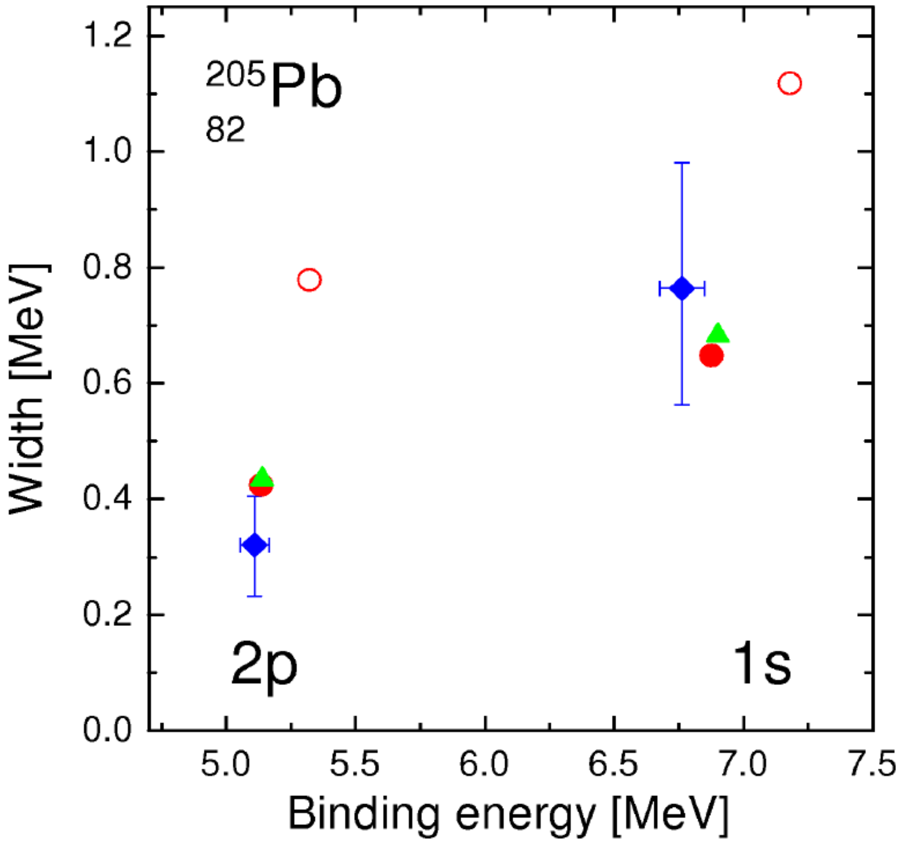
Missing repulsion problem

$$T^{(-)}(\omega) \approx \frac{\omega}{2 f_\pi^2}$$

energy independent potential ○

energy dependent potential
- ● phenomenological model [Weise 2001] $f_\pi^{*2}(\rho) = f_\pi^2 - \frac{\sigma_{\pi N}}{m_\pi^2}\rho$
- ▲ in-medium chiral perturbation theory [EEK,Kaiser,Weise 2003] $T^{(+)}(\omega - V_{\rm C})$





Reveals a new scale of πN interaction

$$\omega^2 - m_\pi^2 - \Pi(\omega) = 0$$

$$\omega^2 = m_\pi^2 + 2\, m_\pi\, U$$

$$U \ll m_\pi$$

$$2\, m_\pi\, U = \frac{\Pi(m_\pi)}{1 - \left.\frac{\partial \Pi}{\partial \omega^2}\right|_{\omega = m_\pi}}$$

Chiral symmetry

see lectures by Evgeny Epelbaum

- *QCD with light quarks*

✔ **quarks** $q_L(x) = \frac{1+\gamma_5}{2} \begin{pmatrix} u(x) \\ d(x) \\ s(x) \end{pmatrix}$

✔ **gluons** $G^a(x) \longleftarrow \mathrm{SU}(N_c)$ gauge bosons

$D_\mu(G) = \partial_\mu - i\,\frac{g}{2}\,G^a_\mu(x)\,\lambda_a$

$$\mathcal{L}_{\rm QCD}(x) = \bar q_L(x)\, i\,\gamma^\mu D_\mu(G) q_L(x) + \bar q_R(x)\, i\,\gamma^\mu D_\mu(G) q_R(x) - \frac{1}{4}\sum_{a=1}^{N_c^2-1} G_a^{\mu\nu}(x)\, G_{\mu\nu,a}(x)$$

$$- \bar q_L(x) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} q_R(x) - \bar q_R(x) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} q_L(x)$$

Accidental symmetries

✔ consider $m_{u,d,s}$ to be small

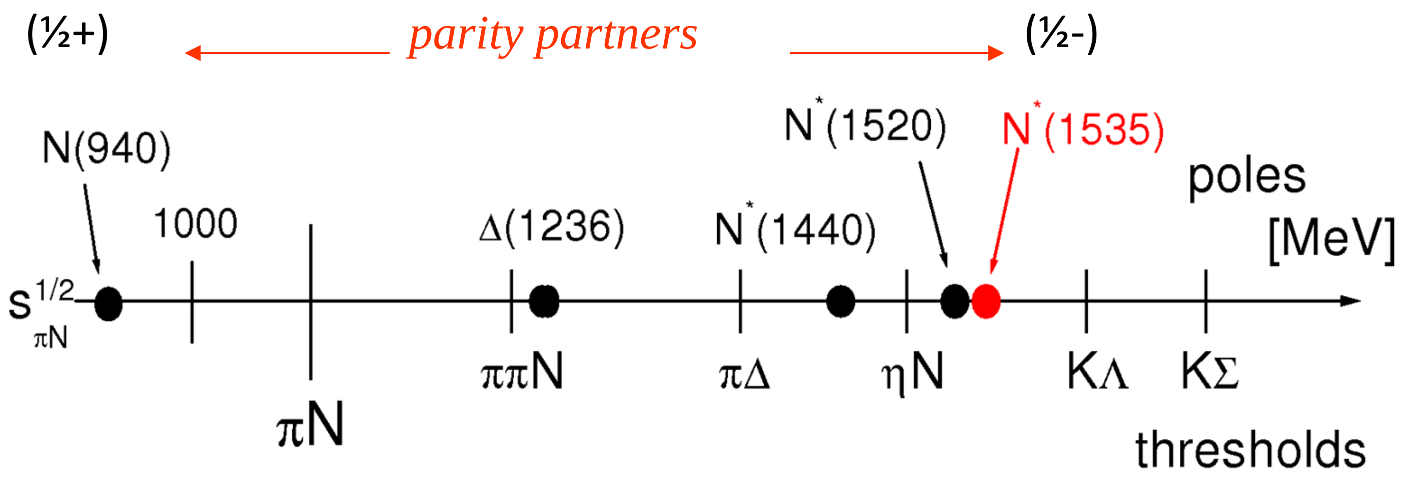
- approximate $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ chiral symmetry
- parity doublets in hadron spectrum (if not broken spontaneously !)

✔ consider number of colors $N_c = 3$ to be large

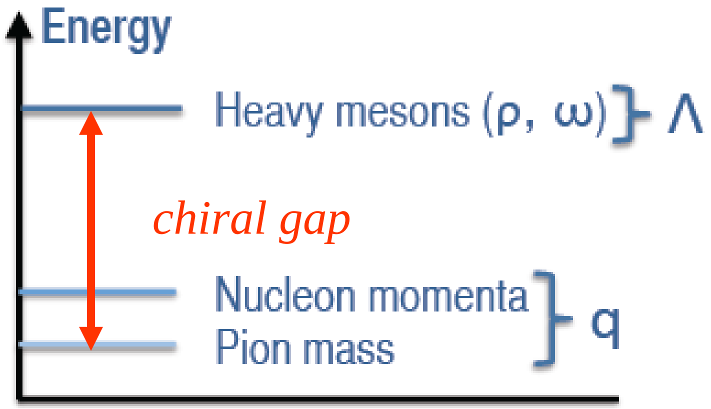
- contracted spin-flavor symmetry SU(6)

- *world of pion-nucleon interaction*

see lectures by Evgeny Epelbaum



Pions – Goldstone bosons of chiral symmetry breaking



- gradient couplings
- expansion in small pion momenta and masses

- *effective chiral Lagrangian* Building blocks: B baryon field matrix

$$U_\mu = \frac{1}{2} e^{-i\frac{\Phi}{2f}} \left(\partial_\mu e^{i\frac{\Phi}{2f}} \right) e^{+i\frac{\Phi}{2f}} = \partial_\mu \Phi + \ldots$$

Φ meson field matrix

Meson mass terms

- *Chiral Lagrangian at Q^2 order*

$$\mathcal{L}_{\rm int} = -\frac{1}{4f^2}\bar{N}\gamma_\mu(\boldsymbol{\tau}\cdot[\boldsymbol{\phi}\times(\partial^\mu\boldsymbol{\phi})])N + \frac{g_A}{2f}\bar{N}\gamma_5\gamma_\mu(\boldsymbol{\tau}\cdot\partial^\mu\boldsymbol{\phi})N$$

LO: Q^1 Weinberg-Tomazawa and πNN

$$-\frac{2c_1}{f^2}m_\pi^2\bar{N}(\boldsymbol{\phi}\cdot\boldsymbol{\phi})N$$

NLO: Q^2 σ-term

$$-\frac{c_2}{2f^2m_N^2}\{\bar{N}(\partial_\mu\boldsymbol{\phi})\cdot(\partial_\nu\boldsymbol{\phi})\partial^\mu\partial^\nu N + \text{h.c.}\} + \frac{c_3}{f^2}\bar{N}(\partial_\mu\boldsymbol{\phi})\cdot(\partial^\mu\boldsymbol{\phi})N - \frac{c_4}{2f^2}\bar{N}\sigma^{\mu\nu}(\boldsymbol{\tau}\cdot[(\partial_\mu\boldsymbol{\phi})\times(\partial_\nu\boldsymbol{\phi})])N$$

NLO: Q^2 range-term

Scattering amplitude $\quad \pi^+(q)+n(p)\to\pi^+(\bar{q})+n(\bar{p})$

$$\hat{T}^{(\pi^+n)} = +\frac{\not{\bar{q}}+\not{q}}{4f^2} - \frac{4c_1}{f^2}m_\pi^2 + \frac{c_2}{f^2m_N^2}[(p\cdot\bar{q})(p\cdot q)+(\bar{p}\cdot\bar{q})(\bar{p}\cdot q)] + 2\frac{c_3}{f^2}(\bar{q}\cdot q) + \frac{c_4}{2f^2}[\not{\bar{q}}\not{q}-\not{q}\not{\bar{q}}] - \frac{g_A^2}{2f^2}\gamma_5\not{\bar{q}}\hat{G}_p^{(0)}(p+q)\gamma_5\not{q}$$

S-wave amplitude (tree level) $\quad \sigma_{\pi N} = -4c_1m_\pi^2 \qquad \beta = -2(c_3+c_2)m_\pi^2 + \frac{g_A^2m_\pi^2}{4m_N} \qquad f = 90\,\text{MeV}$

$$T_{\pi N,\rm s}^{(1/2)}(\sqrt{s}=m_N+\omega) = \frac{\omega}{f^2} + \frac{\sigma_{\pi N}}{f^2} - \frac{\beta}{f^2}\frac{\omega^2}{m_\pi^2}$$

$$T_{\pi N,\rm s}^{(3/2)}(\sqrt{s}=m_N+\omega) = -\frac{\omega}{2f^2} + \frac{\sigma_{\pi N}}{f^2} - \frac{\beta}{f^2}\frac{\omega^2}{m_\pi^2}$$

$$T^+ = \frac{1}{3}T^{(1/2)} + \frac{2}{3}T^{(3/2)} = \frac{\sigma_{\pi N}}{f^2} - \frac{\beta}{f^2}\frac{\omega^2}{m_\pi^2}$$

$$T^- = \frac{1}{3}\left(T^{(1/2)} - T^{(3/2)}\right) = \frac{\omega}{2f^2}$$

Exp: $\beta \approx \sigma_{\pi N}$

Pion polarization operator from the scattering amplitude

Polarization operator of the $\pi^{\pm}$ meson is equal to

Vector mean fields

$$\Pi^{(+)}(q) = \Pi_n^{(+)}(q) + \Pi_p^{(+)}(q)$$

$$\Pi_n^{(+)}(q) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} i\mathrm{Tr}\{\hat{G}_n^{(\mathrm{m})}(p - v_n u; m_N^*)\hat{T}_{\mathrm{forw}}^{(\pi^+ n)}\},$$

$$\Pi_p^{(+)}(q) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} i\mathrm{Tr}\{\hat{G}_p^{(\mathrm{m})}(p - v_p u; m_N^*)\hat{T}_{\mathrm{forw}}^{(\pi^+ p)}\}$$

Effective mass

In-medium nucleon propagator

$$\hat{G}_a^*(p) = \hat{G}^{(0)}(p - v_a u; m_N^*) + \hat{G}_a^{(\mathrm{m})}(p - v_a u; m_N^*)$$

$$\hat{G}_a^{(\mathrm{m})}(p; m) = 2\pi i n_a(p)\hat{S}(p; m)\delta(p^2 - m^2)\theta(p_0)$$

$$n_a(p) = \theta(p_{\mathrm{F},a}^2 + p^2 - (p \cdot u)^2)$$

$$u = (1, 0, 0, 0)$$

$$n_a = p_{\mathrm{F},a}^3/(3\pi^2)$$

- *S-wave pion polarization operator at chiral Q^2 order* (approximated) $O(p_F^3)$

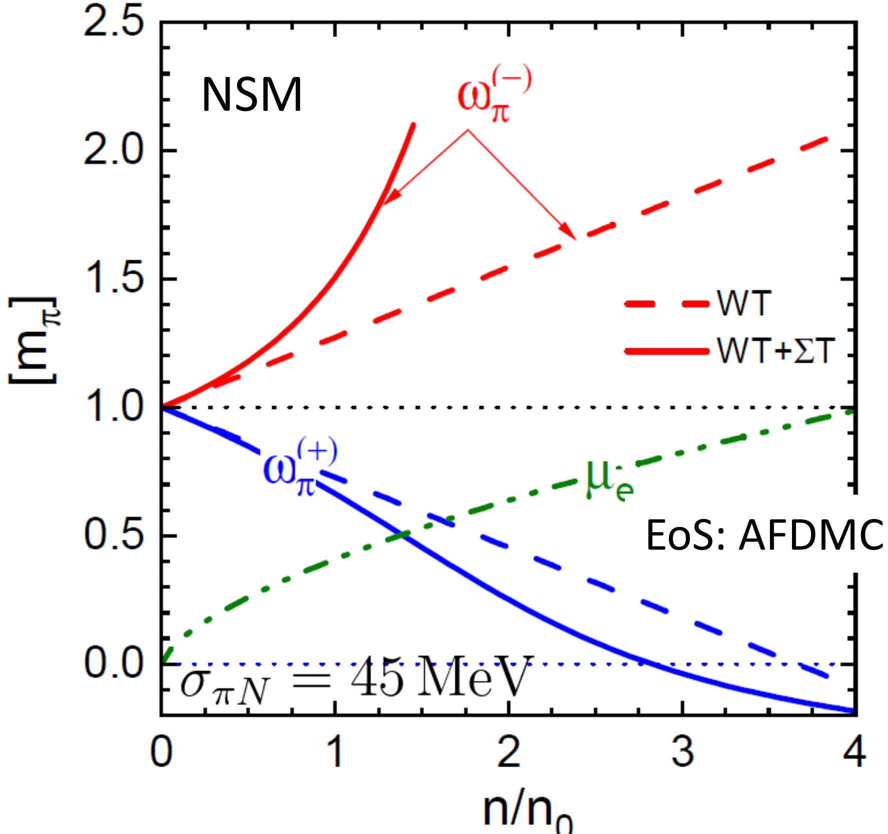
$$\Pi^{(\pm)}_{1,S}(\omega) = \mp\frac{\omega}{2f^2}(n_n - n_p) - \frac{\sigma_{\pi N}}{f^2}(n_p + n_n) + \frac{\beta\omega^2}{m_\pi^2 f^2}(n_p + n_n)$$

$$\omega_\pi^{(\pm)} = \frac{1}{1-\frac{n}{n_{c,\beta}}}\left\{\sqrt{m_\pi^2\left(1-\frac{n}{n_{c,\sigma}}\right)^2 + \frac{(n_n-n_p)^2}{16f^4}} \mp \frac{n_n-n_p}{4f^2}\right\}, \quad n = n_p + n_n$$

$$n_{c,\beta} = \frac{f^2 m_\pi^2}{\beta}$$

From experiment we know: $\beta \approx \sigma_{\pi N}$ $\rightarrow$ $n_{c,\beta} \approx n_{c,\sigma}$

$$n_{c,\sigma} = \frac{f^2 m_\pi^2}{\sigma_{\pi N}}$$



EoS: AFDMC [Gandolfi]

Small densities $\omega_\pi^{(\pm)} = m_\pi \mp \frac{n_n - n_p}{4f^2} + O(n^2)$

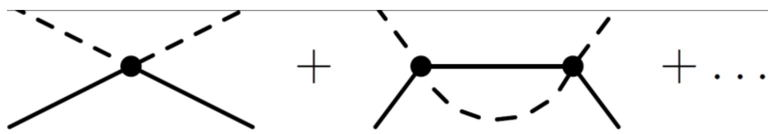
In isospin symmetric matter $\omega^\pm = m_\pi$

At $n \rightarrow n_{c,\beta}$, the pion masses behave as

$$\omega_\pi^{(+)} \rightarrow 0, \quad \text{and} \quad \omega_\pi^{(-)} \rightarrow \infty.$$

See also Onishi, Jido et al, PRC 80, 038202 (2009)

- *Pion–Nucleon scattering amplitude*



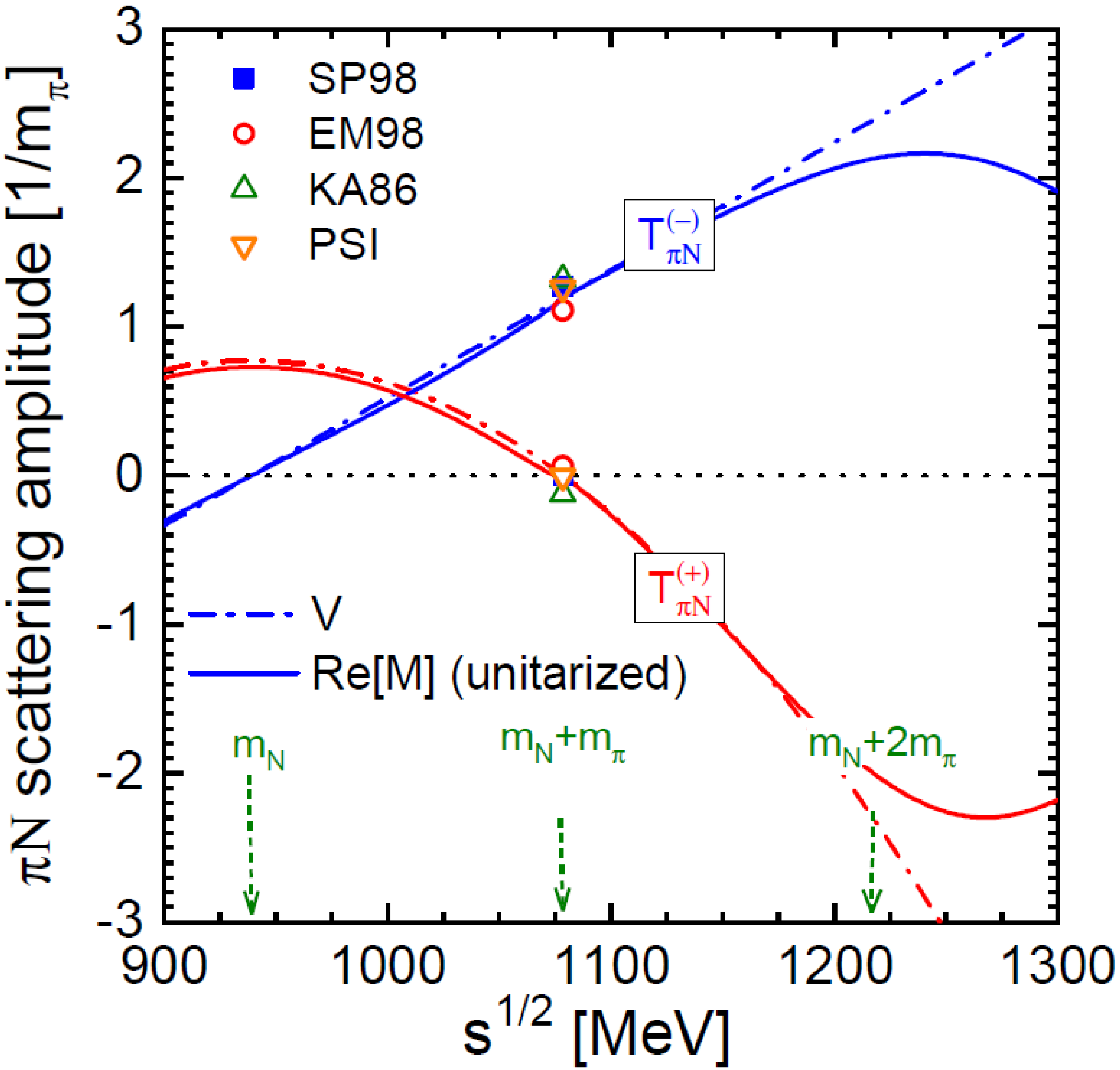
$$T^{(I)}(\sqrt{s}) = \frac{1}{[V^{(I)}(\sqrt{s})]^{-1} - J(\sqrt{s})} \qquad I = \frac{1}{2}, \frac{3}{2}$$

$$V^{(1/2)}(\sqrt{s}) = \frac{1}{f^2}\,(\sqrt{s} - m_N) + \frac{\sigma_{\pi N}}{f^2} - \frac{\beta}{f2}\,\frac{(\sqrt{s} - m_N)^2}{m_\pi^2}$$

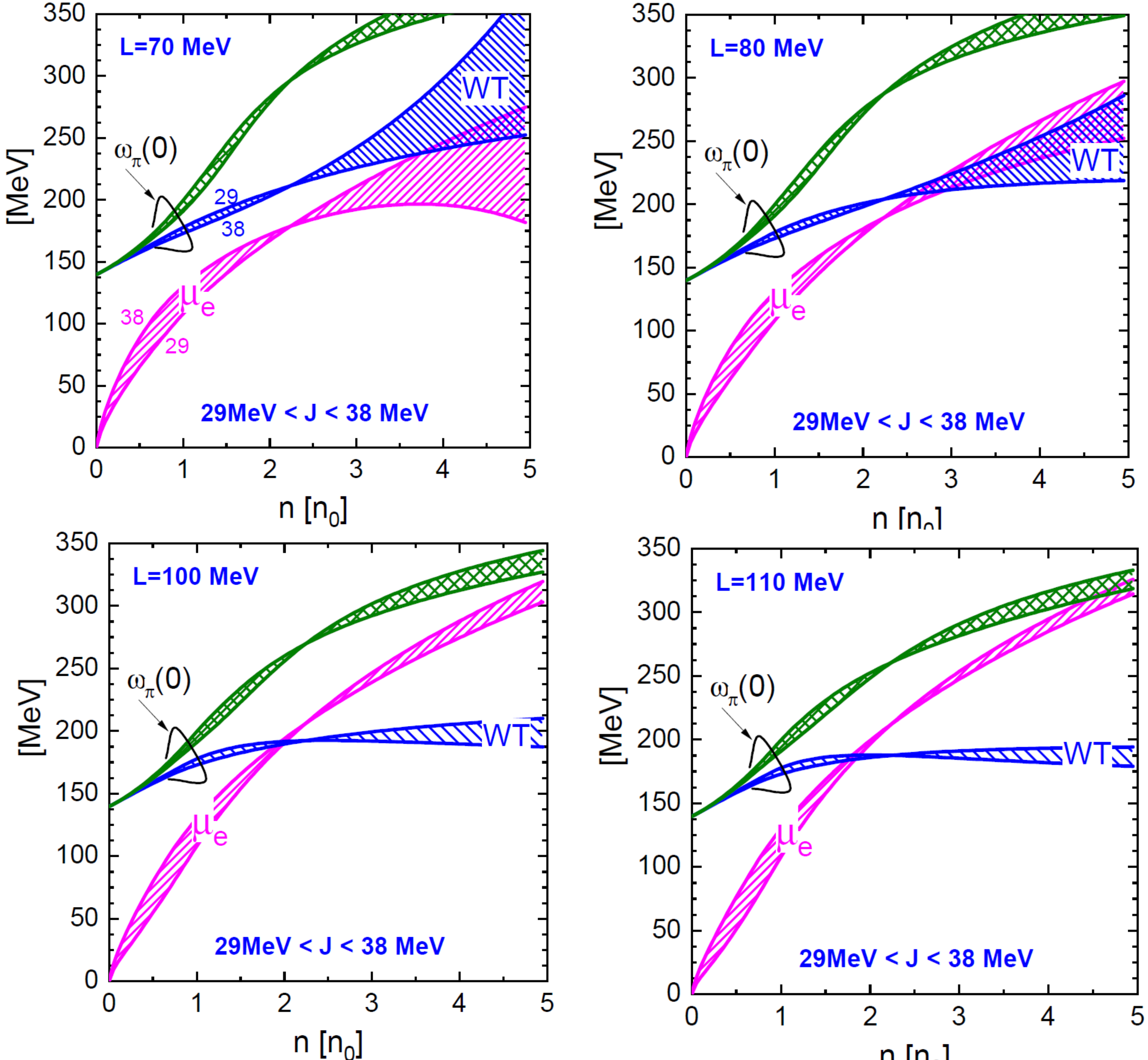
$$V^{(3/2)}(\sqrt{s}) = -\frac{1}{2f^2}\,(\sqrt{s} - m_N) + \frac{\sigma_{\pi N}}{f^2} - \frac{\beta}{f^2}\,\frac{(\sqrt{s} - m_N)^2}{m_\pi^2}$$

$$J(\sqrt{s}) = (E_{\rm cm} + m_N)\,\big(I(\sqrt{s}) - I(\mu_M)\big)$$

[Lutz, EEK, NPA700]



$$I(\sqrt{s}) = \frac{1}{16\,\pi^2}\left[\frac{p_{\rm cm}}{\sqrt{s}}\left(\ln\left(1 - \frac{s - 2\,p_{\rm cm}\sqrt{s}}{m_\pi^2 + m_N^2}\right) - \ln\left(1 - \frac{s + 2\,p_{\rm cm}\sqrt{s}}{m_\pi^2 + m_N^2}\right)\right) + \left(\frac{1}{2}\,\frac{m_\pi^2 + m_N^2}{m_\pi^2 - m_N^2} - \frac{m_\pi^2 - m_N^2}{2\,s}\right)\ln\left(\frac{m_\pi^2}{m_N^2}\right) + 1\right] + I(0)$$



Iterated pN amplitude, including Sigma-and range terms, could provide a shield against pionization

New s-wave pion condensation

We are interested now in $\omega < m_\pi$

$$\Pi^{(\pm)}_{1,S}(\omega) = -\frac{\sigma_{\pi N}}{f^2}(n_p + n_n) \mp \frac{\omega}{2f^2}(n_n - n_p) + \frac{\beta\omega^2}{m_\pi^2 f^2}(n_p + n_n)$$

Iteration is not important

Pion propagator $$D^{(-)}_\pi(\omega, \boldsymbol{q}) = \omega^2 - \boldsymbol{q}^2 - m_\pi^2 - \Pi^{(-)}_{1,S}(\omega)$$

Effective pion gap $$\widetilde{\omega}_\pi^2 = -D^{(-)}_\pi(0,0) = m_\pi^2 + \Pi^{(-)}_{1,S}(0) = m_\pi^2 - \frac{\sigma_{\pi N}}{f^2}(n_p + n_n) = m_\pi\left(1 - \frac{n}{n_{c,\sigma}}\right)$$

It vanishes at $$n_{c,\sigma} = \frac{f^2 m_\pi^2}{\sigma_{\pi N}} = 2.83\, n_0$$ for $$\sigma_{\pi N} = 45\,\mathrm{MeV}$$

It was argued by D.N. Voskresensky that at the density $n = n_{c,\sigma}$ there appears the spatially constant pion field varying with time as

$$\phi(t) = e^{i\alpha}\theta\big(n_{c,\beta} - n\big)\frac{m_\pi}{\sqrt{\Lambda}}\big(n/n_{c,\beta} - 1\big)^{1/2}\tanh\frac{m_\pi t}{\sqrt{2}}$$

[D.N. Voskresensky, S-wave pion condensation in symmetric nuclear matter, Phys. Rev. D 105 (2022) 116007]

- p-wave pion condensation

Stability of Vacuum and Limiting Fields

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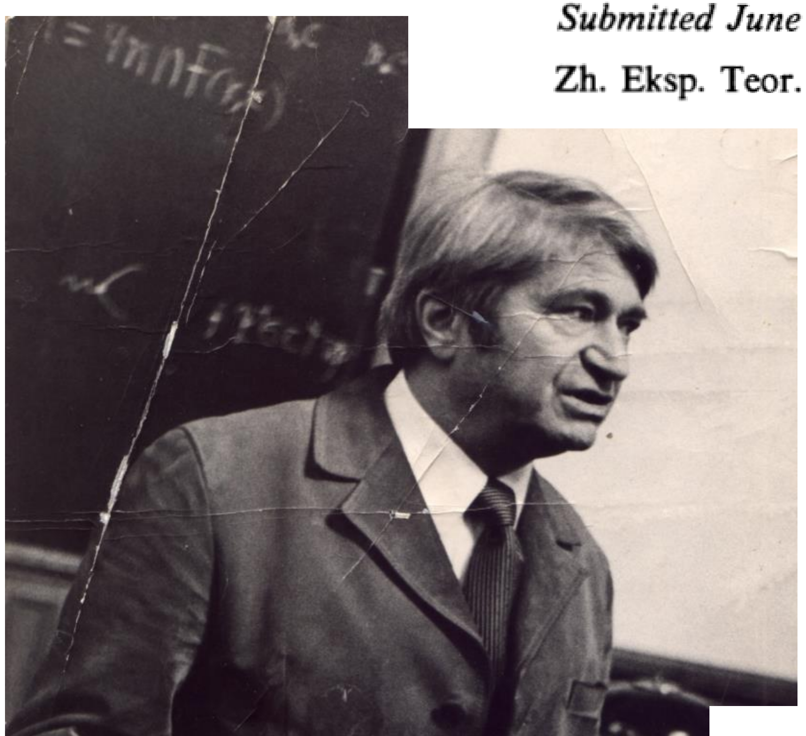

Condensed π^- Phase in Neutron-Star Matter*

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π^- Condensate in Dense Nuclear Matter*

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Pion Condensation in Nuclear and Neutron Star Matter*

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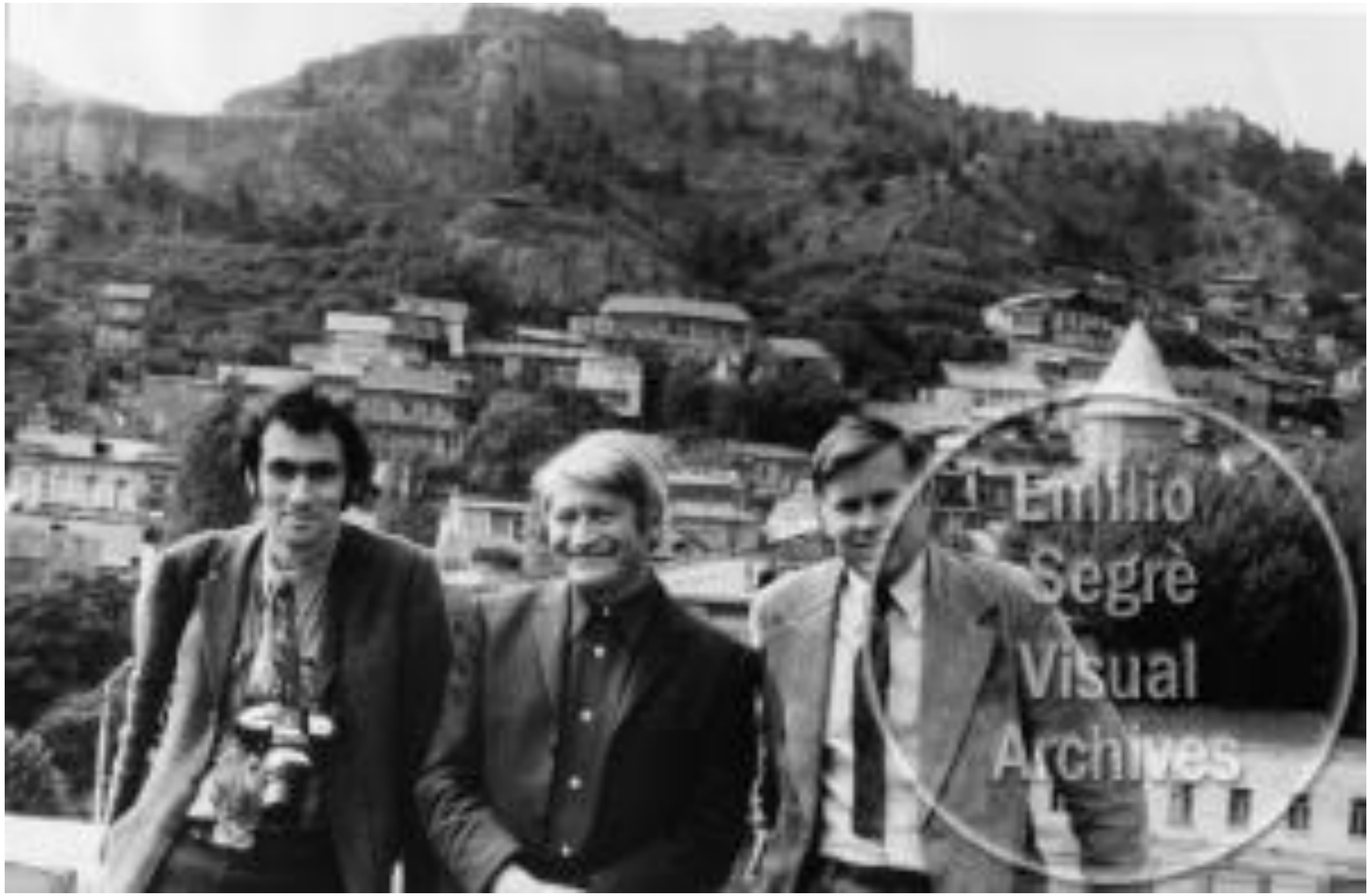
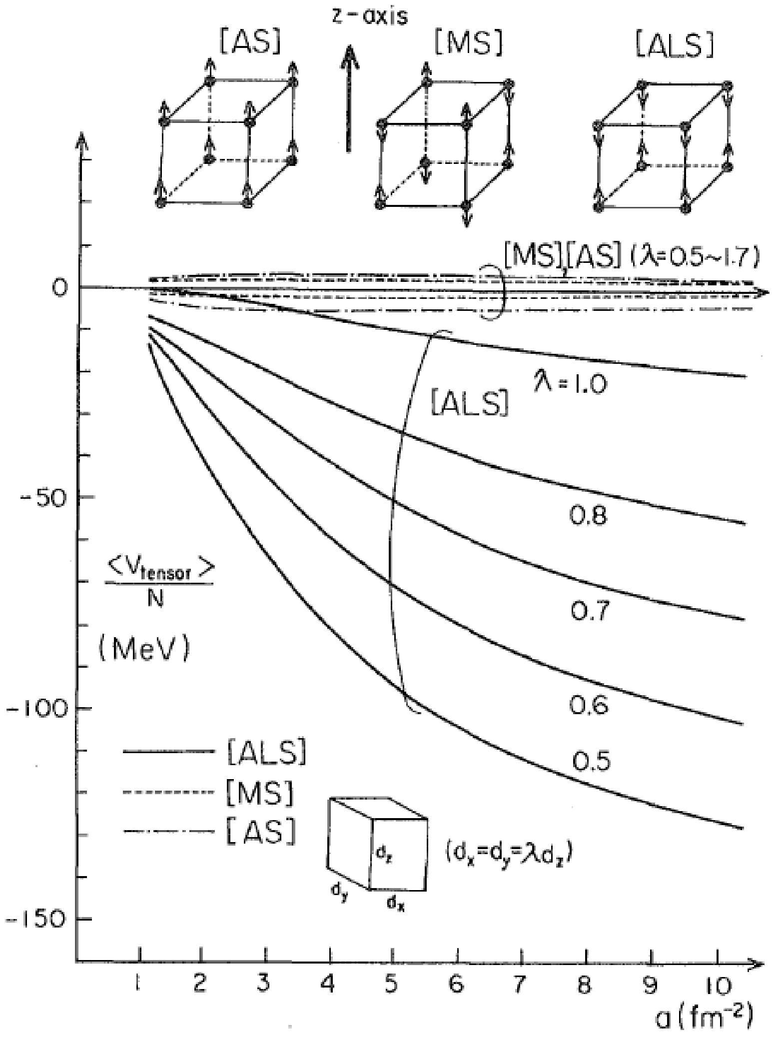


Baym Migdal Scalapino

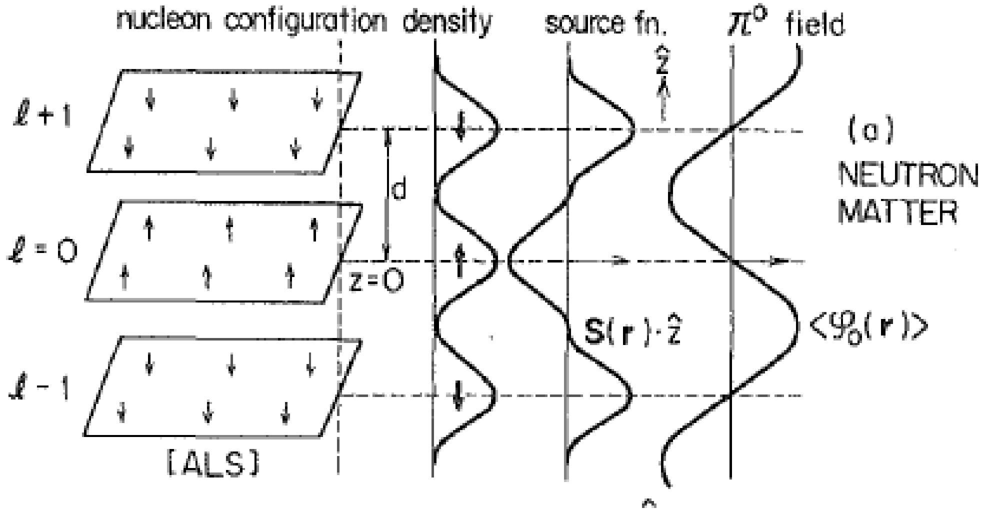
1974 Tbilisi

Alternating-layer-spin configurations

[R. Tamagaki]



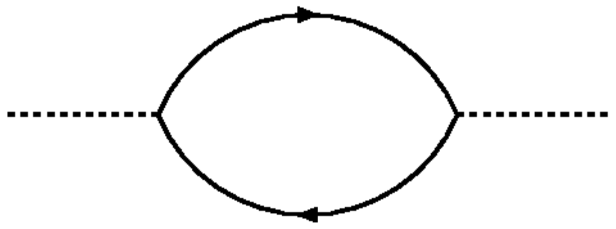
tensor force contribution to the energy



π^0 condensate

Nucleon particle-hole polarization operator

$$\mathcal{L} = \frac{g_A}{2\,f}\bar{N}\gamma_5\gamma_\mu(\boldsymbol{\tau}\cdot\partial^\mu\boldsymbol{\phi}\,)N$$



π^+ polarization operator

$$\Pi^{(+)}_{\mathrm{ph},a}(q) = 2f^2_{\pi NN}\Big\{(\omega-\lambda_a\Delta v)n_a + (\boldsymbol{q}^{\,2}+(\Delta v)^2-\omega^2)A(\omega+\lambda_a\Delta v,\boldsymbol{q},p_{\mathrm{F},a}) + (\Delta v)^2 B(\omega+\lambda_a\Delta v,\boldsymbol{q},p_{\mathrm{F},a})\Big\}$$
$$+\frac{(\Delta v)^2}{2m^*_N}2f^2_{\pi NN}\Big\{-\chi_{p^2}(\xi_a)n_a + (\omega+2\lambda_a\Delta v)C(\omega+\lambda_a\Delta v,\boldsymbol{q},p_{\mathrm{F},a}) + \frac{(\omega+\lambda_a\Delta v)^2-\boldsymbol{q}^{\,2}}{2m^*_N}A(\omega+\lambda_a\Delta v,\boldsymbol{q},p_{\mathrm{F},a})\Big\},$$

$$f_{\pi NN}=\frac{g_A}{2f}\,,\quad \Delta v = v_n - v_p,\quad \lambda_n=+1,\ \lambda_p=-1\,,\quad a=n,p$$

$$A(\omega,\boldsymbol{q},p_{\mathrm{F}}) = \int_0^{p_{\mathrm{F}}}\frac{2\mathrm{d}^3p}{(2\pi)^3}\frac{m^{*2}_N}{E^2_{\boldsymbol{p}}}\frac{1}{\omega-\frac{\boldsymbol{p}\boldsymbol{q}}{E_{\boldsymbol{p}}}+\frac{\omega^2-\boldsymbol{q}^{\,2}}{2E_{\boldsymbol{p}}}}\,,$$

$$B(\omega,\boldsymbol{q},p_{\mathrm{F}}) = \int_0^{p_{\mathrm{F}}}\frac{2\mathrm{d}^3p}{(2\pi)^3}\frac{\boldsymbol{p}^{\,2}}{E^2_{\boldsymbol{p}}}\frac{1}{\omega-\frac{\boldsymbol{p}\boldsymbol{q}}{E_{\boldsymbol{p}}}+\frac{\omega^2-\boldsymbol{q}^{\,2}}{2E_{\boldsymbol{p}}}}\,.$$

$$C(\omega,\boldsymbol{q},p_{\mathrm{F}}) = \int_0^{p_{\mathrm{F}}}\frac{2\mathrm{d}^3p}{(2\pi)^3}\frac{m^{*}_N}{E_{\boldsymbol{p}}}\frac{1}{\omega-\frac{\boldsymbol{p}\boldsymbol{q}}{E_{\boldsymbol{p}}}+\frac{\omega^2-\boldsymbol{q}^{\,2}}{2E_{\boldsymbol{p}}}}\,,$$

$$E_{\boldsymbol{p}} = \sqrt{m^{*2}_N+\boldsymbol{p}^{\,2}}\,.$$

$$A(\omega,\boldsymbol{q},p_{\mathrm{F}}) \approx -\frac{m_N p_{\mathrm{F}}}{\pi^2}\phi_1\Big(\omega+\frac{\omega^2-\boldsymbol{q}^{\,2}}{2m_N},\boldsymbol{q},p_{\mathrm{F}}\Big)$$

$$\phi_1(\omega,\boldsymbol{q},p_{\mathrm{F}}) = \frac{m_N}{2|\boldsymbol{q}\,|^3 v_{\mathrm{F}}}\Big(\frac{\omega^2-\boldsymbol{q}^{\,2}v^2_{\mathrm{F}}}{2}\log\frac{\omega+|\boldsymbol{q}\,|v_{\mathrm{F}}}{\omega-|\boldsymbol{q}\,|v_{\mathrm{F}}}-\omega|\boldsymbol{q}\,|v_{\mathrm{F}}\Big)$$

$$v_{\mathrm{F}} = \frac{p_{\mathrm{F}}}{m_N}$$

Migdal function

Short-range correlations has to be included as in MFM Lutz PLB 552, 159

Nucleon particle-hole polarization operator

No vector potentials $\Pi^{(+)}_{\mathrm{ph},a}(q) = 2f^2_{\pi NN}\left\{\omega n_a + (\boldsymbol{q}^{\,2} - \omega^2)A(\omega, \boldsymbol{q}, p_{\mathrm{F},a})\right\}$

$$A(\omega, 0, p_{\mathrm{F},a}) \approx \frac{n_a}{\omega + \frac{\omega^2}{2m^*_N}}$$

Migdal model

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π Condensation in Nuclear Matter

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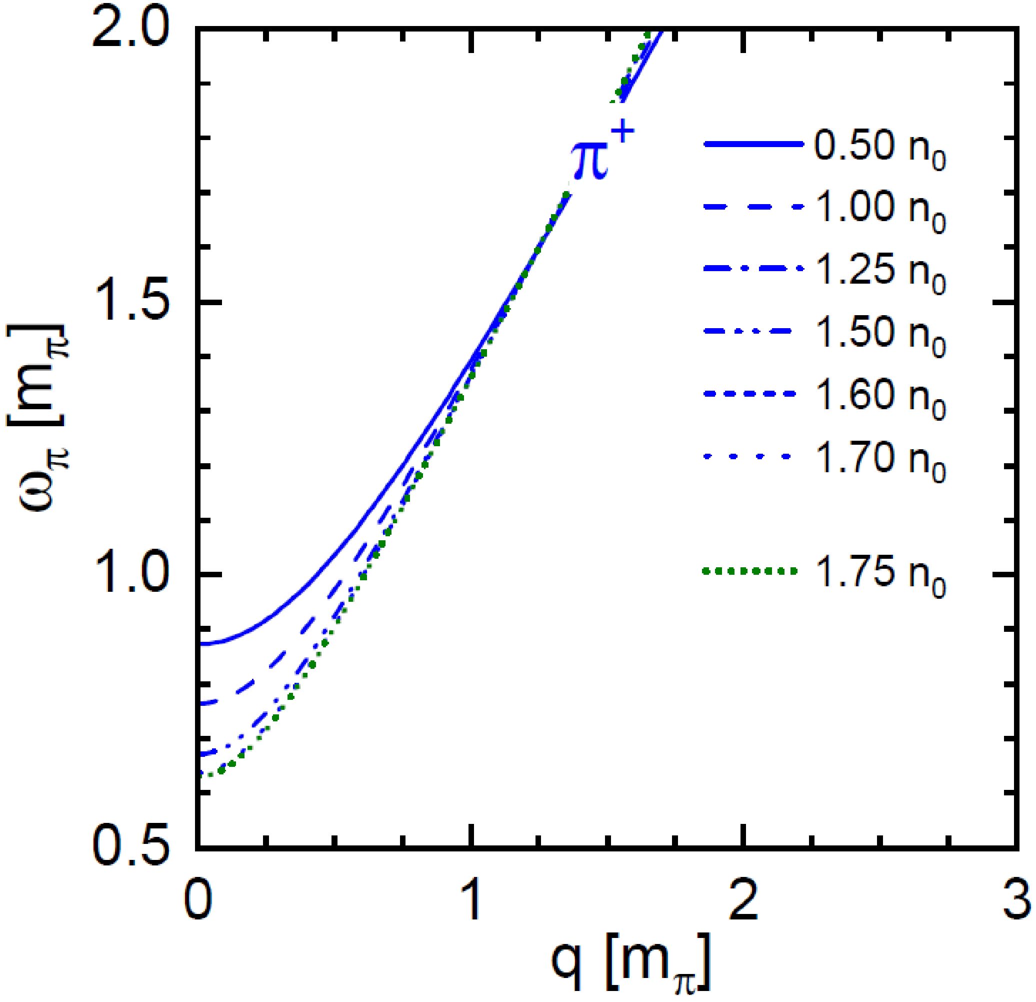
It is shown that in nuclear matter at $Z=0$ (neutron star) at a density $n_1 < n_{\mathrm{nucl}}$ a π^0 condensate appears. Nearly at the same density an electrically neutral π^+, π^- condensate arises. The π^- condensate assumed by other workers apparently does not arise even at very high densities.

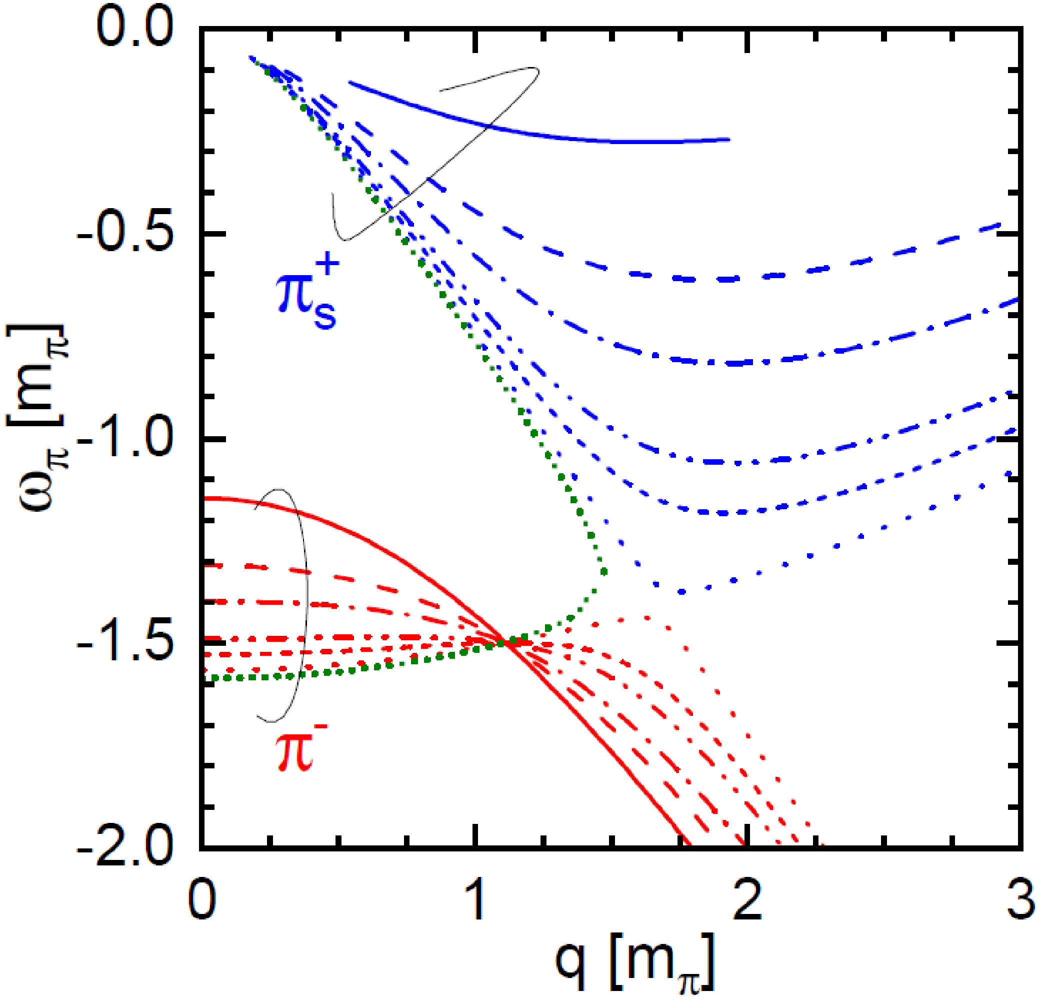
$$\Pi^{(+)}_{\mathrm{n.g.,Mig}}(\omega, \boldsymbol{q},) = -\frac{\omega}{2f^2}n_n + 2f^2_{\pi NN}\boldsymbol{q}^{\,2}A(\omega, \boldsymbol{q}, p_{\mathrm{F},n})$$

$$\approx -\frac{\omega}{2f^2}n_n - 2f^2_{\pi NN}\boldsymbol{q}^{\,2}\frac{m_N p_{\mathrm{F},n}}{\pi^2}\phi_1\left(\omega - \frac{\boldsymbol{q}^{\,2} - \omega^2}{2m_N}, \boldsymbol{q}, p_{\mathrm{F},n}\right)$$

- *Pion condensation in the simplified Migdal model*

Pure neutron gas





- *Pion condensation in the simplified Migdal model*

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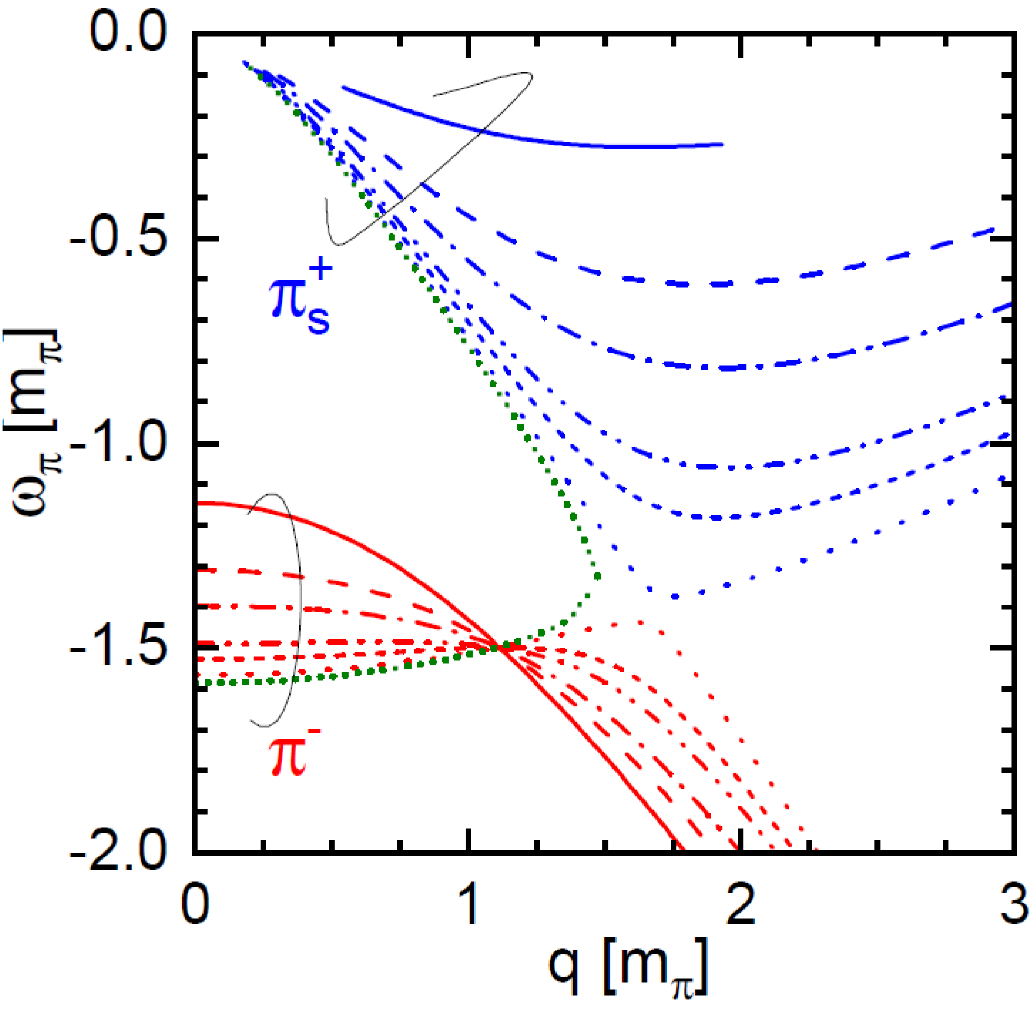
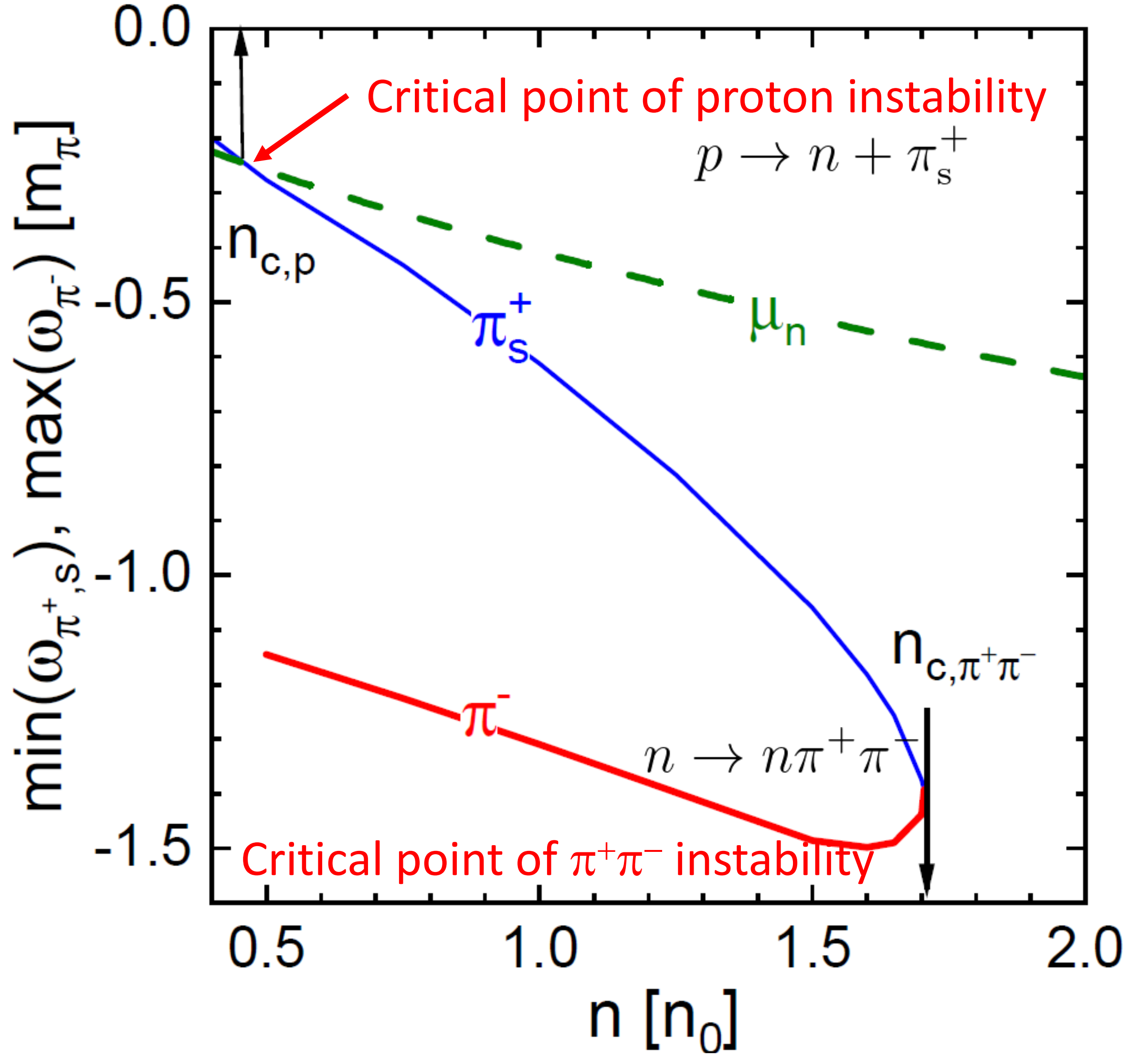


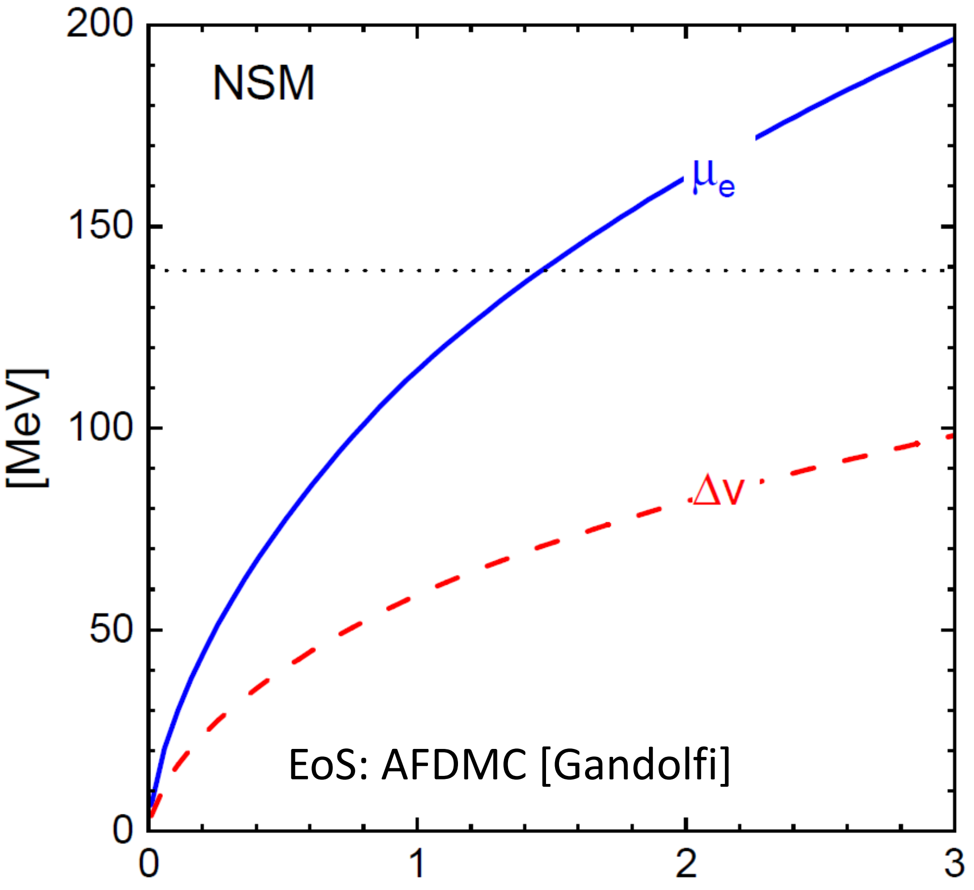


New branches in pion spectrum

B. Fore, N. Kaiser, S. Reddy, N.C. Warrington, The mass of charged pions in neutron star matter, arXiv:2301.07226.

Vector potentials (Vector part of the nucleon self-energy) are important

$$\text{EoS: } E(n,x) \qquad \mu_n = \frac{\partial E(n,x)}{\partial n} - \frac{x}{n}\frac{\partial E(n,x)}{\partial x} \qquad \mu_p = \frac{\partial E(n,x)}{\partial n} + \frac{1-x}{n}\frac{\partial E(n,x)}{\partial x}$$



$$\mu_a = \sqrt{m_N^{*2} + p_{\mathrm{F},a}^2} + v_a$$

$$\Delta v = v_n - v_p = \mu_n - \mu_p - \sqrt{m_N^{*2} + p_{\mathrm{F},n}^2} + \sqrt{m_N^{*2} + p_{\mathrm{F},p}^2}$$

assumed $$m_N^*(n) = \frac{m_N}{1 + \left(\frac{1}{0.85} - 1\right)\sqrt{\frac{n}{n_0}}}$$

S-wave pion polarization operator at chiral order Q^2 including $p_F^{5/3}$ corrections

Pure neutron matter

We will keep the correction terms of the order $\xi_n^2 = p_{\mathrm{F},n}^2/m_N^{*2}$ and also will assume $\omega/m_N^* \ll 1$, and $\Delta v/m_N^* \ll 1$. So we will drop terms proportional to $\xi_n^2\omega/m_N^*$ and $\xi_n\Delta v/m_N^*$.

$$\Pi^{(+)}_{\mathrm{n.m.},S}(\omega) = -\frac{\omega}{2f^2}n_n - \left(\frac{\sigma_{\pi N}}{f^2} - \frac{\beta\omega^2}{m_\pi^2 f^2}\right)\left(n_n - \frac{3\xi_n^2}{10}n_n\right) - \frac{2c_2}{f^2}\omega^2\frac{3}{5}\xi_n^2 n_n$$
$$+ 2f_{\pi NN}^2\left\{(\omega-\Delta v)n_n + \left((\Delta v)^2-\omega^2\right)A(\omega+\Delta v, 0, p_{\mathrm{F},n}) + (\Delta v)^2 B(\omega+\Delta v, 0, p_{\mathrm{F},n}) - \omega^2\frac{n_n}{2m_N}\right\}$$

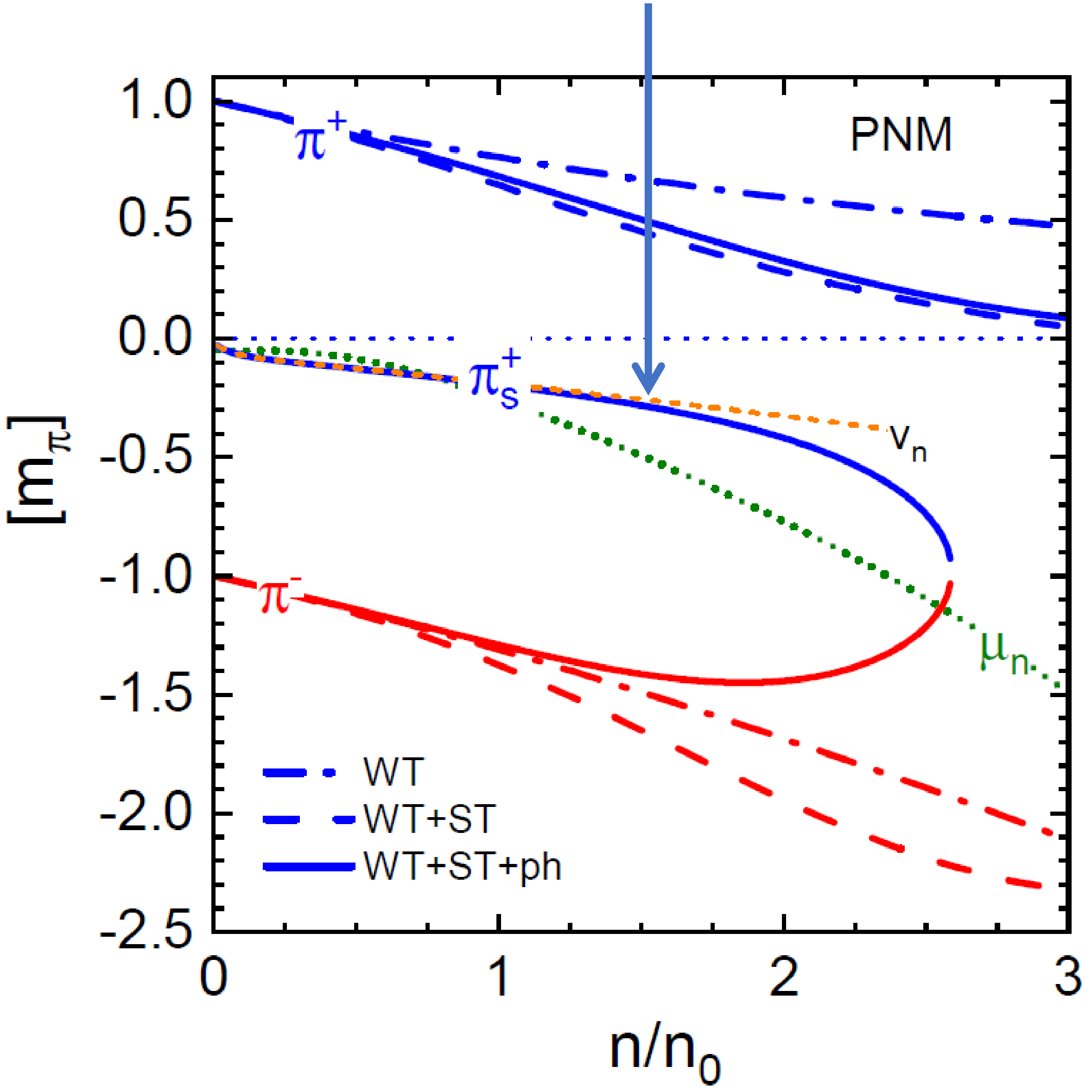
$$A(\omega, 0, p_{\mathrm{F},a}) \approx \frac{n_a - \frac{3}{5}\xi_a^2 n_a}{\omega + \frac{\omega^2}{2m_N^*}}, \quad B(\omega, 0, p_{\mathrm{F},a}) \approx \frac{\frac{3}{5}\xi_a^2 n_a}{\omega + \frac{\omega^2}{2m_N^*}}$$

$$\Pi^{(+)}_{\mathrm{n.m.},S}(\omega) = -m_\pi^2\frac{n_n}{n_{c,\sigma}}\left(1 - \frac{3\xi_n^2}{10}\right) - \frac{\omega}{2f^2}n_n + \omega^2\frac{n}{n_{c,\beta}}\left(1 - \frac{3\xi_n^2}{10} - \frac{2c_2 m_\pi^2}{\beta}\frac{3}{5}\xi_n^2\right)$$
$$+ 2f_{\pi NN}^2\left\{\frac{(\Delta v)^2 n_n\left(\frac{3}{5}\xi_n^2 + \frac{\Delta v}{2m_N^*}\right)}{\omega+\Delta v} + n_n(\omega-\Delta v)\frac{3}{5}\xi_n^2 + \omega^2 n_n\frac{m_N - m_N^*}{2m_N^* m_N}\right\}$$

Pole term! → new branch [Fore et all.]

No term of the order n, only higher orders in density

New branch at vanishing momentum



Sigma-term instability is shifted to higher densities

$$\widetilde{\omega}_\pi^2 = -D_\pi(0,0)$$
$$= m_\pi^2 - m_\pi^2 \frac{n_n}{n_{c,\sigma}}\left(1 - \frac{3\xi_n^2}{10}\right) + 2f_{\pi NN}^2 \frac{v_n^2 n_n}{2m_N^*}$$

Conclusion

- For non-interacting pions, the pionization of the NS matter occurs at very low density ~1.3n_0
- The Weinber-Tomazawa term in the πN interaction alone does not protect against pionization for the stiff symmetry energy
- Sigma-term and range term (Next-to-leading chiral order) has to be taken into account
- π^- mass diverges at $n_{c,\beta} = \dfrac{f^2 m_\pi^2}{\beta} \approx 2.8 n_0$ and π^+ mass vanishes
- One has to use the unitarized amplitudes. Pionization could be prevented.
- New type of the s-wave condensation occurs at density ~2.8n_0
- In the particle-hole term one has to take into account the nucleon vector potential. New spectral branch at the vanishing momentum which connects to the Migdal branch at finite moments.
- New instabilities!