The three-body DD^*K system on the lattice EFT

Qian Wang

South China Normal University

YITP long-term and Nishinomiya-Yukawa memorial workshop

Hadrons and Hadron Interactions in QCD 2024 (HHIQCD 2024)

-- Experiments, Effective theories, and Lattice --

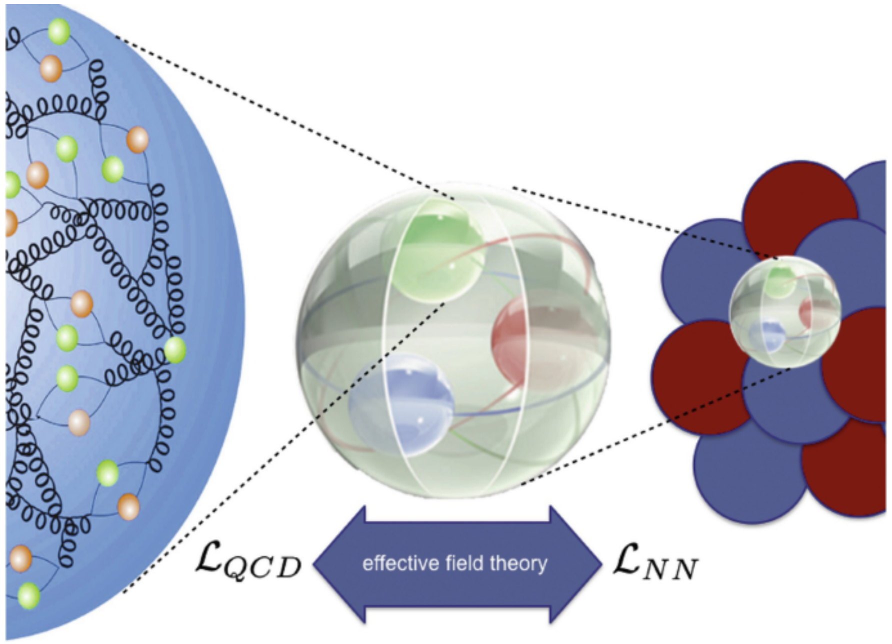
14th Oct. - 15th Nov., 2024

Yukawa Institute for Theoretical Physics, Kyoto University, Japan

Introduction to LEFT

Lattice EFT=Chiral EFT+Lattice+Monte Carlo

- Hadrons are effective degrees of freedom
- EFT describes hadron interactions
- Lattice spacing is set around 1 fm



	LQCD	LEFT
Degrees of freedom	quarks & gluons	hadrons
lattice spacing	~0.1fm	~1fm
dispersion relation	relativistic	non-relativistic
continuum limit	✓	✗
model	Lagrangian	Schrödinger
solver	path integral	matrix compute

Dean Lee, Prog.Part.Nucl.Phys., 63(2009)117,

Lähde, Meißner, “Nuclear Lattice Effective Field Theory”, Springer (2019).

Applications of LEFT

Applications in Nuclear Physics

Neutron-proton scattering	Phys.Rev.C, 98(2018)044002
Nuclear binding	Phys.Rev.Lett., 117(2016)132501
Alpha–alpha scattering	Phys.Rev.Lett., 111(2013)032502, Nature, 528(2015)111
Nuclear thermodynamics	Phys.Rev.Lett., 125(2020)192502
Properties of nuclei	Phys.Lett.B, 797(2019)134863
Hoyle state	Phys.Rev.Lett., 109(2012)252501, Nat. Commun., 14(2023)2777

Applications of LEFT

Applications in Hadron Physics

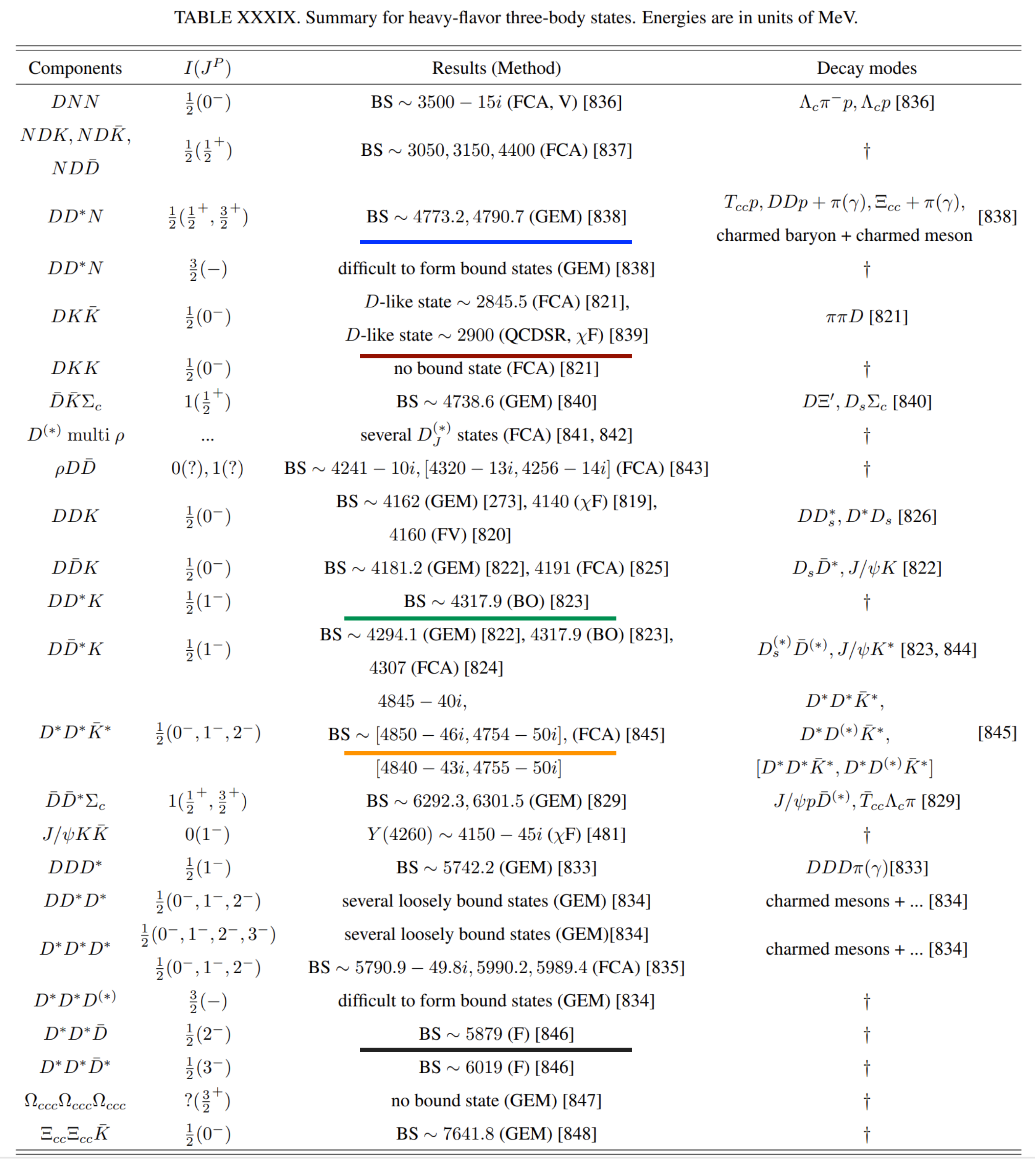
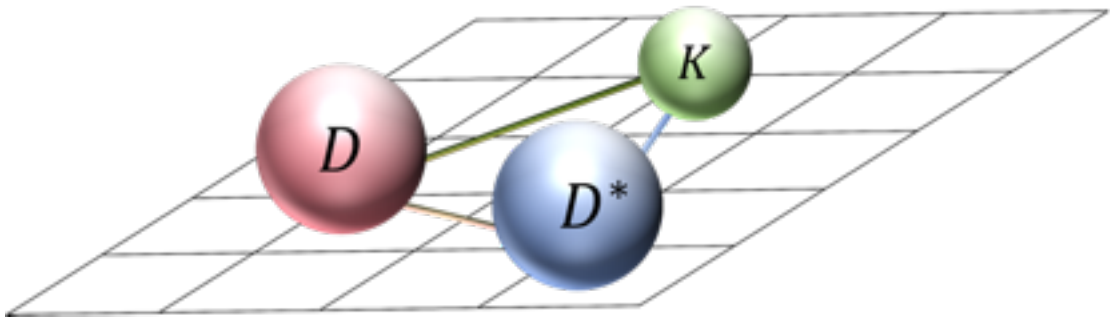


TABLE XXXIX. Summary for heavy-flavor three-body states. Energies are in units of MeV.

Components	$I(J^P)$	Results (Method)	Decay modes
DNN	$\frac{1}{2}(0^-)$	BS $\sim 3500 - 15i$ (FCA, V) [836]	$\Lambda_c\pi^- p, \Lambda_c p$ [836]
$NDK, ND\bar{K}$, $ND\bar{D}$	$\frac{1}{2}(\frac{1}{2}^+)$	BS $\sim 3050, 3150, 4400$ (FCA) [837]	†
DD^*N	$\frac{1}{2}(\frac{1}{2}^+, \frac{3}{2}^+)$	BS $\sim 4773.2, 4790.7$ (GEM) [838]	$T_{cc}p, DDp + \pi(\gamma), \Xi_{cc} + \pi(\gamma)$, charmed baryon + charmed meson [838]
DD^*N	$\frac{3}{2}(-)$	difficult to form bound states (GEM) [838]	†
$DK\bar{K}$	$\frac{1}{2}(0^-)$	D-like state ~ 2845.5 (FCA) [821], D-like state ~ 2900 (QCDSR, χF) [839]	$\pi\pi D$ [821]
DKK	$\frac{1}{2}(0^-)$	no bound state (FCA) [821]	†
$\bar{D}\bar{K}\Sigma_c$	$1(\frac{1}{2}^+)$	BS ~ 4738.6 (GEM) [840]	$D\Xi', D_s\Sigma_c$ [840]
$D^{(*)}$ multi ρ	...	several $D_J^{(*)}$ states (FCA) [841, 842]	†
$\rho D\bar{D}$	$0(?), 1(?)$	BS $\sim 4241 - 10i, [4320 - 13i, 4256 - 14i]$ (FCA) [843]	†
DDK	$\frac{1}{2}(0^-)$	BS ~ 4162 (GEM) [273], 4140 (χF) [819], 4160 (FV) [820]	DD_s^*, D^*D_s [826]
$D\bar{D}K$	$\frac{1}{2}(0^-)$	BS ~ 4181.2 (GEM) [822], 4191 (FCA) [825]	$D_s\bar{D}^*, J/\psi K$ [822]
DD^*K	$\frac{1}{2}(1^-)$	BS ~ 4317.9 (BO) [823]	†
$D\bar{D}^*K$	$\frac{1}{2}(1^-)$	BS ~ 4294.1 (GEM) [822], 4317.9 (BO) [823], 4307 (FCA) [824]	$D_s^{(*)}\bar{D}^{(*)}, J/\psi K^*$ [823, 844]
$D^*D^*\bar{K}^*$	$\frac{1}{2}(0^-, 1^-, 2^-)$	$4845 - 40i$, BS $\sim [4850 - 46i, 4754 - 50i]$, (FCA) [845] $[4840 - 43i, 4755 - 50i]$	$D^*D^*\bar{K}^*$, $D^*D^{(*)}\bar{K}^*$, $[D^*D^*\bar{K}^*, D^*D^{(*)}\bar{K}^*]$ [845]
$\bar{D}\bar{D}^*\Sigma_c$	$1(\frac{1}{2}^+, \frac{3}{2}^+)$	BS $\sim 6292.3, 6301.5$ (GEM) [829]	$J/\psi p\bar{D}^{(*)}, \bar{T}_{cc}\Lambda_c\pi$ [829]
$J/\psi K\bar{K}$	$0(1^-)$	$Y(4260) \sim 4150 - 45i$ (χF) [481]	†
DDD^*	$\frac{1}{2}(1^-)$	BS ~ 5742.2 (GEM) [833]	$DDD\pi(\gamma)$[833]
DD^*D^*	$\frac{1}{2}(0^-, 1^-, 2^-)$	several loosely bound states (GEM) [834]	charmed mesons + ... [834]
$D^*D^*D^*$	$\frac{1}{2}(0^-, 1^-, 2^-, 3^-)$	several loosely bound states (GEM)[834]	charmed mesons + ... [834]
	$\frac{1}{2}(0^-, 1^-, 2^-)$	BS $\sim 5790.9 - 49.8i, 5990.2, 5989.4$ (FCA) [835]	
$D^*D^*D^{(*)}$	$\frac{3}{2}(-)$	difficult to form bound states (GEM) [834]	†
$D^*D^*\bar{D}$	$\frac{1}{2}(2^-)$	BS ~ 5879 (F) [846]	†
$D^*D^*\bar{D}^*$	$\frac{1}{2}(3^-)$	BS ~ 6019 (F) [846]	†
$\Omega_{ccc}\Omega_{ccc}\Omega_{ccc}$	$?(\frac{3}{2}^+)$	no bound state (GEM) [847]	†
$\Xi_{cc}\Xi_{cc}\bar{K}$	$\frac{1}{2}(0^-)$	BS ~ 7641.8 (GEM) [848]	†



- Gaussian expansion method (GEM)
- QCD sum rule (QCDSR)
- Born-Oppenheimer approximation
- Fixed center approximation (FCA)
- Faddeev equation (F)

Without 3-body force!

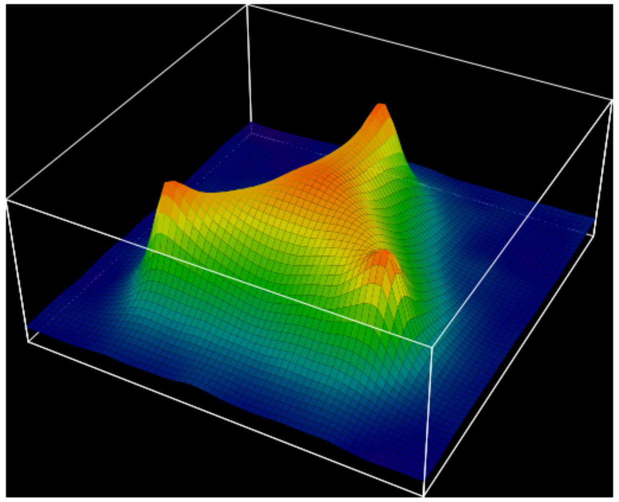
Talk by Lisheng Geng this morning

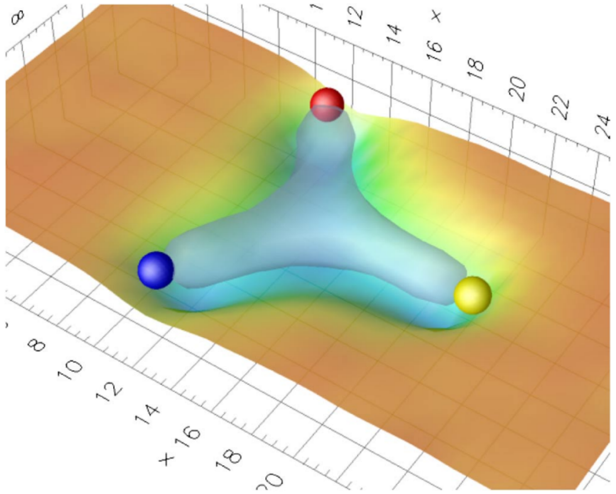
Liu et al., arXiv:2404.06399

Advantages of LEFT

- Include three-body force directly

- Three-body force in baryon from LQCD





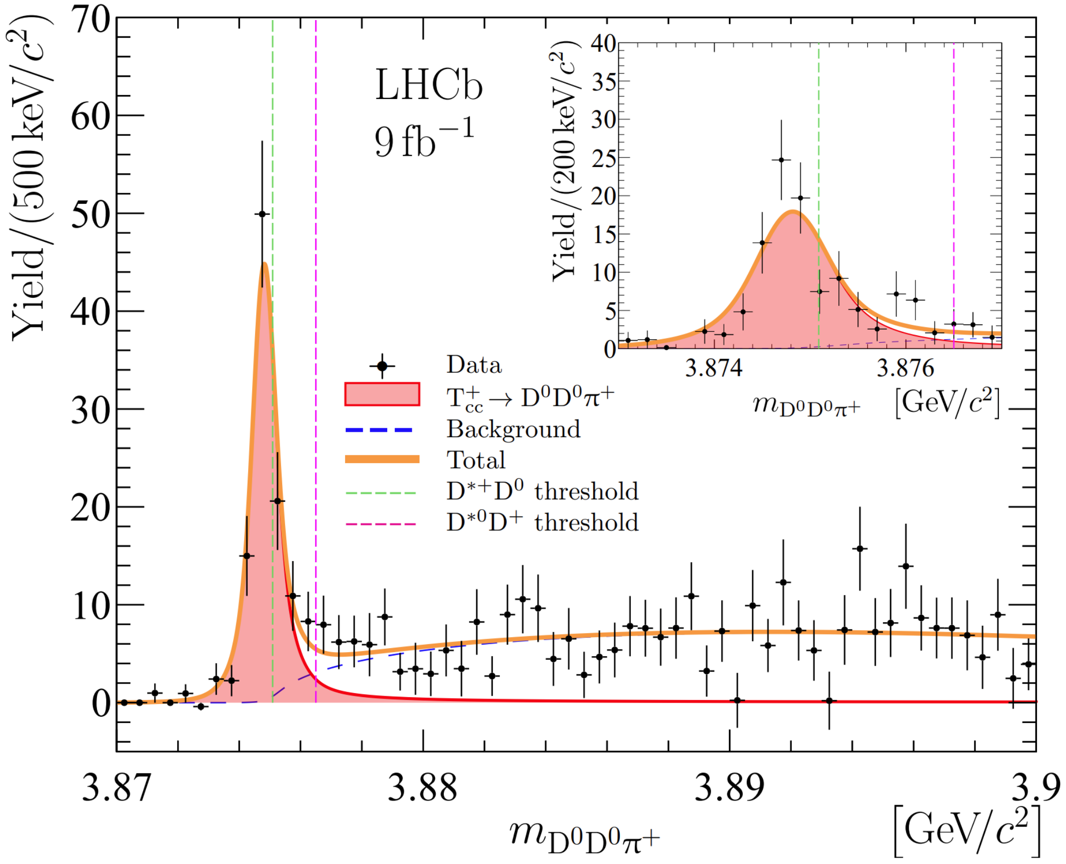
H. Ichie et al., Nucl. Phys.A 721(2003)C899-C902, F. Bissey et al., Phys.Rev.D 76(2007)114512

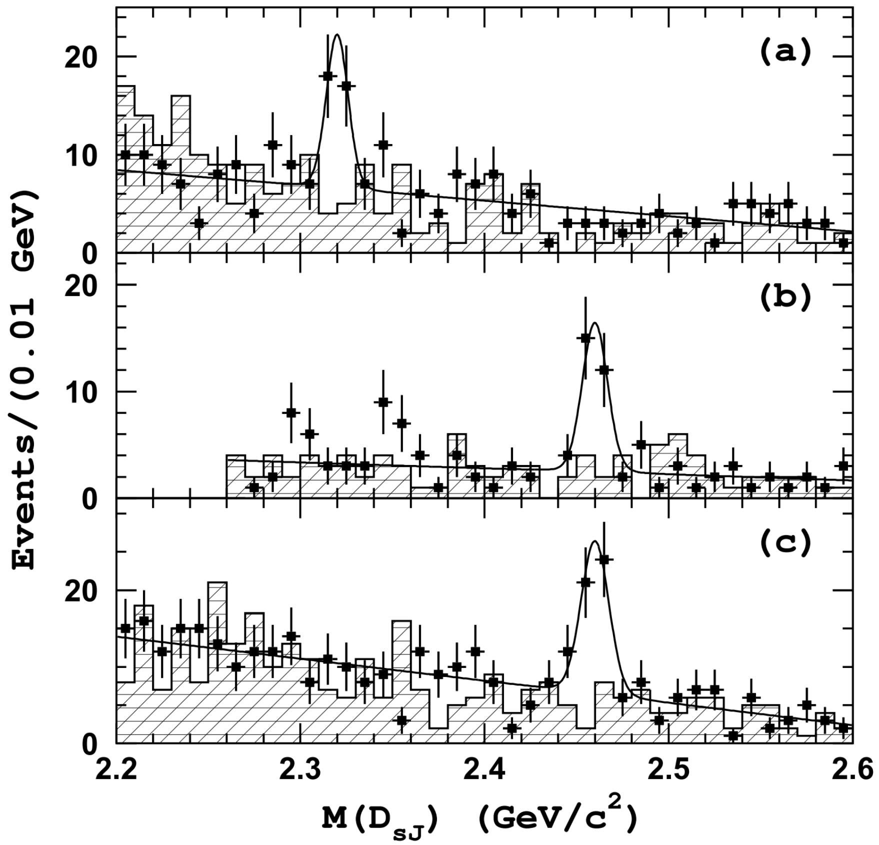
- Three-body force makes nucleus system ren. group invariant

Λ (MeV)	250	275	300	325	350	375	400	Exp.
c_E	5.170	2.763	1.538	0.890	0.561	0.412	0.380	
$E_{2NF}\left({}^3H\right)$	-6.17(4)	-6.63(4)	-7.05(2)	-7.39(2)	-7.64(1)	-7.77(1)	-7.78(1)	-8.482
$E_{2NF+3NF}\left({}^3H\right)$	-8.482	-8.482	-8.489	-8.485	-8.483	-8.483	-8.483	-8.482
$E_{2NF}\left({}^4He\right)$	-30.6(7)	-30.3(6)	-30.7(4)	-30.0(4)	-29.8(4)	-29.4(4)	-29.2(4)	-28.34
$E_{2NF+3NF}\left({}^4He\right)$	-29.8(7)	-29.5(6)	-29.9(4)	-29.2(4)	-29.0(4)	-28.6(4)	-28.4(4)	-28.34
$E_{2NF}\left({}^{16}O\right)$	-144.0(21)	-135.1(14)	-136.3(11)	-139.1(9)	-140.6(8)	-141.7(8)	-141.8(9)	-127.6
$E_{2NF+3NF}\left({}^{16}O\right)$	-135.8(20)	-124.8(14)	-124.5(11)	-126.3(9)	-127.3(8)	-128.1(8)	-128.1(8)	-127.6

The motivation of the DD^*K system

- The observation of $T_{cc}^+, D_{s0}^*(2317), D_{s1}(2460)$ in experiment





R. Aaij et al. (LHCb), Nature Phys. 18 (2022)751 P. Krokovny et al. (Belle), Phys. Rev. Lett. 91(2003) 262002

- Close to the DD^*, DK, D^*K thresholds
- Could be hadronic molecular candidates
- Be used to constraint the two-body force

The motivation of the DD^*K system

- The observation of $T_{cc}^{+}, D_{s0}^{*}(2317), D_{s1}(2460)$ in experiment

Meng et al., PRD104(2021)051502,

Agaev et al., NPB975(2022)115650,

Feijoo et al., PRD104(2021)114015,

Yan et al., PRD105(2022)014007,

Albaladejo et al., PLB829(2022)137052,

Du et al., PRD105(2022)014024,

Padmanath et al., PRL129(2022)032002,

Chen et al., PLB833(2022)137391,

Lyu et al., PRL131(2023)161901,……

Guo et al, EPJA40(2009)171,

Liu et al., PRD87(2013)014508,

Guo et al., PRD98(2018)014510,

Liu et al., PRD109(2024)5,

Kim et al., PTEP2024(2024)073D01,

Gil-Dominguez et al., PLB843(2023)137997,

Asokan et al., EPJC83(2023)850,

Fu et al., EPJA58(2022)70,

Kong et al., PRD104(2021)094012,……

R. Aaij et al. (LHCb), Nature Phys. 18 (2022)751 P. Krokovny et al. (Belle), Phys. Rev. Lett. 91(2003) 262002

- Close to the DD^*, DK, D^*K thresholds
- Could be hadronic molecular candidates
- Be used to constraint the two-body force

Two-body interactions

- The observation of $T_{cc}^{+}, D_{s0}^{*}(2317), D_{s1}(2460)$ in experiment

- DD^* interaction: LO+OPE

Du et al, PRD105(2022)014024

- DK interaction: LO+NLO

Guo et al, EPJA40(2009)171

- D^*K interaction: LO+NLO

- Single particle regulator is used to obtain a better ren. Group invariant

Lu et al, arXiv:2308.14559

Two-body interactions

- The observation of $T_{cc}^{+}, D_{s0}^{*}(2317), D_{s1}(2460)$ in experiment

◆ DD^* interaction: LO+OPE

Du et al, PRD105(2022)014024

$$V_{DD^*}^{\text{Con}} = v_0 \epsilon \cdot \epsilon^* \qquad V_{DD^*}^{OPE}(\boldsymbol{q}) = -\frac{3g^2}{4f_\pi^2}\frac{\boldsymbol{\epsilon}\cdot\boldsymbol{q}\boldsymbol{\epsilon}^*\cdot\boldsymbol{q}}{\boldsymbol{q}^2+\mu^2}$$

◆ DK interaction: LO+NLO

Guo et al, EPJA40(2009)171

$$V_{\text{LO}}^{DK}\left(p_i\right) = \frac{-1}{2f_\pi^2}\left(p_1\cdot p_2 + p_1'\cdot p_2' + p_1\cdot p_2' + p_2\cdot p_1'\right)$$

$$V_{\text{NLO}}^{DK}\left(p_i\right) = -\frac{8M_K^2}{3f_\pi^2}h_1 + \frac{4}{f_\pi^2}h_3 p_2\cdot p_2' + h_5\left(p_1\cdot p_2 p_1'\cdot p_2' + p_1\cdot p_2' p_2\cdot p_1'\right)\Bigg)$$

◆ D^*K interaction: LO+NLO

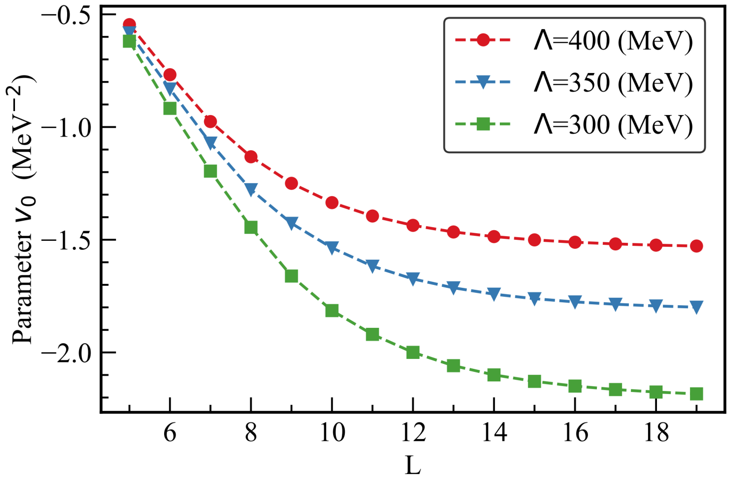
$$V_{\text{NLO}}^{D^*K}\left(p_i\right) = \left(-\frac{8M_K^2}{3f_\pi^2}h_1^* + \frac{4}{f_\pi^2}\left(h_3' p_2\cdot p_2' + h_5^*\left(p_1\cdot p_2 p_1'\cdot p_2' + p_1\cdot p_2' p_2\cdot p_1'\right)\right)\right)\boldsymbol{\epsilon}\cdot\epsilon^*$$

◆ Single particle regulator is used to obtain a better ren. Group invariant

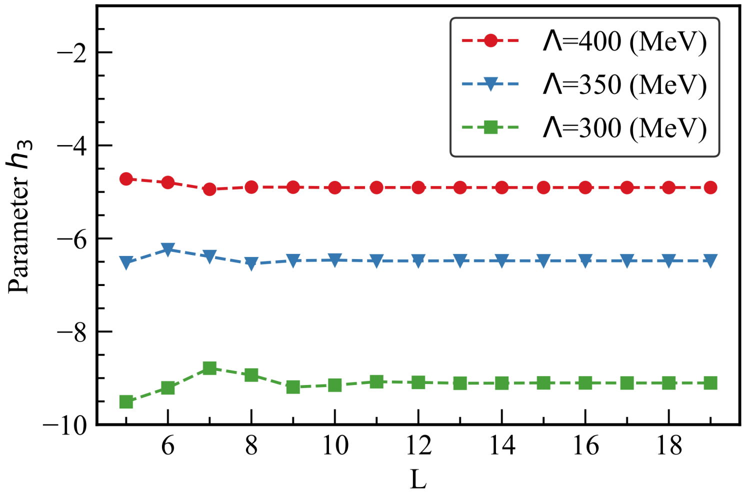
Lu et al, arXiv:2308.14559

The two-body parameters

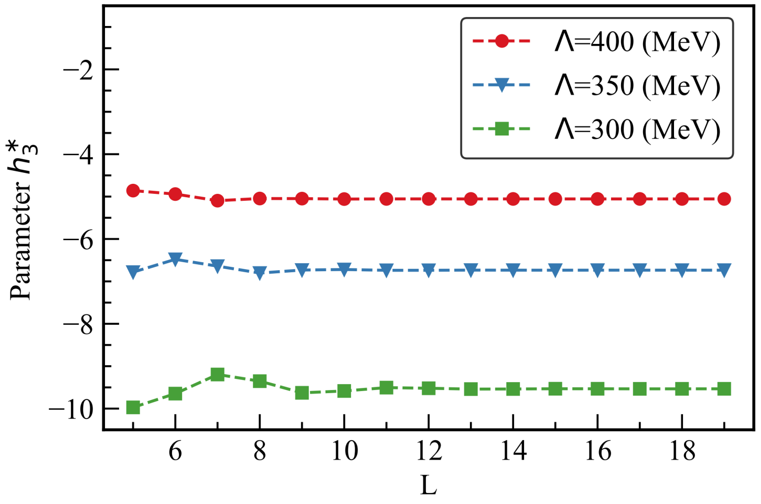
$T_{cc}^{+} \Rightarrow v_0$



$D_{s0}^{*}(2317) \Rightarrow h_3$



$D_{s1}(2460) \Rightarrow h_3^{*}$



- Cubic lattice $L^3 = 5^3 \cdots 19^3$
- Cutoff $\Lambda = 300,\ 350,\ 400\ \mathrm{MeV}$ in the regulator
- Lattice spacing $a = 1/200\ \mathrm{MeV} \sim 0.99\ \mathrm{fm}$
- v_0 converges slow $\Leftarrow$ long-ranged force
- $\Lambda = 400\ \mathrm{MeV}$ converges quickly

Zhang et al., arXiv:2409.01325

The three-body interactions

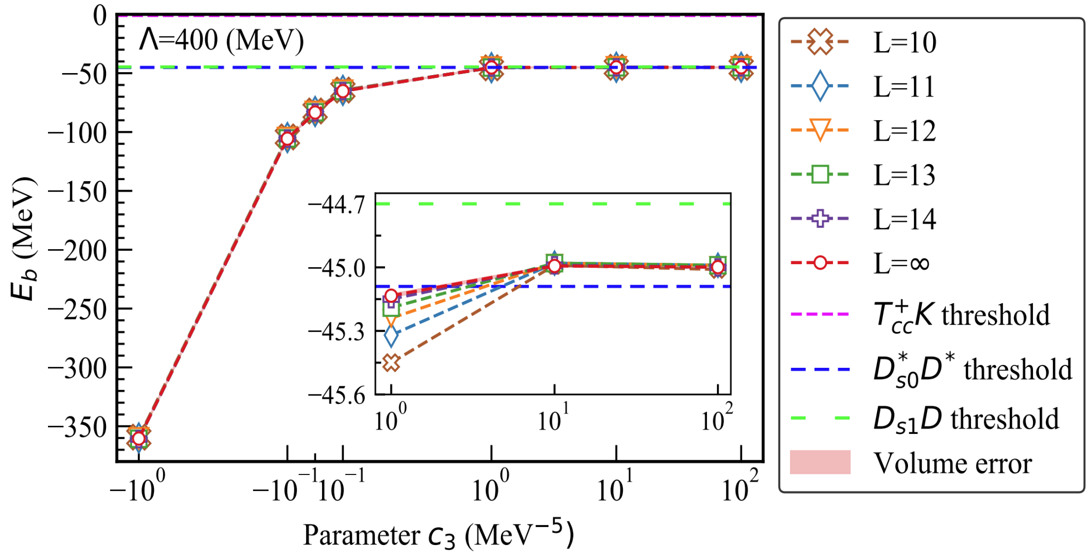
- DD^*K Lag. $\mathscr{L} = c_3 \left\langle H\mathscr{D}_\mu H^\dagger H\mathscr{D}^\mu H^\dagger \right\rangle + c_3' \left\langle H\mathscr{A}_\mu H^\dagger H\mathscr{A}^\mu H^\dagger \right\rangle$

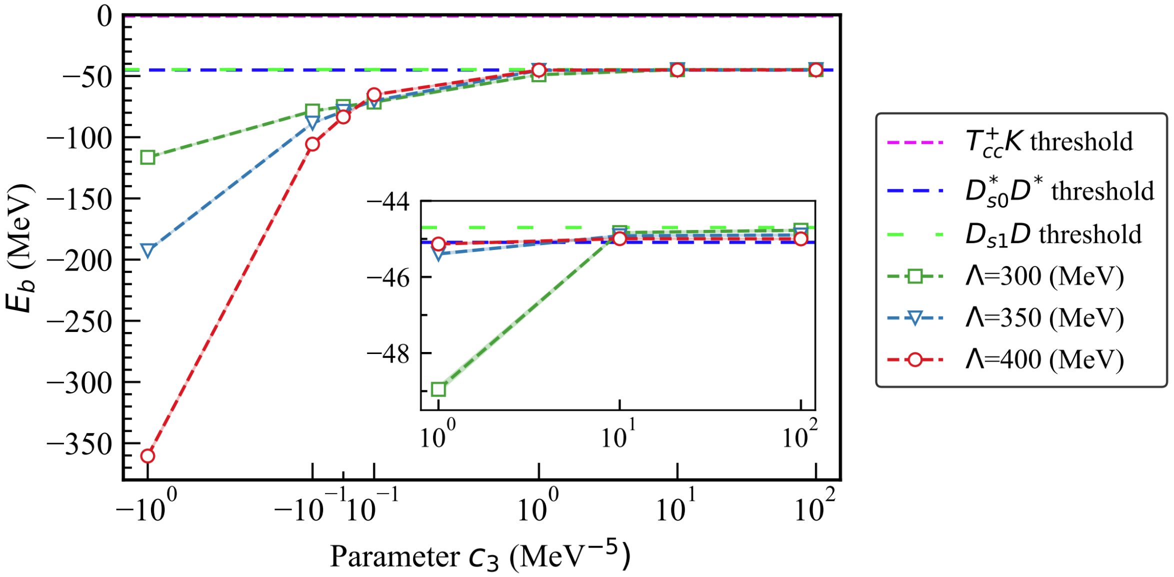
- DD^*K three-body force

$$V_{DD^*K}(p_i) = \frac{c_3}{4f_\pi^2}\left(p_1\cdot p_3 + p_1\cdot p_3' + p_2\cdot p_3 + p_2\cdot p_3' + p_1'\cdot p_3 + p_1'\cdot p_3' + p_2'\cdot p_3 + p_2'\cdot p_3'\right)\epsilon\cdot\epsilon^*$$

- DD^*K binding energy

Zhang et al., arXiv:2409.01325





- Extrapolate to infinite volume

Meng et al., PRD98(2018)014508

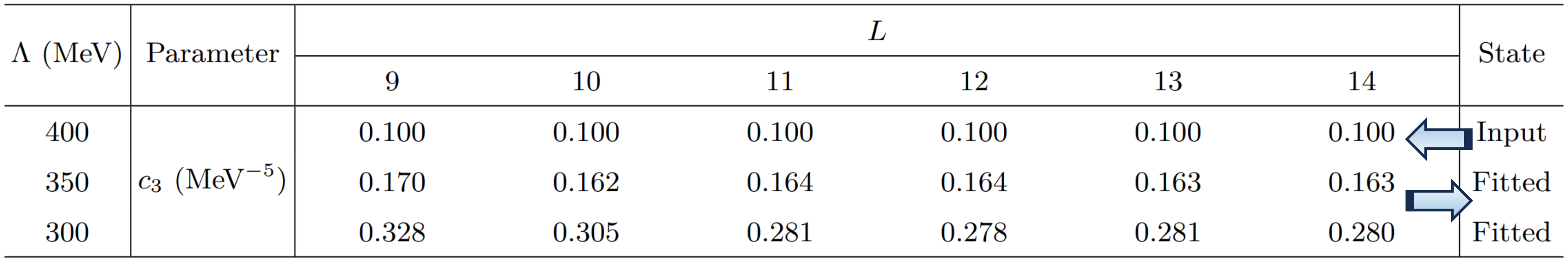
$$\frac{\Delta E}{E_T} = -(\kappa L)^{-3/2}\sum_{i=1}^{3} C_i \exp\left(-\mu_i \kappa L\right)$$

- Switch off three-body force, the result is consistent with that in

Ma et al., CPC43(2019)014102

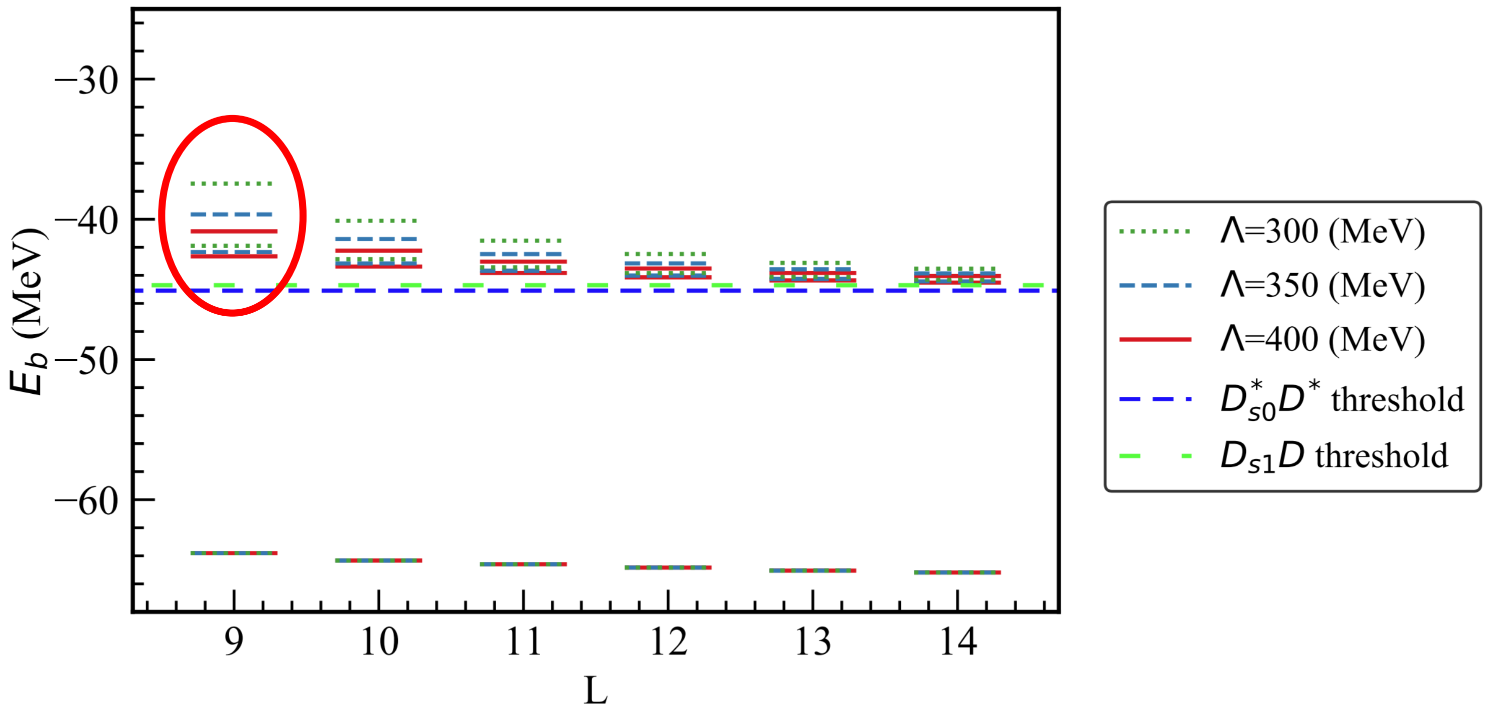
The first excited state

- No experimental data ⇒ binding energy with $\Lambda = 400$ MeV as input
- The parameter c_3 at various cubic



Λ (MeV)	Parameter	L						State
		9	10	11	12	13	14	
400	c_3 (MeV^{-5})	0.100	0.100	0.100	0.100	0.100	0.100	Input
350		0.170	0.162	0.164	0.164	0.163	0.163	Fitted
300		0.328	0.305	0.281	0.278	0.281	0.280	Fitted

Zhang et al., arXiv:2409.01325



- $E^{\text{excited}}_{\Lambda_1} - E^{\text{excited}}_{\Lambda_2}$ decreases
- ρ-type and λ-type excitation
- The standard angular momentum and parity projection technique is used

- $J^P = 1^-$ $\Leftarrow$

$$\left|\Psi_A\right\rangle = \frac{d_n}{24}\sum_{i=1}^{24}\chi_n\left(\Omega_i\right) R\left(\Omega_i\right)\left|\Psi_0\right\rangle$$

Lu et al., PRD90(2014)034507

Summary and outlook

- The number of particles is conserved
- Find a $J^P = 1^-$ DD^*K bound state with binding energy in the $(-84, -44)$ MeV region
- We checked the ren. group invariance
- Two energy levels of excited state are ρ-type and λ-type
- Lattice EFT has more advantage for many hadrons system

Thank you very much for your attention!