iTHEMS RIKEN

Doubly charmed tetraquark T_{cc}^{+} and left-hand cut from lattice QCD

Yan Lyu

iTHEMS, RIKEN

Oct. 18, 2024

Outline

- Introduction
- LQCD study on T_{cc}^{+}
- Left-hand cut of the scattering amplitude
- Summary and Outlook

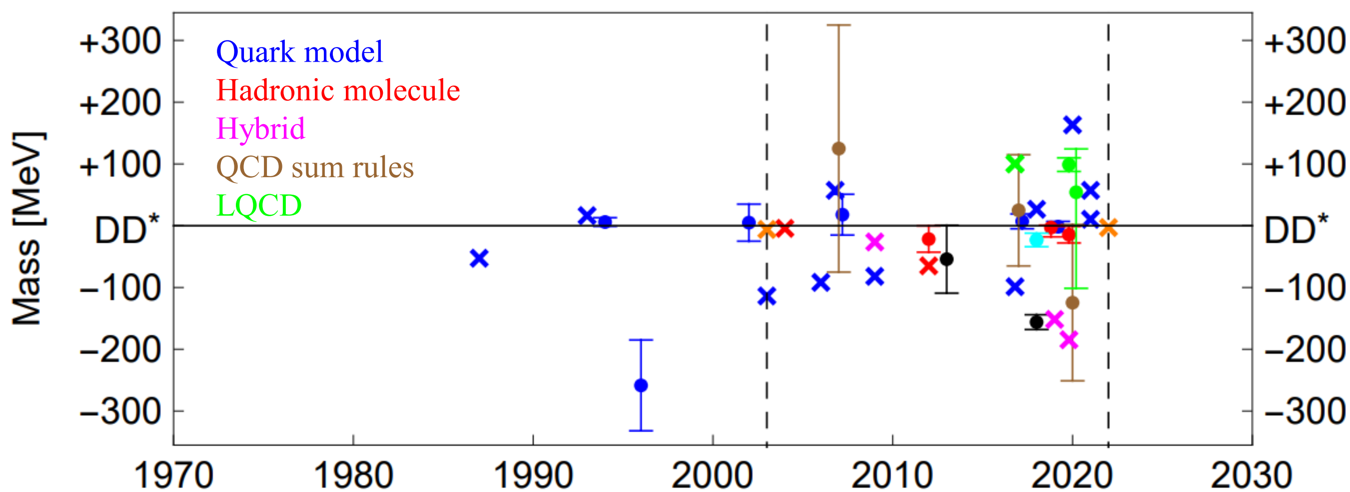
Doubly heavy exotics

- Intriguing aspects on $QQ\bar{q}\bar{q}'$
  - Open flavor, once observed its minimal quark content contains four quarks
  - Likely to be bound in the limit of $m_Q \to \infty$ ($QQ \sim \bar{Q}$)

A. Manohar and M. Wise, Nucl. Phys. B 339, 17 (1993)

$bb\bar{q}\bar{q}'$ (√)　　$cc\bar{q}\bar{q}'$ (?)　　$ss\bar{q}\bar{q}'$ (×)

- A long history of theoretical prediction on $cc\bar{u}\bar{d}$ $(IJ^P = 01^+)$



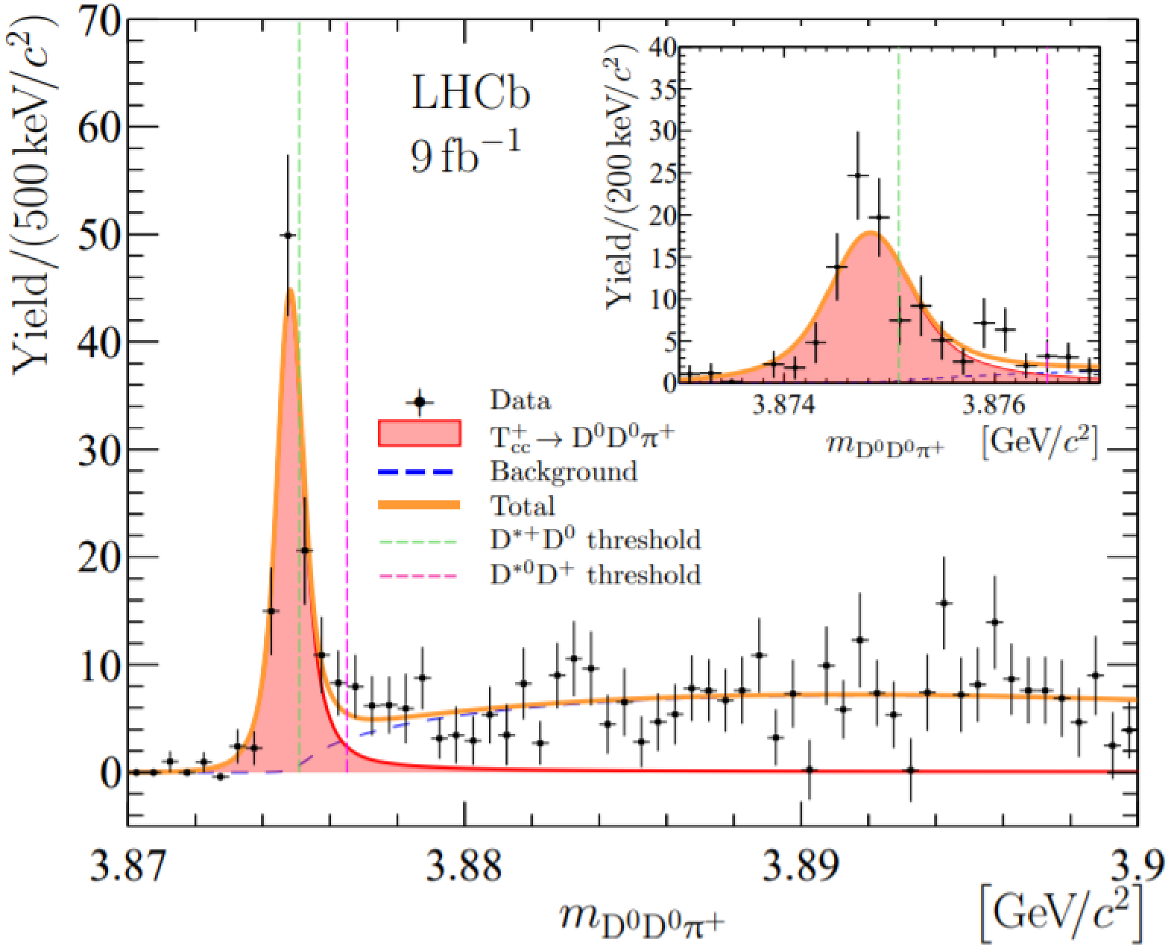
H.-X. Chen *et al.*, Rept. Prog. Phys. 86, 026201 (2023)

  - Mass varies within ± 300 MeV with respect to $m_{D^*} + m_D$

First doubly charmed tetraquark T_{cc}^{+}

- 2022, LHCb discovered T_{cc}^{+} in the $D^0D^0\pi^+$ spectrum

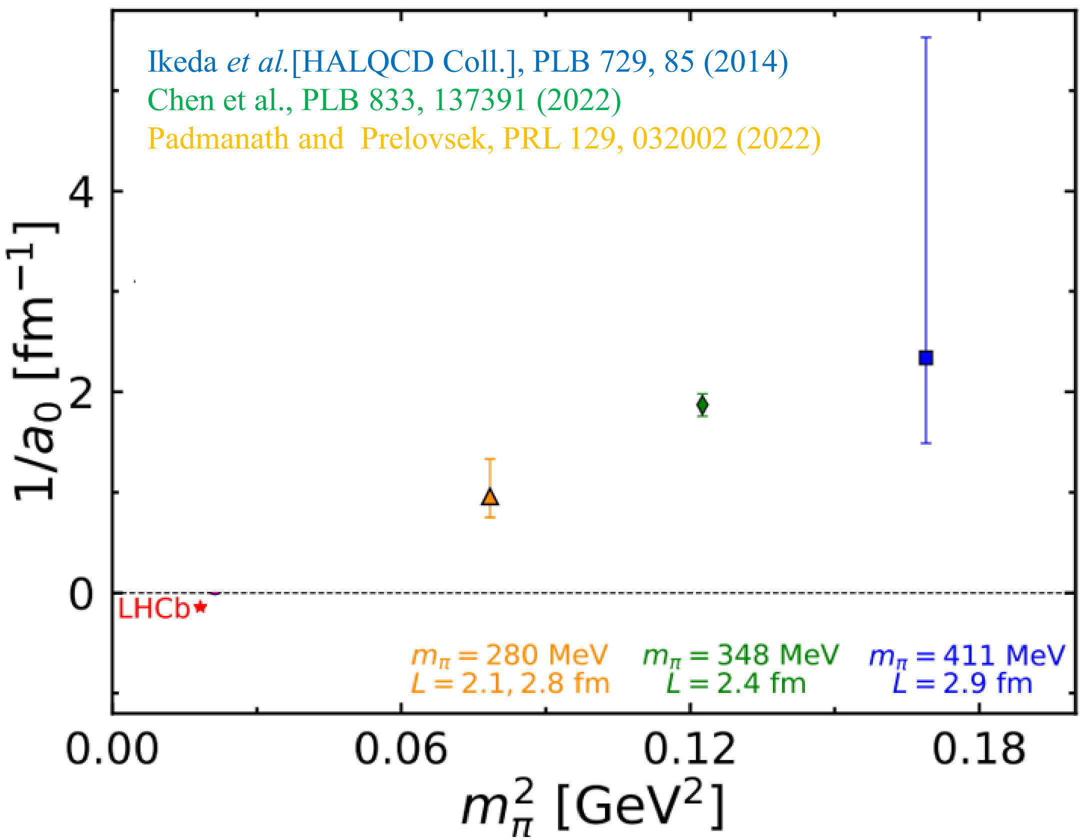
LHCb Coll., Nature Phys. 18, 751 (2022); Nature Comm. 13, 3351 (2022)

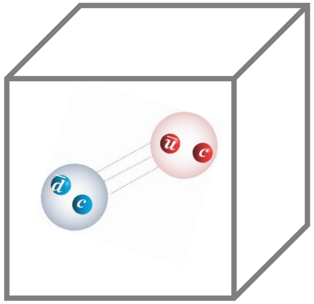


$I(J^P)$	δm_{pole}	Γ_{pole}	$\text{Re}(a_0)$	$\text{Im}(a_0)$
$0(1^+)$	-360±40 keV	48±2 keV	7.16 ± 0.51 fm	-1.85 ± 0.28 fm

T_{cc}^+ from first-principle lattice QCD

- Limited to heavy quark masses ($m_\pi \geq 280$ MeV)

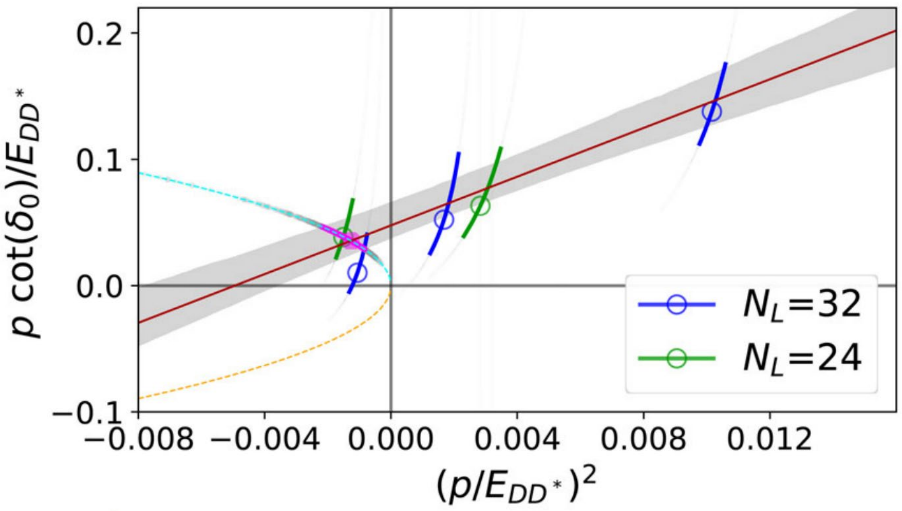


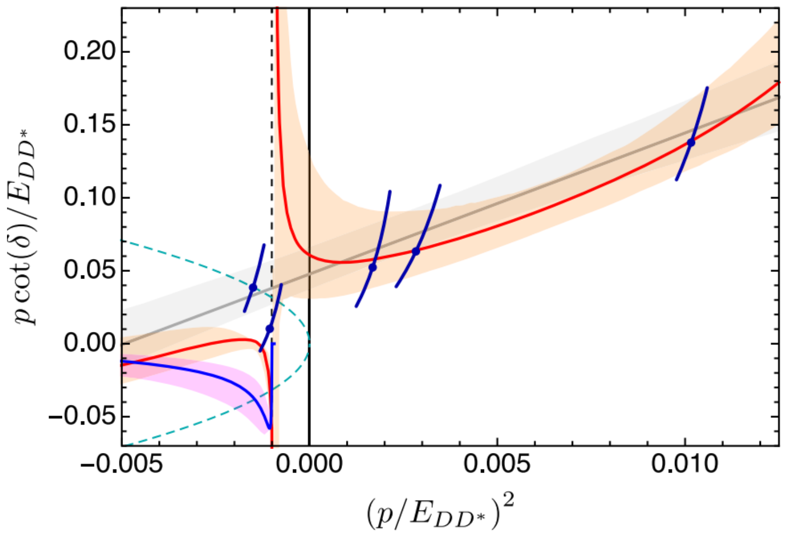


- A huge gap between experimental and lattice results due to unphysical pion mass used in the studies

Left-hand cut

- The left-hand cut invalidates the analysis





M.-L. Du, A. Filin, V. Baru, X.-K. Dong, E. Epelbaum, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves, and Q. Wang, Phys. Rev. Lett. 131, 131903 (2023)

- Standard Lüscher quantization condition fails
- Effective-range expansion fails

- Modified finite-volume formula

A. Raposo and M. Hansen, JHEP 08, 075 (2024)

- New effective-range expansion → M.-L. Du's talk

M.-L. Du, F.-K. Guo, and B. Wu, arXiv: 2408.09375

The purpose of this talk

- Bridge the gap between lattice and experimental data
  - What does T_{cc}^{+} look like if m_π^{lat} down to just a few MeV above m_π^{phy}?
  - How far/close are we from explaining/confirming the experimental results?

- Revisit the left-hand cut
  - What is the origin?
  - Another remedy

[1] YL, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng, PRL 131, 161901 (2023)

[2] YL, S. Aoki, T. Doi, T. Hatsuda, W. Yamada *et al*, In preparation

Outline

- Introduction
- LQCD study on T_{cc}^{+}
- Left-hand cut of the scattering amplitude
- Summary and Outlook

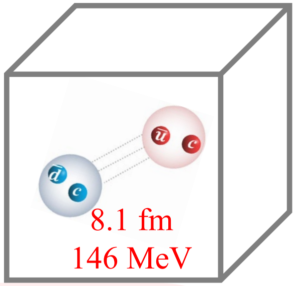
(2+1) flavor lattice QCD at m_π =146 MeV

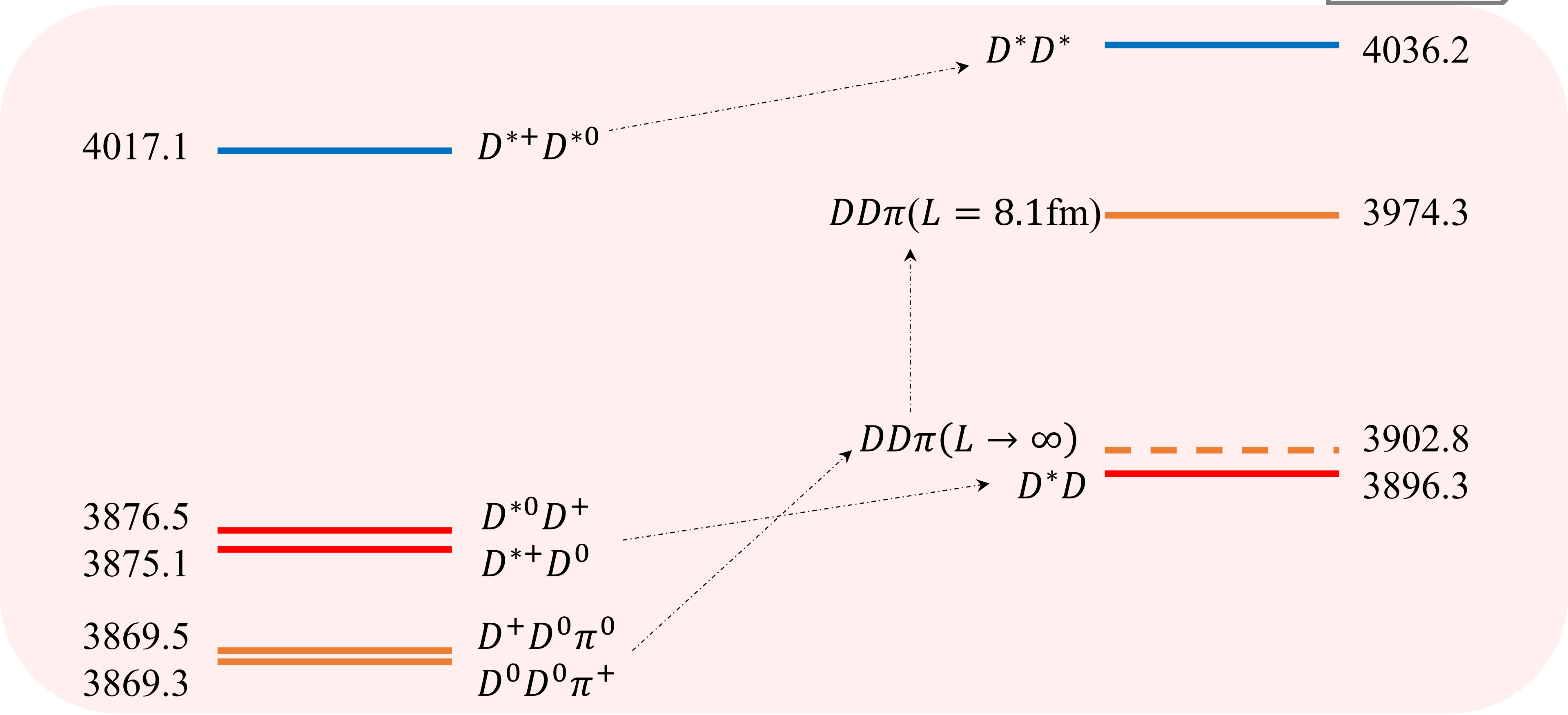
Nature

$\pi^0(134.98)$ $\pi^+(139.57)$
$D^0(1864.84)$ $D^+(1869.66)$
$D^{*0}(2006.85)$ $D^{*+}(2010.26)$

Lattice

$\pi(146.4)$
$D(1878.2)$
$D^*(2018.1)$

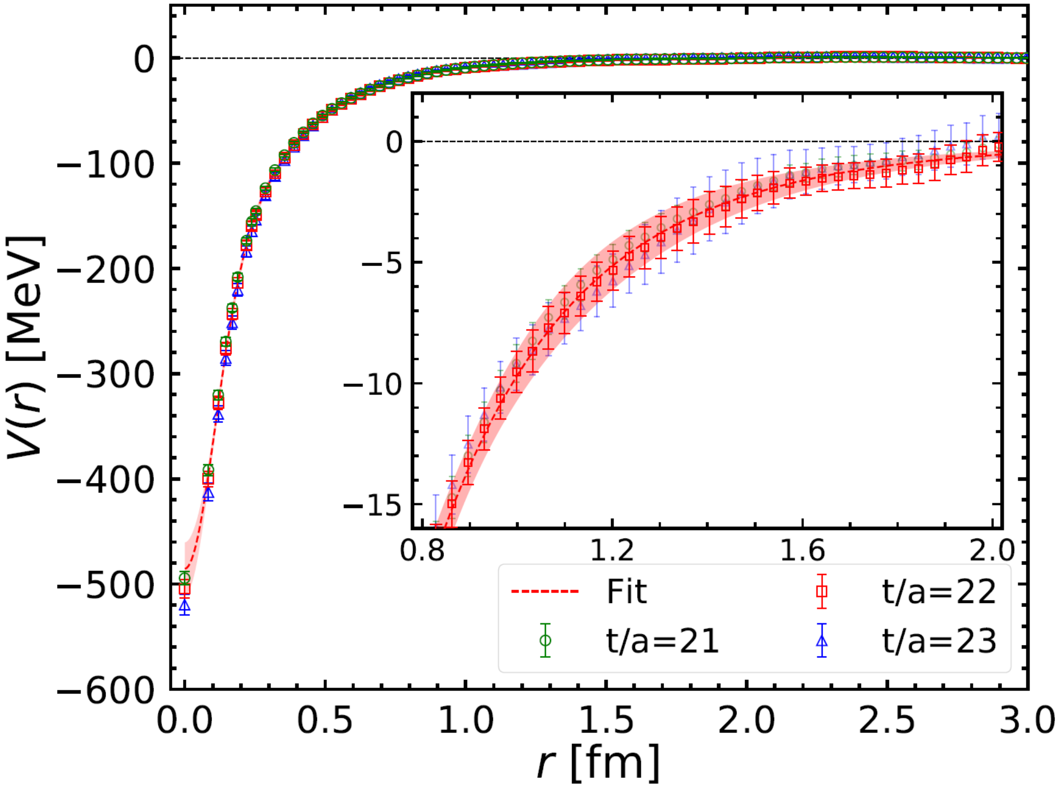




- The lowest energy level of $DD\pi$ (D^*D^*) is around 78 (140) MeV above on the lattice

D^*D interaction from HAL QCD method

- D^*D potential in the $(I, J^P) = (0, 1^+)$ channel ($t \simeq 1.9$ fm)



- Short range: antidiquark-diquark $[\bar{u}\bar{d}]_{3_c, I=J=0}[cc]_{\bar{3}_c, J=1}$

M. Karliner and H. Lipkin, arXiv: 0307243

R. Jaffe and F. Wilczek, Phys. Rev. Lett. 91 232003 (2003)

- Long range: attraction from pion-exchange interaction

Long-range potential

- One-pion exchange

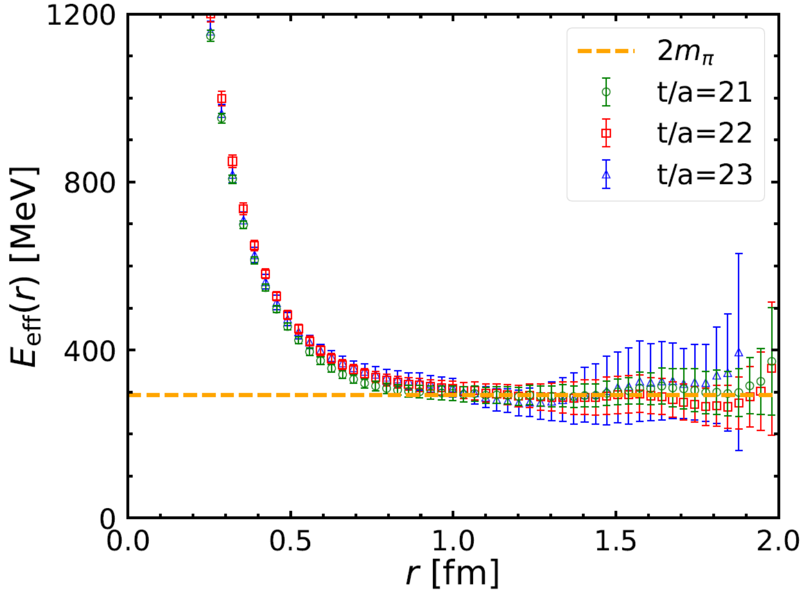
S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh, and A. Hosaka, Phys. Rev. D 86, 034019 (2012)
Ning Li, Zhi-Feng Sun, Xiang Liu, and Shi-Lin Zhu, Phys. Rev. D 88, 114008 (2013)

$$V(r) = -\alpha \frac{e^{-\mu r}}{r}, \quad \mu = m_\pi \text{ or } \sqrt{m_\pi^2 - (m_{D^*} - m_D)^2}$$

- Fail to describe long-range potential (why?)

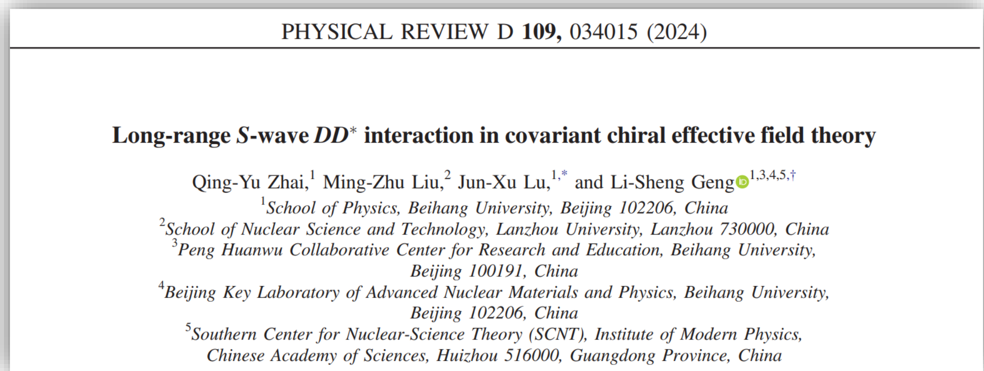
- Two-pion exchange

$$V(r) = -\alpha \frac{e^{-2m_\pi r}}{r^2} \quad \Rightarrow \quad E_{\text{eff}}(r) = -\frac{\ln\left[-V(r)r^2/\alpha\right]}{r}$$



- Long range potential ($1 \leq r \leq 2$ fm) is consistent with two-pion exchange

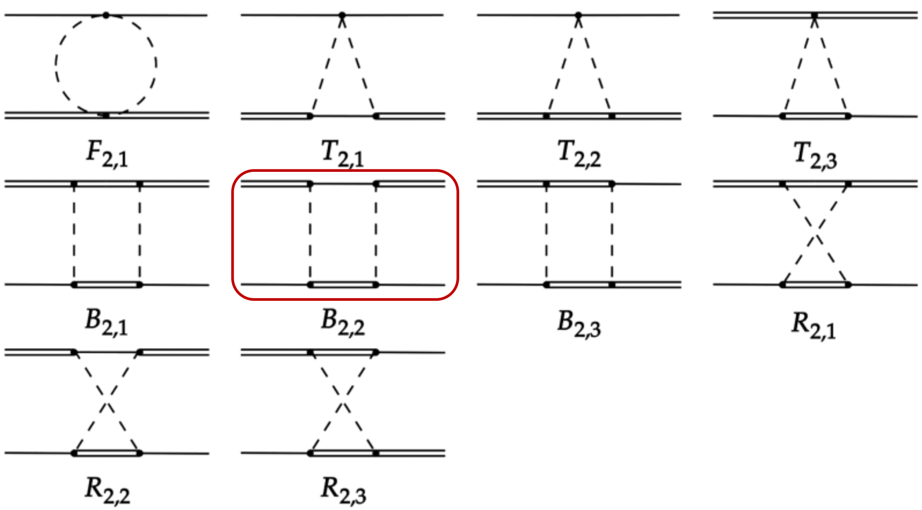
An explanation based on covariant chiral EFT


PHYSICAL REVIEW D **109**, 034015 (2024)

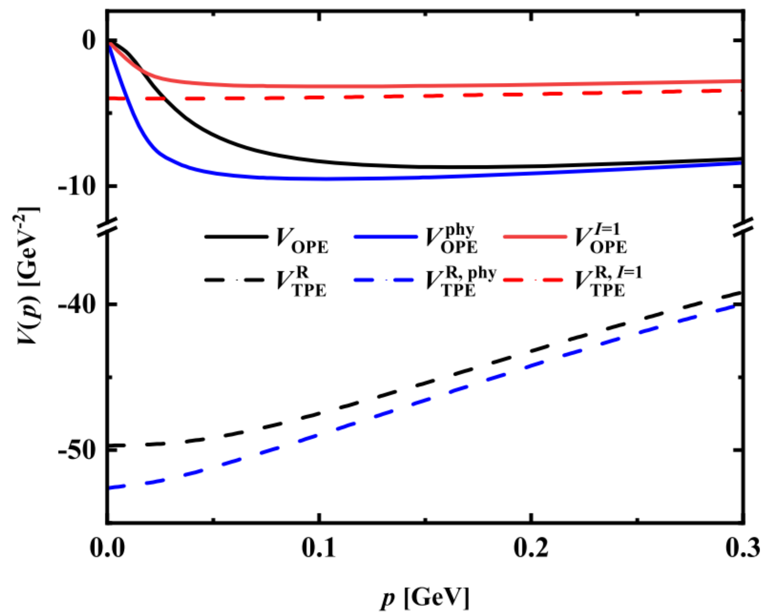
Long-range *S*-wave DD^* interaction in covariant chiral effective field theory

Qing-Yu Zhai,[1] Ming-Zhu Liu,[2] Jun-Xu Lu,[1,*] and Li-Sheng Geng[1,3,4,5,†]
[1]*School of Physics, Beihang University, Beijing 102206, China*
[2]*School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China*
[3]*Peng Huanwu Collaborative Center for Research and Education, Beihang University, Beijing 100191, China*
[4]*Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 102206, China*
[5]*Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, Guangdong Province, China*

- TPE diagrams @NLO



- TPE vs OPE



- Box diagram $B_{2,2}$ play a dominate role due to 4 propagators are almost on-shell
- TPE is much strong than OPE around $p \simeq 0$

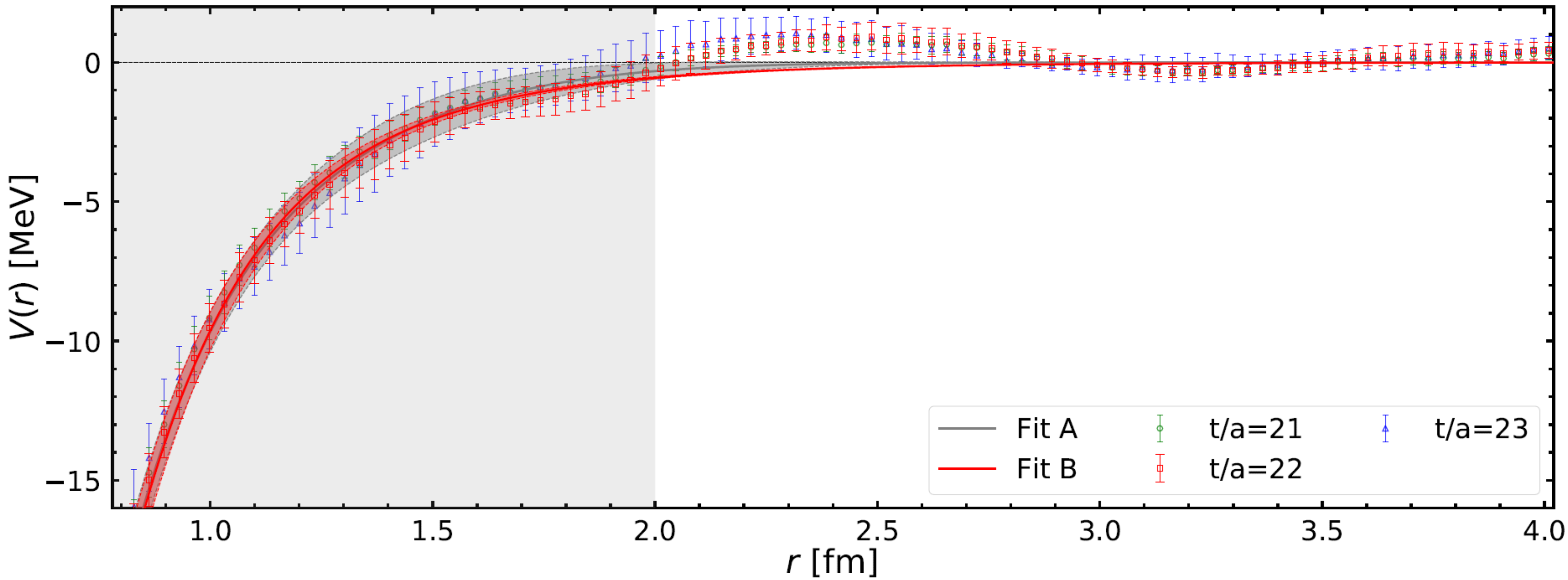
Fit

- Fit A: purely phenomenological fit ($\chi^2/\text{dof} = 1.01$)

$$V_{\text{fit}}(r) = \sum_{i=1,\cdots,4} a_i e^{-(r/b_i)^2}$$

- Fit B: TPE-motivated fit ($\chi^2/\text{dof} = 0.96$)

$$V_{\text{fit}}(r; m_\pi) = \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 (1 - e^{-(r/b_3)^2})^2 \frac{e^{-2m_\pi r}}{r^2}$$

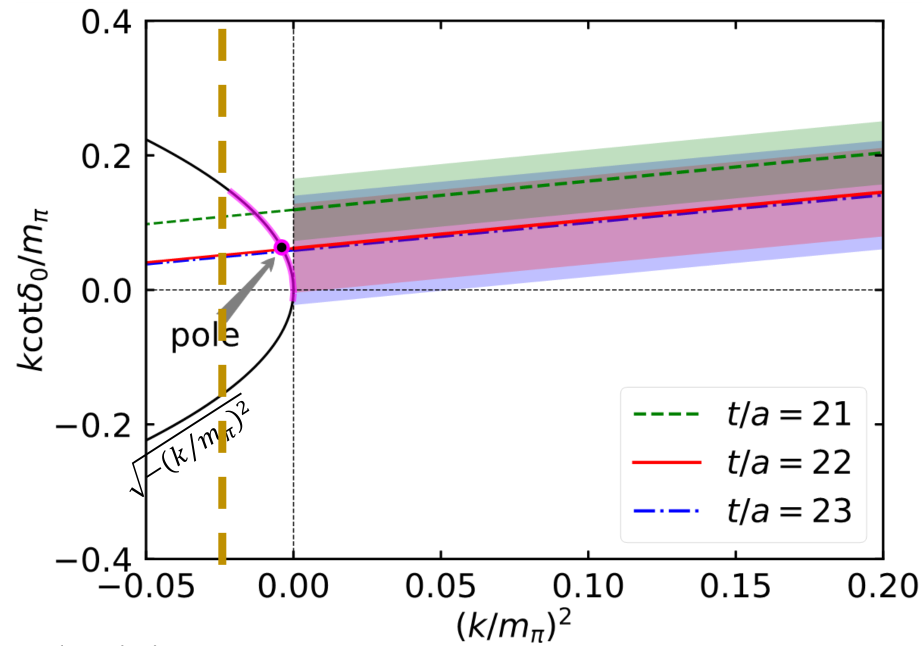


Scattering properties

- Scattering phase shift
  - ERE expansion

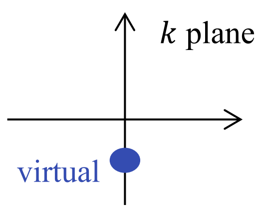
$$S(k) = \frac{k\cot\delta_0 + ik}{k\cot\delta_0 - ik}$$

$$k\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2} r_{\mathrm{eff}} k^2$$



- Scattering parameters and pole singularities

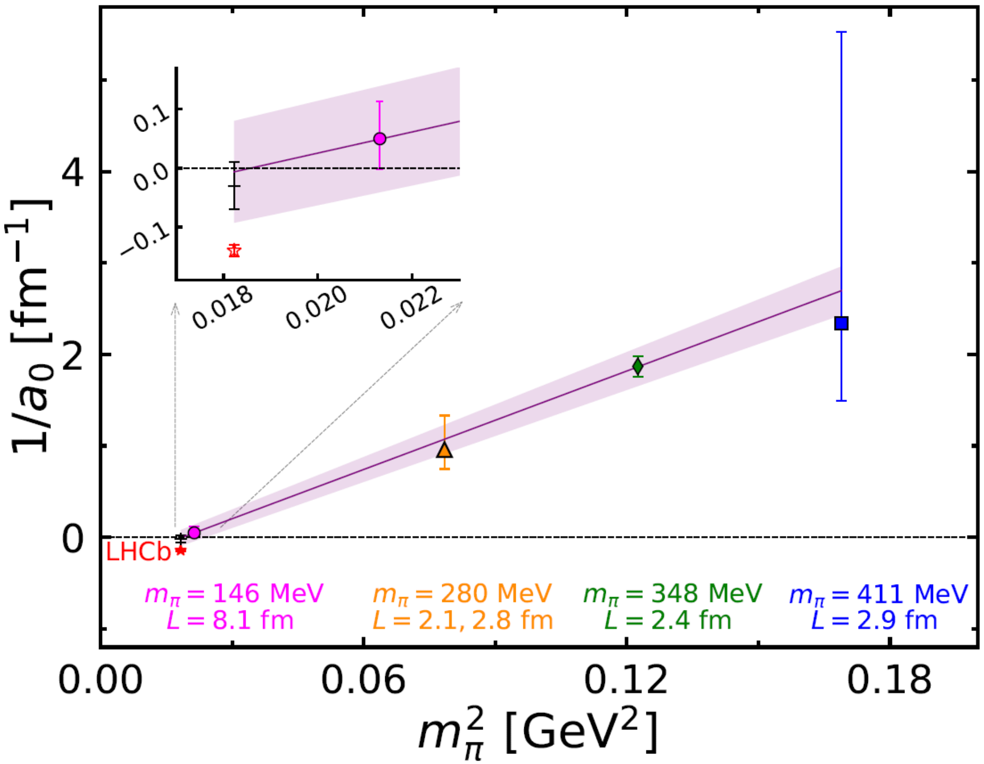
m_π (MeV)	146.4
$1/a_0$ (fm^{-1})	$0.05(5)(^{+2}_{-2})$
r_{eff} (fm)	$1.12(3)(^{+3}_{-8})$
$k = i\kappa_{\mathrm{pole}}$ κ_{pole} (MeV)	$-8(8)(^{+3}_{-5})$
E_{pole} (keV)	$-59(^{+53}_{-99})(^{+2}_{-67})$



- T_{cc}^{+} appears as a near-threshold virtual state at $m_\pi = 146.4$ MeV
- The pole position is above the possible left-hand cut singularity

Comparison

- $1/a_0$



Ikeda *et al.*[HALQCD Coll.], Phys. Lett. B 729, 85 (2014)

Chen et al., Phys. Lett. B 833, 137391 (2022)

Padmanath and Prelovsek, Phys. Rev. Lett. 129, 032002 (2022)

- $1/a_0$ from current study with $m_\pi = 146$ MeV is extremely close to LHCb data
- As m_π decreases, LQCD results approach to the LHCb data

Extrapolate to physical point based on TPE

➢ Extrapolation

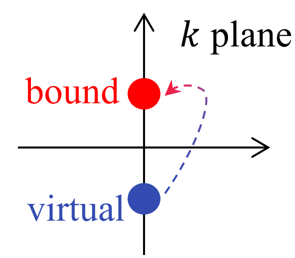
- Extrapolate TPE interaction to physical point

$$V_{\mathrm{fit}}(r; m_\pi = 146 \to 135\ \mathrm{MeV})$$

- Adopt physical values for $m_{D^{*+}}$ and m_{D^0}
- Do NOT consider isospin breaking nor opening of $DD\pi$ channel

➢ Scattering parameters and pole singularities

m_π (MeV)	146.4	135.0
$1/a_0$ (fm^{-1})	$0.05(5)(^{+2}_{-2})$	$-0.03(4)$
r_{eff} (fm)	$1.12(3)(^{+3}_{-8})$	$1.12(3)$
$k = i\kappa_{\mathrm{pole}}$ κ_{pole} (MeV)	$-8(8)(^{+3}_{-5})$	$+5(8)$
E_{pole} (keV)	$-59(^{+53}_{-99})(^{+2}_{-67})$	$-45(^{+41}_{-78})$



- $m_\pi = 146 \to 135$ MeV, T_{cc}^+ evolves from a near-threshold virtual state into a loosely bound state

Construction of $D^0 D^0 \pi^+$ spetrum

- Production amplitude of $D^{*+}D^0$ from a source function P

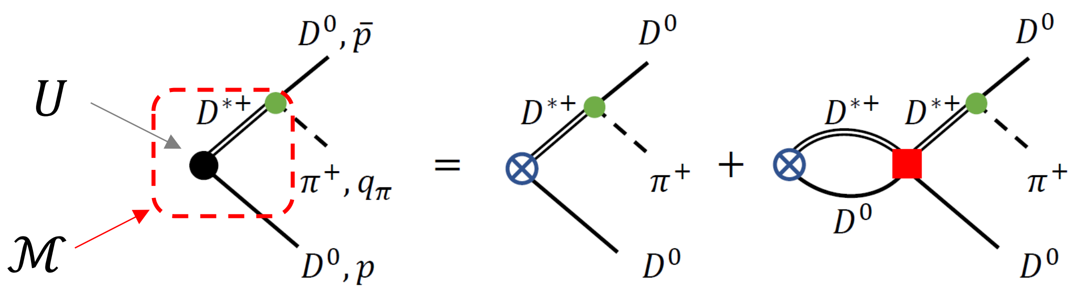
$$U(M,p) = P + \int \frac{d^3q}{(2\pi)^3} T(M,p,q) G(M,q) P$$


PHYSICAL REVIEW D **105,** 014024 (2022)

Coupled-channel approach to T_{cc}^+ including three-body effects

Meng-Lin Du,[1,*] Vadim Baru,[2,3,†] Xiang-Kun Dong,[4,5,‡] Arseniy Filin,[2] Feng-Kun Guo,[4,5,§] Christoph Hanhart,[6,∥] Alexey Nefediev,[7,8,¶] Juan Nieves,[1,**] and Qian Wang[9,10,11,††]

- For simplicity, consider a pointlike source (constant in p-space, $P = \mathcal{N}$)
- Only S-wave production at low energies



- Adopt experimental values for m_{D^{*+},D^0,π^+} and $\Gamma_{D^{*+}}$ in the kinematics to keep the same phase space with the experiment

- Three-body mass spectrum for $D^0 D^0 \pi^+$

$$\mathcal{M}(U \to D^0 D^0 \pi^+) = U(M,p)G(M,p)q_\pi + U(M,\bar{p})G(M,\bar{p})\bar{q}_\pi$$

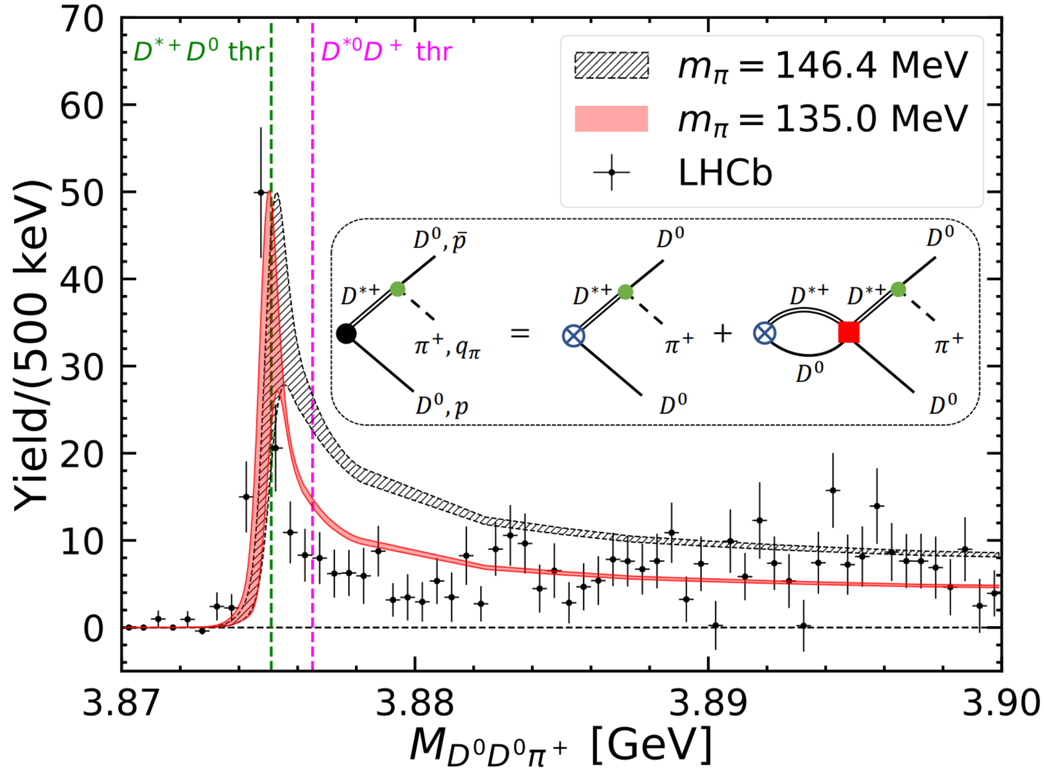
$$\frac{d\mathrm{Br}}{dM} = \mathcal{N}' \int_0^{p_{\max}} p dp \int_{\bar{p}_{\min}}^{\bar{p}_{\max}} \bar{p} d\bar{p} |\mathcal{M}(U \to D^0 D^0 \pi^+)|^2$$

- A known energy resolution function needs to considered for comparison w/ exp. data

LHCb Coll., Nature Comm. 13, 3351 (2022)

$D^0 D^0 \pi^+$ spetrum

➢ Results at different m_π



- A peak around $D^{*+}D^0$ threshold
- m_π =146 MeV → 135 MeV, peak position shifts to the left, better description to LHCb data

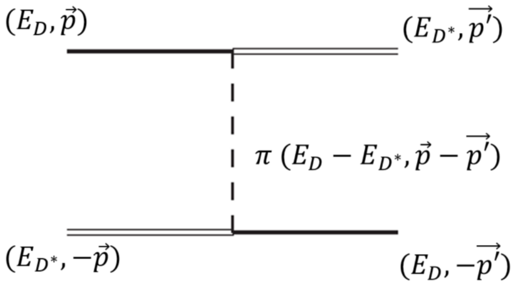
Outline

- Introduction
- LQCD study on T_{cc}^{+}
- Left-hand cut of the scattering amplitude
- Summary and Discussion

Left-hand cut from momentum space

- One-pion exchange between D and D^*

$$V(\boldsymbol{p}', \boldsymbol{p}) = \frac{g^2}{(E_{D^*} - E_D)^2 - (\boldsymbol{p}' - \boldsymbol{p})^2 - m_\pi^2} = \frac{-g^2}{(\boldsymbol{p}' - \boldsymbol{p})^2 + \mu^2},$$



  - The effective mass is defined as $\mu^2 = m_\pi^2 - (E_{D^*} - E_D)^2 \simeq m_\pi^2 - (m_{D^*} - m_D)^2$
  - A pole appears when $(\boldsymbol{p}' - \boldsymbol{p})^2 = -\mu^2$

- Partial-wave projection

$$V_s(p) = \frac{1}{2}\int_0^\pi d\theta \frac{-g^2}{(\boldsymbol{p}' - \boldsymbol{p})^2 + \mu^2} P_0(\cos\theta) = \frac{g^2}{4p^2}\int_{-1}^{1} dx \frac{1}{x - (1 + \mu^2/2p^2)} = -\frac{g^2}{4p^2}\log\left(1 + \frac{4p^2}{\mu^2}\right)$$

  - A branch cut appears along $p^2 < -\mu^2/4$ in $V_s(p)$
  - The scattering matrix inherits the cut from the potential

$$T(p', p; E) = V(p', p) + \int \frac{d^3q}{(2\pi)^3} V(p', q) G(E, q) T(q, p; E)$$

Regular solution and S matrix

- S-wave Schrödinger equation

$$\left[\frac{d^2}{dr^2} - U(r) + k^2\right]\varphi(k,r) = 0$$

  - The regular solution is obtained w/ b.c. $\varphi(k, r=0) = 0,\ \frac{d}{dr}\varphi(k, r=0) = 1$

- The integral equation for the regular solution $\varphi(k, r)$

$$\varphi(k,r) = k^{-1}\sin kr + k^{-1}\int_0^r dr' \sin k(r-r')U(r')\varphi(k,r')$$
$$= \frac{1}{2ik}[\mathcal{F}(-k,r)e^{ikr} - \mathcal{F}(k,r)e^{-ikr}]$$

  - where $\mathcal{F}(k,r) = 1 + \int_0^r dr' e^{ikr'}U(r')\varphi(k,r')$

- The scattering matrix is defined from the asymptotic behavior of $\varphi(k, r)$

$$\varphi(k,r) \xrightarrow{r\to\infty} ae^{ikr} - be^{-ikr}$$
$$S(k) \equiv \frac{a}{b} = \frac{\mathcal{F}(-k, r=\infty)}{\mathcal{F}(k, r=\infty)}$$

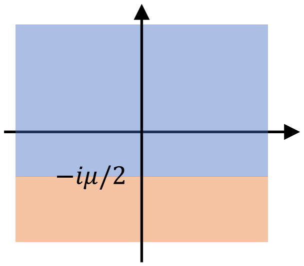
Left-hand cut from coordinate space

- One-pion exchange leads to a Yukawa potential in coordinate space

$$V(\boldsymbol{r}) = \int \frac{d^3q}{(2\pi)^3} \frac{-g^2}{\boldsymbol{q}^2 + \mu^2} = -\frac{g^2}{4\pi} \frac{e^{-\mu r}}{r}$$

- Asymptotic behavior of $\mathcal{F}(k, r)$ w/ Yukawa potential

$$\lim_{r\to\infty} \mathcal{F}(k,r) = 1 + \lim_{r\to\infty} \int_0^r dr' e^{ikr'} U(r') \varphi(k, r')$$
$$\propto \int^{\infty} dr' e^{(ik+|\mathrm{Im}k|)r'} \frac{e^{-\mu r}}{r}$$



  - The integration is divergent when $\mathrm{Im}k \leq -\mu/2$ due to *integration to infinity*
  - It can be further shown $\mathcal{F}(k, \infty)$ has a branch cut starting from $-i\mu/2$ to $-\infty$

- Left-hand cut for scattering matrix

$$S(k) = \frac{\mathcal{F}(-k, \infty)}{\mathcal{F}(k, \infty)}$$

  - A branch cut $k^2 \leq -\mu^2/4$, which is *dictated by the infinitely long-range potential*

Scattering matrix from cutoff potential

- Given that the left-hand cut comes from the infinitely long-range potential, what if the Yukawa potential is truncated at arbitrarily large R

$$V_{\text{cut}}(r) = \begin{cases} V(r), & r < R \\ 0, & r > R \end{cases}$$

  - Then, left-hand cut singularity disappears

$$\varphi(k,r) \xrightarrow{r\to\infty} \frac{1}{2ik}[\mathcal{F}(-k,R)e^{ikr} - \mathcal{F}(k,R)e^{-ikr}], \quad S(k) = \frac{\mathcal{F}(-k,R)}{\mathcal{F}(k,R)}$$

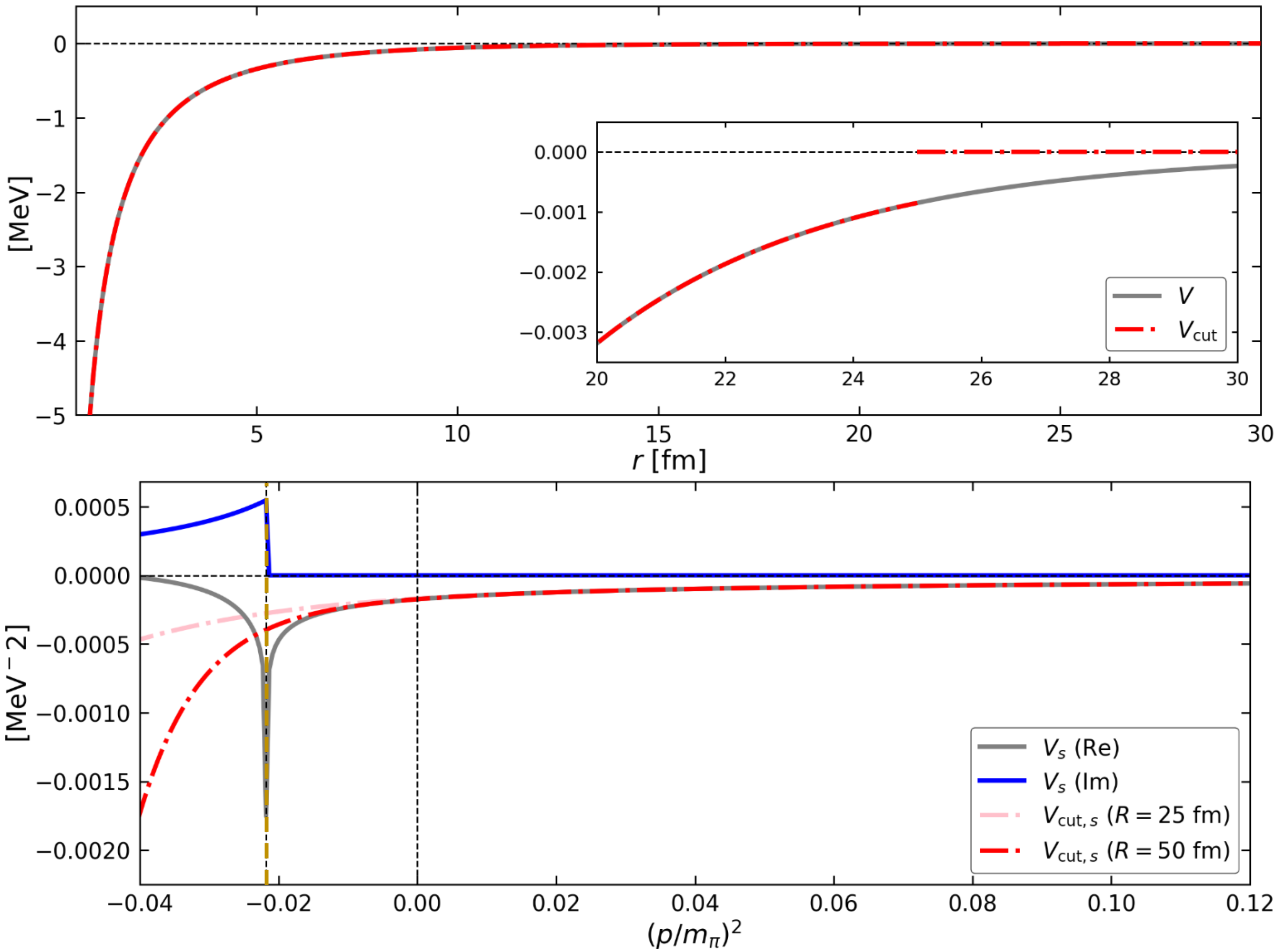
- In momentum space, it means the following modified pion propagator

$$V_{\text{cut}}(\boldsymbol{q}) = \iint_0^R V(\boldsymbol{r})e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} r^2 dr d\Omega = -\frac{g^2}{\boldsymbol{q}^2+\mu^2}\mathcal{G}(q;R)$$

$$\mathcal{G}(q;R) = 1 - e^{-\mu R}\left[\cos(qR) + \sin(qR)\frac{\mu}{q}\right]$$

  - $\mathcal{G}(q;R)$ has zeros at $q^2 = -\mu^2$, such that V_{cut} *does not have poles*, and therefore partial-wave projection does not lead to the branch cut

Cutoff potential

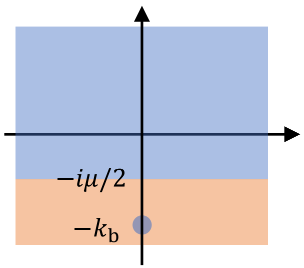


- $V_{\mathrm{cut},s} - V_s$ can be arbitrarily small as R increases for $p^2 > -\mu^2/4$
- $V_{\mathrm{cut},s}$ is real even for $p^2 < -\mu^2/4$, no left-hand cut

$g = 0.57$

- The regular solution has the following asymptotic behavior
  - Fore real k, $\varphi(k,r)$ oscillates at long range $\sim\sin(kr+\delta)$
  - For imaginary k, $\varphi(k,r)$ at long range exponentially increases, $\sim -\frac{f(k)}{2ik} e^{|\mathrm{Im}k|r}$
  - At binding k_b, $\varphi(k_\mathrm{b},r)$ at long range exponentially decreases, $\sim$ Const $\times\, e^{-|k_b|r}$
- $\mathcal{F}(k,R)$ as $R\to\infty$,

$$\begin{aligned}\mathcal{F}(k,R) &= 1 + \int_0^R dr' e^{ikr'} U(r')\varphi(k,r') \\ &= \mathcal{F}(k,\tilde{R}) + \int_{\tilde{R}}^R dr' e^{ikr'} U(r')\varphi(k,r') \\ &\simeq \mathcal{F}(k,\tilde{R}) - \frac{f(k)}{2ik}\int_{\tilde{R}}^R dr' e^{(ik+|\mathrm{Im}k|)r'}\frac{e^{-\mu r'}}{r'}\end{aligned}$$



  - $\mathcal{F}(k,R)$ is convergent (divergent) as $R\to\infty$ if $\mathrm{Im}k > -m/2$ ($< -m/2$)
  - At binding k_b, even if $\mathrm{Im}k_\mathrm{b} > m/2$, $\mathcal{F}(-k_\mathrm{b},R)$ is always convergent
- For scattering matrix, as $R\to\infty$,
  - $S(k)$ is convergent (divergent), if $k^2 > -\mu^2/4$ ($< -\mu^2/4$)
  - For the bound state, the pole position from $S(k)$ is convergent

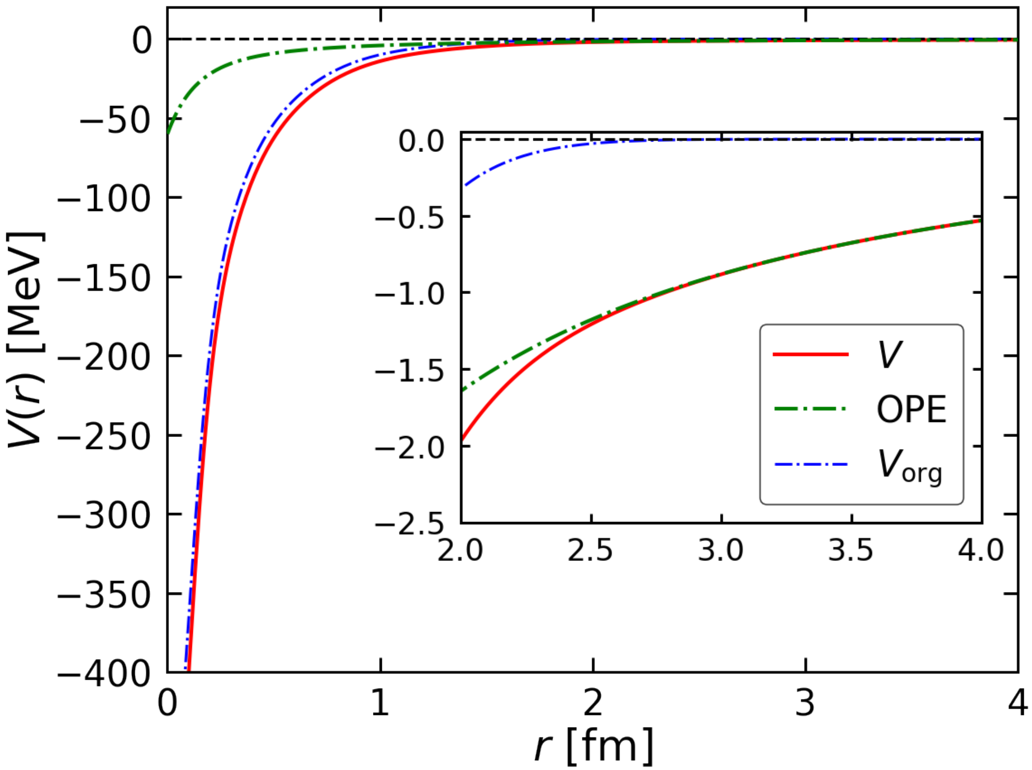
$$S(k) = \frac{\mathcal{F}(-k,R)}{\mathcal{F}(k,R)}$$

Case study

➢ D^*-D scattering with the following interaction

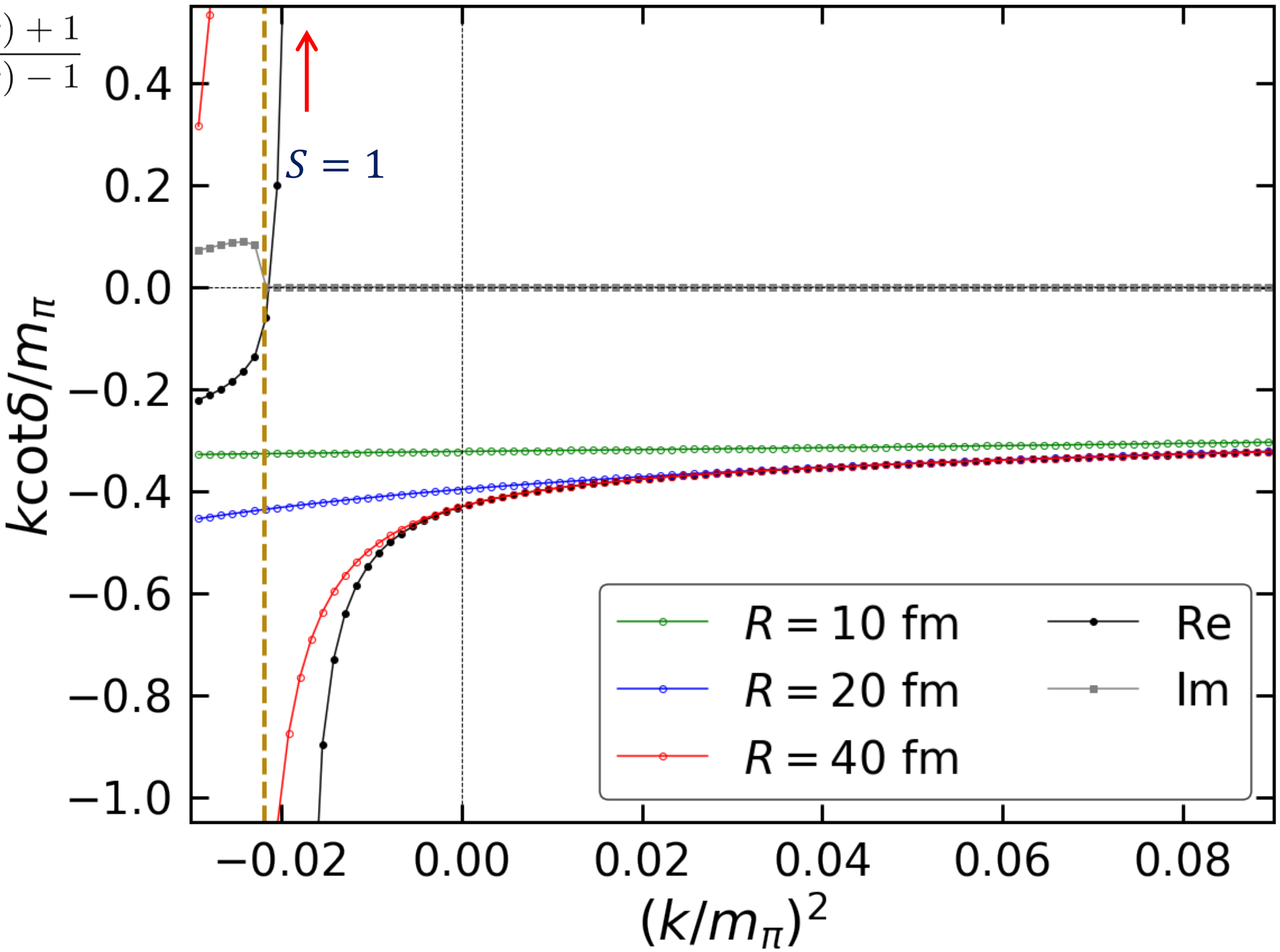
$$V = V_{\rm org} + V_{\rm OPE}$$

- The long-range part of the interaction is $\sim \frac{g^2}{4\pi}\frac{e^{-\mu r}}{r}$ with $\mu \simeq 43$ MeV
- The interaction strength is tuned to have a bound state (on the left-hand cut)



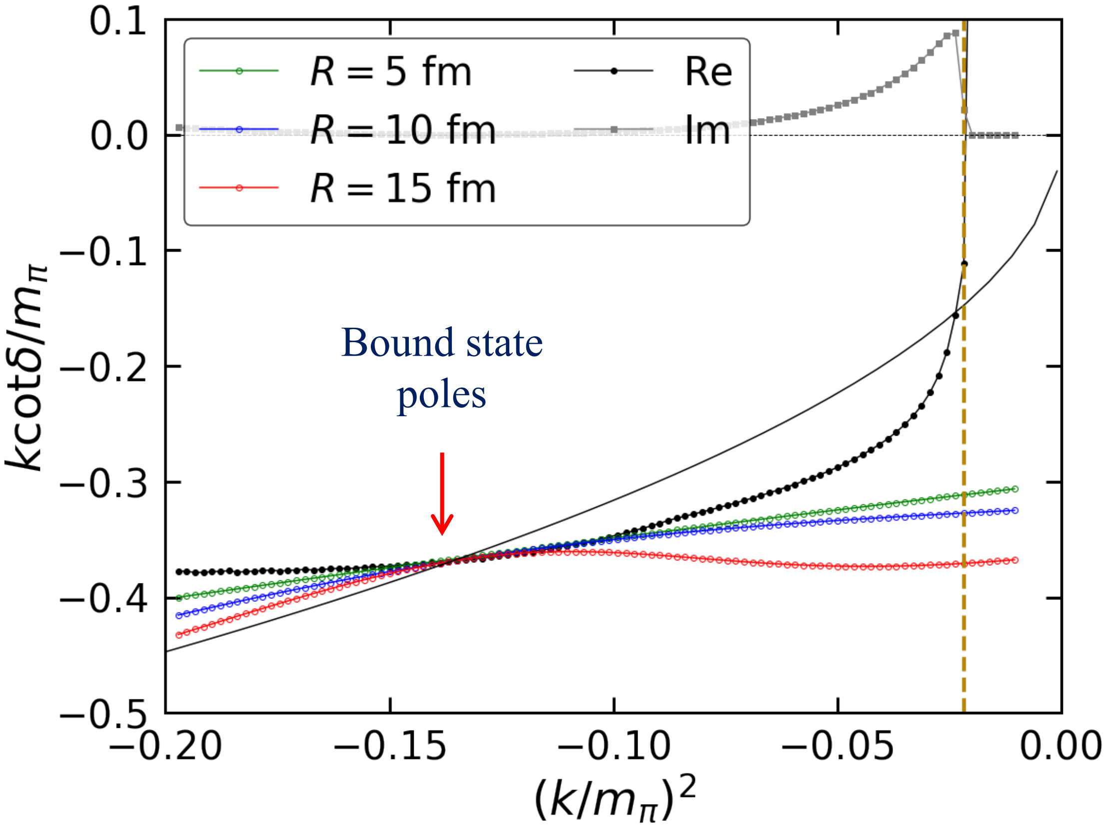
Scattering phase shifts from the cutoff potential

$$k \cot \delta = ik \frac{S(k)+1}{S(k)-1}$$



- The difference on phase shifts of cutoff potential and original potential can be *arbitrarily small*
- The scattering phase shifts from cutoff potential is aways *real, no left-hand cut*

The bound state pole from the cutoff potential



R [fm]	E_{pole} [MeV]
5	-1.476963
10	-1.498553
15	-1.498833
∞	**-1.498837**

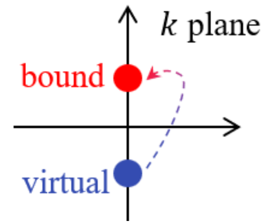
- The difference on bound state pole position of cutoff potential and original potential can be *arbitrarily small*
- The S matrix tends to be divergent as $R \to \infty$ $\quad k\cot\delta = ik\frac{S(k)+1}{S(k)-1}$

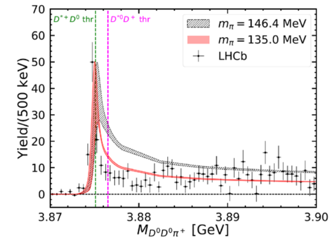
Outline

- Introduction
- LQCD study on T_{cc}^{+}
- Left-hand cut of the scattering amplitude
- Summary and Discussion

Summary and Discussion

- Lattice study on T_{cc}^{+} with almost physical quark masses
  - T_{cc}^{+} appears a near-threshold virtual state
  - T_{cc}^{+} evolves into a loosely bound state as m_π =146→135 MeV
  - $1/a_0$ is extremely close to the experimental data
  - LHCb $D^0D^0\pi^+$ spectrum can be explained semiquantitatively





- Left-hand cut singularity
  - originated from the infinitely long-range interaction
  - disappears once the long-range potential is truncated
  - effects on physical observables from cutoff potential can be arbitrarily small

Summary and Discussion

- From the study of the cutoff potential, it may indicate
  - in order to get correct physical observables, lattice box should be large enough to include important interactions
  - as long as lattice box is large enough, one can obtain correct phase shift and bound state pole even without considering left-hand cut

- In practice, it is also important to explicitly obtain/check the long-range behavior of the interaction which leads to left-hand cut singularity

Thanks for your attention!