Analysis of bound and resonant states of doubly heavy tetraquarks as hadronic molecules

M Sakai, Y Yamaguchi, Phys. Rev. D **109**, 054016, on progress

Manato Sakai

in collaboration with Yasuhiro Yamaguchi

Nagoya University

October 17, 2024

Hadron and Exotic hadron

- Hadron: Composite particle of quarks and gluons
  Type of hadrons
  - **meson**$\sim q\bar{q}$ (e.g. $\pi, \rho, \omega, \sigma$)
  - **baryon**$\sim qqq$ (e.g. proton, neutron)

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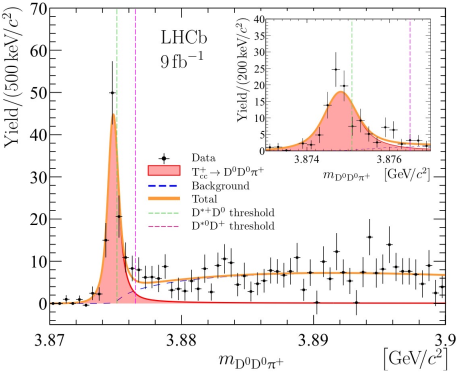
  Many exotic hadrons are reported by the some experiments.
  - X, Y, Z N. Brambilla Eur.Phys.J.C **71** (2011)1534
    $X(3872)$
    reported in 2003, $c\bar{c}q\bar{q}$ tetraquark

    Belle Collaboration S.K. Choi et al Phys.Rev.Lett **91** (2003) 262001
  - $P_c(4312), P_c(4440), P_c(4457)$R. Aaji, et al. PRL112(2019)222001
    reported in 2019, $c\bar{c}qqq$ pentaquarks
  - $T_{cc}(cc\bar{u}\bar{d})^+$ LHCb, Nature Phys. 18 (2022) 751-754, Nature Commun. 13 (2022) 3351
    reported in 2022, $cc\bar{u}\bar{d}$ tetraquark
    We analyze T_{cc} in our work.

$T_{cc}(cc\bar{u}\bar{d})^+$



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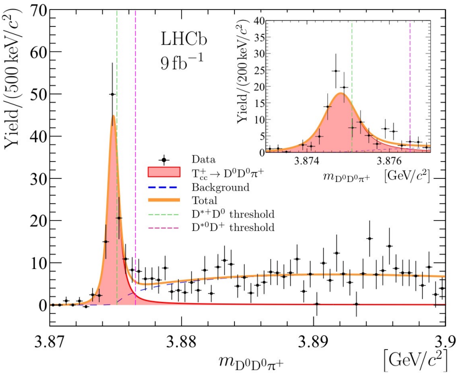

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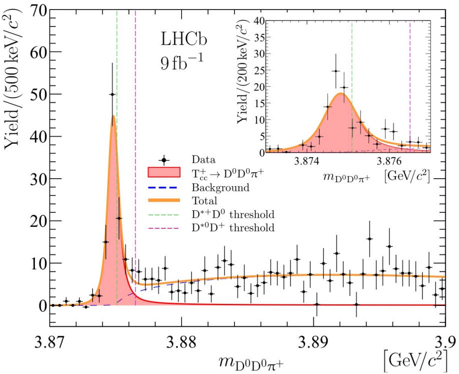

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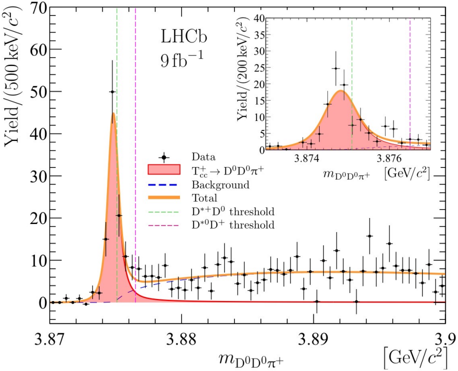

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- $cc\bar{u}\bar{d}$ tetraquark
- $J^P = 1^+$
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- Mass differences from the $D^{*+}D^0$ threshold
  - Breit-Wigner fitting $\delta m_{\rm BW} = -273 \pm 61\,\mathrm{keV}/c^2$
  - pole $\delta m_{\rm pole} = -360 \pm 40\,\mathrm{keV}/c^2$

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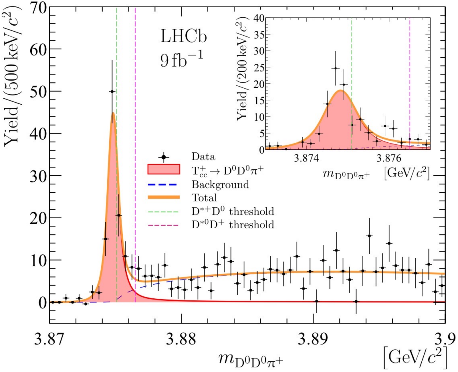

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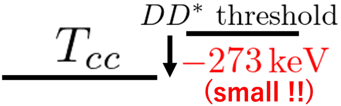

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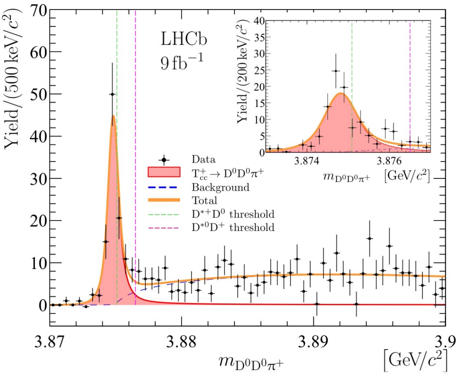

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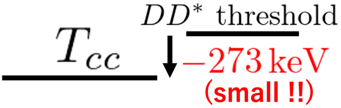

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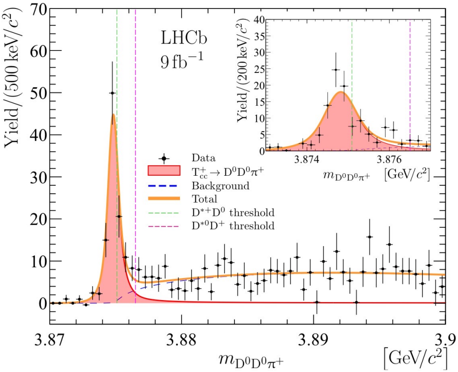

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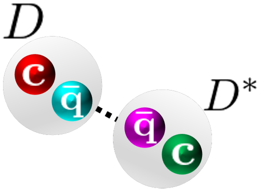


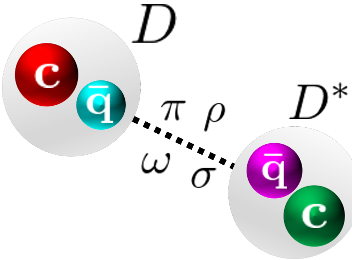
- T_{cc} lies slightly below the $D^{*+}D^0$ threshold.
  $\Rightarrow$ T_{cc} is considered as DD^* **bound state (hadronic molecule)**.

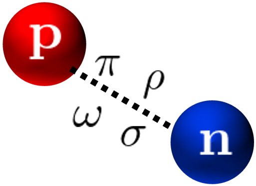
Theoretical research of T_{cc}

- Non relativestic quark model
  - J.I.Ballot and J.M.Richard, Phys.Lett. B **123**, 449 (1983)
  - S.Zouzou *et al*, Z. Phys. C **30**,457 (1986)
  - Q. Meng *et al*, Phys. Lett. B **814**, 136095 (2019)
- Hadronic molecule
  - S.Ohkoda et al, Phys. Rev. D **86**, 034019 (2012)
    Bound and resonant states of T_{cc} and T_{bb}.
    Interaction:
    One pion exchange potential, One boson (π, ρ, ω) exchange potential.
- Lattice QCD
  - Y. Ikeda *et al*, Phys. Lett. B **729**, 85 (2014)
  - M. Padmanath and S. Prelovsek, Phys. Rev. Lett. **129**, 032002 (2022)
  - Y. Lyu *et al*, Phys. Rev. Lett. **131**, 161901

The studies of T_{cc} are summarized in [1]
[1] Hua-Xing Chen *et al*, Rept. Prog. Phys. **86**, 026201 (2023)

Hadronic molecule in our study





$B.E. \simeq 2.2\,\mathrm{MeV}$

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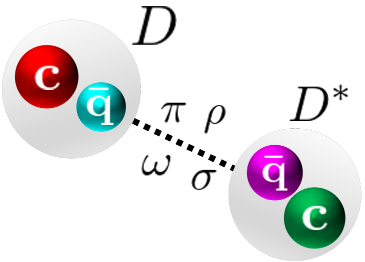

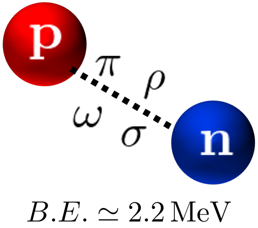

- Structure
  Bound and resonant states of DD^* (= Hadronic molecules)
  $\leftrightarrow$ Deuteron (= Bound state of a proton and a neutron)

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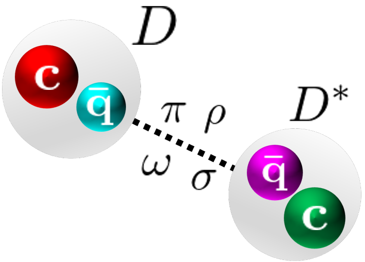

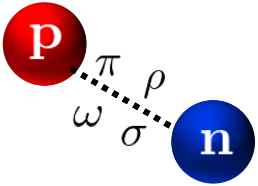

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- Structure
  Bound and resonant states of DD^* (= Hadronic molecules)
  $\leftrightarrow$ Deuteron (= Bound state of a proton and a neutron)
- Interaction
  One boson exchange potetinal (OBEP) $\pi, \rho, \omega,$ σ
  respecting the chiral symmetry and heavy quark spin symmetry (HQS)

Heavy quark spin symmetry (HQS)

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  $\rightarrow$ The masses of heavy hadrons with the different angular momentum are degenerate

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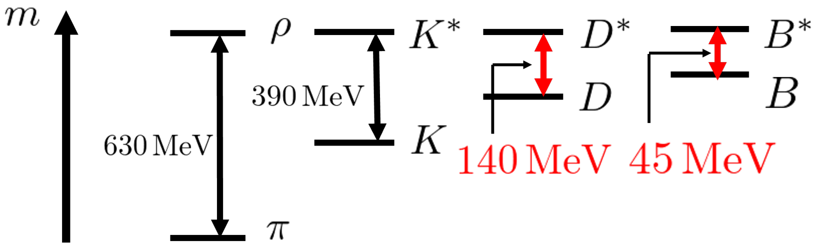
- Mass difference between pseudoscalar and vector mesons



- The mass differences between heavy mesons are smaller than those of light mesons.

Heavy quark spin symmetry (HQS)

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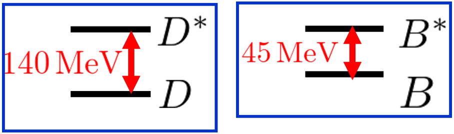
  $\rightarrow$ The masses of heavy hadrons with the different angular momentum are degenerate
- HQS doublet



HQS doublet

▶ $(D,\ D^*)$ and (B, B^*) are considered as the **HQS doublet**

Heavy quark spin symmetry (HQS)

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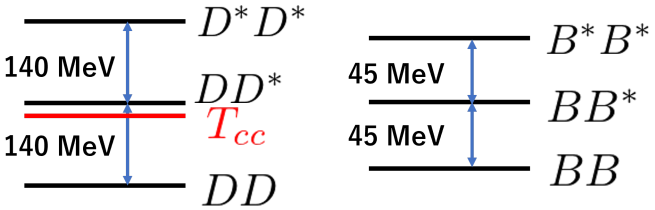
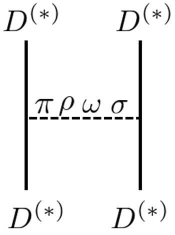
- Coupled channel



► PP, PP^* and P^*P^* are coupled by the OBEP ($P^{(*)} = D^{(*)}, B^{(*)}$).

OBEP of $D^{(*)}D^{(*)}$

- Potentials



$$\pi:\ V^{\pi}(r)=\frac{1}{3}\left(\frac{g_\pi}{2f_\pi}\right)^2[\vec{\mathcal{O}}_1\cdot\vec{\mathcal{O}}_2C(r;m_\pi)+S_{12}(r)T(r;m_\pi)]\vec{\tau}_1\cdot\vec{\tau}_2$$

$$\text{vector}:\ V^{v}(r)=\left(\frac{\beta g_V}{2m_v}\right)^2C(r;m_v)\vec{\tau}_1\cdot\vec{\tau}_2 +\frac{1}{3}(\lambda g_V)^2[2\vec{\mathcal{O}}_1\cdot\vec{\mathcal{O}}_2C(r;m_v)-S_{12}(r)T(r;m_v)]\vec{\tau}_1\cdot\vec{\tau}_2$$

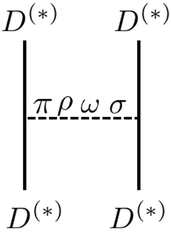
$$\sigma:\ V^{\sigma}(r)=-\left(\frac{g_\sigma}{m_\sigma}\right)^2C(r;m_\sigma)$$

- Form factor

$$F(\vec{q};m)=\frac{\Lambda^2-m^2}{\Lambda^2+\vec{q}^{\,2}}$$

OBEP of $D^{(*)}D^{(*)}$

- Potentials



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$$\sigma: \ V^{\sigma}(r) = -\left(\frac{g_\sigma}{m_\sigma}\right)^2 C(r;m_\sigma)$$

- Form factor

$$F(\vec{q};m) = \frac{\Lambda^2 - m^2}{\Lambda^2 + \vec{q}^{\,2}}$$

- Parameter

  $g_\pi = 0.59$ ($D^* \to D\pi$ decay), $\beta = 0.9$ (Lattice QCD),
  $\lambda = 0.56\,\mathrm{GeV}^{-1}$ (B decay), $g_\sigma = g_{\sigma NN}/3 = 3.4$, Λ: Free parameter

S. Ahmed et al., CLEO Collaboration, Phys. Rev. Lett. 87, 251801 (2001)
Ming-Zhu Liu et al., Phys. Rev. D 99 094018 (2019), C. Isola, Phys. Rev. D 68, 114001(2003)

OBEP of $D^{(*)}D^{(*)}$

- Possible channels of $T_{cc}(0(1^+))$

$$\begin{pmatrix} |[DD^*]_-({}^3S_1, {}^3D_1)\rangle \\ |D^*D^*({}^3S_1, {}^3D_1)\rangle \end{pmatrix}$$

- Heavy quark spin symmetry
  $\rightarrow DD^* - D^*D^*$ mixing
- $S - D$ mixing (Tensor force)

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- One pion exchange potential matrix of $T_{cc}(0(1^+))$

$$V^{\mathrm{HM}}_{\pi,0(1^+)} = \begin{pmatrix} -C_\pi & \sqrt{2}T_\pi & 2C_\pi & \sqrt{2}T_\pi \\ \sqrt{2}T_\pi & -C_\pi - T_\pi & \sqrt{2}T_\pi & 2C_\pi - T_\pi \\ 2C_\pi & \sqrt{2}T_\pi & -C_\pi & \sqrt{2}T_\pi \\ \sqrt{2}T_\pi & 2C_\pi - T_\pi & \sqrt{2}T_\pi & -C_\pi - T_\pi \end{pmatrix}$$

$$C_\pi = \frac{1}{3}\left(\frac{g_\pi}{2f_\pi}\right)^2 C(r; m_\pi)\vec{\tau}_1 \cdot \vec{\tau}_2, \quad T_\pi = \frac{1}{3}\left(\frac{g_\pi}{2f_\pi}\right)^2 T(r; m_\pi)\vec{\tau}_1 \cdot \vec{\tau}_2$$

Research Process

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   - $T_{cc}(0(1^+))$ lies slightly below the DD^* threshold.
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   - Analysis of the bound state of $T_{bb}(0(1^+))$

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3. T_{QQ} in the heavy quark limit ($m_Q \rightarrow \infty$)

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3. T_{QQ} in the heavy quark limit ($m_Q \to \infty$)
   - Introduction of the light cloud basis
   - HQS multiplet structures of obtained T_{cc} and T_{bb}

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- Solving the coupled channel Schrödinger equation:

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- Gaussian expansion method E. Hiyama *et al*, Prog.Part.Nucl.Phys., 51 (2003) 223-307,
  E. Hiyama *et al*, Front.Phys.(Beijing) 13 (2018) 6, 132106

  Expand the wavefunction using the Gaussian:

$$\psi = \sum_{n=1}^{n_{\max}} c_n N_{nl} \mathbf{e^{-r^2/r_n^2}} Y_{nl}(\psi, \varphi)$$

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- Complex scaling method
  R. Suzuki *et al*, AIP Conf.Proc. 768 (2005) 1, 455, Prog.Theor.Phys. 113 (2005) 1273
  T. Myo *et al*, Prog.Part.Nucl.Phys. 79 (2014) 1-56, T. Myo *et al*, Prog.Theor.Exp.Phys., 2020, 12A101

  Rotate $\vec{r}$ on the complex plane: $\vec{r} \to \vec{r}e^{i\theta}$

  The Hamiltonian is non hermitian $\to$ Complex energy eigenvalue
  $e^{i(\kappa_r - i\gamma_r)re^{i\theta}} \propto e^{(-\kappa_r \sin\theta + \gamma_r \cos\theta)r} \to 0$ for $\theta > \mathrm{Arctan}(\gamma_r/\kappa_r)$
  $\to$ Bound state: $E_B = -B$
  Resonance: $E_{\mathrm{reso}} = E_r - i\frac{\Gamma}{2}$

$T_{cc}(cc\bar{q}\bar{q}) \leftrightarrow D^{(*)}D^{(*)}$molecule

- Analysis of T_{cc} with $(0(1^+))$
- Analysis of T_{cc} with other quantum numbers

T_{cc}

- Analysis of $T_{cc}(0(1^+))$

T_{cc}

- Analysis of $T_{cc}(0(1^+))$
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  The experimental data of T_{cc} are reproduced for $\Lambda = 1069.8\,\mathrm{MeV}$

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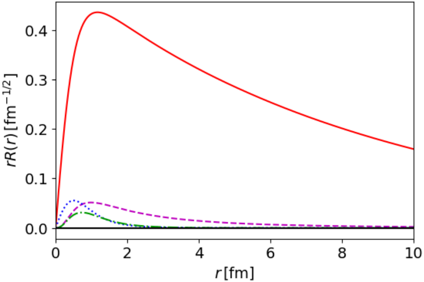

B.E.	$0.273\,\mathrm{MeV}$
$[\mathbf{DD^*}]_-(\mathbf{^3S_1})$	$\mathbf{99.2}\,\%$
$[DD^*]_-(^3D_1)$	$0.467\,\%$
$D^*D^*(^3S_1)$	$0.229\,\%$
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$\sqrt{\langle r^2 \rangle}$	$6.43\,\mathrm{fm}$

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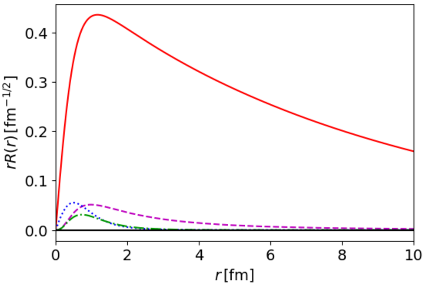

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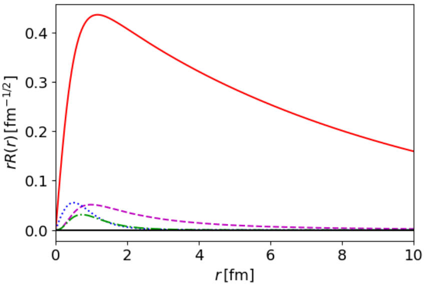

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	$DD^*(^3S_1)$	$DD^*(^3D_1)$	$D^*D^*(^3S_1)$	$D^*D^*(^3D_1)$
$DD^*(^3S_1)$	$-4.7(\sigma)$	$-0.55(\pi)$	$-0.31(\rho)$	$-0.34(\pi)$
$DD^*(^3D_1)$	$-0.55(\pi)$	-0.0048	-0.061	0.01
$D^*D^*(^3S_1)$	$-0.31(\rho)$	-0.061	-0.11	-0.042
$D^*D^*(^3D_1)$	$-0.34(\pi)$	0.01	-0.042	-0.0051

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  - Important component
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    - Tensor force of π exchange

T_{cc}

- T_{cc} with other quantum numbers
  The same parameters for T_{cc} with $0(1^+)$ are used.

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T_{cc} are not found.

This result is consistent with the fact that only T_{cc} with $0(1^+)$ has been reported.

$T_{bb}(bb\bar{q}\bar{q}) \leftrightarrow B^{(*)}B^{(*)}$molecule

- Analysis of T_{bb} with $0(1^+)$
- Analysis of T_{bb} with other quantum numbers

Bound state of $T_{bb}(0(1^+))$

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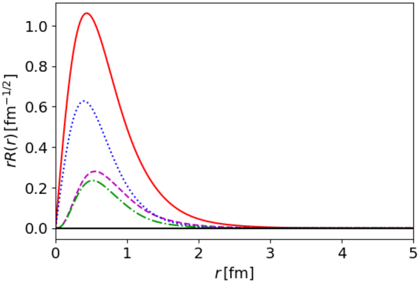

B.E.	$46.0\,\mathrm{MeV}$
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$[BB^*]_-(^3D_1)$	$4.71\,\%$
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$\sqrt{\langle r^2 \rangle}$	$0.620\,\mathrm{fm}$

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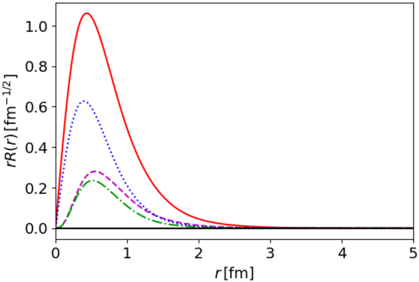

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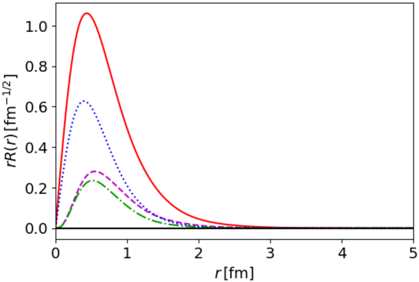

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  (D^*Dß* channel is not important in the case of T_{cc}.)
- $[BB^*]_-(^3S_1) - B^*B^*(^3S_1)$ are more coupled.
  $(\Delta m_B = m_{B^*} - m_B < \Delta m_D = m_{D^*} - m_D)$

Bound state of $T_{bb}(0(1^+))$

- $T_{bb} \leftrightarrow B^{(*)}B^{(*)}$ molecule (Undiscovered)
  Binding energy $46.0\,\mathrm{MeV}$
  Mixing ratio:
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	$BB^*(^3S_1)$	$BB^*(^3D_1)$	$B^*B^*(^3S_1)$	$B^*B^*(^3D_1)$
$BB^*(^3S_1)$	$-40(\sigma)$	$-7.3(\pi)$	$-11(\rho)$	$-6.2(\pi)$
$BB^*(^3D_1)$	$-7.3(\pi)$	-0.67	-4.1	0.33
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  - **Center force of** ρ $(BB^*(^3S) - B^*B^*(^3S))$

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  - **Tensor force of π**
  - **Center force of ρ** ($BB^*(^3S) - B^*B^*(^3S)$)
  - Comparison with T_{cc}: σ exchange ($B^*B^* - B^*B^*$), Center force of ρ

Bound and Resonant states of T_{bb}

Bound state:

$I(J^P)$	Lowest threshold	$-E_B$ [MeV]
$0(0^-)$	BB^*	-24.4
$0(1^+)$	BB^*	-46.0
$0(1^-)$	BB	-
$0(2^+)$	BB^*	-
$0(2^-)$	BB^*	-6.11
$1(0^+)$	BB	-7.23
$1(0^-)$	BB^*	-
$1(1^+)$	BB^*	-2.46
$1(1^-)$	BB^*	-
$1(2^+)$	BB	-
$1(2^-)$	BB^*	-

- Many bound states were found.
  (Only bound state of T_{cc} with $0(1^+)$ was found).

Bound and Resonant states of T_{bb}

$I(J^P)$	Lowest threshold	$-E_B$ [MeV] $E_r - i\frac{\Gamma}{2}$ [MeV]
$0(0^-)$	BB^*	-24.4
$0(1^+)$	BB^*	-46.0 $35.9 - i\frac{8.02}{2}$
$0(1^-)$	BB	$1.52 - i\frac{1.13}{2}$
$0(2^-)$	BB^*	-6.11 $33.3 - i\frac{8.74}{2}$
$1(0^+)$	BB	-7.23 $76.4 - i\frac{4.99}{2}$
$1(1^+)$	BB^*	-2.46
$1(2^+)$	BB	$89.2 - i\frac{3.09}{2}$

- We can find resonant states of T_{bb} with $0(1^+),\ 0(1^-),\ 0(2^-),\ 1(0^+),\ 1(2^+)$.
- Feshbach resonance (Bound state of $B^{(*)}B^*$)

T_{QQ} in the heavy quark limit

- Introduction of the light cloud basis
- HQS multiplet structures of obtained T_{cc} and T_{bb}

HQS multiplet structure

- Heavy quark spin symmetry (HQS)

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  - $J = L \otimes S \to S_Q \otimes j_\ell$ (S_Q: heavy quark spin, j_ℓ: light cloud)

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  - $J = L \otimes S \to S_Q \otimes j_\ell$ (S_Q: heavy quark spin, j_ℓ: light cloud)
    $\because$ Transition between (S_Q, j_ℓ) and (S'_Q, j'_ℓ) stats is 0 in the heavy quark limit.

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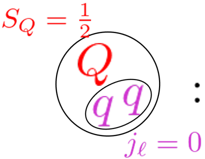
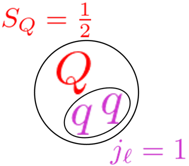
    $\because$ Transition between (S_Q, j_ℓ) and (S'_Q, j'_ℓ) stats is 0 in the heavy quark limit.
- **HQS multiplet** (e.g. Baryons $\Lambda_c(cqq), \Sigma_c(cqq), \Sigma_c^*(cqq)$)

- $S_Q = \frac{1}{2},\ j_\ell = 0$ (HQS singlet)



$$: \left(\frac{1}{2}\right)_{S_Q} \otimes (0)_{j_\ell} = \frac{1}{2}$$

- $S_Q = \frac{1}{2},\ j_\ell = 1$ (HQS doublet)

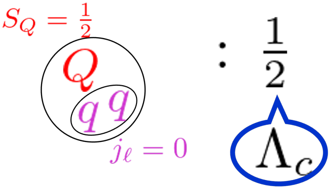


$$: \left(\frac{1}{2}\right)_{S_Q} \otimes (1)_{j_\ell} = \frac{1}{2} \oplus \frac{3}{2}$$

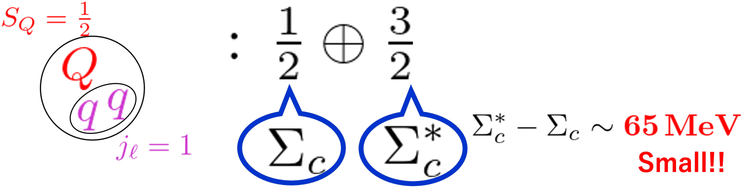
HQS multiplet structure

- Heavy quark spin symmetry (HQS)
  - $J = L \otimes S \to S_Q \otimes j_\ell$ (S_Q: heavy quark spin, j_ℓ: light cloud)
- **HQS multiplet**

▶ $S_Q = \frac{1}{2},\ j_\ell = 0$ (HQS singlet)



▶ $S_Q = \frac{1}{2},\ j_\ell = 1$ (HQS doublet)

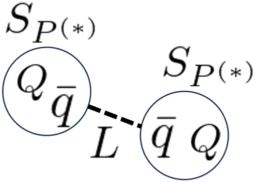


Light cloud basis in the heavy quark limit

Light cloud basis in the heavy quark limit

- Hadronic molecule basis $(L \otimes S)$ → Light cloud basis $(S_{QQ} \otimes j_\ell)$

$$\left|\left[L\ \left[[Q\bar{q}]_{S_{P_1}}\ [Q\bar{q}]_{S_{P_2}}\right]_S\right]_J\right\rangle \to \left|\left[[QQ]_{\mathbf{S_{QQ}}}\left[L\ [\bar{q}\bar{q}]_{S_q}\right]_{\mathbf{j_\ell}}\right]_J\right\rangle$$

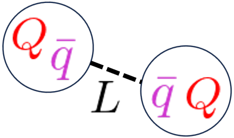


$S = S_{P(*)} \otimes S_{P(*)}$

Hadronic molecule basis

$J = L \otimes S$

$S_{QQ} = S_Q \otimes S_Q$



$j_\ell = L \otimes (S_q \otimes S_q)$

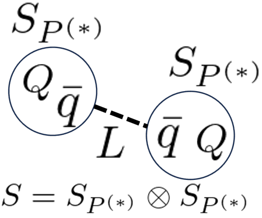
Light cloud basis

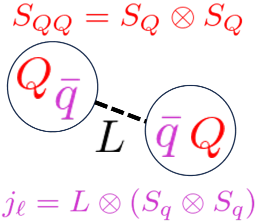
$J = S_{QQ} \otimes j_\ell$

Light cloud basis in the heavy quark limit

- Hadronic molecule basis $(L \otimes S)$ → Light cloud basis $(S_{QQ} \otimes j_\ell)$

$$\left| \left[L \left[[Q\bar{q}]_{S_{P_1}} \ [Q\bar{q}]_{S_{P_2}} \right]_S \right]_J \right\rangle \to \left| \left[[QQ]_{\mathbf{S_{QQ}}} \left[L \ [\bar{q}\bar{q}]_{S_q} \right]_{\mathbf{j_\ell}} \right]_J \right\rangle$$





Hadronic molecule basis

$$J = L \otimes S$$

Light cloud basis

$$J = S_{QQ} \otimes j_\ell$$

- Spin structures $(\mathbf{S_{QQ}}, \mathbf{j_\ell})$ can be found.
  → HQS multiplet structures of T_{cc}, T_{bb} can be examined.

HQS multiplet structure

- Channel and One pion exchange potential matrix
  (Hadronic molecule: $J = S_{QQ} \otimes j_\ell$)

$$\left| \left[L \left[[Q\bar{q}]_{S_{P_1}} \; [Q\bar{q}]_{S_{P_2}} \right]_S \right]_J \right\rangle$$

$$0(1^+) : \begin{pmatrix} |[PP^*]_-({}^3S_1)\rangle \\ |[PP^*]_-({}^3D_1)\rangle \\ |P^*P^*({}^3S_1)\rangle \\ |P^*P^*({}^3D_1)\rangle \end{pmatrix}, \begin{pmatrix} -C_\pi & \sqrt{2}T_\pi & 2C_\pi & \sqrt{2}T_\pi \\ \sqrt{2}T_\pi & -C_\pi - T_\pi & \sqrt{2}T_\pi & 2C_\pi - T_\pi \\ 2C_\pi & \sqrt{2}T_\pi & -C_\pi & \sqrt{2}T_\pi \\ \sqrt{2}T_\pi & 2C_\pi - T_\pi & \sqrt{2}T_\pi & -C_\pi - T_\pi \end{pmatrix}$$

$$0(2^+) : \begin{pmatrix} |[PP^*]_-({}^3D_2)\rangle \\ |P^*P^*({}^3D_2)\rangle \end{pmatrix}, \begin{pmatrix} -C_\pi + T_\pi & 2C_\pi + T_\pi \\ 2C_\pi + T_\pi & -C_\pi + T_\pi \end{pmatrix}$$

HQS multiplet structure

- Channel and One pion exchange potential matrix
  (Light cloud basis: $J = S_{QQ} \otimes j_\ell$)

$$\left|\left[[QQ]_{S_{QQ}}\left[L\ [\bar{q}\bar{q}]_{S_q}\right]_{j_\ell}\right]_J\right\rangle = U^{-1}_{I(J^P)}\left|\left[L\ \left[[Q\bar{q}]_{S_{P_1}}\ [Q\bar{q}]_{S_{P_2}}\right]_S\right]_J\right\rangle$$

$$0(1^+):\ \begin{pmatrix} \left|\left[[QQ]_1\ [S\ [\bar{q}\bar{q}]_0]_0\right]_1\right\rangle \\ \left|\left[[QQ]_0\ [S\ [\bar{q}\bar{q}]_1]_1\right]_1\right\rangle \\ \left|\left[[QQ]_0\ [D\ [\bar{q}\bar{q}]_1]_1\right]_1\right\rangle \\ \left|\left[[QQ]_1\ [D\ [\bar{q}\bar{q}]_0]_2\right]_1\right\rangle \end{pmatrix},\ \left(\begin{array}{c|cc|c} -3C_\pi & 0 & 0 & 0 \\ \hline 0 & C_\pi & 2\sqrt{2}T_\pi & 0 \\ 0 & 2\sqrt{2}T_\pi & C_\pi - 2T_\pi & 0 \\ \hline 0 & 0 & 0 & -3C_\pi \end{array}\right)$$

$$0(2^+):\ \begin{pmatrix} \left|\left[[QQ]_0\ [D\ [\bar{q}\bar{q}]_1]_2\right]_2\right\rangle \\ \left|\left[[QQ]_1\ [D\ [\bar{q}\bar{q}]_0]_2\right]_2\right\rangle \end{pmatrix},\ \left(\begin{array}{c|c} C_\pi + 2T_\pi & 0 \\ \hline 0 & -3C_\pi \end{array}\right)$$

HQS multiplet structure

- Channel and One pion exchange potential matrix
  (Light cloud basis: $J = S_{QQ} \otimes j_\ell$)

$$\left|\left[[QQ]_{S_{QQ}}\left[L\ [\bar{q}\bar{q}]_{S_q}\right]_{j_\ell}\right]_J\right\rangle = U^{-1}_{I(J^P)}\left|\left[L\ \left[[Q\bar{q}]_{S_{P_1}}\ [Q\bar{q}]_{S_{P_2}}\right]_S\right]_J\right\rangle$$

$$0(1^+):\ \begin{pmatrix} \left|\left[[QQ]_1\ [S\ [\bar{q}\bar{q}]_0]_0\right]_1\right\rangle \\ \left|\left[[QQ]_0\ [S\ [\bar{q}\bar{q}]_1]_1\right]_1\right\rangle \\ \left|\left[[QQ]_0\ [D\ [\bar{q}\bar{q}]_1]_1\right]_1\right\rangle \\ \left|\left[[QQ]_1\ [D\ [\bar{q}\bar{q}]_0]_2\right]_1\right\rangle \end{pmatrix},\ \left(\begin{array}{c|cc|c} -3C_\pi & 0 & 0 & 0 \\ \hline 0 & C_\pi & 2\sqrt{2}T_\pi & 0 \\ 0 & 2\sqrt{2}T_\pi & C_\pi - 2T_\pi & 0 \\ \hline 0 & 0 & 0 & -3C_\pi \end{array}\right)$$

$$0(2^+):\ \begin{pmatrix} \left|\left[[QQ]_0\ [D\ [\bar{q}\bar{q}]_1]_2\right]_2\right\rangle \\ \left|\left[[QQ]_1\ [D\ [\bar{q}\bar{q}]_0]_2\right]_2\right\rangle \end{pmatrix},\ \left(\begin{array}{c|c} C_\pi + 2T_\pi & 0 \\ \hline 0 & -3C_\pi \end{array}\right)$$

- ▶ Block diagonalized

HQS multiplet structure

- Channel and One pion exchange potential matrix
  (Light cloud basis: $J = S_{QQ} \otimes j_\ell$)

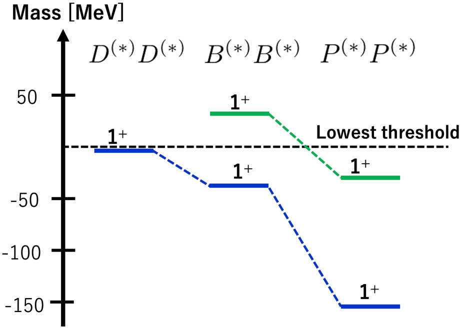
$$\left|\left[[QQ]_{S_{QQ}}\left[L\ [\bar{q}\bar{q}]_{S_q}\right]_{j_\ell}\right]_J\right\rangle = U^{-1}_{I(J^P)}\left|\left[L\ \left[[Q\bar{q}]_{S_{P_1}}\ [Q\bar{q}]_{S_{P_2}}\right]_S\right]_J\right\rangle$$

$$0(1^+):\quad \begin{pmatrix} \left|\left[[QQ]_1\ \left[S\ [\bar{q}\bar{q}]_0\right]_0\right]_1\right\rangle \\ \left|\left[[QQ]_0\ \left[S\ [\bar{q}\bar{q}]_1\right]_1\right]_1\right\rangle \\ \left|\left[[QQ]_0\ \left[D\ [\bar{q}\bar{q}]_1\right]_1\right]_1\right\rangle \\ \left|\left[[QQ]_1\ \left[D\ [\bar{q}\bar{q}]_0\right]_2\right]_1\right\rangle \end{pmatrix},\quad \left(\begin{array}{c|cc|c} -3C_\pi & 0 & 0 & 0 \\ \hline 0 & C_\pi & 2\sqrt{2}T_\pi & 0 \\ 0 & 2\sqrt{2}T_\pi & C_\pi - 2T_\pi & 0 \\ \hline 0 & 0 & 0 & -3C_\pi \end{array}\right)$$

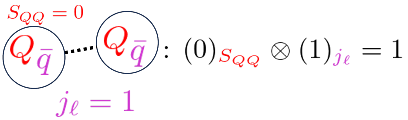
$$0(2^+):\quad \begin{pmatrix} \left|\left[[QQ]_0\ \left[D\ [\bar{q}\bar{q}]_1\right]_2\right]_2\right\rangle \\ \left|\left[[QQ]_1\ \left[D\ [\bar{q}\bar{q}]_0\right]_2\right]_2\right\rangle \end{pmatrix},\quad \left(\begin{array}{c|c} C_\pi + 2T_\pi & 0 \\ \hline 0 & -3C_\pi \end{array}\right)$$

  - Block diagonalized
  - $0(1^+)$ and $0(2^+)$ have the same component $-3C_\pi$ (Spin structure: $(S_{QQ}, j_\ell) = (1, 2)$)
    $\rightarrow$ HQS triplet structures: $(1)_{S_{QQ}} \otimes (2)_{j_\ell} = 1 \oplus 2 \oplus 3$

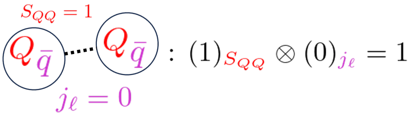
HQS multiplet structures of T_{cc} and T_{bb} with $0(1^+)$



The bound state belongs to the HQS singlet.

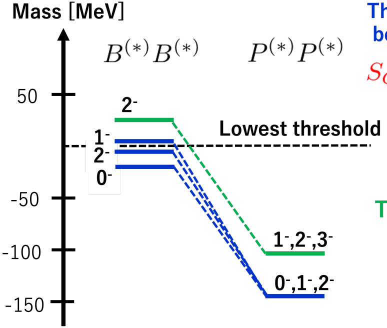


The resonance belongs to the HQS singlet.

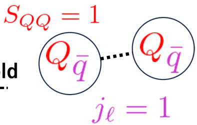


- Bound states of $T_{cc},\ T_{bb}(0(1^+))$ belong to the HQS singlet.
- Resonance of $T_{bb}(0(1^+))$ belong to the (other) HQS singlet.
  These states do not have partners.
  $\rightarrow$ As for T_{cc}, this result is consistent with the fact that only T_{cc} with $0(1^+)$ have been reported.

HQS multiplet structures of T_{bb} with $0(J^-)$

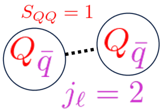


The bound states of $T_{bb}(0(0^-,2^-))$ and resonance of T_{bb} $(0(1^-)$ belong to the HQS triplet



$: \ (1)_{S_{QQ}} \otimes (1)_{j_\ell} = 0 \oplus 1 \oplus 2$

The resonance of $T_{bb}(0(2^-))$ belong to the HQS triplet



$: \ (1)_{S_{QQ}} \otimes (2)_{j_\ell} = 1 \oplus 2 \oplus 3$

- The bound states of $T_{bb}(0(0^-, 2^-))$ and resonance belong to the HQS triplet
- The resonance of $T_{bb}(0(2^-))$ belongs to the HQS triplet.
  We cannot obtain the partner of the resonance of T_{bb} with $0(2^-)$.

Summary

- Analysis of the bound and resonant states of T_{cc}、T_{bb}.
  - T_{cc}
    The only bound state of $T_{cc}(0(1^+))$ was found.
  - T_{bb}
    The bound states of $T_{bb}(0(1^+, 0^-, 2^-))$, $T_{bb}(1(0^+, 1^+))$ were found.
    The resonances of $T_{bb}(0(1^+, 1^-, 2^-))$, $T_{bb}(1(1^+, 2^+))$ were found.
- Analysis of HQS partners using the light cloud basis
  - $0(1^+)$ states
    The bound and resonant states each belong to the different HQS singlet.
  - $0(0^-, 2^-)$ states
    The ground states of $T_{bb}(0(0^-, 1^-, 2^-))$ belong to the same HQS triplet.
    The other state of $T_{bb}(0(2^-))$ belongs to the different HQS triplet.