$\chi_{cJ}(2P)$ with hadronic molecules

Kotaro Miyake[1]
Collaborator: Yasuhiro Yamaguchi[1 2]

[1]Department of Physics, Nagoya University [2]KMI, Nagoya University

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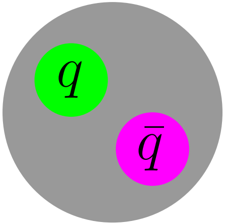
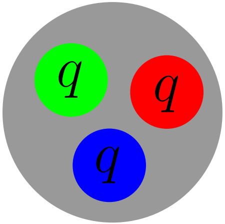
Hadron

- Composite particles of quarks and gluons.
- QCD cannot be solved by perturbation theory except in the high energy regime.
- Effective theory is important.
- Normally $q\bar{q}$ or qqq.

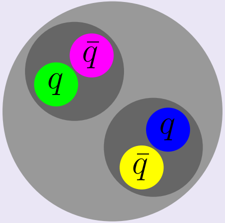




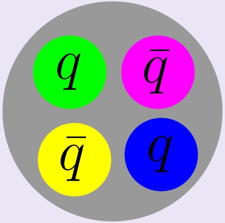
Exotic Hadron

- Difficult to explain by $q\bar{q}$ or qqq.
- More complex structure than normal hadrons.
- Various states are considered, such as hadronic molecular states and compact states.
- In recent years, new exotic hadrons have been reported almost every year.

Hadronic molecular state

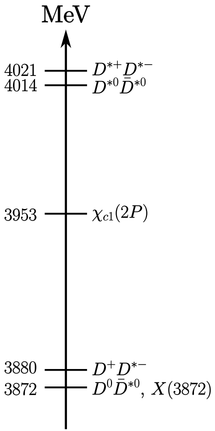


Compact state



What is $X(3872)$?

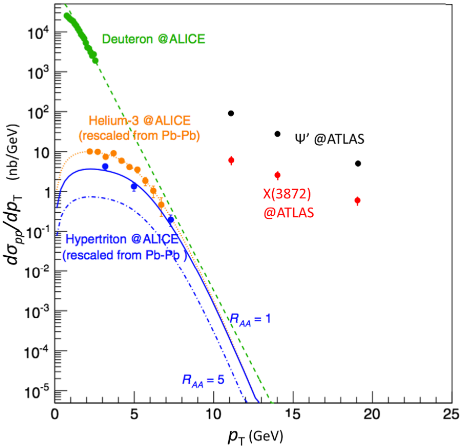
- One of the best known exotic hadrons.
- Reported by the Belle experiment in 2003, and later reported in various experiments.
  (CDF(2004), D0(2004), BaBar(2005), LHCb(2012), CMS(2013), BESIII(2014), ATLAS,(2017))
- $J^{PC} = 1^{++}$, $c\bar{c}q\bar{q}(q = u, d)$?
- Hadronic molecular state? Compact state?
- Very close to the threshold of $D^0\bar{D}^{*0}$.
  $m_{X(3872)} - m_{D^0\bar{D}^{*0}} = -0.05$ MeV
  (Particle Data Group, Phys. Rev. D **110**(2024)030001)



Is $X(3872)$ a hadronic molecular state?

- Close to $\chi_{c1}(2P)$ quark model mass prediction 3953 MeV.

  (S. Godfrey and N. Isgur, Phys. Rev. D **32**(1985)189)
- $\chi_{c1}(2P)$ is a $c\bar{c}$ meson with $J^{PC} = 1^{++}$ same as $X(3872)$.
- Some experimental data suggest that $X(3872)$ has a different structure than hadronic molecules.
- **Can $X(3872)$ be explained by "hadronic molecules + $\chi_{c1}(2P)$"?**

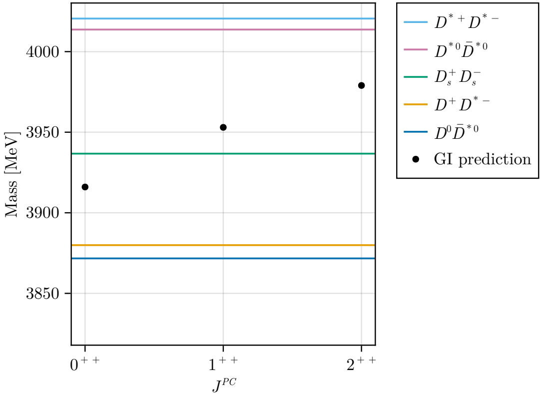


Esposito et al, Phys. Rev. D **92**(2015)034028

Olsen et al, Rev. Mod. Phys. **90**(2018)015003

What is the situation for $\chi_{c0}(2P)$ and $\chi_{c2}(2P)$?

- In the 0^{++} case, two particles are reported.
- In the 2^{++} case, quark model predictions and experimental data are inconsistent.



Particle Data Group, Phys. Rev. D **110**(2024)030001
S. Godfrey and N. Isgur, Phys. Rev. D **32**(1985)189

- **Can we explain not only $X(3872)$ but also 0^{++} and 2^{++} experimental data in a consistent way?**

Schrödinger equation

- Consider the superposition state of hadronic molecules ($D^{(*)}\bar{D}^{(*)}$) and charmonium core ($\chi_{cJ}(2P)$).

$J^{PC} = 1^{++}$ case

$$\mathcal{H}\Psi = E\Psi \tag{1}$$

$$\mathcal{H} = \begin{pmatrix} H_0 + V_{\mathrm{OBE}} & \mathcal{U}^\dagger \\ \mathcal{U} & m_{\chi_{c1}} - (m_{D^0} + m_{D^{*0}}) \end{pmatrix} \tag{2}$$

$$V_{\mathrm{OBE}} = V_\pi(\Lambda_\pi, \boldsymbol{r}) + V_\eta(\Lambda_\eta, \boldsymbol{r}) + V_\rho(\Lambda_\rho, \boldsymbol{r}) + V_\omega(\Lambda_\omega, \boldsymbol{r}) \tag{3}$$

$$\Lambda_{\mathrm{meson}} = 220\,\mathrm{MeV} \cdot \alpha + m_{\mathrm{meson}} \tag{4}$$

(H.-Y. Cheng et al, Phys. Rev. D **71**(2005)014030)

$$\Psi = \begin{pmatrix} c_1\left|[D^0\bar{D}^{*0}](S)\right\rangle \\ c_2\left|[D^+D^{*-}](S)\right\rangle \\ c_3\left|[D^0\bar{D}^{*0}](D)\right\rangle \\ c_4\left|[D^+D^{*-}](D)\right\rangle \\ c_5\left|D^{*0}\bar{D}^{*0}(D)\right\rangle \\ c_6\left|D^{*+}D^{*-}(D)\right\rangle \\ c_7\left|\chi_{c1}(2P)\right\rangle \end{pmatrix} \tag{5}$$

One boson exchange

$$\mathcal{L} = g\,\mathrm{Tr}\left[H_b^{(Q)}\gamma^\mu\gamma^5 A_{ba\mu}\bar{H}_a^{(Q)}\right] + \beta\,\mathrm{Tr}\left[H_b^{(Q)}v_\mu(V_{ba}^\mu - \rho_{ba}^\mu)\bar{H}_a^{(Q)}\right] + \lambda\,\mathrm{Tr}\left[H_b^{(Q)}\sigma_{\mu\nu}F_{ba}^{\mu\nu}\bar{H}_a^{(Q)}\right] + c.c. \tag{6}$$

$$H_a^{(Q)} = \frac{1+\not{v}}{2}(P_a^{*\mu}\gamma_\mu - P_a\gamma^5), \quad \bar{H}_a^{(Q)} = (P_a^{*\mu\dagger}\gamma_\mu + P_a^\dagger\gamma^5)\frac{1+\not{v}}{2} \tag{7}$$

$$A_\mu = \frac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger) \simeq -\frac{\partial_\mu(\boldsymbol{\pi}\cdot\boldsymbol{\tau})}{2f_\pi}, \quad \frac{\boldsymbol{\pi}\cdot\boldsymbol{\tau}}{\sqrt{2}} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix} \tag{8}$$

$$F^{\mu\nu} = \partial^\mu\rho^\nu - \partial^\nu\rho^\mu - i[\rho^\mu, \rho^\nu], \quad \rho^\mu = -\frac{g_v}{\sqrt{2}}\hat{\rho}^\mu, \quad \hat{\rho}^\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} \tag{9}$$

Schrödinger equation

- Consider $\chi_{cJ}(2P)$ couples to the S wave of $D^{(*)}\bar{D}^{(*)}$.
  (M. Takizawa and S. Takeuchi, PTEP **2013**(2013)093D01)

$J^{PC} = 1^{++}$ case

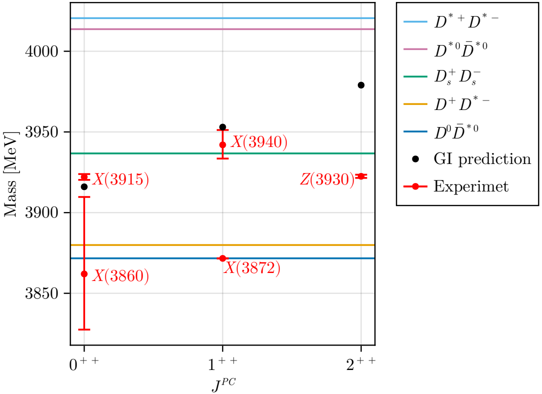
$$\mathcal{H} = \begin{pmatrix} H_0 + V_{\mathrm{OBE}} & \mathcal{U}^\dagger \\ \mathcal{U} & m_{\chi_{c1}(2P)} - (m_{D^0} + m_{D^{*0}}) \end{pmatrix} \tag{10}$$

$$\mathcal{U} = \begin{pmatrix} U & U & 0 & 0 & 0 & 0 \end{pmatrix} \tag{11}$$

$$\begin{aligned} \langle \chi_{c1}(2P) | U | [D^0\bar{D}^{*0}](S) \rangle &= \int d^3x \, \langle \chi_{c1}(2P) | U | \boldsymbol{x} \rangle \langle \boldsymbol{x} | [D^0\bar{D}^{*0}](S) \rangle \\ &\equiv \int d^3x \, f_{\mathrm{spin}} g_{c\bar{c}} \Lambda_q^{\frac{3}{2}} \frac{e^{-\Lambda_q r}}{r} Y_l^m(\theta, \phi) \langle \boldsymbol{x} | [D^0\bar{D}^{*0}](S) \rangle \end{aligned} \tag{12}$$

Set up

- $X(3872)$ and $Z(3930)$ have been reported in several experiments.
- $X(3860)$ was not found in LHCb.
- $X(3860)$ has large error.
- Consider only the bound state.



Particle Data Group, Phys. Rev. D **110**(2024)030001
S. Godfrey and N. Isgur, Phys. Rev. D **32**(1985)189

- Determine parameters to reproduce the mass of $X(3872)$ and $Z(3930)$.
- Check if the ground state of 0^{++} is consistent with $X(3860)$.

Parameters

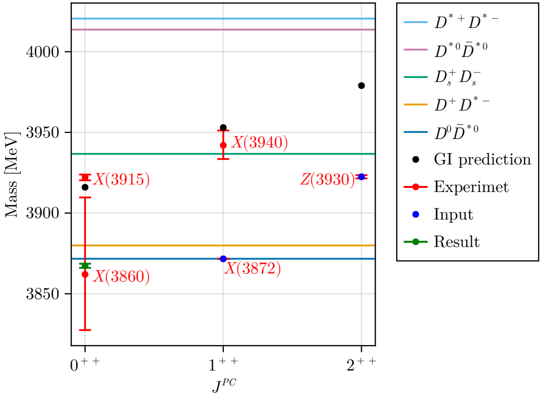
Λ_{meson} (MeV)	$220\,\text{MeV} \cdot \alpha + m_{\text{meson}}$		
α	0.7	1.0	1.3
$g_{c\bar{c}}$	0.0448	0.0427	0.0409
Λ_q (MeV)	2260	3089	4647

- α : size of the OBE cutoff. (H.-Y. Cheng et al, Phys. Rev. D **71**(2005)014030)
- $g_{c\bar{c}}$: strength of the coupling with $\chi_{cJ}(2P)$.
  (M. Takizawa and S. Takeuchi, PTEP **2013**(2013)093D01)
- Λ_q : spread of $\chi_{cJ}(2P)$ and U. (M. Takizawa and S. Takeuchi, PTEP **2013**(2013)093D01)
- Fix α and determine the other two parameters to reproduce the mass of $X(3872)$ and $Z(3930)$.
- Compute bound state of 0^{++} with three parameter sets.

Result : A bound state of 0++ was obtained.

α	Mass (MeV)
0.7	3866.07
1.0	3867.31
1.3	3868.62

- Ground state of 0^{++} is consistent with $X(3860)$.
- Error bars are values in each of the three parameter sets.
- The mass of 0^{++} is almost the same for the three parameter sets.



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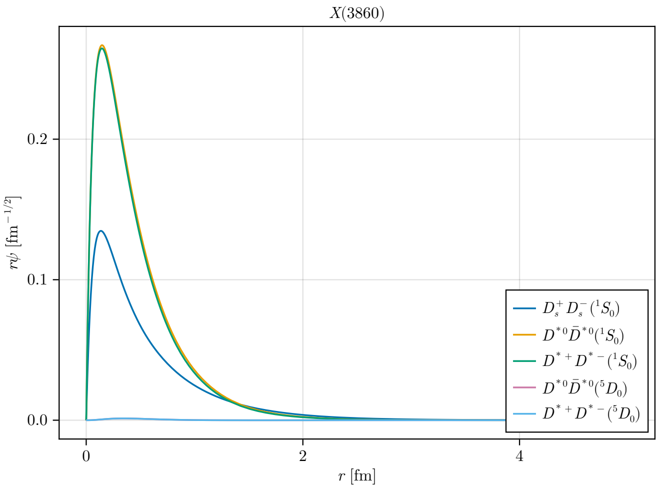
Mixing ratio of $X(3860)$ (0^{++})

	$D_s^+ D_s^- ({}^1S_0)$	$D^{*0}\bar{D}^{*0}({}^1S_0)$	$D^{*+}D^{*-}({}^1S_0)$	$D^*\bar{D}^*({}^5D_0)$	$\chi_{c0}(2P)$
Mass (MeV)	3937	4014	4021	4017	3916
$\alpha = 0.7$	$1.11\,\%$	$2.81\,\%$	$2.71\,\%$	$0.00\,\%$	$93.37\,\%$
$\alpha = 1.0$	$0.66\,\%$	$2.53\,\%$	$2.45\,\%$	$0.00\,\%$	$94.36\,\%$
$\alpha = 1.3$	$0.32\,\%$	$2.16\,\%$	$2.10\,\%$	$0.00\,\%$	$95.42\,\%$

- $\chi_{c0}(2P)$ is the main component.
  - $\chi_{c0}(2P)$ is closest.
- $D^*\bar{D}^*({}^1S_0)$ is the second main component.
- $D_s^+D_s^-({}^1S_0)$ is small but not zero.
  - Considered $\chi_{cJ}(2P)$ couples only to the S wave of hadronic molecules.
  - χ_{c0} coupling with $D_s^+D_s^-({}^1S_0)$ is smaller than the one with $D^*\bar{D}^*({}^1S_0)$ due to the spin factor.

Wave function of $X(3860)$ (0^{++})

- In hadroninc molecules, $D^*\bar{D}^*(^1S_0)$ is the main component and $D_s^+D_s^-(^1S_0)$ is the second main component.
- $D^*\bar{D}^*(^5D_0)$ is very small.



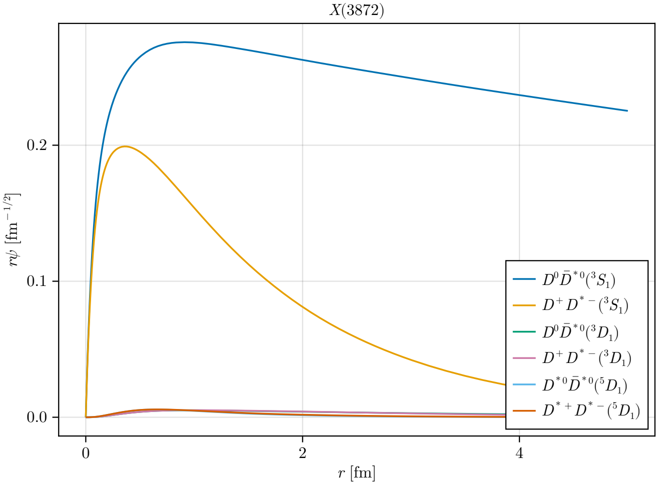
Mixing ratio of $X(3872)\ (1^{++})$

	$D^0\bar{D}^{*0}(^3S_1)$	$D^+D^{*-}(^3S_1)$	$D\bar{D}^*(^3D_1)$	$D^*\bar{D}^*(^5D_1)$	$\chi_{c1}(2P)$
Mass (MeV)	3872	3880	3876	4017	3953
$\alpha = 0.7$	$80.49\,\%$	$5.05\,\%$	$0.00\,\%$	$0.00\,\%$	$14.46\,\%$
$\alpha = 1.0$	$82.03\,\%$	$4.97\,\%$	$0.01\,\%$	$0.01\,\%$	$12.99\,\%$
$\alpha = 1.3$	$84.76\,\%$	$4.86\,\%$	$0.02\,\%$	$0.02\,\%$	$10.33\,\%$

- $D^0\bar{D}^{*0}$ is the main component.
  - Much closer than any other channel.
- $\chi_{c1}(2P)$ is the second main component.
  - $\chi_{cJ}(2P)$ must be large to reproduce $Z(3930)$ as well.
- Different structure from 0^{++}.
- Isospin symmetry is broken.

Wave function of $X(3872)$ (1^{++})

- $D^0\bar{D}^{*0}$ is very large and is the main component.
- Isospin symmetry is broken.
- Different structure from 0^{++}.



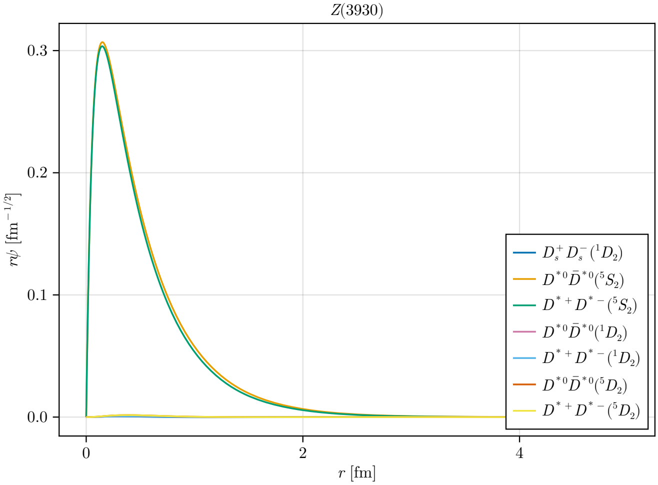
Mixing ratio of $Z(3930)$ (2^{++})

	$D_s^+ D_s^- ({}^1D_2)$	$D^{*0}\bar{D}^{*0} ({}^5S_2)$	$D^{*+}D^{*-} ({}^5S_2)$	$D^*\bar{D}^* ({}^1D_2)$	$D^*\bar{D}^* ({}^5D_2)$	$\chi_{c2}(2P)$
Mass (MeV)	3937	4014	4021	4017	4017	3979
$\alpha = 0.7$	$0.00\,\%$	$4.61\,\%$	$4.38\,\%$	$0.00\,\%$	$0.00\,\%$	$91.00\,\%$
$\alpha = 1.0$	$0.00\,\%$	$3.67\,\%$	$3.50\,\%$	$0.00\,\%$	$0.00\,\%$	$92.83\,\%$
$\alpha = 1.3$	$0.00\,\%$	$2.67\,\%$	$2.55\,\%$	$0.00\,\%$	$0.00\,\%$	$94.78\,\%$

- $\chi_{c2}(2P)$ is the main component.
- $D^*\bar{D}^*({}^5S_2)$ is the second main component.
- $D_s^+D_s^-({}^1D_2)$ is almost zero, unlike 0^{++}.
    - Considered $\chi_{cJ}(2P)$ couples only to the S wave of hadronic molecules.

Wave function of $Z(3930)$ (2^{++})

- It has almost the same form as the 0^{++} case except that $D_s^+ D_s^- (^5S_0)$ is almost zero.



Summary

- Theoretical predictions of $\chi_{c1}(2P)$, $\chi_{c0}(2P)$ and $\chi_{c2}(2P)$ are not consistent with the experimental data.
- Considered the superposition of $\chi_{cJ}(2P)$ and hadronic molecules.
- **$\boldsymbol{X(3872)}$, $\boldsymbol{Z(3930)}$ and $\boldsymbol{X(3860)}$ could be explained in a consistent way.**
- In the case of $Z(3930)$ and $X(3860)$, $\chi_{cJ}(2P)$ are the main component.
- In the case of $X(3872)$, $D^0\bar{D}^{*0}$ hadronic molecule is the main component.
- $X(3860)$ contains a small amount of $D_s^+D_s^-$, but in $Z(3930)$, it is almost zero.
- However, these mixing ratios are highly model dependent.

Outlook

- Use $^3P_0$ model for the potential between $\chi_{cJ}(2P)$ and hadronic molecules.
- Calculate resonance states using the complex scaling method.
- Study the decay and compare it to the experiment.
- Calculate $\chi_{bJ}(3P)$ using the same model.

Back up

Parameters

Parameter	Value
g	0.55
g_v	$\frac{m_\rho}{\sqrt{2}f_\pi}$
β	0.9
λ	$0.56\,\mathrm{GeV}^{-1}$

Spin factor

$$\begin{pmatrix} \left| \left[[Q\bar{Q}]_0 [S[q\bar{q}]_0]_0 \right]_0 \right\rangle \\ \left| \left[[Q\bar{Q}]_1 [S[q\bar{q}]_1]_1 \right]_0 \right\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \left| D\bar{D}(^1S_0) \right\rangle \\ \left| D^*\bar{D}^*(^1S_0) \right\rangle \end{pmatrix} \tag{13}$$