# Diquark mass and quark-diquark potential of the $1^+$diquark in $\Sigma_c$ from Lattice QCD

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# Introduction

# Diquark is a composite particle made of two quarks



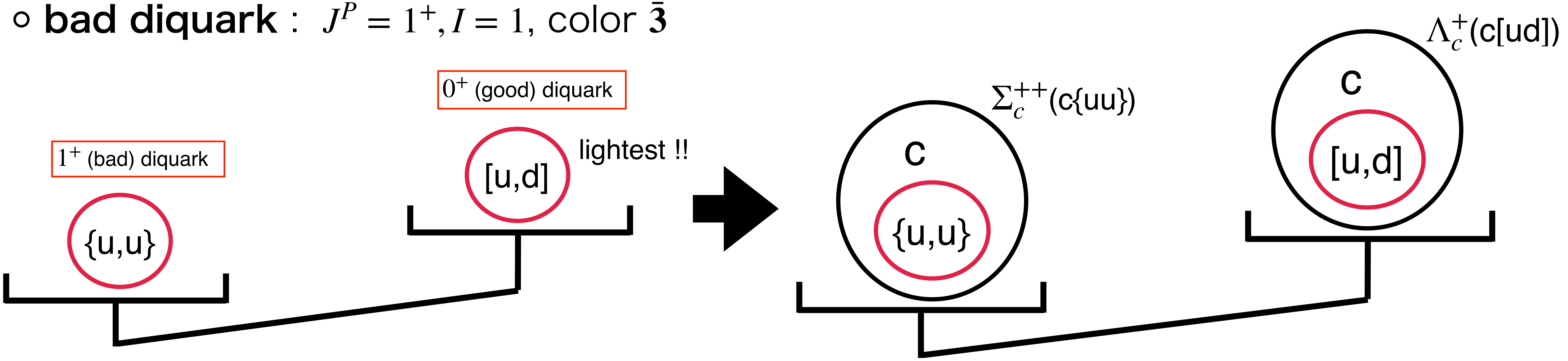
- Diquarks are colored particles

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \cancel{\mathbf{6}}$$

Color $\mathbf{6}$ diquarks are considered to be much heavy.

- **Phenomenologically important diquarks**:
  - good diquark : $J^P = 0^+, I = 0$, color $\bar{\mathbf{3}}$
  - **bad diquark** : $J^P = 1^+, I = 1$, color $\bar{\mathbf{3}}$

favored by (1) one-gluon exchange interaction
(2) instanton-induced interaction




- **Experimental study** : Difficult because of color confinement

# Lattice QCD studies of diquarks



Conventional method **may not work** in obtaining diquark mass due to **color confinement**

—- We should not assume that 2-point functions have a pole for colored particles.

$$\boxed{\mathrm{F.T.}\left\langle D(x)D^{\dagger}(0)\right\rangle = \cancel{\frac{i}{q^2 - m^2 + i\epsilon}} + \cdots}$$

cf. Review on quark mass in PDG2020

Diquark masses have been treated as follows.

- M.Hess et al.,PRD.58.11502

  Landau gauge fixing is employed

  Diquark mass is naively obtained from two-point function as a pole-mass.

- C.Alexandrou et al., PRL.97.222002

  A static quark is added to neutralize the system.

  Diquark mass is obtained by neglecting interaction energy between a diquark and a static quark.

- K.Watanabe, PRD.105.074510

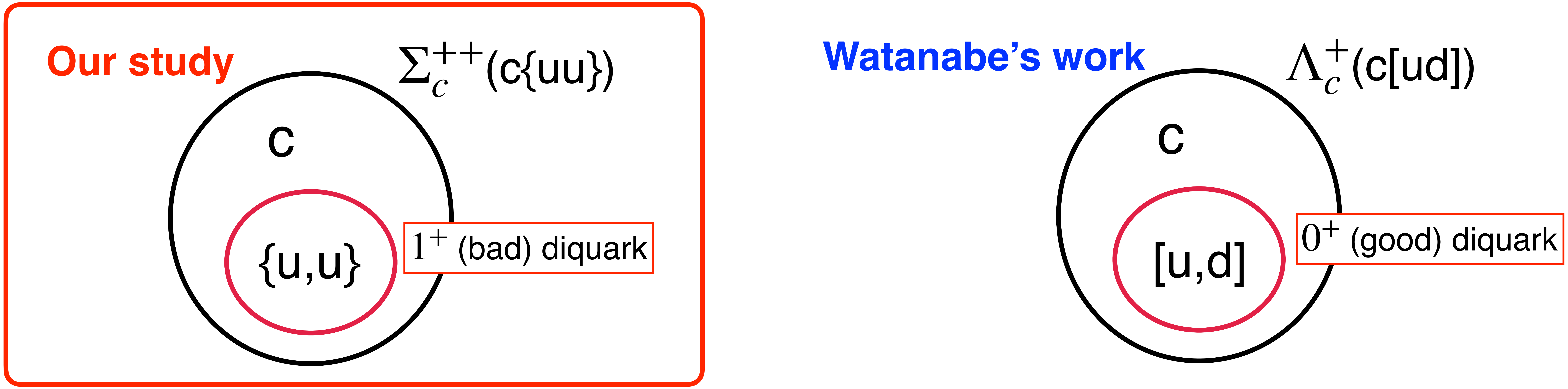
  $0^+$ diquark mass is treated as a mass parameter of a quark-diquark model

  which is constructed by an extended HAL QCD's potential method.

# Our goal



- **We study $1^+$ diquark in $\Sigma_c^{++}(c\{uu\})$.**
  We employ a similar strategy as K.Watanabe, PRD105
  where $0^+$ diquark in $\Lambda_c(c[ud])$ was studied.




We obtain

- **quark-diquark potential**
  by an extended HAL QCD's potential method from equal-time NBS wave function.
- $1^+$ **diquark mass** as a mass parameter of a quark-diquark model.
  Diquark mass is determined by employing a similar prescription
  which was proposed by T.Kawanai and S.Sasaki in ccbar sector.

# Formalism

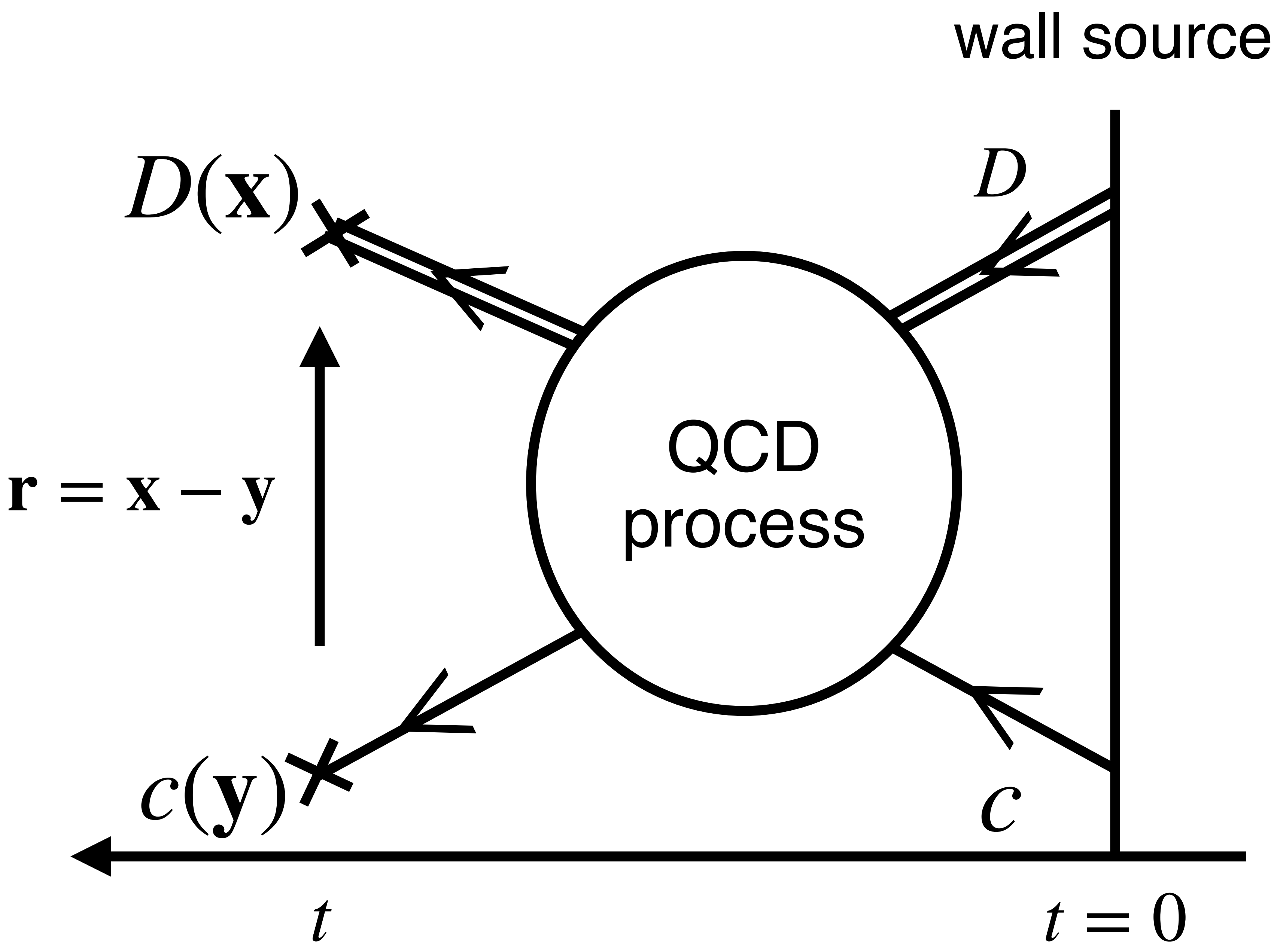
# Quark-diquark wave function



- **Quark-diquark 4-point function** and its spectral decomposition

$$C(\mathbf{x}-\mathbf{y},t) \equiv \left\langle 0 \left| \mathrm{T}\left[ D(\mathbf{x},t)\, c(\mathbf{y},t) \cdot \bar{c}(t=0)\, D^{\dagger}(t=0) \right] \right| 0 \right\rangle$$

$$= \sum_n \psi_n(\mathbf{x}-\mathbf{y})\, a_n \exp(-M_n t)$$




- $1^+$ diquark operator

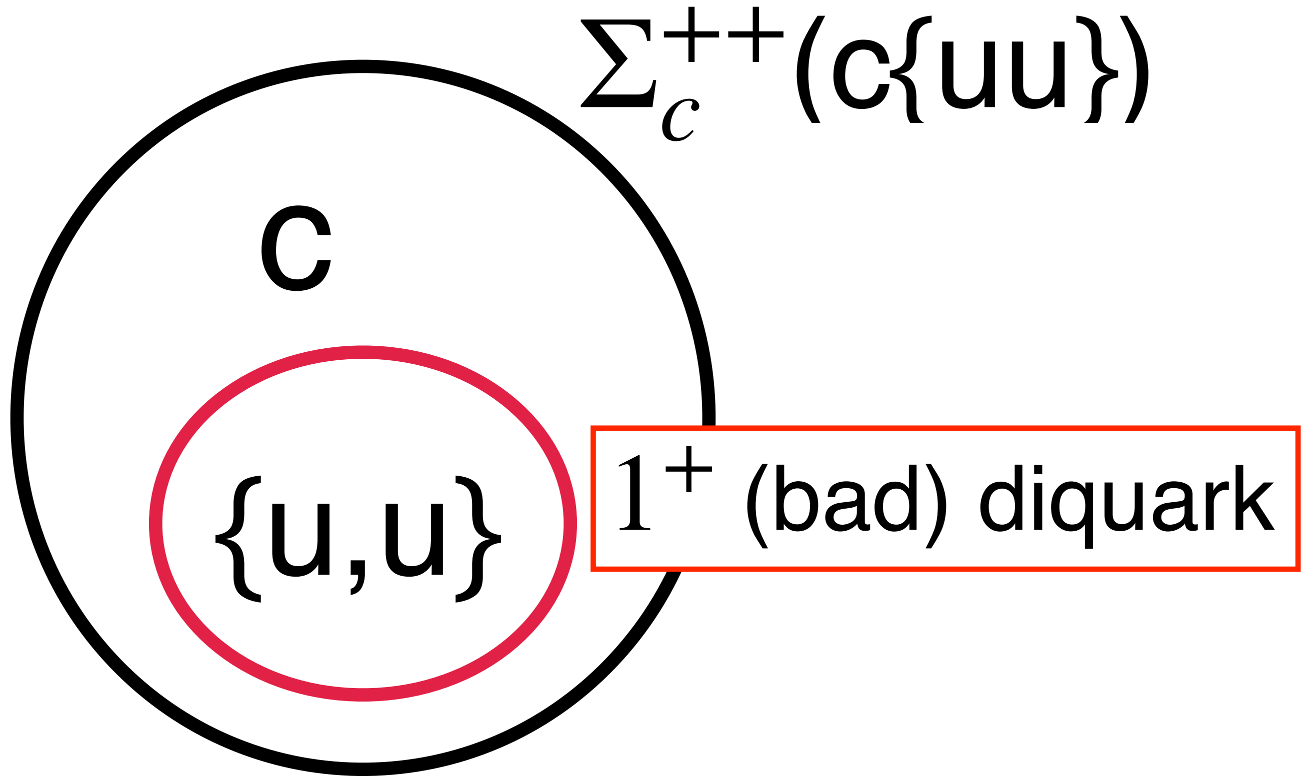
$$D_{ai}(x) \equiv \epsilon_{abc} u_b^T(x) C\gamma_5\gamma_i u_c(x)$$

- **Equal time quark-diquark Nambu-Bethe-Salpeter (NBS) wave function** for $\Sigma_c^{++}$

$$\psi_{i\alpha}(\mathbf{x}-\mathbf{y}) \equiv \langle 0 \,|\, D_{ai}(\mathbf{x})\, c_{a\alpha}(\mathbf{y}) \,|\, \Sigma_c \rangle$$

**Rarita-Schwinger form** is used for $\psi$.
Spin of swave $\Sigma_c$ is 1/2 or 3/2.

$$\boxed{\mathbf{1} \otimes \frac{\mathbf{1}}{\mathbf{2}} = \frac{\mathbf{1}}{\mathbf{2}} \oplus \frac{\mathbf{3}}{\mathbf{2}}}$$



# Conventional HAL QCD's potential method

We demand that **NBS wave function** should satisfy Schroedinger eq.

$$\left(-\frac{\nabla^2}{2\mu}+\hat{V}\right)\psi(\mathbf{r}) = E\,\psi(\mathbf{r}) \quad \text{with} \quad \hat{V} \simeq V_0(r) + V_S(r)\mathbf{s}_c\cdot\mathbf{s}_D$$

which split into

**Schroedinger eq. in each channel**

$$\left(-\frac{\nabla^2}{2\mu}+V_0(\mathbf{r})-V_s(\mathbf{r})\right)\psi_{1/2}(\mathbf{r}) = \left(M_{1/2}-m_c-m_D\right)\psi_{1/2}(\mathbf{r}) \qquad (J=1/2)$$

$$\left(-\frac{\nabla^2}{2\mu}+V_0(\mathbf{r})+\frac{1}{2}V_s(\mathbf{r})\right)\psi_{3/2}(\mathbf{r}) = \left(M_{3/2}-m_c-m_D\right)\psi_{3/2}(\mathbf{r}) \qquad (J=3/2)$$

We solve them inversely for potentials

- **Central potential**

$$V_0(\mathbf{r}) = \frac{2M_{3/2}+M_{1/2}}{3} - m_c - m_D + \frac{1}{2\mu}\left(\frac{2}{3}\frac{\nabla^2\psi_{3/2}(\mathbf{r})}{\psi_{3/2}(\mathbf{r})} + \frac{1}{3}\frac{\nabla^2\psi_{1/2}(\mathbf{r})}{\psi_{1/2}(\mathbf{r})}\right)$$

- **Spin dependent potential**

$$V_s(\mathbf{r}) = \frac{2}{3}(M_{3/2}-M_{1/2}) - \frac{1}{3\mu}\left(\frac{\nabla^2\psi_{3/2}(\mathbf{r})}{\psi_{3/2}(\mathbf{r})} - \frac{\nabla^2\psi_{1/2}(\mathbf{r})}{\psi_{1/2}(\mathbf{r})}\right)$$

We encounter a **PROBLEM**:

**Choice of $m_c$ and $m_D$ is not obvious.** (2-point function should not be used due to confinement.)

$$\mathbf{s}_c\cdot\mathbf{s}_D = \begin{cases} -1 & (J=1/2) \\ 1/2 & (J=3/2) \end{cases}$$

- charm quark mass $m_c$
- $1^+$ diquark mass $m_D$
- "binding energy" $E \equiv m_{\Sigma_c} - m_c - m_D$
- reduced mass $\mu \equiv \dfrac{1}{1/m_c + 1/m_D}$
- for $J = 1/2, 3/2$
  baryon mass $M_J$
  wave function $\psi_J$

# Kawanai-Sasaki prescription to determine diquark mass



Kawanai and Sasaki proposed a self-consistent method to determine quark mass. (PRL107.091601)

By applying their prescription to quark-diquark system, we demand

$$V_s(\mathbf{r}) = \frac{2}{3}(M_{3/2} - M_{1/2}) - \frac{1}{3\mu}\left(\frac{\nabla^2\psi_{3/2}(\mathbf{r})}{\psi_{3/2}(\mathbf{r})} - \frac{\nabla^2\psi_{1/2}(\mathbf{r})}{\psi_{1/2}(\mathbf{r})}\right) \to 0 \quad \text{as } |\mathbf{r}| \to \infty$$

This leads to

**Kawanai-Sasaki condition for quark-diquark system**

$$\mu = -\lim_{r\to\infty} \frac{1}{2(M_{3/2} - M_{1/2})}\left(\frac{\nabla^2\psi_{3/2}(\mathbf{r})}{\psi_{3/2}(\mathbf{r})} - \frac{\nabla^2\psi_{1/2}(\mathbf{r})}{\psi_{1/2}(\mathbf{r})}\right)$$

$\Rightarrow$

Using $m_c$ from Kawanai-Sasaki prescription in $c\bar{c}$ sector.

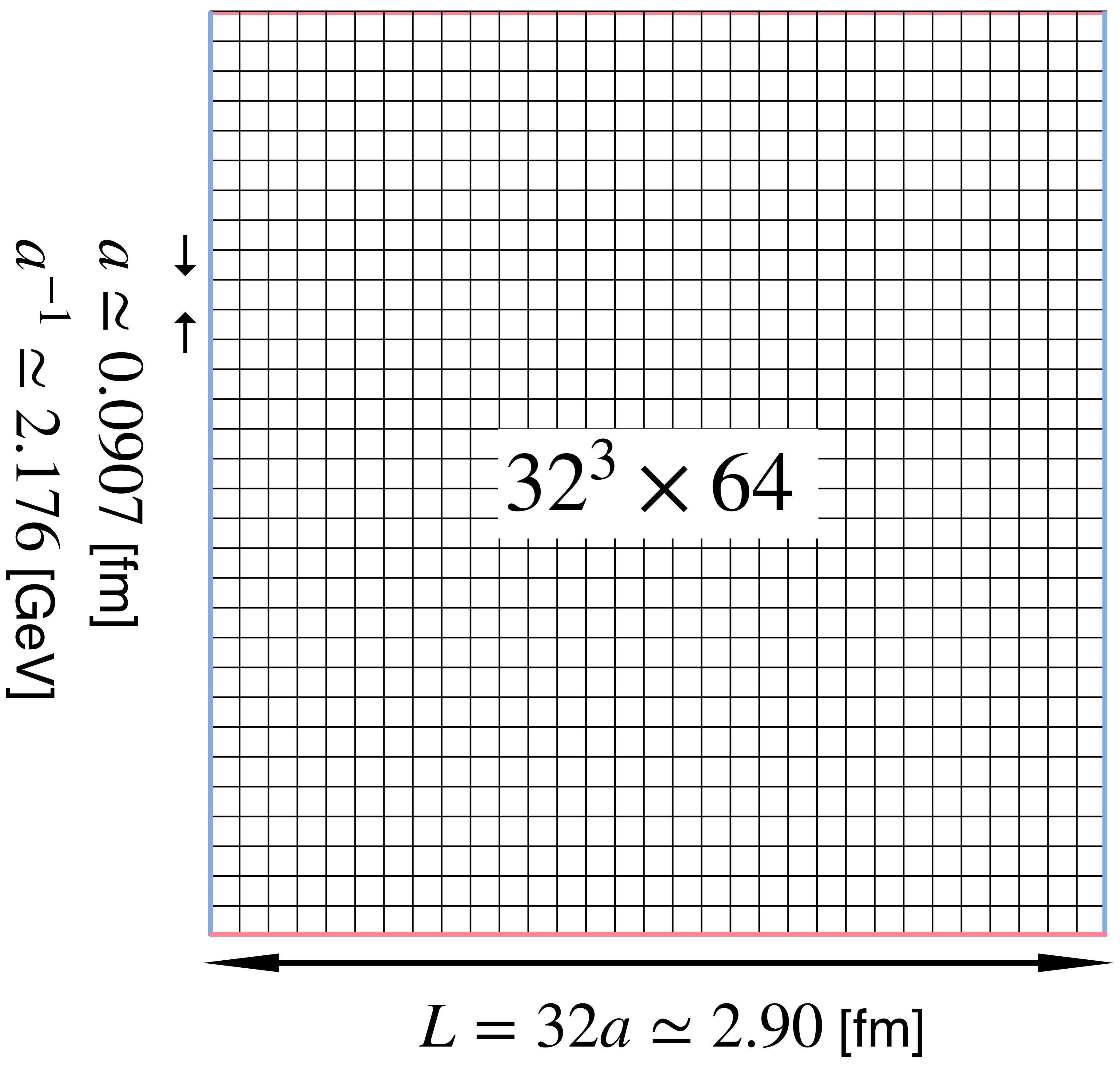
$$m_D = \frac{1}{1/\mu - 1/m_c}$$

- Combining HAL QCD's potential method with Kawanai-Sasaki prescription provides a **self-consistent way** to obtain **diquark mass** and **quark-diquark potential**
- This approach avoids the issue of pole-mass in 2-point function of diquark. (Diquark mass is obtained as a mass parameter of quark-diquark model)

# Numerical Results

# Lattice QCD setup

- 2+1 flavor QCD gauge config. on $32^3 \times 64$ lattice [Ukita et al., PACS-CS Coll., PRD.79.034503]
  - RG improved Iwasaki gauge action $(\beta = 1.90)$
    $O(a)$-improved Wilson quark action $(\kappa_{ud} = 0.137,$
    $C_{SW} = 1.715)$
  - Lattice spacing $a = 0.0907$ fm
    $1/a = 2.176$ GeV
    Spatial extension $L = 2.90$ fm
- Charm quark added with quenched approx.
  Relativistic heavy quark action
  [Namekawa et al., PACS-CS Coll., PRD.84.074505]
- Coulomb gauge fixing is employed




These setup reproduce typical hadron mass

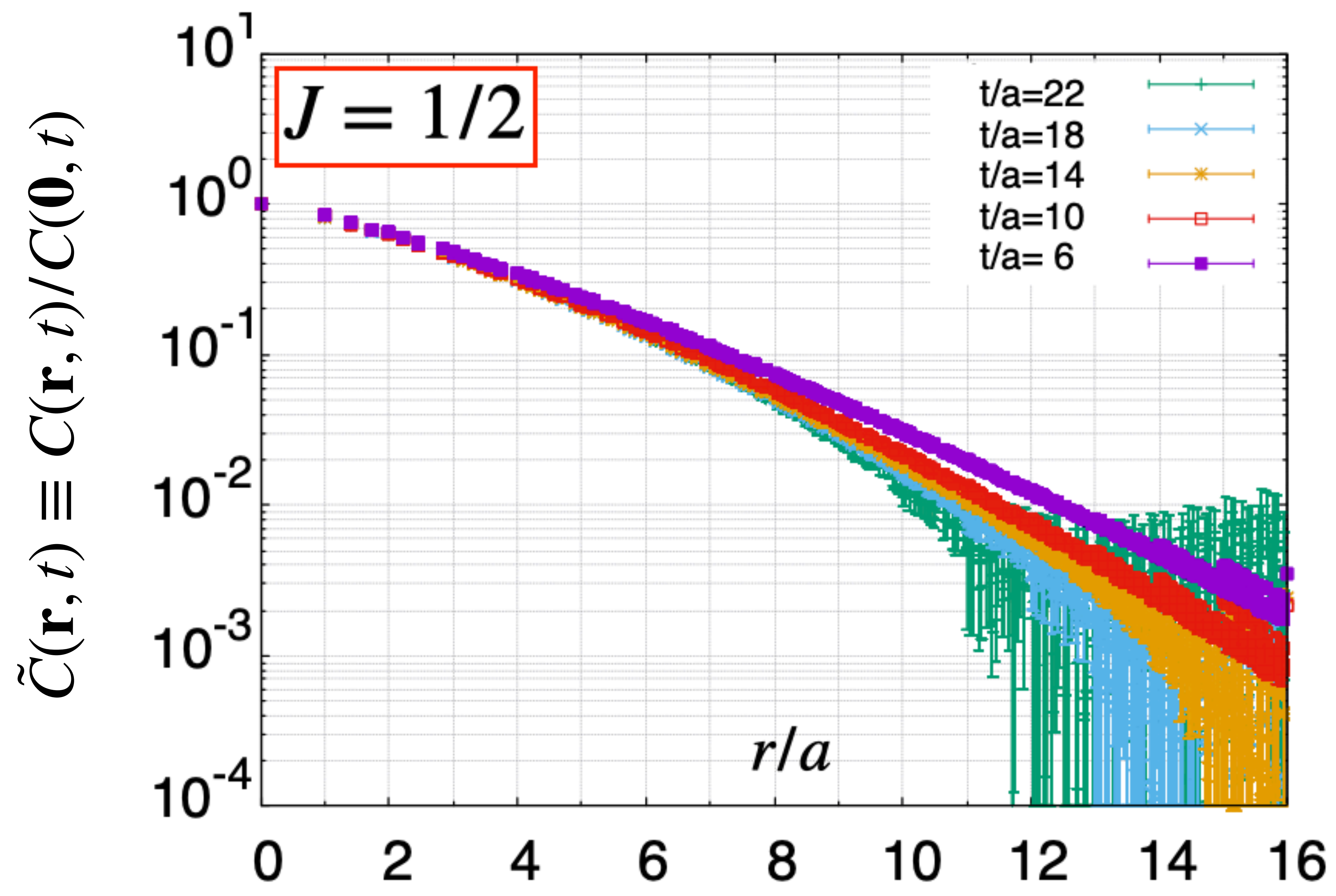
$m_\pi \sim 700\,\mathrm{MeV}$, $m_N \sim 1600\,\mathrm{MeV}$

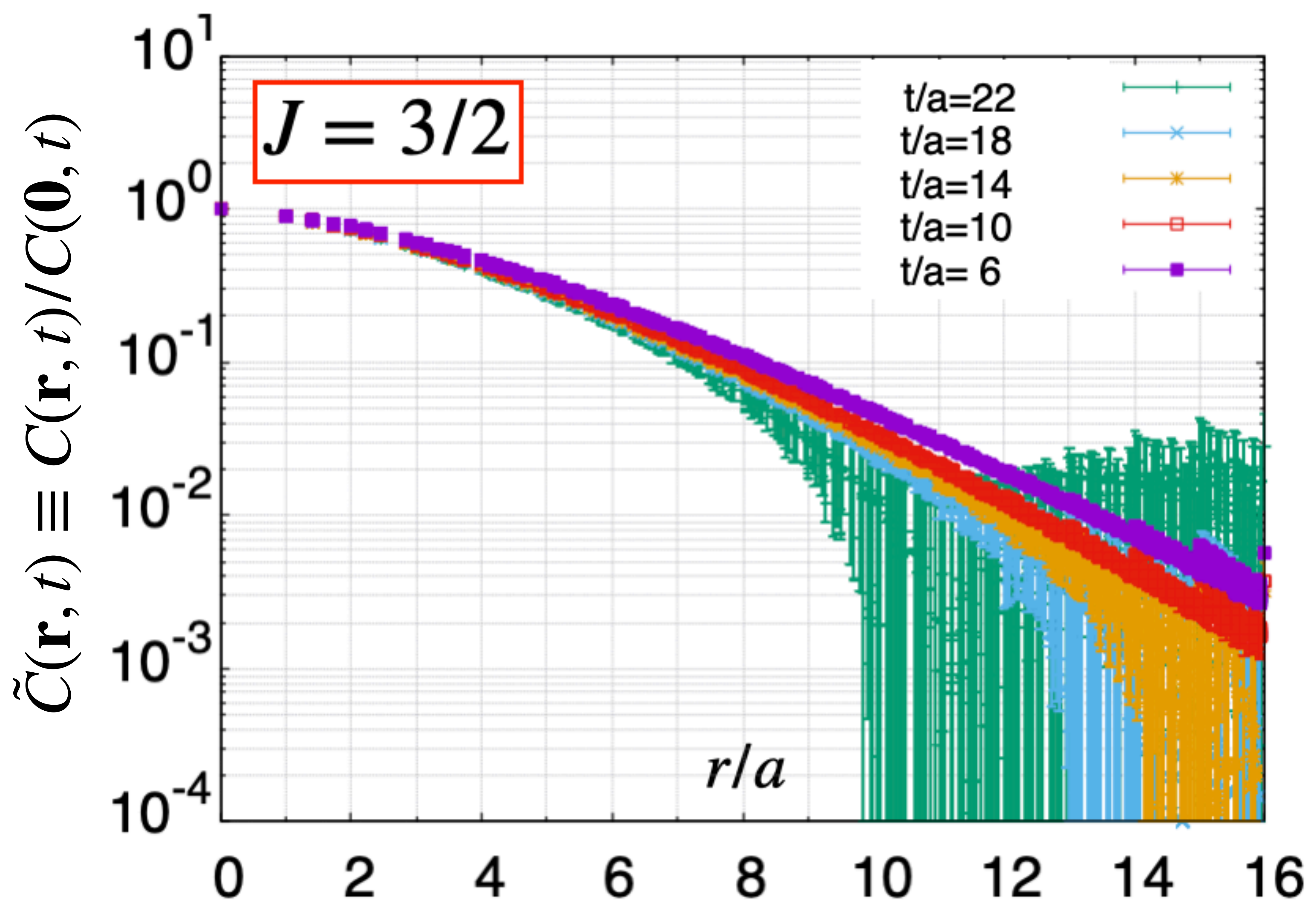
$m_{\eta_c} \sim 3025\,\mathrm{MeV}$, $m_{J/\psi} \sim 3144\,\mathrm{MeV}$

$m_{\Lambda_c} \sim 2691\,\mathrm{MeV}$

$m_{\Sigma_c(J=1/2)} \sim 2777\,\mathrm{MeV}$, $m_{\Sigma_c(J=3/2)} \sim 2859\,\mathrm{MeV}$

# 4-point function and NBS wave function



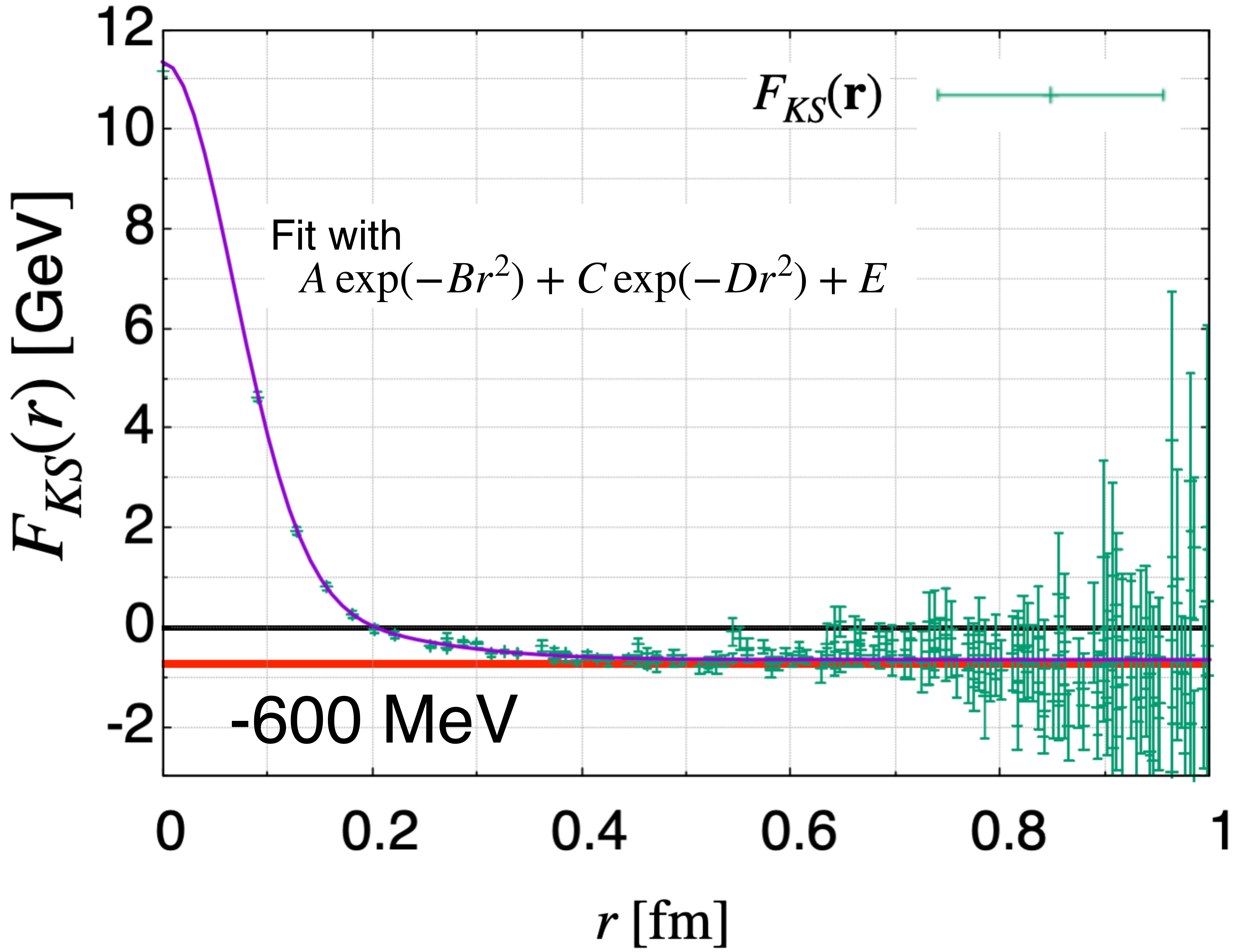

$$C(\mathbf{r},t) = \sum_n \psi_n(\mathbf{r})\, a_n e^{-M_n t}$$

$$\to \psi_0(\mathbf{r})\, a_0 e^{-M_0 t} \quad \text{for large } t$$

Rough convergence is achieved at t/a = 18 in the region $r < 10a \sim 0.9\,\mathrm{fm}$.

**4-point func. at t/a=18** is accepted as a converged **NBS wave func.**
(t/a=22 has too large error bar to be accepted)

# Determination of diquark mass




Kawanai-Sasaki function

$$F_{\rm KS}(\mathbf{r}) \equiv \frac{1}{2(M_{3/2} - M_{1/2})}\left(\frac{\nabla^2\psi_{3/2}(\mathbf{r})}{\psi_{3/2}(\mathbf{r})} - \frac{\nabla^2\psi_{1/2}(\mathbf{r})}{\psi_{1/2}(\mathbf{r})}\right)$$

**Reduced mass** is determined by

**Kawanai-Sasaki condition**

$$\mu = -\lim_{r\to\infty} F_{\rm KS}(\mathbf{r}) \sim 600\,\mathrm{MeV}$$

**Diquark mass**

$$m_D = \frac{1}{1/\mu - 1/m_c} \simeq 867\,\mathrm{MeV}$$

where charm quark mass

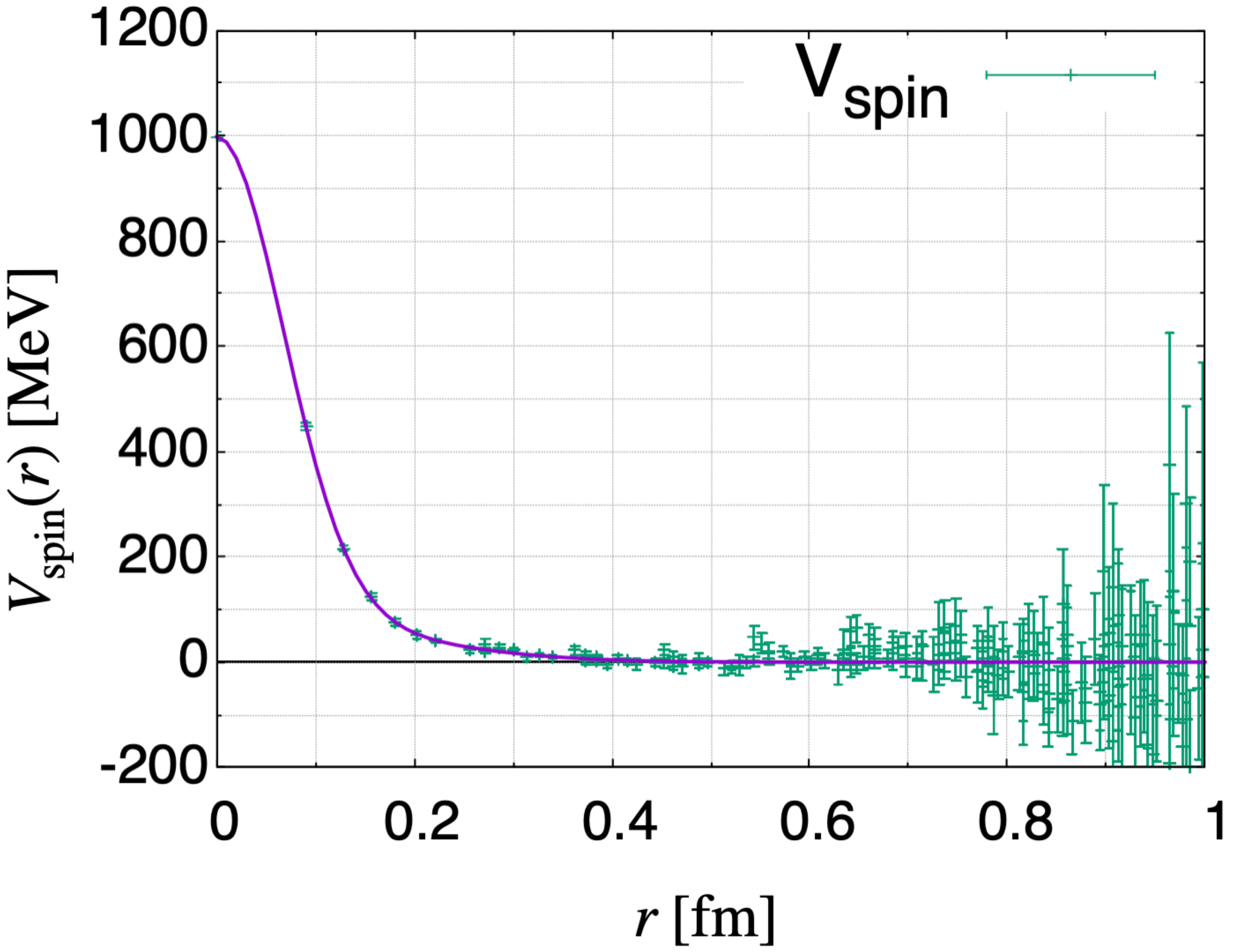
$m_c \simeq 1950\,\mathrm{MeV}$ (from ccbar sector)

We feel that $m_D$ might be a bit underestimated because of large noise of $F_{KS}$ at large $r$

# Spin-dependent (quark-diquark) potential



## $V_{\mathrm{spin}}$ is short-ranged

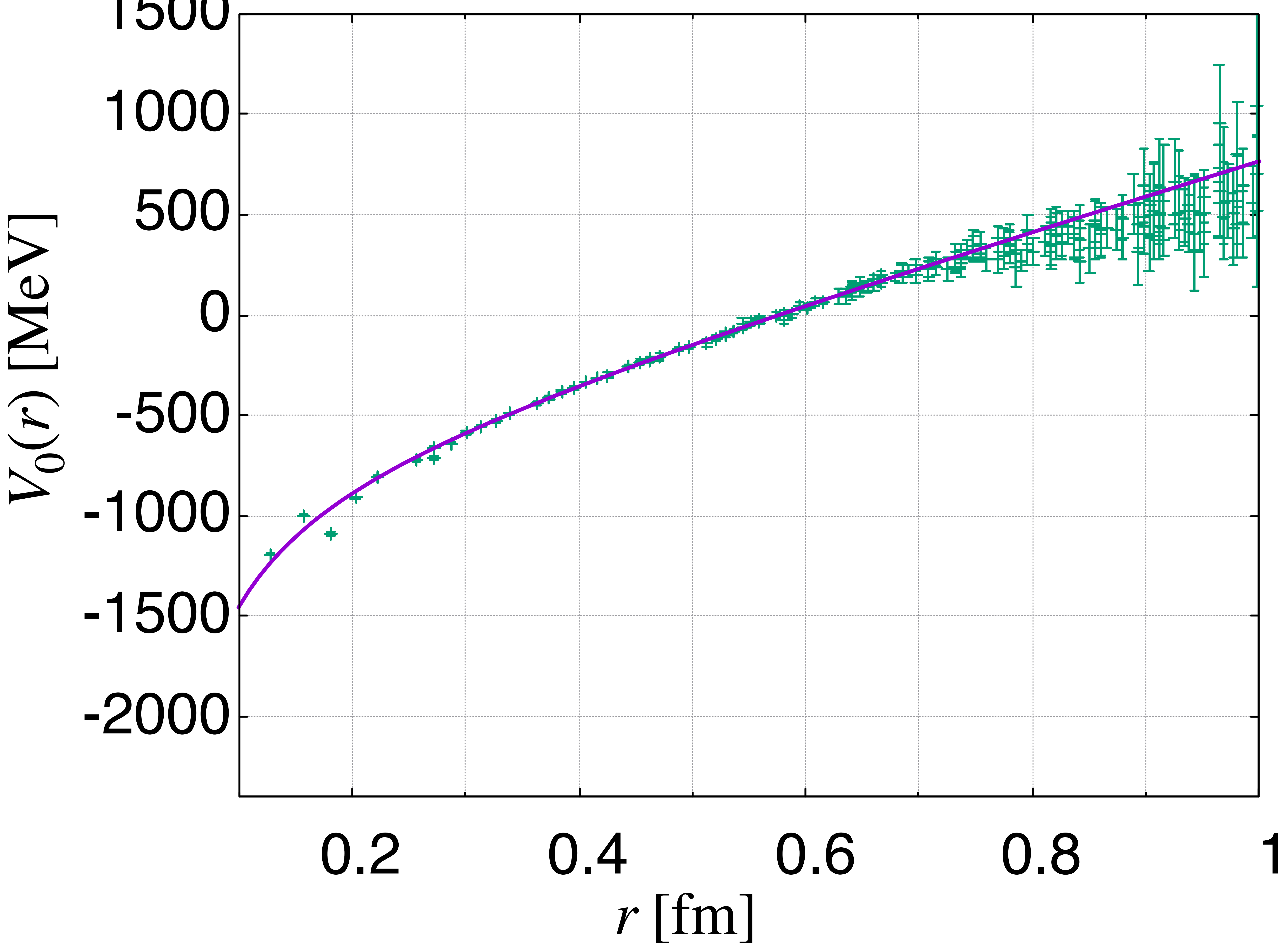



fit with 2-gaussian func. form

$$A\exp(-Br^2) + C\exp(-Dr^2)$$

# Spin-indep. (quark-diquark) potential

## $V_0$ is of Cornell-type




Fit with Cornell type func. form

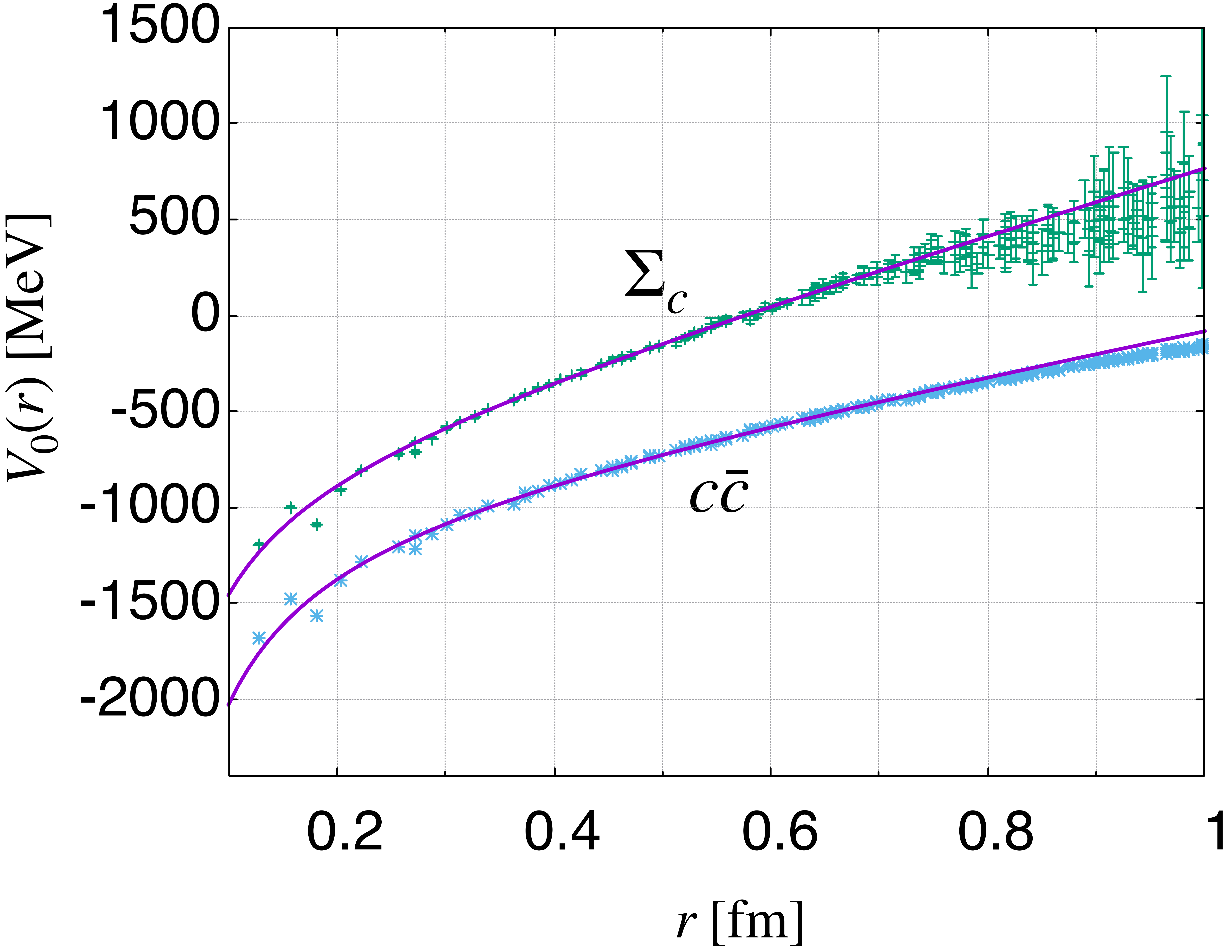
$$-A/r + \sigma r + \text{const}$$

$$A = 86 \text{ MeV/fm}$$

$$\sqrt{\sigma} = 565 \text{ MeV}$$

Our feeling:
$A$ and $\sigma$ may be overestimated
due to possible underestimate of $m_D$.

# Quark diquark potential vs $c\bar{c}$ potential




Fitting function

$$-A/r + \sigma r + \text{const}$$

Coulomb coefficient

$\Sigma_c$ $A = 86$ MeV/fm

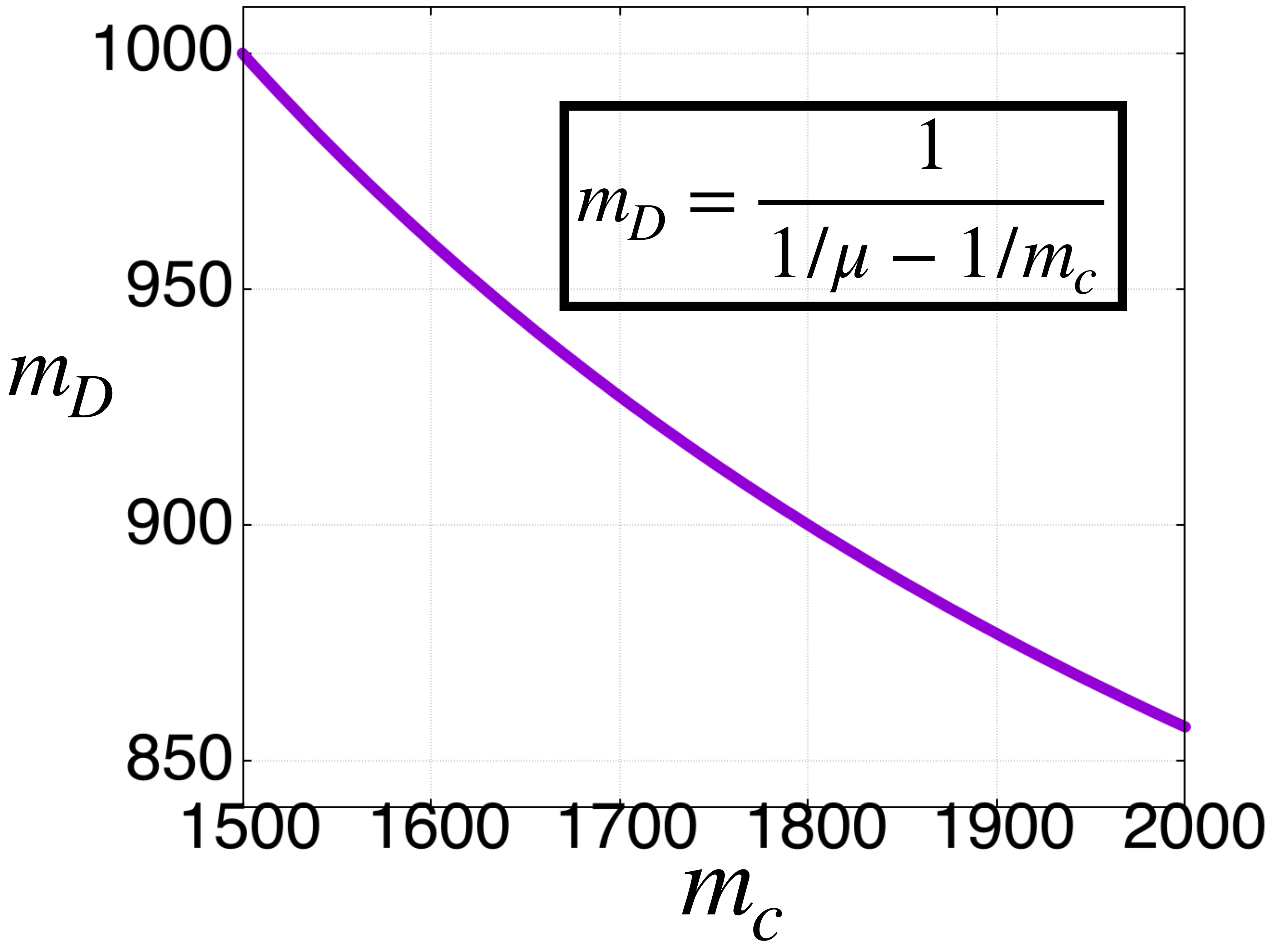
$c\bar{c}$ $A = 103$ MeV/fm

String tension

$\Sigma_c$ $\sqrt{\sigma} = 565$ MeV

$c\bar{c}$ $\sqrt{\sigma} = 459$ MeV

# $1^+$ diquark (this work) vs $0^+$ diquark (Watanabe's work)




Naive comparison

$$m_{1^+} = 867 \text{ MeV} \quad < \quad m_{0^+} = 1273 \text{ MeV}$$

This is contrary to our expectations !

Possible reasons:

- Watanabe uses **p-wave spectrum** to determine diquark mass and charm quark mass, whereas we use **Kawanai-Sasaki method** for this determination.

- Watanabe employs charm quark mass $m_c \sim 1840$ MeV
  ours is $m_c \sim 1950$ MeV

  Note: Larger charm quark mass results in smaller diquark mass.

- $F_{KS}(\mathbf{r})$ has large error at long distance
  leading to an uncertainty in determining the constant part.

# $1^+$ diquark (this work) vs $0^+$ diquark (Watanabe's work) [cont'd]



If we employ Watanabe's value of the charm quark mass $m_c = 1840$

$m_{1^+} = 867$ MeV → about $900$ MeV    (< 1273 MeV)

Mass increases slightly but it is not enough.

## Possible way out in the future:

- Improve the statistical noise of $F_{\mathrm{KS}}(\mathbf{r})$ at long distance to improve the value of $\mu$.
- Use p-wave spectrum to determine also the $1^+$ diquark mass

# Summary

- **An extended HAL QCD method** was applied to $\mathbf{\Sigma_c}$ (c-{uu}) in the heavy quark mass region to study $1^+$ **diquark mass** and **quark-diquark potentials** by 2+1 flavor lattice QCD
  - $1^+$**diquark mass** was obtained by using **Kawanai-Sasaki prescription.**
  - **Central potential is of Cornell type**
  - **Spin dependent potential is short-ranged**
  - $m_D \sim 867\,\mathrm{MeV}$  $\sqrt{\sigma} = 565$ MeV  $A = 86$ MeV/fm

    Precise evaluation is needed.
    We feel that Diquark mass may be a bit underestimated. $\sigma$ and $A$ may be a bit overestimated.
    $F_{\mathrm{KS}}(r)$ is noisy at long distance

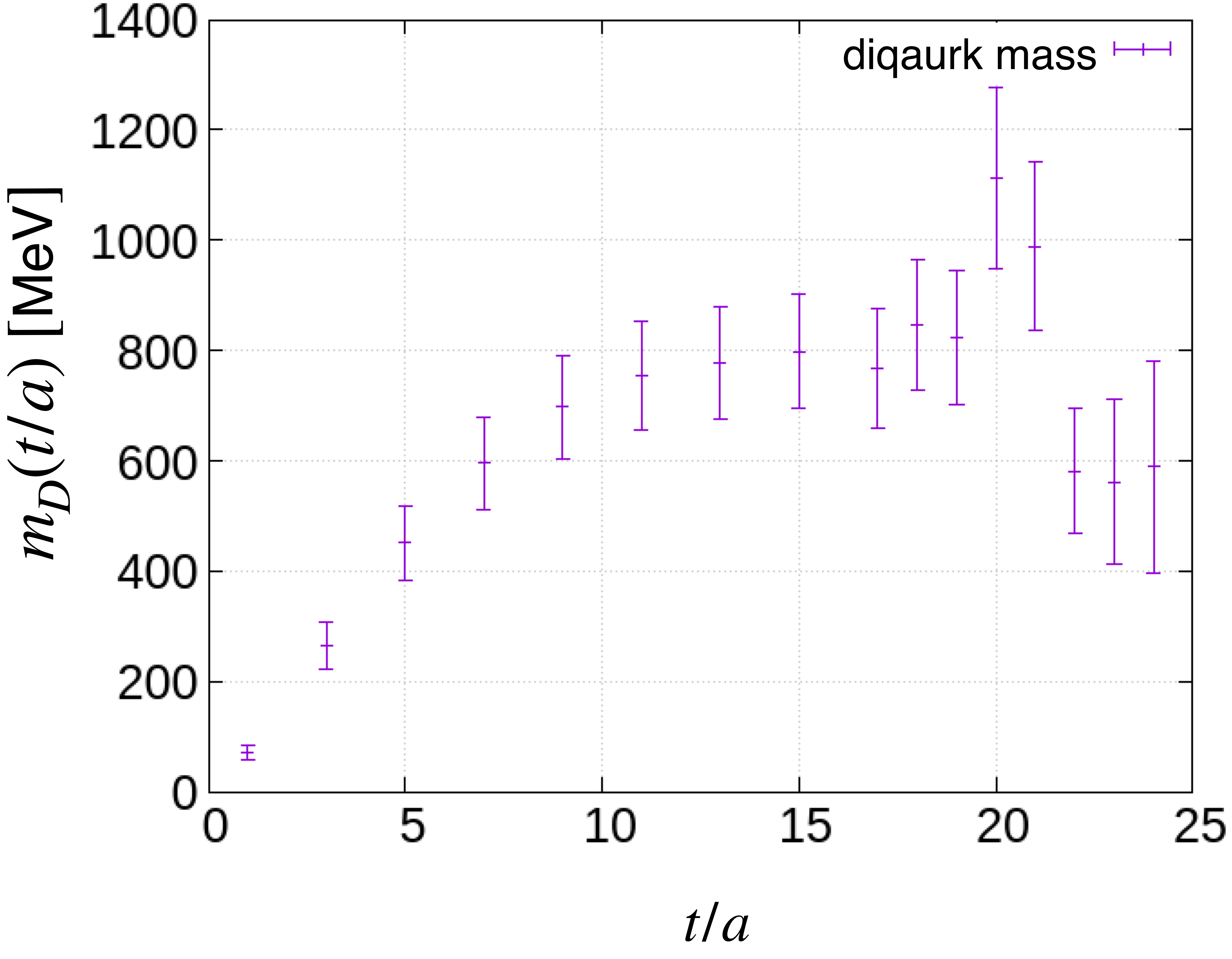
- **Future plans**
  - Quark mass dependence
  - Comparison of our $1^+$ diquark result with $0^+$ diquark in K.Watanabe, PRD.105.074510

    (He obtained $m_{0^+} \sim 1.27$ GeV by a similar but different method)
    Several things have to be fixed before the comparison
    - Watanabe employs $m_c \sim 1.840\mathrm{GeV}$ whereas ours is $m_c \sim 1.950\mathrm{GeV}$.
      (This is due to different formalism employed to obtain charm quark mass)
    - Large statistical noise of Kawanai-Sasaki function at long distance has to be improved for precise determination of $1^+$ diquark mass.

# Backup

# Determination of diquark mass

**$t$-dependence of the diquark mass $m_D$**




- $m_D$ increases with t approaching a constant value $m_D \sim 800$ MeV in the region $t \geq 15$.
- In the following slides, we employ $t/a \sim 18$, where sufficient convergence is achieved.

# 2 point function -> mass split of $\Sigma_c$

- 2-point function

$$G_{i\alpha j\beta}(t) \equiv \sum_{\vec{x}} \langle B_{i\alpha}(\vec{x}, t)\bar{B}_{j\beta}(0)\rangle$$

(Baryon operator $B_{i\alpha}(\vec{x}, \vec{y}, t) \equiv D_i^a(\vec{x}, t)c_{a\alpha}(\vec{y}, t)$)

- Let $g_J(t)$ be projection of this operator onto the spin $J$ state

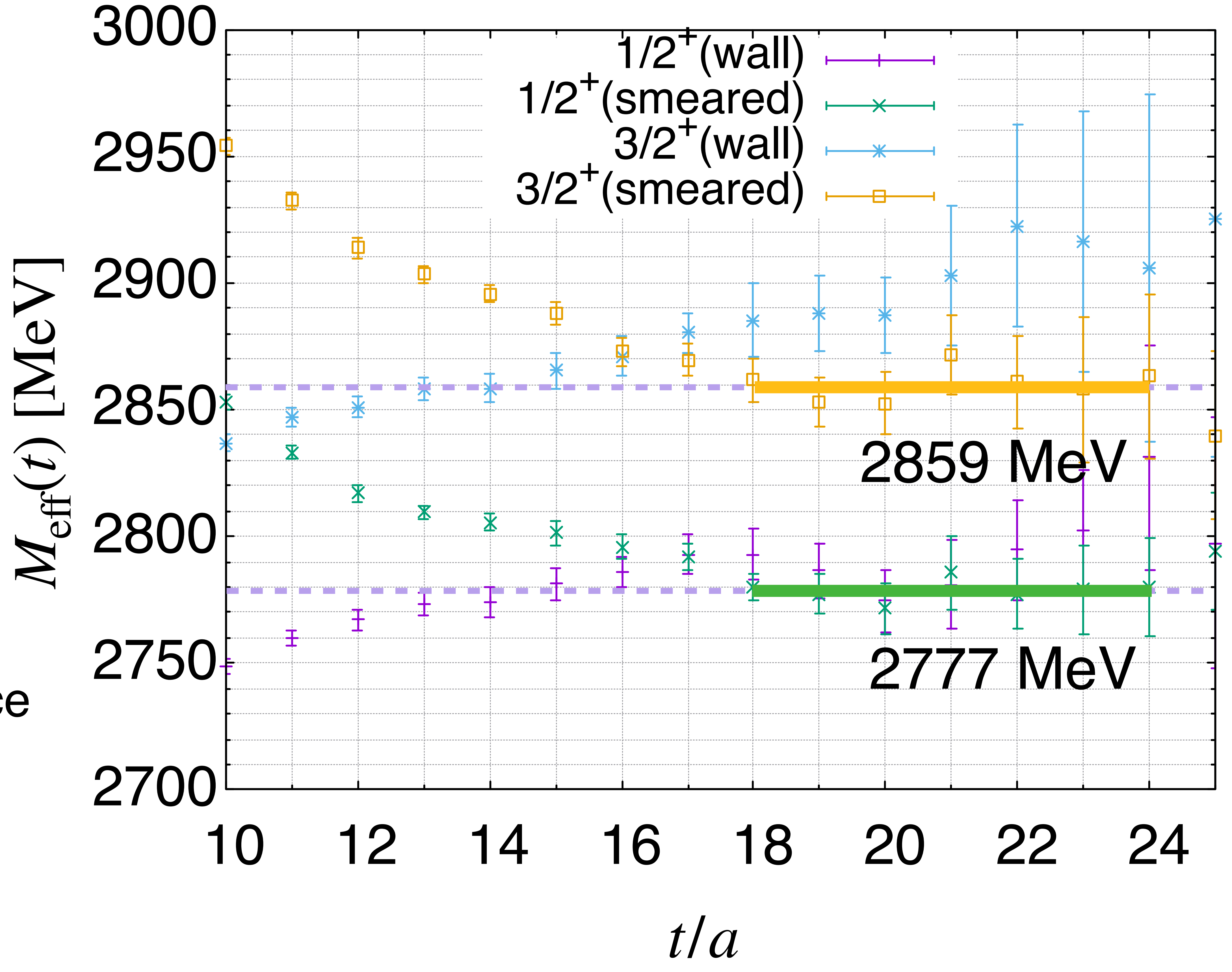
$$g_J(t) \xrightarrow{\text{large t}} Ae^{-M_J t}$$

- Take $M_J$ from the effective mass to obtain the mass difference

$$M^J_{\text{eff}}(t) \equiv \log \frac{g_J(t)}{g_J(t+1)}$$

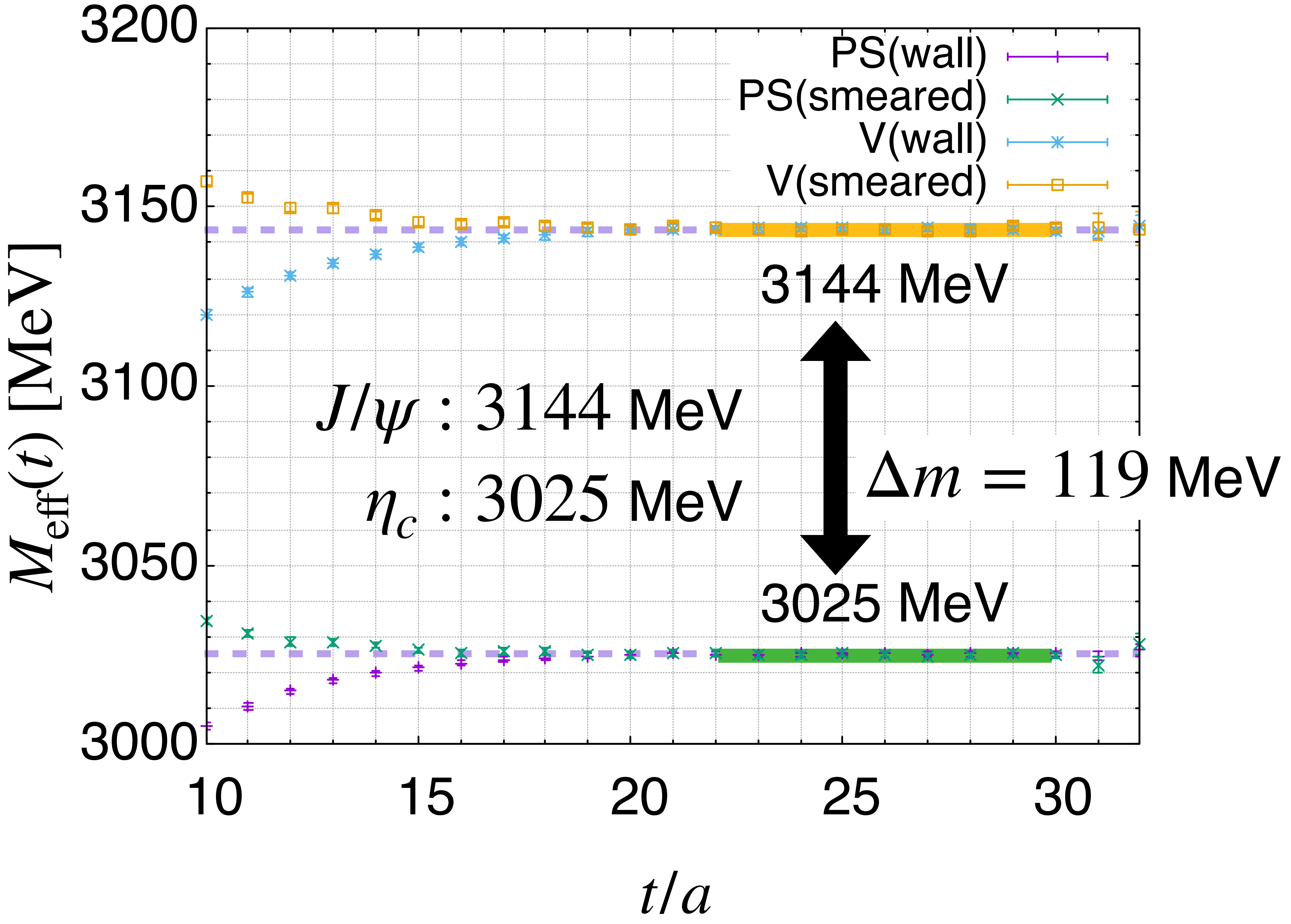
This is called the effective mass,
which asymptotically approaches $M_J$ in the large $t$ region.

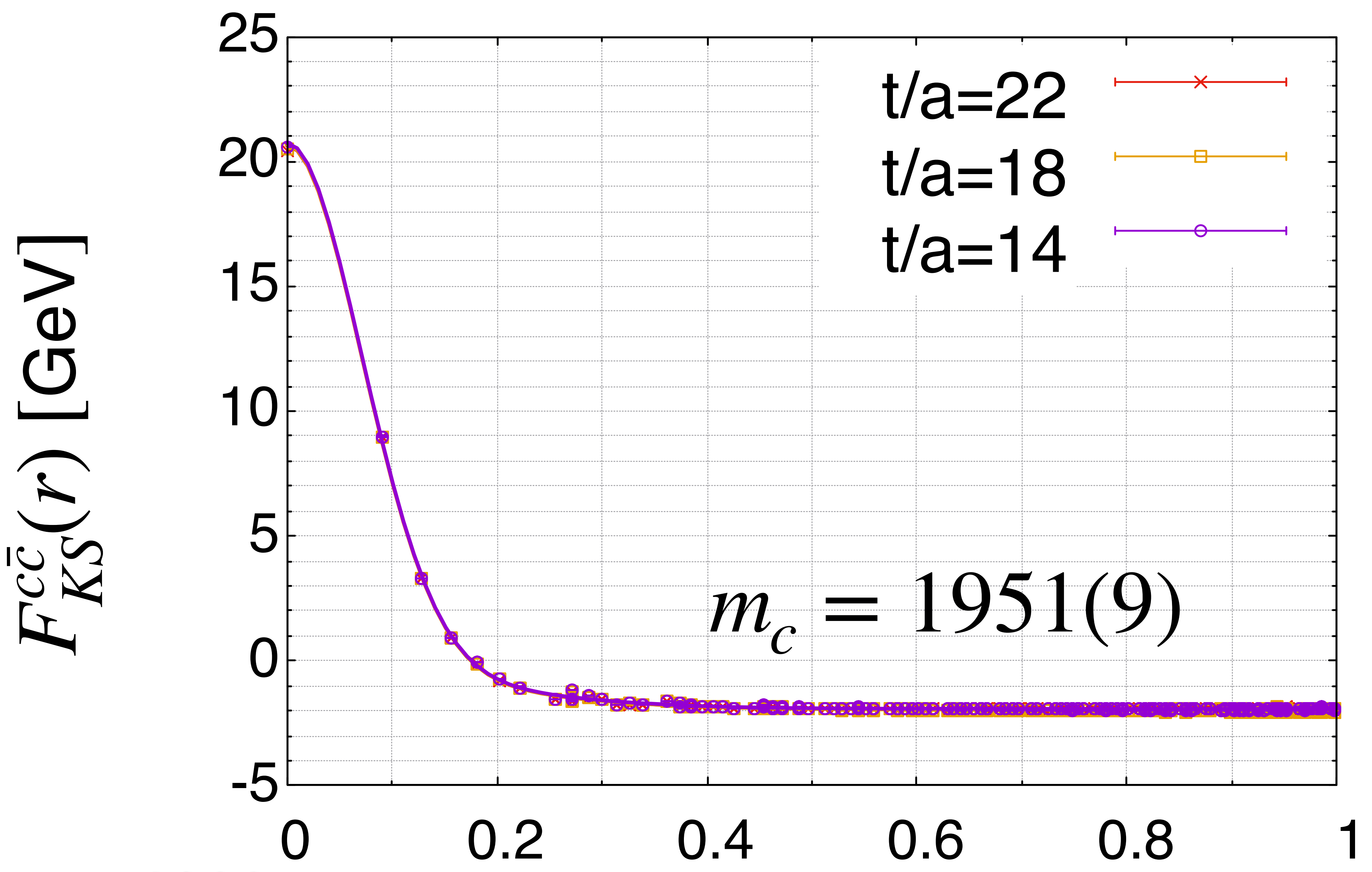
effective mass plot

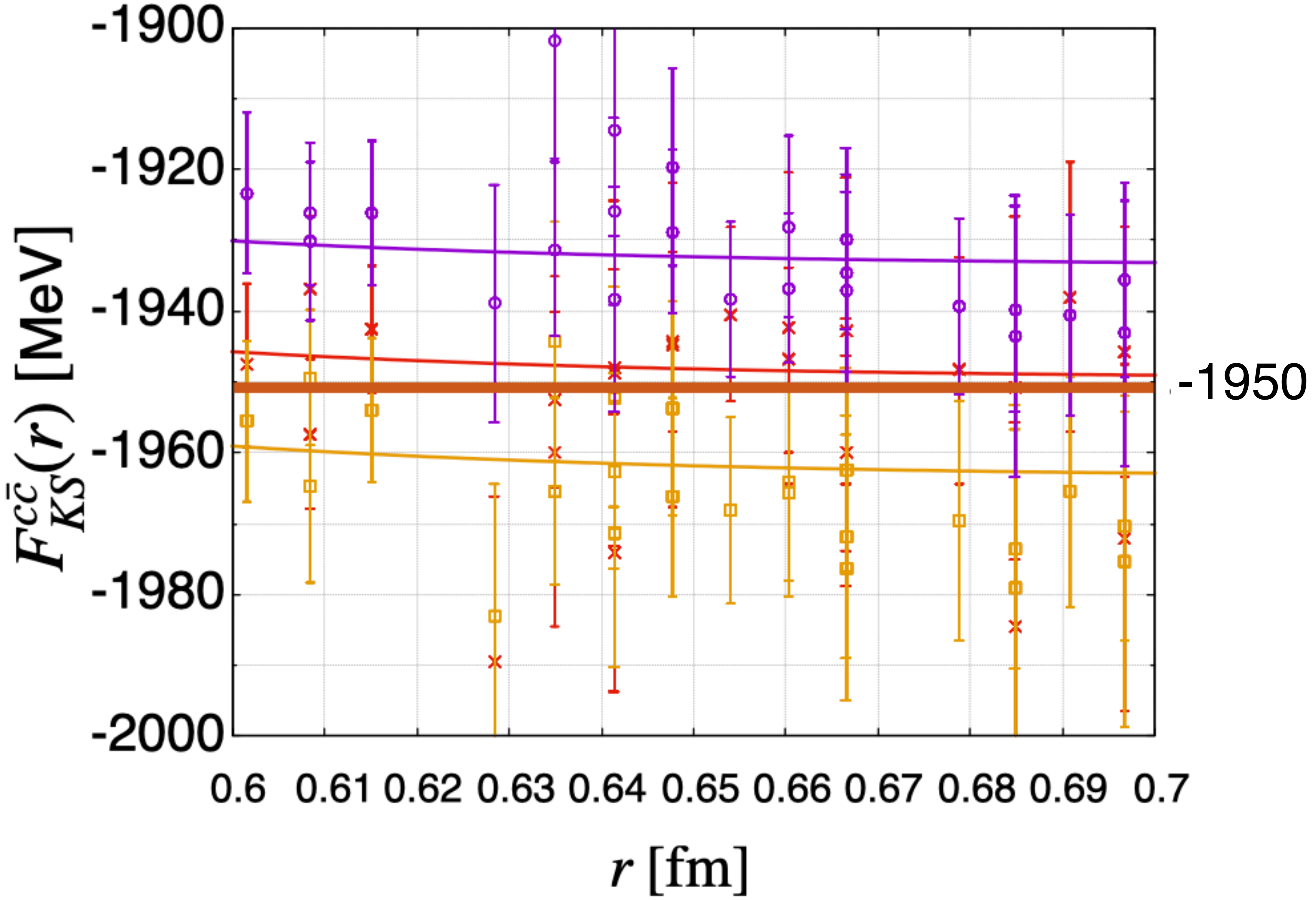



$$\boxed{M_{3/2} - M_{1/2} = 82(8)\ \text{MeV}}$$

# $c\bar{c}$ sector




**Kawanai-Sasaki condition**

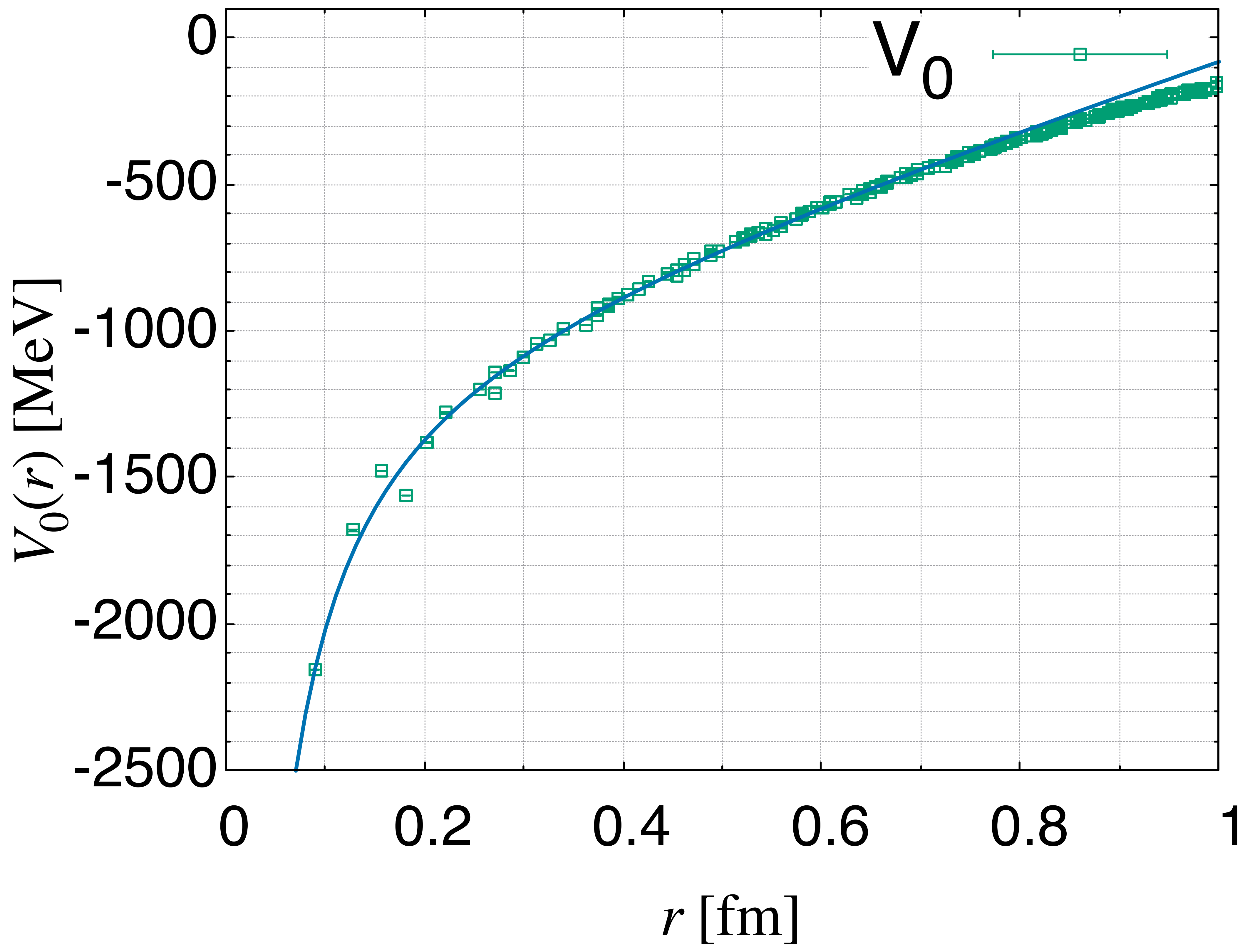
$$m_c = -\lim_{r\to\infty} F^{c\bar{c}}_{\text{KS}}(\mathbf{r})$$

$$F^{c\bar{c}}_{\text{KS}}(\mathbf{r}) \equiv \frac{1}{\Delta m}\left(\frac{\nabla^2\psi_{J/\psi}(\mathbf{r})}{\psi_{J/\psi}(\mathbf{r})} - \frac{\nabla^2\psi_{\eta_c}(\mathbf{r})}{\psi_{\eta_c}(\mathbf{r})}\right)$$

➡ **Charm quark mass** : $1950(9)$ MeV

# $c\bar{c}$ sector

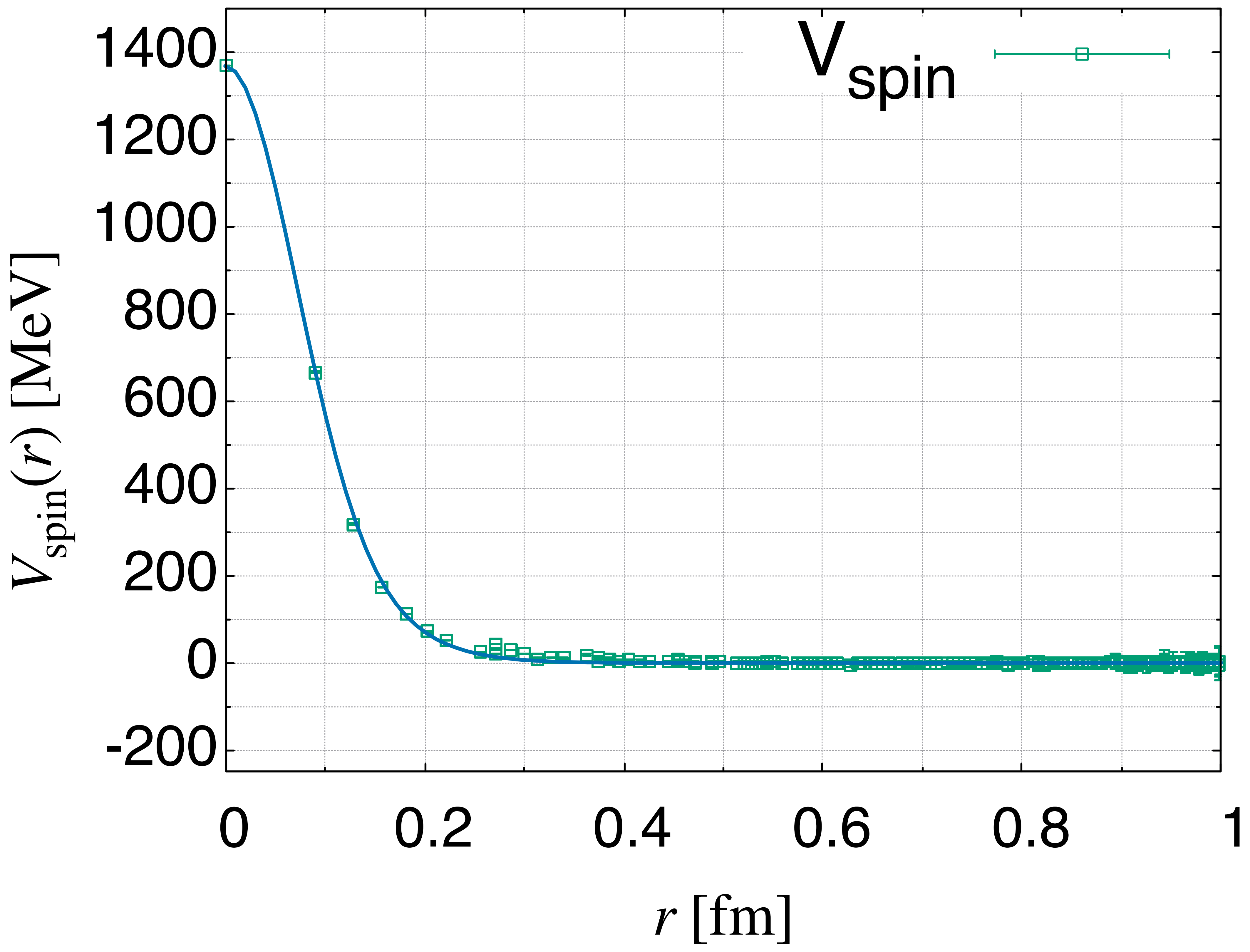
$$-A/r + \sigma r + \text{const}$$




$A \sim 0.526$ (Lattice unit)

$\sqrt{\sigma} = 459$ MeV

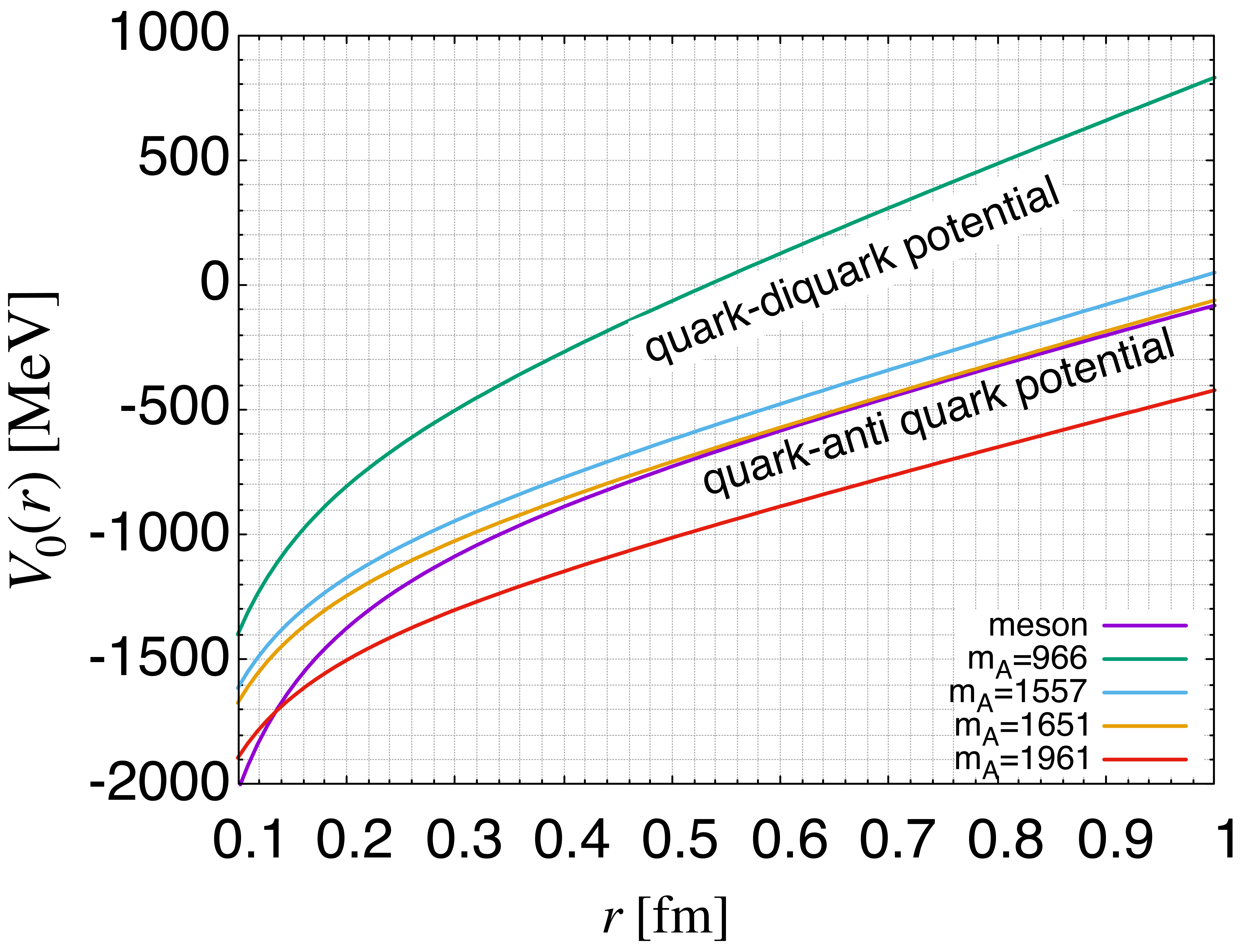
$$A\exp(-Br^2) + C\exp(-Dr^2)$$



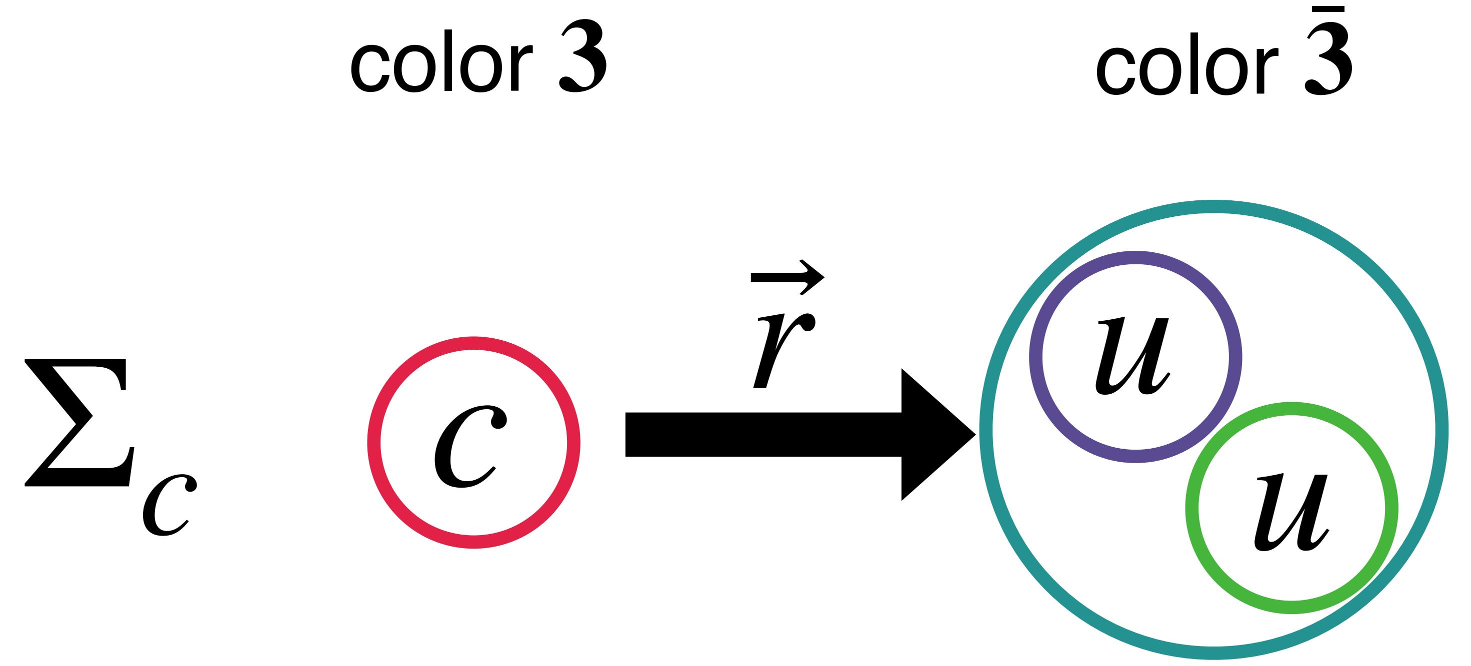

$A = 1283$ MeV, $B = 0.80$(Lattice unit)

$C = 90$ MeV, $D = 0.14$(Lattice unit)

# $m_D$ vs spin independent potential




| $m_A$ | 966 MeV | 1557 MeV | 1651 MeV | 1961 MeV |
|---|---|---|---|---|
| $\sqrt{\sigma}$ | 565 MeV | 488 MeV | 480 MeV | 459 MeV |




Far enough away, the behavior is the same.


Distance-dependent part of spin-independent potential

$$-A/r + \sigma r = \frac{1}{6\mu}\left(\tilde{V}(r) - \tilde{V}\left(\sqrt{\frac{A}{\sigma}}\right)\right)$$

where、$\tilde{V}(r) = \frac{\nabla^2 \psi_{1/2}}{\psi_{1/2}} + 2\frac{\nabla^2 \psi_{3/2}}{\psi_{3/2}}$

$m_D$ increase ➡ $\mu$ increase ➡ $\sigma$ decrease

Larger $m_D$ may be more natural