Diquark Mass and Quark-Diquark Potential from Lattice QCD

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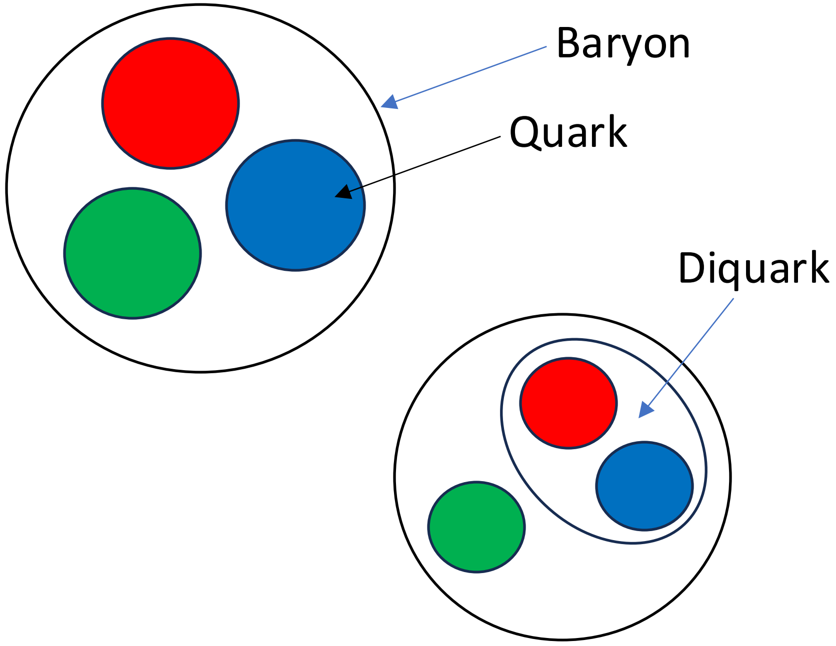
Introduction

Introduction: What is a Diquark?

- A coloured-object, so it cannot be isolated as asymptotic state due to color confinement in QCD.
- 2 (major) types of diquarks [1]:

- "Good" diquark: $\bar{3}_F,\ \bar{3}_c,\ J^P = 0^+$ (scalar)
- "Bad" diquark: $6_F,\ \bar{3}_c,\ J^P = 1^+$ (axial-vector)



Why diquark?

- It is a useful building blocks for phenomenological descriptions of hadronic states. Diquark pictures is very useful in understanding baryons and even exotics.
- "Good" diquark condenses in high density region, forming color superconductor.

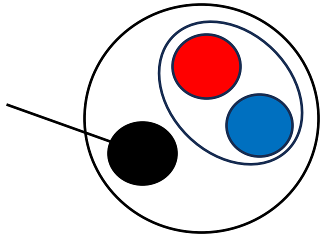
Ref:

1. R.L. Jaffe, Phys. Rep. 409 (2005)

Background

Background:

Previous work 0: *M. Hess et al, PRD58 (1998), Y. Bi et al,* Chin. Phys. C 40 (2016) :



They naively calculate diquark mass from two-point correlator in Landau gauge.

- They do no seem to take account the color confinement seriously.
- We shall not assume the existence of particle pole in two-point correlator. (R.L. Workman, PTEP 2022)

Previous work 1: *C Alexandrou et al, PRL 2006, A. Francis*, JHEP 2022 :

They calculated the *mass differences* between various diquark channels with a spectator static quark.

- The binding energy between the static quark and the diquark is neglected contaminating the diquark mass.

Previous work 2: *K. Watanabe*, PRD 2022 :

An extended HALQCD potential method is applied to Λ_c baryon as a charm – scalar diquark system (*cD*) with a self-consistent determination of scalar diquark mass.

- Diquark mass is separated from the binding energy
- Charm quark mass --> way of determination is not unique --> affects the diquark mass

This work:

- We modify K. Watanabe, PRD2022 by considering a bound system of a static quark and a scalar diquark to avoid the uncertainty in the charm quark mass.

Extended HALQCD potential method

Equal-time Nambu-Bethe-Salpeter (NBS) wavefunction and the HALQCD method

- Equal-time Nambu-Bethe-Salpeter (NBS) wavefunction

$$\psi_{J^P}(\mathbf{r}) = \sum_x \langle 0| D_c(\mathbf{x}) Q_c(\mathbf{y}) \left|B(J^P)\right\rangle ,$$

where $D_c(x) = \varepsilon_{abc} u_a^T C\gamma_5 d_b(x)$ is the scalar diquark interpolator and $Q_c(x)$ is the static quark field.

- We demand the equal-time NBS wavefunction shall satisfy the following Schrodinger equation:

$$\left(-\frac{\nabla^2}{2m_D} + \hat{V}_0(r)\right)\psi_{J^P}(\mathbf{r}) \quad = \quad (\varepsilon_B - m_D)\psi_{J^P}(\mathbf{r})$$

Where

ε_B = total relativistic energy for $\left|B(J^P)\right\rangle$

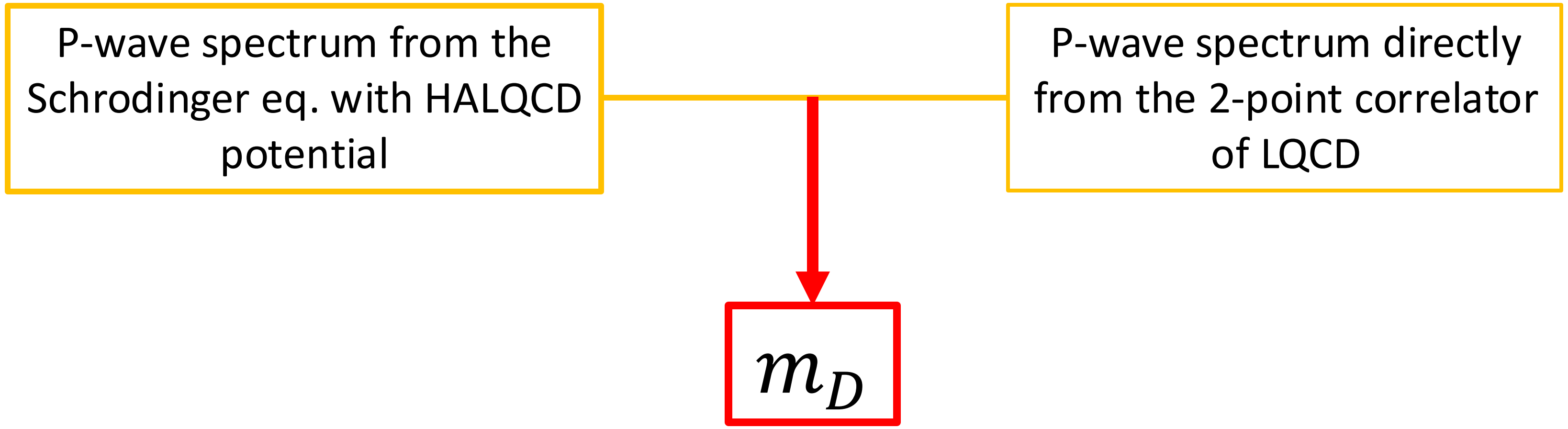
$\hat{V}_0(\boldsymbol{r}) = V_0(\boldsymbol{r}) + O(\nabla^2)$ = HALQCD potential (unknown)

m_D = mass of scalar diquark (unknown)

A self-consistent determination of HALQCD potential and the diquark mass

Strategy:

- $V_0(\boldsymbol{r})$ is determined from S-wave sector with unknown parameter, m_D
- m_D, is determined by demanding that $V_0(\boldsymbol{r})$ should reproduce p-wave spectrum obtained from the 2-point correlator.



Result

Lattice Setup

- (2+1)-flavors gauge configuration generated by CP-PACS and JLQCD Collab. [1]
  - Renormalization-group-improved Iwasaki gauge action $\beta = \frac{6}{g^2} = 1.83$.
- Lattice size = $L^3 \times T = 16^3 \times 32$
- Nonperturbatively *O(a)*-improved Wilson quark action
  - $\kappa_{ud} = \kappa_s = 0.13710, \;\; c_{sw} = 1.761$
- Lattice spacing: $a \simeq 0.121(2)\text{fm}$ ($a^{-1} \simeq 1.63$ GeV), $\mathrm{L} \simeq 1.93(3)\text{fm}$
- $m_\pi \simeq 1014$ MeV, $m_N \simeq 2026$ MeV
- Quark propagators from Wall-source and Gaussian-smeared source with Coulomb gauge fixing

To reduce statistical noise, we consider

➢ HYP smearing on gauge links to compute Wilson lines for static quark propagator

➢ 4 source points at t/a = 0, 8, 16, …

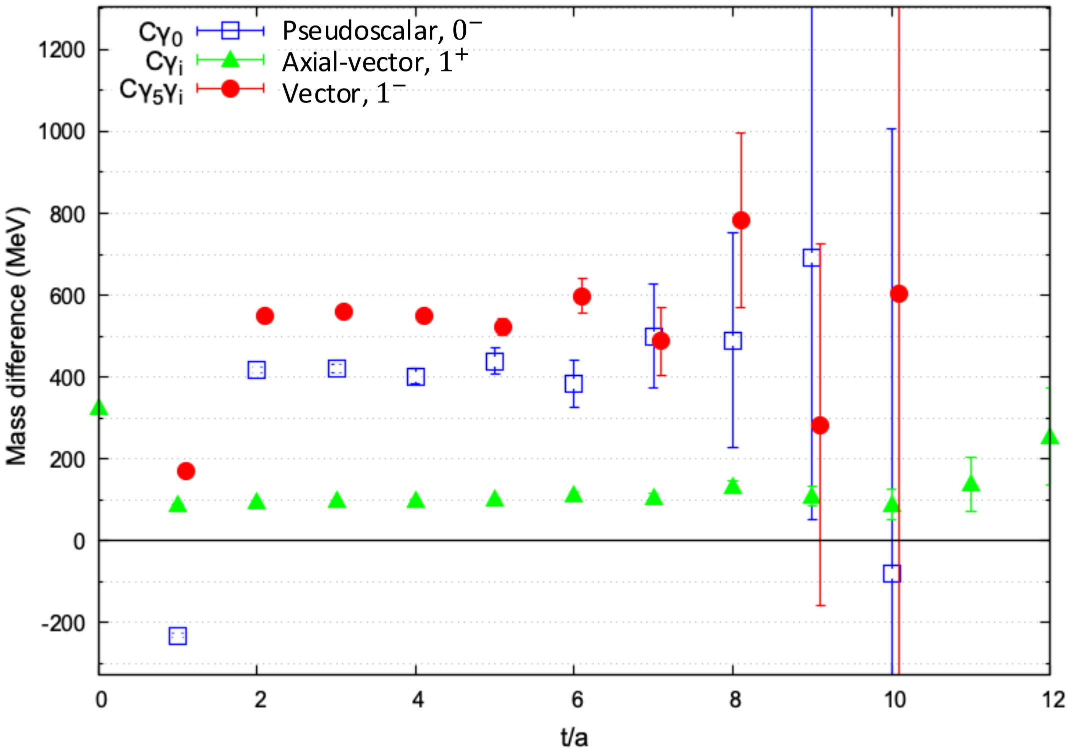
➢ Cubic group (Rotational symmetry of the lattice)

Ref:
[1] T. Ishikawa et. al., PRD **78**, 011502(R) (2008)

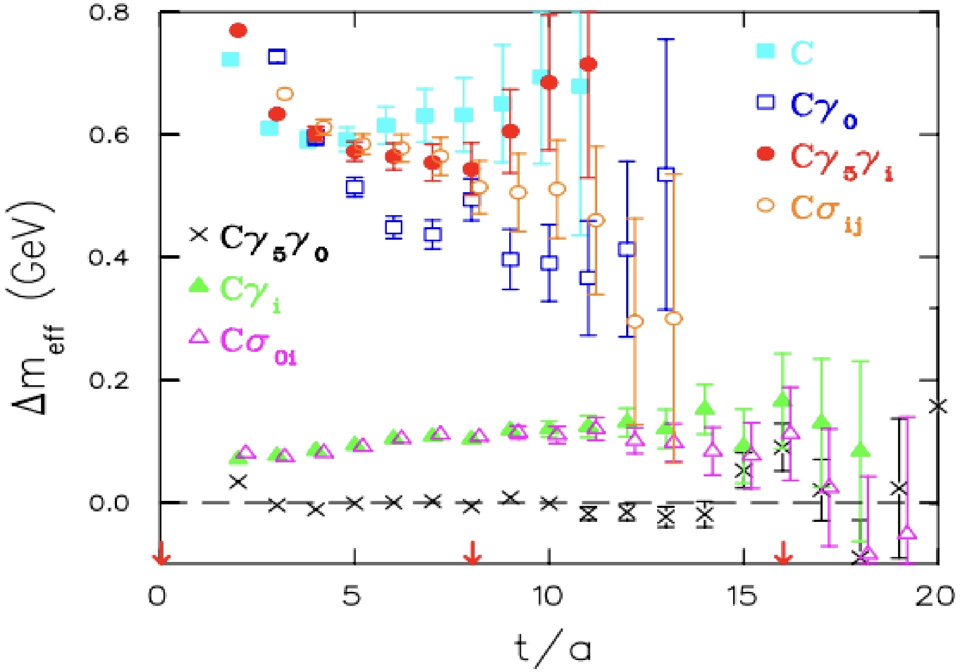
Effective mass difference

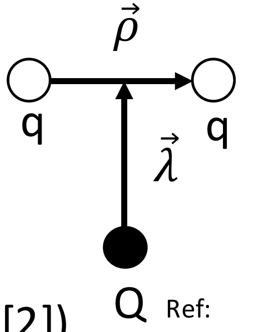
$$\Delta m_{C\Gamma}(n_t) = m_{C\Gamma}(n_t) - m_{C\gamma_5}(n_t)$$

Our Results



Results from *C. Alexandrou* **PRL** 2006





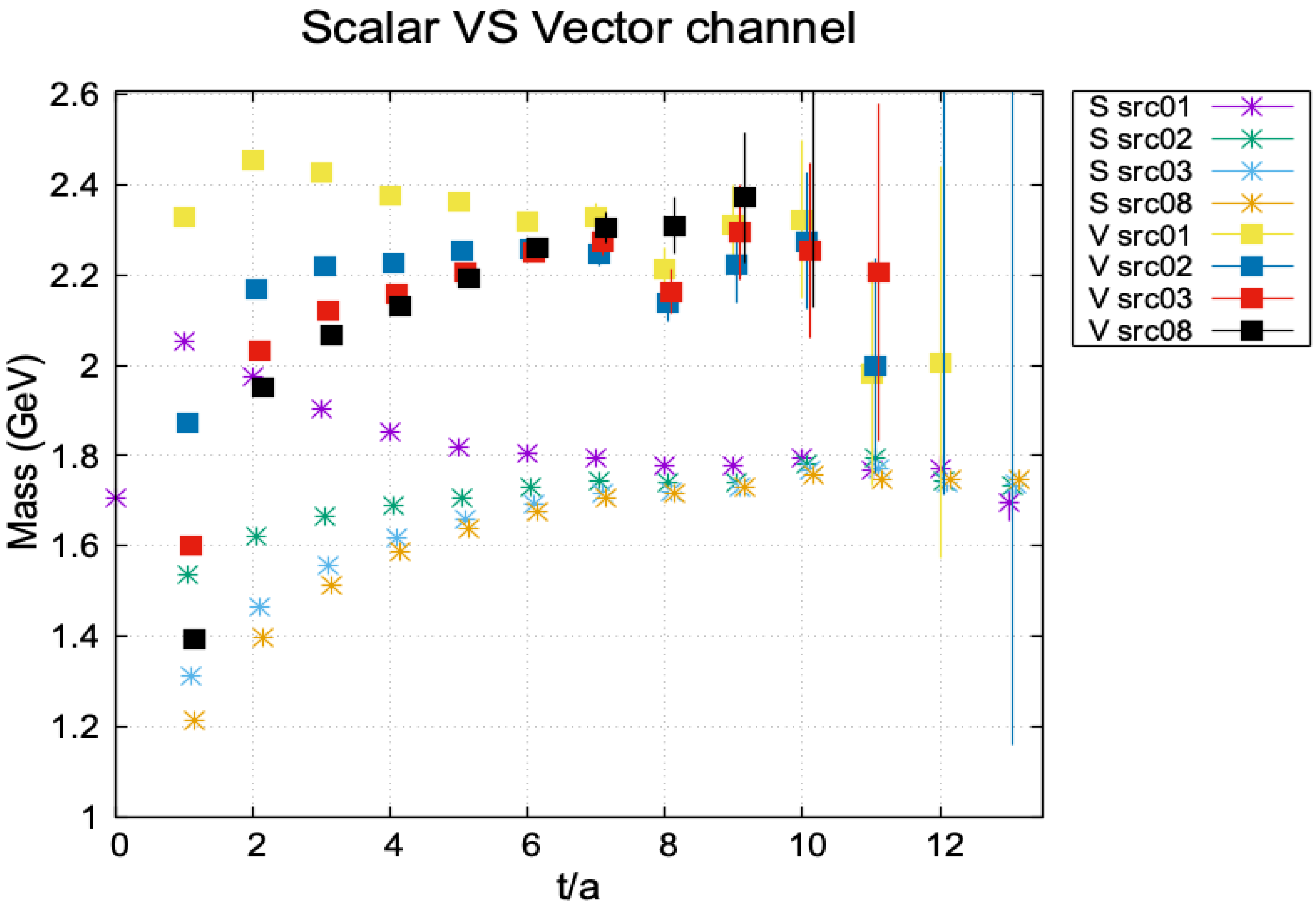
- Our results agree with C. Alexandrou's result.
  - The odd-parity baryons (0^-, 1^-) are heavier and 1^+ are heavier than 0^+.
- The 1^- state is identified as:
  - A. 1^- diquark in S-wave (C. Alexandrou's work [1])
  - B. 0^+ diquark in P-wave (this work) (Phenomenologically supported by Nagahiro's paper[2])

which is used to determine the scalar diquark mass, m_D

Ref:

1. *C Alexandrou et al, PRL 2006*
2. *H. Nagahiro et al, PRD 95 2017*

Effective Mass Plot (Scalar vs Vector)



Note:

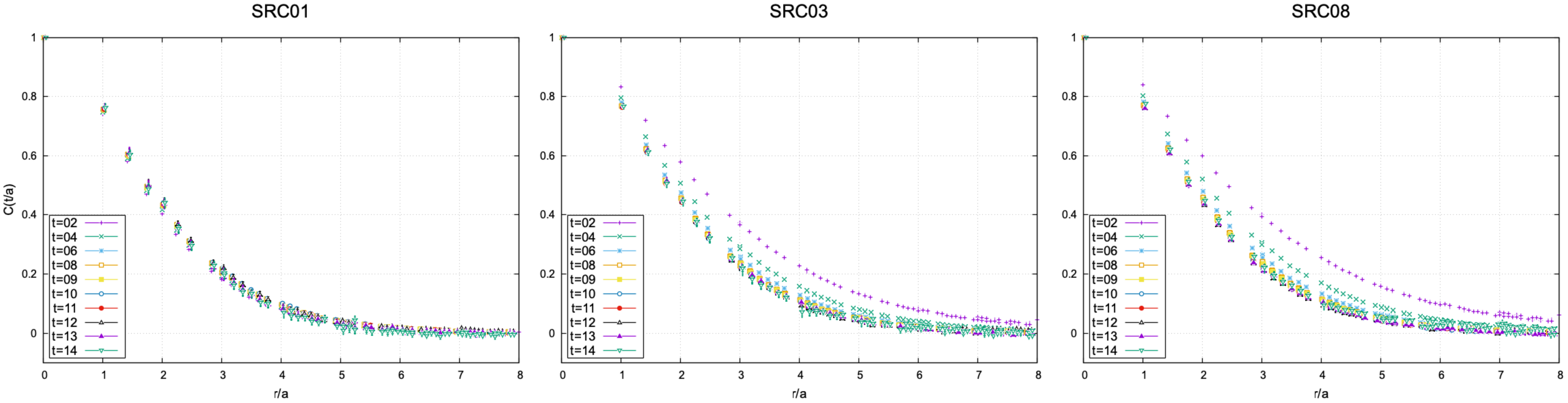
We consider two-point correlator with:

- Smeared-source (exp. type), $e^{-\frac{r}{r_0}}$
  $\frac{r_0}{a} = 1$ (SRC01)
  $\frac{r_0}{a} = 2$ (SRC02)
  $\frac{r_0}{a} = 3$ (SRC03)
- Wall-source (SRC08)

- The effective mass plot by all source operators, SRC(01,02,03,08) give **consistent plateau for the scalar channel**.
- For vector channel, the statistical noise is large. However, we can roughly identify the plateau region.
- We shall improve the result by implementing the variational method in future.

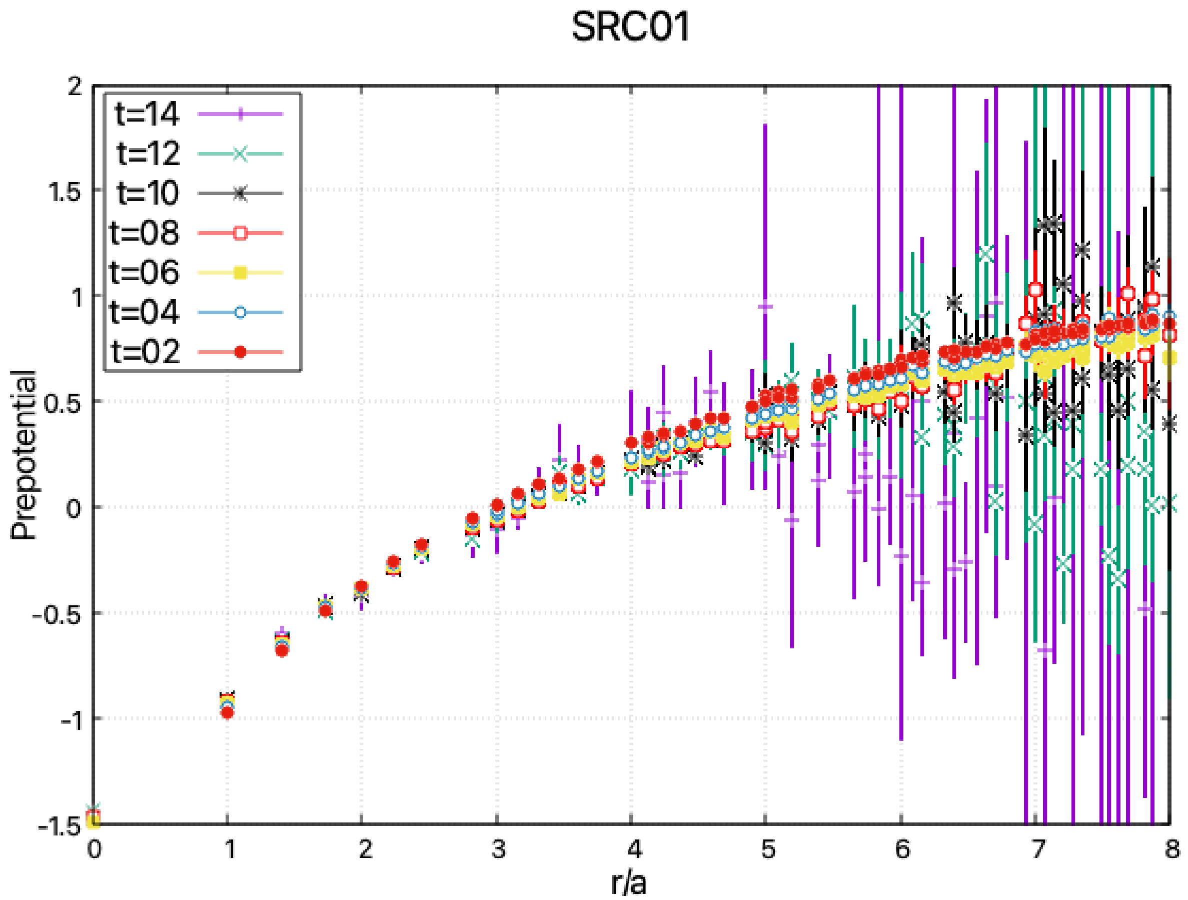
Four-point function and equal-time NBS wavefunction

$$
\begin{aligned}
&C(\mathbf{r}, t; t_{src}) \\
&= \sum_{\mathbf{x},\mathbf{x}',\mathbf{y}'} \langle 0| D_c(\mathbf{x}, t) Q_c(\mathbf{x}+\mathbf{r}, t) \Big(D_{c'}(\mathbf{x}', t_{\mathrm{src}}) Q_{c'}(\mathbf{y}', t_{\mathrm{src}}) \Big)^\dagger |0\rangle \\
&= \sum_n A_n \psi_n(\mathbf{r}) e^{-E_n t} \\
&\simeq A_0 \psi_0(\mathbf{r}) e^{-E_0 t} \quad \text{(for large t)}
\end{aligned}
$$



- Speed of convergence: SRC01 > SRC03 > SRC08
- Hence, we choose the source operator to be SRC01 (Gaussian-smeared source)

Prepotential, $\tilde{V}_0(r) = \frac{\nabla^2 \psi_S(\mathbf{r})}{\psi_S(\mathbf{r})} = 2m_D(V_0(r) + m_D - \varepsilon_B)$



- From the plot, we can see that the potential converged at t/a = 10.
- We then fit the potential to a Cornell-type potential at t/a = 10.

$$\tilde{V}_0^{\text{fit}}(r) = -A/r + Br + v_0$$

Determination of m_D by comparing P-wave spectrum

$$\left(-\frac{\nabla^2}{2m_D}+V_0(r)\right)\psi_S(\mathbf{r}) = (\varepsilon_S - m_D)\psi_S(\mathbf{r})$$
$$\left(-\frac{\nabla^2}{2m_D}+V_0(r)\right)\psi_P(\mathbf{r}) = (\varepsilon_P - m_D)\psi_P(\mathbf{r})$$

$$\left(-\nabla^2+\tilde{V}_0(r)\right)\psi_S(\mathbf{r}) = 0$$
$$\left(-\nabla^2+\tilde{V}_0(r)\right)\psi_P(\mathbf{r}) = \Delta\tilde{E}\,\psi_P(\mathbf{r}),$$

with $\Delta\tilde{E} = 2m_D(\varepsilon_P - \varepsilon_S)$

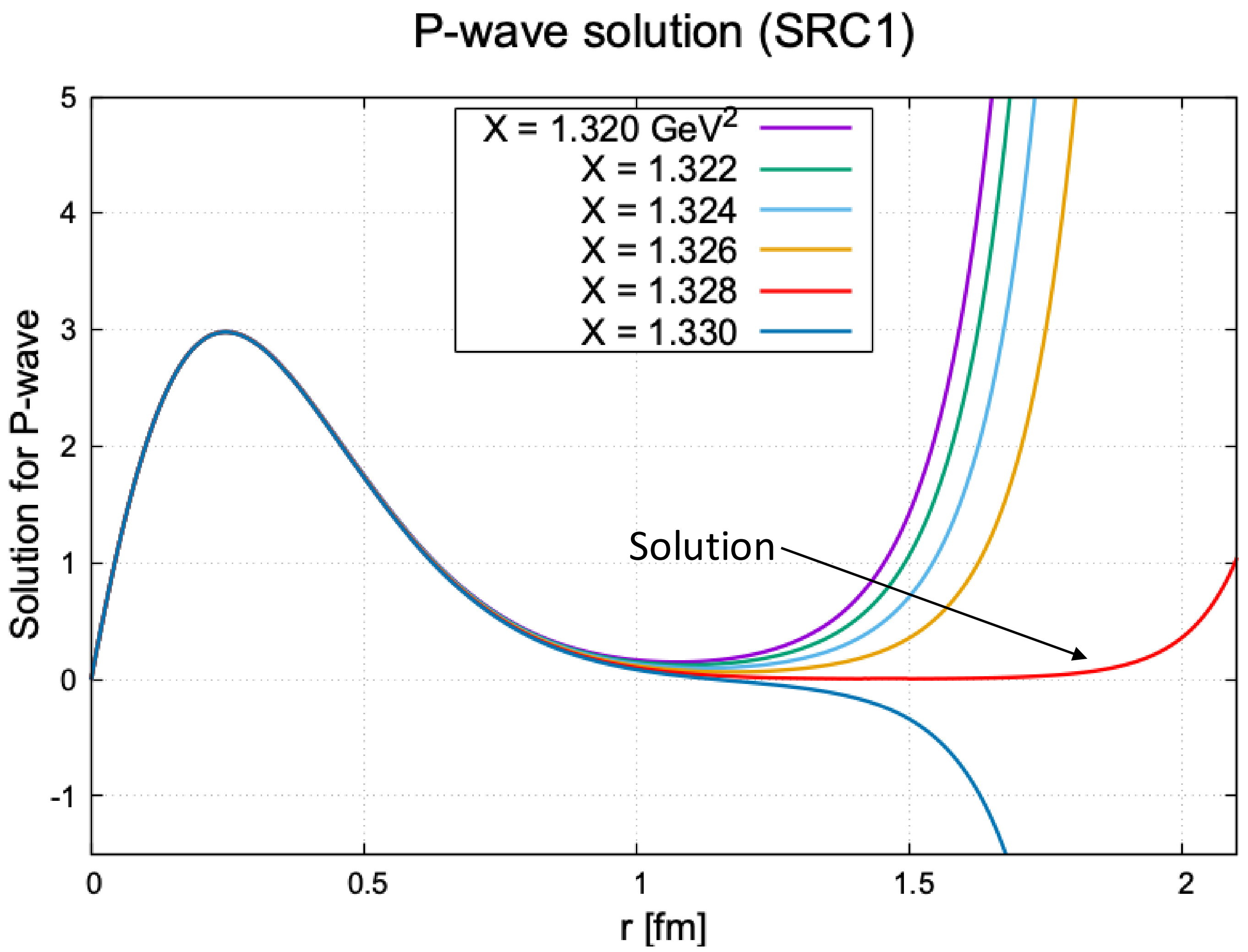
$\Delta\tilde{E} = 1.328\ \mathrm{GeV}^2$ From potential

$(\varepsilon_P - \varepsilon_S) = 0.535\ \mathrm{GeV}^2$ From 2-point correlator

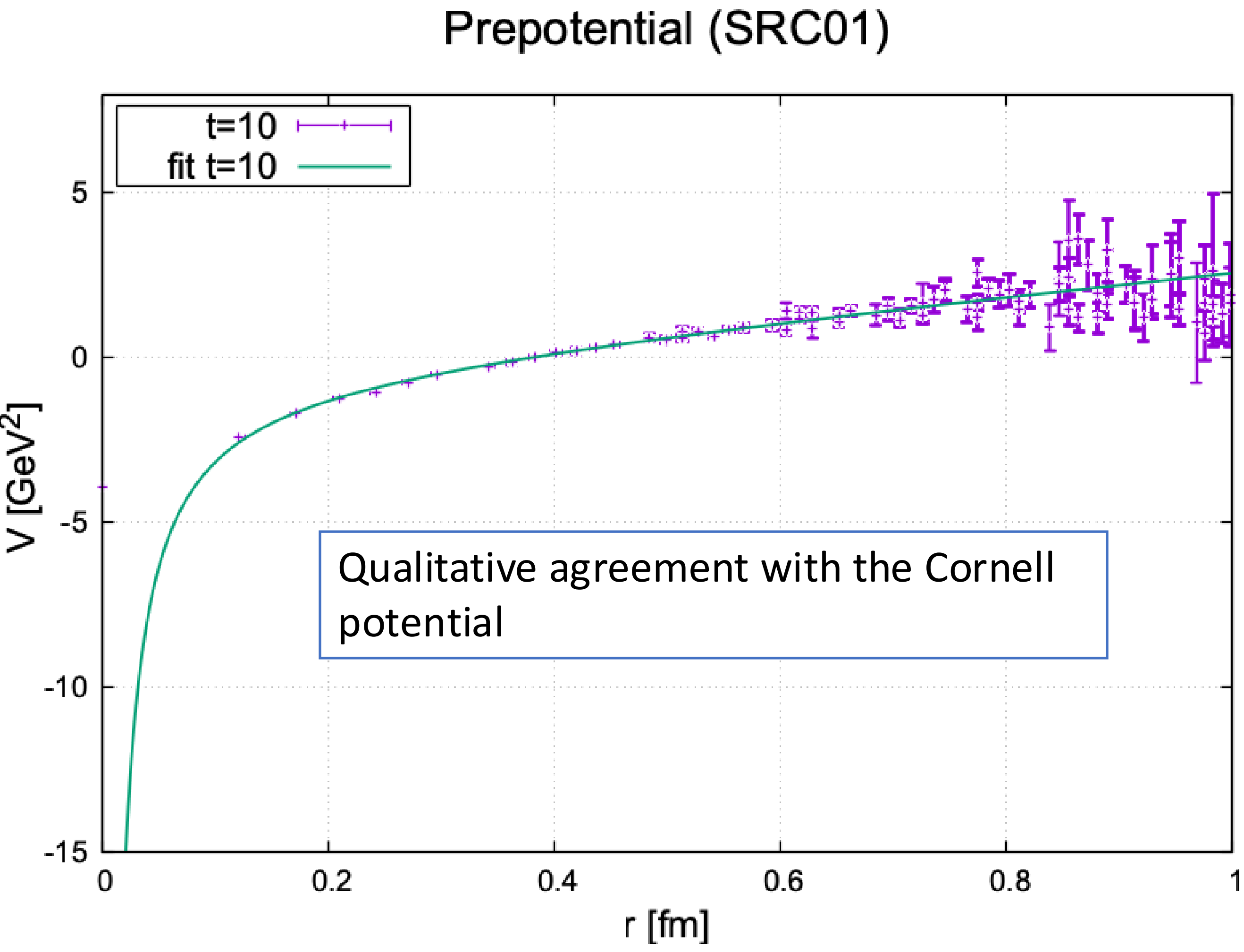
$$m_D = \frac{\Delta\tilde{E}}{2(\varepsilon_P - \varepsilon_S)} = 1.241\ \mathrm{GeV}$$

2/3 proton mass = 1.351 GeV

V_0 = potential
$\tilde{V}_0$ = pre-potential
$\Delta\tilde{E}$ = P-wave solution
ϵ_P, ϵ_S = energy of P-wave and S-wave
m_D = Diquark mass



Quark-diquark potential



We compute the potential of this baryon from the prepotential:

$$V_0(r) = \frac{1}{2m_D}\tilde{V}_0(r) + \mathrm{E}_{\mathrm{j^P=0^+}}$$

with $\mathrm{E}_{\mathrm{j^P=0^+}} = \mathrm{M}_{\mathrm{j^P=0^+}} - m_D$

Quark-diquark potential:

$$V_0(r) = -\frac{A}{r} + Br + v_0$$

- $A = 0.121(4)$ GeV fm
- $\sqrt{B} = 510(5)$ MeV
- $v_0 = 0.141(15)\ GeV$

- Value of A (Coul. Coeff.) might be affected by the HYP smearing[1].
- Our $\sqrt{B}$ (sqrt. of string tension) are slight larger than the static qqbar potential, i.e 510 MeV > 440 MeV.

Ref:
[1] A. Hasenfratz, F. Knechtli, PRD **64** 034504, 2001

Summary and Future outlook

- We calculated the energy spectrum of system made up from diquark and static quark. Our results are consistent with C Alexandrou's result.
- Then, in order to get the scalar diquark mass, we employ the extended HALQCD potential method.
- We obtained the scalar diquark mass to be 1.241 GeV, which is roughly close to twice the naïve constituent quark mass, i.e. 2/3 mass of proton: 1.351 GeV.
- We also constructed the potential for the scalar diquark- static quark baryon, which behave like a Cornell-type potential.
- We also obtained the Coulomb coefficient = $0.121(4)$ GeV fm and $\sqrt{string\ tension}$ = 510 MeV.

Relation between our diquark mass and Alexandrou's diquark mass:

Energy of 0+ baryon	= Scalar diquark mass + interaction energy	
(Alexandrou's diquark mass)	(our diquark mass)	
1.766 GeV	= 1.241 GeV.	+ 0.525 GeV

Future work:

- We will also compare their Coulomb coefficient and string tension to the static quark-antiquark potential in the future.
- Variational method will be considered to improve the convergence of the effective mass plot.
- To improve the result, we shall consider gauge configuration with larger lattice volume.
- Axial-vector diquark.
- Quark mass dependence of diquark and their potential.