Examination of the ϕ-NN bound-state problem with lattice QCD N-ϕ potentials

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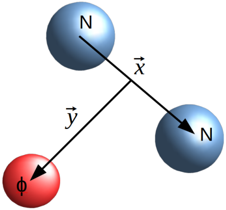

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Introduction

1. This presentation is based on [F. Etminan and A. Aalimi PRC 109 2024]

- Possibility of the formation of ϕ-mesic bound states with N is important due to the quark content of the ϕ-meson as $s\bar{s}$. [J. J. Cobos-Martınezet et al PRC 96, 2017] [I. Filikhin et al PRD 110 2024]

- One of the simplest candidates for ϕ-mesic nuclei can be the $\phi-NN$ system. [V. B. Belyaev et al Few-Body Syst. 44, 2008]

- The two-body bound states or resonances may become robust in few-body systems,
  e.g. there is not seen any bound for strangeness -1 dibaryon states but the hypertriton ${}^{3}_{\Lambda}H\,(I)\,J^P = (0)\,1/2^+$, is bound with a separation energy of 130 ± 50 keV. [H. Garcilazo et al PRC 99 2019]

- In the older work: using the folding method, Faddeev equations mostly attractive phenomenological $N-\phi$ interaction by central binding energy of 9.47 MeV and the semi-realistic Malfliet-Tjon (MT) NN potential are employed. [S. A. Sofianos et al. J. Phys. G Nucl. Part. Phys. 37 2010]

- They concluded that $\phi-NN$ is bound with $40\,(23)$ MeV for triplet(singlet) NN interaction.

Introduction

- from the experimental point of view,
- ALICE collaboration measured the correlation function of proton-ϕ in heavy-ion collisions, together by indicating a $p-\phi$ bound state using two-particle correlation functions [ALICE PRL 127,2021, E. Chizzali et al PLB 848,2024]
- Femtoscopic analysis of hadron-deuteron (hd) correlation functions could play a crucial role in understanding structure of the atomic nuclei. [ALICE Phys. Rev. X 14 2024]
- These request the proton$-\phi$ bound state hypothesis.
- Recently, HAL QCD Collaboration has derived $N-\phi$ potential in ${}^4S_{3/2}$ channel from lattice QCD at nearly physical quark masses, [Y. Lyu et al PRD 106,2022]

✦ We examined hypothetical multi-strangeness nucleus $\phi-NN$ by using HAL $N-\phi$ potential

3B bound states in HH expansion method

✦ **Jacobi coordinates** (x_i, y_i)

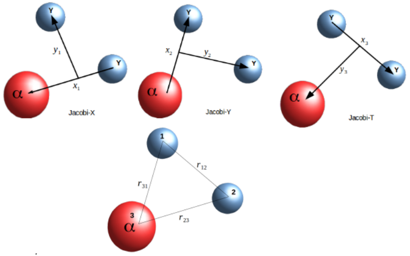
- $x_i = \sqrt{A_{jk}} r_{jk} = \sqrt{A_{jk}}(r_j - r_k),,$
- $y_i = \sqrt{A_{(jk)i}} r_{(jk)i} = \sqrt{A_{(jk)i}} \left(r_i - \frac{A_j r_j + A_k r_k}{A_j + A_k} \right),$

✦ Reduced masses:

- $A_{jk} = \frac{A_j A_k}{A_j + A_k},$
- $A_{(jk)i} = \frac{(A_j + A_k)A_i}{A_i + A_j + A_k},$

where

- $i, j, k \in (1, 2, 3),$
- $A_i = \frac{m_i}{m}$, m_i the mass of particle i in a.



3B bound states in HH expansion method

3B wave function expansion on hyperspherical harmonics (HH): [I. Thompson et al Comput. Phys. Commun 161, 87 2004]

$$\Psi_{j\mu}(\rho,\Omega) = \sum_{\beta} \mathcal{R}^{j}_{\beta}(\rho)\, \mathcal{Y}_{\beta j\mu}(\Omega)$$

- j: total angular momentum
- μ: projection of total angular momentum
- ρ: hyperradius ($\rho^2 = x^2 + y^2$)
- $\Omega = (\alpha, \hat{x}, \hat{y})$: a functions of five hyperspherical polar angles
- α: hyperangle ($\alpha = \arctan(x/y)$)
- $\beta \equiv \{K, l_x, l_y, l, S_x, j_{ab}\}$: a set of quantum numbers coupled to j
  - K: hypermomentum
  - l_x: orbital angular momentum related to Jacobi coordinate x
  - l_y: orbital angular momentum related to Jacobi coordinate y
  - $l = l_x + l_y$: total orbital angular momentum
  - S_x: spin of the particles associated with the x coordinate
  - $j_{ab} = l + S_x$

3B bound states in HH expansion method

The expansion of the 3B wave function's angular part $\mathcal{Y}_{\beta j\mu}(\Omega)$ on the hyperspherical harmonics $\Upsilon^{l_x l_y}_{Klm_l}$:

$$\mathcal{Y}_{\beta j\mu}(\Omega) = \sum_{\nu\iota} \langle j_{ab}\nu I\iota | j\mu\rangle \kappa_I^\iota = \sum_{m_l\sigma} \langle l m_l S_x\sigma | j_{ab}\nu\rangle \Upsilon^{l_x l_y}_{Klm_l}(\Omega)\, \chi^\sigma_{S_x}$$

- $\chi^\sigma_{S_x}$: spin wave function of two particles in the Jacobi coordinate x
- κ_I^ι: spin wave function of third particle
- HH are eigenstates of $\hat{K}$
- $\Upsilon^{l_x l_y}_{Klm_l}(\Omega) = \varphi_K^{l_x l_y}(\alpha)\left[Y_{l_x}(\hat{x}) \otimes Y_{l_y}(\hat{y})\right]_{lm_l}$
  - $\varphi_K^{l_x l_y}(\alpha) = N_K^{l_x l_y}(\sin\alpha)^{l_x}(\cos\alpha)^{l_y} P_n^{l_x+\frac{1}{2}, l_y+\frac{1}{2}}(\cos 2\alpha)$
  - $N_K^{l_x l_y}$: normalization constant
  - $P_n^{a,b}$: Jacobi polynomial by order $n = (K - l_x - l_y)/2$

Two-Body Potentials:

✦ For NN interactions, we use the Yukawa-type Malfliet-Tjon (MT) [R. Malfliet and J. Tjon NPA 127 1969]

$$V_{NN}(r) = \sum_{i=1}^{2} C_i \frac{e^{-\mu_i r}}{r}, \tag{1}$$

This potential supports a deuteron binding energy of -2.2307 MeV.

(I, J)	a_0 (fm)	$\mathrm{r_{eff}}$ (fm)	C_1 (MeV · fm)	μ_1 (fm^{-1})	C_2 (MeV · fm)	μ_2(fm^{-1})
$(1, 0)$	-23.56	2.88	-513.968	1.55	14.38.72	3.11
$(0, 1)$	5.51	1.89	-626.885	1.55	1438.72	3.11

Two-Body Potentials:

$N{-}\phi\,({}^4S_{3/2})$ potential

✦ concrete parameterizations, are taken straight from [Y. Lyu et al PRD 106,2022] which is published recently by the HAL QCD collaboration.

✦ composed of attractive Gaussian and long-range Yukawa squared attractions

$$V_A(r) = \sum_{i=1}^{2} \alpha_i e^{-(r/\beta_i)^2} + a_3 m_\pi^4 f(r;\beta_3)\left(\frac{e^{-m_\pi r}}{r}\right)^2. \tag{2}$$

✦ long-range part of $N{-}\phi$ dominated by two-pion exchange (TPE).

✦This behavior suggests the $V_A(r)$ has a TPE tail at long range with a strength coefficient m_π^4.

✦ for comparison, a purely phenomenological Gaussian form is considered,

$$V_B(r) = \sum_{i=1}^{3} \alpha_i e^{-(r/\beta_i)^2}. \tag{3}$$

$N\!-\!\phi\left({}^4S_{3/2}\right)$ potential

✦ for $f\left(r;b_3\right)$ two different types commonly used in the NN potential, is applied a Nijmegen-type form factor

$$f_{erfc}\left(r;\beta_3\right)=\left[erfc\left(\frac{m_\pi}{\Lambda}-\frac{\Lambda r}{2}\right)-e^{2m_\pi r}erfc\left(\frac{m_\pi}{\Lambda}+\frac{\Lambda r}{2}\right)\right]^2/4, \quad (4)$$

✦ and the Argonne-type form factor,

$$f_{exp}\left(r;\beta_3\right)=\left(1-e^{-\left(r/\beta_3\right)^2}\right)^2, \quad (5)$$

✦ $m_\pi=146.4$ MeV, $\Lambda=2/\beta_3$ and $erfc\left(z\right)=\frac{2}{\sqrt{\pi}}\int_z^\infty e^{-t^2}dt$ is the complementary error function.

✦ we refer to $V_A\left(r\right)$ with $f_{erfc}\left(f_{exp}\right)$ form factor as $A_{erfc}\left(A_{exp}\right)$ model, ✦ and model B is applied to $V_B\left(r\right)$.

✦ by A_{erfc} potential are the scattering length $a_0^{N-\phi}=-1.43(23)$ fm, and the effective range $r_{eff}^{N-\phi}=2.36(10)$ fm and no binding energy is observed for this interaction.

Two-Body Potentials:

$N{-}\phi\ \left({}^4S_{3/2}\right)$ potential

Table: The parameters of $N{-}\phi\ \left({}^4S_{3/2}\right)$ potential at lattice Euclidean time 14. The numbers in parentheses indicate statistical errors.

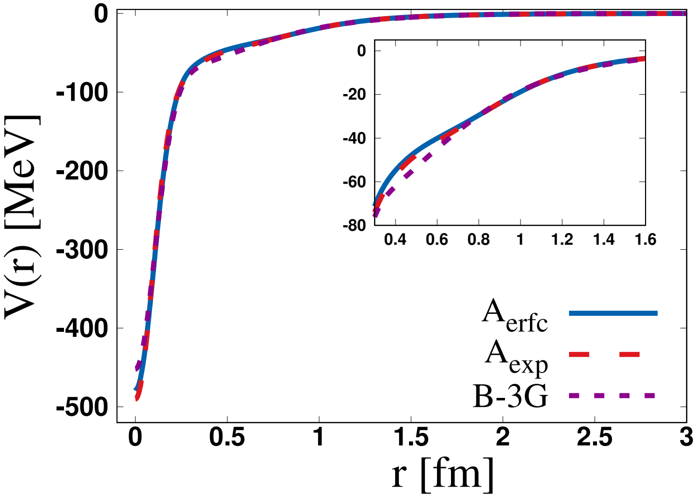
	α_1 (MeV)	β_1 (fm)	α_2 (MeV)	β_2 (fm)	$\alpha_3 m_\pi^{4n}$ (MeV · fm^{2n})	β_3(fm)
A_{erfc}	−376(20)	0.14(1)	306(122)	0.46(4)	−95(13)	0.41(7)
A_{exp}	−371(27)	0.13(1)	−119(39)	0.30(5)	−97(14)	0.63(4)
$B-3G$	−371(19)	0.15(3)	−50(35)	0.66(61)	−31(53)	1.09(41)

[Y. Lyu et al PRD 106,2022]

Two-Body Potentials:

$N-\phi\,(^4S_{3/2})$ potential

✦ for three model, i.e. A_{erfc} (blue solid line), A_{exp} (red dashed lines), and $B-3G$ (purple dot line) using the parameters given in Table.



[Y. Lyu et al PRD 106,2022]

Nuclear matter radius

The common feature of all HH calculations is a very rapid convergence of the WF and a relatively slow one for the binding energy. The geometrical properties of these systems can be extracted by [B. Danilin et al. NPA 1998] The nuclear matter radius of A-nucleon system:

$$r_{mat} = \sqrt{\langle r^2 \rangle} = \sqrt{\frac{1}{A}\left[\left(\sum_{q=1}^{3} A_q \left\langle r_{A_q}^2 \right\rangle\right) + \langle \rho^2 \rangle\right]}; \quad r^2 = \frac{1}{A}\sum_{i=1}^{A} r_i^2$$

- r_i: position of $i-$th nucleon with respect to the C.M. of the system
- A_q: Mass number
- $\left\langle r_{A_q}^2 \right\rangle$ squared radius of each cluster
- $\langle \rho^2 \rangle = \sum_\beta \int d\rho \rho^2 \left| \mathcal{R}_\beta^j(\rho) \right|^2$
- To calculate the r.m.s. matter radius of the $\phi-d$ system, strong interaction radius of proton, neutron and ϕ-meson $0.82, 0.80$ fm and 0.46 fm, respectively, are used as input. [X.-Y. Wang et al PRC 108 2023]

Numerical Results

- B_3: Three-body binding energies (in MeV) for $(I)J^\pi = (0)2^- \ \phi{-}d$ by $K_{max} = 80$
- Nuclear matter radius (r_{mat})
- By experimental values of masses, $m_N = 938.9\ MeV/c^2$ and $m_\phi = 1019.5\ MeV/c^2$,
- And masses obtained from $(2+1)$-flavor lattice QCD simulations, $m_N = 954.0\ MeV/c^2$ and $m_\phi = 1048.0\ MeV/c^2$
- no bound state found for $(I)J^\pi = (1)1^- \ \phi{-}NN$ state, i.e. $\phi{-}nn$ and $\phi{-}pp$

	A_{erfc}		A_{exp}		$B-3G$	
	B_3 (MeV)	r_{mat} (fm)	B_3 (MeV)	r_{mat} (fm)	B_3 (MeV)	r_{mat} (fm)
Expt.	6.9	8.33	6.8	8.24	6.7	8.08
Lattice	7.3	8.35	7.2	8.25	7.1	8.05

- B_3 by lattice masses are a bit larger than by the experimental masses. Increasing the masses, repulsive kinetic energy contribution will decrease which in turn leads to an increment in binding energies.

Summary

- Binding energy of the three-body system $\phi - NN$ is examined using the
- HAL lattice QCD $N - \phi$ potential in the ${}^4S_{3/2}$ channel in three different analytical forms, i.e. A_{erfc}, A_{exp} and $B - 3G$
- And semi-realistic Malfliet-Tjon NN interactions.
- Coupled Faddeev equations in the coordinate space are solved within the hyperspherical harmonics expansion method.
- No bound state or resonances found for $(I)J^{\pi} = (1)1^{-}$ $\phi{-}nn$ and $\phi{-}pp$ systems.
- $(I)J^{\pi} = (0)2^{-}$, $\phi{-}d$ system in the maximal spin presents a bound state about 7 MeV and a nuclear matter radius of about 8 fm.
- $\phi{-}d$ system in the maximal spin channel cannot couple to the lower three-body open channels ΛKN and ΣKN because D wave subsystems ΛK and ΣK are kinematically suppressed at low energies.
- Because of the small phase space, the decay to final states by four or more particles e.g. $\Sigma\pi KN, \Lambda\pi KN$, and $\Lambda\pi\pi KN$ are supposed to be suppressed
- Last but not least, in original HAL $N - \phi$ potential the OZI(Okubo-Zweig-Iizuka) violating $s\bar{s}$ annihilation is not considered in their simulations. Nevertheless, considering the coupling to channels like ρN or $\pi\Delta$ could change the results obtained here significantly.

Acknowledgments

Thanks **HHIQCD2024** organizers for organizing such wonderful event!

Thank You for your attention!

Backup

Backup

Hyperradial wave function are expanded in a discrete orthonormal basis up to i_{max} hyperradial excitations in each channel,

$$\mathcal{R}^{j}_{\beta}(\rho) = \sum_{i=0}^{i_{max}} C_i^{\beta j} R_{i\beta}(\rho),$$

$C_i^{\beta j}$: diagonalization coefficients,
Hyperradial functions is

$$R_{i\beta}(\rho) = \rho_0^{-3} \left[i!/(i+5)!\right]^{1/2} L_i^5(z) \exp(-z/2),$$

$L_i^5(z)$: Laguerre polynomial, $z = \rho/\rho_o$ with scaling radius ρ_0.