Structural analysis of Pion and Kaon with Distribution Functions

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working
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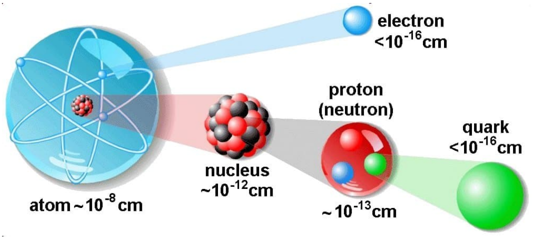
Introduction



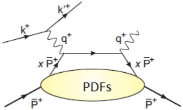
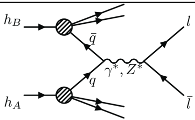
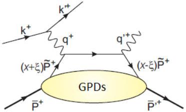
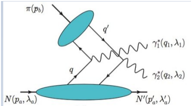
Hadrons are the bound state of quarks and gluons by strong interaction quantum chromodynamics (QCD).

- How actually quarks are distributed within hadrons or what is the internal structure of hadrons ?
  - Distribution functions

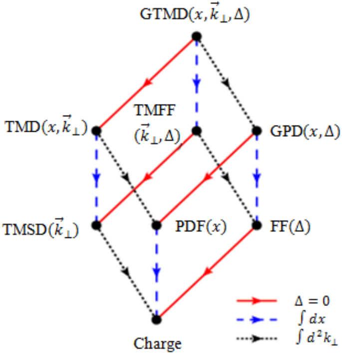
Distribution Functions (DFs)

- Describes the probability distribution of finding the constituents (quarks and gluons) within a hadron with physical obesrvables.
- Information about longitudinal momentum, spatial location, momenta distributions, mechanical properties, structure of hadron etc.
- Used to study different decays of the the exclusive processes through some observables and compared with experimental data with evolutions.
- Some important DFs are generalized transverse momentum dependent distributions **(GTMDs)**, transverse momentum dependent distribution functions **(TMDs)**, generalized parton distributions **(GPDs)**, parton distribution functions **(PDFs)** etc.

Distribution Functions

PDFs	TMDs	GPDs	GTMDs
1-D structure	3-D structure	3-D structure	6-D structure
longitudinal distributions (x)	Both longitudinal (x) and transverse distribution ($k_\perp$)	longitudinal (x) and spatial distribution (ξ and Δ)	longitudinal (x), transverse ($k_\perp$), spatial distribution(ξ and Δ) with all DOF
 DIS	 Drell-Yan process	 DVCS	 Double Drell - Yan

Distribution Functions

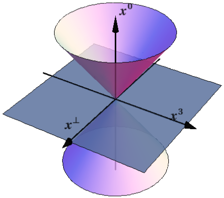


- $x = \frac{k^+}{P^+}$ is the momentum fraction carried by the quark.
- $k_\perp$ is the transverse momentum of quark.
- Δ is the momentum difference of the final and initial state of hadron.
- $\xi = -\frac{\Delta^+}{2P^+}$ is the skewness.

Light-front dynamics

Equal time quantization

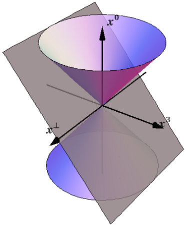
- time, $t = x^0$



- $x = (x^0, x^1, x^2, x^3)$ and $p = (p^0, p^1, p^2, p^3)$
- $i\frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi(t)\rangle$
- Energy, $p^0 = \sqrt{m^2 + p^2}$

Light-front quantization

- time, $t = x^+ = x^0 + x^3$



- $x = (x^+, x^-, x^1, x^2)$ and $p = (p^+, p^-, p^1, p^2)$
- $i\frac{\partial|\psi(x^+)\rangle}{\partial x^+} = \frac{1}{2}p^-|\psi(x^+)\rangle$
- Energy, $p^- = \frac{m^2+p_\perp^2}{p^+}$

Why light-front ?

Ideal Framework to describe the hadronic structure. It can overcome many obstacles with many advantages.

- Simple vacume state
- Boost invariant frame
- Frame independent wave functions
- Hamiltonial formlism for relativistic bound state
- No square root in Hamiltonial p^-
- Maximum number of kinematic variables

TMDs for Different Spin Particles

- Depending upon the spin polarization, there are different TMDs for different spin particles.
- For the case of spin-1/2 nucleons, there are 6 TMDs at the leading twist. However for spin-1 particles, there are 18 TMDs at the leading twist.

Hadron \ Quark	U (γ^+)		L ($\gamma^+\gamma_5$)		T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$)	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1], [h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}], [h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}], [h_{1TT}^\perp]$

Spin-1 leading twist TMDs

Source- S. Kumano and Q. T. Song, PLB **826** (2022)

S. Puhan and H. Dahiya, PRD, **109** (2024).

S. Sharma, S. Puhan, et. al, PTEP, **150** (2024).

TMDs Correlator ror spin-0 mesons

- In case of spin-0, pseudoscalar mesons, there are total 8 TMDs upto twist-4, which can be expressed in terms of quark-quark correlator as [S. Meissner, A. Metz, M. Schlegel and K. Goeke, JHEP 08, 038, (2008]

$$\Phi_q^{[\Gamma]}(x, \mathbf{k}_\perp) = \frac{1}{2} \int \frac{\mathrm{d}z^- \mathrm{d}^2 \vec{z}_\perp}{2(2\pi)^3} e^{i\mathbf{k}\cdot z} \langle \pi(K)|\bar{\psi}(0)\Gamma \mathcal{W}(0,z)\psi(z)|\pi(K)\rangle|_{z^+=0}.$$

Where $|\pi(K)\rangle$ is the LF bound state of pion/kaon with masses and momentum $M_{\pi(K)}$ and $(P^+, P_\perp)$ respectively.
$\mathcal{W}(0,z)$ is the Wilson line which preserves the gauge invariance of the bilocal quark field operators in the correlation functions. For this work, we have taken it as unity.

All TMDs for spin-0 mesons

Γ is the Dirac matrix which determines the Lorentz structure of the correlator $\Phi_q^{[\Gamma]}$. Depending upon the Γ, the TMDs are given as

	T-even	T-odd
Twist-2	$\Phi^{[\gamma^+]} = f_1^q(x, \mathbf{k}_\perp^2)$	$\Phi^{[\iota\sigma^{j+}\gamma_5]} = \iota \frac{\epsilon^{iJ} k_\perp^j}{M} h_1^\perp(x, \mathbf{k}_\perp^2)$
Twist-3	$\Phi^{[\gamma^1]} = \frac{M}{P^+} e^q(x, \mathbf{k}_\perp^2)$ $\Phi^{[\gamma^j]} = \frac{k_\perp^j}{P^+} f^{\perp q}(x, \mathbf{k}_\perp^2)$	$\Phi^{[\gamma^j\gamma_5]} = \iota \frac{\epsilon^{iJ} k_\perp^i}{P^+} g^\perp(x, \mathbf{k}_\perp^2)$ $\Phi^{[\iota\sigma^{ij}\gamma_5]} = \iota \frac{M\epsilon^{iJ}}{P^+} h(x, \mathbf{k}_\perp^2)$
Twist-4	$\Phi^{[\gamma^-]} = \frac{M^2}{(P^+)^2} f_3^q(x, \mathbf{k}_\perp^2)$	$\Phi^{[\iota\sigma^{j-}\gamma_5]} = \iota \frac{M\epsilon^{iJ} k_\perp^i}{(P^+)^2} h_3^\perp(x, \mathbf{k}_\perp^2)$

- There are 4 T-even and 4 T-odd TMDs for spin-0 particles.
- All these T-even TMDs are unpolarized quark distributions.

S. Puhan, et.al, JHEP, **02**, (2024).

Relations Among TMDs

- All the higher twist TMDs are related with leading twist TMD by

$$
\begin{aligned}
x\, e^q(x, \mathbf{k}_\perp^2) &= x\, \tilde{e}^q(x, \mathbf{k}_\perp^2) + \frac{m_{q(\bar{q})}}{M}\, f_1^q(x, \mathbf{k}_\perp^2), \\
x\, f^{\perp q}(x, \mathbf{k}_\perp^2) &= x\, \tilde{f}^{\perp q}(x, \mathbf{k}_\perp^2) + f_1^q(x, \mathbf{k}_\perp^2), \\
x^2 f_3^q(x, \mathbf{k}_\perp^2) &= x^2 \tilde{f}_3^q(x, \mathbf{k}_\perp^2) + \frac{\mathbf{k}_\perp^2 + m_{q(\bar{q})}^2}{M^2}\, f_1^q(x, \mathbf{k}_\perp^2), \\
x\, g^\perp(x, \mathbf{k}_\perp^2) &= x\, \tilde{g}^\perp(x, \mathbf{k}_\perp^2) + \frac{m_{q(\bar{q})}}{M}\, h_1^\perp(x, \mathbf{k}_\perp^2), \\
x\, h(x, \mathbf{k}_\perp^2) &= x\, \tilde{h}(x, \mathbf{k}_\perp^2) + \frac{k_\perp^2}{M^2} h_1^\perp(x, \mathbf{k}_\perp^2), \\
x^2\, h_3^\perp(x, \mathbf{k}_\perp^2) &= x^2\, \tilde{h_3^\perp}(x, \mathbf{k}_\perp^2) + AF(\mathbf{k}_\perp^2) h_1^\perp(x, \mathbf{k}_\perp^2)
\end{aligned}
$$

- In this work, we have adopted Wandzura-Wilczek approximation for valence quark calculations of T-even TMDs.

Light Mesons

Pion

- Bound state of *u*- quark and *d*- antiquark.
- Having mass about 140 MeV. Exists only if mass is dynamically generated.

Kaon

- Bound state of *u*- quark and *s*- antiquark.
- Having mass about 490 MeV. Boundary between emergentand Higgs-mass mechanisms.

For the pion and the kaon the EIC will allow determination of the quark and gluon contributions to mass and internal structure through Sullivan Process.

[A.C. Aguilar et al., Pion and Kaon structure at the EIC, EPJA 55 (2019) 190.
J. Arrington et al., Revealing the structure of light pseudoscalar mesons at the EIC, J. Phys. G 48 (2021) 7 075106.]

Sullivan process

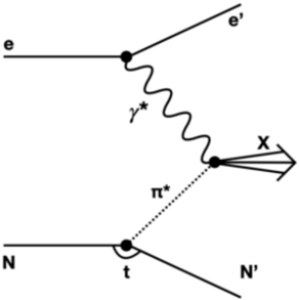


Diagram for the Sullivan process used to probe the structure of the pion.

Yellow EIC Report (2022)

Light cone quark model

The minimal Fock-state description of mesons in the form quark helicity λ for quark-antiquark is given by

$$|M(P, S_Z)\rangle = \sum_{\lambda_1,\lambda_2} \int \frac{\mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp}{\sqrt{x(1-x)}16\pi^3} \psi_{S_Z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)|x, \mathbf{k}_\perp, \lambda_1, \lambda_2\rangle.$$

- ψ_{S_Z} is the spin improved meson wave function written in the form of

$$\psi_{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = \chi_{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)\varphi(x, \mathbf{k}_\perp).$$

With $\varphi(x, \mathbf{k}_\perp)$ is the momentum space wave function defined as

$$\varphi(x, \mathbf{k}_\perp) = A \exp\left[-\frac{\frac{\mathbf{k}_\perp^2+m_q^2}{x} + \frac{\mathbf{k}_\perp^2+m_{\bar{q}}^2}{1-x}}{8\beta} - \frac{(m_q^2 - m_{\bar{q}}^2)^2}{8\beta^2\left(\frac{\mathbf{k}_\perp^2+m_q^2}{x} + \frac{\mathbf{k}_\perp^2+m_{\bar{q}}^2}{1-x}\right)} + \frac{m_q^2 + m_{\bar{q}}^2}{4\beta^2}\right].$$

Here β is the harmonic scale parameters.

Light cone quark model

$\chi_{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)$ is the spin wave function with quark helicities represented as [W. Qian and B. Q. Ma, Phys. Rev. D 78, 074002, (2008)]

$$\chi_{S_z}(x, \mathbf{k}_\perp, \uparrow, \downarrow) = \frac{[(xM + m_q)((1-x)M + m_{\bar{q}}) - k_\perp^2]}{(\sqrt{2}\,\omega_1\omega_2)},$$

$$\chi_{S_z}(x, \mathbf{k}_\perp, \downarrow, \uparrow) = -\frac{[(xM + m_q)((1-x)M + m_{\bar{q}}) - k_\perp^2]}{(\sqrt{2}\,\omega_1\omega_2)},$$

$$\chi_{S_z}(x, \mathbf{k}_\perp, \uparrow, \uparrow) = \frac{[(xM + m_q)q_2^L - ((1-x)M + m_{\bar{q}})q_1^L]}{(\sqrt{2}\,\omega_1\omega_2)},$$

$$\chi_{S_z}(x, \mathbf{k}_\perp, \downarrow, \downarrow) = \frac{[(xM + m_q)q_2^R - ((1-x)M + m_{\bar{q}})q_1^R]}{(\sqrt{2}\,\omega_1\omega_2)}.$$

With $\omega_1 = [(xM + m_q)^2 + \mathbf{k}_\perp^2]^{\frac{1}{2}}, \omega_2 = [((1-x)M + m_{\bar{q}})^2 + \mathbf{k}_\perp^2]^{\frac{1}{2}}$.

Leading Twist TMDs in LCQM

The leading twist f_1^q can be represent in the overlap form as

$$f_1^q(x, \mathbf{k}_\perp^2) = \frac{1}{16\pi^3}\Big[\mid \psi_0(x, \mathbf{k}_\perp, \uparrow, \uparrow) \mid^2 + \mid \psi_0(x, \mathbf{k}_\perp, \downarrow, \downarrow) \mid^2 + \mid \psi_0(x, \mathbf{k}_\perp, \downarrow, \uparrow) \mid^2 + \mid \psi_0(x, \mathbf{k}_\perp, \uparrow, \downarrow) \mid^2 \Big].$$

With spin improved meson wave function, it is calculated as

$$f_1^q(x, \mathbf{k}_\perp^2) = \frac{1}{16\pi^3}\left[\left((xM + m)((1-x)M + m) - \mathbf{k}_\perp^2\right)^2 + \left(M + 2m\right)^2 \right] \frac{\mid \varphi(x, \mathbf{k}_\perp) \mid^2}{\omega_1^2 \omega_2^2}.$$

Light-Front Holographic Model

In case of LFHM, the meson state is taken as the overlap form of of 0 and 1 quark orbital angular momentum (QOAM) [L_z] with momentum P as [B. Pasquini and P. Schweitzer, Phys. Rev. D 90, 014050, (2014)]

$$|M(P)\rangle = |M(P)\rangle_{L_z=0} + |M(P)\rangle_{|L_z|=1}.$$

With

$$|M(P)\rangle_{L_z=0} = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{dx}{\sqrt{6x(1-x)}} \Psi^{(0)}(x,\mathbf{k}_\perp) \sum_{a=1}^{3} \Big[b_\uparrow^{\dagger a}(1) d_\downarrow^{\dagger a}(2) - b_\downarrow^{\dagger a}(1) d_\uparrow^{\dagger a}(2) \Big] |0\rangle,$$

$$|M(P)\rangle_{|L_z|=1} = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \frac{dx}{\sqrt{2x(1-x)}} \Psi^{(1)}(x,\mathbf{k}_\perp) \Big(\frac{\mathbf{k}_\perp^+}{\sqrt{3}} \sum_{a=1}^{3} b_\uparrow^{\dagger a}(1) d_\uparrow^{\dagger a}(2)|0\rangle + \frac{\mathbf{k}_\perp^-}{\sqrt{3}} \sum_{a=1}^{3} b_\downarrow^{\dagger a}(1) d_\downarrow^{\dagger a}(2)|0\rangle \Big).$$

Light-front Holographic Model

Where $(1) = (x, \mathbf{k}_\perp)$ and $(2) = ((1-x), -\mathbf{k}_\perp)$. Ψ being the spin improved wave function written as [N. Kaur and H. Dahiya, Int. J. Mod. Phys. A 36, 2150052, (2021)]

$$\Psi^{(0)}(x, \mathbf{k}_\perp) = -\frac{m_{q(\bar{q})} + Mx(1-x)}{\sqrt{2}x(1-x)}\psi(x, \mathbf{k}_\perp),$$

$$\Psi^{(1)}(x, \mathbf{k}_\perp) = -\frac{1}{\sqrt{2}x(1-x)}\psi(x, \mathbf{k}_\perp).$$

With

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi N}{\kappa\sqrt{x(1-x)}}\exp\left[-\frac{(\mathbf{k}_\perp^2 + (1-x)m_q^2 + xm_{\bar{q}}^2)}{2\kappa^2 x(1-x)}\right].$$

Leading twist TMD in LFHM

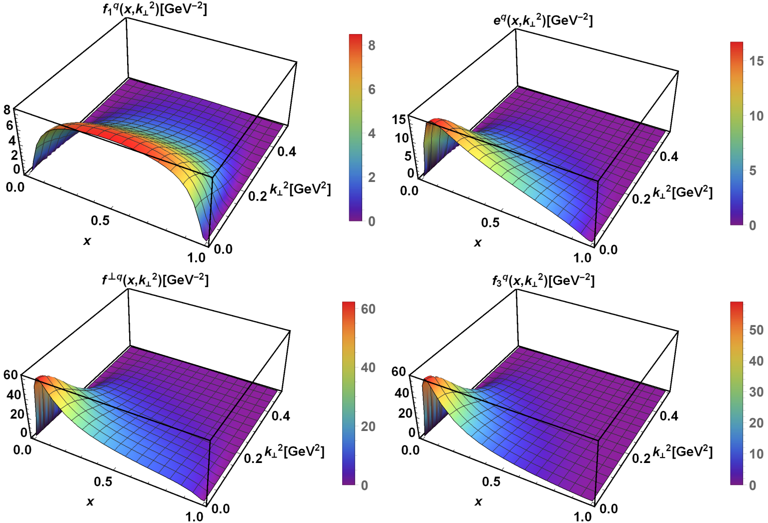
The overlap form of leading twist TMD is found to be

$$f_1^q(x, \mathbf{k}_\perp^2) \quad = \quad \frac{1}{(16\pi^3)}\left[|\Psi^{(0)}(x, \mathbf{k}_\perp)|^2 +^2 \; |\Psi^{(1)}(x, \mathbf{k}_\perp)|^2\right].$$

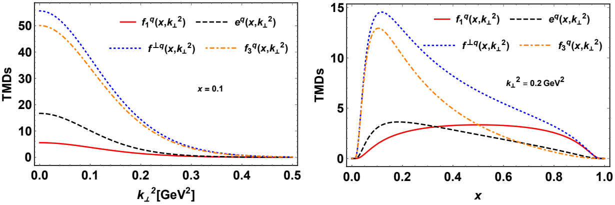
The form of f_1^q TMD is found to be

$$f_1^q(x, \mathbf{k}_\perp^2) = \frac{N^2}{2\pi}\, \frac{[\mathbf{k}_\perp^2 + (m_q + x(1-x)M)^2]}{\kappa^2\, x^3\, (1-x)^3} \exp\left[-\frac{(\mathbf{k}_\perp^2 + (1-x)m_q^2 + xm_{\bar{q}}^2)}{\kappa^2 x(1-x)}\right].$$

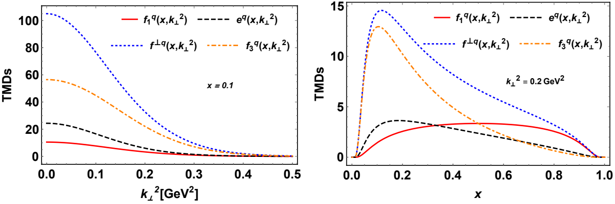
Pion u-quark in LCQM



u-quark TMDs comparison

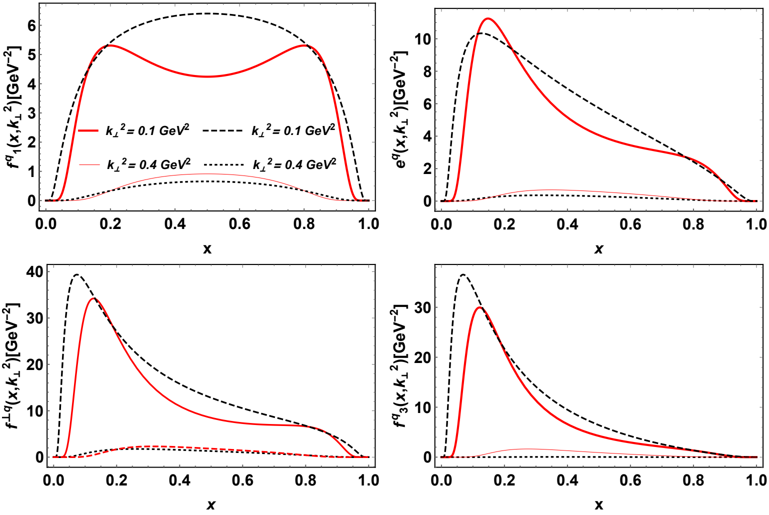


Pion u-quark TMDs at a fixed x and in LCQM.



Kaon u-quark TMDs at a fixed x and in LCQM.

Both model comparison



Pion u- quark comparison in both models.

Average transverse momentum

- The x-dependent mean transverse momenta $(n = 1)$ and mean squared transverse momenta $(n = 2)$ can be given as

$$\langle \mathbf{k}_\perp^n \rangle = \frac{\int dx \int d^2\mathbf{k}_\perp^n \, TMDs}{\int dx \int d^2\mathbf{k}_\perp \, TMDs}.$$

- The Gaussian transverse dependence ratio of these TMDs to find the extent upto which the model supports Gaussian behavior as

$$R_G = \frac{2}{\sqrt{\pi}} \frac{\langle \mathbf{k}_\perp \rangle}{\langle \mathbf{k}_\perp^2 \rangle^{1/2}}.$$

pion	LCQM			LFHM			LFCM			BLFQ	
	$\langle \mathbf{k}_\perp \rangle$	$\langle \mathbf{k}_\perp^2 \rangle^{\frac{1}{2}}$	R_G	$\langle \mathbf{k}_\perp \rangle$	$\langle \mathbf{k}_\perp^2 \rangle^{\frac{1}{2}}$	R_G	$\langle \mathbf{k}_\perp \rangle$	$\langle \mathbf{k}_\perp^2 \rangle^{\frac{1}{2}}$	R_G	$\langle \mathbf{k}_\perp \rangle$	$\langle \mathbf{k}_\perp^2 \rangle^{\frac{1}{2}}$
f_1	0.22	0.26	0.96	0.24	0.27	1.00	0.28	0.32	0.99	0.26	0.30
e	0.18	0.22	0.95	0.21	0.24	0.99	0.26	0.30	0.99	0.26	0.30
$f_\perp$	0.21	0.25	0.96	0.23	0.26	0.99	0.26	0.30	0.99	0.25	0.29
f_3	0.21	0.24	0.95	0.22	0.25	0.99	0.30	0.33	0.98	-	-

Average transverse momentum

Kaon Average Momenta

	LCQM			LFHM		
u-quark	$\langle \mathbf{k}_\perp \rangle$	$\langle \mathbf{k}_\perp^2 \rangle^{\frac{1}{2}}$	R_G	$\langle \mathbf{k}_\perp \rangle$	$\langle \mathbf{k}_\perp^2 \rangle^{\frac{1}{2}}$	R_G
f_1	0.26	0.30	0.98	0.24	0.27	1
e	0.21	0.25	0.96	0.21	0.24	0.99
$f^\perp$	0.23	0.27	0.96	0.22	0.25	0.99
f_3	0.23	0.27	0.96	0.22	0.25	0.99
s-quark						
f_1	0.26	0.30	0.98	0.23	0.26	1
e	0.21	0.25	0.95	0.20	0.23	0.98
$f^\perp$	0.23	0.27	0.96	0.21	0.24	0.99
f_3	0.18	0.21	0.97	0.19	0.22	0.97

The R_G is aproximate 5 percentage deviate from unity for few TMDs.

Parton distribution functions

- The PDFs are obtained by integrating the TMDs over transverse momenta of the quark as

$$f_1^q(x) = \int d^2\mathbf{k}_\perp f_1^q(x, \mathbf{k}_\perp^2)$$

- Similarly the other PDFs are $e^q(x)$, $f^\perp(x)$ and $f_3(x)$.
- These PDFs follows the following sum rule

$$\int dx f_1^q(x) = 1\,,$$

$$\int dx x f_1^q(x) + \int dx(1-x) f_1^{\bar{q}}(x) = 1\,,$$

$$\sum_q \int dx e^q(x) = \frac{\sigma}{m_{q(\bar{q})}}\,,$$

$$\int dx x e^q(x) = \frac{m_{q(\bar{q})}}{M}.$$

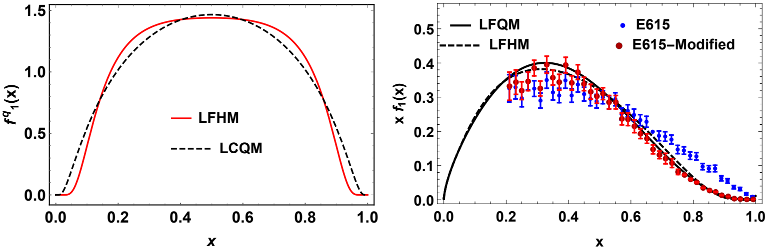
Parton Distribution Functions

- The other relations which are found to be

$$f_3^q(x) = \frac{f_1^q(x)}{2} + \frac{d}{dx}\int d^2\mathbf{k}_\perp \frac{\mathbf{k}_\perp^2}{2M_{\pi(K)}^2} f^{\perp q}(x, \mathbf{k}_\perp^2).$$

The above equation is called as "quark-model Lorentz-invariance relations (qLIRs)" for spin-0 PDFs.

Pion u-quark PDFs

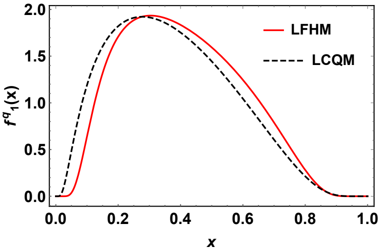


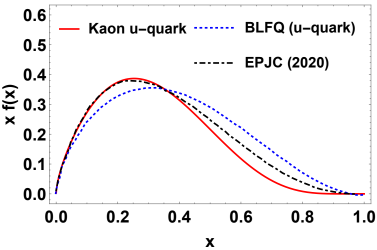
Pion u- quark comparison in both models along with experimental data.

- The evolution has been carried out by NLO DGLAP equation from an initial model scale 0.20 GeV^2 to 16 GeV^2.

E615 - J. S. Conway et al., Phys. Rev. D 39, 92 (1989).
E615-Modified - M. Aicher et al., Phys. Rev. Lett. 105, 252003 (2010).

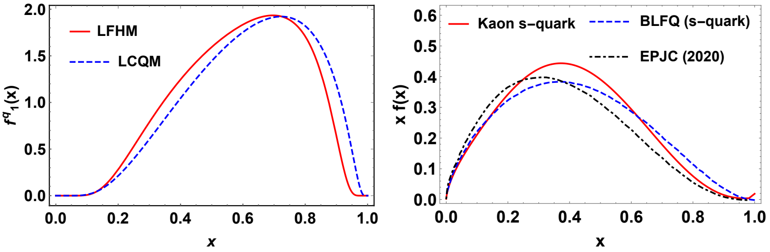
Kaon u-quark PDFs





Kaon u-quark unpolarized PDFs evolved from 0.20 GeV^2 to 20 GeV^2 compared with Phys.Rev.D 101 (2020)[BLFQ] and Eur. Phys. J. C 80, 1064 (2020).

Kaon s-antiquark PDFs



Kaon s- antiquark comparison in both models along with other model predictions.

- Ongoing COMPASS++/AMBER will provide more information very soon along with the upcoming EIC.

Average x Value

$\langle x \rangle$	$f_1^q(x)$		$e^q(x)$		$f^{\perp}(x)$		$f_3^q(x)$	
	LCQM	LFHM	LCQM	LFHM	LCQM	LFHM	LCQM	LFHM
$u^{(\pi^+)}$	0.5	0.5	0.38	0.37	0.36	0.36	0.28	0.28
$u^{(K^+)}$	0.37	0.40	0.26	0.32	0.25	0.31	0.19	0.26

Table: Average longitudinal momentum fraction of pion and kaon

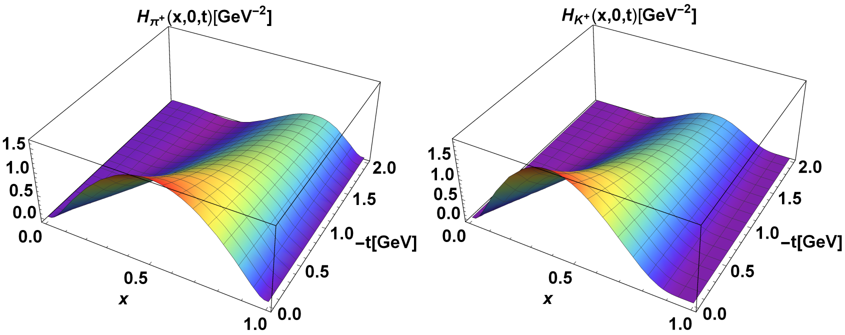
- The leading twist PDFs carry higher longitudinal momentum fration than higher twist PDFs.
- In case of Kaon, the $\bar{s}$ antiquark carry higher longitudinal momentum fraction than the u-quark.
- At $Q^2 = 4$ GeV^2, $\langle x \rangle$ is found to be 0.30 in LCQM which have very similar results with Lattice data and other models data.

Generalized Parton Distribution Function

- In case of spin-0 mesons, there is only $H^q(x, \xi, t)$ GPD at the leading twist, which can be represented in the overlap form as

$$
\begin{aligned}
H_M(x, \xi, -t) \;=\; & \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\big[\psi_0^*(x'', \mathbf{k}''_\perp, \uparrow, \uparrow)\psi_0(x', \mathbf{k}'_\perp, \uparrow, \uparrow) \\
& +\psi_0^*(x'', \mathbf{k}''_\perp, \uparrow, \downarrow)\psi_0(x', \mathbf{k}'_\perp, \uparrow, \downarrow) \\
& +\psi_0^*(x'', \mathbf{k}''_\perp, \downarrow, \uparrow)\psi_0(x', \mathbf{k}'_\perp, \downarrow, \uparrow) \\
& +\psi_0^*(x'', \mathbf{k}''_\perp, \downarrow, \downarrow)\psi_0(x', \mathbf{k}'_\perp, \downarrow, \downarrow)\big] .
\end{aligned}
$$

With $x'' = (x+\xi)/(1+\xi)$, $x' = (x-\xi)/(1-\xi)$, $\mathbf{k}'_\perp = \mathbf{k}_\perp - (1-x)\Delta_\perp/2$ and $\mathbf{k}''_\perp = \mathbf{k}_\perp + (1-x)\Delta_\perp/2$.....
are the initial and final active quark longitudinal and transverse momentum.



Pion and kaon GPDs at skewness $\xi = 0$.

S. Puhan, et.al, Arxiv: 2410.07596

Electromagnetic form factors

- The zeroth moment of the unpolarized GPD $H_M(x, 0, -t)$ provides an insight to the contribution of q-quark flavor to the total elastic EMFF of meson and can be described as

$$F_{M_q}(-t) = \int dx \; H_M(x, 0, -t) \, ,$$

where $t = -Q^2$.

- The meson form factor can be written as sum of quark and antiquark,

$$F_M(-t) = e_q F_{M_q}(-t) + e_{\bar{q}} F_{M_{\bar{q}}}(-t).$$

- These form factors obey the sum rule,

$$F_M(Q^2 = 0) = 1.$$

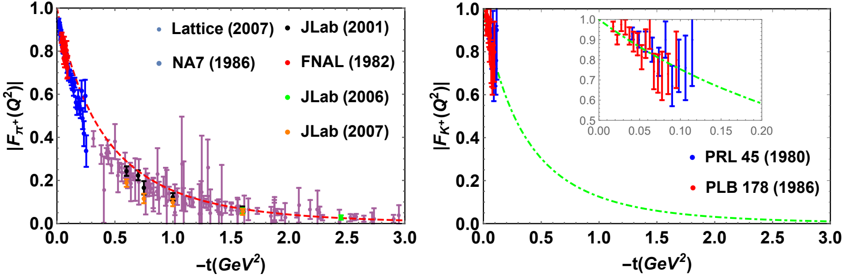
- The radius of the meson can also be calculated from EMFFs

$$\langle r^2 \rangle = -6\hbar^2 \frac{dF_M(Q^2)}{dQ^2} \, ,$$

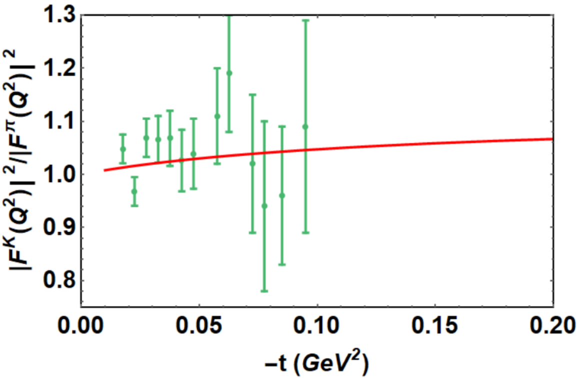
at $Q^2 = 0$.

Electromagnetic form factors

The EMFFs derived from GPDs



- Pion and kaon EMFFs have been compared with Experimental Data and Lattice Data Points.



Kaon to pion EMFFs ratio.
[Data Points:-PLB 178 (1986)]

- The charge radius of pion is found to be 0.297 $\mathrm{fm}^2$and for kaon, it is 0.287 fm^2.

Theoeretical Extraction of TMDs and PDFs

- Extraction of pion transverse momentum distributions from Drell-Yan data [MAP (Multi-dimensional Analyses of Partonic distributions) Phys. Rev. D **107** (2023)]
- Pion-induced Drell-Yan processes within TMD factorization[Alexey Vladimirov, JHEP 10(2019).]
- Constraining kaon PDFs from Drell-Yan and J/ production [Wen-Chen Chang, Jen-Chieh Peng, Stephane Platchkov, Takahiro Sawada, Arxiv:2402.02860]

Medium Effect on DFs

Nuclear Medium Effect

on

Pion TMDs and PDFs.

Chiral SU(3) Quark Mean Field Model

- For medium modifications, we have adopted the chiral SU(3) quark mean field model.
  - The quarks are bound inside a hadron through confining potential and interact with each other via scalar fields.
  - Low energy properties: Chiral symmetry and its spontaneous breaking are incorporated in this model.
  - Broken- Scale invariance of QCD is also stimulated through the dilation field χ.
- The general effective Lagrangian density of CQMF model is expressed as

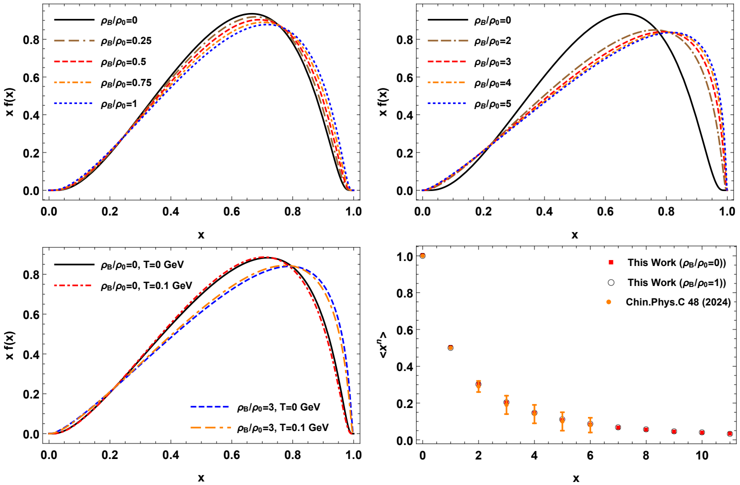
$$\mathcal{L}_{\rm eff} = \mathcal{L}_{q0} + \mathcal{L}_{qm} + \mathcal{L}_{\Sigma\Sigma} + \mathcal{L}_{VV} + \mathcal{L}_{\chi SB} + \mathcal{L}_{c}.$$

Chiral SU(3) Quark Mean Field Model

- $\mathcal{L}_{q0} = \bar{q}\, i\gamma^\mu \partial_\mu\, q$ is the kinetic term for quarks.
- $\mathcal{L}_{qm}$ describes the interactions of constituent quarks with the scalar and vector mesons.
- The self-interactions of scalar mesons σ, ζ and δ and the dilaton field χ are described by the third term $\mathcal{L}_{\Sigma\Sigma}$.
- The fourth term $\mathcal{L}_{VV}$ gives the self interactions of vector mesons ω and ρ.
- The term $\mathcal{L}_{\chi SB}$ representing the explicit symmetry breaking term.
- $\mathcal{L}_c$ corresponds to the confinement of quarks inside the hadrons.

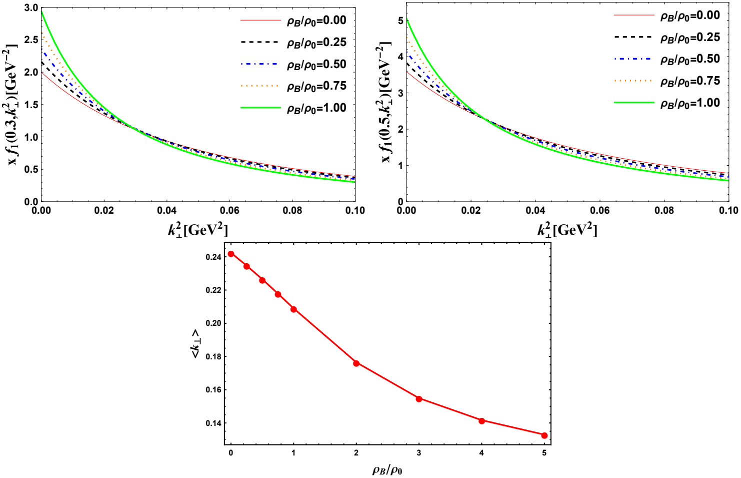
For More details- S. Puhan, et.al, PRD, **110** (2024).

Baryonic Density and Temperature effects on PDFs



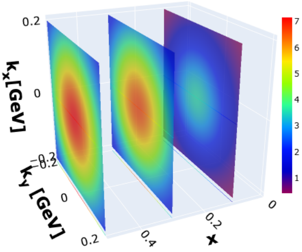
S. Puhan, et.al, PRD, **110** (2024).

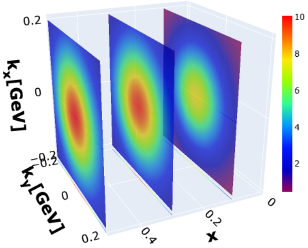
TMDs in Medium

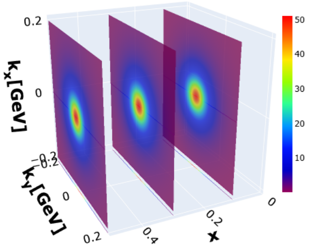


N. Kaur, S. Puhan, et.al, Arxiv:2409.05394.

Spin Densities in Medium





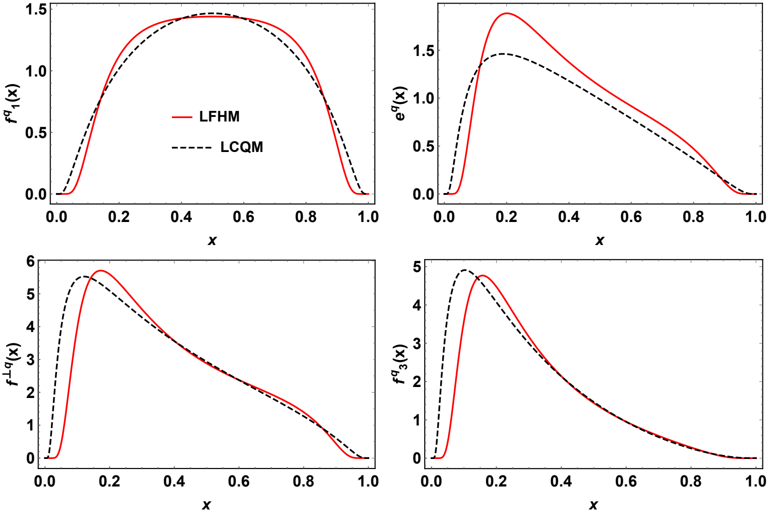


Thank You

Backup Slides

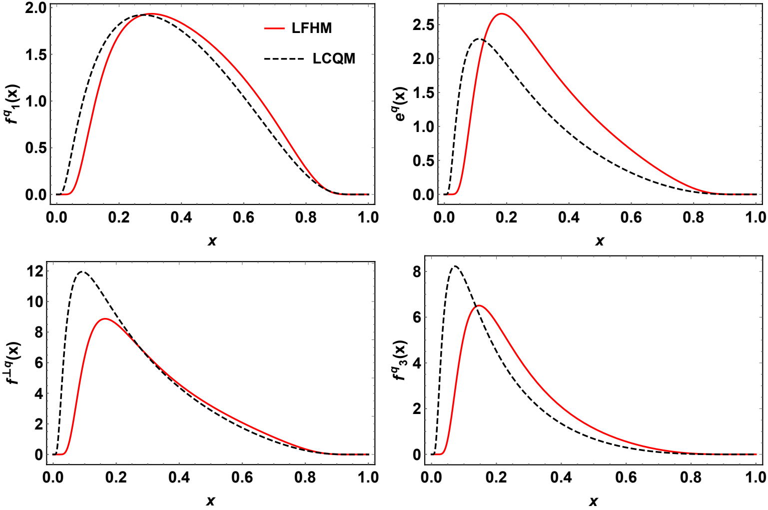
Backup Slides

Pion u-quark PDFs



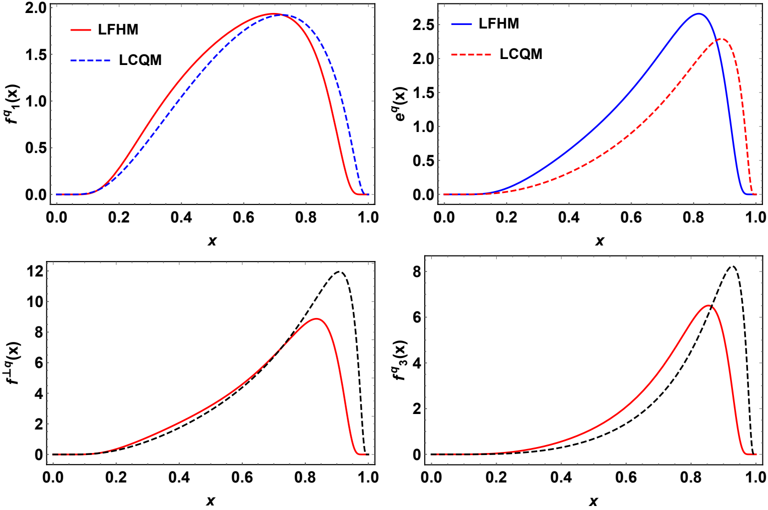
Pion u- quark comparison in both models.

Kaon u-quark PDFs



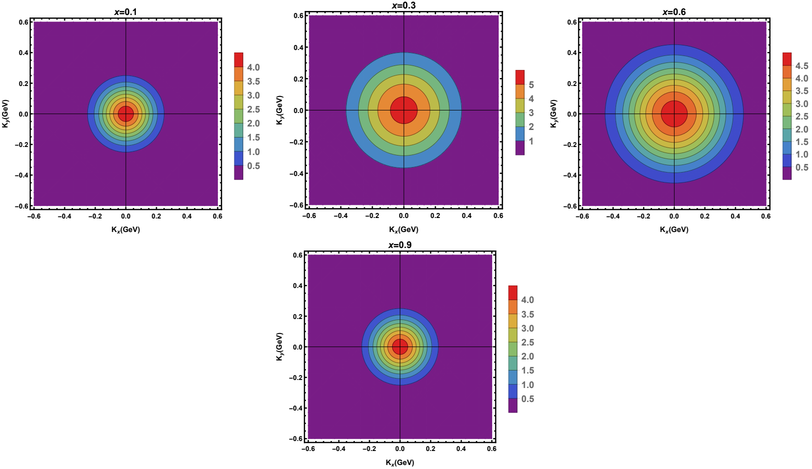
Kaon u- quark comparison in both models.

Kaon s-antiquark PDFs



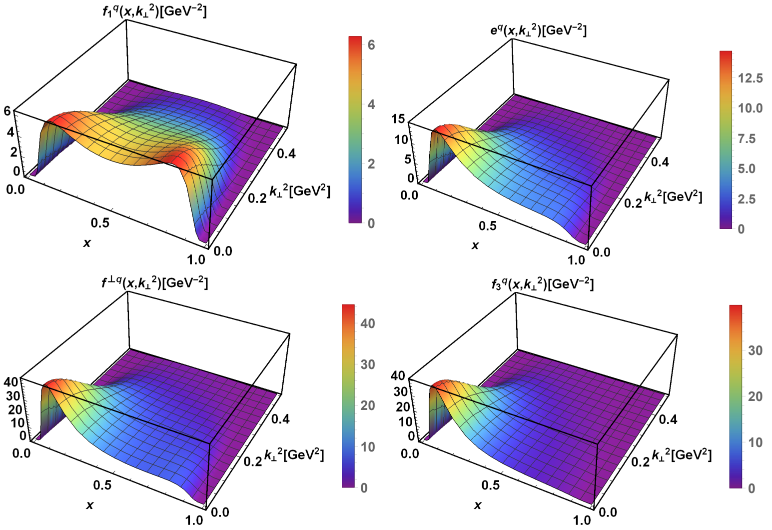
Kaon s- antiquark comparison in both models.

TMSDs

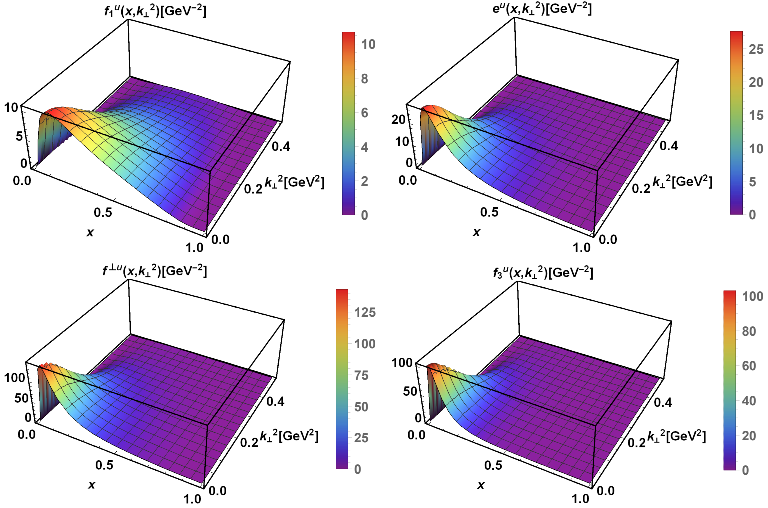


Pion u- quark f_1^q TMSDs in LFHM model.

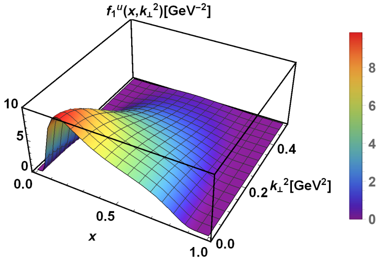
Pion u-quark in LFHM

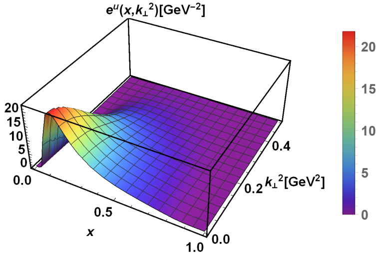


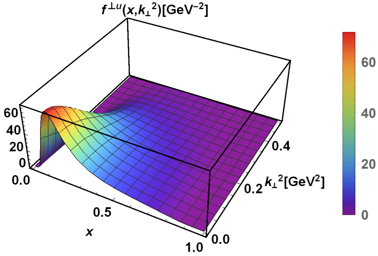
Kaon u-quark in LCQM

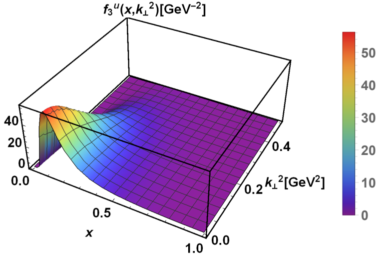


Kaon u-quark in LFHM









PDF

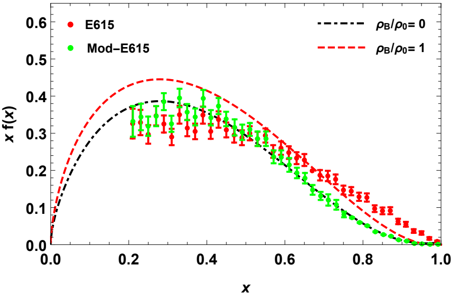


Figure: Caption