Lambda(1405) in the flavor SU(3) limit from lattice QCD

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Hadrons and Hadron Interactions in QCD 2024 (HHIQCD 2024)

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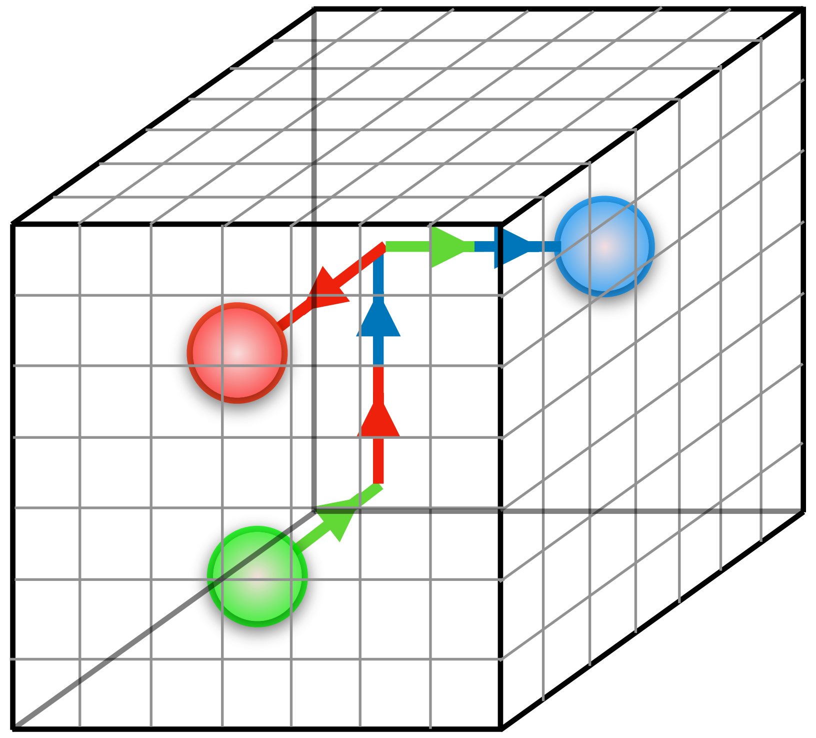
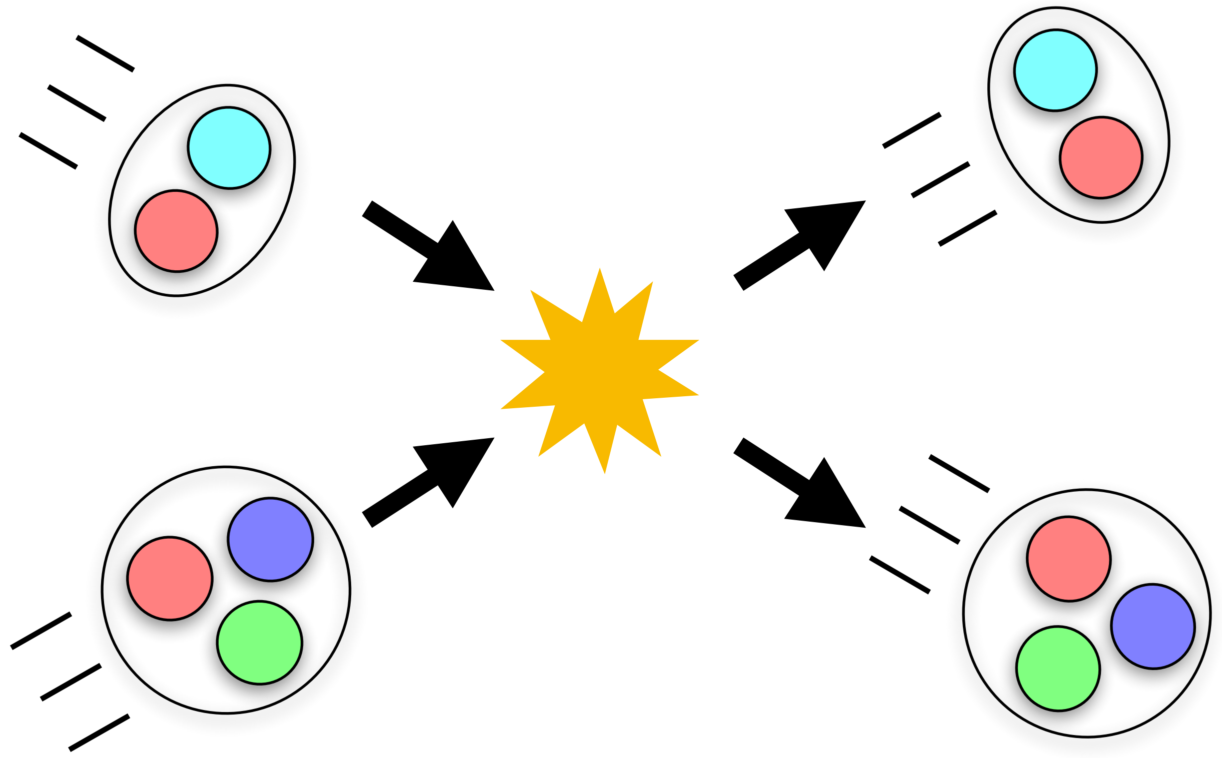
Introduction

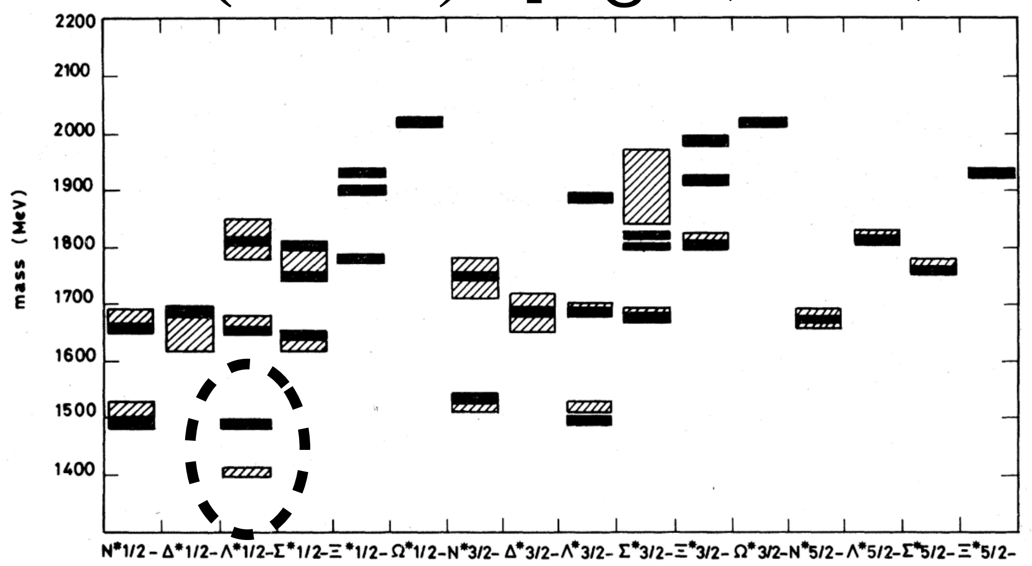
- ultimate goal: understand the exotic hadrons from lattice QCD
- key: hadron scatterings (interactions)

Lattice QCD | **hadron scatterings** | **exotic hadrons**

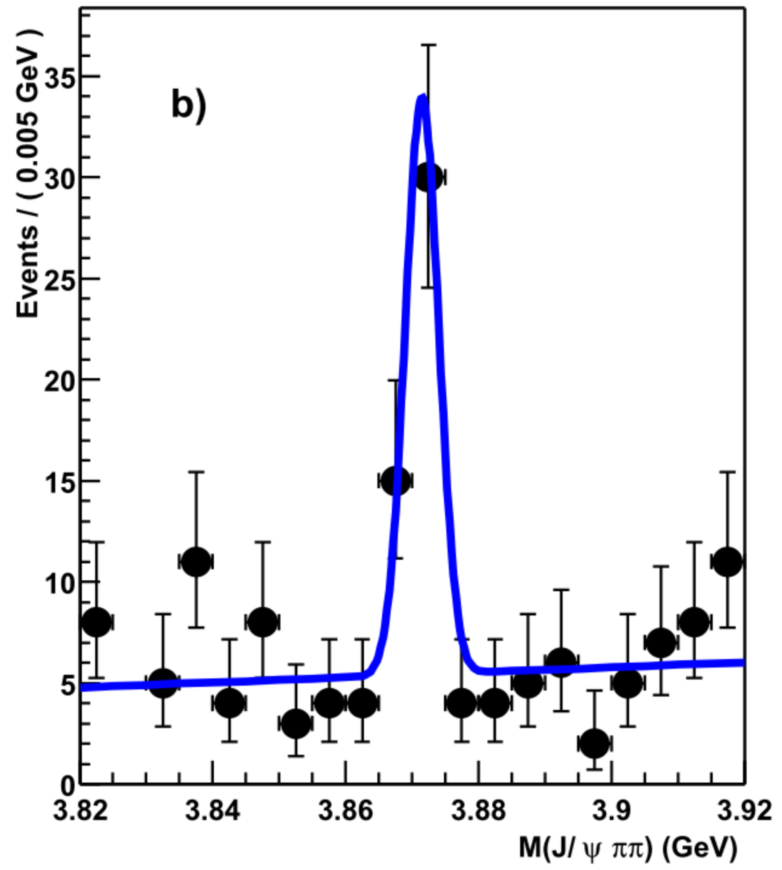




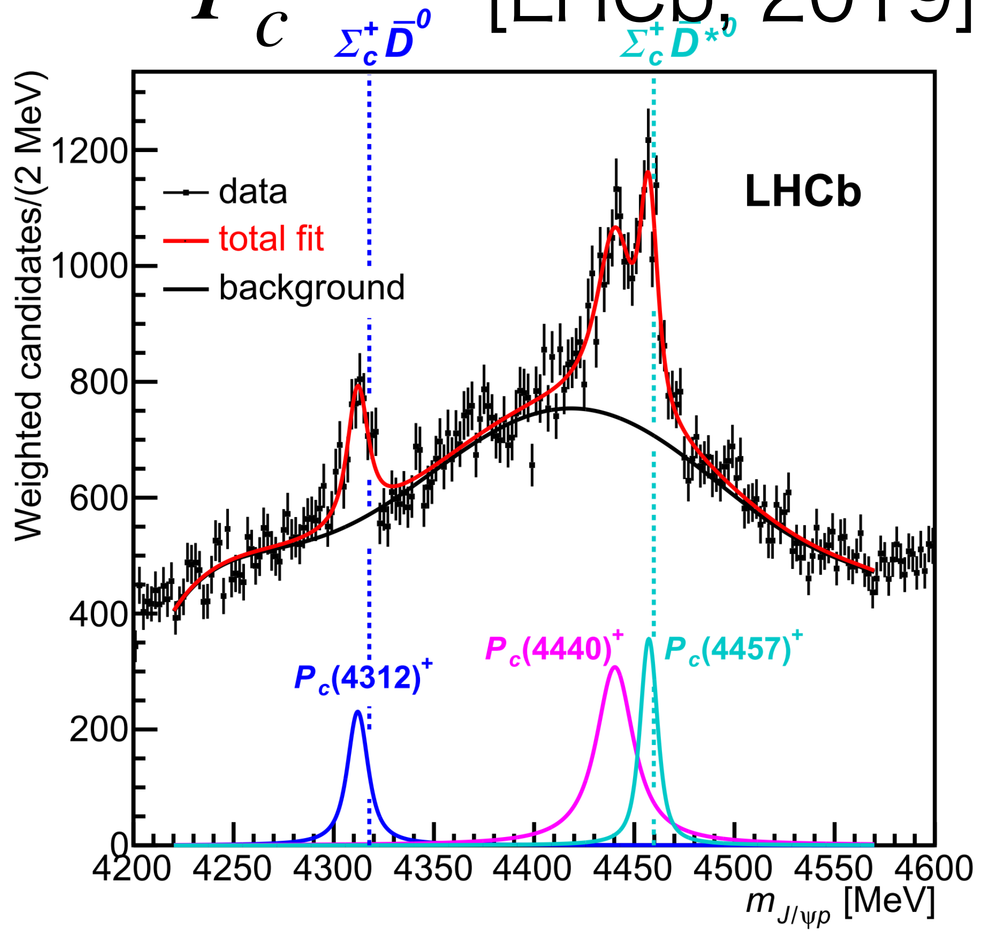
$\Lambda(1405)$ [Isgur, Karl, 1978]



$X(3872)$ [Belle, 2003]



P_c [LHCb, 2019]



- Finite-volume method [Lüscher, 1991]

  (c.f., talks by M. Nagatsuka, J. R. Green, M. Tomii)

- HAL QCD method [Ishii, Aoki, Hatsuda 2007]

  (c.f., talks by T. Doi, Y. Lyu, W. Yamada)

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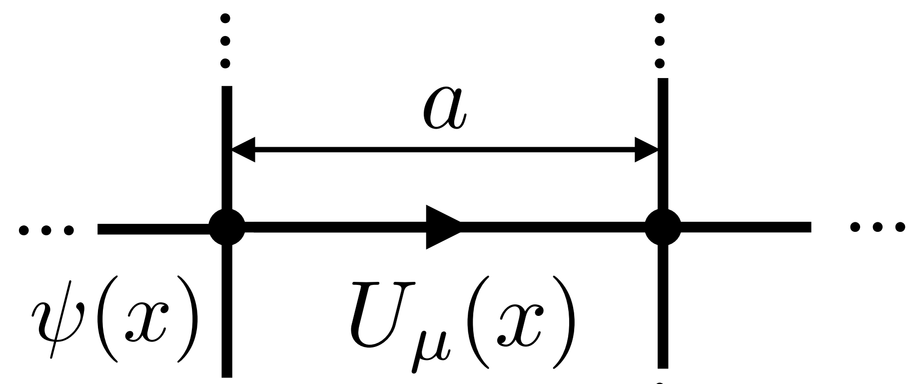
Lattice QCD

- lattice QCD: **QCD in discrete Euclidean spacetime**



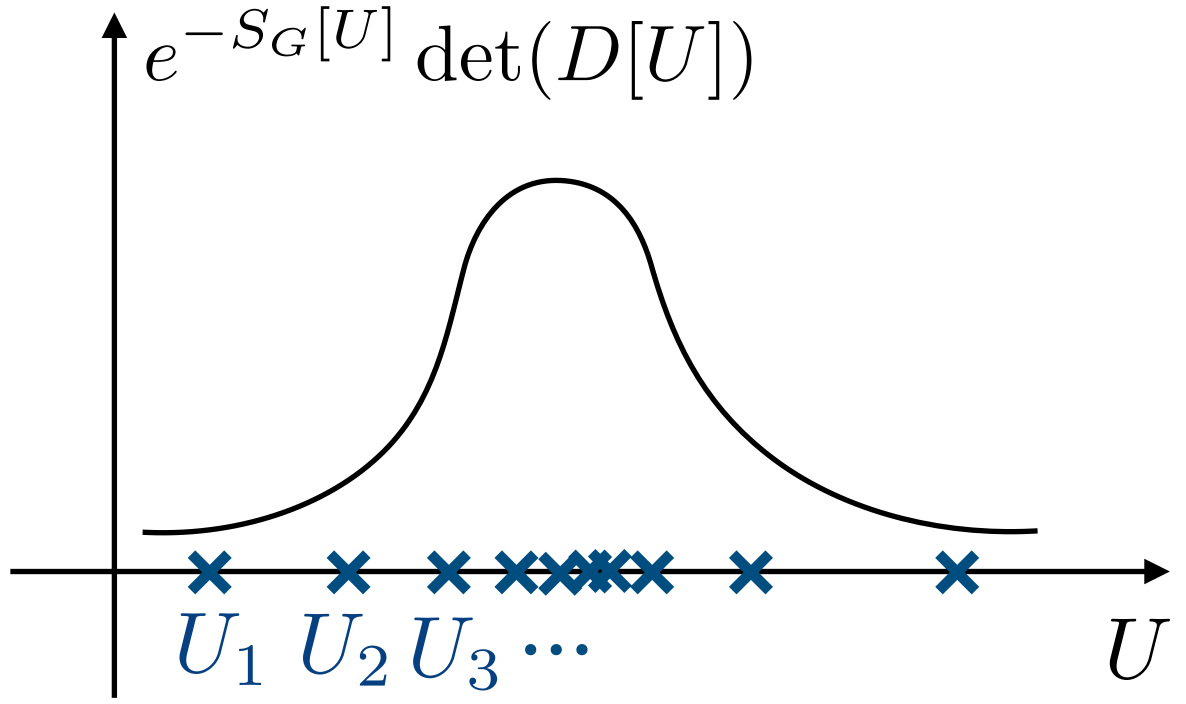
quark field

gluon field (link variable)

$$\langle O \rangle = \frac{1}{Z}\int \mathcal{D}[U] e^{-S_G[U]} \det(D[U])\ \tilde{O}[D^{-1}[U], U]$$

$$= \lim_{N\to\infty} \frac{1}{N}\sum_n \tilde{O}[D^{-1}[U_n], U_n]$$

Monte Carlo method



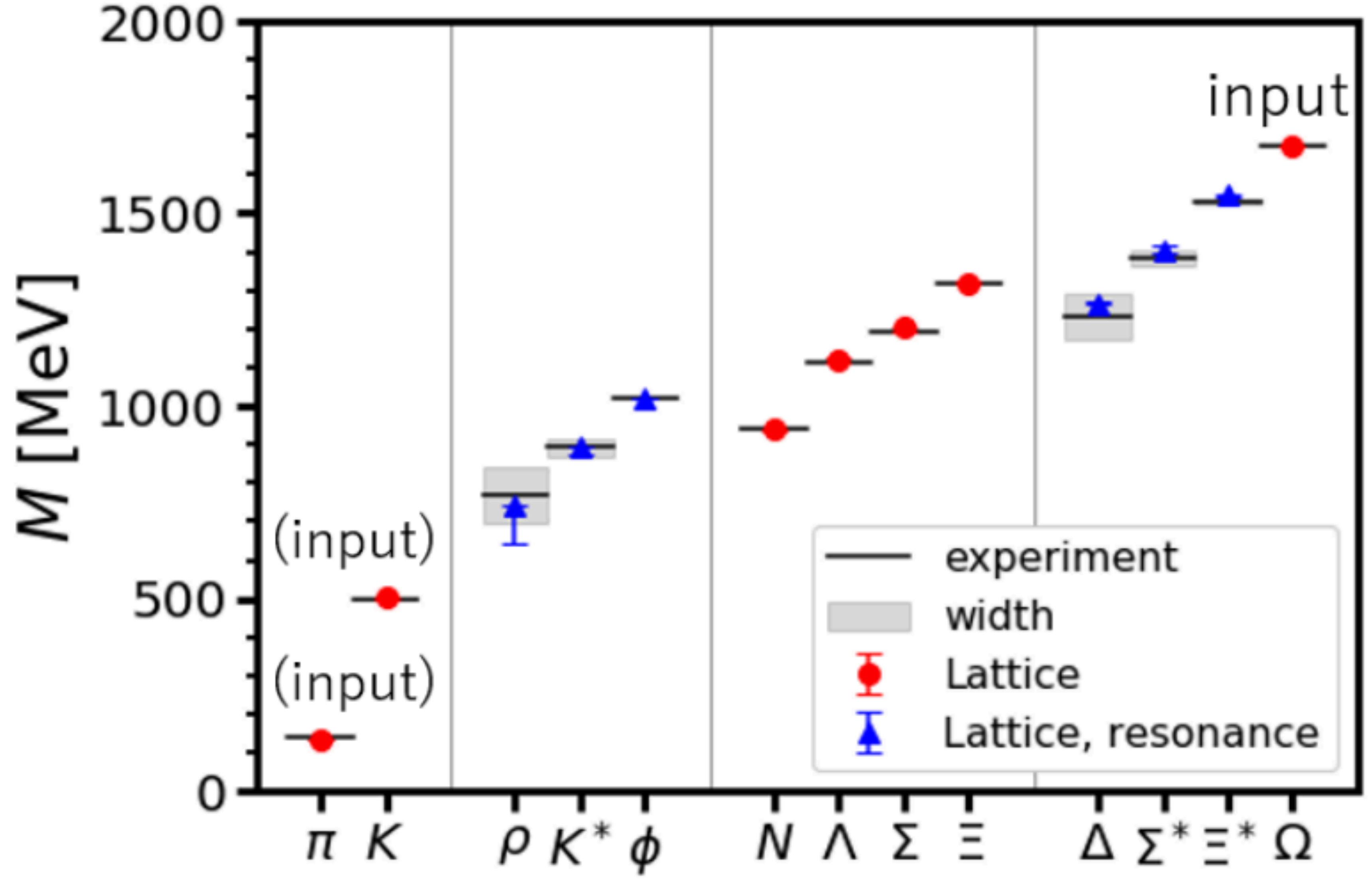
- example: hadron masses from 2pt correlation functions

meson: $\bar{\psi}(t)\Gamma\psi(t)$, baryon: $\psi(t)\psi(t)\psi(t)$

$$C_O(\tau) = \langle O(\tau) O^\dagger(0) \rangle$$

$$\underset{t\to\infty}{\to} (\text{const.}) \times e^{-m_O \tau}$$

hadron masses at $m_\pi \approx 137$ MeV



[HAL QCD Collab., 2024]

Hadron scatterings in lattice QCD

- key quantity: Equal-time Nambu-Bethe-Salpeter (NBS) wave function

$$\Psi^W(\mathbf{r}) = \langle 0 | \underbrace{O_1(\mathbf{r},0) O_2(\mathbf{0},0)}_{\text{hadron operators}} \underbrace{|1,2;W\rangle}_{\text{2-body hadron state with energy } W}$$

hadron operators

2-body hadron state with energy W

$(W = \sqrt{k^2+m_1^2} + \sqrt{k^2+m_2^2})$

$$\xrightarrow[r\to\infty]{} A \frac{\sin(kr - \frac{l}{2}\pi + \delta^l(k))}{kr} e^{i\delta^l(k)} Y_{l,m}(\Omega)$$

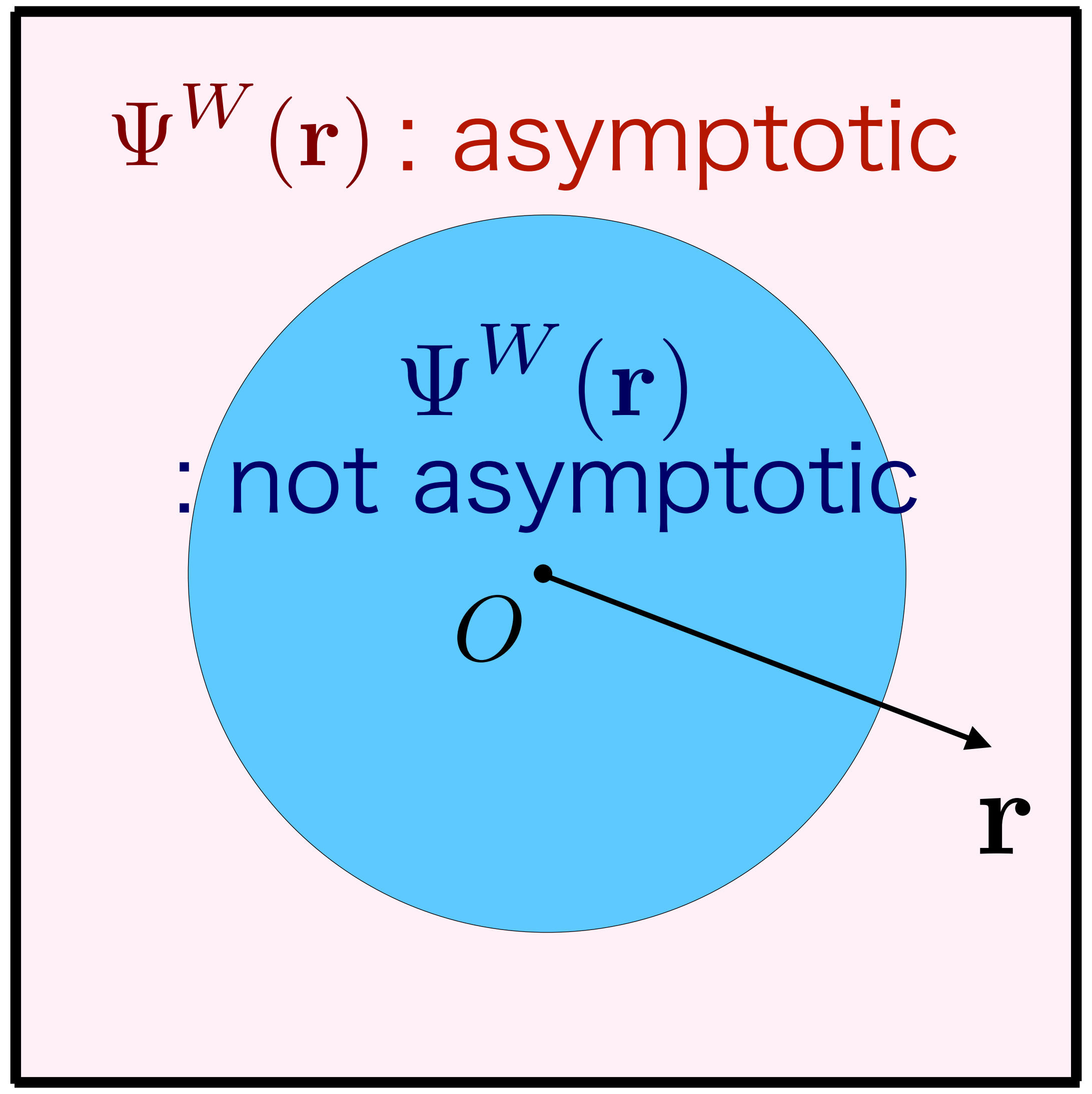
phase shift

[Lin, Martinelli, Sachrajda, Testa, 2001]

- methods based on NBS wave function:
  - Finite-volume method [Lüscher, 1991]

    finite-volume energy ⟶ quantization condition ⟶ phase shift
  - HAL QCD method [Ishii, Aoki, Hatsuda 2007]

    NBS wave function ⟶ **interaction potential** ⟶ phase shift



HAL QCD method

[Ishii, Aoki, Hatsuda 2007]
[Ishii et al. 2011]

- R-correlator:

$$R(\mathbf{r},t) = \frac{\langle O_1(\mathbf{r},t) O_2(\mathbf{0},t) \bar{J}(0)\rangle}{\langle O_1(t)\bar{O}_1(0)\rangle\langle O_2(t)\bar{O}_2(0)\rangle} \approx \sum_n C_{\bar{J},n} \underbrace{\Psi^{W_n}(\mathbf{r})}_{\text{NBS wave function}} e^{-(W_n - m_1 + m_2)t}$$

- time-dependent HAL QCD method

(μ: reduced mass)

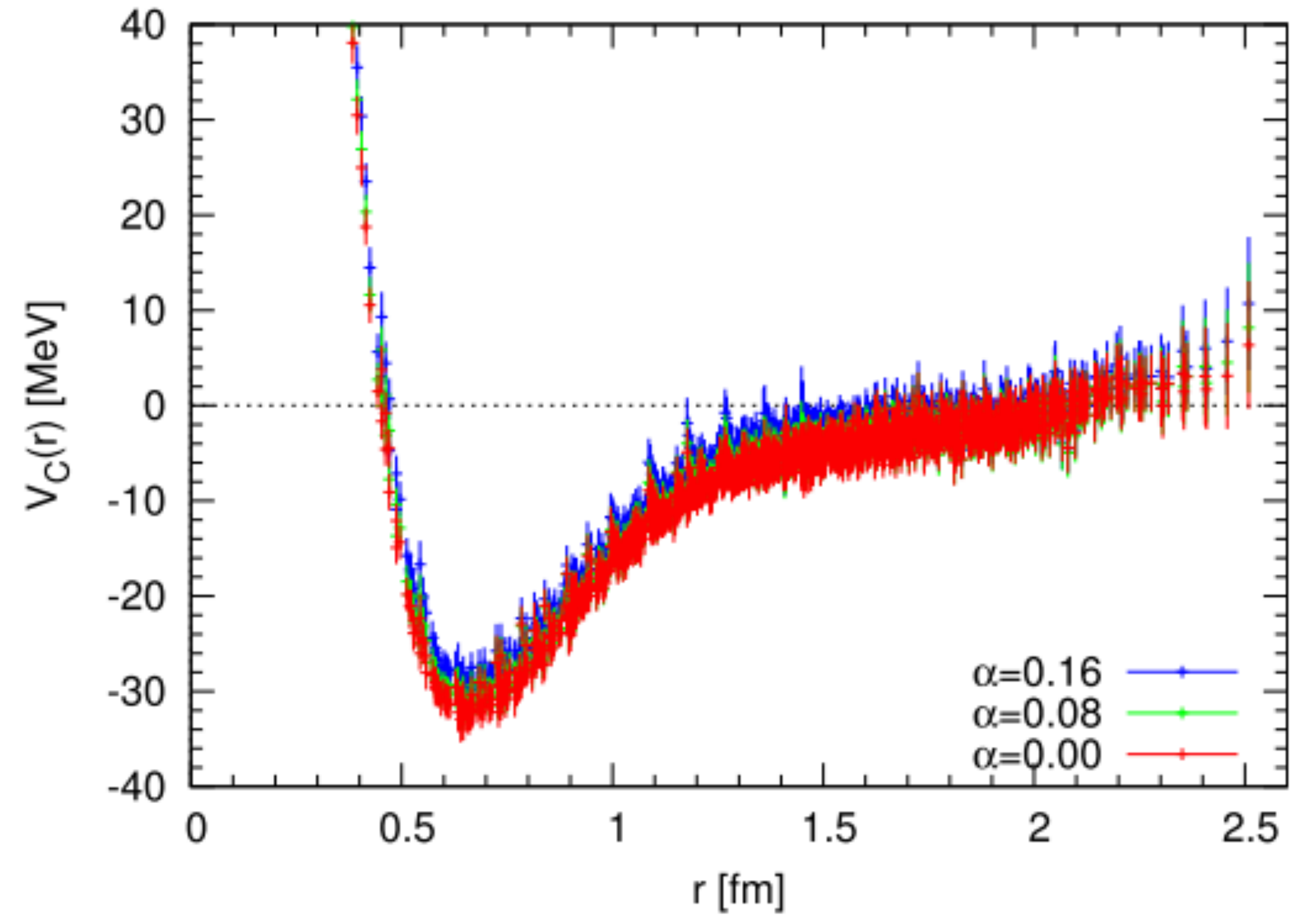
$$\int d^3 r' \, U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t) \simeq \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right) R(\mathbf{r},t)$$

$$\approx V(r)\delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

(local (leading-order) approximation)

$$\rightarrow \quad V(\mathbf{r}) \simeq \frac{1}{R(\mathbf{r},t)}\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right) R(\mathbf{r},t)$$

e.g. NN potential



[Ishii et al. 2011]

Exotic hadrons and quark pair annihilations

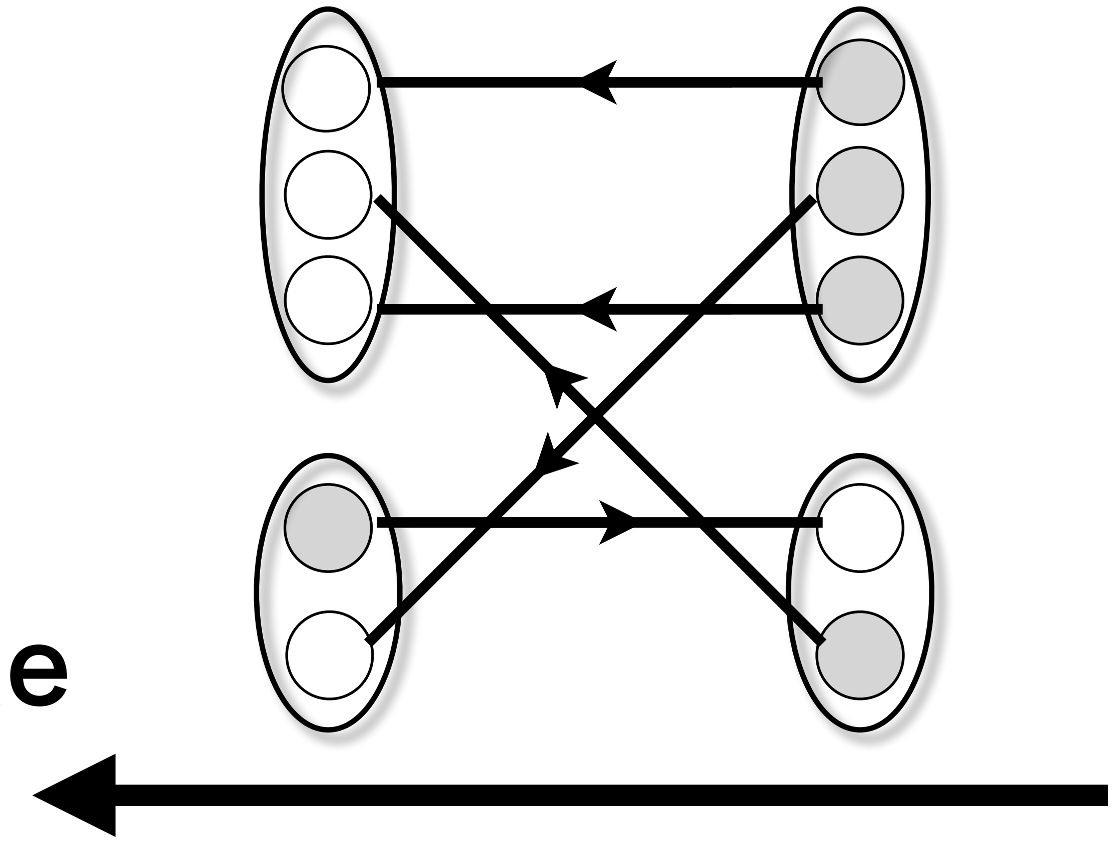
- situation of HAL QCD studies much depends on whether the system has **quark pair annihilations**

(neglect $Q\bar{Q}$ annihilation)

- system **w/o** quark pair annihilations:

$QQ\bar{q}\bar{q}$, $Q\bar{Q}q\bar{q}'$, $Q\bar{Q}qqq$, $q\bar{q}'qqq$

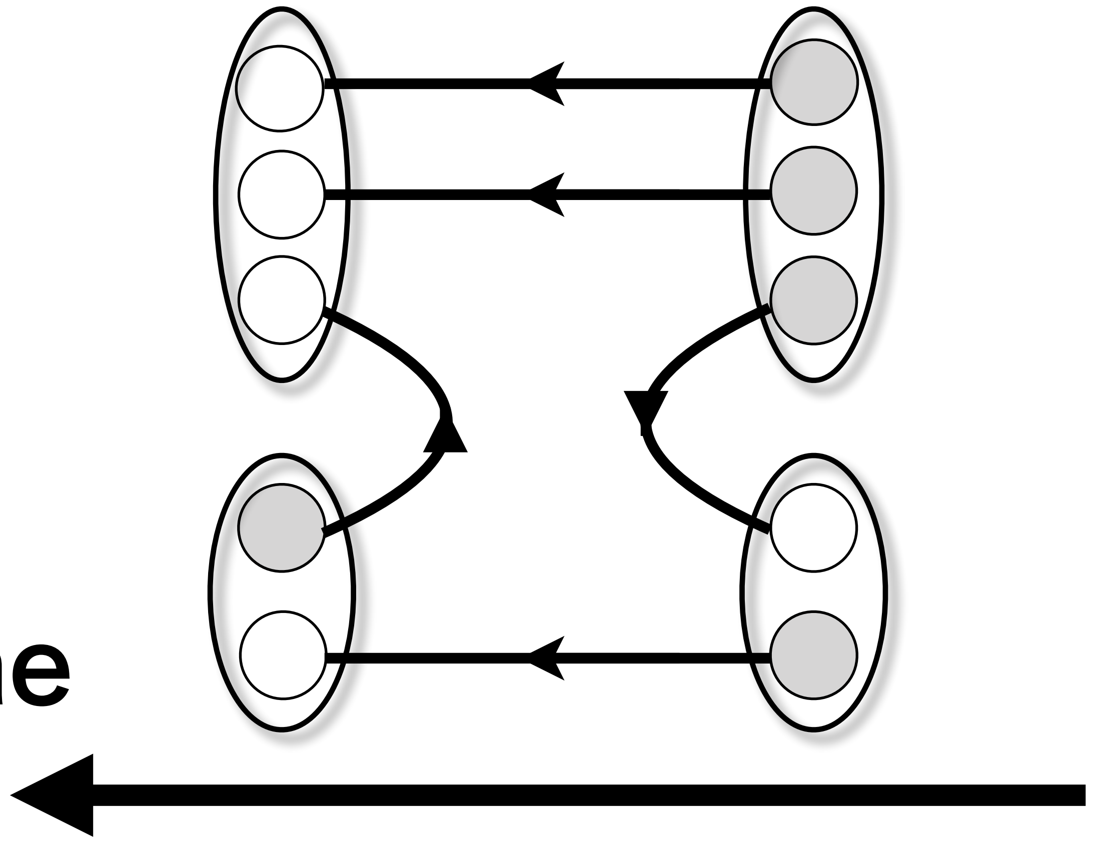
T_{cc} Z P_c Θ^+



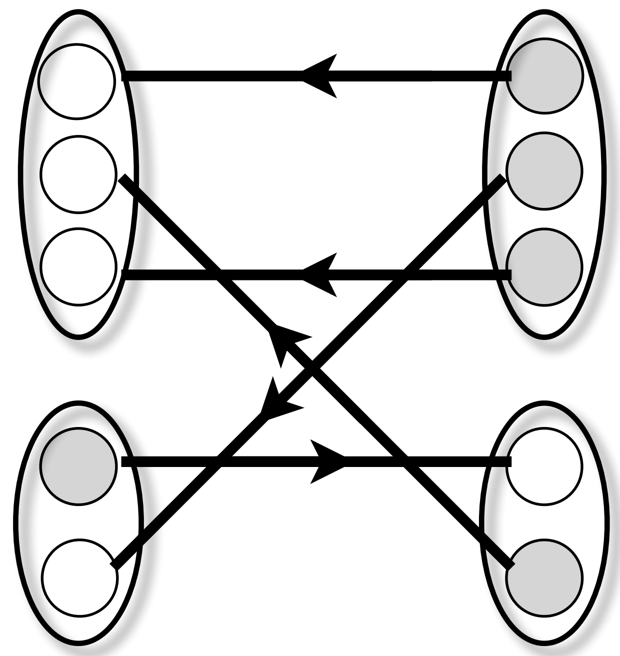
- system **w/** quark pair annihilations:

resonances, $Q\bar{Q}q\bar{q}$, $q\bar{q}q\bar{q}$, $q\bar{q}qqq$

X f_0/σ $\Lambda(1405)$



Exotic hadrons w/o quark pair annihilations



- HAL QCD studies have been done **in almost realistic setups**

m_π

$\gtrsim$ 400 MeV

- $Z_c(3900)$ [Ikeda et al., 2016]
- P_c pentaquark [Sugiura et al., 2018]
- $\Theta^+(1540)$ [Ikeda, 2011 (PoS)] [**KM**, Akahoshi, Aoki, 2020]
- T_{bb} tetraquark [T. Aoki, S. Aoki, Inoue, 2023]

146 MeV near-physical point

- $\Omega\Omega$ dibaryon [Gongyo et al., 2018]
- $N\Omega$ dibaryon [Iritani et al., 2019]
- H-dibaryon [Sasaki et al., 2020]
- $\Omega_{ccc}\Omega_{ccc}$ dibaryon [Lyu et al., 2021]
- T_{cc}^+ tetraquark [Lyu et al., 2023]
- $N\phi$ + femtoscopy [Lyu et al., 2022] [Chizzali et al., 2022]

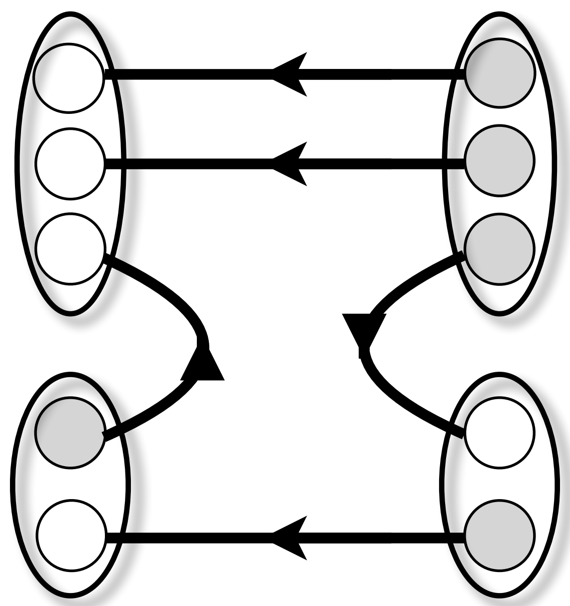
138 MeV physical point

configuration generated using Supercomputer Fugaku (Japan) [HAL QCD Collab., 2024] → hadron interactions ongoing

$H, KN, \bar{D}N, \Omega_{ccc}N, \Lambda_c N, J/\psi N, \cdots$

HAL QCD method in quark pair annihilations

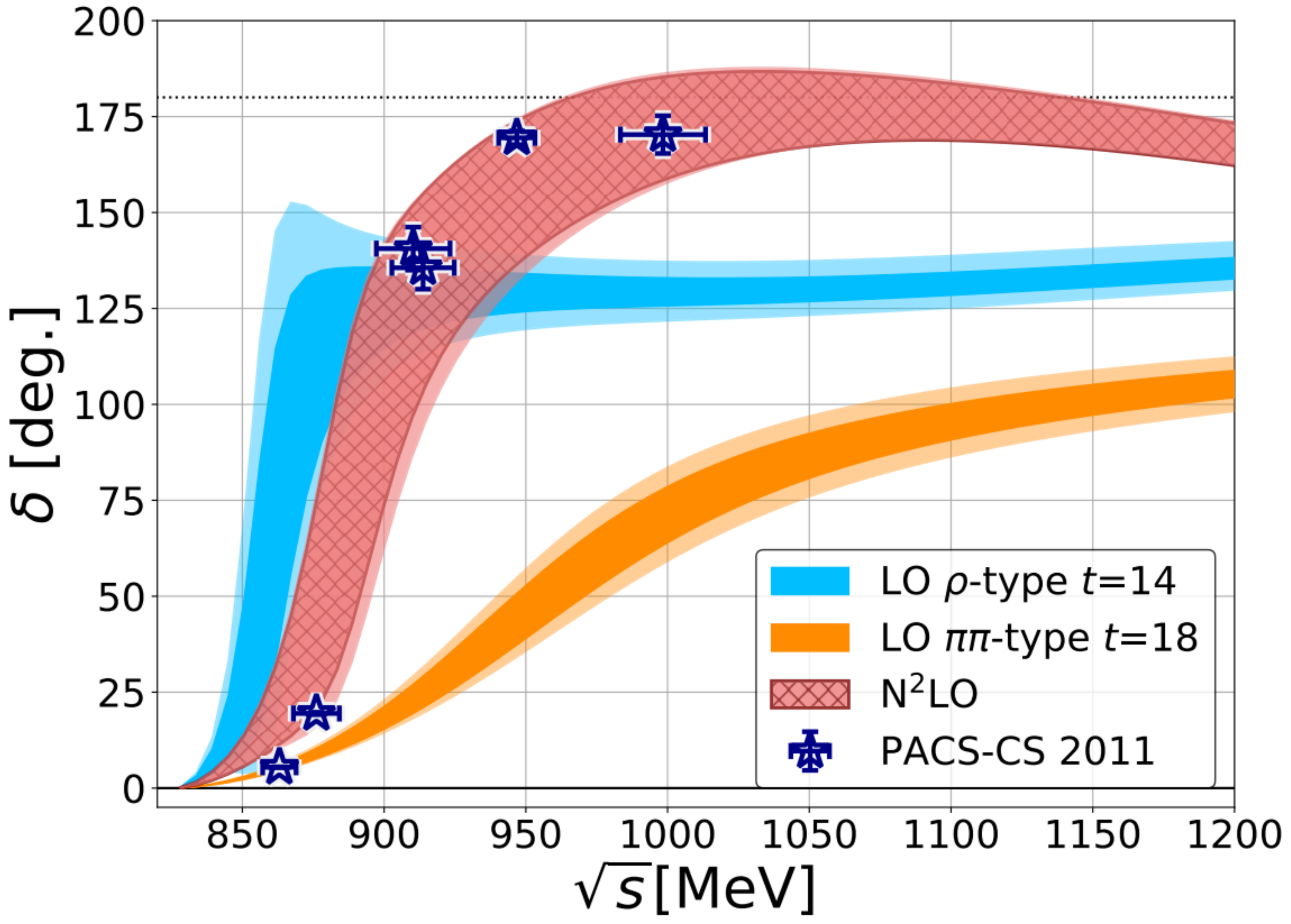
- hadron resonances/some exotic hadrons:
  quark-pair annihilation diagrams appear
  ➡ computational cost is very high ← $\times O(L^4)$ larger



- **New calculation technique** to suppress the cost in HAL QCD

[Akahoshi, Aoki, Doi, 2021]

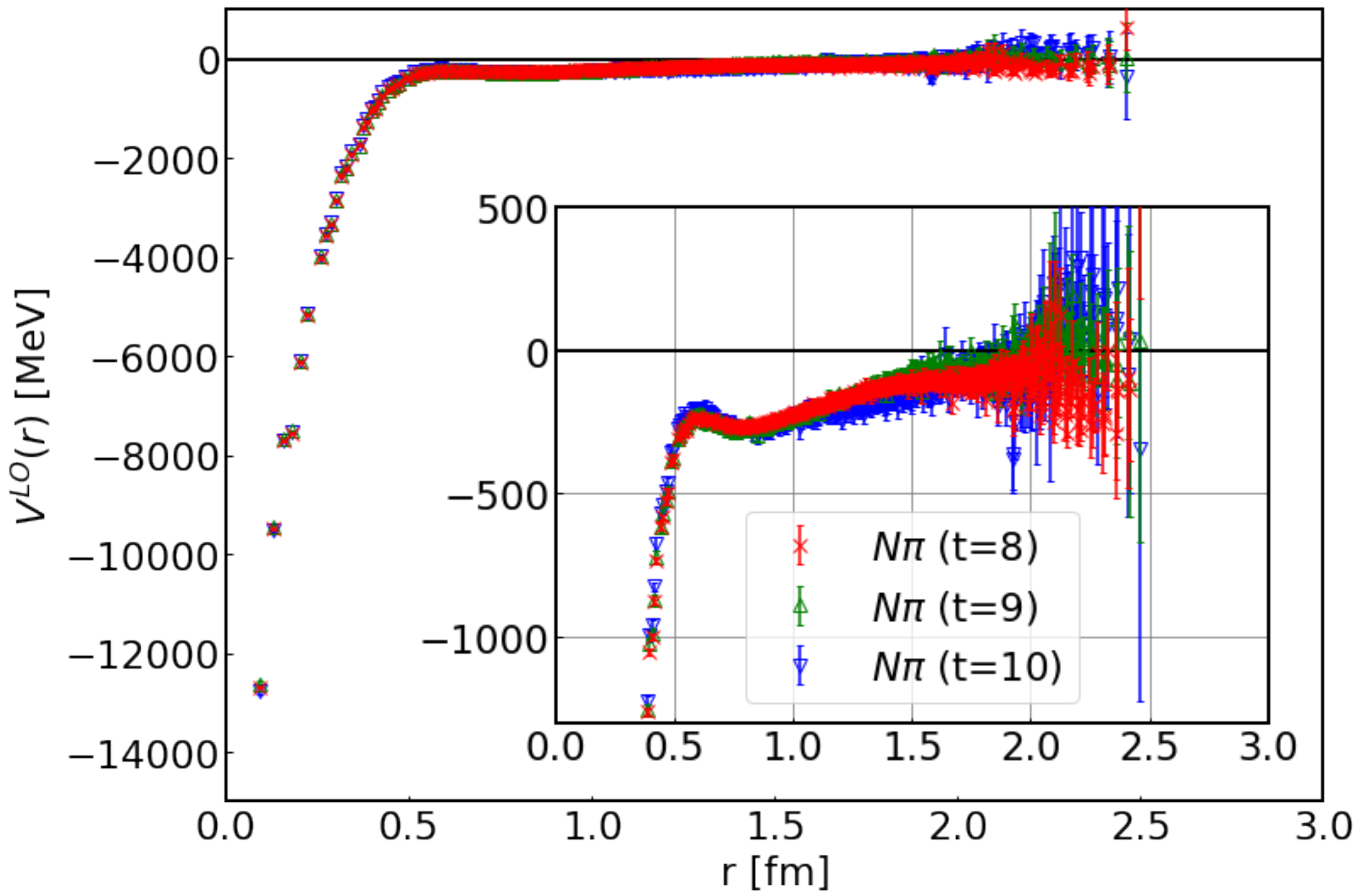
- $\pi\pi \to \rho$ (resonance)
  [Akahoshi, Aoki, Doi, 2021]



one-end trick + All-mode averaging (AMA)

[M. Foster, C. Michael, 1999]
[Bali, Collins, Schäfer 2010]
[Blum, Izubuchi, Shintani 2013]

- $N\pi$ $(\Xi\bar{K}) \to \Delta$ (Ω) (stable)
  [KM, Akahoshi, Aoki, Doi, Sasaki, 2023]



noise + AMA (w/o low lode)

- next step:
  exotic hadrons
  ($\Lambda(1405)$ etc.)

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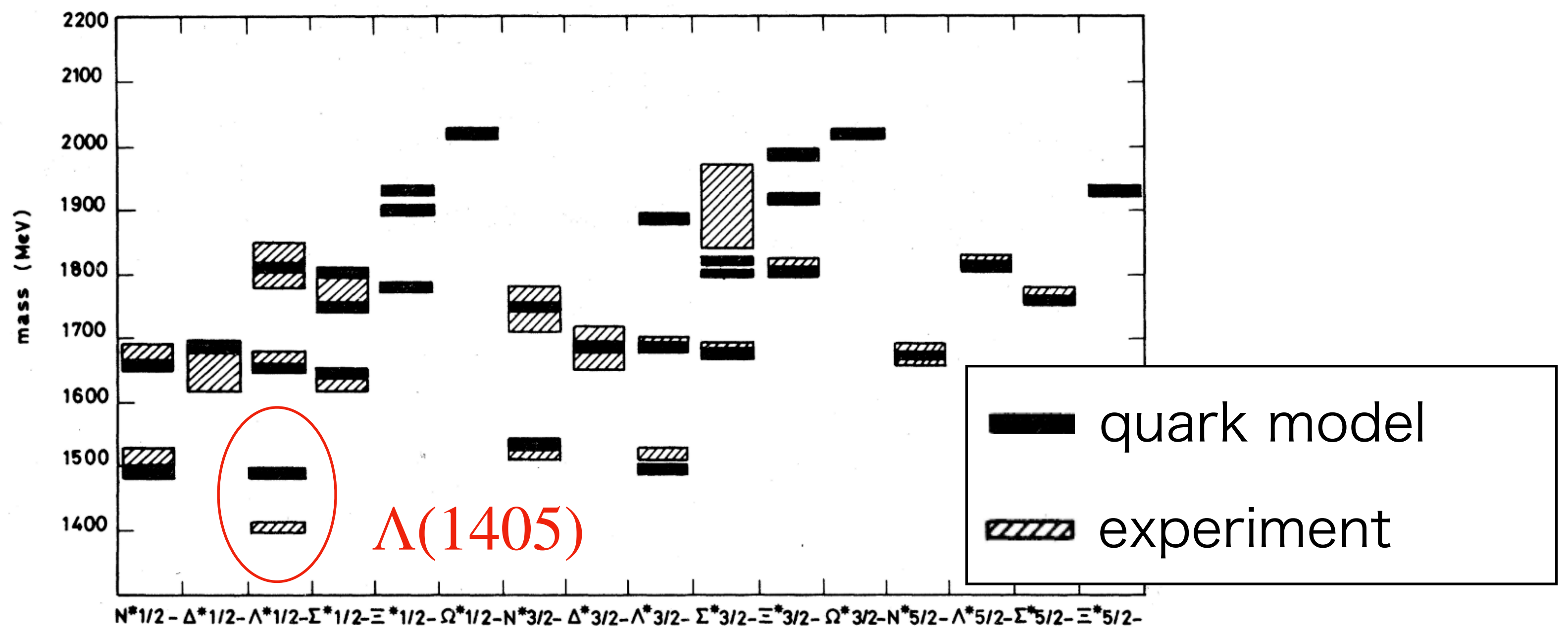
$\Lambda(1405)$

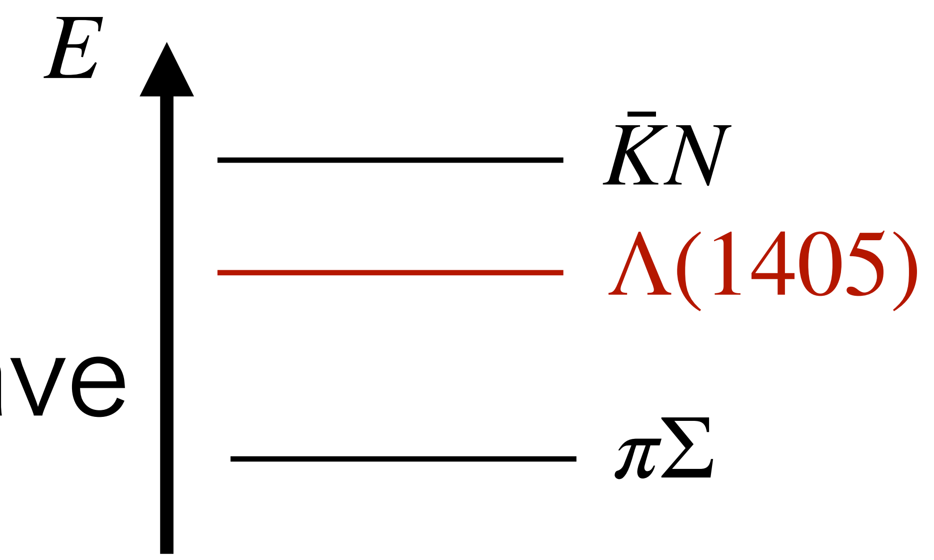
- $I = 0, J^P = 1/2^-$ baryon known from 1950s
  [Dalitz, Tuan, 1959]

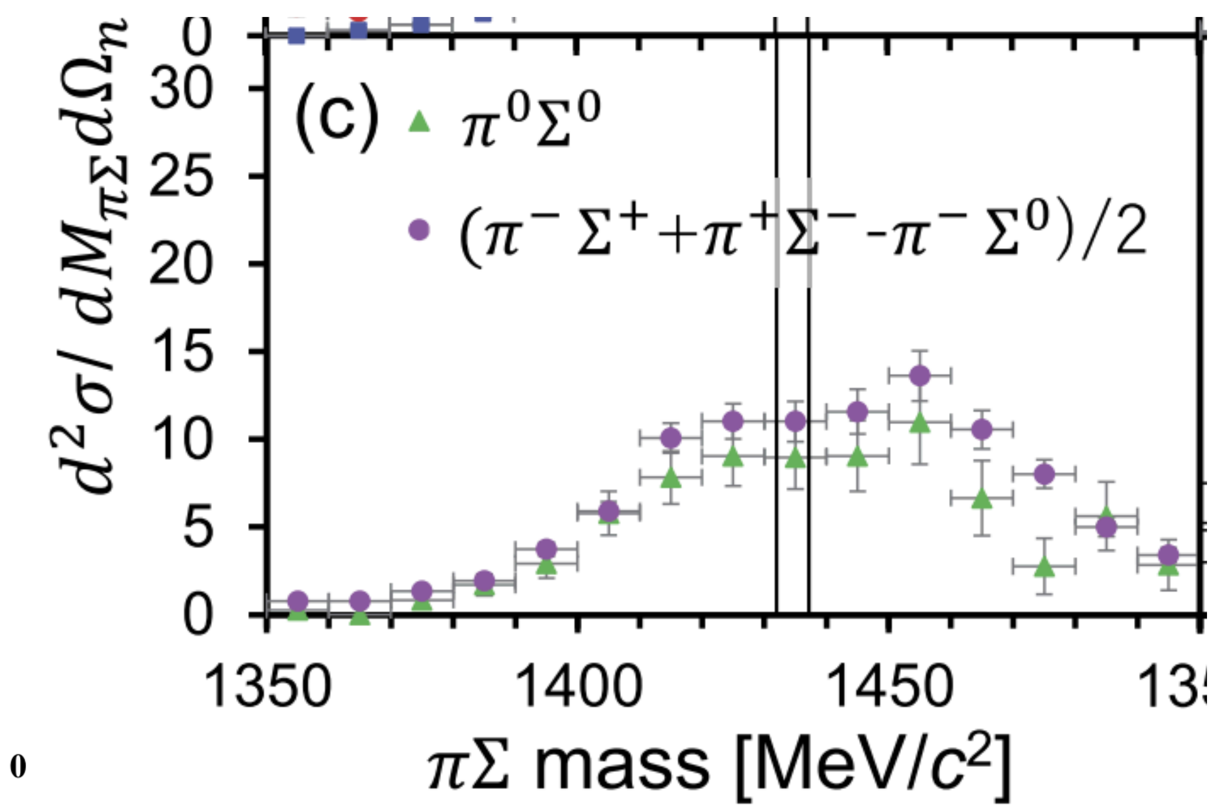


[Isgur and Karl, 1978]

- not a simple Λ baryon

- (mainly) couple to $\bar{K}N$ and $\pi\Sigma$
  - both $\bar{K}N$ and $\pi\Sigma$ scatterings have been experimentally measured

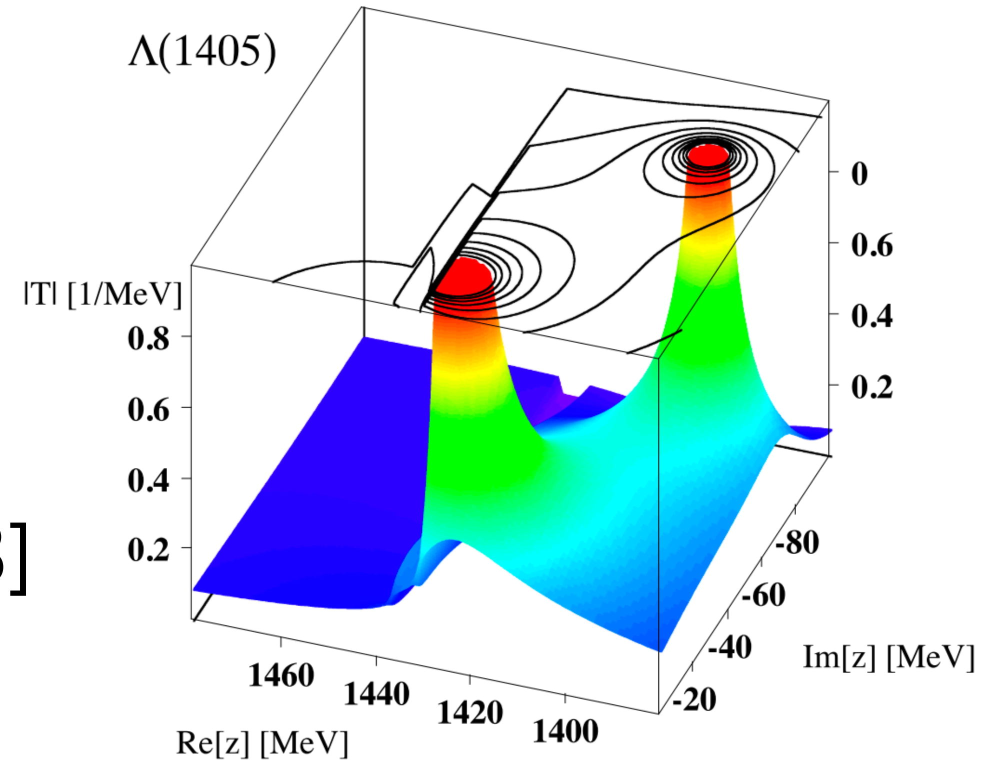




[J-PARC E31,2023]

- **two pole structure?**
  - suggested by chiral unitary model

[Oller and Meissner, 2001]
[Jido, Oller, Oset, Ramos, Meissner, 2003]



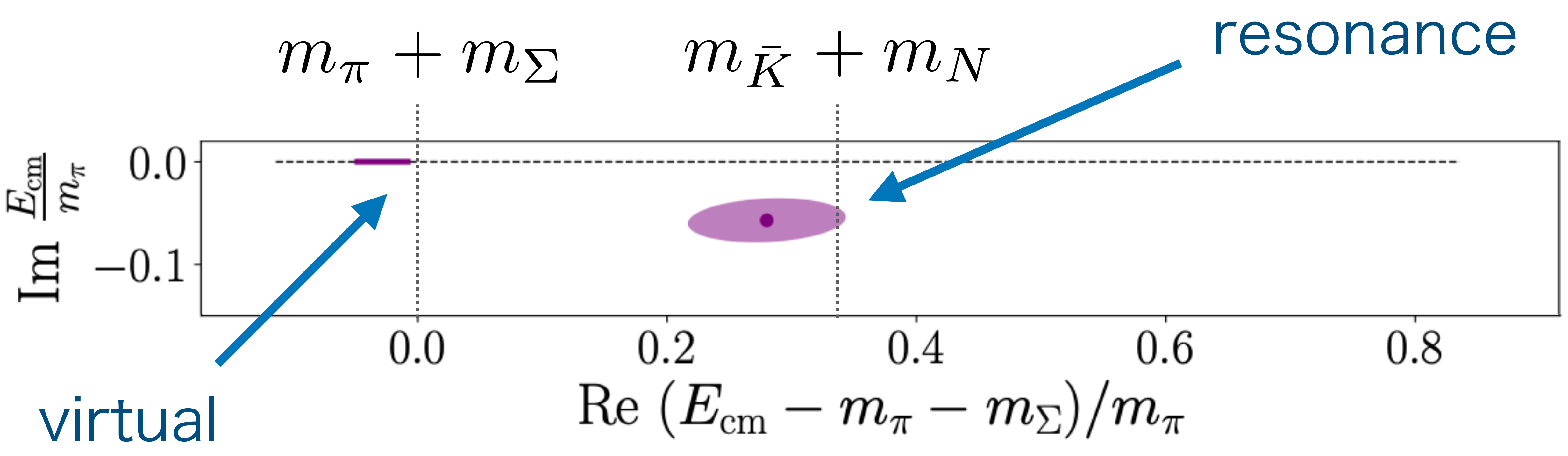
[Hyodo and Jido 2012]

first-principle calculation is needed

$\Lambda(1405)$ from lattice QCD

- early studies of $\Lambda(1405)$ spectrum from two-point functions [Melnitchouk et al., 2003; Nemoto et al., 2003; BGR Collab., 2006; Ishii et al., 2007; Takahashi, Oka, 2010; Menadue et al., 2012; Engel et al., 2013; Gubler et al., 2016] and EM form factor [Hall et al., 2014]

- study from $\bar{K}N-\pi\Sigma$ scatterings in finite-volume method at $m_\pi \approx 200$ MeV



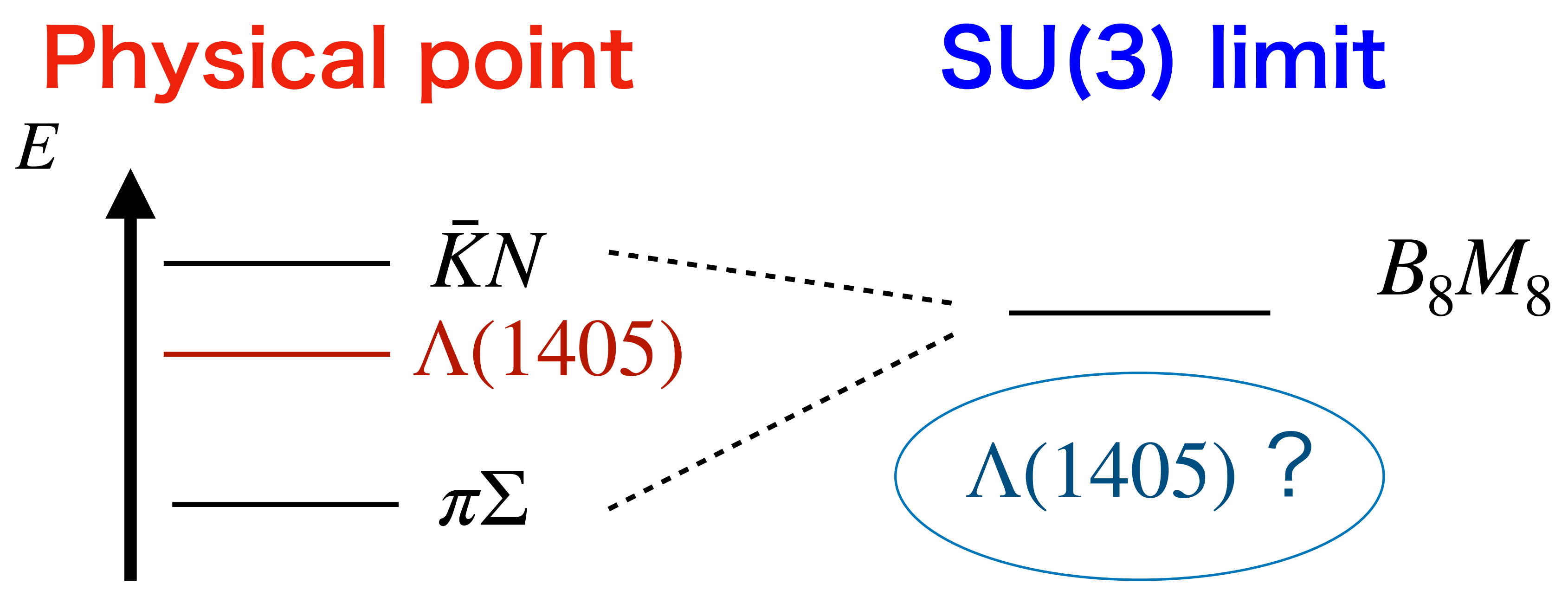
[Bulava et al. (BaSc Collab.), 2024]

- our goal: **study from HAL QCD approach**
  - couple-channel scattering amplitude can be determined uniquely
  - how does the interaction behave to generate the $\Lambda(1405)$ pole(s)?

$\Lambda(1405)$ in flavor SU(3) limit

- we study $\Lambda(1405)$

in **flavor SU(3) limit** $m_u = m_d = m_s$



- channels: $\underset{\text{meson}}{\underline{8}} \otimes \underset{\text{baryon}}{\underline{8}} = 27 \oplus 10 \oplus 10^* \oplus \underline{8_s \oplus 8_a \oplus 1}$

- studies in chiral unitary model [Jido, Oller, Oset, Ramos, Meissner, 2003; Bruns, Cieplý, 2022; Guo, Kamiya, Mai, Meißner, 2023]

- physical point

two poles of $\Lambda(1405)$

one pole of $\Lambda(1680)$

- SU(3) limit

one pole in 1 channel

$m_1 < m_8, m_{8'}$

two poles in $8_s \oplus 8_a$ channel

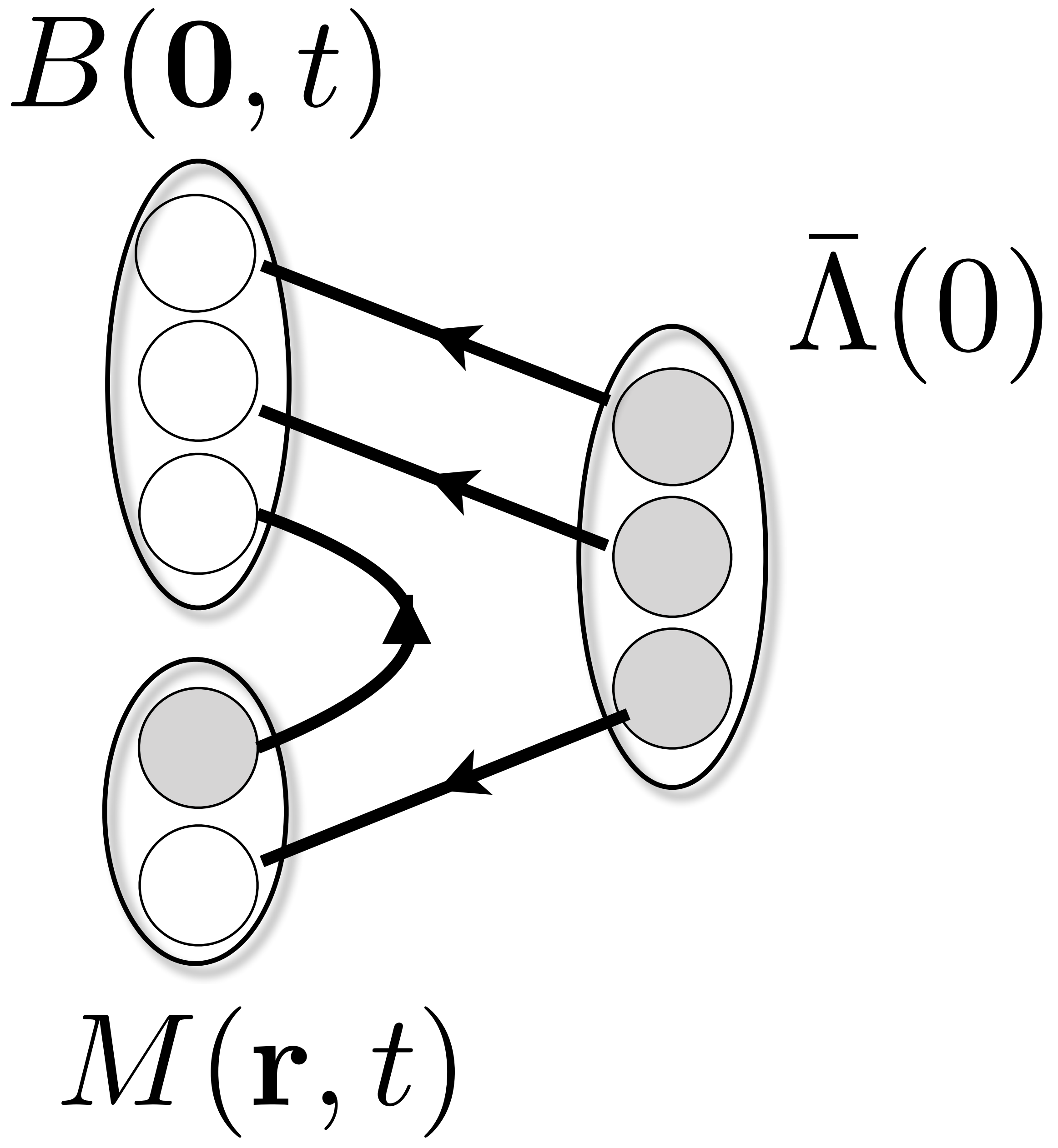
Setups

- SU(3) conf. w/ $a \approx 0.12$ fm, 32^4 lattices (Inoue (HAL QCD), PoS **CD15** (2016), 020)

- R-correlators

$(\mathrm{rep} = 1, 8_s, 8_a)$

$$R^{(\mathrm{rep})}(\mathbf{r}, t) = \frac{\langle (M(\mathbf{r}, t) B(\mathbf{0}, t))_{(\mathrm{rep})} \underline{\bar{\Lambda}(0) \rangle}}{\langle M(t) \bar{M}(0) \rangle \langle B(t) \bar{B}(0) \rangle} \sim \sum_{\mathbf{z}} \bar{u}(\mathbf{z}) \bar{d}(\mathbf{z}) \bar{s}(\mathbf{z})$$



- **bound state in each channel from**
$\langle \Lambda^{(8)}(t) \bar{\Lambda}^{(8)}(0) \rangle$ **and** $\langle \Lambda^{(1)}(t) \bar{\Lambda}^{(1)}(0) \rangle$**:**

- **neglect coupling between** 8_s **and** 8_a in this work

$$\begin{pmatrix} V_{8_s 8_s}(r) & V_{8_s 8_a}(r) \\ V_{8_a 8_s}(r) & V_{8_a 8_a}(r) \end{pmatrix} \approx \begin{pmatrix} V_{8_s 8_s}(r) & 0 \\ 0 & V_{8_a 8_a}(r) \end{pmatrix}$$

cf. chiral perturbation theory with Weinberg-Tomozawa term:

- **no coupling between** 8_s **and** 8_a
- interactions for 8_s and 8_a are the same

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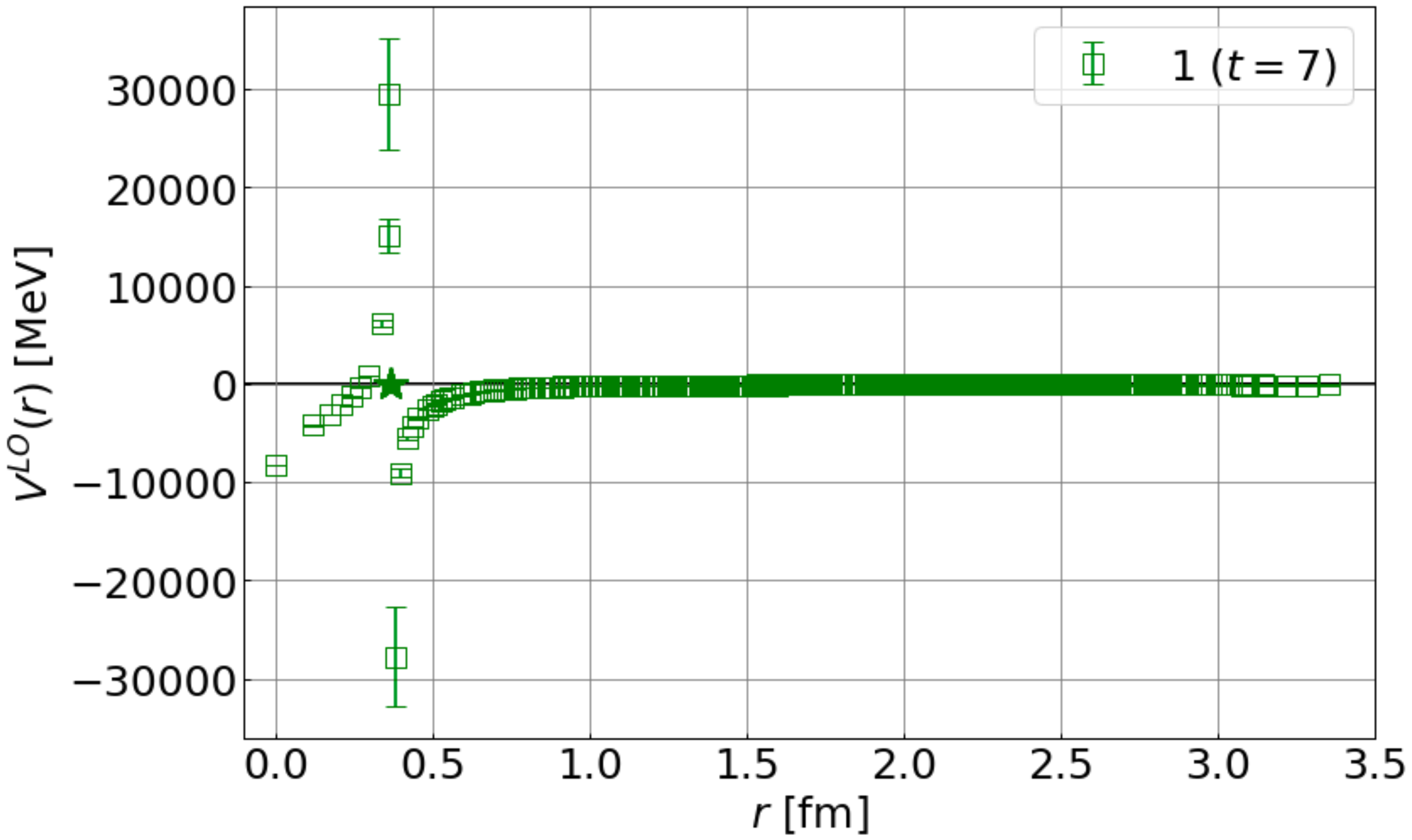
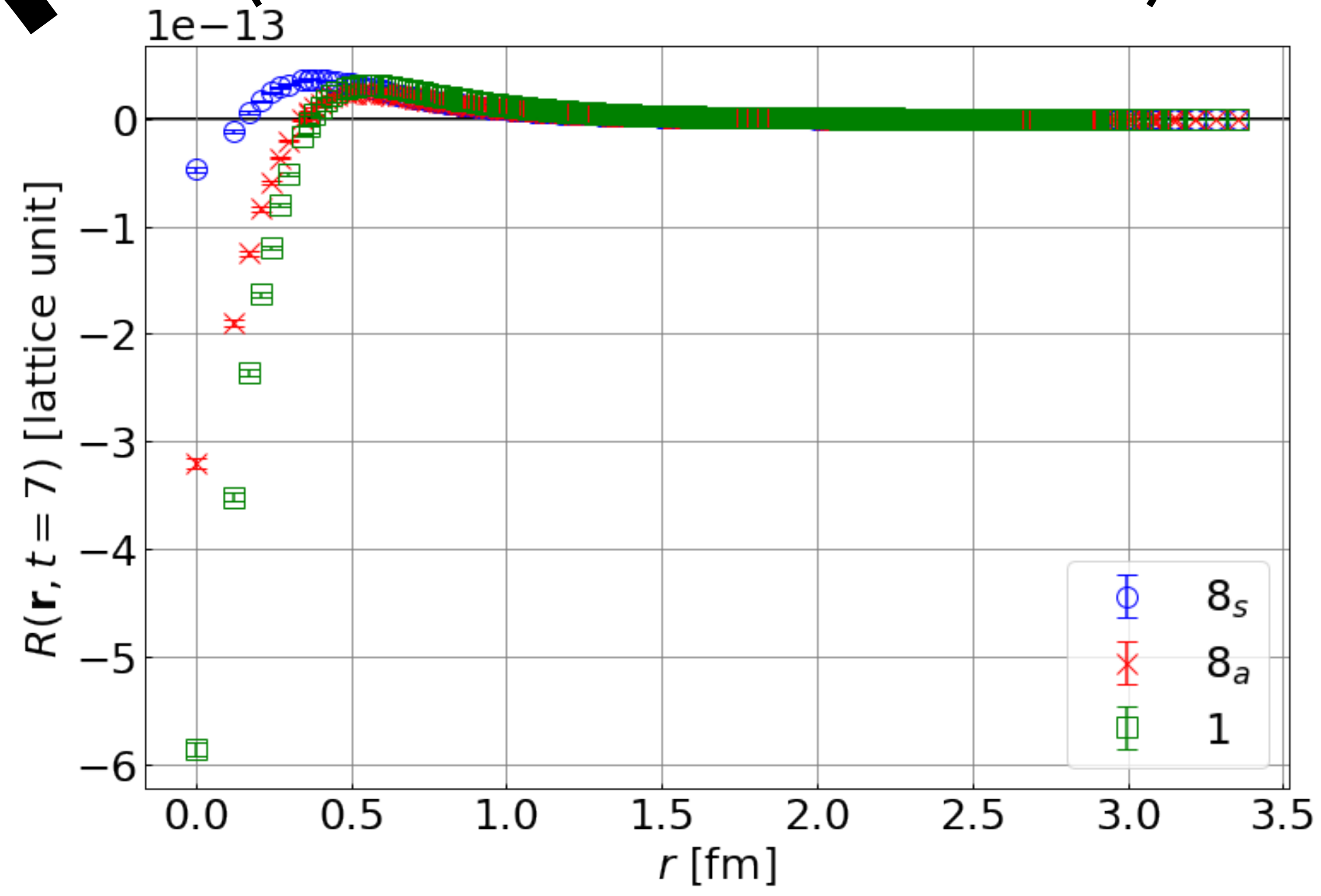
Local potentials

$$V(\mathbf{r}) \simeq \frac{1}{R(\mathbf{r},t)}\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right)R(\mathbf{r},t)$$

- local potential in singlet channel $V_1(r)$



- R-correlators $R(\mathbf{r},t)$ (NBS wave functions)



- singular behavior in all channels because of R-correlators crossing zero

no problematic in principle,
but difficult to obtain reliable results

- **take alternative approach**
  - mixed operator (only in octet channel)
  - separable potential

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    [KM, S. Aoki, PoS LATTICE2023, 063 (2024)]
  - Analysis 2: separable-potential approach
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Utilizing the two octet R-correlators

- **assume 8_s and 8_a are degenerated** in this work

cf. chiral perturbation theory w/ WT interaction:
- no coupling between 8_s and 8_a
- **interactions for 8_s and 8_a are the same**

➡ $R^{(8_s)}(\mathbf{r},t)$, $R^{(8_a)}(\mathbf{r},t)$: **different potentials, but produce the same scattering amplitude**

➡ same situation for $R^{(8_\mathrm{mix})}(\mathbf{r},t) = R^{(8_s)}(\mathbf{r},t) - cR^{(8_a)}(\mathbf{r},t)$ at any c

$$R^{(8_s)}(\mathbf{r},t) \rightarrow V(r) \searrow$$

different

$$R^{(8_a)}(\mathbf{r},t) \rightarrow V'(r) \rightarrow \mathcal{M}(s)$$

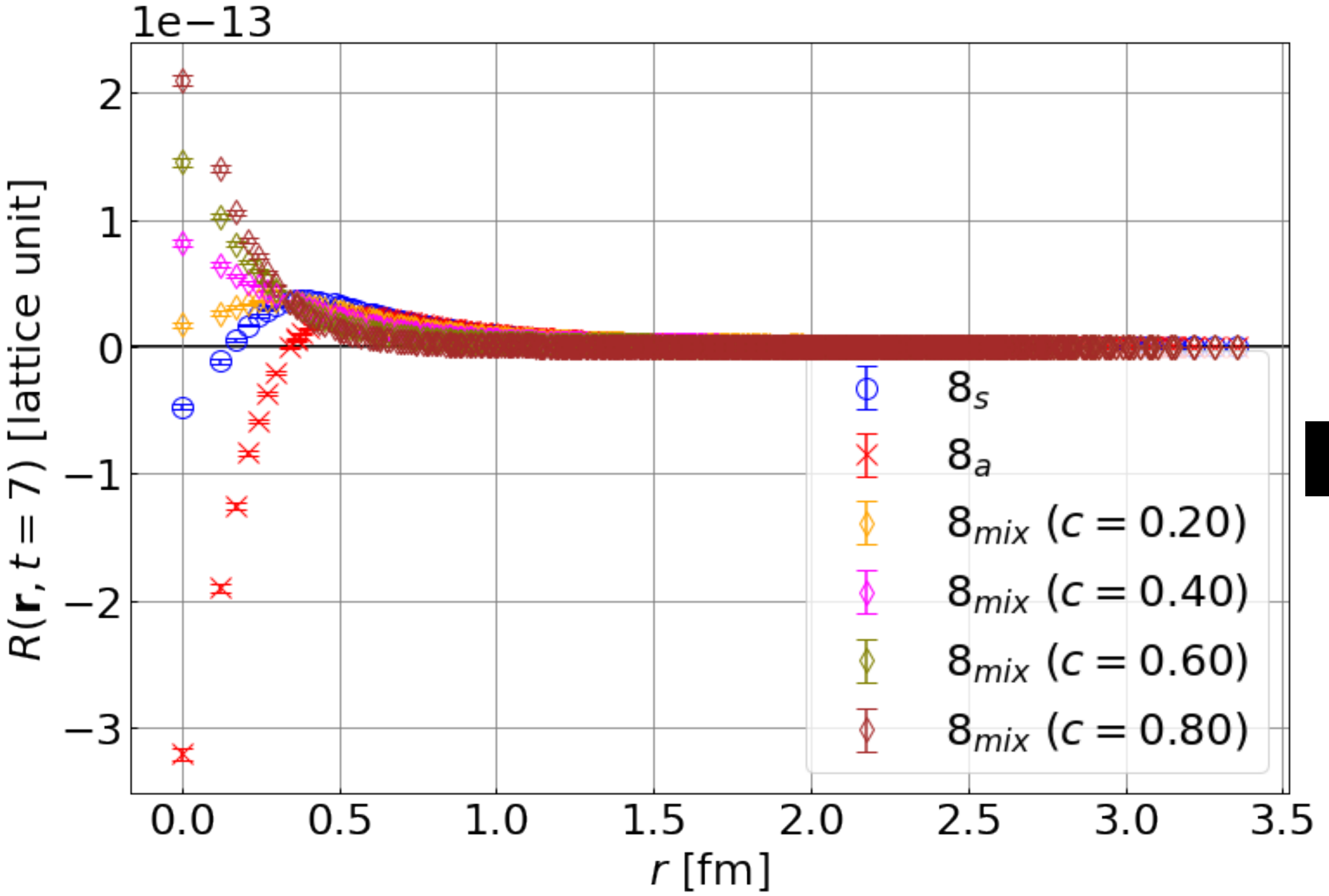
$$R^{(8_\mathrm{mix})}(\mathbf{r},t) \equiv R^{(8_s)}(\mathbf{r},t) - cR^{(8_a)}(\mathbf{r},t) \rightarrow V^{(c)}(r) \nearrow$$

- c is set such that $R^{(8_\mathrm{mix})}(\mathbf{r},t)$ **does not cross zero**

LO potentials from mixed R-correlators

$(m_M \approx 670 \text{ MeV}, m_B \approx 1489 \text{ MeV})$

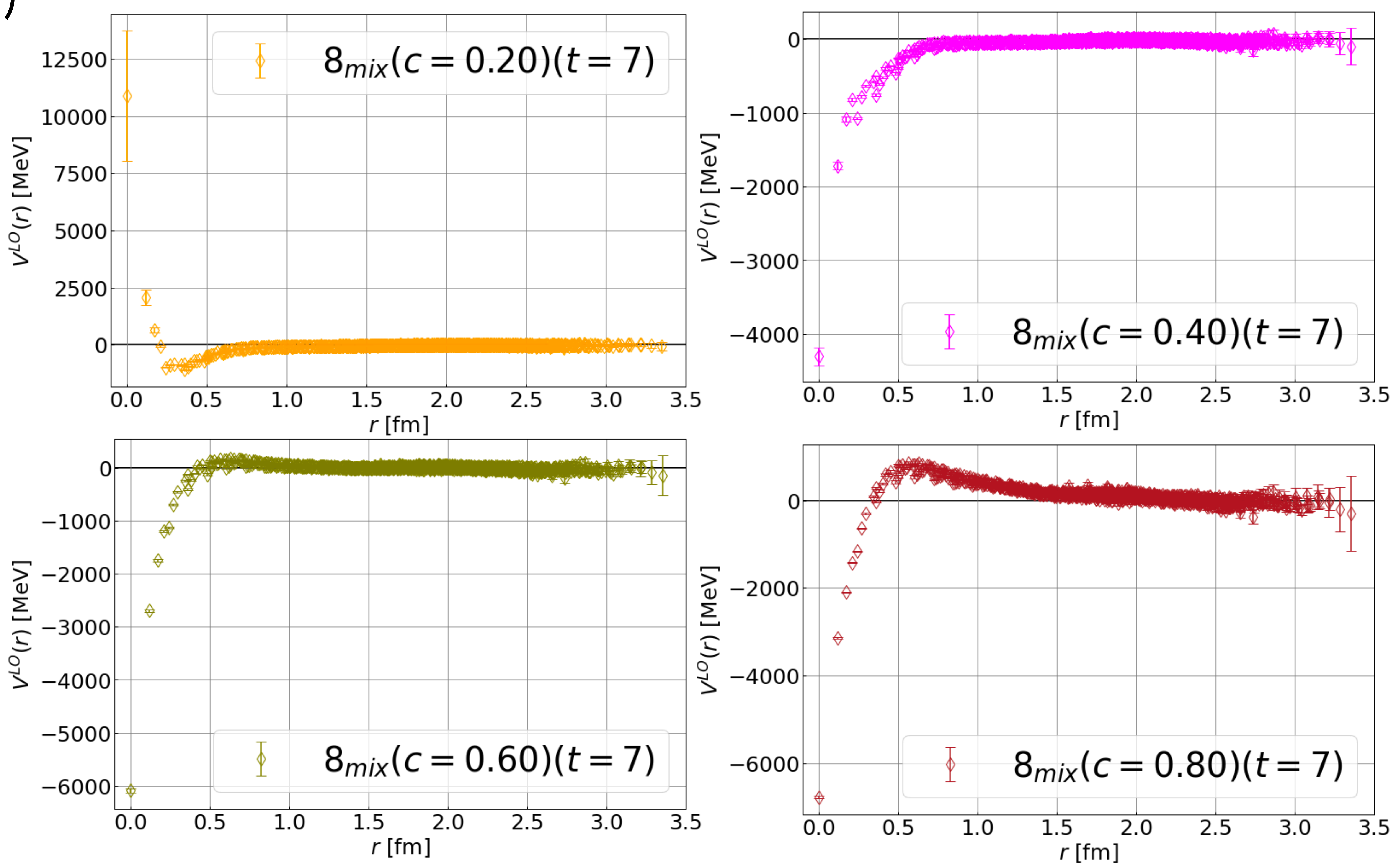
$$R^{(8_{\rm mix})}(\mathbf{r},t) = R^{(8_s)}(\mathbf{r},t) - cR^{(8_a)}(\mathbf{r},t)$$



- the zero points disappear

in $0.2 \lesssim c \lesssim 0.8$

LO potentials



- **attractive for all c**

- the shape drastically changes for different c

➡ **physical observables?**

Binding energy in octet channel

($m_M \approx 670$ MeV,
$m_B \approx 1489$ MeV)

- solve Schrödinger equation

➡ binding energy for each c

c	0.2	0.25	0.3	0.4	0.6	0.8
$E_{\rm bind}^{\rm (octet)}$ [MeV]	179(4)	177(5)	177(5)	163(7)	132(13)	99(15)

$$\Rightarrow \quad E_{\rm bind}^{\rm (octet)} = 163(7)_{\rm stat} \begin{pmatrix} +16 \\ -64 \end{pmatrix}_{\rm sys} \text{ MeV}$$

- consistent with the value from $\langle \Lambda_{\rm octet}(t) \bar{\Lambda}_{\rm octet}(0) \rangle$ ($156(8)_{\rm stat}$ MeV)

➡ **our analysis (and assumption)** is more or less **reliable**

- systematic error possibly comes from:
  - difference between 8_s, 8_a
  - effect of the 8_s-8_a coupling
  - non-locality effect

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Separable potentials in the HAL QCD method

- time-dependent equation

$$\left(R(\mathbf{r},t) = \frac{\langle O_1(\mathbf{r},t)O_2(\mathbf{0},t)\bar{J}(0)\rangle}{\langle O_1(t)\bar{O}_1(0)\rangle\langle O_2(t)\bar{O}_2(0)\rangle} \right)$$

$$\int d^3r' \; U(\mathbf{r},\mathbf{r}')R(\mathbf{r}',t) \simeq \Big(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\Big)R(\mathbf{r},t)$$

$$\approx \eta v(\mathbf{r})v(\mathbf{r}'), \quad (\eta = \pm 1)$$

(separable potential approximation)

$$\rightarrow \quad \eta v(\mathbf{r}) \underline{\int d^3r' \; v(\mathbf{r}')R(\mathbf{r}',t)} \approx \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right) R(\mathbf{r},t)$$

constant (indep. of $\mathbf{r}$)

→ no singular behavior for $v(\mathbf{r})$

How to extract separable potentials

- time-dependent (TD) equation for separable potential: $(\eta = \pm 1)$

$$\eta v(\mathbf{r}) \underbrace{\int d^3r' \; v(\mathbf{r}')R(\mathbf{r}',t)}_{= A[R,v]\text{: constant (indep. of } \mathbf{r})} = \underbrace{\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right) R(\mathbf{r},t)}_{= \mathcal{D}R(\mathbf{r},t)}$$

$$\times \int d^3r \; R(\mathbf{r},t)$$

$$\eta (A[R,v])^2 = \int d^3r \; \underline{R(\mathbf{r},t)\mathcal{D}R(\mathbf{r},t)}$$

real

$$\eta = \mathrm{sgn}[\eta(A[R,v])^2] = \mathrm{sgn}\left[\int d^3r \; R(\mathbf{r},t)\mathcal{D}R(\mathbf{r},t)\right]$$

$$A[R,v] = \sqrt{|\eta(A[R,v])^2|} = \sqrt{\left|\int d^3r \; R(\mathbf{r},t)\mathcal{D}R(\mathbf{r},t)\right|}$$

$$\boxed{v(\mathbf{r}) = \frac{\mathcal{D}R(\mathbf{r},t)}{\eta A[R,v]}}$$

Setups for separable potentials

- neglect coupling between 8_s and 8_a

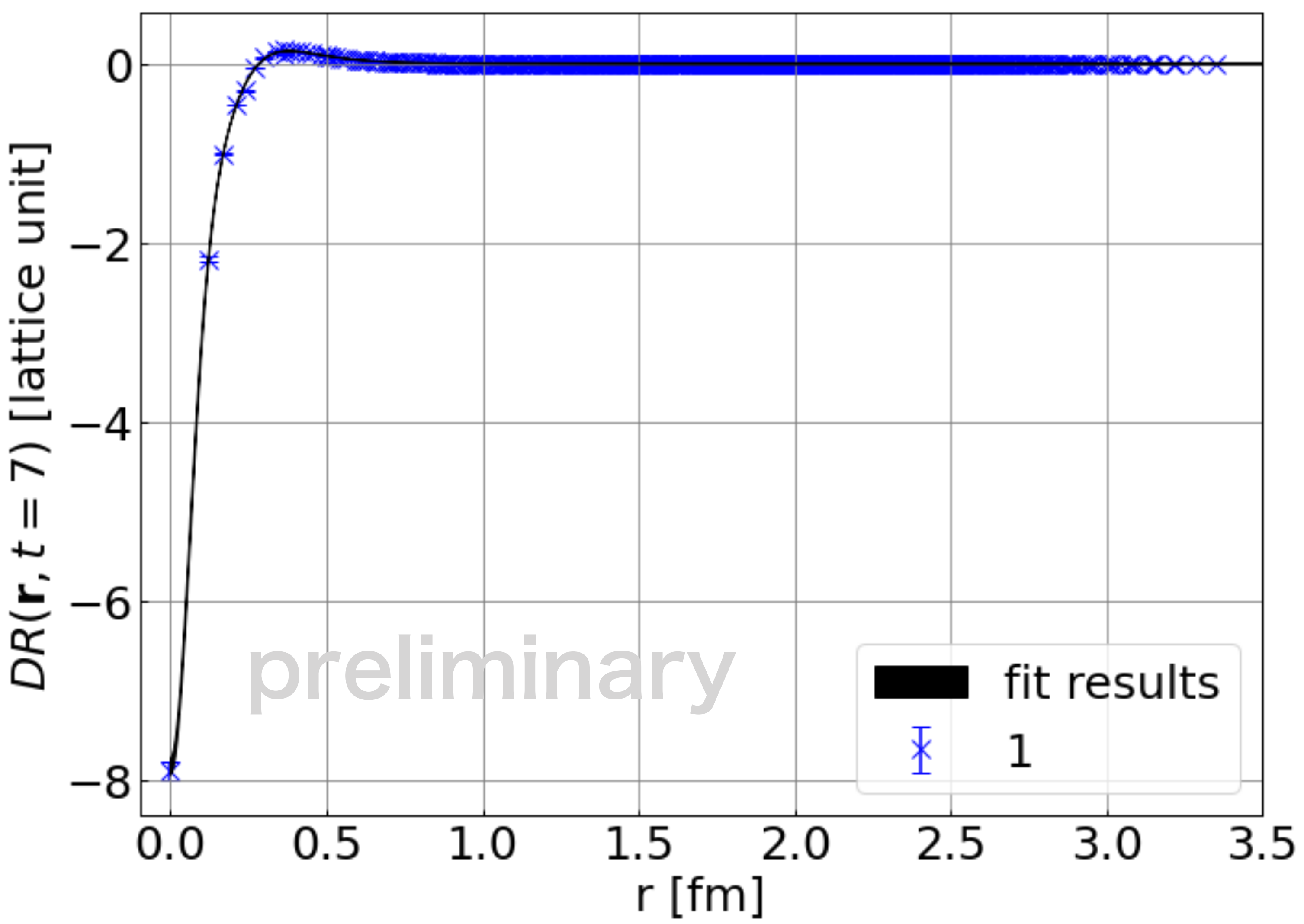
$$U_1(\mathbf{r},\mathbf{r}') \approx \eta_1 v_1(\mathbf{r}) v_1(\mathbf{r}')$$

$$\begin{pmatrix} U_{8_s8_s}(\mathbf{r},\mathbf{r}') & U_{8_s8_a}(\mathbf{r},\mathbf{r}') \\ U_{8_a8_s}(\mathbf{r},\mathbf{r}') & U_{8_a8_a}(\mathbf{r},\mathbf{r}') \end{pmatrix} \approx \begin{pmatrix} \eta_{8_s} v_{8_s}(\mathbf{r}) v_{8_s}(\mathbf{r}') & 0 \\ 0 & \eta_{8_a} v_{8_a}(\mathbf{r}) v_{8_a}(\mathbf{r}') \end{pmatrix}$$

- fitting for $\mathcal{D}R(\mathbf{r},t)$ using multi-Gaussians to obtain potentials in continuum

$$\left(\mathcal{D}R(\mathbf{r},t) = \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu} \frac{\partial^2}{\partial t^2} \right) R(\mathbf{r},t) \right)$$

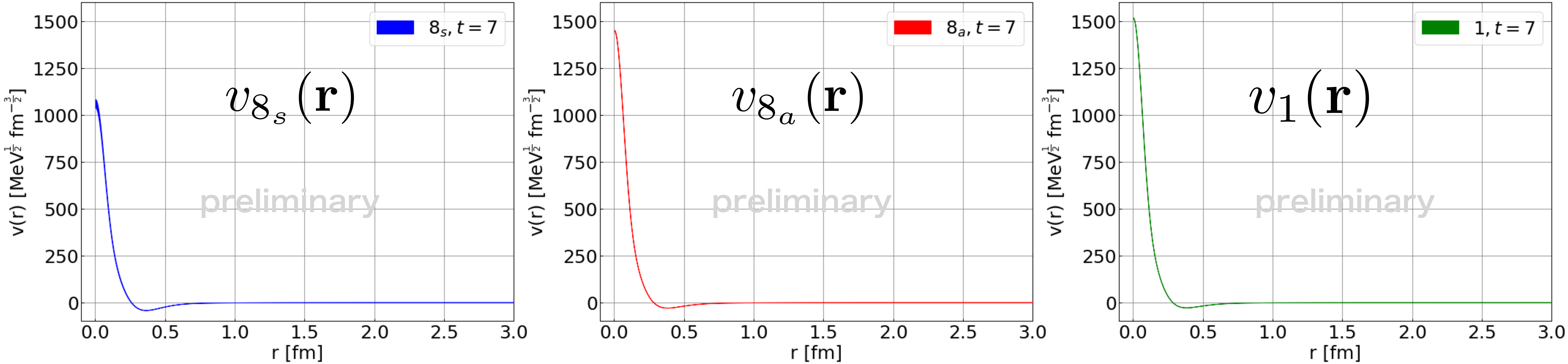
$\mathcal{D}R(\mathbf{r},t)$ in singlet channel



Results of separable potentials

$(m_M \approx 460\ \mathrm{MeV},\ m_B \approx 1166\ \mathrm{MeV})$

- Results of $v(\mathbf{r})$, η



	8_s channel	8_a channel	1 channel
η	-1	-1	-1

- $\eta = -1$ for all three channels ➡ attractive interactions
- magnitude of $v(\mathbf{r})$ in short distance is larger for singlet channel

Binding energies ($m_M \approx 460$ MeV, $m_B \approx 1166$ MeV)

- solve Schrödinger equation in the Gaussian expansion method with separable potentials [Hiyama, Kino, Kamimura, 2003]

- systematic error includes
  - timeslice dependence
  - finite-volume effects

- our results (preliminary)

	8_s channel	8_a channel	1 channel
$E_{\rm bind}$ [MeV]	$57.9(5.5)_{\rm stat}(^{+5.0}_{-2.9})_{\rm syst}$	$55.9(3.5)_{\rm stat}(^{+5.0}_{-7.4})_{\rm syst}$	$72.4(6.2)_{\rm stat}(^{+11.4}_{-2.4})_{\rm syst}$

- c.f. estimates from $\langle \Lambda^{(X)}(t)\bar{\Lambda}^{(X)}(0)\rangle$ $(X=1,8)$:

	$8_s(8_a)$ channel	1 channel
$E_{\rm bind}$ [MeV]	$24.5(17.3)_{\rm stat}$	$87.6(7.0)_{\rm stat}$

  - (2σ) consistent with the results from $\langle \Lambda^{(X)}(t)\bar{\Lambda}^{(X)}(0)\rangle$ within (large) errors
  - $E^{1}_{\rm bind} > E^{8_s}_{\rm bind}, E^{8_a}_{\rm bind}$ ← same as chiral unitary model ($m_1 < m_8, m_{8'}$)

Discussions

- singular behavior:
  - it appears due to the zeros of $R(\mathbf{r}, t)$ (wave functions)
  - such behavior does not happen in the usual QM
    ➡ singular behavior: effects beyond QM (effects from QFT)

    nonlocality of the HAL QCD potential
  - HAL QCD method with separable potentials allow us to avoid singular behavior
- systematic error of binding energy:
  - mainly comes from timeslice dependence of the potentials in short distance ($r \lesssim 0.15\ \mathrm{fm}$)

  ➡ binding energy is sensitive to short-distance behavior of separable potentials

Contents

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- HAL QCD method
- $\Lambda(1405)$ in SU(3) limit
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- Summary

Summary

- we study $\Lambda(1405)$ in flavor SU(3) limit from the **meson-baryon scatterings in lattice QCD** using the HAL QCD method
- R-correlator has a zero point, which leads to singular behavior for the local potential
- we utilize the **mixed R-correlator in the octet channel**, from which the potential gives similar binding energies regardless of the mixture
- we employ a **separable potential** and the results show **attractive interactions** and produce consistent binding energies within (large) errors

Future work

- more realistic and precise setups
  - include coupling between 8_s and 8_a
  - beyond SU(3) limit, simulation w/ lighter pion mass
  - more complicated separable form of potentials (application of [Ernst, Shakin, Thaler, 1973]?) [Meng, Epelbaum, in preparation]

← coupled-channel analysis & 4pt correlation function are required

- this work: first time application of the HAL QCD method with separable potentials
  - other application (κ resonance?)
  - useful to check systematics from non-locality effect