Study of multiquark states based on effective models

Masayasu Harada (Nagoya University)

@ Hadrons and Hadron Interactions in QCD 2024 (HHIQCD 2024)

(October 28, 2024)

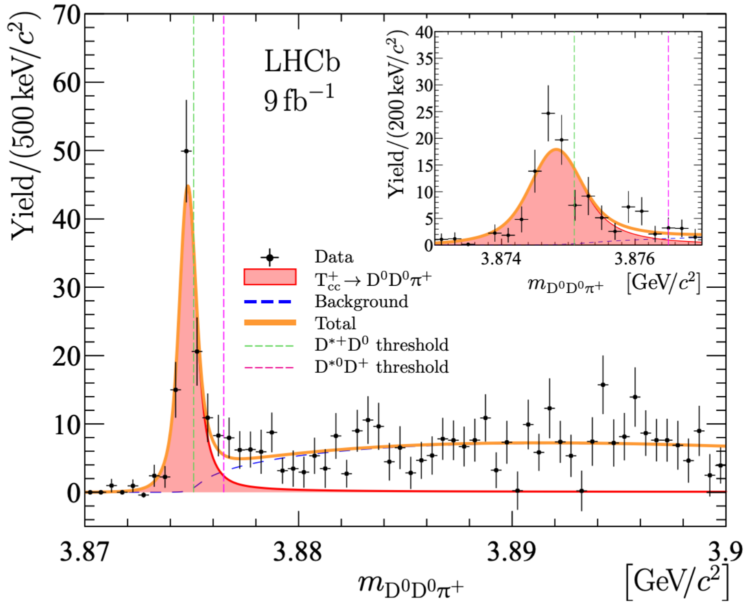
Based on

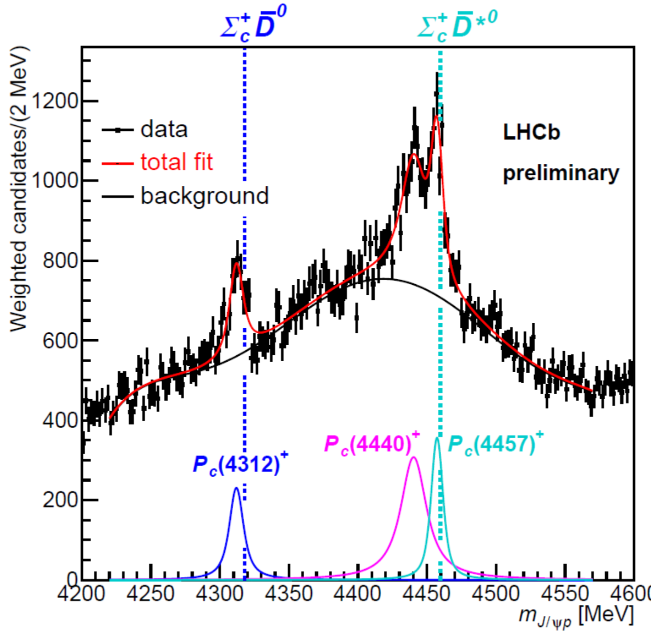
- B.-R. He, M. Harada and B.-S. Zou, Phys. Rev. D 108, 054025 (2023)
- B.-R. He, M. Harada and B.-S. Zou, Eur. Phys. J. C 83, 1159 (2023)
- T. Asanuma, Y. Yamaguchi and M. Harada, arXiv:2311.04695, to appear in Phys. Rev. D.
- M. Tanaka, Y. Yamaguchi and M. Harada, Phys. Rev. D 110, 016024 (2024).

Introduction

Exotic Hadrons

- Tetraquarks
  - X(3872) @ Bell (2003) …
  - X(6900) @ LHCb (2020)
  - Zcs(4681) @ BES III (2021)
  - Tcc(3875) @ LHCb (2022)
  - Tcs(2900) @ LHCb (2022)
  - …
- Pentaquarks
  - Pc(4380), Pc(4450) @ LHCb (2015)
  - Pc(4312), Pc(4440), Pc(4457) @ LHCb (2017)
  - Pc(4337) @ LHCb (2022)
  - Pcs(4338) @ LHCb (2022)
  - …
- more





Structure of Exotic Hadrons ?

- It is useful to treat exotic hadrons based on quarks.
  - Conventional quark model lacks the chiral symmetry.
  - The chiral symmetry and its spontaneous breaking is one of the most important features in the low-energy region for hadrons including light quarks.
- It is well-known that the heavy quark symmetry plays an important role to study hadrons including heavy quarks.
  - The superflavor symmetry plays an important role to study hadrons including multi-heavy quarks.

Outline

I. Introduction

II. Study of hadron spectra based on a chiral quark model with hidden local symmetry

1. Motivation for Part II
2. Formulation of a new chiral quark model
3. Numerical results
4. Extension to include strange quark based on $SU(3)_L \times SU(3)_R$ chiral symmetry
5. Summary of Part II

III. Study of multiquark states based on the superflavor symmetry

1. Motivation for Part III
2. Superflavor symmetry for heavy hadrons
3. Molecular Tetraquark and Pentaquark based on the superflavor symmetry
4. EFT Analysis of Doubly Heavy Tetraquarks based on the superflavor symmetry
5. Summary of Part III

IV. Summary

Part II : Study of hadron spectrum based on a chiral quark model

B.-R. He, M. Harada and B.-S. Zou, Phys. Rev. D 108, 054025 (2023)

B.-R. He, M. Harada and B.-S. Zou, Eur. Phys. J. C 83, 1159 (2023)

Outline of PART II

1. Problem of an old chiral quark model
2. Formulation of a new chiral quark model
3. Numerical results
4. Extension to include strange quark based on $SU(3)_L \times SU(3)_R$ chiral symmetry
5. Summary of Part II

II.1 : Problem of an old chiral quark model

Chiral quark model

A. Manohar and H. Georgi, Nucl. Phys. B 234, 189 (1984)

- EFT in $\Lambda_\chi \gtrsim E \gtrsim \Lambda_c$
  - Λ_χ: chiral symmetry breaking scale ~ 1 GeV
  - Λ_c: confinement scale ~ a few 100 MeV
- includes
  - quarks with masses generated by SχSB
  - pions as Nambu-Goldstone bosons
  - gluons mediating the color force

Past works

- K. Shimizu, Phys. Lett. B 148, 418-422 (1984)
  - pseudo-scalar mesons + confining potential (CON)

- I. T. Obukhovsky and A. M. Kusainov, Phys. Lett. B 238, 142-148 (1990).
  - scalar and pseudo-scalar mesons + one-gluon exchange (OGE) + CON

- L. Y. Glozman and D. O. Riska, Phys. Rept. 268, 263-303 (1996); L. Y. Glozman, Nucl. Phys. A 663, 103-112 (2000).
  - pseudo-scalar and vector mesons + CON to study baryon spectrum

- L. R. Dai, Z. Y. Zhang, Y. W. Yu and P. Wang, Nucl. Phys. A 727, 321-332 (2003).
  - scalar, pseudo-scalar, vector mesons + OGE + CON to study phase shift of NN scattering

- J. Vijande, F. Fernandez and A. Valcarce, J. Phys. G 31, 481 (2005); J. Vijande and A. Valcarce, Phys. Lett. B 677, 36-38 (2009); A. Valcarce, H. Garcilazo, F. Fernandez and P. Gonzalez, Rept. Prog. Phys. 68 (2005), 965-1042.
  - scalar, pseudo-scalar mesons + OGE + CON to study meson and baryon spectra.
  - did not include the vector mesons for avoiding the double counting.

Problem and Proposal

- A chiral quark model with π and σ provides too much strong attractive force between two quarks which form a good diquark:
  - $M_N^{(exp)} - M_N^{(theo)} = 262\ MeV$
  - $M_{\Lambda_c}^{(exp)} - M_{\Lambda_c}^{(theo)} = 322\ MeV$
  - $M_{\Lambda_b}^{(exp)} - M_{\Lambda_b}^{(theo)} = 359\ MeV$

We use best fitted parameters in [J. Vijande, F. Fernandez and A. Valcarce, J. Phys. G 31, 481 (2005)].

B.-R. He, M. Harada and B.-S. Zou, Phys. Rev. D 108, 054025 (2023)

- New chiral quark model with vector mesons
  - ρ and ω are included based on the Hidden Local Symmetry(HLS)
  - $m_\rho,\ m_\omega \sim 780\ MeV < \Lambda_\chi$!

II.2 : Formulation of a new chiral quark model

Hamiltonian for the new chiral quark model with HLS

B.-R. He, M. Harada and B.-S. Zou, Phys. Rev. D 108, 054025 (2023)

$$H = \sum_{i=1} \left(m_i + \frac{p_i^2}{2m_i} \right) - T_{CM} + \sum_{j>i=1} \left(V_{ij}^{\mathrm{CON}} + V_{ij}^{\mathrm{OGE}} + V_{ij}^{\sigma} + V_{ij}^{\pi} + V_{ij}^{\omega} + V_{ij}^{\rho} \right)$$

m_i, p_i : mass and momentum of quark

T_{CM}: kinetic energy of the center of mass of the system

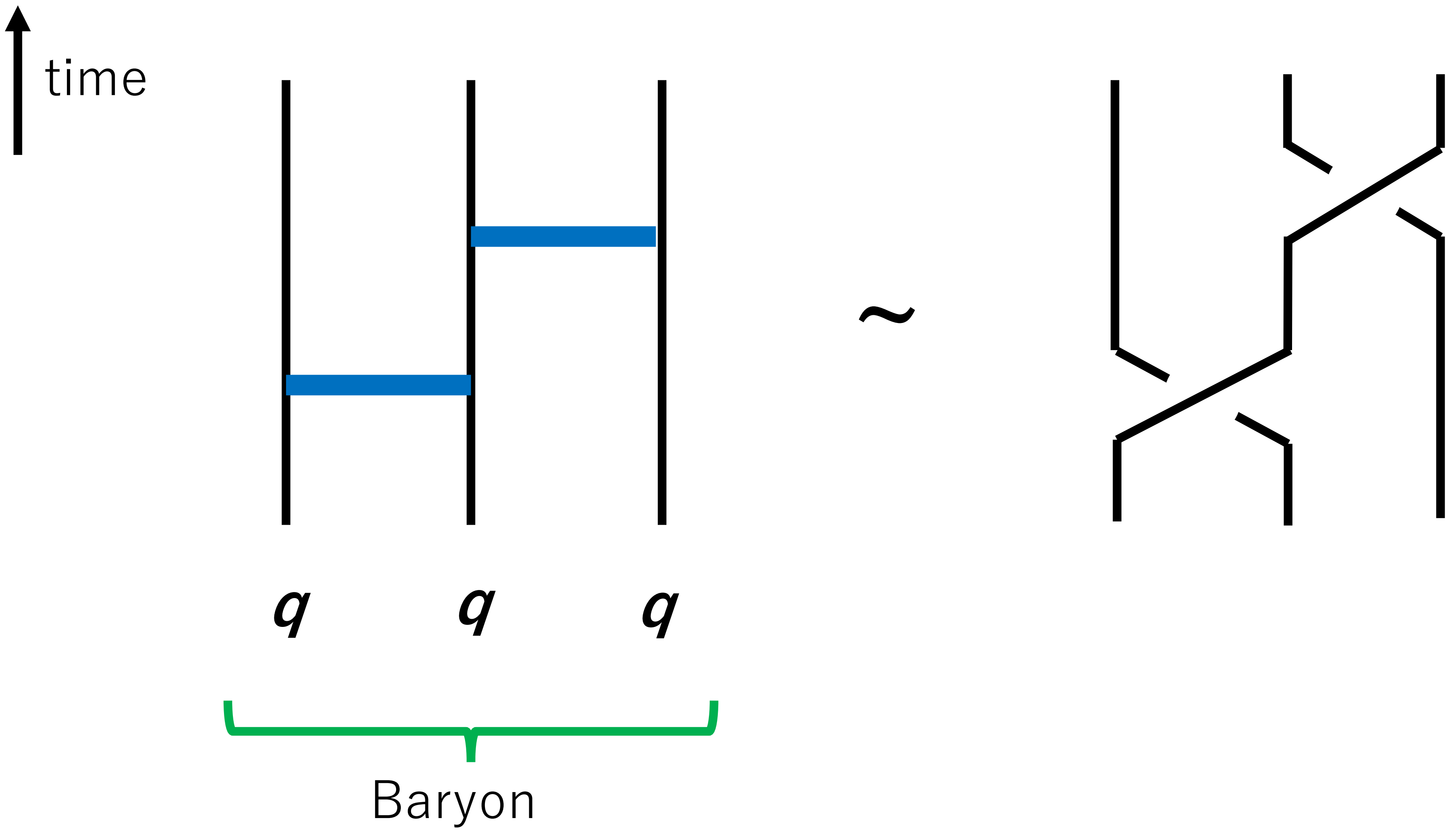
V_{ij}^{CON}: confining potential

V_{ij}^{OGE}: one-gluon exchange potential

$V_{ij}^{\sigma}, V_{ij}^{\pi}$: σ and π exchange potentials

$V_{ij}^{\omega}, V_{ij}^{\rho}$: ω and ρ exchange potentials

Meson exchange ~ quark exchange effects



II.3 : Numerical results

fitting

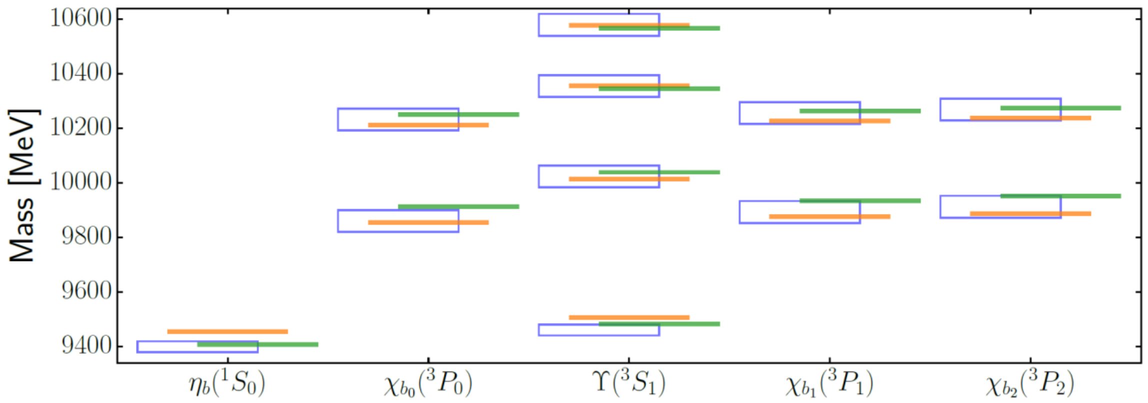
- model parameters : 20 (quark masses, meson couplings, etc.)
- inputs: 51 masses (42 mesons & 9 baryons)
- $\chi^2 = \sum \left(\frac{m(theo) - m(exp)}{Err(sys)} \right)^2$
  -
  - $Err(sys) = \sqrt{(Err(exp))^2 + (Err(theo))^2}$
    - ✓ $Err(theo) = \begin{cases} 40\ MeV \text{ for ground states} \\ 80\ MeV \text{ for excited states} \end{cases}$

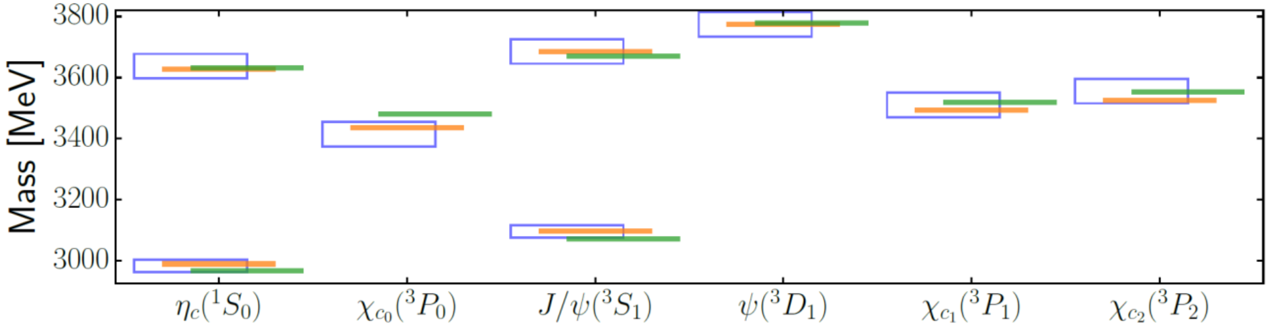
$$\chi^2/dof = 15/31 \simeq 0.49$$

very good fitting !

$m_u = m_d(\mathrm{MeV})$	303.3	$m_\sigma(\mathrm{fm}^{-1})$	3.42
$m_c(\mathrm{MeV})$	1696.0	$m_\pi(\mathrm{fm}^{-1})$	0.7
$m_b(\mathrm{MeV})$	5039.6	$m_\omega(\mathrm{fm}^{-1})$	3.97
$a_c(\mathrm{MeV})$	364.5	$m_\rho(\mathrm{fm}^{-1})$	3.93
$\mu_c(\mathrm{fm}^{-1})$	0.6	$\Lambda_\sigma = \Lambda_\pi(\mathrm{fm}^{-1})$	4.2
$\Delta(\mathrm{MeV})$	100.28	$\Lambda_\omega = \Lambda_\rho(\mathrm{fm}^{-1})$	7.2
a_s	0.731	$g_{ch}^2/(4\pi)$	0.54
α_0	2.742	g_ω	3.841
$\Lambda_0(\mathrm{fm}^{-1})$	0.304	g_ρ	0.675
$\mu_0(\mathrm{MeV})$	241.205	f_ω	-1.416
$\hat{r}_0(\mathrm{MeV} \cdot \mathrm{fm})$	95.621	f_ρ	0.581
$\hat{r}_g(\mathrm{MeV} \cdot \mathrm{fm})$	155.066		

Masses of $b\bar{b}$ & $c\bar{c}$ mesons

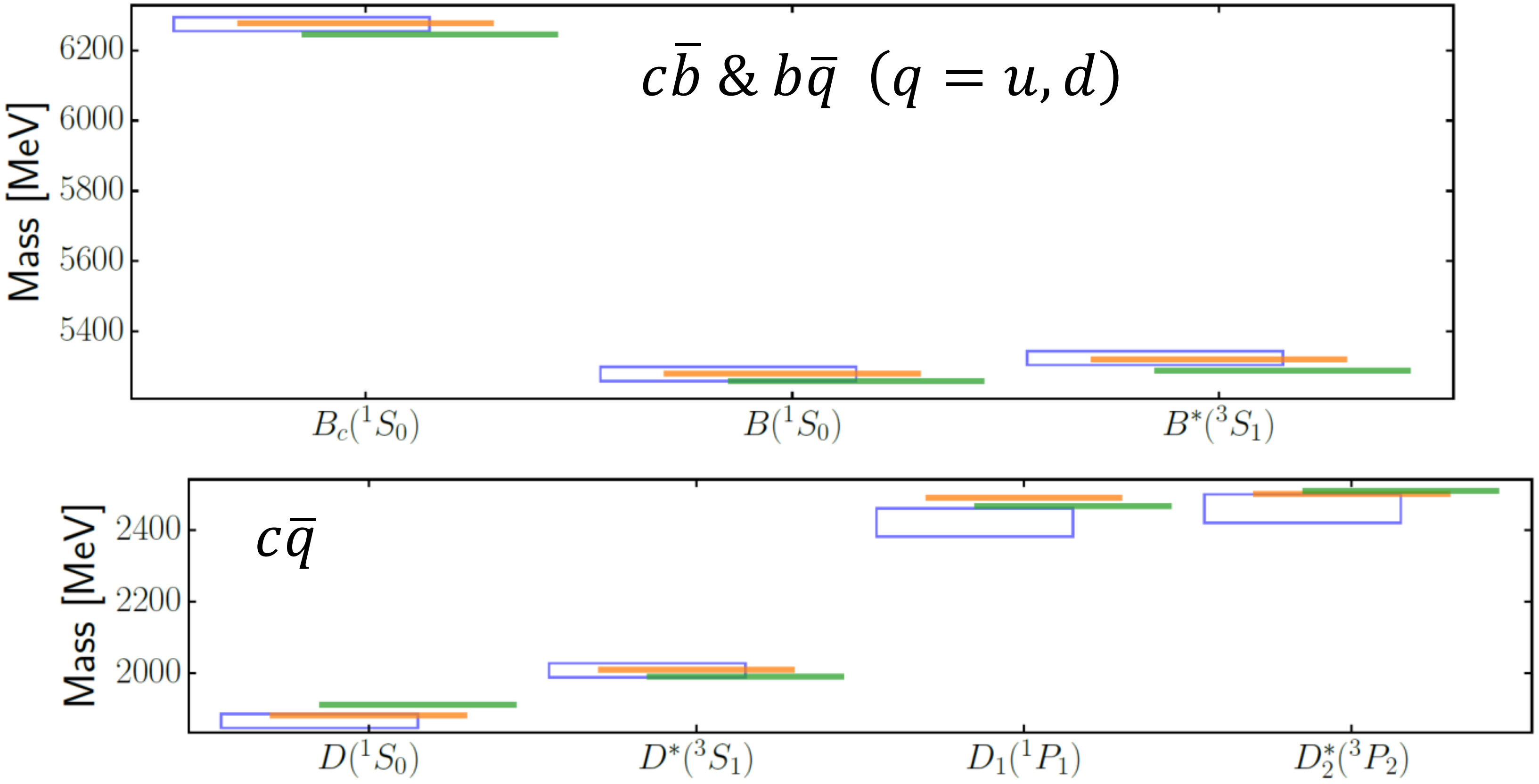




green: present best fitted results
orange: results with NO vector mesons
blue blocks: experimental values with Err(sys)

Only color force (OGE & CON) contributes
⇒ No essential difference between two models

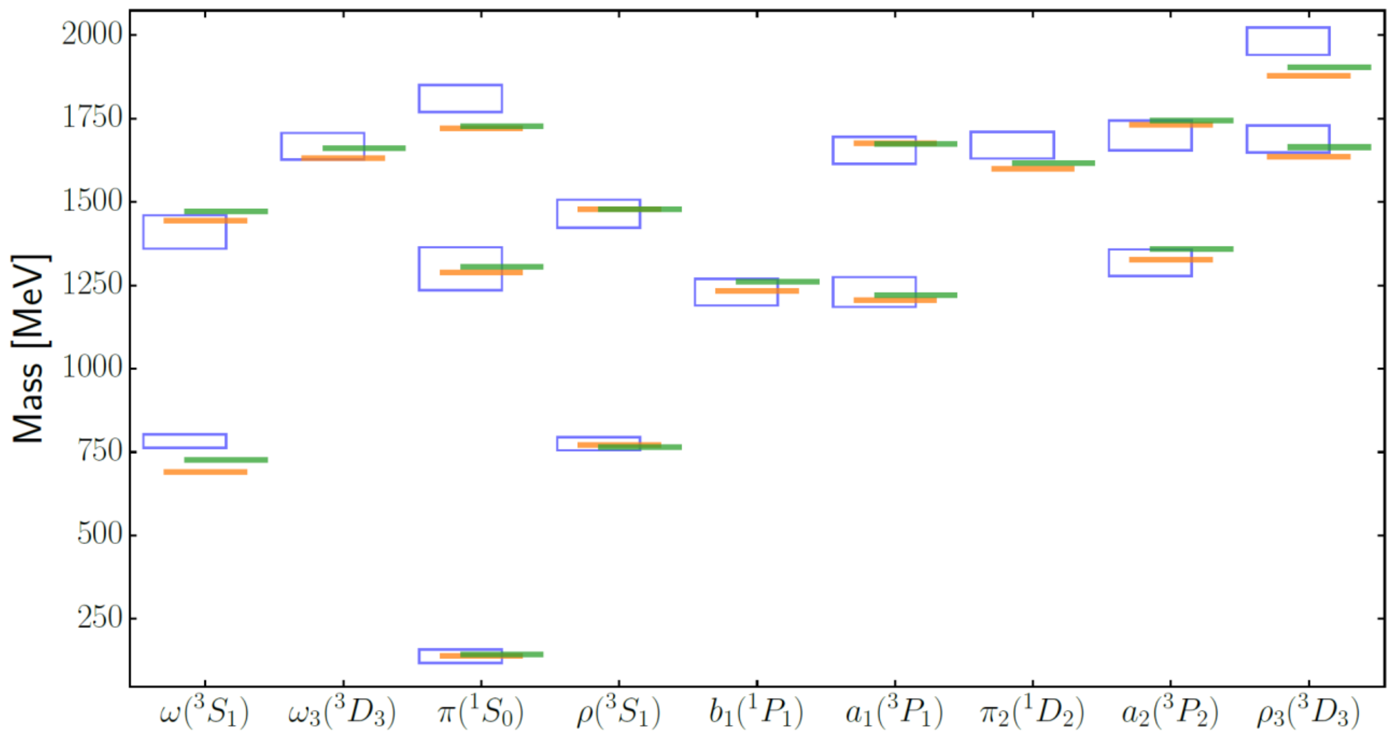
Masses of heavy mesons



Only color force (OGE & CON) contributes

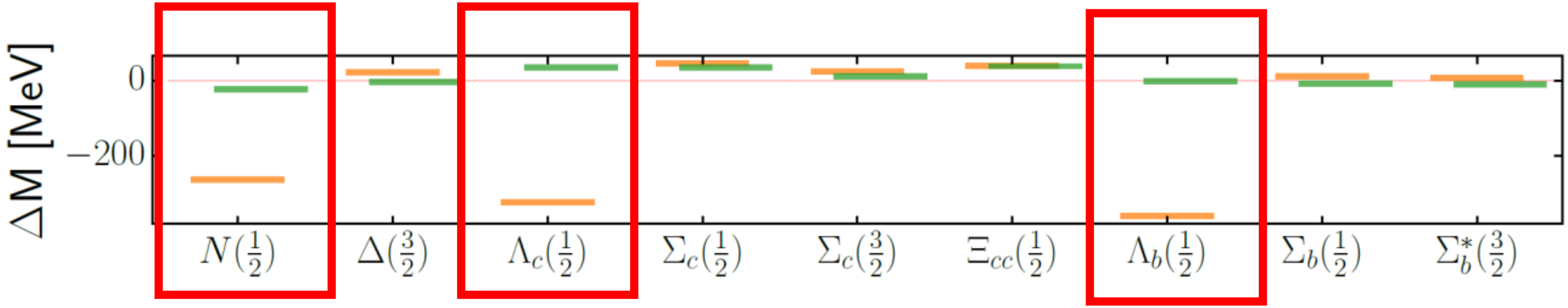
⇒ No essential difference between two models

Masses of $q\bar{q}$ mesons $(q = u, d)$



- Effects of meson exchange are included.
  - NO essential difference
- Model parameters are adjusted to reproduce meson spectra.

Masses of Baryons



green: present best fitted results
orange: results with NO vector mesons

- Effects of meson exchange are included.
- Masses of N, Λ_c, Λ_b are dramatically improved.
  - Effects of ω meson is important.

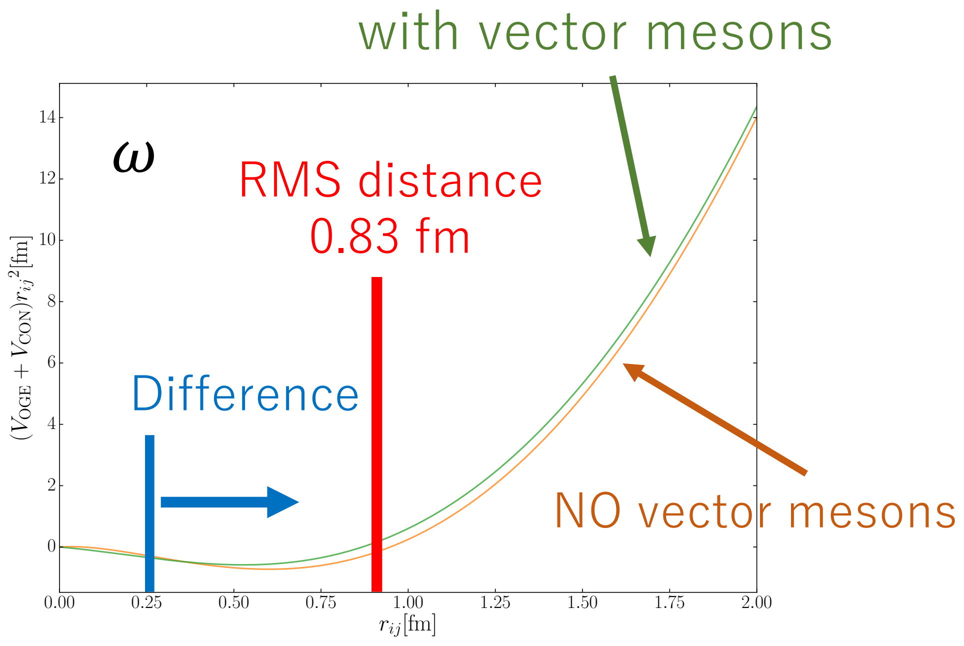
Operators in the potentials

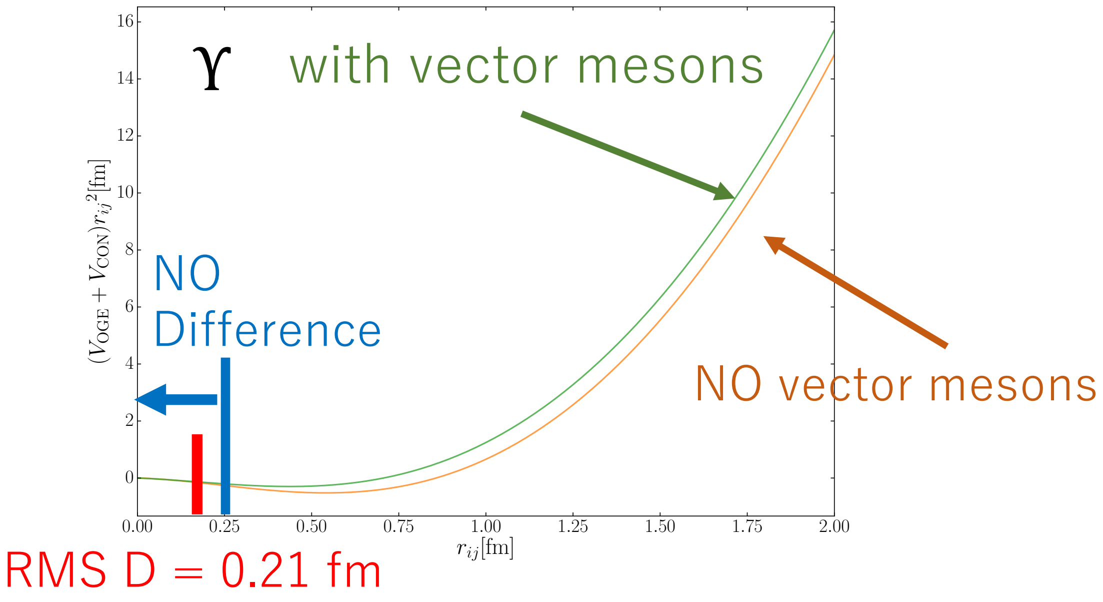
signs in the potential for qq / $q\bar{q}$

	1	$\tau_i \tau_j$	$\sigma_i \sigma_j$	$\sigma_i \sigma_j \cdot \tau_i \tau_j$
σ [15]	$-/-$			
π [15]				$+/-$
a_0 [16]		$-/+$		
OGE [15]	$-/-$		$+/+$	
CON [15]	$+/+$			
ω (This work)	$+/-$		$+/-$	
ρ (This work)		$+/+$		$+/+$

- σ meson + OGE give negative potential for both qq and $q\bar{q}$.
- ω meson gives negative potential for $q\bar{q}$ but positive potential for qq.

Potentials for color force



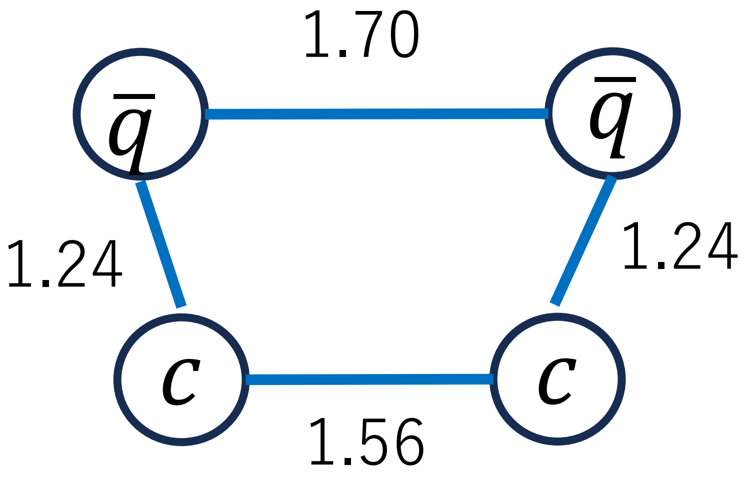


- Color potential is slightly enhanced when vector meson exchange effect is included.
  - Effects from color force + vector meson exchange give suitable binding energy for $q\bar{q}$ mesons.

Study of T_{cc} & T_{bb}

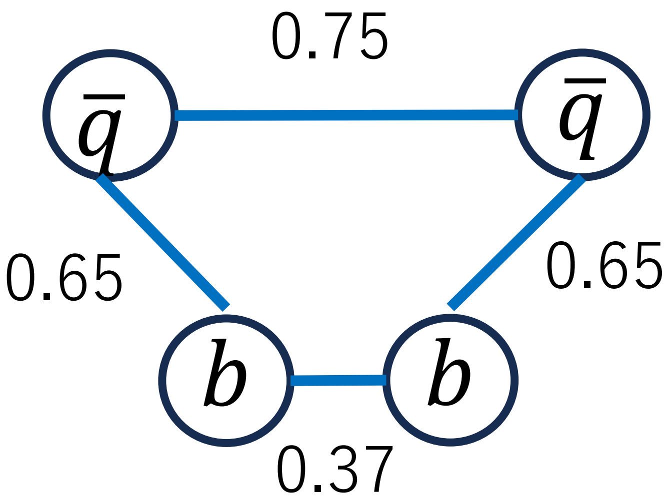
- T_{cc} (prediction)
  $M = DD^* - 4.9\ MeV$

RMS distance (fm)



molecule structure

- T_{bb} (prediction)
  $M = BB^* - 88.2\ MeV$



diquark structure

II.4 : Extension to include strange quark based on $SU(3)_L \times SU(3)_R$ chiral symmetry

B.-R. He, M. Harada and B.-S. Zou, 2307.16280

Hamiltonian

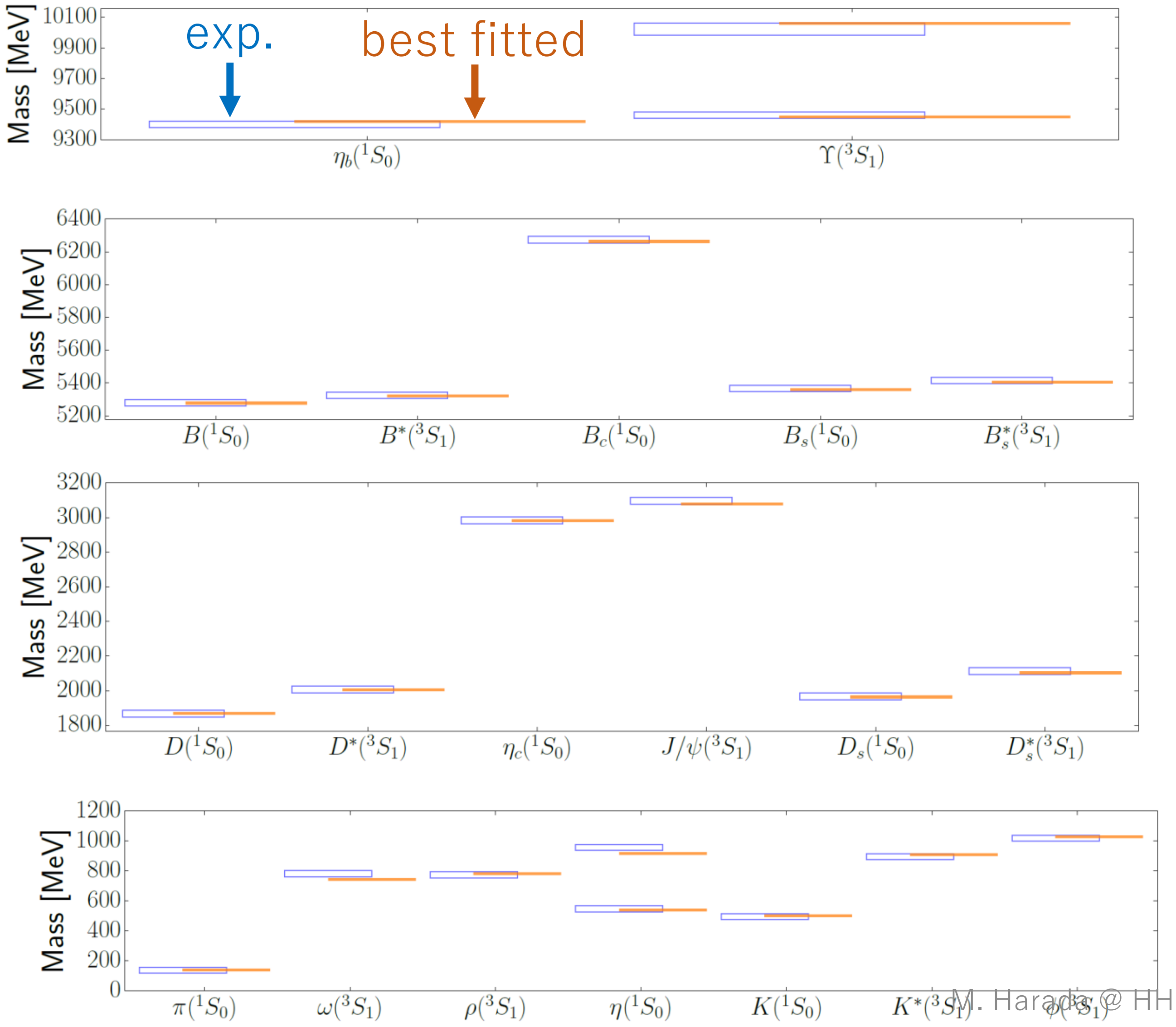
$$H = \sum_{i=1}\left(m_i + \frac{p_i^2}{2m_i}\right) - T_{CM} + \sum_{j>i=1}\left(V_{ij}^{\mathrm{CON}} + V_{ij}^{\mathrm{OGE}} + V_{ij}^{\bar{\sigma}} + V_{ij}^{PS} + V_{ij}^{V}\right)$$

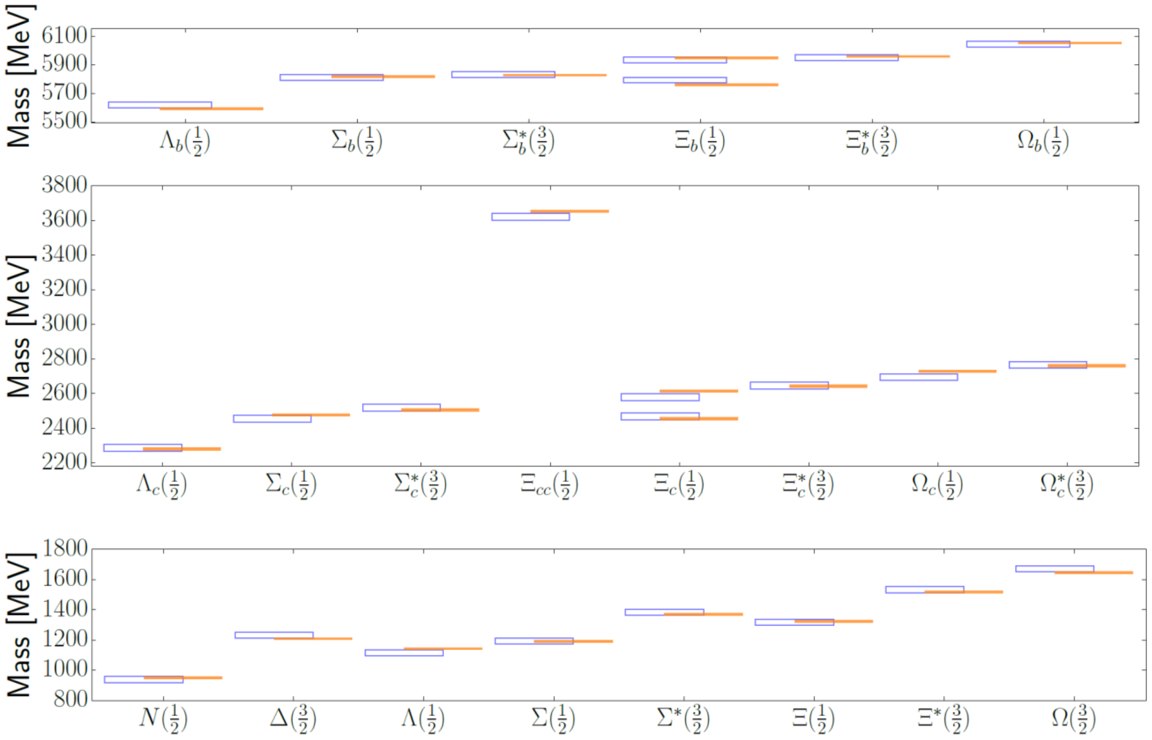
- $\bar{\sigma}$: SU(3) flavor singlet scalar meson
- PS: π, K, η, η'
- V: ρ, ω, K^*, ϕ

$m_u = m_d(\mathrm{MeV})$	381.1	$m_{\bar{\sigma}}(\mathrm{fm}^{-1})$	2.53
$m_s(\mathrm{MeV})$	551.6	$m_\eta(\mathrm{fm}^{-1})$	2.78
$m_c(\mathrm{MeV})$	1735.0	$m_{\eta'}(\mathrm{fm}^{-1})$	4.85
$m_b(\mathrm{MeV})$	5094.8	$m_\pi(\mathrm{fm}^{-1})$	0.7
$a_c(\mathrm{MeV})$	352.7	$m_K(\mathrm{fm}^{-1})$	2.51
$\mu_c(\mathrm{fm}^{-1})$	3.3	$m_\omega(\mathrm{fm}^{-1})$	3.97
$\Delta(\mathrm{MeV})$	327.6	$m_\phi(\mathrm{fm}^{-1})$	5.17
α_0	0.703	$m_\rho(\mathrm{fm}^{-1})$	3.93
$\Lambda_0(\mathrm{fm}^{-1})$	0.835	$m_{K^*}(\mathrm{fm}^{-1})$	4.54
$\mu_0(\mathrm{MeV})$	300.688	$\theta_p(^\circ)$	-11.3
$\hat{r}_0(\mathrm{MeV}\cdot\mathrm{fm})$	23.067	$\Lambda_{\bar{\sigma}} = \Lambda_\pi = \Lambda_\eta = \Lambda_K(\mathrm{fm}^{-1})$	4.2
$g_{\bar{\sigma}q} = g_{\bar{\sigma}s}$	-0.003	$\Lambda_\omega = \Lambda_\rho = \Lambda_{K^*}(\mathrm{fm}^{-1})$	5.2
$g_\pi = g_K$	2.912	$\Lambda_{\eta'} = \Lambda_\phi(\mathrm{fm}^{-1})$	6.2
$g_{\eta q}$	1.177	$f_{\omega q}$	0.42
$g_{\omega q}$	1.118	$f_\rho = f_{K^*}$	-0.222
$g_\rho = g_{K^*}$	-0.323		

- Fitting: $\chi^2 = \sum\left(\frac{m(theo) - m(exp)}{Err(sys)}\right)^2$
  - $\chi^2/_{dof} = 8.8/_{(46-22)} \simeq 0.37$

Masses of Mesons and Baryons

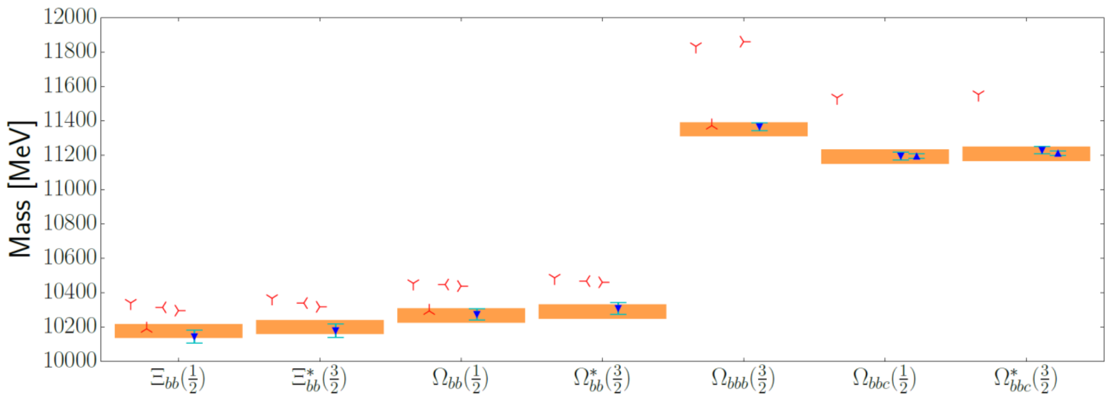


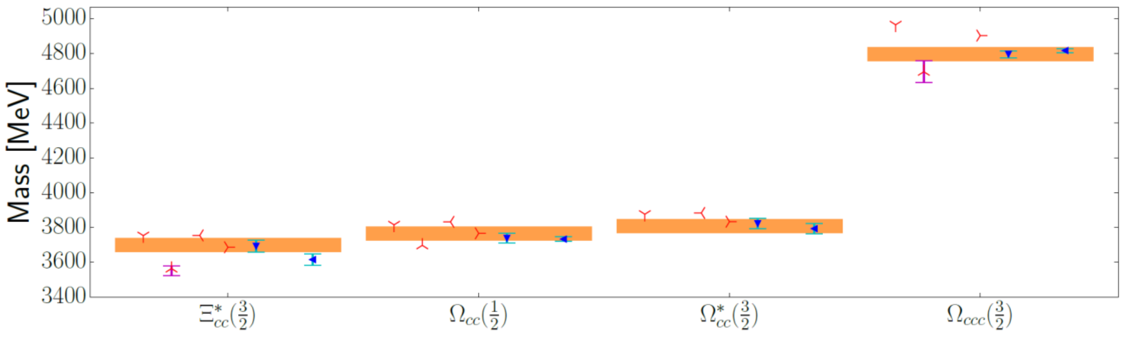


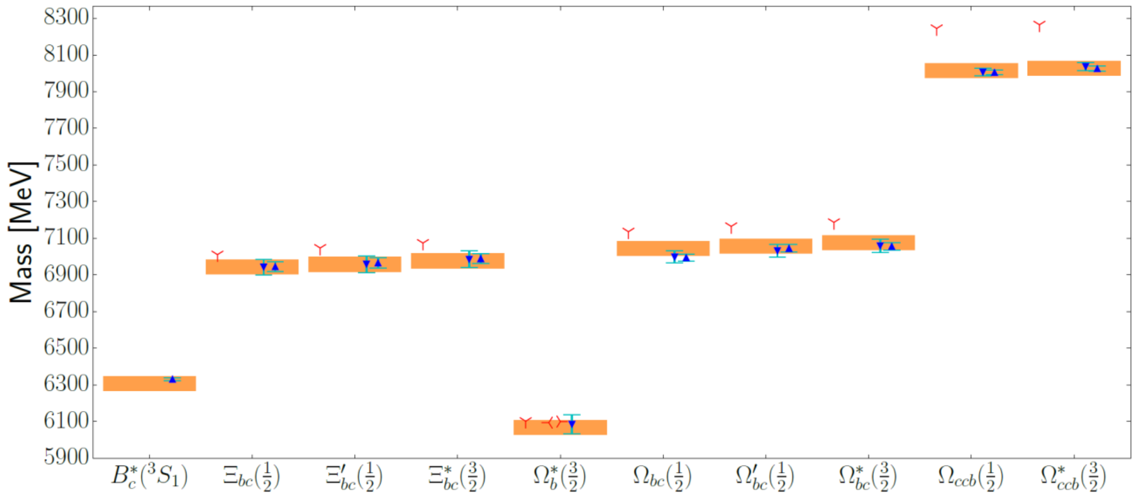
Masses of 46 hadrons are beautifully fitted by 22 parameters:

$$\chi^2/_{dof} = {}^{8.8}/_{24} \simeq 0.37$$

Predictions for unobserved hadrons







Predictions with 40MeV errors: ▬

lattice QCD (▼ [24], ▲ [25], ◀ [26])

Other quark models

(Y [18], ⅄ [19–21], ≺ [22], ≻ [23])

- Most predictions are consistent with lattice results.
- Some are different with some quark models.

II.5 : Summary of Part II

- We proposed a chiral quark model with hidden local symmetry (HLS), which includes light vector mesons in addition to pseudoscalar and scalar mesons consistently with the chiral symmetry.
- Resultant masses of N, Λ_c, Λ_b are dramatically improved, mainly owing to the effects of ω meson.
- Masses of 46 hadrons are beautifully fitted by a single parameter set for 22 parameters: $\chi^2/_{dof} = {}^{8.8}/_{24} \simeq 0.37$
- Unobserved ground-state baryons are predicted, which agree with lattice QCD restuls.
- Our analysis indicates that Tcc takes molecule-like structure, while Tbb takes diquark-like structure.

III：Study of Multiquark states based on the superflavor symmetry

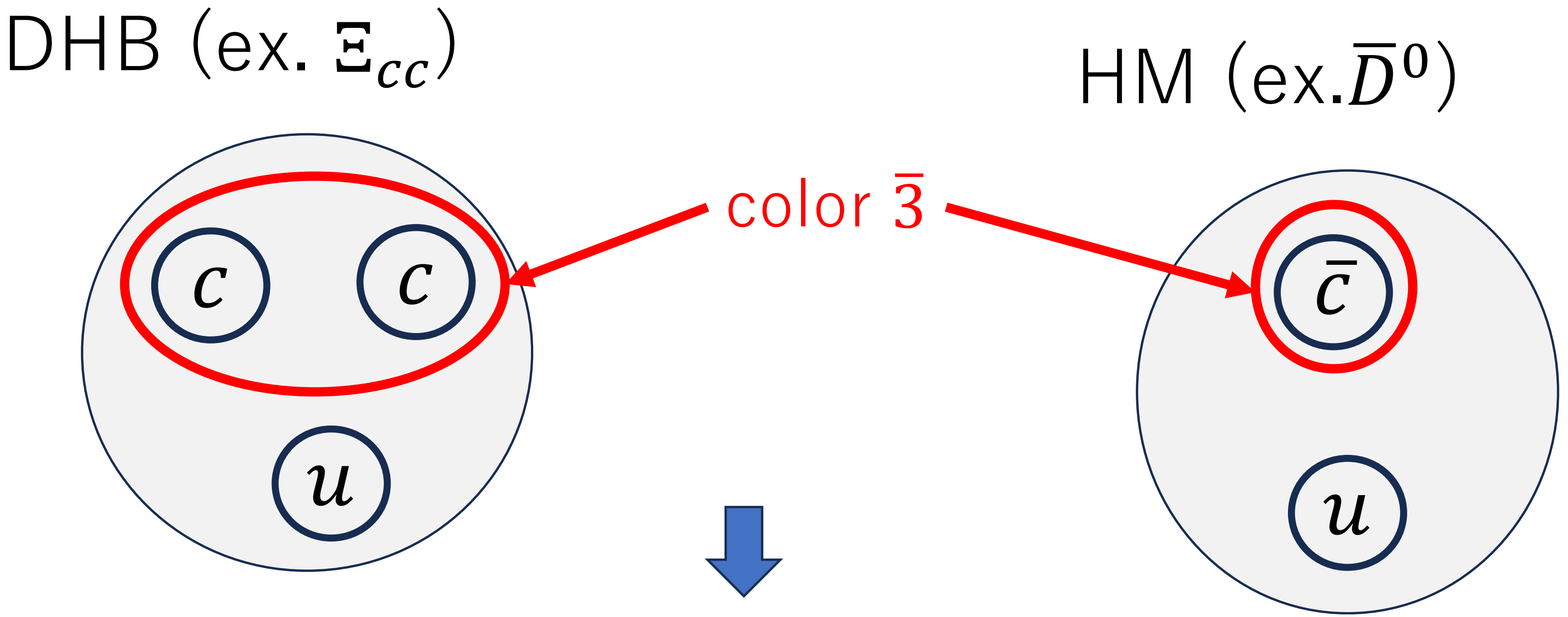
Outline of PART III

1. Superflavor symmetry for heavy hadrons
2. Molecular Tetraquark and Pentaquark based on the superflavor symmetry
3. EFT Analysis of Doubly Heavy Tetraquarks based on the superflavor symmetry
4. Summary of Part III

III.1 : Superflavor symmetry for heavy hadrons

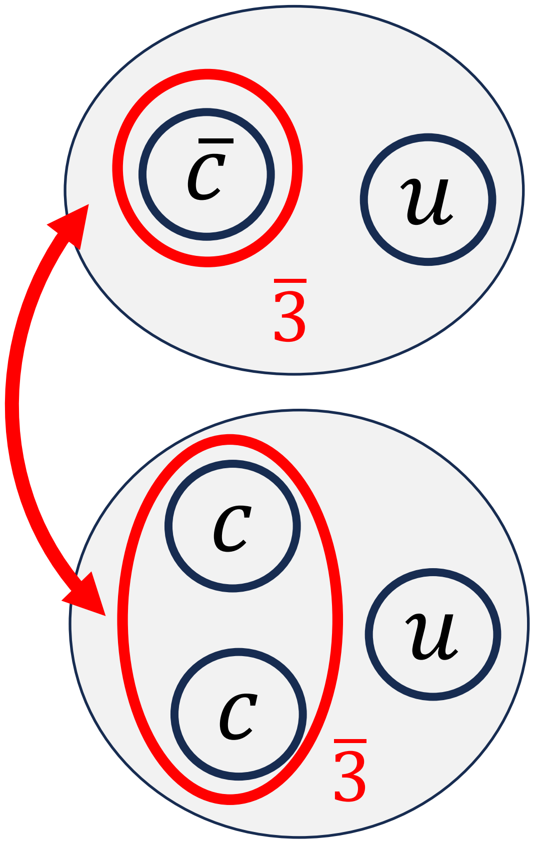
Superflavor symmetry

H. Georgi and M. B. Wise, Phys. Lett. B 243, 279 (1990).
M. J. Savage and M. B. Wise, Phys. Lett. B 248, 177 (1990).



- In the heavy quark limit ($m_c \to \infty$),
  the same color electric force applies to cc and $\bar{c}$.
- The color-magnetic force is suppressed by $1/m_c$.
- The properties of DHBs and HMs are governed by the same dynamics of the same light-quark cloud.

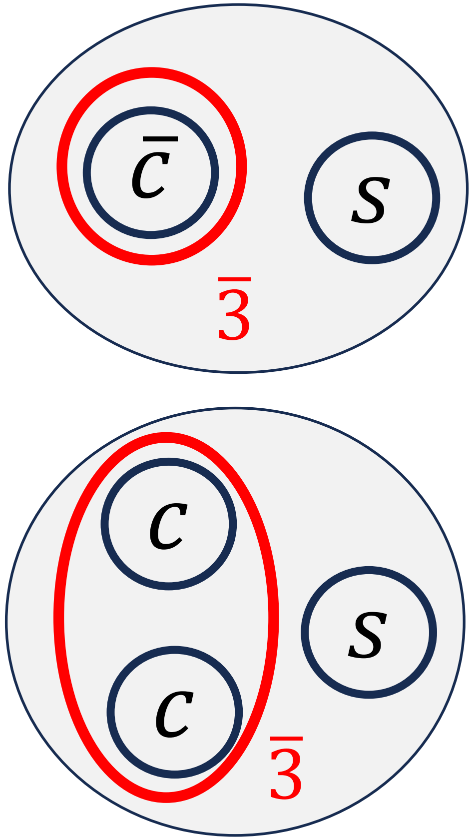
Superflavor symmetry for masses of HMs & DHBs



$$M_{\bar{c}u} = m_c + E_u$$

common

$$M_{ccu} = m_{cc} + E_u$$



$$M_{\bar{c}s} = m_c + E_s$$

common

$$M_{ccs} = m_{cc} + E_s$$

$$M_{ccs} - M_{ccu} = M_{\bar{c}s} - M_{\bar{c}u} = E_s - E_u$$

Validity of Superflavor symmetry

$$M_{ccs} - M_{ccu} = M_{\bar{c}s} - M_{\bar{c}u}$$

	Mass(MeV)
Ξ_{cc}	3615
Ξ^*_{cc}	3703
Ω_{cc}	3733
Ω^*_{cc}	3793
$D^{\pm}$	1870
$D^{*\pm}$	2010
$D_s^{\pm}$	1968
$D_s^{*\pm}$	2112

lattice (for Ξ_{cc}, Ξ^*_{cc}, Ω_{cc}, Ω^*_{cc})

cf: $M(\Xi_{cc}^{++}) = 3621\ MeV$ (LHCb)

H. Bahtiyar et al. (TRJQCD Collaboration), PRD102, 054513 (2020)

$$\bar{M}(\Omega_{cc}) - \bar{M}(\Xi_{cc}) = \boxed{100\ MeV}$$

Superflavor symmetry is good !

PDG (for $D^{\pm}$, $D^{*\pm}$, $D_s^{\pm}$, $D_s^{*\pm}$)

$$\bar{M}(D_s) - \bar{M}(D^{\pm}) = \boxed{101\ MeV}$$

$\bar{M}$: spin average

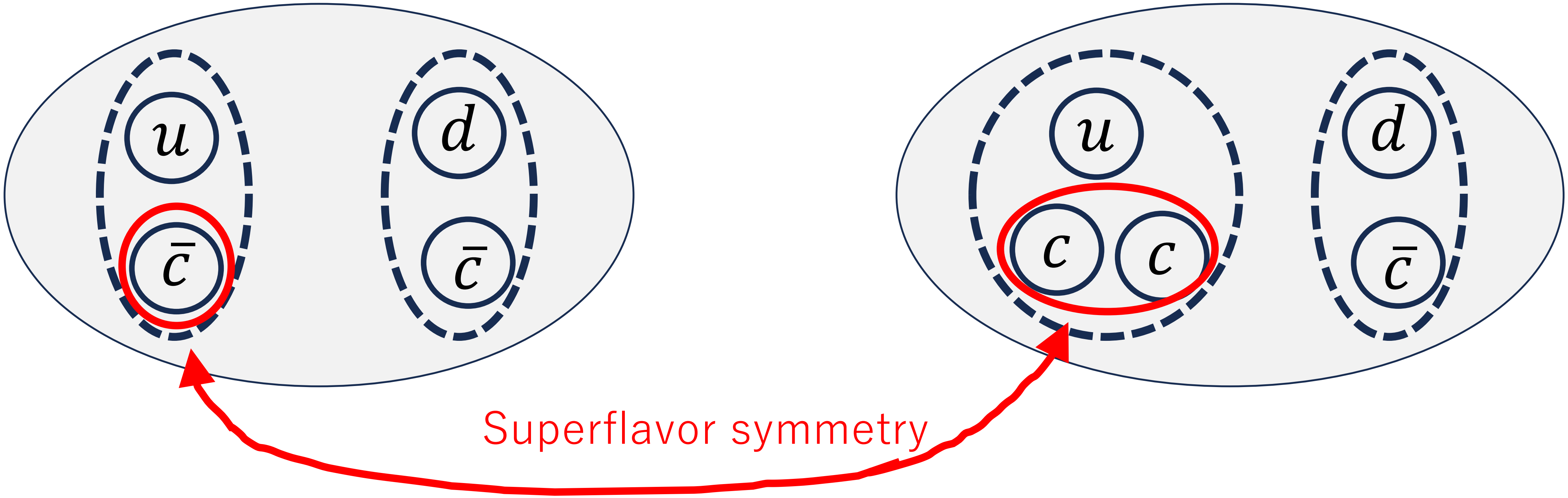
III.2 : Molecular Tetraquark and Pentaquark based on the superflavor symmetry

T. Asanuma, Y. Yamaguchi and M. Harada, arXiv:2311.04695, to appear in Phys. Rev. D.

Outlook

$\bar{T}_{cc} \sim \bar{D}^{(*)}\bar{D}^{(*)}$ molecule

$\Xi_{cc}^{(*)}\bar{D}^{(*)}$ molecule

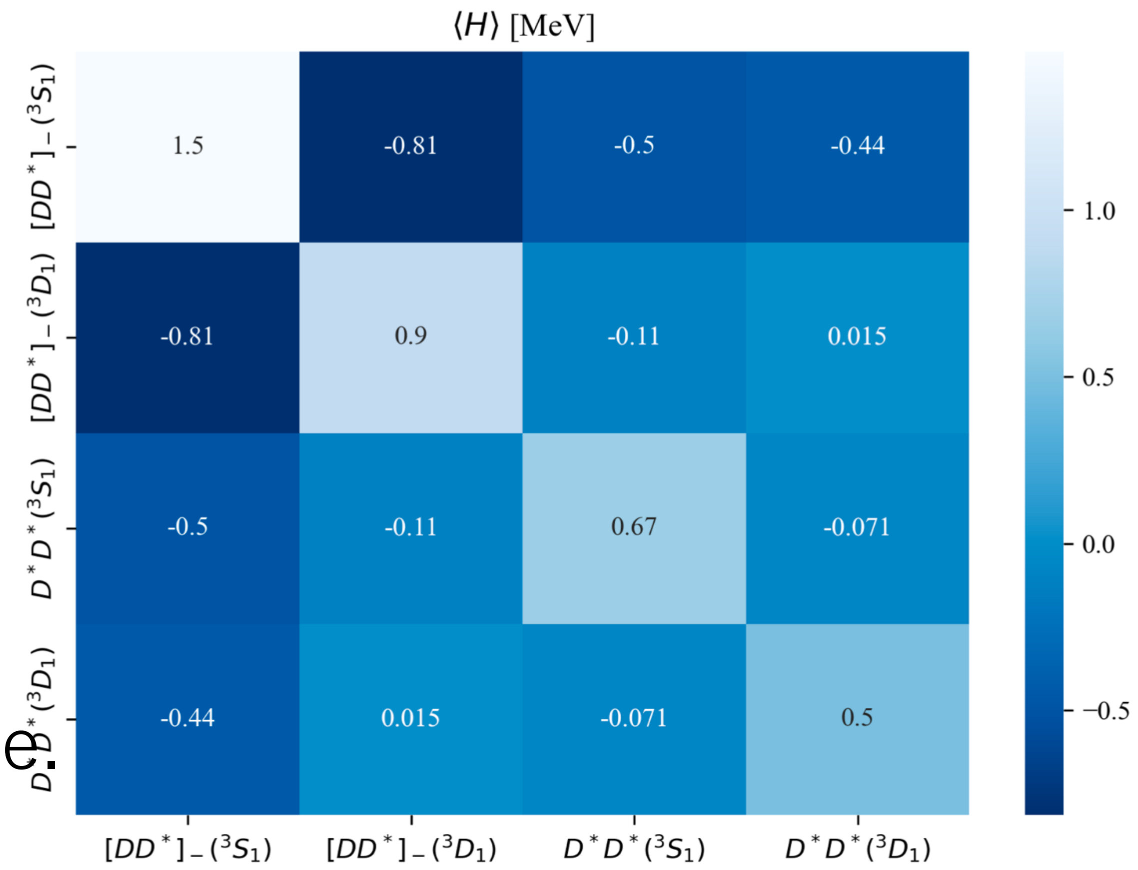


$D^{(*)}D^{(*)}$molecule from One Boson Exchange Potential (OBEP)

- $\pi, \sigma, \rho, \omega$ mesons
- coupling constants
  - π: $g_\pi = 0.59$ (from $D^* \to D\pi$ decay)
  - ρ, ω: $\beta g_V(\text{electric type}) = 5.31$; $\lambda g_V(\text{magnetic type}) = 0.504\text{GeV}^{-1}$
  - σ: 2 choices ; $g_\sigma^{(1)} = 3.4$ $\left[g_\sigma^{(2)} = 0.76\right]$
- Form factor
  - $F(q) = \frac{\Lambda^2 - \mu_{eff}^2}{\Lambda^2 + \boldsymbol{q}^2}$, $\mu_{eff}^2 = m_{ex}^2 - (q^0)^2$, $q^0 \cong \frac{m_2^2 - m_1^2 + m_3^2 - m_4^2}{2(m_3 + m_4)}$

Results for T_{cc} (Λ is tuned to reproduced $m(T_{cc})|_{\text{exp}}$)

Λ [MeV]	1160	1182	1200
$\mathrm{B_{in}}$ [MeV]	0.074	0.273	0.549
$P_{[DD^*]_-}(^3S_1)$	0.992	0.987	0.983
$P_{[DD^*]_-}(^3D_1)$	0.00545	0.00840	0.0103
$P_{D^*D^*}(^3S_1)$	0.00159	0.00348	0.00541
$P_{D^*D^*}(^3D_1)$	0.000542	0.00106	0.00151
$\sqrt{\langle r^2\rangle}$ [fm]	11.33	6.42	4.70



- very large size $R \approx 6.4$ fm

- Diagonal part ($DD^*[{}^3S_1]$) is repulsive.
  - $\langle DD^*[^3S_1]|H|DD^*[^3S_1]\rangle = 1.5$ MeV
- Off-diagonal ($DD^*[{}^3S_1]$ & $DD^*[{}^3D_1]$) is important.
  - $\langle DD^*[^3S_1]|H|DD^*[^3D_1]\rangle = -0.81$ MeV

$\bar{D}^{(*)}\Xi_{cc}^{(*)}$ molecule $I(J^P) = 0(1/2^-)$

- Coupled channel analysis (7 channels)
  - $\bar{D}\Xi_{cc}(^2S_{\frac{1}{2}}), \bar{D}\Xi_{cc}^*(^4D_{\frac{1}{2}}), \bar{D}^*\Xi_{cc}(^2S_{\frac{1}{2}}, ^4D_{\frac{1}{2}}),$ $\bar{D}^*\Xi_{cc}^*(^2S_{\frac{1}{2}}, ^4D_{\frac{1}{2}}, ^6D_{\frac{1}{2}})$
    - ✓ note: (Ξ_{cc}, Ξ_{cc}^*) form HQS doublet
- OBEP: same as DD^*
  - $\bar{D}$ and Ξ_{cc} have the same light cloud (superflavor symmetry)
  - use of the same $\Lambda = 1182$ MeV.

- Result

Λ [MeV]	1160	1182	1200
$\mathrm{B_{in}}$ [MeV]	5.65	7.46	9.10
$P_{\bar{D}\Xi_{cc}}(^2S_{\frac{1}{2}})$	0.99	0.98	0.98
$P_{\bar{D}\Xi_{cc}^*}(^4D_{\frac{1}{2}})$	0.000008	0.000012	0.000017
$P_{\bar{D}^*\Xi_{cc}}(^2S_{\frac{1}{2}})$	0.00056	0.00076	0.00095
$P_{\bar{D}^*\Xi_{cc}}(^4D_{\frac{1}{2}})$	0.0026	0.0030	0.0033
$P_{\bar{D}^*\Xi_{cc}^*}(^2S_{\frac{1}{2}})$	0.0021	0.0028	0.0035
$P_{\bar{D}^*\Xi_{cc}^*}(^4D_{\frac{1}{2}})$	0.00068	0.00074	0.00079
$P_{\bar{D}^*\Xi_{cc}^*}(^6D_{\frac{1}{2}})$	0.0085	0.0097	0.011
$\sqrt{\langle r^2 \rangle}$ [fm]	1.54	1.38	1.28

- Boundstate exists!
  - Binding energy ≈ 7.5 MeV
  - Rather compact size : $R \approx 1.4$ fm

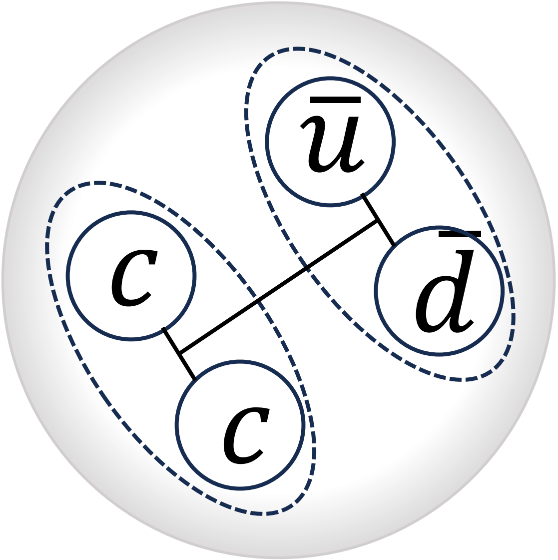
III.3 : EFT Analysis of Doubly Heavy Tetraquarks based on the superflavor symmetry

M. Tanaka, Y. Yamaguchi and M. Harada, Phys. Rev. D 110, 016024 (2024).

Assumption

T_{cc}^{+} (LHCb,2021)

Mass(MeV)	3875
Width(keV)	48
Flavor	$cc\bar{u}\bar{d}$
J^P	1^+



In our study,

- Assumption
  T_{cc} : cc diquark + $\bar{q}\bar{q}$ light quark cloud
  $q = u, d, s$
- No radial excitation between c and c
- Color rep. of cc diquark : $\bar{3}$

Color factor
$C_F^{\bar{3}} = -\frac{2}{3}, C_F^{6} = \frac{1}{3}$

$3 \times \bar{3} = 1 + 8$

$6 \times \bar{6} = 1 + 8 + 27$

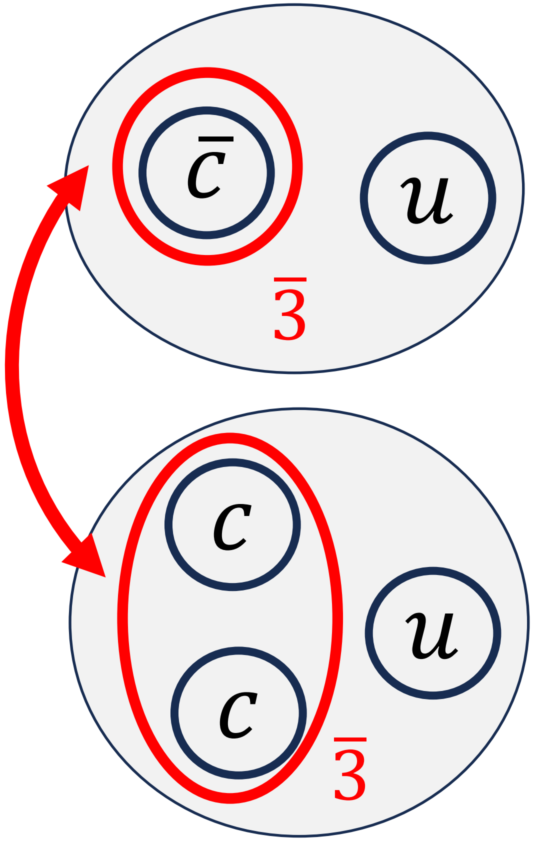
We assume $\bar{3}$ is dominant

More stable?

Comparison with future experiment

We can study whether the color anti-triplet state of diquark is dominant in the DHTs or not.

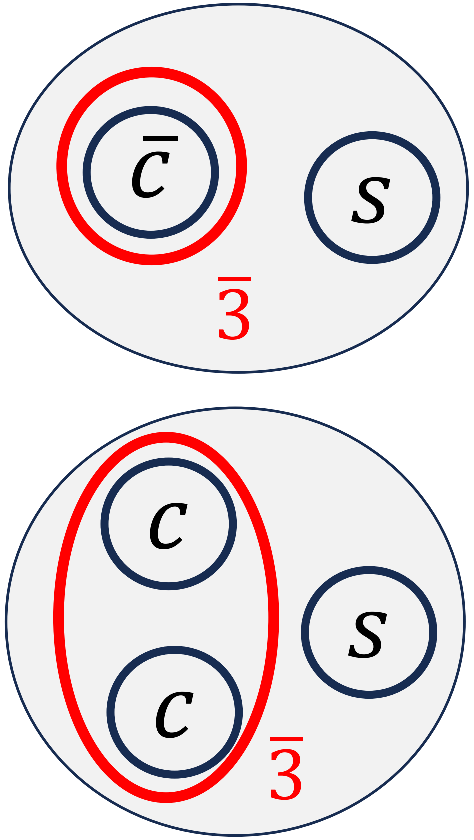
A simple mass relation for mesons & baryons from superflavor symmetry



$$M_{\bar{c}u} = m_c + E_u$$

Common

$$M_{ccu} = m_{cc} + E_u$$



$$M_{\bar{c}s} = m_c + E_s$$

Common

$$M_{ccs} = m_{cc} + E_s$$

$$M_{ccs} - M_{ccu} = M_{\bar{c}s} - M_{\bar{c}u} = E_s - E_u$$

Symmetry breaking terms based on the effective Lagrangian approach 1

(1) Invariant under superflavor and SU(3) light-flavor symmetry

$$\mathcal{L} = -\mathrm{tr}\left[\bar{\Psi}^i iv\cdot\partial\Psi_i + \bar{\Phi}_{ij} iv\cdot\partial\Phi^{ji}\right] - \Lambda_\Psi \mathrm{tr}\left[\bar{\Psi}^i\Psi_i\right] - \Lambda_\Phi tr\left[\bar{\Phi}_{ij}\Phi^{ji}\right]$$

$\bar{\Psi} = \begin{pmatrix} \bar{H} \\ B \end{pmatrix}$ Heavy Mesons $\bar{Q}q$, Doubly Heavy Baryons QQq

$\Phi = \begin{pmatrix} \bar{S} \\ T^{(\bar{3})} \end{pmatrix}$ Singly Heavy Baryons $\overline{Qqq}$, Doubly Heavy Tetraquarks $QQ\overline{qq}$

(2) Break SU(3) light-flavor symmetry but are invariant under superflavor symmetry

$$-c_\Psi \mathrm{tr}\left[\bar{\Psi}^i\Psi_j\right](\mathcal{M})^j{}_i - c_\Phi \mathrm{tr}\left[\bar{\Phi}_{ij}(\mathcal{M})^j{}_k\Phi^{ki}\right]$$

$\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$: current quark masses

The above 2 contributions are already included in the previous slides

Symmetry breaking terms based on the effective Lagrangian approach 2

(3) Break SU(3) light-flavor symmetry and superflavor symmetry

$$-\frac{\Lambda_f}{m_Q}\mathrm{tr}\left[H_i(\mathcal{M})^i{}_j\overline{H^j}\right]-\frac{\Lambda_f'}{2m_Q}\mathrm{tr}\left[\overline{\mathcal{B}}_i(\mathcal{M})^i{}_j\mathcal{B}^j\right]-\frac{\Lambda_{ff}}{m_Q}\mathrm{tr}[\bar{S}_{ij}(\mathcal{M})^j{}_kS^{ki}]-\frac{\Lambda_{ff}'}{2m_Q}\mathrm{tr}\left[(\bar{T}^{(\bar{3})})_{ij}(\mathcal{M})^j{}_k(\bar{T}^{(\bar{3})})^{ki}\right]$$

(4) Break superflavor symmetry and heavy-quark spin symmetry

$$-\frac{\Lambda_\sigma^2}{8m_Q}\mathrm{tr}\left[H(\sigma^{\mu\nu}_{\mathrm{heavy}})^T\bar{H}\sigma_{\mu\nu}^{\mathrm{light}}\right]-\frac{\Lambda_{\sigma f}}{8m_Q}\mathrm{tr}\left[H_i(\sigma^{\mu\nu}_{\mathrm{heavy}})^T(\mathcal{M})^i{}_j\bar{H}^j\sigma_{\mu\nu}^{\mathrm{light}}\right]$$

$$-\frac{(\Lambda_\sigma')^2}{8m_Q}\left[[\bar{\mathcal{B}}]_{h_1h_2l_1}(\sigma^{\mu\nu}_{\mathrm{heavy}})_{h_2h_3}[\mathcal{B}]_{h_3h_1l_2}(\sigma_{\mu\nu}^{\mathrm{light}})_{l_2l_1}\right]-\frac{\Lambda_{\sigma f}'}{8m_Q}\left[[\bar{\mathcal{B}}_i]_{h_1h_2l_1}(\sigma^{\mu\nu}_{\mathrm{heavy}})_{h_2h_3}(\mathcal{M})^i{}_j[\mathcal{B}^j]_{h_3h_2l_2}(\sigma_{\mu\nu}^{\mathrm{light}})_{l_2l_1}\right]$$

(5) mixing between $T^{(\bar{3})}$ and $T_\mu^{(6)}$

$$-\frac{\Lambda_{\mathrm{mix}}^2}{2m_Q}\mathrm{Tr}\left[(\bar{T}_\mu^{(6)})\gamma_5\gamma^\mu(T^{(\bar{3})})+h.c.\right]$$

$T^{(\bar{3})}$: QQ belongs to color $\bar{3}$ representation

$T_\mu^{(6)}$: QQ belongs to color 6 representation

Determination of model parameters

e.g.) $[M_{\text{ave}}(D_s) - M_{\text{ave}}(D)] - [M_{\text{ave}}(B_s) - M_{\text{ave}}(B)]$

$$= (m_s - m_{u,d})\left(\frac{1}{m_c} - \frac{1}{m_b}\right)\Lambda_f$$

⇒ $\Lambda_f = 272.2 \pm 70.9$ (MeV)

Similarly, $\Lambda_{ff} = 157.4 \pm 70.9$ (MeV)

$\Lambda_\sigma = 423.5 \pm 25.1$ (MeV)

$\Lambda_{\sigma f} = 36.70 \pm 70.87$ (MeV)

	Mass(MeV)
B	5280
B^*	5325
$\mathrm{B_s}$	5367
$\mathrm{B_s^*}$	5416
D	1867
D^*	2009
D_s	1968
D_s^*	2112

From PDG

We assume that $(\Lambda'_f, \Lambda'_{ff}, \Lambda'_\sigma, \Lambda'_{\sigma f})$ are of the same order as $(\Lambda_f, \Lambda_{ff}, \Lambda_\sigma, \Lambda_{\sigma f})$.

$\Lambda'_f = 0 \pm 378.4$ (MeV), $\Lambda'_{ff} = 0 \pm 263.6$ (MeV)

$\Lambda'_\sigma = 0 + 448.6$ (MeV), $\Lambda'_{\sigma f} = 0 + 107.9$ (MeV)

$$M_{QQ\bar{s}\bar{q}} - M_{QQ\bar{q}\bar{q}} - \frac{(m_s - m_q)}{2m_Q}\Lambda'_{ff} = M_{\bar{Q}\bar{s}\bar{q}} - M_{\bar{Q}\bar{q}\bar{q}} - \frac{(m_s - m_q)}{m_Q}\Lambda_{ff}$$

$$M_{QQ\bar{q}\bar{q}} - M_{\bar{Q}\bar{q}\bar{q}} + \frac{M_6}{2({M_6}^2 - {M_3}^2)}\left(\frac{\Lambda_{mix}^2}{2m_Q}\right)^2 - \frac{m_q}{m_Q}(\Lambda'_{ff} - 2\Lambda_{ff})$$

$$= M_{QQq} - M_{\bar{Q}q} - \frac{1}{2}\frac{m_q}{m_Q}(\Lambda'_f - 2\Lambda_f)$$

Mass of T_{ccs}

M Tanaka et al, PRD110, 016024 (2024).

$$M(T_{ccs}) - M(T_{cc}) - \frac{(m_s - m_{u,d})}{2m_c}\Lambda'_{ff} = M(\Xi_c) - M(\Lambda_c) - \frac{(m_s - m_{u,d})}{m_c}\Lambda_{ff}$$

Input Values

	Mass(MeV)
T_{cc}	3875
Ξ_c	2469
Λ_c	2286

LHCb and PDG

$$M(T_{ccs}) = 4047 \pm 11\ (\mathrm{MeV})$$

※This value is isospin averaged.

$$M_{\mathrm{QM}}(T_{ccs}) = 4106\ (\mathrm{MeV})$$

M. Karliner and J. L. Rosner, Phys. Rev. D 105, 034020 (2022).

$$M_{\mathrm{lattice}}(T_{ccs}) = 3969 \pm 8\ (\mathrm{MeV})$$

P. Junnarkar, N. Mathur, and M. Padmanath, Phys. Rev. D 99, 034507 (2019).

Agreement of our prediction with future experiment indicates that the heavy diquark in T_{cc} and T_{ccs} is dominated by the one with color $\bar{3}$ representation.

III.4 : Summary of Part III

- Study of hadrons including heavy quark(s) based on
  - heavy quark symmetry
  - superflavor symmetry
  - chiral symmetry
- Triple-heavy Pentaquark
  - Boundstate of $\bar{D}^{(*)}\Xi_{cc}^{(*)}$ molecule related to $D^{(*)}D^{(*)}(T_{cc})$ by the superflavor symmetry.
  - $B.E \approx 7.5\ \mathrm{MeV}$
- Doubly-heavy Tetraquark
  - Mass of Tccs is predicted based on the superflavor symmetry
  - $M(T_{ccs}) = 4047 \pm 11\ (\mathrm{MeV})$
  - Comparison of our predictions with future experimental results will provide a clue to understand the spatial and color structures of Doubly Heavy Tetraquarks.

VI : Overall Summary

- We proposed a new chiral quark model, in which the vector mesons are included based on the hidden local symmetry.
  - We showed that masses of ground states of mesons and baryons are well reproduced with a single set of the parameters.
- We used the superflavor symmetry as a novel key to study hadrons including multi-heavy quarks.
  - We predicted the existence of a boundstate of $\bar{D}^{(*)}\Xi_{cc}^{(*)}$ molecule.
  - We predicted the mass of T_{ccs} as $M(T_{ccs}) = 4047 \pm 11\ (\mathrm{MeV})$
- Future prospect
  - Study of decay process in quark models
  - Study of scattering processes of hadrons including heavy quarks based on the superflavor symmetry.

Thank you very much for your attention !