Femtoscopic study of the $\Omega\alpha$ interaction in heavy-ion collisions

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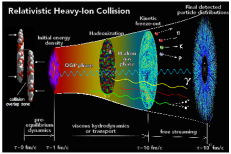

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Introduction

1. This presentation is based on [F. Etminan arXiv:2409.19705v2]

➤

The spin-2 Omega-nucleon (ΩN) state with $S = -3$ is expected to lack a repulsive core. [F. Etminan et al. NPA 928, 89 2014] [T. Iritani et al.,PLB 792,284 2019]

➤ Strongly attractive interaction in the $^5S_2$ channel reported by HAL QCD Collaboration and Sekihara, Kamiya and Hyodo based on the meson exchanges with effective Lagrangians [T. Sekihara, Y. Kamiya, and T. Hyodo,PRC 98, 015205 2018.]

➤ One method for studying the hh interaction that is hard to investigate in scattering experiment is measuring the momentum correlation functions in high-energy collisions

➤ It provides information for effective emission source and interaction potential.

➤ First measurement of the p-Ω correlation function in heavy-ion collisions is done by STAR experiment at RHIC [K. Morita et al., PRC 94, 031901 2016 [M. P. i Méndez, et al. (2024), arXiv:2409.16747; J. Adam et al. (STAR Coll.), PLB 790, 490 2019

Introduction

➤ As the next step in the femtoscopic analyses, h-deuteron correlation functions would be promising [S. Acharya et al. (ALICE Coll.),PRX 14, 031051 2024]

➤ The production of ΩNN and $\Omega\Omega N$ in ultra-relativistic heavy-ion collisions using the Lattice QCD ΩN, $\Omega\Omega$ potentials studied. [L. Zhang, et al., EPJC 82,1 2022]

➤ And very recently, the momentum correlation between $\Lambda\alpha$ and $\Xi\alpha$ are examined to shed light on the interaction between a hyperon and nucleons (N). [A. Jinno, et al, PRC 110, 014001 2024; Y. Kamiya, et al, arXiv:2409.13207]

✦ Therefore I want to explore the $\Omega\alpha$ correlation function in the relativistic heavy ion collisions to probe the nature of ΩN interactions as an independent source of information.

✦ Also, $\phi\alpha$ is in preparation.

HAL QCD ΩN potential

- From Ref. [T. Iritani et al.,PLB 792,284 2019]

  S-wave and spin $2\Omega N$ potential in configuration space at nearly physical quark masses.

- The discrete lattice potential is fitted by an analytic function

$$V_{\Omega N}^{HAL}(r) = b_1 e^{\left(-b_2 r^2\right)} + b_3 \left(1 - e^{-b_4 r^2}\right) \left(\frac{e^{-m_\pi r}}{r}\right)^2,$$

- Gauss functions describe the short-range and the Yukawa functions explain the meson exchange picture at medium to long-range distances of the potential.

- Discrete lattice results are fitted reasonably well, $\chi^2/d.o.f \simeq 1$, with four different sets of parameters $b_{1,2,3}$ and b_4, they are given in Table 1 of [T. Iritani et al.,PLB 792,284 2019]

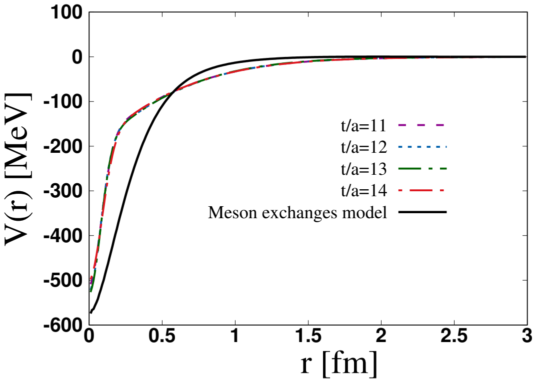
- For imaginary-time slices $t/a = 11, 12, 13, 14$ and $a = 0.0846$ fm is the lattice spacing. Pion mass is taken from the lattice simulation, $m_\pi = 146$ MeV.

- For this potential, the scattering length, effective range and binding energy, respectively are $a_0^{\Omega N} = 5.30$ fm, $r_0^{\Omega N} = 1.26$ fm and $B^{\Omega N} = 1.54$ MeV.

HAL QCD ΩN potential

✦ HAL QCD ΩN potential, $V_{\Omega N}^{HAL}$ at the imaginary-time distances $t/a = 11, 12, 13, 14$ from Ref.[T. Iritani et al.,PLB 792,284 2019]

✦ ΩN potential based on the meson exchanges model (solid black line), Ref.[T. Sekihara, Y. Kamiya, and T. Hyodo,PRC 98, 015205 2018.]



✦ It should be mentioned that the HAL potentials are only in $^5S_2$ channel but in the case of ME potential model the coupled channels effects to $\Omega N\,^5S_2$ interaction are imposed. [T. Sekihara, Y. Kamiya, and T. Hyodo,PRC 98, 015205 2018.]

Meson Excange (ME) ΩN potential

- Developed by Sekihara, Kamiya and Hyodo Ref based the meson exchanges (MS) potential, [T. Sekihara, Y. Kamiya, and T. Hyodo,PRC 98, 015205 2018.]
-

$$V_{\Omega N}^{ME}(r) = \frac{1}{4\pi r} \sum_{n=1}^{9} C_n \left(\frac{\Lambda^2}{\Lambda^2 - \mu_n^2} \right)^2 \left(\exp(-\mu_n r) - \frac{(\Lambda^2 - \mu_n^2) r + 2\Lambda}{2\Lambda} \exp(-\Lambda r) \right) \quad (1)$$

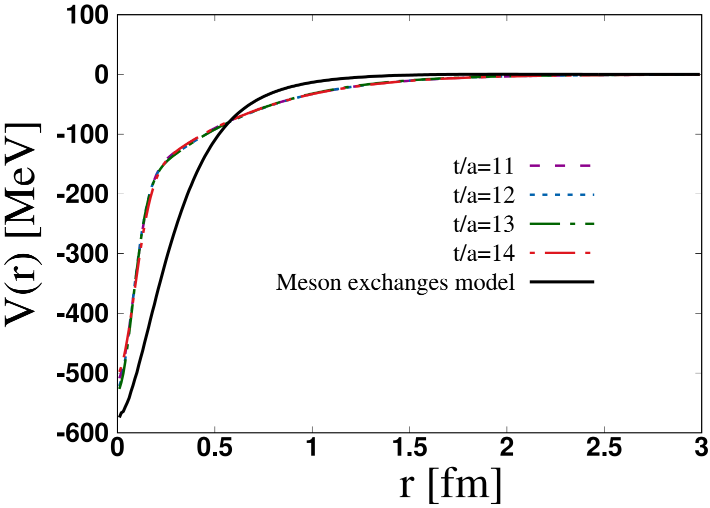
- Cutoff parameter $\Lambda = 1$ GeV, $\mu_n = 100\, n$ MeV and parameters C_n are taken directly from Table V of [T. Sekihara, Y. Kamiya, and T. Hyodo,PRC 98, 015205 2018.].
- They formulated an equivalent local potential that reproduces $\Omega N\, {}^5S_2$ scattering length 7.4 ± 1.6 fm at the time range $t/a = 11$ of the lattice simulations.

Meson Excange (ME) ΩN potential

- Long range part is build on the exchanges of the η meson and correlated two mesons in the scalar-isoscalar channel (Known as "σ") The short-range part is constructed by the contact interaction.
- Coupled channels effect to $\Omega N\ {}^5S_2$ interaction by adding the box diagrams with intermediate $\Lambda\Xi, \Sigma\Xi$ and $\Lambda\Xi$ channels.
- They concluded that even though the elimination of these channels induces the energy dependence of the single-channel ΩN interaction, this effect is not significant.

HAL QCD and ME ΩN potential

- HAL QCD ΩN potential, $V_{\Omega N}^{HAL}$ at the imaginary-time distances $t/a = 11, 12, 13, 14$ from Ref.[T. Iritani et al.,PLB 792,284 2019]
- ΩN potential based on the meson exchanges model (solid black line), Ref. [T. Sekihara, Y. Kamiya, and T. Hyodo,PRC 98, 015205 2018.]



$\Omega\alpha$ potential from single-folding potential approach

- The effective $\Omega + \alpha$ nuclear potential is approximated by the single-folding model [G. Satchler and W. Love,Phys. Rep. 55, 183 (1979)]

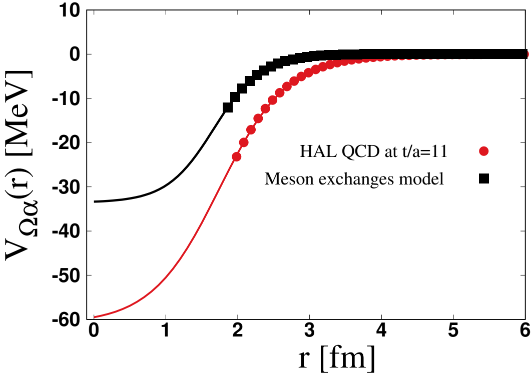
$$V_{\Omega\alpha}(r) = \int \rho(r')\, V_{\Omega N}(|\mathbf{r} - \mathbf{r}'|)\, d\mathbf{r}', \tag{2}$$

- $V_{\Omega N}(|\mathbf{r} - \mathbf{r}'|)$ is ΩN potential between the Ω particle at $\mathbf{r}'$ and the nucleon at $\mathbf{r}$ [F. Etminan, Chin. Phys. C 44, 054106 (2020)]
- And $\rho(r')$ is the nucleon density function in α-particle at a distance $\mathbf{r}'$ from its center-of-mass where can be taken as[K. S. Myint and Y. Akaishi, PTP. 117,251 1994],

$$\rho(r') = 4\left(\frac{4\beta}{3\pi}\right)^{3/2} \exp\left(-\frac{4}{3}\beta r'^2\right), \tag{3}$$

- β is a constant and it is defined by the rms radius of $^4\mathrm{He}$, i.e, $r_{r.m.s} = \frac{3}{\sqrt{8\beta}} = 1.47\ \mathrm{fm}$.

$\Omega\alpha$ potential from single-folding potential approach



✦ $V_{\Omega\alpha}(r)$, the resultant single-folding potential as functions of distance between Ω and α. Potentials are obtained using ΩN potential models of HAL QCD at $t/a = 11$ (red circle) and meson exchanges model (black square).

$\Omega\alpha$ Correlation function KP formula

- The two-particle momentum correlation function C_q is defined by Koonin-Pratt (KP) formula[A. Ohnishi, et al., NPA 954, 294 2016]

$$C(q) = \int d\boldsymbol{r} S(\boldsymbol{r}) \left|\Psi^{(-)}(\boldsymbol{r}, \boldsymbol{q})\right|^2, \tag{4}$$

- $S(r) = \exp\left(-\frac{r^2}{4R^2}\right) / \left(4\pi R^2\right)^{3/2}$ is a single particle source function that it is assumed to be spherical and static Gaussian with radius (or source size) R.
- The relative wave function $\Psi^{(-)}$ contains only the S-wave interaction effect. The resulting correlation function can be written as

$$C(q) = 1 + \int_0^\infty 4\pi r^2 \, dr \, S(r) \left[|\psi(q, r)|^2 - |j_0(qr)|^2\right], \tag{5}$$

- $j_{l=0}(qr) = \sin(qr)/qr$ is the spherical Bessel function and $\psi(k, r)$ is the S- wave scattering wave function. For a given two-body $\Omega\alpha$ potential it can be obtained straightforwardly by solving the Schrödinger equation.

$\Omega\alpha$ Correlation function LL formula

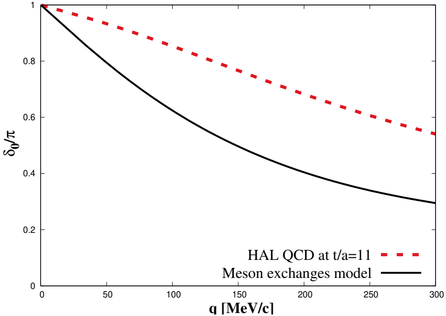
- When the source size is much larger than the interaction range, it is possible to employ the asymptotic behaviour of the wave function,
- $\psi(q,r) \to j_0(qr) + f(q)\exp(iqr)/r$, which leads to much more simple formula the Lednicky-Lyuboshits (LL) approach, [A. Ohnishi, et al., NPA 954, 294 2016] [R. Lednicky and V. L. Lyuboshits,Yad. Fiz. 35, 1316 1981]

$$C_{LL}(q) = 1 + \frac{|f(q)|^2}{2R^2} F_0\left(\frac{r_0}{R}\right) + \frac{2\mathrm{Re}\, f(q)}{\sqrt{\pi}R} F_1(2qR) - \frac{\mathrm{Im}\, f(q)}{R} F_2(2qR), \tag{6}$$

-
    - $f(q) \approx 1/\left(-1/a_0 + r_0 q^2/2 - iq\right)$ is scattering amplitude which can be calculated with the effective range expansion (ERE) formula
    - $F_1(x) = \int_0^x dt e^{t^2-x^2}/x$, $F_2(x) = \left(1 - e^{-x^2}\right)/x$,
    - the factor $F_0(x) = 1 - x/\left(2\sqrt{\pi}\right)$ is a correction for the deviation of the true wave function from the asymptotic form.

Numerical Results

- For phenomenological application I fit $V_{\Omega\alpha}(r)$ to a Wood-Saxon motivated by common Dover-Gal model of potential[C. Dover and A. Gal, Ann. Phys. 146, 309 1983.]
  $V^{fit}_{\Omega\alpha}(r) = -V_0\left[1 + \exp\left(\frac{r-R}{c}\right)\right]^{-1}$,
- V_0 is known as the depth parameter, $R = 1.1A^{1/3}$ with $A = 4$ being the mass number of α and c is known as the surface diffuseness.
- By using the fit functions as input, the Schrödinger equation were solved in the infinite volume to extract binding energy and scattering observables.



✦ $q = \sqrt{2\mu E}$ and μ is the reduced mass of $\Omega\alpha$ system

Numerical Results

- Low-energy part of $\Omega\alpha$ phase shifts provides the scattering length and the effective range by employing the ERE formula up to the next-leading-order (NLO),

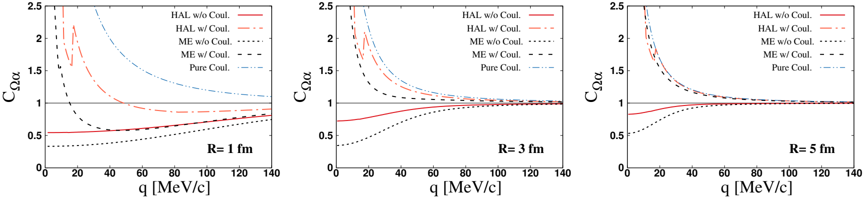
$$q \cot \delta_0 = -\frac{1}{a_0} + \frac{1}{2} r_0 q^2 + \mathcal{O}\left(q^4\right) .. \quad (7)$$

Model	V_0 (MeV)	R(fm)	c (fm)	a_0(fm)	r_0(fm)	$B_{\Omega\alpha}$(MeV)	$B_{\Omega^- \alpha^{++}}$ (MeV)
$t/a = 11$	61.0	1.74	0.47	0.79(0.63)	2.81(5.80)	22.9(23.3)	24.2(24.6)
Meson exchange	33.6	1.67	0.33	2.65(2.60)	1.30(1.28)	6.4(6.6)	7.5(7.7)

✦ The experimental masses of α and Ω, 3727.38 MeV/c and 1672.45 MeV/c respectively.

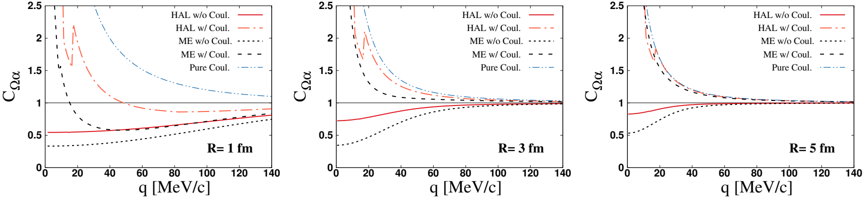
✦ The results corresponding to Ω mass value derived by the HAL QCD Collaboration 1711.5 MeV/c are given between parenthesis.

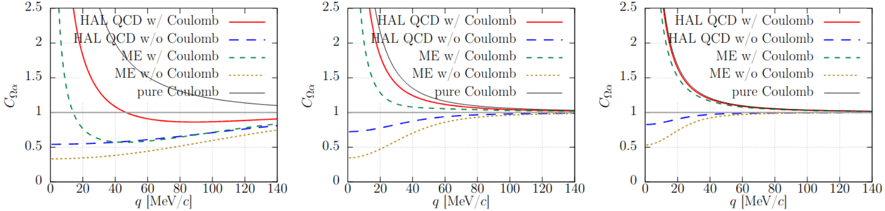
Numerical Results



- in the area of low momentum $q \lesssim 100$ MeV/c, the results for two potentials are different.
- the HAL potential model, is more attractive than ME potential model, thus it gives enhancement of $C_{\Omega\alpha}\ (q)$.
- Nevertheless, with the increase of the source size , the difference between the $C_{\Omega\alpha}\ (q)$s decreases until they are almost same for $R = 5$ fm.
- Therefore, the future measurement of $\Omega\alpha$ correlation function from a small source at small relative momentum, substantially can be constrained by ΩN interaction at high densities.

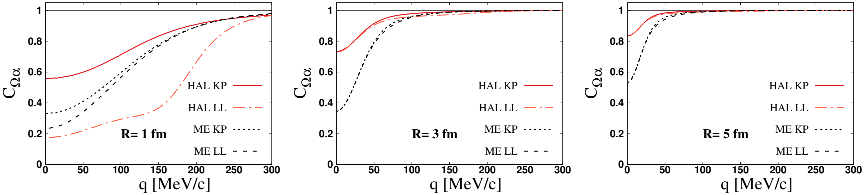
Numerical Results





✦ Corrected by Yuki Kamiya-san

Numerical Results, KP vs. LL formula



✦ In principle, the LL formula can not be a good approximation where the source size is smaller than the interaction range (for interactions that include nuclei can be about $\gtrsim 3$ fm) [A. Jinno, et al, PRC 110, 014001 2024; Y. Kamiya, et al, arXiv:2409.13207].

✦ On the other hand, the LL approximation is consistent with KP formula for relatively large source size, i.e. $R \geqq 3$ fm.

✦ Note that here, only the strong interaction is switched on.

Summary

- $\Omega\alpha$ potential is made from two available ΩN interactions,
  - First principles HAL QCD HAL
  - and developed meson exchange potentials, in the latter potential coupled channels effect are considered.
- $\Omega + \alpha$ potentials were obtained by using SFP model and they were fitted by a Woods-Saxon type functions.
- Employing two $\Omega\alpha$ potentials, correlation functions are calculated using KP formula for three different source sizes, $R = 1, 3$ fm and 5 fm.
- The difference of potentials is appear in the correlation functions by small source size around $1 - 3$ fm, with and without considering the Coulomb interactions,
- while for source size $R \gtrsim 5$ fm the correlation functions tended to become same for both $\Omega\alpha$ potentials with Coulomb interactions.

Summary

- correlation functions is examined within the Lednicky-Lyuboshits approach
- It was seen as expected, the LL formula by small source (1 fm) seriously differ from KP formula in the low-momentum region.
- In conclusion, since the correlation functions are sensitive to $\Omega\alpha$ potentials behavior and Coulomb interactions, we could get important information about effects of Ω particle in dense nuclear medium.

Summary

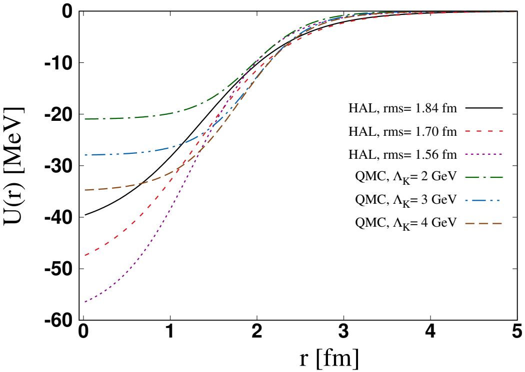
- The Koonin-Pratt formula is valid while the two correlated particles can be considered as well separated point-like particle.
- In the case of composite particle like α, since there is a possibility of simultaneous formation of alpha particle the effective source size must be larger than those with single hadron emissions[S. Mrówczyński and P. Słoń,PRC 104, 024909 2021].
- So basically we are facing a 5-body problem of two protons, two neutrons and Ω and a emergence of alpha particle and production of Ω-α correlation take place at same time,
- This effect should be considered in future works.

Exploring the ϕ-α interaction via femtoscopic study

Exploring the ϕ-α interaction via femtoscopic study

$\phi\alpha$ potential from single-folding potential approach

✦ From [I. Filikhin, et al.,2024 arXiv:2408.13415]



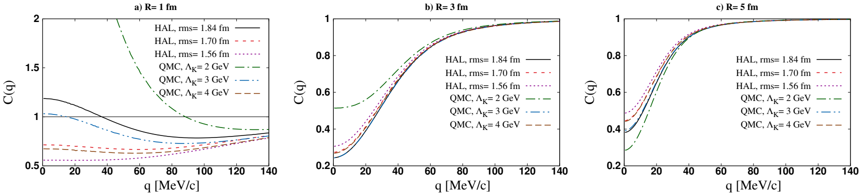
✦ The rms matter radius of ^4He is measured to be 1.70 ± 0.14 fm [R. Wang, C.Han, and X. Chen, PRC 109, L012201 2024] [Y. Lyu, et al. PRD 106, 074507 2022]

✦ And the second three based on the QMC model of ϕ-N potentials for three values of the cutoff parameter Λ_K : 2 GeV (dash-dotted green line), 3 GeV (dash-dot-dotted blue line) and 4 GeV (dashed brown line), where the rms matter radius of ^4He is chosen to be 1.56 fm. [J. J. Cobos-Martinez, et al. PRC 96, 035201 2017]

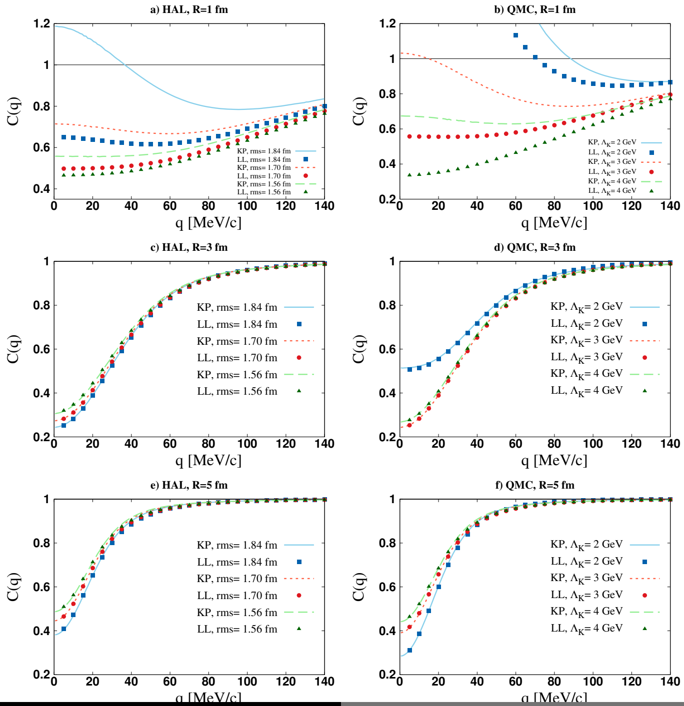
Exploring the ϕ-α interaction via femtoscopic study

Model		rms (fm)	V_0 (MeV)	R (fm)	c (fm)	a_0 (fm)	r_0 (fm)	$B_{\phi\text{-}\alpha}$ (MeV)
$U^{HAL}_{\phi\text{-}\alpha}$		1.84	43	1.36	0.55	4.0	1.7	2.97
		1.70	52	1.30	0.55	3.4	1.5	4.78
		1.56	60	1.26	0.45	3.0	1.3	5.98
$U^{QMC}_{\phi\text{-}\alpha}$	$\Lambda_K = 2$ (GeV)	1.56	21	1.94	0.33	6.7	2.0	0.80
	$\Lambda_K = 3$ (GeV)	1.56	28	1.94	0.33	3.9	1.7	3.19
	$\Lambda_K = 4$ (GeV)	1.56	35	1.80	0.37	3.4	1.6	4.71

ϕ-α Numerical Results



ϕ-α Numerical Results, KP vs. LL formula



Acknowledgments

Thanks **HHIQCD2024** organizers for organizing such wonderful event!

Thank You for your attention!